1.

P2:

Let T be a TM that decides P2:

“On input w:

1. If w is not of type <M> where M is a DFA over {a,b}, reject.
2. Construct a TM T_{EDFA} where $L(T_{EDFA}) = E_{DFA}$
3. Run T_{EDFA} on M.
4. If T_{EDFA} accepts, reject.
5. Determine if the accept state of M has only one string that leads to it, by beginning to find strings in M. If at any point we find more than one string, reject.
6. Otherwise, accept.

All our steps are finite and for every TM x, if x is in P2, T accepts. If x is not in P2, T rejects. We are able to determine if $|L(M)| = 1$ in step 2 because DFAs have a finite number of states, and finite number. Notice that if M was a regular expression, and there is a * or U, T will reject finitely. Otherwise, we find the only string in M.

Thus, P2 is decidable.

P4:

Let T be a TM that decides P4:

“On input w:

1. If w is not of type <M> where M is a DFA over {a, b}, reject.
2. Construct a TM T_{ADFA} where $L(T_{ADFA}) = A_{DFA}$
3. Run T_{ADFA} on <M, ab>
4. If T_{ADFA} accepts, accepts. Otherwise reject.”

All our steps are finite and A_{DFA} is decidable, and for every TM x, if x is in P4, T accepts. If x is not in P4, T rejects.

Thus, P4 is decidable.

P5:

Let T be a TM that decides P5:

“On input w:

1. If w is not of type <M, M’> where M and M’ are both DFA over {a, b}, reject.
2. Construct a TM T_{EQDFA} where $L(T_{EQDFA}) = EQ_{DFA}$
3. Run T_{EQDFA} on w.
4. If T_{EQDFA} accepts, reject. Otherwise accept. “

All our steps are finite and EQ_{DFA} is decidable, and for every TM x, if x is in P5, T accepts. If x is not in P5, T rejects.

Thus, P5 is decidable.

P6:

Let T be a TM that decides P6:

“On input w:

1. If w is not of type <M, M’> where M and M’ are both DFA over {a, b}, reject.
2. In parallel, find the list of reachable states M and M’s transition functions, and if they are accept or reject states. At any point, if M has an accept state and M’ has a reject state, reject.
3. After all the states are ran, accept.

All our steps are finite and for every TM x, if x is in P6, T accepts. If x is not in P6, T rejects. Notice a DFA has a finite amount of states, so step 2 is finite.

Thus, P6 is decidable.

2.

a) No such language exists. Proof by contradiction:

Suppose there is an unrecognizable language whose complement is finite. Its complement must be a decidable language. A language is decidable iff both it and its complement are recognizable, which means the original unrecognizable language is recognizable, which is a contradiction.

b) No such language exists. Proof by contradiction:

Suppose there is a context-free language that is undecidable. However, we know that every context-free language is decidable by Theorem 4.9 of the textbook. It cannot be undecidable and decidable.

c) Σ^*

We know Σ^* is recognizable, as we can construct a TM that accepts every string. We also know every language is a subset of Σ^*, and it has unrecognizable subsets, such as E_{TM}.

d) A_{TM}

A_{TM} is recognizable as we can construct a TM for it. It is undecidable as defined in class, as we cannot compute D on D via the diagonalization proof.

e) A_{TM}^C

We know a language is decidable if and only it and its complement is recognizable. As A_{TM} as seen above is undecidable, A_{TM}^C must be unrecognizable. If it is unrecognizable, it is also undecidable.

3.

a)

Let T be a TM that recognizes $4st_{TM}$: (also decides)

"On input w:

1. If w is not of type <M> where M is a TM, reject.
2. Read the number of states in M.
3. If $|Q|$ of M is 4, accept. Otherwise reject."

We know there are Turing Machines that can be called as a subroutine to decode string representations of objects and interact with them, so we are able to count the number of states in M. Turing Machines have a finite set of states, so this is possible. Also, every step in this definition is finite.

Thus, $4st_{TM}$ is recognizable.

b)

Let T be a TM that recognizes $NonE_{TM}$:

"On input w:

1. If w is not of type <M> where M is a TM, reject.
2. Run E_{TM}^C on w.
3. If E_{TM}^C accepts, accept. Otherwise reject."

Notice that E_{TM}^C is recognizable. For every TM x, if x is in $NonE_{TM}$. T accepts. If x is not, T rejects or does not halt.

Thus, $NonE_{TM}$ is recognizable.