

Name: _____

PID: _____

On your desk you should have only the exam paper, writing tools, and the cheat-sheet. The cheat-sheet is one page handwritten on both sides.

No hats or hoods allowed (unless religious items).

Be clear and concise. Write your answers **in the space provided** after each question. Whenever possible, **write your answers as expressions, not as numbers**. If an expression is reused later in the problem, you are encouraged to assign it to a letter variable and use the variable in the later expressions. For example, suppose in the first part of the problem you find the answer to be $C(52, 5)$ and that in a later part the answer is $22/C(52, 5)$. You can define in the first part $a = C(52, 5)$ and write the second expression as $22/a$. This will save you time and space and is easier for us to grade.

Do your work at the empty space below each question. If your answer is incorrect, we will look at your work and give you credit if you were going in the right direction. You can also use the blank page opposite the problem page as scratch space, but **we will not consider that scratch space in the evaluation**. We have extra scratch paper if you need it.

Good Luck!

1) Uniform Distributions (10 pt)

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On which of the following sets is it possible to define a uniform distribution?

1. $\{1, 2, \dots, n\}$ Possible / Impossible
2. The Natural numbers $\{1, 2, \dots\}$ Possible / Impossible
3. A segment of the real line: $5 \leq x \leq 7$ Possible / Impossible

2) Set Algebra (10pt)

Let A, B be events such that $P(A) = 0.5, P(B) = 0.4$
and $P(A \cap B) = 0.3$ What is $P((A \cup B)^c)$?
Show your work.

3) Bayesian Inference (20 pt)

You watch a match between two tennis players: Bill and Tom. You know that one of them is ranked first in the district and the other is ranked 3rd but you don't know which is which. You also know that the probability that the 3rd ranked player wins any game is $p < 1/2$, independently of the outcome of other games. You watch ten games. Bill wins eight and Tom wins two. What is the probability that Bill is the first ranked player? Show your work.

4) Covariance Calculation (30 points)

Covariance Calculation

Suppose X, Y are discrete random variables whose joint distribution is

	$X=1$	$X=2$	$X=3$
$Y=1$	0.25	0	0
$Y=2$	0	0	0.5
$Y=3$	0.25	0	0

1. What is the distribution over X ?

$$P(X = 1) = \underline{\hspace{2cm}} \quad P(X = 2) = \underline{\hspace{2cm}}$$

$$P(X = 3) = \underline{\hspace{2cm}}$$

$$\text{What is } E(X) ? \underline{\hspace{2cm}}$$

$$\text{What is } Var(X) ? \underline{\hspace{2cm}}$$

2. What is the distribution over Y ?

$$P(Y = 1) = \underline{\hspace{2cm}} \quad P(Y = 2) = \underline{\hspace{2cm}}$$

$$P(Y = 3) = \underline{\hspace{2cm}}$$

$$\text{What is } E(Y) ? \underline{\hspace{2cm}}$$

$$\text{What is } Var(Y) ? \underline{\hspace{2cm}}$$

3. What is the relationship between X and Y ?

$$\text{What is } E(XY) ? \underline{\hspace{2cm}}$$

$$\text{What is } Cov(X, Y) ? \underline{\hspace{2cm}}$$

$$\text{What is } Corr(X, Y) ? \underline{\hspace{2cm}}$$

$$\text{Are } X \text{ and } Y \text{ independent (1=independent, 0=dependent)?} \underline{\hspace{2cm}}$$

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5) Poisson Process (30 points)

A professor schedules office hours for his students. He finds that the arrivals of the students follow a Poisson process with an average rate of $\gamma = 1.8$

- a. What is the probability that the number of students that arrive during a particular hour is 4?

- b. What is the probability that the time between consecutive arrivals is exactly 13 minutes?

- c. What is the probability that the time between consecutive student arrivals is between 13 and 28 ?
