Web Mining and Recommender Systems

Recommender Systems: Introduction

Learning Goals

 Introduced the topic of recommender systems and explain how they relate to supervised and unsupervised learning

The goal of recommender systems is...

To help people discover new content

Recommendations for You in Amazon Instant Video See more







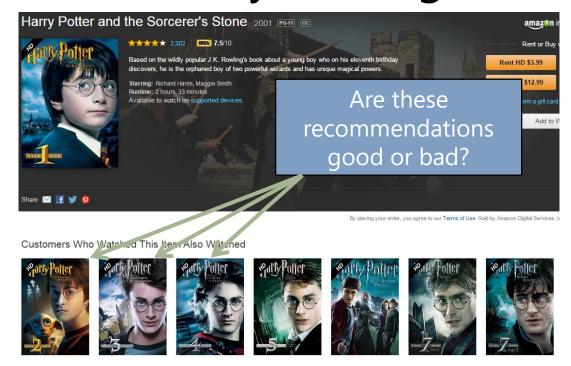






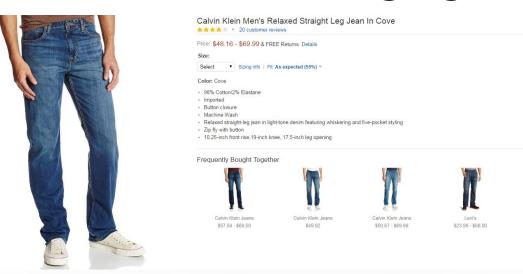


- The goal of recommender systems is...
- To help us find the content we were already looking for



The goal of recommender systems is...

To discover which things go together



Customers Who Bought This Item Also Bought













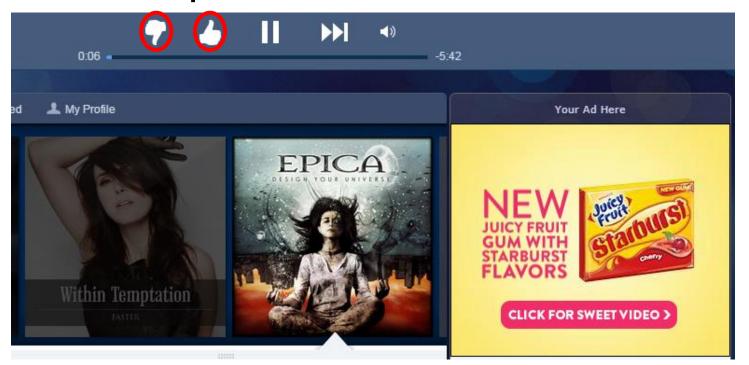




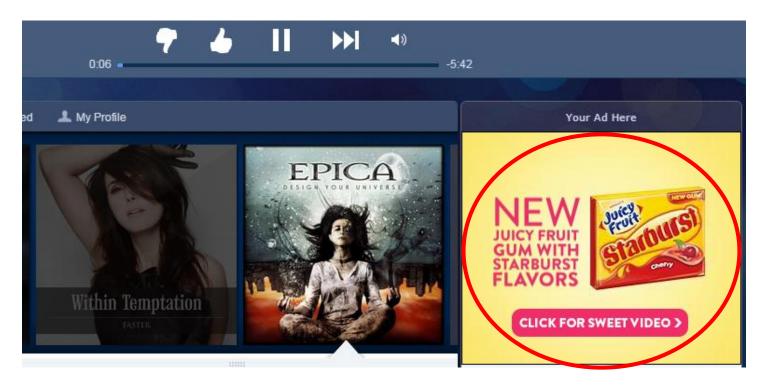


The goal of recommender systems is...

To personalize user experiences in response to user feedback



- The goal of recommender systems is...
- To recommend incredible products that are relevant to our interests



The goal of recommender systems is...

To identify things that we like



The goal of recommender systems is...

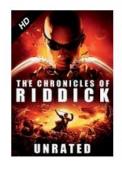
- To help people discover new content

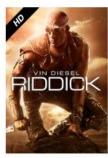
To model people's preferences, opinions, and behavior ces in character about the composition of the composition of

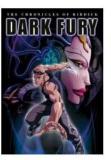
To identify things that we like

Suppose we want to build a movie recommender

e.g. which of these films will I rate highest?









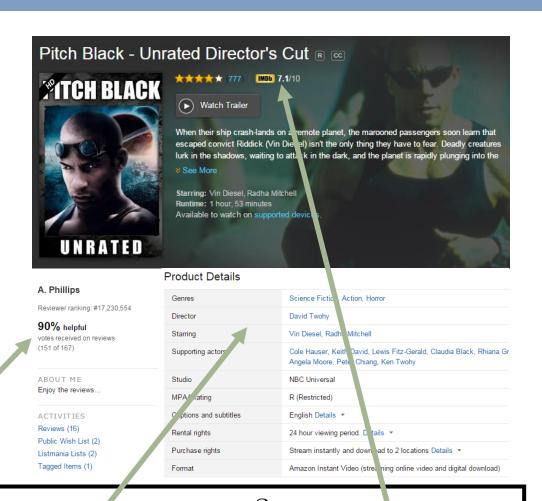






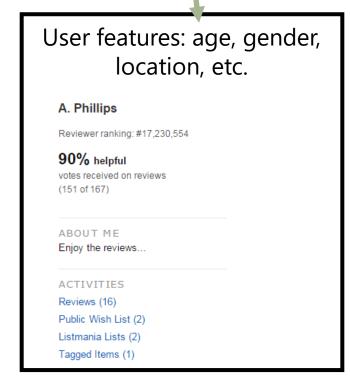


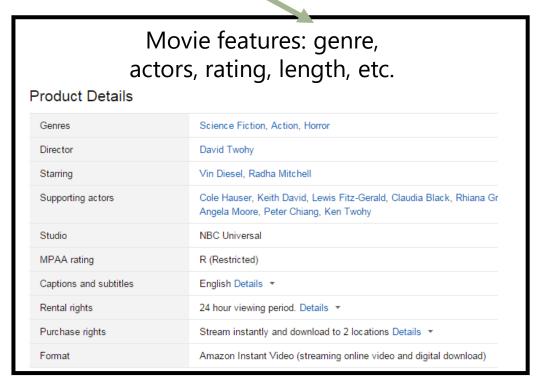
We already have a few tools in our "supervised learning" toolbox that may help us



 $f(\text{user features}, \text{movie features}) \xrightarrow{?} \text{star rating}$

 $f(\text{user features}, \text{movie features}) \stackrel{?}{\rightarrow} \text{star rating}$





 $f(\text{user features}, \text{movie features}) \xrightarrow{?} \text{star rating}$

With the models we've seen so far, we can build predictors that account for...

- Do women give higher ratings than men?
- Do Americans give higher ratings than Australians?
- Do people give higher ratings to action movies?
- Are ratings higher in the summer or winter?
- Do people give high ratings to movies with Vin Diesel?

So what can't we do yet?

 $f(\text{user features}, \text{movie features}) \xrightarrow{?} \text{star rating}$

Consider the following linear predictor (e.g. from week 1):

f(user features, movie features) =

$$\langle \phi(\text{user features}); \phi(\text{movie features}), \theta \rangle$$

$$= \langle \phi(\text{user}), \phi(\text{movie}), \phi(\text{novie}), \phi$$

But this is essentially just two separate predictors!

```
f(\text{user features}, \text{movie features}) =
= \langle \phi(\text{user features}), \theta_{\text{user}} \rangle + \langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle
= \langle \phi(\text{user features}), \theta_{\text{user}} \rangle + \langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle
= \langle \phi(\text{user features}), \theta_{\text{user}} \rangle + \langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle
```

That is, we're treating user and movie features as though they're **independent!**

But these predictors should (obviously?) not be independent

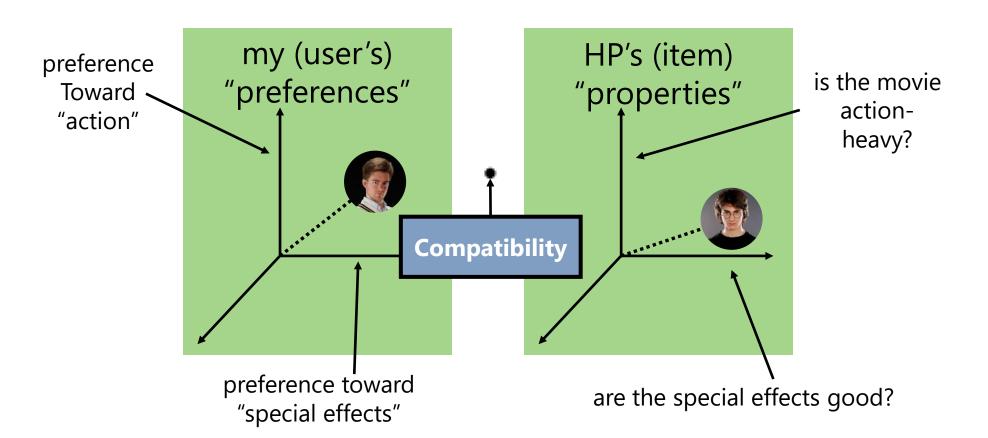
f(user features, movie features) = f(user) + f(movie)

do I tend to give high ratings?

does the population tend to give high ratings to this genre of movie?

But what about a feature like "do I give high ratings to **this genre** of movie"?

Recommender Systems go beyond the methods we've seen so far by trying to model the **relationships** between people and the items they're evaluating



This section

Recommender Systems

- 1. (next) Collaborative filtering
- (performs recommendation in terms of user/user and item/item similarity)
 - 2. (later) Latent-factor models
- (performs recommendation by projecting users and items into some low-dimensional space)
 - 3. (later) The Netflix Prize

Web Mining and Recommender Systems

Similarity-based Recommender Systems

Learning Goals

 Introduced some simple recommendation strategies based on the notions of user or item similarity

Defining similarity between users & items

Q: How can we measure the similarity between two users?
A: In terms of the items they purchased!

Q: How can we measure the similarity between two items?A: In terms of the users who purchased them!

Defining similarity between users & items

e.g.: Amazon



Calvin Klein Men's Relaxed Straight Leg Jean In Cove

Size:

Select ▼ Sizing info | Fit: As expected (55%) ▼

Color: Cove

- 98% Cotton/2% Elastane
- Imported
- Button closure
- Machine Wash
- · Relaxed straight-leg jean in light-tone denim featuring whiskering and five-pocket styling
- Zip fly with butto
- 10.25-inch front rise, 19-inch knee, 17.5-inch leg opening

Frequently Bought Together















\$23.99 - \$68.00

Customers Who Viewed This Item Also Viewed





















Customers Who Bought This Item Also Bought





















D.

Definitions

Definitions

 I_u = set of items purchased by user u

 U_i = set of users who purchased item i

Definitions

Or equivalently...
$$R = \left(\begin{array}{ccc} 1 & 0 & \cdots & 1 \\ 0 & 0 & & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{array}\right) \text{ users}$$

 R_u = binary representation of items purchased by u $R_{\cdot,i}$ = binary representation of users who purchased i

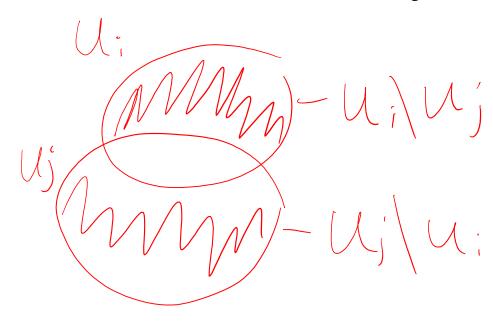
$$I_u = \left\{ \begin{array}{c|c} i & \text{Rei-1} \end{array} \right\} \quad U_i = \left\{ u \mid \text{Rei-1} \right\}$$

0. Euclidean distance

Euclidean distance:

e.g. between two items i,j (similarly defined between two users)

$$|U_i \setminus U_j| + |U_j \setminus U_i| = ||R_i - R_j||$$



0. Euclidean distance

Euclidean distance:

e.g.: U_1 = {1,4,8,9,11,23,25,34}
U_2 = {1,4,6,8,9,11,23,25,34,35,38}
U_3 = {4}
U_4 = {5}

$$|U_1 \setminus U_2| + |U_2 \setminus U_1| = 3$$

 $|U_3 \setminus U_4| + |U_3 \setminus U_4| = 3$

Problem: favors small sets, even if they have few elements in common

1. Jaccard similarity

$$\operatorname{Jaccard}(A, B) = \operatorname{AAB}$$

$$\operatorname{Jaccard}(U_i, U_j) = \operatorname{JAAB}$$

Mindy Mindy

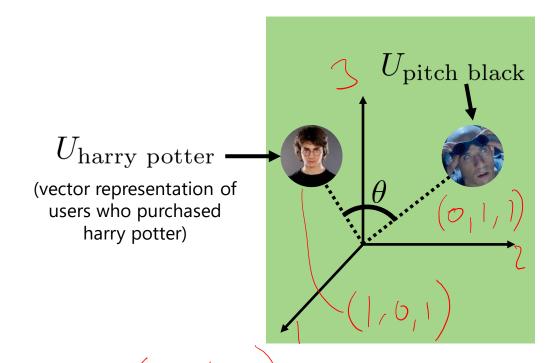
→ Maximum of 1 if the two users purchased **exactly the same** set of items

(or if two items were purchased by the same set of users)

→ Minimum of 0 if the two users purchased **completely disjoint** sets of items

(or if the two items were purchased by completely disjoint sets of users)

2. Cosine similarity



$$\cos(\theta) = 1$$

(theta = 0) \rightarrow A and B point in exactly the same direction

$$\cos(\theta) = -1$$

(theta = 180) → A and B point in opposite directions (won't actually happen for 0/1 vectors)

$$\cos(\theta) = 0$$
 (theta = 90) \rightarrow A and B are orthogonal

$$O = (us-1) \left(\frac{A \cdot T}{|A| |B|}\right)$$

$$Cos(0) = \frac{U_i \cdot U_j}{|U : ||U : ||U : ||}$$

$$biliary intractions ||U : ||U : ||}{\sqrt{|U : ||U : ||}}$$

2. Cosine similarity

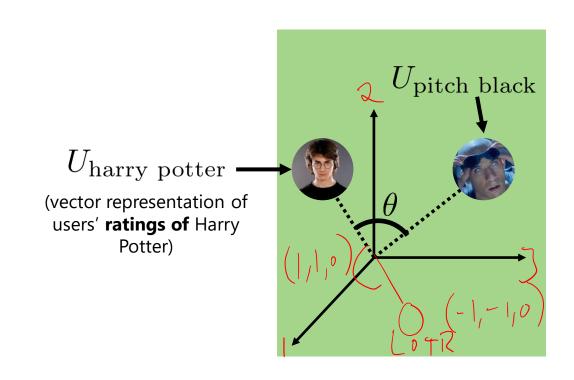
Why cosine?

- Unlike Jaccard, works for arbitrary vectors
- E.g. what if we have **opinions** in addition to purchases?

$$R = \begin{pmatrix} 1 & 0 & \cdots & 1 \\ 0 & 0 & & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{pmatrix} \xrightarrow{\begin{pmatrix} -1 & 0 & \cdots & 1 \\ 0 & 0 & & -1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & -1 \end{pmatrix}}$$
 bought and **liked** didn't buy

2. Cosine similarity

E.g. our previous example, now with "thumbs-up/thumbs-down" ratings



$$\cos(\theta) = 1$$

(theta = 0) \rightarrow Rated by the same users, and they all agree

$$\cos(\theta) = -1$$
 (theta = 180) \rightarrow Rated by the same users, but they **completely disagree** about it

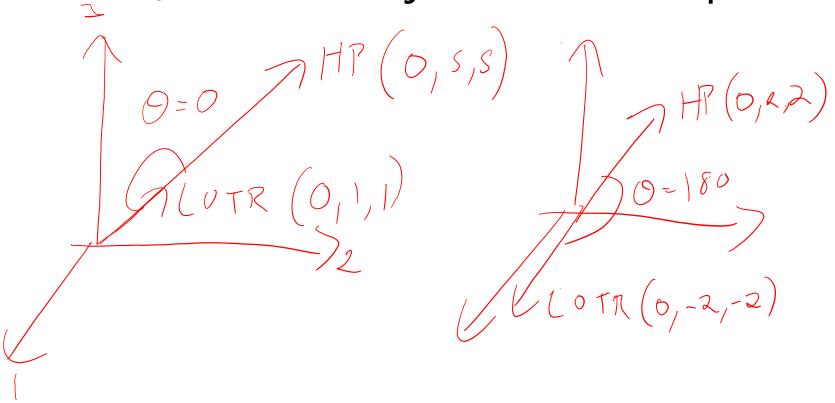
$$cos(\theta) = 0$$

(theta = 90) \rightarrow Rated by different sets of users

What if we have numerical ratings (rather than just thumbs-up/down)?

$$R = \begin{pmatrix} -1 & 0 & \cdots & 1 \\ 0 & 0 & & -1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 4 & 0 & \cdots & 2 \\ 0 & 0 & & 3 \\ \vdots & & \ddots & \vdots \\ 5 & 0 & \cdots & 1 \end{pmatrix}$$
 bought and **liked** didn't buy

What if we have numerical ratings (rather than just thumbs-up/down)?



What if we have numerical ratings (rather than just thumbs-up/down)?

- We wouldn't want 1-star ratings to be parallel to 5star ratings
 - So we can subtract the average values are then negative for below-average ratings and positive for above-average ratings

items rated by both users average rating by user *v*

$$Sim(u, v) = \frac{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R_u})(R_{v,i} - \bar{R_v})}{\sqrt{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R_u})^2 \sum_{i \in I_u \cap I_v} (R_{v,i} - \bar{R_v})^2}}$$

Compare to the cosine similarity:

Pearson similarity (between users):

items rated by both users average rating by user v

$$Sim(u,v) = \frac{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R_u})(R_{v,i} - \bar{R_v})}{\sqrt{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R_u})^2 \sum_{i \in I_u \cap I_v} (R_{v,i} - \bar{R_v})^2}}$$

Cosine similarity (between users):

$$Sim(u, v) = \frac{\sum_{i \in I_u \cap I_v} R_{u,i} R_{v,i}}{\sqrt{\sum_{i \in I_u \cap I_v} R_{u,i}^2 \sum_{i \in I_u \cap I_v} R_{v,i}^2}}$$

Note: slightly different from previous definition. Here similarity is determined only based on items *both* users have consumed

$$Sim(u, v) = \frac{\sum_{i \in I_u \cap I_v} R_{u, i} R_{v, i}}{\sqrt{\sum_{i \in I_u \cap I_v} R_{u, i}^2 \sum_{i \in I_u \cap I_v} R_{v, i}^2}}$$

$$Cosine(A, B) = \frac{A \cdot B}{\|A\| \|B\|}$$

Consider **all items** in the denominator, or just shared items?

Just shared: two users should be considered maximally similar if they've rated shared items the same way. If only one user has rated an item, we have no evidence that the other user is different.

All: Two users who've rated items the same way and only rated the same items should be more similar than two users who've rated some different items.

Ultimately, these are *heuristics*, and either definition could be used depending on the situation

Collaborative filtering in practice

How does amazon generate their recommendations?

Given a product:



Let U_i be the set of users who viewed it

Rank products according to: $\frac{|U_i\cap U_j|}{|U_i\cup U_j|}$ (or cosine/pearson)





















.84

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•••

Collaborative filtering in practice

Can also use similarity functions to estimate ratings:

$$\Gamma(u;i) = \frac{1}{Z} \sum_{j \in I_n \setminus \{i\}} \Gamma(u;j) \Gamma(u;j)$$

$$\sum_{j \in I_n \setminus \{i\}} \sum_{j \in I_n \setminus \{i\}} \Gamma(u;j)$$

Collaborative filtering in practice

Note: (surprisingly) that we built something pretty useful out of **nothing but rating data** – we didn't look at any features of the products whatsoever

Collaborative filtering in practice

But: we still have a few problems left to address...

- 1. This is actually kind of slow given a huge enough dataset if one user purchases one item, this will change the rankings of every other item that was purchased by at least one user in common
- Of no use for **new users** and **new items** ("coldstart" problems
 - 3. Won't necessarily encourage diverse results

Learning Outcomes

- Introduced several similarity measures for different types of data (interactions, likes, ratings)
- Showed how recommender systems can operate purely based on interactions, without observed features

Web Mining and Recommender Systems

Similarity based recommender – implementation

Learning Goals

• Walk through a quick implementation of a similarity-based recommender

Code

Code on course webpage

Uses Amazon "Musical Instrument" data from

https://s3.amazonaws.com/amazon-reviewspds/tsv/index.txt

Code: Reading the data

Read the data:

```
In [1]: import gzip
    from collections import defaultdict
    import random
    import numpy
    import scipy.optimize

In [2]: path = "/home/jmcauley/datasets/mooc/amazon/amazon_reviews_ux_Musical_Instruments 1_00.tsv.gz"

In [3]: f = gzip.open(path, 'rt', encoding="utf8")

In [4]: header = f.readline()
    header = header.strip().split('\t')
```

Code: Reading the data

Our goal is to make recommendations of products based on users' purchase histories. The only information needed to do so is **user and item IDs**

```
In [5]: dataset = []
In [6]: for line in f:
            fields = line.strip().split('\t')
             d = dict(zip(header, fields))
             d['star_rating'] = int(d['star_rating'])
             d['helpful votes'] = int(d['helpful votes'])
             d['total_votes'] = int(d['total_votes'])
             dataset.append(d)
In [7]: dataset[0]
Out[7]: {'marketplace': 'US'
          'customer_id': (45610553')
         'review id': 'RMDC=wyy50Z9',
          'product id': (B00HH62VB6)
          'product parent': '618218723',
          'product title': 'AGPtek® 10 Isolated Output 9V 12V 18V Guitar Pedal Board Power Supply Effect Pedals
        with Isolated Short Cricuit / Overcurrent Protection',
```

Code: Useful data structures

Build data structures representing the set of items for each user and users for each item:

Code: Jaccard similarity

The Jaccard similarity implementation follows the definition directly:

$$\operatorname{Jaccard}(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

```
In [12]: def Jaccard(s1, s2):
    numer = len(s1.intersection(s2))
    denom = len(s1.union(s2))
    return numer / denom
```

Recommendation

We want a recommendation function that return **items similar to a candidate item** *i***.** Our strategy will be as follows:

- Find the set of users who purchased i
- Iterate over all other items other than i
- For all other items, compute their similarity with *i* (and store it)
 - Sort all other items by (Jaccard) similarity
 - Return the most similar

Now we can implement the recommendation function itself:

Next, let's use the code to make a recommendation. The query is just a product ID:

```
In [14]: dataset[2]
Out[14]: {'marketplace': 'US',
           'customer id': '6111003',
           'review id': 'RIZR67JKUDBI0',
           'product id': 'B0006VMBHI',
           'product parent': '603261968'
           'product_title': AudioQuest LP record clean brush'
           'product_category': 'Musical instruments ,
           'star rating': 3,
           'helpful votes': 0,
           'total votes': 1,
           'vine': 'N',
           'verified_purchase': 'Y',
           'review_headline': 'Three Stars',
           'review_body': 'removes dust. does not clean',
           'review date': '2015-08-31'}
In [15]: query = dataset[2]['product id']
```

Next, let's use the code to make a recommendation. The query is just a product ID:

Items that were recommended:

Recommending more efficiently

Our implementation was not very efficient. The slowest component is the iteration over all other items:

- Find the set of users who purchased i
- Iterate over all other items other than i
- For all other items, compute their similarity with *i* (and store it)
 - Sort all other items by (Jaccard) similarity
 - Return the most similar

This can be done more efficiently as most items will have no overlap

Recommending more efficiently

In fact it is sufficient to iterate over **those items** purchased by one of the users who purchased *i*

- Find the set of users who purchased i
- Iterate over all users who purchased i
- Build a candidate set from all items those users consumed
- For items in this set, compute their similarity with i
 (and store it)
 - Sort all other items by (Jaccard) similarity
 - Return the most similar

Code: Faster implementation

Our more efficient implementation works as follows:

```
In [19]:
    def mostSimilarFast(i):
        similarities = []
        users = usersPerItem[i]
        CandidateItems = set()
        for u in users:
            candidateItems = candidateItems.union(itemsPerUser[u])
        for i2 in candidateItems:
            if i2 == i: continue
            sim = Jaccard(users, usersPerItem[i2])
            similarities.append((sim,i2))
        similarities.sort(reverse=True)
        return similarities[:10]
```

Code: Faster recommendation

Which ought to recommend the same set of items, but **much** more quickly:

Learning Outcomes

 Walked through an implementation of a similarity-based recommender, and discussed some of the computational challenges involved

Web Mining and Recommender Systems

Similarity-based rating prediction

Learning Goals

 Show how a similarity-based recommender can be used for rating prediction

In the previous section we provided code to make recommendations based on the **Jaccard similarity**

How can the same ideas be used for rating prediction?

A simple heuristic for rating prediction works as follows:

- The user (u)'s rating for an item i is a weighted combination of all of their previous ratings for items j
- The weight for each rating is given by the Jaccard similarity between *i* and *j*

This can be written as:

$$r(u,i) = \frac{1}{Z} \sum_{j \in I_u \setminus \{i\}} r_{u,j} \cdot \mathrm{sim}(i,j)$$
 Normalization constant All items the user has rated other than i
$$Z = \sum_{j \in I_u \setminus \{i\}} \mathrm{sim}(i,j)$$

Now we can adapt our previous recommendation code to predict ratings

Our rating prediction code works as follows:

```
In [27]:  \begin{array}{l} \text{def predictRating(user,item):} \\ \text{ratings = []} \\ \text{similarities = []} \\ \text{for d in reviewsPerUser[user]:} \\ \text{i2 = d['product_id']} \\ \text{if i2 == item: continue} \\ \text{ratings.append(d['star_rating'])} \\ \text{similarities.append(Jaccard(usersPerItem[item],usersPerItem[i2]))} \\ \text{if (sum(similarities) > 0):} \\ \text{weightedRatings = [(x*y) for x,y in zip(ratings,similarities)]} \\ \text{return sum(weightedRatings) / sum(similarities)} \\ \text{else:} \\ \# \textit{User hasn't rated any similar items} \\ \text{return ratingMean}  \end{array}
```

As an example, select a rating for prediction:

```
In [28]: dataset[1]
Out[28]: {'marketplace': 'US',
           'customer id': '14640079',
           'review id': 'RZSL0BALIYUNU',
           'product_id': 'B003LRN53I',
           'product parent': '986692292',
           'product title': 'Sennheiser HD203 Closed-Back DJ Headphones',
           'product_category': 'Musical Instruments',
           'star rating': 5,
           'helpful votes': 0,
           'total votes': 0,
           'vine': 'N',
           'verified_purchase': 'Y',
           'review headline': 'Five Stars',
           'review_body': 'Nice headphones at a reasonable price.',
           'review_date': '2015-08-31'}
In [29]: u,i = dataset[1]['customer_id'], dataset[1]['product_id']
In [30]: predictRating(u, i)
Out[30]: 5.0
```

Similarly, we can evaluate accuracy across the entire corpus:

```
In [31]: def MSE(predictions, labels):
    differences = [(x-y)**2 for x,y in zip(predictions, labels)]
    return sum(differences) / len(differences)

In [32]: alwaysPredictMean = [ratingMean for d in dataset]

In [33]: cfPredictions = [predictRating(d['customer_id'], d['product_id']) for d in dataset]

In [34]: labels = [d['star_rating'] for d in dataset]

In [35]: MSE(alwaysPredictMean, labels)

Out[35]: 1.4796142779564334

In [36]: MSE(cfPredictions, labels)

Out[36]: 1.6146130004291603
```

Note that this is just a **heuristic** for rating prediction

- In fact in this case it did worse (in terms of the MSE) than always predicting the mean
 - We could adapt this to use:
- 1. A different similarity function (e.g. cosine)
- 2. Similarity based on users rather than items
 - 3. A different weighting scheme

Learning Outcomes

 Examined the use of a similaritybased recommender for rating prediction

Web Mining and Recommender Systems

Latent-factor models

Learning Goals

- Show how recommendation can be cast as a supervised learning problem
- (Start to) introduce latent factor models

Summary so far

Recap

- 1. Measuring similarity between users/items for **binary** prediction

 Jaccard similarity
- 2. Measuring similarity between users/items for **real-valued** prediction cosine/Pearson similarity

Now: Dimensionality reduction for **real-valued** prediction *latent-factor models*

Latent factor models

So far we've looked at approaches that try to define some definition of user/user and item/item **similarity**

Recommendation then consists of

- Finding an item i that a user likes (gives a high rating)
- Recommending items that are similar to it (i.e., items j
 with a similar rating profile to i)

What we've seen so far are unsupervised approaches and whether the work depends highly on whether we chose a "good" notion of similarity

So, can we perform recommendations via **supervised** learning?

e.g. if we can model

 $f(\text{user features}, \text{movie features}) \rightarrow \text{star rating}$

Then recommendation will consist of identifying

 $recommendation(u) = \arg\max_{i \in \text{unseen items}} f(u, i)$

The Netflix prize

In 2006, Netflix created a dataset of **100,000,000** movie ratings Data looked like:

The goal was to reduce the (R)MSE at predicting ratings:

$$\mathrm{RMSE}(f) = \sqrt{\frac{1}{N} \sum_{u,i,t \in \mathrm{test \ set}} (f(u,i,t) - r_{u,i,t})^2}$$
 model's prediction ground-truth

Whoever first manages to reduce the RMSE by **10%** versus Netflix's solution wins **\$1,000,000**

The Netflix prize

This led to **a lot** of research on rating prediction by minimizing the Mean-Squared Error

NETFLIX

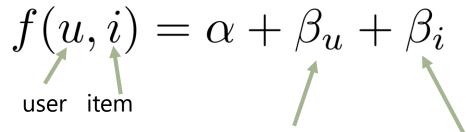
(it also led to a lawsuit against Netflix, once somebody managed to de-anonymize their data)

We'll look at a few of the main approaches

Let's start with the simplest possible model:

$$f(u,i) = \alpha$$
 user item

What about the **2nd** simplest model?



how much does this user tend to rate things above the mean?

does this item tend to receive higher ratings than others

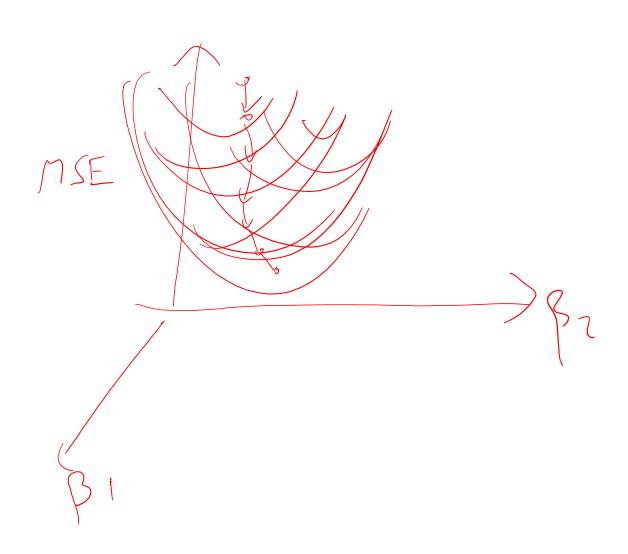
PITCH BLACK
$$eta_{
m pitch\ black}=-0.1$$
 $lpha=4.2$ $eta_{
m julian}=-0.2$

The optimization problem becomes:

$$\underset{\text{error}}{\arg\min_{\alpha,\beta} \sum_{u,i} (\alpha + \beta_u + \beta_i - R_{u,i})^2 + \lambda \left[\sum_{u} \beta_u^2 + \sum_{i} \beta_i^2 \right]}}$$

Jointly convex in \beta_i, \beta_u. Can be solved by iteratively removing the mean and solving for beta

Jointly convex?



Differentiate:

$$\underset{\beta}{\operatorname{arg\,min}}_{\alpha,\beta} \sum_{u,i} (\alpha + \beta_u + \beta_i - R_{u,i})^2 + \lambda \left[\sum_{u} \beta_u^2 + \sum_{i} \beta_i^2 \right]$$

$$\underset{\beta}{\operatorname{bu}} \qquad \qquad \underbrace{\sum_{u} \left(\times \beta_u + \beta_i - R_{u,i} \right)}_{i \in I_u} + 2 \beta_u$$

Differentiate:

$$\frac{\partial \text{obj}}{\partial \beta_u} = \sum_{i \in I_u} 2(\alpha + \beta_u + \beta_i - R_{u,i}) + 2\lambda \beta_u$$

Two ways to solve:

1. "Regular" gradient descent 2. Solve $\frac{\partial \text{obj}}{\partial \beta_u} = 0$ (sim. for beta_i, alpha)

Differentiate:

Iterative procedure – repeat the following updates until convergence:

$$\alpha = \frac{\sum_{u,i \in \text{train}} (R_{u,i} - (\beta_u + \beta_i))}{N_{\text{train}}}$$

$$\beta_u = \frac{\sum_{i \in I_u} R_{u,i} - (\alpha + \beta_i)}{\lambda + |I_u|}$$

$$\beta_i = \frac{\sum_{u \in U_i} R_{u,i} - (\alpha + \beta_u)}{\lambda + |U_i|}$$

(exercise: write down derivatives and convince yourself of these update equations!)

Looks good (and actually works surprisingly well), but doesn't solve the basic issue that we started with

```
f(\text{user features}, \text{movie features}) =
= \langle \phi(\text{user features}), \theta_{\text{user}} \rangle + \langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle
= \langle \phi(\text{user features}), \theta_{\text{user}} \rangle + \langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle
= \langle \phi(\text{user features}), \theta_{\text{user}} \rangle + \langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle
```

That is, we're **still** fitting a function that treats users and items independently

Learning Outcomes

- Introduced (some of) the latent factor model
- Thought about how describe rating prediction as a regression/supervised learning task
- Discussed the history of this type of recommendation system

Web Mining and Recommender Systems

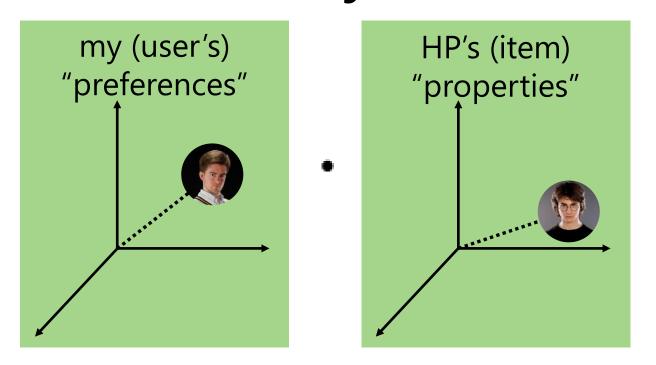
Latent-factor models (part 2)

Learning Goals

 Complete our presentation of the latent factor model

Recommending things to people

How about an approach based on dimensionality reduction?



i.e., let's come up with low-dimensional representations of the users and the items so as to best explain the data

Dimensionality reduction

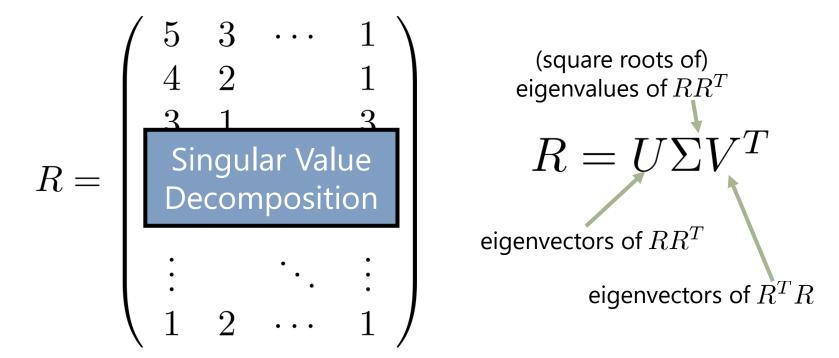
We already have some tools that ought to help us, e.g. from dimensionality reduction:

$$R = \begin{pmatrix} 5 & 3 & \cdots & 1 \\ 4 & 2 & & 1 \\ 3 & 1 & & 3 \\ 2 & 2 & & 4 \\ 1 & 5 & & 2 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 1 \end{pmatrix}$$

What is the best lowrank approximation of *R* in terms of the meansquared error?

Dimensionality reduction

We already have some tools that ought to help us, e.g. from dimensionality reduction:



The "best" rank-K approximation (in terms of the MSE) consists of taking the eigenvectors with the highest eigenvalues

Dimensionality reduction

But! Our matrix of ratings is only partially observed; and it's **really big!**

$$R = \begin{pmatrix} 5 & 3 & \cdots & \cdot \\ 4 & 2 & & 1 \\ 3 & \cdot & & 3 \\ \cdot & 2 & & 4 \\ 1 & 5 & & & \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & \cdot \end{pmatrix}$$
 Missing ratings

SVD is **not defined** for partially observed matrices, and it is **not practical** for matrices with 1Mx1M+ dimensions

Instead, let's solve approximately using gradient descent

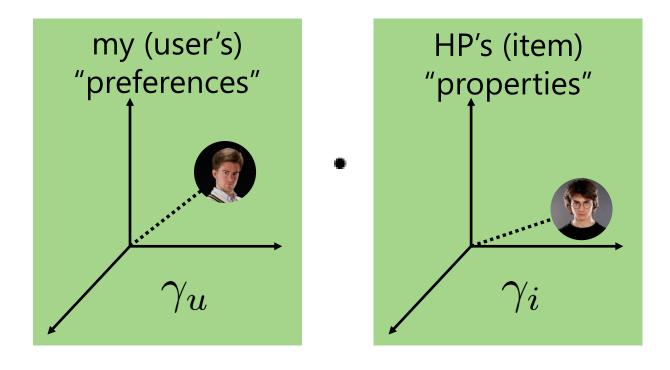
$$R = \begin{pmatrix} 5 & 3 & \cdots & \cdot \\ 4 & 2 & & 1 \\ 3 & \cdot & & 3 \\ \cdot & 2 & & 4 \\ 1 & 5 & & \cdot \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & \cdot \end{pmatrix} \text{ users } \begin{pmatrix} \text{K-dimensional representation of each item} \\ R \simeq UV^T \\ \text{K-dimensional representation of each user} \\ \text{K-dimensional representation of each user} \\ \text{Items}$$

Instead, let's solve approximately using gradient descent

$$R = \begin{pmatrix} 5 & 3 & \cdots & \cdot \\ 4 & 2 & & 1 \\ 3 & \cdot & & 3 \\ \cdot & 2 & & 4 \\ 1 & 5 & & \cdot \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & \cdot \end{pmatrix}$$

Let's write this as:

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$



Let's write this as:

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$

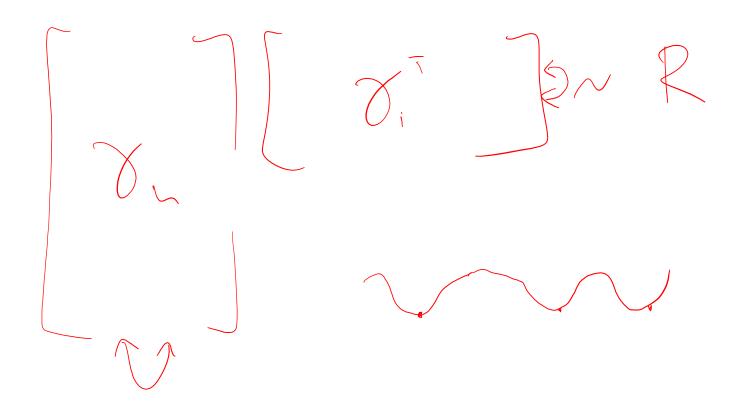
Our optimization problem is then

$$\arg\min_{\alpha,\beta,\gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[\sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \|\gamma_i\|_2^2 + \sum_u \|\gamma_u\|_2^2 \right]$$

error

regularizer

Problem: this is certainly not convex



Oh well. We'll just solve it approximately Again, two ways to solve:

1. "Regular" gradient descent 2. Solve $\frac{\partial \text{obj}}{\partial \gamma_u} = 0$ (sim. For beta_i, alpha, etc.)

(**Solution 1** is much easier to implement, though **Solution 2** might converge more quickly/easily)

Latent-factor models (Solution 1)

 $\arg\min_{\alpha,\beta,\gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[\sum_{u} \beta_u^2 + \sum_{i} \beta_i^2 + \sum_{i} \|\gamma_i\|_2^2 + \sum_{u} \|\gamma_u\|_2^2 \right]$ $\sum_{i \in I_u} \gamma_{i,i} \left(\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i} \right)$ $i \in I_u$

Latent-factor models (Solution 2)

Observation: if we know either the user or the item parameters, the problem becomes "easy"

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$

e.g. fix gamma_i – pretend we're fitting parameters for features

(Harder solution): iteratively solve the following subproblems

objective:

$$\arg\min_{\alpha,\beta,\gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[\sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \|\gamma_i\|_2^2 + \sum_u \|\gamma_u\|_2^2 \right]$$

$$= \arg\min_{\alpha,\beta,\gamma} objective(\alpha,\beta,\gamma)$$

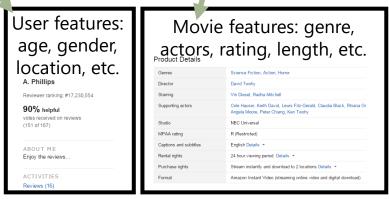
- 1) fix γ_i . Solve $\arg\min_{\alpha,\beta,\gamma_u} objective(\alpha,\beta,\gamma)$
- 2) fix γ_u . Solve $\arg\min_{\alpha,\beta,\gamma_i} objective(\alpha,\beta,\gamma)$

3,4,5...) repeat until convergence

Each of these subproblems is "easy" – just regularized least-squares, like we've been doing since we studied regression. This procedure is called **alternating least squares.**

Observation: we went from a method which uses **only** features:

 $f(\text{user features}, \text{movie features}) \rightarrow \text{star rating}$



to one which **completely ignores** them:

$$\arg\min_{\alpha,\beta,\gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[\sum_{u} \beta_u^2 + \sum_{i} \beta_i^2 + \sum_{i} \|\gamma_i\|_2^2 + \sum_{u} \|\gamma_u\|_2^2 \right]$$

Should we use features or not? 1) Argument **against** features:

In principle, the addition of features adds **no expressive power** to the model. We **could** have a feature like "is this an action movie?", but if this feature were useful, the model would "discover" a latent dimension corresponding to action movies, and we wouldn't need the feature anyway

In the limit, this argument is valid: as we add more ratings per user, and more ratings per item, the latent-factor model should automatically discover any useful dimensions of variation, so the influence of observed features will disappear

Should we use features or not? 2) Argument **for** features:

But! Sometimes we **don't** have many ratings per user/item

Latent-factor models are next-to-useless if **either** the user or the item was never observed before

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$

reverts to zero if we've never seen the user before (because of the regularizer)

Should we use features or not? 2) Argument **for** features:

This is known as the **cold-start** problem in recommender systems. Features are not useful if we have many observations about users/items, but are useful for **new** users and items.

We also need some way to handle users who are **active**, but don't necessarily rate anything, e.g. through **implicit feedback**

Overview & recap

Recently we've followed the programme below:

- Measuring similarity between users/items for binary prediction (e.g. Jaccard similarity)
- 2. Measuring similarity between users/items for **real-valued** prediction (e.g. cosine/Pearson similarity)
 - 3. Dimensionality reduction for **real-valued** prediction (latent-factor models)
 - **4. Finally** dimensionality reduction for **binary** prediction

Learning Outcomes

- Completed our presentation of the latent factor model
- Revisited the relationship between recommendation and other types of learning

Web Mining and Recommender Systems

One-class recommendation

Learning Goals

- (Briefly) discuss how latent factor models might be adapted for interaction data (advanced)
- Summarize our discussion of recommender systems so far

How can we use **dimensionality** reduction to predict **binary** outcomes?

- Previously we saw regression and logistic regression.
 These two approaches use the same type of linear function to predict real-valued and binary outputs
 - We can apply an analogous approach to binary recommendation tasks

This is referred to as "one-class" recommendation

Suppose we have binary (0/1) observations (e.g. purchases) or pos./neg. feedback (thumbs-up/down)

$$R = \begin{pmatrix} 1 & 0 & \cdots & 1 \\ 0 & 0 & & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{pmatrix} \text{ or } \begin{pmatrix} -1 & ? & \cdots & 1 \\ ? & ? & & -1 \\ \vdots & & \ddots & \vdots \\ 1 & ? & \cdots & -1 \end{pmatrix}$$
 purchased didn't purchase

So far, we've been fitting functions of the form

$$R \simeq UV^T$$

- Let's change this so that we maximize the difference in predictions between positive and negative items
- E.g. for a user who likes an item *i* and dislikes an item *j* we want to maximize:

$$\max \ln \sigma(\gamma_u \cdot \gamma_i - \gamma_u \cdot \gamma_j)$$

We can think of this as maximizing the probability of correctly predicting pairwise preferences, i.e.,

$$p(i \text{ is preferred over } j) = \sigma(\gamma_u \cdot \gamma_i - \gamma_u \cdot \gamma_j)$$

- As with logistic regression, we can now maximize the likelihood associated with such a model by gradient ascent
- In practice it isn't feasible to consider all pairs of positive/negative items, so we proceed by stochastic gradient ascent i.e., randomly sample a (positive, negative) pair and update the model according to the gradient w.r.t. that pair

Summary

Recap

- 1. Measuring similarity between users/items for **binary** prediction

 Jaccard similarity
- 2. Measuring similarity between users/items for **real-valued** prediction cosine/Pearson similarity
- 3. Dimensionality reduction for **real-valued** prediction *latent-factor models*
 - 4. Dimensionality reduction for **binary** prediction one-class recommender systems

References

Further reading:

One-class recommendation:

http://goo.gl/08Rh59

Amazon's solution to collaborative filtering at scale:

http://www.cs.umd.edu/~samir/498/Amazon-Recommendations.pdf

An (expensive) textbook about recommender systems:

http://www.springer.com/computer/ai/book/978-0-387-85819-7

Cold-start recommendation (e.g.):

http://wanlab.poly.edu/recsys12/recsys/p115.pdf

Web Mining and Recommender Systems

Extensions of latent-factor models, (and more on the Netflix prize)

Learning Goals

- Discuss several extensions of the latent factor model
- Further discuss the history of the Netflix Prize

So far we have a model that looks like:

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$

How might we extend this to:

- Incorporate features about users and items
 - Handle implicit feedback
 - Change over time

See **Yehuda Koren** (+Bell & Volinsky)'s magazine article: "Matrix Factorization Techniques for Recommender Systems" IEEE Computer, 2009

1) Features about users and/or items

(simplest case) Suppose we have **binary attributes** to describe users or items

$$A(u) = [1,0,1,1,0,0,0,0,0,1,0,1]$$
 attribute vector for user u e.g. is female is male is between 18-24yo

1) Features about users and/or items

(simplest case) Suppose we have **binary attributes** to describe users or items

- Associate a parameter vector with each attribute
- Each vector encodes how much a particular feature "offsets" the given latent dimensions

$$A(u) = [1,0,1,1,0,0,0,0,0,1,0,1]$$

attribute vector for user u

e.g. $y_0 = [-0.2,0.3,0.1,-0.4,0.8]$ ~ "how does being male impact gamma_u"

1) Features about users and/or items

(simplest case) Suppose we have **binary attributes** to describe users or items

- Associate a parameter vector with each attribute
- Each vector encodes how much a particular feature "offsets" the given latent dimensions
 - Model looks like:

$$f(u,i) = \alpha + \beta_u + \beta_i + (\gamma_u + \sum_{a \in A(u)} \rho_a) \cdot \gamma_i$$

• Fit as usual:

$$\operatorname{arg\,min}_{\alpha,\beta,\gamma,\rho} \sum_{u,i \in \operatorname{train}} (f(u,i) - r_{u,i})^2 + \lambda \Omega(\beta,\gamma)$$

2) Implicit feedback

Perhaps many users will never actually rate things, but may still interact with the system, e.g. through the movies they view, or the products they purchase (but never rate)

 Adopt a similar approach – introduce a binary vector describing a user's actions

$$N(u) = [1,0,0,0,1,0,...,0,1]$$

implicit feedback vector for user u

e.g. $y_0 = [-0.1,0.2,0.3,-0.1,0.5]$ Clicked on "Love Actually" but didn't watch

2) Implicit feedback

Perhaps many users will never actually rate things, but may still interact with the system, e.g. through the movies they view, or the products they purchase (but never rate)

- Adopt a similar approach introduce a binary vector describing a user's actions
 - Model looks like:

$$f(u,i) = \alpha + \beta_u + \beta_i + (\gamma_u + \frac{1}{\|N(u)\|} \sum_{a \in N(u)} \rho_a) \cdot \gamma_i$$

normalize by the number of actions the user performed

3) Change over time

There are a number of reasons why rating data might be subject to temporal effects...

3) Change over time

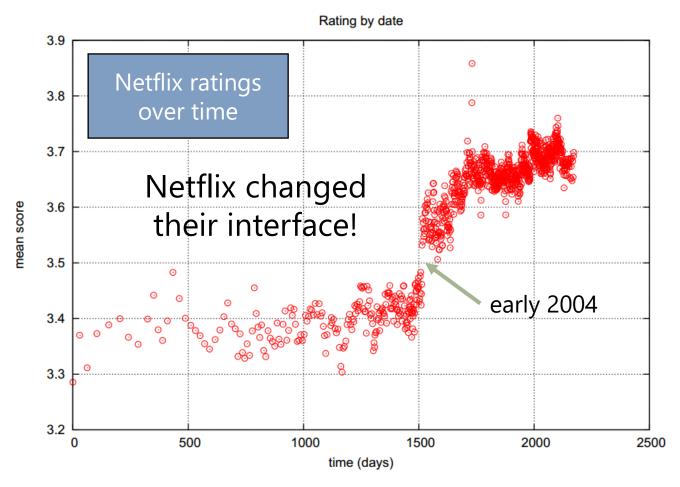


Figure from Koren: "Collaborative Filtering with Temporal Dynamics" (KDD 2009)

3) Change over time

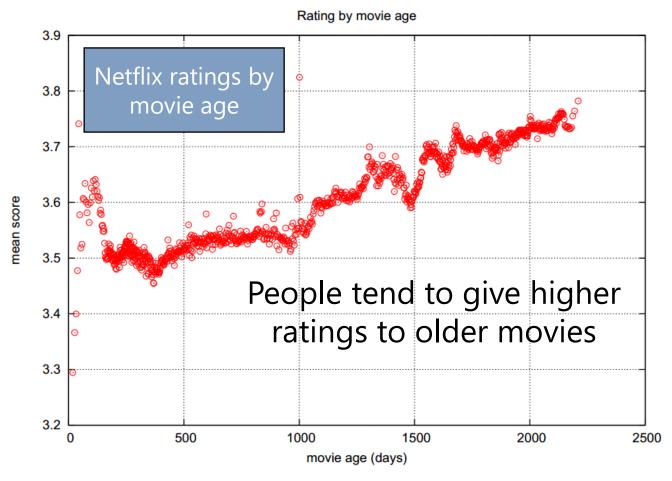
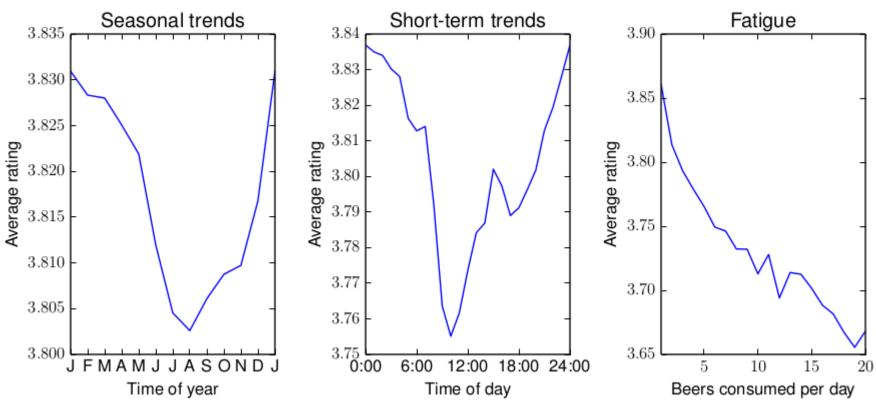


Figure from Koren: "Collaborative Filtering with Temporal Dynamics" (KDD 2009)

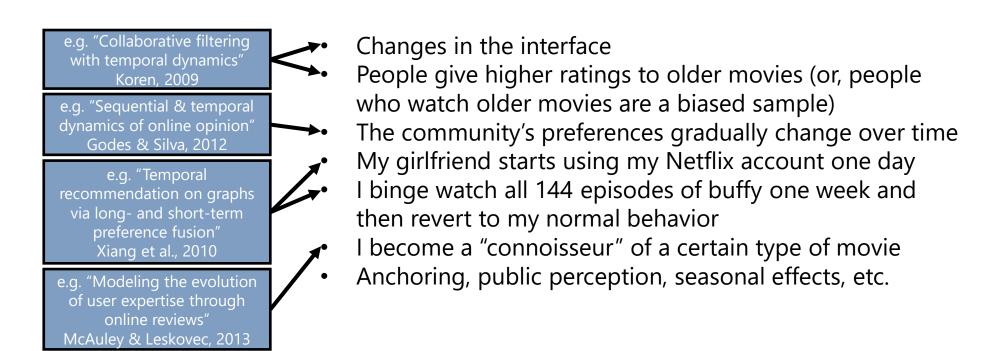
3) Change over time



A few temporal effects from beer reviews

3) Change over time

There are a number of reasons why rating data might be subject to temporal effects...



3) Change over time

Each definition of temporal evolution demands a slightly different model assumption (we'll see some in more detail later tonight!) but the basic idea is the following:

1) Start with our original model:

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$

2) And define some of the parameters as a function of time:

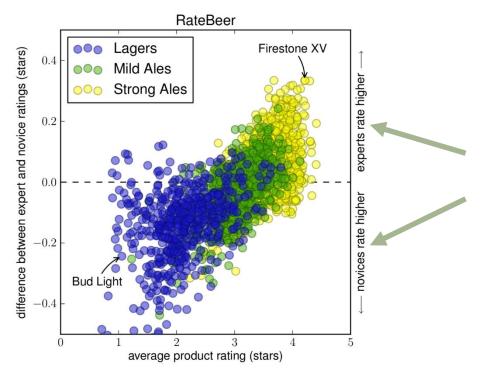
$$f(u, i, t) = \alpha + \beta_u(t) + \beta_i(t) + \gamma_u(t) \cdot \gamma_i$$

3) Add a regularizer to constrain the time-varying terms:

$$\arg\min_{\alpha,\beta,\gamma} \sum_{u,i,t \in \text{train}} (f(u,i,t) - r_{u,i,t})^2 + \lambda_1 \Omega(\beta,\gamma) + \lambda_2 \|\gamma(t) - \gamma(t+\delta)\|$$

3) Change over time

Case study: how do people acquire tastes for beers (and potentially for other things) over time?



Differences between "beginner" and "expert" preferences for different beer styles

4) Missing-not-at-random

- Our decision about whether to purchase a movie (or item etc.) is a function of how we expect to rate it
- Even for items we've purchased, our decision to enter a rating or write a review is a function of our rating
 - e.g. some rating distribution from a few datasets:

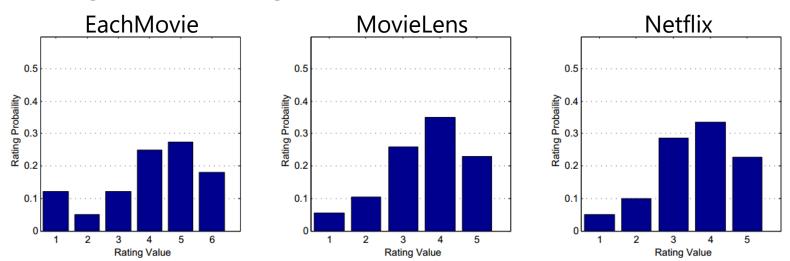
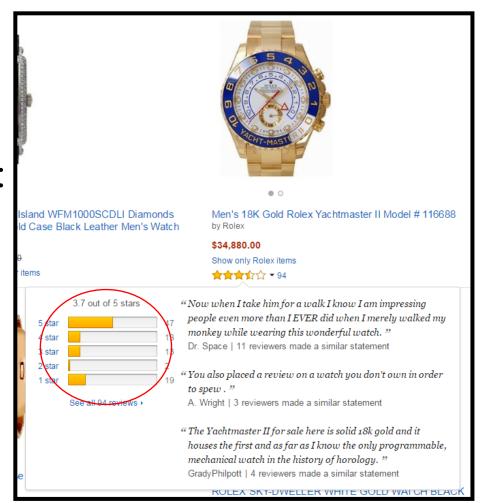


Figure from Marlin et al. "Collaborative Filtering and the Missing at Random Assumption" (UAI 2007)

4) Missing-not-at-random

e.g. Men's watches:

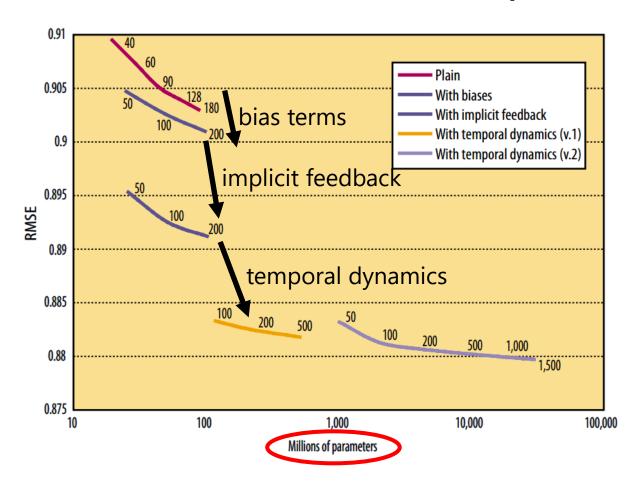


4) Missing-not-at-random

- Our decision about whether to purchase a movie (or item etc.) is a function of how we expect to rate it
- Even for items we've purchased, our decision to enter a rating or write a review is a function of our rating
 - So we can predict ratings more accurately by building models that account for these differences
 - 1. Not-purchased items have a different prior on ratings than purchased ones
- 2. Purchased-but-not-rated items have a different prior on ratings than rated ones

How much do these extension help?

Moral: increasing complexity helps a bit, but changing the model can help **a lot**



So what actually happened with Netflix?

- The AT&T team "BellKor", consisting of Yehuda Koren, Robert Bell, and Chris Volinsky were early leaders. Their main insight was how to effectively incorporate temporal dynamics into recommendation on Netflix.
- Before long, it was clear that no one team would build the winning solution, and Frankenstein efforts started to merge. Two frontrunners emerged, "BellKor's Pragmatic Chaos", and "The Ensemble".
- The BellKor team was the first to achieve a 10% improvement in RMSE, putting the competition in "last call" mode. The winner would be decided after 30 days.
- After 30 days, performance was evaluated on the hidden part of the test set.
- Both of the frontrunning teams had **the same** RMSE (up to some precision) but BellKor's team submitted their solution 20 minutes earlier and won \$1,000,000

For a less rough summary, see the Wikipedia page about the Netflix prize, and the nytimes article about the competition: http://goo.gl/WNpy7o

Afterword

- Netflix had a class-action lawsuit filed against them after somebody deanonymized the competition data
- \$1,000,000 seems to be **incredibly cheap** for a company the size of Netflix in terms of the amount of research that was devoted to the task, and the potential benefit to Netflix of having their recommendation algorithm improved by 10%
- Other similar competitions have emerged, such as the Heritage Health Prize (\$3,000,000 to predict the length of future hospital visits)
- But... the winning solution never made it into production at Netflix it's a monolithic algorithm that is very expensive to update as new data comes in*

^{*}source: a friend of mine told me and I have no actual evidence of this claim

Finally...

Q: Is the RMSE really the right approach? Will improving rating prediction by 10% actually improve the user experience by a significant amount?

A: Not clear. Even a solution that only changes the RMSE slightly could drastically change which items are top-ranked and ultimately suggested to the user.

Q: But... are the following recommendations actually any good?

A1: Yes, these are my favorite movies!

or A2: No! There's no diversity, so how will I discover new content?



5.0 stars



5.0 stars



5.0 stars



5.0 stars



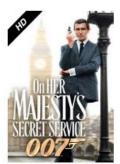
4.9 stars



4.9 stars



4.8 stars



4.8 stars

predicted rating

Summary

Various extensions of latent factor models:

- Incorporating features e.g. for cold-start recommendation
 - Implicit feedback
- e.g. when ratings aren't available, but other actions are
- Incorporating temporal information into latent factor models seasonal effects, short-term "bursts", long-term trends, etc.
 - Missing-not-at-random
 - incorporating priors about items that were not bought or rated
 - The Netflix prize

Learning Outcomes

- Discussed several extensions of latent factor models
- Described what types of solutions worked on the Netflix Prize
- Thought about potential limitations of the solutions we've seen so far

References

Further reading:

Yehuda Koren's, Robert Bell, and Chris Volinsky's IEEE computer article:

http://www2.research.att.com/~volinsky/papers/ieeecomputer.pdf

Paper about the "Missing-at-Random" assumption, and how to address it:

http://www.cs.toronto.edu/~marlin/research/papers/cfmar-uai2007.pdf

Collaborative filtering with temporal dynamics:

http://research.yahoo.com/files/kdd-fp074-koren.pdf

Recommender systems and sales diversity:

http://papers.ssrn.com/sol3/papers.cfm?abstract_id=955984