CHAPTER 8
Equations of Lines and Planes

Review of Prerequisite Skills, pp. 424–425

1. a. $(3, -2, 1) - (1, 7, -5)$
$= (3 - 1, -2 - 7, 1 - (-5))$
$= (2, -9, 6)$
b. $5(2, -3, -4) + 3(1, 1, -7)$
$= (5 \times 2, 5 \times (-3), 5 \times (-4))$
$+ (3 \times 1, 3 \times 1, 3 \times (-7))$
$= (10, -15, -20) + (3, 3, -21)$
$= (13, -12, -41)$
2. a. The points A, B, and C are collinear if and only if the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear.
$\overrightarrow{AB} = (4, 2) - (1, -3)$
$= (3, 5)$
$\overrightarrow{AC} = (-8, -18) - (1, -3)$
$= (-9, -15)$
$= -3(3, 5)$
So $\overrightarrow{AC} = -3\overrightarrow{AB}$, and so $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear.
b. The points J, K, and L are collinear if and only if the vectors $\overrightarrow{JK}$ and $\overrightarrow{JL}$ are collinear.
$\overrightarrow{JK} = (4, 5) - (-4, 3)$
$= (8, 2)$
$\overrightarrow{JL} = (0, 4) - (-4, 3)$
$= (4, 1)$
$= \frac{1}{2}(8, 2)$
So $\overrightarrow{JK} = \frac{1}{2}\overrightarrow{JL}$, and so $\overrightarrow{JK}$ and $\overrightarrow{JL}$ are collinear.
c. The points A, B, and C are collinear if and only if the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear.
$\overrightarrow{AB} = (4, 7, 0) - (1, 2, 1)$
$= (3, 5, -1)$
$\overrightarrow{AC} = (7, 12, -1) - (1, 2, 1)$
$= (6, 10, -2)$
$= 2(3, 5, -1)$
So $\overrightarrow{AC} = 2\overrightarrow{AB}$, and so $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear.
d. The points R, S, and T are collinear if and only if the vectors $\overrightarrow{RS}$ and $\overrightarrow{RT}$ are collinear.
$\overrightarrow{RS} = (4, 1, 3) - (1, 2, -3)$
$= (3, -1, 6)$
$\overrightarrow{RT} = (2, 4, 0) - (1, 2, -3)$
$= (1, 2, 3)$
Since the ratios of the components are not equal, $\overrightarrow{RS}$ and $\overrightarrow{RT}$ are not collinear. So R, S, and T do not lie on the same line.
3. ABC is a right triangle if its sides, $|\overrightarrow{AB}|$, $|\overrightarrow{AC}|$, and $|\overrightarrow{BC}|$, satisfy the Pythagorean theorem.
$\overrightarrow{AB} = (2, 5, 3) - (1, 6, -2)$
$= (1, -1, 5)$
So $|\overrightarrow{AB}| = \sqrt{1^2 + (-1)^2 + 5^2}$
$= \sqrt{27}.$
$\overrightarrow{AC} = (5, 3, 2) - (1, 6, -2)$
$= (4, -3, 4)$
So $|\overrightarrow{AC}| = \sqrt{4^2 + (-3)^2 + 4^2}$
$= \sqrt{41}.$
$\overrightarrow{BC} = (5, 3, 2) - (2, 5, 3)$
$= (3, -2, -1)$
So $|\overrightarrow{BC}| = \sqrt{3^2 + (-2)^2 + (-1)^2}$
$= \sqrt{14}.$
Since $|\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 = |\overrightarrow{AC}|^2$, ABC is a right triangle.
4. The vectors $\vec{u}$ and $\vec{v}$ are perpendicular if $\vec{u} \cdot \vec{v} = 0$.
$\vec{u} \cdot \vec{v} = (t, -1, 3) \cdot (2, t, -6)$
$= 2t + (-1)t + 3(-6)$
$= t - 18$
So if $t = 18$, then $\vec{u} \cdot \vec{v} = 0$.
5. a. A vector, (t_1, t_2), is perpendicular to $\vec{a}$ if $\vec{a} \cdot (t_1, t_2) = 0$ and t_1 and t_2 are not both zero.
$\vec{a} \cdot (t_1, t_2) = (1, -3) \cdot (t_1, t_2)$
$= 1(t_1) - 3(t_2)$
So if $t_1 = 3$ and $t_2 = 1$, then $\vec{a} \cdot (t_1, t_2) = 0$. So $(3, 1)$ is perpendicular to $\vec{a}$.
b. A vector, (t_1, t_2), is perpendicular to $\vec{b}$ if $\vec{b} \cdot (t_1, t_2) = 0$ and t_1 and t_2 are not both zero.
$\vec{b} \cdot (t_1, t_2) = (6, -5) \cdot (t_1, t_2)$
$= 6(t_1) - 5(t_2)$
So if $t_1 = 5$ and $t_2 = 6$, then $\vec{a} \cdot (t_1, t_2) = 0$. So $(5, 6)$ is perpendicular to $\vec{b}$.
c. A vector, (t_1, t_2, t_3), is perpendicular to $\vec{c}$ if $\vec{c} \cdot (t_1, t_2, t_3) = 0$ and t_1, t_2, t_3 are not all zero.
$\vec{c} \cdot (t_1, t_2, t_3) = (-7, -4, 0) \cdot (t_1, t_2, t_3)$
$= -7(t_1) - 4(t_2) + 0(t_3)$

So if $t_1 = -4$, $t_2 = 7$, and $t_3 = 0$, then $\vec{c} \cdot (t_1, t_2, t_3) = 0$. So $(-4, 7, 0)$ is perpendicular to $\vec{c}$.

6. The area of a parallelogram formed by two vectors is determined by the magnitude of the cross product of the vectors.

$$\begin{aligned}&(4, 10, 9) \times (3, 1, -2)\\ &= ((10)(-2) - (9)(1), (9)(3) - (4)(2),\\ &\quad (4)(1) - (10)(3))\\ &= (-29, 35, -26)\end{aligned}$$

$$\begin{aligned}A &= \text{area of parallelogram}\\ &= |(-29, 35, -26)|\\ &= \sqrt{(-29)^2 + 35^2 + (-26)^2}\\ &= \sqrt{2802}\end{aligned}$$

7. a. $\vec{a} \times \vec{b} = ((1)(2) - (-4)(-5), (-4)(3) - (2)(-2), (2)(-5) - (1)(3))$
$= (-22, -8, -13)$

$$\begin{aligned}\vec{a} \cdot (-22, -8, -13) &= (2)(-22) + (1)(-8) + (-4)(-13)\\ &= -44 - 8 + 52\\ &= 0\end{aligned}$$

$$\begin{aligned}\vec{b} \cdot (-22, -8, -13) &= (3)(-22) + (-5)(-8) + (-2)(-13)\\ &= -66 + 40 + 26\\ &= 0\end{aligned}$$

So $(-22, -8, -13)$ is a vector perpendicular to both vectors.

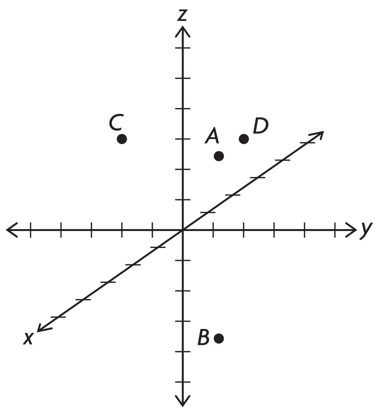
b. $\vec{a} \times \vec{b} = ((-2)(0) - (0)(-1), (0)(-2) - (-1)(0), (-1)(-1) - (-2)(-2))$
$= (0, 0, -3)$

$$\begin{aligned}\vec{a} \cdot (0, 0, -3) &= (-1)(0) + (-2)(0) + (0)(-3)\\ &= 0\end{aligned}$$

$$\begin{aligned}&\vec{b} \cdot (-22, -8, -13)\\ &= (-2)(0) + (-1)(0) + (0)(-3)\\ &= 0\end{aligned}$$

So $(0, 0, -3)$ is a vector perpendicular to both vectors.

8.



9. a. $\vec{p} = (-3, 5) - (4, 8)$
$= (-7, -3)$

b. $\vec{p} = (3, 8) - (-7, -6)$
$= (10, 14)$

c. $\vec{p} = (3, -6, 9) - (1, 2, 4)$
$= (2, -8, 5)$

d. $\vec{p} = (0, 5, 0) - (4, 0, -4)$
$= (-4, 5, 4)$

10. a. $\vec{p} = (4, 8) - (-3, 5)$
$= (7, 3)$

b. $\vec{p} = (-7, -6) - (3, 8)$
$= (-10, -14)$

c. $\vec{p} = (1, 2, 4) - (3, -6, 9)$
$= (-2, 8, -5)$

d. $\vec{p} = (4, 0, -4) - (0, 5, 0)$
$= (4, -5, -4)$

11. a. The y-intercept occurs when $x = 0$.

$$\begin{aligned}y &= -2x - 5\\ &= -2(0) - 5\\ &= -5\end{aligned}$$

So the y-intercept is -5. The slope is equal to -2.

b. $4x - 8y = 8$
$4(0) - 8y = 8$

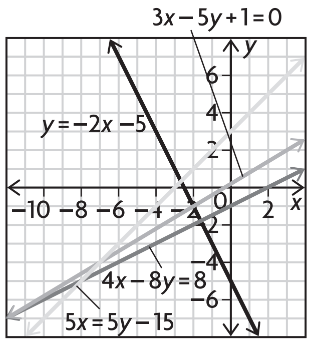
$$\begin{aligned}y &= \frac{8}{-8}\\ &= -1\end{aligned}$$

So the y-intercept is -1. The slope is equal to $\frac{4}{8} = \frac{1}{2}$.

c. $3x - 5y + 1 = 0$
$3(0) - 5y + 1 = 0$
So $-5y = -1$
So the y-intercept is $\frac{-1}{-5} = \frac{1}{5}$. The slope is equal to $\frac{3}{5}$.

d. $5x = 5y - 15$
$5(0) = 5y - 15$
So $5y = 15$
So the y–intercept is 3. The slope is equal to $\frac{5}{5} = 1$.

a. –d.



12. Any positive scalar multiple of a vector is a collinear vector in the same direction. Answers may vary. For example:

a. $2(4, 7) = (8, 14)$

b. $3(-5, 4, 3) = (-15, 12, 9)$

c. $\frac{1}{2}(2\vec{i} + 6\vec{j} - 4\vec{k}) = \vec{i} + 3\vec{j} - 2\vec{k}$

d. $4(-5\vec{i} + 8\vec{j} + 2\vec{k}) = -20\vec{i} + 32\vec{j} + 8\vec{k}$

13. To simplify $\vec{v}$ can be written in algebraic notation. So $\vec{v} = (4, -2, 1)$

a. $\vec{u} \cdot \vec{v} = (4)(4) + (-9)(-2) + (-1)(1)$
$= 16 + 18 - 1$
$= 33$

b. $-\vec{v} = -1(4, -2, 1)$
$= (-4, 2, -1)$

So $-\vec{v} \cdot \vec{u} = (-4)(4) + (2)(-9) + (-1)(-1)$
$= -16 - 18 + 1$
$= -33$

c. $\vec{u} + \vec{v} = (4, -9, -1) + (4, -2, 1)$
$= (8, -11, 0)$

$\vec{u} - \vec{v} = (4, -9, -1) - (4, -2, 1)$
$= (0, -7, -2)$

$(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v}) = (8)(0) + (-11)(-7) + (0)(-2)$
$= 77$

d. $\vec{u} \times \vec{v} = ((-9)(1) - (-1)(-2), (-1)(4) - (4)(1), (4)(-2) - (-9)(4))$
$= (-11, -8, 28)$

e. $\vec{v} \times \vec{u}$ is merely the negative of $\vec{u} \times \vec{v}$.

So $\vec{v} \times \vec{u} = -(-11, -8, 28)$
$= (11, 8, -28)$

f. $2\vec{u} + \vec{v} = 2(4, -9, -1) + (4, -2, 1)$
$= (8, -18, -2) + (4, -2, 1)$
$= (12, -20, -1)$

$\vec{u} - 2\vec{v} = (4, -9, -1) - 2(4, -2, 1)$
$= (4, -9, -1) - (8, -4, 2)$
$= (-4, -5, -3)$

$(2\vec{u} + \vec{v}) \times (\vec{u} - 2\vec{v}) = ((-20)(-3) - (-1)(-5), (-1)(-4) - (12)(-3), (12)(-5) - (-20)(-4))$
$= (55, 40, -140)$

14. The dot product of two vectors yields a real number, while the cross product of two vectors gives another vector.

8.1 Vector and Parametric Equations of a Line in R^2, pp. 433–434

1. Direction vectors for a line are unique only up to scalar multiplication. So since each of the given vectors is just a scalar multiple of $\left(\frac{1}{3}, \frac{1}{6}\right)$ each is an acceptable direction vectors for the line.

2. a. Simply find x and y coordinates for three values of t. Three possible values are $t = -1$, $t = 0$, and $t = 1$. At $t = -1$, $x = 1 + 3(-1) = -2$ and $y = 5 - 2(-1) = 7$. At $t = 0$, $x = 1 + 3(0) = 1$ and $y = 5 - 2(0) = 5$. At $t = 1$, $x = 1 + 3(1) = 4$ and $y = 5 - 2(1) = 3$. So $(-2, 7)$, $(1, 5)$, and $(4, 3)$ are three points on the line.

b. Find the t value when the y-coordinate is 15. So solve $15 = 5 - 2t$ for t.
$-2t = 10$
$t = -5$

If $t = -5$, the $x = 1 + 3(-5) = -14$. So $P(-14, 15)$ is a point on the line.

3. Answers may vary. For example:

a. $(3, 4)$ is a point on the line and $(2, 1)$ is a direction vector for the line.

b. $(1, 3)$ is a point on the line and $(2, -7)$ is a direction vector for the line.

c. $(4, 1)$ is a point on the line and $(0, 2)$ is a direction vector for the line.

d. $(0, 6)$ is a point on the line and $(-5, 0)$ is a direction vector for the line.

4. Answers may vary. For example: One possible line has A(2, 1) as its origin point and $\overrightarrow{AB}$ as its direction vector, while another has $B(-3, 5)$ as its origin point and $\overrightarrow{BA}$ as its direction vector.

$\overrightarrow{AB} = (-3, 5) - (2, 1) = (-5, 4)$

So the first case is $\vec{r} = (2, 1) + t(-5, 4), t \in \mathbf{R}$.

$\overrightarrow{BA} = (2, 1) - (-3, 5) = (5, -4)$

The second case is $\vec{q} = (-3, 5) + s(5, -4), s \in \mathbf{R}$.

5. a. Find the t value when the y-coordinate is 18. So solve $18 = 4 + 2t$ for t.
$2t = 14$
$t = 7$

If $t = 7$, the $x = -2 - 7 = -9$. So $R(-9, 18)$ is a point on the line.

b. Answers may vary. For example: A directional vector for the line is $(-1, 2)$. Since $R(-9, 18)$ is a point on the line, a possible vector equation is $\vec{r} = (-9, 18) + t(-1, 2), t \in \mathbf{R}$.

c. Answers may vary. For example: We may take $t = 0$ to find another point on the line. So $x = -2 - 0 = -2$ and $y = 4 + 2(0) = 4$. Hence $(-2, 4)$ is a point on the line. So another vector equation is $\vec{r} = (-2, 4) + t(-1, 2), t \in \mathbf{R}$.

6. Answers may vary. For example:

a. Three different s values will yield three different points on the line. If $s = -1$, then

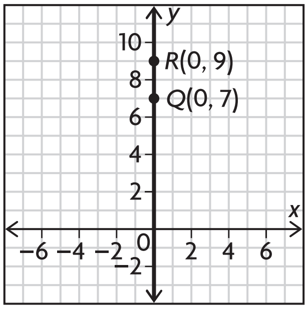
$s(3, 4) = (-3, -4)$. If $s = 0$, then $s(3, 4) = (0, 0)$ and if $s = 1$, then $s(3, 4) = (3, 4)$. Hence $(-3, -4)$, $(0, 0)$, and $(3, 4)$ are three points on the line.

b. $\vec{r} = t(1, 1)$, $t \in \mathbf{R}$ is a line that passes through the origin different from the line in part **a.**

c. If $t = -3$, then $(9, 12) + t(3, 4) = (0, 0)$. So $\vec{r} = (9, 12) + t(3, 4)$, $t \in \mathbf{R}$, is a line that passes through the origin with a direction vector of $(3, 4)$. Hence this describes the same line as part **a.**

7. One can multiply a direction vector by a constant to keep the same line, but multiplying the point yields a different line.

8. a.



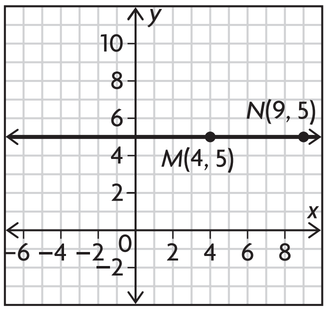
b. $\overrightarrow{QR}$ is a possible direction vector for this line and $Q(0, 7)$ is a point on the line.

$\overrightarrow{QR} = (0, 9) - (0, 7) = (0, 2)$

So a vector equation for the line is

$\vec{r} = (0, 7) + t(0, 2)$, $t \in \mathbf{R}$. The corresponding parametric equation is $x = 0$, $y = 7 + 2t$, $t \in \mathbf{R}$.

9. a.



b. $\overrightarrow{MN}$ is a possible direction vector for this line and $M(4, 5)$ is a point on the line.

$\overrightarrow{MN} = (9, 5) - (4, 5) = (5, 0)$

So a vector equation for the line is

$\vec{r} = (4, 5) + t(5, 0)$, $t \in \mathbf{R}$. The corresponding parametric equation is $x = 4 + 5t$, $y = 5$, $t \in \mathbf{R}$.

10. a. A line perpendicular to L would have a direction vector that is perpendicular to the direction vector of L. (t_1, t_2) is perpendicular to $(3, 5)$ if $(3, 5) \cdot (t_1, t_2) = 0$ and t_1 and t_2 are not both zero.

$(3, 5) \cdot (t_1, t_2) = 3(t_1) + 5(t_2)$

So if $t_1 = 5$ and $t_2 = -3$, then $(3, 5) \cdot (t_1, t_2) = 0$. So an equation for a line with $(5, -3)$ as a direction vector and $P(2, 0)$ as a line is $\vec{r} = (2, 0) + t(5, -3)$, $t \in \mathbf{R}$.

b. The line intersects the y-axis when the x coordinate is zero. The x coordinate is zero, when $2 + 5t = 0$, or $t = -0.4$. The y coordinate at this point is $0 + 3t$ or $y = -1.2$. So the line intersects the y-axis at the point $(0, -1.2)$.

11. The line crosses the x-axis, when $y = 0$, so $8 + s = 0$, or $s = -8$. So the x coordinate at this point is $x = -10 - 2(-8) = 6$. The line crosses the y-axis, when $x = 0$, so $-10 - 2s = 0$, or $s = -5$. So the y coordinate at this point is $y = 8 + (-5) = 3$. So the triangle formed by the origin, $A(6, 0)$, and $B(0, 3)$ is a right triangle with a base of six units and a height of three units. So the area is $\frac{1}{2}(3)(6) = 9$.

12. First all the relevant vectors are found.

$$\begin{aligned}\overrightarrow{AB} &= ((1, 2) + 1(-2, 3)) - ((1, 2) + 0(-2, 3)) \\ &= (-2, 3)\end{aligned}$$

$$\begin{aligned}\overrightarrow{AC} &= ((1, 2) + 2(-2, 3)) - ((1, 2) + 0(-2, 3)) \\ &= (-4, 6)\end{aligned}$$

$$\begin{aligned}\overrightarrow{AD} &= ((1, 2) + 3(-2, 3)) - ((1, 2) + 0(-2, 3)) \\ &= (-6, 9)\end{aligned}$$

a. $\overrightarrow{AC} = (-4, 6) = 2(-2, 3) = 2\overrightarrow{AB}$

b. $\overrightarrow{AD} = (-6, 9) = 3(-2, 3) = 3\overrightarrow{AB}$

c. $\overrightarrow{AC} = (-4, 6) = \dfrac{2}{3}(-6, 9) = \dfrac{2}{3}\overrightarrow{AD}$

13. a. Find the t values such that x and y coordinates satisfy $x^2 + y^2 = 169$ or similarly $x^2 + y^2 - 169 = 0$.

$$\begin{aligned}x^2 + y^2 - 169 &= (2 + t)^2 + (9 + t)^2 - 169 \\ &= 4 + 4t + t^2 + 81 + 18t \\ &\quad + t^2 - 169 \\ &= t^2 + 11t - 42 \\ &= (t - 3)(t + 14)\end{aligned}$$

So $x^2 + y^2 - 169 = 0$, when $t = 3$ or $t = -14$. Let A be the point where $t = 3$. So x coordinate of A is $2 + 3 = 5$, and the y coordinate is $9 + 3 = 12$. Let B be the point where $t = -14$. So x coordinate of B is $2 - 14 = -12$, and the y coordinate is $9 - 14 = -5$. So A is $(5, 12)$ and B is $(-12, -5)$

b. $\overrightarrow{AB} = (-12, -5) - (5, 12) = (-17, -17)$ and hence the length of $AB = |\overrightarrow{AB}|$

$= \sqrt{(-17)^2 + (-17)^2}$

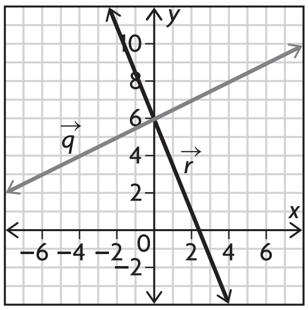
$= \sqrt{578}$, or about 24.04

14. In the parametric form, the second equation becomes $x = 1 + 6t$, $y = 6 + 4t$, $t \in \mathbf{R}$. If t is solved for in this equation, we obtain $t = \frac{x - 1}{6}$ and $t = \frac{y - 6}{4}$.

Setting these two expressions equal to each other, the line is described by $\frac{x - 1}{6} = \frac{y - 6}{4}$, or by simplifying, $y - 6 = \frac{2}{3}x - \frac{2}{3}$. So the second equation describes a line with a slope of $\frac{2}{3}$. If y is solved for in the first expression, we see that $y = \frac{2}{3}x + 5$. $(1, 6)$ is on the second line but not the first. Hence both equations are lines with slope of $\frac{2}{3}$ with no point in common and must be parallel.

8.2 Cartesian Equation of a Line, pp. 443–444

1. a. $\vec{m} = (6, -5)$ is a direction vector parallel to the line.
b. For a vector perpendicular to the line, a suitable $\vec{n}$ has to be found, such that $\vec{m} \cdot \vec{n} = 0$. $\vec{n} = (5, 6)$ is a such a vector.
c. If $x = 0$, then $y = 9$, so $(0, 9)$ is a point on the given line.
d. A direction vector was found in part **a.,** so a vector equation for a parallel line passing through $A(7, 9)$ is $\vec{r} = (7, 9) + t(6, -5)$, $t \in \mathbf{R}$. The corresponding parametric equation is $x = 7 + 6t$, $y = 9 - 5t$, $t \in \mathbf{R}$.
e. A direction vector was found in part **b.,** so a vector equation for a perpendicular line passing through $B(-2, 1)$ is $\vec{r} = (-2, 1) + t(5, 6)$, $t \in \mathbf{R}$. The corresponding parametric equation is $x = -2 + 5t$, $y = 1 + 6t$, $t \in \mathbf{R}$.
2. a.-b.



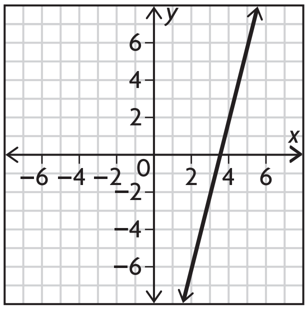
c. Switching the components of the direction vector with the coordinates of the point on the line produces a different line.

3. a. A direction vector parallel to the line is $(8, 7)$, and if $x = 0$, then $y = -6$. So $(0, -6)$ is a point on the line. So a vector equation for the line is $\vec{r} = (0, -6) + t(8, 7)$, $t \in \mathbf{R}$. The corresponding parametric equation is $x = 8t$, $y = -6 + 7t$, $t \in \mathbf{R}$.
b. A direction vector parallel to the line is $(2, 3)$, and if $x = 0$, then $y = 5$. So $(0, 5)$ is a point on the line. So a vector equation for the line is $\vec{r} = (0, 5) + t(2, 3)$, $t \in \mathbf{R}$. The corresponding parametric equation is $x = 2t$, $y = 5 + 3t$, $t \in \mathbf{R}$.
c. The equation $y = -1$ describes a horizontal line in the xy-plane, so a direction vector parallel to this line is $(1, 0)$. Also $(0, -1)$ is a point on this line, so a vector equation for the line is $\vec{r} = (0, -1) + t(1, 0)$, $t \in \mathbf{R}$, which gives a parametric equation of $x = t$, $y = -1$, $t \in \mathbf{R}$.
d. The equation $x = 4$ describes a vertical line in the xy-plane, so a direction vector parallel to this line is $(0, 1)$. Also $(4, 0)$ is a point on this line, so a vector equation for the line is $\vec{r} = (4, 0) + t(0, 1)$, $t \in \mathbf{R}$, which gives a parametric equation of $x = 4$, $y = t$, $t \in \mathbf{R}$.
4. If the two lines have direction vectors that are collinear and share a point in common, then the two lines are coincident. In this example, both have $(3, 2)$ as a parallel direction vector and both have $(-4, 0)$ as a point on the line. Hence the two lines are coincident.
5. a. The normal vectors for the lines are $(2, -3)$ and $(4, -6)$, which are collinear. Since in two dimensions, any two direction vector perpendicular to $(2, -3)$ are collinear, the lines have collinear direction vectors. Hence the lines are parallel.
b. The lines will be coincident if they share a common point. $(0, 2)$ is a point in the first line. So the lines are coincident if and only if $4(0) - 6(2) + k = 0$, or equivalently $k = 12$. So only if $k = 12$, are the lines coincident.
6. Since the normal vector is $(4, 5)$, the Cartesian equation of the line is $4x + 5y + k = 0$, for some constant k. Since $A(-1, 5)$ is a point on the graph, $4(-1) + 5(5) + k = 0$.
So $k = 4 - 25 = -21$.
So the equation of the line is $4x + 5y - 21 = 0$.
7. So the slope of this line is equal to $\frac{4 - 5}{-2 - (-3)} = -1$. Hence the equation for the line satisfies $\frac{y - 5}{x - (-3)} = -1$, or by multiplying both sides by

$x - (-3)$, $y - 5 = -1(x + 3)$. Moving everything the left hand side yields $y - 5 + x + 3 = 0$, or $x + y - 2 = 0$, which is the equation in Cartesian form.

8. So the directional vector of the line is collinear with the normal vector $(2, -4)$, and so has slope equal to -2. Furthermore $P(7, 2)$ is a point on the line. Hence the equation for the line satisfies $\frac{y - 2}{x - 7} = -2$, or by multiplying both sides by $x - 7$, $y - 2 = -2(x - 7)$. Moving everything to the left side yields $y - 2 + 2x - 14 = 0$, or $2x + y - 16 = 0$, which is the equation in Cartesian form.

9. a.



b. First solve for t in both coordinates. So $t = 3 - x$ and $t = \frac{y + 2}{-4}$. Then set these two sides equal to each other to obtain $3 - x = \frac{y + 2}{-4}$, or simply $-4(3 - x) = y + 2$. So $-12 + 4x = y + 2$ or $4x - y - 14 = 0$.

10. The acute angle of the intersection between two vectors $\vec{a}$ and $\vec{b}$ is found by taking the inverse cosine of the absolute value of $\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$.

a. $(2, -5) \cdot (-4, -1) = -3$, $|(2, -5)| = \sqrt{29}$, and $|(-4, -1)| = \sqrt{17}$. So the acute angle is $\cos^{-1}\left(\frac{3}{\sqrt{29}\sqrt{17}}\right) \doteq 82°$.

b. $(-5, 4) \cdot (1, -6) = -29$, $|(-5, 4)| = \sqrt{41}$, and $|(1, -6)| = \sqrt{37}$. So the acute angle is $\cos^{-1}\left(\frac{29}{\sqrt{41}\sqrt{37}}\right) \doteq 42°$.

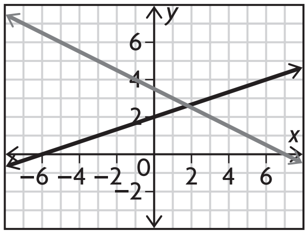
c. The direction vector for the first line is $(2, 1)$ and a direction vector for the second is $(4, -3)$. $(2, 1) \cdot (4, -3) = 5$, $|(2, 1)| = \sqrt{5}$, and $|(4, -3)| = \sqrt{25}$. So the acute angle is $\cos^{-1}\left(\frac{5}{\sqrt{25}\sqrt{5}}\right) \doteq 63°$.

d. A direction vector for the second line is $(2, 1)$. $(2, 4) \cdot (2, 1) = 8$, $|(2, 4)| = \sqrt{20}$, and $|(2, 1)| = \sqrt{5}$. So the acute angle is $\cos^{-1}\left(\frac{8}{\sqrt{20}\sqrt{5}}\right) \doteq 37°$.

e. $(2, -5) \cdot (-4, 1) = -13$, $|(2, -5)| = \sqrt{29}$, and $|(-4, 1)| = \sqrt{17}$. So the acute angle is $\cos^{-1}\left(\frac{13}{\sqrt{29}\sqrt{17}}\right) \doteq 54°$.

f. $x = 3$ has a direction vector of $(0, 1)$ and the direction vector for the second line is $(2, 1)$. $(0, 1) \cdot (2, 1) = 1$, $|(0, 1)| = \sqrt{1}$, and $|(2, 1)| = \sqrt{5}$. So the acute angle is $\cos^{-1}\left(\frac{1}{\sqrt{1}\sqrt{5}}\right) \doteq 63°$.

11. a.



b. The normal vectors are $(1, -3)$ and $(1, 2)$. $(1, -3) \cdot (1, 2) = -5$, $|(1, -3)| = \sqrt{10}$, and $|(1, 2)| = \sqrt{5}$. So the acute angle is $\cos^{-1}\left(\frac{5}{\sqrt{10}\sqrt{5}}\right) = 45°$ and the obtuse angle is $180° - 45° = 135°$.

12. a. Let the coordinates of C be (x, y). They must satisfy the equation $(x, y) = (-6, 6) + t(3, -4)$. Rewrite this equation in Cartesian form. The slope is $m = -\frac{4}{3}$. The equation is of the form $y = -\frac{4}{3}x + b$. Substitute $(-6, 6)$ into the equation to solve for b.

$$6 = -\frac{4}{3}(-6) + b$$
$$6 = 8 + b$$
$$-2 = b$$

The equation of the line is $y = -\frac{4}{3}x - 2$.

If C is the vertex of the right triangle, $\overrightarrow{CA}$ and $\overrightarrow{CB}$ must be perpendicular, meaning that their dot product must be 0.

$\overrightarrow{CA} = (-3 - x, 2 - y)$

$\overrightarrow{CB} = (8 - x, 4 - y)$

$\overrightarrow{CA} \cdot \overrightarrow{CB} = (-3 - x)(8 - x) + (2 - y)(4 - y)$

$(-3 - x)(8 - x) + (2 - y)(4 - y) = 0$

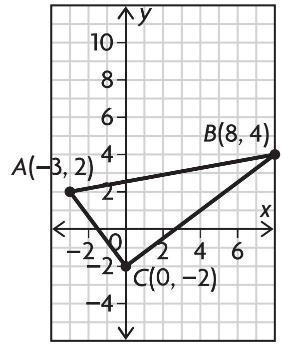
So $-24 - 5x + x^2 + 8 - 6y + y^2 = 0$.

Substitute $-\frac{4}{3}x - 2$ for y.

$$
\begin{aligned}
-24 - 5x + x^2 + 8 - 6y + y^2 &= 0 \\
-16 - 5x + x^2 - 6\left(-\frac{4}{3}x - 2\right) + \left(-\frac{4}{3}x - 2\right)^2 &= 0 \\
-16 - 5x + x^2 + 8x + 12 + \frac{16}{9}x^2 + \frac{16}{3}x + 4 &= 0 \\
\frac{25}{9}x^2 + 3x + \frac{16}{3}x &= 0 \\
25x^2 + 75x &= 0 \\
25x(x + 3) &= 0
\end{aligned}
$$

So $x = 0$ or $x = -3$.
When $x = 0$, $y = -2$.
When $x = -3$, $y = 2$. But then C would have the same coordinates as A. This would not produce a right triangle. So the coordinates of C are $(0, -2)$.

b.



c.
$$
\begin{aligned}
\overrightarrow{CA} &= (-3 - 0, 2 - (-2)) \\
&= (-3, 4) \\
\overrightarrow{CB} &= (8 - 0, 4 - (-2)) \\
&= (8, 6) \\
\overrightarrow{CA} \cdot \overrightarrow{CB} &= (-3)(8) + (4)(6) \\
&= -24 + 24 \\
&= 0
\end{aligned}
$$

Since the dot product of the vectors is 0, the vectors are perpendicular, and $\angle ACB = 90°$.

13. The sum of the interior angles of a quadrilateral is $360°$. The normals make $90°$ angles with their respective lines at A and C. The angle of the quadrilateral at B is $180° - \theta$. Let x represent the measure of the interior angle of the quadrilateral at O.

$$
\begin{aligned}
90° + 90° + 180° - \theta + x &= 360° \\
360° - \theta + x &= 360° \\
x &= \theta
\end{aligned}
$$

Therefore, the angle between the normals is the same as the angle between the lines.

14. The normal vector for the first line is $(1, -1)$ and $(1, k)$ for the second. $(1, -1) \cdot (1, k) = 1 - k$, $|(1, -1)| = \sqrt{2}$, and $|(1, k)| = \sqrt{1 + k^2}$.
So $\cos(60°) = \frac{1 - k}{\sqrt{2}\sqrt{1 + k^2}}$ and $\cos(60°) = 0.5$. We obtain after squaring both sides that $\frac{1 - 2k + k^2}{2(1 + k^2)} = \frac{1}{4}$.
So $2 - 4k + 2k^2 = 1 + k^2$ or simply $k^2 - 4k + 1 = 0$. Solving by the quadratic equation gives $k = 2 \pm \sqrt{3}$.

8.3 Vector, Parametric, and Symmetric Equations of a Line in R^3, pp. 449–450

1. a. A point on this line is $(-3, 1, 8)$.
b. A point on this line is $(1, -1, 3)$.
c. A point on this line is $(-2, 1, 3)$.
d. A point on this line is $(-2, -3, 1)$.
e. A point on this line is $(3, -2, -1)$.
f. A point on this line is $\left(\frac{1}{3}, -\frac{3}{4}, \frac{2}{5}\right)$.
2. a. A direction vector is $(-1, 1, 9)$.
b. A direction vector is $(2, 1, -1)$.
c. A direction vector is $(3, -4, -1)$.
d. A direction vector is $(-1, 0, 2)$.
e. A direction vector is (0, 0, 2).
f. A direction vector is $\left(\frac{1}{2}, -\frac{1}{4}, \frac{1}{2}\right)$, which if multiplied by the least common denominator, 4, yields a vector of $(2, -1, 2)$.
3. a. $\overrightarrow{AB} = (3, -3, 5) - (-1, 2, 4) = (4, -5, 1)$ is a direction vector, as well as $\overrightarrow{BA} = (-1, 2, 4) - (3, -3, 5) = (-4, 5, -1)$. So $\vec{r} = (-1, 2, 4) + t(4, -5, 1), t \in \mathbf{R}$, is one possible vector equation $\vec{q} = (3, -3, 5) + s(-4, 5, -1)$, $s \in \mathbf{R}$ is another.
b. The parametric equation corresponding with the first vector equation is $x = -1 + 4t$, $y = 2 - 5t$, $z = 4 + t$, $t \in \mathbf{R}$. The second parametric equation is $x = 3 - 4s$, $y = -3 + 5s$, $z = 5 - s$, $s \in \mathbf{R}$.
4. a. $\overrightarrow{AB} = (2, 5, -4) - (-1, 5, -4) = (3, 0, 0)$. So $(3, 0, 0)$ is a direction vector for the equation, and so $(1, 0, 0)$ may be used as the direction vector. Hence $\vec{r} = (-1, 5, 4) + t(1, 0, 0), t \in \mathbf{R}$, is a vector equation for a line containing the points $A(\text{A}-1, 5, -4)$ and $B(2, 5, -4)$.
b. The corresponding parametric equation is $x = -1 + t$, $y = 5$, $z = -4$, $t \in \mathbf{R}$.
c. Since two of the coordinates in the direction vector are zero, a symmetric equation cannot exist.

5. a. So $\vec{r} = (-1, 2, 1) + t(3, -2, 1), t \in \mathbf{R}$, is a vector equation for the line and the corresponding parametric equation is $x = -1 + 3t, y = 2 - 2t, z = 1 + t, t \in \mathbf{R}$. So the symmetric equation is $\frac{x + 1}{3} = \frac{y - 2}{-2} = \frac{z - 1}{1}$.

b. $\overrightarrow{AB} = (-1, 2, 1) - (-1, 1, 0) = (0, 1, 1)$ is a direction vector for the line. So $\vec{r} = (-1, 1, 0) + t(0, 1, 1), t \in \mathbf{R}$ is a vector equation for the line and the corresponding parametric equation is $x = -1, y = 1 + t, z = t, t \in \mathbf{R}$. So the symmetric equation is $\frac{y - 1}{1} = \frac{z}{1}, x = -1$.

c. $\overrightarrow{MN} = (-2, 4, 7) - (-2, -2, 1) = (0, -6, -6)$ is a direction vector for the line. Since $(0, -6, -6) = -6(0, 1, 1)$, $(0, 1, 1)$ is also a direction vector for this line. So $\vec{r} = (-2, 3, 0) + t(0, 1, 1), t \in \mathbf{R}$ is a vector equation for the line and the corresponding parametric equation is $x = -2, y = 3 + t, z = t, t \in \mathbf{R}$. So the symmetric equation is $\frac{y - 3}{1} = \frac{z}{1}, x = -2$.

d. $\overrightarrow{DE} = (-1, 1, 0) - (-1, 0, 0) = (0, 1, 0)$ is a direction vector for the line. So $\vec{r} = (-1, 0, 0) + t(0, 1, 0), t \in \mathbf{R}$ is a vector equation for the line and the corresponding parametric equation is $x = -1, y = t, z = 0, t \in \mathbf{R}$. Since two of the coordinates in the direction vector are zero, there is no symmetric equation for this line.

e. $\overrightarrow{XO} = (-4, 3, 0) - (0, 0, 0) = (-4, 3, 0)$ is a direction vector for the line. So $\vec{r} = t(-4, 3, 0), t \in \mathbf{R}$ is a vector equation for the line and the corresponding parametric equation is $x = -4t, y = 3t, z = 0, t \in \mathbf{R}$. So the symmetric equation is $\frac{x}{-4} = \frac{y}{3}, z = 0$.

f. The direction vector for the z-axis is $(0, 0, 1)$, so a line parallel to the z-axis has $(0, 0, 1)$ as a direction vector. So $\vec{r} = (1, 2, 4) + t(0, 0, 1), t \in \mathbf{R}$ is a vector equation for the line and the corresponding parametric equation is $x = 1, y = 2, z = 4 + t, t \in \mathbf{R}$. Since two of the coordinates in the direction vector are zero, there is no symmetric equation for this line.

6. a. So the first line is given by $\frac{x + 6}{1} = \frac{y - 10}{-1} = \frac{z - 7}{1}(=t)$. If x, y, and z are solved for in terms of t, the corresponding parametric equations is $x = -6 + t, y = 10 - t, z = 7 + t, t \in \mathbf{R}$. So the first line has a direction vector of $(1, -1, 1)$. The second line is given by $\frac{x + 7}{1} = \frac{y - 11}{-1}(=s), z = 5$. If x and y are solved for in terms of s, $x = -7 + s$, and $y = 11 - s$ are obtained. So the parametric equation for the second line is $x = -7 + s, y = 11 - s, z = 5, s \in \mathbf{R}$, and so has a direction vector of $(1, -1, 0)$.

b. $(1, -1, 1) \cdot (1, -1, 0)$
$= 1(1) - 1(-1) + 0(1) = 2$. $|(1, -1, 0)| = \sqrt{2}$ and $|(1, -1, 1)| = \sqrt{3}$. So the angle between the two lines is $\cos^{-1}\left(\frac{2}{\sqrt{2}\sqrt{3}}\right) \doteq 35.3°$.

7. The directional vector of the first line is $(8, 2, -2) = -2(-4, -1, 1)$. So $(-4, -1, 1)$ is a directional vector for the first line as well. Since $(-4, -1, 1)$ is also the directional vector of the second line, the lines are the same if the lines share a point. $(1, 1, 3)$ is a point on the second line. Since $1 = \frac{1 + 7}{8} = \frac{1 + 1}{2} = \frac{3 - 5}{-2}$, $(1, 1, 3)$ is a point on the first line as well. Hence the lines are the same.

8. a. The line that passes through $(0, 0, 3)$ with a directional vector of $(-3, 1, -6)$ is given by the parametric equation is $x = -3t, y = t, z = 3 - 6t, t \in \mathbf{R}$. So the y coordinate is equal to -2 only when $t = -2$. At $t = -2$, $x = -3(-2) = 6$ and $z = 3 - 6(-2) = 15$. So $A(6, -2, 15)$ is a point on the line. So the y coordinate is equal to 5 only when $t = 5$. At $t = 5$, $x = -3(5) = -15$ and $z = 3 - 6(5) = -27$. So $B(-15, 5, -27)$ is a point on the line.

b. Since the point A occurs when $t = -2$, and point B occurs when $t = 5$, the line segment connecting the two points is precisely all the t values between -2 and 5. So the equation is $x = -3t, y = t, z = 3 - 6t, -2 \le t \le 5$.

9. The direction vector for the first line is $(k, 2, k - 1)$ and for the second line is $(-2, 0, 1)$. The lines are perpendicular precisely when $(k, 2, k - 1) \cdot (-2, 0, 1) = 0$. So $(k, 2, k - 1) \cdot (-2, 0, 1)$
$= -2(k) + 0(2) + 1(k - 1) = -k - 1$. So if $k = -1$, then $(k, 2, k - 1) \cdot (-2, 0, 1) = 0$, and the lines are perpendicular.

10. a. Three different points occur at three different values of t. At $t = -1$, the corresponding point on the line is $(4, -2, 5) - (-4, -6, 8) = (8, 4, -3)$. At $t = 1$, the corresponding point on the line is

$(4, -2, 5) + (-4, -6, 8) = (0, -8, 13)$. The point at the origin is $(4, -2, 5)$.
b. Three different points occur at three different values of s. At $s = -1$, the corresponding point on the line is when $x = -4 + 5(-1) = -9$, $y = 2 - (-1) = 3$, and $z = 9 - 6(-1) = 15$. At $s = 1$, the corresponding point on the line is when $x = -4 + 5(1) = 1$, $y = 2 - (1) = 1$, and $z = 9 - 6(1) = 3$. So $(-9, 3, 15)$ and $(1, 1, 3)$ are two points on the line. The point at the origin is $(-4, 2, 9)$.
c. $\frac{x+1}{3} = \frac{y-2}{-1} = \frac{z}{4}$ is actually equal to $\frac{x+1}{3} = \frac{y-2}{-1} = \frac{z}{4}(=t)$, for any $t \in \mathbf{R}$. So we can pick different t values to obtain different points on the lines. At $t = -1$, the corresponding point on the line is found by solving for x, y, and z, in the equation $\frac{x+1}{3} = \frac{y-2}{-1} = \frac{z}{4}(=-1)$. So $x = (-1)3 - 1 = -4$, $y = (-1)(-1) + 2 = 3$, and $z = (-1)4 = -4$. So $(-4, 3, -4)$ is a point on the line. At $t = 1$ and solving for x, y, and z, in the equation $\frac{x+1}{3} = \frac{y-2}{-1} = \frac{z}{4}(=1)$, yields $x = (1)3 - 1 = 2$, $y = (1)(-1) + 2 = 1$, and $z = (1)4 = 4$. So $(2, 1, 4)$ is a point on the line. Also the point at the origin is $(-1, 2, 0)$.
d. $x = -4, \frac{y-2}{3} = \frac{z-3}{5}$ is actually equal to $x = -4, \frac{y-2}{3} = \frac{z-3}{5}(=t)$, for any $t \in \mathbf{R}$. So we can pick different t values to obtain different points on the lines. At $t = -1$, the corresponding point on the line is found by solving for x, y, and z, in the equation $x = -4, \frac{y-2}{3} = \frac{z-3}{5}(=-1)$. So $x = -4$, $y = (-1)(3) + 2 = -1$, and $z = (-1)5 + 3 = -2$. So $(-4, -1, -2)$ is a point on the line. At $t = 1$ and solving for x, y, and z, in the equation $x = -4, \frac{y-2}{3} = \frac{z-3}{5}(=1)$, yields $x = -4$, $y = (1)(3) + 2 = 5$, and $z = (1)5 + 3 = 8$. So $(-4, 5, 8)$ is a point on the line. Also the point at the origin is $(-4, 2, 3)$.
11. For part **a.** the corresponding parametric equation is $x = 4 - 4t$, $y = -2 - 6t$, $z = 5 + 8t$, $t \in \mathbf{R}$. The corresponding symmetric equation is $\frac{x-4}{-4} = \frac{y+2}{-6} = \frac{z-5}{8}$.
For part **b.** the corresponding vector equation is $\vec{r} = (-4, 2, 9) + s(5, -1, -6), s \in \mathbf{R}$. The corresponding symmetric equation is $\frac{x+4}{5} = \frac{y-2}{-1} = \frac{z-9}{-6}$.
For part **c.** the point at the origin is $(-1, 2, 0)$ and the direction vector is $(3, -1, 4)$. So the corresponding vector equation is $\vec{r} = (-1, 2, 0) + t(3, -1, 4)\ t \in \mathbf{R}$, and parametric equation $x = -1 + 3t$, $y = 2 - t$, $z = 4t$, $t \in \mathbf{R}$.
For part **d.** the point at the origin is $(-4, 2, 3)$ and a direction vector is $(0, 3, 5)$. So the corresponding vector equation is $\vec{r} = (-4, 2, 3) + t(0, 3, 5), t \in \mathbf{R}$, and parametric equation $x = -4$, $y = 2 + 3t$, $z = 3 + 5t$, $t \in \mathbf{R}$.
12. The direction vector of the first line is $(-4, -7, 3)$ and the direction vector of the second line is $(3, 2, 4)$. The cross product of these two vectors gives a vector that is perpendicular to both direction vectors.
$$\begin{aligned}
&(-4, -7, 3) \times (3, 2, 4) \\
&= ((-7)4 - (3)2, (3)3 - (-4)4, (-4)2 - (-7)3) \\
&= (-28 - 6, 9 + 16, -8 + 21) \\
&= (-34, 25, 13)
\end{aligned}$$
So a line with a direction vector of $(-34, 25, 13)$ is perpendicular to the two initial lines. A parametric equation of such a line passing through the point $(2, -5, 0)$ is $x = 2 - 34t$, $y = -5 + 25t$, $z = 13t$, $t \in \mathbf{R}$.
13. Since $x = 10 + 2s$, $y = 5 + s$, and $z = 2$, if $x^2 + y^2 + z^2 = 9$, then $(10 + 2s)^2 + (5 + s)^2 + (2)^2 = 9$ or equivalently $(10 + 2s)^2 + (5 + s)^2 + (2)^2 - 9 = 0$.
$$\begin{aligned}
&(10 + 2s)^2 + (5 + s)^2 + (2)^2 - 9 \\
&= 5s^2 + 50s + 120 \\
&= 5(s + 6)(s + 4).
\end{aligned}$$
So if $x^2 + y^2 + z^2 = 9$, then $s = -6$ or $s = -4$. Also if $s = -6$ or $s = -4$, then $x^2 + y^2 + z^2 = 9$. So the only two points occur at $s = -6$ and $s = -4$. At $s = -6$, $x = 10 + 2(-6) = -2$, $y = 5 + (-6) = -1$, and $z = 2$, or $(-2, -1, 2)$. At $s = -4$, $x = 10 + 2(-4) = 2$, $y = 5 + (-4) = 1$, and $z = 2$, or $(2, 1, 2)$.
14. Let $P_1(4 + 2t, 4 + t, -3 - t)$ and $P_2(-2 + 3s, -7 + 2s, 2 - 3s)$ be two such points for some real numbers s and t. So $\overrightarrow{P_1P_2}$ is perpendicular to the lines L_1 and L_2, and so since

the direction vectors for the lines are $(2, 1, -1)$ and $(3, 2, -3)$, respectively, $\overrightarrow{P_1P_2} \cdot (2, 1, -1) = 0$ and $\overrightarrow{P_1P_2} \cdot (3, 2, -3) = 0$.

$$\begin{aligned}\overrightarrow{P_1P_2} &= (-2 + 3s, -7 + 2s, 2 - 3s) \\ &\quad - (4 + 2t, 4 + t, -3 - t) \\ &= (-6 + 3s - 2t, -11 + 2s - t, 5 - 3s + t)\end{aligned}$$

So $\overrightarrow{P_1P_2} \cdot (2, 1, -1) = 2(-6 + 3s - 2t) + 1(-11 + 2s - t) + (-1)(5 - 3s + t) = -28 + 11s - 6t = 0$.

So $\overrightarrow{P_1P_2} \cdot (3, 2, -3) = 3(-6 + 3s - 2t) + 2(-11 + 2s - t) + (-3)(5 - 3s + t) = -55 + 22s - 11t = 0$

So $(-2)[\overrightarrow{P_1P_2} \cdot (2, 1, -1)] + [\overrightarrow{P_1P_2} \cdot (3, 2, -3)] = -2(0) + 0 = 0$.

Yet $(-2)[\overrightarrow{P_1P_2} \cdot (2, 1, -1)] + [\overrightarrow{P_1P_2} \cdot (3, 2, -3)]$
$= (-2)(-28 + 11s - 6t) + (-55 + 22s - 11t)$
$= 1 + t$. So $1 + t = 0$, or $t = -1$.

Since $-28 + 11s - 6t = 0$,
$-28 + 11s - 6(-1) = 0$, or $11s = 22$. So $s = 2$.
At $t = -1$, $x = 4 + 2(-1) = 2$,
$y = 4 + (-1) = 3$, and $z = -3 - (-1) = -2$. At $s = 2$, $x = -2 + 3(2) = 4$, $y = -7 + 2(2) = -3$, and $z = 2 - 3(2) = -4$. So $P_1(2, 3, -2)$ and $P_2(4, -3, -4)$ are the points that work.

15. The direction vector for the first line is $(2, 1, 0)$ and the direction vector for the second line is $(3, 2, 1)$. $(2, 1, 0) \cdot (3, 2, 1) = 2(3) + 1(2) + 0(1) = 8$. $|(2, 1, 0)| = \sqrt{5}$ and $|(3, 2, 1)| = \sqrt{14}$. So the angle between the two lines is
$\cos^{-1}\left(\frac{8}{\sqrt{5}\sqrt{14}}\right) \doteq 17°$.

Chapter 8 Mid-Chapter Review, pp. 451–452

1. a. Any three different t values yield three different points. At $t = -1$, $x = 2(-1) - 5 = -7$, $y = 3(-1) + 1 = -2$. At $t = 0$, $x = 2(0) - 5 = -5$, $y = 3(0) + 1 = 1$, and at $t = 1$, $x = 2(1) - 5 = -3$, $y = 3(1) + 1 = 4$. So $(-7, -2)$, $(-5, 1)$, and $(-3, 4)$ are three points on the line.

b. Pick any three s values. At
$s = -1$, $(2, 3) + (-1)(3, -2) = (-1, 5)$. At
$s = 0$, $(2, 3) + (0)(3, -2) = (2, 3)$, and at
$s = 1$, $(2, 3) + (1)(3, -2) = (5, 1)$. So
$(-1, 5)$, $(2, 3)$, and $(5, 1)$ are three points on the line.

c. Pick three different x values and solve for y to obtain the three points. At $x = -1$,
$3(-1) + 5y - 8 = 0$, or $5y = 11$. So $y = \frac{11}{5}$, when $x = -1$. Similarly at $x = 0$, $3(0) + 5y - 8 = 0$, or $y = \frac{8}{5}$. At $x = 1$, $3(1) + 5y - 8 = 0$, or $y = \frac{5}{5} = 1$. So three points on the line are $\left(-1, \frac{11}{5}\right)$, $\left(0, \frac{8}{5}\right)$, and $(1, 1)$.

d. $\frac{x - 1}{3} = \frac{y + 2}{2} = \frac{z - 5}{1}$ is actually equal to $\frac{x - 1}{3} = \frac{y + 2}{2} = \frac{z - 5}{1} (= t)$, for any $t \in \mathbf{R}$. So we can pick different t values to obtain different points on the lines. At $t = -1$, the corresponding point on the line is found by solving for x, y, and z, in the equation $\frac{x - 1}{3} = \frac{y + 2}{2} = \frac{z - 5}{1} (= -1)$. So $x = (-1)3 + 1 = -2$, $y = (-1)2 - 2 = -4$, and $z = (-1)1 + 5 = 4$. So $(-2, -4, 4)$ is a point on the line. At $t = 1$ and solving for x, y, and z, in the equation $\frac{x - 1}{3} = \frac{y + 2}{2} = \frac{z - 5}{1} (= 1)$, yields
$x = (1)3 + 1 = 4$, $y = (1)2 - 2 = 0$, and
$z = (1)1 + 5 = 6$. So $(4, 0, 6)$ is another point on the line. Also the point at the origin is $(1, -2, 5)$.

2. a. The x-intercept occurs when $y = 0$, so solve for the t values when $y = 0$, to find the point. At $y = 0$, $1 + 5t = 0$, so $t = \frac{-1}{5}$. So
$x = 3 - 3\left(\frac{-1}{5}\right) = \left(\frac{18}{5}\right)$. So the x-intercept is at $\left(\frac{18}{5}, 0\right)$. The y-intercept occurs when $x = 0$, or $3 - 3t = 0$. So at the y-intercept, $t = 1$. So $y = 1 + 5(1) = 6$. So the y-intercept is at $(0, 6)$.

b. The x-intercept occurs when $y = 0$, so solve for the s values when $y = 0$, to find the point. At $y = 0$, $3 - 2s = 0$, so $s = \frac{2}{3}$. So $x = -6 + 2\left(\frac{2}{3}\right) = -\frac{14}{3}$.
So the x-intercept is at $\left(-\frac{14}{3}, 0\right)$. The y-intercept occurs when $x = 0$, or $-6 + 2s = 0$. So at the y-intercept, $t = 3$. So $y = 3 - 2(3) = -3$. So the y-intercept is at $(0, -3)$.

3. The direction vector for the first line is $(-4, 7)$ and the direction vector for the second is $(2, 1)$.
$(-4, 7) \cdot (2, 1) = -1$, $|(-4, 7)| = \sqrt{65}$, and $|(2, 1)| = \sqrt{5}$. So the angle between the lines is $\cos^{-1}\left(\frac{-1}{\sqrt{5}\sqrt{65}}\right) \doteq 93.2°$. The acute angle between the lines is approximately $180° - 93.2° = 86.8°$.

4. The direction vector for the x-axis is $(1, 0)$ and the direction vector for the y-axis is $(0, 1)$. The direction vector of the line is $(4, -5)$. $(4, -5) \cdot (1, 0) = 4$, $|(4, -5)| = \sqrt{41}$, and $|(1, 0)| = \sqrt{1} = 1$.
So the angle the line makes with the x-axis is
$\cos^{-1}\left(\frac{4}{1\sqrt{41}}\right) \doteq 51°$. $(4, -5) \cdot (0, 1) = -5$,

$|(4, -5)| = \sqrt{41}$, and $|(0, 1)| = \sqrt{1} = 1$. So the angle the line makes with the y-axis is $\cos^{-1}\left(\frac{-5}{1\sqrt{41}}\right) \doteq 141°$. The acute angle between them is approximately $180° - 141° = 39°$.

5. Since the perpendicular line has $(5, -7)$ as a direction vector, $(5, -7)$ is a normal vector for the desired line. So a Cartesian equation for this line is $5x - 7y + C = 0$, for some constant C. C is found by knowing that $(4, -3)$ is a point on the line. So $5(4) - 7(-3) + C = 0$ or $41 + C = 0$. Hence $C = -41$, and the Cartesian equation is $5x - 7y - 41 = 0$.

6. Parallel lines have collinear direction vectors. Since the direction vector for the first line is $(3, -4, 4)$, it may also be the direction vector for the desired line. The symmetric equation for this line having $(0, 0, 2)$ as its origin point is $\frac{x}{3} = \frac{y}{-4} = \frac{z-2}{4}$.

7. $\overrightarrow{KL} = (3, -5, 6) - (2, 4, 5) = (1, -9, 1)$. Since parallel lines have collinear direction vectors, $(1, -9, 1)$ may be taken to be the direction vector for the parallel line. So the parametric equation with $(1, 2, 5)$ as its origin point is $x = 1 + t$, $y = 2 - 9t$, $z = 5 + t$, $t \in \mathbf{R}$.

8. The direction vector for this line is $(2, -8, 7)$. The direction angles are found by finding the angles this vector makes with the coordinate axes. The direction vectors for the x-axis, y-axis, and z-axis are $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, respectively.
$|(2, -8, 7)| = \sqrt{2^2 + (-8)^2 + 7^2} = \sqrt{117}$, and $|(1, 0, 0)| = |(0, 1, 0)| = |(0, 0, 1)| = \sqrt{1} = 1$.
$(2, -8, 7) \cdot (1, 0, 0) = 2$, so the angle the line makes with the x-axis is $\cos^{-1}\left(\frac{2}{1\sqrt{117}}\right) \doteq 79.3°$.
$(2, -8, 7) \cdot (0, 1, 0) = -8$, so the angle the line makes with the y-axis is $\cos^{-1}\left(\frac{-8}{1\sqrt{117}}\right) \doteq 137.7°$.
$(2, -8, 7) \cdot (0, 0, 1) = 7$, so the angle the line makes with the z-axis is $\cos^{-1}\left(\frac{7}{1\sqrt{117}}\right) \doteq 49.7°$.
So the direction angles are approximately $79.3°$, $137.7°$, and $49.7°$.

9. If (a, b, c) is a unit vector with direction vectors $60°$, $90°$, and $30°$, then $\cos(60°) = (a, b, c) \cdot (1, 0, 0)$. Yet $\cos(60°) = \frac{1}{2}$ and $(a, b, c) \cdot (1, 0, 0) = a$. So $a = \frac{1}{2}$. Similarly $\cos(90°) = (a, b, c) \cdot (0, 1, 0)$, so $0 = b$. Also $\cos(30°) = (a, b, c) \cdot (0, 0, 1)$, so $\frac{\sqrt{3}}{2} = c$. So $\left(\frac{1}{2}, 0, \frac{\sqrt{3}}{2}\right)$ is a direction vector for the line, as well as $(1, 0, \sqrt{3})$.
So the symmetric equation of the line with this direction vector and $P(3, -4, 6)$ as an origin point is $y = -4$, $\frac{x-3}{1} = \frac{z-6}{\sqrt{3}}$.

10. The direction vectors for the x-axis, y-axis, and z-axis are $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, respectively. The origin is a point on each of the axes, so it may be taken as the origin point for each equation. So a parametric equation for the x-axis is $x = t$, $y = 0$, $z = 0$, $t \in \mathbf{R}$. Similarly a parametric equation for the y-axis is $x = 0$, $y = t$, $z = 0$, $t \in \mathbf{R}$, and a parametric equation for the z-axis is $x = 0$, $y = 0$, $z = t$, $t \in \mathbf{R}$.

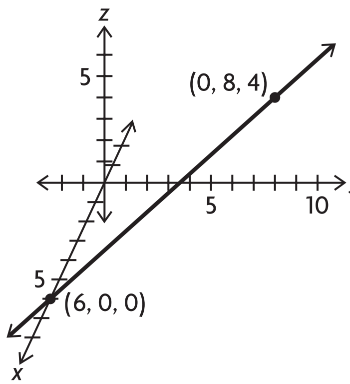
11. a. The direction vector for the first line is $(k + 1, 3k + 1, k - 3)$ and the direction vector for the second line is $(-3, -10, -5)$. The lines are parallel if and only if the direction vectors are collinear. The vectors are collinear only when one is a multiple of the other, which happens only when the ratio between the coordinates is constant. So the direction vectors are parallel if and only if $\frac{k+1}{-3} = \frac{3k+1}{-10} = \frac{k-3}{-5}$. If $\frac{k+1}{-3} = \frac{3k+1}{-10}$, then $-10(k + 1) = -3(3k + 1)$ or $-10k - 10 = -9k - 3$. So $k = -7$. If $k = -7$, then $\frac{k+1}{-3} = \frac{3k+1}{-10} = 2$, and since $\frac{-7-3}{-5} = 2$ as well, the ratios are a constant. So the lines are parallel if $k = -7$. The lines are perpendicular if and only if the dot product of the direction vectors is zero.
$(k + 1, 3k + 1, k - 3) \cdot (-3, -10, -5) =$
$(k + 1)(-3) + (3k + 1)(-10) + (k - 3)(-5)$
$= -38k + 2$.
So the dot product is zero when $-38k + 2 = 0$, or simply $k = \frac{1}{19}$. So if $k = \frac{1}{19}$, then the lines are perpendicular.

12. The x-intercept occurs when $y = 0$, so solve for the x value when $y = 0$, to find the point. At $y = 0$, $\frac{x-6}{3} = \frac{0+8}{-2} = -4$, so $x = (-4)3 + 6 = -6$. So the x-intercept is at $(-6, 0)$. The y-intercept occurs when $x = 0$. So at the y-intercept, $\frac{y+8}{-2} = \frac{0-6}{3} = -2$, so $y = (-2)(-2) - 8 = -4$. So the y-intercept is at $(0, -4)$. So the triangle with the origin has a base of 6 units and a height of 4 units.
Hence the hypotenuse has a length of $\sqrt{4^2 + 6^2} = \sqrt{52}$. So the perimeter is equal to $4 + 6 + \sqrt{52} \doteq 17.2$ units. The area of the triangle is $\frac{1}{2} \times 4 \times 6 = 12$.

13. a. Solving the Cartesian equation for y yields $y = \frac{-3}{4}x + 6$. So the direction of the line is $(4, -3)$

and the y-intercept, (0, 6), may be the origin point of the line. So a vector equation is $\vec{r} = (0, 6) + t(4, -3), t \in \mathbf{R}$.
b. The corresponding parametric equation for the vector equation in part **a.** is $x = 4t$, $y = 6 - 3t$, $t \in \mathbf{R}$.
c. A direction vector for the x-axis is (1, 0).
$(4, -3) \cdot (1, 0) = 4$, $|(4, -3)| = \sqrt{25} = 5$, and $|(1, 0)| = 1$. So the angle between the line and the x-axis is $\cos^{-1}\left(\frac{4}{5}\right) \doteq 36.9°$.
d. The normal vector for the line is (3, 4), which is a vector perpendicular to the line. So a line with the origin as its origin point with a direction vector of (3, 4) is $\vec{r} = t(3, 4), t \in \mathbf{R}$.
14. $\frac{4 - 6}{8 - (-4)} = \frac{-2}{12} = -\frac{1}{6}$, is the slope of the line connecting $A(-4, 6)$ and $B(8, 4)$. Since $A(-4, 6)$ is a point, the scalar equation can be found from $\frac{y - 6}{x - (-4)} = -\frac{1}{6}$. So $6(y - 6) = -1(x + 4)$, which gives $x + 6y - 32 = 0$ as the scalar equation. $\overrightarrow{AB} = (8, 4) - (-4, 6) = (12, -2)$ is a direction vector for the line and we may take $A(-4, 6)$ to be the origin point for the line. So a vector equation for the line is $\vec{r} = (-4, 6) + t(12, -2), t \in \mathbf{R}$. The corresponding parametric equation is $x = -4 + 12t$, $y = 6 - 2t$, $t \in \mathbf{R}$.
15. The direction vector for the given line is $(2, -4)$. So a vector (t_1, t_2), is normal to $(2, -4)$ if $(2, -4) \cdot (t_1, t_2) = 0$ and t_1 and t_2 are not both zero.
$(2, -4) \cdot (t_1, t_2) = 2(t_1) - 4(t_2)$
So if $t_1 = 2$ and $t_2 = 1$, then $(2, -4) \cdot (t_1, t_2) = 0$.
So $(2, 1)$ is normal to the line. Since $|(2, 1)| = \sqrt{5}$, $\frac{1}{\sqrt{5}}(2, 1) = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$ is a unit vector normal to the given line.
16. a. Since the slope is $-\frac{2}{3}$, a direction vector for the line is $(3, -2)$. A parametric equation with an origin point of $(-5, 10)$ is $x = -5 + 3t$, $y = 10 - 2t$, $t \in \mathbf{R}$.
b. The direction vector for the given line is $(2, -2)$. A vector, (t_1, t_2), perpendicular to $(2, -2)$ satisfies $(2, -2) \cdot (t_1, t_2) = 0$ and t_1 and t_2 are not both zero. $(2, -2) \cdot (t_1, t_2) = 2(t_1) - 2(t_2)$
So if $t_1 = 1$ and $t_2 = 1$, then $(2, -4) \cdot (t_1, t_2) = 0$.
So (1, 1) is a direction vector for a line perpendicular to the given line. A parametric equation for this perpendicular line with an origin point of $(1, -1)$ is $x = 1 + t$, $y = -1 + t$, $t \in \mathbf{R}$.
c. A direction vector for the line is $(0, 10) - (0, 7) = (0, 3)$. So we may use (0, 1) as a direction vector. Also since the line connecting (0, 10) and (0, 7) has the origin as a point, the origin may be used as the origin point for the parametric equation. Hence a parametric equation for the line is $x = 0$, $y = t$, $t \in \mathbf{R}$.
17. a. The coordinate planes are $x = 0$, $y = 0$, and $z = 0$. If the x coordinate of the line is zero, then $12 - 3t = 0$, so $t = 4$. So the line intersects the yz-plane when $t = 4$. Since $(12, -8, -4) + 4(-3, 4, 2) = (0, 8, 4)$, the line intersects the yz-plane at (0, 8, 4). Similarly if the y coordinate of the line is zero, $-8 + 4t = 0$, so $t = 2$. Since $(12, -8, -4) + 2(-3, 4, 2) = (6, 0, 0)$, the line intersects the xz-plane at (6, 0, 0). If the z coordinate of the line is zero, t is also 2. Hence the line intersects the xy-plane at (6, 0, 0).
b. Since any line intersecting a coordinate axis intersects two coordinate planes at the same point, the only possible points for intersection with an axis are (0, 8, 4) and (6, 0, 0). (0, 8, 4) does not lie on a coordinate axis, but (6, 0, 0) lies on the x-axis. So the line intersects the x-axis at (6, 0, 0).

c.



18. a. Choose P_0 to be the origin point for the equations. So the vector equation is $\vec{r} = (1, -2, 8) + t(-5, -2, 1), t \in \mathbf{R}$. The corresponding parametric equation is $x = 1 - 5t$, $y = -2 - 2t$, $z = 8 + t$, $t \in \mathbf{R}$, and the symmetric equation is $\frac{x - 1}{-5} = \frac{y + 2}{-2} = \frac{z - 8}{1}$.
b. Choose P_0 to be the origin point for the equations. So the vector equation is $\vec{r} = (3, 6, 9) + (2, 4, 6)$, $t \in \mathbf{R}$. The corresponding parametric equation is $x = 3 + 2t$, $y = 6 + 4t$, $z = 9 + 6t$, $t \in \mathbf{R}$, and the symmetric equation is $\frac{x - 3}{2} = \frac{y - 6}{4} = \frac{z - 9}{6}$.
c. Choose P_0 to be the origin point for the equations. So the vector equation is $\vec{r} = (0, 0, 6) + t(-1, 5, 1)$, $t \in \mathbf{R}$. The corresponding parametric equation is

$x = -t$, $y = 5t$, $z = 6 + t$, $t \in \mathbf{R}$, and the symmetric equation is $\frac{x}{-1} = \frac{y}{5} = \frac{z-6}{1}$.

d. Choose P_0 to be the origin point for the equations. So the vector equation is $\vec{r} = (2, 0, 0) + t(0, 0, -2)$, $t \in \mathbf{R}$. The corresponding parametric equation is $x = 2$, $y = 0$, $z = -2t$, $t \in \mathbf{R}$. Since the direction vector has two zero coordinates, there is no symmetric equation for this line.

19. A line parallel to the line connecting the points $(-4, 5, 6)$ and $(6, -5, 4)$ has a direction vector of $(6, -5, 4) - (-4, 5, 6) = (10, -10, -2)$. Since collinear vectors of $(10, -10, -2)$ are also direction vectors for the line, $(5, -5, -1)$ is a direction vector. So the vector equation for a line with a direction vector of $(5, -5, -1)$ passing through the origin is $\vec{r} = t(5, -5, -1)$, $t \in \mathbf{R}$.

20. The midpoint between (2, 6, 10) and $(-4, 4, -8)$ is precisely $\frac{1}{2}[(2, 6, 10) + (-4, 4, -8)] = (-1, 5, 1)$. The line connecting the midpoint and the given point has a direction vector of $(0, -8, 1) - (-1, 5, 1) = (1, -13, 0)$. So the parametric equations of the line through the desired points is $x = t$, $y = -8 - 13t$, $z = 1$, $t \in \mathbf{R}$.

21. The direction vector for the first line is $(1, 3, -5)$, and the direction vector for the second line is $(-3, -9, 15) = -3(1, 3, -5)$. So the direction vectors are collinear. The direction vectors are collinear if and only if the lines are parallel, so the equations describe parallel lines.

22. Since $\frac{7-4}{3} = \frac{-1+2}{1} = \frac{8-6}{2} = 1$, the point $(7, -1, 8)$ lies on the line.

8.4 Vector and Parametric Equations of a Plane, pp. 459–460

1. a. plane; This is a vector equation of a plane in R^3.
b. line; This is a vector equation of a line in R^3.
c. line; This is a parametric equation for a line in R^3.
d. plane; This is a parametric equation of a plane in R^3 using (0, 0, 0) as $\vec{r}_0$.

2. a. The first direction vector can be expressed with integers as follows:
$$\left(\frac{1}{3}, -2, \frac{3}{4}\right) \times 12 = (4, -24, 9).$$
b. The second direction vector can be reduced as follows:
$$(6, -12, 30) \times \frac{1}{6} = (1, -2, 5)$$
c. The resulting equation of the plane using the two new direction vectors is:
$\vec{r} = (2, 1, 3) + s(4, -24, 9) + t(1, -2, 5)$, $t, s \in \mathbf{R}$

3. a. By inspection, if we choose $n = m = 0$, we get the point $(0, 0, -1)$.
b. Collecting the vector components of the n, and m, multiples we can rewrite the equation of the plane in vector form as:
$\vec{r} = (0, 0, -1) + m(2, -3, -3) + n(0, 5, -2)$; $m, n \in \mathbf{R}$
Thus our direction vectors are:
$(2, -3, -3)$ and $(0, 5, -2)$
c. $\vec{r} = (0, 0, -1) + m(2, -3, -3) + n(0, 5, -2)$; $m, n \in \mathbf{R}$
Letting $m = -1$ and $n = -4$ we get:
$$\begin{aligned}\vec{r} &= (0, 0, -1) + (-1)(2, -3, -3) + (-4)(0, 5, -2)\\ &= (0, 0, -1) + (-2, 3, 3) + (0, -20, 8)\\ &= (-2, -17, 10)\end{aligned}$$
d. Letting $\vec{r} = A(0, 15, 17)$
$$A(0, 15, -7) = (0, 0, -1) + m(2, -3, -3) + n(0, 5, -2)$$
We get the following parametric equations:
$$\begin{aligned}0 &= 0 + 2m + 0n;\\ 0 &= m.\\ 15 &= 0 + (-3)m + 5n\\ 15 &= 5n\\ 3 &= n.\\ -7 &= -1 - 3m - 2n; \text{ for } m = 0 \text{ and } n = 3 \text{ we get:}\\ -7 &= -1 - 3(0) - 2(3)\\ -7 &= -7\end{aligned}$$
So our solution is $m = 0$ and $n = 3$.
e. For the point $B(0, 15, -8)$ the first two parametric equations are the same; yielding $m = 0$ and $n = 3$, however the third equation would then give:
$-8 = -1 - 3m - 2n$
$-8 = -1 - 3(0) - 2(3)$
$-8 = -7$ which is not true. So there can be no solution.

4. a. $P(-2, 3, 1)$, $Q(-2, 3, 2)$, $R(1, 0, 1)$
$$\begin{aligned}\overrightarrow{PQ} &= Q - P = (-2 - (-2), 3 - 3, 2 - 1)\\ &= (0, 0, 1)\\ \overrightarrow{PR} &= R - P = (1 - (-2), 0 - 3, 1 - 1)\\ &= (3, -3, 0)\\ \vec{r} &= (-2, 3, 1) + t(0, 0, 1) + s(3, -3, 0)\end{aligned}$$
b. $$\begin{aligned}\overrightarrow{QR} &= R - Q = (1 - (-2), 0 - 3, 1 - 2)\\ &= (3, -3, -1)\end{aligned}$$

Using $\overrightarrow{PQ}$ as the other direction vector:
$\vec{r} = (-2, 3, -2) + t(0, 0, 1) + s(3, -3, -1)$,
$t, s \in \mathbf{R}$
Using $\overrightarrow{PR}$ as the other direction vector:
$\vec{r} = (1, 0, 1) + t(3, -3, 0) + s(3, -3, -1), t, s \in \mathbf{R}$
5. a. $\vec{r} = (1, 0, -1) + s(2, 3, -4) + t(4, 6, -8)$,
$t, s \in \mathbf{R}$, does not represent a plane because the direction vectors are the same. We can rewrite the second direction vector as:
$(2)(2, 3, -4)$
And so we can rewrite the equation as:
$\vec{r} = (1, 0, -1) + s(2, 3, -4) + 2t(2, 3, -4)$
$= (1, 0, -1) + (s + 2t)(2, 3, -4)$
$= (1, 0, -1) + n(2, 3, 4), n \in \mathbf{R}$
This is an equation of a line in R^3.
6. a. The plane with direction vectors $\vec{a} = (4, 1, 0)$ and $\vec{b} = (3, 4, -1)$, that passes through the point $A(-1, 2, 7)$ has a vector equation of:
$\vec{r} = (-1, 2, 7) + t(4, 1, 0) + s(3, 4, -1), t, s \in \mathbf{R}$
The parametric equations are then:
$x = -1 + 4t + 3s$
$y = 2 + t + 4s$
$z = 7 - s; t, s \in \mathbf{R}$
b. $\overrightarrow{AB} = (0, 1, 0) - (1, 0, 0) = (-1, 1, 0)$
$\overrightarrow{AC} = (0, 0, 1) - (1, 0, 0) = (-1, 0, 1)$
Using A(1, 0, 0) as our point with $\overrightarrow{AB}$ and $\overrightarrow{AC}$ as our direction vectors, our vector equation is:
$\vec{r} = (1, 0, 0) + t(-1, 1, 0) + s(-1, 0, 1), t, s \in \mathbf{R}$
And thus our parametric equations are:
$x = 1 - t - s$
$y = t$
$z = s, t, s \in \mathbf{R}$
c. $\overrightarrow{AB} = B - A = (3, 4, -6)$ using this and $\vec{a} = (7, 1, 2)$ as our direction vectors and $A(1, 1, 0)$ as our point, the vector equation is:
$\vec{r} = (1, 1, 0) + t(3, 4, -6) + s(7, 1, 2), t, s \in \mathbf{R}$
The parametric equations are:
$x = 1 + 3t + 7s$
$y = 1 + 4t + s$
$z = -6t + 2s, t, s \in \mathbf{R}$
7. a. $(5, 3, 2) = (2, 0, 1) + s(4, 2, -1) + t(-1, 1, 2)$
This gives the parametric equations:
$5 = 2 + 4s - t \Rightarrow t = -3 + 4s$.
$3 = 2s + t$. Substituting for t gives:
$3 = 2s + (-3 + 4s)$
$6 = 6s$
$1 = s$.
$t = -3 + 4(1) = 1$.
$2 = 1 - s + 2t$
$2 = 1 - 1 + 2(1)$
$2 = 2$; which is true so $s = 1$ and $t = 1$.
b. $(0, 5, -4) = (2, 0, 1) + s(4, 2, -1) + t(-1, 1, 2)$
Gives the following parametric equations:
$0 = 2 + 4s - t \Rightarrow t = 2 + 4s$.
$5 = 2s + t$
$5 = 2s + (2 + 4s)$
$3 = 6s$
$\frac{1}{2} = s$.
$t = 2 + 4\left(\frac{1}{2}\right)$
$t = 2 + 2 = 4$.
The third equation then says:
$-4 = 1 - s + 2t$
$-4 = 1 - \frac{1}{2} + 2(4)$
$-4 = \frac{17}{2}$, which is a false statement. So the point $A(0, 5, -4)$ is not on the plane.
8. a. Using the direction vectors $\vec{a} = (-1, 1, 2)$, $\vec{b} = (2, 1, -3)$ and the point $A(-3, 5, 6)$, two equations of intersecting lines on the plane in vector form are:
$\vec{l} = (-3, 5, 6) + s(-1, 1, 2); s \in \mathbf{R}$
$\vec{p} = (-3, 5, 6) + t(2, 1, -3); t \in \mathbf{R}$
b. When $s = 0$ and $t = 0$ it is easily seen that these two lines both have the point $(-3, 5, 6)$ in common.
9. $\vec{r} = (4, 1, 6) + s(11, -1, 3) + t(-7, 2, -2)$ has parametric equations:
$x = 4 + 11s - 7t$
$y = 1 - s + 2t$
$z = 6 + 3s - 2t$
The plane crosses the z-axis when both x and y equal 0.
$0 = 1 - s + 2t \Rightarrow s = 1 + 2t$
$0 = 4 + 11s - 7t$
$0 = 4 + 11(1 + 2t) - 7t$
$0 = 15 + 15t$
$t = -1$.
$s = 1 + 2(-1) = -1$. And so the z-coordinate is:
$z = 6 + 3(-1) - 2(-1) = 5$. The plane crosses the z-axis at the point (0, 0, 5)
10. Using the point $Q(2, 1, 3)$ on the line and the point $P(-1, 2, 1)$, we get another direction vector: $\vec{a} = Q - P = (3, -1, 2)$. The equation of the plane having the given properties is then:
$\vec{r} = (2, 1, 3) + s(4, 1, 5) + t(3, -1, 2), t, s \in \mathbf{R}$

11. Using the point $A(-2, 2, 3)$ and the point (0, 0, 0) on the line we get another direction vector of: $\vec{a} = (-2, 2, 3)$. So the equation of the plane with the given properties is:
$\vec{r} = m(2, -1, 7) + n(-2, 2, 3), m, n \in \mathbf{R}$.
12. a. The xy-plane in R^3 has no z-coordinate so two sets of direction vectors are: (1, 0, 0), (0, 1, 0) and (1, 1, 0), $(-1, 1, 0)$.
b. A vector equation for the xy-plane in R^3 is:
$\vec{r} = s(1, 0, 0) + t(0, 1, 0), t, s \in \mathbf{R}$.
The parametric equations are:
$x = s$
$y = t$
$z = 0, t, s \in \mathbf{R}$
13. a. We can use the direction vectors $\overrightarrow{OA} = (-1, 2, 5)$ and $\overrightarrow{OC} = (3, -1, 7)$ and the origin to write the vector equation of the plane:
$\vec{r} = s(-1, 2, 5) + t(3, -1, 7), t, s \in \mathbf{R}$
b. Using $\overrightarrow{PQ} = Q - P = (-1, 2, 5)$ and $\overrightarrow{PR} = R - P = (3, -1, 7)$ as direction vectors, the vector equation of the plane is:
$\vec{r} = (-2, 2, 3) + s(-1, 2, 5) + t(3, -1, 7), t, s \in \mathbf{R}$
c. The two planes in parts **a.** and **b.** are parallel since they have the same direction vectors.
14. We simply need to show that the direction vectors can be expressed as a linear combination of the other two:

$$(-4, 7, 1) - (-3, 2, 4) = (-1, 5, -3)$$
$$\frac{27}{13}(-3, 2, 4) - \frac{17}{13}(-4, 7, 1) = (-1, -5, 7).$$

15. The plane
$\vec{r} = (1, 2, 3) + m(1, 2, 5) + n(1, -1, 3)$ has parametric equations:
$x = 1 + m + n$
$y = 2 + 2m - n$
$z = 3 + 5m + 3n$
Solving for the y-intercept:
$0 = 1 + m + n \Rightarrow n = -1 - m$
$0 = 3 + 5m + 3n$
$0 = 3 + 5m + 3(-1 - m)$
$0 = 4m$
$0 = m; n = -1$
$y = 2 + 2(0) - (-1) = 3$
Solving for the z-intercept:
$n = -1 - m$
$0 = 2 + 2m - (-1 - m)$
$0 = 3 + 3m$
$-1 = m; n = 0$
$z = 3 + 5(-1) + 3(0) = -2.$
The direction vector between the two points is then:
$(0, 3, 0) - (0, 0, -2) = (0, 3, 2)$.
And the equation of the line between them is:
$\vec{r} = (0, 3, 0) + t(0, 3, 2), t \in \mathbf{R}$
16. The fact that the plane
$\vec{r} = \overrightarrow{OP_0} + s\vec{a} + t\vec{b}$ contains both of the given lines is easily seen when letting $s = 0$ and $t = 0$ respectively.

8.5 The Cartesian Equation of a Plane, pp. 468–469

1. a. $\vec{n} = (A, B, C) = (1, -7, -18)$
b. In the Cartesian equation:
$Ax + By + Cz + D = 0$, If $D = 0$ the plane passes through the origin.
c. Three coordinates: $(0, 0, 0)$, $(11, -1, 1)$, $(11, -1, 1)$,
2. a. $\vec{n} = (A, B, C) = (2, -5, 0)$
b. In the Cartesian equation: $D = 0$. So the plane passes through the origin.
c. Three coordinates: $(0, 0, 0)$, $(5, 2, 0)$, $(5, 2, 1)$
3. a. $\vec{n} = (A, B, C) = (1, 0, 0)$
b. In the Cartesian equation: $D = 0$. So the plane passes through the origin.
c. Three coordinates: $(0, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$
4. a. $\vec{n} = (15, 75, -105)$ which is equivalent to $\vec{n} = (1, 5, -7)$. The Cartesian equation is:
$x + 5y - 7z + D = 0$. Since the plane passes through the origin $D = 0$. So the equation is:
$x + 5y - 7z = 0$.
b. $\vec{n} = (-\frac{1}{2}, \frac{3}{4}, \frac{7}{16})$ is equivalent to
$\vec{n} = (-8, 12, 7)$, so the Cartesian equation is:
$-8x + 12y + 7z + D = 0$, and since the plane passes through the origin $D = 0$. $-8x + 12y + 7z = 0$
5. *Method 1:* Let $A(x, y, z)$ be a point on the plane. Then $\overrightarrow{PA} = (x + 3, y - 3, z - 5)$ is a vector on the plane.

$$\vec{n} \cdot \overrightarrow{PA} = 0$$
$$(x + 3) + 7(y - 3) + 5(z - 5) = 0$$
$$x + 7y + 5z - 43 = 0$$

Method 2: $\vec{n} = (1, 7, 5)$ so the Cartesian equation is: $x + 7y + 5z + D = 0$.
We know the point $(-3, 3, 5)$ is on the plane and must satisfy the equation, so:

$$(-3) + 7(3) + 5(5) + D = 0$$
$$43 + D = 0$$
$$D = -43.$$

This also gives the equation:

$$x + 7y + 5z - 43 = 0.$$

6. a. $\overrightarrow{PQ} = (3 - (-1), 1 - 2, 4 - 1)$
$= (4, -1, 3)$
$\overrightarrow{QR} = (-2 - 3, 3 - 1, 5 - 4) = (-5, 2, 1)$
$\overrightarrow{PQ} \times \overrightarrow{QR} = ((-1)(1) - (3)(2), (3)(-5) - (4)(1), (4)(2) - (-1)(-5))$
$= (-7, -19, 3)$
$= -1(7, 19, -3).$
Using $\vec{n} = (7, 19, -3)$ the Cartesian equation is:
$7x + 19y - 3z + D = 0.$
Using the point $R(-2, 3, 5)$ on the plane to solve for D:
$7(-2) + 19(3) - 3(5) + D = 0$
$-14 + 57 - 15 + D = 0$
$28 + D = 0$
$D = -28.$
$7x + 19y - 3z - 28 = 0$

b. $\overrightarrow{QP} = (-1 - 3, 2 - 1, 1 - 4)$
$= (-4, 1, -3)$
$\overrightarrow{PR} = (-2 - (-1), 3 - 2, 5 - 1)$
$= (-1, 1, 4)$
$\overrightarrow{QP} \times \overrightarrow{PR} = ((1)(4) - (-3)(1), (-3)(-1) - (-4)(4), (-4)(1) - (1)(-1))$
$= (7, 19, -3).$
Using $\vec{n} = (7, 19, -3)$ the Cartesian equation is:
$7x + 19y - 3z + D = 0.$
Using the point $P(-1, 2, 1)$ on the plane to solve for D:
$7(-1) + 19(2) - (3)(1) + D = 0$
$-7 + 38 - 3 + D = 0$
$28 + D = 0$
$D = -28.$
$7x + 19y - 3z - 28 = 0$

c. There is only one simplified Cartesian equation that satisfies the given information, so the equations must be the same.

7. $\overrightarrow{AB} = (5, 1, 4).$
$\overrightarrow{AC} = (3, -2, -1).$
$\overrightarrow{AB} \times \overrightarrow{AC} = ((1)(-1) - (4)(-2), (4)(3) - (5)(-1), (5)(-2) - (1)(3))$
$= (7, 17, -13)$
Using $\vec{n} = (7, 17, -13)$ the Cartesian equation is:
$7x + 17y - 13z + D = 0.$
Using the point $(1, 1, 0)$ on the plane to solve for D:
$7(1) + 17(1) - 13(0) + D = 0$
$24 + D = 0$
$D = -24.$
$7x + 17y - 13z - 24 = 0$

8. The point $Q(2, 0, 1)$ is on the line and thus also on the plane and we can get another direction vector from:
$\overrightarrow{PQ} = (1, -3, 1)$. Using $\vec{a} = (-4, 5, 5)$ as the other direction vector we can find the normal vector:
$\vec{n} = \overrightarrow{PQ} \times \vec{a} = ((-3)(5) - (1)(5), (1)(-4) - (1)(5), (1)(5) - (-3)(-4))$
$= (-20, -9, -7) = -1(20, 9, 7).$
Our Cartesian equation is thus:
$20x + 9y + 7z + D = 0.$
Using the point $(1, 3, 0)$ to determine D:
$20(1) + 9(3) + 7(0) + D = 0.$
$47 + D = 0$
$D = -47.$
$20x + 9y + 7z - 47 = 0$

9. a. $2x + 2y - z - 1 = 0$
$\vec{n} = (2, 2, -1)$
$|\vec{n}| = \sqrt{4 + 4 + 1}$
$= 3$
So the unit normal vector is:
$$\frac{\vec{n}}{|\vec{n}|} = \left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$$
b. $4x - 3y + z - 3 = 0$
$\vec{n} = (4, -3, 1)$
$|\vec{n}| = \sqrt{16 + 9 + 1}$
$= \sqrt{26}$
So the unit normal vector is:
$$\frac{\vec{n}}{|\vec{n}|} = \left(\frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}\right)$$
c. $3x - 4y + 12z - 1 = 0$
$\vec{n} = (3, -4, 12)$
$|\vec{n}| = \sqrt{9 + 16 + 144}$
$= \sqrt{169}$
$= 13$
So the unit normal vector is:
$$\frac{\vec{n}}{|\vec{n}|} = \left(\frac{3}{13}, -\frac{4}{13}, \frac{12}{13}\right)$$

10. We know the point $P(1, 1, 5)$ is on the plane, and can obtain another direction vector from:
$\overrightarrow{AP} = (1, 1, -6)$. Let $\vec{a} = (2, 1, 3)$ be our other direction vector.
$\vec{n} = \overrightarrow{AP} \times \vec{a} = ((1)(3) - (-6)(3), (-6)(2) - (1)(3), (1)(1) - (1)(2))$
$= (21, -15, -1)$
The Cartesian equation is then:
$21x - 15y - z + D = 0.$

Using the point $(1, 1, 5)$ to solve for D:

$$21(1) - 15(1) - (5) + D = 0$$
$$1 + D = 0$$
$$D = -1.$$
$$21x - 15y - z - 1 = 0.$$

11. Since the normal vector is perpendicular to the plane, we can use the direction vector of the line as our normal vector:

$\vec{n} = (3, -2, 0) - (1, 2, 1) = (2, -4, -1).$

The Cartesian equation is then:

$2x - 4y - z + D = 0.$

We need the point $(-1, 1, 0)$ to be on the plane so:

$$2(-1) - 4(1) - (0) + D = 0.$$
$$-6 + D = 0$$

$D = 6$. And the Cartesian equation of the plane satisfying the given conditions is:

$2x - 4y - z + 6 = 0.$

12. a. To determine the angle between two planes, first determine their normal vectors. This is easily done if the equations given are in Cartesian form. Once the normal vectors are known, $\vec{n_1}$ and $\vec{n_2}$, then the angle between the two planes can be determined from the formula:

$$\cos(\theta) = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}.$$

b. $\vec{n_1} = (1, 0, -1)$. $\vec{n_2} = (2, 1, -1)$.

$$\vec{n_1} \cdot \vec{n_2} = 2 + 0 + 1$$
$$= 3$$
$$\vec{n_1} \cdot \vec{n_2} = \sqrt{2} \cdot \sqrt{6}$$
$$= \sqrt{12}.$$
$$\cos(\theta) = \frac{3}{\sqrt{12}}$$
$$= \frac{\sqrt{3}}{2}$$
$$\theta = \frac{\pi}{6}$$
$$= 30°$$

13. a. $\vec{n_1} = (1, 2, -3)$. $\vec{n_2} = (1, 2, 0)$

$$\vec{n_1} \cdot \vec{n_2} = 1 + 4$$
$$= 5$$
$$|\vec{n_1}||\vec{n_2}| = \sqrt{14} \cdot \sqrt{5}$$
$$= \sqrt{70}$$
$$\cos(\theta) = \frac{5}{\sqrt{70}}.$$
$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{70}}\right) = 53.3°$$

b. The parametric equations for the line are:

$x = 3 - 2t$
$y = -1 + 3t$
$z = -4 + t,$

which give the following vector equation:

$\vec{r} = (3, -1, -4) + t(-2, 3, 1)$. Since the line and normal vector are both perpendicular to the plane we may take:

$\vec{n} = (2, -3, -1).$

The Cartesian equation for the plane is then:

$2x - 3y - z + D = 0.$

Using the point $P(1, 2, 1)$ to solve for D:

$$2(1) - (3)(2) - (1)(1) + D = 0$$
$$-5 + D = 0$$

$D = 5$. And the Cartesian equation becomes:

$2x - 3y - z + 5 = 0.$

14. a. $\vec{n_1} = (4, k, -2)$ and $\vec{n_2} = (2, 4, -1)$.

When $k = 8$, $\vec{n_1}$ is equivalent to: $\vec{n_1} = 2(2, 4, -1)$, so the planes are parallel when $k = 8$.

b. When the planes are perpendicular

$$\vec{n_1} \cdot \vec{n_2} = 0.$$
$$\vec{n_1} \cdot \vec{n_2} = 8 + 4k + 2 = 0$$
$$10 + 4k = 0$$
$$k = -\frac{10}{4} = -\frac{5}{2}$$

c. No the planes cannot ever be coincident. If they were then they would also be parallel, so $k = 8$, and we would have the two equations:

$$4x + 8y - 2z + 1 = 0.$$
$$2x + 4y - z + 4 = 0 \Rightarrow 4x + 8y - 2z + 8 = 0.$$

Here all of the coefficients are equal except for the D values, which means that they don't coincide.

15. Since the plane passes through the points $(1, 4, 5)$ and $(3, 2, 1)$ it contains the line and the direction vector between them. The direction vector is:

$\vec{r} = (2, -2, -4).$

The normal vector, $\vec{n_1}$, must be perpendicular to the direction vector and to the normal vector, $\vec{n_2} = (2, -1, 1)$, of the other plane, so:

$$\vec{n_1} = \vec{r} \times \vec{n_2} = ((-2)(1) - (-4)(-1), (-4)(2) - (2)(1), (2)(-1) - (-2)(2))$$
$$= (-6, -10, 2) = -2(3, 5, -1)$$

Take $\vec{n_1} = (3, 5, -1)$ and the Cartesian equation of the plane is:

$3x + 5y - z + D = 0$

Use the point (1, 4, 5) to determine D:

$$3(1) + 5(4) - 5 + D = 0$$
$$18 + D = 0$$
$$D = -18.$$
$$3x + 5y - z - 18 = 0$$

16. Let $\vec{n_1} = (A, B, C)$, be the normal vector of the unknown plane, and $\vec{n_2} = (1, 2, 0)$ be the normal vector to the perpendicular plane. $\vec{n_1} \cdot \vec{n_2} = 0$ so we get:

$A + 2B = 0.$

$A = -2B$

We also know that the z-axis has the direction vector $\vec{r} = (0, 0, 1)$. So:

$$\cos(30°) = \frac{\sqrt{3}}{2} = \frac{\vec{n_1} \cdot \vec{r}}{|\vec{n_1}||\vec{r}|} = \frac{C}{\sqrt{A^2 + B^2 + C^2}}.$$

The other constraint which we can choose is the length of $\vec{n_1}$. Since this is arbitrary (multiplication by any scalar will give an equivalent normal vector) choose $|\vec{n_1}| = 2$. We have:

$$\frac{\sqrt{3}}{2} = \frac{C}{2} \Rightarrow C = \sqrt{3}.$$

$$A^2 + B^2 + C^2 = 4$$

$$4B^2 + B^2 + 3 = 4$$

$$B^2 = \frac{1}{5}$$

$$B = \frac{1}{\sqrt{5}}; A = -\frac{2}{\sqrt{5}}.$$

The equation of the plane is then:

$$-\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y + \sqrt{3}z = 0.$$

17. The point equidistant from $(-1, 2, 4)$ and $(3, 1, -4)$ is the point $\frac{1}{2}((-1, 2, 4) + (3, 1, -4)) = (1, \frac{3}{2}, 0)$. If every point in the plane is equidistant from these two point than the normal to the plane must point in the same direction as the line connecting them:

$\vec{n} = (3, 1, -4) - (-1, 2, 4) = (4, -1, -8).$

The equation of the plane is thus:

$4x - y - 8z + D = 0.$

Using the point $(1, \frac{3}{2}, 0)$ to solve for D:

$$4(1) - \frac{3}{2} - 0 + D = 0$$

$$\frac{5}{2} + D = 0 \Rightarrow D = -\frac{5}{2}$$

We now have the equation of the plane:

$$4x - y - 8z - \frac{5}{2} = 0.$$

Or equivalently:

$8x - 2y - 16z - 5 = 0.$

8.6 Sketching Planes in R^3, pp. 476–477

1. a. A plane parallel to the yz-axis but two units away, in the negative x direction.

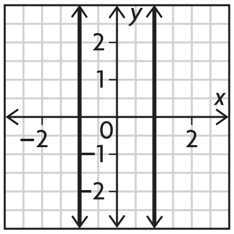
b. A plane parallel to the xz-axis but three units away, in the positive y direction.

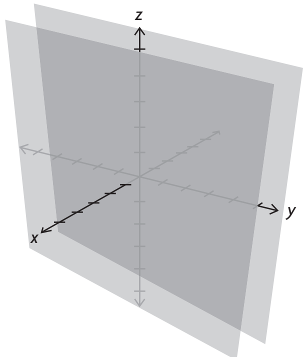
c. A plane parallel to the xy-axis but 4 units away, in the positive z direction.

2. The point of intersection of the three planes in problem 1 must lie in every plane. Therefore the point of intersection is: $(-2, 3, 4)$

3. The point $P(5, -3, -3)$ must lie on the plane π_1: $x = 5$, since the point has an x-coordinate of 5, and doesn't have a y-coordinate of 6.

4. In R^2, $x^2 - 1 = 0$ represents two lines, $x = -1$ and $x = 1$. In R^3, $x^2 - 1 = 0$ represents two planes with the same equations.





5. a. i. x-intercept is when $y = z = 0$.

$2x = 18$

$x = 9$

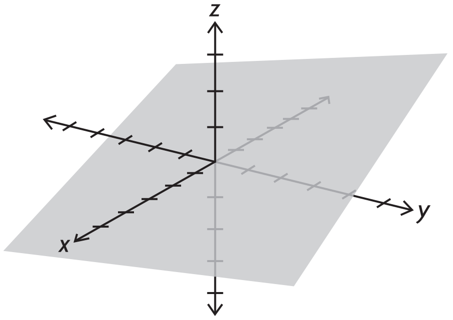
Similarly the y-intercept is:

$3y = 18$

$y = 6$

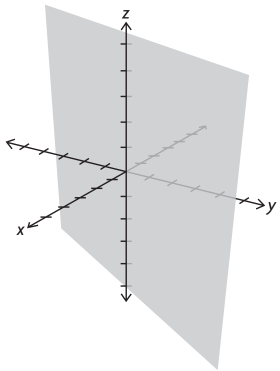
Since x and y cannot both be zero at the same time there is no z-intercept. The plane is parallel to the z-axis.

ii. x-intercept:
$3x = 120$
$x = 40$
y-intercept:
$-4y = 120$
$y = -30$
z-intercept:
$5z = 120$
$z = 24$
iii. There is no x-intercept since y and z cannot both be simultaneously zero.
y-intercept:
$13y = 39$
$y = 3$
z-intercept:
$-z = 39$
$z = -39$
b. i. Since the plane is parallel to the z-axis one directional vector is: $(0, 0, 1)$. The other lies along the line $2x + 3y = 18$, so $(3, -2, 0)$.
ii. We can find directional vectors by taking the difference between two points, namely the intercepts we found in **a.** : $(40, 0, 0) - (0, -30, 0) = (40, 30, 0)$ or equivalently $(4, 3, 0)$.
$(40, 0, 0) - (0, 0, 24) = (40, 0, -24)$ or equivalently $(5, 0, -3)$.
iii. Since the plane is parallel to the x-axis $(1, 0, 0)$ is one directional vector.
$(0, 3, 0) - (0, 0, -39) = (0, 3, 39)$. Or equivalently $(0, 1, 13)$.
6. a. i. π: $2x - y + 5z = 0$. Three points satisfying this equation are: $(0, 0, 0)$, $(1, 2, 0)$, $(0, 5, 1)$.
ii. The line where this plane intersects the xy-plane is simply the line when $z = 0$:
$2x - y = 0$.
b.

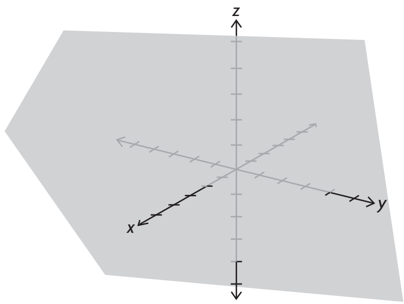


7. $xyz = 0$ has the solutions: $x = 0$, $y = 0$, $z = 0$. So the three planes are the yz-plane, xz-plane, and the xy-plane.

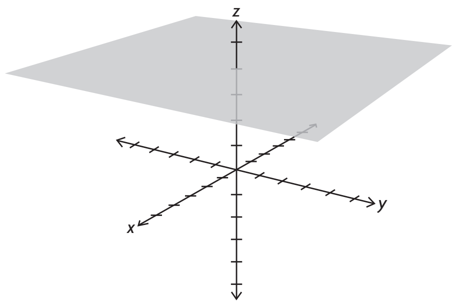
8. a.



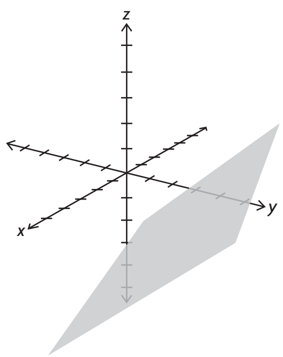
b.



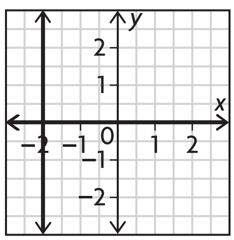
c.



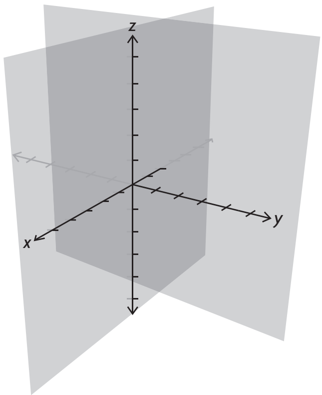
d.



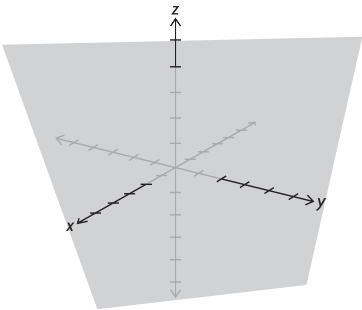
9. a. $xy + 2y = 0$
$y(x + 2) = 0$
b.



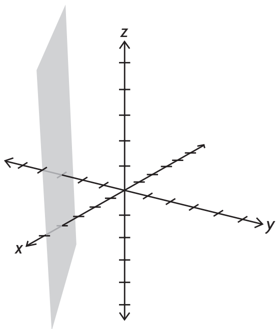
c.



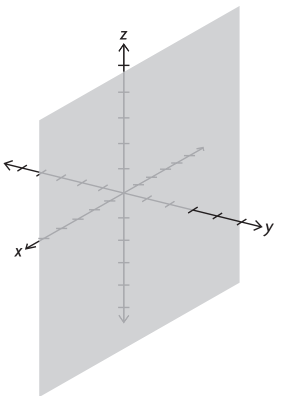
10. a.



b.



c.



11. a. The plane with x-, y-, z- intercepts of 3, 4, and 6, respectively is $\frac{x}{3} + \frac{y}{4} + \frac{z}{6} = 1$.

b. The plane with x- and z-intercepts of 5 and -7, respectively, and which is parallel to the y-axis is $\frac{x}{5} - \frac{z}{7} = 1$.

c. No x- or y-intercepts but with a z-intercept of 8 has the equation $\frac{z}{8} = 1$.

Review Exercise, pp. 480–483

1. Answers may vary. For example:
$A(1, 2, -1)$, $B(2, 1, 1)$, $C(3, 1, 4)$

$\overrightarrow{AB} = (1, -1, 2) = \vec{a}$

$\overrightarrow{BC} = (1, 0, 3) = \vec{b}$

$\vec{r} = \vec{r_0} + s\vec{a} + t\vec{b}$

$\vec{r} = (1, 2, -1) + s(1, -1, 2) + t(1, 0, 3), s, t \in \mathbf{R}$

$x = 1 + s + t$

$y = 2 - s$

$z = -1 + 2s + 3t$

2. $A(1, 2, -1)$, $B(2, 1, 1)$, $C(3, 1, 4)$

$\overrightarrow{AB} = (1, -1, 2) = \vec{a}$

$\overrightarrow{BC} = (1, 0, 3) = \vec{b}$

$\vec{r} = (1, 2, -1) + s(1, -1, 2) + t(1, 0, 3), s, t \in \mathbf{R}$

$\vec{b} \times \vec{a} = (1, 0, 3) \times (1, -1, 2) = (3, 1, -1)$

$$\begin{aligned} Ax + By + Cz + D &= 0 \\ (3)x + (1)y + (-1)z + D &= 0 \\ 3(1) + (2) - 1(-1) + D &= 0 \\ D &= -6 \end{aligned}$$

$3x + y - z - 6 = 0$

$\overrightarrow{AC} = (2, -1, 5) = \vec{c}$

$\overrightarrow{BC} = (1, 0, 3) = \vec{b}$

$\vec{r} = (1, 2, -1) + s(2, -1, 5) + t(1, 0, 3), t, s \in \mathbf{R}$

$\vec{b} \times \vec{c} = (1, 0, 3) \times (2, -1, 5) = (3, 1, -1)$

$$\begin{aligned} Ax + By + Cz + D &= 0 \\ (3)x + (1)y + (-1)z + D &= 0 \\ 3(1) + (2) - 1(-1) + D &= 0 \\ D &= -6 \end{aligned}$$

$3x + y - z - 6 = 0$

Both Cartesian equations are the same regardless of which vectors are used.

3. a. Answers may vary. For example:
$A(-3, 2, 8)$, $B(4, 3, 9)$

$\overrightarrow{AB} = (7, 1, 1) = \vec{a}$

$\vec{r} = (4, 3, 9) + t(7, 1, 1), t \in \mathbf{R}$

$x = x_0 + ta, y = y_0 + tb, z = z_0 + tc$

$x = 4 + 7t, y = 3 + t, z = 9 + t, t \in \mathbf{R}$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

$$\frac{x - 4}{7} = \frac{y - 3}{1} = \frac{z - 9}{1}$$

b. Answers may vary. For example:
$A(-3, 2, 8), B(4, 3, 9), C(-2, -1, 3)$

$\overrightarrow{AB} = (7, 1, 1) = \vec{a}$

$\overrightarrow{CB} = (6, 4, 6) = (3, 2, 3) = \vec{b}$

$\vec{r} = (4, 3, 9) + t(7, 1, 1) + s(3, 2, 3), t, s \in \mathbf{R}$

$x = x_0 + ta_1 + tb_1, y = y_0 + ta_2 + tb_2,$
$z = z_0 + ta_3 + tb_3$

$x = 4 + 7t + 3s, y = 3 + t + 2s,$
$z = 9 + t + 3s, t, s \in \mathbf{R}$

c. There are no symmetric equations, because there are two parameters.

4. A line passing through $A(7, 1, -2)$ and perpendicular to the plane with the equation $2x - 3y + z - 1 = 0$. Since the line is perpendicular to the plane, the normal of the plane is the lines vector.

$\vec{m} = (2, -3, 1)$

$\vec{r} = \vec{r_0} + t\vec{m}$

$\vec{r} = (7, 1, -2) + t(2, -3, 1), t \in \mathbf{R}$

$x = 7 + 2t, y = 1 - 3t, z = -2 + t$

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

$$\frac{x - 7}{2} = \frac{y - 1}{-3} = \frac{z + 2}{1}$$

5. a. $P(0, 1, -2)$

$\vec{n} = (-1, 3, 3)$

$$Ax + By + Cz + D = 0$$
$$(-1)(x - 0) + (3)(y - 1) + (3)(z + 2) = 0$$
$$-x + 3y - 3 + 3z + 6 = 0$$
$$x - 3y - 3z - 3 = 0$$

b. $A(3, 0, 1), B(0, 1, -1)$

$\overrightarrow{AB} = (-3, 1, -2)$

$\vec{n} = (1, -1, -1)$

$\vec{n} \times \overrightarrow{AB} = (3, 5, -2)$

$$Ax + By + Cz + D = 0$$
$$(3)x + (5)y + (-2)z + D = 0$$
$$(3)(3) + (5)(0) + (-2)(1) + D = 0$$
$$D = -7$$

$3x + 5y - 2z - 7 = 0$

c. $A(1, 2, 1), B(2, 1, 4)$

$\overrightarrow{AB} = (1, -1, 3)$

$\vec{n} = (1, 0, 0)$

$\overrightarrow{AB} \times \vec{n} = (0, 3, 1)$

$$Ax + By + Cz + D = 0$$
$$(0)x + (3)y + (1)z + D = 0$$
$$(0)(1) + (3)(2) + (1)(1) + D = 0$$
$$D = -7$$

$3y + z - 7 = 0$

6. $\vec{r} = (3, 7, 1) + t(2, 2, 3), t \in \mathbf{R}$

$(2, 2, 3) \cdot (a, b, c) = \vec{n}$

$2a + 2b + 3c = 0$

$a = 19, b = -7, c = -8$

$\vec{n} = (19, -7, -8)$

$$Ax + By + Cz + D = 0$$
$$19x - 7y - 8z + D = 0$$
$$19(0) - 7(0) - 8(0) + D = 0$$
$$D = 0$$

$19x - 7y - 8z = 0$

7. Since the plane is parallel to the yz-plane, its direction vectors are (0, 1, 0) and (0, 0, 1).

$A = (-1, 2, 1)$

$\vec{r} = \vec{r_0} + t\vec{a} + s\vec{b}$

$\vec{a} = (0, 1, 0), \ \vec{b} = (0, 0, 1)$

$\vec{r} = (-1, 2, 1) + t(0, 1, 0) + s(0, 0, 1)t, s \in \mathbf{R}$

$x = -1, y = 2 + t, z = 1 + s$

8. $A = (4, -3, 2)$

$\vec{r} = (2, 3, 2) + t(1, 1, 4), t \in \mathbf{R}$

$\vec{a} = (1, 1, 4), \vec{b} = [(4 - 2), (-3 - 3), (2 - 2)]$

$\vec{a} = (1, 1, 4), \vec{b} = (2, -6, 0)$

$\vec{a} \times \vec{b} = (24, 8, -8) = (3, 1, -1)$

$$Ax + By + Cz + D = 0$$
$$(3)x + (1)y + (-1)z + D = 0$$
$$3(4) + 1(-3) - 1(2) + D = 0$$
$$D = -7$$

$3x + y - z - 7 = 0$

9. $L_1: \vec{r} = (4, 4, 5) + s(5, -4, 6), s \in \mathbf{R}$

$L_2: \vec{r} = (4, 4, 5) + s(2, -3, -4), s \in \mathbf{R}$

$\vec{a} = (5, -4, 6), \vec{b} = (2, -3, -4)$

$\vec{a} \times \vec{b} = (34, 32, -7)$

$$Ax + By + Cz + D = 0$$
$$(34)x + (32)y + (-7)z + D = 0$$
$$34(4) + 32(4) - 7(5) + D = 0$$
$$D = -229$$

$34x + 32y - 7z - 229 = 0$

10. Answers may vary. For example: Since the line is perpendicular to the plane. The normal of the plane is the directional vector of the line.

$A(2, 3, -3)$

$3x - 2y + z = 0$

$\vec{n} = (3, -2, 1)$

$\vec{r} = (2, 3, -3) + s(3, -2, 1), s \in \mathbf{R}$

$x = 2 + 3s, y = 3 - 2s, z = -3 + s$

$$\frac{x-2}{3} = \frac{y-3}{-2} = \frac{z+3}{1}$$

11. Answers may vary. For example: Use the dot product and cross product to find two points that are orthogonal to the normal of the plane. Then use any point from the plane.

$$\begin{aligned} 3x + 2y - z + 6 &= 0 \\ \vec{a} \cdot (3, 2, -1) &= 0 \\ (a, b, c) \cdot (3, 2, -1) &= 0 \\ 3a + 2b - c &= 0 \end{aligned}$$

$\vec{a} = (1, 0, 3)$

$$\begin{aligned} (3, 2, -1) \times (1, 0, 3) &= (6, -10, -2) \\ &= (3, -5, -1) \end{aligned}$$

$\vec{r} = (0, 0, 6) + s(1, 0, 3) + t(3, -5, -1), s, t \in \mathbf{R}$

$x = s + 3t, y = -5t, z = 6 + 3s - t$

12. Answers may vary. For example: The x-intercept is $(-3.5, 0, 0)$ and z-intercept is (0, 0, 7). Find the directional vector from these points and use a point one of the intercepts.

$A = (-3.5, 0, 0), B = (0, 0, 7)$

$\vec{v} = [(0 - 3.5), (0 - 0), (7 - 0)]$

$\vec{v} = (3.5, 0, 7) = (1, 0, 2)$

$\vec{r} = r_0 + ta, t \in \mathbf{R}$

$\vec{r} = (0, 0, 7) + t(1, 0, 2), t \in \mathbf{R}$

$x = t, y = 0, z = 7 + 2t$

13. The two direction vectors for these lines are

$\vec{a} = (1, -3, -5)$

$\vec{b} = (2, -6, -10) = 2\vec{a}$

So the lines L_1 and L_2 are parallel (they aren't the same line, as $(3, -4, 1)$, a point on L_1, is not a point on L_2). Take one of the direction vectors for the plane to be the vector $\vec{a} = (1, -3, -5)$, and find another by computing the vector with tail at $(3, -4, 1)$ (a point on L_1) and head at $(7, -1, 0)$ (a point on L_2). This is the vector

$$\begin{aligned} \vec{v} &= (7, -1, 0) - (3, -4, 1) \\ &= (4, 3, -1) \end{aligned}$$

The point $(3, -4, 1)$ is on the plane, so the vector equation of the plane is

$\vec{r} = (3, -4, 1) + s(1, -3, -5) + t(4, 3, -1),$
$s, t \in \mathbf{R}$.

The parametric form for the plane is

$x = 3 + s + 4t,$
$y = -4 - 3s + 3t$
$z = 1 - 5s - t, s, t \in \mathbf{R}$

Finally, to find the Cartesian equation of the plane, compute the cross product of the direction vectors.

$$\begin{aligned} \vec{a} \times \vec{v} &= (1, -3, -5) \times (4, 3, -1) \\ &= (-3(-1) - (3)(-5), 4(-5) \\ &\quad - 1(-1), 1(3) - 4(-3)) \\ &= (18, -19, 15) \end{aligned}$$

So the Cartesian equation is of the form

$18x - 19y + 15z + D = 0.$

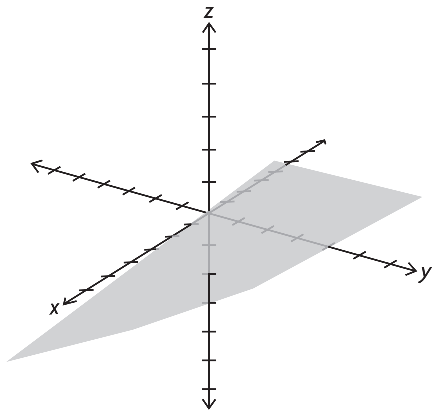
To find the value of D, substitute in the point on the plane $(3, -4, 1)$.

$$\begin{aligned} 18(3) - 19(-4) + 15(1) + D &= 0 \\ D &= -145 \end{aligned}$$

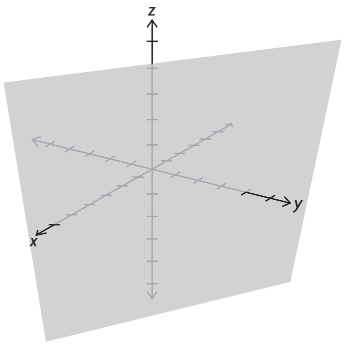
So the Cartesian equation is

$18x - 19y + 15z - 145 = 0$

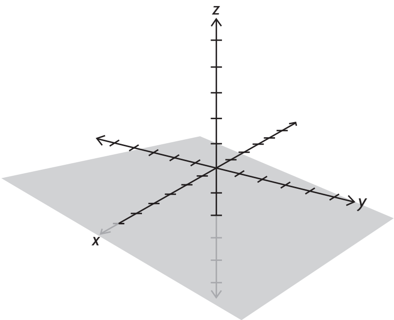
14. a.



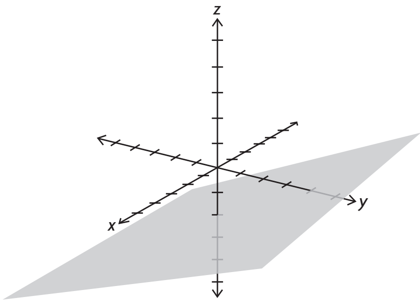
b.



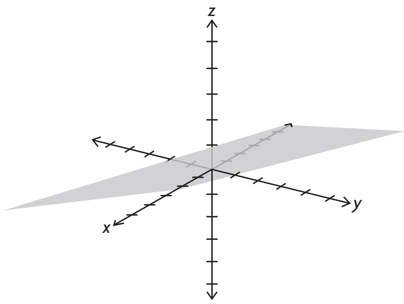
c.



d.



e.



15. a. Answers may vary. For example:

$P(1, -2, 5)$

$Q(3, 1, 2)$

$\overrightarrow{PQ} = \vec{p} = (2, 3, -3)$

$L: \vec{r} = 2ti + (4t + 3)j + (t + 1)k$

$= (0, 3, 1) + t(2, 4, 1)$

$\vec{a} = (2, 4, 1)$

$\vec{r} = (3, 1, 2) + t(2, 4, 1) + s(2, 3, -3), t, s \in \mathbf{R}$

$x = 3 + 2t + 2s, y = 1 + 4t + 3s, z = 2 + t - 3s$

$\vec{a} \times \vec{p} = (2, 3, -3) \times (2, 4, 1) = (15, -8, 2)$

$Ax + By + Cz + D = 0$

$(15)x + (-8)y + (2)z + D = 0$

$15(3) - 8(1) + 2(2) + D = 0$

$D = -41$

$15x - 8y + 2z - 41 = 0$

b. Answers may vary. For example: The normal of the plane is the direction vector of the line, since it is perpendicular to the plane. Then find using the Cartesian form of a plane.

$A(1, 1, 2)\ B = (2, 1, -6), C = (-2, 1, 5)$

$\overrightarrow{BC} = (-4, 0, 11)$

$Ax + Bx + Cx + D = 0$

$(-4)x + (0)y + (11)z + D = 0$

$-4(1) + 11(2) + D = 0$

$D = -18$

$-4x + 11z - 18 = 0$

c. Answers may vary. For example: Since the plane is parallel to the z-axis, one of its direction vectors is (0, 0, 1).

$A(4, 1, -1), B(5, -2, 4)$

$\overrightarrow{AB} = (1, -3, 5)$

$\vec{r} = (4, 1, -1) + t(1, -3, 5) + s(0, 01), t, s \in \mathbf{R}$

$x = 4 + t, y = 1 - 3t, z = -1 + 5t + s$

$(1, -3, 5) \cdot (0, 0, 1) = (-3, -1, 0) = (3, 1, 0)$

$Ax + By + Cz + D = 0$

$(3)x + (1)y + (0)z + D = 0$

$3(4) + 1(1) + D = 0$

$D = -13$

$3x + y - 13 = 0$

d. Answers may vary. For example:

$A(1, 3, -5), B(2, 6, 4), C(3, -3, 3)$

$\overrightarrow{AB} = (1, 3, 9)$

$\overrightarrow{BC} = (1, -9, -1)$

$\vec{r} = r_0 + t\vec{a} + s\vec{b}$

$\vec{r} = (1, 3, -5) + t(1, 3, 9) + s(1, -9, -1), t, s \in \mathbf{R}$

$x = 1 + t + s, y = 3 + 3t - 9s, z = -5 + 9t - s$

$\overrightarrow{AB} \times \overrightarrow{BC} = (78, 10, -12)$

$Ax + By + Cz + D = 0$

$(78)x + (10)y + (-12)z + D = 0$

$78(1) + 10(3) - 12(-5) + D = 0$

$D = -168$

$78x + 10y - 12z - 168 = 0$

16. They are in the same plane because both planes have the same normal vectors and Cartesian equations.

$L_1: \vec{r} = (1, 2, 3) + s(-3, 5, 21) + t(0, 1, 3), s, t \in \mathbf{R}$

$L_2: \vec{r} = (1, -1, -6) + u(1, 1, 1) + v(2, 5, 11),$
$u, v \in \mathbf{R}$

$(-3, 5, 21) \times (0, 1, 3) = (-6, 9, -3) = (2, -3, 1)$

$(1, 1, 1) \times (2, 5, 11) = (6, -9, 3) = (2, -3, 1)$

$Ax + By + Cz + D = 0$

$2x - 3y + z + D = 0$

$2(1) - 3(2) + (3) + D = 0$

$D = 1$

$2(1) - 3(-1) + (-6) + D = 0$

$D = 1$

$2x - 3y + z + 1 = 0$

17. A point B on the line L_2 will have coordinates $B(2 + 2t, 1 + t, 2 - t)$, $t \in \mathbf{R}$ Then

$$\overrightarrow{AB} = (2 + 2t, 1 + t, 2 - t) - (5, 4, -3)$$
$$= (-3 + 2t, -3 + t, 5 - t)$$

For this vector to be perpendicular to L_2, it would have zero dot product with the direction vector for L_2, $\vec{v} = (2, 1, -1)$. So

$$0 = \vec{v} \cdot \overrightarrow{AB}$$
$$= (2, 1, -1) \cdot (-3 + 2t, -3 + t, 5 - t)$$
$$= -6 + 4t - 3 + t - 5 + t$$
$$= -14 + 6t$$

So $t = \frac{14}{6} = \frac{7}{3}$, and the point B is

$$B\left(2 + 2\left(\frac{7}{3}\right), 1 + \frac{7}{3}, 2 - \frac{7}{3}\right) = B\left(\frac{20}{3}, \frac{10}{3}, -\frac{1}{3}\right).$$

18. a. The plane is parallel to the z-axis through the points (3, 0, 0) and (0, −2, 0).

b. The plane is parallel to the y-axis through the points (6, 0, 0) and (0, 0, −2).

c. The plane is parallel to the x-axis through the points (0, 3, 0) and (0, 0, −6).

19. a. To determine which points lie on the line, just see if there is a t-value such that the coordinate works.

$x = 2t, y = 3 + t, z = 1 + t$

$A(2, 4, 2)$

$2 = 2t, 4 = 3 + t, 2 = 1 + t$

$t = 1$

$B(-2, 2, 1)$

$-2 = 2t, 2 = 3 + t, 1 = 1 + t$

There is no value of t that satisfies the equations.

$C(4, 5, 2)$

$4 = 2t, 5 = 3 + t, 2 = 1 + t$

There is no value of t that satisfies the equations.

$D(6, 6, 2)$

$6 = 2t, 6 = 3 + t, 2 = 1 + t$

There is no value of t that satisfies the equations.

Only A lies on the line.

b. $x = 2t, y = 3 + t, z = 1 + t$

$a = 2t, b = 3 + t, -3 = 1 + t$

$t = -4$

$a = 2t = -8$

$b = 3 + t = -1$

20. a. $L_1: \dfrac{x - 1}{1} = \dfrac{y - 3}{5}$

$L_2: \dfrac{x - 2}{2} = \dfrac{1 - y}{3}$

$$\cos\theta = \frac{|n_1 \cdot n_2|}{|n_1||n_2|}$$

$n_1 = (1, 5),\ n_2 = (2, -3)$

$$\cos\theta = \frac{13}{(\sqrt{26})(\sqrt{13})}$$

$\theta = 45.0°$

b. $y = 4x + 2, y = -x + 3$

$n_1 = (1, 4)$

$n_2 = (1, -1)$

$$\cos\theta = \frac{|n_1 \cdot n_2|}{|n_1||n_2|}$$

$$\cos\theta = \frac{3}{(\sqrt{17})(\sqrt{2})}$$

$\theta = 59.0°$

c. $L_1: x = -1 + 3t, y = 1 + 4t, z = -2t$

$L_2: x = -1 + 2s, y = 3s, z = -7 + s$

$$\cos\theta = \frac{|n_1 \cdot n_2|}{|n_1||n_2|}$$

$n_1 = (3, 4, -2), n_2 = (2, 3, 1)$

$$\cos\theta = \frac{16}{(\sqrt{29})(\sqrt{14})}$$

$\theta = 37.4°$

d. $L_1: (x, y, z) = (4, 7, -1) + t(4, 8, -4)$

$L_2: (x, y, z) = (1, 5, -4) + t(-1, 2, 3)$

$$\cos\theta = \frac{|n_1 \cdot n_2|}{|n_1||n_2|}$$

$n_1 = (4, 8, -4), n_2 = (-1, 2, 3)$

$$\cos\theta = \frac{0}{(\sqrt{96})(\sqrt{14})}$$

$\theta = 90°$

21. a. $L_1: 2x + 3y - z + 9 = 0$

$L_2: x + 2y + 4 = 0$

$$\cos\theta = \frac{|n_1 \cdot n_2|}{|n_1||n_2|}$$

$n_1 = (2, 3, -1),$

$n_2 = (1, 2, 0)$

$$\cos\theta = \frac{6}{(\sqrt{14})(\sqrt{5})}$$

$\theta = 44.2°$

b. $L_1: x - y - z - 1 = 0$

$L_2: 2x + 3y - z + 4 = 0$

$$\cos\theta = \frac{|n_1 \cdot n_2|}{|n_1||n_2|}$$

$n_1 = (1, -1, -1), n_2 = (2, 3, -1)$

$$\cos\theta = \frac{0}{(\sqrt{3})(\sqrt{14})}$$

$\theta = 90°$

22. a. i. The given line is not parallel to the plane because (3, 0, 2) is a point on the line and the plane.

ii. Substitute the expressions for the components of the parametric equation of the line into the equation of the plane.

$$4(-3t) + (-5 + 2t) - (-10t) - 10 = 0$$
$$-12t - 5 + 2t + 10t - 10 = 0$$
$$-15 = 0$$

This last statement is never true. So the line and the plane have no points in common. Therefore, the line is parallel to the plane.

iii. Use the symmetric equation to rewrite x and z in terms of y.

$x = -4y - 23$

$z = -y - 6$

Substitute into the equation of the plane.

$$4(-4y - 23) + y - (-y - 6) - 10 = 0$$
$$-16y - 92 + y + y + 6 - 10 = 0$$
$$-14y - 96 = 0$$

This equation has a solution. Therefore, the line and plane have a point in common and are not parallel.

b. i. Substitute the expressions for the components of the parametric equation of the line into the equation of the plane.

$$4(3 + t) + (-2t) - (2 + 2t) - 10 = 0$$
$$12 + 4t - 2t - 2 - 2t - 10 = 0$$
$$0 = 0$$

This last statement is always true. So every point on the line is also in the plane. Therefore, the line lies in the plane.

ii. The line is parallel to the plane, and so does not lie in it.

iii. $(5, -7, 1)$ is a point that lies on the line that does not lie in the plane. Therefore, the line does not lie in the plane.

23. $(x, y, z) = (4, 1, 6) + p(3, -2, 1) + q(-6, 6, -1)$

$$(x, y, z) = (4, 1, 6) + 4(3, -2, 1) + 2(-6, 6, -1)$$
$$(x, y, z) = (4, 5, 8) \neq (4, 5, 6)$$

24. One direction vector for the plane is $(3, -1, 1)$. (2, 4, 1) and (1, 4, 4) are on the plane, so another direction vector is $(2, 4, 1) - (1, 4, 4) = (1, 0, -3)$. So the parametric equations are $x = 1 + s + 3t$, $y = 4 - t$, $z = 4 - 3s + t$, $s, t \in \mathbf{R}$.

25. A plane has two parameters, because a plane goes in two different directions unlike a line that only goes in one direction.

26. This equation will always pass through the origin, because you can always set $s = 0$ and $t = -1$ to obtain (0, 0, 0).

$$(x, y, z) = (a, b, c) + s(d, e, f) + t(a, b, c)$$
$$s = 0, t = -1$$
$$(x, y, z) = (a, b, c) + 0(d, e, f) - 1(a, b, c)$$
$$(x, y, z) = (a - a, b - b, c - c) = (0, 0, 0)$$

27. a. They do not form a plane, because these three points are collinear.

$\vec{r} = (-1, 2, 1) + t(3, 1, -2)$

b. They do not form a plane, because the point lies on the line.

$\vec{r} = (4, 9, -3) + t(1, -4, 2)$

$$\vec{r} = (4, 9, -3) + 4(1, -4, 2) = (8, -7, 5)$$

28. If a is the x-intercept, b is the y-intercept, and c is the z-intercept, this means that $(a, 0, 0)$, $(0, b, 0)$, and $(0, 0, c)$ are points on the plane. So

$$\vec{u} = (a, 0, 0) - (0, 0, c) = (a, 0, -c)$$
$$\vec{v} = (0, b, 0) - (0, 0, c) = (0, b, -c)$$

are direction vectors for the plane. So a normal for this plane is

$$\vec{u} \times \vec{v} = (a, 0, -c) \times (0, b, -c)$$
$$= (0(-c) - b(-c), 0(-c) - a(-c), a(b) - 0(0))$$
$$= (bc, ac, ab)$$

So the Cartesian equation of the plane is of the form

$bcx + acy + abz + D = 0$

Substitute the x-intercept, $(a, 0, 0)$, into this equation to determine the value of D.

$$bc(a) + ac(0) + ab(0) + D = 0$$
$$D = -abc$$

So the Cartesian equation of this plane is

$bcx + acy + abz - abc = 0$ or

$bcx + acy + abz = abc$

29. If the normal vector is $(6, -5, 12)$, then the Cartesian equation of the plane will be of the form

$6x - 5y + 12z + D = 0$

To determine the value of D, substitute the point $(5, 8, -3)$ (which is on the plane) into this equation.

$$6(5) - 5(8) + 12(-3) + D = 0$$
$$D = 46$$

So the Cartesian equation of the plane is

$6x - 5y + 12z + 46 = 0$.

30. a., b. $A(1, -3, 2)$, $B(-2, 4, -2)$, $C(3, 2, 1)$

$\overrightarrow{AB} = (-3, 7, -4)$

$\overrightarrow{BC} = (5, -2, 3)$

$$\vec{r} = \vec{r}_0 + t\vec{a} + s\vec{b}, t, s \in \mathbf{R}$$
$$\vec{r} = (1, -3, 2) + t(-3, 7, -4) + s(5, -2, 3), t, s \in \mathbf{R}$$
$$x = 1 - 3t + 5s, y = -3 + 7t - 2s, z = 2 - 4t + 3s$$

c. To find the Cartesian equation of the plane, a normal vector is needed. This can be found by computing the cross product of the direction vectors found in parts a. and b.

$$\begin{aligned}\overrightarrow{AB} \times \overrightarrow{BC} &= (-3, 7, -4) \times (5, -2, 3)\\ &= (7(3) - (-2)(-4), 5(-4)\\ &\quad - (-3)(3), (-3)(-2) - 5(7))\\ &= (13, -11, -29)\end{aligned}$$

So the Cartesian equation has the form
$13x - 11y - 29z + D = 0.$
Since $(1, -3, 2)$ is a point on this plane, we can substitute it in to determine the value of D.

$$\begin{aligned}13(1) - 11(-3) - 29(2) + D &= 0\\ D &= 12\end{aligned}$$

So the Cartesian equation of this plane is
$13x - 11y - 29z + 12 = 0.$

d. Substituting $(3, 5, -4)$ into the Cartesian equation found in part **c.**, we get
$13(3) - 11(5) - 29(-4) + 12 = 100 \neq 0$
This means that $(3, 5, -4)$ is not on the plane.

31. a. The normal vector to the given plane is $(4, -2, 5)$, so any plane parallel to this one must have this same normal vector. So if a parallel plane contains the point (0, 0, 0), it will have the form
$4x - 2y + 5z + D = 0.$
Substitute in the point (0, 0, 0) to determine the value of D.

$$\begin{aligned}4(0) - 2(0) + 5(0) + D &= 0\\ D &= 0\end{aligned}$$

So the Cartesian equation of this plane is
$4x - 2y + 5z = 0.$

b. Reasoning as in part **a.**, if we want the point $(-1, 5, -1)$ to be in our parallel plane we find D in the following way:

$$\begin{aligned}4(-1) - 2(5) + 5(-1) + D &= 0\\ D &= 19\end{aligned}$$

So the Cartesian equation of the plane in this case is
$4x - 2y + 5z + 19 = 0.$

c. Reasoning as in parts a. and b., if we want the point $(2, -2, 2)$ to be in our parallel plane we find D in the following way:

$$\begin{aligned}4(2) - 2(-2) + 5(2) + D &= 0\\ D &= -22\end{aligned}$$

So the Cartesian equation of the plane in this case is
$4x - 2y + 5z - 22 = 0.$

32. a. The direction vector for L_1 is (2, 1) and for L_2 is $(-2, -1) = -1(2, 1)$. This means that L_1 and L_2 are parallel, and since they have the point (11, 0) in common (take $t = 3$ in L_1 and $s = 6$ in L_2), these lines are coincident. So the angle between them is $\theta = 0°$.

b. The parametric equations of these lines are
L_1: $x = -3 + 3t$, $y = -1 + 4t$, $t \in \mathbf{R}$
L_2: $x = 6 + 3s$, $y = 2 - 2s$, $s \in \mathbf{R}$
So a point of intersection satisfies
$-3 + 3t = 6 + 3s$
$-1 + 4t = 2 - 2s$
or
$3t - 3s = 9$
$4t - 2s = 3$
or

$$\begin{aligned}t &= s + 3\\ 4t + 2s &= 3\\ 4(s + 3) + 2s &= 3\\ 6s &= -9\\ s &= -\frac{3}{2}\\ t &= s + 3\\ &= -\frac{3}{2} + 3\\ &= \frac{3}{2}\end{aligned}$$

So the point of intersection is

$$\begin{aligned}x &= -3 + 3\left(\frac{3}{2}\right)\\ &= \frac{3}{2}\\ y &= -1 + 4\left(\frac{3}{2}\right)\\ &= 5\end{aligned}$$

The point of intersection is $\left(\frac{3}{2}, 5\right)$ at $s = -\frac{3}{2}$ (for L_2) and $t = \frac{3}{2}$ (for L_1).
The direction vector for L_1 is (3, 4), and for L_2 is $(3, -2)$. So the angle θ between these lines satisfies

$$\begin{aligned}\cos\theta &= \frac{(3, 4) \cdot (3, -2)}{|(3, 4)||(3, -2)|}\\ \theta &= \cos^{-1}\left(\frac{(3, 4) \cdot (3, -2)}{|(3, 4)||(3, -2)|}\right)\\ &= \cos^{-1}\left(\frac{1}{5\sqrt{3}}\right)\\ &\doteq 86.82°\end{aligned}$$

It would also have been correct to report the supplement of this angle, or roughly 93.18°, as the answer in this case.

33. a. $P(1, 3, 5)$

$$\begin{aligned}\vec{r} &= \vec{r}_0 + t\vec{a}\\ \vec{r} &= (1, 3, 5) + t(-2, -4, -10), t \in \mathbf{R}\end{aligned}$$

$x = 1 - 2t, y = 3 - 4t, z = 5 - 10t$

$\frac{x-1}{-2} = \frac{y-3}{-4} = \frac{z-5}{-10}$

b. $P(1, 3, 5), Q(-7, 9, 3)$

$\overrightarrow{PQ} = (-8, 6, -2)$

$\vec{r} = \vec{r}_0 + t\vec{a}$

$\vec{r} = (1, 3, 5) + t(-8, 6, -2), t \in \mathbf{R}$

$x = 1 - 8t, y = 3 + 6t, z = 5 - 2t$

$\frac{x-1}{-8} = \frac{x-3}{6} = \frac{x-5}{-2}$

c. $P(1, 3, 5)$

$\overrightarrow{RS} = (-6, -13, 14)$

$\vec{r} = \vec{r}_0 + t\vec{a}$

$\vec{r} = (1, 3, 5) + t(-6, -13, 14), t \in \mathbf{R}$

$x = 1 - 6t, y = 3 - 13t, z = 5 + 14t$

$\frac{x-1}{-6} = \frac{x-3}{-13} = \frac{x-5}{14}$

d. Since its parallel to the x-axis, its direction vector is (1, 0, 0).

$P(1, 3, 5),$

$\vec{n} = (1, 0, 0)$

$\vec{r} = \vec{r}_0 + t\vec{a}$

$\vec{r} = (1, 3, 5) + t(1, 0, 0), t \in \mathbf{R}$

$x = 1 + t, y = 3, z = 5$

e. Find a perpendicular vector use the dot product.

$(-3, 4, -6) \cdot (a, b, c) = 0$

$-3a + 4b - 6c + 0$

$a = 0, b = 6, c = 4$

$\vec{n} = (0, 6, 4)$

$\vec{r} = (1, 3, 5) + t(0, 6, 4), t \in \mathbf{R}$

f. Since the line is perpendicular to the plane, the line's directional vector is the normal of the plane. Use the cross product to find the vector.

$A(4, 2, 1), B(3, -4, 2), C(-3, 2, 1)$

$\overrightarrow{AB} = (-1, -6, 1)$

$\overrightarrow{BC} = (-6, 6, -1)$

$\overrightarrow{AB} \times \overrightarrow{BC} = (0, -7, -42) = (0, 1, 6) = \vec{n}$

$\vec{r} = (1, 3, 5) + t(0, 1, 6)$

$x = 1, y = 3 + t, z = 5 + 6t$

34. a. This plane will be of the form $2x - 4y + 5z + D = 0.$

To find D, substitute in $P(-2, 6, 1)$.

$2(-2) - 4(6) + 5(1) + D = 0$

$D = 23$

So the Cartesian equation of the plane is $2x - 4y + 5z + 23 = 0.$

b. The direction vector for this line is $(3, -5, 2)$ (we can use this as one of the direction vectors for the plane), and a point on this line is $(4, -2, 1)$. So a second direction vector for the plane will be

$\vec{v} = (4, -2, 1) - P(-2, 0, 6)$

$= (6, -2, -5)$

So a normal vector for this plane is

$$\begin{aligned}(3, -5, 2) \times (6, -2, -5) &= ((-5)(-5) \\ &\quad - (-2)(2), 6(2) \\ &\quad - 3(-5), 3(-2) \\ &\quad - 6(-5)) \\ &= (29, 27, 24)\end{aligned}$$

The Cartesian equation of this plane has the form $29x + 27y + 24z + D = 0.$

Substitute in $P(-2, 0, 6)$ to determine D.

$29(-2) + 27(0) + 24(6) + D = 0$

$D = -86$

The Cartesian equation of this plane is $29x + 27y + 24z - 86 = 0.$

c. This plane, being parallel to the xy-plane, is completely determined by a fixed z-coordinate (the x- and y- coordinates are allowed to be anything at all). Since it passes through the point $P(3, 3, 3)$, the equation of this plane is $z = 3$. Written in Cartesian form, this is $z - 3 = 0$.

d. Since this plane is to be parallel to $3x + y - 4z + 8 = 0$, it will have the same normal vector, $(3, 1, -4)$. So this plane will be of the form $3x + y - 4z + D = 0$.

Since $P(-4, 2, 4)$ is on this plane, we can substitute this in to determine the value of D.

$3(-4) + 2 - 4(4) + D = 0$

$D = 26$

So the Cartesian equation of this plane is $3x + y - 4z + 26 = 0.$

e. Since this plane is perpendicular to the yz-plane, it is completely determined by its intersection with the yz-plane, which will be a line with y-intercept 4 and z-intercept -2. This means that y and z are related by $y = mz + 4$ because of the y-intercept of 4. We can find the value of m by using the z-intercept of -2.

$0 = m(-2) + 4$

$m = 2$

So y and z are related via $y = 2z + 4$, and the Cartesian equation of the plane is $y - 2z - 4 = 0$. (x is allowed to be anything here.)

f. A normal vector, (A, B, C), for this plane will be perpendicular to the normal vector for the plane

$x - 2y + z = 6$, which is $(1, -2, 1)$. Also, (A, B, C) will be perpendicular to the direction vector for the line contained in the plane we seek. This direction vector is (3, 1, 2), and so this means we can take

$$\begin{aligned}(A, B, C) &= (1, -2, 1) \times (3, 1, 2) = ((-2)(2) \\ &\quad - (1)(1), 3(1) - 1(2), 1(1) - 3(-2)) \\ &= (-5, 1, 7)\end{aligned}$$

So the Cartesian equation will have the form $-5x + y + 7z + D = 0$.

Since is on this plane (take in the line this plane is to contain), we can substitute this in to determine the value of D.

$$\begin{aligned}-5(2) + (-1) + 7(-1) + D &= 0 \\ D &= 18\end{aligned}$$

So the Cartesian equation of this plane is $-5x + y + 7z + 18 = 0$.

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1. a. i $\overrightarrow{AB}$ and $\overrightarrow{AC}$ can be the direction vectors for this plane and $A(1, 2, 4)$ can be the origin point.

$$\begin{aligned}\overrightarrow{AB} &= (2, 0, 3) - (1, 2, 4) \\ &= (1, -2, -1)\end{aligned}$$

$$\begin{aligned}\overrightarrow{AC} &= (4, 4, 4) - (1, 2, 4) \\ &= (3, 2, 0)\end{aligned}$$

This gives a vector equation of $\vec{r} = (1, 2, 4) + s(1, -2, -1) + t(3, 2, 0), s, t \in \mathbf{R}$.

The corresponding parametric equation for this plane is $x = 1 + s + 3t$, $y = 2 - 2s + 2t$, $z = 4 - s$, $s, t \in \mathbf{R}$.

ii. The corresponding Cartesian equation is found by taking the cross product of the two direction vectors.

$$\begin{aligned}\overrightarrow{AB} \times \overrightarrow{AC} &= ((-2)0 - (-1)2, (-1)3 \\ &\quad - (1)0, (1)2 - (-2)3) \\ &= (2, -3, 8)\end{aligned}$$

So $(2, -3, 8)$ is a normal vector for the plane, so the plane has the form $2x - 3y + 8z + D = 0$, for some constant D. To find D, we know that A(1, 2, 4) is a point on the plane, so

$2(1) - 3(2) + 8(4) + D = 0$. So $28 + D = 0$, or $D = -28$. So the Cartesian equation for the plane is $2x - 3y + 8z - 28 = 0$.

b. A point (x, y, z) is on the plane if and only if $2x - 3y + 8z - 28 = 0$. Since $2(1) - 3(-1) + 8\left(-\frac{1}{2}\right) - 28 = -27 \neq 0$, the point $\left(1, -1, -\frac{1}{2}\right)$ is not on the plane.

2. a. Since $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ for all (x, y, z) on the plane, it holds true for the given points. So $\frac{2}{a} + \frac{0}{b} + \frac{0}{c} = 1$ or $a = 2$. Similarly $\frac{0}{a} + \frac{3}{b} + \frac{0}{c} = 1$ and $\frac{0}{a} + \frac{0}{b} + \frac{4}{c} = 1$ implies that $b = 3$ and $c = 4$. So the equation of the plane is $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$.

b. If both sides are multiplied by the least common multiple of the denominators, then an equivalent equation for the plane is $6x + 4y + 3z = 12$. Hence (6, 4, 3) is a normal vector for this plane.

3. a. Since the origin is a point on the plane and $(2, 1, 3) + 0(1, 2, 5) = (2, 1, 3)$ is a point on the plane, (2, 1, 3) is a direction vector for the plane. $(2, 1, 3) + 1(1, 2, 5) = (3, 3, 8)$ is a point on the plane and (2, 1, 3) is another point on the plane, so $(3, 3, 8) - (2, 1, 3) = (1, 2, 5)$ is a directional vector for the plane as well. (2, 1, 3) and (1, 2, 5) are not collinear, because the ratios between the coordinates are not equal. Since the origin is a point on the plane, a vector equation for the plane is $\vec{r} = s(2, 1, 3) + t(1, 2, 5), s, t \in \mathbf{R}$.

b. To find the Cartesian equation for the plane, the normal vector is determined by the cross product of the two direction vectors from part **a**.

$$\begin{aligned}(2, 1, 3) \times (1, 2, 5) &= ((1)5 - (3)2, (3)1 \\ &\quad - (2)5, (2)2 - (1)1) \\ &= (-1, -7, 3)\end{aligned}$$

So the Cartesian equation for the plane has the form $-x - 7y + 3z + D = 0$, for some constant D. Since the origin is a point on the plane, $-(0) - 7(0) + 3(0) + D = 0$, so $D = 0$. Thus the equation is $-x - 7y + 3z = 0$.

4. a. $(2, 0, -3)$ and $(5, 1, -1)$ are each direction vectors for the planes. The vectors are not collinear since the ratios of the coordinates are not equal. $(4, -3, 5)$ is a point on the plane, so a vector equation for the plane is $\vec{r} = (4, -3, 5) + s(2, 0, -3) + t(5, 1, -1), s, t \in \mathbf{R}$.

b. To find the Cartesian equation for the plane, the normal vector is determined by the cross product of the two direction vectors from part **a**.

$$\begin{aligned}(2, 0, -3) \times (5, 1, -1) &= ((0)(-1) \\ &\quad - (-3)1, (-3)5 \\ &\quad - (2)(-1), (2)1 - (0)5) \\ &= (3, -13, 2)\end{aligned}$$

So the Cartesian equation for the plane has the form $3x - 13y + 2z + D = 0$, for some constant D.

Since the $(4, -3, 5)$ is a point on the plane,
$3(4) - 13(-3) + 2(5) + D = 0$, so
$61 + D = 0$. So $D = -61$. Thus the equation is
$3x - 13y + 2z - 61 = 0$.

5. a. The line intersects the yz-plane when $x = 0$.
If $x = 0$, then $\frac{y - 4}{-2} = z = \frac{0 - 2}{4} = -\frac{1}{2}$, so $y = \left(-\frac{1}{2}\right)(-2) + 4 = 5$ and $z = -\frac{1}{2}$. Thus the point is $\left(0, 5, -\frac{1}{2}\right)$.

b. The direction vector $(4, -2, 1)$ is the same, so the equivalent symmetric equation for the line is
$\frac{x}{4} = \frac{y - 5}{-2} = z + \frac{1}{2}$.

6. a. The angle between two planes is determined by the dot product of their normal vectors. The normal vector of the first plane is $(1, 1, -1)$ and the normal vector of the second plane is $(1, -1, 1)$.
$(1, 1, -1) \cdot (1, -1, 1) = -1$ and
$|(1, 1, -1)| = \sqrt{3}$. So the angle between the planes is $\cos^{-1}\left(\frac{-1}{\sqrt{3}\sqrt{3}}\right) \doteq 109.5°$ The acute angle is $70.5°$.

b. i. The planes are parallel if and only if the corresponding normal vectors are parallel. The normal vector of the first plane is $(2, -1, k)$ and the normal vector of the second plane is $(k, -2, 8)$. The vectors are parallel if and only if the ratios between the coordinates are equal. Suppose $\frac{k}{2} = \frac{-2}{-1} = 2$, so then $k = 4$. So the vectors can be parallel only when $k = 4$. Since $\frac{8}{4} = 2$ as well, the vectors are parallel at $k = 4$.

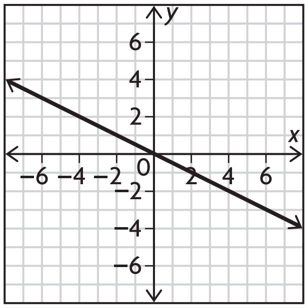
ii. The planes are perpendicular when their normal vectors are perpendicular. The vectors are perpendicular when their dot product is equal to zero.

$$\begin{aligned}(2, -1, k) \cdot (k, -2, 8) &= 2k - 1(-2) + 8k \\ &= 10k + 2\end{aligned}$$

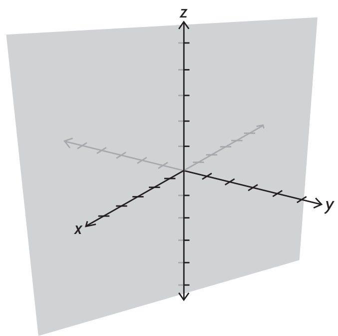
So if $k = -\frac{1}{5}$, then the dot product of the two normal vectors is equal to zero. Hence the planes are perpendicular at $k = -\frac{1}{5}$.

c. The first plane in **b**. intersects the y-axis at the point $(0, d, 0)$, where d satisfies $2(0) - d + k(0) = 5$. So $d = -5$. The second plane in **b**. intersects the y-axis at the point $(0, e, 0)$, where e satisfies $k(0) - 2e + 8(0) = 9$. So $e = -4.5$. Since the planes intersect the y-axis only once and the points are different, the equations can never represent the same plane.

7. a.



b. The equation for the plane can be written as $2x + y + 0z = 0$. So for any real number t, $2(0) + \ (0) + 0(t) = 0$, so the point $(0, 0, t)$ is on the graph. So the z-axis is on the plane. Also the plane cuts across the xy-plane along the line $2x + y = 0$. So the origin is a point, as well as $(-2, 1, 0)$.



c. The equation for the plane can be written as $Ax + By + 0z = 0$. For any real number t, $A(0) + B(0) + 0(t) = 0$, so $(0, 0, t)$ is on the plane. Since this is true for all real numbers, the z-axis is on the plane.