1)

TRUTH TABLE:

cin	a	b	b'	(a, b, cin) t = 0		(a, b', cin) t = 1	
				cout	s	cout	s
0	0	0	1	0	0	0	1
0	0	1	0	0	1	0	0
0	1	0	1	0	1	1	0
0	1	1	0	1	0	0	1
1	0	0	1	0	1	1	0
1	0	1	0	1	0	0	1
1	1	0	1	1	0	1	1
1	1	1	0	1	1	1	0
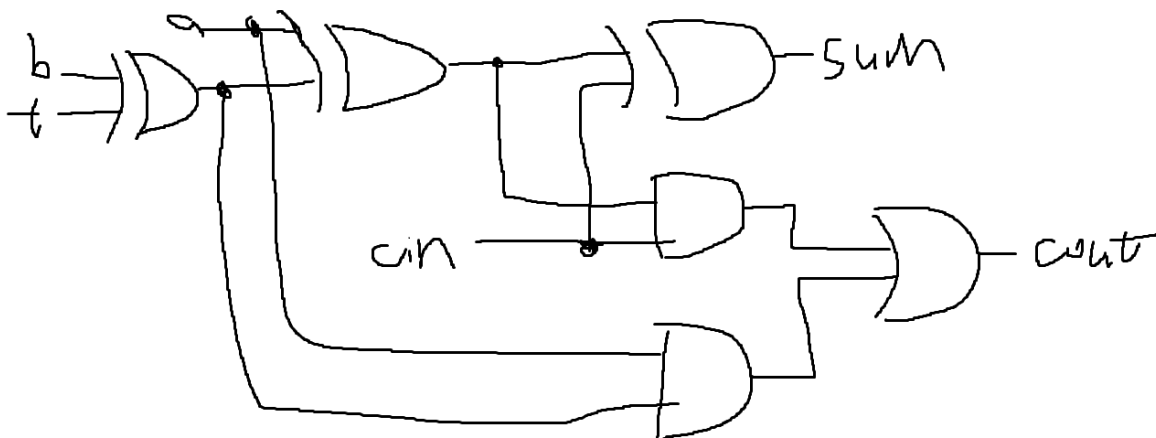
At most this travels through four gates, which is minimal. With only one gate, the adder becomes a subtractor as well.

2)

2.1

Column i	15-10	9	8	7	6	5	4	3	2	1	0	-1
238	0	0	0	1	1	1	0	1	1	1	0	0
675	0	1	0	1	0	1	0	0	0	1	1	0
$(G_{i:i}, P_{i:i})$	(0, 0)	(0,1)	(0,0)	(1,1)	(1,0)	(1,1)	(0,0)	(1,0)	(1,0)	(1,1)	(0,1)	(0, 0)

1st Stage:

$G_{0:-1} = G_{0:0} + P_{0:0}G_{-1:-1} = 0$
$G_{2:1} = G_{2:2} + P_{2:2}G_{1:1} = 1$
$G_{4:3} = G_{4:4} + P_{4:4}G_{3:3} = 0$
$G_{6:-5} = G_{6:6} + P_{6:6}G_{5:5} = 1$
$G_{8:7} = G_{8:8} + P_{8:8}G_{7:7} = 0$
$G_{10:9} = G_{10:10} + P_{10:10}G_{9:9} = 0$
$G_{12:11} = G_{12:12} + P_{12:12}G_{11:11} = 0$
$G_{14:13} = G_{14:14} + P_{14:14}G_{13:13} = 0$

$P_{0:-1} = P_{0:0}P_{-1:-1} = 0$
$P_{2:1} = P_{2:2}P_{1:1} = 1$
$P_{4:3} = P_{4:4}P_{3:3} = 0$
$P_{6:5} = P_{6:6}P_{5:5} = 1$
$P_{8:7} = P_{8:8}P_{7:7} = 0$
$P_{10:9} = P_{10:10}P_{9:9} = 0$
$P_{12:11} = P_{12:12}P_{11:11} = 0$
$P_{14:13} = P_{14:14}P_{13:13} = 0$

Column i:j	14:13	12:11	10:9	8:7	6:5	4:3	2:1	0:-1
$(G_{i:j}, P_{i:j})$	(0, 0)	(0, 0)	(0,0)	(0,0)	(1,1)	(0,0)	(1,1)	(0,0)

2nd stage:

$$G_{1:-1} = G_{1:1} + P_{1:1}G_{0:-1} = 1$$
$$G_{2:-1} = G_{2:1} + P_{2:1}G_{0:-1} = 1$$
$$G_{5:3} = G_{5:5} + P_{5:5}G_{4:3} = 1$$
$$G_{6:3} = G_{6:5} + P_{6:5}G_{4:3} = 1$$
$$G_{9:7} = G_{9:9} + P_{9:9}G_{8:7} = 0$$
$$G_{10:7} = G_{10:9} + P_{10:9}G_{8:7} = 0$$
$$G_{13:11} = G_{13:13} + P_{13:13}G_{12:11} = 0$$
$$G_{14:11} = G_{14:13} + P_{14:13}G_{12:11} = 0$$

$$P_{1:-1} = P_{1:1}P_{0:-1} = 0$$
$$P_{2:-1} = P_{2:1}P_{0:-1} = 0$$
$$P_{5:3} = P_{5:5}P_{4:3} = 0$$
$$P_{6:3} = P_{6:5}P_{4:3} = 0$$
$$P_{9:7} = P_{9:9}P_{8:7} = 0$$
$$P_{10:7} = P_{10:P}G_{8:7} = 0$$
$$P_{13:11} = P_{13:13}P_{12:11} = 0$$
$$P_{14:11} = P_{14:13}P_{12:11} = 0$$

Column i:j	14:11	13:11	10:7	9:7	6:3	5:3	2:-1	1:-1
$(G_{i:j}, P_{i:j})$	(0, 0)	(0, 0)	(0,0)	(0,0)	(1,0)	(1,0)	(1,0)	(1,0)

3rd stage:

$$G_{3:-1} = G_{3:3} + P_{3:3}G_{2:-1} = 1$$
$$G_{4:-1} = G_{4:3} + P_{4:3}G_{2:-1} = 0$$
$$G_{5:-1} = G_{5:3} + P_{5:3}G_{2:-1} = 1$$
$$G_{6:-1} = G_{6:3} + P_{6:3}G_{2:-1} = 1$$
$$G_{11:7} = G_{11:11} + P_{11:11}G_{10:7} = 0$$
$$G_{12:7} = G_{12:11} + P_{12:11}G_{10:7} = 0$$
$$G_{13:7} = G_{13:11} + P_{13:11}G_{10:7} = 0$$
$$G_{14:7} = G_{14:11} + P_{14:11}G_{10:7} = 0$$

$$P_{3:-1} = P_{3:3}P_{2:-1} = 0$$
$$P_{4:-1} = P_{4:3}P_{2:-1} = 0$$
$$P_{5:-1} = P_{5:3}P_{2:-1} = 0$$
$$P_{6:-1} = P_{6:3}P_{2:-1} = 0$$
$$P_{11:7} = P_{11:11}P_{10:7} = 0$$
$$P_{12:7} = P_{12:11}P_{10:7} = 0$$
$$P_{13:7} = P_{13:11}P_{10:7} = 0$$
$$P_{14:7} = P_{14:11}P_{10:7} = 0$$

Column i:j	14:7	13:7	12:7	11:7	6:-1	5:-1	4:-1	3:-1
$(G_{i:j}, P_{i:j})$	(0, 0)	(0, 0)	(0,0)	(0,0)	(1,0)	(1,0)	(1,0)	(1,0)

4th stage:

$G_{7:-1} = G_{7:7} + P_{7:7}G_{6:-1} = 1$
$G_{8:-1} = G_{8:7} + P_{8:7}G_{6:-1} = 0$
$G_{9:-1} = G_{9:7} + P_{9:7}G_{6:-1} = 0$
$G_{10:-1} = G_{9:7} + P_{9:7}G_{6:-1} = 0$
$G_{11:-1} = G_{11:7} + P_{11:7}G_{6:-1} = 0$
$G_{12:-1} = G_{12:7} + P_{12:7}G_{6:-1} = 0$
$G_{13:-1} = G_{13:7} + P_{13:7}G_{6:-1} = 0$
$G_{14:-1} = G_{14:7} + P_{14:7}G_{6:-1} = 0$

$P_{7:-1} = P_{7:7}P_{6:-1} = 1$
$P_{8:-1} = P_{8:7}P_{6:-1} = 0$
$P_{9:-1} = P_{9:7}P_{6:-1} = 0$
$P_{10:-1} = P_{9:7}P_{6:-1} = 0$
$P_{11:-1} = P_{11:7}P_{6:-1} = 0$
$P_{12:-1} = P_{12:7}P_{6:-1} = 0$
$P_{13:-1} = P_{13:7}P_{6:-1} = 0$
$P_{14:-1} = P_{14:7}P_{6:-1} = 0$

Column i:j	14:-1	13:-1	12:-1	11:-1	10:-1	9:-1	8:-1	7:-1
$(G_{i:j}, P_{i:j})$	(0, 0)	(0, 0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)

Column i	15-11	10	9	8	7	6	5	4	3	2	1	0
238	0	0	0	0	1	1	1	0	1	1	1	0
675	0	0	1	0	1	0	1	0	0	0	1	1
$G_{i-1:-1}$	0	0	0	1	1	1	0	1	1	1	0	0
Sum	0	0	1	1	1	0	0	1	0	0	0	1

$= 01110010001 = 913$

2.2

Column i	15-10	9	8	7	6	5	4	3	2	1	0	-1
432	0	0	1	1	0	1	1	0	0	0	0	
521	0	1	0	0	0	0	0	1	0	0	1	
$(G_{i:i}, P_{i:i})$	(0, 0)	(0,1)	(1,0)	(1,0)	(0,0)	(1,0)	(1,0)	(0,1)	(0,0)	(0,0)	(0,1)	(0,0)

1st stage:

$G_{0:-1} = G_{0:0} + P_{0:0}G_{-1:-1} = 0$
$G_{2:1} = G_{2:2} + P_{2:2}G_{1:1} = 0$
$G_{4:3} = G_{4:4} + P_{4:4}G_{3:3} = 0$
$G_{6:5} = G_{6:6} + P_{6:6}G_{5:5} = 0$
$G_{8:7} = G_{8:8} + P_{8:8}G_{7:7} = 0$
$G_{10:9} = G_{10:10} + P_{10:10}G_{9:9} = 0$
$G_{12:11} = G_{12:12} + P_{12:12}G_{11:11} = 0$
$G_{14:13} = G_{14:14} + P_{14:14}G_{13:13} = 0$

$P_{0:-1} = P_{0:0}P_{-1:-1} = 0$
$P_{2:1} = P_{2:2}P_{1:1} = 0$
$P_{4:3} = P_{4:4}P_{3:3} = 1$
$P_{6:5} = P_{6:6}P_{5:5} = 0$
$P_{8:7} = P_{8:8}P_{7:7} = 1$
$P_{10:9} = P_{10:10}P_{9:9} = 0$
$P_{12:11} = P_{12:12}P_{11:11} = 0$
$P_{14:13} = P_{14:14}P_{13:13} = 0$

Column i:j	14:13	12:11	10:9	8:7	6:5	4:3	2:1	0:-1
$(G_{i:j}, P_{i:j})$	(0, 0)	(0, 0)	(0,0)	(0,1)	(0,0)	(0,1)	(0,0)	(0,0)

2nd stage:

$G_{1:-1} = G_{1:1} + P_{1:1}G_{0:-1} = 0$
$G_{2:-1} = G_{2:1} + P_{2:1}G_{0:-1} = 0$
$G_{5:3} = G_{5:5} + P_{5:5}G_{4:3} = 0$
$G_{6:3} = G_{6:5} + P_{6:5}G_{4:3} = 0$
$G_{9:7} = G_{9:9} + P_{9:9}G_{8:7} = 0$
$G_{10:7} = G_{10:9} + P_{10:9}G_{8:7} = 0$
$G_{13:11} = G_{13:13} + P_{13:13}G_{12:11} = 0$

$G_{14:11} = G_{14:13} + P_{14:13}G_{12:11} = 0$

$P_{1:-1} = P_{1:1}P_{0:-1} = 0$
$P_{2:-1} = P_{2:1}P_{0:-1} = 0$
$P_{5:3} = P_{5:5}P_{4:3} = 1$
$P_{6:3} = P_{6:5}P_{4:3} = 0$
$P_{9:7} = P_{9:9}P_{8:7} = 1$
$P_{10:7} = P_{10:P}G_{8:7} = 0$
$P_{13:11} = P_{13:13}P_{12:11} = 0$
$P_{14:11} = P_{14:13}P_{12:11} = 0$

Column i:j	14:11	13:11	10:7	9:7	6:3	5:3	2:-1	1:-1
$(G_{i:j}, P_{i:j})$	(0, 0)	(0, 0)	(0,0)	(0,1)	(0,0)	(0,1)	(0,0)	(0,0)

3rd stage:

$G_{3:-1} = G_{3:3} + P_{3:3}G_{2:-1} = 0$
$G_{4:-1} = G_{4:3} + P_{4:3}G_{2:-1} = 0$
$G_{5:-1} = G_{5:3} + P_{5:3}G_{2:-1} = 0$
$G_{6:-1} = G_{6:3} + P_{6:3}G_{2:-1} = 0$
$G_{11:7} = G_{11:11} + P_{11:11}G_{10:7} = 0$
$G_{12:7} = G_{12:11} + P_{12:11}G_{10:7} = 0$
$G_{13:7} = G_{13:11} + P_{13:11}G_{10:7} = 0$
$G_{14:7} = G_{14:11} + P_{14:11}G_{10:7} = 0$

$P_{3:-1} = P_{3:3}P_{2:-1} = 0$
$P_{4:-1} = P_{4:3}P_{2:-1} = 0$
$P_{5:-1} = P_{5:3}P_{2:-1} = 0$
$P_{6:-1} = P_{6:3}P_{2:-1} = 0$
$P_{11:7} = P_{11:11}P_{10:7} = 0$
$P_{12:7} = P_{12:11}P_{10:7} = 0$
$P_{13:7} = P_{13:11}P_{10:7} = 0$
$P_{14:7} = P_{14:11}P_{10:7} = 0$

Column i:j	14:7	13:7	12:7	11:7	6:-1	5:-1	4:-1	3:-1
$(G_{i:j}, P_{i:j})$	(0, 0)	(0, 0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

4th stage:

$G_{7:-1} = G_{7:7} + P_{7:7}G_{6:-1} = 0$
$G_{8:-1} = G_{8:7} + P_{8:7}G_{6:-1} = 0$
$G_{9:-1} = G_{9:7} + P_{9:7}G_{6:-1} = 0$
$G_{10:-1} = G_{9:7} + P_{9:7}G_{6:-1} = 0$
$G_{11:-1} = G_{11:7} + P_{11:7}G_{6:-1} = 0$
$G_{12:-1} = G_{12:7} + P_{12:7}G_{6:-1} = 0$
$G_{13:-1} = G_{13:7} + P_{13:7}G_{6:-1} = 0$
$G_{14:-1} = G_{14:7} + P_{14:7}G_{6:-1} = 0$

$P_{7:-1} = P_{7:7}P_{6:-1} = 0$
$P_{8:-1} = P_{8:7}P_{6:-1} = 0$
$P_{9:-1} = P_{9:7}P_{6:-1} = 0$
$P_{10:-1} = P_{9:7}P_{6:-1} = 0$
$P_{11:-1} = P_{11:7}P_{6:-1} = 0$
$P_{12:-1} = P_{12:7}P_{6:-1} = 0$
$P_{13:-1} = P_{13:7}P_{6:-1} = 0$
$P_{14:-1} = P_{14:7}P_{6:-1} = 0$

Column i:j	14:-1	13:-1	12:-1	11:-1	10:-1	9:-1	8:-1	7:-1
$(G_{i:j}, P_{i:j})$	(0, 0)	(0, 0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

Column i	15-11	9	8	7	6	5	4	3	2	1	0
432	0	0	1	1	0	1	1	0	0	0	0
521	0	1	0	0	0	0	0	1	0	0	1
$G_{i-1:-1}$	0	0	0	0	0	0	0	0	0	0	0
Sum	0	1	1	1	0	1	1	1	0	0	1

= 1110111001 = 953

3)

3.1

For each digit, we have 3 bits

Weight 0: (0, 0, 0)

Weight 1: (0, 0, 1), (0, 1, 0), (1, 0, 0)

Weight 2: (0, 1, 1), (1, 0, 1), (1, 1, 0)

Weight 3: (1, 1, 1)

Algorithm:

1. Convert the 1st addend to the redundant number system.
2. From the 5 numbers, convert them to carry save additions. Convert back to binary.
3. Convert the 1st of the remaining binary values into its redundant number system.
4. Add those two numbers.

3.2

Weight	16	8	4	2	1
12	0	1	1	0	0
7	0	0	1	1	1
9	0	1	0	0	1
10	0	1	0	1	0
17	1	0	0	0	1

0	1	1	0	0	1
0	1	1	1	1	0
0	0	0	0	0	0
1	1	0	1	1	1

= 110111 = 55

Weight	16	8	4	2	1
11	0	1	0	1	1
5	0	0	1	0	1
14	0	1	1	1	0
16	1	0	0	0	0

0	1	0	0	0	0
0	1	1	1	1	0
0	0	0	0	0	0
1	**0**	**1**	**1**	**1**	**0**

= 101110 = 46

Weight	32	16	8	4	2	1
46	1	0	1	1	1	0

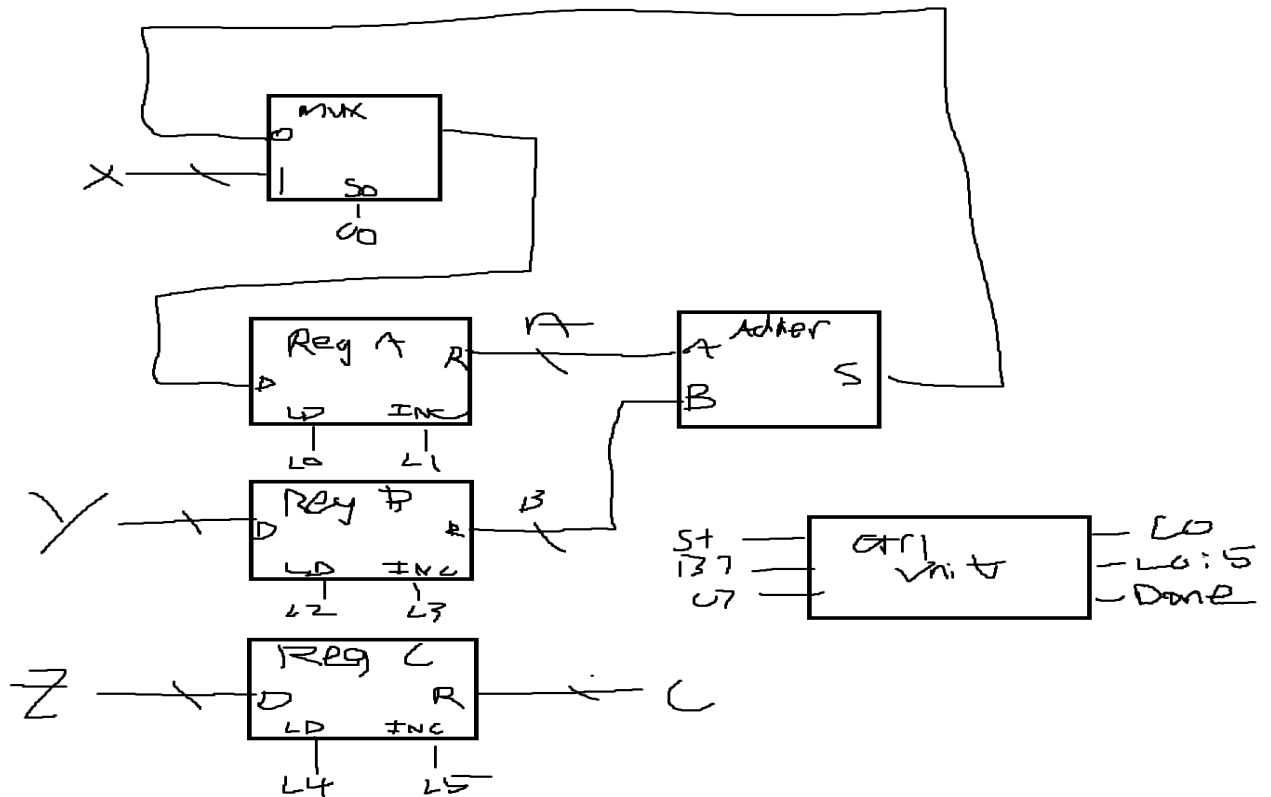| 0	0	1	1	0	0	1
1	0	0	1	1	0	0
0	0	0	0	0	0	0
1	**1**	**0**	**0**	**1**	**0**	**1**

= 1100101 = 101

3.3

Because we have 5 numbers, that is the case, as we will never go past 3 bits to hold 5 1s. So using 3 bits is all we need.

4)

4.1

State	Instr	Op
S1	done ← 1	
S2	A ← X	A ← Load(X)
S2	B ← Y	B ← Load(Y)
S2	C ← Z	C ← Load(Z)
S3	A ← A + B	A ← Add(A, B)
S3	B ← Inc(B)	B ← Inc(B)
S4	C ← Inc(C)	C ← Inc(C)
S5	A ← Inc(A)	A ← Inc(A)
S6	U ← A	Wire
S6	done ← 1	

4.2

	L0	L1	L2	L3	L4	L5	c0	done
S0	0	0	0	0	0	0	X	1
S1	1	0	1	0	1	0	1	0
S2	1	0	0	1	0	0	0	0
S3	0	0	0	0	0	1	X	0
S4	0	1	0	0	0	0	X	0
S5	0	0	0	0	0	0	X	1

4.3