

CHAPTER 2

Derivatives

Review of Prerequisite Skills, pp. 62–63

1. a. $a^5 \times a^3 = a^{5+3}$
 $= a^8$

b. $(-2a^2)^3 = (-2)^3(a^2)^3$
 $= -8(a^{2 \times 3})$
 $= -8a^6$

c. $\frac{4p^7 \times 6p^9}{12p^{15}} = \frac{24p^{7+9}}{12p^{15}}$
 $= 2p^{16-15}$
 $= 2p$

d. $(a^4b^{-5})(a^{-6}b^{-2}) = (a^{4-6})(b^{-5-2})$
 $= a^{-2}b^{-7}$
 $= \frac{1}{a^2b^7}$

e. $(3e^6)(2e^3)^4 = (3)(e^6)(2^4)(e^3)^4$
 $= (3)(2^4)(e^6)(e^{3 \times 4})$
 $= (3)(16)(e^{6+12})$
 $= 48e^{18}$

f. $\frac{(3a^{-4})[2a^3(-b)^3]}{12a^5b^2} = \frac{(3)(2)(-1)^3(a^{-4+3})(b^3)}{12a^5b^2}$
 $= \frac{-6(a^{-1-5})(b^{3-2})}{12}$
 $= \frac{-1(a^{-6})(b)}{2}$
 $= -\frac{b}{2a^6}$

2. a. $(x^{\frac{1}{2}})(x^{\frac{2}{3}}) = x^{\frac{1}{2} + \frac{2}{3}}$
 $= x^{\frac{7}{6}}$

b. $(8x^6)^{\frac{2}{3}} = 8^{\frac{2}{3}}x^{6 \times \frac{2}{3}}$
 $= 4x^4$

c. $\frac{\sqrt{a}\sqrt[3]{a}}{\sqrt{a}} = \frac{(a^{\frac{1}{2}})(a^{\frac{1}{3}})}{a^{\frac{1}{2}}}$
 $= a^{\frac{1}{3}}$

3. A perpendicular line will have a slope that is the negative reciprocal of the slope of the given line:

a. slope = $\frac{-1}{\frac{3}{2}}$
 $= -\frac{3}{2}$

b. slope = $\frac{-1}{-\frac{1}{2}}$
 $= 2$

c. slope = $\frac{-1}{\frac{5}{3}}$
 $= -\frac{3}{5}$

d. slope = $\frac{-1}{-1}$
 $= 1$

4. a. This line has slope $m = \frac{-4 - (-2)}{-3 - 9}$
 $= \frac{-2}{-12}$
 $= \frac{1}{6}$

The equation of the desired line is therefore
 $y + 4 = \frac{1}{6}(x + 3)$ or $x - 6y - 21 = 0$.

b. The equation $3x - 2y = 5$ can be rewritten as
 $2y = 3x - 5$ or $y = \frac{3}{2}x - \frac{5}{2}$, which has slope $\frac{3}{2}$.

The equation of the desired line is therefore
 $y + 5 = \frac{3}{2}(x + 2)$ or $3x - 2y - 4 = 0$.

c. The line perpendicular to $y = \frac{3}{4}x - 6$ will have

slope $m = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$. The equation of the desired line

is therefore $y + 3 = -\frac{4}{3}(x - 4)$ or $4x + 3y - 7 = 0$.

5. a. $(x - 3y)(2x + y) = 2x^2 + xy - 6xy - 3y^2$
 $= 2x^2 - 5xy - 3y^2$

b. $(x - 2)(x^2 - 3x + 4)$
 $= x^3 - 3x^2 + 4x - 2x^2 + 6x - 8$
 $= x^3 - 5x^2 + 10x - 8$

c. $(6x - 3)(2x + 7) = 12x^2 + 42x - 6x - 21$
 $= 12x^2 + 36x - 21$

d. $2(x + y) - 5(3x - 8y) = 2x + 2y - 15x + 40y$
 $= -13x + 42y$

e. $(2x - 3y)^2 + (5x + y)^2$
 $= 4x^2 - 12xy + 9y^2 + 25x^2 + 10xy + y^2$
 $= 29x^2 - 2xy + 10y^2$

f. $3x(2x - y)^2 - x(5x - y)(5x + y)$
 $= 3x(4x^2 - 4xy + y^2) - x(25x^2 - y^2)$
 $= 12x^3 - 12x^2y + 3xy^2 - 25x^3 + xy^2$
 $= -13x^3 - 12x^2y + 4xy^2$

6. a.
$$\frac{3x(x+2)}{x^2} \times \frac{5x^3}{2x(x+2)} = \frac{15x^4(x+2)}{2x^3(x+2)}$$

$$= \frac{15}{2}x^{4-3}$$

$$= \frac{15}{2}x$$

$x \neq 0, -2$

b.
$$\frac{y}{(y+2)(y-5)} \times \frac{(y-5)^2}{4y^3}$$

$$= \frac{y(y-5)(y-5)}{4y^3(y+2)(y-5)}$$

$$= \frac{y-5}{4y^2(y+2)}$$

$y \neq -2, 0, 5$

c.
$$\frac{4}{h+k} \div \frac{9}{2(h+k)} = \frac{4}{h+k} \times \frac{2(h+k)}{9}$$

$$= \frac{8(h+k)}{9(h+k)}$$

$$= \frac{8}{9}$$

$h \neq -k$

d.
$$\frac{(x+y)(x-y)}{5(x-y)} \div \frac{(x+y)^3}{10}$$

$$= \frac{(x+y)(x-y)}{5(x-y)} \times \frac{10}{(x+y)^3}$$

$$= \frac{10(x+y)(x-y)}{5(x-y)(x+y)^3}$$

$$= \frac{2}{(x+y)^2}$$

$x \neq -y, +y$

e.
$$\frac{x-7}{2x} + \frac{5x}{x-1} = \frac{(x-7)(x-1)}{2x(x-1)} + \frac{(5x)(2x)}{2x(x-1)}$$

$$= \frac{x^2 - 7x - x + 7 + 10x^2}{2x(x-1)}$$

$$= \frac{11x^2 - 8x + 7}{2x(x-1)}$$

$x \neq 0, 1$

f.
$$\frac{x+1}{x-2} - \frac{x+2}{x+3}$$

$$= \frac{(x+1)(x+3)}{(x-2)(x+3)} - \frac{(x+2)(x-2)}{(x+3)(x-2)}$$

$$= \frac{x^2 + x + 3x + 3 - x^2 + 4}{(x+3)(x-2)}$$

$$= \frac{(x+3)(x-2)}{4x+7}$$

$$= \frac{4x+7}{(x+3)(x-2)}$$

$x \neq -3, 2$

7. a. $4k^2 - 9 = (2k+3)(2k-3)$

b. $x^2 + 4x - 32 = x^2 + 8x - 4x - 32$
 $= x(x+8) - 4(x+8)$
 $= (x-4)(x+8)$

c. $3a^2 - 4a - 7 = 3a^2 - 7a + 3a - 7$
 $= a(3a-7) + 1(3a-7)$
 $= (a+1)(3a-7)$

d. $x^4 - 1 = (x^2 + 1)(x^2 - 1)$
 $= (x^2 + 1)(x + 1)(x - 1)$

e. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

f. $r^4 - 5r^2 + 4 = r^4 - 4r^2 - r^2 + 4$

$$= r^2(r^2 - 4) - 1(r^2 - 4)$$

$$= (r^2 - 1)(r^2 - 4)$$

$$= (r+1)(r-1)(r+2)(r-2)$$

8. a. Letting $f(a) = a^3 - b^3$, $f(b) = b^3 - b^3$

$$= 0$$

So b is a root of $f(a)$, and so by the factor theorem, $a - b$ is a factor of $a^3 - b^3$. Polynomial long division provides the other factor:

$$\begin{array}{r} a^2 + ab + b^2 \\ a - b \overline{) a^3 + 0a^2 + 0a - b^3} \\ \underline{a^3 - a^2b} \\ a^2b + 0a - b^3 \\ \underline{a^2b - ab^2} \\ ab^2 - b^3 \\ \underline{ab^2 - b^3} \\ 0 \end{array}$$

So $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

b. Using long division or recognizing a pattern from the work in part a.:

$$a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4).$$

c. Using long division or recognizing a pattern from the work in part a.: $a^7 - b^7$

$$= (a-b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6).$$

d. Using the pattern from the previous parts:

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + a^2b^{n-3} + ab^{n-2} + b^{n-1}).$$

9. a. $f(2) = -2(2^4) + 3(2^2) + 7 - 2(2)$
 $= -32 + 12 + 7 - 4$
 $= -17$

b. $f(-1) = -2(-1)^4 + 3(-1)^2 + 7 - 2(-1)$
 $= -2 + 3 + 7 + 2$
 $= 10$

c. $f\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^4 + 3\left(\frac{1}{2}\right)^2 + 7 - 2\left(\frac{1}{2}\right)$
 $= -\frac{1}{8} + \frac{3}{4} + 7 - 1$
 $= \frac{53}{8}$

$$\begin{aligned}
\text{d. } f(-0.25) &= f\left(-\frac{1}{4}\right) \\
&= 2\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^2 + 7 - 2\left(-\frac{1}{4}\right) \\
&= -\frac{1}{128} + \frac{3}{16} + 7 + \frac{1}{2} \\
&= \frac{983}{128} \\
&\doteq 7.68
\end{aligned}$$

$$\begin{aligned}
\text{10. a. } \frac{3}{\sqrt{2}} &= \frac{3\sqrt{2}}{(\sqrt{2})(\sqrt{2})} \\
&= \frac{3\sqrt{2}}{2}
\end{aligned}$$

$$\begin{aligned}
\text{b. } \frac{4 - \sqrt{2}}{\sqrt{3}} &= \frac{(4 - \sqrt{2})(\sqrt{3})}{(\sqrt{3})(\sqrt{3})} \\
&= \frac{4\sqrt{3} - \sqrt{6}}{3}
\end{aligned}$$

$$\begin{aligned}
\text{c. } \frac{2 + 3\sqrt{2}}{3 - 4\sqrt{2}} &= \frac{(2 + 3\sqrt{2})(3 + 4\sqrt{2})}{(3 - 4\sqrt{2})(3 + 4\sqrt{2})} \\
&= \frac{6 + 9\sqrt{2} + 8\sqrt{2} + 12(2)}{3^2 - (4\sqrt{2})^2} \\
&= \frac{30 + 17\sqrt{2}}{9 - 16(2)} \\
&= -\frac{30 + 17\sqrt{2}}{23}
\end{aligned}$$

$$\begin{aligned}
\text{d. } \frac{3\sqrt{2} - 4\sqrt{3}}{3\sqrt{2} + 4\sqrt{3}} &= \frac{(3\sqrt{2} - 4\sqrt{3})(3\sqrt{2} - 4\sqrt{3})}{(3\sqrt{2} + 4\sqrt{3})(3\sqrt{2} - 4\sqrt{3})} \\
&= \frac{(3\sqrt{2})^2 - 24\sqrt{6} + (4\sqrt{3})^2}{(3\sqrt{2})^2 - (4\sqrt{3})^2} \\
&= \frac{9(2) - 24\sqrt{6} + 16(3)}{9(2) - 16(3)} \\
&= -\frac{66 - 24\sqrt{6}}{30} \\
&= -\frac{11 - 4\sqrt{6}}{5}
\end{aligned}$$

$$\text{11. a. } f(x) = 3x^2 - 2x$$

When $a = 2$,

$$\begin{aligned}
\frac{f(a+h) - f(a)}{h} &= \frac{f(2+h) - f(2)}{h} \\
&= \frac{3(2+h)^2 - 2(2+h) - [3(2)^2 - 2(2)]}{h} \\
&= \frac{3(4+4h+h^2) - 4 - 2h - 8}{h} \\
&= \frac{12+12h+3h^2-2h-12}{h}
\end{aligned}$$

$$\begin{aligned}
&= \frac{3h^2 + 10h}{h} \\
&= 3h + 10
\end{aligned}$$

This expression can be used to determine the slope of the secant line between $(2, 8)$ and $(2+h, f(2+h))$.

b. For $h = 0.01$: $3(0.01) + 10 = 10.03$

c. The value in part b. represents the slope of the secant line through $(2, 8)$ and $(2.01, 8.1003)$.

2.1 The Derivative Function, pp. 73–75

1. A function is not differentiable at a point where its graph has a cusp, a discontinuity, or a vertical tangent:

a. The graph has a cusp at $x = -2$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq -2\}$.

b. The graph is discontinuous at $x = 2$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq 2\}$.

c. The graph has no cusps, discontinuities, or vertical tangents, so f is differentiable on $\{x \in \mathbf{R}\}$.

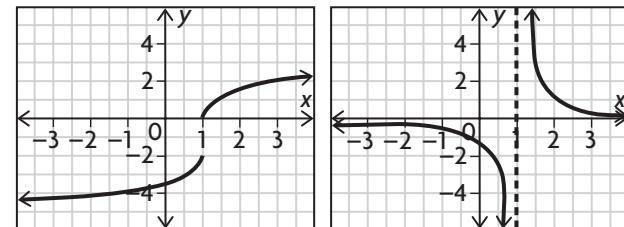
d. The graph has a cusp at $x = 1$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq 1\}$.

e. The graph has no cusps, discontinuities, or vertical tangents, so f is differentiable on $\{x \in \mathbf{R}\}$.

f. The function does not exist for $x < 2$, but has no cusps, discontinuities, or vertical tangents elsewhere, so f is differentiable on $\{x \in \mathbf{R} \mid x > 2\}$.

2. The derivative of a function represents the slope of the tangent line at a given value of the independent variable or the instantaneous rate of change of the function at a given value of the independent variable.

3.



$$\begin{aligned}
\text{4. a. } f(x) &= 5x - 2 \\
f(a+h) &= 5(a+h) - 2 \\
&= 5a + 5h - 2
\end{aligned}$$

$$\begin{aligned}
f(a+h) - f(a) &= 5a + 5h - 2 - (5a - 2) \\
&= 5h
\end{aligned}$$

$$\begin{aligned}
\text{b. } f(x) &= x^2 + 3x - 1 \\
f(a+h) &= (a+h)^2 + 3(a+h) - 1 \\
&= a^2 + 2ah + h^2 + 3a + 3h \\
&\quad + 3h - 1 \\
f(a+h) - f(a) &= a^2 + 2ah + h^2 + 3a + 3h \\
&\quad - 1 - (a^2 + 3a - 1) \\
&= 2ah + h^2 + 3h
\end{aligned}$$

c. $f(x) = x^3 - 4x + 1$
 $f(a + h) = (a + h)^3 - 4(a + h) + 1$
 $= a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h + 1$

$$f(a + h) - f(a) = a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h + 1 - (a^3 - 4a + 1)$$
 $= 3a^2h + 3ah^2 + h^3 - 4h$

d. $f(x) = x^2 + x - 6$
 $f(a + h) = (a + h)^2 + (a + h) - 6$
 $= a^2 + 2ah + h^2 + a + h - 6$
 $f(a + h) - f(a) = a^2 + 2ah + h^2 + a + h - 6 - (a^2 + a - 6)$
 $= 2ah + h^2 + h$

e. $f(x) = -7x + 4$
 $f(a + h) = -7(a + h) + 4$
 $= -7a - 7h + 4$
 $f(a + h) - f(a) = -7a - 7h + 4 - (-7a + 4)$
 $= -7h$

f. $f(x) = 4 - 2x - x^2$
 $f(a + h) = 4 - 2(a + h) - (a + h)^2$
 $= 4 - 2a - 2h - a^2 - 2ah - h^2$
 $f(a + h) - f(a) = 4 - 2a - 2h - a^2 - 2ah - h^2 - 4 + 2a + a^2$
 $= -2h - h^2 - 2ah$

5. a. $f'(1) = \lim_{h \rightarrow 0} \frac{f(1 + h) - f(1)}{h}$
 $= \lim_{h \rightarrow 0} \frac{(1 + h)^2 - 1^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 - 1}{h}$
 $= \lim_{h \rightarrow 0} \frac{2h + h^2}{h}$
 $= \lim_{h \rightarrow 0} (2 + h)$
 $= 2$

b. $f'(3) = \lim_{h \rightarrow 0} \frac{f(3 + h) - f(3)}{h}$
 $= \lim_{h \rightarrow 0} \left[\frac{(3 + h)^2 + 3(3 + h) + 1}{h} - \frac{(3^2 + 3(3) + 1)}{h} \right]$
 $= \lim_{h \rightarrow 0} \frac{9 + 6h + h^2 + 9 + 3h + 1 - 19}{h}$
 $= \lim_{h \rightarrow 0} \frac{9h + h^2}{h}$
 $= \lim_{h \rightarrow 0} (9 + h)$
 $= 9$

c. $f'(0) = \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{h}$
 $= \lim_{h \rightarrow 0} \frac{\sqrt{h + 1} - \sqrt{0 + 1}}{h}$
 $= \lim_{h \rightarrow 0} \frac{\sqrt{h + 1} - 1}{h}$
 $= \lim_{h \rightarrow 0} \frac{(\sqrt{h + 1} - 1)(\sqrt{h + 1} + 1)}{h(\sqrt{h + 1} + 1)}$
 $= \lim_{h \rightarrow 0} \frac{(\sqrt{h + 1})^2 - 1}{h(\sqrt{h + 1} + 1)}$
 $= \lim_{h \rightarrow 0} \frac{h + 1 - 1}{h(\sqrt{h + 1} + 1)}$
 $= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{h + 1} + 1)}$
 $= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{1} + 1)}$
 $= \frac{1}{2}$

d. $f'(-1) = \lim_{h \rightarrow 0} \frac{f(-1 + h) - f(-1)}{h}$
 $= \lim_{h \rightarrow 0} \frac{\frac{5}{-1 + h} - \frac{5}{-1}}{h}$
 $= \lim_{h \rightarrow 0} \frac{\frac{5}{-1 + h} + 5}{h}$
 $= \lim_{h \rightarrow 0} \frac{\frac{5}{-1 + h} + \frac{5(-1 + h)}{-1 + h}}{h}$
 $= \lim_{h \rightarrow 0} \frac{5 - 5 + 5h}{h(-1 + h)}$
 $= \lim_{h \rightarrow 0} \frac{5h}{h(-1 + h)}$
 $= \lim_{h \rightarrow 0} \frac{5}{(-1 + h)}$
 $= \frac{5}{-1}$
 $= -5$

6. a. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$
 $= \lim_{h \rightarrow 0} \frac{-5(x + h) - 8 - (-5x - 8)}{h}$
 $= \lim_{h \rightarrow 0} \frac{-5x - 5h - 8 + 5x + 8}{h}$

$$= \lim_{h \rightarrow 0} \frac{-5h}{h}$$

$$= \lim_{h \rightarrow 0} -5$$

$$= -5$$

b. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \left[\frac{2(x+h)^2 + 4(x+h)}{h} - \frac{(2x^2 + 4x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{2x^2 + 4xh + 2h^2 + 4x}{h} + \frac{4h - 2x^2 - 4x}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 + 4h}{h}$$

$$= \lim_{h \rightarrow 0} (4x + 2h + 4)$$

$$= 4x + 4$$

c. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \left[\frac{6(x+h)^3 - 7(x+h)}{h} - \frac{(6x^3 - 7x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{6x^3 + 18x^2h + 18xh^2 + 6h^3}{h} + \frac{-7x - 7h - 6x^3 + 7x}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{18x^2h + 18xh^2 + 6h^3 - 7h}{h}$$

$$= \lim_{h \rightarrow 0} (18x^2 + 18xh + 6h^2 - 7)$$

$$= 18x^2 - 7$$

d. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3(x+h)+2} - \sqrt{3x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3x+3h+2} - \sqrt{3x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{(\sqrt{3x+3h+2} - \sqrt{3x+2})}{h} \times \frac{(\sqrt{3x+3h+2} + \sqrt{3x+2})}{(\sqrt{3x+3h+2} + \sqrt{3x+2})} \right]$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{3x+3h+2})^2 - (\sqrt{3x+2})^2}{h(\sqrt{3x+3h+2} + \sqrt{3x+2})}$$

$$= \lim_{h \rightarrow 0} \frac{3x+3h+2 - 3x-2}{h(\sqrt{3x+3h+2} + \sqrt{3x+2})}$$

$$= \lim_{h \rightarrow 0} \frac{3}{\sqrt{3x+3h+2} + \sqrt{3x+2}}$$

$$= \frac{3}{2\sqrt{3x+2}}$$

7. a. Let $y = f(x)$, then

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6 - 7(x+h) - (6 - 7x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6 - 7x - 7h - 6 + 7x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-7h}{h}$$

$$= \lim_{h \rightarrow 0} -7$$

$$= -7$$

b. Let $y = f(x)$, then

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{(x+h+1)(x-1)}{(x+h-1)(x-1)}}{h} - \frac{\frac{(x+1)(x+h-1)}{(x-1)(x+h-1)}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{x^2+hx+x-x-h-1}{(x+h-1)(x-1)}}{h} - \frac{\frac{x^2+hx-x+x+h-1}{(x+h-1)(x-1)}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{\frac{-2h}{(x+h-1)(x-1)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{(x+h-1)(x-1)}$$

$$= -\frac{2}{(x-1)^2}$$

c. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 3x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \\ &= \lim_{h \rightarrow 0} 6x + 3h \\ &= 6x\end{aligned}$$

8. Let $y = f(x)$, then the slope of the tangent at each point x can be found by calculating $f'(x)$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 4(x+h) - 2x^2 + 4x}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{2x^2 + 4xh + 2h^2 - 4x - 4h}{h} + \frac{-2x^2 + 4x}{h} \right] \\ &= \lim_{h \rightarrow 0} \frac{4xh + h^2 - 4h}{h} \\ &= \lim_{h \rightarrow 0} 4x + h - 4 \\ &= 4x - 4\end{aligned}$$

So the slope of the tangent at $x = 0$ is

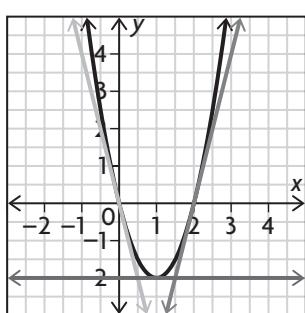
$$\begin{aligned}f'(0) &= 4(0) - 4 \\ &= -4\end{aligned}$$

At $x = 1$, the slope of the tangent is

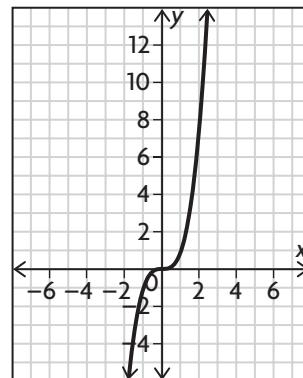
$$\begin{aligned}f'(1) &= 4(1) - 4 \\ &= 0\end{aligned}$$

At $x = 2$, the slope of the tangent is

$$\begin{aligned}f'(2) &= 4(2) - 4 \\ &= 4\end{aligned}$$



9. a.



b. Let $y = f(x)$, then the slope of the tangent at each point x can be found by calculating $f'(x)$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\ &= 3x^2\end{aligned}$$

So the slope of the tangent at $x = -2$ is

$$\begin{aligned}f'(-2) &= 3(-2)^2 \\ &= 12\end{aligned}$$

At $x = -1$, the slope of the tangent is

$$\begin{aligned}f'(-1) &= 3(-1)^2 \\ &= 3\end{aligned}$$

At $x = 0$, the slope of the tangent is

$$\begin{aligned}f'(0) &= 3(0)^2 \\ &= 0\end{aligned}$$

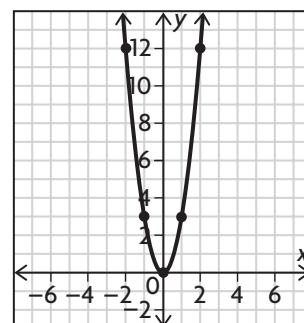
At $x = 1$, the slope of the tangent is

$$\begin{aligned}f'(1) &= 3(1)^2 \\ &= 3\end{aligned}$$

At $x = 2$, the slope of the tangent is

$$\begin{aligned}f'(2) &= 3(2)^2 \\ &= 12\end{aligned}$$

c.



d. The graph of $f(x)$ is a cubic. The graph of $f'(x)$ seems to be a parabola.

10. The velocity the particle at time t is given by $s'(t)$

$$\begin{aligned}s'(t) &= \lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h} \\&= \lim_{h \rightarrow 0} \frac{-(t+h)^2 + 8(t+h) - (-t^2 + 8t)}{h} \\&= \lim_{h \rightarrow 0} \frac{-t^2 - 2th - h^2 + 8t + 8h + t^2 - 8t}{h} \\&= \lim_{h \rightarrow 0} \frac{-2th - h^2 + 8h}{h} \\&= \lim_{h \rightarrow 0} -2t - h + 8 \\&= -2t + 8\end{aligned}$$

So the velocity at $t = 0$ is

$$\begin{aligned}s'(0) &= -2(0) + 8 \\&= 8 \text{ m/s}\end{aligned}$$

At $t = 4$, the velocity is

$$\begin{aligned}s'(4) &= -2(4) + 8 \\&= 0 \text{ m/s}\end{aligned}$$

At $t = 6$, the velocity is

$$\begin{aligned}s'(6) &= -2(6) + 8 \\&= -4 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\text{11. } f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\&= \lim_{h \rightarrow 0} \left[\frac{(\sqrt{x+h+1} - \sqrt{x+1})}{h} \right. \\&\quad \times \left. \frac{(\sqrt{x+h+1} + \sqrt{x+1})}{(\sqrt{x+h+1} + \sqrt{x+1})} \right] \\&= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h+1})^2 - (\sqrt{x+1})^2}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \lim_{h \rightarrow 0} \frac{x+h+1 - x-1}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \frac{1}{2\sqrt{x+1}}\end{aligned}$$

The equation $x - 6y + 4 = 0$ can be rewritten as $y = \frac{1}{6}x + \frac{2}{3}$, so this line has slope $\frac{1}{6}$. The value of x where the tangent to $f(x)$ has slope $\frac{1}{6}$ will satisfy $f'(x) = \frac{1}{6}$.

$$\begin{aligned}\frac{1}{2\sqrt{x+1}} &= \frac{1}{6} \\6 &= 2\sqrt{x+1} \\3^2 &= (\sqrt{x+1})^2 \\9 &= x+1 \\8 &= x \\f(8) &= \sqrt{8+1} \\&= \sqrt{9} \\&= 3\end{aligned}$$

So the tangent passes through the point $(8, 3)$, and its equation is $y - 3 = \frac{1}{6}(x - 8)$ or $x - 6y + 10 = 0$.

12. a. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{c - c}{h} \\&= \lim_{h \rightarrow 0} \frac{0}{h} \\&= 0\end{aligned}$$

b. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h) - x}{h} \\&= \lim_{h \rightarrow 0} \frac{h}{h} \\&= \lim_{h \rightarrow 0} 1 \\&= 1\end{aligned}$$

c. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{m(x+h) + b - mx - b}{h} \\&= \lim_{h \rightarrow 0} \frac{mx + mh + b - mx - b}{h} \\&= \lim_{h \rightarrow 0} \frac{mh}{h} \\&= \lim_{h \rightarrow 0} m \\&= m\end{aligned}$$

d. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \left[\frac{a(x+h)^2 + b(x+h) + c}{h} \right. \\&\quad \left. - \frac{(ax^2 + bx + c)}{h} \right]\end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \left[\frac{ax^2 + 2axh + ah^2 + bx + bh}{h} \right. \\
&\quad \left. + \frac{-ax^2 - bx - c}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{2axh + ah^2 + bh}{h} \\
&= \lim_{h \rightarrow 0} (2ax + ah + b) \\
&= 2ax + b
\end{aligned}$$

13. The slope of the function at a point x is given by

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\
&= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\
&= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\
&= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\
&= 3x^2
\end{aligned}$$

Since $3x^2$ is nonnegative for all x , the original function never has a negative slope.

14. $h(t) = 18t - 4.9t^2$

$$\begin{aligned}
\text{a. } h'(t) &= \lim_{k \rightarrow 0} \frac{h(t+k) - h(t)}{k} \\
&= \lim_{k \rightarrow 0} \frac{18(t+k) - 4.9(t+k)^2}{k} \\
&\quad - \frac{(18t - 4.9t^2)}{k} \\
&= \lim_{k \rightarrow 0} \frac{18t + 18k - 4.9t^2 - 9.8tk - 4.9k^2}{k} \\
&\quad - \frac{18t + 4.9t^2}{k} \\
&= \lim_{k \rightarrow 0} \frac{18k - 9.8tk - 4.9k^2}{k} \\
&= \lim_{k \rightarrow 0} (18 - 9.8t - 4.9k) \\
&= 18 - 9.8t - 4.9(0) \\
&= 18 - 9.8t
\end{aligned}$$

Then $h'(2) = 18 - 9.8(2) = -1.6$ m/s.

b. $h'(2)$ measures the rate of change in the height of the ball with respect to time when $t = 2$.

15. a. This graph has positive slope for $x < 0$, zero slope at $x = 0$, and negative slope for $x > 0$, which corresponds to graph e.

b. This graph has positive slope for $x < 0$, zero slope at $x = 0$, and positive slope for $x > 0$, which corresponds to graph f.

c. This graph has negative slope for $x < -2$, positive slope for $-2 < x < 0$, negative slope for $0 < x < 2$, positive slope for $x > 2$, and zero slope at $x = -2$, $x = 0$, and $x = 2$, which corresponds to graph d.

16. This function is defined piecewise as $f(x) = -x^2$ for $x < 0$, and $f(x) = x^2$ for $x \geq 0$. The derivative will exist if the left-side and right-side derivatives are the same at $x = 0$:

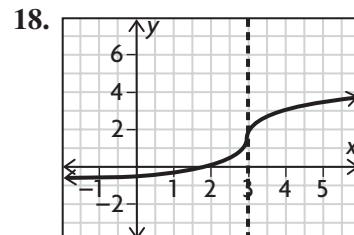
$$\begin{aligned}
\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^-} \frac{-(0+h)^2 - (-0^2)}{h} \\
&= \lim_{h \rightarrow 0^-} \frac{-h^2}{h} \\
&= \lim_{h \rightarrow 0^-} (-h) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^+} \frac{(0+h)^2 - (0^2)}{h} \\
&= \lim_{h \rightarrow 0^+} \frac{h^2}{h} \\
&= \lim_{h \rightarrow 0^+} (h) \\
&= 0
\end{aligned}$$

Since the limits are equal for both sides, the derivative exists and $f'(0) = 0$.

17. Since $f'(a) = 6$ and $f(a) = 0$,

$$\begin{aligned}
6 &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\
6 &= \lim_{h \rightarrow 0} \frac{f(a+h) - 0}{h} \\
3 &= \lim_{h \rightarrow 0} \frac{f(a+h)}{2h}
\end{aligned}$$



$f(x)$ is continuous.

$$f(3) = 2$$

But $f'(3) = \infty$.

(Vertical tangent)

19. $y = x^2 - 4x - 5$ has a tangent parallel to $2x - y = 1$.

Let $f(x) = x^2 - 4x - 5$. First, calculate

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \left[\frac{(x+h)^2 - 4(x+h) - 5}{h} \right. \\
&\quad \left. - \frac{(x^2 - 4x - 5)}{h} \right] \\
&= \lim_{h \rightarrow 0} \left[\frac{x^2 + 2xh + h^2 - 4x - 4h - 5}{h} \right. \\
&\quad \left. + \frac{-x^2 + 4x + 5}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 4h}{h} \\
&= \lim_{h \rightarrow 0} (2x + h - 4) \\
&= 2x + 0 - 4 \\
&= 2x - 4
\end{aligned}$$

Thus, $2x - 4$ is the slope of the tangent to the curve at x . We want the tangent parallel to $2x - y = 1$. Rearranging, $y = 2x - 1$.

If the tangent is parallel to this line,

$$2x - 4 = 2$$

$$x = 3$$

When $x = 3$, $y = (3)^2 - 4(3) - 5 = -8$.

The point is $(3, -8)$.

$$\mathbf{20. } f(x) = x^2$$

The slope of the tangent at any point (x, x^2) is

$$\begin{aligned}
f' &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h+x)(x+h-x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} \\
&= \lim_{h \rightarrow 0} (2x+h) \\
&= 2x + 0 \\
&= 2x
\end{aligned}$$

Let (a, a^2) be a point of tangency. The equation of the tangent is

$$y - a^2 = (2a)(x - a)$$

$$y = (2a)x - a^2$$

Suppose the tangent passes through $(1, -3)$.

Substitute $x = 1$ and $y = -3$ into the equation of the tangent:

$$\begin{aligned}
-3 &= (2a)(1) - a^2 \\
a^2 - 2a - 3 &= 0
\end{aligned}$$

$$(a-3)(a+1) = 0$$

$$a = -1, 3$$

So the two tangents are $y = -2x - 1$ or

$$2x + y + 1 = 0 \text{ and } y = 6x - 9 \text{ or } 6x - y - 9 = 0.$$

2.2 The Derivatives of Polynomial Functions, pp. 82–84

1. Answers may vary. For example:

constant function rule: $\frac{d}{dx}(5) = 0$

power rule: $\frac{d}{dx}(x^3) = 3x^2$

constant multiple rule: $\frac{d}{dx}(4x^3) = 12x^2$

sum rule: $\frac{d}{dx}(x^2 + x) = 2x + 1$

difference rule: $\frac{d}{dx}(x^3 - x^2 + 3x) = 3x^2 - 2x + 3$

$$\begin{aligned}
\mathbf{2. a. } f'(x) &= \frac{d}{dx}(4x) - \frac{d}{dx}(7) \\
&= 4 \frac{d}{dx}(x) - \frac{d}{dx}(7) \\
&= 4(x^0) - 0 \\
&= 4
\end{aligned}$$

$$\begin{aligned}
\mathbf{b. } f'(x) &= \frac{d}{dx}(x^3) - \frac{d}{dx}(x^2) \\
&= 3x^2 - 2x
\end{aligned}$$

$$\begin{aligned}
\mathbf{c. } f'(x) &= \frac{d}{dx}(-x^2) + \frac{d}{dx}(5x) + \frac{d}{dx}(8) \\
&= -\frac{d}{dx}(x^2) + 5 \frac{d}{dx}(x) + \frac{d}{dx}(8) \\
&= -(2x) + 5 + 0 \\
&= -2x + 5
\end{aligned}$$

$$\begin{aligned}
\mathbf{d. } f'(x) &= \frac{d}{dx}(\sqrt[3]{x}) \\
&= \frac{d}{dx}(x^{\frac{1}{3}}) \\
&= \frac{1}{3}(x^{(\frac{1}{3}-1)}) \\
&= \frac{1}{3}(x^{-\frac{2}{3}}) \\
&= \frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}} \\
&= \frac{1}{3\sqrt[3]{x^2}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{e. } f'(x) &= \frac{d}{dx}\left(\left(\frac{x}{2}\right)^4\right) \\
&= \left(\frac{1}{2}\right)^4 \frac{d}{dx}(x^4) \\
&= \frac{1}{16}(4x^3) \\
&= \frac{x^3}{4}
\end{aligned}$$

$$\mathbf{f. } f'(x) = \frac{d}{dx}(x^{-3}) \\ = (-3)(x^{-3-1}) \\ = -3x^{-4}$$

$$\mathbf{3. a. } h'(x) = \frac{d}{dx}((2x+3)(x+4)) \\ = \frac{d}{dx}(2x^2 + 8x + 3x + 12) \\ = \frac{d}{dx}(2x^2) + \frac{d}{dx}(11x) + \frac{d}{dx}(12) \\ = 2\frac{d}{dx}(x^2) + 11\frac{d}{dx}(x) + \frac{d}{dx}(12) \\ = 2(2x) + 11(1) + 0 \\ = 4x + 11$$

$$\mathbf{b. } f'(x) = \frac{d}{dx}(2x^3 + 5x^2 - 4x - 3.75) \\ = \frac{d}{dx}(2x^3) + \frac{d}{dx}(5x^2) - \frac{d}{dx}(4x) \\ - \frac{d}{dx}(3.75) \\ = 2\frac{d}{dx}(x^3) + 5\frac{d}{dx}(x^2) - 4\frac{d}{dx}(x) \\ - \frac{d}{dx}(3.75) \\ = 2(3x^2) + 5(2x) - 4(1) - 0 \\ = 6x^2 + 10x - 4$$

$$\mathbf{c. } \frac{ds}{dt} = \frac{d}{dt}(t^2(t^2 - 2t)) \\ = \frac{d}{dt}(t^4 - 2t^3) \\ = \frac{d}{dt}(t^4) - \frac{d}{dt}(2t^3) \\ = \frac{d}{dt}(t^4) - 2\frac{d}{dt}(t^3) \\ = 4t^3 - 2(3t^2) \\ = 4t^3 - 6t^2$$

$$\mathbf{d. } \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{5}x^5 + \frac{1}{3}x^3 - \frac{1}{2}x^2 + 1\right) \\ = \frac{d}{dx}\left(\frac{1}{5}x^5\right) + \frac{d}{dx}\left(\frac{1}{3}x^3\right) - \frac{d}{dx}\left(\frac{1}{2}x^2\right) + \frac{d}{dx}(1) \\ = \left(\frac{1}{5}\right)\frac{d}{dx}(x^5) + \left(\frac{1}{3}\right)\frac{d}{dx}(x^3) - \left(\frac{1}{2}\right)\frac{d}{dx}(x^2) \\ + \frac{d}{dx}(1) \\ = \frac{1}{5}(5x^4) + \frac{1}{3}(3x^2) - \frac{1}{2}(2x) + 0 \\ = x^4 + x^2 - x$$

$$\mathbf{e. } g'(x) = \frac{d}{dx}(5(x^2)^4)$$

$$= 5\frac{d}{dx}(x^{2 \times 4}) \\ = 5\frac{d}{dx}(x^8) \\ = 5(8x^7) \\ = 40x^7$$

$$\mathbf{f. } s'(t) = \frac{d}{dt}\left(\frac{t^5 - 3t^2}{2t}\right) \\ = \left(\frac{1}{2}\right)\frac{d}{dt}(t^4 - 3t) \\ = \left(\frac{1}{2}\right)\left(\frac{d}{dt}(t^4) - \frac{d}{dt}(3t)\right) \\ = \left(\frac{1}{2}\right)\left(\frac{d}{dt}(t^4) - 3\frac{d}{dt}(t)\right) \\ = \left(\frac{1}{2}\right)(4t^3 - 3(1)) \\ = 2t^3 - \frac{3}{2}$$

$$\mathbf{4. a. } \frac{dy}{dx} = \frac{d}{dx}(3x^{\frac{5}{3}}) \\ = 3\frac{d}{dx}(x^{\frac{5}{3}}) \\ = \left(\frac{5}{3}\right)3(x^{(\frac{5}{3}-1)}) \\ = 5x^{\frac{2}{3}}$$

$$\mathbf{b. } \frac{dy}{dx} = \frac{d}{dx}\left(4x^{-\frac{1}{2}} - \frac{6}{x}\right) \\ = 4\frac{d}{dx}(x^{-\frac{1}{2}}) - 6\frac{d}{dx}(x^{-1}) \\ = 4\left(\frac{-1}{2}\right)(x^{-\frac{1}{2}-1}) - 6(-1)(x^{-1-1}) \\ = -2x^{-\frac{3}{2}} + 6x^{-2}$$

$$\mathbf{c. } \frac{dy}{dx} = \frac{d}{dx}\left(\frac{6}{x^3} + \frac{2}{x^2} - 3\right) \\ = 6\frac{d}{dx}(x^{-3}) + 2\frac{d}{dx}(x^{-2}) - \frac{d}{dx}(3) \\ = 6(-3)(x^{-3-1}) + 2(-2)(x^{-2-1}) - 0 \\ = -18x^{-4} - 4x^{-3} \\ = \frac{-18}{x^4} - \frac{4}{x^3}$$

$$\begin{aligned}\mathbf{d.} \frac{dy}{dx} &= \frac{d}{dx}(9x^{-2} + 3\sqrt{x}) \\&= 9\frac{d}{dx}(x^{-2}) + 3\frac{d}{dx}(x^{\frac{1}{2}}) \\&= 9(-2)(x^{-2-1}) + 3\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) \\&= -18x^{-3} + \frac{3}{2}x^{-\frac{1}{2}}\end{aligned}$$

$$\begin{aligned}\mathbf{e.} \frac{dy}{dx} &= \frac{d}{dx}(\sqrt{x} + 6\sqrt{x^3} + \sqrt{2}) \\&= \frac{d}{dx}(x^{\frac{1}{2}}) + 6\frac{d}{dx}(x^{\frac{3}{2}}) + \frac{d}{dx}(\sqrt{2}) \\&= \frac{1}{2}(x^{\frac{1}{2}-1}) + 6\left(\frac{3}{2}\right)(x^{\frac{3}{2}-1}) + 0 \\&= \frac{1}{2}(x^{-\frac{1}{2}}) + 9x^{\frac{1}{2}}\end{aligned}$$

$$\begin{aligned}\mathbf{f.} \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1+\sqrt{x}}{x}\right) \\&= \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}\left(\frac{x^{\frac{1}{2}}}{x}\right) \\&= \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(x^{-\frac{1}{2}}) \\&= (-1)x^{-1-1} + \frac{-1}{2}(x^{-\frac{1}{2}-1}) \\&= -x^{-2} - \frac{1}{2}x^{-\frac{3}{2}}\end{aligned}$$

$$\begin{aligned}\mathbf{5. a.} \frac{ds}{dt} &= \frac{d}{dt}(-2t^2 + 7t) \\&= (-2)\left(\frac{d}{dt}(t^2)\right) + 7\left(\frac{d}{dt}(t)\right) \\&= (-2)(2t) + 7(1) \\&= -4t + 7\end{aligned}$$

$$\begin{aligned}\mathbf{b.} \frac{ds}{dt} &= \frac{d}{dt}\left(18 + 5t - \frac{1}{3}t^3\right) \\&= \frac{d}{dt}(18) + 5\frac{d}{dt}(t) - \left(\frac{1}{3}\right)\frac{d}{dt}(t^3) \\&= 0 + 5(1) - \left(\frac{1}{3}\right)(3t^2) \\&= 5 - t^2\end{aligned}$$

$$\begin{aligned}\mathbf{c.} \frac{ds}{dt} &= \frac{d}{dt}((t-3)^2) \\&= \frac{d}{dt}(t^2 - 6t + 9) \\&= \frac{d}{dt}(t^2) - (6)\frac{d}{dt}(t) + \frac{d}{dt}(9)\end{aligned}$$

$$\begin{aligned}&= 2t - 6(1) + 0 \\&= 2t - 6\end{aligned}$$

$$\begin{aligned}\mathbf{6. a.} f'(x) &= \frac{d}{dx}(x^3 - \sqrt{x}) \\&= \frac{d}{dx}(x^3) - \frac{d}{dx}(x^{\frac{1}{2}}) \\&= 3x^2 - \frac{1}{2}(x^{\frac{1}{2}-1}) \\&= 3x^2 - \frac{1}{2}x^{-\frac{1}{2}}\end{aligned}$$

$$\begin{aligned}\text{so } f'(a) &= f'(4) = 3(4)^2 - \frac{1}{2}(4)^{-\frac{1}{2}} \\&= 3(16) - \frac{1}{2}\frac{1}{\sqrt{4}} \\&= 48 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\&= 47.75\end{aligned}$$

$$\begin{aligned}\mathbf{b.} f'(x) &= \frac{d}{dx}(7 - 6\sqrt{x} + 5x^{\frac{2}{3}}) \\&= \frac{d}{dx}(7) - 6\frac{d}{dx}(x^{\frac{1}{2}}) + 5\frac{d}{dx}(x^{\frac{2}{3}}) \\&= 0 - 6\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) + 5\left(\frac{2}{3}\right)(x^{\frac{2}{3}-1}) \\&= -3x^{-\frac{1}{2}} + \left(\frac{10}{3}\right)(x^{-\frac{1}{3}})\end{aligned}$$

$$\begin{aligned}\text{so } f'(a) &= f'(64) = -3(64^{-\frac{1}{2}}) + \left(\frac{10}{3}\right)(64^{-\frac{1}{3}}) \\&= -3\left(\frac{1}{8}\right) + \frac{10}{3}\left(\frac{1}{4}\right) \\&= \frac{11}{24}\end{aligned}$$

$$\begin{aligned}\mathbf{7. a.} \frac{dy}{dx} &= \frac{d}{dx}(3x^4) \\&= 3\frac{d}{dx}(x^4) \\&= 3(4x^3) \\&= 12x^3\end{aligned}$$

The slope at (1, 3) is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$. So the slope = $12(1)^3 = 12$

$$\begin{aligned}\mathbf{b.} \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{x^{-5}}\right) \\&= \frac{d}{dx}(x^5) \\&= 5x^4\end{aligned}$$

The slope at $(-1, -1)$ is found by substituting $x = -1$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= 5(-1)^4 \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{c. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{2}{x}\right) \\ &= 2\frac{d}{dx}(x^{-1}) \\ &= 2(-1)x^{-1-1} \\ &= -2x^{-2}\end{aligned}$$

The slope at $(-2, -1)$ is found by substituting $x = -2$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= -2(-2)^{-2} \\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{d. } \frac{dy}{dx} &= \frac{d}{dx}(\sqrt{16x^3}) \\ &= \sqrt{16}\frac{d}{dx}(x^{\frac{3}{2}}) \\ &= 4\left(\frac{3}{2}\right)x^{\frac{3}{2}-1} \\ &= 6x^{\frac{1}{2}}\end{aligned}$$

The slope at $(4, 32)$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= 6(4)^{\frac{1}{2}} \\ &= 12\end{aligned}$$

$$8. \text{ a. } y = 2x^3 + 3x$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(2x^3 + 3x) \\ &= 2\frac{d}{dx}(x^3) + 3\frac{d}{dx}(x) \\ &= 2(3x^2) + 3(1) \\ &= 6x^2 + 3\end{aligned}$$

The slope at $x = 1$ is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$. So the slope is

$$6(1)^2 + 3 = 9.$$

$$\text{b. } y = 2\sqrt{x} + 5$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(2\sqrt{x} + 5) \\ &= 2\frac{d}{dx}(x^{\frac{1}{2}}) + \frac{d}{dx}(5) \\ &= 2\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) + 0 \\ &= x^{-\frac{1}{2}}\end{aligned}$$

The slope at $x = 4$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$. So the slope is $(4)^{\frac{-1}{2}} = \frac{1}{2}$.

$$\text{c. } y = \frac{16}{x^2}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{16}{x^2}\right) \\ &= 16\frac{d}{dx}(x^{-2}) \\ &= 16(-2)x^{-2-1} \\ &= -32x^{-3}\end{aligned}$$

The slope at $x = -2$ is found by substituting

$x = -2$ into the equation for $\frac{dy}{dx}$. So the slope is $-32(-2)^{-3} = \frac{(-32)}{(-2)^3} = 4$.

$$\text{d. } y = x^{-3}(x^{-1} + 1)$$

$$= x^{-4} + x^{-3}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x^{-4} + x^{-3}) \\ &= -4x^{-5} - 3x^{-4} \\ &= -\frac{4}{x^5} - \frac{3}{x^4}\end{aligned}$$

The slope at $x = 1$ is found by substituting

$x = 1$ into the equation for $\frac{dy}{dx}$. So the slope is $-\frac{4}{1^5} - \frac{3}{1^4} = -7$.

$$9. \text{ a. } \frac{dy}{dx} = \frac{d}{dx}\left(2x - \frac{1}{x}\right)$$

$$\begin{aligned}&= 2\frac{d}{dx}(x) - \frac{d}{dx}(x^{-1}) \\ &= 2(1) - (-1)x^{-1-1} \\ &= 2 + x^{-2}\end{aligned}$$

The slope at $x = 0.5$ is found by substituting

$x = 0.5$ into the equation for $\frac{dy}{dx}$.

So the slope is $2 + (0.5)^{-2} = 6$.

The equation of the tangent line is therefore $y + 1 = 6(x - 0.5)$ or $6x - y - 4 = 0$.

$$\text{b. } \frac{dy}{dx} = \frac{d}{dx}\left(\frac{3}{x^2} - \frac{4}{x^3}\right)$$

$$\begin{aligned}&= 3\frac{d}{dx}(x^{-2}) - 4\frac{d}{dx}(x^{-3}) \\ &= 3(-2)x^{-2-1} - 4(-3)x^{-3-1} \\ &= 12x^{-4} - 6x^{-3}\end{aligned}$$

The slope at $x = -1$ is found by substituting

$x = -1$ into the equation for $\frac{dy}{dx}$. So the slope is $12(-1)^{-4} - 6(-1)^{-3} = 18$.

The equation of the tangent line is therefore $y - 7 = 18(x + 1)$ or $18x - y + 25 = 0$.

$$\begin{aligned} \text{c. } \frac{dy}{dx} &= \frac{d}{dx}(\sqrt{3x^3}) \\ &= \sqrt{3} \frac{d}{dx}(x^{\frac{3}{2}}) \\ &= \sqrt{3} \left(\frac{3}{2}\right) x^{\frac{3}{2}-1} \\ &= \frac{3\sqrt{3}x^{\frac{1}{2}}}{2} \end{aligned}$$

The slope at $x = 3$ is found by substituting $x = 3$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } \frac{3\sqrt{3}(3)^{\frac{1}{2}}}{2} = \frac{9}{2}.$$

The equation of the tangent line is therefore $y - 9 = \frac{9}{2}(x - 3)$ or $9x - 2y - 9 = 0$.

$$\begin{aligned} \text{d. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{x}\left(x^2 + \frac{1}{x}\right)\right) \\ &= \frac{d}{dx}\left(x + \frac{1}{x^2}\right) \\ &= \frac{d}{dx}(x) + \frac{d}{dx}(x^{-2}) \\ &= 1 + (-2)x^{-2-1} \\ &= 1 - 2x^{-3} \end{aligned}$$

The slope at $x = 1$ is found by substituting into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } 1 - 2(1)^{-3} = -1.$$

The equation of the tangent line is therefore $y - 2 = -(x - 1)$ or $x + y - 3 = 0$.

$$\begin{aligned} \text{e. } \frac{dy}{dx} &= \frac{d}{dx}((\sqrt{x} - 2)(3\sqrt{x} + 8)) \\ &= \frac{d}{dx}(3(\sqrt{x})^2 + 8\sqrt{x} - 6\sqrt{x} - 16) \\ &= \frac{d}{dx}(3x + 2\sqrt{x} - 16) \\ &= \frac{d}{dx}(3x) + 2\frac{d}{dx}(x^{\frac{1}{2}}) - \frac{d}{dx}(16) \\ &= 3(1) + 2\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} - 0 \\ &= 3 + x^{-\frac{1}{2}} \end{aligned}$$

The slope at $x = 4$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } 3 + (4)^{-\frac{1}{2}} = 3.5.$$

The equation of the tangent line is therefore $y = 3.5(x - 4)$ or $7x - 2y - 28 = 0$.

$$\begin{aligned} \text{f. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{\sqrt{x} - 2}{\sqrt[3]{x}}\right) \\ &= \frac{d}{dx}\left(\frac{x^{\frac{1}{2}} - 2}{x^{\frac{1}{3}}}\right) \\ &= \frac{d}{dx}(x^{\frac{1}{2}-\frac{1}{3}} - 2x^{-\frac{1}{3}}) \\ &= \frac{d}{dx}(x^{\frac{1}{6}}) - 2\frac{d}{dx}(x^{-\frac{1}{3}}) \\ &= \frac{1}{6}(x^{\frac{1}{6}-1}) - 2\left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1} - 0 \\ &= \frac{1}{6}(x^{-\frac{5}{6}}) + \frac{2}{3}x^{-\frac{4}{3}} \end{aligned}$$

The slope at $x = 1$ is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } \frac{1}{6}(1)^{-\frac{5}{6}} + \frac{2}{3}(1)^{-\frac{4}{3}} = \frac{5}{6}.$$

The equation of the tangent line is therefore $y + 1 = \frac{5}{6}(x - 1)$ or $5x - 6y - 11 = 0$.

10. A normal to the graph of a function at a point is a line that is perpendicular to the tangent at the given point.

$$y = \frac{3}{x^2} - \frac{4}{x^3} \text{ at } P(-1, 7)$$

Slope of the tangent is 18, therefore, the slope of the normal is $-\frac{1}{18}$.

$$\text{Equation is } y - 7 = -\frac{1}{18}(x + 1).$$

$$x + 18y - 125 = 0$$

$$\text{11. } y = \frac{3}{\sqrt[3]{x}} = 3x^{-\frac{1}{3}}$$

Parallel to $x + 16y + 3 = 0$

Slope of the line is $-\frac{1}{16}$.

$$\frac{dy}{dx} = -x^{-\frac{4}{3}}$$

$$x^{-\frac{4}{3}} = \frac{1}{16}$$

$$\frac{1}{x^{\frac{4}{3}}} = \frac{1}{16}$$

$$x^{\frac{4}{3}} = 16$$

$$x = (16)^{\frac{3}{4}} = 8$$

12. $y = \frac{1}{x} = x^{-1}$; $y = x^3$

$$\frac{dy}{dx} = -\frac{1}{x^2}; \frac{dy}{dx} = 3x^2$$

Now, $-\frac{1}{x^2} = 3x^2$

$$x^4 = -\frac{1}{3}$$

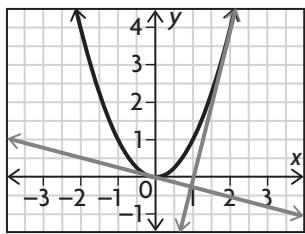
No real solution. They never have the same slope.

13. $y = x^2$, $\frac{dy}{dx} = 2x$

The slope of the tangent at $A(2, 4)$ is 4 and at

$$B\left(-\frac{1}{8}, \frac{1}{64}\right)$$

Since the product of the slopes is -1 , the tangents at $A(2, 4)$ and $B\left(-\frac{1}{8}, \frac{1}{64}\right)$ will be perpendicular.



14. $y = -x^2 + 3x + 4$

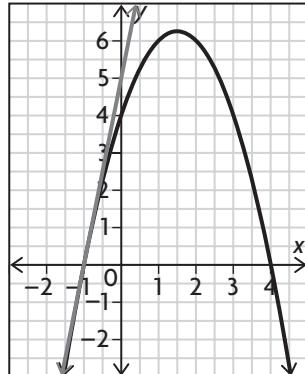
$$\frac{dy}{dx} = -2x + 3$$

For $\frac{dy}{dx} = 5$

$$5 = -2x + 3$$

$$x = -1.$$

The point is $(-1, 0)$.



15. $y = x^3 + 2$

$$\frac{dy}{dx} = 3x^2, \text{slope is } 12$$

$$x^2 = 4$$

$$x = 2 \text{ or } x = -2$$

Points are $(2, 10)$ and $(-2, -6)$.

16. $y = \frac{1}{5}x^5 - 10x$, slope is 6

$$\frac{dy}{dx} = x^4 - 10 = 6$$

$$x^4 = 16$$

$$x^2 = 4 \text{ or } x^2 = -4$$

$$x = \pm 2 \text{ non-real}$$

Tangents with slope 6 are at the points $(2, -\frac{68}{5})$ and $(-2, \frac{68}{5})$.

17. $y = 2x^2 + 3$

a. Equation of tangent from $A(2, 3)$:

$$\text{If } x = a, y = 2x^2 + 3.$$

Let the point of tangency be $P(a, 2a^2 + 3)$.

$$\text{Now, } \frac{dy}{dx} = 4x \text{ and when } x = a, \frac{dy}{dx} = 4a.$$

The slope of the tangent is the slope of AP .

$$\frac{2a^2}{a-2} = 4a$$

$$2a^2 = 4a^2 - 8a$$

$$2a^2 - 8a = 0$$

$$2a(a - 4) = 0$$

$$a = 0 \text{ or } a = 4.$$

Point $(2, 3)$:

Slope is 0.

Equation of tangent is

$$y - 3 = 0.$$

Slope is 16.

Equation of tangent is

$$y - 3 = 16(x - 2) \text{ or}$$

$$16x - y - 29 = 0.$$

b. From the point $B(2, -7)$:

$$\text{Slope of } BP: \frac{2a^2 + 10}{a - 2} = 4a$$

$$2a^2 + 10 = 4a^2 - 8a$$

$$2a^2 - 8a - 10 = 0$$

$$a^2 - 4a - 5 = 0$$

$$(a - 5)(a + 1) = 0$$

$$a = 5$$

$$a = -1$$

Slope is $4a = 20$.

Equation is

$$y + 7 = 20(x - 2)$$

$$\text{or } 20x - y - 47 = 0.$$

Slope is $4a = -4$.

Equation is

$$y + 7 = -4(x - 2)$$

$$\text{or } 4x + y - 1 = 0.$$

18. $ax - 4y + 21 = 0$ is tangent to $y = \frac{a}{x^2}$ at $x = -2$.

Therefore, the point of tangency is $(-2, \frac{a}{4})$,

This point lies on the line, therefore,

$$a(-2) - 4\left(\frac{a}{4}\right) + 21 = 0$$

$$-3a + 21 = 0$$

$$a = 7.$$

19. a. When $h = 200$,

$$d = 3.53\sqrt{200} \\ \doteq 49.9$$

Passengers can see about 49.9 km.

b. $d = 3.53\sqrt{h} = 3.53h^{\frac{1}{2}}$

$$d' = 3.53\left(\frac{1}{2}h^{-\frac{1}{2}}\right) \\ = \frac{3.53}{2\sqrt{h}}$$

When $h = 200$,

$$d' = \frac{3.53}{2\sqrt{200}} \\ \doteq 0.12$$

The rate of change is about 0.12 km/m.

20. $d(t) = 4.9t^2$

a. $d(2) = 4.9(2)^2 = 19.6 \text{ m}$
 $d(5) = 4.9(5)^2 = 122.5 \text{ m}$

The average rate of change of distance with respect to time from 2 s to 5 s is

$$\frac{\Delta d}{\Delta t} = \frac{122.5 - 19.6}{5 - 2} \\ = 34.3 \text{ m/s}$$

b. $d'(t) = 9.8t$

Thus, $d'(4) = 9.8(4) = 39.2 \text{ m/s}$.

c. When the object hits the ground, $d = 150$.

Set $d(t) = 150$:

$$4.9t^2 = 150$$

$$t^2 = \frac{1500}{49}$$

$$t = \pm \frac{10}{7}\sqrt{15}$$

Since $t \geq 0$, $t = \frac{10}{7}\sqrt{15}$

Then,

$$d'\left(\frac{10}{7}\sqrt{15}\right) = 9.8\left(\frac{10}{7}\sqrt{15}\right) \\ \doteq 54.2 \text{ m/s}$$

21. $v(t) = s'(t) = 2t - t^2$

$$0.5 = 2t - t^2$$

$$t^2 - 2t + 0.5 = 0$$

$$2t^2 - 4t + 1 = 0$$

$$t = \frac{4 \pm \sqrt{8}}{4}$$

$$t \doteq 1.71, 0.29$$

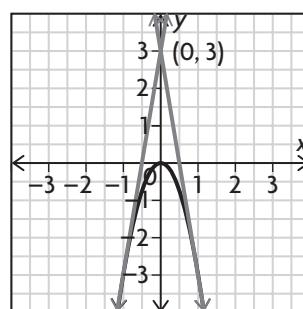
The train has a velocity of 0.5 km/min at about 0.29 min and 1.71 min.

22. $v(t) = R'(t) = -10t$

$$v(2) = -20$$

The velocity of the bolt at $t = 2$ is -20 m/s .

23.



Let the coordinates of the points of tangency be $A(a, -3a^2)$.

$$\frac{dy}{dx} = -6x, \text{ slope of the tangent at } A \text{ is } -6a$$

$$\text{Slope of } PA: \frac{-3a^2 - 3}{a} = -6a$$

$$-3a^2 - 3 = -6a^2$$

$$3a^2 = 3$$

$$a = 1 \text{ or } a = -1$$

Coordinates of the points at which the tangents touch the curve are $(1, -3)$ and $(-1, -3)$.

24. $y = x^3 - 6x^2 + 8x$, tangent at $A(3, -3)$

$$\frac{dy}{dx} = 3x^2 - 12x + 8$$

When $x = 3$,

$$\frac{dy}{dx} = 27 - 36 + 8 = -1$$

The slope of the tangent at $A(3, -3)$ is -1 .

Equation will be

$$y + 3 = -1(x - 3)$$

$$y = -x$$

$$-x = x^3 - 6x^2 + 8x$$

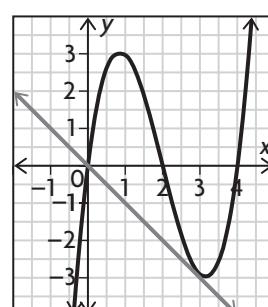
$$x^3 - 6x^2 + 9x = 0$$

$$x(x^2 - 6x + 9) = 0$$

$$x(x - 3)^2 = 0$$

$$x = 0 \text{ or } x = 3$$

Coordinates are $B(0, 0)$.



25. a. i. $f(x) = 2x - 5x^2$

$$f'(x) = 2 - 10x$$

Set $f'(x) = 0$:

$$2 - 10x = 0$$

$$10x = 2$$

$$x = \frac{1}{5}$$

Then,

$$\begin{aligned} f\left(\frac{1}{5}\right) &= 2\left(\frac{1}{5}\right) - 5\left(\frac{1}{5}\right)^2 \\ &= \frac{2}{5} - \frac{1}{5} \\ &= \frac{1}{5} \end{aligned}$$

Thus the point is $\left(\frac{1}{5}, \frac{1}{5}\right)$.

ii. $f(x) = 4x^2 + 2x - 3$

$$f'(x) = 8x + 2$$

Set $f'(x) = 0$:

$$8x + 2 = 0$$

$$8x = -2$$

$$x = -\frac{1}{4}$$

Then,

$$\begin{aligned} f\left(-\frac{1}{4}\right) &= 4\left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right) - 3 \\ &= \frac{1}{4} - \frac{2}{4} - \frac{12}{4} \\ &= -\frac{13}{4} \end{aligned}$$

Thus the point is $\left(-\frac{1}{4}, -\frac{13}{4}\right)$.

iii. $f(x) = x^3 - 8x^2 + 5x + 3$

$$f'(x) = 3x^2 - 16x + 5$$

Set $f'(x) = 0$:

$$3x^2 - 16x + 5 = 0$$

$$3x^2 - 15x - x + 5 = 0$$

$$3x(x - 5) - (x - 5) = 0$$

$$(3x - 1)(x - 5) = 0$$

$$x = \frac{1}{3}, 5$$

$$\begin{aligned} f\left(\frac{1}{3}\right) &= \left(\frac{1}{3}\right)^3 - 8\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) + 3 \\ &= \frac{1}{27} - \frac{24}{27} + \frac{45}{27} + \frac{81}{27} \\ &= \frac{103}{27} \end{aligned}$$

$$\begin{aligned} f(5) &= (5)^3 - 8(5)^2 + 5(5) + 3 \\ &= 25 - 200 + 25 + 3 \\ &= -47 \end{aligned}$$

Thus the two points are $\left(\frac{1}{3}, \frac{103}{27}\right)$ and $(5, -47)$.

b. At these points, the slopes of the tangents are zero, meaning that the rate of change of the value of the function with respect to the domain is zero. These points are also local maximum or minimum points.

26. $\sqrt{x} + \sqrt{y} = 1$

$P(a, b)$ is on the curve, therefore $a \geq 0, b \geq 0$.

$$\sqrt{y} = 1 - \sqrt{x}$$

$$y = 1 - 2\sqrt{x} + x$$

$$\frac{dy}{dx} = -\frac{1}{2} \cdot 2x^{-\frac{1}{2}} + 1$$

$$\text{At } x = a, \text{slope is } -\frac{1}{\sqrt{a}} + 1 = \frac{-1 + \sqrt{a}}{\sqrt{a}}.$$

But $\sqrt{a} + \sqrt{b} = 1$

$$-\sqrt{b} = \sqrt{a} - 1.$$

$$\text{Therefore, slope is } -\frac{\sqrt{b}}{\sqrt{a}} = -\sqrt{\frac{b}{a}}.$$

27. $f(x) = x^n, f'(x) = nx^{n-1}$

Slope of the tangent at $x = 1$ is $f'(1) = n$,

The equation of the tangent at $(1, 1)$ is:

$$y - 1 = n(x - 1)$$

$$nx - y - n + 1 = 0$$

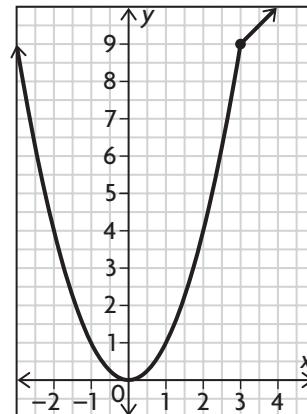
$$\text{Let } y = 0, nx = n - 1$$

$$x = \frac{n-1}{n} = 1 - \frac{1}{n}.$$

The x -intercept is $1 - \frac{1}{n}$; as $n \rightarrow \infty$, $\frac{1}{n} \rightarrow 0$, and

the x -intercept approaches 1. As $n \rightarrow \infty$, the slope of the tangent at $(1, 1)$ increases without bound, and the tangent approaches a vertical line having equation $x - 1 = 0$.

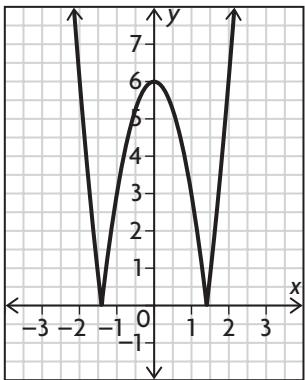
28. a.



$$f(x) = \begin{cases} x^2, & \text{if } x < 3 \\ x + 6, & \text{if } x \geq 3 \end{cases} \quad f'(x) = \begin{cases} 2x, & \text{if } x < 3 \\ 1, & \text{if } x \geq 3 \end{cases}$$

$f'(3)$ does not exist.

b.

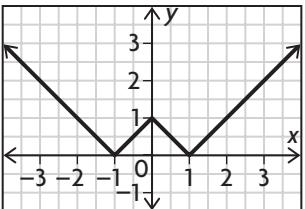


$$f(x) = \begin{cases} 3x^2 - 6, & \text{if } x < -\sqrt{2} \text{ or } x > \sqrt{2} \\ 6 - 3x^2, & \text{if } -\sqrt{2} < x < \sqrt{2} \end{cases}$$

$$f'(x) = \begin{cases} 6x, & \text{if } x < -\sqrt{2} \text{ or } x > \sqrt{2} \\ -6x, & \text{if } -\sqrt{2} \leq x \leq \sqrt{2} \end{cases}$$

$f'(\sqrt{2})$ and $f'(-\sqrt{2})$ do not exist.

c.



$$f(x) = \begin{cases} x - 1, & \text{if } x \geq 1 \\ 1 - x, & \text{if } 0 \leq x < 1 \\ x + 1, & \text{if } -1 < x < 0 \\ -x - 1, & \text{if } x \leq -1 \end{cases}$$

since $|x - 1| = x - 1$
since $|x - 1| = 1 - x$
since $|-x - 1| = x + 1$
since $|-x - 1| = -x - 1$

$$f'(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } 0 < x < 1 \\ 1, & \text{if } -1 < x < 0 \\ -1, & \text{if } x < -1 \end{cases}$$

$f'(0)$, $f'(-1)$, and $f'(1)$ do not exist.

2.3 The Product Rule, pp. 90–91

1. a. $h(x) = x(x - 4)$

$$h'(x) = x(1) + (1)(x - 4)$$

$$= 2x - 4$$

b. $h(x) = x^2(2x - 1)$

$$h'(x) = x^2(2) + (2x)(2x - 1)$$

$$= 6x^2 - 2x$$

c. $h(x) = (3x + 2)(2x - 7)$

$$h'(x) = (3x + 2)(2) + (3)(2x - 7)$$

$$= 12x - 17$$

d. $h(x) = (5x^7 + 1)(x^2 - 2x)$

$$h'(x) = (5x^7 + 1)(2x - 2) + (35x^6)(x^2 - 2x)$$

$$= 45x^8 - 80x^7 + 2x - 2$$

e. $s(t) = (t^2 + 1)(3 - 2t^2)$

$$s'(t) = (t^2 + 1)(-4t) + (2t)(3 - 2t^2)$$

$$= -8t^3 + 2t$$

f. $f(x) = \frac{x - 3}{x + 3}$

$$f(x) = (x - 3)(x + 3)^{-1}$$

$$f'(x) = (x - 3)(-1)(x + 3)^{-2} + (1)(x + 3)^{-1}$$

$$= (x + 3)^{-2}(-x + 3 + x + 3)$$

$$= \frac{6}{(x + 3)^2}$$

2. a. $y = (5x + 1)^3(x - 4)$

$$\frac{dy}{dx} = (5x + 1)^3(1) + 3(5x + 1)^2(5)(x - 4)$$

$$= (5x + 1)^3 + 15(5x + 1)^2(x - 4)$$

b. $y = (3x^2 + 4)(3 + x^3)^5$

$$\frac{dy}{dx} = (3x^2 + 4)(5)(3 + x^3)^4(3x^2)$$

$$+ (6x)(3 + x^3)^5$$

$$= 15x^2(3x^2 + 4)(3 + x^3)^4 + 6x(3 + x^3)^5$$

c. $y = (1 - x^2)^4(2x + 6)^3$

$$\frac{dy}{dx} = 4(1 - x^2)^3(-2x)(2x + 6)^3$$

$$+ (1 - x^2)^4 3(2x + 6)^2(2)$$

$$= -8x(1 - x^2)^3(2x + 6)^3$$

$$+ 6(1 - x^2)^4(2x + 6)^2$$

d. $y = (x^2 - 9)^4(2x - 1)^3$

$$\frac{dy}{dx} = (x^2 - 9)^4(3)(2x - 1)^2(2)$$

$$+ 4(x^2 - 9)^3(2x)(2x - 1)^3$$

$$= 6(x^2 - 9)^4(2x - 1)^2$$

$$+ 8x(x^2 - 9)^3(2x - 1)^3$$

3. It is not appropriate or necessary to use the product rule when one of the factors is a constant or when it would be easier to first determine the product of the factors and then use other rules to determine the derivative. For example, it would not be best to use the product rule for $f(x) = 3(x^2 + 1)$ or $g(x) = (x + 1)(x - 1)$.

4. $F(x) = [b(x)][c(x)]$

$$F'(x) = [b(x)][c'(x)] + [b'(x)][c(x)]$$

5. a. $y = (2 + 7x)(x - 3)$

$$\frac{dy}{dx} = (2 + 7x)(1) + 7(x - 3)$$

At $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= (2 + 14) + 7(-1) \\ &= 16 - 7 \\ &= 9\end{aligned}$$

b. $y = (1 - 2x)(1 + 2x)$

$$\frac{dy}{dx} = (1 - 2x)(2) + (-2)(1 + 2x)$$

At $x = \frac{1}{2}$,

$$\begin{aligned}\frac{dy}{dx} &= (0)(2) - 2(2) \\ &= -4\end{aligned}$$

c. $y = (3 - 2x - x^2)(x^2 + x - 2)$

$$\begin{aligned}\frac{dy}{dx} &= (3 - 2x - x^2)(2x + 1) \\ &\quad + (-2 - 2x)(x^2 + x - 2)\end{aligned}$$

At $x = -2$,

$$\begin{aligned}\frac{dy}{dx} &= (3 + 4 - 4)(-4 + 1) \\ &\quad + (-2 + 4)(4 - 2 - 2) \\ &= (3)(-3) + (2)(0) \\ &= -9\end{aligned}$$

d. $y = x^3(3x + 7)^2$

$$\frac{dy}{dx} = 3x^2(3x + 7)^2 + x^3 \cdot 6(3x + 7)$$

At $x = -2$,

$$\begin{aligned}\frac{dy}{dx} &= 12(1)^2 + (-8)(6)(1) \\ &= 12 - 48 \\ &= -36\end{aligned}$$

e. $y = (2x + 1)^5(3x + 2)^4, x = -1$

$$\begin{aligned}\frac{dy}{dx} &= 5(2x + 1)^4(2)(3x + 2)^4 \\ &\quad + (2x + 1)^5 \cdot 4(3x + 2)^3(3)\end{aligned}$$

At $x = -1$,

$$\begin{aligned}\frac{dy}{dx} &= 5(-1)^4(2)(-1)^4 \\ &\quad + (-1)^5(4)(-1)^3(3) \\ &= 10 + 12 \\ &= 22\end{aligned}$$

f. $y = x(5x - 2)(5x + 2)$

$$\frac{dy}{dx} = x(50x) + (25x^2 - 4)(1)$$

At $x = 3$,

$$\begin{aligned}\frac{dy}{dx} &= 3(150) + (25 \cdot 9 - 4) \\ &= 450 + 221 \\ &= 671\end{aligned}$$

6. Tangent to $y = (x^3 - 5x + 2)(3x^2 - 2x)$ at $(1, -2)$

$$\begin{aligned}\frac{dy}{dx} &= (3x^2 - 5)(3x^2 - 2x) \\ &\quad + (x^3 - 5x + 2)(6x - 2)\end{aligned}$$

when $x = 1$,

$$\begin{aligned}\frac{dy}{dx} &= (-2)(1) + (-2)(4) \\ &= -2 + -8 \\ &= -10\end{aligned}$$

Slope of the tangent at $(1, -2)$ is -10 .

The equation is $y + 2 = -10(x - 1)$;
 $10x + y - 8 = 0$.

7. a. $y = 2(x - 29)(x + 1)$

$$\begin{aligned}\frac{dy}{dx} &= 2(x - 29)(1) + 2(1)(x + 1) \\ 2x - 58 + 2x + 2 &= 0 \\ 4x - 56 &= 0 \\ 4x &= 56 \\ x &= 14\end{aligned}$$

Point of horizontal tangency is $(14, -450)$.

b. $y = (x^2 + 2x + 1)(x^2 + 2x + 1)$

$$\begin{aligned}&= (x^2 + 2x + 1)^2 \\ \frac{dy}{dx} &= 2(x^2 + 2x + 1)(2x + 2) \\ (x^2 + 2x + 1)(2x + 2) &= 0 \\ 2(x + 1)(x + 1)(x + 1) &= 0 \\ x &= -1\end{aligned}$$

Point of horizontal tangency is $(-1, 0)$.

8. a. $y = (x + 1)^3(x + 4)(x - 3)^2$

$$\begin{aligned}\frac{dy}{dx} &= 3(x + 1)^2(x + 4)(x - 3)^2 \\ &\quad + (x + 1)^3(1)(x - 3)^2 \\ &\quad + (x + 1)^3(x + 4)[2(x - 3)]\end{aligned}$$

b. $y = x^2(3x^2 + 4)^2(3 - x^3)^4$

$$\begin{aligned}\frac{dy}{dx} &= 2x(3x^2 + 4)^2(3 - x^3)^4 \\ &\quad + x^2[2(3x^2 + 4)(6x)](3 - x^3)^4 \\ &\quad + x^2(3x^2 + 4)^2[4(3 - x^3)^3(-3x^2)]\end{aligned}$$

9. $V(t) = 75\left(1 - \frac{t}{24}\right)^2, 0 \leq t \leq 24$

$$75 \text{ L} \times 60\% = 45 \text{ L}$$

$$\text{Set } \frac{45}{75} = \left(1 - \frac{t}{24}\right)^2$$

$$\pm \sqrt{\frac{3}{5}} = 1 - \frac{t}{24}$$

$$t = \left(\pm \sqrt{\frac{3}{5}} - 1\right)(-24)$$

$$t \doteq 42.590 \text{ (inadmissible)} \text{ or } t \doteq 5.4097$$

$$V(t) = 75\left(1 - \frac{t}{24}\right)^2$$

$$V(t) = 75\left(1 - \frac{t}{24}\right)\left(1 - \frac{t}{24}\right)$$

$$\begin{aligned}V'(t) &= 75\left[\left(1 - \frac{t}{24}\right)\left(-\frac{1}{24}\right) \right. \\ &\quad \left. + \left(-\frac{1}{24}\right)\left(1 - \frac{t}{24}\right)\right] \\ &= (75)(2)\left(1 - \frac{t}{24}\right)\left(-\frac{1}{24}\right)\end{aligned}$$

$$V'(5.4097) = -4.84 \text{ L/h}$$

10. Determine the point of tangency, and then find the negative reciprocal of the slope of the tangent. Use this information to find the equation of the normal.

$$\begin{aligned}h(x) &= 2x(x + 1)^3(x^2 + 2x + 1)^2 \\ h'(x) &= 2(x + 1)^3(x^2 + 2x + 1)^2 \\ &\quad + (2x)(3)(x + 1)^2(x^2 + 2x + 1)^2 \\ &\quad + 2x(x + 1)^32(x^2 + 2x + 1)(2x + 2)\end{aligned}$$

$$\begin{aligned}h'(-2) &= 2(-1)^3(1)^2 \\ &\quad + 2(-2)(3)(-1)^2(1)^2 \\ &\quad + 2(-2)(-1)^3(2)(1)(-2) \\ &= -2 - 12 - 16 \\ &= -30\end{aligned}$$

11.

a. $f(x) = g_1(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n(x)$

$$\begin{aligned}f'(x) &= g_1'(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n(x) \\ &\quad + g_1(x)g_2'(x)g_3(x) \dots g_{n-1}(x)g_n(x) \\ &\quad + g_1(x)g_2(x)g_3'(x) \dots g_{n-1}(x)g_n(x) \\ &\quad + \dots + g_1(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n'(x)\end{aligned}$$

b. $f(x) = (1 + x)(1 + 2x)(1 + 3x) \dots (1 + nx)$

$$\begin{aligned}f'(x) &= 1(1 + 2x)(1 + 3x) \dots (1 + nx) \\ &\quad + (1 + x)(2)(1 + 3x) \dots (1 + nx) \\ &\quad + (1 + x)(1 + 2x)(3) \dots (1 + nx) \\ &\quad + \dots + (1 + x)(1 + 2x)(1 + 3x) \\ &\quad \dots (n)\end{aligned}$$

$$\begin{aligned}f'(0) &= 1(1)(1)(1) \dots (1) \\ &\quad + 1(2)(1)(1) \dots (1) \\ &\quad + 1(1)(3)(1) \dots (1) \\ &\quad + \dots + (1)(1)(1) \dots (n) \\ &= 1 + 2 + 3 + \dots + n\end{aligned}$$

$$f'(0) = \frac{n(n + 1)}{2}$$

12. $f(x) = ax^2 + bx + c$

$$f'(x) = 2ax + b \quad (1)$$

Horizontal tangent at $(-1, -8)$

$$f'(x) = 0 \text{ at } x = -1$$

$$-2a + b = 0$$

Since $(2, 19)$ lies on the curve,

$$4a + 2b + c = 19 \quad (2)$$

Since $(-1, -8)$ lies on the curve,

$$a - b + c = -8 \quad (3)$$

$$4a + 2b + c = 19$$

$$-3a - 3b = -27$$

$$a + b = 9$$

$$-2a + b = 0$$

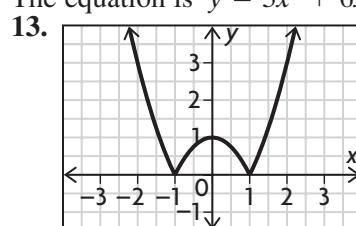
$$3a = 9$$

$$a = 3, b = 6$$

$$3 - 6 + c = -8$$

$$c = -5$$

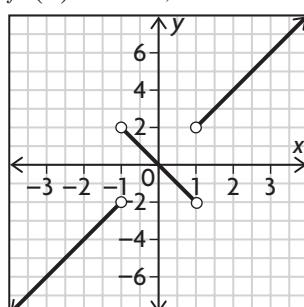
The equation is $y = 3x^2 + 6x - 5$.



a. $x = 1$ or $x = -1$

b. $f'(x) = 2x, x < -1$ or $x > 1$

$$f'(x) = -2x, -1 < x < 1$$



c. $f'(-2) = 2(-2) = -4$
 $f'(0) = -2(0) = 0$
 $f'(3) = 2(3) = 6$

14. $y = \frac{16}{x^2} - 1$

$$\frac{dy}{dx} = -\frac{32}{x^3}$$

Slope of the line is 4.

$$-\frac{32}{x^3} = 4$$

$$4x^3 = -32$$

$$x^3 = -8$$

$$x = -2$$

$$y = \frac{16}{4} - 1$$

$$= 3$$

Point is at $(-2, 3)$.

Find intersection of line and curve:

$$4x - y + 11 = 0$$

$$y = 4x + 11$$

Substitute,

$$4x + 11 = \frac{16}{x^2} - 1$$

$$4x^3 + 11x^2 = 16 - x^2 \text{ or } 4x^3 + 12x^2 - 16 = 0.$$

Let $x = -2$

$$\text{RS} = 4(-2)^3 + 12(-2)^2 - 16$$

$$= 0$$

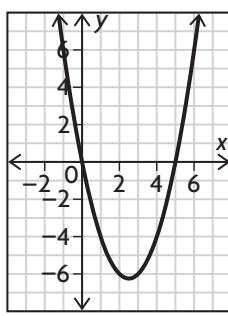
Since $x = -2$ satisfies the equation, therefore it is a solution.

When $x = -2$, $y = 4(-2) + 11 = 3$.

Intersection point is $(-2, 3)$. Therefore, the line is tangent to the curve.

Mid-Chapter Review, pp. 92–93

1. a.



b. $f'(x) = \lim_{h \rightarrow 0} \frac{((x+h)^2 - 5(x+h)) - (x^2 - 5x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - 5x - 5h - x^2 + 5x}{h}$$

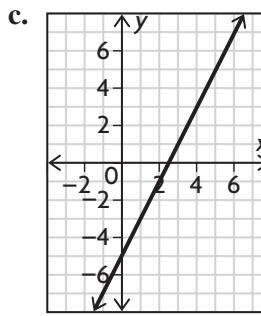
$$= \lim_{h \rightarrow 0} \frac{h^2 + 2hx - 5h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(h + 2x - 5)}{h}$$

$$= 2x - 5$$

Use the derivative function to calculate the slopes of the tangents.

x	Slope of Tangent $f'(x)$
0	-5
1	-3
2	-1
3	1
4	3
5	5



c. $f(x)$ is quadratic; $f'(x)$ is linear.

2. a. $f'(x) = \lim_{h \rightarrow 0} \frac{(6(x+h) + 15) - (6x + 15)}{h}$

$$= \lim_{h \rightarrow 0} \frac{6h}{h}$$

$$= \lim_{h \rightarrow 0} 6$$

$$= 6$$

b. $f'(x) = \lim_{h \rightarrow 0} \frac{(2(x+h)^2 - 4) - (2x^2 - 4)}{h}$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{((x+h) - x)((x+h) + x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \rightarrow 0} 2(2x+h)$$

$$= 4x$$

$$c. f'(x) = \lim_{h \rightarrow 0} \frac{\frac{5}{(x+h)+5} - \frac{5}{x+5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5(x+5) - 5((x+h)+5)}{((x+h)+5)(x+5)h}$$

$$= \lim_{h \rightarrow 0} \frac{-5h}{((x+h)+5)(x+5)h}$$

$$= \lim_{h \rightarrow 0} \frac{-5}{((x+h)+5)(x+5)}$$

$$= \frac{-5}{(x+5)^2}$$

$$\mathbf{d. } f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)-2} - \sqrt{x-2}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{\sqrt{(x+h)-2} - \sqrt{x-2}}{h} \right]$$

$$\quad \times \frac{\sqrt{(x+h)-2} + \sqrt{x-2}}{\sqrt{(x+h)-2} + \sqrt{x-2}}$$

$$= \lim_{h \rightarrow 0} \frac{((x+h)-2) - (x-2)}{h(\sqrt{(x+h)-2} + \sqrt{x-2})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{(x+h)-2} + \sqrt{x-2})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{(x+h)-2} + \sqrt{x-2}}$$

$$= \frac{1}{2\sqrt{x-2}}$$

3. a. $y' = 2x - 4$

When $x = 1$,

$$y' = 2(1) - 4$$

$$= -2.$$

When $x = 1$,

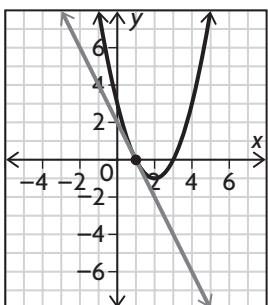
$$y = (1)^2 - 4(1) + 3$$

$$= 0.$$

Equation of the tangent line:

$$y - 0 = -2(x - 1), \text{ or } y = -2x + 2$$

b.



4. a. $\frac{dy}{dx} = 24x^3$

b. $\frac{dy}{dx} = 5x^{-\frac{1}{2}}$

$$= \frac{5}{\sqrt{x}}$$

c. $g'(x) = -6x^{-4}$

$$= -\frac{6}{x^4}$$

d. $\frac{dy}{dx} = 5 - 6x^{-3}$

$$= 5 - \frac{6}{x^3}$$

e. $\frac{dy}{dt} = 2(11t + 1)(11)$

$$= 242t + 22$$

f. $y = 1 - \frac{1}{x}$

$$= 1 - x^{-1}$$

$$\frac{dy}{dx} = x^{-2}$$

$$= \frac{1}{x^2}$$

5. f'(x) = 8x^3

$$8x^3 = 1$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^4$$

$$= \frac{1}{8}$$

Equation of the tangent line:

$$y - \frac{1}{8} = 1\left(x - \frac{1}{2}\right), \text{ or } y = x - \frac{3}{8}$$

6. a. $f'(x) = 8x - 7$

b. $f'(x) = -6x^2 + 8x + 5$

c. $f(x) = 5x^{-2} - 3x^{-3}$

$$f'(x) = -10x^{-3} + 9x^{-4}$$

$$= -\frac{10}{x^3} + \frac{9}{x^4}$$

d. $f(x) = x^{\frac{1}{3}} + x^{\frac{1}{3}}$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{3x^{\frac{2}{3}}}$$

e. $f(x) = 7x^{-2} - 3x^{\frac{1}{2}}$

$$f'(x) = -14x^{-3} - \frac{3}{2}x^{-\frac{1}{2}}$$

$$= -\frac{14}{x^3} - \frac{3}{2x^{\frac{1}{2}}}$$

f. $f'(x) = 4x^{-2} + 5$

$$= \frac{4}{x^2} + 5$$

7. a. $y' = -6x + 6$

When $x = 1$,

$$\begin{aligned}y' &= -6(1) + 6 \\&= 0.\end{aligned}$$

When $x = 1$,

$$\begin{aligned}y &= -3(1^2) + 6(1) + 4 \\&= 7.\end{aligned}$$

Equation of the tangent line:

$$y - 7 = 0(x - 1), \text{ or}$$

$$y = 7$$

b. $y = 3 - 2x^{\frac{1}{2}}$

$$\begin{aligned}y' &= -x^{-\frac{1}{2}} \\&= \frac{-1}{\sqrt{x}}\end{aligned}$$

When $x = 9$,

$$\begin{aligned}y' &= \frac{-1}{\sqrt{9}} \\&= -\frac{1}{3}.\end{aligned}$$

When $x = 9$,

$$\begin{aligned}y &= 3 - 2\sqrt{9} \\&= -3.\end{aligned}$$

Equation of the tangent line:

$$y - (-3) = -\frac{1}{3}(x - 9), \text{ or } y = -\frac{1}{3}x$$

c. $f'(x) = -8x^3 + 12x^2 - 4x - 8$

$$\begin{aligned}f'(3) &= -8(3)^3 + 12(3)^2 - 4(3) - 8 \\&= -216 + 108 - 12 - 8 \\&= -218 \\f(3) &= -2(3)^4 + 4(3)^3 - 2(3)^2 - 8(3) + 9 \\&= -162 + 108 - 18 - 24 + 9 \\&= -87\end{aligned}$$

Equation of the tangent line:

$$y - (-87) = -128(x - 3), \text{ or}$$

$$y = -128x + 297$$

8. a. $f'(x) = \frac{d}{dx}(4x^2 - 9x)(3x^2 + 5)$

$$\begin{aligned}&+ (4x^2 - 9x)\frac{d}{dx}(3x^2 + 5) \\&= (8x - 9)(3x^2 + 5) + (4x^2 - 9x)(6x) \\&= 24x^3 - 27x^2 + 40x - 45 \\&\quad + 24x^3 - 54x^2 \\&= 48x^3 - 81x^2 + 40x - 45\end{aligned}$$

b. $f'(t) = \frac{d}{dt}(-3t^2 - 7t + 8)(4t - 1)$

$$\begin{aligned}&+ (-3t^2 - 7t + 8)\frac{d}{dt}(4t - 1) \\&= (-6t - 7)(4t - 1) \\&\quad + (-3t^2 - 7t + 8)(4)\end{aligned}$$

$$\begin{aligned}&= -24t^2 - 28t + 6t + 7 - 12t^2 - 28t + 32 \\&= -36t^2 - 50t + 39\end{aligned}$$

c. $\frac{dy}{dx} = \frac{d}{dx}(3x^2 + 4x - 6)(2x^2 - 9)$

$$\begin{aligned}&+ (3x^2 + 4x - 6)\frac{d}{dx}(2x^2 - 9) \\&= (6x + 4)(2x^2 - 9) + (3x^2 + 4x - 6)(4x) \\&= 12x^3 - 54x + 8x^2 - 36 + 12x^3 \\&\quad + 16x^2 - 24x \\&= 24x^3 + 24x^2 - 78x - 36\end{aligned}$$

d. $\frac{dy}{dx} = \frac{d}{dx}(3 - 2x^3)^2(3 - 2x^3)$

$$+ (3 - 2x^3)^2\frac{d}{dx}(3 - 2x^3)$$

$$\begin{aligned}&= \left[\frac{d}{dx}(3 - 2x^3)(3 - 2x^3) \right. \\&\quad \left. + (3 - 2x^3)\frac{d}{dx}(3 - 2x^3) \right](3 - 2x^3) \\&+ (3 - 2x^3)^2(-6x^2) \\&= [2(-6x^2)(3 - 2x^3)](3 - 2x^3) \\&\quad + (3 - 2x^3)^2(-6x^2) \\&= 3(3 - 2x^3)^2(-6x^2) \\&= (3 - 2x^3)^2(-18x^2) \\&= (9 - 12x^3 + 4x^6)(-18x^2) \\&= -162x^2 + 216x^5 - 72x^8\end{aligned}$$

9. $y' = \frac{d}{dx}(5x^2 + 9x - 2)(-x^2 + 2x + 3)$

$$\begin{aligned}&+ (5x^2 + 9x - 2)\frac{d}{dx}(-x^2 + 2x + 3) \\&= (10x + 9)(-x^2 + 2x + 3) \\&\quad + (5x^2 + 9x - 2)(2 - 2x)\end{aligned}$$

$$\begin{aligned}y'(1) &= (10(1) + 9)(-(1)^2 + 2(1) + 3) \\&\quad + (5(1)^2 + 9(1) - 2)(2 - 2(1)) \\&= (19)(4) \\&= 76\end{aligned}$$

Equation of the tangent line:

$$y - 76 = 76(x - 1), \text{ or } 76x - y - 28 = 0$$

10. $\frac{dy}{dx} = 2\frac{d}{dx}(x - 1)(5 - x)$

$$\begin{aligned}&+ 2(x - 1)\frac{d}{dx}(5 - x) \\&= 2(5 - x) - 2(x - 1) \\&= 12 - 4x\end{aligned}$$

The tangent line is horizontal when $\frac{dy}{dx} = 0$.

$$12 - 4x = 0$$

$$12 = 4x$$

$$x = 3$$

When $x = 3$,

$$y = 2((3) - 1)(5 - (3)) \\ = 8.$$

Point where tangent line is horizontal: $(3, 8)$

$$\begin{aligned} \text{11. } \frac{dy}{dx} &= \lim_{h \rightarrow 0} \left[\frac{(5(x+h)^2 - 8(x+h) + 4)}{h} \right. \\ &\quad \left. - \frac{(5x^2 - 8x + 4)}{h} \right] \\ &= \lim_{h \rightarrow 0} \frac{5(x+h)^2 - 5x^2 - 8h}{h} \\ &= \lim_{h \rightarrow 0} \frac{5((x+h)-x)((x+h)+x) - 8h}{h} \\ &= \lim_{h \rightarrow 0} \frac{5h(2x+h) - 8h}{h} \\ &= \lim_{h \rightarrow 0} (5(2x+h) - 8) \\ &= 10x - 8 \end{aligned}$$

$$\text{12. } V(t) = 500 \left(1 - \frac{t}{90}\right)^2. 0 \leq t \leq 90$$

a. After 1 h, $t = 60$, and the volume is

$$\begin{aligned} V(60) &= 500 \left(1 - \frac{60}{90}\right)^2 \\ &= 500 \left(\frac{30}{90}\right)^2 \\ &= 500 \left(\frac{1}{3}\right)^2 \\ &= \frac{500}{9} \text{ L} \end{aligned}$$

$$\text{b. } V(0) = 500(1 - 0)^2 = 500 \text{ L}$$

$$V(60) = \frac{500}{9} \text{ L}$$

The average rate of change of volume with respect to time from 0 min to 60 min is

$$\begin{aligned} \frac{\Delta V}{\Delta t} &= \frac{\frac{500}{9} - 500}{60 - 0} \\ &= \frac{-\frac{8}{9}(500)}{60} \\ &= -\frac{200}{27} \text{ L/min} \end{aligned}$$

c. Calculate $V'(t)$:

$$\begin{aligned} V'(t) &= \lim_{h \rightarrow 0} \frac{V(t+h) - V(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{500 \left(1 - \frac{t+h}{90}\right)^2 - 500 \left(1 - \frac{t}{90}\right)^2}{h} \end{aligned}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{500 \left(1 - \frac{t+h}{90} - 1 + \frac{t}{90}\right)}{h} \\ &\quad \times \frac{\left(1 - \frac{t+h}{90} + 1 - \frac{t}{90}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{500 \left(-\frac{h}{90}\right) \left(2 - \frac{2t+h}{90}\right)}{h} \\ &= \lim_{h \rightarrow 0} -\frac{500}{90} \left(2 - \frac{2t+h}{90}\right) \\ &= \frac{-50}{9} \left(2 - \frac{2t}{90}\right) \\ &= \frac{-900 + 10t}{81} \end{aligned}$$

Then,

$$\begin{aligned} V'(30) &= \frac{-900 + 10(30)}{81} \\ &= -\frac{200}{27} \text{ L/min} \end{aligned}$$

$$\text{13. } V(r) = \frac{4}{3}\pi r^3$$

$$\begin{aligned} \text{a. } V(10) &= \frac{4}{3}\pi(10)^3 & V(15) &= \frac{4}{3}\pi(15)^3 \\ &= \frac{4}{3}\pi(1000) & &= \frac{4}{3}\pi(3375) \\ &= \frac{4000}{3}\pi & &= 4500\pi \end{aligned}$$

Then, the average rate of change of volume with respect to radius is

$$\begin{aligned} \frac{\Delta V}{\Delta r} &= \frac{4500\pi - \frac{4000}{3}\pi}{15 - 10} \\ &= \frac{500\pi(9 - \frac{8}{3})}{5} \\ &= 100\pi \left(\frac{19}{3}\right) \\ &= \frac{1900}{3}\pi \text{ cm}^3/\text{cm} \end{aligned}$$

b. First calculate $V'(r)$:

$$\begin{aligned} V'(r) &= \lim_{h \rightarrow 0} \frac{V(r+h) - V(r)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi[(r+h)^3 - r^3]}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi(r^3 + 3r^2h + 3rh^2 + h^3 - r^3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi(3r^2h + 3rh^2 + h^3)}{h} \end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{4}{3} \pi (3r^2 + 3rh + h^2) \\
&= \frac{4}{3} \pi (3r^2 + 3r(0) + (0)^2) \\
&= 4\pi r^2
\end{aligned}$$

$$\begin{aligned}
\text{Then, } V'(8) &= 4\pi(8)^2 \\
&= 4\pi(64) \\
&= 256\pi \text{ cm}^3/\text{cm}
\end{aligned}$$

14. This statement is always true. A cubic polynomial function will have the form $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$. So the derivative of this cubic is $f'(x) = 3ax^2 + 2bx + c$, and since $3a \neq 0$, this derivative is a quadratic polynomial function. For example, if $f(x) = x^3 + x^2 + 1$, we get

$$f'(x) = 3x^2 + 2x,$$

and if

$$f(x) = 2x^3 + 3x^2 + 6x + 2,$$

we get

$$f'(x) = 6x^2 + 6x + 6$$

$$\mathbf{15. } y = \frac{x^{2a+3b}}{x^{a-b}}, a, b \in \mathbb{I}$$

Simplifying,

$$y = x^{2a+3b-(a-b)} = x^{a+4b}$$

Then,

$$y' = (a+4b)^{a+4b-1}$$

$$\mathbf{16. a. } f(x) = -6x^3 + 4x - 5x^2 + 10$$

$$f'(x) = -18x^2 + 4 - 10x$$

$$\text{Then, } f'(x) = -18(3)^2 + 4 - 10(3)$$

$$= -188$$

b. $f'(3)$ is the slope of the tangent line to $f(x)$ at $x = 3$ and the rate of change in the value of $f(x)$ with respect to x at $x = 3$.

$$\mathbf{17. a. } P(t) = 100 + 120t + 10t^2 + 2t^3$$

$$P(t) = 100 + 120t + 10t^2 + 2t^3$$

$$\begin{aligned}
P(0) &= 100 + 120(0) + 10(0)^2 + 2(0)^3 \\
&= 100 \text{ bacteria}
\end{aligned}$$

b. At 5 h, the population is

$$\begin{aligned}
P(5) &= 100 + 120(5) + 10(5)^2 + 2(5)^3 \\
&= 1200 \text{ bacteria}
\end{aligned}$$

$$\mathbf{c. } P'(t) = 120 + 20t + 6t^2$$

At 5 h, the colony is growing at

$$\begin{aligned}
P'(5) &= 120 + 20(5) + 6(5)^2 \\
&= 370 \text{ bacteria/h}
\end{aligned}$$

$$\mathbf{18. } C(t) = \frac{100}{t}, t > 2$$

Simplifying, $C(t) = 100t^{-1}$.

$$\text{Then, } C'(t) = -100t^{-2} = -\frac{100}{t^2}.$$

$$\begin{array}{lll}
C'(5) & C'(50) & C'(100) \\
= -\frac{100}{(5)^2} & = -\frac{100}{(50)^2} & = -\frac{100}{(100)^2} \\
= -\frac{100}{25} & = -\frac{100}{2500} & = -\frac{1}{100} \\
= -4 & = -0.04 & = -0.01
\end{array}$$

These are the rates of change of the percentage with respect to time at 5, 50, and 100 min. The percentage of carbon dioxide that is released per unit time from the pop is decreasing. The pop is getting flat.

2.4 The Quotient Rule, pp. 97–98

1. For x, a, b real numbers,

$$x^a x^b = x^{a+b}$$

For example,

$$x^9 x^{-6} = x^3$$

Also,

$$(x^a)^b = x^{ab}$$

For example,

$$(x^2)^3 = x^6$$

Also,

$$\frac{x^a}{x^b} = x^{a-b}, x \neq 0$$

For example,

$$\frac{x^5}{x^3} = x^2$$

2.

Function	Rewrite	Differentiate and Simplify, If Necessary
$f(x) = \frac{x^2 + 3x}{x}, x \neq 0$	$f(x) = x + 3$	$f'(x) = 1$
$g(x) = \frac{3x^{\frac{5}{3}}}{x}, x \neq 0$	$g(x) = 3x^{\frac{2}{3}}$	$g'(x) = 2x^{-\frac{1}{3}}$
$h(x) = \frac{1}{10x^5}, x \neq 0$	$h(x) = \frac{1}{10}x^{-5}$	$h'(x) = \frac{-1}{2}x^{-6}$
$y = \frac{8x^3 + 6x}{2x}, x \neq 0$	$y = 4x^2 + 3$	$\frac{dy}{dx} = 8x$
$s = \frac{t^2 - 9}{t - 3}, t \neq 3$	$s = t + 3$	$\frac{ds}{dt} = 1$

3. In the previous problem, all of these rational examples could be differentiated via the power rule after a minor algebraic simplification.

A second approach would be to rewrite a rational example

$$h(x) = \frac{f(x)}{g(x)}$$

using the exponent rules as

$$h(x) = f(x)(g(x))^{-1},$$

and then apply the product rule for differentiation (together with the power of a function rule to find $h'(x)$).

A third (and perhaps easiest) approach would be to just apply the quotient rule to find $h'(x)$.

$$\textbf{4. a. } h'(x) = \frac{(x+1)(1) - x(1)}{(x+1)^2}$$

$$= \frac{1}{(x+1)^2}$$

$$\textbf{b. } h'(t) = \frac{(t+5)(2) - (2t-3)(1)}{(t+5)^2}$$

$$= \frac{13}{(t+5)^2}$$

$$\textbf{c. } h'(x) = \frac{(2x^2-1)(3x^2) - x^3(4x)}{(2x^2-1)^2}$$

$$= \frac{2x^4 - 3x^2}{(2x^2-1)^2}$$

$$\textbf{d. } h'(x) = \frac{(x^2+3)(0) - 1(2x)}{(x^2+3)^2}$$

$$= \frac{-2x}{(x^2+3)^2}$$

$$\textbf{e. } y = \frac{x(3x+5)}{(1-x^2)} = \frac{3x^2 + 5x}{1-x^2}$$

$$\frac{dy}{dx} = \frac{(6x+5)(1-x^2) - (3x^2+5x)(-2x)}{(1-x^2)^2}$$

$$= \frac{6x+5 - 6x^3 - 5x^2 + 6x^3 + 10x^2}{(1-x^2)^2}$$

$$= \frac{5x^2 + 6x + 5}{(1-x^2)^2}$$

$$\textbf{f. } \frac{dy}{dx} = \frac{(x^2+3)(2x-1) - (x^2-x+1)(2x)}{(x^2+3)^2}$$

$$= \frac{2x^3 + 6x - x^2 - 3 - 2x^3 + 2x^2 - 2x}{(x^2+3)^2}$$

$$= \frac{x^2 + 4x - 3}{(x^2+3)^2}$$

$$\textbf{5. a. } y = \frac{3x+2}{x+5}, x = -3$$

$$\frac{dy}{dx} = \frac{(x+5)(3) - (3x+2)(1)}{(x+5)^2}$$

At $x = -3$:

$$\frac{dy}{dx} = \frac{(2)(3) - (-7)(1)}{(2)^2}$$

$$= \frac{13}{4}$$

$$\textbf{b. } y = \frac{x^3}{x^2+9}, x = 1$$

$$\frac{dy}{dx} = \frac{(x^2+9)(3x^2) - (x^3)(2x)}{(x^2+9)^2}$$

At $x = 1$:

$$\frac{dy}{dx} = \frac{(10)(3) - (1)(2)}{(10)^2}$$

$$= \frac{28}{100}$$

$$= \frac{7}{25}$$

$$\textbf{c. } y = \frac{x^2-25}{x^2+25}, x = 2$$

$$\frac{dy}{dx} = \frac{2x(x^2+25) - (x^2-25)(2x)}{(x^2+25)^2}$$

At $x = 2$:

$$\frac{dy}{dx} = \frac{4(29) - (-21)(4)}{(29)^2}$$

$$= \frac{116 + 84}{29^2}$$

$$= \frac{200}{841}$$

$$\textbf{d. } y = \frac{(x+1)(x+2)}{(x-1)(x-2)}, x = 4$$

$$= \frac{x^2 + 3x + 2}{x^2 - 3x + 2}$$

$$\frac{dy}{dx} = \frac{(2x+3)(x^2-3x+2)}{(x-1)^2(x-2)^2}$$

$$- \frac{(x^2+3x+2)(2x-3)}{(x-1)^2(x-2)^2}$$

At $x = 4$:

$$\frac{dy}{dx} = \frac{(11)(6) - (30)(5)}{(9)(4)}$$

$$= -\frac{84}{36}$$

$$= -\frac{7}{3}$$

6. $y = \frac{x^3}{x^2 - 6}$

$$\frac{dy}{dx} = \frac{3x^2(x^2 - 6) - x^3(2x)}{(x^2 - 6)^2}$$

At $(3, 9)$:

$$\begin{aligned}\frac{dy}{dx} &= \frac{3(9)(3) - (27)(6)}{(3)^2} \\ &= 9 - 18 \\ &= -9\end{aligned}$$

The slope of the tangent to the curve at $(3, 9)$ is -9 .

7. $y = \frac{3x}{x - 4}$

$$\frac{dy}{dx} = \frac{3(x - 4) - 3x}{(x - 4)^2} = -\frac{12}{(x - 4)^2}$$

Slope of the tangent is $-\frac{12}{25}$.

Therefore, $\frac{12}{(x - 4)^2} = \frac{12}{25}$

$$x - 4 = 5 \text{ or } x - 4 = -5$$

$$x = 9 \text{ or } x = -1$$

Points are $(9, \frac{27}{5})$ and $(-1, \frac{3}{5})$.

8. $f(x) = \frac{5x + 2}{x + 2}$

$$f'(x) = \frac{(x + 2)(5) - (5x + 2)(1)}{(x + 2)^2}$$

$$f'(x) = \frac{8}{(x + 2)^2}$$

Since $(x + 2)^2$ is positive or zero for all $x \in \mathbf{R}$,

$\frac{8}{(x + 2)^2} > 0$ for $x \neq -2$. Therefore, tangents to

the graph of $f(x) = \frac{5x + 2}{x + 2}$ do not have a negative slope.

9. a. $y = \frac{2x^2}{x - 4}, x \neq 4$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x - 4)(4x) - (2x^2)(1)}{(x - 4)^2} \\ &= \frac{4x^2 - 16x - 2x^2}{(x - 4)^2} \\ &= \frac{2x^2 - 16x}{(x - 4)^2} \\ &= \frac{2x(x - 8)}{(x - 4)^2}\end{aligned}$$

Curve has horizontal tangents when $\frac{dy}{dx} = 0$, or when $x = 0$ or 8 . At $x = 0$:

$$\begin{aligned}y &= \frac{0}{-4} \\ &= 0\end{aligned}$$

At $x = 8$:

$$\begin{aligned}y &= \frac{2(8)^2}{4} \\ &= 32\end{aligned}$$

So the curve has horizontal tangents at the points $(0, 0)$ and $(8, 32)$.

b. $y = \frac{x^2 - 1}{x^2 + x - 2}$

$$\begin{aligned}&= \frac{(x - 1)(x + 1)}{(x + 2)(x - 1)} \\ &= \frac{x + 1}{x + 2}, x \neq 1\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x + 2) - (x + 1)}{(x + 2)^2} \\ &= \frac{1}{(x + 2)^2}\end{aligned}$$

Curve has horizontal tangents when $\frac{dy}{dx} = 0$.

No value of x will produce a slope of 0, so there are no horizontal tangents.

10. $p(t) = 1000 \left(1 + \frac{4t}{t^2 + 50}\right)$

$$\begin{aligned}p'(t) &= 1000 \left(\frac{4(t^2 + 50) - 4t(2t)}{(t^2 + 50)^2}\right) \\ &= \frac{1000(200 - 4t^2)}{(t^2 + 50)^2}\end{aligned}$$

$$p'(1) = \frac{1000(196)}{(51)^2} = 75.36$$

$$p'(2) = \frac{1000(184)}{(54)^2} = 63.10$$

Population is growing at a rate of 75.4 bacteria per hour at $t = 1$ and at 63.1 bacteria per hour at $t = 2$.

11. $y = \frac{x^2 - 1}{3x}$

$$= \frac{1}{3}x - \frac{1}{3}x^{-1}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{3} + \frac{1}{3}x^{-2} \\ &= \frac{1}{3} + \frac{1}{3x^2}\end{aligned}$$

At $x = 2$:

$$\begin{aligned}y &= \frac{(2)^2 - 1}{3(2)} \\ &= \frac{1}{2}\end{aligned}$$

and

$$\frac{dy}{dx} = \frac{1}{3} + \frac{1}{3(2)^2}$$

$$= \frac{1}{3} + \frac{1}{12}$$

$$= \frac{5}{12}$$

So the equation of the tangent to the curve at $x = 2$ is:

$$y - \frac{1}{2} = \frac{5}{12}(x - 2), \text{ or } 5x - 12y - 4 = 0.$$

12. a. $s(t) = \frac{10(6-t)}{t+3}, 0 \leq t \leq 6, t = 0,$

$$s(0) = 20$$

The boat is initially 20 m from the dock.

b. $v(t) = s'(t) = 10 \left[\frac{(t+3)(-1) - (6-t)(1)}{(t+3)^2} \right]$

$$v(t) = \frac{-90}{(t+3)^2}$$

At $t = 0$, $v(0) = -10$, the boat is moving towards the dock at a speed of 10 m/s. When $s(t) = 0$, the boat will be at the dock.

$$\frac{10(6-t)}{t+3} = 0, t = 6.$$

$$v(6) = \frac{-90}{9^2} = -\frac{10}{9}$$

The speed of the boat when it bumps into the dock is $\frac{10}{9}$ m/s.

13. a. i. $t = 0$

$$r(0) = \frac{1 + 2(0)}{1 + 0}$$

$$= 1 \text{ cm}$$

ii. $\frac{1 + 2t}{1 + t} = 1.5$

$$1 + 2t = 1.5(1 + t)$$

$$1 + 2t = 1.5 + 1.5t$$

$$0.5t = 0.5$$

$$t = 1 \text{ s}$$

iii. $r'(t) = \frac{(1+t)(2) - (1+2t)(1)}{(1+t)^2}$

$$= \frac{2 + 2t - 1 - 2t}{(1+t)^2}$$

$$= \frac{1}{(1+t)^2}$$

$$r'(1.5) = \frac{1}{(1+1)^2}$$

$$= \frac{1}{4}$$

$$= 0.25 \text{ cm/s}$$

b. No, the radius will never reach 2 cm, because $y = 2$ is a horizontal asymptote of the graph of the function. Therefore, the radius approaches but never equals 2 cm.

$$14. f(x) = \frac{ax + b}{(x-1)(x-4)}$$

$$f'(x) = \frac{(x-1)(x-4)(a)}{(x-1)^2(x-4)^2}$$

$$= \frac{(ax+b)\frac{d}{dx}[(x-1)(x-4)]}{(x-1)^2(x-4)^2}$$

$$= \frac{(x-1)(x-4)(a)}{(x-1)^2(x-4)^2}$$

$$= \frac{(ax+b)[(x-1) + (x-4)]}{(x-1)^2(x-4)^2}$$

$$= \frac{(x^2 - 5x + 4)(a) - (ax+b)(2x-5)}{(x-1)^2(x-4)^2}$$

$$= \frac{-ax^2 - 2bx + 4a + 5b}{(x-1)^2(x-4)^2}$$

Since the point $(2, -1)$ is on the graph (as it's on the tangent line) we know that

$$-1 = f(2)$$

$$= \frac{2a + b}{(1)(-2)}$$

$$2 = 2a + b$$

$$b = 2 - 2a$$

Also, since the tangent line is horizontal at $(2, -1)$, we know that

$$0 = f'(2)$$

$$= \frac{-a(2)^2 - 2b(2) + 4a + 5b}{(1)^2(-2)^2}$$

$$b = 0$$

$$0 = 2 - 2a$$

$$a = 1$$

So we get

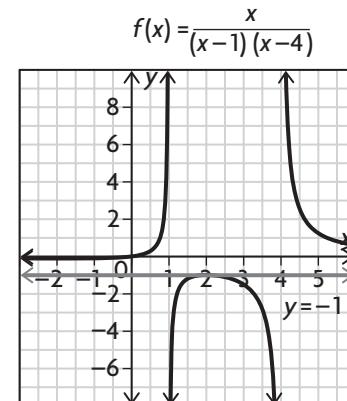
$$f(x) = \frac{x}{(x-1)(x-4)}$$

Since the tangent line is horizontal at the point

$(2, -1)$, the equation of this tangent line is

$$y - (-1) = 0(x - 2), \text{ or } y = -1$$

Here are the graphs of both $f(x)$ and this horizontal tangent line:



$$15. c'(t) = \frac{(2t^2 + 7)(5) - (5t)(4t)}{(2t^2 + 7)^2}$$

$$= \frac{10t^2 + 35 - 20t^2}{(2t^2 + 7)^2}$$

$$= \frac{-10t^2 + 35}{(2t^2 + 7)^2}$$

Set $c'(t) = 0$ and solve for t .

$$\begin{aligned} \frac{-10t^2 + 35}{(2t^2 + 7)^2} &= 0 \\ -10t^2 + 35 &= 0 \\ 10t^2 &= 35 \\ t^2 &= 3.5 \\ t &= \pm\sqrt{3.5} \\ t &\doteq \pm 1.87 \end{aligned}$$

To two decimal places, $t = -1.87$ or $t = 1.87$, because $s'(t) = 0$ for these values. Reject the negative root in this case because time is positive ($t \geq 0$). Therefore, the concentration reaches its maximum value at $t = 1.87$ hours.

16. When the object changes direction, its velocity changes sign.

$$\begin{aligned} s'(t) &= \frac{(t^2 + 8)(1) - t(2t)}{(t^2 + 8)^2} \\ &= \frac{t^2 + 8 - 2t^2}{(t^2 + 8)^2} \\ &= \frac{-t^2 + 8}{(t^2 + 8)^2} \end{aligned}$$

solve for t when $s'(t) = 0$.

$$\begin{aligned} \frac{-t^2 + 8}{(t^2 + 8)^2} &= 0 \\ -t^2 + 8 &= 0 \\ t^2 &= 8 \\ t &= \pm\sqrt{8} \\ t &\doteq \pm 2.83 \end{aligned}$$

To two decimal places, $t = 2.83$ or $t = -2.83$, because $s'(t) = 0$ for these values. Reject the negative root because time is positive ($t \geq 0$). The object changes direction when $t = 2.83$ s.

$$\begin{aligned} 17. f(x) &= \frac{ax + b}{cx + d}, x \neq -\frac{d}{c} \\ f'(x) &= \frac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} \\ f'(x) &= \frac{ad - bc}{(cx + d)^2} \end{aligned}$$

For the tangents to the graph of $y = f(x)$ to have positive slopes, $f'(x) > 0$. $(cx + d)^2$ is positive for all $x \in \mathbf{R}$. $ad - bc > 0$ will ensure each tangent has a positive slope.

2.5 The Derivatives of Composite Functions, pp. 105–106

1. $f(x) = \sqrt{x}$, $g(x) = x^2 - 1$

a. $f(g(1)) = f(1 - 1)$
 $= f(0)$
 $= 0$

b. $g(f(1)) = g(1)$
 $= 0$

c. $g(f(0)) = g(0)$
 $= 0 - 1$
 $= -1$

d. $f(g(-4)) = f(16 - 1)$
 $= f(15)$
 $= \sqrt{15}$

e. $f(g(x)) = f(x^2 - 1)$
 $= \sqrt{x^2 - 1}$

f. $g(f(x)) = g(\sqrt{x})$
 $= (\sqrt{x})^2 - 1$
 $= x - 1$

2. a. $f(x) = x^2$, $g(x) = \sqrt{x}$

$(f \circ g)(x) = f(g(x))$
 $= f(\sqrt{x})$
 $= (\sqrt{x})^2$
 $= x$

Domain = $\{x \geq 0\}$

$(g \circ f)(x) = g(f(x))$
 $= g(x^2)$
 $= \sqrt{x^2}$
 $= |x|$

Domain = $\{x \in \mathbf{R}\}$

The composite functions are not equal for negative x -values (as $(f \circ g)$ is not defined for these x), but are equal for non-negative x -values.

b. $f(x) = \frac{1}{x}$, $g(x) = x^2 + 1$

$(f \circ g)(x) = f(g(x))$
 $= f(x^2 + 1)$
 $= \frac{1}{x^2 + 1}$

Domain = $\{x \in \mathbf{R}\}$

$(g \circ f)(x) = g(f(x))$
 $= g\left(\frac{1}{x}\right)$
 $= \left(\frac{1}{x}\right)^2 + 1$

$$= \frac{1}{x^2} + 1$$

$$\text{Domain} = \{x \neq 0\}$$

The composite functions are not equal here. For instance, $(f \circ g)(1) = \frac{1}{2}$ and $(g \circ f)(1) = 2$.

c. $f(x) = \frac{1}{x}, g(x) = \sqrt{x+2}$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(\sqrt{x+2}) \\ &= \frac{1}{\sqrt{x+2}}\end{aligned}$$

$$\text{Domain} = \{x > -2\}$$

$$(g \circ f)(x) = g(f(x))$$

$$\begin{aligned}&= g\left(\frac{1}{x}\right) \\ &= \sqrt{\frac{1}{x} + 2}\end{aligned}$$

The domain is all x such that

$$\frac{1}{x} + 2 \geq 0 \text{ and } x \neq 0, \text{ or equivalently}$$

$$\text{Domain} = \{x \leq -\frac{1}{2} \text{ or } x > 0\}$$

The composite functions are not equal here. For

instance, $(f \circ g)(2) = \frac{1}{2}$ and $(g \circ f)(2) = \sqrt{\frac{5}{2}}$.

3. If $f(x)$ and $g(x)$ are two differentiable functions of x , and

$$\begin{aligned}h(x) &= (f \circ g)(x) \\ &= f(g(x))\end{aligned}$$

is the composition of these two functions, then

$$h'(x) = f'(g(x)) \cdot g'(x)$$

This is known as the “chain rule” for differentiation of composite functions. For example, if $f(x) = x^{10}$ and $g(x) = x^2 + 3x + 5$, then $h(x) = (x^2 + 3x + 5)^{10}$, and so

$$\begin{aligned}h'(x) &= f'(g(x)) \cdot g'(x) \\ &= 10(x^2 + 3x + 5)^9(2x + 3)\end{aligned}$$

As another example, if $f(x) = x^{\frac{2}{3}}$ and $g(x) = x^2 + 1$, then $h(x) = (x^2 + 1)^{\frac{2}{3}}$, and so

$$h'(x) = \frac{2}{3}(x^2 + 1)^{-\frac{1}{3}}(2x)$$

4. a. $f(x) = (2x + 3)^4$
 $f'(x) = 4(2x + 3)^3(2)$
 $= 8(2x + 3)^3$

b. $g(x) = (x^2 - 4)^3$
 $g'(x) = 3(x^2 - 4)^2(2x)$
 $= 6x(x^2 - 4)^2$

c. $h(x) = (2x^2 + 3x - 5)^4$
 $h'(x) = 4(2x^2 + 3x - 5)^3(4x + 3)$

d. $f(x) = (\pi^2 - x^2)^3$
 $f'(x) = 3(\pi^2 - x^2)^2(-2x)$
 $= -6x(\pi^2 - x^2)^2$

e. $y = \sqrt{x^2 - 3}$
 $= (x^2 - 3)^{\frac{1}{2}}$
 $y' = \frac{1}{2}(x^2 - 3)^{\frac{1}{2}}(2x)$
 $= \frac{x}{\sqrt{x^2 - 3}}$

f. $f(x) = \frac{1}{(x^2 - 16)^5}$
 $= (x^2 - 16)^{-5}$
 $f'(x) = -5(x^2 - 16)^{-6}(2x)$
 $= \frac{-10x}{(x^2 - 16)^6}$

5. a. $y = -\frac{2}{x^3}$
 $= -2x^{-3}$

$$\begin{aligned}\frac{dy}{dx} &= (-2)(-3)x^{-4} \\ &= \frac{6}{x^4}\end{aligned}$$

b. $y = \frac{1}{x+1}$
 $= (x+1)^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= (-1)(x+1)^{-2}(1) \\ &= \frac{-1}{(x+1)^2}\end{aligned}$$

c. $y = \frac{1}{x^2 - 4}$
 $= (x^2 - 4)^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= (-1)(x^2 - 4)^{-2}(2x) \\ &= \frac{-2x}{(x^2 - 4)^2}\end{aligned}$$

d. $y = \frac{3}{9 - x^2} = 3(9 - x^2)^{-1}$
 $\frac{dy}{dx} = \frac{6x}{(9 - x^2)^2}$

e. $y = \frac{1}{5x^2 + x}$
 $= (5x^2 + x)^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= (-1)(5x^2 + x)^{-2}(10x + 1) \\ &= -\frac{10x + 1}{(5x^2 + x)^2}\end{aligned}$$

f. $y = \frac{1}{(x^2 + x + 1)^4}$
 $= (x^2 + x + 1)^{-4}$

$$\begin{aligned}\frac{dy}{dx} &= (-4)(x^2 + x + 1)^{-5}(2x + 1) \\ &= -\frac{8x + 4}{(x^2 + x + 1)^5}\end{aligned}$$

6. $h = g \circ f$
 $= g(f(x))$
 $h(-1) = g(f(-1))$
 $= g(1)$
 $= -4$

$$\begin{aligned}h(x) &= g(f(x)) \\ h'(x) &= g'(f(x))f'(x) \\ h'(-1) &= g'(f(-1))f'(-1) \\ &= g'(1)(-5) \\ &= (-7)(-5) \\ &= 35\end{aligned}$$

7. $f(x) = (x - 3)^2$, $g(x) = \frac{1}{x}$, $h(x) = f(g(x))$,
 $f'(x) = 2(x - 3)$, $g'(x) = -\frac{1}{x^2}$

$$\begin{aligned}h'(x) &= f'(g(x))g'(x) \\ &= f'\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) \\ &= 2\left(\frac{1}{x} - 3\right)\left(-\frac{1}{x^2}\right) \\ &= -\frac{2}{x^2}\left(\frac{1}{x} - 3\right)\end{aligned}$$

8. a. $f(x) = (x + 4)^3(x - 3)^6$

$$\begin{aligned}f'(x) &= \frac{d}{dx}[(x + 4)^3] \cdot (x - 3)^6 \\ &\quad + (x + 4)^3 \frac{d}{dx}[(x - 3)^6] \\ &= 3(x + 4)^2(x - 3)^6 \\ &\quad + (x + 4)^3(6)(x - 3)^5 \\ &= (x + 4)^2(x - 3)^5 \\ &\quad \times [3(x - 3) + 6(x + 4)] \\ &= (x + 4)^2(x - 3)^5(9x + 15)\end{aligned}$$

b. $y = (x^2 + 3)^3(x^3 + 3)^2$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}[(x^2 + 3)^3] \cdot (x^3 + 3)^2 \\ &\quad + (x^2 + 3)^3 \cdot \frac{d}{dx}[(x^3 + 3)^2] \\ &= 3(x^2 + 3)^2(2x)(x^3 + 3)^2 \\ &\quad + (x^2 + 3)^3(2)(x^3 + 3)(3x^2) \\ &= 6x(x^2 + 3)^2(x^3 + 3)[(x^3 + 3) + x(x^2 + 3)] \\ &= 6x(x^2 + 3)^2(x^3 + 3)(2x^3 + 3x + 3)\end{aligned}$$

c. $y = \frac{3x^2 + 2x}{x^2 + 1}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(6x + 2)(x^2 + 1) - (3x^2 + 2x)(2x)}{(x^2 + 1)^2} \\ &= \frac{6x^3 + 2x^2 + 6x + 2 - 6x^3 - 4x^2}{(x^2 + 1)^2} \\ &= \frac{-2x^2 + 6x + 2}{(x^2 + 1)^2}\end{aligned}$$

d. $h(x) = x^3(3x - 5)^2$

$$\begin{aligned}h'(x) &= \frac{d}{dx}[x^3] \cdot (3x - 5)^2 + x^3 \frac{d}{dx}[(3x - 5)^2] \\ &= 3x^2(3x - 5)^2 + x^3(2)(3x - 5)(3) \\ &= 3x^2(3x - 5)[(3x - 5) + 2x] \\ &= 3x^2(3x - 5)(5x - 5) \\ &= 15x^2(3x - 5)(x - 1)\end{aligned}$$

e. $y = x^4(1 - 4x^2)^3$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}[x^4](1 - 4x^2)^3 + x^4 \cdot \frac{d}{dx}[(1 - 4x^2)^3] \\ &= 4x^3(1 - 4x^2)^3 + x^4(3)(1 - 4x^2)^2(-8x) \\ &= 4x^3(1 - 4x^2)^2[(1 - 4x^2) - 6x^2] \\ &= 4x^3(1 - 4x^2)^2(1 - 10x^2)\end{aligned}$$

f. $y = \left(\frac{x^2 - 3}{x^2 + 3}\right)^4$

$$\begin{aligned}\frac{dy}{dx} &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \frac{d}{dx}\left[\frac{x^2 - 3}{x^2 + 3}\right] \\ &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \cdot \frac{(x^2 + 3)(2x) - (x^2 - 3)(2x)}{(x^2 + 3)^2} \\ &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \cdot \frac{12x}{(x^2 + 3)^2} \\ &= \frac{48x(x^2 - 3)^3}{(x^2 + 3)^5}\end{aligned}$$

9. a. $s(t) = t^{\frac{1}{3}}(4t - 5)^{\frac{2}{3}}$

$$\begin{aligned}&= t^{\frac{1}{3}}[(4t - 5)^2]^{\frac{1}{3}} \\ &= [t(4t - 5)^2]^{\frac{1}{3}} \\ &= [t(16t^2 - 40t + 25)]^{\frac{1}{3}} \\ &= (16t^3 - 40t^2 + 25t)^{\frac{1}{3}}, t = 8\end{aligned}$$

$$\begin{aligned}s'(t) &= \frac{1}{3}(16t^3 - 40t^2 + 25t)^{-\frac{2}{3}} \\&\quad \times (48t^2 - 80t + 25) \\&= \frac{(48t^2 - 80t + 25)}{3(16t^3 - 40t^2 + 25t)^{\frac{2}{3}}}\end{aligned}$$

Rate of change at $t = 8$:

$$\begin{aligned}s'(8) &= \frac{(48(8)^2 - 80(8) + 25)}{3(16(8)^3 - 40(8)^2 + 25(8))^{\frac{2}{3}}} \\&= \frac{2457}{972} \\&= \frac{91}{36}\end{aligned}$$

b. $s(t) = \left(\frac{t-\pi}{t-6\pi}\right)^{\frac{1}{3}}, t = 2\pi$

$$\begin{aligned}s'(t) &= \frac{1}{3}\left(\frac{t-\pi}{t-6\pi}\right)^{-\frac{2}{3}} \cdot \frac{d}{dt}\left[\frac{t-\pi}{t-6\pi}\right] \\&= \frac{1}{3}\left(\frac{t-6\pi}{t-\pi}\right)^{\frac{2}{3}} \cdot \frac{(t-6\pi)-(t-\pi)}{(t-6\pi)^2} \\&= \frac{1}{3}\left(\frac{t-6\pi}{t-\pi}\right)^{\frac{2}{3}} \cdot \frac{-5\pi}{(t-6\pi)^2}\end{aligned}$$

Rate of change at $t = 2\pi$:

$$\begin{aligned}s'(2\pi) &= \frac{1}{3}(-4)^{\frac{2}{3}} \cdot \frac{-5\pi}{16\pi^2} \\&= -\frac{5\sqrt[3]{2}}{24\pi}\end{aligned}$$

10. $y = (1 + x^3)^2 \quad y = 2x^6$

$$\frac{dy}{dx} = 2(1 + x^3)(3x^2) \quad \frac{dy}{dx} = 12x^5$$

For the same slope,

$$\begin{aligned}6x^2(1 + x^3) &= 12x^5 \\6x^2 + 6x^5 &= 12x^5 \\6x^2 - 6x^5 &= 0 \\6x^2(x^3 - 1) &= 0 \\x = 0 \text{ or } x &= 1.\end{aligned}$$

Curves have the same slope at $x = 0$ and $x = 1$.

11. $y = (3x - x^2)^{-2}$

$$\frac{dy}{dx} = -2(3x - x^2)^{-3}(3 - 2x)$$

At $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= -2[6 - 4]^{-3}(3 - 4) \\&= 2(2)^{-3} \\&= \frac{1}{4}\end{aligned}$$

The slope of the tangent line at $x = 2$ is $\frac{1}{4}$.

12. $y = (x^3 - 7)^5$ at $x = 2$

$$\frac{dy}{dx} = 5(x^3 - 7)^4(3x^2)$$

When $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= 5(1)^4(12) \\&= 60\end{aligned}$$

Slope of the tangent is 60.

Equation of the tangent at $(2, 1)$ is

$$y - 1 = 60(x - 2)$$

$$60x - y - 119 = 0.$$

13. a. $y = 3u^2 - 5u + 2$

$$u = x^2 - 1, x = 2$$

$$u = 3$$

$$\frac{dy}{du} = 6u - 5, \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= (6u - 5)(2x)$$

$$= (18 - 5)(4)$$

$$= 13(4)$$

$$= 52$$

b. $y = 2u^3 + 3u^2, u = x + x^{\frac{1}{2}}, x = 1$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (6u^2 + 6u)\left(1 + \frac{1}{2\sqrt{x}}\right)$$

At $x = 1$:

$$\begin{aligned}u &= 1 + 1^{\frac{1}{2}} \\&= 2\end{aligned}$$

$$\frac{dy}{dx} = (6(2)^2 + 6(2))\left(1 + \frac{1}{2\sqrt{1}}\right)$$

$$= 36 \times \frac{3}{2}$$

$$= 54$$

c. $y = u(u^2 + 3)^3, u = (x + 3)^2, x = -2$

$$\frac{dy}{du} = (u^2 + 3)^3 + 6u^2(u^2 + 3)^2, \frac{du}{dx} = 2(x + 3)$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = [7^3 + 6(4)^2][2(1)]$$

$$= 439 \times 2$$

$$= 878$$

d. $y = u^3 - 5(u^3 - 7u)^2,$

$$\begin{aligned}u &= \sqrt{x} \\&= x^{\frac{1}{2}}, x = 4\end{aligned}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 3[3u^2 - 10(u^3 - 7u)(3u^2 - 7)] \cdot \left(\frac{1}{2}x^{\frac{1}{2}}\right)$$

$$= [3u^2 - 10(u^3 - 7u)(3u^2 - 7)] \cdot \frac{1}{2\sqrt{x}}$$

At $x = 4$:

$$\begin{aligned} u &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= [3(2)^2 - 10((2)^3 - 7(2))(3(2)^2 - 7)] \frac{1}{2(2)} \\ &= 78 \end{aligned}$$

- 14.** $h(x) = f(g(x))$, therefore

$$\begin{aligned} h'(x) &= f'(g(x)) \times g'(x) \\ f(u) &= u^2 - 1, g(2) = 3, g'(2) = -1 \end{aligned}$$

$$\begin{aligned} \text{Now, } h'(2) &= f'(g(2)) \times g'(2) \\ &= f'(3) \times g'(2). \end{aligned}$$

Since $f(u) = u^2 - 1$, $f'(u) = 2u$, and $f'(3) = 6$,

$$\begin{aligned} h'(2) &= 6(-1) \\ &= -6. \end{aligned}$$

15. $V(t) = 50000 \left(1 - \frac{t}{30}\right)^2$

$$V'(t) = 50000 \left[2 \left(1 - \frac{t}{30}\right) \left(-\frac{1}{30}\right)\right]$$

$$\begin{aligned} V'(10) &= 50000 \left[2 \left(1 - \frac{10}{30}\right) \left(-\frac{1}{30}\right)\right] \\ &= 50000 \left[2 \left(\frac{2}{3}\right) \left(-\frac{1}{30}\right)\right] \\ &\doteq 2222 \end{aligned}$$

At $t = 10$ minutes, the water is flowing out of the tank at a rate of 2222 L/min.

- 16.** The velocity function is the derivative of the position function.

$$s(t) = (t^3 + t^2)^{\frac{1}{2}}$$

$$v(t) = s'(t) = \frac{1}{2}(t^3 + t^2)^{-\frac{1}{2}}(3t^2 + 2t)$$

$$= \frac{3t^2 + 2t}{2\sqrt{t^3 + t^2}}$$

$$\begin{aligned} v(3) &= \frac{3(3)^2 + 2(3)}{2\sqrt{3^3 + 3^2}} \\ &= \frac{27 + 6}{2\sqrt{36}} \\ &= \frac{33}{12} \\ &= 2.75 \end{aligned}$$

The particle is moving at 2.75 m/s.

- 17. a.** $h(x) = p(x)q(x)r(x)$

$$\begin{aligned} h'(x) &= p'(x)q(x)r(x) + p(x)q'(x)r(x) \\ &\quad + p(x)q(x)r'(x) \end{aligned}$$

b. $h(x) = x(2x + 7)^4(x - 1)^2$

Using the result from part a.,

$$\begin{aligned} h'(x) &= (1)(2x + 7)^4(x - 1)^2 \\ &\quad + x[4(2x + 7)^3(2)](x - 1)^2 \\ &\quad + x(2x + 7)^4[2(x - 1)] \end{aligned}$$

$$\begin{aligned} h'(-3) &= 1(16) + (-3)[4(1)(2)](16) \\ &\quad + (-3)(1)[2(-4)] \\ &= 16 - 384 + 24 \\ &= -344 \end{aligned}$$

18. $y = (x^2 + x - 2)^3 + 3$

$$\frac{dy}{dx} = 3(x^2 + x - 2)^2(2x + 1)$$

At the point $(1, 3)$, $x = 1$ and the slope of the tangent will be $3(1 + 1 - 2)^2(2 + 1) = 0$.

Equation of the tangent at $(1, 3)$ is $y - 3 = 0$.

Solving this equation with the function, we have

$$(x^2 + x - 2)^3 + 3 = 3$$

$$(x + 2)^3(x - 1)^3 = 0$$

$$x = -2 \text{ or } x = 1$$

Since -2 and 1 are both triple roots, the line with equation $y - 3 = 0$ will be a tangent at both $x = 1$ and $x = -2$. Therefore, $y - 3 = 0$ is also a tangent at $(-2, 3)$.

$$\begin{aligned} \mathbf{19.} \quad y &= \frac{x^2(1-x)^3}{(1+x)^3} \\ &= x^2 \left[\left(\frac{1-x}{1+x} \right) \right]^3 \\ \frac{dy}{dx} &= 2x \left(\frac{1-x}{1+x} \right)^3 + 3x^2 \left(\frac{1-x}{1+x} \right)^2 \\ &\quad \times \left[\frac{-(1+x) - (1-x)(1)}{(1+x)^2} \right] \\ &= 2x \left(\frac{1-x}{1+x} \right)^3 + 3x^2 \left(\frac{1-x}{1+x} \right)^2 \left[\frac{-2}{(1+x)^2} \right] \\ &= 2x \left(\frac{1-x}{1+x} \right)^2 \left[\frac{1-x}{1+x} - \frac{3x}{(1+x)^2} \right] \\ &= 2x \left(\frac{1-x}{1+x} \right)^2 \left[\frac{1-x^2-3x}{(1+x)^2} \right] \\ &= -\frac{2x(x^2+3x-1)(1-x)^2}{(1+x)^4} \end{aligned}$$

Review Exercise, pp. 110–113

1. To find the derivative $f'(x)$, the limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

must be computed, provided it exists. If this limit does not exist, then the derivative of $f(x)$ does not

exist at this particular value of x . As an alternative to this limit, we could also find $f'(x)$ from the definition by computing the equivalent limit

$$f'(x) = \lim_{z \rightarrow x} \frac{f(z) - f(x)}{z - x}.$$

These two limits are seen to be equivalent by substituting $z = x + h$.

2. a. $y = 2x^2 - 5x$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{(2(x+h)^2 - 5(x+h)) - (2x^2 - 5x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2((x+h)^2 - x^2) - 5h}{h} \\ &= \lim_{h \rightarrow 0} \frac{2((x+h) - x)((x+h) + x) - 5h}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h(2x+h) - 5h}{h} \\ &= \lim_{h \rightarrow 0} (2(2x+h) - 5) \\ &= 4x - 5 \end{aligned}$$

b. $y = \sqrt{x-6}$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)-6} - \sqrt{x-6}}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{\sqrt{(x+h)-6} - \sqrt{x-6}}{h} \right. \\ &\quad \times \left. \frac{\sqrt{(x+h)-6} + \sqrt{x-6}}{\sqrt{(x+h)-6} + \sqrt{x-6}} \right] \\ &= \lim_{h \rightarrow 0} \frac{((x+h)-6) - (x-6)}{h(\sqrt{(x+h)-6} + \sqrt{x-6})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{(x+h)-6} + \sqrt{x-6}} \\ &= \frac{1}{2\sqrt{x-6}} \end{aligned}$$

c. $y = \frac{x}{4-x}$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{\frac{x+h}{4-(x+h)} - \frac{x}{4-x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+h)(4-x) - x(4-(x+h))}{4-(x+h)(4-x)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{(4-(x+h))(4-x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{4h}{h(4-(x+h))(4-x)} \\ &= \lim_{h \rightarrow 0} \frac{4}{(4-(x+h))(4-x)} \\ &= \frac{4}{(4-x)^2} \end{aligned}$$

3. a. $y = x^2 - 5x + 4$

$$\frac{dy}{dx} = 2x - 5$$

b. $f(x) = x^{\frac{3}{4}}$

$$\begin{aligned} f'(x) &= \frac{3}{4}x^{-\frac{1}{4}} \\ &= \frac{3}{4x^{\frac{1}{4}}} \end{aligned}$$

c. $y = \frac{7}{3x^4}$

$$= \frac{7}{3}x^{-4}$$

$$\frac{dy}{dx} = \frac{-28}{3}x^{-5}$$

$$= -\frac{28}{3x^5}$$

d. $y = \frac{1}{x^2 + 5}$

$$= (x^2 + 5)^{-1}$$

$$\frac{dy}{dx} = (-1)(x^2 + 5)^{-2} \cdot (2x)$$

$$= -\frac{2x}{(x^2 + 5)^2}$$

e. $y = \frac{3}{(3-x^2)^2}$

$$= 3(3-x^2)^{-2}$$

$$\frac{dy}{dx} = (-6)(3-x^2)^{-3} \cdot (-2x)$$

$$= \frac{12x}{(3-x^2)^3}$$

f. $y = \sqrt{7x^2 + 4x + 1}$

$$\frac{dy}{dx} = \frac{1}{2}(7x^2 + 4x + 1)^{-\frac{1}{2}}(14x + 4)$$

$$= \frac{7x + 2}{\sqrt{7x^2 + 4x + 1}}$$

4. a. $f(x) = \frac{2x^3 - 1}{x^2}$

$$= 2x - \frac{1}{x^2}$$

$$= 2x - x^{-2}$$

$$f'(x) = 2 + 2x^{-3}$$

$$= 2 + \frac{2}{x^3}$$

b. $g(x) = \sqrt{x}(x^3 - x)$

$$= x^{\frac{1}{2}}(x^3 - x)$$

$$= x^{\frac{7}{2}} - x^{\frac{3}{2}}$$

$$g'(x) = \frac{7}{2}x^{\frac{5}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$

$$= \frac{\sqrt{x}}{2}(7x^2 - 3)$$

c. $y = \frac{x}{3x - 5}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(3x - 5)(1) - (x)(3)}{(3x - 5)^2} \\ &= -\frac{5}{(3x - 5)^2} \end{aligned}$$

d. $y = (x - 1)^{\frac{1}{2}}(x + 1)$

$$\begin{aligned} y' &= (x - 1)^{\frac{1}{2}} + (x + 1)\left(\frac{1}{2}\right)(x - 1)^{-\frac{1}{2}} \\ &= \sqrt{x - 1} + \frac{x + 1}{2\sqrt{x - 1}} \\ &= \frac{2x - 2 + x + 1}{2\sqrt{x - 1}} \\ &= \frac{3x - 1}{2\sqrt{x - 1}} \end{aligned}$$

e. $f(x) = (\sqrt{x} + 2)^{-\frac{2}{3}}$

$$= (x^{\frac{1}{2}} + 2)^{-\frac{2}{3}}$$

$$\begin{aligned} f'(x) &= \frac{-2}{3}(x^{\frac{1}{2}} + 2)^{-\frac{5}{3}} \cdot \frac{1}{2}x^{-\frac{1}{2}} \\ &= -\frac{1}{3\sqrt{x}(\sqrt{x} + 2)^{\frac{5}{3}}} \end{aligned}$$

f. $y = \frac{x^2 + 5x + 4}{x + 4}$

$$= \frac{(x + 4)(x + 1)}{x + 4}$$

$$= x + 1, x \neq -4$$

$$\frac{dy}{dx} = 1$$

5. a. $y = x^4(2x - 5)^6$

$$\begin{aligned} y' &= x^4[6(2x - 5)^5(2)] + 4x^3(2x - 5)^6 \\ &= 4x^3(2x - 5)^5[3x + (2x - 5)] \\ &= 4x^3(2x - 5)^5(5x - 5) \\ &= 20x^3(2x - 5)^5(x - 1) \end{aligned}$$

b. $y = x\sqrt{x^2 + 1}$

$$\begin{aligned} y' &= x\left[\frac{1}{2}(x^2 + 1)^{-\frac{1}{2}}(2x)\right] + (1)\sqrt{x^2 + 1} \\ &= \frac{x^2}{\sqrt{x^2 + 1}} + \sqrt{x^2 + 1} \end{aligned}$$

c. $y = \frac{(2x - 5)^4}{(x + 1)^3}$

$$y' = \frac{(x + 1)^3 4(2x - 5)^3(2)}{(x + 1)^6}$$

$$- \frac{3(2x - 5)^4(x + 1)^2}{(x + 1)^6}$$

$$= \frac{(x + 1)^2(2x - 5)^3[8x + 8 - 6x + 15]}{(x + 1)^6}$$

$$y' = \frac{(2x - 5)^3(2x + 23)}{(x + 1)^4}$$

d. $y = \left(\frac{10x - 1}{3x + 5}\right)^6 = (10x - 1)^6(3x + 5)^{-6}$

$$y' = (10x - 1)^6[-6(3x + 5)^{-7}(3)]$$

$$+ 6(10x - 1)^5(10)(3x + 5)^{-6}$$

$$= (10x - 1)^5(3x + 5)^{-7}[x - 18(10x - 1)]$$

$$+ 60(3x + 5)$$

$$= (10x - 1)^5(3x + 5)^{-7}$$

$$\times (-180x + 18 + 180x + 300)$$

$$= \frac{318(10x - 1)^5}{(3x + 5)^7}$$

e. $y = (x - 2)^3(x^2 + 9)^4$

$$y' = (x - 2)^3[4(x^2 + 9)^3(2x)]$$

$$+ 3(x - 2)^2(1)(x^2 + 9)^4$$

$$= (x - 2)^2(x^2 + 9)^3[8x(x - 2) + 3(x^2 + 9)]$$

$$= (x - 2)^2(x^2 + 9)^3(11x^2 - 16x + 27)$$

f. $y = (1 - x^2)^3(6 + 2x)^{-3}$

$$= \left(\frac{1 - x^2}{6 + 2x}\right)^3$$

$$y' = 3\left(\frac{1 - x^2}{6 + 2x}\right)^2$$

$$\times \left[\frac{(6 + 2x)(-2x) - (1 - x^2)(2)}{(6 + 2x)^2} \right]$$

$$= \frac{3(1 - x^2)^2(-12x - 4x^2 - 2 + 2x^2)}{(6 + 2x)^4}$$

$$= -\frac{3(1 - x^2)^2(2x^2 + 12x + 2)}{(6 + 2x)^4}$$

$$= -\frac{3(1 - x^2)^2(x^2 + 6x + 1)}{8(3 - x)^4}$$

6. a. $g(x) = f(x^2)$

$$g'(x) = f(x^2) \times 2x$$

b. $h(x) = 2xf(x)$

$$h'(x) = 2xf'(x) + 2f(x)$$

7. a. $y = 5u^2 + 3u - 1, u = \frac{18}{x^2 + 5}$

$$x = 2$$

$$u = 2$$

$$\frac{dy}{du} = 10u + 3$$

$$\frac{du}{dx} = -\frac{36x}{(x^2 + 5)^2}$$

When $x = 2$,

$$\frac{du}{dx} = -\frac{72}{81} = -\frac{8}{9}$$

When $u = 2$,

$$\frac{dy}{du} = 20 + 3$$

$$= 23$$

$$\frac{dy}{dx} = 23 \left(-\frac{8}{9} \right)$$

$$= -\frac{184}{9}$$

b. $y = \frac{u+4}{u-4}$, $u = \frac{\sqrt{x}+x}{10}$,

$$x = 4$$

$$u = \frac{3}{5}$$

$$\frac{dy}{du} = \frac{(u-4) - (u+4)}{(u-4)^2}$$

$$\frac{du}{dx} = \frac{1}{10} \left(\frac{1}{2}x^{-\frac{1}{2}} + 1 \right)$$

When $x = 4$,

$$= -\frac{8}{(u-4)^2} \frac{du}{dx} = \frac{1}{10} \left(\frac{5}{4} \right)$$

$$= \frac{1}{8}$$

When $u = \frac{3}{5}$,

$$\frac{dy}{du} = -\frac{8}{\left(\frac{3}{5} - \frac{20}{5} \right)^2}$$

$$= -\frac{8(25)}{(-17)^2}$$

When $x = 4$,

$$\frac{dy}{dx} = \frac{8(25)}{17^2} \times \frac{1}{8}$$

$$= \frac{25}{289}$$

c. $y = f(\sqrt{x^2 + 9})$, $f'(5) = -2$, $x = 4$

$$\frac{dy}{dx} = f'(\sqrt{x^2 + 9}) \times \frac{1}{2}(x^2 + 9)^{-\frac{1}{2}}(2x)$$

$$\frac{dy}{dx} = f'(5) \cdot \frac{1}{2} \cdot \frac{1}{5} \cdot 8$$

$$= -2 \cdot \frac{4}{5}$$

$$= -\frac{8}{5}$$

8. $f(x) = (9 - x^2)^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3}(9 - x^2)^{-\frac{1}{3}}(-2x)$$

$$= \frac{-4x}{3(9 - x^2)^{\frac{1}{3}}}$$

$$f'(1) = -\frac{2}{3}$$

The slope of the tangent line at $(1, 4)$ is $-\frac{2}{3}$.

9. $y = -x^3 + 6x^2$

$$y' = -3x^2 + 12x$$

$$-3x^2 + 12x = -12$$

$$x^2 - 4x - 4 = 0$$

$$-3x^2 + 12x = -15$$

$$x^2 - 4x - 5 = 0$$

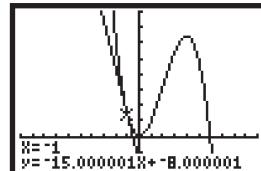
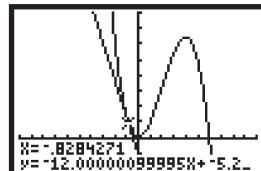
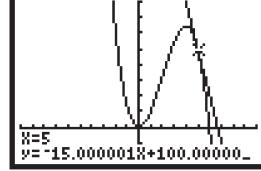
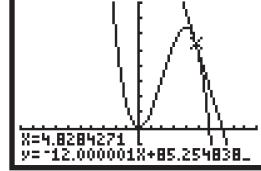
$$x = \frac{4 \pm \sqrt{16 + 16}}{2}$$

$$(x - 5)(x + 1) = 0$$

$$= \frac{4 \pm 4\sqrt{2}}{2}$$

$$x = 5, x = -1$$

$$x = 2 \pm 2\sqrt{2}$$



10. a. i. $y = (x^2 - 4)^5$
 $y' = 5(x^2 - 4)^4(2x)$

Horizontal tangent,

$$10x(x^2 - 4)^4 = 0$$

$$x = 0, x = \pm 2$$

ii. $y = (x^3 - x)^2$

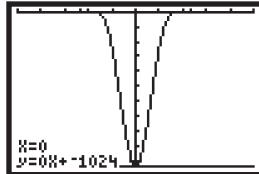
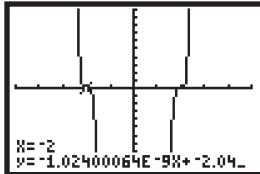
$$y' = 2(x^3 - x)(3x^2 - 1)$$

Horizontal tangent,

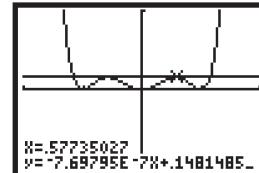
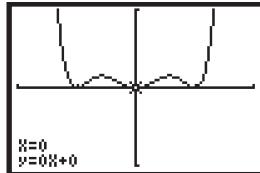
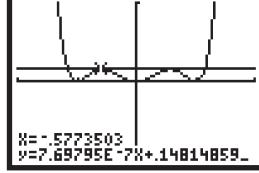
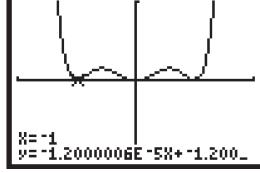
$$2x(x^2 - 1)(3x^2 - 1) = 0$$

$$x = 0, x = \pm 1, x = \pm \frac{\sqrt{3}}{3}$$

b. i.



ii.



11. a. $y = (x^2 + 5x + 2)^4$ at $(0, 16)$

$$y' = 4(x^2 + 5x + 2)^3(2x + 5)$$

At $x = 0$,

$$\begin{aligned} y' &= 4(2)^3(5) \\ &= 160 \end{aligned}$$

Equation of the tangent at $(0, 16)$ is

$$y - 16 = 160(x - 0)$$

$$y = 160x + 16$$

or $160x - y + 16 = 0$

b. $y = (3x^{-2} - 2x^3)^5$ at $(1, 1)$

$$y' = 5(3x^{-2} - 2x^3)^4(-6x^{-3} - 6x^2)$$

At $x = 1$,

$$\begin{aligned} y' &= 5(1)^4(-6 - 6) \\ &= -60 \end{aligned}$$

Equation of the tangent at $(1, 1)$ is

$$y - 1 = -60(x - 1)$$

$$60x + y - 61 = 0$$

12. $y = 3x^2 - 7x + 5$

$$\frac{dy}{dx} = 6x - 7$$

Slope of $x + 5y - 10 = 0$ is $-\frac{1}{5}$.

Since perpendicular, $6x - 7 = 5$

$$x = 2$$

$$\begin{aligned} y &= 3(4) - 14 + 5 \\ &= 3. \end{aligned}$$

Equation of the tangent at $(2, 3)$ is

$$y - 3 = 5(x - 2)$$

$$5x - y - 7 = 0.$$

13. $y = 8x + b$ is tangent to $y = 2x^2$

$$\frac{dy}{dx} = 4x$$

Slope of the tangent is 8, therefore $4x = 8$, $x = 2$.

Point of tangency is $(2, 8)$.

Therefore, $8 = 16 + b$, $b = -8$.

$$\text{Or } 8x + b = 2x^2$$

$$2x^2 - 8x - b = 0$$

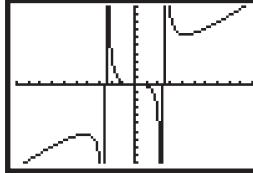
$$x = \frac{8 \pm \sqrt{64 + 8b}}{2(2)}.$$

For tangents, the roots are equal, therefore

$$64 + 8b = 0, b = -8.$$

Point of tangency is $(2, 8)$, $b = -8$.

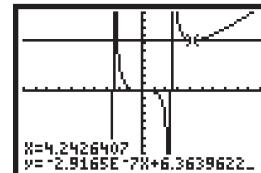
14. a.



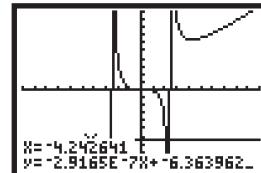
b.



The equation of the tangent is $y = 0$.



The equation of the tangent is $y = 6.36$.



The equation of the tangent is $y = -6.36$.

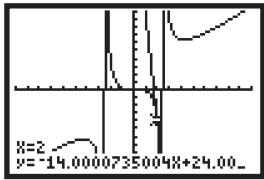
c. $f'(x) = \frac{(x^2 - 6)(3x^2) - x^3(2x)}{(x^2 - 6)^2}$
 $= \frac{x^4 - 18x^2}{(x^2 - 6)^2}$

$$\frac{x^4 - 18x^2}{(x^2 - 6)^2} = 0$$
 $x^2(x^2 - 18) = 0$
 $x^2 = 0 \text{ or } x^2 - 18 = 0$
 $x = 0 \quad x = \pm 3\sqrt{2}$

The coordinates of the points where the slope is 0 are $(0, 0)$, $(3\sqrt{2}, \frac{9\sqrt{2}}{2})$, and $(-3\sqrt{2}, -\frac{9\sqrt{2}}{2})$.

d. Substitute into the expression for $f'(x)$ from part b.

$$f'(2) = \frac{16 - 72}{(-2)^2}$$
 $= \frac{-56}{4}$
 $= -14$



15. a. $f(x) = 2x^{5/3} - 5x^{1/3}$
 $f'(x) = 2 \times \frac{5}{3}x^{2/3} - 5 \times \frac{2}{3}x^{-2/3}$

$$= \frac{10}{3}x^{2/3} - \frac{10}{3x^{2/3}}$$

$$f(x) = 0 \quad \therefore x^{2/3}[2x - 5] = 0$$

$$x = 0 \text{ or } x = \frac{5}{2}$$

$y = f(x)$ crosses the x -axis at $x = \frac{5}{2}$, and

$$f'(x) = \frac{10}{3}\left(\frac{x - 1}{x^{1/3}}\right)$$

$$f'\left(\frac{5}{2}\right) = \frac{10}{3} \times \frac{3}{2} \times \frac{1}{\left(\frac{5}{2}\right)^{1/3}}$$

$$= 5 \times \frac{\sqrt[3]{2}}{\sqrt[3]{5}} = 5^{2/3} \times 2^{1/3}$$

$$= (25 \times 2)^{1/3}$$

$$= \sqrt[3]{50}$$

b. To find a , let $f(x) = 0$.

$$\frac{10}{3}x^{2/3} - \frac{10}{3x^{1/3}} = 0$$

$$30x = 30$$
 $x = 1$

Therefore $a = 1$.

16. $M = 0.1t^2 - 0.001t^3$

a. When $t = 10$,

$$M = 0.1(100) - 0.001(1000)$$
 $= 9$

When $t = 15$,

$$M = 0.1(225) - 0.001(3375)$$
 $= 19.125$

One cannot memorize partial words, so 19 words are memorized after 15 minutes.

b. $M' = 0.2t - 0.003t^2$

When $t = 10$,

$$M' = 0.2(10) - 0.003(100)$$
 $= 1.7$

The number of words memorized is increasing by 1.7 words/min.

When $t = 15$,

$$M' = 0.2(15) - 0.003(225)$$
 $= 2.325$

The number of words memorized is increasing by 2.325 words/min.

17. a. $N(t) = 20 - \frac{30}{\sqrt{9 + t^2}}$

$$N'(t) = \frac{30t}{(9 + t^2)^{3/2}}$$

b. No, according to this model, the cashier never stops improving. Since $t > 0$, the derivative is always positive, meaning that the rate of change in the cashier's productivity is always increasing. However, these increases must be small, since, according to the model, the cashier's productivity can never exceed 20.

18. $C(x) = \frac{1}{3}x^3 + 40x + 700$

a. $C'(x) = x^2 + 40$

b. $C'(x) = 76$

$$x^2 + 40 = 76$$

$$x^2 = 36$$

$$x = 6$$

Production level is 6 gloves/week.

19. $R(x) = 750x - \frac{x^2}{6} - \frac{2}{3}x^3$

a. Marginal Revenue

$$R'(x) = 750 - \frac{x}{3} - 2x^2$$

b. $R'(10) = 750 - \frac{10}{3} - 2(100)$
 $= \$546.67$

20. $D(p) = \frac{20}{\sqrt{p-1}}, p > 1$
 $D'(p) = 20\left(-\frac{1}{2}\right)(p-1)^{-\frac{3}{2}}$
 $= -\frac{10}{(p-1)^{\frac{3}{2}}}$
 $D'(5) = \frac{10}{\sqrt{4^3}} = -\frac{10}{8}$
 $= -\frac{5}{4}$

Slope of demand curve at $(5, 10)$ is $-\frac{5}{4}$.

21. $B(x) = -0.2x^2 + 500, 0 \leq x \leq 40$

a. $B(0) = -0.2(0)^2 + 500 = 500$

$B(30) = -0.2(30)^2 + 500 = 320$

b. $B'(x) = -0.4x$

$B'(0) = -0.4(0) = 0$

$B'(30) = -0.4(30) = -12$

c. $B(0)$ = blood sugar level with no insulin

$B(30)$ = blood sugar level with 30 mg of insulin

$B'(0)$ = rate of change in blood sugar level
with no insulin

$B'(30)$ = rate of change in blood sugar level
with 30 mg of insulin

d. $B'(50) = -0.4(50) = -20$

$B(50) = -0.2(50)^2 + 500 = 0$

$B'(50) = -20$ means that the patient's blood sugar level is decreasing at 20 units per mg of insulin 1 h after 50 mg of insulin is injected.

$B(50) = 0$ means that the patient's blood sugar level is zero 1 h after 50 mg of insulin is injected. These values are not logical because a person's blood sugar level can never reach zero and continue to decrease.

22. a. $f(x) = \frac{3x}{1-x^2}$
 $= \frac{3x}{(1-x)(1+x)}$

$f(x)$ is not differentiable at $x = 1$ because it is not defined there (vertical asymptote at $x = 1$).

b. $g(x) = \frac{x-1}{x^2+5x-6}$
 $= \frac{x-1}{(x+6)(x-1)}$
 $= \frac{1}{(x+6)}$ for $x \neq 1$

$g(x)$ is not differentiable at $x = 1$ because it is not defined there (hole at $x = 1$).

c. $h(x) = \sqrt[3]{(x-2)^2}$

The graph has a cusp at $(2, 0)$ but it is differentiable at $x = 1$.

d. $m(x) = |3x-3| - 1$

The graph has a corner at $x = 1$, so $m(x)$ is not differentiable at $x = 1$.

23. a. $f(x) = \frac{3}{4x^2-x}$
 $= \frac{3}{x(4x-1)}$

$f(x)$ is not defined at $x = 0$ and $x = 0.25$. The graph has vertical asymptotes at $x = 0$ and $x = 0.25$. Therefore, $f(x)$ is not differentiable at $x = 0$ and $x = 0.25$.

b. $f(x) = \frac{x^2-x-6}{x^2-9}$
 $= \frac{(x-3)(x+2)}{(x-3)(x+3)}$
 $= \frac{(x+2)}{(x+3)}$ for $x \neq 3$

$f(x)$ is not defined at $x = 3$ and $x = -3$. At $x = -3$, the graph has a vertical asymptote and at $x = 3$ it has a hole. Therefore, $f(x)$ is not differentiable at $x = 3$ and $x = -3$.

c. $f(x) = \sqrt{x^2-7x+6}$
 $= \sqrt{(x-6)(x-1)}$

$f(x)$ is not defined for $1 < x < 6$. Therefore, $f(x)$ is not differentiable for $1 < x < 6$.

24. $p'(t) = \frac{(t+1)(25) - (25t)(t)}{(t+1)^2}$
 $= \frac{25t+25-25t}{(t+1)^2}$
 $= \frac{25}{(t+1)^2}$

25. Answers may vary. For example,
 $f(x) = 2x + 3$

$$y = \frac{1}{2x+3}$$

$$y' = \frac{(2x+3)(0) - (1)(2)}{(2x+3)^2}$$

$$= -\frac{2}{(2x+3)^2}$$

$f(x) = 5x + 10$

$$y = \frac{1}{5x+10}$$

$$y' = \frac{(5x+10)(0) - (1)(5)}{(5x+10)^2}$$

$$= -\frac{5}{(5x+10)^2}$$

Rule: If $f(x) = ax + b$ and $y = \frac{1}{f(x)}$, then

$$y' = \frac{-a}{(ax+b)^2}$$

$$\begin{aligned} y' &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{1}{a(x+h)+b} - \frac{1}{ax+b} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{ax+b - [a(x+h)+b]}{[a(x+h)+b](ax+h)} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{ax+b - ax-ah-b}{[a(x+h)+b](ax+b)} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-ah}{[a(x+h)+b](ax+b)} \right] \\ &= \lim_{h \rightarrow 0} \frac{-a}{[a(x+h)+b](ax+b)} \\ &= \frac{-a}{(ax+b)^2} \end{aligned}$$

26. a. Let $y = f(x)$

$$y = \frac{(2x-3)^2 + 5}{2x-3}$$

Let $u = 2x-3$.

$$\text{Then } y = \frac{u^2 + 5}{u}.$$

$$y = u + 5u^{-1}$$

$$\text{b. } f'(x) = \frac{dy}{dx}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= (1-5u^{-2})(2) \\ &= 2(1-5(2x-3)^{-2}) \end{aligned}$$

$$\text{27. } g(x) = \sqrt{2x-3} + 5(2x-3)$$

a. Let $y = g(x)$.

$$y = \sqrt{2x-3} + 5(2x-3)$$

Let $u = 2x-3$.

Then $y = \sqrt{u} + 5u$.

$$\begin{aligned} \text{b. } g'(x) &= \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \\ &= \left(\frac{1}{2}u^{-\frac{1}{2}} + 5 \right)(2) \\ &= u^{-\frac{1}{2}} + 10 \\ &= (2x-3)^{-\frac{1}{2}} + 10 \end{aligned}$$

$$\text{28. a. } f(x) = (2x-5)^3(3x^2+4)^5$$

$$\begin{aligned} f'(x) &= (2x-5)^3(5)(3x^2+4)^4(6x) \\ &\quad + (3x^2+4)^5(3)(2x-5)^2(2) \\ &= 30x(2x-5)^3(3x^2+4)^4 \\ &\quad + 6(3x^2+4)^5(2x-5)^2 \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times [5x(2x-5) + (3x^2+4)] \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times (10x^2-25x+3x^2+4) \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times (13x^2-25x+4) \end{aligned}$$

$$\text{b. } g(x) = (8x^3)(4x^2+2x-3)^5$$

$$\begin{aligned} g'(x) &= (8x^3)(5)(4x^2+2x-3)^4(8x+2) \\ &\quad + (4x^2+2x-3)^5(24x^2) \\ &= 40x^3(4x^2+2x-3)^4(8x+2) \\ &\quad + 24x^2(4x^2+2x-3)^5 \\ &= 8x^2(4x^2+2x-3)^4[5x(8x+2) \\ &\quad + 3(4x^2+2x-3)] \\ &= 8x^2(4x^2+2x-3)^4 \\ &\quad (40x^2+10x+12x^2+6x-9) \\ &= 8x^2(4x^2+2x-3)^4(52x^2+16x-9) \end{aligned}$$

$$\text{c. } y = (5+x)^2(4-7x^3)^6$$

$$\begin{aligned} y' &= (5+x)^2(6)(4-7x^3)^5(-21x^2) \\ &\quad + (4-7x^3)^6(2)(5+x) \\ &= -126x^2(5+x)^2(4-7x^3)^5 \\ &\quad + 2(5+x)(4-7x^3)^6 \\ &= 2(5+x)(4-7x^3)^5[-63x^2(5+x) \\ &\quad + 4-7x^3] \\ &= 2(5+x)(4-7x^3)^5(4-315x^2-70x^3) \end{aligned}$$

$$\text{d. } h(x) = \frac{6x-1}{(3x+5)^4}$$

$$\begin{aligned} h'(x) &= \frac{(3x+5)^4(6) - (6x-1)(4)(3x+5)^3(3)}{((3x+5)^4)^2} \\ &= \frac{6(3x+5)^3[(3x+5)-2(6x-1)]}{(3x+5)^8} \\ &= \frac{6(-9x+7)}{(3x+5)^5} \end{aligned}$$

$$\text{e. } y = \frac{(2x^2-5)^3}{(x+8)^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+8)^2(3)(2x^2-5)^2(4x)}{((x+8)^2)^2} \\ &\quad - \frac{(2x^2-5)^3(2)(x+8)}{((x+8)^2)^2} \\ &= \frac{2(x+8)(2x^2-5)^2[6x(x+8)-(2x^2-5)]}{(x+8)^4} \\ &= \frac{2(2x^2-5)^2(4x^2+48x+5)}{(x+8)^3} \end{aligned}$$

f. $f(x) = \frac{-3x^4}{\sqrt{4x - 8}}$

$$= \frac{-3x^4}{(4x - 8)^{\frac{1}{2}}}$$

$$f'(x) = \frac{(4x - 8)^{\frac{1}{2}}(-12x^3)}{((4x - 8)^{\frac{1}{2}})^2}$$

$$- \frac{(-3x^4)\left(\frac{1}{2}\right)(4x - 8)^{-\frac{1}{2}}(4)}{((4x - 8)^{\frac{1}{2}})^2}$$

$$= \frac{-6x^3(4x - 8)^{-\frac{1}{2}}[2(4x - 8) - x]}{4x - 8}$$

$$= \frac{-6x^3(7x - 16)}{(4x - 8)^{\frac{3}{2}}}$$

$$= \frac{-3x^3(7x - 16)}{(4x - 8)^{\frac{3}{2}}}$$

g. $g(x) = \left(\frac{2x + 5}{6 - x^2}\right)^4$

$$g'(x) = 4\left(\frac{2x + 5}{6 - x^2}\right)^3$$

$$\times \left(\frac{(6 - x^2)(2) - (2x + 5)(-2x)}{(6 - x^2)^2}\right)$$

$$= 4\left(\frac{2x + 5}{6 - x^2}\right)^3 \left(\frac{2(6 + x^2 + 5x)}{(6 - x^2)^2}\right)$$

$$= 8\left(\frac{2x + 5}{6 - x^2}\right)^3 \left(\frac{(x + 2)(x + 3)}{(6 - x^2)^2}\right)$$

h. $y = \left[\frac{1}{(4x + x^2)^3}\right]^3$

$$= (4x + x^2)^{-9}$$

$$\frac{dy}{dx} = -9(4x + x^2)^{-10}(4 + 2x)$$

29. $f(x) = ax^2 + bx + c$,

It is given that $(0, 0)$ and $(8, 0)$ are on the curve, and $f'(2) = 16$.

Calculate $f'(x) = 2ax + b$.

Then,

$$16 = 2a(2) + b$$

$$4a + b = 16 \quad (1)$$

Since $(0, 0)$ is on the curve,

$$0 = a(0)^2 + b(0) + c$$

$$c = 0$$

Since $(8, 0)$ is on the curve,

$$0 = a(8)^2 + b(8) + c$$

$$0 = 64a + 8b + 0$$

$$8a + b = 0 \quad (2)$$

Solve (1) and (2):

$$\text{From (2), } b = -8a \quad (1)$$

In (1),

$$4a - 8a = 16$$

$$-4a = 16$$

$$a = -4$$

Using (1),

$$b = -8(-4) = 32$$

$$a = -4, b = 32, c = 0, f(x) = -4x^2 + 32x$$

30. a. $A(t) = -t^3 + 5t + 750$

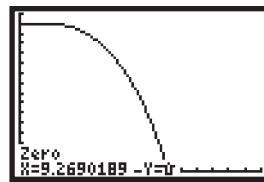
$$A'(t) = -3t^2 + 5$$

b. $A'(5) = -3(25) + 5$
 $= -70$

At 5 h, the number of ants living in the colony is decreasing by 7000 ants/h.

c. $A(0) = 750$, so there were $750(100)$ or 75 000 ants living in the colony before it was treated with insecticide.

d. Determine t so that $A(t) = 0$. $-t^3 + 5t + 750$ cannot easily be factored, so find the zeros by using a graphing calculator.



All of the ants have been killed after about 9.27 h.

Chapter 2 Test, p. 114

1. You need to use the chain rule when the derivative for a given function cannot be found using the sum, difference, product, or quotient rules or when writing the function in a form that would allow the use of these rules is tedious. The chain rule is used when a given function is a composition of two or more functions.

2. f is the blue graph (it's a cubic). f' is the red graph (it is quadratic). The derivative of a polynomial function has degree one less than the derivative of the function. Since the red graph is a quadratic (degree 2) and the blue graph is cubic (degree 3), the blue graph is f and the red graph is f' .

3. $f(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{x + h - (x + h)^2 - (x - x^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x + h - (x^2 + 2hx + h^2) - x + x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h - 2hx - h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(1 - 2x - h)}{h}$$

$$= \lim_{h \rightarrow 0} (1 - 2x - h)$$

$$= 1 - 2x$$

Therefore, $\frac{d}{dx}(x - x^2) = 1 - 2x$.

4. a. $y = \frac{1}{3}x^3 - 3x^{-5} + 4\pi$

$$\frac{dy}{dx} = x^2 + 15x^{-6}$$

b. $y = 6(2x - 9)^5$

$$\frac{dy}{dx} = 30(2x - 9)^4(2)$$

$$= 60(2x - 9)^4$$

c. $y = \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{3}} + 6\sqrt[3]{x}$

$$= 2x^{-\frac{1}{2}} + \frac{1}{\sqrt{3}}x + 6x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = -x^{-\frac{3}{2}} + \frac{1}{\sqrt{3}} + 2x^{-\frac{2}{3}}$$

d. $y = \left(\frac{x^2 + 6}{3x + 4}\right)^5$

$$\frac{dy}{dx} = 5\left(\frac{x^2 + 6}{3x + 4}\right)^4 \frac{2x(3x + 4) - (x^2 + 6)3}{(3x + 4)^2}$$

$$= \frac{5(x^2 + 6)^4(3x^2 + 8x - 18)}{(3x + 4)^6}$$

e. $y = x^2 \sqrt[3]{6x^2 - 7}$

$$\frac{dy}{dx} = 2x(6x^2 - 7)^{\frac{1}{3}} + x^2 \frac{1}{3}(6x^2 - 7)^{-\frac{2}{3}}(12x)$$

$$= 2x(6x^2 - 7)^{-\frac{2}{3}}((6x^2 - 7) + 2x^2)$$

$$= 2x(6x^2 - 7)^{-\frac{2}{3}}(8x^2 - 7)$$

f. $y = \frac{4x^5 - 5x^4 + 6x - 2}{x^4}$

$$= 4x - 5 + 6x^{-3} - 2x^{-4}$$

$$\frac{dy}{dx} = 4 - 18x^{-4} + 8x^{-5}$$

$$= \frac{4x^5 - 18x + 8}{x^5}$$

5. $y = (x^2 + 3x - 2)(7 - 3x)$

$$\frac{dy}{dx} = (2x + 3)(7 - 3x) + (x^2 + 3x - 2)(-3)$$

At $(1, 8)$,

$$\frac{dy}{dx} = (5)(4) + (2)(-3)$$

$$= 14.$$

The slope of the tangent to $y = (x^2 + 3x - 2)(7 - 3x)$ at $(1, 8)$ is 14.

6. $y = 3u^2 + 2u$

$$\frac{dy}{du} = 6u + 2$$

$$u = \sqrt{x^2 + 5}$$

$$\frac{du}{dy} = \frac{1}{2}(x^2 + 5)^{-\frac{1}{2}}2x$$

$$\frac{dy}{dx} = (6u + 2)\left(\frac{x}{\sqrt{x^2 + 5}}\right)$$

At $x = -2, u = 3$.

$$\frac{dy}{dx} = (20)\left(-\frac{2}{3}\right)$$

$$= -\frac{40}{3}$$

7. $y = (3x^{-2} - 2x^3)^5$

$$\frac{dy}{dx} = 5(3x^{-2} - 2x^3)^4(-6x^{-3} - 6x^2)$$

At $(1, 1)$,

$$\frac{dy}{dx} = 5(1)^4(-6 - 6)$$

$$= -60.$$

Equation of tangent line at $(1, 1)$ is $y - 1 = 60(x - 1)$

$$y - 1 = -60x + 60$$

$$60x + y - 61 = 0.$$

8. $P(t) = (t^{\frac{1}{4}} + 3)^3$

$$P'(t) = 3(t^{\frac{1}{4}} + 3)^2\left(\frac{1}{4}t^{-\frac{3}{4}}\right)$$

$$P'(16) = 3(16^{\frac{1}{4}} + 3)^2\left(\frac{1}{4} \times 16^{-\frac{3}{4}}\right)$$

$$= 3(2 + 3)^2\left(\frac{1}{4} \times \frac{1}{8}\right)$$

$$= \frac{75}{32}$$

The amount of pollution is increasing at a rate of $\frac{75}{32}$ ppm/year.

9. $y = x^4$

$$\frac{dy}{dx} = 4x^3$$

$$-\frac{1}{16} = 4x^3$$

Normal line has a slope of 16. Therefore,

$$\frac{dy}{dx} = -\frac{1}{16}.$$

$$x^3 = -\frac{1}{64}$$

$$x = -\frac{1}{4}$$

$$y = \frac{1}{256}$$

Therefore, $y = x^4$ has a normal line with a slope of 16 at $(-\frac{1}{4}, \frac{1}{256})$.

10. $y = x^3 - x^2 - x + 1$

$$\frac{dy}{dx} = 3x^2 - 2x - 1$$

For a horizontal tangent line, $\frac{dy}{dx} = 0$.

$$3x^2 - 2x - 1 = 0$$

$$(3x + 1)(x - 1) = 0$$

$$x = -\frac{1}{3} \quad \text{or} \quad x = 1$$

$$\begin{aligned} y &= -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 1 & y &= 1 - 1 - 1 + 1 \\ &= \frac{-1 - 3 + 9 + 27}{27} & &= 0 \end{aligned}$$

$$= \frac{32}{27}$$

The required points are $(-\frac{1}{3}, \frac{32}{27}), (1, 0)$.

11. $y = x^2 + ax + b$

$$\frac{dy}{dx} = 2x + a$$

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

Since the parabola and cubic function are tangent at $(1, 1)$, then $2x + a = 3x^2$.

$$\text{At } (1, 1) \quad 2(1) + a = 3(1)^2$$

$$a = 1.$$

Since $(1, 1)$ is on the graph of
 $y = x^2 + x + b$, $1 = 1^2 + 1 + b$
 $b = -1$.

The required values are 1 and -1 for a and b , respectively.

CHAPTER 2

Derivatives

Review of Prerequisite Skills, pp. 62–63

1. a. $a^5 \times a^3 = a^{5+3}$
 $= a^8$

b. $(-2a^2)^3 = (-2)^3(a^2)^3$
 $= -8(a^{2 \times 3})$
 $= -8a^6$

c. $\frac{4p^7 \times 6p^9}{12p^{15}} = \frac{24p^{7+9}}{12p^{15}}$
 $= 2p^{16-15}$
 $= 2p$

d. $(a^4b^{-5})(a^{-6}b^{-2}) = (a^{4-6})(b^{-5-2})$
 $= a^{-2}b^{-7}$
 $= \frac{1}{a^2b^7}$

e. $(3e^6)(2e^3)^4 = (3)(e^6)(2^4)(e^3)^4$
 $= (3)(2^4)(e^6)(e^{3 \times 4})$
 $= (3)(16)(e^{6+12})$
 $= 48e^{18}$

f. $\frac{(3a^{-4})[2a^3(-b)^3]}{12a^5b^2} = \frac{(3)(2)(-1)^3(a^{-4+3})(b^3)}{12a^5b^2}$
 $= \frac{-6(a^{-1-5})(b^{3-2})}{12}$
 $= \frac{-1(a^{-6})(b)}{2}$
 $= -\frac{b}{2a^6}$

2. a. $(x^{\frac{1}{2}})(x^{\frac{2}{3}}) = x^{\frac{1}{2} + \frac{2}{3}}$
 $= x^{\frac{7}{6}}$

b. $(8x^6)^{\frac{2}{3}} = 8^{\frac{2}{3}}x^{6 \times \frac{2}{3}}$
 $= 4x^4$

c. $\frac{\sqrt{a}\sqrt[3]{a}}{\sqrt{a}} = \frac{(a^{\frac{1}{2}})(a^{\frac{1}{3}})}{a^{\frac{1}{2}}}$
 $= a^{\frac{1}{3}}$

3. A perpendicular line will have a slope that is the negative reciprocal of the slope of the given line:

a. slope = $\frac{-1}{\frac{3}{2}}$
 $= -\frac{3}{2}$

b. slope = $\frac{-1}{-\frac{1}{2}}$
 $= 2$

c. slope = $\frac{-1}{\frac{5}{3}}$
 $= -\frac{3}{5}$

d. slope = $\frac{-1}{-1}$
 $= 1$

4. a. This line has slope $m = \frac{-4 - (-2)}{-3 - 9}$
 $= \frac{-2}{-12}$
 $= \frac{1}{6}$

The equation of the desired line is therefore $y + 4 = \frac{1}{6}(x + 3)$ or $x - 6y - 21 = 0$.

b. The equation $3x - 2y = 5$ can be rewritten as $2y = 3x - 5$ or $y = \frac{3}{2}x - \frac{5}{2}$, which has slope $\frac{3}{2}$.

The equation of the desired line is therefore $y + 5 = \frac{3}{2}(x + 2)$ or $3x - 2y - 4 = 0$.

c. The line perpendicular to $y = \frac{3}{4}x - 6$ will have

slope $m = -\frac{1}{\frac{3}{4}} = -\frac{4}{3}$. The equation of the desired line is therefore $y + 3 = -\frac{4}{3}(x - 4)$ or $4x + 3y - 7 = 0$.

5. a. $(x - 3y)(2x + y) = 2x^2 + xy - 6xy - 3y^2$
 $= 2x^2 - 5xy - 3y^2$

b. $(x - 2)(x^2 - 3x + 4)$
 $= x^3 - 3x^2 + 4x - 2x^2 + 6x - 8$
 $= x^3 - 5x^2 + 10x - 8$

c. $(6x - 3)(2x + 7) = 12x^2 + 42x - 6x - 21$
 $= 12x^2 + 36x - 21$

d. $2(x + y) - 5(3x - 8y) = 2x + 2y - 15x + 40y$
 $= -13x + 42y$

e. $(2x - 3y)^2 + (5x + y)^2$
 $= 4x^2 - 12xy + 9y^2 + 25x^2 + 10xy + y^2$
 $= 29x^2 - 2xy + 10y^2$

f. $3x(2x - y)^2 - x(5x - y)(5x + y)$
 $= 3x(4x^2 - 4xy + y^2) - x(25x^2 - y^2)$
 $= 12x^3 - 12x^2y + 3xy^2 - 25x^3 + xy^2$
 $= -13x^3 - 12x^2y + 4xy^2$

6. a.
$$\frac{3x(x+2)}{x^2} \times \frac{5x^3}{2x(x+2)} = \frac{15x^4(x+2)}{2x^3(x+2)}$$

$$= \frac{15}{2}x^{4-3}$$

$$= \frac{15}{2}x$$

$x \neq 0, -2$

b.
$$\frac{y}{(y+2)(y-5)} \times \frac{(y-5)^2}{4y^3}$$

$$= \frac{y(y-5)(y-5)}{4y^3(y+2)(y-5)}$$

$$= \frac{y-5}{4y^2(y+2)}$$

$y \neq -2, 0, 5$

c.
$$\frac{4}{h+k} \div \frac{9}{2(h+k)} = \frac{4}{h+k} \times \frac{2(h+k)}{9}$$

$$= \frac{8(h+k)}{9(h+k)}$$

$$= \frac{8}{9}$$

$h \neq -k$

d.
$$\frac{(x+y)(x-y)}{5(x-y)} \div \frac{(x+y)^3}{10}$$

$$= \frac{(x+y)(x-y)}{5(x-y)} \times \frac{10}{(x+y)^3}$$

$$= \frac{10(x+y)(x-y)}{5(x-y)(x+y)^3}$$

$$= \frac{2}{(x+y)^2}$$

$x \neq -y, +y$

e.
$$\frac{x-7}{2x} + \frac{5x}{x-1} = \frac{(x-7)(x-1)}{2x(x-1)} + \frac{(5x)(2x)}{2x(x-1)}$$

$$= \frac{x^2 - 7x - x + 7 + 10x^2}{2x(x-1)}$$

$$= \frac{11x^2 - 8x + 7}{2x(x-1)}$$

$x \neq 0, 1$

f.
$$\frac{x+1}{x-2} - \frac{x+2}{x+3}$$

$$= \frac{(x+1)(x+3)}{(x-2)(x+3)} - \frac{(x+2)(x-2)}{(x+3)(x-2)}$$

$$= \frac{x^2 + x + 3x + 3 - x^2 + 4}{(x+3)(x-2)}$$

$$= \frac{(x+3)(x-2)}{4x+7}$$

$$= \frac{4x+7}{(x+3)(x-2)}$$

$x \neq -3, 2$

7. a. $4k^2 - 9 = (2k+3)(2k-3)$

b. $x^2 + 4x - 32 = x^2 + 8x - 4x - 32$
 $= x(x+8) - 4(x+8)$
 $= (x-4)(x+8)$

c. $3a^2 - 4a - 7 = 3a^2 - 7a + 3a - 7$
 $= a(3a-7) + 1(3a-7)$
 $= (a+1)(3a-7)$

d. $x^4 - 1 = (x^2 + 1)(x^2 - 1)$
 $= (x^2 + 1)(x + 1)(x - 1)$

e. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

f. $r^4 - 5r^2 + 4 = r^4 - 4r^2 - r^2 + 4$

$$= r^2(r^2 - 4) - 1(r^2 - 4)$$

$$= (r^2 - 1)(r^2 - 4)$$

$$= (r+1)(r-1)(r+2)(r-2)$$

8. a. Letting $f(a) = a^3 - b^3, f(b) = b^3 - b^3$

$$= 0$$

So b is a root of $f(a)$, and so by the factor theorem, $a - b$ is a factor of $a^3 - b^3$. Polynomial long division provides the other factor:

$$\begin{array}{r} a^2 + ab + b^2 \\ a - b \overline{) a^3 + 0a^2 + 0a - b^3} \\ \underline{a^3 - a^2b} \\ a^2b + 0a - b^3 \\ \underline{a^2b - ab^2} \\ ab^2 - b^3 \\ \underline{ab^2 - b^3} \\ 0 \end{array}$$

So $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$.

b. Using long division or recognizing a pattern from the work in part a.:

$$a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4).$$

c. Using long division or recognizing a pattern from the work in part a.: $a^7 - b^7$

$$= (a-b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6).$$

d. Using the pattern from the previous parts:

$$a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + a^2b^{n-3} + ab^{n-2} + b^{n-1}).$$

9. a. $f(2) = -2(2^4) + 3(2^2) + 7 - 2(2)$
 $= -32 + 12 + 7 - 4$
 $= -17$

b. $f(-1) = -2(-1)^4 + 3(-1)^2 + 7 - 2(-1)$
 $= -2 + 3 + 7 + 2$
 $= 10$

c. $f\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^4 + 3\left(\frac{1}{2}\right)^2 + 7 - 2\left(\frac{1}{2}\right)$
 $= -\frac{1}{8} + \frac{3}{4} + 7 - 1$
 $= \frac{53}{8}$

$$\begin{aligned}
\text{d. } f(-0.25) &= f\left(-\frac{1}{4}\right) \\
&= 2\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^2 + 7 - 2\left(-\frac{1}{4}\right) \\
&= -\frac{1}{128} + \frac{3}{16} + 7 + \frac{1}{2} \\
&= \frac{983}{128} \\
&\doteq 7.68
\end{aligned}$$

$$\begin{aligned}
\text{10. a. } \frac{3}{\sqrt{2}} &= \frac{3\sqrt{2}}{(\sqrt{2})(\sqrt{2})} \\
&= \frac{3\sqrt{2}}{2}
\end{aligned}$$

$$\begin{aligned}
\text{b. } \frac{4 - \sqrt{2}}{\sqrt{3}} &= \frac{(4 - \sqrt{2})(\sqrt{3})}{(\sqrt{3})(\sqrt{3})} \\
&= \frac{4\sqrt{3} - \sqrt{6}}{3}
\end{aligned}$$

$$\begin{aligned}
\text{c. } \frac{2 + 3\sqrt{2}}{3 - 4\sqrt{2}} &= \frac{(2 + 3\sqrt{2})(3 + 4\sqrt{2})}{(3 - 4\sqrt{2})(3 + 4\sqrt{2})} \\
&= \frac{6 + 9\sqrt{2} + 8\sqrt{2} + 12(2)}{3^2 - (4\sqrt{2})^2} \\
&= \frac{30 + 17\sqrt{2}}{9 - 16(2)} \\
&= -\frac{30 + 17\sqrt{2}}{23}
\end{aligned}$$

$$\begin{aligned}
\text{d. } \frac{3\sqrt{2} - 4\sqrt{3}}{3\sqrt{2} + 4\sqrt{3}} &= \frac{(3\sqrt{2} - 4\sqrt{3})(3\sqrt{2} - 4\sqrt{3})}{(3\sqrt{2} + 4\sqrt{3})(3\sqrt{2} - 4\sqrt{3})} \\
&= \frac{(3\sqrt{2})^2 - 24\sqrt{6} + (4\sqrt{3})^2}{(3\sqrt{2})^2 - (4\sqrt{3})^2} \\
&= \frac{9(2) - 24\sqrt{6} + 16(3)}{9(2) - 16(3)} \\
&= -\frac{66 - 24\sqrt{6}}{30} \\
&= -\frac{11 - 4\sqrt{6}}{5}
\end{aligned}$$

$$\text{11. a. } f(x) = 3x^2 - 2x$$

When $a = 2$,

$$\begin{aligned}
\frac{f(a+h) - f(a)}{h} &= \frac{f(2+h) - f(2)}{h} \\
&= \frac{3(2+h)^2 - 2(2+h) - [3(2)^2 - 2(2)]}{h} \\
&= \frac{3(4+4h+h^2) - 4 - 2h - 8}{h} \\
&= \frac{12+12h+3h^2-2h-12}{h}
\end{aligned}$$

$$\begin{aligned}
&= \frac{3h^2 + 10h}{h} \\
&= 3h + 10
\end{aligned}$$

This expression can be used to determine the slope of the secant line between $(2, 8)$ and $(2+h, f(2+h))$.

b. For $h = 0.01$: $3(0.01) + 10 = 10.03$

c. The value in part b. represents the slope of the secant line through $(2, 8)$ and $(2.01, 8.1003)$.

2.1 The Derivative Function, pp. 73–75

1. A function is not differentiable at a point where its graph has a cusp, a discontinuity, or a vertical tangent:

a. The graph has a cusp at $x = -2$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq -2\}$.

b. The graph is discontinuous at $x = 2$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq 2\}$.

c. The graph has no cusps, discontinuities, or vertical tangents, so f is differentiable on $\{x \in \mathbf{R}\}$.

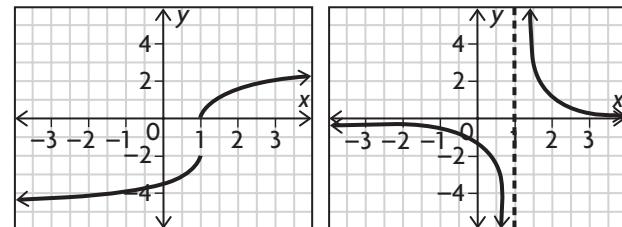
d. The graph has a cusp at $x = 1$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq 1\}$.

e. The graph has no cusps, discontinuities, or vertical tangents, so f is differentiable on $\{x \in \mathbf{R}\}$.

f. The function does not exist for $x < 2$, but has no cusps, discontinuities, or vertical tangents elsewhere, so f is differentiable on $\{x \in \mathbf{R} \mid x > 2\}$.

2. The derivative of a function represents the slope of the tangent line at a given value of the independent variable or the instantaneous rate of change of the function at a given value of the independent variable.

3.



$$\begin{aligned}
\text{4. a. } f(x) &= 5x - 2 \\
f(a+h) &= 5(a+h) - 2 \\
&= 5a + 5h - 2
\end{aligned}$$

$$\begin{aligned}
f(a+h) - f(a) &= 5a + 5h - 2 - (5a - 2) \\
&= 5h
\end{aligned}$$

$$\begin{aligned}
\text{b. } f(x) &= x^2 + 3x - 1 \\
f(a+h) &= (a+h)^2 + 3(a+h) - 1 \\
&= a^2 + 2ah + h^2 + 3a + 3h \\
&\quad + 3h - 1 \\
f(a+h) - f(a) &= a^2 + 2ah + h^2 + 3a + 3h \\
&\quad - 1 - (a^2 + 3a - 1) \\
&= 2ah + h^2 + 3h
\end{aligned}$$

c. $f(x) = x^3 - 4x + 1$
 $f(a + h) = (a + h)^3 - 4(a + h) + 1$
 $= a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h + 1$

$$f(a + h) - f(a) = a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h + 1 - (a^3 - 4a + 1)$$
 $= 3a^2h + 3ah^2 + h^3 - 4h$

d. $f(x) = x^2 + x - 6$
 $f(a + h) = (a + h)^2 + (a + h) - 6$
 $= a^2 + 2ah + h^2 + a + h - 6$
 $f(a + h) - f(a) = a^2 + 2ah + h^2 + a + h - 6 - (a^2 + a - 6)$
 $= 2ah + h^2 + h$

e. $f(x) = -7x + 4$
 $f(a + h) = -7(a + h) + 4$
 $= -7a - 7h + 4$
 $f(a + h) - f(a) = -7a - 7h + 4 - (-7a + 4)$
 $= -7h$

f. $f(x) = 4 - 2x - x^2$
 $f(a + h) = 4 - 2(a + h) - (a + h)^2$
 $= 4 - 2a - 2h - a^2 - 2ah - h^2$
 $f(a + h) - f(a) = 4 - 2a - 2h - a^2 - 2ah - h^2 - 4 + 2a + a^2$
 $= -2h - h^2 - 2ah$

5. a. $f'(1) = \lim_{h \rightarrow 0} \frac{f(1 + h) - f(1)}{h}$
 $= \lim_{h \rightarrow 0} \frac{(1 + h)^2 - 1^2}{h}$
 $= \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 - 1}{h}$
 $= \lim_{h \rightarrow 0} \frac{2h + h^2}{h}$
 $= \lim_{h \rightarrow 0} (2 + h)$
 $= 2$

b. $f'(3) = \lim_{h \rightarrow 0} \frac{f(3 + h) - f(3)}{h}$
 $= \lim_{h \rightarrow 0} \left[\frac{(3 + h)^2 + 3(3 + h) + 1}{h} - \frac{(3^2 + 3(3) + 1)}{h} \right]$
 $= \lim_{h \rightarrow 0} \frac{9 + 6h + h^2 + 9 + 3h + 1 - 19}{h}$
 $= \lim_{h \rightarrow 0} \frac{9h + h^2}{h}$
 $= \lim_{h \rightarrow 0} (9 + h)$
 $= 9$

c. $f'(0) = \lim_{h \rightarrow 0} \frac{f(0 + h) - f(0)}{h}$
 $= \lim_{h \rightarrow 0} \frac{\sqrt{h + 1} - \sqrt{0 + 1}}{h}$
 $= \lim_{h \rightarrow 0} \frac{\sqrt{h + 1} - 1}{h}$
 $= \lim_{h \rightarrow 0} \frac{(\sqrt{h + 1} - 1)(\sqrt{h + 1} + 1)}{h(\sqrt{h + 1} + 1)}$
 $= \lim_{h \rightarrow 0} \frac{(\sqrt{h + 1})^2 - 1}{h(\sqrt{h + 1} + 1)}$
 $= \lim_{h \rightarrow 0} \frac{h + 1 - 1}{h(\sqrt{h + 1} + 1)}$
 $= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{h + 1} + 1)}$
 $= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{1} + 1)}$
 $= \frac{1}{2}$

d. $f'(-1) = \lim_{h \rightarrow 0} \frac{f(-1 + h) - f(-1)}{h}$
 $= \lim_{h \rightarrow 0} \frac{\frac{5}{-1 + h} - \frac{5}{-1}}{h}$
 $= \lim_{h \rightarrow 0} \frac{\frac{5}{-1 + h} + 5}{h}$
 $= \lim_{h \rightarrow 0} \frac{\frac{5}{-1 + h} + \frac{5(-1 + h)}{-1 + h}}{h}$
 $= \lim_{h \rightarrow 0} \frac{5 - 5 + 5h}{h(-1 + h)}$
 $= \lim_{h \rightarrow 0} \frac{5h}{h(-1 + h)}$
 $= \lim_{h \rightarrow 0} \frac{5}{(-1 + h)}$
 $= \frac{5}{-1}$
 $= -5$

6. a. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$
 $= \lim_{h \rightarrow 0} \frac{-5(x + h) - 8 - (-5x - 8)}{h}$
 $= \lim_{h \rightarrow 0} \frac{-5x - 5h - 8 + 5x + 8}{h}$

$$= \lim_{h \rightarrow 0} \frac{-5h}{h}$$

$$= \lim_{h \rightarrow 0} -5$$

$$= -5$$

b. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \left[\frac{2(x+h)^2 + 4(x+h)}{h} - \frac{(2x^2 + 4x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{2x^2 + 4xh + 2h^2 + 4x}{h} + \frac{4h - 2x^2 - 4x}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 + 4h}{h}$$

$$= \lim_{h \rightarrow 0} (4x + 2h + 4)$$

$$= 4x + 4$$

c. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \left[\frac{6(x+h)^3 - 7(x+h)}{h} - \frac{(6x^3 - 7x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{6x^3 + 18x^2h + 18xh^2 + 6h^3}{h} + \frac{-7x - 7h - 6x^3 + 7x}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{18x^2h + 18xh^2 + 6h^3 - 7h}{h}$$

$$= \lim_{h \rightarrow 0} (18x^2 + 18xh + 6h^2 - 7)$$

$$= 18x^2 - 7$$

d. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3(x+h)+2} - \sqrt{3x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3x+3h+2} - \sqrt{3x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{(\sqrt{3x+3h+2} - \sqrt{3x+2})}{h} \times \frac{(\sqrt{3x+3h+2} + \sqrt{3x+2})}{(\sqrt{3x+3h+2} + \sqrt{3x+2})} \right]$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{3x+3h+2})^2 - (\sqrt{3x+2})^2}{h(\sqrt{3x+3h+2} + \sqrt{3x+2})}$$

$$= \lim_{h \rightarrow 0} \frac{3x+3h+2 - 3x-2}{h(\sqrt{3x+3h+2} + \sqrt{3x+2})}$$

$$= \lim_{h \rightarrow 0} \frac{3}{\sqrt{3x+3h+2} + \sqrt{3x+2}}$$

$$= \frac{3}{2\sqrt{3x+2}}$$

7. a. Let $y = f(x)$, then

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6 - 7(x+h) - (6 - 7x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6 - 7x - 7h - 6 + 7x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-7h}{h}$$

$$= \lim_{h \rightarrow 0} -7$$

$$= -7$$

b. Let $y = f(x)$, then

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{(x+h+1)(x-1)}{(x+h-1)(x-1)}}{h} - \frac{\frac{(x+1)(x+h-1)}{(x-1)(x+h-1)}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{x^2+hx+x-x-h-1}{(x+h-1)(x-1)}}{h} - \frac{\frac{x^2+hx-x+x+h-1}{(x+h-1)(x-1)}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{\frac{-2h}{(x+h-1)(x-1)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{(x+h-1)(x-1)}$$

$$= -\frac{2}{(x-1)^2}$$

c. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 3x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \\ &= \lim_{h \rightarrow 0} 6x + 3h \\ &= 6x\end{aligned}$$

8. Let $y = f(x)$, then the slope of the tangent at each point x can be found by calculating $f'(x)$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 4(x+h) - 2x^2 + 4x}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{2x^2 + 4xh + 2h^2 - 4x - 4h}{h} + \frac{-2x^2 + 4x}{h} \right] \\ &= \lim_{h \rightarrow 0} \frac{4xh + h^2 - 4h}{h} \\ &= \lim_{h \rightarrow 0} 4x + h - 4 \\ &= 4x - 4\end{aligned}$$

So the slope of the tangent at $x = 0$ is

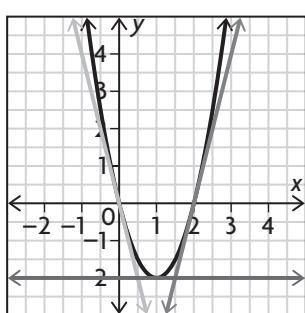
$$\begin{aligned}f'(0) &= 4(0) - 4 \\ &= -4\end{aligned}$$

At $x = 1$, the slope of the tangent is

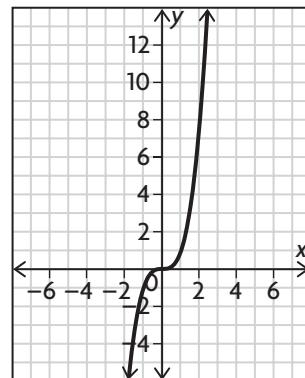
$$\begin{aligned}f'(1) &= 4(1) - 4 \\ &= 0\end{aligned}$$

At $x = 2$, the slope of the tangent is

$$\begin{aligned}f'(2) &= 4(2) - 4 \\ &= 4\end{aligned}$$



9. a.



b. Let $y = f(x)$, then the slope of the tangent at each point x can be found by calculating $f'(x)$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\ &= 3x^2\end{aligned}$$

So the slope of the tangent at $x = -2$ is

$$\begin{aligned}f'(-2) &= 3(-2)^2 \\ &= 12\end{aligned}$$

At $x = -1$, the slope of the tangent is

$$\begin{aligned}f'(-1) &= 3(-1)^2 \\ &= 3\end{aligned}$$

At $x = 0$, the slope of the tangent is

$$\begin{aligned}f'(0) &= 3(0)^2 \\ &= 0\end{aligned}$$

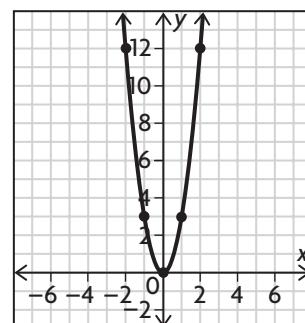
At $x = 1$, the slope of the tangent is

$$\begin{aligned}f'(1) &= 3(1)^2 \\ &= 3\end{aligned}$$

At $x = 2$, the slope of the tangent is

$$\begin{aligned}f'(2) &= 3(2)^2 \\ &= 12\end{aligned}$$

c.



d. The graph of $f(x)$ is a cubic. The graph of $f'(x)$ seems to be a parabola.

10. The velocity the particle at time t is given by $s'(t)$

$$\begin{aligned}s'(t) &= \lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h} \\&= \lim_{h \rightarrow 0} \frac{-(t+h)^2 + 8(t+h) - (-t^2 + 8t)}{h} \\&= \lim_{h \rightarrow 0} \frac{-t^2 - 2th - h^2 + 8t + 8h + t^2 - 8t}{h} \\&= \lim_{h \rightarrow 0} \frac{-2th - h^2 + 8h}{h} \\&= \lim_{h \rightarrow 0} -2t - h + 8 \\&= -2t + 8\end{aligned}$$

So the velocity at $t = 0$ is

$$\begin{aligned}s'(0) &= -2(0) + 8 \\&= 8 \text{ m/s}\end{aligned}$$

At $t = 4$, the velocity is

$$\begin{aligned}s'(4) &= -2(4) + 8 \\&= 0 \text{ m/s}\end{aligned}$$

At $t = 6$, the velocity is

$$\begin{aligned}s'(6) &= -2(6) + 8 \\&= -4 \text{ m/s}\end{aligned}$$

$$\begin{aligned}\text{11. } f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\&= \lim_{h \rightarrow 0} \left[\frac{(\sqrt{x+h+1} - \sqrt{x+1})}{h} \right. \\&\quad \times \left. \frac{(\sqrt{x+h+1} + \sqrt{x+1})}{(\sqrt{x+h+1} + \sqrt{x+1})} \right] \\&= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h+1})^2 - (\sqrt{x+1})^2}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \lim_{h \rightarrow 0} \frac{x+h+1 - x-1}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{x+h+1} + \sqrt{x+1})} \\&= \frac{1}{2\sqrt{x+1}}\end{aligned}$$

The equation $x - 6y + 4 = 0$ can be rewritten as $y = \frac{1}{6}x + \frac{2}{3}$, so this line has slope $\frac{1}{6}$. The value of x where the tangent to $f(x)$ has slope $\frac{1}{6}$ will satisfy $f'(x) = \frac{1}{6}$.

$$\begin{aligned}\frac{1}{2\sqrt{x+1}} &= \frac{1}{6} \\6 &= 2\sqrt{x+1} \\3^2 &= (\sqrt{x+1})^2 \\9 &= x+1 \\8 &= x \\f(8) &= \sqrt{8+1} \\&= \sqrt{9} \\&= 3\end{aligned}$$

So the tangent passes through the point $(8, 3)$, and its equation is $y - 3 = \frac{1}{6}(x - 8)$ or $x - 6y + 10 = 0$.

12. a. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{c - c}{h} \\&= \lim_{h \rightarrow 0} \frac{0}{h} \\&= 0\end{aligned}$$

b. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h) - x}{h} \\&= \lim_{h \rightarrow 0} \frac{h}{h} \\&= \lim_{h \rightarrow 0} 1 \\&= 1\end{aligned}$$

c. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \frac{m(x+h) + b - mx - b}{h} \\&= \lim_{h \rightarrow 0} \frac{mx + mh + b - mx - b}{h} \\&= \lim_{h \rightarrow 0} \frac{mh}{h} \\&= \lim_{h \rightarrow 0} m \\&= m\end{aligned}$$

d. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\&= \lim_{h \rightarrow 0} \left[\frac{a(x+h)^2 + b(x+h) + c}{h} \right. \\&\quad \left. - \frac{(ax^2 + bx + c)}{h} \right]\end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \left[\frac{ax^2 + 2axh + ah^2 + bx + bh}{h} \right. \\
&\quad \left. + \frac{-ax^2 - bx - c}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{2axh + ah^2 + bh}{h} \\
&= \lim_{h \rightarrow 0} (2ax + ah + b) \\
&= 2ax + b
\end{aligned}$$

13. The slope of the function at a point x is given by

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\
&= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\
&= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\
&= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\
&= 3x^2
\end{aligned}$$

Since $3x^2$ is nonnegative for all x , the original function never has a negative slope.

14. $h(t) = 18t - 4.9t^2$

$$\begin{aligned}
\text{a. } h'(t) &= \lim_{k \rightarrow 0} \frac{h(t+k) - h(t)}{k} \\
&= \lim_{k \rightarrow 0} \frac{18(t+k) - 4.9(t+k)^2}{k} \\
&\quad - \frac{(18t - 4.9t^2)}{k} \\
&= \lim_{k \rightarrow 0} \frac{18t + 18k - 4.9t^2 - 9.8tk - 4.9k^2}{k} \\
&\quad - \frac{18t + 4.9t^2}{k} \\
&= \lim_{k \rightarrow 0} \frac{18k - 9.8tk - 4.9k^2}{k} \\
&= \lim_{k \rightarrow 0} (18 - 9.8t - 4.9k) \\
&= 18 - 9.8t - 4.9(0) \\
&= 18 - 9.8t
\end{aligned}$$

Then $h'(2) = 18 - 9.8(2) = -1.6$ m/s.

b. $h'(2)$ measures the rate of change in the height of the ball with respect to time when $t = 2$.

15. a. This graph has positive slope for $x < 0$, zero slope at $x = 0$, and negative slope for $x > 0$, which corresponds to graph e.

b. This graph has positive slope for $x < 0$, zero slope at $x = 0$, and positive slope for $x > 0$, which corresponds to graph f.

c. This graph has negative slope for $x < -2$, positive slope for $-2 < x < 0$, negative slope for $0 < x < 2$, positive slope for $x > 2$, and zero slope at $x = -2$, $x = 0$, and $x = 2$, which corresponds to graph d.

16. This function is defined piecewise as $f(x) = -x^2$ for $x < 0$, and $f(x) = x^2$ for $x \geq 0$. The derivative will exist if the left-side and right-side derivatives are the same at $x = 0$:

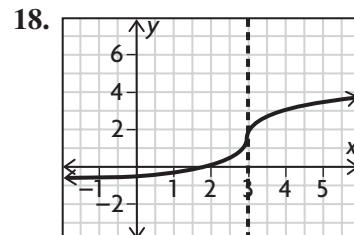
$$\begin{aligned}
\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^-} \frac{-(0+h)^2 - (-0^2)}{h} \\
&= \lim_{h \rightarrow 0^-} \frac{-h^2}{h} \\
&= \lim_{h \rightarrow 0^-} (-h) \\
&= 0
\end{aligned}$$

$$\begin{aligned}
\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^+} \frac{(0+h)^2 - (0^2)}{h} \\
&= \lim_{h \rightarrow 0^+} \frac{h^2}{h} \\
&= \lim_{h \rightarrow 0^+} (h) \\
&= 0
\end{aligned}$$

Since the limits are equal for both sides, the derivative exists and $f'(0) = 0$.

17. Since $f'(a) = 6$ and $f(a) = 0$,

$$\begin{aligned}
6 &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\
6 &= \lim_{h \rightarrow 0} \frac{f(a+h) - 0}{h} \\
3 &= \lim_{h \rightarrow 0} \frac{f(a+h)}{2h}
\end{aligned}$$



$f(x)$ is continuous.

$$f(3) = 2$$

But $f'(3) = \infty$.

(Vertical tangent)

19. $y = x^2 - 4x - 5$ has a tangent parallel to $2x - y = 1$.

Let $f(x) = x^2 - 4x - 5$. First, calculate

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \left[\frac{(x+h)^2 - 4(x+h) - 5}{h} \right. \\
&\quad \left. - \frac{(x^2 - 4x - 5)}{h} \right] \\
&= \lim_{h \rightarrow 0} \left[\frac{x^2 + 2xh + h^2 - 4x - 4h - 5}{h} \right. \\
&\quad \left. + \frac{-x^2 + 4x + 5}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 4h}{h} \\
&= \lim_{h \rightarrow 0} (2x + h - 4) \\
&= 2x + 0 - 4 \\
&= 2x - 4
\end{aligned}$$

Thus, $2x - 4$ is the slope of the tangent to the curve at x . We want the tangent parallel to $2x - y = 1$. Rearranging, $y = 2x - 1$.

If the tangent is parallel to this line,

$$2x - 4 = 2$$

$$x = 3$$

When $x = 3$, $y = (3)^2 - 4(3) - 5 = -8$.

The point is $(3, -8)$.

$$\mathbf{20. } f(x) = x^2$$

The slope of the tangent at any point (x, x^2) is

$$\begin{aligned}
f' &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h+x)(x+h-x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} \\
&= \lim_{h \rightarrow 0} (2x+h) \\
&= 2x + 0 \\
&= 2x
\end{aligned}$$

Let (a, a^2) be a point of tangency. The equation of the tangent is

$$y - a^2 = (2a)(x - a)$$

$$y = (2a)x - a^2$$

Suppose the tangent passes through $(1, -3)$.

Substitute $x = 1$ and $y = -3$ into the equation of the tangent:

$$\begin{aligned}
-3 &= (2a)(1) - a^2 \\
a^2 - 2a - 3 &= 0
\end{aligned}$$

$$(a-3)(a+1) = 0$$

$$a = -1, 3$$

So the two tangents are $y = -2x - 1$ or

$2x + y + 1 = 0$ and $y = 6x - 9$ or $6x - y - 9 = 0$.

2.2 The Derivatives of Polynomial Functions, pp. 82–84

1. Answers may vary. For example:

constant function rule: $\frac{d}{dx}(5) = 0$

power rule: $\frac{d}{dx}(x^3) = 3x^2$

constant multiple rule: $\frac{d}{dx}(4x^3) = 12x^2$

sum rule: $\frac{d}{dx}(x^2 + x) = 2x + 1$

difference rule: $\frac{d}{dx}(x^3 - x^2 + 3x) = 3x^2 - 2x + 3$

$$\begin{aligned}
\mathbf{2. a. } f'(x) &= \frac{d}{dx}(4x) - \frac{d}{dx}(7) \\
&= 4 \frac{d}{dx}(x) - \frac{d}{dx}(7) \\
&= 4(x^0) - 0 \\
&= 4
\end{aligned}$$

$$\begin{aligned}
\mathbf{b. } f'(x) &= \frac{d}{dx}(x^3) - \frac{d}{dx}(x^2) \\
&= 3x^2 - 2x
\end{aligned}$$

$$\begin{aligned}
\mathbf{c. } f'(x) &= \frac{d}{dx}(-x^2) + \frac{d}{dx}(5x) + \frac{d}{dx}(8) \\
&= -\frac{d}{dx}(x^2) + 5 \frac{d}{dx}(x) + \frac{d}{dx}(8) \\
&= -(2x) + 5 + 0 \\
&= -2x + 5
\end{aligned}$$

$$\begin{aligned}
\mathbf{d. } f'(x) &= \frac{d}{dx}(\sqrt[3]{x}) \\
&= \frac{d}{dx}(x^{\frac{1}{3}}) \\
&= \frac{1}{3}(x^{(\frac{1}{3}-1)}) \\
&= \frac{1}{3}(x^{-\frac{2}{3}}) \\
&= \frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}} \\
&= \frac{1}{3\sqrt[3]{x^2}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{e. } f'(x) &= \frac{d}{dx}\left(\left(\frac{x}{2}\right)^4\right) \\
&= \left(\frac{1}{2}\right)^4 \frac{d}{dx}(x^4) \\
&= \frac{1}{16}(4x^3) \\
&= \frac{x^3}{4}
\end{aligned}$$

$$\mathbf{f. } f'(x) = \frac{d}{dx}(x^{-3}) \\ = (-3)(x^{-3-1}) \\ = -3x^{-4}$$

$$\mathbf{3. a. } h'(x) = \frac{d}{dx}((2x+3)(x+4)) \\ = \frac{d}{dx}(2x^2 + 8x + 3x + 12) \\ = \frac{d}{dx}(2x^2) + \frac{d}{dx}(11x) + \frac{d}{dx}(12) \\ = 2\frac{d}{dx}(x^2) + 11\frac{d}{dx}(x) + \frac{d}{dx}(12) \\ = 2(2x) + 11(1) + 0 \\ = 4x + 11$$

$$\mathbf{b. } f'(x) = \frac{d}{dx}(2x^3 + 5x^2 - 4x - 3.75) \\ = \frac{d}{dx}(2x^3) + \frac{d}{dx}(5x^2) - \frac{d}{dx}(4x) \\ - \frac{d}{dx}(3.75) \\ = 2\frac{d}{dx}(x^3) + 5\frac{d}{dx}(x^2) - 4\frac{d}{dx}(x) \\ - \frac{d}{dx}(3.75) \\ = 2(3x^2) + 5(2x) - 4(1) - 0 \\ = 6x^2 + 10x - 4$$

$$\mathbf{c. } \frac{ds}{dt} = \frac{d}{dt}(t^2(t^2 - 2t)) \\ = \frac{d}{dt}(t^4 - 2t^3) \\ = \frac{d}{dt}(t^4) - \frac{d}{dt}(2t^3) \\ = \frac{d}{dt}(t^4) - 2\frac{d}{dt}(t^3) \\ = 4t^3 - 2(3t^2) \\ = 4t^3 - 6t^2$$

$$\mathbf{d. } \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{5}x^5 + \frac{1}{3}x^3 - \frac{1}{2}x^2 + 1\right) \\ = \frac{d}{dx}\left(\frac{1}{5}x^5\right) + \frac{d}{dx}\left(\frac{1}{3}x^3\right) - \frac{d}{dx}\left(\frac{1}{2}x^2\right) + \frac{d}{dx}(1) \\ = \left(\frac{1}{5}\right)\frac{d}{dx}(x^5) + \left(\frac{1}{3}\right)\frac{d}{dx}(x^3) - \left(\frac{1}{2}\right)\frac{d}{dx}(x^2) \\ + \frac{d}{dx}(1) \\ = \frac{1}{5}(5x^4) + \frac{1}{3}(3x^2) - \frac{1}{2}(2x) + 0 \\ = x^4 + x^2 - x$$

$$\mathbf{e. } g'(x) = \frac{d}{dx}(5(x^2)^4)$$

$$= 5\frac{d}{dx}(x^{2 \times 4}) \\ = 5\frac{d}{dx}(x^8) \\ = 5(8x^7) \\ = 40x^7$$

$$\mathbf{f. } s'(t) = \frac{d}{dt}\left(\frac{t^5 - 3t^2}{2t}\right) \\ = \left(\frac{1}{2}\right)\frac{d}{dt}(t^4 - 3t) \\ = \left(\frac{1}{2}\right)\left(\frac{d}{dt}(t^4) - \frac{d}{dt}(3t)\right) \\ = \left(\frac{1}{2}\right)\left(\frac{d}{dt}(t^4) - 3\frac{d}{dt}(t)\right) \\ = \left(\frac{1}{2}\right)(4t^3 - 3(1)) \\ = 2t^3 - \frac{3}{2}$$

$$\mathbf{4. a. } \frac{dy}{dx} = \frac{d}{dx}(3x^{\frac{5}{3}}) \\ = 3\frac{d}{dx}(x^{\frac{5}{3}}) \\ = \left(\frac{5}{3}\right)3(x^{(\frac{5}{3}-1)}) \\ = 5x^{\frac{2}{3}}$$

$$\mathbf{b. } \frac{dy}{dx} = \frac{d}{dx}\left(4x^{-\frac{1}{2}} - \frac{6}{x}\right) \\ = 4\frac{d}{dx}(x^{-\frac{1}{2}}) - 6\frac{d}{dx}(x^{-1}) \\ = 4\left(\frac{-1}{2}\right)(x^{-\frac{1}{2}-1}) - 6(-1)(x^{-1-1}) \\ = -2x^{-\frac{3}{2}} + 6x^{-2}$$

$$\mathbf{c. } \frac{dy}{dx} = \frac{d}{dx}\left(\frac{6}{x^3} + \frac{2}{x^2} - 3\right) \\ = 6\frac{d}{dx}(x^{-3}) + 2\frac{d}{dx}(x^{-2}) - \frac{d}{dx}(3) \\ = 6(-3)(x^{-3-1}) + 2(-2)(x^{-2-1}) - 0 \\ = -18x^{-4} - 4x^{-3} \\ = \frac{-18}{x^4} - \frac{4}{x^3}$$

$$\begin{aligned}\mathbf{d.} \frac{dy}{dx} &= \frac{d}{dx}(9x^{-2} + 3\sqrt{x}) \\&= 9\frac{d}{dx}(x^{-2}) + 3\frac{d}{dx}(x^{\frac{1}{2}}) \\&= 9(-2)(x^{-2-1}) + 3\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) \\&= -18x^{-3} + \frac{3}{2}x^{-\frac{1}{2}}\end{aligned}$$

$$\begin{aligned}\mathbf{e.} \frac{dy}{dx} &= \frac{d}{dx}(\sqrt{x} + 6\sqrt{x^3} + \sqrt{2}) \\&= \frac{d}{dx}(x^{\frac{1}{2}}) + 6\frac{d}{dx}(x^{\frac{3}{2}}) + \frac{d}{dx}(\sqrt{2}) \\&= \frac{1}{2}(x^{\frac{1}{2}-1}) + 6\left(\frac{3}{2}\right)(x^{\frac{3}{2}-1}) + 0 \\&= \frac{1}{2}(x^{-\frac{1}{2}}) + 9x^{\frac{1}{2}}\end{aligned}$$

$$\begin{aligned}\mathbf{f.} \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1+\sqrt{x}}{x}\right) \\&= \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}\left(\frac{x^{\frac{1}{2}}}{x}\right) \\&= \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(x^{-\frac{1}{2}}) \\&= (-1)x^{-1-1} + \frac{-1}{2}(x^{-\frac{1}{2}-1}) \\&= -x^{-2} - \frac{1}{2}x^{-\frac{3}{2}}\end{aligned}$$

$$\begin{aligned}\mathbf{5. a.} \frac{ds}{dt} &= \frac{d}{dt}(-2t^2 + 7t) \\&= (-2)\left(\frac{d}{dt}(t^2)\right) + 7\left(\frac{d}{dt}(t)\right) \\&= (-2)(2t) + 7(1) \\&= -4t + 7\end{aligned}$$

$$\begin{aligned}\mathbf{b.} \frac{ds}{dt} &= \frac{d}{dt}\left(18 + 5t - \frac{1}{3}t^3\right) \\&= \frac{d}{dt}(18) + 5\frac{d}{dt}(t) - \left(\frac{1}{3}\right)\frac{d}{dt}(t^3) \\&= 0 + 5(1) - \left(\frac{1}{3}\right)(3t^2) \\&= 5 - t^2\end{aligned}$$

$$\begin{aligned}\mathbf{c.} \frac{ds}{dt} &= \frac{d}{dt}((t-3)^2) \\&= \frac{d}{dt}(t^2 - 6t + 9) \\&= \frac{d}{dt}(t^2) - (6)\frac{d}{dt}(t) + \frac{d}{dt}(9)\end{aligned}$$

$$\begin{aligned}&= 2t - 6(1) + 0 \\&= 2t - 6\end{aligned}$$

$$\begin{aligned}\mathbf{6. a.} f'(x) &= \frac{d}{dx}(x^3 - \sqrt{x}) \\&= \frac{d}{dx}(x^3) - \frac{d}{dx}(x^{\frac{1}{2}}) \\&= 3x^2 - \frac{1}{2}(x^{\frac{1}{2}-1}) \\&= 3x^2 - \frac{1}{2}x^{-\frac{1}{2}}\end{aligned}$$

$$\begin{aligned}\text{so } f'(a) &= f'(4) = 3(4)^2 - \frac{1}{2}(4)^{-\frac{1}{2}} \\&= 3(16) - \frac{1}{2}\frac{1}{\sqrt{4}} \\&= 48 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\&= 47.75\end{aligned}$$

$$\begin{aligned}\mathbf{b.} f'(x) &= \frac{d}{dx}(7 - 6\sqrt{x} + 5x^{\frac{2}{3}}) \\&= \frac{d}{dx}(7) - 6\frac{d}{dx}(x^{\frac{1}{2}}) + 5\frac{d}{dx}(x^{\frac{2}{3}}) \\&= 0 - 6\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) + 5\left(\frac{2}{3}\right)(x^{\frac{2}{3}-1}) \\&= -3x^{-\frac{1}{2}} + \left(\frac{10}{3}\right)(x^{-\frac{1}{3}})\end{aligned}$$

$$\begin{aligned}\text{so } f'(a) &= f'(64) = -3(64^{-\frac{1}{2}}) + \left(\frac{10}{3}\right)(64^{-\frac{1}{3}}) \\&= -3\left(\frac{1}{8}\right) + \frac{10}{3}\left(\frac{1}{4}\right) \\&= \frac{11}{24}\end{aligned}$$

$$\begin{aligned}\mathbf{7. a.} \frac{dy}{dx} &= \frac{d}{dx}(3x^4) \\&= 3\frac{d}{dx}(x^4) \\&= 3(4x^3) \\&= 12x^3\end{aligned}$$

The slope at (1, 3) is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$. So the slope = $12(1)^3 = 12$

$$\begin{aligned}\mathbf{b.} \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{x^{-5}}\right) \\&= \frac{d}{dx}(x^5) \\&= 5x^4\end{aligned}$$

The slope at $(-1, -1)$ is found by substituting $x = -1$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= 5(-1)^4 \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{c. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{2}{x}\right) \\ &= 2\frac{d}{dx}(x^{-1}) \\ &= 2(-1)x^{-1-1} \\ &= -2x^{-2}\end{aligned}$$

The slope at $(-2, -1)$ is found by substituting $x = -2$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= -2(-2)^{-2} \\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{d. } \frac{dy}{dx} &= \frac{d}{dx}(\sqrt{16x^3}) \\ &= \sqrt{16}\frac{d}{dx}(x^{\frac{3}{2}}) \\ &= 4\left(\frac{3}{2}\right)x^{\frac{3}{2}-1} \\ &= 6x^{\frac{1}{2}}\end{aligned}$$

The slope at $(4, 32)$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= 6(4)^{\frac{1}{2}} \\ &= 12\end{aligned}$$

$$8. \text{ a. } y = 2x^3 + 3x$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(2x^3 + 3x) \\ &= 2\frac{d}{dx}(x^3) + 3\frac{d}{dx}(x) \\ &= 2(3x^2) + 3(1) \\ &= 6x^2 + 3\end{aligned}$$

The slope at $x = 1$ is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$. So the slope is

$$6(1)^2 + 3 = 9.$$

$$\text{b. } y = 2\sqrt{x} + 5$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(2\sqrt{x} + 5) \\ &= 2\frac{d}{dx}(x^{\frac{1}{2}}) + \frac{d}{dx}(5) \\ &= 2\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) + 0 \\ &= x^{-\frac{1}{2}}\end{aligned}$$

The slope at $x = 4$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$. So the slope is $(4)^{\frac{-1}{2}} = \frac{1}{2}$.

$$\text{c. } y = \frac{16}{x^2}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{16}{x^2}\right) \\ &= 16\frac{d}{dx}(x^{-2}) \\ &= 16(-2)x^{-2-1} \\ &= -32x^{-3}\end{aligned}$$

The slope at $x = -2$ is found by substituting

$x = -2$ into the equation for $\frac{dy}{dx}$. So the slope is $-32(-2)^{-3} = \frac{(-32)}{(-2)^3} = 4$.

$$\text{d. } y = x^{-3}(x^{-1} + 1)$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(x^{-4} + x^{-3}) \\ &= -4x^{-5} - 3x^{-4} \\ &= -\frac{4}{x^5} - \frac{3}{x^4}\end{aligned}$$

The slope at $x = 1$ is found by substituting

$x = 1$ into the equation for $\frac{dy}{dx}$. So the slope is $-\frac{4}{1^5} - \frac{3}{1^4} = -7$.

$$9. \text{ a. } \frac{dy}{dx} = \frac{d}{dx}\left(2x - \frac{1}{x}\right)$$

$$\begin{aligned}&= 2\frac{d}{dx}(x) - \frac{d}{dx}(x^{-1}) \\ &= 2(1) - (-1)x^{-1-1} \\ &= 2 + x^{-2}\end{aligned}$$

The slope at $x = 0.5$ is found by substituting

$x = 0.5$ into the equation for $\frac{dy}{dx}$.

So the slope is $2 + (0.5)^{-2} = 6$.

The equation of the tangent line is therefore $y + 1 = 6(x - 0.5)$ or $6x - y - 4 = 0$.

$$\text{b. } \frac{dy}{dx} = \frac{d}{dx}\left(\frac{3}{x^2} - \frac{4}{x^3}\right)$$

$$\begin{aligned}&= 3\frac{d}{dx}(x^{-2}) - 4\frac{d}{dx}(x^{-3}) \\ &= 3(-2)x^{-2-1} - 4(-3)x^{-3-1} \\ &= 12x^{-4} - 6x^{-3}\end{aligned}$$

The slope at $x = -1$ is found by substituting

$x = -1$ into the equation for $\frac{dy}{dx}$. So the slope is $12(-1)^{-4} - 6(-1)^{-3} = 18$.

The equation of the tangent line is therefore $y - 7 = 18(x + 1)$ or $18x - y + 25 = 0$.

$$\begin{aligned} \text{c. } \frac{dy}{dx} &= \frac{d}{dx}(\sqrt{3x^3}) \\ &= \sqrt{3} \frac{d}{dx}(x^{\frac{3}{2}}) \\ &= \sqrt{3} \left(\frac{3}{2}\right) x^{\frac{3}{2}-1} \\ &= \frac{3\sqrt{3}x^{\frac{1}{2}}}{2} \end{aligned}$$

The slope at $x = 3$ is found by substituting $x = 3$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } \frac{3\sqrt{3}(3)^{\frac{1}{2}}}{2} = \frac{9}{2}.$$

The equation of the tangent line is therefore $y - 9 = \frac{9}{2}(x - 3)$ or $9x - 2y - 9 = 0$.

$$\begin{aligned} \text{d. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{x}\left(x^2 + \frac{1}{x}\right)\right) \\ &= \frac{d}{dx}\left(x + \frac{1}{x^2}\right) \\ &= \frac{d}{dx}(x) + \frac{d}{dx}(x^{-2}) \\ &= 1 + (-2)x^{-2-1} \\ &= 1 - 2x^{-3} \end{aligned}$$

The slope at $x = 1$ is found by substituting into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } 1 - 2(1)^{-3} = -1.$$

The equation of the tangent line is therefore $y - 2 = -(x - 1)$ or $x + y - 3 = 0$.

$$\begin{aligned} \text{e. } \frac{dy}{dx} &= \frac{d}{dx}((\sqrt{x} - 2)(3\sqrt{x} + 8)) \\ &= \frac{d}{dx}(3(\sqrt{x})^2 + 8\sqrt{x} - 6\sqrt{x} - 16) \\ &= \frac{d}{dx}(3x + 2\sqrt{x} - 16) \\ &= \frac{d}{dx}(3x) + 2\frac{d}{dx}(x^{\frac{1}{2}}) - \frac{d}{dx}(16) \\ &= 3(1) + 2\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} - 0 \\ &= 3 + x^{-\frac{1}{2}} \end{aligned}$$

The slope at $x = 4$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } 3 + (4)^{-\frac{1}{2}} = 3.5.$$

The equation of the tangent line is therefore $y = 3.5(x - 4)$ or $7x - 2y - 28 = 0$.

$$\begin{aligned} \text{f. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{\sqrt{x} - 2}{\sqrt[3]{x}}\right) \\ &= \frac{d}{dx}\left(\frac{x^{\frac{1}{2}} - 2}{x^{\frac{1}{3}}}\right) \\ &= \frac{d}{dx}(x^{\frac{1}{2}-\frac{1}{3}} - 2x^{-\frac{1}{3}}) \\ &= \frac{d}{dx}(x^{\frac{1}{6}}) - 2\frac{d}{dx}(x^{-\frac{1}{3}}) \\ &= \frac{1}{6}(x^{\frac{1}{6}-1}) - 2\left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1} - 0 \\ &= \frac{1}{6}(x^{-\frac{5}{6}}) + \frac{2}{3}x^{-\frac{4}{3}} \end{aligned}$$

The slope at $x = 1$ is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } \frac{1}{6}(1)^{-\frac{5}{6}} + \frac{2}{3}(1)^{-\frac{4}{3}} = \frac{5}{6}.$$

The equation of the tangent line is therefore $y + 1 = \frac{5}{6}(x - 1)$ or $5x - 6y - 11 = 0$.

10. A normal to the graph of a function at a point is a line that is perpendicular to the tangent at the given point.

$$y = \frac{3}{x^2} - \frac{4}{x^3} \text{ at } P(-1, 7)$$

Slope of the tangent is 18, therefore, the slope of the normal is $-\frac{1}{18}$.

$$\text{Equation is } y - 7 = -\frac{1}{18}(x + 1).$$

$$x + 18y - 125 = 0$$

$$\text{11. } y = \frac{3}{\sqrt[3]{x}} = 3x^{-\frac{1}{3}}$$

Parallel to $x + 16y + 3 = 0$

Slope of the line is $-\frac{1}{16}$.

$$\frac{dy}{dx} = -x^{-\frac{4}{3}}$$

$$x^{-\frac{4}{3}} = \frac{1}{16}$$

$$\frac{1}{x^{\frac{4}{3}}} = \frac{1}{16}$$

$$x^{\frac{4}{3}} = 16$$

$$x = (16)^{\frac{3}{4}} = 8$$

12. $y = \frac{1}{x} = x^{-1}$; $y = x^3$

$$\frac{dy}{dx} = -\frac{1}{x^2}; \frac{dy}{dx} = 3x^2$$

Now, $-\frac{1}{x^2} = 3x^2$

$$x^4 = -\frac{1}{3}.$$

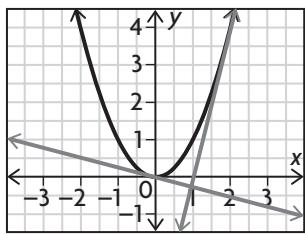
No real solution. They never have the same slope.

13. $y = x^2$, $\frac{dy}{dx} = 2x$

The slope of the tangent at $A(2, 4)$ is 4 and at

$$B\left(-\frac{1}{8}, \frac{1}{64}\right)$$

Since the product of the slopes is -1 , the tangents at $A(2, 4)$ and $B\left(-\frac{1}{8}, \frac{1}{64}\right)$ will be perpendicular.



14. $y = -x^2 + 3x + 4$

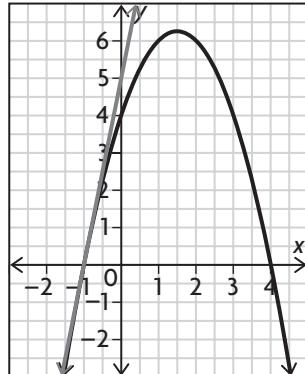
$$\frac{dy}{dx} = -2x + 3$$

For $\frac{dy}{dx} = 5$

$$5 = -2x + 3$$

$$x = -1.$$

The point is $(-1, 0)$.



15. $y = x^3 + 2$

$$\frac{dy}{dx} = 3x^2, \text{slope is } 12$$

$$x^2 = 4$$

$$x = 2 \text{ or } x = -2$$

Points are $(2, 10)$ and $(-2, -6)$.

16. $y = \frac{1}{5}x^5 - 10x$, slope is 6

$$\frac{dy}{dx} = x^4 - 10 = 6$$

$$x^4 = 16$$

$$x^2 = 4 \text{ or } x^2 = -4$$

$$x = \pm 2 \text{ non-real}$$

Tangents with slope 6 are at the points $(2, -\frac{68}{5})$ and $(-2, \frac{68}{5})$.

17. $y = 2x^2 + 3$

a. Equation of tangent from $A(2, 3)$:

$$\text{If } x = a, y = 2a^2 + 3.$$

Let the point of tangency be $P(a, 2a^2 + 3)$.

$$\text{Now, } \frac{dy}{dx} = 4x \text{ and when } x = a, \frac{dy}{dx} = 4a.$$

The slope of the tangent is the slope of AP .

$$\frac{2a^2}{a - 2} = 4a.$$

$$2a^2 = 4a^2 - 8a$$

$$2a^2 - 8a = 0$$

$$2a(a - 4) = 0$$

$$a = 0 \text{ or } a = 4.$$

Point $(2, 3)$:

Slope is 0.

Equation of tangent is

$$y - 3 = 0.$$

Slope is 16.

Equation of tangent is

$$y - 3 = 16(x - 2) \text{ or}$$

$$16x - y - 29 = 0.$$

b. From the point $B(2, -7)$:

$$\text{Slope of } BP: \frac{2a^2 + 10}{a - 2} = 4a$$

$$2a^2 + 10 = 4a^2 - 8a$$

$$2a^2 - 8a - 10 = 0$$

$$a^2 - 4a - 5 = 0$$

$$(a - 5)(a + 1) = 0$$

$$a = 5$$

$$a = -1$$

Slope is $4a = 20$.

Equation is

$$y + 7 = 20(x - 2)$$

$$\text{or } 20x - y - 47 = 0.$$

Slope is $4a = -4$.

Equation is

$$y + 7 = -4(x - 2)$$

$$\text{or } 4x + y - 1 = 0.$$

18. $ax - 4y + 21 = 0$ is tangent to $y = \frac{a}{x^2}$ at $x = -2$.

Therefore, the point of tangency is $(-2, \frac{a}{4})$,

This point lies on the line, therefore,

$$a(-2) - 4\left(\frac{a}{4}\right) + 21 = 0$$

$$-3a + 21 = 0$$

$$a = 7.$$

19. a. When $h = 200$,

$$d = 3.53\sqrt{200}$$

$$\doteq 49.9$$

Passengers can see about 49.9 km.

b. $d = 3.53\sqrt{h} = 3.53h^{\frac{1}{2}}$

$$d' = 3.53\left(\frac{1}{2}h^{-\frac{1}{2}}\right)$$

$$= \frac{3.53}{2\sqrt{h}}$$

When $h = 200$,

$$d' = \frac{3.53}{2\sqrt{200}}$$

$$\doteq 0.12$$

The rate of change is about 0.12 km/m.

20. $d(t) = 4.9t^2$

a. $d(2) = 4.9(2)^2 = 19.6$ m

$d(5) = 4.9(5)^2 = 122.5$ m

The average rate of change of distance with respect to time from 2 s to 5 s is

$$\frac{\Delta d}{\Delta t} = \frac{122.5 - 19.6}{5 - 2}$$

$$= 34.3 \text{ m/s}$$

b. $d'(t) = 9.8t$

Thus, $d'(4) = 9.8(4) = 39.2$ m/s.

c. When the object hits the ground, $d = 150$.

Set $d(t) = 150$:

$$4.9t^2 = 150$$

$$t^2 = \frac{1500}{49}$$

$$t = \pm \frac{10}{7}\sqrt{15}$$

$$\text{Since } t \geq 0, t = \frac{10}{7}\sqrt{15}$$

Then,

$$d'\left(\frac{10}{7}\sqrt{15}\right) = 9.8\left(\frac{10}{7}\sqrt{15}\right)$$

$$\doteq 54.2 \text{ m/s}$$

21. $v(t) = s'(t) = 2t - t^2$

$$0.5 = 2t - t^2$$

$$t^2 - 2t + 0.5 = 0$$

$$2t^2 - 4t + 1 = 0$$

$$t = \frac{4 \pm \sqrt{8}}{4}$$

$$t \doteq 1.71, 0.29$$

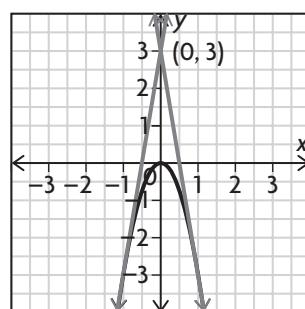
The train has a velocity of 0.5 km/min at about 0.29 min and 1.71 min.

22. $v(t) = R'(t) = -10t$

$$v(2) = -20$$

The velocity of the bolt at $t = 2$ is -20 m/s.

23.



Let the coordinates of the points of tangency be $A(a, -3a^2)$.

$$\frac{dy}{dx} = -6x, \text{ slope of the tangent at } A \text{ is } -6a$$

$$\text{Slope of } PA: \frac{-3a^2 - 3}{a} = -6a$$

$$-3a^2 - 3 = -6a^2$$

$$3a^2 = 3$$

$$a = 1 \text{ or } a = -1$$

Coordinates of the points at which the tangents touch the curve are $(1, -3)$ and $(-1, -3)$.

24. $y = x^3 - 6x^2 + 8x$, tangent at $A(3, -3)$

$$\frac{dy}{dx} = 3x^2 - 12x + 8$$

When $x = 3$,

$$\frac{dy}{dx} = 27 - 36 + 8 = -1$$

The slope of the tangent at $A(3, -3)$ is -1 .

Equation will be

$$y + 3 = -1(x - 3)$$

$$y = -x$$

$$-x = x^3 - 6x^2 + 8x$$

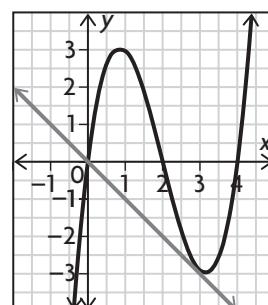
$$x^3 - 6x^2 + 9x = 0$$

$$x(x^2 - 6x + 9) = 0$$

$$x(x - 3)^2 = 0$$

$$x = 0 \text{ or } x = 3$$

Coordinates are $B(0, 0)$.



25. a. i. $f(x) = 2x - 5x^2$

$$f'(x) = 2 - 10x$$

Set $f'(x) = 0$:

$$2 - 10x = 0$$

$$10x = 2$$

$$x = \frac{1}{5}$$

Then,

$$\begin{aligned} f\left(\frac{1}{5}\right) &= 2\left(\frac{1}{5}\right) - 5\left(\frac{1}{5}\right)^2 \\ &= \frac{2}{5} - \frac{1}{5} \\ &= \frac{1}{5} \end{aligned}$$

Thus the point is $\left(\frac{1}{5}, \frac{1}{5}\right)$.

ii. $f(x) = 4x^2 + 2x - 3$

$$f'(x) = 8x + 2$$

Set $f'(x) = 0$:

$$8x + 2 = 0$$

$$8x = -2$$

$$x = -\frac{1}{4}$$

Then,

$$\begin{aligned} f\left(-\frac{1}{4}\right) &= 4\left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right) - 3 \\ &= \frac{1}{4} - \frac{2}{4} - \frac{12}{4} \\ &= -\frac{13}{4} \end{aligned}$$

Thus the point is $\left(-\frac{1}{4}, -\frac{13}{4}\right)$.

iii. $f(x) = x^3 - 8x^2 + 5x + 3$

$$f'(x) = 3x^2 - 16x + 5$$

Set $f'(x) = 0$:

$$3x^2 - 16x + 5 = 0$$

$$3x^2 - 15x - x + 5 = 0$$

$$3x(x - 5) - (x - 5) = 0$$

$$(3x - 1)(x - 5) = 0$$

$$x = \frac{1}{3}, 5$$

$$\begin{aligned} f\left(\frac{1}{3}\right) &= \left(\frac{1}{3}\right)^3 - 8\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) + 3 \\ &= \frac{1}{27} - \frac{24}{27} + \frac{45}{27} + \frac{81}{27} \\ &= \frac{103}{27} \end{aligned}$$

$$\begin{aligned} f(5) &= (5)^3 - 8(5)^2 + 5(5) + 3 \\ &= 25 - 200 + 25 + 3 \\ &= -47 \end{aligned}$$

Thus the two points are $\left(\frac{1}{3}, \frac{103}{27}\right)$ and $(5, -47)$.

b. At these points, the slopes of the tangents are zero, meaning that the rate of change of the value of the function with respect to the domain is zero. These points are also local maximum or minimum points.

26. $\sqrt{x} + \sqrt{y} = 1$

$P(a, b)$ is on the curve, therefore $a \geq 0, b \geq 0$.

$$\sqrt{y} = 1 - \sqrt{x}$$

$$y = 1 - 2\sqrt{x} + x$$

$$\frac{dy}{dx} = -\frac{1}{2} \cdot 2x^{-\frac{1}{2}} + 1$$

$$\text{At } x = a, \text{slope is } -\frac{1}{\sqrt{a}} + 1 = \frac{-1 + \sqrt{a}}{\sqrt{a}}.$$

But $\sqrt{a} + \sqrt{b} = 1$

$$-\sqrt{b} = \sqrt{a} - 1.$$

$$\text{Therefore, slope is } -\frac{\sqrt{b}}{\sqrt{a}} = -\sqrt{\frac{b}{a}}.$$

27. $f(x) = x^n, f'(x) = nx^{n-1}$

Slope of the tangent at $x = 1$ is $f'(1) = n$,

The equation of the tangent at $(1, 1)$ is:

$$y - 1 = n(x - 1)$$

$$nx - y - n + 1 = 0$$

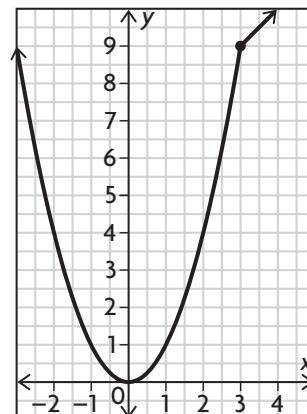
$$\text{Let } y = 0, nx = n - 1$$

$$x = \frac{n-1}{n} = 1 - \frac{1}{n}.$$

The x -intercept is $1 - \frac{1}{n}$; as $n \rightarrow \infty$, $\frac{1}{n} \rightarrow 0$, and

the x -intercept approaches 1. As $n \rightarrow \infty$, the slope of the tangent at $(1, 1)$ increases without bound, and the tangent approaches a vertical line having equation $x - 1 = 0$.

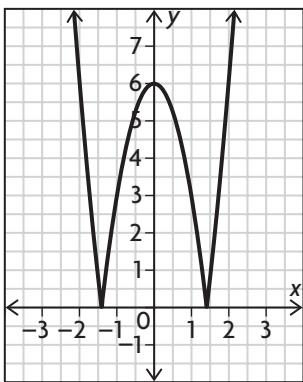
28. a.



$$f(x) = \begin{cases} x^2, & \text{if } x < 3 \\ x + 6, & \text{if } x \geq 3 \end{cases} \quad f'(x) = \begin{cases} 2x, & \text{if } x < 3 \\ 1, & \text{if } x \geq 3 \end{cases}$$

$f'(3)$ does not exist.

b.

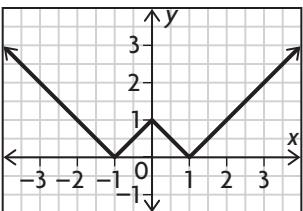


$$f(x) = \begin{cases} 3x^2 - 6, & \text{if } x < -\sqrt{2} \text{ or } x > \sqrt{2} \\ 6 - 3x^2, & \text{if } -\sqrt{2} < x < \sqrt{2} \end{cases}$$

$$f'(x) = \begin{cases} 6x, & \text{if } x < -\sqrt{2} \text{ or } x > \sqrt{2} \\ -6x, & \text{if } -\sqrt{2} \leq x \leq \sqrt{2} \end{cases}$$

$f'(\sqrt{2})$ and $f'(-\sqrt{2})$ do not exist.

c.



$$f(x) = \begin{cases} x - 1, & \text{if } x \geq 1 \\ 1 - x, & \text{if } 0 \leq x < 1 \\ x + 1, & \text{if } -1 < x < 0 \\ -x - 1, & \text{if } x \leq -1 \end{cases}$$

since $|x - 1| = x - 1$
since $|x - 1| = 1 - x$
since $|-x - 1| = x + 1$
since $|-x - 1| = -x - 1$

$$f'(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } 0 < x < 1 \\ 1, & \text{if } -1 < x < 0 \\ -1, & \text{if } x < -1 \end{cases}$$

$f'(0)$, $f'(-1)$, and $f'(1)$ do not exist.

2.3 The Product Rule, pp. 90–91

1. a. $h(x) = x(x - 4)$
 $\quad h'(x) = x(1) + (1)(x - 4)$
 $\quad \quad \quad = 2x - 4$

b. $h(x) = x^2(2x - 1)$
 $\quad h'(x) = x^2(2) + (2x)(2x - 1)$
 $\quad \quad \quad = 6x^2 - 2x$

c. $h(x) = (3x + 2)(2x - 7)$
 $\quad h'(x) = (3x + 2)(2) + (3)(2x - 7)$
 $\quad \quad \quad = 12x - 17$

d. $h(x) = (5x^7 + 1)(x^2 - 2x)$
 $\quad h'(x) = (5x^7 + 1)(2x - 2) + (35x^6)(x^2 - 2x)$
 $\quad \quad \quad = 45x^8 - 80x^7 + 2x - 2$

e. $s(t) = (t^2 + 1)(3 - 2t^2)$
 $\quad s'(t) = (t^2 + 1)(-4t) + (2t)(3 - 2t^2)$
 $\quad \quad \quad = -8t^3 + 2t$

f. $f(x) = \frac{x - 3}{x + 3}$

$$\begin{aligned} f(x) &= (x - 3)(x + 3)^{-1} \\ f'(x) &= (x - 3)(-1)(x + 3)^{-2} + (1)(x + 3)^{-1} \\ &= (x + 3)^{-2}(-x + 3 + x + 3) \\ &= \frac{6}{(x + 3)^2} \end{aligned}$$

2. a. $y = (5x + 1)^3(x - 4)$

$$\begin{aligned} \frac{dy}{dx} &= (5x + 1)^3(1) + 3(5x + 1)^2(5)(x - 4) \\ &= (5x + 1)^3 + 15(5x + 1)^2(x - 4) \end{aligned}$$

b. $y = (3x^2 + 4)(3 + x^3)^5$

$$\begin{aligned} \frac{dy}{dx} &= (3x^2 + 4)(5)(3 + x^3)^4(3x^2) \\ &\quad + (6x)(3 + x^3)^5 \\ &= 15x^2(3x^2 + 4)(3 + x^3)^4 + 6x(3 + x^3)^5 \end{aligned}$$

c. $y = (1 - x^2)^4(2x + 6)^3$

$$\begin{aligned} \frac{dy}{dx} &= 4(1 - x^2)^3(-2x)(2x + 6)^3 \\ &\quad + (1 - x^2)^4 3(2x + 6)^2(2) \\ &= -8x(1 - x^2)^3(2x + 6)^3 \\ &\quad + 6(1 - x^2)^4(2x + 6)^2 \end{aligned}$$

d. $y = (x^2 - 9)^4(2x - 1)^3$

$$\begin{aligned} \frac{dy}{dx} &= (x^2 - 9)^4(3)(2x - 1)^2(2) \\ &\quad + 4(x^2 - 9)^3(2x)(2x - 1)^3 \\ &= 6(x^2 - 9)^4(2x - 1)^2 \\ &\quad + 8x(x^2 - 9)^3(2x - 1)^3 \end{aligned}$$

3. It is not appropriate or necessary to use the product rule when one of the factors is a constant or when it would be easier to first determine the product of the factors and then use other rules to determine the derivative. For example, it would not be best to use the product rule for $f(x) = 3(x^2 + 1)$ or $g(x) = (x + 1)(x - 1)$.

4. $F(x) = [b(x)][c(x)]$

$$F'(x) = [b(x)][c'(x)] + [b'(x)][c(x)]$$

5. a. $y = (2 + 7x)(x - 3)$

$$\frac{dy}{dx} = (2 + 7x)(1) + 7(x - 3)$$

At $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= (2 + 14) + 7(-1) \\ &= 16 - 7 \\ &= 9\end{aligned}$$

b. $y = (1 - 2x)(1 + 2x)$

$$\frac{dy}{dx} = (1 - 2x)(2) + (-2)(1 + 2x)$$

At $x = \frac{1}{2}$,

$$\begin{aligned}\frac{dy}{dx} &= (0)(2) - 2(2) \\ &= -4\end{aligned}$$

c. $y = (3 - 2x - x^2)(x^2 + x - 2)$

$$\begin{aligned}\frac{dy}{dx} &= (3 - 2x - x^2)(2x + 1) \\ &\quad + (-2 - 2x)(x^2 + x - 2)\end{aligned}$$

At $x = -2$,

$$\begin{aligned}\frac{dy}{dx} &= (3 + 4 - 4)(-4 + 1) \\ &\quad + (-2 + 4)(4 - 2 - 2) \\ &= (3)(-3) + (2)(0) \\ &= -9\end{aligned}$$

d. $y = x^3(3x + 7)^2$

$$\frac{dy}{dx} = 3x^2(3x + 7)^2 + x^3 \cdot 6(3x + 7)$$

At $x = -2$,

$$\begin{aligned}\frac{dy}{dx} &= 12(1)^2 + (-8)(6)(1) \\ &= 12 - 48 \\ &= -36\end{aligned}$$

e. $y = (2x + 1)^5(3x + 2)^4, x = -1$

$$\begin{aligned}\frac{dy}{dx} &= 5(2x + 1)^4(2)(3x + 2)^4 \\ &\quad + (2x + 1)^5 \cdot 4(3x + 2)^3(3)\end{aligned}$$

At $x = -1$,

$$\begin{aligned}\frac{dy}{dx} &= 5(-1)^4(2)(-1)^4 \\ &\quad + (-1)^5(4)(-1)^3(3) \\ &= 10 + 12 \\ &= 22\end{aligned}$$

f. $y = x(5x - 2)(5x + 2)$

$$\frac{dy}{dx} = x(50x) + (25x^2 - 4)(1)$$

At $x = 3$,

$$\begin{aligned}\frac{dy}{dx} &= 3(150) + (25 \cdot 9 - 4) \\ &= 450 + 221 \\ &= 671\end{aligned}$$

6. Tangent to $y = (x^3 - 5x + 2)(3x^2 - 2x)$ at $(1, -2)$

$$\begin{aligned}\frac{dy}{dx} &= (3x^2 - 5)(3x^2 - 2x) \\ &\quad + (x^3 - 5x + 2)(6x - 2)\end{aligned}$$

when $x = 1$,

$$\begin{aligned}\frac{dy}{dx} &= (-2)(1) + (-2)(4) \\ &= -2 + -8 \\ &= -10\end{aligned}$$

Slope of the tangent at $(1, -2)$ is -10 .

The equation is $y + 2 = -10(x - 1)$;
 $10x + y - 8 = 0$.

7. a. $y = 2(x - 29)(x + 1)$

$$\begin{aligned}\frac{dy}{dx} &= 2(x - 29)(1) + 2(1)(x + 1) \\ 2x - 58 + 2x + 2 &= 0 \\ 4x - 56 &= 0 \\ 4x &= 56 \\ x &= 14\end{aligned}$$

Point of horizontal tangency is $(14, -450)$.

b. $y = (x^2 + 2x + 1)(x^2 + 2x + 1)$

$$\begin{aligned}&= (x^2 + 2x + 1)^2 \\ \frac{dy}{dx} &= 2(x^2 + 2x + 1)(2x + 2) \\ (x^2 + 2x + 1)(2x + 2) &= 0 \\ 2(x + 1)(x + 1)(x + 1) &= 0 \\ x &= -1\end{aligned}$$

Point of horizontal tangency is $(-1, 0)$.

8. a. $y = (x + 1)^3(x + 4)(x - 3)^2$

$$\begin{aligned}\frac{dy}{dx} &= 3(x + 1)^2(x + 4)(x - 3)^2 \\ &\quad + (x + 1)^3(1)(x - 3)^2 \\ &\quad + (x + 1)^3(x + 4)[2(x - 3)]\end{aligned}$$

b. $y = x^2(3x^2 + 4)^2(3 - x^3)^4$

$$\begin{aligned}\frac{dy}{dx} &= 2x(3x^2 + 4)^2(3 - x^3)^4 \\ &\quad + x^2[2(3x^2 + 4)(6x)](3 - x^3)^4 \\ &\quad + x^2(3x^2 + 4)^2[4(3 - x^3)^3(-3x^2)]\end{aligned}$$

9. $V(t) = 75\left(1 - \frac{t}{24}\right)^2, 0 \leq t \leq 24$

$$75 \text{ L} \times 60\% = 45 \text{ L}$$

$$\text{Set } \frac{45}{75} = \left(1 - \frac{t}{24}\right)^2$$

$$\pm \sqrt{\frac{3}{5}} = 1 - \frac{t}{24}$$

$$t = \left(\pm \sqrt{\frac{3}{5}} - 1\right)(-24)$$

$$t \doteq 42.590 \text{ (inadmissible)} \text{ or } t \doteq 5.4097$$

$$V(t) = 75\left(1 - \frac{t}{24}\right)^2$$

$$V(t) = 75\left(1 - \frac{t}{24}\right)\left(1 - \frac{t}{24}\right)$$

$$\begin{aligned}V'(t) &= 75\left[\left(1 - \frac{t}{24}\right)\left(-\frac{1}{24}\right)\right. \\ &\quad \left. + \left(-\frac{1}{24}\right)\left(1 - \frac{t}{24}\right)\right] \\ &= (75)(2)\left(1 - \frac{t}{24}\right)\left(-\frac{1}{24}\right)\end{aligned}$$

$$V'(5.4097) = -4.84 \text{ L/h}$$

10. Determine the point of tangency, and then find the negative reciprocal of the slope of the tangent. Use this information to find the equation of the normal.

$$\begin{aligned}h(x) &= 2x(x + 1)^3(x^2 + 2x + 1)^2 \\ h'(x) &= 2(x + 1)^3(x^2 + 2x + 1)^2 \\ &\quad + (2x)(3)(x + 1)^2(x^2 + 2x + 1)^2 \\ &\quad + 2x(x + 1)^32(x^2 + 2x + 1)(2x + 2)\end{aligned}$$

$$\begin{aligned}h'(-2) &= 2(-1)^3(1)^2 \\ &\quad + 2(-2)(3)(-1)^2(1)^2 \\ &\quad + 2(-2)(-1)^3(2)(1)(-2) \\ &= -2 - 12 - 16 \\ &= -30\end{aligned}$$

11.

a. $f(x) = g_1(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n(x)$

$$\begin{aligned}f'(x) &= g_1'(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n(x) \\ &\quad + g_1(x)g_2'(x)g_3(x) \dots g_{n-1}(x)g_n(x) \\ &\quad + g_1(x)g_2(x)g_3'(x) \dots g_{n-1}(x)g_n(x) \\ &\quad + \dots + g_1(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n'(x)\end{aligned}$$

b. $f(x) = (1 + x)(1 + 2x)(1 + 3x) \dots$

$$(1 + nx)$$

$$f'(x) = 1(1 + 2x)(1 + 3x) \dots (1 + nx)$$

$$+ (1 + x)(2)(1 + 3x) \dots (1 + nx)$$

$$+ (1 + x)(1 + 2x)(3) \dots (1 + nx)$$

$$+ \dots + (1 + x)(1 + 2x)(1 + 3x)$$

$$\dots (n)$$

$$f'(0) = 1(1)(1)(1) \dots (1)$$

$$+ 1(2)(1)(1) \dots (1)$$

$$+ 1(1)(3)(1) \dots (1)$$

$$+ \dots + (1)(1)(1) \dots (n)$$

$$= 1 + 2 + 3 + \dots + n$$

$$f'(0) = \frac{n(n + 1)}{2}$$

12. $f(x) = ax^2 + bx + c$

$$f'(x) = 2ax + b \quad (1)$$

Horizontal tangent at $(-1, -8)$

$$f'(x) = 0 \text{ at } x = -1$$

$$-2a + b = 0$$

Since $(2, 19)$ lies on the curve,

$$4a + 2b + c = 19 \quad (2)$$

Since $(-1, -8)$ lies on the curve,

$$a - b + c = -8 \quad (3)$$

$$4a + 2b + c = 19$$

$$-3a - 3b = -27$$

$$a + b = 9$$

$$-2a + b = 0$$

$$3a = 9$$

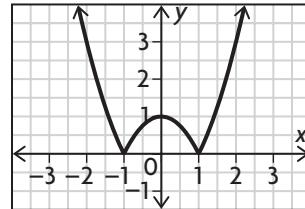
$$a = 3, b = 6$$

$$3 - 6 + c = -8$$

$$c = -5$$

The equation is $y = 3x^2 + 6x - 5$.

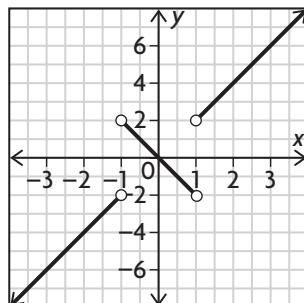
13.



a. $x = 1$ or $x = -1$

b. $f'(x) = 2x, x < -1$ or $x > 1$

$$f'(x) = -2x, -1 < x < 1$$



c. $f'(-2) = 2(-2) = -4$
 $f'(0) = -2(0) = 0$
 $f'(3) = 2(3) = 6$

14. $y = \frac{16}{x^2} - 1$

$$\frac{dy}{dx} = -\frac{32}{x^3}$$

Slope of the line is 4.

$$-\frac{32}{x^3} = 4$$

$$4x^3 = -32$$

$$x^3 = -8$$

$$x = -2$$

$$y = \frac{16}{4} - 1$$

$$= 3$$

Point is at $(-2, 3)$.

Find intersection of line and curve:

$$4x - y + 11 = 0$$

$$y = 4x + 11$$

Substitute,

$$4x + 11 = \frac{16}{x^2} - 1$$

$$4x^3 + 11x^2 = 16 - x^2 \text{ or } 4x^3 + 12x^2 - 16 = 0.$$

Let $x = -2$

$$\text{RS} = 4(-2)^3 + 12(-2)^2 - 16$$

$$= 0$$

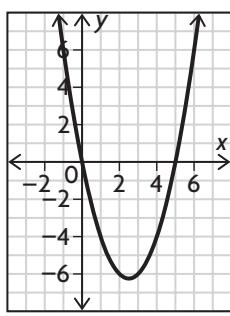
Since $x = -2$ satisfies the equation, therefore it is a solution.

When $x = -2$, $y = 4(-2) + 11 = 3$.

Intersection point is $(-2, 3)$. Therefore, the line is tangent to the curve.

Mid-Chapter Review, pp. 92–93

1. a.



b. $f'(x) = \lim_{h \rightarrow 0} \frac{((x+h)^2 - 5(x+h)) - (x^2 - 5x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - 5x - 5h - x^2 + 5x}{h}$$

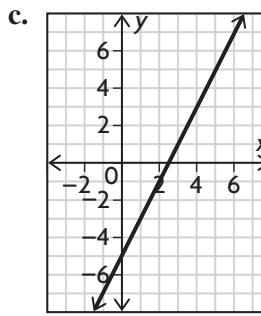
$$= \lim_{h \rightarrow 0} \frac{h^2 + 2hx - 5h}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(h + 2x - 5)}{h}$$

$$= 2x - 5$$

Use the derivative function to calculate the slopes of the tangents.

x	Slope of Tangent $f'(x)$
0	-5
1	-3
2	-1
3	1
4	3
5	5



c. $f(x)$ is quadratic; $f'(x)$ is linear.

2. a. $f'(x) = \lim_{h \rightarrow 0} \frac{(6(x+h) + 15) - (6x + 15)}{h}$

$$= \lim_{h \rightarrow 0} \frac{6h}{h}$$

$$= \lim_{h \rightarrow 0} 6$$

$$= 6$$

b. $f'(x) = \lim_{h \rightarrow 0} \frac{(2(x+h)^2 - 4) - (2x^2 - 4)}{h}$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{((x+h) - x)((x+h) + x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \rightarrow 0} 2(2x+h)$$

$$= 4x$$

$$c. f'(x) = \lim_{h \rightarrow 0} \frac{\frac{5}{(x+h)+5} - \frac{5}{x+5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5(x+5) - 5((x+h)+5)}{((x+h)+5)(x+5)h}$$

$$= \lim_{h \rightarrow 0} \frac{-5h}{((x+h)+5)(x+5)h}$$

$$= \lim_{h \rightarrow 0} \frac{-5}{((x+h)+5)(x+5)}$$

$$= \frac{-5}{(x+5)^2}$$

$$\mathbf{d. } f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)-2} - \sqrt{x-2}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{\sqrt{(x+h)-2} - \sqrt{x-2}}{h} \right]$$

$$\quad \times \frac{\sqrt{(x+h)-2} + \sqrt{x-2}}{\sqrt{(x+h)-2} + \sqrt{x-2}}$$

$$= \lim_{h \rightarrow 0} \frac{((x+h)-2) - (x-2)}{h(\sqrt{(x+h)-2} + \sqrt{x-2})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{(x+h)-2} + \sqrt{x-2})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{(x+h)-2} + \sqrt{x-2}}$$

$$= \frac{1}{2\sqrt{x-2}}$$

3. a. $y' = 2x - 4$

When $x = 1$,

$$y' = 2(1) - 4$$

$$= -2.$$

When $x = 1$,

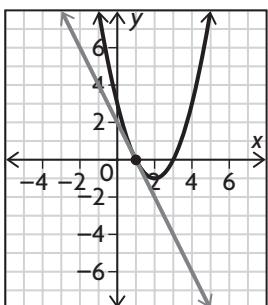
$$y = (1)^2 - 4(1) + 3$$

$$= 0.$$

Equation of the tangent line:

$$y - 0 = -2(x - 1), \text{ or } y = -2x + 2$$

b.



4. a. $\frac{dy}{dx} = 24x^3$

b. $\frac{dy}{dx} = 5x^{-\frac{1}{2}}$

$$= \frac{5}{\sqrt{x}}$$

c. $g'(x) = -6x^{-4}$

$$= -\frac{6}{x^4}$$

d. $\frac{dy}{dx} = 5 - 6x^{-3}$

$$= 5 - \frac{6}{x^3}$$

e. $\frac{dy}{dt} = 2(11t + 1)(11)$

$$= 242t + 22$$

f. $y = 1 - \frac{1}{x}$

$$= 1 - x^{-1}$$

$$\frac{dy}{dx} = x^{-2}$$

$$= \frac{1}{x^2}$$

5. f'(x) = 8x^3

$$8x^3 = 1$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^4$$

$$= \frac{1}{8}$$

Equation of the tangent line:

$$y - \frac{1}{8} = 1\left(x - \frac{1}{2}\right), \text{ or } y = x - \frac{3}{8}$$

6. a. $f'(x) = 8x - 7$

b. $f'(x) = -6x^2 + 8x + 5$

c. $f(x) = 5x^{-2} - 3x^{-3}$

$$f'(x) = -10x^{-3} + 9x^{-4}$$

$$= -\frac{10}{x^3} + \frac{9}{x^4}$$

d. $f(x) = x^{\frac{1}{3}} + x^{\frac{1}{3}}$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{3x^{\frac{2}{3}}}$$

e. $f(x) = 7x^{-2} - 3x^{\frac{1}{2}}$

$$f'(x) = -14x^{-3} - \frac{3}{2}x^{-\frac{1}{2}}$$

$$= -\frac{14}{x^3} - \frac{3}{2x^{\frac{1}{2}}}$$

f. $f'(x) = 4x^{-2} + 5$

$$= \frac{4}{x^2} + 5$$

7. a. $y' = -6x + 6$

When $x = 1$,

$$\begin{aligned}y' &= -6(1) + 6 \\&= 0.\end{aligned}$$

When $x = 1$,

$$\begin{aligned}y &= -3(1^2) + 6(1) + 4 \\&= 7.\end{aligned}$$

Equation of the tangent line:

$$y - 7 = 0(x - 1), \text{ or}$$

$$y = 7$$

b. $y = 3 - 2x^{\frac{1}{2}}$

$$\begin{aligned}y' &= -x^{-\frac{1}{2}} \\&= \frac{-1}{\sqrt{x}}\end{aligned}$$

When $x = 9$,

$$\begin{aligned}y' &= \frac{-1}{\sqrt{9}} \\&= -\frac{1}{3}.\end{aligned}$$

When $x = 9$,

$$\begin{aligned}y &= 3 - 2\sqrt{9} \\&= -3.\end{aligned}$$

Equation of the tangent line:

$$y - (-3) = -\frac{1}{3}(x - 9), \text{ or } y = -\frac{1}{3}x$$

c. $f'(x) = -8x^3 + 12x^2 - 4x - 8$

$$\begin{aligned}f'(3) &= -8(3)^3 + 12(3)^2 - 4(3) - 8 \\&= -216 + 108 - 12 - 8 \\&= -218 \\f(3) &= -2(3)^4 + 4(3)^3 - 2(3)^2 - 8(3) + 9 \\&= -162 + 108 - 18 - 24 + 9 \\&= -87\end{aligned}$$

Equation of the tangent line:

$$y - (-87) = -128(x - 3), \text{ or}$$

$$y = -128x + 297$$

8. a. $f'(x) = \frac{d}{dx}(4x^2 - 9x)(3x^2 + 5)$

$$\begin{aligned}&+ (4x^2 - 9x)\frac{d}{dx}(3x^2 + 5) \\&= (8x - 9)(3x^2 + 5) + (4x^2 - 9x)(6x) \\&= 24x^3 - 27x^2 + 40x - 45 \\&\quad + 24x^3 - 54x^2 \\&= 48x^3 - 81x^2 + 40x - 45\end{aligned}$$

b. $f'(t) = \frac{d}{dt}(-3t^2 - 7t + 8)(4t - 1)$

$$\begin{aligned}&+ (-3t^2 - 7t + 8)\frac{d}{dt}(4t - 1) \\&= (-6t - 7)(4t - 1) \\&\quad + (-3t^2 - 7t + 8)(4)\end{aligned}$$

$$\begin{aligned}&= -24t^2 - 28t + 6t + 7 - 12t^2 - 28t + 32 \\&= -36t^2 - 50t + 39\end{aligned}$$

c. $\frac{dy}{dx} = \frac{d}{dx}(3x^2 + 4x - 6)(2x^2 - 9)$

$$\begin{aligned}&+ (3x^2 + 4x - 6)\frac{d}{dx}(2x^2 - 9) \\&= (6x + 4)(2x^2 - 9) + (3x^2 + 4x - 6)(4x) \\&= 12x^3 - 54x + 8x^2 - 36 + 12x^3 \\&\quad + 16x^2 - 24x \\&= 24x^3 + 24x^2 - 78x - 36\end{aligned}$$

d. $\frac{dy}{dx} = \frac{d}{dx}(3 - 2x^3)^2(3 - 2x^3)$

$$+ (3 - 2x^3)^2\frac{d}{dx}(3 - 2x^3)$$

$$\begin{aligned}&= \left[\frac{d}{dx}(3 - 2x^3)(3 - 2x^3) \right. \\&\quad \left. + (3 - 2x^3)\frac{d}{dx}(3 - 2x^3) \right](3 - 2x^3) \\&+ (3 - 2x^3)^2(-6x^2) \\&= [2(-6x^2)(3 - 2x^3)](3 - 2x^3) \\&\quad + (3 - 2x^3)^2(-6x^2) \\&= 3(3 - 2x^3)^2(-6x^2) \\&= (3 - 2x^3)^2(-18x^2) \\&= (9 - 12x^3 + 4x^6)(-18x^2) \\&= -162x^2 + 216x^5 - 72x^8\end{aligned}$$

9. $y' = \frac{d}{dx}(5x^2 + 9x - 2)(-x^2 + 2x + 3)$

$$\begin{aligned}&+ (5x^2 + 9x - 2)\frac{d}{dx}(-x^2 + 2x + 3) \\&= (10x + 9)(-x^2 + 2x + 3) \\&\quad + (5x^2 + 9x - 2)(2 - 2x)\end{aligned}$$

$$\begin{aligned}y'(1) &= (10(1) + 9)(-(1)^2 + 2(1) + 3) \\&\quad + (5(1)^2 + 9(1) - 2)(2 - 2(1)) \\&= (19)(4) \\&= 76\end{aligned}$$

Equation of the tangent line:

$$y - 76 = 76(x - 1), \text{ or } 76x - y - 28 = 0$$

10. $\frac{dy}{dx} = 2\frac{d}{dx}(x - 1)(5 - x)$

$$\begin{aligned}&+ 2(x - 1)\frac{d}{dx}(5 - x) \\&= 2(5 - x) - 2(x - 1) \\&= 12 - 4x\end{aligned}$$

The tangent line is horizontal when $\frac{dy}{dx} = 0$.

$$12 - 4x = 0$$

$$12 = 4x$$

$$x = 3$$

When $x = 3$,

$$y = 2((3) - 1)(5 - (3)) \\ = 8.$$

Point where tangent line is horizontal: $(3, 8)$

$$\begin{aligned} \mathbf{11.} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \left[\frac{(5(x+h)^2 - 8(x+h) + 4)}{h} \right. \\ &\quad \left. - \frac{(5x^2 - 8x + 4)}{h} \right] \\ &= \lim_{h \rightarrow 0} \frac{5(x+h)^2 - 5x^2 - 8h}{h} \\ &= \lim_{h \rightarrow 0} \frac{5((x+h) - x)((x+h) + x) - 8h}{h} \\ &= \lim_{h \rightarrow 0} \frac{5h(2x + h) - 8h}{h} \\ &= \lim_{h \rightarrow 0} (5(2x + h) - 8) \\ &= 10x - 8 \end{aligned}$$

$$\mathbf{12.} V(t) = 500 \left(1 - \frac{t}{90}\right)^2. 0 \leq t \leq 90$$

a. After 1 h, $t = 60$, and the volume is

$$\begin{aligned} V(60) &= 500 \left(1 - \frac{60}{90}\right)^2 \\ &= 500 \left(\frac{30}{90}\right)^2 \\ &= 500 \left(\frac{1}{3}\right)^2 \\ &= \frac{500}{9} \text{ L} \end{aligned}$$

$$\mathbf{b.} V(0) = 500(1 - 0)^2 = 500 \text{ L}$$

$$V(60) = \frac{500}{9} \text{ L}$$

The average rate of change of volume with respect to time from 0 min to 60 min is

$$\begin{aligned} \frac{\Delta V}{\Delta t} &= \frac{\frac{500}{9} - 500}{60 - 0} \\ &= \frac{-\frac{8}{9}(500)}{60} \\ &= -\frac{200}{27} \text{ L/min} \end{aligned}$$

c. Calculate $V'(t)$:

$$\begin{aligned} V'(t) &= \lim_{h \rightarrow 0} \frac{V(t+h) - V(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{500 \left(1 - \frac{t+h}{90}\right)^2 - 500 \left(1 - \frac{t}{90}\right)^2}{h} \end{aligned}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{500 \left(1 - \frac{t+h}{90} - 1 + \frac{t}{90}\right)}{h} \\ &\quad \times \frac{\left(1 - \frac{t+h}{90} + 1 - \frac{t}{90}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{500 \left(-\frac{h}{90}\right) \left(2 - \frac{2t+h}{90}\right)}{h} \\ &= \lim_{h \rightarrow 0} -\frac{500}{90} \left(2 - \frac{2t+h}{90}\right) \\ &= \frac{-50}{9} \left(2 - \frac{2t}{90}\right) \\ &= \frac{-900 + 10t}{81} \end{aligned}$$

Then,

$$\begin{aligned} V'(30) &= \frac{-900 + 10(30)}{81} \\ &= -\frac{200}{27} \text{ L/min} \end{aligned}$$

$$\mathbf{13.} V(r) = \frac{4}{3}\pi r^3$$

$$\begin{aligned} \mathbf{a.} V(10) &= \frac{4}{3}\pi(10)^3 & V(15) &= \frac{4}{3}\pi(15)^3 \\ &= \frac{4}{3}\pi(1000) & &= \frac{4}{3}\pi(3375) \\ &= \frac{4000}{3}\pi & &= 4500\pi \end{aligned}$$

Then, the average rate of change of volume with respect to radius is

$$\begin{aligned} \frac{\Delta V}{\Delta r} &= \frac{4500\pi - \frac{4000}{3}\pi}{15 - 10} \\ &= \frac{500\pi(9 - \frac{8}{3})}{5} \\ &= 100\pi \left(\frac{19}{3}\right) \\ &= \frac{1900}{3}\pi \text{ cm}^3/\text{cm} \end{aligned}$$

b. First calculate $V'(r)$:

$$\begin{aligned} V'(r) &= \lim_{h \rightarrow 0} \frac{V(r+h) - V(r)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi[(r+h)^3 - r^3]}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi(r^3 + 3r^2h + 3rh^2 + h^3 - r^3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi(3r^2h + 3rh^2 + h^3)}{h} \end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{4}{3} \pi (3r^2 + 3rh + h^2) \\
&= \frac{4}{3} \pi (3r^2 + 3r(0) + (0)^2) \\
&= 4\pi r^2
\end{aligned}$$

$$\begin{aligned}
\text{Then, } V'(8) &= 4\pi(8)^2 \\
&= 4\pi(64) \\
&= 256\pi \text{ cm}^3/\text{cm}
\end{aligned}$$

14. This statement is always true. A cubic polynomial function will have the form $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$. So the derivative of this cubic is $f'(x) = 3ax^2 + 2bx + c$, and since $3a \neq 0$, this derivative is a quadratic polynomial function. For example, if $f(x) = x^3 + x^2 + 1$, we get

$$f'(x) = 3x^2 + 2x,$$

and if

$$f(x) = 2x^3 + 3x^2 + 6x + 2,$$

we get

$$f'(x) = 6x^2 + 6x + 6$$

$$\mathbf{15. } y = \frac{x^{2a+3b}}{x^{a-b}}, a, b \in \mathbb{I}$$

Simplifying,

$$y = x^{2a+3b-(a-b)} = x^{a+4b}$$

Then,

$$y' = (a+4b)^{a+4b-1}$$

$$\mathbf{16. a. } f(x) = -6x^3 + 4x - 5x^2 + 10$$

$$f'(x) = -18x^2 + 4 - 10x$$

$$\text{Then, } f'(x) = -18(3)^2 + 4 - 10(3)$$

$$= -188$$

b. $f'(3)$ is the slope of the tangent line to $f(x)$ at $x = 3$ and the rate of change in the value of $f(x)$ with respect to x at $x = 3$.

$$\mathbf{17. a. } P(t) = 100 + 120t + 10t^2 + 2t^3$$

$$P(t) = 100 + 120t + 10t^2 + 2t^3$$

$$\begin{aligned}
P(0) &= 100 + 120(0) + 10(0)^2 + 2(0)^3 \\
&= 100 \text{ bacteria}
\end{aligned}$$

b. At 5 h, the population is

$$\begin{aligned}
P(5) &= 100 + 120(5) + 10(5)^2 + 2(5)^3 \\
&= 1200 \text{ bacteria}
\end{aligned}$$

$$\mathbf{c. } P'(t) = 120 + 20t + 6t^2$$

At 5 h, the colony is growing at

$$\begin{aligned}
P'(5) &= 120 + 20(5) + 6(5)^2 \\
&= 370 \text{ bacteria/h}
\end{aligned}$$

$$\mathbf{18. } C(t) = \frac{100}{t}, t > 2$$

Simplifying, $C(t) = 100t^{-1}$.

$$\text{Then, } C'(t) = -100t^{-2} = -\frac{100}{t^2}.$$

$$\begin{array}{lll}
C'(5) & C'(50) & C'(100) \\
= -\frac{100}{(5)^2} & = -\frac{100}{(50)^2} & = -\frac{100}{(100)^2} \\
= -\frac{100}{25} & = -\frac{100}{2500} & = -\frac{1}{100} \\
= -4 & = -0.04 & = -0.01
\end{array}$$

These are the rates of change of the percentage with respect to time at 5, 50, and 100 min. The percentage of carbon dioxide that is released per unit time from the pop is decreasing. The pop is getting flat.

2.4 The Quotient Rule, pp. 97–98

1. For x, a, b real numbers,

$$x^a x^b = x^{a+b}$$

For example,

$$x^9 x^{-6} = x^3$$

Also,

$$(x^a)^b = x^{ab}$$

For example,

$$(x^2)^3 = x^6$$

Also,

$$\frac{x^a}{x^b} = x^{a-b}, x \neq 0$$

For example,

$$\frac{x^5}{x^3} = x^2$$

2.

Function	Rewrite	Differentiate and Simplify, If Necessary
$f(x) = \frac{x^2 + 3x}{x}, x \neq 0$	$f(x) = x + 3$	$f'(x) = 1$
$g(x) = \frac{3x^{\frac{5}{3}}}{x}, x \neq 0$	$g(x) = 3x^{\frac{2}{3}}$	$g'(x) = 2x^{-\frac{1}{3}}$
$h(x) = \frac{1}{10x^5}, x \neq 0$	$h(x) = \frac{1}{10}x^{-5}$	$h'(x) = \frac{-1}{2}x^{-6}$
$y = \frac{8x^3 + 6x}{2x}, x \neq 0$	$y = 4x^2 + 3$	$\frac{dy}{dx} = 8x$
$s = \frac{t^2 - 9}{t - 3}, t \neq 3$	$s = t + 3$	$\frac{ds}{dt} = 1$

3. In the previous problem, all of these rational examples could be differentiated via the power rule after a minor algebraic simplification.

A second approach would be to rewrite a rational example

$$h(x) = \frac{f(x)}{g(x)}$$

using the exponent rules as

$$h(x) = f(x)(g(x))^{-1},$$

and then apply the product rule for differentiation (together with the power of a function rule to find $h'(x)$).

A third (and perhaps easiest) approach would be to just apply the quotient rule to find $h'(x)$.

$$\begin{aligned} \textbf{4. a. } h'(x) &= \frac{(x+1)(1) - x(1)}{(x+1)^2} \\ &= \frac{1}{(x+1)^2} \end{aligned}$$

$$\begin{aligned} \textbf{b. } h'(t) &= \frac{(t+5)(2) - (2t-3)(1)}{(t+5)^2} \\ &= \frac{13}{(t+5)^2} \end{aligned}$$

$$\begin{aligned} \textbf{c. } h'(x) &= \frac{(2x^2-1)(3x^2) - x^3(4x)}{(2x^2-1)^2} \\ &= \frac{2x^4 - 3x^2}{(2x^2-1)^2} \end{aligned}$$

$$\begin{aligned} \textbf{d. } h'(x) &= \frac{(x^2+3)(0) - 1(2x)}{(x^2+3)^2} \\ &= \frac{-2x}{(x^2+3)^2} \end{aligned}$$

$$\begin{aligned} \textbf{e. } y &= \frac{x(3x+5)}{(1-x^2)} = \frac{3x^2 + 5x}{1-x^2} \\ \frac{dy}{dx} &= \frac{(6x+5)(1-x^2) - (3x^2+5x)(-2x)}{(1-x^2)^2} \\ &= \frac{6x+5 - 6x^3 - 5x^2 + 6x^3 + 10x^2}{(1-x^2)^2} \\ &= \frac{5x^2 + 6x + 5}{(1-x^2)^2} \end{aligned}$$

$$\begin{aligned} \textbf{f. } \frac{dy}{dx} &= \frac{(x^2+3)(2x-1) - (x^2-x+1)(2x)}{(x^2+3)^2} \\ &= \frac{2x^3 + 6x - x^2 - 3 - 2x^3 + 2x^2 - 2x}{(x^2+3)^2} \\ &= \frac{x^2 + 4x - 3}{(x^2+3)^2} \end{aligned}$$

$$\textbf{5. a. } y = \frac{3x+2}{x+5}, x = -3$$

$$\frac{dy}{dx} = \frac{(x+5)(3) - (3x+2)(1)}{(x+5)^2}$$

At $x = -3$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{(2)(3) - (-7)(1)}{(2)^2} \\ &= \frac{13}{4} \end{aligned}$$

$$\textbf{b. } y = \frac{x^3}{x^2 + 9}, x = 1$$

$$\frac{dy}{dx} = \frac{(x^2+9)(3x^2) - (x^3)(2x)}{(x^2+9)^2}$$

At $x = 1$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{(10)(3) - (1)(2)}{(10)^2} \\ &= \frac{28}{100} \\ &= \frac{7}{25} \end{aligned}$$

$$\textbf{c. } y = \frac{x^2 - 25}{x^2 + 25}, x = 2$$

$$\frac{dy}{dx} = \frac{2x(x^2+25) - (x^2-25)(2x)}{(x^2+25)^2}$$

At $x = 2$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{4(29) - (-21)(4)}{(29)^2} \\ &= \frac{116 + 84}{29^2} \\ &= \frac{200}{841} \end{aligned}$$

$$\textbf{d. } y = \frac{(x+1)(x+2)}{(x-1)(x-2)}, x = 4$$

$$\begin{aligned} &= \frac{x^2 + 3x + 2}{x^2 - 3x + 2} \\ \frac{dy}{dx} &= \frac{(2x+3)(x^2-3x+2)}{(x-1)^2(x-2)^2} \\ &\quad - \frac{(x^2+3x+2)(2x-3)}{(x-1)^2(x-2)^2} \end{aligned}$$

At $x = 4$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{(11)(6) - (30)(5)}{(9)(4)} \\ &= -\frac{84}{36} \\ &= -\frac{7}{3} \end{aligned}$$

6. $y = \frac{x^3}{x^2 - 6}$

$$\frac{dy}{dx} = \frac{3x^2(x^2 - 6) - x^3(2x)}{(x^2 - 6)^2}$$

At $(3, 9)$:

$$\begin{aligned}\frac{dy}{dx} &= \frac{3(9)(3) - (27)(6)}{(3)^2} \\ &= 9 - 18 \\ &= -9\end{aligned}$$

The slope of the tangent to the curve at $(3, 9)$ is -9 .

7. $y = \frac{3x}{x - 4}$

$$\frac{dy}{dx} = \frac{3(x - 4) - 3x}{(x - 4)^2} = -\frac{12}{(x - 4)^2}$$

Slope of the tangent is $-\frac{12}{25}$.

Therefore, $\frac{12}{(x - 4)^2} = \frac{12}{25}$

$$x - 4 = 5 \text{ or } x - 4 = -5$$

$$x = 9 \text{ or } x = -1$$

Points are $(9, \frac{27}{5})$ and $(-1, \frac{3}{5})$.

8. $f(x) = \frac{5x + 2}{x + 2}$

$$f'(x) = \frac{(x + 2)(5) - (5x + 2)(1)}{(x + 2)^2}$$

$$f'(x) = \frac{8}{(x + 2)^2}$$

Since $(x + 2)^2$ is positive or zero for all $x \in \mathbf{R}$,

$\frac{8}{(x + 2)^2} > 0$ for $x \neq -2$. Therefore, tangents to

the graph of $f(x) = \frac{5x + 2}{x + 2}$ do not have a negative slope.

9. a. $y = \frac{2x^2}{x - 4}, x \neq 4$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x - 4)(4x) - (2x^2)(1)}{(x - 4)^2} \\ &= \frac{4x^2 - 16x - 2x^2}{(x - 4)^2} \\ &= \frac{2x^2 - 16x}{(x - 4)^2} \\ &= \frac{2x(x - 8)}{(x - 4)^2}\end{aligned}$$

Curve has horizontal tangents when $\frac{dy}{dx} = 0$, or when $x = 0$ or 8 . At $x = 0$:

$$\begin{aligned}y &= \frac{0}{-4} \\ &= 0\end{aligned}$$

At $x = 8$:

$$\begin{aligned}y &= \frac{2(8)^2}{4} \\ &= 32\end{aligned}$$

So the curve has horizontal tangents at the points $(0, 0)$ and $(8, 32)$.

b. $y = \frac{x^2 - 1}{x^2 + x - 2}$

$$\begin{aligned}&= \frac{(x - 1)(x + 1)}{(x + 2)(x - 1)} \\ &= \frac{x + 1}{x + 2}, x \neq 1\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x + 2) - (x + 1)}{(x + 2)^2} \\ &= \frac{1}{(x + 2)^2}\end{aligned}$$

Curve has horizontal tangents when $\frac{dy}{dx} = 0$.

No value of x will produce a slope of 0, so there are no horizontal tangents.

10. $p(t) = 1000 \left(1 + \frac{4t}{t^2 + 50}\right)$

$$\begin{aligned}p'(t) &= 1000 \left(\frac{4(t^2 + 50) - 4t(2t)}{(t^2 + 50)^2}\right) \\ &= \frac{1000(200 - 4t^2)}{(t^2 + 50)^2}\end{aligned}$$

$$p'(1) = \frac{1000(196)}{(51)^2} = 75.36$$

$$p'(2) = \frac{1000(184)}{(54)^2} = 63.10$$

Population is growing at a rate of 75.4 bacteria per hour at $t = 1$ and at 63.1 bacteria per hour at $t = 2$.

11. $y = \frac{x^2 - 1}{3x}$

$$= \frac{1}{3}x - \frac{1}{3}x^{-1}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{3} + \frac{1}{3}x^{-2} \\ &= \frac{1}{3} + \frac{1}{3x^2}\end{aligned}$$

At $x = 2$:

$$\begin{aligned}y &= \frac{(2)^2 - 1}{3(2)} \\ &= \frac{1}{2}\end{aligned}$$

and

$$\frac{dy}{dx} = \frac{1}{3} + \frac{1}{3(2)^2}$$

$$= \frac{1}{3} + \frac{1}{12}$$

$$= \frac{5}{12}$$

So the equation of the tangent to the curve at $x = 2$ is:

$$y - \frac{1}{2} = \frac{5}{12}(x - 2), \text{ or } 5x - 12y - 4 = 0.$$

12. a. $s(t) = \frac{10(6-t)}{t+3}, 0 \leq t \leq 6, t = 0,$

$$s(0) = 20$$

The boat is initially 20 m from the dock.

b. $v(t) = s'(t) = 10 \left[\frac{(t+3)(-1) - (6-t)(1)}{(t+3)^2} \right]$

$$v(t) = \frac{-90}{(t+3)^2}$$

At $t = 0$, $v(0) = -10$, the boat is moving towards the dock at a speed of 10 m/s. When $s(t) = 0$, the boat will be at the dock.

$$\frac{10(6-t)}{t+3} = 0, t = 6.$$

$$v(6) = \frac{-90}{9^2} = -\frac{10}{9}$$

The speed of the boat when it bumps into the dock is $\frac{10}{9}$ m/s.

13. a. i. $t = 0$

$$r(0) = \frac{1 + 2(0)}{1 + 0}$$

$$= 1 \text{ cm}$$

ii. $\frac{1 + 2t}{1 + t} = 1.5$

$$1 + 2t = 1.5(1 + t)$$

$$1 + 2t = 1.5 + 1.5t$$

$$0.5t = 0.5$$

$$t = 1 \text{ s}$$

iii. $r'(t) = \frac{(1+t)(2) - (1+2t)(1)}{(1+t)^2}$

$$= \frac{2 + 2t - 1 - 2t}{(1+t)^2}$$

$$= \frac{1}{(1+t)^2}$$

$$r'(1.5) = \frac{1}{(1+1)^2}$$

$$= \frac{1}{4}$$

$$= 0.25 \text{ cm/s}$$

b. No, the radius will never reach 2 cm, because $y = 2$ is a horizontal asymptote of the graph of the function. Therefore, the radius approaches but never equals 2 cm.

$$14. f(x) = \frac{ax + b}{(x-1)(x-4)}$$

$$f'(x) = \frac{(x-1)(x-4)(a)}{(x-1)^2(x-4)^2}$$

$$= \frac{(ax+b)\frac{d}{dx}[(x-1)(x-4)]}{(x-1)^2(x-4)^2}$$

$$= \frac{(x-1)(x-4)(a)}{(x-1)^2(x-4)^2}$$

$$= \frac{(ax+b)[(x-1) + (x-4)]}{(x-1)^2(x-4)^2}$$

$$= \frac{(x^2 - 5x + 4)(a) - (ax+b)(2x-5)}{(x-1)^2(x-4)^2}$$

$$= \frac{-ax^2 - 2bx + 4a + 5b}{(x-1)^2(x-4)^2}$$

Since the point $(2, -1)$ is on the graph (as it's on the tangent line) we know that

$$-1 = f(2)$$

$$= \frac{2a + b}{(1)(-2)}$$

$$2 = 2a + b$$

$$b = 2 - 2a$$

Also, since the tangent line is horizontal at $(2, -1)$, we know that

$$0 = f'(2)$$

$$= \frac{-a(2)^2 - 2b(2) + 4a + 5b}{(1)^2(-2)^2}$$

$$b = 0$$

$$0 = 2 - 2a$$

$$a = 1$$

So we get

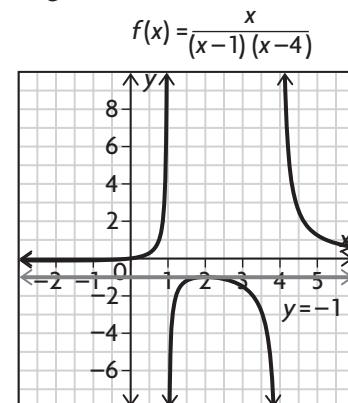
$$f(x) = \frac{x}{(x-1)(x-4)}$$

Since the tangent line is horizontal at the point

$(2, -1)$, the equation of this tangent line is

$$y - (-1) = 0(x - 2), \text{ or } y = -1$$

Here are the graphs of both $f(x)$ and this horizontal tangent line:



$$15. c'(t) = \frac{(2t^2 + 7)(5) - (5t)(4t)}{(2t^2 + 7)^2}$$

$$= \frac{10t^2 + 35 - 20t^2}{(2t^2 + 7)^2}$$

$$= \frac{-10t^2 + 35}{(2t^2 + 7)^2}$$

Set $c'(t) = 0$ and solve for t .

$$\begin{aligned} \frac{-10t^2 + 35}{(2t^2 + 7)^2} &= 0 \\ -10t^2 + 35 &= 0 \\ 10t^2 &= 35 \\ t^2 &= 3.5 \\ t &= \pm\sqrt{3.5} \\ t &\doteq \pm 1.87 \end{aligned}$$

To two decimal places, $t = -1.87$ or $t = 1.87$, because $s'(t) = 0$ for these values. Reject the negative root in this case because time is positive ($t \geq 0$). Therefore, the concentration reaches its maximum value at $t = 1.87$ hours.

16. When the object changes direction, its velocity changes sign.

$$\begin{aligned} s'(t) &= \frac{(t^2 + 8)(1) - t(2t)}{(t^2 + 8)^2} \\ &= \frac{t^2 + 8 - 2t^2}{(t^2 + 8)^2} \\ &= \frac{-t^2 + 8}{(t^2 + 8)^2} \end{aligned}$$

solve for t when $s'(t) = 0$.

$$\begin{aligned} \frac{-t^2 + 8}{(t^2 + 8)^2} &= 0 \\ -t^2 + 8 &= 0 \\ t^2 &= 8 \\ t &= \pm\sqrt{8} \\ t &\doteq \pm 2.83 \end{aligned}$$

To two decimal places, $t = 2.83$ or $t = -2.83$, because $s'(t) = 0$ for these values. Reject the negative root because time is positive ($t \geq 0$). The object changes direction when $t = 2.83$ s.

$$\begin{aligned} 17. f(x) &= \frac{ax + b}{cx + d}, x \neq -\frac{d}{c} \\ f'(x) &= \frac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} \\ f'(x) &= \frac{ad - bc}{(cx + d)^2} \end{aligned}$$

For the tangents to the graph of $y = f(x)$ to have positive slopes, $f'(x) > 0$. $(cx + d)^2$ is positive for all $x \in \mathbf{R}$. $ad - bc > 0$ will ensure each tangent has a positive slope.

2.5 The Derivatives of Composite Functions, pp. 105–106

1. $f(x) = \sqrt{x}$, $g(x) = x^2 - 1$

$$\begin{aligned} \mathbf{a. } f(g(1)) &= f(1 - 1) \\ &= f(0) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \mathbf{b. } g(f(1)) &= g(1) \\ &= 0 \end{aligned}$$

$$\begin{aligned} \mathbf{c. } g(f(0)) &= g(0) \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

$$\begin{aligned} \mathbf{d. } f(g(-4)) &= f(16 - 1) \\ &= f(15) \\ &= \sqrt{15} \end{aligned}$$

$$\begin{aligned} \mathbf{e. } f(g(x)) &= f(x^2 - 1) \\ &= \sqrt{x^2 - 1} \end{aligned}$$

$$\begin{aligned} \mathbf{f. } g(f(x)) &= g(\sqrt{x}) \\ &= (\sqrt{x})^2 - 1 \\ &= x - 1 \end{aligned}$$

2. **a.** $f(x) = x^2$, $g(x) = \sqrt{x}$

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(\sqrt{x}) \\ &= (\sqrt{x})^2 \\ &= x \end{aligned}$$

Domain = $\{x \geq 0\}$

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g(x^2) \\ &= \sqrt{x^2} \\ &= |x| \end{aligned}$$

Domain = $\{x \in \mathbf{R}\}$

The composite functions are not equal for negative x -values (as $(f \circ g)$ is not defined for these x), but are equal for non-negative x -values.

b. $f(x) = \frac{1}{x}$, $g(x) = x^2 + 1$

$$\begin{aligned} (f \circ g)(x) &= f(g(x)) \\ &= f(x^2 + 1) \\ &= \frac{1}{x^2 + 1} \end{aligned}$$

Domain = $\{x \in \mathbf{R}\}$

$$\begin{aligned} (g \circ f)(x) &= g(f(x)) \\ &= g\left(\frac{1}{x}\right) \\ &= \left(\frac{1}{x}\right)^2 + 1 \end{aligned}$$

$$= \frac{1}{x^2} + 1$$

$$\text{Domain} = \{x \neq 0\}$$

The composite functions are not equal here. For instance, $(f \circ g)(1) = \frac{1}{2}$ and $(g \circ f)(1) = 2$.

c. $f(x) = \frac{1}{x}, g(x) = \sqrt{x+2}$

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\&= f(\sqrt{x+2}) \\&= \frac{1}{\sqrt{x+2}}\end{aligned}$$

$$\text{Domain} = \{x > -2\}$$

$$(g \circ f)(x) = g(f(x))$$

$$\begin{aligned}&= g\left(\frac{1}{x}\right) \\&= \sqrt{\frac{1}{x} + 2}\end{aligned}$$

The domain is all x such that

$$\frac{1}{x} + 2 \geq 0 \text{ and } x \neq 0, \text{ or equivalently}$$

$$\text{Domain} = \{x \leq -\frac{1}{2} \text{ or } x > 0\}$$

The composite functions are not equal here. For

instance, $(f \circ g)(2) = \frac{1}{2}$ and $(g \circ f)(2) = \sqrt{\frac{5}{2}}$.

3. If $f(x)$ and $g(x)$ are two differentiable functions of x , and

$$\begin{aligned}h(x) &= (f \circ g)(x) \\&= f(g(x))\end{aligned}$$

is the composition of these two functions, then

$$h'(x) = f'(g(x)) \cdot g'(x)$$

This is known as the “chain rule” for differentiation of composite functions. For example, if $f(x) = x^{10}$ and $g(x) = x^2 + 3x + 5$, then $h(x) = (x^2 + 3x + 5)^{10}$, and so

$$\begin{aligned}h'(x) &= f'(g(x)) \cdot g'(x) \\&= 10(x^2 + 3x + 5)^9(2x + 3)\end{aligned}$$

As another example, if $f(x) = x^{\frac{2}{3}}$ and $g(x) = x^2 + 1$, then $h(x) = (x^2 + 1)^{\frac{2}{3}}$, and so

$$h'(x) = \frac{2}{3}(x^2 + 1)^{-\frac{1}{3}}(2x)$$

4. a. $f(x) = (2x + 3)^4$
 $f'(x) = 4(2x + 3)^3(2)$
 $= 8(2x + 3)^3$

b. $g(x) = (x^2 - 4)^3$
 $g'(x) = 3(x^2 - 4)^2(2x)$
 $= 6x(x^2 - 4)^2$

c. $h(x) = (2x^2 + 3x - 5)^4$
 $h'(x) = 4(2x^2 + 3x - 5)^3(4x + 3)$

d. $f(x) = (\pi^2 - x^2)^3$
 $f'(x) = 3(\pi^2 - x^2)^2(-2x)$
 $= -6x(\pi^2 - x^2)^2$

e. $y = \sqrt{x^2 - 3}$
 $= (x^2 - 3)^{\frac{1}{2}}$
 $y' = \frac{1}{2}(x^2 - 3)^{\frac{1}{2}}(2x)$
 $= \frac{x}{\sqrt{x^2 - 3}}$

f. $f(x) = \frac{1}{(x^2 - 16)^5}$
 $= (x^2 - 16)^{-5}$
 $f'(x) = -5(x^2 - 16)^{-6}(2x)$
 $= \frac{-10x}{(x^2 - 16)^6}$

5. a. $y = -\frac{2}{x^3}$
 $= -2x^{-3}$

$$\begin{aligned}\frac{dy}{dx} &= (-2)(-3)x^{-4} \\&= \frac{6}{x^4}\end{aligned}$$

b. $y = \frac{1}{x+1}$
 $= (x+1)^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= (-1)(x+1)^{-2}(1) \\&= \frac{-1}{(x+1)^2}\end{aligned}$$

c. $y = \frac{1}{x^2 - 4}$
 $= (x^2 - 4)^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= (-1)(x^2 - 4)^{-2}(2x) \\&= \frac{-2x}{(x^2 - 4)^2}\end{aligned}$$

d. $y = \frac{3}{9 - x^2} = 3(9 - x^2)^{-1}$
 $\frac{dy}{dx} = \frac{6x}{(9 - x^2)^2}$

e. $y = \frac{1}{5x^2 + x}$
 $= (5x^2 + x)^{-1}$

$$\begin{aligned}\frac{dy}{dx} &= (-1)(5x^2 + x)^{-2}(10x + 1) \\ &= -\frac{10x + 1}{(5x^2 + x)^2}\end{aligned}$$

f. $y = \frac{1}{(x^2 + x + 1)^4}$
 $= (x^2 + x + 1)^{-4}$

$$\begin{aligned}\frac{dy}{dx} &= (-4)(x^2 + x + 1)^{-5}(2x + 1) \\ &= -\frac{8x + 4}{(x^2 + x + 1)^5}\end{aligned}$$

6. $h = g \circ f$
 $= g(f(x))$
 $h(-1) = g(f(-1))$
 $= g(1)$
 $= -4$

$$\begin{aligned}h(x) &= g(f(x)) \\ h'(x) &= g'(f(x))f'(x) \\ h'(-1) &= g'(f(-1))f'(-1) \\ &= g'(1)(-5) \\ &= (-7)(-5) \\ &= 35\end{aligned}$$

7. $f(x) = (x - 3)^2$, $g(x) = \frac{1}{x}$, $h(x) = f(g(x))$,
 $f'(x) = 2(x - 3)$, $g'(x) = -\frac{1}{x^2}$

$$\begin{aligned}h'(x) &= f'(g(x))g'(x) \\ &= f'\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) \\ &= 2\left(\frac{1}{x} - 3\right)\left(-\frac{1}{x^2}\right) \\ &= -\frac{2}{x^2}\left(\frac{1}{x} - 3\right)\end{aligned}$$

8. a. $f(x) = (x + 4)^3(x - 3)^6$

$$\begin{aligned}f'(x) &= \frac{d}{dx}[(x + 4)^3] \cdot (x - 3)^6 \\ &\quad + (x + 4)^3 \frac{d}{dx}[(x - 3)^6] \\ &= 3(x + 4)^2(x - 3)^6 \\ &\quad + (x + 4)^3(6)(x - 3)^5 \\ &= (x + 4)^2(x - 3)^5 \\ &\quad \times [3(x - 3) + 6(x + 4)] \\ &= (x + 4)^2(x - 3)^5(9x + 15)\end{aligned}$$

b. $y = (x^2 + 3)^3(x^3 + 3)^2$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}[(x^2 + 3)^3] \cdot (x^3 + 3)^2 \\ &\quad + (x^2 + 3)^3 \cdot \frac{d}{dx}[(x^3 + 3)^2] \\ &= 3(x^2 + 3)^2(2x)(x^3 + 3)^2 \\ &\quad + (x^2 + 3)^3(2)(x^3 + 3)(3x^2) \\ &= 6x(x^2 + 3)^2(x^3 + 3)[(x^3 + 3) + x(x^2 + 3)] \\ &= 6x(x^2 + 3)^2(x^3 + 3)(2x^3 + 3x + 3)\end{aligned}$$

c. $y = \frac{3x^2 + 2x}{x^2 + 1}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(6x + 2)(x^2 + 1) - (3x^2 + 2x)(2x)}{(x^2 + 1)^2} \\ &= \frac{6x^3 + 2x^2 + 6x + 2 - 6x^3 - 4x^2}{(x^2 + 1)^2} \\ &= \frac{-2x^2 + 6x + 2}{(x^2 + 1)^2}\end{aligned}$$

d. $h(x) = x^3(3x - 5)^2$

$$\begin{aligned}h'(x) &= \frac{d}{dx}[x^3] \cdot (3x - 5)^2 + x^3 \frac{d}{dx}[(3x - 5)^2] \\ &= 3x^2(3x - 5)^2 + x^3(2)(3x - 5)(3) \\ &= 3x^2(3x - 5)[(3x - 5) + 2x] \\ &= 3x^2(3x - 5)(5x - 5) \\ &= 15x^2(3x - 5)(x - 1)\end{aligned}$$

e. $y = x^4(1 - 4x^2)^3$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}[x^4](1 - 4x^2)^3 + x^4 \cdot \frac{d}{dx}[(1 - 4x^2)^3] \\ &= 4x^3(1 - 4x^2)^3 + x^4(3)(1 - 4x^2)^2(-8x) \\ &= 4x^3(1 - 4x^2)^2[(1 - 4x^2) - 6x^2] \\ &= 4x^3(1 - 4x^2)^2(1 - 10x^2)\end{aligned}$$

f. $y = \left(\frac{x^2 - 3}{x^2 + 3}\right)^4$

$$\begin{aligned}\frac{dy}{dx} &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \frac{d}{dx}\left[\frac{x^2 - 3}{x^2 + 3}\right] \\ &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \cdot \frac{(x^2 + 3)(2x) - (x^2 - 3)(2x)}{(x^2 + 3)^2} \\ &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \cdot \frac{12x}{(x^2 + 3)^2} \\ &= \frac{48x(x^2 - 3)^3}{(x^2 + 3)^5}\end{aligned}$$

9. a. $s(t) = t^{\frac{1}{3}}(4t - 5)^{\frac{2}{3}}$

$$\begin{aligned}&= t^{\frac{1}{3}}[(4t - 5)^2]^{\frac{1}{3}} \\ &= [t(4t - 5)^2]^{\frac{1}{3}} \\ &= [t(16t^2 - 40t + 25)]^{\frac{1}{3}} \\ &= (16t^3 - 40t^2 + 25t)^{\frac{1}{3}}, t = 8\end{aligned}$$

$$\begin{aligned}s'(t) &= \frac{1}{3}(16t^3 - 40t^2 + 25t)^{-\frac{2}{3}} \\&\quad \times (48t^2 - 80t + 25) \\&= \frac{(48t^2 - 80t + 25)}{3(16t^3 - 40t^2 + 25t)^{\frac{2}{3}}}\end{aligned}$$

Rate of change at $t = 8$:

$$\begin{aligned}s'(8) &= \frac{(48(8)^2 - 80(8) + 25)}{3(16(8)^3 - 40(8)^2 + 25(8))^{\frac{2}{3}}} \\&= \frac{2457}{972} \\&= \frac{91}{36}\end{aligned}$$

b. $s(t) = \left(\frac{t-\pi}{t-6\pi}\right)^{\frac{1}{3}}, t = 2\pi$

$$\begin{aligned}s'(t) &= \frac{1}{3}\left(\frac{t-\pi}{t-6\pi}\right)^{-\frac{2}{3}} \cdot \frac{d}{dt}\left[\frac{t-\pi}{t-6\pi}\right] \\&= \frac{1}{3}\left(\frac{t-6\pi}{t-\pi}\right)^{\frac{2}{3}} \cdot \frac{(t-6\pi)-(t-\pi)}{(t-6\pi)^2} \\&= \frac{1}{3}\left(\frac{t-6\pi}{t-\pi}\right)^{\frac{2}{3}} \cdot \frac{-5\pi}{(t-6\pi)^2}\end{aligned}$$

Rate of change at $t = 2\pi$:

$$\begin{aligned}s'(2\pi) &= \frac{1}{3}(-4)^{\frac{2}{3}} \cdot \frac{-5\pi}{16\pi^2} \\&= -\frac{5\sqrt[3]{2}}{24\pi}\end{aligned}$$

10. $y = (1 + x^3)^2 \quad y = 2x^6$

$$\frac{dy}{dx} = 2(1 + x^3)(3x^2) \quad \frac{dy}{dx} = 12x^5$$

For the same slope,

$$\begin{aligned}6x^2(1 + x^3) &= 12x^5 \\6x^2 + 6x^5 &= 12x^5 \\6x^2 - 6x^5 &= 0 \\6x^2(x^3 - 1) &= 0 \\x = 0 \text{ or } x &= 1.\end{aligned}$$

Curves have the same slope at $x = 0$ and $x = 1$.

11. $y = (3x - x^2)^{-2}$

$$\frac{dy}{dx} = -2(3x - x^2)^{-3}(3 - 2x)$$

At $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= -2[6 - 4]^{-3}(3 - 4) \\&= 2(2)^{-3} \\&= \frac{1}{4}\end{aligned}$$

The slope of the tangent line at $x = 2$ is $\frac{1}{4}$.

12. $y = (x^3 - 7)^5$ at $x = 2$

$$\frac{dy}{dx} = 5(x^3 - 7)^4(3x^2)$$

When $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= 5(1)^4(12) \\&= 60\end{aligned}$$

Slope of the tangent is 60.

Equation of the tangent at $(2, 1)$ is

$$y - 1 = 60(x - 2)$$

$$60x - y - 119 = 0.$$

13. a. $y = 3u^2 - 5u + 2$

$$u = x^2 - 1, x = 2$$

$$u = 3$$

$$\frac{dy}{du} = 6u - 5, \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= (6u - 5)(2x)$$

$$= (18 - 5)(4)$$

$$= 13(4)$$

$$= 52$$

b. $y = 2u^3 + 3u^2, u = x + x^{\frac{1}{2}}, x = 1$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (6u^2 + 6u)\left(1 + \frac{1}{2\sqrt{x}}\right)$$

At $x = 1$:

$$\begin{aligned}u &= 1 + 1^{\frac{1}{2}} \\&= 2\end{aligned}$$

$$\frac{dy}{dx} = (6(2)^2 + 6(2))\left(1 + \frac{1}{2\sqrt{1}}\right)$$

$$= 36 \times \frac{3}{2}$$

$$= 54$$

c. $y = u(u^2 + 3)^3, u = (x + 3)^2, x = -2$

$$\frac{dy}{du} = (u^2 + 3)^3 + 6u^2(u^2 + 3)^2, \frac{du}{dx} = 2(x + 3)$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = [7^3 + 6(4)^2][2(1)]$$

$$= 439 \times 2$$

$$= 878$$

d. $y = u^3 - 5(u^3 - 7u)^2,$

$$\begin{aligned}u &= \sqrt{x} \\&= x^{\frac{1}{2}}, x = 4\end{aligned}$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 3[3u^2 - 10(u^3 - 7u)(3u^2 - 7)] \cdot \left(\frac{1}{2}x^{\frac{1}{2}}\right)$$

$$= [3u^2 - 10(u^3 - 7u)(3u^2 - 7)] \cdot \frac{1}{2\sqrt{x}}$$

At $x = 4$:

$$\begin{aligned} u &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= [3(2)^2 - 10((2)^3 - 7(2))(3(2)^2 - 7)] \frac{1}{2(2)} \\ &= 78 \end{aligned}$$

- 14.** $h(x) = f(g(x))$, therefore

$$\begin{aligned} h'(x) &= f'(g(x)) \times g'(x) \\ f(u) &= u^2 - 1, g(2) = 3, g'(2) = -1 \end{aligned}$$

$$\begin{aligned} \text{Now, } h'(2) &= f'(g(2)) \times g'(2) \\ &= f'(3) \times g'(2). \end{aligned}$$

Since $f(u) = u^2 - 1$, $f'(u) = 2u$, and $f'(3) = 6$,

$$\begin{aligned} h'(2) &= 6(-1) \\ &= -6. \end{aligned}$$

15. $V(t) = 50000 \left(1 - \frac{t}{30}\right)^2$

$$V'(t) = 50000 \left[2 \left(1 - \frac{t}{30}\right) \left(-\frac{1}{30}\right)\right]$$

$$\begin{aligned} V'(10) &= 50000 \left[2 \left(1 - \frac{10}{30}\right) \left(-\frac{1}{30}\right)\right] \\ &= 50000 \left[2 \left(\frac{2}{3}\right) \left(-\frac{1}{30}\right)\right] \\ &\doteq 2222 \end{aligned}$$

At $t = 10$ minutes, the water is flowing out of the tank at a rate of 2222 L/min.

- 16.** The velocity function is the derivative of the position function.

$$s(t) = (t^3 + t^2)^{\frac{1}{2}}$$

$$v(t) = s'(t) = \frac{1}{2}(t^3 + t^2)^{-\frac{1}{2}}(3t^2 + 2t)$$

$$= \frac{3t^2 + 2t}{2\sqrt{t^3 + t^2}}$$

$$\begin{aligned} v(3) &= \frac{3(3)^2 + 2(3)}{2\sqrt{3^3 + 3^2}} \\ &= \frac{27 + 6}{2\sqrt{36}} \\ &= \frac{33}{12} \\ &= 2.75 \end{aligned}$$

The particle is moving at 2.75 m/s.

- 17. a.** $h(x) = p(x)q(x)r(x)$

$$\begin{aligned} h'(x) &= p'(x)q(x)r(x) + p(x)q'(x)r(x) \\ &\quad + p(x)q(x)r'(x) \end{aligned}$$

b. $h(x) = x(2x + 7)^4(x - 1)^2$

Using the result from part a.,

$$\begin{aligned} h'(x) &= (1)(2x + 7)^4(x - 1)^2 \\ &\quad + x[4(2x + 7)^3(2)](x - 1)^2 \\ &\quad + x(2x + 7)^4[2(x - 1)] \end{aligned}$$

$$\begin{aligned} h'(-3) &= 1(16) + (-3)[4(1)(2)](16) \\ &\quad + (-3)(1)[2(-4)] \\ &= 16 - 384 + 24 \\ &= -344 \end{aligned}$$

18. $y = (x^2 + x - 2)^3 + 3$

$$\frac{dy}{dx} = 3(x^2 + x - 2)^2(2x + 1)$$

At the point $(1, 3)$, $x = 1$ and the slope of the tangent will be $3(1 + 1 - 2)^2(2 + 1) = 0$.

Equation of the tangent at $(1, 3)$ is $y - 3 = 0$.

Solving this equation with the function, we have

$$(x^2 + x - 2)^3 + 3 = 3$$

$$(x + 2)^3(x - 1)^3 = 0$$

$$x = -2 \text{ or } x = 1$$

Since -2 and 1 are both triple roots, the line with equation $y - 3 = 0$ will be a tangent at both $x = 1$ and $x = -2$. Therefore, $y - 3 = 0$ is also a tangent at $(-2, 3)$.

$$\begin{aligned} \mathbf{19.} \quad y &= \frac{x^2(1-x)^3}{(1+x)^3} \\ &= x^2 \left[\left(\frac{1-x}{1+x} \right) \right]^3 \\ \frac{dy}{dx} &= 2x \left(\frac{1-x}{1+x} \right)^3 + 3x^2 \left(\frac{1-x}{1+x} \right)^2 \\ &\quad \times \left[\frac{-(1+x) - (1-x)(1)}{(1+x)^2} \right] \\ &= 2x \left(\frac{1-x}{1+x} \right)^3 + 3x^2 \left(\frac{1-x}{1+x} \right)^2 \left[\frac{-2}{(1+x)^2} \right] \\ &= 2x \left(\frac{1-x}{1+x} \right)^2 \left[\frac{1-x}{1+x} - \frac{3x}{(1+x)^2} \right] \\ &= 2x \left(\frac{1-x}{1+x} \right)^2 \left[\frac{1-x^2-3x}{(1+x)^2} \right] \\ &= -\frac{2x(x^2+3x-1)(1-x)^2}{(1+x)^4} \end{aligned}$$

Review Exercise, pp. 110–113

1. To find the derivative $f'(x)$, the limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

must be computed, provided it exists. If this limit does not exist, then the derivative of $f(x)$ does not

exist at this particular value of x . As an alternative to this limit, we could also find $f'(x)$ from the definition by computing the equivalent limit

$$f'(x) = \lim_{z \rightarrow x} \frac{f(z) - f(x)}{z - x}.$$

These two limits are seen to be equivalent by substituting $z = x + h$.

2. a. $y = 2x^2 - 5x$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{(2(x+h)^2 - 5(x+h)) - (2x^2 - 5x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2((x+h)^2 - x^2) - 5h}{h} \\ &= \lim_{h \rightarrow 0} \frac{2((x+h) - x)((x+h) + x) - 5h}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h(2x+h) - 5h}{h} \\ &= \lim_{h \rightarrow 0} (2(2x+h) - 5) \\ &= 4x - 5 \end{aligned}$$

b. $y = \sqrt{x-6}$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)-6} - \sqrt{x-6}}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{\sqrt{(x+h)-6} - \sqrt{x-6}}{h} \right. \\ &\quad \times \left. \frac{\sqrt{(x+h)-6} + \sqrt{x-6}}{\sqrt{(x+h)-6} + \sqrt{x-6}} \right] \\ &= \lim_{h \rightarrow 0} \frac{((x+h)-6) - (x-6)}{h(\sqrt{(x+h)-6} + \sqrt{x-6})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{(x+h)-6} + \sqrt{x-6}} \\ &= \frac{1}{2\sqrt{x-6}} \end{aligned}$$

c. $y = \frac{x}{4-x}$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{\frac{x+h}{4-(x+h)} - \frac{x}{4-x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+h)(4-x) - x(4-(x+h))}{(4-(x+h))(4-x)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{4h}{h(4-(x+h))(4-x)} \\ &= \lim_{h \rightarrow 0} \frac{4}{(4-(x+h))(4-x)} \\ &= \frac{4}{(4-x)^2} \end{aligned}$$

3. a. $y = x^2 - 5x + 4$

$$\frac{dy}{dx} = 2x - 5$$

b. $f(x) = x^{\frac{3}{4}}$

$$\begin{aligned} f'(x) &= \frac{3}{4}x^{-\frac{1}{4}} \\ &= \frac{3}{4x^{\frac{1}{4}}} \end{aligned}$$

c. $y = \frac{7}{3x^4}$

$$= \frac{7}{3}x^{-4}$$

$$\frac{dy}{dx} = \frac{-28}{3}x^{-5}$$

$$= -\frac{28}{3x^5}$$

d. $y = \frac{1}{x^2 + 5}$

$$= (x^2 + 5)^{-1}$$

$$\frac{dy}{dx} = (-1)(x^2 + 5)^{-2} \cdot (2x)$$

$$= -\frac{2x}{(x^2 + 5)^2}$$

e. $y = \frac{3}{(3-x^2)^2}$

$$= 3(3-x^2)^{-2}$$

$$\frac{dy}{dx} = (-6)(3-x^2)^{-3} \cdot (-2x)$$

$$= \frac{12x}{(3-x^2)^3}$$

f. $y = \sqrt{7x^2 + 4x + 1}$

$$\frac{dy}{dx} = \frac{1}{2}(7x^2 + 4x + 1)^{-\frac{1}{2}}(14x + 4)$$

$$= \frac{7x + 2}{\sqrt{7x^2 + 4x + 1}}$$

4. a. $f(x) = \frac{2x^3 - 1}{x^2}$

$$= 2x - \frac{1}{x^2}$$

$$= 2x - x^{-2}$$

$$f'(x) = 2 + 2x^{-3}$$

$$= 2 + \frac{2}{x^3}$$

b. $g(x) = \sqrt{x}(x^3 - x)$

$$= x^{\frac{1}{2}}(x^3 - x)$$

$$= x^{\frac{7}{2}} - x^{\frac{3}{2}}$$

$$g'(x) = \frac{7}{2}x^{\frac{5}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$

$$= \frac{\sqrt{x}}{2}(7x^2 - 3)$$

c. $y = \frac{x}{3x - 5}$

$$\frac{dy}{dx} = \frac{(3x - 5)(1) - (x)(3)}{(3x - 5)^2}$$

$$= -\frac{5}{(3x - 5)^2}$$

d. $y = (x - 1)^{\frac{1}{2}}(x + 1)$

$$y' = (x - 1)^{\frac{1}{2}} + (x + 1)\left(\frac{1}{2}\right)(x - 1)^{-\frac{1}{2}}$$

$$= \sqrt{x - 1} + \frac{x + 1}{2\sqrt{x - 1}}$$

$$= \frac{2x - 2 + x + 1}{2\sqrt{x - 1}}$$

$$= \frac{3x - 1}{2\sqrt{x - 1}}$$

e. $f(x) = (\sqrt{x} + 2)^{-\frac{2}{3}}$

$$= (x^{\frac{1}{2}} + 2)^{-\frac{2}{3}}$$

$$f'(x) = \frac{-2}{3}(x^{\frac{1}{2}} + 2)^{-\frac{5}{3}} \cdot \frac{1}{2}x^{-\frac{1}{2}}$$

$$= -\frac{1}{3\sqrt{x}(\sqrt{x} + 2)^{\frac{5}{3}}}$$

f. $y = \frac{x^2 + 5x + 4}{x + 4}$

$$= \frac{(x + 4)(x + 1)}{x + 4}$$

$$= x + 1, x \neq -4$$

$$\frac{dy}{dx} = 1$$

5. a. $y = x^4(2x - 5)^6$

$$y' = x^4[6(2x - 5)^5(2)] + 4x^3(2x - 5)^6$$

$$= 4x^3(2x - 5)^5[3x + (2x - 5)]$$

$$= 4x^3(2x - 5)^5(5x - 5)$$

$$= 20x^3(2x - 5)^5(x - 1)$$

b. $y = x\sqrt{x^2 + 1}$

$$y' = x\left[\frac{1}{2}(x^2 + 1)^{-\frac{1}{2}}(2x)\right] + (1)\sqrt{x^2 + 1}$$

$$= \frac{x^2}{\sqrt{x^2 + 1}} + \sqrt{x^2 + 1}$$

c. $y = \frac{(2x - 5)^4}{(x + 1)^3}$

$$y' = \frac{(x + 1)^3 4(2x - 5)^3(2)}{(x + 1)^6}$$

$$- \frac{3(2x - 5)^4(x + 1)^2}{(x + 1)^6}$$

$$= \frac{(x + 1)^2(2x - 5)^3[8x + 8 - 6x + 15]}{(x + 1)^6}$$

$$y' = \frac{(2x - 5)^3(2x + 23)}{(x + 1)^4}$$

d. $y = \left(\frac{10x - 1}{3x + 5}\right)^6 = (10x - 1)^6(3x + 5)^{-6}$

$$y' = (10x - 1)^6[-6(3x + 5)^{-7}(3)]$$

$$+ 6(10x - 1)^5(10)(3x + 5)^{-6}$$

$$= (10x - 1)^5(3x + 5)^{-7}[x - 18(10x - 1)]$$

$$+ 60(3x + 5)$$

$$= (10x - 1)^5(3x + 5)^{-7}$$

$$\times (-180x + 18 + 180x + 300)$$

$$= \frac{318(10x - 1)^5}{(3x + 5)^7}$$

e. $y = (x - 2)^3(x^2 + 9)^4$

$$y' = (x - 2)^3[4(x^2 + 9)^3(2x)]$$

$$+ 3(x - 2)^2(1)(x^2 + 9)^4$$

$$= (x - 2)^2(x^2 + 9)^3[8x(x - 2) + 3(x^2 + 9)]$$

$$= (x - 2)^2(x^2 + 9)^3(11x^2 - 16x + 27)$$

f. $y = (1 - x^2)^3(6 + 2x)^{-3}$

$$= \left(\frac{1 - x^2}{6 + 2x}\right)^3$$

$$y' = 3\left(\frac{1 - x^2}{6 + 2x}\right)^2$$

$$\times \left[\frac{(6 + 2x)(-2x) - (1 - x^2)(2)}{(6 + 2x)^2} \right]$$

$$= \frac{3(1 - x^2)^2(-12x - 4x^2 - 2 + 2x^2)}{(6 + 2x)^4}$$

$$= -\frac{3(1 - x^2)^2(2x^2 + 12x + 2)}{(6 + 2x)^4}$$

$$= -\frac{3(1 - x^2)^2(x^2 + 6x + 1)}{8(3 - x)^4}$$

6. a. $g(x) = f(x^2)$

$$g'(x) = f(x^2) \times 2x$$

b. $h(x) = 2xf(x)$

$$h'(x) = 2xf'(x) + 2f(x)$$

7. a. $y = 5u^2 + 3u - 1, u = \frac{18}{x^2 + 5}$

$$x = 2$$

$$u = 2$$

$$\frac{dy}{du} = 10u + 3$$

$$\frac{du}{dx} = -\frac{36x}{(x^2 + 5)^2}$$

When $x = 2$,

$$\frac{du}{dx} = -\frac{72}{81} = -\frac{8}{9}$$

When $u = 2$,

$$\frac{dy}{du} = 20 + 3$$

$$= 23$$

$$\frac{dy}{dx} = 23 \left(-\frac{8}{9} \right)$$

$$= -\frac{184}{9}$$

b. $y = \frac{u+4}{u-4}$, $u = \frac{\sqrt{x}+x}{10}$,

$$x = 4$$

$$u = \frac{3}{5}$$

$$\frac{dy}{du} = \frac{(u-4) - (u+4)}{(u-4)^2}$$

$$\frac{du}{dx} = \frac{1}{10} \left(\frac{1}{2}x^{-\frac{1}{2}} + 1 \right)$$

When $x = 4$,

$$= -\frac{8}{(u-4)^2} \frac{du}{dx} = \frac{1}{10} \left(\frac{5}{4} \right)$$

$$= \frac{1}{8}$$

When $u = \frac{3}{5}$,

$$\frac{dy}{du} = -\frac{8}{\left(\frac{3}{5} - \frac{20}{5} \right)^2}$$

$$= -\frac{8(25)}{(-17)^2}$$

When $x = 4$,

$$\frac{dy}{dx} = \frac{8(25)}{17^2} \times \frac{1}{8}$$

$$= \frac{25}{289}$$

c. $y = f(\sqrt{x^2 + 9})$, $f'(5) = -2$, $x = 4$

$$\frac{dy}{dx} = f'(\sqrt{x^2 + 9}) \times \frac{1}{2}(x^2 + 9)^{-\frac{1}{2}}(2x)$$

$$\frac{dy}{dx} = f'(5) \cdot \frac{1}{2} \cdot \frac{1}{5} \cdot 8$$

$$= -2 \cdot \frac{4}{5}$$

$$= -\frac{8}{5}$$

8. $f(x) = (9 - x^2)^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3}(9 - x^2)^{-\frac{1}{3}}(-2x)$$

$$= \frac{-4x}{3(9 - x^2)^{\frac{1}{3}}}$$

$$f'(1) = -\frac{2}{3}$$

The slope of the tangent line at $(1, 4)$ is $-\frac{2}{3}$.

9. $y = -x^3 + 6x^2$

$$y' = -3x^2 + 12x$$

$$-3x^2 + 12x = -12$$

$$x^2 - 4x - 4 = 0$$

$$-3x^2 + 12x = -15$$

$$x^2 - 4x - 5 = 0$$

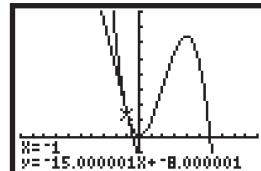
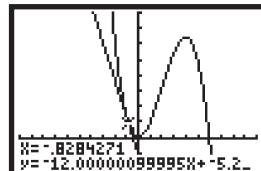
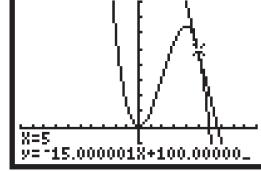
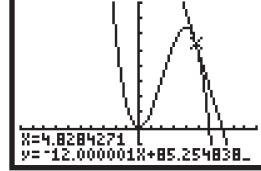
$$x = \frac{4 \pm \sqrt{16 + 16}}{2}$$

$$(x - 5)(x + 1) = 0$$

$$= \frac{4 \pm 4\sqrt{2}}{2}$$

$$x = 5, x = -1$$

$$x = 2 \pm 2\sqrt{2}$$



10. a. i. $y = (x^2 - 4)^5$
 $y' = 5(x^2 - 4)^4(2x)$

Horizontal tangent,

$$10x(x^2 - 4)^4 = 0$$

$$x = 0, x = \pm 2$$

ii. $y = (x^3 - x)^2$

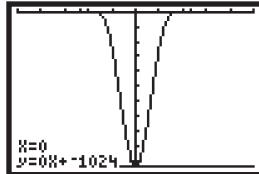
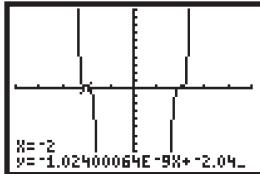
$$y' = 2(x^3 - x)(3x^2 - 1)$$

Horizontal tangent,

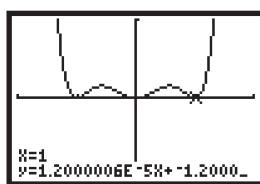
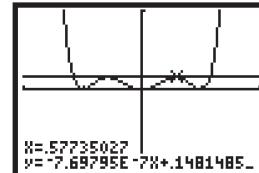
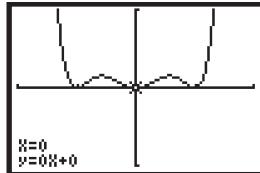
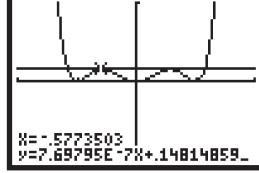
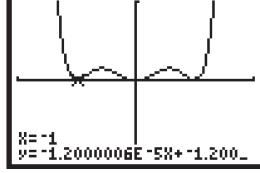
$$2x(x^2 - 1)(3x^2 - 1) = 0$$

$$x = 0, x = \pm 1, x = \pm \frac{\sqrt{3}}{3}$$

b. i.



ii.



11. a. $y = (x^2 + 5x + 2)^4$ at $(0, 16)$

$$y' = 4(x^2 + 5x + 2)^3(2x + 5)$$

$$\text{At } x = 0,$$

$$y' = 4(2)^3(5) \\ = 160$$

Equation of the tangent at $(0, 16)$ is

$$y - 16 = 160(x - 0)$$

$$y = 160x + 16$$

$$\text{or } 160x - y + 16 = 0$$

b. $y = (3x^{-2} - 2x^3)^5$ at $(1, 1)$

$$y' = 5(3x^{-2} - 2x^3)^4(-6x^{-3} - 6x^2)$$

$$\text{At } x = 1,$$

$$y' = 5(1)^4(-6 - 6) \\ = -60$$

Equation of the tangent at $(1, 1)$ is

$$y - 1 = -60(x - 1)$$

$$60x + y - 61 = 0.$$

12. $y = 3x^2 - 7x + 5$

$$\frac{dy}{dx} = 6x - 7$$

Slope of $x + 5y - 10 = 0$ is $-\frac{1}{5}$.

Since perpendicular, $6x - 7 = 5$

$$x = 2$$

$$y = 3(4) - 14 + 5 \\ = 3.$$

Equation of the tangent at $(2, 3)$ is

$$y - 3 = 5(x - 2)$$

$$5x - y - 7 = 0.$$

13. $y = 8x + b$ is tangent to $y = 2x^2$

$$\frac{dy}{dx} = 4x$$

Slope of the tangent is 8, therefore $4x = 8$, $x = 2$.

Point of tangency is $(2, 8)$.

Therefore, $8 = 16 + b$, $b = -8$.

$$\text{Or } 8x + b = 2x^2$$

$$2x^2 - 8x - b = 0$$

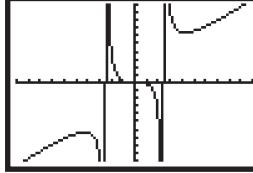
$$x = \frac{8 \pm \sqrt{64 + 8b}}{2(2)}.$$

For tangents, the roots are equal, therefore

$$64 + 8b = 0, b = -8.$$

Point of tangency is $(2, 8)$, $b = -8$.

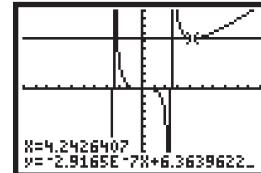
14. a.



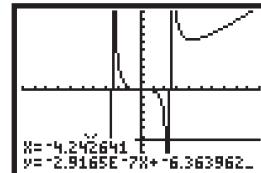
b.



The equation of the tangent is $y = 0$.



The equation of the tangent is $y = 6.36$.



The equation of the tangent is $y = -6.36$.

c. $f'(x) = \frac{(x^2 - 6)(3x^2) - x^3(2x)}{(x^2 - 6)^2}$

$$= \frac{x^4 - 18x^2}{(x^2 - 6)^2}$$

$$\frac{x^4 - 18x^2}{(x^2 - 6)^2} = 0$$

$$x^2(x^2 - 18) = 0$$

$$x^2 = 0 \text{ or } x^2 - 18 = 0$$

$$x = 0 \quad x = \pm 3\sqrt{2}$$

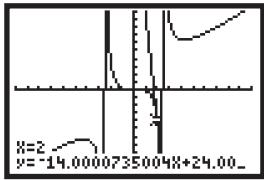
The coordinates of the points where the slope is 0 are $(0, 0)$, $(3\sqrt{2}, \frac{9\sqrt{2}}{2})$, and $(-3\sqrt{2}, -\frac{9\sqrt{2}}{2})$.

d. Substitute into the expression for $f'(x)$ from part b.

$$f'(2) = \frac{16 - 72}{(-2)^2}$$

$$= \frac{-56}{4}$$

$$= -14$$



15. a. $f(x) = 2x^{5/3} - 5x^{1/3}$

$$f'(x) = 2 \times \frac{5}{3}x^{2/3} - 5 \times \frac{2}{3}x^{-2/3}$$

$$= \frac{10}{3}x^{2/3} - \frac{10}{3x^{2/3}}$$

$$f(x) = 0 \quad \therefore x^{2/3}[2x - 5] = 0$$

$$x = 0 \text{ or } x = \frac{5}{2}$$

$y = f(x)$ crosses the x -axis at $x = \frac{5}{2}$, and

$$f'(x) = \frac{10}{3}\left(\frac{x - 1}{x^{1/3}}\right)$$

$$f'\left(\frac{5}{2}\right) = \frac{10}{3} \times \frac{3}{2} \times \frac{1}{\left(\frac{5}{2}\right)^{1/3}}$$

$$= 5 \times \frac{\sqrt[3]{2}}{\sqrt[3]{5}} = 5^{2/3} \times 2^{1/3}$$

$$= (25 \times 2)^{1/3}$$

$$= \sqrt[3]{50}$$

b. To find a , let $f(x) = 0$.

$$\frac{10}{3}x^{2/3} - \frac{10}{3x^{1/3}} = 0$$

$$30x = 30$$

$$x = 1$$

Therefore $a = 1$.

16. $M = 0.1t^2 - 0.001t^3$

a. When $t = 10$,

$$M = 0.1(100) - 0.001(1000)$$

$$= 9$$

When $t = 15$,

$$M = 0.1(225) - 0.001(3375)$$

$$= 19.125$$

One cannot memorize partial words, so 19 words are memorized after 15 minutes.

b. $M' = 0.2t - 0.003t^2$

When $t = 10$,

$$M' = 0.2(10) - 0.003(100)$$

$$= 1.7$$

The number of words memorized is increasing by 1.7 words/min.

When $t = 15$,

$$M' = 0.2(15) - 0.003(225)$$

$$= 2.325$$

The number of words memorized is increasing by 2.325 words/min.

17. a. $N(t) = 20 - \frac{30}{\sqrt{9 + t^2}}$

$$N'(t) = \frac{30t}{(9 + t^2)^{3/2}}$$

b. No, according to this model, the cashier never stops improving. Since $t > 0$, the derivative is always positive, meaning that the rate of change in the cashier's productivity is always increasing. However, these increases must be small, since, according to the model, the cashier's productivity can never exceed 20.

18. $C(x) = \frac{1}{3}x^3 + 40x + 700$

a. $C'(x) = x^2 + 40$

b. $C'(x) = 76$

$$x^2 + 40 = 76$$

$$x^2 = 36$$

$$x = 6$$

Production level is 6 gloves/week.

19. $R(x) = 750x - \frac{x^2}{6} - \frac{2}{3}x^3$

a. Marginal Revenue

$$R'(x) = 750 - \frac{x}{3} - 2x^2$$

b. $R'(10) = 750 - \frac{10}{3} - 2(100)$
 $= \$546.67$

20. $D(p) = \frac{20}{\sqrt{p-1}}, p > 1$
 $D'(p) = 20\left(-\frac{1}{2}\right)(p-1)^{-\frac{3}{2}}$
 $= -\frac{10}{(p-1)^{\frac{3}{2}}}$
 $D'(5) = \frac{10}{\sqrt{4^3}} = -\frac{10}{8}$
 $= -\frac{5}{4}$

Slope of demand curve at $(5, 10)$ is $-\frac{5}{4}$.

21. $B(x) = -0.2x^2 + 500, 0 \leq x \leq 40$

a. $B(0) = -0.2(0)^2 + 500 = 500$

$B(30) = -0.2(30)^2 + 500 = 320$

b. $B'(x) = -0.4x$

$B'(0) = -0.4(0) = 0$

$B'(30) = -0.4(30) = -12$

c. $B(0)$ = blood sugar level with no insulin

$B(30)$ = blood sugar level with 30 mg of insulin

$B'(0)$ = rate of change in blood sugar level
with no insulin

$B'(30)$ = rate of change in blood sugar level
with 30 mg of insulin

d. $B'(50) = -0.4(50) = -20$

$B(50) = -0.2(50)^2 + 500 = 0$

$B'(50) = -20$ means that the patient's blood sugar level is decreasing at 20 units per mg of insulin 1 h after 50 mg of insulin is injected.

$B(50) = 0$ means that the patient's blood sugar level is zero 1 h after 50 mg of insulin is injected. These values are not logical because a person's blood sugar level can never reach zero and continue to decrease.

22. a. $f(x) = \frac{3x}{1-x^2}$
 $= \frac{3x}{(1-x)(1+x)}$

$f(x)$ is not differentiable at $x = 1$ because it is not defined there (vertical asymptote at $x = 1$).

b. $g(x) = \frac{x-1}{x^2+5x-6}$
 $= \frac{x-1}{(x+6)(x-1)}$
 $= \frac{1}{(x+6)}$ for $x \neq 1$

$g(x)$ is not differentiable at $x = 1$ because it is not defined there (hole at $x = 1$).

c. $h(x) = \sqrt[3]{(x-2)^2}$

The graph has a cusp at $(2, 0)$ but it is differentiable at $x = 1$.

d. $m(x) = |3x-3| - 1$

The graph has a corner at $x = 1$, so $m(x)$ is not differentiable at $x = 1$.

23. a. $f(x) = \frac{3}{4x^2-x}$
 $= \frac{3}{x(4x-1)}$

$f(x)$ is not defined at $x = 0$ and $x = 0.25$. The graph has vertical asymptotes at $x = 0$ and $x = 0.25$. Therefore, $f(x)$ is not differentiable at $x = 0$ and $x = 0.25$.

b. $f(x) = \frac{x^2-x-6}{x^2-9}$
 $= \frac{(x-3)(x+2)}{(x-3)(x+3)}$
 $= \frac{(x+2)}{(x+3)}$ for $x \neq 3$

$f(x)$ is not defined at $x = 3$ and $x = -3$. At $x = -3$, the graph has a vertical asymptote and at $x = 3$ it has a hole. Therefore, $f(x)$ is not differentiable at $x = 3$ and $x = -3$.

c. $f(x) = \sqrt{x^2-7x+6}$
 $= \sqrt{(x-6)(x-1)}$

$f(x)$ is not defined for $1 < x < 6$. Therefore, $f(x)$ is not differentiable for $1 < x < 6$.

24. $p'(t) = \frac{(t+1)(25) - (25t)(t)}{(t+1)^2}$
 $= \frac{25t+25-25t}{(t+1)^2}$
 $= \frac{25}{(t+1)^2}$

25. Answers may vary. For example,
 $f(x) = 2x + 3$

$$y = \frac{1}{2x+3}$$

$$y' = \frac{(2x+3)(0) - (1)(2)}{(2x+3)^2}$$

$$= -\frac{2}{(2x+3)^2}$$

$f(x) = 5x + 10$

$$y = \frac{1}{5x+10}$$

$$y' = \frac{(5x+10)(0) - (1)(5)}{(5x+10)^2}$$

$$= -\frac{5}{(5x+10)^2}$$

Rule: If $f(x) = ax + b$ and $y = \frac{1}{f(x)}$, then

$$y' = \frac{-a}{(ax+b)^2}$$

$$\begin{aligned} y' &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{1}{a(x+h)+b} - \frac{1}{ax+b} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{ax+b - [a(x+h)+b]}{[a(x+h)+b](ax+h)} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{ax+b - ax - ah - b}{[a(x+h)+b](ax+b)} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-ah}{[a(x+h)+b](ax+b)} \right] \\ &= \lim_{h \rightarrow 0} \frac{-a}{[a(x+h)+b](ax+b)} \\ &= \frac{-a}{(ax+b)^2} \end{aligned}$$

26. a. Let $y = f(x)$

$$y = \frac{(2x-3)^2 + 5}{2x-3}$$

Let $u = 2x-3$.

$$\text{Then } y = \frac{u^2 + 5}{u}.$$

$$y = u + 5u^{-1}$$

$$\text{b. } f'(x) = \frac{dy}{dx}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= (1 - 5u^{-2})(2) \\ &= 2(1 - 5(2x-3)^{-2}) \end{aligned}$$

$$27. g(x) = \sqrt{2x-3} + 5(2x-3)$$

a. Let $y = g(x)$.

$$y = \sqrt{2x-3} + 5(2x-3)$$

Let $u = 2x-3$.

Then $y = \sqrt{u} + 5u$.

$$\begin{aligned} \text{b. } g'(x) &= \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \\ &= \left(\frac{1}{2}u^{-\frac{1}{2}} + 5 \right)(2) \\ &= u^{-\frac{1}{2}} + 10 \\ &= (2x-3)^{-\frac{1}{2}} + 10 \end{aligned}$$

$$\text{28. a. } f(x) = (2x-5)^3(3x^2+4)^5$$

$$\begin{aligned} f'(x) &= (2x-5)^3(5)(3x^2+4)^4(6x) \\ &\quad + (3x^2+4)^5(3)(2x-5)^2(2) \\ &= 30x(2x-5)^3(3x^2+4)^4 \\ &\quad + 6(3x^2+4)^5(2x-5)^2 \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times [5x(2x-5) + (3x^2+4)] \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times (10x^2 - 25x + 3x^2 + 4) \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times (13x^2 - 25x + 4) \end{aligned}$$

$$\text{b. } g(x) = (8x^3)(4x^2+2x-3)^5$$

$$\begin{aligned} g'(x) &= (8x^3)(5)(4x^2+2x-3)^4(8x+2) \\ &\quad + (4x^2+2x-3)^5(24x^2) \\ &= 40x^3(4x^2+2x-3)^4(8x+2) \\ &\quad + 24x^2(4x^2+2x-3)^5 \\ &= 8x^2(4x^2+2x-3)^4[5x(8x+2) \\ &\quad + 3(4x^2+2x-3)] \\ &= 8x^2(4x^2+2x-3)^4 \\ &\quad (40x^2 + 10x + 12x^2 + 6x - 9) \\ &= 8x^2(4x^2+2x-3)^4(52x^2 + 16x - 9) \end{aligned}$$

$$\text{c. } y = (5+x)^2(4-7x^3)^6$$

$$\begin{aligned} y' &= (5+x)^2(6)(4-7x^3)^5(-21x^2) \\ &\quad + (4-7x^3)^6(2)(5+x) \\ &= -126x^2(5+x)^2(4-7x^3)^5 \\ &\quad + 2(5+x)(4-7x^3)^6 \\ &= 2(5+x)(4-7x^3)^5[-63x^2(5+x) \\ &\quad + 4-7x^3] \\ &= 2(5+x)(4-7x^3)^5(4-315x^2-70x^3) \end{aligned}$$

$$\text{d. } h(x) = \frac{6x-1}{(3x+5)^4}$$

$$\begin{aligned} h'(x) &= \frac{(3x+5)^4(6) - (6x-1)(4)(3x+5)^3(3)}{((3x+5)^4)^2} \\ &= \frac{6(3x+5)^3[(3x+5) - 2(6x-1)]}{(3x+5)^8} \\ &= \frac{6(-9x+7)}{(3x+5)^5} \end{aligned}$$

$$\text{e. } y = \frac{(2x^2-5)^3}{(x+8)^2}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{(x+8)^2(3)(2x^2-5)^2(4x)}{((x+8)^2)^2} \\ &\quad - \frac{(2x^2-5)^3(2)(x+8)}{((x+8)^2)^2} \\ &= \frac{2(x+8)(2x^2-5)^2[6x(x+8) - (2x^2-5)]}{(x+8)^4} \\ &= \frac{2(2x^2-5)^2(4x^2+48x+5)}{(x+8)^3} \end{aligned}$$

f. $f(x) = \frac{-3x^4}{\sqrt{4x - 8}}$

$$= \frac{-3x^4}{(4x - 8)^{\frac{1}{2}}}$$

$$f'(x) = \frac{(4x - 8)^{\frac{1}{2}}(-12x^3)}{((4x - 8)^{\frac{1}{2}})^2}$$

$$- \frac{(-3x^4)\left(\frac{1}{2}\right)(4x - 8)^{-\frac{1}{2}}(4)}{((4x - 8)^{\frac{1}{2}})^2}$$

$$= \frac{-6x^3(4x - 8)^{-\frac{1}{2}}[2(4x - 8) - x]}{4x - 8}$$

$$= \frac{-6x^3(7x - 16)}{(4x - 8)^{\frac{3}{2}}}$$

$$= \frac{-3x^3(7x - 16)}{(4x - 8)^{\frac{3}{2}}}$$

g. $g(x) = \left(\frac{2x + 5}{6 - x^2}\right)^4$

$$g'(x) = 4\left(\frac{2x + 5}{6 - x^2}\right)^3$$

$$\times \left(\frac{(6 - x^2)(2) - (2x + 5)(-2x)}{(6 - x^2)^2}\right)$$

$$= 4\left(\frac{2x + 5}{6 - x^2}\right)^3 \left(\frac{2(6 + x^2 + 5x)}{(6 - x^2)^2}\right)$$

$$= 8\left(\frac{2x + 5}{6 - x^2}\right)^3 \left(\frac{(x + 2)(x + 3)}{(6 - x^2)^2}\right)$$

h. $y = \left[\frac{1}{(4x + x^2)^3}\right]^3$

$$= (4x + x^2)^{-9}$$

$$\frac{dy}{dx} = -9(4x + x^2)^{-10}(4 + 2x)$$

29. $f(x) = ax^2 + bx + c$,

It is given that $(0, 0)$ and $(8, 0)$ are on the curve, and $f'(2) = 16$.

Calculate $f'(x) = 2ax + b$.

Then,

$$16 = 2a(2) + b$$

$$4a + b = 16 \quad (1)$$

Since $(0, 0)$ is on the curve,

$$0 = a(0)^2 + b(0) + c$$

$$c = 0$$

Since $(8, 0)$ is on the curve,

$$0 = a(8)^2 + b(8) + c$$

$$0 = 64a + 8b + 0$$

$$8a + b = 0 \quad (2)$$

Solve (1) and (2):

$$\text{From (2), } b = -8a \quad (1)$$

In (1),

$$4a - 8a = 16$$

$$-4a = 16$$

$$a = -4$$

Using (1),

$$b = -8(-4) = 32$$

$$a = -4, b = 32, c = 0, f(x) = -4x^2 + 32x$$

30. a. $A(t) = -t^3 + 5t + 750$

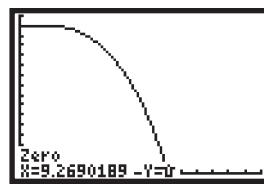
$$A'(t) = -3t^2 + 5$$

b. $A'(5) = -3(25) + 5$
 $= -70$

At 5 h, the number of ants living in the colony is decreasing by 7000 ants/h.

c. $A(0) = 750$, so there were $750(100)$ or 75 000 ants living in the colony before it was treated with insecticide.

d. Determine t so that $A(t) = 0$. $-t^3 + 5t + 750$ cannot easily be factored, so find the zeros by using a graphing calculator.



All of the ants have been killed after about 9.27 h.

Chapter 2 Test, p. 114

1. You need to use the chain rule when the derivative for a given function cannot be found using the sum, difference, product, or quotient rules or when writing the function in a form that would allow the use of these rules is tedious. The chain rule is used when a given function is a composition of two or more functions.

2. f is the blue graph (it's a cubic). f' is the red graph (it is quadratic). The derivative of a polynomial function has degree one less than the derivative of the function. Since the red graph is a quadratic (degree 2) and the blue graph is cubic (degree 3), the blue graph is f and the red graph is f' .

3. $f(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{x + h - (x + h)^2 - (x - x^2)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{x + h - (x^2 + 2hx + h^2) - x + x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h - 2hx - h^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(1 - 2x - h)}{h}$$

$$= \lim_{h \rightarrow 0} (1 - 2x - h)$$

$$= 1 - 2x$$

Therefore, $\frac{d}{dx}(x - x^2) = 1 - 2x$.

4. a. $y = \frac{1}{3}x^3 - 3x^{-5} + 4\pi$

$$\frac{dy}{dx} = x^2 + 15x^{-6}$$

b. $y = 6(2x - 9)^5$

$$\begin{aligned}\frac{dy}{dx} &= 30(2x - 9)^4(2) \\ &= 60(2x - 9)^4\end{aligned}$$

c. $y = \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{3}} + 6\sqrt[3]{x}$

$$= 2x^{-\frac{1}{2}} + \frac{1}{\sqrt{3}}x + 6x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = -x^{-\frac{3}{2}} + \frac{1}{\sqrt{3}} + 2x^{-\frac{2}{3}}$$

d. $y = \left(\frac{x^2 + 6}{3x + 4}\right)^5$

$$\begin{aligned}\frac{dy}{dx} &= 5\left(\frac{x^2 + 6}{3x + 4}\right)^4 \frac{2x(3x + 4) - (x^2 + 6)3}{(3x + 4)^2} \\ &= \frac{5(x^2 + 6)^4(3x^2 + 8x - 18)}{(3x + 4)^6}\end{aligned}$$

e. $y = x^2 \sqrt[3]{6x^2 - 7}$

$$\begin{aligned}\frac{dy}{dx} &= 2x(6x^2 - 7)^{\frac{1}{3}} + x^2 \frac{1}{3}(6x^2 - 7)^{-\frac{2}{3}}(12x) \\ &= 2x(6x^2 - 7)^{-\frac{2}{3}}((6x^2 - 7) + 2x^2) \\ &= 2x(6x^2 - 7)^{-\frac{2}{3}}(8x^2 - 7)\end{aligned}$$

f. $y = \frac{4x^5 - 5x^4 + 6x - 2}{x^4}$

$$= 4x - 5 + 6x^{-3} - 2x^{-4}$$

$$\frac{dy}{dx} = 4 - 18x^{-4} + 8x^{-5}$$

$$= \frac{4x^5 - 18x + 8}{x^5}$$

5. $y = (x^2 + 3x - 2)(7 - 3x)$

$$\frac{dy}{dx} = (2x + 3)(7 - 3x) + (x^2 + 3x - 2)(-3)$$

At $(1, 8)$,

$$\begin{aligned}\frac{dy}{dx} &= (5)(4) + (2)(-3) \\ &= 14.\end{aligned}$$

The slope of the tangent to $y = (x^2 + 3x - 2)(7 - 3x)$ at $(1, 8)$ is 14.

6. $y = 3u^2 + 2u$

$$\frac{dy}{du} = 6u + 2$$

$$u = \sqrt{x^2 + 5}$$

$$\frac{du}{dy} = \frac{1}{2}(x^2 + 5)^{-\frac{1}{2}}2x$$

$$\frac{dy}{dx} = (6u + 2)\left(\frac{x}{\sqrt{x^2 + 5}}\right)$$

At $x = -2, u = 3$.

$$\begin{aligned}\frac{dy}{dx} &= (20)\left(-\frac{2}{3}\right) \\ &= -\frac{40}{3}\end{aligned}$$

7. $y = (3x^{-2} - 2x^3)^5$

$$\frac{dy}{dx} = 5(3x^{-2} - 2x^3)^4(-6x^{-3} - 6x^2)$$

At $(1, 1)$,

$$\begin{aligned}\frac{dy}{dx} &= 5(1)^4(-6 - 6) \\ &= -60.\end{aligned}$$

Equation of tangent line at $(1, 1)$ is $y - 1 = 60(x - 1)$

$$y - 1 = -60x + 60$$

$$60x + y - 61 = 0.$$

8. $P(t) = (t^{\frac{1}{4}} + 3)^3$

$$P'(t) = 3(t^{\frac{1}{4}} + 3)^2\left(\frac{1}{4}t^{-\frac{3}{4}}\right)$$

$$P'(16) = 3(16^{\frac{1}{4}} + 3)^2\left(\frac{1}{4} \times 16^{-\frac{3}{4}}\right)$$

$$= 3(2 + 3)^2\left(\frac{1}{4} \times \frac{1}{8}\right)$$

$$= \frac{75}{32}$$

The amount of pollution is increasing at a rate of $\frac{75}{32}$ ppm/year.

9. $y = x^4$

$$\frac{dy}{dx} = 4x^3$$

$$-\frac{1}{16} = 4x^3$$

Normal line has a slope of 16. Therefore,

$$\frac{dy}{dx} = -\frac{1}{16}.$$

$$x^3 = -\frac{1}{64}$$

$$x = -\frac{1}{4}$$

$$y = \frac{1}{256}$$

Therefore, $y = x^4$ has a normal line with a slope of 16 at $(-\frac{1}{4}, \frac{1}{256})$.

10. $y = x^3 - x^2 - x + 1$

$$\frac{dy}{dx} = 3x^2 - 2x - 1$$

For a horizontal tangent line, $\frac{dy}{dx} = 0$.

$$3x^2 - 2x - 1 = 0$$

$$(3x + 1)(x - 1) = 0$$

$$x = -\frac{1}{3} \quad \text{or} \quad x = 1$$

$$\begin{aligned} y &= -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 1 & y &= 1 - 1 - 1 + 1 \\ &= \frac{-1 - 3 + 9 + 27}{27} & &= 0 \end{aligned}$$

$$= \frac{32}{27}$$

The required points are $(-\frac{1}{3}, \frac{32}{27}), (1, 0)$.

11. $y = x^2 + ax + b$

$$\frac{dy}{dx} = 2x + a$$

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

Since the parabola and cubic function are tangent at $(1, 1)$, then $2x + a = 3x^2$.

$$\text{At } (1, 1) \quad 2(1) + a = 3(1)^2$$

$$a = 1.$$

Since $(1, 1)$ is on the graph of
 $y = x^2 + x + b$, $1 = 1^2 + 1 + b$
 $b = -1$.

The required values are 1 and -1 for a and b , respectively.

CHAPTER 2

Derivatives

Review of Prerequisite Skills, pp. 62–63

1. a. $a^5 \times a^3 = a^{5+3}$
 $= a^8$

b. $(-2a^2)^3 = (-2)^3(a^2)^3$
 $= -8(a^{2 \times 3})$
 $= -8a^6$

c. $\frac{4p^7 \times 6p^9}{12p^{15}} = \frac{24p^{7+9}}{12p^{15}}$
 $= \frac{2p}{p^{16-15}}$
 $= 2p$

d. $(a^4b^{-5})(a^{-6}b^{-2}) = (a^{4-6})(b^{-5-2})$
 $= a^{-2}b^{-7}$
 $= \frac{1}{a^2b^7}$

e. $(3e^6)(2e^3)^4 = (3)(e^6)(2^4)(e^3)^4$
 $= (3)(2^4)(e^6)(e^{3 \times 4})$
 $= (3)(16)(e^{6+12})$
 $= 48e^{18}$

f. $\frac{(3a^{-4})[2a^3(-b)^3]}{12a^5b^2} = \frac{(3)(2)(-1)^3(a^{-4+3})(b^3)}{12a^5b^2}$
 $= \frac{-6(a^{-1-5})(b^{3-2})}{12}$
 $= \frac{-1(a^{-6})(b)}{2}$
 $= -\frac{b}{2a^6}$

2. a. $(x^{\frac{1}{2}})(x^{\frac{2}{3}}) = x^{\frac{1}{2} + \frac{2}{3}}$
 $= x^{\frac{7}{6}}$

b. $(8x^6)^{\frac{2}{3}} = 8^{\frac{2}{3}}x^{6 \times \frac{2}{3}}$
 $= 4x^4$

c. $\frac{\sqrt{a}\sqrt[3]{a}}{\sqrt{a}} = \frac{(a^{\frac{1}{2}})(a^{\frac{1}{3}})}{a^{\frac{1}{2}}}$
 $= a^{\frac{1}{3}}$

3. A perpendicular line will have a slope that is the negative reciprocal of the slope of the given line:

a. slope = $\frac{-1}{\frac{3}{2}}$
 $= -\frac{2}{3}$

b. slope = $\frac{-1}{-\frac{1}{2}}$
 $= 2$

c. slope = $\frac{-1}{\frac{5}{3}}$
 $= -\frac{3}{5}$

d. slope = $\frac{-1}{-1}$
 $= 1$

4. a. This line has slope $m = \frac{-4 - (-2)}{-3 - 9}$
 $= \frac{-2}{-12}$
 $= \frac{1}{6}$

The equation of the desired line is therefore
 $y + 4 = \frac{1}{6}(x + 3)$ or $x - 6y - 21 = 0$.

b. The equation $3x - 2y = 5$ can be rewritten as
 $2y = 3x - 5$ or $y = \frac{3}{2}x - \frac{5}{2}$, which has slope $\frac{3}{2}$.

The equation of the desired line is therefore
 $y + 5 = \frac{3}{2}(x + 2)$ or $3x - 2y - 4 = 0$.

c. The line perpendicular to $y = \frac{3}{4}x - 6$ will have
slope $m = \frac{-1}{\frac{3}{4}} = -\frac{4}{3}$. The equation of the desired line

is therefore $y + 3 = -\frac{4}{3}(x - 4)$ or $4x + 3y - 7 = 0$.

5. a. $(x - 3y)(2x + y) = 2x^2 + xy - 6xy - 3y^2$
 $= 2x^2 - 5xy - 3y^2$

b. $(x - 2)(x^2 - 3x + 4)$
 $= x^3 - 3x^2 + 4x - 2x^2 + 6x - 8$
 $= x^3 - 5x^2 + 10x - 8$

c. $(6x - 3)(2x + 7) = 12x^2 + 42x - 6x - 21$
 $= 12x^2 + 36x - 21$

d. $2(x + y) - 5(3x - 8y) = 2x + 2y - 15x + 40y$
 $= -13x + 42y$

e. $(2x - 3y)^2 + (5x + y)^2$
 $= 4x^2 - 12xy + 9y^2 + 25x^2 + 10xy + y^2$
 $= 29x^2 - 2xy + 10y^2$

f. $3x(2x - y)^2 - x(5x - y)(5x + y)$
 $= 3x(4x^2 - 4xy + y^2) - x(25x^2 - y^2)$
 $= 12x^3 - 12x^2y + 3xy^2 - 25x^3 + xy^2$
 $= -13x^3 - 12x^2y + 4xy^2$

6. a.
$$\frac{3x(x+2)}{x^2} \times \frac{5x^3}{2x(x+2)} = \frac{15x^4(x+2)}{2x^3(x+2)}$$
$$= \frac{15}{2}x^{4-3}$$
$$= \frac{15}{2}x$$

$x \neq 0, -2$

b.
$$\frac{y}{(y+2)(y-5)} \times \frac{(y-5)^2}{4y^3}$$
$$= \frac{y(y-5)(y-5)}{4y^3(y+2)(y-5)}$$
$$= \frac{y-5}{4y^2(y+2)}$$

$y \neq -2, 0, 5$

c.
$$\frac{4}{h+k} \div \frac{9}{2(h+k)} = \frac{4}{h+k} \times \frac{2(h+k)}{9}$$
$$= \frac{8(h+k)}{9(h+k)}$$
$$= \frac{8}{9}$$

$h \neq -k$

d.
$$\frac{(x+y)(x-y)}{5(x-y)} \div \frac{(x+y)^3}{10}$$
$$= \frac{(x+y)(x-y)}{5(x-y)} \times \frac{10}{(x+y)^3}$$
$$= \frac{10(x+y)(x-y)}{5(x-y)(x+y)^3}$$
$$= \frac{2}{(x+y)^2}$$

$x \neq -y, +y$

e.
$$\frac{x-7}{2x} + \frac{5x}{x-1} = \frac{(x-7)(x-1)}{2x(x-1)} + \frac{(5x)(2x)}{2x(x-1)}$$
$$= \frac{x^2 - 7x - x + 7 + 10x^2}{2x(x-1)}$$
$$= \frac{11x^2 - 8x + 7}{2x(x-1)}$$

$x \neq 0, 1$

f.
$$\frac{x+1}{x-2} - \frac{x+2}{x+3}$$
$$= \frac{(x+1)(x+3)}{(x-2)(x+3)} - \frac{(x+2)(x-2)}{(x+3)(x-2)}$$
$$= \frac{x^2 + x + 3x + 3 - x^2 + 4}{(x+3)(x-2)}$$
$$= \frac{(x+3)(x-2)}{4x+7}$$
$$= \frac{4x+7}{(x+3)(x-2)}$$

$x \neq -3, 2$

7. a. $4k^2 - 9 = (2k+3)(2k-3)$

b. $x^2 + 4x - 32 = x^2 + 8x - 4x - 32$
$$= x(x+8) - 4(x+8)$$
$$= (x-4)(x+8)$$

c. $3a^2 - 4a - 7 = 3a^2 - 7a + 3a - 7$
$$= a(3a-7) + 1(3a-7)$$
$$= (a+1)(3a-7)$$

d. $x^4 - 1 = (x^2 + 1)(x^2 - 1)$
$$= (x^2 + 1)(x + 1)(x - 1)$$

e. $x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

f. $r^4 - 5r^2 + 4 = r^4 - 4r^2 - r^2 + 4$
$$= r^2(r^2 - 4) - 1(r^2 - 4)$$
$$= (r^2 - 1)(r^2 - 4)$$
$$= (r+1)(r-1)(r+2)(r-2)$$

8. a. Letting $f(a) = a^3 - b^3$, $f(b) = b^3 - b^3$
 $= 0$

So b is a root of $f(a)$, and so by the factor theorem, $a - b$ is a factor of $a^3 - b^3$. Polynomial long division provides the other factor:

$$\begin{array}{r} a^2 + ab + b^2 \\ a - b \overline{)a^3 + 0a^2 + 0a - b^3} \\ \underline{a^3 - a^2b} \\ a^2b + 0a - b^3 \\ \underline{a^2b - ab^2} \\ ab^2 - b^3 \\ \underline{ab^2 - b^3} \\ 0 \end{array}$$

So $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$.

b. Using long division or recognizing a pattern from the work in part a.:

$a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$.

c. Using long division or recognizing a pattern from the work in part a.: $a^7 - b^7$

$$(a - b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$$

d. Using the pattern from the previous parts:

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + a^2b^{n-3} + ab^{n-2} + b^{n-1})$$

9. a. $f(2) = -2(2^4) + 3(2^2) + 7 - 2(2)$
$$= -32 + 12 + 7 - 4$$
$$= -17$$

b. $f(-1) = -2(-1)^4 + 3(-1)^2 + 7 - 2(-1)$
$$= -2 + 3 + 7 + 2$$
$$= 10$$

c. $f\left(\frac{1}{2}\right) = -2\left(\frac{1}{2}\right)^4 + 3\left(\frac{1}{2}\right)^2 + 7 - 2\left(\frac{1}{2}\right)$
$$= -\frac{1}{8} + \frac{3}{4} + 7 - 1$$
$$= \frac{53}{8}$$

$$\begin{aligned}
 \text{d. } f(-0.25) &= f\left(-\frac{1}{4}\right) \\
 &= 2\left(-\frac{1}{4}\right)^4 + 3\left(-\frac{1}{4}\right)^2 + 7 - 2\left(-\frac{1}{4}\right) \\
 &= -\frac{1}{128} + \frac{3}{16} + 7 + \frac{1}{2} \\
 &= \frac{983}{128} \\
 &\doteq 7.68
 \end{aligned}$$

$$\begin{aligned}
 \text{10. a. } \frac{3}{\sqrt{2}} &= \frac{3\sqrt{2}}{(\sqrt{2})(\sqrt{2})} \\
 &= \frac{3\sqrt{2}}{2}
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } \frac{4 - \sqrt{2}}{\sqrt{3}} &= \frac{(4 - \sqrt{2})(\sqrt{3})}{(\sqrt{3})(\sqrt{3})} \\
 &= \frac{4\sqrt{3} - \sqrt{6}}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{c. } \frac{2 + 3\sqrt{2}}{3 - 4\sqrt{2}} &= \frac{(2 + 3\sqrt{2})(3 + 4\sqrt{2})}{(3 - 4\sqrt{2})(3 + 4\sqrt{2})} \\
 &= \frac{6 + 9\sqrt{2} + 8\sqrt{2} + 12(2)}{3^2 - (4\sqrt{2})^2} \\
 &= \frac{30 + 17\sqrt{2}}{9 - 16(2)} \\
 &= -\frac{30 + 17\sqrt{2}}{23}
 \end{aligned}$$

$$\begin{aligned}
 \text{d. } \frac{3\sqrt{2} - 4\sqrt{3}}{3\sqrt{2} + 4\sqrt{3}} &= \frac{(3\sqrt{2} - 4\sqrt{3})(3\sqrt{2} - 4\sqrt{3})}{(3\sqrt{2} + 4\sqrt{3})(3\sqrt{2} - 4\sqrt{3})} \\
 &= \frac{(3\sqrt{2})^2 - 24\sqrt{6} + (4\sqrt{3})^2}{(3\sqrt{2})^2 - (4\sqrt{3})^2} \\
 &= \frac{9(2) - 24\sqrt{6} + 16(3)}{9(2) - 16(3)} \\
 &= -\frac{66 - 24\sqrt{6}}{30} \\
 &= -\frac{11 - 4\sqrt{6}}{5}
 \end{aligned}$$

11. a. $f(x) = 3x^2 - 2x$

When $a = 2$,

$$\begin{aligned}
 \frac{f(a+h) - f(a)}{h} &= \frac{f(2+h) - f(2)}{h} \\
 &= \frac{3(2+h)^2 - 2(2+h) - [3(2)^2 - 2(2)]}{h} \\
 &= \frac{3(4+4h+h^2) - 4 - 2h - 8}{h} \\
 &= \frac{12+12h+3h^2-2h-12}{h}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{3h^2 + 10h}{h} \\
 &= 3h + 10
 \end{aligned}$$

This expression can be used to determine the slope of the secant line between $(2, 8)$ and $(2+h, f(2+h))$.

b. For $h = 0.01$: $3(0.01) + 10 = 10.03$

c. The value in part b. represents the slope of the secant line through $(2, 8)$ and $(2.01, 8.1003)$.

2.1 The Derivative Function, pp. 73–75

1. A function is not differentiable at a point where its graph has a cusp, a discontinuity, or a vertical tangent:

a. The graph has a cusp at $x = -2$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq -2\}$.

b. The graph is discontinuous at $x = 2$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq 2\}$.

c. The graph has no cusps, discontinuities, or vertical tangents, so f is differentiable on $\{x \in \mathbf{R}\}$.

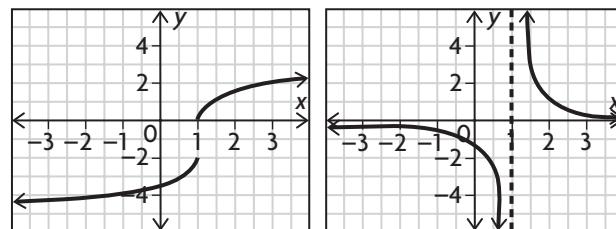
d. The graph has a cusp at $x = 1$, so f is differentiable on $\{x \in \mathbf{R} \mid x \neq 1\}$.

e. The graph has no cusps, discontinuities, or vertical tangents, so f is differentiable on $\{x \in \mathbf{R}\}$.

f. The function does not exist for $x < 2$, but has no cusps, discontinuities, or vertical tangents elsewhere, so f is differentiable on $\{x \in \mathbf{R} \mid x > 2\}$.

2. The derivative of a function represents the slope of the tangent line at a given value of the independent variable or the instantaneous rate of change of the function at a given value of the independent variable.

3.



$$\begin{aligned}
 \text{4. a. } f(x) &= 5x - 2 \\
 f(a+h) &= 5(a+h) - 2 \\
 &= 5a + 5h - 2
 \end{aligned}$$

$$\begin{aligned}
 f(a+h) - f(a) &= 5a + 5h - 2 - (5a - 2) \\
 &= 5h
 \end{aligned}$$

$$\begin{aligned}
 \text{b. } f(x) &= x^2 + 3x - 1 \\
 f(a+h) &= (a+h)^2 + 3(a+h) - 1 \\
 &= a^2 + 2ah + h^2 + 3a + 3h - 1
 \end{aligned}$$

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When $a = 2$,

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$$\begin{aligned}
&= \frac{3h^2 + 10h}{h} \\
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This expression can be used to determine the slope of the secant line between $(2, 8)$ and $(2+h, f(2+h))$.

b. For $h = 0.01$: $3(0.01) + 10 = 10.03$

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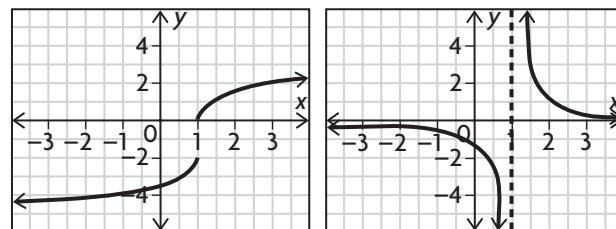
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3.



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\text{4. a. } f(x) &= 5x - 2 \\
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&= 5a + 5h - 2
\end{aligned}$$

$$\begin{aligned}
f(a+h) - f(a) &= 5a + 5h - 2 - (5a - 2) \\
&= 5h
\end{aligned}$$

$$\begin{aligned}
\text{b. } f(x) &= x^2 + 3x - 1 \\
f(a+h) &= (a+h)^2 + 3(a+h) - 1 \\
&= a^2 + 2ah + h^2 + 3a + 3h - 1
\end{aligned}$$

$$\begin{aligned}
f(a+h) - f(a) &= a^2 + 2ah + h^2 + 3a + 3h - 1 - (a^2 + 3a - 1) \\
&= 2ah + h^2 + 3h
\end{aligned}$$

c.

$$\begin{aligned} f(x) &= x^3 - 4x + 1 \\ f(a+h) &= (a+h)^3 - 4(a+h) + 1 \\ &= a^3 + 3a^2h + 3ah^2 + h^3 \\ &\quad - 4a - 4h + 1 \\ f(a+h) - f(a) &= a^3 + 3a^2h + 3ah^2 + h^3 - 4a \\ &\quad - 4h + 1 - (a^3 - 4a + 1) \\ &= 3a^2h + 3ah^2 + h^3 - 4h \end{aligned}$$

d.

$$\begin{aligned} f(x) &= x^2 + x - 6 \\ f(a+h) &= (a+h)^2 + (a+h) - 6 \\ &= a^2 + 2ah + h^2 + a + h - 6 \\ f(a+h) - f(a) &= a^2 + 2ah + h^2 + a + h - 6 \\ &\quad - (a^2 + a - 6) \\ &= 2ah + h^2 + h \end{aligned}$$

e.

$$\begin{aligned} f(x) &= -7x + 4 \\ f(a+h) &= -7(a+h) + 4 \\ &= -7a - 7h + 4 \\ f(a+h) - f(a) &= -7a - 7h + 4 - (-7a + 4) \\ &= -7h \end{aligned}$$

f.

$$\begin{aligned} f(x) &= 4 - 2x - x^2 \\ f(a+h) &= 4 - 2(a+h) - (a+h)^2 \\ &= 4 - 2a - 2h - a^2 - 2ah - h^2 \\ f(a+h) - f(a) &= 4 - 2a - 2h - a^2 - 2ah \\ &\quad - h^2 - 4 + 2a + a^2 \\ &= -2h - h^2 - 2ah \end{aligned}$$

5. a.

$$\begin{aligned} f'(1) &= \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(1+h)^2 - 1^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{1 + 2h + h^2 - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h + h^2}{h} \\ &= \lim_{h \rightarrow 0} (2 + h) \\ &= 2 \end{aligned}$$

b.

$$\begin{aligned} f'(3) &= \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{(3+h)^2 + 3(3+h) + 1}{h} \right. \\ &\quad \left. - \frac{(3^2 + 3(3) + 1)}{h} \right] \\ &= \lim_{h \rightarrow 0} \frac{9 + 6h + h^2 + 9 + 3h + 1 - 19}{h} \\ &= \lim_{h \rightarrow 0} \frac{9h + h^2}{h} \\ &= \lim_{h \rightarrow 0} (9 + h) \\ &= 9 \end{aligned}$$

c.

$$\begin{aligned} f'(0) &= \lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{h+1} - \sqrt{0+1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{h+1} - 1}{h} \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{h+1} - 1)(\sqrt{h+1} + 1)}{h(\sqrt{h+1} + 1)} \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{h+1})^2 - 1}{h(\sqrt{h+1} + 1)} \\ &= \lim_{h \rightarrow 0} \frac{h+1 - 1}{h(\sqrt{h+1} + 1)} \\ &= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{h+1} + 1)} \\ &= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{1} + 1)} \\ &= \frac{1}{2} \end{aligned}$$

d.

$$\begin{aligned} f'(-1) &= \lim_{h \rightarrow 0} \frac{f(-1+h) - f(-1)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{5}{-1+h} - \frac{5}{-1}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{5}{-1+h} + 5}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{5}{-1+h} + \frac{5(-1+h)}{-1+h}}{h} \\ &= \lim_{h \rightarrow 0} \frac{5 - 5 + 5h}{h(-1+h)} \\ &= \lim_{h \rightarrow 0} \frac{5h}{h(-1+h)} \\ &= \lim_{h \rightarrow 0} \frac{5}{(-1+h)} \\ &= \frac{5}{-1} \\ &= -5 \end{aligned}$$

6. a.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-5(x+h) - 8 - (-5x - 8)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-5x - 5h - 8 + 5x + 8}{h} \end{aligned}$$

$$= \lim_{h \rightarrow 0} \frac{-5h}{h}$$

$$= \lim_{h \rightarrow 0} -5$$

$$= -5$$

b. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \left[\frac{2(x+h)^2 + 4(x+h)}{h} \right. \\ \left. - \frac{(2x^2 + 4x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{2x^2 + 4xh + 2h^2 + 4x}{h} \right. \\ \left. + \frac{4h - 2x^2 - 4x}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 + 4h}{h}$$

$$= \lim_{h \rightarrow 0} (4x + 2h + 4)$$

$$= 4x + 4$$

c. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \left[\frac{6(x+h)^3 - 7(x+h)}{h} \right. \\ \left. - \frac{(6x^3 - 7x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{6x^3 + 18x^2h + 18xh^2 + 6h^3}{h} \right. \\ \left. + \frac{-7x - 7h - 6x^3 + 7x}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{18x^2h + 18xh^2 + 6h^3 - 7h}{h}$$

$$= \lim_{h \rightarrow 0} (18x^2 + 18xh + 6h^2 - 7)$$

$$= 18x^2 - 7$$

d. $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3(x+h)+2} - \sqrt{3x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{3x+3h+2} - \sqrt{3x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{(\sqrt{3x+3h+2} - \sqrt{3x+2})}{h} \right. \\ \left. \times \frac{(\sqrt{3x+3h+2} + \sqrt{3x+2})}{(\sqrt{3x+3h+2} + \sqrt{3x+2})} \right]$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{3x+3h+2})^2 - (\sqrt{3x+2})^2}{h(\sqrt{3x+3h+2} + \sqrt{3x+2})}$$

$$= \lim_{h \rightarrow 0} \frac{3x+3h+2 - 3x-2}{h(\sqrt{3x+3h+2} + \sqrt{3x+2})}$$

$$= \lim_{h \rightarrow 0} \frac{3}{\sqrt{3x+3h+2} + \sqrt{3x+2}}$$

$$= \frac{3}{2\sqrt{3x+2}}$$

7. a. Let $y = f(x)$, then

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6 - 7(x+h) - (6 - 7x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{6 - 7x - 7h - 6 + 7x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-7h}{h}$$

$$= \lim_{h \rightarrow 0} -7$$

$$= -7$$

b. Let $y = f(x)$, then

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{(x+h+1)(x-1)}{(x+h-1)(x-1)}}{h} \right. \\ \left. - \frac{\frac{(x+1)(x+h-1)}{(x-1)(x+h-1)}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{x^2+hx+x-x-h-1}{(x+h-1)(x-1)}}{h} \right. \\ \left. - \frac{\frac{x^2+hx-x+x+h-1}{(x+h-1)(x-1)}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \frac{\frac{-2h}{(x+h-1)(x-1)}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2}{(x+h-1)(x-1)}$$

$$= -\frac{2}{(x-1)^2}$$

c. Let $y = f(x)$, then

$$\begin{aligned}\frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{3(x+h)^2 - 3x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{6xh + 3h^2}{h} \\ &= \lim_{h \rightarrow 0} 6x + 3h \\ &= 6x\end{aligned}$$

8. Let $y = f(x)$, then the slope of the tangent at each point x can be found by calculating $f'(x)$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 4(x+h) - 2x^2 + 4x}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{2x^2 + 4xh + 2h^2 - 4x - 4h}{h} + \frac{-2x^2 + 4x}{h} \right] \\ &= \lim_{h \rightarrow 0} \frac{4xh + h^2 - 4h}{h} \\ &= \lim_{h \rightarrow 0} 4x + h - 4 \\ &= 4x - 4\end{aligned}$$

So the slope of the tangent at $x = 0$ is

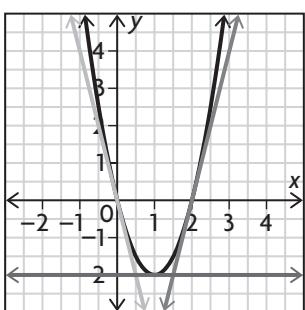
$$\begin{aligned}f'(0) &= 4(0) - 4 \\ &= -4\end{aligned}$$

At $x = 1$, the slope of the tangent is

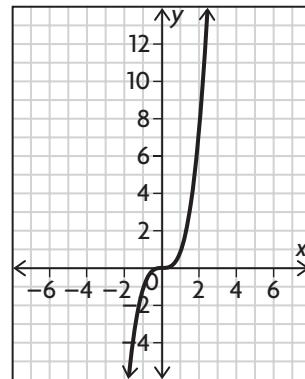
$$\begin{aligned}f'(1) &= 4(1) - 4 \\ &= 0\end{aligned}$$

At $x = 2$, the slope of the tangent is

$$\begin{aligned}f'(2) &= 4(2) - 4 \\ &= 4\end{aligned}$$



9. a.



b. Let $y = f(x)$, then the slope of the tangent at each point x can be found by calculating $f(x)$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\ &= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ &= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\ &= 3x^2\end{aligned}$$

So the slope of the tangent at $x = -2$ is

$$\begin{aligned}f'(-2) &= 3(-2)^2 \\ &= 12\end{aligned}$$

At $x = -1$, the slope of the tangent is

$$\begin{aligned}f'(-1) &= 3(-1)^2 \\ &= 3\end{aligned}$$

At $x = 0$, the slope of the tangent is

$$\begin{aligned}f'(0) &= 3(0)^2 \\ &= 0\end{aligned}$$

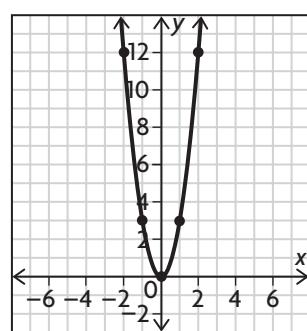
At $x = 1$, the slope of the tangent is

$$\begin{aligned}f'(1) &= 3(1)^2 \\ &= 3\end{aligned}$$

At $x = 2$, the slope of the tangent is

$$\begin{aligned}f'(2) &= 3(2)^2 \\ &= 12\end{aligned}$$

c.



d. The graph of $f(x)$ is a cubic. The graph of $f'(x)$ seems to be a parabola.

10. The velocity the particle at time t is given by $s'(t)$

$$\begin{aligned} s'(t) &= \lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-(t+h)^2 + 8(t+h) - (-t^2 + 8t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{-t^2 - 2th - h^2 + 8t + 8h + t^2 - 8t}{h} \\ &= \lim_{h \rightarrow 0} \frac{-2th - h^2 + 8h}{h} \\ &= \lim_{h \rightarrow 0} -2t - h + 8 \\ &= -2t + 8 \end{aligned}$$

So the velocity at $t = 0$ is

$$\begin{aligned} s'(0) &= -2(0) + 8 \\ &= 8 \text{ m/s} \end{aligned}$$

At $t = 4$, the velocity is

$$\begin{aligned} s'(4) &= -2(4) + 8 \\ &= 0 \text{ m/s} \end{aligned}$$

At $t = 6$, the velocity is

$$\begin{aligned} s'(6) &= -2(6) + 8 \\ &= -4 \text{ m/s} \end{aligned}$$

$$\begin{aligned} 11. f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{(\sqrt{x+h+1} - \sqrt{x+1})}{h} \right. \\ &\quad \times \left. \frac{(\sqrt{x+h+1} + \sqrt{x+1})}{(\sqrt{x+h+1} + \sqrt{x+1})} \right] \\ &= \lim_{h \rightarrow 0} \frac{(\sqrt{x+h+1})^2 - (\sqrt{x+1})^2}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \lim_{h \rightarrow 0} \frac{x+h+1 - x-1}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \lim_{h \rightarrow 0} \frac{1}{(\sqrt{x+h+1} + \sqrt{x+1})} \\ &= \frac{1}{2\sqrt{x+1}} \end{aligned}$$

The equation $x - 6y + 4 = 0$ can be rewritten as $y = \frac{1}{6}x + \frac{2}{3}$, so this line has slope $\frac{1}{6}$. The value of x where the tangent to $f(x)$ has slope $\frac{1}{6}$ will satisfy $f'(x) = \frac{1}{6}$.

$$\begin{aligned} \frac{1}{2\sqrt{x+1}} &= \frac{1}{6} \\ 6 &= 2\sqrt{x+1} \\ 3^2 &= (\sqrt{x+1})^2 \\ 9 &= x+1 \\ 8 &= x \\ f(8) &= \sqrt{8+1} \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

So the tangent passes through the point $(8, 3)$, and its equation is $y - 3 = \frac{1}{6}(x - 8)$ or $x - 6y + 10 = 0$.

12. a. Let $y = f(x)$, then

$$\begin{aligned} \frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{c - c}{h} \\ &= \lim_{h \rightarrow 0} \frac{0}{h} \\ &= 0 \end{aligned}$$

b. Let $y = f(x)$, then

$$\begin{aligned} \frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{(x+h) - x}{h} \\ &= \lim_{h \rightarrow 0} \frac{h}{h} \\ &= \lim_{h \rightarrow 0} 1 \\ &= 1 \end{aligned}$$

c. Let $y = f(x)$, then

$$\begin{aligned} \frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{m(x+h) + b - mx - b}{h} \\ &= \lim_{h \rightarrow 0} \frac{mx + mh + b - mx - b}{h} \\ &= \lim_{h \rightarrow 0} \frac{mh}{h} \\ &= \lim_{h \rightarrow 0} m \\ &= m \end{aligned}$$

d. Let $y = f(x)$, then

$$\begin{aligned} \frac{dy}{dx} = f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{a(x+h)^2 + b(x+h) + c}{h} \right. \\ &\quad \left. - \frac{(ax^2 + bx + c)}{h} \right] \end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \left[\frac{ax^2 + 2axh + ah^2 + bx + bh}{h} \right. \\
&\quad \left. + \frac{-ax^2 - bx - c}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{2axh + ah^2 + bh}{h} \\
&= \lim_{h \rightarrow 0} (2ax + ah + b) \\
&= 2ax + b
\end{aligned}$$

13. The slope of the function at a point x is given by

$$\begin{aligned}
f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\
&= \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} \\
&= \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\
&= \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 \\
&= 3x^2
\end{aligned}$$

Since $3x^2$ is nonnegative for all x , the original function never has a negative slope.

14. $h(t) = 18t - 4.9t^2$

$$\begin{aligned}
\text{a. } h'(t) &= \lim_{k \rightarrow 0} \frac{h(t+k) - h(t)}{k} \\
&= \lim_{k \rightarrow 0} \frac{18(t+k) - 4.9(t+k)^2}{k} \\
&\quad - \frac{(18t - 4.9t^2)}{k} \\
&= \lim_{k \rightarrow 0} \frac{18t + 18k - 4.9t^2 - 9.8tk - 4.9k^2}{k} \\
&\quad - \frac{18t + 4.9t^2}{k} \\
&= \lim_{k \rightarrow 0} \frac{18k - 9.8tk - 4.9k^2}{k} \\
&= \lim_{k \rightarrow 0} (18 - 9.8t - 4.9k) \\
&= 18 - 9.8t - 4.9(0) \\
&= 18 - 9.8t
\end{aligned}$$

Then $h'(2) = 18 - 9.8(2) = -1.6$ m/s.

b. $h'(2)$ measures the rate of change in the height of the ball with respect to time when $t = 2$.

15. a. This graph has positive slope for $x < 0$, zero slope at $x = 0$, and negative slope for $x > 0$, which corresponds to graph e.

b. This graph has positive slope for $x < 0$, zero slope at $x = 0$, and positive slope for $x > 0$, which corresponds to graph f.

c. This graph has negative slope for $x < -2$, positive slope for $-2 < x < 0$, negative slope for $0 < x < 2$, positive slope for $x > 2$, and zero slope at $x = -2$, $x = 0$, and $x = 2$, which corresponds to graph d.

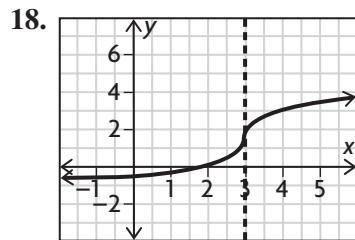
16. This function is defined piecewise as $f(x) = -x^2$ for $x < 0$, and $f(x) = x^2$ for $x \geq 0$. The derivative will exist if the left-side and right-side derivatives are the same at $x = 0$:

$$\begin{aligned}
\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^-} \frac{-(0+h)^2 - (-0^2)}{h} \\
&= \lim_{h \rightarrow 0^-} \frac{-h^2}{h} \\
&= \lim_{h \rightarrow 0^-} (-h) \\
&= 0 \\
\lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h} &= \lim_{h \rightarrow 0^+} \frac{(0+h)^2 - (0^2)}{h} \\
&= \lim_{h \rightarrow 0^+} \frac{h^2}{h} \\
&= \lim_{h \rightarrow 0^+} (h) \\
&= 0
\end{aligned}$$

Since the limits are equal for both sides, the derivative exists and $f'(0) = 0$.

17. Since $f'(a) = 6$ and $f(a) = 0$,

$$\begin{aligned}
6 &= \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} \\
6 &= \lim_{h \rightarrow 0} \frac{f(a+h) - 0}{h} \\
3 &= \lim_{h \rightarrow 0} \frac{f(a+h)}{2h}
\end{aligned}$$



$f(x)$ is continuous.

$$f(3) = 2$$

But $f'(3) = \infty$.

(Vertical tangent)

19. $y = x^2 - 4x - 5$ has a tangent parallel to $2x - y = 1$.

Let $f(x) = x^2 - 4x - 5$. First, calculate

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \left[\frac{(x+h)^2 - 4(x+h) - 5}{h} \right. \\
&\quad \left. - \frac{(x^2 - 4x - 5)}{h} \right] \\
&= \lim_{h \rightarrow 0} \left[\frac{x^2 + 2xh + h^2 - 4x - 4h - 5}{h} \right. \\
&\quad \left. + \frac{-x^2 + 4x + 5}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 4h}{h} \\
&= \lim_{h \rightarrow 0} (2x + h - 4) \\
&= 2x + 0 - 4 \\
&= 2x - 4
\end{aligned}$$

Thus, $2x - 4$ is the slope of the tangent to the curve at x . We want the tangent parallel to $2x - y = 1$.

Rearranging, $y = 2x - 1$.

If the tangent is parallel to this line,

$$2x - 4 = 2$$

$$x = 3$$

When $x = 3$, $y = (3)^2 - 4(3) - 5 = -8$.

The point is $(3, -8)$.

$$\mathbf{20. } f(x) = x^2$$

The slope of the tangent at any point (x, x^2) is

$$\begin{aligned}
f' &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\
&= \lim_{h \rightarrow 0} \frac{(x+h+x)(x+h-x)}{h} \\
&= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h} \\
&= \lim_{h \rightarrow 0} (2x+h) \\
&= 2x + 0 \\
&= 2x
\end{aligned}$$

Let (a, a^2) be a point of tangency. The equation of the tangent is

$$y - a^2 = (2a)(x - a)$$

$$y = (2a)x - a^2$$

Suppose the tangent passes through $(1, -3)$.

Substitute $x = 1$ and $y = -3$ into the equation of the tangent:

$$\begin{aligned}
-3 &= (2a)(1) - a^2 \\
a^2 - 2a - 3 &= 0 \\
(a-3)(a+1) &= 0 \\
a &= -1, 3
\end{aligned}$$

So the two tangents are $y = -2x - 1$ or

$$2x + y + 1 = 0 \text{ and } y = 6x - 9 \text{ or } 6x - y - 9 = 0.$$

2.2 The Derivatives of Polynomial Functions, pp. 82–84

1. Answers may vary. For example:

constant function rule: $\frac{d}{dx}(5) = 0$

power rule: $\frac{d}{dx}(x^3) = 3x^2$

constant multiple rule: $\frac{d}{dx}(4x^3) = 12x^2$

sum rule: $\frac{d}{dx}(x^2 + x) = 2x + 1$

difference rule: $\frac{d}{dx}(x^3 - x^2 + 3x) = 3x^2 - 2x + 3$

$$\begin{aligned}
\mathbf{2. a. } f'(x) &= \frac{d}{dx}(4x) - \frac{d}{dx}(7) \\
&= 4 \frac{d}{dx}(x) - \frac{d}{dx}(7) \\
&= 4(x^0) - 0 \\
&= 4
\end{aligned}$$

$$\begin{aligned}
\mathbf{b. } f'(x) &= \frac{d}{dx}(x^3) - \frac{d}{dx}(x^2) \\
&= 3x^2 - 2x
\end{aligned}$$

$$\begin{aligned}
\mathbf{c. } f'(x) &= \frac{d}{dx}(-x^2) + \frac{d}{dx}(5x) + \frac{d}{dx}(8) \\
&= -\frac{d}{dx}(x^2) + 5 \frac{d}{dx}(x) + \frac{d}{dx}(8) \\
&= -(2x) + 5 + 0 \\
&= -2x + 5
\end{aligned}$$

$$\begin{aligned}
\mathbf{d. } f'(x) &= \frac{d}{dx}(\sqrt[3]{x}) \\
&= \frac{d}{dx}(x^{\frac{1}{3}}) \\
&= \frac{1}{3}(x^{(\frac{1}{3}-1)}) \\
&= \frac{1}{3}(x^{-\frac{2}{3}}) \\
&= \frac{1}{3} \cdot \frac{1}{x^{\frac{2}{3}}} \\
&= \frac{1}{3\sqrt[3]{x^2}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{e. } f'(x) &= \frac{d}{dx}\left(\left(\frac{x}{2}\right)^4\right) \\
&= \left(\frac{1}{2}\right)^4 \frac{d}{dx}(x^4) \\
&= \frac{1}{16}(4x^3) \\
&= \frac{x^3}{4}
\end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \left[\frac{(x+h)^2 - 4(x+h) - 5}{h} \right. \\
&\quad \left. - \frac{(x^2 - 4x - 5)}{h} \right] \\
&= \lim_{h \rightarrow 0} \left[\frac{x^2 + 2xh + h^2 - 4x - 4h - 5}{h} \right. \\
&\quad \left. + \frac{-x^2 + 4x + 5}{h} \right] \\
&= \lim_{h \rightarrow 0} \frac{2xh + h^2 - 4h}{h} \\
&= \lim_{h \rightarrow 0} (2x + h - 4) \\
&= 2x + 0 - 4 \\
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\end{aligned}$$

Thus, $2x - 4$ is the slope of the tangent to the curve at x . We want the tangent parallel to $2x - y = 1$.

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&= 2x + 0 \\
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\end{aligned}$$

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\end{aligned}$$

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constant multiple rule: $\frac{d}{dx}(4x^3) = 12x^2$

sum rule: $\frac{d}{dx}(x^2 + x) = 2x + 1$

difference rule: $\frac{d}{dx}(x^3 - x^2 + 3x) = 3x^2 - 2x + 3$

$$\begin{aligned}
\mathbf{2. a. } f'(x) &= \frac{d}{dx}(4x) - \frac{d}{dx}(7) \\
&= 4 \frac{d}{dx}(x) - \frac{d}{dx}(7) \\
&= 4(x^0) - 0 \\
&= 4
\end{aligned}$$

$$\begin{aligned}
\mathbf{b. } f'(x) &= \frac{d}{dx}(x^3) - \frac{d}{dx}(x^2) \\
&= 3x^2 - 2x
\end{aligned}$$

$$\begin{aligned}
\mathbf{c. } f'(x) &= \frac{d}{dx}(-x^2) + \frac{d}{dx}(5x) + \frac{d}{dx}(8) \\
&= -\frac{d}{dx}(x^2) + 5 \frac{d}{dx}(x) + \frac{d}{dx}(8) \\
&= -(2x) + 5 + 0 \\
&= -2x + 5
\end{aligned}$$

$$\begin{aligned}
\mathbf{d. } f'(x) &= \frac{d}{dx}(\sqrt[3]{x}) \\
&= \frac{d}{dx}(x^{\frac{1}{3}}) \\
&= \frac{1}{3}(x^{(\frac{1}{3}-1)}) \\
&= \frac{1}{3}(x^{-\frac{2}{3}}) \\
&= \frac{1}{3} \frac{1}{x^{\frac{2}{3}}} \\
&= \frac{1}{3\sqrt[3]{x^2}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{e. } f'(x) &= \frac{d}{dx}\left(\left(\frac{x}{2}\right)^4\right) \\
&= \left(\frac{1}{2}\right)^4 \frac{d}{dx}(x^4) \\
&= \frac{1}{16}(4x^3) \\
&= \frac{x^3}{4}
\end{aligned}$$

$$\begin{aligned}\mathbf{f. } f'(x) &= \frac{d}{dx}(x^{-3}) \\ &= (-3)(x^{-3-1}) \\ &= -3x^{-4}\end{aligned}$$

$$\begin{aligned}\mathbf{3. a. } h'(x) &= \frac{d}{dx}((2x+3)(x+4)) \\ &= \frac{d}{dx}(2x^2 + 8x + 3x + 12) \\ &= \frac{d}{dx}(2x^2) + \frac{d}{dx}(11x) + \frac{d}{dx}(12) \\ &= 2\frac{d}{dx}(x^2) + 11\frac{d}{dx}(x) + \frac{d}{dx}(12) \\ &= 2(2x) + 11(1) + 0 \\ &= 4x + 11\end{aligned}$$

$$\begin{aligned}\mathbf{b. } f'(x) &= \frac{d}{dx}(2x^3 + 5x^2 - 4x - 3.75) \\ &= \frac{d}{dx}(2x^3) + \frac{d}{dx}(5x^2) - \frac{d}{dx}(4x) \\ &\quad - \frac{d}{dx}(3.75) \\ &= 2\frac{d}{dx}(x^3) + 5\frac{d}{dx}(x^2) - 4\frac{d}{dx}(x) \\ &\quad - \frac{d}{dx}(3.75) \\ &= 2(3x^2) + 5(2x) - 4(1) - 0 \\ &= 6x^2 + 10x - 4\end{aligned}$$

$$\begin{aligned}\mathbf{c. } \frac{ds}{dt} &= \frac{d}{dt}(t^2(t^2 - 2t)) \\ &= \frac{d}{dt}(t^4 - 2t^3) \\ &= \frac{d}{dt}(t^4) - \frac{d}{dt}(2t^3) \\ &= \frac{d}{dt}(t^4) - 2\frac{d}{dt}(t^3) \\ &= 4t^3 - 2(3t^2) \\ &= 4t^3 - 6t^2\end{aligned}$$

$$\begin{aligned}\mathbf{d. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{5}x^5 + \frac{1}{3}x^3 - \frac{1}{2}x^2 + 1\right) \\ &= \frac{d}{dx}\left(\frac{1}{5}x^5\right) + \frac{d}{dx}\left(\frac{1}{3}x^3\right) - \frac{d}{dx}\left(\frac{1}{2}x^2\right) + \frac{d}{dx}(1) \\ &= \left(\frac{1}{5}\right)\frac{d}{dx}(x^5) + \left(\frac{1}{3}\right)\frac{d}{dx}(x^3) - \left(\frac{1}{2}\right)\frac{d}{dx}(x^2) \\ &\quad + \frac{d}{dx}(1) \\ &= \frac{1}{5}(5x^4) + \frac{1}{3}(3x^2) - \frac{1}{2}(2x) + 0 \\ &= x^4 + x^2 - x\end{aligned}$$

$$\begin{aligned}\mathbf{e. } g'(x) &= \frac{d}{dx}(5(x^2)^4) \\ &= 5\frac{d}{dx}(x^{2 \times 4})\end{aligned}$$

$$\begin{aligned}&= 5\frac{d}{dx}(x^8) \\ &= 5(8x^7) \\ &= 40x^7\end{aligned}$$

$$\begin{aligned}\mathbf{f. } s'(t) &= \frac{d}{dt}\left(\frac{t^5 - 3t^2}{2t}\right) \\ &= \left(\frac{1}{2}\right)\frac{d}{dt}(t^4 - 3t) \\ &= \left(\frac{1}{2}\right)\left(\frac{d}{dt}(t^4) - \frac{d}{dt}(3t)\right) \\ &= \left(\frac{1}{2}\right)\left(\frac{d}{dt}(t^4) - 3\frac{d}{dt}(t)\right) \\ &= \left(\frac{1}{2}\right)(4t^3 - 3(1)) \\ &= 2t^3 - \frac{3}{2}\end{aligned}$$

$$\begin{aligned}\mathbf{4. a. } \frac{dy}{dx} &= \frac{d}{dx}(3x^{\frac{5}{3}}) \\ &= 3\frac{d}{dx}(x^{\frac{5}{3}}) \\ &= \left(\frac{5}{3}\right)3(x^{\frac{5}{3}-1}) \\ &= 5x^{\frac{2}{3}}\end{aligned}$$

$$\begin{aligned}\mathbf{b. } \frac{dy}{dx} &= \frac{d}{dx}\left(4x^{-\frac{1}{2}} - \frac{6}{x}\right) \\ &= 4\frac{d}{dx}(x^{-\frac{1}{2}}) - 6\frac{d}{dx}(x^{-1}) \\ &= 4\left(\frac{-1}{2}\right)(x^{-\frac{1}{2}-1}) - 6(-1)(x^{-1-1}) \\ &= -2x^{-\frac{3}{2}} + 6x^{-2}\end{aligned}$$

$$\begin{aligned}\mathbf{c. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{6}{x^3} + \frac{2}{x^2} - 3\right) \\ &= 6\frac{d}{dx}(x^{-3}) + 2\frac{d}{dx}(x^{-2}) - \frac{d}{dx}(3) \\ &= 6(-3)(x^{-3-1}) + 2(-2)(x^{-2-1}) - 0 \\ &= -18x^{-4} - 4x^{-3} \\ &= \frac{-18}{x^4} - \frac{4}{x^3}\end{aligned}$$

$$\begin{aligned}
\mathbf{d.} \quad & \frac{dy}{dx} = \frac{d}{dx}(9x^{-2} + 3\sqrt{x}) \\
& = 9\frac{d}{dx}(x^{-2}) + 3\frac{d}{dx}(x^{\frac{1}{2}}) \\
& = 9(-2)(x^{-2-1}) + 3\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) \\
& = -18x^{-3} + \frac{3}{2}x^{-\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{e.} \quad & \frac{dy}{dx} = \frac{d}{dx}(\sqrt{x} + 6\sqrt{x^3} + \sqrt{2}) \\
& = \frac{d}{dx}(x^{\frac{1}{2}}) + 6\frac{d}{dx}(x^{\frac{3}{2}}) + \frac{d}{dx}(\sqrt{2}) \\
& = \frac{1}{2}(x^{\frac{1}{2}-1}) + 6\left(\frac{3}{2}\right)(x^{\frac{3}{2}-1}) + 0 \\
& = \frac{1}{2}(x^{-\frac{1}{2}}) + 9x^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{f.} \quad & \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1+\sqrt{x}}{x}\right) \\
& = \frac{d}{dx}\left(\frac{1}{x}\right) + \frac{d}{dx}\left(\frac{x^{\frac{1}{2}}}{x}\right) \\
& = \frac{d}{dx}(x^{-1}) + \frac{d}{dx}(x^{-\frac{1}{2}}) \\
& = (-1)x^{-1-1} + \frac{-1}{2}(x^{-\frac{1}{2}-1}) \\
& = -x^{-2} - \frac{1}{2}x^{-\frac{3}{2}}
\end{aligned}$$

$$\begin{aligned}
\mathbf{5. a.} \quad & \frac{ds}{dt} = \frac{d}{dt}(-2t^2 + 7t) \\
& = (-2)\left(\frac{d}{dt}(t^2)\right) + 7\left(\frac{d}{dt}(t)\right) \\
& = (-2)(2t) + 7(1) \\
& = -4t + 7
\end{aligned}$$

$$\begin{aligned}
\mathbf{b.} \quad & \frac{ds}{dt} = \frac{d}{dt}\left(18 + 5t - \frac{1}{3}t^3\right) \\
& = \frac{d}{dt}(18) + 5\frac{d}{dt}(t) - \left(\frac{1}{3}\right)\frac{d}{dt}(t^3) \\
& = 0 + 5(1) - \left(\frac{1}{3}\right)(3t^2) \\
& = 5 - t^2
\end{aligned}$$

$$\begin{aligned}
\mathbf{c.} \quad & \frac{ds}{dt} = \frac{d}{dt}((t-3)^2) \\
& = \frac{d}{dt}(t^2 - 6t + 9) \\
& = \frac{d}{dt}(t^2) - (6)\frac{d}{dt}(t) + \frac{d}{dt}(9)
\end{aligned}$$

$$\begin{aligned}
& = 2t - 6(1) + 0 \\
& = 2t - 6
\end{aligned}$$

$$\begin{aligned}
\mathbf{6. a.} \quad & f'(x) = \frac{d}{dx}(x^3 - \sqrt{x}) \\
& = \frac{d}{dx}(x^3) - \frac{d}{dx}(x^{\frac{1}{2}}) \\
& = 3x^2 - \frac{1}{2}(x^{\frac{1}{2}-1}) \\
& = 3x^2 - \frac{1}{2}x^{-\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\text{so } f'(a) &= f'(4) = 3(4)^2 - \frac{1}{2}(4)^{-\frac{1}{2}} \\
&= 3(16) - \frac{1}{2}\frac{1}{\sqrt{4}} \\
&= 48 - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \\
&= 47.75
\end{aligned}$$

$$\begin{aligned}
\mathbf{b.} \quad & f'(x) = \frac{d}{dx}(7 - 6\sqrt{x} + 5x^{\frac{2}{3}}) \\
& = \frac{d}{dx}(7) - 6\frac{d}{dx}(x^{\frac{1}{2}}) + 5\frac{d}{dx}(x^{\frac{2}{3}}) \\
& = 0 - 6\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) + 5\left(\frac{2}{3}\right)(x^{\frac{2}{3}-1}) \\
& = -3x^{-\frac{1}{2}} + \left(\frac{10}{3}\right)(x^{-\frac{1}{3}})
\end{aligned}$$

$$\begin{aligned}
\text{so } f'(a) &= f'(64) = -3(64^{-\frac{1}{2}}) + \left(\frac{10}{3}\right)(64^{-\frac{1}{3}}) \\
&= -3\left(\frac{1}{8}\right) + \frac{10}{3}\left(\frac{1}{4}\right) \\
&= \frac{11}{24}
\end{aligned}$$

$$\begin{aligned}
\mathbf{7. a.} \quad & \frac{dy}{dx} = \frac{d}{dx}(3x^4) \\
& = 3\frac{d}{dx}(x^4) \\
& = 3(4x^3) \\
& = 12x^3
\end{aligned}$$

The slope at $(1, 3)$ is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$. So the slope = $12(1)^3 = 12$

$$\begin{aligned}
\mathbf{b.} \quad & \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{x^{-5}}\right) \\
& = \frac{d}{dx}(x^5) \\
& = 5x^4
\end{aligned}$$

The slope at $(-1, -1)$ is found by substituting $x = -1$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= 5(-1)^4 \\ &= 5\end{aligned}$$

$$\begin{aligned}\text{c. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{2}{x}\right) \\ &= 2\frac{d}{dx}(x^{-1}) \\ &= 2(-1)x^{-1-1} \\ &= -2x^{-2}\end{aligned}$$

The slope at $(-2, -1)$ is found by substituting $x = -2$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= -2(-2)^{-2} \\ &= -\frac{1}{2}\end{aligned}$$

$$\begin{aligned}\text{d. } \frac{dy}{dx} &= \frac{d}{dx}\left(\sqrt{16x^3}\right) \\ &= \sqrt{16}\frac{d}{dx}(x^{\frac{3}{2}}) \\ &= 4\left(\frac{3}{2}\right)x^{\frac{3}{2}-1} \\ &= 6x^{\frac{1}{2}}\end{aligned}$$

The slope at $(4, 32)$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$. So the

$$\begin{aligned}\text{slope} &= 6(4)^{\frac{1}{2}} \\ &= 12\end{aligned}$$

8. a. $y = 2x^3 + 3x$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(2x^3 + 3x) \\ &= 2\frac{d}{dx}(x^3) + 3\frac{d}{dx}(x) \\ &= 2(3x^2) + 3(1) \\ &= 6x^2 + 3\end{aligned}$$

The slope at $x = 1$ is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$. So the slope is

$$6(1)^2 + 3 = 9.$$

b. $y = 2\sqrt{x} + 5$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(2\sqrt{x} + 5) \\ &= 2\frac{d}{dx}(x^{\frac{1}{2}}) + \frac{d}{dx}(5) \\ &= 2\left(\frac{1}{2}\right)(x^{\frac{1}{2}-1}) + 0 \\ &= x^{-\frac{1}{2}}\end{aligned}$$

The slope at $x = 4$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$. So the slope is $(4)^{\frac{-1}{2}} = \frac{1}{2}$.

$$\text{c. } y = \frac{16}{x^2}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}\left(\frac{16}{x^2}\right) \\ &= 16\frac{d}{dx}(x^{-2}) \\ &= 16(-2)x^{-2-1} \\ &= -32x^{-3}\end{aligned}$$

The slope at $x = -2$ is found by substituting

$x = -2$ into the equation for $\frac{dy}{dx}$. So the slope is $-32(-2)^{-3} = \frac{(-32)}{(-2)^3} = 4$.

$$\text{d. } y = x^{-3}(x^{-1} + 1)$$

$$\begin{aligned}&= x^{-4} + x^{-3} \\ \frac{dy}{dx} &= \frac{d}{dx}(x^{-4} + x^{-3}) \\ &= -4x^{-5} - 3x^{-4} \\ &= -\frac{4}{x^5} - \frac{3}{x^4}\end{aligned}$$

The slope at $x = 1$ is found by substituting

$x = 1$ into the equation for $\frac{dy}{dx}$. So the slope is $-\frac{4}{1^5} - \frac{3}{1^4} = -7$.

$$\text{9. a. } \frac{dy}{dx} = \frac{d}{dx}\left(2x - \frac{1}{x}\right)$$

$$\begin{aligned}&= 2\frac{d}{dx}(x) - \frac{d}{dx}(x^{-1}) \\ &= 2(1) - (-1)x^{-1-1} \\ &= 2 + x^{-2}\end{aligned}$$

The slope at $x = 0.5$ is found by substituting

$x = 0.5$ into the equation for $\frac{dy}{dx}$.

So the slope is $2 + (0.5)^{-2} = 6$.

The equation of the tangent line is therefore $y + 1 = 6(x - 0.5)$ or $6x - y - 4 = 0$.

$$\text{b. } \frac{dy}{dx} = \frac{d}{dx}\left(\frac{3}{x^2} - \frac{4}{x^3}\right)$$

$$\begin{aligned}&= 3\frac{d}{dx}(x^{-2}) - 4\frac{d}{dx}(x^{-3}) \\ &= 3(-2)x^{-2-1} - 4(-3)x^{-3-1} \\ &= 12x^{-4} - 6x^{-3}\end{aligned}$$

The slope at $x = -1$ is found by substituting

$x = -1$ into the equation for $\frac{dy}{dx}$. So the slope is $12(-1)^{-4} - 6(-1)^{-3} = 18$.

The equation of the tangent line is therefore $y - 7 = 18(x + 1)$ or $18x - y + 25 = 0$.

$$\begin{aligned} \text{c. } \frac{dy}{dx} &= \frac{d}{dx}(\sqrt{3x^3}) \\ &= \sqrt{3} \frac{d}{dx}(x^{\frac{3}{2}}) \\ &= \sqrt{3} \left(\frac{3}{2} \right) x^{\frac{3}{2}-1} \\ &= \frac{3\sqrt{3}x^{\frac{1}{2}}}{2} \end{aligned}$$

The slope at $x = 3$ is found by substituting $x = 3$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } \frac{3\sqrt{3}(3)^{\frac{1}{2}}}{2} = \frac{9}{2}.$$

The equation of the tangent line is therefore $y - 9 = \frac{9}{2}(x - 3)$ or $9x - 2y - 9 = 0$.

$$\begin{aligned} \text{d. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{1}{x}(x^2 + \frac{1}{x})\right) \\ &= \frac{d}{dx}\left(x + \frac{1}{x^2}\right) \\ &= \frac{d}{dx}(x) + \frac{d}{dx}(x^{-2}) \\ &= 1 + (-2)x^{-2-1} \\ &= 1 - 2x^{-3} \end{aligned}$$

The slope at $x = 1$ is found by substituting into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } 1 - 2(1)^{-3} = -1.$$

The equation of the tangent line is therefore $y - 2 = -(x - 1)$ or $x + y - 3 = 0$.

$$\begin{aligned} \text{e. } \frac{dy}{dx} &= \frac{d}{dx}((\sqrt{x} - 2)(3\sqrt{x} + 8)) \\ &= \frac{d}{dx}(3(\sqrt{x})^2 + 8\sqrt{x} - 6\sqrt{x} - 16) \\ &= \frac{d}{dx}(3x + 2\sqrt{x} - 16) \\ &= \frac{d}{dx}(3x) + 2\frac{d}{dx}(x^{\frac{1}{2}}) - \frac{d}{dx}(16) \\ &= 3(1) + 2\left(\frac{1}{2}\right)x^{\frac{1}{2}-1} - 0 \\ &= 3 + x^{-\frac{1}{2}} \end{aligned}$$

The slope at $x = 4$ is found by substituting $x = 4$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } 3 + (4)^{-\frac{1}{2}} = 3.5.$$

The equation of the tangent line is therefore $y = 3.5(x - 4)$ or $7x - 2y - 28 = 0$.

$$\begin{aligned} \text{f. } \frac{dy}{dx} &= \frac{d}{dx}\left(\frac{\sqrt{x} - 2}{\sqrt[3]{x}}\right) \\ &= \frac{d}{dx}\left(\frac{x^{\frac{1}{2}} - 2}{x^{\frac{1}{3}}}\right) \\ &= \frac{d}{dx}(x^{\frac{1}{2}-\frac{1}{3}} - 2x^{-\frac{1}{3}}) \\ &= \frac{d}{dx}(x^{\frac{1}{6}}) - 2\frac{d}{dx}(x^{-\frac{1}{3}}) \\ &= \frac{1}{6}(x^{\frac{1}{6}-1}) - 2\left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1} - 0 \\ &= \frac{1}{6}(x^{-\frac{5}{6}}) + \frac{2}{3}x^{-\frac{4}{3}} \end{aligned}$$

The slope at $x = 1$ is found by substituting $x = 1$ into the equation for $\frac{dy}{dx}$.

$$\text{So the slope is } \frac{1}{6}(1)^{-\frac{5}{6}} + \frac{2}{3}(1)^{-\frac{4}{3}} = \frac{5}{6}.$$

The equation of the tangent line is therefore $y + 1 = \frac{5}{6}(x - 1)$ or $5x - 6y - 11 = 0$.

10. A normal to the graph of a function at a point is a line that is perpendicular to the tangent at the given point.

$$y = \frac{3}{x^2} - \frac{4}{x^3} \text{ at } P(-1, 7)$$

Slope of the tangent is 18, therefore, the slope of the normal is $-\frac{1}{18}$.

$$\text{Equation is } y - 7 = -\frac{1}{18}(x + 1).$$

$$x + 18y - 125 = 0$$

$$\text{11. } y = \frac{3}{\sqrt[3]{x}} = 3x^{-\frac{1}{3}}$$

Parallel to $x + 16y + 3 = 0$

Slope of the line is $-\frac{1}{16}$.

$$\frac{dy}{dx} = -x^{-\frac{4}{3}}$$

$$x^{-\frac{4}{3}} = \frac{1}{16}$$

$$\frac{1}{x^{\frac{4}{3}}} = \frac{1}{16}$$

$$x^{\frac{4}{3}} = 16$$

$$x = (16)^{\frac{3}{4}} = 8$$

12. $y = \frac{1}{x} = x^{-1}$: $y = x^3$

$$\frac{dy}{dx} = -\frac{1}{x^2} : \frac{dy}{dx} = 3x^2$$

Now, $-\frac{1}{x^2} = 3x^2$

$$x^4 = -\frac{1}{3}.$$

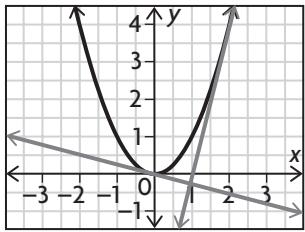
No real solution. They never have the same slope.

13. $y = x^2$, $\frac{dy}{dx} = 2x$

The slope of the tangent at $A(2, 4)$ is 4 and at

$$B\left(-\frac{1}{8}, \frac{1}{64}\right)$$

Since the product of the slopes is -1 , the tangents at $A(2, 4)$ and $B\left(-\frac{1}{8}, \frac{1}{64}\right)$ will be perpendicular.



14. $y = -x^2 + 3x + 4$

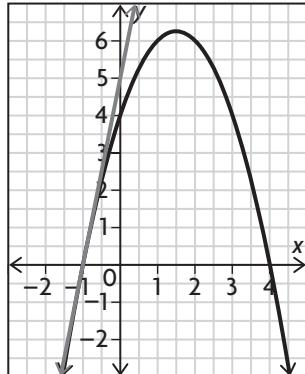
$$\frac{dy}{dx} = -2x + 3$$

For $\frac{dy}{dx} = 5$

$$5 = -2x + 3$$

$$x = -1.$$

The point is $(-1, 0)$.



15. $y = x^3 + 2$

$$\frac{dy}{dx} = 3x^2, \text{slope is } 12$$

$$x^2 = 4$$

$$x = 2 \text{ or } x = -2$$

Points are $(2, 10)$ and $(-2, -6)$.

16. $y = \frac{1}{5}x^5 - 10x$, slope is 6

$$\frac{dy}{dx} = x^4 - 10 = 6$$

$$x^4 = 16$$

$$x^2 = 4 \text{ or } x^2 = -4$$

$$x = \pm 2 \text{ non-real}$$

Tangents with slope 6 are at the points $(2, -\frac{68}{5})$ and $(-2, \frac{68}{5})$.

17. $y = 2x^2 + 3$

a. Equation of tangent from $A(2, 3)$:

$$\text{If } x = a, y = 2a^2 + 3.$$

Let the point of tangency be $P(a, 2a^2 + 3)$.

$$\text{Now, } \frac{dy}{dx} = 4x \text{ and when } x = a, \frac{dy}{dx} = 4a.$$

The slope of the tangent is the slope of AP .

$$\frac{2a^2}{a - 2} = 4a.$$

$$2a^2 = 4a^2 - 8a$$

$$2a^2 - 8a = 0$$

$$2a(a - 4) = 0$$

$$a = 0 \text{ or } a = 4.$$

Point $(2, 3)$:

Slope is 0.

Equation of tangent is

$$y - 3 = 0. \quad \text{Equation of tangent is } y - 3 = 16(x - 2) \text{ or } 16x - y - 29 = 0.$$

b. From the point $B(2, -7)$:

$$\text{Slope of } BP: \frac{2a^2 + 10}{a - 2} = 4a$$

$$2a^2 + 10 = 4a^2 - 8a$$

$$2a^2 - 8a - 10 = 0$$

$$a^2 - 4a - 5 = 0$$

$$(a - 5)(a + 1) = 0$$

$$a = 5$$

$$a = -1$$

Slope is $4a = 20$.

Slope is $4a = -4$.

Equation is

$$y + 7 = 20(x - 2) \quad \text{Equation is } y + 7 = -4(x - 2) \\ \text{or } 20x - y - 47 = 0. \quad \text{or } 4x + y - 1 = 0.$$

18. $ax - 4y + 21 = 0$ is tangent to $y = \frac{a}{x^2}$ at $x = -2$.

Therefore, the point of tangency is $(-2, \frac{a}{4})$,

This point lies on the line, therefore,

$$a(-2) - 4\left(\frac{a}{4}\right) + 21 = 0$$

$$-3a + 21 = 0$$

$$a = 7.$$

19. a. When $h = 200$,

$$d = 3.53\sqrt{200}$$

$$\doteq 49.9$$

Passengers can see about 49.9 km.

b. $d = 3.53\sqrt{h} = 3.53h^{\frac{1}{2}}$

$$d' = 3.53\left(\frac{1}{2}h^{-\frac{1}{2}}\right)$$

$$= \frac{3.53}{2\sqrt{h}}$$

When $h = 200$,

$$d' = \frac{3.53}{2\sqrt{200}}$$

$$\doteq 0.12$$

The rate of change is about 0.12 km/m.

20. $d(t) = 4.9t^2$

a. $d(2) = 4.9(2)^2 = 19.6$ m
 $d(5) = 4.9(5)^2 = 122.5$ m

The average rate of change of distance with respect to time from 2 s to 5 s is

$$\frac{\Delta d}{\Delta t} = \frac{122.5 - 19.6}{5 - 2}$$

$$= 34.3$$
 m/s

b. $d'(t) = 9.8t$

Thus, $d'(4) = 9.8(4) = 39.2$ m/s.

c. When the object hits the ground, $d = 150$.

Set $d(t) = 150$:

$$4.9t^2 = 150$$

$$t^2 = \frac{1500}{49}$$

$$t = \pm \frac{10}{7}\sqrt{15}$$

Since $t \geq 0$, $t = \frac{10}{7}\sqrt{15}$

Then,

$$d'\left(\frac{10}{7}\sqrt{15}\right) = 9.8\left(\frac{10}{7}\sqrt{15}\right)$$

$$\doteq 54.2$$
 m/s

21. $v(t) = s'(t) = 2t - t^2$

$$0.5 = 2t - t^2$$

$$t^2 - 2t + 0.5 = 0$$

$$2t^2 - 4t + 1 = 0$$

$$t = \frac{4 \pm \sqrt{8}}{4}$$

$$t \doteq 1.71, 0.29$$

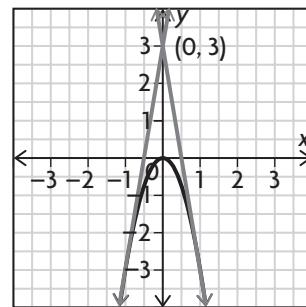
The train has a velocity of 0.5 km/min at about 0.29 min and 1.71 min.

22. $v(t) = R'(t) = -10t$

$$v(2) = -20$$

The velocity of the bolt at $t = 2$ is -20 m/s.

23.



Let the coordinates of the points of tangency be $A(a, -3a^2)$.

$$\frac{dy}{dx} = -6x, \text{ slope of the tangent at } A \text{ is } -6a$$

$$\text{Slope of PA: } \frac{-3a^2 - 3}{a} = -6a$$

$$-3a^2 - 3 = -6a^2$$

$$3a^2 = 3$$

$$a = 1 \text{ or } a = -1$$

Coordinates of the points at which the tangents touch the curve are $(1, -3)$ and $(-1, -3)$.

24. $y = x^3 - 6x^2 + 8x$, tangent at $A(3, -3)$

$$\frac{dy}{dx} = 3x^2 - 12x + 8$$

When $x = 3$,

$$\frac{dy}{dx} = 27 - 36 + 8 = -1$$

The slope of the tangent at $A(3, -3)$ is -1 .

Equation will be

$$y + 3 = -1(x - 3)$$

$$y = -x$$

$$-x = x^3 - 6x^2 + 8x$$

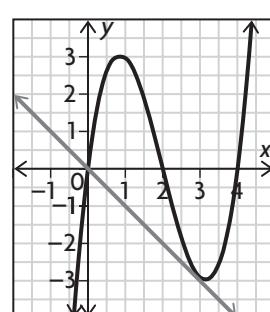
$$x^3 - 6x^2 + 9x = 0$$

$$x(x^2 - 6x + 9) = 0$$

$$x(x - 3)^2 = 0$$

$$x = 0 \text{ or } x = 3$$

Coordinates are $B(0, 0)$.



25. a. i. $f(x) = 2x - 5x^2$

$$f'(x) = 2 - 10x$$

Set $f'(x) = 0$:

$$2 - 10x = 0$$

$$10x = 2$$

$$x = \frac{1}{5}$$

Then,

$$\begin{aligned} f\left(\frac{1}{5}\right) &= 2\left(\frac{1}{5}\right) - 5\left(\frac{1}{5}\right)^2 \\ &= \frac{2}{5} - \frac{1}{5} \\ &= \frac{1}{5} \end{aligned}$$

Thus the point is $\left(\frac{1}{5}, \frac{1}{5}\right)$.

ii. $f(x) = 4x^2 + 2x - 3$

$$f'(x) = 8x + 2$$

Set $f'(x) = 0$:

$$8x + 2 = 0$$

$$8x = -2$$

$$x = -\frac{1}{4}$$

Then,

$$\begin{aligned} f\left(-\frac{1}{4}\right) &= 4\left(-\frac{1}{4}\right)^2 + 2\left(-\frac{1}{4}\right) - 3 \\ &= \frac{1}{4} - \frac{2}{4} - \frac{12}{4} \\ &= -\frac{13}{4} \end{aligned}$$

Thus the point is $\left(-\frac{1}{4}, -\frac{13}{4}\right)$.

iii. $f(x) = x^3 - 8x^2 + 5x + 3$

$$f'(x) = 3x^2 - 16x + 5$$

Set $f'(x) = 0$:

$$3x^2 - 16x + 5 = 0$$

$$3x^2 - 15x - x + 5 = 0$$

$$3x(x - 5) - (x - 5) = 0$$

$$(3x - 1)(x - 5) = 0$$

$$x = \frac{1}{3}, 5$$

$$\begin{aligned} f\left(\frac{1}{3}\right) &= \left(\frac{1}{3}\right)^3 - 8\left(\frac{1}{3}\right)^2 + 5\left(\frac{1}{3}\right) + 3 \\ &= \frac{1}{27} - \frac{24}{27} + \frac{45}{27} + \frac{81}{27} \\ &= \frac{103}{27} \end{aligned}$$

$$\begin{aligned} f(5) &= (5)^3 - 8(5)^2 + 5(5) + 3 \\ &= 25 - 200 + 25 + 3 \\ &= -47 \end{aligned}$$

Thus the two points are $\left(\frac{1}{3}, \frac{103}{27}\right)$ and $(5, -47)$.

b. At these points, the slopes of the tangents are zero, meaning that the rate of change of the value of the function with respect to the domain is zero. These points are also local maximum or minimum points.

26. $\sqrt{x} + \sqrt{y} = 1$

$P(a, b)$ is on the curve, therefore $a \geq 0, b \geq 0$.

$$\sqrt{y} = 1 - \sqrt{x}$$

$$y = 1 - 2\sqrt{x} + x$$

$$\frac{dy}{dx} = -\frac{1}{2} \cdot 2x^{-\frac{1}{2}} + 1$$

$$\text{At } x = a, \text{slope is } -\frac{1}{\sqrt{a}} + 1 = \frac{-1 + \sqrt{a}}{\sqrt{a}}.$$

$$\text{But } \sqrt{a} + \sqrt{b} = 1$$

$$-\sqrt{b} = \sqrt{a} - 1.$$

$$\text{Therefore, slope is } -\frac{\sqrt{b}}{\sqrt{a}} = -\sqrt{\frac{b}{a}}.$$

27. $f(x) = x^n, f'(x) = nx^{n-1}$

Slope of the tangent at $x = 1$ is $f'(1) = n$,

The equation of the tangent at $(1, 1)$ is:

$$y - 1 = n(x - 1)$$

$$nx - y - n + 1 = 0$$

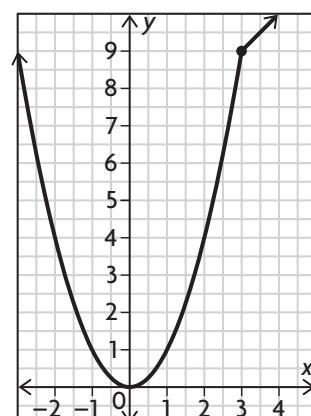
Let $y = 0, nx = n - 1$

$$x = \frac{n-1}{n} = 1 - \frac{1}{n}.$$

The x -intercept is $1 - \frac{1}{n}$; as $n \rightarrow \infty$, $\frac{1}{n} \rightarrow 0$, and

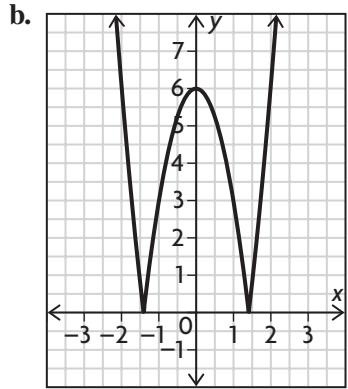
the x -intercept approaches 1. As $n \rightarrow \infty$, the slope of the tangent at $(1, 1)$ increases without bound, and the tangent approaches a vertical line having equation $x - 1 = 0$.

28. a.



$$f(x) = \begin{cases} x^2, & \text{if } x < 3 \\ x + 6, & \text{if } x \geq 3 \end{cases} \quad f'(x) = \begin{cases} 2x, & \text{if } x < 3 \\ 1, & \text{if } x \geq 3 \end{cases}$$

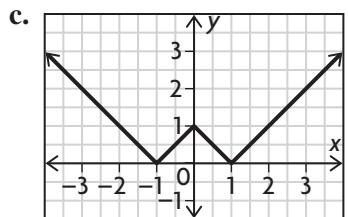
$f'(3)$ does not exist.



$$f(x) = \begin{cases} 3x^2 - 6, & \text{if } x < -\sqrt{2} \text{ or } x > \sqrt{2} \\ 6 - 3x^2, & \text{if } -\sqrt{2} < x < \sqrt{2} \end{cases}$$

$$f'(x) = \begin{cases} 6x, & \text{if } x < -\sqrt{2} \text{ or } x > \sqrt{2} \\ -6x, & \text{if } -\sqrt{2} \leq x \leq \sqrt{2} \end{cases}$$

$f'(\sqrt{2})$ and $f'(-\sqrt{2})$ do not exist.



$$f(x) = \begin{cases} x - 1, & \text{if } x \geq 1 \\ 1 - x, & \text{if } 0 \leq x < 1 \\ x + 1, & \text{if } -1 < x < 0 \\ -x - 1, & \text{if } x \leq -1 \end{cases}$$

since $|x - 1| = x - 1$
since $|x - 1| = 1 - x$
since $|-x - 1| = x + 1$
since $|-x - 1| = -x - 1$

$$f'(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } 0 < x < 1 \\ 1, & \text{if } -1 < x < 0 \\ -1, & \text{if } x < -1 \end{cases}$$

$f'(0)$, $f'(-1)$, and $f'(1)$ do not exist.

2.3 The Product Rule, pp. 90–91

1. a. $h(x) = x(x - 4)$

$$h'(x) = x(1) + (1)(x - 4)$$

$$= 2x - 4$$

b. $h(x) = x^2(2x - 1)$

$$h'(x) = x^2(2) + (2x)(2x - 1)$$

$$= 6x^2 - 2x$$

c. $h(x) = (3x + 2)(2x - 7)$

$$h'(x) = (3x + 2)(2) + (3)(2x - 7)$$

$$= 12x - 17$$

d. $h(x) = (5x^7 + 1)(x^2 - 2x)$

$$h'(x) = (5x^7 + 1)(2x - 2) + (35x^6)(x^2 - 2x)$$

$$= 45x^8 - 80x^7 + 2x - 2$$

e. $s(t) = (t^2 + 1)(3 - 2t^2)$

$$s'(t) = (t^2 + 1)(-4t) + (2t)(3 - 2t^2)$$

$$= -8t^3 + 2t$$

f. $f(x) = \frac{x - 3}{x + 3}$

$$f(x) = (x - 3)(x + 3)^{-1}$$

$$f'(x) = (x - 3)(-1)(x + 3)^{-2} + (1)(x + 3)^{-1}$$

$$= (x + 3)^{-2}(-x + 3 + x + 3)$$

$$= \frac{6}{(x + 3)^2}$$

2. a. $y = (5x + 1)^3(x - 4)$

$$\frac{dy}{dx} = (5x + 1)^3(1) + 3(5x + 1)^2(5)(x - 4)$$

$$= (5x + 1)^3 + 15(5x + 1)^2(x - 4)$$

b. $y = (3x^2 + 4)(3 + x^3)^5$

$$\frac{dy}{dx} = (3x^2 + 4)(5)(3 + x^3)^4(3x^2)$$

$$+ (6x)(3 + x^3)^5$$

$$= 15x^2(3x^2 + 4)(3 + x^3)^4 + 6x(3 + x^3)^5$$

c. $y = (1 - x^2)^4(2x + 6)^3$

$$\frac{dy}{dx} = 4(1 - x^2)^3(-2x)(2x + 6)^3$$

$$+ (1 - x^2)^4 3(2x + 6)^2(2)$$

$$= -8x(1 - x^2)^3(2x + 6)^3$$

$$+ 6(1 - x^2)^4(2x + 6)^2$$

d. $y = (x^2 - 9)^4(2x - 1)^3$

$$\frac{dy}{dx} = (x^2 - 9)^4(3)(2x - 1)^2(2)$$

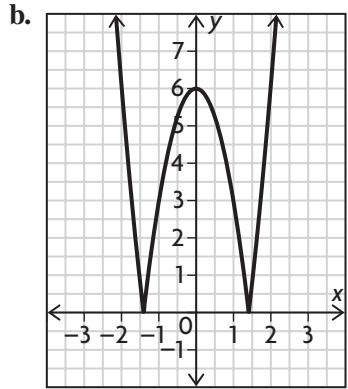
$$+ 4(x^2 - 9)^3(2x)(2x - 1)^3$$

$$= 6(x^2 - 9)^4(2x - 1)^2$$

$$+ 8x(x^2 - 9)^3(2x - 1)^3$$

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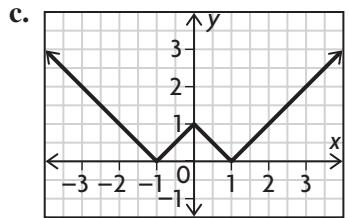
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since $|x - 1| = x - 1$
 since $|x - 1| = 1 - x$
 since $|-x - 1| = x + 1$
 since $|-x - 1| = -x - 1$

$$f'(x) = \begin{cases} 1, & \text{if } x > 1 \\ -1, & \text{if } 0 < x < 1 \\ 1, & \text{if } -1 < x < 0 \\ -1, & \text{if } x < -1 \end{cases}$$

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$$\frac{dy}{dx} = 4(1 - x^2)^3(-2x)(2x + 6)^3$$

$$+ (1 - x^2)^4 3(2x + 6)^2(2)$$

$$= -8x(1 - x^2)^3(2x + 6)^3$$

$$+ 6(1 - x^2)^4(2x + 6)^2$$

d. $y = (x^2 - 9)^4(2x - 1)^3$

$$\frac{dy}{dx} = (x^2 - 9)^4(3)(2x - 1)^2(2)$$

$$+ 4(x^2 - 9)^3(2x)(2x - 1)^3$$

$$= 6(x^2 - 9)^4(2x - 1)^2$$

$$+ 8x(x^2 - 9)^3(2x - 1)^3$$

3. It is not appropriate or necessary to use the product rule when one of the factors is a constant or when it would be easier to first determine the product of the factors and then use other rules to determine the derivative. For example, it would not be best to use the product rule for $f(x) = 3(x^2 + 1)$ or $g(x) = (x + 1)(x - 1)$.

4. $F(x) = [b(x)][c(x)]$

$$F'(x) = [b(x)][c'(x)] + [b'(x)][c(x)]$$

5. a. $y = (2 + 7x)(x - 3)$

$$\frac{dy}{dx} = (2 + 7x)(1) + 7(x - 3)$$

At $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= (2 + 14) + 7(-1) \\ &= 16 - 7 \\ &= 9\end{aligned}$$

b. $y = (1 - 2x)(1 + 2x)$

$$\frac{dy}{dx} = (1 - 2x)(2) + (-2)(1 + 2x)$$

At $x = \frac{1}{2}$,

$$\begin{aligned}\frac{dy}{dx} &= (0)(2) - 2(2) \\ &= -4\end{aligned}$$

c. $y = (3 - 2x - x^2)(x^2 + x - 2)$

$$\begin{aligned}\frac{dy}{dx} &= (3 - 2x - x^2)(2x + 1) \\ &\quad + (-2 - 2x)(x^2 + x - 2)\end{aligned}$$

At $x = -2$,

$$\begin{aligned}\frac{dy}{dx} &= (3 + 4 - 4)(-4 + 1) \\ &\quad + (-2 + 4)(4 - 2 - 2) \\ &= (3)(-3) + (2)(0) \\ &= -9\end{aligned}$$

d. $y = x^3(3x + 7)^2$

$$\frac{dy}{dx} = 3x^2(3x + 7)^2 + x^36(3x + 7)$$

At $x = -2$,

$$\begin{aligned}\frac{dy}{dx} &= 12(1)^2 + (-8)(6)(1) \\ &= 12 - 48 \\ &= -36\end{aligned}$$

e. $y = (2x + 1)^5(3x + 2)^4, x = -1$

$$\begin{aligned}\frac{dy}{dx} &= 5(2x + 1)^4(2)(3x + 2)^4 \\ &\quad + (2x + 1)^54(3x + 2)^3(3)\end{aligned}$$

At $x = -1$,

$$\begin{aligned}\frac{dy}{dx} &= 5(-1)^4(2)(-1)^4 \\ &\quad + (-1)^5(4)(-1)^3(3) \\ &= 10 + 12 \\ &= 22\end{aligned}$$

f. $y = x(5x - 2)(5x + 2)$

$$\frac{dy}{dx} = x(50x) + (25x^2 - 4)(1)$$

At $x = 3$,

$$\begin{aligned}\frac{dy}{dx} &= 3(150) + (25 \cdot 9 - 4) \\ &= 450 + 221 \\ &= 671\end{aligned}$$

6. Tangent to $y = (x^3 - 5x + 2)(3x^2 - 2x)$

at $(1, -2)$

$$\begin{aligned}\frac{dy}{dx} &= (3x^2 - 5)(3x^2 - 2x) \\ &\quad + (x^3 - 5x + 2)(6x - 2)\end{aligned}$$

when $x = 1$,

$$\begin{aligned}\frac{dy}{dx} &= (-2)(1) + (-2)(4) \\ &= -2 + -8 \\ &= -10\end{aligned}$$

Slope of the tangent at $(1, -2)$ is -10 .

The equation is $y + 2 = -10(x - 1)$;

$$10x + y - 8 = 0.$$

7. a. $y = 2(x - 29)(x + 1)$

$$\begin{aligned}\frac{dy}{dx} &= 2(x - 29)(1) + 2(1)(x + 1) \\ 2x - 58 + 2x + 2 &= 0 \\ 4x - 56 &= 0 \\ 4x &= 56 \\ x &= 14\end{aligned}$$

Point of horizontal tangency is $(14, -450)$.

b. $y = (x^2 + 2x + 1)(x^2 + 2x + 1)$

$$\begin{aligned}&= (x^2 + 2x + 1)^2 \\ \frac{dy}{dx} &= 2(x^2 + 2x + 1)(2x + 2) \\ (x^2 + 2x + 1)(2x + 2) &= 0 \\ 2(x + 1)(x + 1)(x + 1) &= 0 \\ x &= -1\end{aligned}$$

Point of horizontal tangency is $(-1, 0)$.

8. a. $y = (x + 1)^3(x + 4)(x - 3)^2$

$$\begin{aligned}\frac{dy}{dx} &= 3(x + 1)^2(x + 4)(x - 3)^2 \\ &\quad + (x + 1)^3(1)(x - 3)^2 \\ &\quad + (x + 1)^3(x + 4)[2(x - 3)]\end{aligned}$$

b. $y = x^2(3x^2 + 4)^2(3 - x^3)^4$

$$\begin{aligned}\frac{dy}{dx} &= 2x(3x^2 + 4)^2(3 - x^3)^4 \\ &\quad + x^2[2(3x^2 + 4)(6x)](3 - x^3)^4 \\ &\quad + x^2(3x^2 + 4)^2[4(3 - x^3)^3(-3x^2)]\end{aligned}$$

9. $V(t) = 75\left(1 - \frac{t}{24}\right)^2, 0 \leq t \leq 24$

$$75 \text{ L} \times 60\% = 45 \text{ L}$$

$$\text{Set } \frac{45}{75} = \left(1 - \frac{t}{24}\right)^2$$

$$\pm \sqrt{\frac{3}{5}} = 1 - \frac{t}{24}$$

$$t = \left(\pm \sqrt{\frac{3}{5}} - 1\right)(-24)$$

$$t \doteq 42.590 \text{ (inadmissible)} \text{ or } t \doteq 5.4097$$

$$V(t) = 75\left(1 - \frac{t}{24}\right)^2$$

$$V(t) = 75\left(1 - \frac{t}{24}\right)\left(1 - \frac{t}{24}\right)$$

$$\begin{aligned}V'(t) &= 75\left[\left(1 - \frac{t}{24}\right)\left(-\frac{1}{24}\right) \right. \\ &\quad \left. + \left(-\frac{1}{24}\right)\left(1 - \frac{t}{24}\right)\right] \\ &= (75)(2)\left(1 - \frac{t}{24}\right)\left(-\frac{1}{24}\right)\end{aligned}$$

$$V'(5.4097) = -4.84 \text{ L/h}$$

10. Determine the point of tangency, and then find the negative reciprocal of the slope of the tangent. Use this information to find the equation of the normal.

$$\begin{aligned}h(x) &= 2x(x + 1)^3(x^2 + 2x + 1)^2 \\ h'(x) &= 2(x + 1)^3(x^2 + 2x + 1)^2 \\ &\quad + (2x)(3)(x + 1)^2(x^2 + 2x + 1)^2 \\ &\quad + 2x(x + 1)^3 2(x^2 + 2x + 1)(2x + 2)\end{aligned}$$

$$\begin{aligned}h'(-2) &= 2(-1)^3(1)^2 \\ &\quad + 2(-2)(3)(-1)^2(1)^2 \\ &\quad + 2(-2)(-1)^3(2)(1)(-2) \\ &= -2 - 12 - 16 \\ &= -30\end{aligned}$$

11.

a. $f(x) = g_1(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n(x)$
 $f'(x) = g_1'(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n(x)$
 $+ g_1(x)g_2'(x)g_3(x) \dots g_{n-1}(x)g_n(x)$
 $+ g_1(x)g_2(x)g_3'(x) \dots g_{n-1}(x)g_n(x)$
 $+ \dots + g_1(x)g_2(x)g_3(x) \dots g_{n-1}(x)g_n'(x)$

b. $f(x) = (1 + x)(1 + 2x)(1 + 3x) \dots$
 $(1 + nx)$

$$\begin{aligned}f'(x) &= 1(1 + 2x)(1 + 3x) \dots (1 + nx) \\ &\quad + (1 + x)(2)(1 + 3x) \dots (1 + nx) \\ &\quad + (1 + x)(1 + 2x)(3) \dots (1 + nx) \\ &\quad + \dots + (1 + x)(1 + 2x)(1 + 3x) \\ &\quad \dots (n)\end{aligned}$$

$$\begin{aligned}f'(0) &= 1(1)(1)(1) \dots (1) \\ &\quad + 1(2)(1)(1) \dots (1) \\ &\quad + 1(1)(3)(1) \dots (1) \\ &\quad + \dots + (1)(1)(1) \dots (n) \\ &= 1 + 2 + 3 + \dots + n\end{aligned}$$

$$f'(0) = \frac{n(n + 1)}{2}$$

12. $f(x) = ax^2 + bx + c$

$$f'(x) = 2ax + b \quad (1)$$

Horizontal tangent at $(-1, -8)$

$$f'(x) = 0 \text{ at } x = -1$$

$$-2a + b = 0$$

Since $(2, 19)$ lies on the curve,

$$4a + 2b + c = 19 \quad (2)$$

Since $(-1, -8)$ lies on the curve,

$$a - b + c = -8 \quad (3)$$

$$4a + 2b + c = 19$$

$$-3a - 3b = -27$$

$$a + b = 9$$

$$-2a + b = 0$$

$$3a = 9$$

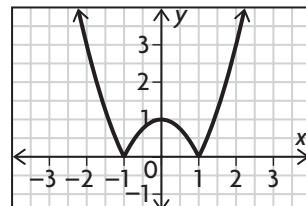
$$a = 3, b = 6$$

$$3 - 6 + c = -8$$

$$c = -5$$

The equation is $y = 3x^2 + 6x - 5$.

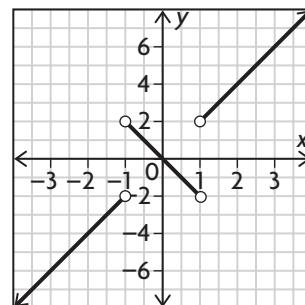
13.



a. $x = 1$ or $x = -1$

b. $f'(x) = 2x, x < -1$ or $x > 1$

$$f'(x) = -2x, -1 < x < 1$$



c. $f'(-2) = 2(-2) = -4$
 $f'(0) = -2(0) = 0$
 $f'(3) = 2(3) = 6$

14. $y = \frac{16}{x^2} - 1$

$$\frac{dy}{dx} = -\frac{32}{x^3}$$

Slope of the line is 4.

$$-\frac{32}{x^3} = 4$$

$$4x^3 = -32$$

$$x^3 = -8$$

$$x = -2$$

$$y = \frac{16}{4} - 1$$

$$= 3$$

Point is at $(-2, 3)$.

Find intersection of line and curve:

$$4x - y + 11 = 0$$

$$y = 4x + 11$$

Substitute,

$$4x + 11 = \frac{16}{x^2} - 1$$

$$4x^3 + 11x^2 = 16 - x^2 \text{ or } 4x^3 + 12x^2 - 16 = 0.$$

Let $x = -2$

$$\begin{aligned} \text{RS} &= 4(-2)^3 + 12(-2)^2 - 16 \\ &= 0 \end{aligned}$$

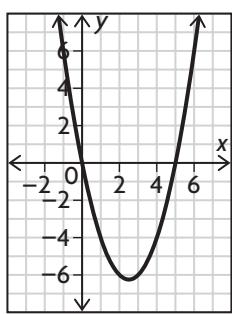
Since $x = -2$ satisfies the equation, therefore it is a solution.

When $x = -2$, $y = 4(-2) + 11 = 3$.

Intersection point is $(-2, 3)$. Therefore, the line is tangent to the curve.

Mid-Chapter Review, pp. 92–93

1. a.



b. $f'(x) = \lim_{h \rightarrow 0} \frac{((x+h)^2 - 5(x+h)) - (x^2 - 5x)}{h}$

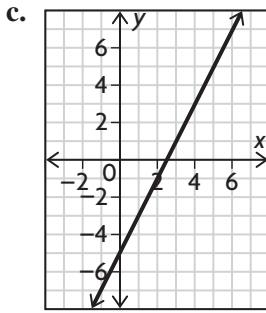
$$= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - 5x - 5h - x^2 + 5x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 + 2hx - 5h}{h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{h(h + 2x - 5)}{h} \\ &= 2x - 5 \end{aligned}$$

Use the derivative function to calculate the slopes of the tangents.

x	Slope of Tangent $f'(x)$
0	-5
1	-3
2	-1
3	1
4	3
5	5



c. $f(x)$ is quadratic; $f'(x)$ is linear.

2. a. $f'(x) = \lim_{h \rightarrow 0} \frac{(6(x+h) + 15) - (6x + 15)}{h}$

$$= \lim_{h \rightarrow 0} \frac{6h}{h}$$

$$= \lim_{h \rightarrow 0} 6$$

$$= 6$$

b. $f'(x) = \lim_{h \rightarrow 0} \frac{(2(x+h)^2 - 4) - (2x^2 - 4)}{h}$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{((x+h) - x)((x+h) + x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \rightarrow 0} (2x+h)$$

$$= 4x$$

$$c. f'(x) = \lim_{h \rightarrow 0} \frac{\frac{5}{(x+h)+5} - \frac{5}{x+5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5(x+5) - 5((x+h)+5)}{((x+h)+5)(x+5)h}$$

$$= \lim_{h \rightarrow 0} \frac{-5h}{((x+h)+5)(x+5)h}$$

c. $f'(-2) = 2(-2) = -4$
 $f'(0) = -2(0) = 0$
 $f'(3) = 2(3) = 6$

14. $y = \frac{16}{x^2} - 1$

$$\frac{dy}{dx} = -\frac{32}{x^3}$$

Slope of the line is 4.

$$-\frac{32}{x^3} = 4$$

$$4x^3 = -32$$

$$x^3 = -8$$

$$x = -2$$

$$y = \frac{16}{4} - 1$$

$$= 3$$

Point is at $(-2, 3)$.

Find intersection of line and curve:

$$4x - y + 11 = 0$$

$$y = 4x + 11$$

Substitute,

$$4x + 11 = \frac{16}{x^2} - 1$$

$$4x^3 + 11x^2 = 16 - x^2 \text{ or } 4x^3 + 12x^2 - 16 = 0.$$

Let $x = -2$

$$\begin{aligned} \text{RS} &= 4(-2)^3 + 12(-2)^2 - 16 \\ &= 0 \end{aligned}$$

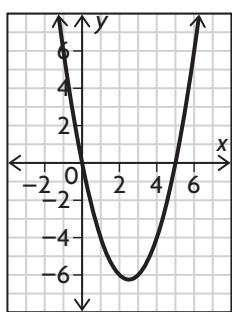
Since $x = -2$ satisfies the equation, therefore it is a solution.

When $x = -2$, $y = 4(-2) + 11 = 3$.

Intersection point is $(-2, 3)$. Therefore, the line is tangent to the curve.

Mid-Chapter Review, pp. 92–93

1. a.



b. $f'(x) = \lim_{h \rightarrow 0} \frac{((x+h)^2 - 5(x+h)) - (x^2 - 5x)}{h}$

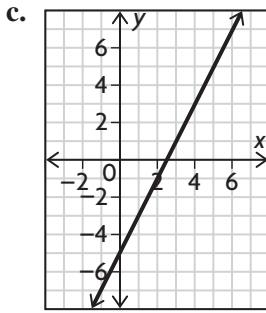
$$= \lim_{h \rightarrow 0} \frac{x^2 + 2hx + h^2 - 5x - 5h - x^2 + 5x}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 + 2hx - 5h}{h}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{h(h + 2x - 5)}{h} \\ &= 2x - 5 \end{aligned}$$

Use the derivative function to calculate the slopes of the tangents.

x	Slope of Tangent $f'(x)$
0	-5
1	-3
2	-1
3	1
4	3
5	5



c. $f(x)$ is quadratic; $f'(x)$ is linear.

2. a. $f'(x) = \lim_{h \rightarrow 0} \frac{(6(x+h) + 15) - (6x + 15)}{h}$

$$= \lim_{h \rightarrow 0} \frac{6h}{h}$$

$$= \lim_{h \rightarrow 0} 6$$

$$= 6$$

b. $f'(x) = \lim_{h \rightarrow 0} \frac{(2(x+h)^2 - 4) - (2x^2 - 4)}{h}$

$$= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{((x+h) - x)((x+h) + x)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h \rightarrow 0} (2x+h)$$

$$= 4x$$

$$c. f'(x) = \lim_{h \rightarrow 0} \frac{\frac{5}{(x+h)+5} - \frac{5}{x+5}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{5(x+5) - 5((x+h)+5)}{((x+h)+5)(x+5)h}$$

$$= \lim_{h \rightarrow 0} \frac{-5h}{((x+h)+5)(x+5)h}$$

$$= \lim_{h \rightarrow 0} \frac{-5}{((x+h)+5)(x+5)}$$

$$= \frac{-5}{(x+5)^2}$$

$$\mathbf{d. } f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)-2} - \sqrt{x-2}}{h}$$

$$= \lim_{h \rightarrow 0} \left[\frac{\sqrt{(x+h)-2} - \sqrt{x-2}}{h} \right]$$

$$\quad \times \frac{\sqrt{(x+h)-2} + \sqrt{x-2}}{\sqrt{(x+h)-2} + \sqrt{x-2}}$$

$$= \lim_{h \rightarrow 0} \frac{((x+h)-2) - (x-2)}{h(\sqrt{(x+h)-2} + \sqrt{x-2})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{(x+h)-2} + \sqrt{x-2})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{(x+h)-2} + \sqrt{x-2}}$$

$$= \frac{1}{2\sqrt{x-2}}$$

3. a. $y' = 2x - 4$

When $x = 1$,

$$y' = 2(1) - 4$$

$$= -2.$$

When $x = 1$,

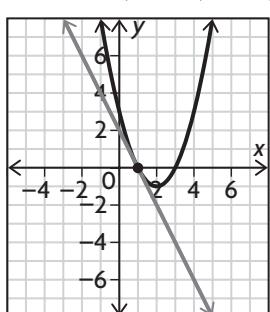
$$y = (1)^2 - 4(1) + 3$$

$$= 0.$$

Equation of the tangent line:

$$y - 0 = -2(x - 1), \text{ or } y = -2x + 2$$

b.



4. a. $\frac{dy}{dx} = 24x^3$

b. $\frac{dy}{dx} = 5x^{-\frac{1}{2}}$

$$= \frac{5}{\sqrt{x}}$$

c. $g'(x) = -6x^{-4}$

$$= -\frac{6}{x^4}$$

d. $\frac{dy}{dx} = 5 - 6x^{-3}$

$$= 5 - \frac{6}{x^3}$$

e. $\frac{dy}{dt} = 2(11t + 1)(11)$

$$= 242t + 22$$

f. $y = 1 - \frac{1}{x}$

$$= 1 - x^{-1}$$

$$\frac{dy}{dx} = x^{-2}$$

$$= \frac{1}{x^2}$$

5. f'(x) = 8x^3

$$8x^3 = 1$$

$$x^3 = \frac{1}{8}$$

$$x = \frac{1}{2}$$

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^4$$

$$= \frac{1}{8}$$

Equation of the tangent line:

$$y - \frac{1}{8} = 1\left(x - \frac{1}{2}\right), \text{ or } y = x - \frac{3}{8}$$

6. a. $f'(x) = 8x - 7$

b. $f'(x) = -6x^2 + 8x + 5$

c. $f(x) = 5x^{-2} - 3x^{-3}$

$$f'(x) = -10x^{-3} + 9x^{-4}$$

$$= -\frac{10}{x^3} + \frac{9}{x^4}$$

d. $f(x) = x^{\frac{1}{2}} + x^{\frac{1}{3}}$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{3}x^{-\frac{2}{3}}$$

$$= \frac{1}{2x^{\frac{1}{2}}} + \frac{1}{3x^{\frac{2}{3}}}$$

e. $f(x) = 7x^{-2} - 3x^{\frac{1}{2}}$

$$f'(x) = -14x^{-3} - \frac{3}{2}x^{-\frac{1}{2}}$$

$$= -\frac{14}{x^3} - \frac{3}{2x^{\frac{1}{2}}}$$

f. $f'(x) = 4x^{-2} + 5$

$$= \frac{4}{x^2} + 5$$

7. a. $y' = -6x + 6$

When $x = 1$,

$$\begin{aligned}y' &= -6(1) + 6 \\&= 0.\end{aligned}$$

When $x = 1$,

$$\begin{aligned}y &= -3(1^2) + 6(1) + 4 \\&= 7.\end{aligned}$$

Equation of the tangent line:

$$y - 7 = 0(x - 1), \text{ or}$$

$$y = 7$$

b. $y = 3 - 2x^{\frac{1}{2}}$

$$\begin{aligned}y' &= -x^{-\frac{1}{2}} \\&= \frac{-1}{\sqrt{x}}\end{aligned}$$

When $x = 9$,

$$\begin{aligned}y' &= \frac{-1}{\sqrt{9}} \\&= -\frac{1}{3}.\end{aligned}$$

When $x = 9$,

$$\begin{aligned}y &= 3 - 2\sqrt{9} \\&= -3.\end{aligned}$$

Equation of the tangent line:

$$y - (-3) = -\frac{1}{3}(x - 9), \text{ or } y = -\frac{1}{3}x$$

c. $f'(x) = -8x^3 + 12x^2 - 4x - 8$

$$\begin{aligned}f'(3) &= -8(3)^3 + 12(3)^2 - 4(3) - 8 \\&= -216 + 108 - 12 - 8 \\&= -218 \\f(3) &= -2(3)^4 + 4(3)^3 - 2(3)^2 - 8(3) + 9 \\&= -162 + 108 - 18 - 24 + 9 \\&= -87\end{aligned}$$

Equation of the tangent line:

$$y - (-87) = -128(x - 3), \text{ or}$$

$$y = -128x + 297$$

8. a. $f'(x) = \frac{d}{dx}(4x^2 - 9x)(3x^2 + 5)$

$$\begin{aligned}&\quad + (4x^2 - 9x)\frac{d}{dx}(3x^2 + 5) \\&= (8x - 9)(3x^2 + 5) + (4x^2 - 9x)(6x) \\&= 24x^3 - 27x^2 + 40x - 45 \\&\quad + 24x^3 - 54x^2 \\&= 48x^3 - 81x^2 + 40x - 45\end{aligned}$$

b. $f'(t) = \frac{d}{dt}(-3t^2 - 7t + 8)(4t - 1)$

$$\begin{aligned}&\quad + (-3t^2 - 7t + 8)\frac{d}{dt}(4t - 1) \\&= (-6t - 7)(4t - 1) \\&\quad + (-3t^2 - 7t + 8)(4)\end{aligned}$$

$$\begin{aligned}&= -24t^2 - 28t + 6t + 7 - 12t^2 - 28t + 32 \\&= -36t^2 - 50t + 39\end{aligned}$$

c. $\frac{dy}{dx} = \frac{d}{dx}(3x^2 + 4x - 6)(2x^2 - 9)$

$$\begin{aligned}&\quad + (3x^2 + 4x - 6)\frac{d}{dx}(2x^2 - 9) \\&= (6x + 4)(2x^2 - 9) + (3x^2 + 4x - 6)(4x) \\&= 12x^3 - 54x + 8x^2 - 36 + 12x^3 \\&\quad + 16x^2 - 24x \\&= 24x^3 + 24x^2 - 78x - 36\end{aligned}$$

d. $\frac{dy}{dx} = \frac{d}{dx}(3 - 2x^3)^2(3 - 2x^3)$

$$\begin{aligned}&\quad + (3 - 2x^3)^2\frac{d}{dx}(3 - 2x^3) \\&= \left[\frac{d}{dx}(3 - 2x^3)(3 - 2x^3) \right.\end{aligned}$$

$$\begin{aligned}&\quad \left. + (3 - 2x^3)\frac{d}{dx}(3 - 2x^3) \right] (3 - 2x^3) \\&\quad + (3 - 2x^3)^2(-6x^2) \\&= [2(-6x^2)(3 - 2x^3)](3 - 2x^3) \\&\quad + (3 - 2x^3)^2(-6x^2) \\&= 3(3 - 2x^3)^2(-6x^2) \\&= (3 - 2x^3)^2(-18x^2) \\&= (9 - 12x^3 + 4x^6)(-18x^2) \\&= -162x^2 + 216x^5 - 72x^8\end{aligned}$$

9. $y' = \frac{d}{dx}(5x^2 + 9x - 2)(-x^2 + 2x + 3)$

$$\begin{aligned}&\quad + (5x^2 + 9x - 2)\frac{d}{dx}(-x^2 + 2x + 3) \\&= (10x + 9)(-x^2 + 2x + 3) \\&\quad + (5x^2 + 9x - 2)(2 - 2x)\end{aligned}$$

$$\begin{aligned}y'(1) &= (10(1) + 9)(-(1)^2 + 2(1) + 3) \\&\quad + (5(1)^2 + 9(1) - 2)(2 - 2(1)) \\&= (19)(4) \\&= 76\end{aligned}$$

Equation of the tangent line:

$$y - 76 = 76(x - 1), \text{ or } 76x - y - 28 = 0$$

10. $\frac{dy}{dx} = 2\frac{d}{dx}(x - 1)(5 - x)$

$$\begin{aligned}&\quad + 2(x - 1)\frac{d}{dx}(5 - x) \\&= 2(5 - x) - 2(x - 1) \\&= 12 - 4x\end{aligned}$$

The tangent line is horizontal when $\frac{dy}{dx} = 0$.

$$12 - 4x = 0$$

$$12 = 4x$$

$$x = 3$$

When $x = 3$,

$$y = 2((3) - 1)(5 - (3)) \\ = 8.$$

Point where tangent line is horizontal: $(3, 8)$

$$\begin{aligned} \mathbf{11.} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \left[\frac{(5(x+h)^2 - 8(x+h) + 4)}{h} \right. \\ &\quad \left. - \frac{(5x^2 - 8x + 4)}{h} \right] \\ &= \lim_{h \rightarrow 0} \frac{5(x+h)^2 - 5x^2 - 8h}{h} \\ &= \lim_{h \rightarrow 0} \frac{5((x+h) - x)((x+h) + x) - 8h}{h} \\ &= \lim_{h \rightarrow 0} \frac{5h(2x+h) - 8h}{h} \\ &= \lim_{h \rightarrow 0} (5(2x+h) - 8) \\ &= 10x - 8 \end{aligned}$$

$$\mathbf{12.} V(t) = 500 \left(1 - \frac{t}{90}\right)^2, 0 \leq t \leq 90$$

a. After 1 h, $t = 60$, and the volume is

$$\begin{aligned} V(60) &= 500 \left(1 - \frac{60}{90}\right)^2 \\ &= 500 \left(\frac{30}{90}\right)^2 \\ &= 500 \left(\frac{1}{3}\right)^2 \\ &= \frac{500}{9} \text{ L} \end{aligned}$$

$$\mathbf{b.} V(0) = 500(1 - 0)^2 = 500 \text{ L}$$

$$V(60) = \frac{500}{9} \text{ L}$$

The average rate of change of volume with respect to time from 0 min to 60 min is

$$\begin{aligned} \frac{\Delta V}{\Delta t} &= \frac{\frac{500}{9} - 500}{60 - 0} \\ &= \frac{\frac{-8}{9}(500)}{60} \\ &= -\frac{200}{27} \text{ L/min} \end{aligned}$$

c. Calculate $V'(t)$:

$$\begin{aligned} V'(t) &= \lim_{h \rightarrow 0} \frac{V(t+h) - V(t)}{h} \\ &= \lim_{h \rightarrow 0} \frac{500 \left(1 - \frac{t+h}{90}\right)^2 - 500 \left(1 - \frac{t}{90}\right)^2}{h} \end{aligned}$$

$$\begin{aligned} &= \lim_{h \rightarrow 0} \frac{500 \left(1 - \frac{t+h}{90} - 1 + \frac{t}{90}\right)}{h} \\ &\quad \times \frac{\left(1 - \frac{t+h}{90} + 1 - \frac{t}{90}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{500 \left(-\frac{h}{90}\right) \left(2 - \frac{2t+h}{90}\right)}{h} \\ &= \lim_{h \rightarrow 0} -\frac{500}{90} \left(2 - \frac{2t+h}{90}\right) \\ &= \frac{-50}{9} \left(2 - \frac{2t}{90}\right) \\ &= \frac{-900 + 10t}{81} \end{aligned}$$

Then,

$$\begin{aligned} V'(30) &= \frac{-900 + 10(30)}{81} \\ &= -\frac{200}{27} \text{ L/min} \end{aligned}$$

$$\mathbf{13.} V(r) = \frac{4}{3}\pi r^3$$

$$\begin{aligned} \mathbf{a.} V(10) &= \frac{4}{3}\pi(10)^3 & V(15) &= \frac{4}{3}\pi(15)^3 \\ &= \frac{4}{3}\pi(1000) & &= \frac{4}{3}\pi(3375) \\ &= \frac{4000}{3}\pi & &= 4500\pi \end{aligned}$$

Then, the average rate of change of volume with respect to radius is

$$\begin{aligned} \frac{\Delta V}{\Delta r} &= \frac{\frac{4500\pi}{3} - \frac{4000\pi}{3}}{15 - 10} \\ &= \frac{500\pi \left(9 - \frac{8}{3}\right)}{5} \\ &= 100\pi \left(\frac{19}{3}\right) \\ &= \frac{1900}{3}\pi \text{ cm}^3/\text{cm} \end{aligned}$$

b. First calculate $V'(r)$:

$$\begin{aligned} V'(r) &= \lim_{h \rightarrow 0} \frac{V(r+h) - V(r)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi[(r+h)^3 - r^3]}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi(r^3 + 3r^2h + 3rh^2 + h^3 - r^3)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{4}{3}\pi(3r^2h + 3rh^2 + h^3)}{h} \end{aligned}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{4}{3} \pi (3r^2 + 3rh + h^2) \\
&= \frac{4}{3} \pi (3r^2 + 3r(0) + (0)^2) \\
&= 4\pi r^2
\end{aligned}$$

$$\begin{aligned}
\text{Then, } V'(8) &= 4\pi(8)^2 \\
&= 4\pi(64) \\
&= 256\pi \text{ cm}^3/\text{cm}
\end{aligned}$$

14. This statement is always true. A cubic polynomial function will have the form $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$. So the derivative of this cubic is $f'(x) = 3ax^2 + 2bx + c$, and since $3a \neq 0$, this derivative is a quadratic polynomial function. For example, if $f(x) = x^3 + x^2 + 1$, we get

$$f'(x) = 3x^2 + 2x,$$

and if

$$f(x) = 2x^3 + 3x^2 + 6x + 2,$$

we get

$$f'(x) = 6x^2 + 6x + 6$$

$$\mathbf{15. } y = \frac{x^{2a+3b}}{x^{a-b}}, a, b \in \mathbb{I}$$

Simplifying,

$$y = x^{2a+3b-(a-b)} = x^{a+4b}$$

Then,

$$y' = (a+4b)^{a+4b-1}$$

$$\mathbf{16. a. } f(x) = -6x^3 + 4x - 5x^2 + 10$$

$$f'(x) = -18x^2 + 4 - 10x$$

$$\text{Then, } f'(x) = -18(3)^2 + 4 - 10(3) \\ = -188$$

b. $f'(3)$ is the slope of the tangent line to $f(x)$ at $x = 3$ and the rate of change in the value of $f(x)$ with respect to x at $x = 3$.

$$\mathbf{17. a. } P(t) = 100 + 120t + 10t^2 + 2t^3$$

$$P(t) = 100 + 120t + 10t^2 + 2t^3$$

$$P(0) = 100 + 120(0) + 10(0)^2 + 2(0)^3 \\ = 100 \text{ bacteria}$$

b. At 5 h, the population is

$$P(5) = 100 + 120(5) + 10(5)^2 + 2(5)^3 \\ = 1200 \text{ bacteria}$$

$$\mathbf{c. } P'(t) = 120 + 20t + 6t^2$$

At 5 h, the colony is growing at

$$P'(5) = 120 + 20(5) + 6(5)^2 \\ = 370 \text{ bacteria/h}$$

$$\mathbf{18. } C(t) = \frac{100}{t}, t > 2$$

$$\text{Simplifying, } C(t) = 100t^{-1}.$$

$$\text{Then, } C'(t) = -100t^{-2} = -\frac{100}{t^2}.$$

$$\begin{array}{lll}
C'(5) & C'(50) & C'(100) \\
= -\frac{100}{(5)^2} & = -\frac{100}{(50)^2} & = -\frac{100}{(100)^2} \\
= -\frac{100}{25} & = -\frac{100}{2500} & = -\frac{1}{100} \\
= -4 & = -0.04 & = -0.01
\end{array}$$

These are the rates of change of the percentage with respect to time at 5, 50, and 100 min. The percentage of carbon dioxide that is released per unit time from the pop is decreasing. The pop is getting flat.

2.4 The Quotient Rule, pp. 97–98

1. For x, a, b real numbers,

$$x^a x^b = x^{a+b}$$

For example,

$$x^9 x^{-6} = x^3$$

Also,

$$(x^a)^b = x^{ab}$$

For example,

$$(x^2)^3 = x^6$$

Also,

$$\frac{x^a}{x^b} = x^{a-b}, x \neq 0$$

For example,

$$\frac{x^5}{x^3} = x^2$$

2.

Function	Rewrite	Differentiate and Simplify, If Necessary
$f(x) = \frac{x^2 + 3x}{x}, x \neq 0$	$f(x) = x + 3$	$f'(x) = 1$
$g(x) = \frac{3x^{\frac{2}{3}}}{x}, x \neq 0$	$g(x) = 3x^{\frac{2}{3}}$	$g'(x) = 2x^{-\frac{1}{3}}$
$h(x) = \frac{1}{10x^5}, x \neq 0$	$h(x) = \frac{1}{10}x^{-5}$	$h'(x) = \frac{-1}{2}x^{-6}$
$y = \frac{8x^3 + 6x}{2x}, x \neq 0$	$y = 4x^2 + 3$	$\frac{dy}{dx} = 8x$
$s = \frac{t^2 - 9}{t - 3}, t \neq 3$	$s = t + 3$	$\frac{ds}{dt} = 1$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{4}{3} \pi (3r^2 + 3rh + h^2) \\
&= \frac{4}{3} \pi (3r^2 + 3r(0) + (0)^2) \\
&= 4\pi r^2
\end{aligned}$$

$$\begin{aligned}
\text{Then, } V'(8) &= 4\pi(8)^2 \\
&= 4\pi(64) \\
&= 256\pi \text{ cm}^3/\text{cm}
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14. This statement is always true. A cubic polynomial function will have the form $f(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$. So the derivative of this cubic is $f'(x) = 3ax^2 + 2bx + c$, and since $3a \neq 0$, this derivative is a quadratic polynomial function. For example, if $f(x) = x^3 + x^2 + 1$, we get

$$f'(x) = 3x^2 + 2x,$$

and if

$$f(x) = 2x^3 + 3x^2 + 6x + 2,$$

we get

$$f'(x) = 6x^2 + 6x + 6$$

$$\mathbf{15. } y = \frac{x^{2a+3b}}{x^{a-b}}, a, b \in \mathbb{I}$$

Simplifying,

$$y = x^{2a+3b-(a-b)} = x^{a+4b}$$

Then,

$$y' = (a+4b)^{a+4b-1}$$

$$\mathbf{16. a. } f(x) = -6x^3 + 4x - 5x^2 + 10$$

$$f'(x) = -18x^2 + 4 - 10x$$

$$\text{Then, } f'(x) = -18(3)^2 + 4 - 10(3) \\ = -188$$

b. $f'(3)$ is the slope of the tangent line to $f(x)$ at $x = 3$ and the rate of change in the value of $f(x)$ with respect to x at $x = 3$.

$$\mathbf{17. a. } P(t) = 100 + 120t + 10t^2 + 2t^3$$

$$P(t) = 100 + 120t + 10t^2 + 2t^3$$

$$P(0) = 100 + 120(0) + 10(0)^2 + 2(0)^3 \\ = 100 \text{ bacteria}$$

b. At 5 h, the population is

$$P(5) = 100 + 120(5) + 10(5)^2 + 2(5)^3 \\ = 1200 \text{ bacteria}$$

$$\mathbf{c. } P'(t) = 120 + 20t + 6t^2$$

At 5 h, the colony is growing at

$$P'(5) = 120 + 20(5) + 6(5)^2 \\ = 370 \text{ bacteria/h}$$

$$\mathbf{18. } C(t) = \frac{100}{t}, t > 2$$

$$\text{Simplifying, } C(t) = 100t^{-1}.$$

$$\text{Then, } C'(t) = -100t^{-2} = -\frac{100}{t^2}.$$

$$\begin{array}{lll}
C'(5) & C'(50) & C'(100) \\
= -\frac{100}{(5)^2} & = -\frac{100}{(50)^2} & = -\frac{100}{(100)^2} \\
= -\frac{100}{25} & = -\frac{100}{2500} & = -\frac{1}{100} \\
= -4 & = -0.04 & = -0.01
\end{array}$$

These are the rates of change of the percentage with respect to time at 5, 50, and 100 min. The percentage of carbon dioxide that is released per unit time from the pop is decreasing. The pop is getting flat.

2.4 The Quotient Rule, pp. 97–98

1. For x, a, b real numbers,

$$x^a x^b = x^{a+b}$$

For example,

$$x^9 x^{-6} = x^3$$

Also,

$$(x^a)^b = x^{ab}$$

For example,

$$(x^2)^3 = x^6$$

Also,

$$\frac{x^a}{x^b} = x^{a-b}, x \neq 0$$

For example,

$$\frac{x^5}{x^3} = x^2$$

2.

Function	Rewrite	Differentiate and Simplify, If Necessary
$f(x) = \frac{x^2 + 3x}{x}, x \neq 0$	$f(x) = x + 3$	$f'(x) = 1$
$g(x) = \frac{3x^{\frac{2}{3}}}{x}, x \neq 0$	$g(x) = 3x^{\frac{2}{3}}$	$g'(x) = 2x^{-\frac{1}{3}}$
$h(x) = \frac{1}{10x^5}, x \neq 0$	$h(x) = \frac{1}{10}x^{-5}$	$h'(x) = \frac{-1}{2}x^{-6}$
$y = \frac{8x^3 + 6x}{2x}, x \neq 0$	$y = 4x^2 + 3$	$\frac{dy}{dx} = 8x$
$s = \frac{t^2 - 9}{t - 3}, t \neq 3$	$s = t + 3$	$\frac{ds}{dt} = 1$

3. In the previous problem, all of these rational examples could be differentiated via the power rule after a minor algebraic simplification.

A second approach would be to rewrite a rational example

$$h(x) = \frac{f(x)}{g(x)}$$

using the exponent rules as

$$h(x) = f(x)(g(x))^{-1},$$

and then apply the product rule for differentiation (together with the power of a function rule to find $h'(x)$).

A third (and perhaps easiest) approach would be to just apply the quotient rule to find $h'(x)$.

$$\begin{aligned} \textbf{4. a. } h'(x) &= \frac{(x+1)(1) - x(1)}{(x+1)^2} \\ &= \frac{1}{(x+1)^2} \end{aligned}$$

$$\begin{aligned} \textbf{b. } h'(t) &= \frac{(t+5)(2) - (2t-3)(1)}{(t+5)^2} \\ &= \frac{13}{(t+5)^2} \end{aligned}$$

$$\begin{aligned} \textbf{c. } h'(x) &= \frac{(2x^2-1)(3x^2) - x^3(4x)}{(2x^2-1)^2} \\ &= \frac{2x^4 - 3x^2}{(2x^2-1)^2} \end{aligned}$$

$$\begin{aligned} \textbf{d. } h'(x) &= \frac{(x^2+3)(0) - 1(2x)}{(x^2+3)^2} \\ &= \frac{-2x}{(x^2+3)^2} \end{aligned}$$

$$\begin{aligned} \textbf{e. } y &= \frac{x(3x+5)}{(1-x^2)} = \frac{3x^2 + 5x}{1-x^2} \\ \frac{dy}{dx} &= \frac{(6x+5)(1-x^2) - (3x^2+5x)(-2x)}{(1-x^2)^2} \\ &= \frac{6x+5 - 6x^3 - 5x^2 + 6x^3 + 10x^2}{(1-x^2)^2} \\ &= \frac{5x^2 + 6x + 5}{(1-x^2)^2} \end{aligned}$$

$$\begin{aligned} \textbf{f. } \frac{dy}{dx} &= \frac{(x^2+3)(2x-1) - (x^2-x+1)(2x)}{(x^2+3)^2} \\ &= \frac{2x^3 + 6x - x^2 - 3 - 2x^3 + 2x^2 - 2x}{(x^2+3)^2} \\ &= \frac{x^2 + 4x - 3}{(x^2+3)^2} \end{aligned}$$

$$\textbf{5. a. } y = \frac{3x+2}{x+5}, x = -3$$

$$\frac{dy}{dx} = \frac{(x+5)(3) - (3x+2)(1)}{(x+5)^2}$$

At $x = -3$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{(2)(3) - (-7)(1)}{(2)^2} \\ &= \frac{13}{4} \end{aligned}$$

$$\textbf{b. } y = \frac{x^3}{x^2 + 9}, x = 1$$

$$\frac{dy}{dx} = \frac{(x^2+9)(3x^2) - (x^3)(2x)}{(x^2+9)^2}$$

At $x = 1$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{(10)(3) - (1)(2)}{(10)^2} \\ &= \frac{28}{100} \\ &= \frac{7}{25} \end{aligned}$$

$$\textbf{c. } y = \frac{x^2 - 25}{x^2 + 25}, x = 2$$

$$\frac{dy}{dx} = \frac{2x(x^2+25) - (x^2-25)(2x)}{(x^2+25)^2}$$

At $x = 2$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{4(29) - (-21)(4)}{(29)^2} \\ &= \frac{116 + 84}{29^2} \\ &= \frac{200}{841} \end{aligned}$$

$$\textbf{d. } y = \frac{(x+1)(x+2)}{(x-1)(x-2)}, x = 4$$

$$\begin{aligned} &= \frac{x^2 + 3x + 2}{x^2 - 3x + 2} \\ \frac{dy}{dx} &= \frac{(2x+3)(x^2-3x+2)}{(x-1)^2(x-2)^2} \\ &\quad - \frac{(x^2+3x+2)(2x-3)}{(x-1)^2(x-2)^2} \end{aligned}$$

At $x = 4$:

$$\begin{aligned} \frac{dy}{dx} &= \frac{(11)(6) - (30)(5)}{(9)(4)} \\ &= -\frac{84}{36} \\ &= -\frac{7}{3} \end{aligned}$$

6. $y = \frac{x^3}{x^2 - 6}$

$$\frac{dy}{dx} = \frac{3x^2(x^2 - 6) - x^3(2x)}{(x^2 - 6)^2}$$

At $(3, 9)$:

$$\begin{aligned}\frac{dy}{dx} &= \frac{3(9)(3) - (27)(6)}{(3)^2} \\ &= 9 - 18 \\ &= -9\end{aligned}$$

The slope of the tangent to the curve at $(3, 9)$ is -9 .

7. $y = \frac{3x}{x - 4}$

$$\frac{dy}{dx} = \frac{3(x - 4) - 3x}{(x - 4)^2} = -\frac{12}{(x - 4)^2}$$

Slope of the tangent is $-\frac{12}{25}$.

Therefore, $\frac{12}{(x - 4)^2} = \frac{12}{25}$

$x - 4 = 5$ or $x - 4 = -5$

$x = 9$ or $x = -1$

Points are $(9, \frac{27}{5})$ and $(-1, \frac{3}{5})$.

8. $f(x) = \frac{5x + 2}{x + 2}$

$$f'(x) = \frac{(x + 2)(5) - (5x + 2)(1)}{(x + 2)^2}$$

$$f'(x) = \frac{8}{(x + 2)^2}$$

Since $(x + 2)^2$ is positive or zero for all $x \in \mathbf{R}$,

$$\frac{8}{(x + 2)^2} > 0 \text{ for } x \neq -2. \text{ Therefore, tangents to}$$

the graph of $f(x) = \frac{5x + 2}{x + 2}$ do not have a negative slope.

9. a. $y = \frac{2x^2}{x - 4}, x \neq 4$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x - 4)(4x) - (2x^2)(1)}{(x - 4)^2} \\ &= \frac{4x^2 - 16x - 2x^2}{(x - 4)^2} \\ &= \frac{2x^2 - 16x}{(x - 4)^2} \\ &= \frac{2x(x - 8)}{(x - 4)^2}\end{aligned}$$

Curve has horizontal tangents when $\frac{dy}{dx} = 0$, or when $x = 0$ or 8 . At $x = 0$:

$$\begin{aligned}y &= \frac{0}{-4} \\ &= 0\end{aligned}$$

At $x = 8$:

$$\begin{aligned}y &= \frac{2(8)^2}{4} \\ &= 32\end{aligned}$$

So the curve has horizontal tangents at the points $(0, 0)$ and $(8, 32)$.

b. $y = \frac{x^2 - 1}{x^2 + x - 2}$

$$\begin{aligned}&= \frac{(x - 1)(x + 1)}{(x + 2)(x - 1)} \\ &= \frac{x + 1}{x + 2}, x \neq 1\end{aligned}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{(x + 2) - (x + 1)}{(x + 2)^2} \\ &= \frac{1}{(x + 2)^2}\end{aligned}$$

Curve has horizontal tangents when $\frac{dy}{dx} = 0$.

No value of x will produce a slope of 0, so there are no horizontal tangents.

10. $p(t) = 1000 \left(1 + \frac{4t}{t^2 + 50}\right)$

$$\begin{aligned}p'(t) &= 1000 \left(\frac{4(t^2 + 50) - 4t(2t)}{(t^2 + 50)^2}\right) \\ &= \frac{1000(200 - 4t^2)}{(t^2 + 50)^2}\end{aligned}$$

$$p'(1) = \frac{1000(196)}{(51)^2} = 75.36$$

$$p'(2) = \frac{1000(184)}{(54)^2} = 63.10$$

Population is growing at a rate of 75.4 bacteria per hour at $t = 1$ and at 63.1 bacteria per hour at $t = 2$.

11. $y = \frac{x^2 - 1}{3x}$

$$= \frac{1}{3}x - \frac{1}{3}x^{-1}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{1}{3} + \frac{1}{3}x^{-2} \\ &= \frac{1}{3} + \frac{1}{3x^2}\end{aligned}$$

At $x = 2$:

$$\begin{aligned}y &= \frac{(2)^2 - 1}{3(2)} \\ &= \frac{1}{2}\end{aligned}$$

and

$$\frac{dy}{dx} = \frac{1}{3} + \frac{1}{3(2)^2}$$

$$= \frac{1}{3} + \frac{1}{12}$$

$$= \frac{5}{12}$$

So the equation of the tangent to the curve at $x = 2$ is:

$$y - \frac{1}{2} = \frac{5}{12}(x - 2), \text{ or } 5x - 12y - 4 = 0.$$

12. a. $s(t) = \frac{10(6-t)}{t+3}, 0 \leq t \leq 6, t = 0,$

$$s(0) = 20$$

The boat is initially 20 m from the dock.

b. $v(t) = s'(t) = 10 \left[\frac{(t+3)(-1) - (6-t)(1)}{(t+3)^2} \right]$

$$v(t) = \frac{-90}{(t+3)^2}$$

At $t = 0$, $v(0) = -10$, the boat is moving towards the dock at a speed of 10 m/s. When $s(t) = 0$, the boat will be at the dock.

$$\frac{10(6-t)}{t+3} = 0, t = 6.$$

$$v(6) = \frac{-90}{9^2} = -\frac{10}{9}$$

The speed of the boat when it bumps into the dock is $\frac{10}{9}$ m/s.

13. a. i. $t = 0$

$$r(0) = \frac{1+2(0)}{1+0}$$

$$= 1 \text{ cm}$$

ii. $\frac{1+2t}{1+t} = 1.5$

$$1+2t = 1.5(1+t)$$

$$1+2t = 1.5 + 1.5t$$

$$0.5t = 0.5$$

$$t = 1 \text{ s}$$

iii. $r'(t) = \frac{(1+t)(2) - (1+2t)(1)}{(1+t)^2}$

$$= \frac{2+2t-1-2t}{(1+t)^2}$$

$$= \frac{1}{(1+t)^2}$$

$$r'(1.5) = \frac{1}{(1+1)^2}$$

$$= \frac{1}{4}$$

$$= 0.25 \text{ cm/s}$$

b. No, the radius will never reach 2 cm, because $y = 2$ is a horizontal asymptote of the graph of the function. Therefore, the radius approaches but never equals 2 cm.

14. $f(x) = \frac{ax+b}{(x-1)(x-4)}$

$$f'(x) = \frac{(x-1)(x-4)(a)}{(x-1)^2(x-4)^2}$$

$$-\frac{(ax+b)\frac{d}{dx}[(x-1)(x-4)]}{(x-1)^2(x-4)^2}$$

$$= \frac{(x-1)(x-4)(a)}{(x-1)^2(x-4)^2}$$

$$-\frac{(ax+b)[(x-1)+(x-4)]}{(x-1)^2(x-4)^2}$$

$$= \frac{(x^2-5x+4)(a)-(ax+b)(2x-5)}{(x-1)^2(x-4)^2}$$

$$= \frac{-ax^2-2bx+4a+5b}{(x-1)^2(x-4)^2}$$

Since the point $(2, -1)$ is on the graph (as it's on the tangent line) we know that

$$-1 = f(2)$$

$$= \frac{2a+b}{(1)(-2)}$$

$$2 = 2a+b$$

$$b = 2 - 2a$$

Also, since the tangent line is horizontal at $(2, -1)$, we know that

$$0 = f'(2)$$

$$= \frac{-a(2)^2 - 2b(2) + 4a + 5b}{(1)^2(-2)^2}$$

$$b = 0$$

$$0 = 2 - 2a$$

$$a = 1$$

So we get

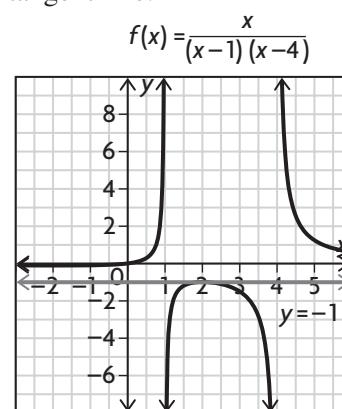
$$f(x) = \frac{x}{(x-1)(x-4)}$$

Since the tangent line is horizontal at the point

$(2, -1)$, the equation of this tangent line is

$$y - (-1) = 0(x - 2), \text{ or } y = -1$$

Here are the graphs of both $f(x)$ and this horizontal tangent line:



$$\begin{aligned}
 15. c'(t) &= \frac{(2t^2 + 7)(5) - (5t)(4t)}{(2t^2 + 7)^2} \\
 &= \frac{10t^2 + 35 - 20t^2}{(2t^2 + 7)^2} \\
 &= \frac{-10t^2 + 35}{(2t^2 + 7)^2}
 \end{aligned}$$

Set $c'(t) = 0$ and solve for t .

$$\begin{aligned}
 \frac{-10t^2 + 35}{(2t^2 + 7)^2} &= 0 \\
 -10t^2 + 35 &= 0 \\
 10t^2 &= 35 \\
 t^2 &= 3.5 \\
 t &= \pm\sqrt{3.5} \\
 t &\doteq \pm 1.87
 \end{aligned}$$

To two decimal places, $t = -1.87$ or $t = 1.87$, because $s'(t) = 0$ for these values. Reject the negative root in this case because time is positive ($t \geq 0$). Therefore, the concentration reaches its maximum value at $t = 1.87$ hours.

16. When the object changes direction, its velocity changes sign.

$$\begin{aligned}
 s'(t) &= \frac{(t^2 + 8)(1) - t(2t)}{(t^2 + 8)^2} \\
 &= \frac{t^2 + 8 - 2t^2}{(t^2 + 8)^2} \\
 &= \frac{-t^2 + 8}{(t^2 + 8)^2}
 \end{aligned}$$

solve for t when $s'(t) = 0$.

$$\begin{aligned}
 \frac{-t^2 + 8}{(t^2 + 8)^2} &= 0 \\
 -t^2 + 8 &= 0 \\
 t^2 &= 8 \\
 t &= \pm\sqrt{8} \\
 t &\doteq \pm 2.83
 \end{aligned}$$

To two decimal places, $t = 2.83$ or $t = -2.83$, because $s'(t) = 0$ for these values. Reject the negative root because time is positive ($t \geq 0$). The object changes direction when $t = 2.83$ s.

$$\begin{aligned}
 17. f(x) &= \frac{ax + b}{cx + d}, x \neq -\frac{d}{c} \\
 f'(x) &= \frac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} \\
 f'(x) &= \frac{ad - bc}{(cx + d)^2}
 \end{aligned}$$

For the tangents to the graph of $y = f(x)$ to have positive slopes, $f'(x) > 0$. $(cx + d)^2$ is positive for all $x \in \mathbf{R}$. $ad - bc > 0$ will ensure each tangent has a positive slope.

2.5 The Derivatives of Composite Functions, pp. 105–106

1. $f(x) = \sqrt{x}$, $g(x) = x^2 - 1$

$$\begin{aligned}
 \mathbf{a. } f(g(1)) &= f(1 - 1) \\
 &= f(0) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{b. } g(f(1)) &= g(1) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c. } g(f(0)) &= g(0) \\
 &= 0 - 1 \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d. } f(g(-4)) &= f(16 - 1) \\
 &= f(15) \\
 &= \sqrt{15}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{e. } f(g(x)) &= f(x^2 - 1) \\
 &= \sqrt{x^2 - 1}
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{f. } g(f(x)) &= g(\sqrt{x}) \\
 &= (\sqrt{x})^2 - 1 \\
 &= x - 1
 \end{aligned}$$

2. a. $f(x) = x^2$, $g(x) = \sqrt{x}$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= f(\sqrt{x}) \\
 &= (\sqrt{x})^2 \\
 &= x
 \end{aligned}$$

Domain = $\{x \geq 0\}$

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= g(x^2) \\
 &= \sqrt{x^2} \\
 &= |x|
 \end{aligned}$$

Domain = $\{x \in \mathbf{R}\}$

The composite functions are not equal for negative x -values (as $(f \circ g)$ is not defined for these x), but are equal for non-negative x -values.

b. $f(x) = \frac{1}{x}$, $g(x) = x^2 + 1$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= f(x^2 + 1) \\
 &= \frac{1}{x^2 + 1}
 \end{aligned}$$

Domain = $\{x \in \mathbf{R}\}$

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= g\left(\frac{1}{x}\right) \\
 &= \left(\frac{1}{x}\right)^2 + 1
 \end{aligned}$$

$$\begin{aligned}
 15. c'(t) &= \frac{(2t^2 + 7)(5) - (5t)(4t)}{(2t^2 + 7)^2} \\
 &= \frac{10t^2 + 35 - 20t^2}{(2t^2 + 7)^2} \\
 &= \frac{-10t^2 + 35}{(2t^2 + 7)^2}
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Set $c'(t) = 0$ and solve for t .

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 t^2 &= 3.5 \\
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 t &\doteq \pm 1.87
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To two decimal places, $t = -1.87$ or $t = 1.87$, because $s'(t) = 0$ for these values. Reject the negative root in this case because time is positive ($t \geq 0$). Therefore, the concentration reaches its maximum value at $t = 1.87$ hours.

16. When the object changes direction, its velocity changes sign.

$$\begin{aligned}
 s'(t) &= \frac{(t^2 + 8)(1) - t(2t)}{(t^2 + 8)^2} \\
 &= \frac{t^2 + 8 - 2t^2}{(t^2 + 8)^2} \\
 &= \frac{-t^2 + 8}{(t^2 + 8)^2}
 \end{aligned}$$

solve for t when $s'(t) = 0$.

$$\begin{aligned}
 \frac{-t^2 + 8}{(t^2 + 8)^2} &= 0 \\
 -t^2 + 8 &= 0 \\
 t^2 &= 8 \\
 t &= \pm\sqrt{8} \\
 t &\doteq \pm 2.83
 \end{aligned}$$

To two decimal places, $t = 2.83$ or $t = -2.83$, because $s'(t) = 0$ for these values. Reject the negative root because time is positive ($t \geq 0$). The object changes direction when $t = 2.83$ s.

$$\begin{aligned}
 17. f(x) &= \frac{ax + b}{cx + d}, x \neq -\frac{d}{c} \\
 f'(x) &= \frac{(cx + d)(a) - (ax + b)(c)}{(cx + d)^2} \\
 f'(x) &= \frac{ad - bc}{(cx + d)^2}
 \end{aligned}$$

For the tangents to the graph of $y = f(x)$ to have positive slopes, $f'(x) > 0$. $(cx + d)^2$ is positive for all $x \in \mathbf{R}$. $ad - bc > 0$ will ensure each tangent has a positive slope.

2.5 The Derivatives of Composite Functions, pp. 105–106

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$$\begin{aligned}
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 &= f(0) \\
 &= 0
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 \mathbf{b. } g(f(1)) &= g(1) \\
 &= 0
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{c. } g(f(0)) &= g(0) \\
 &= 0 - 1 \\
 &= -1
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d. } f(g(-4)) &= f(16 - 1) \\
 &= f(15) \\
 &= \sqrt{15}
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$$\begin{aligned}
 \mathbf{e. } f(g(x)) &= f(x^2 - 1) \\
 &= \sqrt{x^2 - 1}
 \end{aligned}$$

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 \mathbf{f. } g(f(x)) &= g(\sqrt{x}) \\
 &= (\sqrt{x})^2 - 1 \\
 &= x - 1
 \end{aligned}$$

2. a. $f(x) = x^2$, $g(x) = \sqrt{x}$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= f(\sqrt{x}) \\
 &= (\sqrt{x})^2 \\
 &= x
 \end{aligned}$$

Domain = $\{x \geq 0\}$

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= g(x^2) \\
 &= \sqrt{x^2} \\
 &= |x|
 \end{aligned}$$

Domain = $\{x \in \mathbf{R}\}$

The composite functions are not equal for negative x -values (as $(f \circ g)$ is not defined for these x), but are equal for non-negative x -values.

b. $f(x) = \frac{1}{x}$, $g(x) = x^2 + 1$

$$\begin{aligned}
 (f \circ g)(x) &= f(g(x)) \\
 &= f(x^2 + 1) \\
 &= \frac{1}{x^2 + 1}
 \end{aligned}$$

Domain = $\{x \in \mathbf{R}\}$

$$\begin{aligned}
 (g \circ f)(x) &= g(f(x)) \\
 &= g\left(\frac{1}{x}\right) \\
 &= \left(\frac{1}{x}\right)^2 + 1
 \end{aligned}$$

$$= \frac{1}{x^2} + 1$$

$$\text{Domain} = \{x \neq 0\}$$

The composite functions are not equal here. For instance, $(f \circ g)(1) = \frac{1}{2}$ and $(g \circ f)(1) = 2$.

c. $f(x) = \frac{1}{x}, g(x) = \sqrt{x+2}$

$$(f \circ g)(x) = f(g(x)) \\ = f(\sqrt{x+2}) \\ = \frac{1}{\sqrt{x+2}}$$

$$\text{Domain} = \{x > -2\}$$

$$(g \circ f)(x) = g(f(x))$$

$$= g\left(\frac{1}{x}\right) \\ = \sqrt{\frac{1}{x} + 2}$$

The domain is all x such that

$$\frac{1}{x} + 2 \geq 0 \text{ and } x \neq 0, \text{ or equivalently}$$

$$\text{Domain} = \{x \leq -\frac{1}{2} \text{ or } x > 0\}$$

The composite functions are not equal here. For

instance, $(f \circ g)(2) = \frac{1}{2}$ and $(g \circ f)(2) = \sqrt{\frac{5}{2}}$.

3. If $f(x)$ and $g(x)$ are two differentiable functions of x , and

$$h(x) = (f \circ g)(x) \\ = f(g(x))$$

is the composition of these two functions, then

$$h'(x) = f'(g(x)) \cdot g'(x)$$

This is known as the “chain rule” for differentiation of composite functions. For example, if $f(x) = x^{10}$ and $g(x) = x^2 + 3x + 5$, then $h(x) = (x^2 + 3x + 5)^{10}$, and so

$$h'(x) = f'(g(x)) \cdot g'(x) \\ = 10(x^2 + 3x + 5)^9(2x + 3)$$

As another example, if $f(x) = x^{\frac{2}{3}}$ and $g(x) = x^2 + 1$, then $h(x) = (x^2 + 1)^{\frac{2}{3}}$, and so

$$h'(x) = \frac{2}{3}(x^2 + 1)^{-\frac{1}{3}}(2x)$$

4. a. $f(x) = (2x + 3)^4$
 $f'(x) = 4(2x + 3)^3(2)$
 $= 8(2x + 3)^3$

b. $g(x) = (x^2 - 4)^3$
 $g'(x) = 3(x^2 - 4)^2(2x)$
 $= 6x(x^2 - 4)^2$

c. $h(x) = (2x^2 + 3x - 5)^4$
 $h'(x) = 4(2x^2 + 3x - 5)^3(4x + 3)$

d. $f(x) = (\pi^2 - x^2)^3$
 $f'(x) = 3(\pi^2 - x^2)^2(-2x)$
 $= -6x(\pi^2 - x^2)^2$

e. $y = \sqrt{x^2 - 3}$
 $= (x^2 - 3)^{\frac{1}{2}}$
 $y' = \frac{1}{2}(x^2 - 3)^{\frac{1}{2}}(2x)$
 $= \frac{x}{\sqrt{x^2 - 3}}$

f. $f(x) = \frac{1}{(x^2 - 16)^5}$
 $= (x^2 - 16)^{-5}$
 $f'(x) = -5(x^2 - 16)^{-6}(2x)$
 $= \frac{-10x}{(x^2 - 16)^6}$

5. a. $y = -\frac{2}{x^3}$
 $= -2x^{-3}$

$$\frac{dy}{dx} = (-2)(-3)x^{-4}$$

 $= \frac{6}{x^4}$

b. $y = \frac{1}{x+1}$
 $= (x+1)^{-1}$

$$\frac{dy}{dx} = (-1)(x+1)^{-2}(1)$$

 $= \frac{-1}{(x+1)^2}$

c. $y = \frac{1}{x^2 - 4}$
 $= (x^2 - 4)^{-1}$

$$\frac{dy}{dx} = (-1)(x^2 - 4)^{-2}(2x)$$

 $= \frac{-2x}{(x^2 - 4)^2}$

d. $y = \frac{3}{9 - x^2} = 3(9 - x^2)^{-1}$

$$\frac{dy}{dx} = \frac{6x}{(9 - x^2)^2}$$

$$\mathbf{e.} \quad y = \frac{1}{5x^2 + x} \\ = (5x^2 + x)^{-1}$$

$$\begin{aligned}\frac{dy}{dx} &= (-1)(5x^2 + x)^{-2}(10x + 1) \\ &= -\frac{10x + 1}{(5x^2 + x)^2}\end{aligned}$$

$$\mathbf{f.} \quad y = \frac{1}{(x^2 + x + 1)^4} \\ = (x^2 + x + 1)^{-4}$$

$$\begin{aligned}\frac{dy}{dx} &= (-4)(x^2 + x + 1)^{-5}(2x + 1) \\ &= -\frac{8x + 4}{(x^2 + x + 1)^5}\end{aligned}$$

$$\begin{aligned}\mathbf{6.} \quad h &= g \circ f \\ &= g(f(x)) \\ h(-1) &= g(f(-1)) \\ &= g(1) \\ &= -4\end{aligned}$$

$$\begin{aligned}h(x) &= g(f(x)) \\ h'(x) &= g'(f(x))f'(x) \\ h'(-1) &= g'(f(-1))f'(-1) \\ &= g'(1)(-5) \\ &= (-7)(-5) \\ &= 35\end{aligned}$$

$$\begin{aligned}\mathbf{7.} \quad f(x) &= (x - 3)^2, g(x) = \frac{1}{x}, h(x) = f(g(x)), \\ f'(x) &= 2(x - 3), g'(x) = -\frac{1}{x^2}\end{aligned}$$

$$\begin{aligned}h'(x) &= f'(g(x))g'(x) \\ &= f'\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) \\ &= 2\left(\frac{1}{x} - 3\right)\left(-\frac{1}{x^2}\right) \\ &= -\frac{2}{x^2}\left(\frac{1}{x} - 3\right)\end{aligned}$$

$$\begin{aligned}\mathbf{8. a.} \quad f(x) &= (x + 4)^3(x - 3)^6 \\ f'(x) &= \frac{d}{dx}[(x + 4)^3] \cdot (x - 3)^6 \\ &\quad + (x + 4)^3 \frac{d}{dx}[(x - 3)^6] \\ &= 3(x + 4)^2(x - 3)^6 \\ &\quad + (x + 4)^3(6)(x - 3)^5 \\ &= (x + 4)^2(x - 3)^5 \\ &\quad \times [3(x - 3) + 6(x + 4)] \\ &= (x + 4)^2(x - 3)^5(9x + 15)\end{aligned}$$

$$\begin{aligned}\mathbf{b.} \quad y &= (x^2 + 3)^3(x^3 + 3)^2 \\ \frac{dy}{dx} &= \frac{d}{dx}[(x^2 + 3)^3] \cdot (x^3 + 3)^2 \\ &\quad + (x^2 + 3)^3 \cdot \frac{d}{dx}[(x^3 + 3)^2] \\ &= 3(x^2 + 3)^2(2x)(x^3 + 3)^2 \\ &\quad + (x^2 + 3)^3(2)(x^3 + 3)(3x^2) \\ &= 6x(x^2 + 3)^2(x^3 + 3)[(x^3 + 3) + x(x^2 + 3)] \\ &= 6x(x^2 + 3)^2(x^3 + 3)(2x^3 + 3x + 3)\end{aligned}$$

$$\begin{aligned}\mathbf{c.} \quad y &= \frac{3x^2 + 2x}{x^2 + 1} \\ \frac{dy}{dx} &= \frac{(6x + 2)(x^2 + 1) - (3x^2 + 2x)(2x)}{(x^2 + 1)^2} \\ &= \frac{6x^3 + 2x^2 + 6x + 2 - 6x^3 - 4x^2}{(x^2 + 1)^2} \\ &= \frac{-2x^2 + 6x + 2}{(x^2 + 1)^2}\end{aligned}$$

$$\begin{aligned}\mathbf{d.} \quad h(x) &= x^3(3x - 5)^2 \\ h'(x) &= \frac{d}{dx}[x^3] \cdot (3x - 5)^2 + x^3 \frac{d}{dx}[(3x - 5)^2] \\ &= 3x^2(3x - 5)^2 + x^3(2)(3x - 5)(3) \\ &= 3x^2(3x - 5)[(3x - 5) + 2x] \\ &= 3x^2(3x - 5)(5x - 5) \\ &= 15x^2(3x - 5)(x - 1)\end{aligned}$$

$$\begin{aligned}\mathbf{e.} \quad y &= x^4(1 - 4x^2)^3 \\ \frac{dy}{dx} &= \frac{d}{dx}[x^4](1 - 4x^2)^3 + x^4 \cdot \frac{d}{dx}[(1 - 4x^2)^3] \\ &= 4x^3(1 - 4x^2)^3 + x^4(3)(1 - 4x^2)^2(-8x) \\ &= 4x^3(1 - 4x^2)^2[(1 - 4x^2) - 6x^2] \\ &= 4x^3(1 - 4x^2)^2(1 - 10x^2)\end{aligned}$$

$$\begin{aligned}\mathbf{f.} \quad y &= \left(\frac{x^2 - 3}{x^2 + 3}\right)^4 \\ \frac{dy}{dx} &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \frac{d}{dx}\left[\frac{x^2 - 3}{x^2 + 3}\right] \\ &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \cdot \frac{(x^2 + 3)(2x) - (x^2 - 3)(2x)}{(x^2 + 3)^2} \\ &= 4\left(\frac{x^2 - 3}{x^2 + 3}\right)^3 \cdot \frac{12x}{(x^2 + 3)^2} \\ &= \frac{48x(x^2 - 3)^3}{(x^2 + 3)^5}\end{aligned}$$

$$\begin{aligned}\mathbf{9. a.} \quad s(t) &= t^{\frac{1}{3}}(4t - 5)^{\frac{2}{3}} \\ &= t^{\frac{1}{3}}[4t - 5]^{\frac{2}{3}} \\ &= [t(4t - 5)^2]^{\frac{1}{3}} \\ &= [t(16t^2 - 40t + 25)]^{\frac{1}{3}} \\ &= (16t^3 - 40t^2 + 25t)^{\frac{1}{3}}, t = 8\end{aligned}$$

$$\begin{aligned}s'(t) &= \frac{1}{3}(16t^3 - 40t^2 + 25t)^{-\frac{2}{3}} \\&\quad \times (48t^2 - 80t + 25) \\&= \frac{(48t^2 - 80t + 25)}{3(16t^3 - 40t^2 + 25t)^{\frac{2}{3}}}\end{aligned}$$

Rate of change at $t = 8$:

$$\begin{aligned}s'(8) &= \frac{(48(8)^2 - 80(8) + 25)}{3(16(8)^3 - 40(8)^2 + 25(8))^{\frac{2}{3}}} \\&= \frac{2457}{972} \\&= \frac{91}{36}\end{aligned}$$

b. $s(t) = \left(\frac{t-\pi}{t-6\pi}\right)^{\frac{1}{3}}, t = 2\pi$

$$\begin{aligned}s'(t) &= \frac{1}{3}\left(\frac{t-\pi}{t-6\pi}\right)^{-\frac{2}{3}} \cdot \frac{d}{dt}\left[\frac{t-\pi}{t-6\pi}\right] \\&= \frac{1}{3}\left(\frac{t-6\pi}{t-\pi}\right)^{\frac{2}{3}} \cdot \frac{(t-6\pi)-(t-\pi)}{(t-6\pi)^2} \\&= \frac{1}{3}\left(\frac{t-6\pi}{t-\pi}\right)^{\frac{2}{3}} \cdot \frac{-5\pi}{(t-6\pi)^2}\end{aligned}$$

Rate of change at $t = 2\pi$:

$$\begin{aligned}s'(2\pi) &= \frac{1}{3}(-4)^{\frac{2}{3}} \cdot \frac{-5\pi}{16\pi^2} \\&= -\frac{5\sqrt[3]{2}}{24\pi}\end{aligned}$$

10. $y = (1+x^3)^2 \quad y = 2x^6$

$$\frac{dy}{dx} = 2(1+x^3)(3x^2) \quad \frac{dy}{dx} = 12x^5$$

For the same slope,

$$6x^2(1+x^3) = 12x^5$$

$$6x^2 + 6x^5 = 12x^5$$

$$6x^2 - 6x^5 = 0$$

$$6x^2(x^3 - 1) = 0$$

$$x = 0 \text{ or } x = 1.$$

Curves have the same slope at $x = 0$ and $x = 1$.

11. $y = (3x-x^2)^{-2}$

$$\frac{dy}{dx} = -2(3x-x^2)^{-3}(3-2x)$$

At $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= -2[6-4]^{-3}(3-4) \\&= 2(2)^{-3} \\&= \frac{1}{4}\end{aligned}$$

The slope of the tangent line at $x = 2$ is $\frac{1}{4}$.

12. $y = (x^3 - 7)^5$ at $x = 2$

$$\frac{dy}{dx} = 5(x^3 - 7)^4(3x^2)$$

When $x = 2$,

$$\begin{aligned}\frac{dy}{dx} &= 5(1)^4(12) \\&= 60\end{aligned}$$

Slope of the tangent is 60.

Equation of the tangent at $(2, 1)$ is

$$y - 1 = 60(x - 2)$$

$$60x - y - 119 = 0.$$

13. a. $y = 3u^2 - 5u + 2$

$$u = x^2 - 1, x = 2$$

$$u = 3$$

$$\frac{dy}{du} = 6u - 5, \frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= (6u - 5)(2x)$$

$$= (18 - 5)(4)$$

$$= 13(4)$$

$$= 52$$

b. $y = 2u^3 + 3u^2, u = x + x^{\frac{1}{2}}, x = 1$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= (6u^2 + 6u)\left(1 + \frac{1}{2\sqrt{x}}\right)$$

At $x = 1$:

$$u = 1 + 1^{\frac{1}{2}}$$

$$= 2$$

$$\frac{dy}{dx} = (6(2)^2 + 6(2))\left(1 + \frac{1}{2\sqrt{1}}\right)$$

$$= 36 \times \frac{3}{2}$$

$$= 54$$

c. $y = u(u^2 + 3)^3, u = (x+3)^2, x = -2$

$$\frac{dy}{du} = (u^2 + 3)^3 + 6u^2(u^2 + 3)^2, \frac{du}{dx} = 2(x+3)$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = [7^3 + 6(4)^2][2(1)]$$

$$= 439 \times 2$$

$$= 878$$

d. $y = u^3 - 5(u^3 - 7u)^2, u = \sqrt{x}$

$$= x^{\frac{1}{2}}, x = 4$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= 3[3u^2 - 10(u^3 - 7u)(3u^2 - 7)] \cdot \left(\frac{1}{2}x^{\frac{1}{2}}\right)$$

$$= [3u^2 - 10(u^3 - 7u)(3u^2 - 7)] \cdot \frac{1}{2\sqrt{x}}$$

At $x = 4$:

$$\begin{aligned} u &= \sqrt{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \frac{dy}{dx} &= [3(2)^2 - 10((2)^3 - 7(2))(3(2)^2 - 7)] \frac{1}{2(2)} \\ &= 78 \end{aligned}$$

14. $h(x) = f(g(x))$, therefore

$$\begin{aligned} h'(x) &= f'(g(x)) \times g'(x) \\ f(u) &= u^2 - 1, g(2) = 3, g'(2) = -1 \end{aligned}$$

$$\begin{aligned} \text{Now, } h'(2) &= f'(g(2)) \times g'(2) \\ &= f'(3) \times g'(2). \end{aligned}$$

$$\begin{aligned} \text{Since } f(u) &= u^2 - 1, f'(u) = 2u, \text{ and } f'(3) = 6, \\ h'(2) &= 6(-1) \\ &= -6. \end{aligned}$$

$$\mathbf{15. } V(t) = 50000 \left(1 - \frac{t}{30}\right)^2$$

$$V'(t) = 50000 \left[2 \left(1 - \frac{t}{30}\right) \left(-\frac{1}{30}\right)\right]$$

$$\begin{aligned} V'(10) &= 50000 \left[2 \left(1 - \frac{10}{30}\right) \left(-\frac{1}{30}\right)\right] \\ &= 50000 \left[2 \left(\frac{2}{3}\right) \left(-\frac{1}{30}\right)\right] \\ &\doteq 2222 \end{aligned}$$

At $t = 10$ minutes, the water is flowing out of the tank at a rate of 2222 L/min.

16. The velocity function is the derivative of the position function.

$$s(t) = (t^3 + t^2)^{\frac{1}{2}}$$

$$v(t) = s'(t) = \frac{1}{2}(t^3 + t^2)^{-\frac{1}{2}}(3t^2 + 2t)$$

$$= \frac{3t^2 + 2t}{2\sqrt{t^3 + t^2}}$$

$$v(3) = \frac{3(3)^2 + 2(3)}{2\sqrt{3^3 + 3^2}}$$

$$= \frac{27 + 6}{2\sqrt{36}}$$

$$= \frac{33}{12}$$

$$= 2.75$$

The particle is moving at 2.75 m/s.

17. a. $h(x) = p(x)q(x)r(x)$

$$\begin{aligned} h'(x) &= p'(x)q(x)r(x) + p(x)q'(x)r(x) \\ &\quad + p(x)q(x)r'(x) \end{aligned}$$

b. $h(x) = x(2x + 7)^4(x - 1)^2$

Using the result from part a.,

$$\begin{aligned} h'(x) &= (1)(2x + 7)^4(x - 1)^2 \\ &\quad + x[4(2x + 7)^3(2)](x - 1)^2 \\ &\quad + x(2x + 7)^4[2(x - 1)] \end{aligned}$$

$$\begin{aligned} h'(-3) &= 1(16) + (-3)[4(1)(2)](16) \\ &\quad + (-3)(1)[2(-4)] \\ &= 16 - 384 + 24 \\ &= -344 \end{aligned}$$

18. $y = (x^2 + x - 2)^3 + 3$

$$\frac{dy}{dx} = 3(x^2 + x - 2)^2(2x + 1)$$

At the point $(1, 3)$, $x = 1$ and the slope of the tangent will be $3(1 + 1 - 2)^2(2 + 1) = 0$.

Equation of the tangent at $(1, 3)$ is $y - 3 = 0$.

Solving this equation with the function, we have

$$\begin{aligned} (x^2 + x - 2)^3 + 3 &= 3 \\ (x + 2)^3(x - 1)^3 &= 0 \end{aligned}$$

$$x = -2 \text{ or } x = 1$$

Since -2 and 1 are both triple roots, the line with equation $y - 3 = 0$ will be a tangent at both $x = 1$ and $x = -2$. Therefore, $y - 3 = 0$ is also a tangent at $(-2, 3)$.

19. $y = \frac{x^2(1-x)^3}{(1+x)^3}$

$$= x^2 \left[\left(\frac{1-x}{1+x} \right) \right]^3$$

$$\frac{dy}{dx} = 2x \left(\frac{1-x}{1+x} \right)^3 + 3x^2 \left(\frac{1-x}{1+x} \right)^2$$

$$\times \left[\frac{-(1+x) - (1-x)(1)}{(1+x)^2} \right]$$

$$= 2x \left(\frac{1-x}{1+x} \right)^3 + 3x^2 \left(\frac{1-x}{1+x} \right)^2 \left[\frac{-2}{(1+x)^2} \right]$$

$$= 2x \left(\frac{1-x}{1+x} \right)^2 \left[\frac{1-x}{1+x} - \frac{3x}{(1+x)^2} \right]$$

$$= 2x \left(\frac{1-x}{1+x} \right)^2 \left[\frac{1-x^2-3x}{(1+x)^2} \right]$$

$$= -\frac{2x(x^2+3x-1)(1-x)^2}{(1+x)^4}$$

Review Exercise, pp. 110–113

1. To find the derivative $f'(x)$, the limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

must be computed, provided it exists. If this limit does not exist, then the derivative of $f(x)$ does not

$$= 3[3u^2 - 10(u^3 - 7u)(3u^2 - 7)] \cdot \left(\frac{1}{2}x^{\frac{1}{2}}\right)$$

$$= [3u^2 - 10(u^3 - 7u)(3u^2 - 7)] \cdot \frac{1}{2\sqrt{x}}$$

At $x = 4$:

$$\begin{aligned} u &= \sqrt{4} \\ &= 2 \end{aligned}$$

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The particle is moving at 2.75 m/s.

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$$\begin{aligned} h'(x) &= p'(x)q(x)r(x) + p(x)q'(x)r(x) \\ &\quad + p(x)q(x)r'(x) \end{aligned}$$

b. $h(x) = x(2x + 7)^4(x - 1)^2$

Using the result from part a.,

$$\begin{aligned} h'(x) &= (1)(2x + 7)^4(x - 1)^2 \\ &\quad + x[4(2x + 7)^3(2)](x - 1)^2 \\ &\quad + x(2x + 7)^4[2(x - 1)] \end{aligned}$$

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$$\frac{dy}{dx} = 3(x^2 + x - 2)^2(2x + 1)$$

At the point $(1, 3)$, $x = 1$ and the slope of the tangent will be $3(1 + 1 - 2)^2(2 + 1) = 0$.

Equation of the tangent at $(1, 3)$ is $y - 3 = 0$.

Solving this equation with the function, we have

$$\begin{aligned} (x^2 + x - 2)^3 + 3 &= 3 \\ (x + 2)^3(x - 1)^3 &= 0 \end{aligned}$$

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$$= x^2 \left[\left(\frac{1-x}{1+x} \right) \right]^3$$

$$\frac{dy}{dx} = 2x \left(\frac{1-x}{1+x} \right)^3 + 3x^2 \left(\frac{1-x}{1+x} \right)^2$$

$$\times \left[\frac{-(1+x) - (1-x)(1)}{(1+x)^2} \right]$$

$$= 2x \left(\frac{1-x}{1+x} \right)^3 + 3x^2 \left(\frac{1-x}{1+x} \right)^2 \left[\frac{-2}{(1+x)^2} \right]$$

$$= 2x \left(\frac{1-x}{1+x} \right)^2 \left[\frac{1-x}{1+x} - \frac{3x}{(1+x)^2} \right]$$

$$= 2x \left(\frac{1-x}{1+x} \right)^2 \left[\frac{1-x^2-3x}{(1+x)^2} \right]$$

$$= -\frac{2x(x^2+3x-1)(1-x)^2}{(1+x)^4}$$

Review Exercise, pp. 110–113

1. To find the derivative $f'(x)$, the limit

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

must be computed, provided it exists. If this limit does not exist, then the derivative of $f(x)$ does not

exist at this particular value of x . As an alternative to this limit, we could also find $f'(x)$ from the definition by computing the equivalent limit

$$f'(x) = \lim_{z \rightarrow x} \frac{f(z) - f(x)}{z - x}.$$

These two limits are seen to be equivalent by substituting $z = x + h$.

2. a. $y = 2x^2 - 5x$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{(2(x+h)^2 - 5(x+h)) - (2x^2 - 5x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2((x+h)^2 - x^2) - 5h}{h} \\ &= \lim_{h \rightarrow 0} \frac{2((x+h) - x)((x+h) + x) - 5h}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h(2x+h) - 5h}{h} \\ &= \lim_{h \rightarrow 0} (2(2x+h) - 5) \\ &= 4x - 5 \end{aligned}$$

b. $y = \sqrt{x-6}$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{\sqrt{(x+h)-6} - \sqrt{x-6}}{h} \\ &= \lim_{h \rightarrow 0} \left[\frac{\sqrt{(x+h)-6} - \sqrt{x-6}}{h} \right. \\ &\quad \times \left. \frac{\sqrt{(x+h)-6} + \sqrt{x-6}}{\sqrt{(x+h)-6} + \sqrt{x-6}} \right] \\ &= \lim_{h \rightarrow 0} \frac{((x+h)-6) - (x-6)}{h(\sqrt{(x+h)-6} + \sqrt{x-6})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{(x+h)-6} + \sqrt{x-6}} \\ &= \frac{1}{2\sqrt{x-6}} \end{aligned}$$

c. $y = \frac{x}{4-x}$

$$\begin{aligned} \frac{dy}{dx} &= \lim_{h \rightarrow 0} \frac{\frac{x+h}{4-(x+h)} - \frac{x}{4-x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\frac{(x+h)(4-x) - x(4-(x+h))}{4-(x+h)(4-x)}}{h} \\ &= \lim_{h \rightarrow 0} \frac{4h}{h(4-(x+h))(4-x)} \\ &= \lim_{h \rightarrow 0} \frac{4}{(4-(x+h))(4-x)} \\ &= \frac{4}{(4-x)^2} \end{aligned}$$

3. a. $y = x^2 - 5x + 4$

$$\frac{dy}{dx} = 2x - 5$$

b. $f(x) = x^{\frac{3}{4}}$

$$\begin{aligned} f'(x) &= \frac{3}{4}x^{-\frac{1}{4}} \\ &= \frac{3}{4x^{\frac{1}{4}}} \end{aligned}$$

c. $y = \frac{7}{3x^4}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{-28}{3}x^{-5} \\ &= -\frac{28}{3x^5} \end{aligned}$$

d. $y = \frac{1}{x^2 + 5}$

$$= (x^2 + 5)^{-1}$$

$$\begin{aligned} \frac{dy}{dx} &= (-1)(x^2 + 5)^{-2} \cdot (2x) \\ &= -\frac{2x}{(x^2 + 5)^2} \end{aligned}$$

e. $y = \frac{3}{(3-x^2)^2}$

$$\begin{aligned} \frac{dy}{dx} &= 3(3-x^2)^{-2} \\ &= (-6)(3-x^2)^{-3} \cdot (-2x) \end{aligned}$$

$$= \frac{12x}{(3-x^2)^3}$$

f. $y = \sqrt{7x^2 + 4x + 1}$

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2}(7x^2 + 4x + 1)^{-\frac{1}{2}}(14x + 4) \\ &= \frac{7x + 2}{\sqrt{7x^2 + 4x + 1}} \end{aligned}$$

4. a. $f(x) = \frac{2x^3 - 1}{x^2}$

$$= 2x - \frac{1}{x^2}$$

$$= 2x - x^{-2}$$

$$f'(x) = 2 + 2x^{-3}$$

$$= 2 + \frac{2}{x^3}$$

b. $g(x) = \sqrt{x}(x^3 - x)$
 $= x^{\frac{1}{2}}(x^3 - x)$
 $= x^{\frac{7}{2}} - x^{\frac{3}{2}}$

$$g'(x) = \frac{7}{2}x^{\frac{5}{2}} - \frac{3}{2}x^{\frac{1}{2}} \\ = \frac{\sqrt{x}}{2}(7x^2 - 3)$$

c. $y = \frac{x}{3x - 5}$
 $\frac{dy}{dx} = \frac{(3x - 5)(1) - (x)(3)}{(3x - 5)^2}$
 $= -\frac{5}{(3x - 5)^2}$

d. $y = (x - 1)^{\frac{1}{2}}(x + 1)$
 $y' = (x - 1)^{\frac{1}{2}} + (x + 1)\left(\frac{1}{2}\right)(x - 1)^{-\frac{1}{2}}$
 $= \sqrt{x - 1} + \frac{x + 1}{2\sqrt{x - 1}}$
 $= \frac{2x - 2 + x + 1}{2\sqrt{x - 1}}$
 $= \frac{3x - 1}{2\sqrt{x - 1}}$

e. $f(x) = (\sqrt{x} + 2)^{-\frac{2}{3}}$
 $= (x^{\frac{1}{2}} + 2)^{-\frac{2}{3}}$

$$f'(x) = \frac{-2}{3}(x^{\frac{1}{2}} + 2)^{-\frac{5}{3}} \cdot \frac{1}{2}x^{-\frac{1}{2}} \\ = -\frac{1}{3\sqrt{x}(\sqrt{x} + 2)^{\frac{5}{3}}}$$

f. $y = \frac{x^2 + 5x + 4}{x + 4}$
 $= \frac{(x + 4)(x + 1)}{x + 4}$
 $= x + 1, x \neq -4$

$$\frac{dy}{dx} = 1$$

5. a. $y = x^4(2x - 5)^6$

$$y' = x^4[6(2x - 5)^5(2)] + 4x^3(2x - 5)^6 \\ = 4x^3(2x - 5)^5[3x + (2x - 5)] \\ = 4x^3(2x - 5)^5(5x - 5) \\ = 20x^3(2x - 5)^5(x - 1)$$

b. $y = x\sqrt{x^2 + 1}$

$$y' = x\left[\frac{1}{2}(x^2 + 1)^{-\frac{1}{2}}(2x)\right] + (1)\sqrt{x^2 + 1} \\ = \frac{x^2}{\sqrt{x^2 + 1}} + \sqrt{x^2 + 1}$$

c. $y = \frac{(2x - 5)^4}{(x + 1)^3}$
 $y' = \frac{(x + 1)^3 4(2x - 5)^3(2)}{(x + 1)^6}$
 $- \frac{3(2x - 5)^4(x + 1)^2}{(x + 1)^6}$
 $= \frac{(x + 1)^2(2x - 5)^3[8x + 8 - 6x + 15]}{(x + 1)^6}$

$$y' = \frac{(2x - 5)^3(2x + 23)}{(x + 1)^4}$$

d. $y = \left(\frac{10x - 1}{3x + 5}\right)^6 = (10x - 1)^6(3x + 5)^{-6}$
 $y' = (10x - 1)^6[-6(3x + 5)^{-7}(3)]$
 $+ 6(10x - 1)^5(10)(3x + 5)^{-6}$
 $= (10x - 1)^5(3x + 5)^{-7}[x - 18(10x - 1)]$
 $+ 60(3x + 5)$
 $= (10x - 1)^5(3x + 5)^{-7}$
 $\times (-180x + 18 + 180x + 300)$
 $= \frac{318(10x - 1)^5}{(3x + 5)^7}$

e. $y = (x - 2)^3(x^2 + 9)^4$
 $y' = (x - 2)^3[4(x^2 + 9)^3(2x)]$
 $+ 3(x - 2)^2(1)(x^2 + 9)^4$
 $= (x - 2)^2(x^2 + 9)^3[8x(x - 2) + 3(x^2 + 9)]$
 $= (x - 2)^2(x^2 + 9)^3(11x^2 - 16x + 27)$

f. $y = (1 - x^2)^3(6 + 2x)^{-3}$
 $= \left(\frac{1 - x^2}{6 + 2x}\right)^3$
 $y' = 3\left(\frac{1 - x^2}{6 + 2x}\right)^2$
 $\times \left[\frac{(6 + 2x)(-2x) - (1 - x^2)(2)}{(6 + 2x)^2}\right]$
 $= \frac{3(1 - x^2)^2(-12x - 4x^2 - 2 + 2x^2)}{(6 + 2x)^4}$
 $= -\frac{3(1 - x^2)^2(2x^2 + 12x + 2)}{(6 + 2x)^4}$
 $= -\frac{3(1 - x^2)^2(x^2 + 6x + 1)}{8(3 - x)^4}$

6. a. $g(x) = f(x^2)$

$$g'(x) = f(x^2) \times 2x$$

b. $h(x) = 2xf(x)$

$$h'(x) = 2xf'(x) + 2f(x)$$

7. a. $y = 5u^2 + 3u - 1, u = \frac{18}{x^2 + 5}$
 $x = 2$
 $u = 2$

$$\frac{dy}{du} = 10u + 3$$

$$\frac{du}{dx} = -\frac{36x}{(x^2 + 5)^2}$$

When $x = 2$,

$$\frac{du}{dx} = -\frac{72}{81} = -\frac{8}{9}$$

When $u = 2$,

$$\frac{dy}{du} = 20 + 3$$

$$= 23$$

$$\frac{dy}{dx} = 23 \left(-\frac{8}{9} \right)$$

$$= -\frac{184}{9}$$

b. $y = \frac{u+4}{u-4}$, $u = \frac{\sqrt{x}+x}{10}$,

$$x = 4$$

$$u = \frac{3}{5}$$

$$\frac{dy}{du} = \frac{(u-4) - (u+4)}{(u-4)^2}$$

$$\frac{du}{dx} = \frac{1}{10} \left(\frac{1}{2}x^{-\frac{1}{2}} + 1 \right)$$

When $x = 4$,

$$= -\frac{8}{(u-4)^2} \frac{du}{dx} = \frac{1}{10} \left(\frac{5}{4} \right)$$

$$= \frac{1}{8}$$

When $u = \frac{3}{5}$,

$$\frac{dy}{du} = -\frac{8}{\left(\frac{3}{5} - \frac{20}{5}\right)^2}$$

$$= -\frac{8(25)}{(-17)^2}$$

When $x = 4$,

$$\frac{dy}{dx} = \frac{8(25)}{17^2} \times \frac{1}{8}$$

$$= \frac{25}{289}$$

c. $y = f(\sqrt{x^2 + 9})$, $f'(5) = -2$, $x = 4$

$$\frac{dy}{dx} = f'(\sqrt{x^2 + 9}) \times \frac{1}{2}(x^2 + 9)^{-\frac{1}{2}}(2x)$$

$$\frac{dy}{dx} = f'(5) \cdot \frac{1}{2} \cdot \frac{1}{5} \cdot 8$$

$$= -2 \cdot \frac{4}{5}$$

$$= -\frac{8}{5}$$

8. $f(x) = (9 - x^2)^{\frac{2}{3}}$

$$f'(x) = \frac{2}{3}(9 - x^2)^{-\frac{1}{3}}(-2x)$$

$$= \frac{-4x}{3(9 - x^2)^{\frac{1}{3}}}$$

$$f'(1) = -\frac{2}{3}$$

The slope of the tangent line at $(1, 4)$ is $-\frac{2}{3}$.

9. $y = -x^3 + 6x^2$

$$y' = -3x^2 + 12x$$

$$-3x^2 + 12x = -12$$

$$-3x^2 + 12x = -15$$

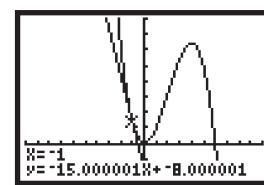
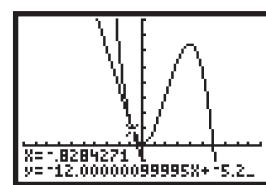
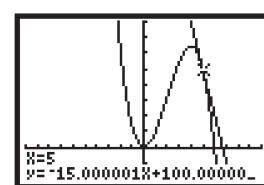
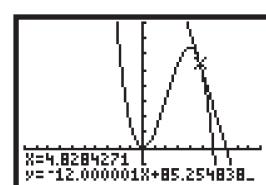
$$x^2 - 4x - 4 = 0$$

$$x^2 - 4x - 5 = 0$$

$$x = \frac{4 \pm \sqrt{16 + 16}}{2} \quad (x - 5)(x + 1) = 0$$

$$= \frac{4 \pm 4\sqrt{2}}{2} \quad x = 5, x = -1$$

$$x = 2 \pm 2\sqrt{2}$$



10. a. i. $y = (x^2 - 4)^5$

$$y' = 5(x^2 - 4)^4(2x)$$

Horizontal tangent,

$$10x(x^2 - 4)^4 = 0$$

$$x = 0, x = \pm 2$$

ii. $y = (x^3 - x)^2$

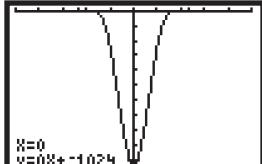
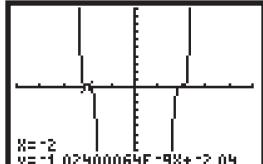
$$y' = 2(x^3 - x)(3x^2 - 1)$$

Horizontal tangent,

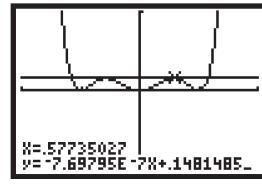
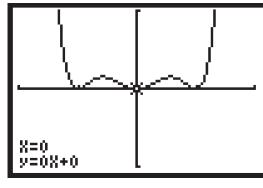
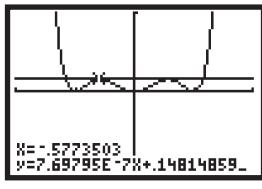
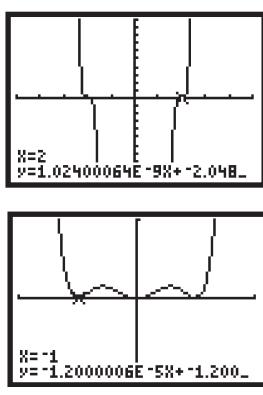
$$2x(x^2 - 1)(3x^2 - 1) = 0$$

$$x = 0, x = \pm 1, x = \pm \frac{\sqrt{3}}{3}$$

b. i.



ii.



11. a. $y = (x^2 + 5x + 2)^4$ at $(0, 16)$

$$y' = 4(x^2 + 5x + 2)^3(2x + 5)$$

At $x = 0$,

$$\begin{aligned} y' &= 4(2)^3(5) \\ &= 160 \end{aligned}$$

Equation of the tangent at $(0, 16)$ is

$$y - 16 = 160(x - 0)$$

$$y = 160x + 16$$

$$\text{or } 160x - y + 16 = 0$$

b. $y = (3x^{-2} - 2x^3)^5$ at $(1, 1)$

$$y' = 5(3x^{-2} - 2x^3)^4(-6x^{-3} - 6x^2)$$

At $x = 1$,

$$\begin{aligned} y' &= 5(1)^4(-6 - 6) \\ &= -60 \end{aligned}$$

Equation of the tangent at $(1, 1)$ is

$$y - 1 = -60(x - 1)$$

$$60x + y - 61 = 0.$$

12. $y = 3x^2 - 7x + 5$

$$\frac{dy}{dx} = 6x - 7$$

Slope of $x + 5y - 10 = 0$ is $-\frac{1}{5}$.

Since perpendicular, $6x - 7 = 5$

$$x = 2$$

$$\begin{aligned} y &= 3(4) - 14 + 5 \\ &= 3. \end{aligned}$$

Equation of the tangent at $(2, 3)$ is

$$y - 3 = 5(x - 2)$$

$$5x - y - 7 = 0.$$

13. $y = 8x + b$ is tangent to $y = 2x^2$

$$\frac{dy}{dx} = 4x$$

Slope of the tangent is 8, therefore $4x = 8$, $x = 2$.

Point of tangency is $(2, 8)$.

Therefore, $8 = 16 + b$, $b = -8$.

$$\text{Or } 8x + b = 2x^2$$

$$2x^2 - 8x - b = 0$$

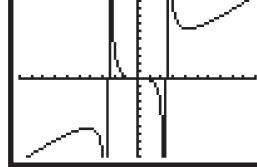
$$x = \frac{8 \pm \sqrt{64 + 8b}}{2(2)}.$$

For tangents, the roots are equal, therefore

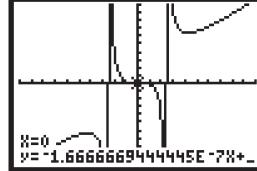
$$64 + 8b = 0, b = -8.$$

Point of tangency is $(2, 8)$, $b = -8$.

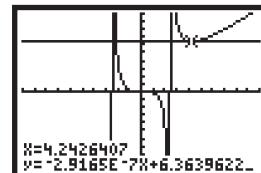
14. a.



b.



The equation of the tangent is $y = 0$.



The equation of the tangent is $y = 6.36$.



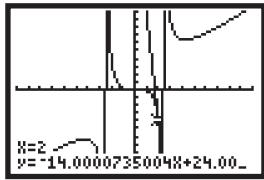
The equation of the tangent is $y = -6.36$.

c.
$$\begin{aligned} f'(x) &= \frac{(x^2 - 6)(3x^2) - x^3(2x)}{(x^2 - 6)^2} \\ &= \frac{x^4 - 18x^2}{(x^2 - 6)^2} \\ \frac{x^4 - 18x^2}{(x^2 - 6)^2} &= 0 \\ x^2(x^2 - 18) &= 0 \\ x^2 = 0 \text{ or } x^2 - 18 &= 0 \\ x = 0 &\quad x = \pm 3\sqrt{2} \end{aligned}$$

The coordinates of the points where the slope is 0 are $(0, 0)$, $(3\sqrt{2}, \frac{9\sqrt{2}}{2})$, and $(-3\sqrt{2}, -\frac{9\sqrt{2}}{2})$.

d. Substitute into the expression for $f'(x)$ from part b.

$$\begin{aligned} f'(2) &= \frac{16 - 72}{(-2)^2} \\ &= \frac{-56}{4} \\ &= -14 \end{aligned}$$



15. a. $f(x) = 2x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$
 $f'(x) = 2 \times \frac{5}{3}x^{\frac{2}{3}} - 5 \times \frac{2}{3}x^{\frac{1}{3}}$
 $= \frac{10}{3}x^{\frac{2}{3}} - \frac{10}{3}x^{\frac{1}{3}}$
 $f(x) = 0 \quad \therefore x^{\frac{2}{3}}[2x - 5] = 0$
 $x = 0 \text{ or } x = \frac{5}{2}$

$y = f(x)$ crosses the x -axis at $x = \frac{5}{2}$, and

$$\begin{aligned} f'(x) &= \frac{10}{3}\left(\frac{x-1}{x^{\frac{1}{3}}}\right) \\ f'\left(\frac{5}{2}\right) &= \frac{10}{3} \times \frac{3}{2} \times \frac{1}{\left(\frac{5}{2}\right)^{\frac{1}{3}}} \\ &= 5 \times \frac{\sqrt[3]{2}}{\sqrt[3]{5}} = 5^{\frac{2}{3}} \times 2^{\frac{1}{3}} \\ &= (25 \times 2)^{\frac{1}{3}} \\ &= \sqrt[3]{50} \end{aligned}$$

b. To find a , let $f(x) = 0$.

$$\frac{10}{3}x^{\frac{2}{3}} - \frac{10}{3}x^{\frac{1}{3}} = 0$$

$$\begin{aligned} 30x &= 30 \\ x &= 1 \end{aligned}$$

Therefore $a = 1$.

16. $M = 0.1t^2 - 0.001t^3$

a. When $t = 10$,

$$\begin{aligned} M &= 0.1(100) - 0.001(1000) \\ &= 9 \end{aligned}$$

When $t = 15$,

$$\begin{aligned} M &= 0.1(225) - 0.001(3375) \\ &= 19.125 \end{aligned}$$

One cannot memorize partial words, so 19 words are memorized after 15 minutes.

b. $M' = 0.2t - 0.003t^2$

When $t = 10$,

$$\begin{aligned} M' &= 0.2(10) - 0.003(100) \\ &= 1.7 \end{aligned}$$

The number of words memorized is increasing by 1.7 words/min.

When $t = 15$,

$$\begin{aligned} M' &= 0.2(15) - 0.003(225) \\ &= 2.325 \end{aligned}$$

The number of words memorized is increasing by 2.325 words/min.

17. a. $N(t) = 20 - \frac{30}{\sqrt{9+t^2}}$

$$N'(t) = \frac{30t}{(9+t^2)^{\frac{3}{2}}}$$

b. No, according to this model, the cashier never stops improving. Since $t > 0$, the derivative is always positive, meaning that the rate of change in the cashier's productivity is always increasing. However, these increases must be small, since, according to the model, the cashier's productivity can never exceed 20.

18. $C(x) = \frac{1}{3}x^3 + 40x + 700$

a. $C'(x) = x^2 + 40$

b. $C'(x) = 76$

$$x^2 + 40 = 76$$

$$x^2 = 36$$

$$x = 6$$

Production level is 6 gloves/week.

19. $R(x) = 750x - \frac{x^2}{6} - \frac{2}{3}x^3$

a. Marginal Revenue

$$R'(x) = 750 - \frac{x}{3} - 2x^2$$

b. $R'(10) = 750 - \frac{10}{3} - 2(100)$
 $= \$546.67$

20. $D(p) = \frac{20}{\sqrt{p-1}}, p > 1$
 $D'(p) = 20 \left(-\frac{1}{2}\right)(p-1)^{-\frac{3}{2}}$
 $= -\frac{10}{(p-1)^{\frac{3}{2}}}$
 $D'(5) = \frac{10}{\sqrt{4^3}} = -\frac{10}{8}$
 $= -\frac{5}{4}$

Slope of demand curve at $(5, 10)$ is $-\frac{5}{4}$.

21. $B(x) = -0.2x^2 + 500, 0 \leq x \leq 40$

a. $B(0) = -0.2(0)^2 + 500 = 500$

$B(30) = -0.2(30)^2 + 500 = 320$

b. $B'(x) = -0.4x$

$B'(0) = -0.4(0) = 0$

$B'(30) = -0.4(30) = -12$

c. $B(0)$ = blood sugar level with no insulin

$B(30)$ = blood sugar level with 30 mg of insulin

$B'(0)$ = rate of change in blood sugar level
with no insulin

$B'(30)$ = rate of change in blood sugar level
with 30 mg of insulin

d. $B'(50) = -0.4(50) = -20$

$B(50) = -0.2(50)^2 + 500 = 0$

$B'(50) = -20$ means that the patient's blood sugar level is decreasing at 20 units per mg of insulin 1 h after 50 mg of insulin is injected.

$B(50) = 0$ means that the patient's blood sugar level is zero 1 h after 50 mg of insulin is injected. These values are not logical because a person's blood sugar level can never reach zero and continue to decrease.

22. a. $f(x) = \frac{3x}{1-x^2}$
 $= \frac{3x}{(1-x)(1+x)}$

$f(x)$ is not differentiable at $x = 1$ because it is not defined there (vertical asymptote at $x = 1$).

b. $g(x) = \frac{x-1}{x^2+5x-6}$
 $= \frac{x-1}{(x+6)(x-1)}$
 $= \frac{1}{(x+6)}$ for $x \neq 1$

$g(x)$ is not differentiable at $x = 1$ because it is not defined there (hole at $x = 1$).

c. $h(x) = \sqrt[3]{(x-2)^2}$

The graph has a cusp at $(2, 0)$ but it is differentiable at $x = 1$.

d. $m(x) = |3x-3| - 1$.

The graph has a corner at $x = 1$, so $m(x)$ is not differentiable at $x = 1$.

23. a. $f(x) = \frac{3}{4x^2-x}$
 $= \frac{3}{x(4x-1)}$

$f(x)$ is not defined at $x = 0$ and $x = 0.25$. The graph has vertical asymptotes at $x = 0$ and $x = 0.25$. Therefore, $f(x)$ is not differentiable at $x = 0$ and $x = 0.25$.

b. $f(x) = \frac{x^2-x-6}{x^2-9}$
 $= \frac{(x-3)(x+2)}{(x-3)(x+3)}$
 $= \frac{(x+2)}{(x+3)}$ for $x \neq 3$

$f(x)$ is not defined at $x = 3$ and $x = -3$. At $x = -3$, the graph has a vertical asymptote and at $x = 3$ it has a hole. Therefore, $f(x)$ is not differentiable at $x = 3$ and $x = -3$.

c. $f(x) = \sqrt{x^2-7x+6}$
 $= \sqrt{(x-6)(x-1)}$

$f(x)$ is not defined for $1 < x < 6$. Therefore, $f(x)$ is not differentiable for $1 < x < 6$.

24. $p'(t) = \frac{(t+1)(25) - (25t)(t)}{(t+1)^2}$
 $= \frac{25t+25-25t}{(t+1)^2}$
 $= \frac{25}{(t+1)^2}$

25. Answers may vary. For example,
 $f(x) = 2x + 3$

$$y = \frac{1}{2x+3}$$

$$y' = \frac{(2x+3)(0) - (1)(2)}{(2x+3)^2}$$

$$= -\frac{2}{(2x+3)^2}$$

$f(x) = 5x + 10$

$$y = \frac{1}{5x+10}$$

$$y' = \frac{(5x+10)(0) - (1)(5)}{(5x+10)^2}$$

$$= -\frac{5}{(5x+10)^2}$$

Rule: If $f(x) = ax + b$ and $y = \frac{1}{f(x)}$, then

$$y' = \frac{-a}{(ax+b)^2}$$

$$\begin{aligned} y' &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{1}{a(x+h)+b} - \frac{1}{ax+b} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{ax+b - [a(x+h)b]}{[a(x+h)+b](ax+h)} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{ax+b - ax - ah - b}{[a(x+h)+b](ax+b)} \right] \\ &= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{-ah}{[a(x+h)+b](ax+b)} \right] \\ &= \lim_{h \rightarrow 0} \frac{-a}{[a(x+h)+b](ax+b)} \\ &= \frac{-a}{(ax+b)^2} \end{aligned}$$

26. a. Let $y = f(x)$

$$y = \frac{(2x-3)^2 + 5}{2x-3}$$

Let $u = 2x-3$.

$$\text{Then } y = \frac{u^2 + 5}{u}.$$

$$y = u + 5u^{-1}$$

$$\text{b. } f'(x) = \frac{dy}{dx}$$

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \times \frac{du}{dx} \\ &= (1 - 5u^{-2})(2) \\ &= 2(1 - 5(2x-3)^{-2}) \end{aligned}$$

$$27. g(x) = \sqrt{2x-3} + 5(2x-3)$$

a. Let $y = g(x)$.

$$y = \sqrt{2x-3} + 5(2x-3)$$

Let $u = 2x-3$.

Then $y = \sqrt{u} + 5u$.

$$\begin{aligned} \text{b. } g'(x) &= \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \\ &= \left(\frac{1}{2}u^{-\frac{1}{2}} + 5 \right)(2) \\ &= u^{-\frac{1}{2}} + 10 \\ &= (2x-3)^{-\frac{1}{2}} + 10 \end{aligned}$$

$$\begin{aligned} \text{28. a. } f(x) &= (2x-5)^3(3x^2+4)^5 \\ f'(x) &= (2x-5)^3(5)(3x^2+4)^4(6x) \\ &\quad + (3x^2+4)^5(3)(2x-5)^2(2) \\ &= 30x(2x-5)^3(3x^2+4)^4 \\ &\quad + 6(3x^2+4)^5(2x-5)^2 \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times [5x(2x-5) + (3x^2+4)] \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times (10x^2 - 25x + 3x^2 + 4) \\ &= 6(2x-5)^2(3x^2+4)^4 \\ &\quad \times (13x^2 - 25x + 4) \end{aligned}$$

$$\begin{aligned} \text{b. } g(x) &= (8x^3)(4x^2+2x-3)^5 \\ g'(x) &= (8x^3)(5)(4x^2+2x-3)^4(8x+2) \\ &\quad + (4x^2+2x-3)^5(24x^2) \\ &= 40x^3(4x^2+2x-3)^4(8x+2) \\ &\quad + 24x^2(4x^2+2x-3)^5 \\ &= 8x^2(4x^2+2x-3)^4[5x(8x+2) \\ &\quad + 3(4x^2+2x-3)] \\ &= 8x^2(4x^2+2x-3)^4 \\ &\quad (40x^2 + 10x + 12x^2 + 6x - 9) \\ &= 8x^2(4x^2+2x-3)^4(52x^2 + 16x - 9) \end{aligned}$$

$$\begin{aligned} \text{c. } y &= (5+x)^2(4-7x^3)^6 \\ y' &= (5+x)^2(6)(4-7x^3)^5(-21x^2) \\ &\quad + (4-7x^3)^6(2)(5+x) \\ &= -126x^2(5+x)^2(4-7x^3)^5 \\ &\quad + 2(5+x)(4-7x^3)^6 \\ &= 2(5+x)(4-7x^3)^5[-63x^2(5+x) \\ &\quad + 4-7x^3] \\ &= 2(5+x)(4-7x^3)^5(4-315x^2-70x^3) \end{aligned}$$

$$\begin{aligned} \text{d. } h(x) &= \frac{6x-1}{(3x+5)^4} \\ h'(x) &= \frac{(3x+5)^4(6) - (6x-1)(4)(3x+5)^3(3)}{(3x+5)^4)^2} \\ &= \frac{6(3x+5)^3[(3x+5)-2(6x-1)]}{(3x+5)^8} \\ &= \frac{6(-9x+7)}{(3x+5)^5} \end{aligned}$$

$$\begin{aligned} \text{e. } y &= \frac{(2x^2-5)^3}{(x+8)^2} \\ \frac{dy}{dx} &= \frac{(x+8)^2(3)(2x^2-5)^2(4x)}{(x+8)^2} \\ &\quad - \frac{(2x^2-5)^3(2)(x+8)}{(x+8)^2} \\ &= \frac{2(x+8)(2x^2-5)^2[6x(x+8) - (2x^2-5)]}{(x+8)^4} \\ &= \frac{2(2x^2-5)^2(4x^2+48x+5)}{(x+8)^3} \end{aligned}$$

$$\mathbf{f. } f(x) = \frac{-3x^4}{\sqrt{4x - 8}}$$

$$= \frac{-3x^4}{(4x - 8)^{\frac{1}{2}}}$$

$$f'(x) = \frac{(4x - 8)^{\frac{1}{2}}(-12x^3)}{((4x - 8)^{\frac{1}{2}})^2}$$

$$- \frac{(-3x^4)\left(\frac{1}{2}\right)(4x - 8)^{-\frac{1}{2}}(4)}{((4x - 8)^{\frac{1}{2}})^2}$$

$$= \frac{-6x^3(4x - 8)^{-\frac{1}{2}}[2(4x - 8) - x]}{4x - 8}$$

$$= \frac{-6x^3(7x - 16)}{(4x - 8)^{\frac{3}{2}}}$$

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$$\mathbf{g. } g(x) = \left(\frac{2x + 5}{6 - x^2}\right)^4$$

$$g'(x) = 4\left(\frac{2x + 5}{6 - x^2}\right)^3$$

$$\times \left(\frac{(6 - x^2)(2) - (2x + 5)(-2x)}{(6 - x^2)^2}\right)$$

$$= 4\left(\frac{2x + 5}{6 - x^2}\right)^3 \left(\frac{2(6 + x^2 + 5x)}{(6 - x^2)^2}\right)$$

$$= 8\left(\frac{2x + 5}{6 - x^2}\right)^3 \left(\frac{(x + 2)(x + 3)}{(6 - x^2)^2}\right)$$

$$\mathbf{h. } y = \left[\frac{1}{(4x + x^2)^3}\right]^3$$

$$= (4x + x^2)^{-9}$$

$$\frac{dy}{dx} = -9(4x + x^2)^{-10}(4 + 2x)$$

$$\mathbf{29. } f(x) = ax^2 + bx + c,$$

It is given that $(0, 0)$ and $(8, 0)$ are on the curve, and $f'(2) = 16$.

Calculate $f'(x) = 2ax + b$.

Then,

$$16 = 2a(2) + b$$

$$4a + b = 16 \quad (1)$$

Since $(0, 0)$ is on the curve,

$$0 = a(0)^2 + b(0) + c$$

$$c = 0$$

Since $(8, 0)$ is on the curve,

$$0 = a(8)^2 + b(8) + c$$

$$0 = 64a + 8b + 0$$

$$8a + b = 0 \quad (2)$$

Solve (1) and (2):

$$\text{From (2), } b = -8a \quad (1)$$

In (1),

$$4a - 8a = 16$$

$$-4a = 16$$

$$a = -4$$

Using (1),

$$b = -8(-4) = 32$$

$$a = -4, b = 32, c = 0, f(x) = -4x^2 + 32x$$

$$\mathbf{30. a. } A(t) = -t^3 + 5t + 750$$

$$A'(t) = -3t^2 + 5$$

$$\mathbf{b. } A'(5) = -3(25) + 5$$

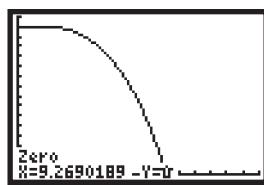
$$= -70$$

At 5 h, the number of ants living in the colony is decreasing by 7000 ants/h.

c. $A(0) = 750$, so there were $750(100)$ or

75 000 ants living in the colony before it was treated with insecticide.

d. Determine t so that $A(t) = 0$. $-t^3 + 5t + 750$ cannot easily be factored, so find the zeros by using a graphing calculator.



All of the ants have been killed after about 9.27 h.

Chapter 2 Test, p. 114

1. You need to use the chain rule when the derivative for a given function cannot be found using the sum, difference, product, or quotient rules or when writing the function in a form that would allow the use of these rules is tedious. The chain rule is used when a given function is a composition of two or more functions.

2. f is the blue graph (it's a cubic). f' is the red graph (it is quadratic). The derivative of a polynomial function has degree one less than the derivative of the function. Since the red graph is a quadratic (degree 2) and the blue graph is cubic (degree 3), the blue graph is f and the red graph is f' .

$$\mathbf{3. } f(x) = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

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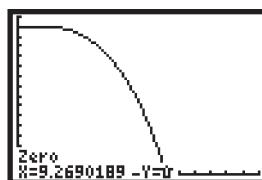
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$$= \lim_{h \rightarrow 0} \frac{h - 2hx - h^2}{h}$$

$$\begin{aligned}
&= \lim_{h \rightarrow 0} \frac{h(1 - 2x - h)}{h} \\
&= \lim_{h \rightarrow 0} (1 - 2x - h) \\
&= 1 - 2x
\end{aligned}$$

Therefore, $\frac{d}{dx}(x - x^2) = 1 - 2x$.

4. a. $y = \frac{1}{3}x^3 - 3x^{-5} + 4\pi$

$$\frac{dy}{dx} = x^2 + 15x^{-6}$$

b. $y = 6(2x - 9)^5$

$$\begin{aligned}
\frac{dy}{dx} &= 30(2x - 9)^4(2) \\
&= 60(2x - 9)^4
\end{aligned}$$

c. $y = \frac{2}{\sqrt{x}} + \frac{x}{\sqrt{3}} + 6\sqrt[3]{x}$

$$= 2x^{-\frac{1}{2}} + \frac{1}{\sqrt{3}}x + 6x^{\frac{1}{3}}$$

$$\frac{dy}{dx} = -x^{-\frac{3}{2}} + \frac{1}{\sqrt{3}} + 2x^{-\frac{2}{3}}$$

d. $y = \left(\frac{x^2 + 6}{3x + 4}\right)^5$

$$\begin{aligned}
\frac{dy}{dx} &= 5\left(\frac{x^2 + 6}{3x + 4}\right)^4 \frac{2x(3x + 4) - (x^2 + 6)3}{(3x + 4)^2} \\
&= \frac{5(x^2 + 6)^4(3x^2 + 8x - 18)}{(3x + 4)^6}
\end{aligned}$$

e. $y = x^2 \sqrt[3]{6x^2 - 7}$

$$\begin{aligned}
\frac{dy}{dx} &= 2x(6x^2 - 7)^{\frac{1}{3}} + x^2 \frac{1}{3}(6x^2 - 7)^{-\frac{2}{3}}(12x) \\
&= 2x(6x^2 - 7)^{-\frac{2}{3}}((6x^2 - 7) + 2x^2) \\
&= 2x(6x^2 - 7)^{-\frac{2}{3}}(8x^2 - 7)
\end{aligned}$$

f. $y = \frac{4x^5 - 5x^4 + 6x - 2}{x^4}$

$$= 4x - 5 + 6x^{-3} - 2x^{-4}$$

$$\begin{aligned}
\frac{dy}{dx} &= 4 - 18x^{-4} + 8x^{-5} \\
&= \frac{4x^5 - 18x + 8}{x^5}
\end{aligned}$$

5. $y = (x^2 + 3x - 2)(7 - 3x)$

$$\frac{dy}{dx} = (2x + 3)(7 - 3x) + (x^2 + 3x - 2)(-3)$$

At $(1, 8)$,

$$\begin{aligned}
\frac{dy}{dx} &= (5)(4) + (2)(-3) \\
&= 14.
\end{aligned}$$

The slope of the tangent to $y = (x^2 + 3x - 2)(7 - 3x)$ at $(1, 8)$ is 14.

6. $y = 3u^2 + 2u$

$$\begin{aligned}
\frac{dy}{du} &= 6u + 2 \\
u &= \sqrt{x^2 + 5} \\
\frac{du}{dy} &= \frac{1}{2}(x^2 + 5)^{-\frac{1}{2}}2x \\
\frac{dy}{dx} &= (6u + 2)\left(\frac{x}{\sqrt{x^2 + 5}}\right)
\end{aligned}$$

At $x = -2, u = 3$.

$$\begin{aligned}
\frac{dy}{dx} &= (20)\left(-\frac{2}{3}\right) \\
&= -\frac{40}{3}
\end{aligned}$$

7. $y = (3x^{-2} - 2x^3)^5$

$$\frac{dy}{dx} = 5(3x^{-2} - 2x^3)^4(-6x^{-3} - 6x^2)$$

At $(1, 1)$,

$$\begin{aligned}
\frac{dy}{dx} &= 5(1)^4(-6 - 6) \\
&= -60.
\end{aligned}$$

Equation of tangent line at $(1, 1)$ is $y - 1 = 60(x - 1)$

$$y - 1 = -60x + 60$$

$$60x + y - 61 = 0.$$

8. $P(t) = (t^{\frac{1}{4}} + 3)^3$

$$\begin{aligned}
P'(t) &= 3(t^{\frac{1}{4}} + 3)^2\left(\frac{1}{4}t^{-\frac{3}{4}}\right) \\
P'(16) &= 3(16^{\frac{1}{4}} + 3)^2\left(\frac{1}{4} \times 16^{-\frac{3}{4}}\right) \\
&= 3(2 + 3)^2\left(\frac{1}{4} \times \frac{1}{8}\right) \\
&= \frac{75}{32}
\end{aligned}$$

The amount of pollution is increasing at a rate of $\frac{75}{32}$ ppm/year.

9. $y = x^4$

$$\begin{aligned}
\frac{dy}{dx} &= 4x^3 \\
-\frac{1}{16} &= 4x^3
\end{aligned}$$

Normal line has a slope of 16. Therefore,

$$\frac{dy}{dx} = -\frac{1}{16}.$$

$$x^3 = -\frac{1}{64}$$

$$x = -\frac{1}{4}$$

$$y = \frac{1}{256}$$

Therefore, $y = x^4$ has a normal line with a slope of 16 at $(-\frac{1}{4}, \frac{1}{256})$.

10. $y = x^3 - x^2 - x + 1$

$$\frac{dy}{dx} = 3x^2 - 2x - 1$$

For a horizontal tangent line, $\frac{dy}{dx} = 0$.

$$3x^2 - 2x - 1 = 0$$

$$(3x + 1)(x - 1) = 0$$

$$x = -\frac{1}{3} \quad \text{or} \quad x = 1$$

$$\begin{aligned} y &= -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 1 & y &= 1 - 1 - 1 + 1 \\ &= \frac{-1 - 3 + 9 + 27}{27} & &= 0 \end{aligned}$$

$$= \frac{32}{27}$$

The required points are $(-\frac{1}{3}, \frac{32}{27}), (1, 0)$.

11. $y = x^2 + ax + b$

$$\frac{dy}{dx} = 2x + a$$

$$y = x^3$$

$$\frac{dy}{dx} = 3x^2$$

Since the parabola and cubic function are tangent at $(1, 1)$, then $2x + a = 3x^2$.

$$\text{At } (1, 1) \quad 2(1) + a = 3(1)^2$$

$$a = 1.$$

Since $(1, 1)$ is on the graph of

$$y = x^2 + x + b, 1 = 1^2 + 1 + b$$

$$b = -1.$$

The required values are 1 and -1 for a and b , respectively.