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Quadrature-based approach for the efficient evaluation of surge hazard

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ABSTRACT

The Joint Probability Method (JPM) has been used for hurricane surge frequency analysis for over three decades, and remains the method of choice owing to the limitations of more direct historical methods. However, use of the JPM approach in conjunction with the modern generation of complex high-resolution numerical models (used to describe winds, waves, and surge) has become highly inefficient, owing to the large number of costly storm simulations that are typically required. This paper describes a new approach to the selection of the storm simulation set that permits reduction of the JPM computational effort by about an order of magnitude (compared to a more conventional approach) while maintaining good accuracy. The method uses an integration scheme called Bayesian or Gaussian-process quadrature (together with conventional integration methods) to evaluate the multi-dimensional joint probability integral over the space of storm parameters (pressure, radius, speed, heading, and any others found to be important) as a weighted summation over a relatively small set of optimally selected nodes (synthetic storms). Examples of an application of the method are shown, drawn from the recent post-Katrina study of coastal Mississippi.

1. Introduction

The Joint Probability Method or JPM (Myers, 1970) has become the standard approach for the evaluation of hurricane surge inundation probabilities. The JPM provides a rigorous, yet practical, mathematical framework for combining the probabilistic characterization of hurricane occurrences and hurricane parameters in the region of interest with the simulations of numerical hydrodynamic models to determine surge-elevation probabilities. In essence, JPM considers all possible combinations of storm characteristics at landfall, calculates the surge effects for each combination, and then combines these results considering the combinations' associated probabilities. The result is the annual probability of exceeding any desired storm stage. Mathematically, this calculation is represented as a multi-dimensional integral (the JPM integral).

Because the JPM must consider the effects of many possible combinations of hurricane parameters, traditional implementations have required numerical wind, wave, and surge calculations for many synthetic storms (perhaps several thousands). At the same time, these numerical simulations have become computationally intensive with the introduction of more accurate algorithms, the ability to incorporate detailed data describing bathymetry and topography at smaller spatial scales, and the need to incorporate wave setup. As a result, it has been necessary to develop schemes that permit use of a smaller number of synthetic storms while maintaining accuracy. These schemes have come to be known as JPM-Optimal Sampling or JPM-OS schemes. In this usage Optimal is meant to suggest a good compromise between accuracy and computational cost in the selection of synthetic storms for a JPM analysis.

Two JPM-OS schemes have been developed and applied in recent hurricane studies performed by Federal Emergency Management Agency (FEMA) and US Army Corps of Engineers (USACE) teams for the north-central Gulf of Mexico coast. We will refer to these schemes as the *Quadrature* and the *Response-Surface* JPM-OS schemes, respectively. The quadrature JPM-OS scheme keeps the number of synthetic storms small by employing an algorithm that selects the parameter combinations in an optimal manner and assigns an appropriate weight to each synthetic storm, transforming the JPM multi-dimensional integral into a weighted summation. The response-surface JPM-OS scheme, on the other hand, interpolates between the surge results obtained for a carefully selected set of synthetic storms. Both approaches take advantage of the smooth variation of the calculated surge η as the hurricane parameters are varied.

This paper first summarizes the hurricane parameters required in most probabilistic surge studies, and provides an overview of the JPM approach as it is commonly applied. This is followed by a detailed discussion of the quadrature JPM-OS approach and other approaches meriting further investigation.

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A companion paper (Toro et al., this issue) provides a detailed comparison of results from the quadrature and response-surface approaches for the coast of Mississippi, using the same probabilistic storm characterization and the same numerical surge model. Another companion paper (Niedoroda et al., this issue) provides background on the overall Mississippi probabilistic surge study where this approach was used, and provides details on the derivation of the storm characteristics considered here. Further details on the data and analysis are available in FEMA (2008). The focus in the present paper is entirely on the quadrature technique, as illustrated by its application in that particular study.

2. Hurricane parameters

For the purposes of the JPM formulation, we describe the storm as it approaches the coast in terms of the following parameters (see Fig. 1): the pressure deficit ΔP (representing hurricane intensity), the radius of the exponential pressure profile R_p (representing hurricane size; the radius to maximum winds, R_{max}, is also frequently used), the storm's forward speed V_f, the storm heading θ (direction to, measured clockwise from north), and the landfall location (or, equivalently, a characteristic distance from the track to a coastal reference point). These parameters represent the primary hurricane characteristics controlling storm surge, and are treated as random variables in the JPM formulation. Other storm characteristics, including parameter B (Holland, 1980), have been treated as constant at landfall or are not considered explicitly, although they could be added as additional parameter dimensions at the cost of increasing the computational burden. Although hurricanes are much more complex than this parameterization allows for, and substantially more information is available for well-studied recent hurricanes, it is most practical at present to utilize this simple storm parameterization for the probabilistic characterization of possible future storms. Nonetheless, the differences between real hurricanes and this simple parameterization are not ignored in the JPM formulation, but have been included in a statistical sense by means of the secondary ε terms described in a later section. The particular probability distributions used here for illustration have been taken from the FEMA study for Mississippi, and are summarized in Niedoroda et al. (this issue).

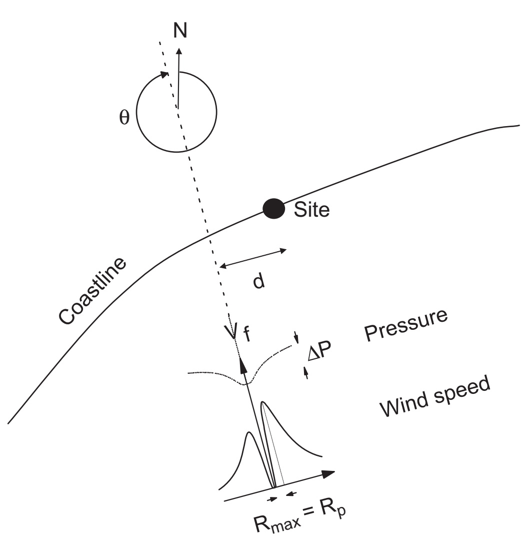

Fig. 1. Characterization of storm as it approaches the coast.

3. The joint probability method

The JPM formulation combines the following inputs:

1. The annual rate of storms of interest λ. Typically, it is also assumed that the occurrences of these storms in time follow a Poisson process (Parzen, 1962). The Poisson assumption is not strictly necessary, however.
2. The joint probability distribution $f_{\underline{X}}(\underline{X})$ of the storm characteristics for storms of interest. These characteristics are defined very broadly at first and narrowed later to make the approach practical.
3. The storm-generated surge $\eta(\underline{X})$ at the site of interest (including all components such as wave setup and tide), given the storm characteristics.

The combined effect of these three inputs is expressed by the multiple integral

$$P[\eta_{max(1\,\mathrm{yr})} > \eta] = \lambda \int \ldots \int_{\underline{x}} f_{\underline{X}}(\underline{x}) P[\eta(\underline{x}) > \eta] d\underline{x} \tag{1}$$

where $P[\eta(\underline{x}) > \eta]$ is the conditional probability that a storm of certain characteristics $\underline{x}$ will generate a flood elevation in excess of an arbitrary value η. This probability would be a Heaviside step function $H[\eta - \eta(\underline{X})]$ if the vector $\underline{X}$ contained a complete characterization of the storm and if we had a perfect tool for the calculation of surge given $\underline{x}$, but these conditions cannot be satisfied in practice. The integral above considers all possible storm characteristics from the population of storms of interest and calculates the fraction of these storms that produce surges in excess of the value of interest η. The right-hand side in Eq. (1) actually represents the mean annual rate of storms that exceed η at the site, but it also provides a good numerical approximation to the annual exceedance probability.

Eq. (1) defines a smooth function of η that can be used to determine the flood levels associated with any annual probability of exceedance. Typical values of interest include the 10%, 2%, 1% and 0.2% annual probabilities, often referred to as the 10-, 50-, 100- and 500 yrs annual exceedance levels, respectively.

As noted by Resio (2007), some approximations are necessary in practice for the evaluation of Eq. (1). Although these are not specific to the JPM-OS quadrature method discussed here, a brief mention of the formulation used in the illustrative Mississippi study is appropriate for clarity. There are two main sources of identifiable errors. First, modeling approximations and data deficiencies make it impossible to calculate the surge exactly. For example, we can't know the details of real wind and pressure fields, and we recognize errors associated with model resolution, description of land cover parameters, and so forth. Second, we may choose not to include a known factor or process in the model if its effects are small and if treating it by detailed modeling methods would be unacceptably costly. In such cases, an alternative is to consider these as (small) error terms, conventionally denoted by ε, and to assess their contributions statistically, writing the actual elevation $\eta(\underline{X})$ in terms of the model-calculated elevation $\eta_m(\underline{X})$ as $\eta(\underline{X}) = \eta_m(\underline{X}) + \varepsilon$. The term ε represents the sum of several error contributions, taken to be a random quantity uncorrelated with $\underline{X}$ and having a mean value of zero. Using this representation, one can write the actual conditional probability $P[\eta(\underline{x}) > \eta]$ as

$$P[\eta(\underline{X}) > \eta] = P[\eta_m(\underline{X}) + \varepsilon > \eta] \tag{2}$$

The error term might include inherent hydrodynamic model errors as well as the contributions of secondary terms not explicitly modeled, such as Holland's B parameter, astronomic tide (when very small compared with the surge values of interest), and so forth. In such a case, denoting these three factors by subscripts m, B, and $tide$, respectively, ε (with units of elevations) would be expressed as $\varepsilon = \varepsilon_m + \varepsilon_B + \varepsilon_{tide} + \cdots$.

Incorporating these simplifications, Eq. (1) transforms to

$$P[\eta_{\max(1\,yr)} > \eta] = \lambda \int \ldots \int_{\underline{x}} f_{\underline{X}}(\underline{x}) P[\eta_m(\underline{x}) + \varepsilon > \eta] d\underline{x} \tag{3}$$

where $\underline{X} = (\Delta P, R_p, V_f, \text{landfall location}, \theta)$.

The determination of the standard deviations of the various components of ε is described elsewhere (see Niedoroda et al., this issue).

3.1. The quadrature JPM-OS approach

As indicated earlier, evaluation of the JPM integral (Eq. (3)) using conventional numerical-integration approaches is impractical for two reasons: (1) each evaluation of the integrand involves evaluation of $\eta_m(\underline{x})$ for one value of $\underline{x}$ (i.e., one synthetic storm), which requires computationally intensive numerical calculations of wind, waves, surge, wave setup, etc.; and (2) numerical evaluation of the 5-dimensional integral in Eq. (3) using conventional approaches requires that the integrand be evaluated a large number of times (this is the so-called curse of dimensionality).

The quadrature JPM-OS approach approximates the integral in Eq. (3) as a weighted summation, i.e.

$$P[\eta_{max(1\,yr)} > \eta] = \lambda \int \ldots \int_{\underline{x}} f_{\underline{X}}(\underline{x}) P[\eta_m(\underline{x}) + \varepsilon > \eta] d\underline{x} \approx \sum_{i=1}^{n} \lambda_i P[\eta_m(\underline{x}_i) + \varepsilon > \eta] \tag{4}$$

where each $\underline{x}_i = (\Delta P_i, R_{p,i}, V_{f,i}, \text{landfall location}_i, \theta_i)$ may be interpreted as a synthetic storm (or, more precisely, the characteristics of the synthetic storm at landfall), $\lambda_i = \lambda p_i$ may be interpreted as the annual occurrence rate for that storm, and $\eta_m(\underline{x}_i)$ may be interpreted as the numerical-model's estimates of the storm elevation generated by that storm. For this approach to be practical, one must be able to specify the storm characteristics $\underline{x}_i$ and their rates λ_i so that the integral can be approximated with sufficient accuracy (for all η values of interest), using a reasonably small value of n (i.e., a reasonably small number of synthetic storms and corresponding numerical-model runs).

The approach used to define the synthetic storms and their rates uses a combination of well-known and more sophisticated lesser known techniques, and may be summarized by the following three steps:

1. Discretize the distribution of ΔP into broad slices. In the illustrative FEMA work (Niedoroda et al., this issue) three slices were used, roughly corresponding to Saffir-Simpson hurricane Categories 3, 4, and 5.
2. Within each ΔP slice, discretize the joint probability distribution of ΔP(*within slice*), R_p, V_f, and θ using the multi-dimensional optimal-sampling procedure known as Bayesian quadrature (Diaconis, 1988; O'Hagan, 1991; Minka, 2000; see later discussion). This procedure represents the response portion of the integrand (i.e., the term $P[\eta_m(\underline{x}) + \varepsilon > \eta]$ as a random function of $\underline{x}$ with certain covariance properties), and determines the optimal values of ΔP_i, $R_{p,i}$, $V_{f,i}$, θ_i, and the associated probability, so that the variance of the integration error is minimized. The covariance properties of the random function (which take the form of correlation distances) depend on how sensitive the response is to each variable (shorter correlation distances for the more important variables). These correlation distances were set based on judgment and on the results of the sensitivity tests described in Niedoroda et al. (this issue).
3. Discretize the distribution of landfall location by replicating each of the synthetic storms defined in the previous two steps at spatial offsets equal to R_p (measured perpendicular to the storm track). Sensitivity studies indicated that a spacing of R_p is small enough to capture the peak surge at all grid locations.

Finally, one computes the probability p_i assigned to each synthetic storm as the product of the probabilities resulting from the three steps. Equivalently, one can compute the rate λ_i assigned to each synthetic storm as the product of the probabilities from the first two steps times the rate per unit length times the storm spacing.

It is useful to discuss some possible variations to this scheme, which may apply to other situations.

- If one were performing calculations for a single site, one could treat location (or distance to the site) as one of the quantities in the Bayesian quadrature (Step 2). To improve the efficiency of this scheme, one could use *importance sampling* (e.g., Melchers, 1999) to sample more heavily at distances near R_p on the strong side of the storm. When performing calculations for a large number of sites, it is considered more convenient to sample distance using constant track spacing (Step 3 above).
- One could include ε as one of the random quantities in the Bayesian quadrature, instead of treating it as part of the effects term. This change would have two detrimental effects on the efficiency of the Bayesian-quadrature scheme, as follows: (1) the number of dimensions would increase, and (2) the $P[\ldots]$ term in the integrand would become more irregular because it has not been integrated over ε, making the Bayesian-quadrature integration more difficult. On the other hand, one can think of situations where this treatment of ε may be required (e.g., if the hydrographs $\eta(t)$ for the synthetic storms are to be used as inputs to a computationally intensive calculation that is nonlinear in $\eta(t)$).

3.2. Bayesian-quadrature approach—background

The word *quadrature* is often used to denote numerical techniques to approximate an integral of the form

$$I = \int_A f(x) p(x) dx \tag{5}$$

over some domain A, as a weighted sum of the form

$$I \approx \sum_{i=1}^{n} w_i p(x_i) \tag{6}$$

where $f(x)$ is typically a probability density function (i.e., it is positive and it integrates to unity) and $p(x)$ represents a function belonging to a certain family of functions.

In our case, A represents a four-dimensional domain, $f(\underline{x})$ represents the joint probability distribution of storm characteristics, and $p(\underline{x}) = P[\eta_m(\underline{x}) + \varepsilon > \eta]$ represents the "surge effects" portion of the JPM integral (for many possible locations and for many possible values of η).

The design of a quadrature involves specification of the number of nodes n, and the selection of both the node values x_i and their associated weights w_i (for $i = 1, \ldots n$). These quantities depend on the functional form of $f(\underline{x})$ and of the $p(\underline{x})$ family, and on the desired characteristics (e.g., accuracy) of the approximation. In our case, each node becomes one synthetic storm.

Classical (Gaussian) quadrature chooses the number of nodes, node values, and weights so that the summation will evaluate the integral exactly if $p(x)$ is a polynomial of a certain degree and $f(x)$ is a particular function (e.g., a standard normal probability density). This technique is used frequently in one dimension (see Miller and Rice, 1983, for details and for results for a variety of probability distributions). Classical quadrature of this type is used later in this paper to construct an accurate, but inefficient, reference-case JPM formulation (which we call JPM-REF). Extension of these zero-error rules to more than one dimension is problematic. The number of required nodes increases rapidly with the number of dimensions (see Minka, 2000). Furthermore, some of the weights often become negative (see Genz and Keister, 1996), which leads to less stable results and makes it impossible to interpret the weights as probabilities.

Bayesian quadrature, in contrast, considers a much broader probabilistically defined family of functions (i.e., realizations of a random process with a certain covariance structure) and minimizes the integration error in a mean-squared sense. Conceptually, it is straightforward to extend Bayesian quadrature to multiple dimensions, although it becomes somewhat computationally demanding for more than six or seven dimensions. According to Diaconis (1988), the approach dates back to the work of Poincare in 1896. It is also closely related to the technique known as *Kriging* (e.g., Journel and Huijbregts, 1978), and even to least-squares regression.

3.3. Derivation

The development of Bayesian quadrature begins by idealizing the function $p(\underline{x})$ in Eq. (5) as a Gaussian random process in m-dimensional space with mean zero and with autocovariance function $k(\underline{x},\underline{y}) = E[p(\underline{x})p(\underline{y})]$, where $E[\cdot]$ denotes mathematical expectation (i.e., $E[z]$ the average value of quantity z over all possible realizations of $p(\underline{x})$), and x and y are any two arbitrary values of x. The dimension, m, is the number of dimensions for the integral in Eq. (5). In this derivation, x and y are m-dimensional vectors, and we will underline them. The autocovariance function contains information about the degree of continuity or smoothness of realizations of $p(\underline{x})$, at both small and large scales (e.g., Vanmarcke, 1983), and will be considered in more detail later; the only requirement at this stage is that the required integrals involving $k(\underline{x},\underline{y})$ and $f(\underline{x})$ do not diverge. Note, too, that the assumption that the process is Gaussian is not strictly necessary; one may obtain the same results using weaker assumptions (namely, by assuming that the conditional mean and variance of the random process $p(\underline{x})$ satisfy the two properties described in the next paragraph; the assumption of a Gaussian distribution for $p(\underline{x})$ is not required).

Let matrix $\mathbf{D} = [\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n]$ denote n nodes for which $p(\underline{x})$ has been evaluated, so that we know the values of $p(\mathbf{D}) = [p(\underline{x}_1), p(\underline{x}_2), \ldots, p(\underline{x}_n)]$. In our m-dimensional integration space, each node location $\underline{x}_i$ is a vector consisting of m nodal coordinates. Because we know the value of $p(\underline{x})$ at these n points, we also know the conditional mean and the conditional variance of $p(\underline{x})$ at all other values of x; this is illustrated in Fig. 2. The mean$E[p(\underline{x})|p(\mathbf{D})]$ is a linear combination of the known nodal values $p(\mathbf{D}) = [p(\underline{x}_1), p(\underline{x}_2), \ldots, p(\underline{x}_n)]$, with coefficients that depend on the values of the covariance function $k(\underline{x},\underline{y})$ between x and each nodal point and between each pair of nodal points. In addition, the conditional variance $Var[p(\underline{x})|p(\mathbf{D})]$depends on the values of the covariance function $k(\underline{x},\underline{y})$, but does not depend on the values of $p(\mathbf{D})$.

O'Hagan (1991) proposes an approximation to Eq. (5) in which one uses the conditional mean $E[p(\underline{x})|p(\mathbf{D})]$ in place of $p(\underline{x})$ (whose value we know at only a few points), i.e.,

$$I = \int_A f(\underline{x})p(\underline{x})d\underline{x} \approx \int_A f(\underline{x})E[p(\underline{x})|p(\mathbf{D})]d\underline{x} \tag{7}$$

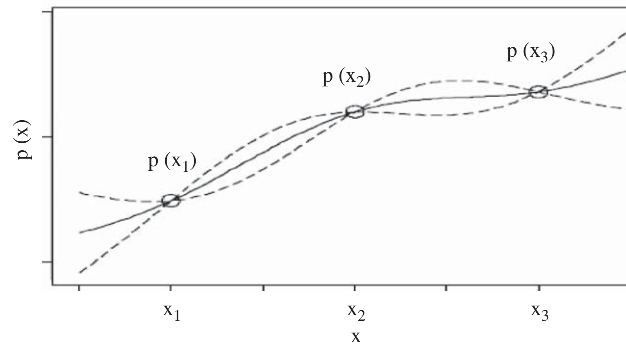

Fig. 2. Illustration of the conditional distribution of random function $p(x)$ at intermediate points between sampling nodes. The function $p(x)$ has been sampled at 3 nodes x_1, x_2, x_3. The solid line displays the conditional mean value. The dashed lines display the conditional mean $\pm$ standard deviation range; the width of this range depends on the distance to the nodes and on the autocovariance function $k(x,y)$.

Because the mean $E[p(\underline{x})|p(\mathbf{D})]$ is a linear combination of the nodal values $p(\mathbf{D}) = [p(\underline{x}_1), p(\underline{x}_2), \ldots, p(\underline{x}_n)]$, the above approximation is also a linear combination of the nodal values, which means that it has the same functional form of Eq. (6) (i.e., a weighted sum of the values of $p(\underline{x})$ at the nodes), with weights that depend on $k(\underline{x},\underline{y})$ and on integrals involving $k(\underline{x},\underline{y})$ and $f(\underline{x})$ (namely, $\underline{W}^T = [w_1, w_2, \ldots, w_n] = \underline{U}(\underline{x},\mathbf{D})\mathbf{K}(\mathbf{D},\mathbf{D})^{-1}$), where

$$\underline{U}(\mathbf{D}) = \begin{bmatrix} \int_A k(\underline{x},\underline{x}_1)f(\underline{x})d\underline{x} \\ \int_A k(\underline{x},\underline{x}_2)f(\underline{x})d\underline{x} \\ \vdots \\ \int_A k(\underline{x},\underline{x}_n)f(\underline{x})d\underline{x} \end{bmatrix} \tag{8}$$

and

$$\mathbf{K}(\mathbf{D},\mathbf{D}) = \begin{bmatrix} k(\underline{x}_1,\underline{x}_1) & k(\underline{x}_1,\underline{x}_2) & \cdots & k(\underline{x}_1,\underline{x}_n) \\ k(\underline{x}_2,\underline{x}_1) & k(\underline{x}_2,\underline{x}_2) & \cdots & k(\underline{x}_2,\underline{x}_n) \\ \vdots & \vdots & \ddots & \vdots \\ k(\underline{x}_n,\underline{x}_1) & k(\underline{x}_n,\underline{x}_2) & \cdots & k(\underline{x}_n,\underline{x}_n) \end{bmatrix} \tag{9}$$

Minka (2000) shows that one can arrive at the same weights with a least-squares formulation, i.e., by determining the weights that minimize the variance (over all possible realizations of $p(\underline{x})|p(\mathbf{D})$) of the difference between the exact integral and the approximation given by Eq. (6).

The resulting weights from this approach may or may not add to unity, depending on the choice of $k(\underline{x},\underline{y})$ and of $\mathbf{D}$. To ensure that the weights always add to unity – which is required because we want to interpret $p(\mathbf{D}) = [p(\underline{x}_1), p(\underline{x}_2), \ldots, p(\underline{x}_n)]$ and the weights as an m-dimensional discrete probability distribution – we introduce the constraint $\sum_{i=1}^{n} w_i = 1$ into Minka's (2000) least-squares representation of the problem. We do this by means of a Lagrange multiplier, obtaining the following system of equations:

$$\begin{bmatrix} k(\underline{x}_1,\underline{x}_1) & k(\underline{x}_1,\underline{x}_2) & \cdots & k(\underline{x}_1,\underline{x}_n) & 1 \\ k(\underline{x}_2,\underline{x}_1) & k(\underline{x}_2,\underline{x}_2) & \cdots & k(\underline{x}_2,\underline{x}_n) & 1 \\ \vdots & \vdots & \ddots & \vdots & 1 \\ k(\underline{x}_n,\underline{x}_1) & k(\underline{x}_n,\underline{x}_2) & \cdots & k(\underline{x}_n,\underline{x}_n) & 1 \\ 1 & 1 & \cdots & 1 & 0 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ -\mu \end{bmatrix} = \begin{bmatrix} \underline{U}(\mathbf{D}) \\ 1 \end{bmatrix} \tag{10}$$

where μ is the Lagrange multiplier (see, e.g., Journel and Huijbregts, 1978). We solve this system of linear equations to obtain weights that add to unity.

The associated estimation variance is given by the expression

$$Var[\int_A f(\underline{x})p(\underline{x})dx - \sum_{i=1}^{n} w_i p(\underline{x}_i)] = u - \underline{W}^T \underline{U}(\mathbf{D}) + \mu \tag{11}$$

where $W^T = [w_1, w_2, \ldots, w_n]$ is the vector of weights obtained by solving Eq. (10) and

$$u = \int_A \int_A k(\underline{x}, \underline{y}) f(\underline{x}) f(\underline{y}) d\underline{x} d\underline{y} \tag{12}$$

It may also be possible to force the weights to sum to unity by assuming that $p(\underline{x})$ has an unknown (but generally non-zero) mean.

So far in this discussion, we have treated the node locations $\mathbf{D} = [\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n]$ as known. The development of an efficient quadrature rule requires finding the optimal node locations $\mathbf{D}$ that minimize the variance in Eq. (11), for a pre-specified value of n. Because each $\underline{x}_i$ in $\mathbf{D}$ represents the coordinates of a node in m-dimensional space, determination of the optimal $\mathbf{D}$ is an $(m \times n)$-dimensional optimization problem. Thus, we have two nested optimizations, both of which seek to minimize the variance of the integration error. At the inner level of nesting, there is the optimization to determine the best weights (for given nodal locations $\mathbf{D}$). This is done analytically, by solving Eq. (10). At the outer level, there is the search for the best set of nodal locations $\mathbf{D}$. This is done numerically (more details will be provided in subsequent sections).

We have not shown that all the weights are non-negative, and in fact negative weights do arise when the nodal vector $\mathbf{D} = [\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n]$ is specified arbitrarily (i.e., without optimization). On the other hand, it is reasonable to expect (and one may be able to prove) that optimization of $\mathbf{D} = [\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n]$, so as to minimize the variance in Eq. (11), forces all the weights to be positive. This argument is related to the concept of quadrature stability factor (i.e., the sum of the absolute values of the weights) employed by Genz and Keister (1996). In our practical applications, the approach introduced here has always led to positive weights when $\mathbf{D}$ is optimized. Negative weights, if they happen to arise, may be eliminated by employing a numerical optimization scheme with inequality constraints on the weights.

3.4. Implementation of Bayesian quadrature for JPM-OS

The prior discussion neglected certain details of the implementation for the sake of generality and simplicity. This section provides some additional information regarding how the Bayesian-quadrature procedure was used as part of the quadrature JPM-OS scheme used in the FEMA Mississippi study.

3.5. Probability distribution: choice for f(x)

The quadrature JPM-OS formulation must be flexible enough to accommodate the joint probability distributions typically used for ΔP, R_p, etc. On the other hand, implementation of the Bayesian-quadrature formulation described above using a general form for $f(\underline{x})$ would require the repeated evaluation of the integrals in Eqs. (8) and (12), which are likely more complex than the JPM integrals we are trying to solve in the first place.

Instead, we formulate the problem in m-dimensional standard normal-distribution space, determine the coordinates of the integration nodes in that space, and then convert these nodal coordinates to the "physical" space of ΔP, R_p, etc., in a manner that takes into account their joint probability distribution. This approach for the transformation of multivariate probability distributions is the so-called Rosenblatt Transformation; it is commonly used in structural reliability theory (e.g., Madsen et al., 1986; Melchers, 1999) and is built into structural reliability software (e.g., Gollwitzer et al., 2006).

One can also achieve the distribution transformations by altering the weights (in a manner similar to acceptance–rejection methods in random-number generation) or by using a combination of both approaches.

3.6. Covariance structure of $p(\underline{x})$: choice of $k(\underline{x}, \underline{y})$ and the specification of correlation distances

We use the covariance structure of $p(\underline{x})$, as represented by the covariance function $k(\underline{x}, \underline{y})$, to specify the importance of the corresponding physical quantity in the surge calculations. If the physical quantity corresponding to the j-th component of x is important, correlation decays faster in that direction than in the direction corresponding to a less-important quantity.

One of the consequences of formulating and solving the Bayesian-quadrature problem in standard normal space is that we must also define $k(\underline{x}, \underline{y})$ in standard normal space. We must also choose a functional form that facilitates analytical evaluation of the integrals in Eqs. (8) and (12).

We choose the double-exponential functional form for the covariance function, i.e.,

$$k(\underline{x}, \underline{y}) = E[p(\underline{x})p(\underline{y})] = \sigma^2 \prod_{j=1}^{m} \exp\left[-\left(\frac{x_j - y_j}{c_j}\right)^2\right] \tag{13}$$

where c_j controls how quickly the correlation decays in the direction of a particular component (in this section, the subscripts denote the coordinates of one point in m-dimensional space $j=1,\ldots m$; x and y denote two points in that m-dimensional space). This covariance model implies that the random field is homogeneous (i.e., correlation depends only on the distance between nodal points) and smooth (more precisely, twice differentiable in a second-order sense). The variance σ^2 cancels out in Eq. (10), and is omitted in the material that follows. The quantity c_j is related to the corresponding correlation distance or scale of fluctuation d_j (Vanmarcke, 1983) by the relation $d_j = \sqrt{\pi} c_j$. Because the covariance function is defined in standard normal space, c_j and d_j have no physical units.

One of the critical steps in the quadrature JPM-OS analysis is the specification of the correlation distances d_j associated with the various hurricane characteristics. This is made more difficult because these correlation distances are specified in normal space, not in physical space. The following discussion provides some guidance to facilitate this step.

In a relative sense, the quadrature JPM-OS algorithm tends to spread the sampling nodes more evenly along those directions for which $p(\underline{x})$ has lower correlation distances, providing a closer match to the marginal probability distributions in those directions. Thus, it is important to specify correlation distances that relate to the importance of the various physical quantities, in order to obtain an optimal allocation of effort among the various directions. In an absolute sense, numerical experiments in one dimension show that low values of the correlation distance cause the algorithm to be more cautious and tend towards equal weights, while high values provide a wide range of weights and nodes that extend further into the tails of the distribution, approaching those obtained by Gaussian quadrature. The ideal choice is between these two extremes. As preliminary guidance for the choice of correlation distances, one might associate sensitive (more important) parameters with d_j values of 1 to 3, and insensitive (or less important) parameters with d_j values of 4 to 6. In principle, one could calculate appropriate values for these correlation distances analytically, using as inputs the results from sensitivity runs such as those described in Niedoroda et al. (this issue), or from the predictions of parametric models (e.g., Irish et al., 2008, 2009); this calculation should take into account the distribution transformation. In our work so far, explicit calculation of correlation distances has not been done, relying instead on choices made on the basis of judgment and on verification against reference JPM results, using an inexpensive numerical surge model.

3.7. Possible refinements in distribution shape and covariance structure

It is also possible to construct $f(\underline{x})$ as a mixed product of probability-distribution shapes (e.g., normal in some directions, uniform in others, exponential or possibly Weibull in others), chosen in such a way that these distribution shapes are closer to the distribution of the corresponding storm characteristics. This may require a somewhat different functional form for the autocovariance function in the non-normal directions, in order to permit analytical evaluation of the required integrals. The effect of this refinement is anticipated to be better performance of the Bayesian quadrature for distribution shapes such as the Weibull, possibly eliminating the need for pre-slicing of the distribution of ΔP.

3.8. Optimization algorithm

As was indicated earlier, determination of n optimal sampling points in m dimensions constitutes an $(m\times n)$-dimensional optimization problem. We have performed this optimization using the NEWUOA algorithm developed by Powell (2004), which does not require derivatives, and choosing the starting points at random. Convergence is fast for the number of dimensions and nodes considered in this work.

3.9. Need for validation

Because we make a number of assumptions regarding the functional form and parameters of the autocovariance function $k(\underline{x},\underline{y})$ of $p(\underline{x})=P[\eta_m(\underline{x})+\varepsilon>\eta]$, it is important to validate the quadrature JPM-OS scheme (i.e., the number of nodes and the correlation distances).

In the illustrative FEMA Mississippi study, we validated the JPM-OS scheme by comparing the cumulative distributions of surge obtained using the JPM-OS scheme with those from a Reference JPM scheme (denoted JPM-REF) using the National Ocean and Atmospheric Administration (NOAA) SLOSH (Sea, Lake and Overland Surges from Hurricanes) program as a diagnostic tool to compute the surge in both schemes. These comparisons are discussed in Niedoroda et al. (this issue). One can also perform such a validation using a parametric surge model (e.g., Irish et al., 2008, 2009).

4. Application to hurricanes in Mississippi

The illustrative work for Mississippi is discussed in some detail in a companion paper (Niedoroda et al., this issue). In this section, we simply summarize key aspects of that work. The example discussed here addresses hurricanes having central pressure depressions, ΔP, exceeding 45 mb (45 HPa).

4.1. Selected JPM-OS scheme

This selected scheme (called JPM-OS 6) was adopted after numerical experiments with a number of schemes having different numbers of nodes and somewhat different correlation distances. Table 1 shows the various slices of the ΔP distribution, their probabilities, and the number of nodes used in the Bayesian-quadrature discretization for each slice. Table 2 shows the correlation distances used in the Bayesian-quadrature procedure. These values were chosen based on extensive sensitivity results and then refined to preserve the marginal moments of the most important quantities.

Table 1
Discretization of ΔP into slices in JPM-OS 6 Scheme for greater storms.

Slice	Cat. 3	Cat. 4	Cat. 5
ΔP range (mb)	45–70	70–95	95–135
Probability	0.657	0.261	0.082
# of points in Bayesian Quadrature	5	7	7

Table 2
Correlation distances in JPM-OS 6 scheme for greater storms and in other candidate JPM-OS schemes considered.

Scheme	Correlation distance (std. normal units)			
	ΔP (within slice)	RP	Vf	Heading
JPM-OS 6	4	2.5	6	5
JPM-OS 3	2	2	4	3
JPM-OS 4a	3	2	5	4
JPM-OS 4b	4	2.5	6	5
JPM-OS 7	2.5/4[a]	2.5	6	5

[a] 2.5 for categories 3 and 4 combined; 4 for category 5.

Fig. 3 provides an illustration of the parameters of the resulting synthetic storms (for one landfall location, i.e., prior to the multiple track-offsetting step). Each chart on the main diagonal shows the probability distribution of the corresponding quantity (in the form of a histogram), as represented in the JPM-OS 6 discretization. Each off-diagonal scatter diagram shows how each pair of quantities (e.g., ΔP and R_p) is jointly distributed in the JPM-OS 6 scheme, with the areas of the circles being proportional to the associated annual rate. After replicating each storm to cover alternative landfall locations, the JPM-OS 6 calculations involved the simulation of 152 storms (parameter combinations).

4.2. Validation of JPM-OS scheme

As indicated earlier, the JPM-OS 6 scheme was validated by comparing against a reference-case JPM scheme (JPM-REF), using the SLOSH software as a diagnostic tool to compute the surge in both schemes. This validation procedure is feasible because the SLOSH surge simulations are quite fast, while retaining the essential characteristics of the surge variation over the study area.

The parameter values and annual rates for the JPM-REF synthetic storms were determined using the one-dimensional quadrature approach described by Miller and Rice (1983), using different numbers of quadrature points according to the importance of the parameter. For R_p (which depends on ΔP), values were drawn from the conditional distribution of radius (given ΔP). In addition, each of the 360 combinations of $\Delta P-R_p-\theta-V_f$ was assigned to multiple parallel tracks with a perpendicular spacing of R_p, as described earlier. For each of the resulting 2967 synthetic storms (i.e., for each combination of parameters and track), the storm event rate was obtained by multiplying the probability associated with each of the storm parameter values (i.e., $p(\Delta P_i)\times p(R_{p,i}|\Delta P_i)\times p(V_{f,i})\times p(\theta_i)$) and then multiplying that result by the storm spacing (equal to R_p) times the annual rate of $\Delta P>45$ mb

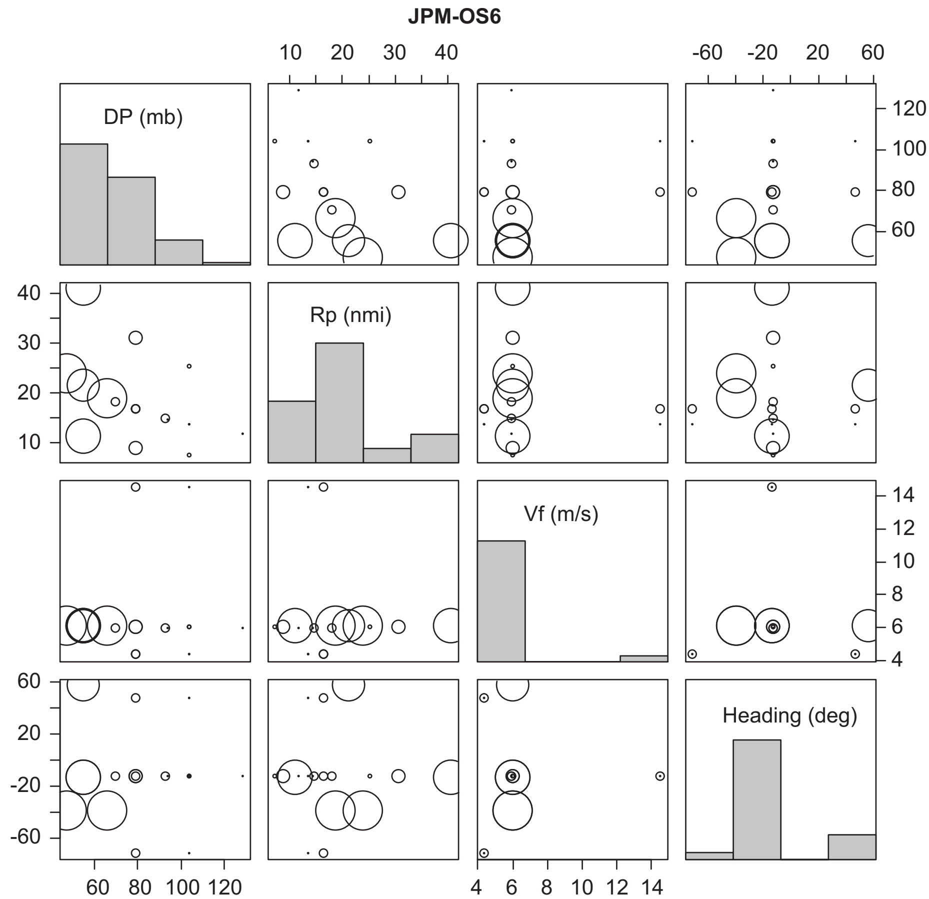

Fig. 3. Graphical representation of the JPM-OS 6 scheme for one landfall location. The areas of the circles are proportional to the associated annual rate.

Table 3
Parameter discretizations for JPM-REF scheme.

ΔP (mb)[a]	45.6	48.6	56.1	69.1	87.6	111.8
Probability	0.0496	0.1661	0.2844	0.2844	0.1661	0.0496
R_p (nmi)[2] for ΔP=45.6 mb	6.94	13.43	24.38	44.28	85.71	
R_p (nmi) for ΔP=48.6 mb	6.63	12.84	23.32	42.34	81.96	
R_p (nmi) for ΔP=56.1 mb	5.99	11.59	21.05	38.23	74.00	
R_p (nmi) for ΔP=69.1 mb	5.16	10.00	18.15	32.96	63.80	
R_p (nmi) for ΔP=87.6 mb	4.36	8.44	15.33	27.84	53.88	
R_p (nmi) for ΔP=111.8 mb	3.67	7.10	12.89	23.41	45.31	
Probability	0.01	0.22	0.53	0.22	0.01	
Heading (θ)	−73.0	−32.7	7.3	49.4		
probability	0.133	0.367	0.367	0.133		
Fwd. velocity (m/s)	2.99	6.04	12.23			
probability	0.1667	0.6666	0.1667			

[a] Unit conversions: 1 mb = 1 HPa; 1 nmi = 1.852 km.

storms occurring in the Gulf coast area. This rate was previously determined to be 3.02E−4 storms/km/yr. Table 3 shows the quadrature nodes and associated probabilities for the various hurricane parameters.

Fig. 4 provides an illustration of the parameters of the resulting synthetic JPM-REF storms (for one landfall location), in a manner similar to Fig. 3. Notice that some of the nodes receive very low weights (i.e., points that are barely visible) because they correspond to combinations of unlikely parameter values. It would be wasteful to spend significant computer resources evaluating the surge for each of these low-weight combinations; this illustrates the inefficiencies that may occur when evaluating the JPM integral using conventional approaches.

For the validation of JPM-OS 6, we calculated surges at 147 test points scattered throughout coastal Mississippi and adjacent portions of Louisiana using both the JPM-OS6 and JPM-REF synthetic storms, and calculated the associated cumulative distributions of surge elevation for each storm set. Fig. 5 shows a comparison at a typical coastal point, confirming the accuracy of the OS-6 findings from 152 simulations, compared to the REF case

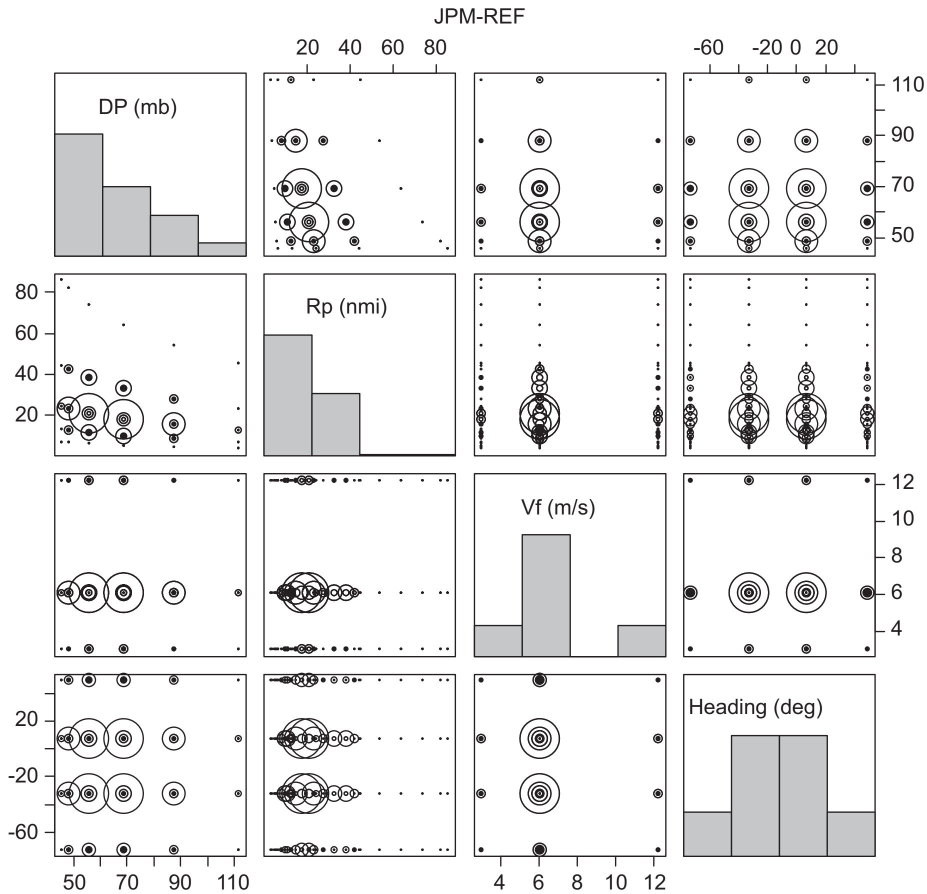

Fig. 4. Graphical representation of the JPM-REF scheme for one landfall location. The areas of the circles are proportional to the associated annual rate. Unit conversions: 1 mb = 1 HPa; 1 nmi = 1.852 km.

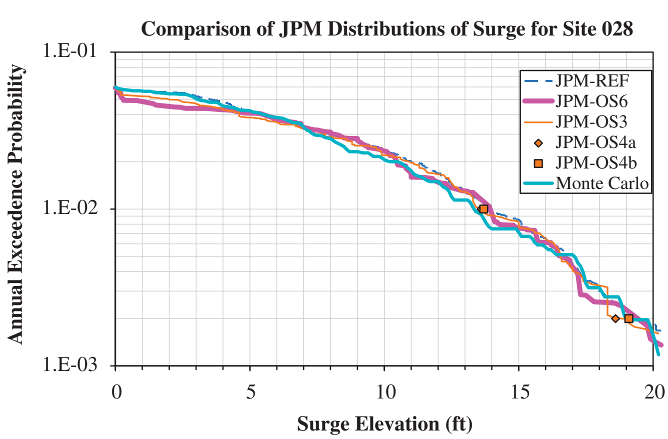

Fig. 5. Comparison of cumulative distribution functions for a test point along the shore near Biloxi, MS. Results for JPM-OS 4a and b are shown only for 100 and 500 years. Unit conversions: 1 ft = 0.3048 m.

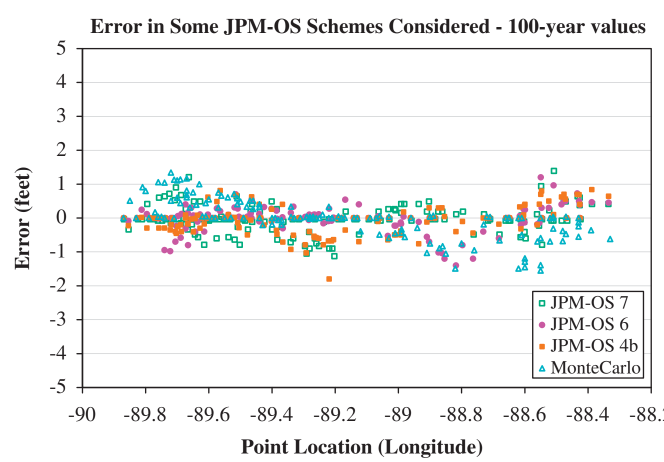

Fig. 6. Comparison of the errors in the 100 yr values (defined relative to the Reference JPM scheme) of various JPM-OS schemes considered, for all test points considered in the validation. Unit conversions: 1 ft = 0.3048 m.

with nearly 3000 simulations. The 100 yr surge corresponds to an annual exceedance probability of 0.01, for which the difference between the two curves is less than 1 foot. Note also the granularity of the cumulative distributions. Fine granularity is generally an indication that the JPM-OS scheme is adequate in the corresponding portion of the curve.

Fig. 5 also shows the results obtained using a different number of simulations and different correlation parameters, as well as results from Monte Carlo simulation. Candidate schemes JPM-OS3, JPM-OS4a, and JPM-OS4b all use 303 simulations, but they use different values for the correlation distances (see Table 2). Comparison of these results provides an indication of the sensitivity to these parameter choices. The Monte Carlo results were obtained using 152 simulations (i.e., the same number of simulations as the JPM-OS6 scheme).

Similarly, Fig. 6 compares the errors in the 100 yr results from candidate schemes JPM-OS 4b, JPM-OS 6, JPM-OS 7 (which uses 146 simulations), and Monte Carlo, for all test points considered in

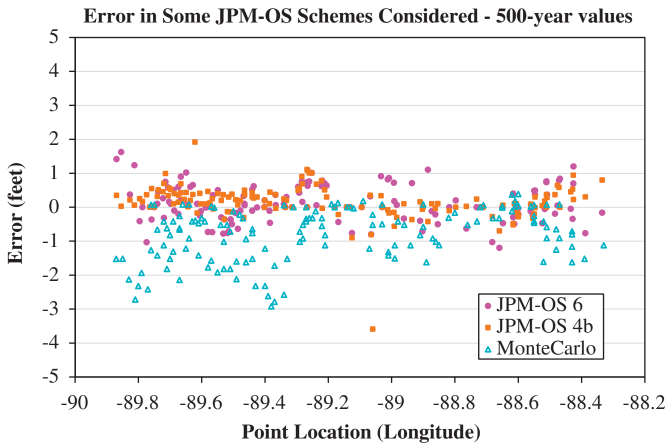

Fig. 7. Comparison of the errors in the 500 yr values (defined relative to the Reference JPM scheme) of various JPM-OS schemes considered, for all test points considered in the validation. Unit conversions: 1 ft = 0.3048 m.

the validation. The error is defined here as the difference between the 100 yr results from the candidate scheme and from JPM-REF. For the sake of scale, the 100 yr values range from 0 to 15 feet (see Toro et al., this issue, for related results). Comparison of the 500 yr results (Fig. 7) shows similar trends. All the candidate JPM-OS schemes, which span a factor of two in the number of synthetic storms and use a moderate range of values for the correlation distances, are sufficiently accurate and are more accurate than Monte Carlo, particularly for the 500 yr values. JPM-OS 6 was chosen because it requires half as many simulations as JPM-OS 3 or 4b and because it has a slightly lower root-mean-square error than JPM-OS 7.

5. Other multi-dimensional quadrature formulations

The simplest way to construct a multi-dimensional quadrature scheme is to apply a Classical quadrature to discretize each hurricane parameter and then generate all possible combinations of parameters. The probability assigned to each combination is the product of the probabilities for the corresponding discrete parameter values. If the parameters are not independent, this is done in a sequential manner. This approach, which is called a "product-rule" quadrature, is precisely what was used above to construct the JPM-REF scheme.

Unfortunately, this approach generates a very large number of parameter combinations as the number of dimensions increases. Some of the combinations receive very low relative probabilities, suggesting that many of these combinations do not warrant separate modeling runs. Smolyak (1966) developed a procedure to reduce the number of parameter combinations needed in multi-dimensional quadratures, but we have not investigated this approach.

Hong (1998) developed an approach that requires $2m+1$ nodes to approximate an integral in m dimensions, while conserving moments up to order 3. Unfortunately, the weight for the central node tends to become negative as the number of dimensions increases. Negative weights lead to some synthetic storms having negative recurrence rates, are usually associated with higher error variances, and may lead to other difficulties such as negative probabilities in cumulative distributions.

5.1. *The response-surface JPM-OS approach*

An alternate OS approach was developed independently as part of the USACE post-Katrina studies of Louisiana. Called the response-surface approach, it involves simulation of a moderate number of synthetic storms – with carefully selected combinations of parameters – to construct a higher-dimensional surface upon which surge elevations for any combination of parameters can be obtained by interpolation. The computational cost for this interpolation is minimal so that, as a result, one can discretize the domain of the JPM integral very finely, even in five dimensions.

The main difficulty in the response-surface JPM-OS scheme resides in the *experimental design*, i.e., selection of the parameter combinations for the synthetic storms in a manner that provides enough points in the five-dimensional $\Delta P - R_p - \theta - V_f - distance$ parameter space, without requiring a very large number of synthetic storms, and then implementing an accurate interpolation scheme that works reliably for all sites of interest. This selection process takes advantage of the weak sensitivity to heading angle θ and forward velocity V_f, concentrating on ΔP, R_p, and track location. Key to the success of the method is the adoption of a characteristic alongshore surge profile shape to control interpolation between adjacent tracks. A discussion of this approach and detailed comparisons with the quadrature approach described here are presented in Toro et al. (this issue). As shown there, the response-surface method has been shown to produce results of comparable accuracy to the quadrature approach, and for the same computational effort. Further details on the response-surface approach are provided in Resio (2007) and in Irish and Resio (2008, 2009).

6. Summary and discussion

This paper has described the quadrature JPM-OS approach, with emphasis on its derivation, application, and verification; a brief description of the response-surface approach has also been given. Experience from the FEMA study for Mississippi, and from the USACE studies for Mississippi and Louisiana, shows that the two approaches are comparable in their accuracy and efficiency. This is confirmed in a companion paper where the two approaches are applied to the same problem and compared to a reference JPM implementation (Toro et al., this issue). In addition, the comparisons shown here for the various candidate JPM-OS schemes indicate that the approach yields accurate results without the need for delicate tuning of the various inputs.

The assumptions introduced in the quadrature JPM-OS approach are (a) that the exceedence probability $p(\underline{x}) = P[\eta_m(\underline{x})+\varepsilon > \eta]$ for any storm with parameters $\underline{x}$ and for any surge elevation of interest has an autocovariance function $k(\underline{x},\underline{y})$ given by Eq. (13) (i.e., that the function is double exponential, with given correlation distances in the various dimensions, and with a separable functional form); (b) that the conditional mean value of $p(\underline{x})$ at an arbitrary point $\underline{x}$, given observations at $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n$ is a linear combination of the values $p(\underline{x}_1), p(\underline{x}_2), \ldots, p(\underline{x}_n)$; and (c) that the conditional variance of $p(\underline{x})$ depends on the coordinates $\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_n$, but not on the actual values $p(\underline{x}_1), p(\underline{x}_2), \ldots, p(\underline{x}_n)$. These assumptions are introduced for the sake of mathematical tractability, as is often the case in engineering models, and they represent an approximation to a more complex probabilistic structure. For this reason, it is important to perform some sort of validation of the resulting synthetic-storm parameters and weights. We have used SLOSH for this purpose, in addition to checks on marginal and joint probabilistic moments.

In this regard, it is important to note that the assumptions described above are made in order to determine the parameters and rates of the synthetic storms. After this is done, it is the synthetic storms and their effects that drive the results. Also, no approximation is made in the JPM-OS calculations regarding the probability distributions of the storm parameters. These parameters are mapped into a multi-dimensional normal distribution using the Rosenblatt transformation, but this mapping is exact.

As indicated earlier, one may be able to improve the performance of the JPM-OS approach described here by mapping the joint distribution of the physical parameters into a mixed product of probability-distribution shapes (e.g., normal along some dimensions, uniform along others, exponential or possibly Weibull along others). These distributions would be chosen in such a way that they are closer in shape to the distribution of the corresponding storm characteristic, while still allowing analytical evaluation of the integrals in Eqs. (8) and (12). The advantage of doing this is that the mapping between probability distributions becomes less nonlinear, which makes the simple autocovariance function in Eq. (13) more realistic. In addition to improved performance, this improvement may streamline the calculations by eliminating the need for pre-slicing of the distribution of ΔP.

This study has not compared the approach presented here to other approaches, such as the approach by Smolyak (1963) mentioned earlier, Latin Hypercubes, and a variety of techniques known loosely as Fast Monte Carlo. A side-by-side comparison, similar to the comparison performed in the companion paper by Toro et al. (this issue), would be worthwhile.

It is not surprising that the conventional Monte Carlo method is less accurate than the quadrature JPM-OS approach described here, when both approaches use the same number of synthetic storms. The 500 yr flood elevations calculated with Monte Carlo are particularly unstable due to sample variability, and one must quadruple the number of synthetic storms in order to cut the Monte Carlo root-mean-square error in half.

It may be possible, and would certainly be worthwhile, to improve the efficiency of the numerical wave and surge calculations for probabilistic surge hazard studies by using somewhat more coarse computational grids and more efficient algorithms – while maintaining the necessary accuracy for studies of this kind. Nonetheless, the need for JPM-OS techniques will remain because the hurricane parameterization for surge hazard studies will likely become more realistic and more complex, thereby increasing the number of dimensions in the JPM integral. As shown here, methods now exist to achieve good accuracy with a JPM-OS approach with an order of magnitude less computational effort than is demanded by the conventional JPM approach.

In this regard, the case of Mississippi may be atypically simple for a variety of reasons. For example, the coastline is nearly straight and nearly perpendicular to the mean hurricane heading, and tides are extremely small compared to significant storm surge. We are in the process of applying this JPM-OS technique for hurricane surge evaluations in other regions, where those simplifying advantages may not obtain. In particular, it may be necessary to add one or more dimensions to the JPM integral in order to represent tide phase and amplitude. Such future applications will provide additional insight into the JPM-OS approach and ways in which it can be improved and extended.

It is worth noting that the so-called JPM-OS schemes described here and in the Toro et al. (this issue) companion paper are optimal in the sense that they seek a good compromise between accuracy and expensive computer resources by reducing the number of computer-intensive synthetic storms simulations needed, while maintaining the desired accuracy. It is also optimal in the sense that it involves two mathematical optimization steps that minimize the variance of the error introduced by the discrete approximation to the JPM integral. This optimization is, of course, an optimization exercise for a given mathematical model. We do not mean to imply that all elements in the JPM model described here and in the companion papers are optimal, although many of the statistical techniques summarized in the Niedoroda et al., (this issue) companion paper and commonly used in practice also happen to be optimal (e.g., maximum likelihood, cross-validation, least squares).

Finally, it is useful to note that the JPM approach is quite general, and is applicable to tropical storms in other regions and to the calculation of other hazards from severe storms, such as winds, waves, and even the response of a structure (see, for example Banon et al., 1994; Wen and Banon (1991), who use the JPM without calling it by that name). Depending on the application, the vector of storm characteristics $\underline{X}$ may include additional quantities such as Holland's B and other storm characteristics. The main requirement is that one must be able to represent the storm and other environmental inputs necessary for the calculation of the peak response η by means of a vector of storm characteristics and that the size of this vector must be manageable.

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References

Banon, H., Toro, G.R., Jefferies, R., De, R.S., 1994. Development of reliability-based global design equations for TLPs. OMAE Proc. Am. Soc. Mech. Eng..

Diaconis, P., 1988. Bayesian numerical analysis. Stat. Decision Theory Relat. Top. IV, 163–175.

Federal Emergency Management Agency (FEMA), 2008. Mississippi Coastal Analysis Project. Project reports prepared by URS Group Inc, (Gaithersburg, MD and Tallahassee, FL) under HMTAP Contract HSFEHQ-06-D-0162, Task Order 06-J-0018.

Genz, A., Keister, B., 1996. Fully symmetric interpolatory rules for multiple integrals over infinite regions with Gaussian weight. J. Comp. Appl. Math. 71, 299–309.

Gollwitzer, S., Kirchgaßner, B., Fischer, R., Rackwitz, R., 2006. PERMAS-RA/STRUREL system of programs for probabilistic reliability analysis. Struct. Saf. 28, 108–129.

Holland, G.J., 1980. An analytic model of the wind and pressure profiles in Hurricanes. Mon. Weather Rev. 108 (8), 1212–1218.

Hong, H.P., 1998. An efficient point estimate method for probabilistic analysis. Reliab. Eng. Sys. Saf. 59 (3), 167–261.

Irish, J., Resio, D., Ratcliff, J., 2008. The influence of storm size on hurricane surge. J. Phys. Oceanogr. 38 (9), 2003–2013.

Irish, J.L., Resio, D., Cialone, M.A., 2009. A surge response function approach to coastal hazard assessment. Part 2: quantification of spatial attributes of response functions. Nat. Hazards doi:10.1007/s11069-009-0381-4.

Journel, A.G., Huijbregts, Ch.J., 1978. Mining Geostatistics. Academic Press, London.

Madsen, H.O., Krenk, S., Lind, N.C., 1986. Methods of Structural Safety. Prentice-Hall, Inc., Englewood Cliffs, NJ.

Melchers, R.E., 1999. Structural Reliability: Analysis and Prediction. Wiley, Chichester, New York.

Miller, A.C., Rice, T.R., 1983. Discrete approximations of probability distributions. Manage. Sci. 29 (3), 352–361.

Minka, T.P., 2000. Deriving quadrature rules from Gaussian processes. Technical Report, Statistics Department, Carnegie Mellon University. Available on-line at 〈http://research.microsoft.com/~minka/papers/minka-quadrature.ps.gz〉.

Myers, V., 1970. Joint probability of tide frequency analysis applied to Atlantic City and long Beach Island, NJ. ESSA Tech Memo WBTM Hydro-11.

Niedoroda, A.W., Resio, D.T., Toro, G.R., Divoky, D., Das, H.S., Reed, C.W., this issue. Analysis of the coastal Mississippi storm surge hazard. Journal of Ocean Engineering, doi:10.1016/j.oceaneng.2009.08.019.

O'Hagan, A., 1991. Bayes-Hermite quadrature. J. Stat. Plann. Inference 29, 245–260.

Parzen, E., 1962. Stochastic Processes. Holden Day, San Francisco, California.

Powell, M.J.D., 2004. The NEWUOA software for unconstrained optimization without derivatives. In: Proceedings of the 40th Workshop on Large Scale Nonlinear Optimization, Erice, Italy.

Resio, D.T., 2007. White Paper on Estimating Hurricane Inundation Probabilities. Engineer Research and Development Center (ERDC), US Army Corps of Engineers, Vicksburg, MS.

Smolyak, S.A., 1963. Quadrature and interpolation formulas for tensor products of certain classes of functions. Sov. Math. Dokl. 4, 240–243.

Toro, G., Resio, D.T., Divoky, D., Niedoroda, A.W., Reed, C.W., this issue. Efficient joint probability methods for hurricane surge frequency analysis. Journal of Ocean Engineering, doi:10.1016/j.oceaneng.2009.09.004.

Vanmarcke, E.H., 1983. Random Fields: Analysis and Synthesis. MIT Press, Cambridge, MA.

Wen, Y.K., Banon, H., 1991. Development of environmental combination design criteria for fixed platforms in the Gulf of Mexico. In: Proceedings of the OTC 6540, Offshore Technology Conference, Houston, TX, May 1991.