I LISTA DE EXERCICIOS INTEGRAL – CALCULO II
CIÊNCIA DA COMPUTAÇÃO – DOM HELDER CAMARA
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5.3 | O TEOREMA FUNDAMENTAL DO CÁLCULO

1. Esboce a área representada por

$$g(x) = \int_{\pi}^{x} (2 + \cos t)\, dt.$$

A seguir, encontre $g'(x)$ de duas formas: (a) utilizando a Parte 1 do Teorema Fundamental do Cálculo e (b) calculando a integral usando a Parte 2 e, então, derivando.

2-12 Use a Parte 1 do Teorema Fundamental do Cálculo para encontrar a derivada da função.

2. $g(x) = \int_{1}^{x} (t^2 - 1)^{20}\, dt$ **3.** $g(x) = \int_{-1}^{x} \sqrt{t^3 + 1}\, dt$

4. $g(u) = \int_{\pi}^{u} \dfrac{1}{1 + t^4}\, dt$ **5.** $g(t) = \int_{0}^{t} \text{sen}(x^2)\, dx$

6. $F(x) = \int_{x}^{4} \left(2 + \sqrt{u}\right)^8 du$ **7.** $h(x) = \int_{2}^{1/x} \text{sen}^4 t\, dt$

8. $h(x) = \int_{1}^{\sqrt{x}} \dfrac{s^2}{s^2 + 1}\, ds$ **9.** $y = \int_{\text{tg}\, x}^{17} \text{sen}(t^4)\, dt$

10. $y = \int_{x^2}^{\pi} \dfrac{\text{sen}\, t}{t}\, dt$ **11.** $y = \int_{0}^{5x+1} \dfrac{1}{u^2 - 5}\, du$

12. $y = \int_{-5}^{\text{sen}\, x} t \cos(t^3)\, dt$

13-35 Use a Parte 2 do Teorema Fundamental do Cálculo para avaliar a integral, ou explique por que ela não existe.

13. $\int_{-2}^{4} (3x - 5)\, dx$ **14.** $\int_{1}^{2} x^{-2}\, dx$

15. $\int_{0}^{4} \sqrt{x}\, dx$ **16.** $\int_{0}^{1} x^{3/7}\, dx$

17. $\int_{-1}^{4} \pi\, dx$ **18.** $\int_{-4}^{-1} \sqrt{3}\, dx$

19. $\int_{3}^{6} (4 - 7x)\, dx$ **20.** $\int_{1}^{4} (2x^2 - 3x + 1)\, dx$

21. $\int_{1}^{3} (x - 2)(x + 3)\, dx$ **22.** $\int_{0}^{\pi/3} (1 - 2\cos x)\, dx$

23. $\int_{0}^{1} (5\cos x + 4x)\, dx$ **24.** $\int_{-1}^{1} \dfrac{3}{t^4}\, dt$

25. $\int_{3}^{3} \sqrt{x^5 + 2}\, dx$ **26.** $\int_{-4}^{2} \dfrac{2}{x^6}\, dx$

27. $\int_{\pi/4}^{\pi/3} \text{sen}\, t\, dt$ **28.** $\int_{1}^{2} \left(x + \dfrac{1}{x}\right)^2 dx$

29. $\int_{0}^{1} \left(\sqrt[4]{x^5} + \sqrt[5]{x^4}\right) dx$ **30.** $\int_{1}^{8} \dfrac{x - 1}{\sqrt[3]{x^2}}\, dx$

31. $\int_{\ln 3}^{\ln 6} 8e^x\, dx$ **32.** $\int_{8}^{9} 2^t\, dt$

33. $\int_{-e^2}^{-e} \dfrac{3}{x}\, dx$ **34.** $\int_{1}^{\sqrt{3}} \dfrac{6}{1 + x^2}\, dx$

35. $\int_{0}^{0,5} \dfrac{dx}{\sqrt{1 - x^2}}$

5.4 | INTEGRAIS INDEFINIDAS E O TEOREMA DA VARIAÇÃO TOTAL

1-2 Verifique, por derivação, que a fórmula está correta.

1. $\int \text{sen}^2 x\, dx = \dfrac{x}{2} - \dfrac{\text{sen}\, 2x}{4} + C$

2. $\int x^2 \,\text{sen}\, x\, dx = -x^2 \cos x + 2 \int x \cos x\, dx$

3-4 Encontre a integral indefinida geral.

3. $\int \sqrt{x}\,(x^2 - 1/x)\, dx$

4. $\int (2x + \sec x \,\text{tg}\, x)\, dx$

5-22 Calcule a integral.

5. $\int_{0}^{1} (1 - 2x - 3x^2)\, dx$ **6.** $\int_{1}^{2} (5x^2 - 4x + 3)\, dx$
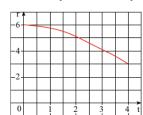
7. $\int_{0}^{1} (y^9 - 2y^5 + 3y)\, dy$ **8.** $\int_{1}^{3} \left(\dfrac{1}{t^2} - \dfrac{1}{t^4}\right) dt$

9. $\int_{1}^{2} \dfrac{t^6 - t^2}{t^4}\, dt$ **10.** $\int_{1}^{2} \dfrac{x^2 + 1}{\sqrt{x}}\, dx$

11. $\int_{0}^{2} (x^3 - 1)^2\, dx$ **12.** $\int_{0}^{1} u\left(\sqrt{u} + \sqrt[3]{u}\right) du$

13. $\int_{-1}^{2} |x - x^2|\, dx$ **14.** $\int_{-2}^{3} |x^2 - 1|\, dx$

15. $\int_{1}^{-1} (x - 1)(3x + 2)\, dx$ **16.** $\int_{1}^{4} \left(\sqrt{t} - \dfrac{2}{\sqrt{t}}\right) dt$

17. $\int_{1}^{8} \left(\sqrt[3]{r} + \dfrac{1}{\sqrt[3]{r}}\right) dr$ **18.** $\int_{-1}^{0} (x + 1)^3\, dx$

19. $\int_{-5}^{-2} \dfrac{x^4 - 1}{x^2 + 1}\, dx$ **20.** $\int_{\pi/6}^{\pi/3} \text{cossec}^2 \theta\, d\theta$

21. $\int_{\pi/3}^{\pi/2} \text{cossec}\, x\, \text{cotg}\, x\, dx$ **22.** $\int_{0}^{2} (x^2 - |x - 1|)\, dx$

23. A água vaza de um tanque a uma taxa de $r(t)$ litros por hora, onde o gráfico de r é como abaixo mostrado. Expresse a quantidade total de água que vazou durante as primeiras quatro horas como uma integral definida. Em seguida, use a Regra do Ponto Médio para estimar esta quantidade.

1-6 Calcule a integral fazendo a substituição dada.

1. $\int x(x^2 - 1)^{99} \, dx, \quad u = x^2 - 1$

2. $\int \dfrac{x^2}{\sqrt{2 + x^3}} \, dx, \quad u = 2 + x^3$

3. $\int \operatorname{sen} 4x \, dx, \quad u = 4x$

4. $\int \dfrac{dx}{(2x + 1)^2}, \quad u = 2x + 1$

5. $\int \dfrac{x + 3}{(x^2 + 6x)^2} \, dx, \quad u = x^2 + 6x$

6. $\int \sec a\theta \, \operatorname{tg} a\theta \, d\theta, \quad u = a\theta$

7-35 Calcule a integral indefinida.

7. $\int (2x + 1)(x^2 + x + 1)^3 \, dx$ **8.** $\int x^3(1 - x^4)^5 \, dx$

9. $\int \sqrt{x - 1} \, dx$ **10.** $\int \sqrt[3]{1 - x} \, dx$

11. $\int x^3 \sqrt{2 + x^4} \, dx$ **12.** $\int x(x^2 + 1)^{3/2} \, dx$

13. $\int \dfrac{2}{(t + 1)^6} \, dt$ **14.** $\int \dfrac{1}{(1 - 3t)^4} \, dt$

15. $\int (1 - 2y)^{1.3} \, dy$ **16.** $\int \sqrt[5]{3 - 5y} \, dy$

17. $\int \cos 2\theta \, d\theta$ **18.** $\int \sec^2 3\theta \, d\theta$

19. $\int \dfrac{3x - 1}{(3x^2 - 2x + 1)^4} \, dx$ **20.** $\int \dfrac{x}{\sqrt{x^2 + 1}} \, dx$

21. $\int \operatorname{sen}^3 x \cos x \, dx$ **22.** $\int \operatorname{tg}^2\theta \, \sec^2\theta \, d\theta$

23. $\int t \operatorname{sen}(t^2) \, dt$ **24.** $\int \dfrac{\left(1 + \sqrt{x}\right)^9}{\sqrt{x}} \, dx$

25. $\int \sec x \operatorname{tg} x \sqrt{1 + \sec x} \, dx$ **26.** $\int t^2 \cos(1 - t^3) \, dt$

27. $\int e^x \operatorname{sen}(e^x) \, dx$ **28.** $\int \cos^4 x \operatorname{sen} x \, dx$

29. $\int \dfrac{x + 1}{x^2 + 2x} \, dx$ **30.** $\int \dfrac{e^x}{e^{2x} + 1} \, dx$

31. $\int x^3(1 - x^2)^{3/2} \, dx$ **32.** $\int \dfrac{\cos \sqrt{x}}{\sqrt{x}} \, dx$

33. $\int \operatorname{sen}(2x + 3) \, dx$ **34.** $\int \cos(7 - 3x) \, dx$

35. $\int (\operatorname{sen} 3\alpha - \operatorname{sen} 3x) \, dx$

36-43 Calcule a integral definida, se existir.

36. $\int_0^1 \cos \pi t \, dt$ **37.** $\int_0^{\pi/4} \operatorname{sen} 4t \, dt$

38. $\int_1^4 \dfrac{1}{x^2} \sqrt{1 + \dfrac{1}{x}} \, dx$ **39.** $\int_0^3 \dfrac{dx}{2x + 3}$

40. $\int_0^1 (2x - 1)^{100} \, dx$ **41.** $\int_0^4 \sqrt{1 - 2x} \, dx$

42. $\int_0^1 (x^4 + x)^5(4x^3 + 1) \, dx$ **43.** $\int_2^3 \dfrac{3x^2 - 1}{(x^3 - x)^2} \, dx$

44. Mostre que a área sob o gráfico de $y = \operatorname{sen} \sqrt{x}$ de 0 a 4 é igual à área sob o gráfico de $y = 2x \operatorname{sen} x$ de 0 a 2.

3. $\frac{2}{7}x^{7/2} - 2x^{1/2} + C$ **4.** $x^2 + \sec x + C$ **5.** -1

6. $\frac{26}{3}$ **7.** $\frac{19}{15}$ **8.** $\frac{28}{21}$ **9.** $\frac{11}{6}$ **10.** $\frac{6}{5}(3\sqrt{2} - 2)$ **11.** $\frac{86}{7}$

12. $\frac{29}{35}$ **13.** $\frac{11}{6}$ **14.** $\frac{28}{3}$ **15.** 2 **16.** $\frac{2}{3}$ **17.** $\frac{63}{4}$

18. $\frac{1}{4}$ **19.** 36 **20.** $\frac{2}{3}\sqrt{3}$ **21.** $-1 + \frac{2}{3}\sqrt{3}$ **22.** $\frac{5}{3}$

23. $\displaystyle\int_0^4 r(t)\, dt \approx 19{,}6\,\text{L}$

1.

(a), (b) $2 + \cos x$

2. $g'(x) = (x^2 - 1)^{20}$ **3.** $g'(x) = \sqrt{x^3 + 1}$

4. $g'(u) = \dfrac{1}{1 + u^4}$ **5.** $g'(t) = \operatorname{sen}(t^2)$

6. $F'(x) = -(2 + \sqrt{x})^8$ **7.** $h'(x) = \dfrac{-\operatorname{sen}^4(1/x)}{x^2}$

8. $h'(x) = \dfrac{\sqrt{x}}{2(x + 1)}$ **9.** $\dfrac{dy}{dx} = -\operatorname{sen}(\operatorname{tg}^4 x)\sec^2 x$

10. $\dfrac{dy}{dx} = -\dfrac{2\operatorname{sen}(x^2)}{x}$ **11.** $\dfrac{dy}{dx} = \dfrac{5}{25x^2 + 10x - 4}$

12. $\dfrac{dy}{dx} = \operatorname{sen} x \cos x \cos(\operatorname{sen}^3 x)$ **13.** -12

14. $\frac{1}{2}$ **15.** $\frac{16}{3}$ **16.** $\frac{7}{10}$ **17.** 5π

18. $3\sqrt{3}$ **19.** $-\frac{165}{2}$ **20.** $\frac{45}{2}$ **21.** $\frac{2}{3}$

22. $\frac{\pi}{3} - \sqrt{3}$ **23.** $5\operatorname{sen} 1 + 2$

24. Não existe   **25.** 0

26. Não existe   **27.** $\frac{\sqrt{2}-1}{2}$

28. $\frac{29}{6}$ **29.** 1 **30.** $\frac{33}{4}$

31. 24 **32.** $\dfrac{2^8}{\ln 2}$ **33.** -3 **34.** $\frac{\pi}{2}$ **35.** $\frac{\pi}{6}$

5.5 RESPOSTAS

1. $\frac{1}{200}\left(x^2-1\right)^{100}+C$ **2.** $\frac{2}{3}\sqrt{2+x^3}+C$

3. $-\frac{1}{4}\cos 4x + C$ **4.** $-\dfrac{1}{2\left(2x+1\right)}+C$

5. $-\dfrac{1}{2\left(x^2+6x\right)}+C$ **6.** $\dfrac{\sec a\theta}{a}+C$

7. $\frac{1}{4}\left(x^2+x+1\right)^4+C$ **8.** $-\frac{1}{24}\left(1-x^4\right)^6+C$

9. $\frac{2}{3}\left(x-1\right)^{3/2}+C$ **10.** $-\frac{3}{4}\left(1-x\right)^{4/3}+C$

11. $\frac{1}{6}\left(2+x^4\right)^{3/2}+C$ **12.** $\frac{1}{5}\left(x^2+1\right)^{5/2}+C$

13. $-\dfrac{2}{5\left(t+1\right)^5}+C$ **14.** $\dfrac{1}{9\left(1-3t\right)^3}+C$

15. $-\dfrac{\left(1-2y\right)^{2,3}}{4,6}+C$ **16.** $-\frac{1}{6}\left(3-5y\right)^{6/5}+C$

17. $\frac{1}{2}\operatorname{sen}2\theta+C$ **18.** $\frac{1}{3}\operatorname{tg}3\theta+C$

19. $-\dfrac{1}{6\left(3x^2-2x+1\right)^3}+C$ **20.** $\sqrt{x^2+1}+C$

21. $\frac{1}{4}\operatorname{sen}^4 x+C$ **22.** $\frac{1}{3}\operatorname{tg}^3\theta+C$

23. $-\frac{1}{2}\cos\left(t^2\right)+C$ **24.** $\dfrac{\left(1+\sqrt{x}\right)^{10}}{5}+C$

25. $\frac{2}{3}\left(1+\sec x\right)^{3/2}+C$ **26.** $-\frac{1}{3}\operatorname{sen}\left(1-t^3\right)+C$

27. $-\cos\left(e^x\right)+C$

28. $-\frac{1}{5}\cos^5 x+C$

29. $\frac{1}{2}\ln\left|x^2+2x\right|+C$

30. $\operatorname{tg}^{-1}\left(e^x\right)+C$

31. $\frac{1}{7}\left(1-x^2\right)^{7/2}-\frac{1}{5}\left(1-x^2\right)^{5/2}+C$

32. $2\operatorname{sen}\sqrt{x}+C$

33. $-\frac{1}{2}\cos\left(2x+3\right)+C$

34. $-\frac{1}{3}\operatorname{sen}\left(7-3x\right)+C$

35. $\left(\operatorname{sen}3a\right)x+\frac{1}{3}\cos 3x+C$

36. 0

37. $\frac{1}{2}$

38. $\frac{4\sqrt{2}}{3}-\frac{5\sqrt{5}}{12}$

39. $\frac{1}{2}\ln 3$

40. $\frac{1}{101}$

41. $-\frac{26}{3}$

42. $\frac{32}{3}$

43. $\frac{1}{8}$