1.

a) **$M' = \{Q \cup \{q_{new}\}, \Sigma, \Gamma, \delta', q0, q_{new}\}$**

where qnew is a new state

$\delta'((q, x, y)) = \{ \delta((q,x,y))$ if $q \in Q \backslash F$, $x \in \Sigma_\varepsilon$, $y \in \Gamma_\varepsilon$

$\delta((q,x,y))$ if $q \in F$, $x \in \Sigma_\varepsilon \setminus \{a\}$, $y \in \Gamma$

$\{(q_{new}, \varepsilon)\} \cup \delta((q,x,y))$ if $q \in F$, $x = a$, $y = \varepsilon$

$\emptyset$ otherwise $\}$

Our set of states is the states of M along with our new accept state

All our old accept states point to our new accept state

Everything else stays the same

b)

$G' = \{V \cup \{W\}, \Sigma \cup a, R \cup \{W \to Sa\}, W\}$

Where W is a variable not in V.

We Σ U a in case a is not included in G, use the same rules as R, but our new rule and start state is the one that appends a to the end of languages defined by G.

2.

a) **$(\{S, T, V\}, \{a, b\}, R, S)$**

with rules $R = \{ S \to aT \mid bT \mid \varepsilon$

$T \to aV \mid bV$

$V \to aS \mid bS \}$

b) **It is not ambiguous as we can't find more than one leftmost derivation. Every leftmost step we take cannot be used ambiguously to create the same string.**

e.g.

If we are trying to form the string aab

if S -> aT -> aaV -> aabS -> aab

Each step is the only step we can take for the sake of our string, as it must be used to form the next symbol required.

c) **({S}, {a, b}, R, S)**

with rules R = { S -> SaSaSbS | SaSbSaS | SbSaSaS | ε }

d) **It is ambiguous as we can find more than one leftmost derivation**

e.g.

S -> SaSaSbS -> aSaSbS -> aaSbS -> aabS -> aabSaSaSbS -> aabaab

S -> SaSaSbS -> SaSaSbSaSaSbS -> aabaab

3.

a) **X = {$0^n1^n2^n$| n >= 0}**

Y = {$0^n1^n2^m$| n, m >=0}

X is not context free, as we can prove it by pumping lemma.

Y is context free and is nonregular, as we can define it with a PDA or CFG (we can concatenate 0^n1^n to 2^m) but not a DFA or NFA.

X is a subset of Y as we can arbitrarily choose m to be n.

b)

A = {0^n1^n | n >= 0}

B = {$0^n1^n2^n$ U 0^n1^n | n >= 0}

A is nonregular as proved in the textbook.

B is context free and is nonregular, as we can define it with a PDA or CFG (we can concatenate 0^n1^n to 2^m) but not a DFA or NFA.

A is a subset of B because it is included in the union of $0^n1^n2^n$ and 0^n1^n