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Efficient joint-probability methods for hurricane surge frequency analysis

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ABSTRACT

The Joint-Probability Method (JPM) was adopted by federal agencies for critical post-Katrina determinations of hurricane surge frequencies. In standard JPM implementations, it is necessary to consider a very large number of combinations of storm parameters, and each such combination (or synthetic storm) requires the simulation of wind, waves, and surge. The tools used to model the wave and surge phenomena have improved greatly in recent years, but this improvement and the use of very large high-resolution grids have made the computations both time-consuming and expensive. In order to ease the computational burden, two independent approaches have been developed to reduce the number of storm surge simulations that are required. Both of these so-called JPM-OS (JPM-Optimal Sampling) methods seek to accurately cover the entire storm parameter space through optimum selection of a small number of parameter values so as to minimize the number of required storm simulations. Tests done for the Mississippi coast showed that the accuracy of the two methods is comparable to that of a full JPM analysis, with a reduction of an order of magnitude or more in the computational effort.

1. Introduction

Estimation of storm surge elevation frequencies is a central component of coastal hazard studies. The design of coastal structures and protection in hurricane zones requires such an assessment, as does FEMA's National Flood Insurance Program for which flood elevations having 10, 2, 1, and 0.2% annual exceedence chances are required. Older approaches to such evaluations have included the use of design storm events, historical methods such as tide gage analysis and the Empirical Simulation Technique (EST), and synthetic simulation methods including, especially, the Joint-Probability Method (JPM) pioneered by Myers (Myers, 1975; Ho and Myers, 1975) for coastal flood estimation.

For the critical post-Katrina studies in the northern Gulf of Mexico, the federal project teams adopted JPM as the preferred method. Design storm methods were not used since no single event can capture the broad range of storm characteristics that determine, say, the 1% flood elevation over an extended geographic region. Historical methods such as tide gage analysis and EST evaluations were not adopted because they have been found highly sensitive to the sample error/variation that exists in local data sets owing to insufficient sample sizes (Divoky and Resio, 2007). The JPM approach, however, has the conceptual advantage of considering all possible storms consistent with the local climatology, each weighted by its appropriate rate of occurrence. In brief, the most basic JPM approach assumes a parametric storm description involving five or six hurricane descriptors such as central pressure, size, translation speed, and so forth. For each of the several parameters, probability distributions (not necessarily mutually independent) are developed through an analysis of the local climatology. These distributions are each discretized into a small number of representative values, and all possible parameter combinations (each defining a synthetic storm) are simulated using a hydrodynamic model constructed to faithfully represent the bathymetry, topography, and ground cover of the study site.

After the disastrous 2005 hurricane season, both FEMA and the US Army Corps of Engineers undertook intensive efforts to update coastal hazard information using detailed methods. Those projects adopted state-of-the-art meteorological and hydrodynamic numerical models to compute local maximum water elevations for each of the synthetic storms required by the JPM approach. The model suite included meteorological, wave, and surge models as required to capture the full range of physical mechanisms controlling the flood levels. This suite included the PBL, WAM, SWAN, STWAVE, and ADCIRC models, and so imposed a heavy computational burden on the analyses. Even with the use of modern parallel computer clusters, a straightforward brute-force

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JPM approach as used in older studies (requiring as many as 3000 or more storm simulations) would have been prohibitively costly. Modeling a single storm with these state-of-the-art tools required five to ten hours on a cluster of 256 CPUs, making a brute-force JPM analysis infeasible.

The two study teams independently tackled this difficulty through development of new strategies for a JPM analysis, aimed at greatly reducing the required number of storm simulations while maintaining accuracy. The two approaches have been designated as *Optimal Sampling* methods (OS), denoting their common intent of choosing a minimum number of storms for simulation in such a way as to accurately cover the entire storm parameter space through optimal parameter selection with associated weighting and interpolation methods. These approaches, both called JPM-OS methods, have been found capable of reducing the brute-force JPM effort by about an order of magnitude. This paper describes the essential features of both approaches and shows validation and comparisons. It is accompanied by two other papers in this same issue of the Journal presenting an overview of FEMA's Mississippi study (Niedoroda et al., 2010), and a more detailed exposition of one of the two JPM-OS approaches summarized here, the Quadrature approach (Toro et al., 2010). To avoid unnecessary duplication of material, the reader is referred to those accompanying papers for additional discussion of matters not primarily bearing on the issue of the efficiency of the two JPM-OS methods, which is the focus here. In addition, supporting Corps and FEMA reports should be consulted wherever full details of the data and analyses are needed (Resio, 2007; FEMA, 2008). Collectively, these papers and reports will be referred to as the *companion documents*.

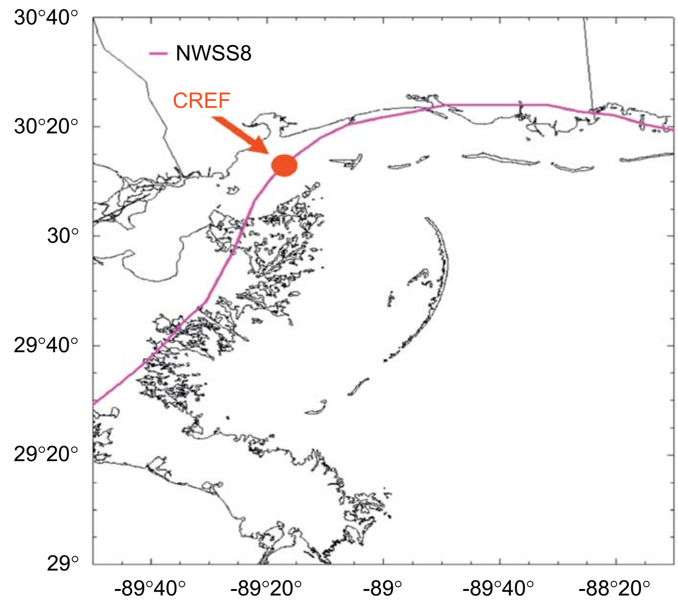

Fig. 1. Simplified shoreline of the Mississippi coast based on one given in NWS #38, indicating the central reference point, CREF.

2. Overview of the evaluation process

In the following sections, we will briefly summarize the two JPM-OS approaches and demonstrate and compare their performance versus a much larger JPM analysis taken as a reference standard. In order to achieve this in a realistic fashion, realistic storm data are required, as is the use of a numerical coastal hydrodynamic model incorporating all of the pertinent storm parameters. We have taken data from the northern Gulf of Mexico for demonstration purposes, as described below; readers should refer to the companion documents for a full discussion of the data and its use. Since the aim here is the comparison of JPM-OS approaches, not the precise determination of flood levels, a simplified and highly efficient hydrodynamic modeling approach was used. This simpler approach is discussed below, followed by sections outlining the two JPM-OS approaches, and showing the results of the validation and comparison tests. All final storm surge determinations for the Corps and FEMA projects were, of course, made using the full suite of state-of-the-art models mentioned above.

2.1. Storm data

Historical data from the period of record between 1940 and 2006 were used to create statistical descriptions of the four basic hurricane parameters that are commonly used in conjunction with numerical models to depict the wind and pressure fields of synthetic storms. The selected period of record corresponds to the modern era of higher quality data obtained through use of improved storm reconnaissance. The parameters of interest are the central pressure deficit, ΔP, the pressure radius, R_p (comparable to the radius to maximum winds), the track heading azimuth, and the storm's forward translation speed, V_f. Values were taken at landfall along the generalized Mississippi shoreline shown in Fig. 1. Only strong storms with pressure deficits greater than 45 mb were considered since these dominate floods at the 100-, and 500-year recurrence intervals of interest. Weaker storms must be added to the simulations to assess more frequent surge elevations.

The storm characterizations were developed from the set of storms that made landfall between 85°W and 95°W. The storm rate and statistical descriptions of the pressure deficit and track azimuth were developed through an application of the Chouinard method (Chouinard and Liu, 1997a; Chouinard et al., 1997b) with distance weighting from the Central Reference Point (CREF) shown in Fig. 1. In the Chouinard method, kernel-smoothing techniques are used to obtain these estimates, rather than the simpler approach of evaluating storms captured within a fixed-size sample window. An optimal kernel size can be determined using split-sample cross-validation methods. For the cases illustrated here, a kernel half width of 200 km was adopted.

The storm parameters derived in this way were fit by conventional probability distribution functions. In particular, the central pressure deficit was described by a three-parameter Weibull distribution. The distribution of storm radius was taken to be conditional upon the central pressure depression in the form of a log-normal distribution with both mean and standard deviation being power functions of ΔP; more intense storms were associated with smaller radii. Forward speed was characterized by a log-normal distribution, independent of other parameters. The storm heading was fit by a beta distribution, with the primary direction being just west of north. The remaining parameter of primary interest is the storm density in both space and time. In this demonstration study, the chosen data sample included about 3E-04 storms per kilometer per year. Additional information regarding data selection, statistical parameter descriptions, and methods, can be found in the companion documents (Niedoroda et al., 2010; Toro et al., 2010). The details of the storm climatology are not of essential interest here, the only requirement being that the choices should be generally consistent with real conditions so as to fairly display the performance of the two JPM-OS schemes.

2.2. Diagnostic surge model (SLOSH)

In order to reveal the JPM performance characteristics, it was not necessary to perform very costly high-resolution hydrodynamic simulations of the sort used for the final determinations. Instead, a highly efficient model of lesser accuracy was preferred, allowing large numbers of simulations to be made quickly and cheaply. For this, we adopted the SLOSH (Sea, Lake and Overland Surges for Hurricanes) model provided to us by NOAA in the Biloxi Basin implementation for the central Mississippi coast. The SLOSH model is routinely used by the National Hurricane Center (NHC) to estimate storm surge heights and winds resulting from historical, hypothetical, and forecast hurricanes, accounting for pressure, radius, forward speed, track, and winds (Jelesnianski et al., 1992). It is a two-dimensional, depth-integrated finite difference code on a curvilinear grid system that allows greater resolution in the area of forecast interest, and computes surges over bays and estuaries, retaining some non-linear terms in the equations of motion, and allowing for the representation of sub-grid scale features such as channels, barriers, and propagation of surge in rivers. This permits inclusion of topographic details such as highway and railroad embankments, causeways, levees, and so forth. The telescoping grid gives good resolution of the topography over a large geographical area. The smallest cell in the Biloxi grid represents an area of about 0.1 square miles, while the largest grid cells are approximately 12 square miles. The model accounts for astronomical tides by specifying an initial tide level, but does not include rainfall amounts, river inflow, or wind-driven waves.

SLOSH was chosen as the diagnostic model for this analysis because it not only incorporates the essential physics and a realistic site description, but is also extremely efficient. A storm simulation typically requires less than two minutes on a standard desktop PC, while accounting for the influence of each storm parameter in a realistic way. That the suite of more complex models used for final surge determinations might give somewhat different results is not critical for the comparison of JPM-OS methods; relative behavior will be preserved. The findings obtained using SLOSH as a diagnostic tool also permit selection of the particular storm set used in the final simulations with the full suite of complex models.

3. Overview of the JPM approach

The JPM method relies on probabilistic descriptions of storm occurrence and storm characteristics to define a set of synthetic storms, together with a numerical method to calculate the coastal flood elevations that would be generated by those storms. The numerical method includes representations of the storm tracks, the evolution of storm characteristics (referenced to the characteristics at landfall), the wind and pressure model, the surge model, and so forth, represented symbolically as

$$\eta_m(\Delta P, R_p, V_f,\ \text{landfall location}, \theta, \ldots) = \eta_m(\underline{X}) \tag{1}$$

where the subscript m denotes a model-calculated value, and the vector $\underline{X}$ represents all storm characteristics; landfall location and track angle determine the proximity of the storm to a specific coastal site. The annual rate of occurrence of a flood elevation at a site in excess of η is defined in terms of the combined probabilities of the storm parameters and is given by the multiple integral

$$P[\eta_{max} > \eta] = \lambda \int \ldots \int_{\underline{x}} f_{\underline{X}}(\underline{x}) P[\eta(\underline{x}) > \eta]\, d\underline{x} \tag{2}$$

where λ is the mean annual rate of all storms of interest for the site, $f_{\underline{X}}(\underline{x})$ is the joint-probability density function of the storm characteristics of these storms, and $P[\eta(\underline{x}) > \eta]$ is the conditional probability that a storm of certain characteristics $\underline{x}$ will generate a flood elevation in excess of η; subscript *max* denotes an annual maximum value. This integral over all possible storms determines the fraction of storms that produce flood elevations in excess of the value of interest η, using the total probability theorem (Benjamin and Cornell, 1970). The entire expression, including λ, is actually a rate with dimensions of events per unit time, but is commonly thought of as annual probability to a good approximation.

At this point, it is useful to compare the JPM approach with other more familiar frequency estimation methods—regarding both the mechanics of calculating the n-year flood elevation and the numerical and statistical stability of the various methods. The more familiar methods generally rely directly on a particular history of flood elevations, observed or synthetic, over a defined period of time. The most direct example of this is the use of a tide gage record covering N years. Elevation frequencies from such a record are commonly estimated by fitting one of many well-known distribution types to the annual peaks of the record. Sometimes a simple plotting position rule is used, relating recurrence interval directly to the magnitude ranks of the annual maxima. Plotting position methods are only meaningful for events that are frequent compared to the length of the record, since they force the largest elevation in N years to approximate the N-year event. This same approach can be used with synthetic data, derived from the simulation of storms over a defined period. A Monte Carlo technique might be used to randomly construct the synthetic sequence of storms by drawing from observed storm parameter distributions such as those also used in the JPM method. An example of this approach is given by Wang et al. (2007). In that study, a 2000-year period was simulated by randomly defining 566 storms (the expected number over the study region during 2000 years), and rank-ordering the flood elevations for each as computed with a hydrodynamic model. The frequency associated with a particular elevation was estimated as 2000/Rank. Since the largest elevation is given Rank = 1, that elevation is taken to be the 2000-year level in this approach. While the tail may not be reliably determined in this way, the frequencies of interest, such as the 100-year level, are represented by about 20 instances in 2000 years, and so are likely to be well-captured. The same 2000-year sequence of 566 storms could have been used to fit an extreme value distribution, for example, achieving similar results for the frequent end of the distribution of prime interest.

The Empirical Simulation Technique (EST; Borgman et al., 1992) has been another alternative, combining some characteristics of gage analysis and Monte Carlo simulation. As applied to storm surge studies, it relies upon the particular storm sequence observed within a region (as does gage analysis) but incorporates a bootstrapping technique with random variation of the storm parameters and tracks, so as to generate a varying synthetic record of arbitrary length. Frequencies are estimated from the synthetic record using plotting position methods.

The fundamental difference between JPM and approaches such as these is that JPM does not work with a particular sequence of storms over a defined period of time. Instead, it bypasses such conceptual histories entirely, and computes the rate of occurrence of flood elevations directly from Eq. (2). The multiple integral in Eq. (2) covers *all possible* values of the storm parameters, using the observed parameter distributions, not discrete storm events. Consequently, Eq. (2) is valid even for extremes, limited only by the accuracy of the estimated parameter distributions. This is unlike the storm history approach of gage analysis and Monte Carlo methods, both of which are based on storm *samples* (as distinct from individual parameter samples) and so introduce the problem of sample variation. Consequently, gage analysis, EST, and

Monte Carlo methods all require a very great number of storms to assure accuracy in estimation of the flood elevation distribution tail. For example, to estimate the 500-year event, one should expect to require several realizations of that level in the record, say 10 or more, suggesting the need for perhaps 5000 years of observed or simulated data. With the JPM approach, the annual probability of exceedence (or, more precisely, the annual rate of exceedance) comes directly from Eq. (2) without the need for any additional steps such as plotting positions or frequency distribution fits. In fact, the left hand side of Eq. (2), when viewed as a function of the flood elevation η, is precisely the *Complementary Cumulative Probability Distribution Function* (CCDF) of the flood elevation, equivalent to a CCDF obtained by more familiar means.

Thus, calculation of the n-year flood elevation in the JPM approach consists of evaluating Eq. (2) for many values of the flood elevation η (e.g., 0–30 ft with a 0.1 ft increment) and then finding the value of η corresponding to an exceedance probability of $1/n$. Although the various JPM implementations work with synthetic storms, it is more appropriate to view these synthetic storms as values of the storm parameters $\underline{x}$ for which $\eta(\underline{x})$ must be calculated in order to obtain a numerical estimate of the integral in Eq. (2), rather than as a sequence of storms occurring over a certain number of years. The number of these synthetic storms is dictated by the required numerical accuracy and by the efficiency of the numerical-integration scheme being used.

Although JPM also depends upon samples from a finite record, they are parameter samples rather than strom samples. They, too, are increasingly poor at their tails, but the 100-year flood at a site is not controlled by the 100-year parameter values in those tails. Instead, the 100-year flood is associated with hurricanes of frequently observed intensity coupled with random track proximity favorable to surge development. Despite its devastation, Katrina, for example, was a Category 3 storm at landfall, with storm parameters that, considered individually, were not exceptional for hurricanes. Track proximity is critical and is usually well-described by the assumption of uniformity over small study regions.

The main problem in the JPM method is one of choosing an efficient method to integrate Eq. (2). The most direct approach to the integration would be to discretize the entire multi-parameter space of Eq. (2) into small segments, and so convert the integral into a simple sum in the manner of elementary numerical-integration rules (analogous to a trapezoidal integration in one dimension, say). This was the approach used in early JPM surge studies involving storm simulations with much simpler models and reduced resolution than is common today. This brute-force numerical integration is problematic since each evaluation of the integrand involves the costly evaluation of $\eta(\underline{x})$ for one set of parameters, $\underline{x}$ (i.e., the costly simulation of one complete storm history, requiring several hours of CPU time on a large parallel cluster), and since this computation must be repeated a very large number of times. If each of the five dimensions in Eq. (2) is represented by a moderate number of discrete values, the so-called *curse of dimensionality* quickly renders the calculations impractical. With each of the five parameters represented by only six discrete values, for example, the entire simulation set would consist of over 7000 storms, clearly a prohibitive number.

The Monte Carlo method may be viewed as another approach for the evaluation of the integral in Eq. (2), relying on random sampling of the entire space of the multiple integral. As discussed above, this gives a particular set of synthetic storms that may be conceptually associated to a synthetic period of record, which is then analyzed by traditional means. As also discussed, many Monte Carlo realizations are necessary in order to obtain stable estimates of the 100- and 500-year values.

It is for this reason that the so-called *Optimal Sampling* (OS) methods discussed here have been devised; simply to reduce the number of evaluations of the integrand (each requiring a storm simulation) that are required to achieve a given level of accuracy in the integration of Eq. (2). Once that integration is done, the desired surge frequencies have been established, subject only to the addition as necessary of any small adjustments to account for secondary parameters and random variability not represented in the integral. There is no additional step involving the frequency analysis of a particular synthetic storm sequence or a particular synthetic period-of-record.

4. The JPM-OS schemes

As noted at the outset, two distinct JPM-OS schemes have been developed to minimize the computational burden represented by Eq. (2). These will be identified as the *Response-Surface* (RS) and the *Quadrature* (Q) JPM-OS schemes. The response-surface scheme (Resio, 2007) is based on the use of a modest number of storms to construct the higher-dimensional surface of surge response vs. the storm parameter values, upon which one can interpolate to obtain an estimate of the surge at any intermediate point (that is, for other storms not explicitly simulated). The quadrature scheme (Toro et al., 2010) reduces the number of synthetic storms by employing an algorithm to select the parameter combinations in an optimal manner and to assign appropriate weights to each, transforming the JPM multi-dimensional integral into a weighted summation. The advantage gained is analogous to integration of an ordinary function by Gaussian quadrature rather than by, say, the trapezoidal rule. Both approaches assume that the surge η varies smoothly with the parameter vector $\underline{x}$.

4.1. The response-surface JPM-OS scheme (JPM-OS-RS)

Conceptually, this method involves interpolation in the higher-dimensional space consisting of the surge response (elevation) and the several storm parameters. Storm simulations provide the response values for each combination of storm parameters, and so define a point in this space. Fig. 2 shows a simple three-dimensional visualization of the multi-dimensional information used in the JPM-OS-RS method. The figure shows three discrete storm tracks each made up of a set of three cubes (one for each of the forward speeds used in the hypothetical simulations). Within each of these cubes, a (relatively small) set of discrete values of central pressure (shown along the x-axis), storm heading angle at the coastline (shown along the y-axis), and storm central pressure (shown along the z-axis) is defined. Each small cube element defines a single storm to be simulated. The response-surface method uses those results to allow interpolation for other surge levels at a much finer resolution within and between the cubes.

For a given location, a major portion of the variation of surge response has been shown to be captured by the variation of ΔP and R_p (Irish et al., 2008). Because of this, the surge response-surface integration method uses (ΔP, R_p) planes as the primary variables within the five-dimensional JPM parameter space. For fixed values of storm landfall location (L), storm speed, V_f, and track angle, θ, a response function at location (χ, ξ) can be defined as

$$\eta_m(\chi,\xi) = \phi_{kmn}(\Delta p, R_p, \theta, \chi - \chi_s, \xi, V_{f_0}) \tag{3}$$

where ϕ_{kmn} is the surge response function with the subscripts k, m, and n denoting a specific landfall location (track), speed, and angle, respectively, and V_{f_0} is a reference value of the forward velocity of the storm. This notation reflects the fact that a separate response function must be defined for each location (χ, ξ), where χ

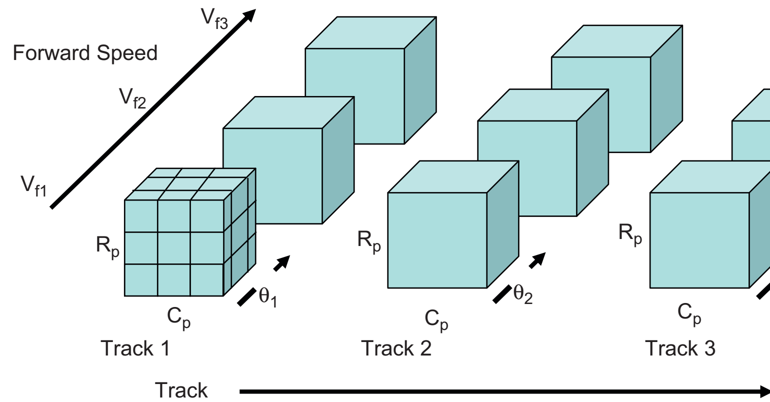

Fig. 2. A three-dimensional visualization of the five-dimensional JPM-OS-RS parameter space and its discretization.
(*Source:* Cialone et al., 2008)

is the alongshore direction, ξ is the cross-shore direction within the study region, and χ_s is the alongshore point of intersection of track *s* with the coast.

Variations in surge response with variations in storm track, speed, and angle are easily estimated. For example, the variation in response caused by a variation in storm speed has been found to be fairly small, approximately linear, and insensitive to R_p and ΔP. Accordingly, only a few storms are needed to characterize the variation with speed, retaining the overall structure of the response function as obtained with a suitable initial choice of forward speed in Eq. (3). Then the value of $\phi_{kmn}(\Delta P, R_p, \theta, \chi-\chi_s, \xi, V_f)$ for an arbitrary forward speed V_f is obtained to first order from the relationship

$$\phi_{kmn}(\Delta P, R_p, \theta, \chi-\chi_s, \xi, V_f) = \phi_{km_0n}(\Delta P, R_p, \theta, \chi-\chi_s, \xi, V_{f_0}) + \psi_{kmn} \quad (4)$$

where

$$\psi_{kmn} = \frac{\partial \phi_{kmn}(\Delta P, R_p, \chi-\chi_s, \xi, V_f)}{\partial V_f}(V_f - V_{f_0}) \quad (5)$$

is the first-order variation due to speed. Similar expressions apply for the other secondary parameter variations. This method can be considered a simple parametric model to interpolate between the discrete parameter categories for which simulations were run, with the important constraint that the parameterizations must exactly equal the simulation results at all computed points within the "storm speed-angle" parameter space.

Whereas numerical tests showed that the functional behavior of the response surface, with respect to variations in storm speed and angle, was quite smooth and simple, this was not found to be true for the variation with respect to the alongshore landfall location χ_s. In this case, as shown by Irish et al. (2009), it is necessary to account in more detail for the variation of surge response along the coast. Failure to do so results in random errors along the coast caused by finite storm track spacing. These errors depend upon the adopted storm track separation and can be made suitably small using a small track spacing. However, since the purpose of the optimal sampling methods is to minimize the total number of runs, the work of Irish et al. (2009) can be adopted to improve the estimate of the surge response function while maintaining moderately large track spacing. In this approach, parametric relationships are developed to approximate the characteristic alongshore variation of surge (see Fig. 3), giving an estimate of surge heights between tracks that is significantly better than simple interpolation.

One sees that by dividing the space into $(\Delta P, R_p)$ planes, and by further subdividing the secondary parameters, the response surface becomes "faceted" with the response variation over a facet being given by the interpolation rules of Eqs. (4) and (5). In other words, the response for any storm (any combination of parameters) can be estimated by simple inspection and interpolation of the response surface, without the need to perform additional hydrodynamic simulations. Consequently, the multidimensional integral of Eq. (2) can be evaluated inexpensively using conventional numerical-integration methods with as many response estimates as needed.

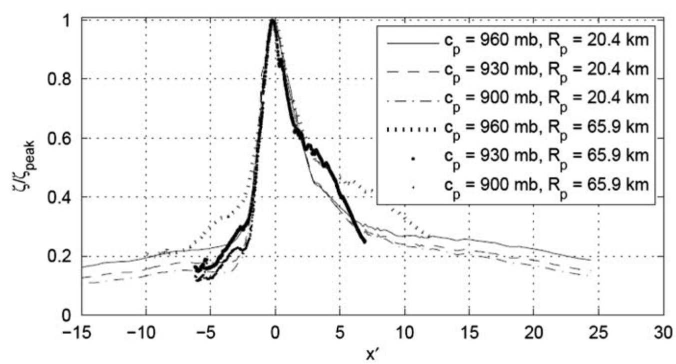

Fig. 3. Characteristic alongshore distribution of surge based on numerical simulations. The heavy line shows the function used by Irish et al., 2009 to estimate the response variation between tracks. Elevation is normalized by the peak value, while dimensionless distance is in units of R_p.

The key to this approach is the intelligent selection of the storm parameter combinations that are used to define the response surface. Although in principle this selection can eventually be automated, for the present work expert judgment was used, based on numerical experiments and on the obvious need to fully span the important regions of the parameter space. Using the distributions discussed earlier, central pressure depressions of 53, 83, and 113 mb were adopted. Although at first sight this seems to be a small number, it has been verified that the surge response varies linearly with intensity to an extremely good approximation so that interpolations within this range and modest extrapolations outside the range are fully acceptable. Multiple values of R_p were selected ranging from 8 to 48 nm, with forward speeds from about 3 to 12 m/s, and heading angles from -58° to $+32^\circ$, positive clockwise from north. The track spacing was taken to be 0.5° longitude, intersecting a latitude 30.5° baseline at eight locations (91.2°, 90.7°, 90.2°, 89.7°, 89.2°, 88.7°, 88.2°, and 87.8°W). Specific combinations were selected to cover the parameter space in a reasonable manner, with primary attention to areas representing the greatest probability mass.

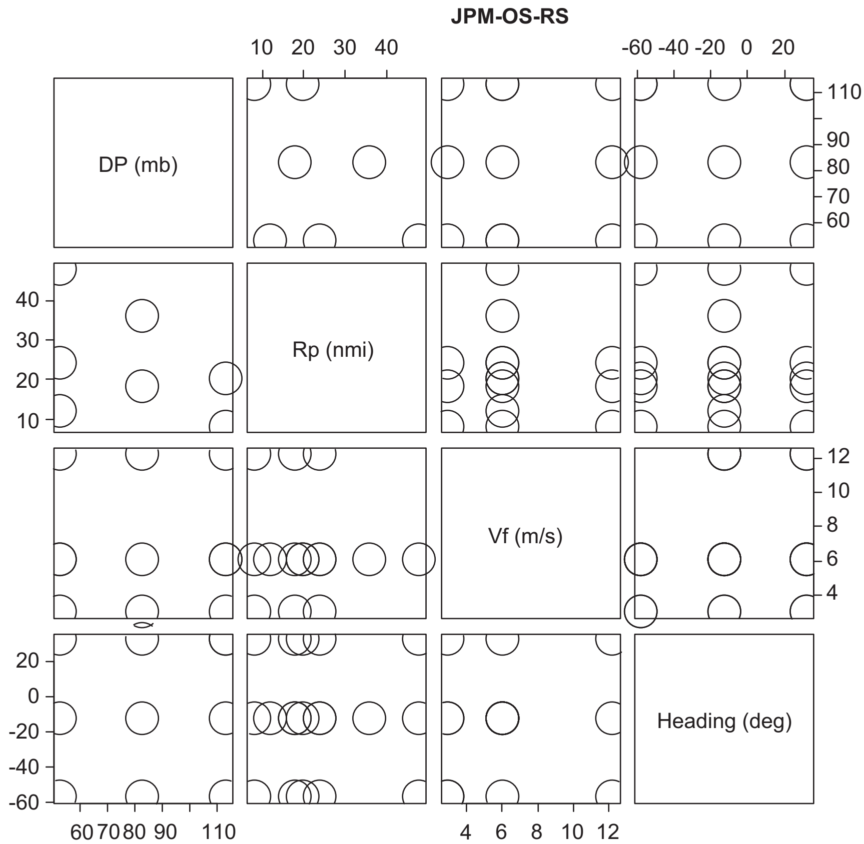

Fig. 4. Graphical representation of the disposition of parameters for the 156 synthetic storms of the RS simulation set.

Since not all combinations of the several parameter values were required to adequately span the parameter space, the net number of storms actually simulated was only 156 (note that this is nearly identical to the number of 158 storms used in the quadrature approach to be discussed next). Fig. 4 shows the pairwise disposition of parameter values (pair values correspond to the positions of the circle centers), and will be compared with a similar figure (Fig. 6) for the quadrature scheme to be described next.

4.2. The quadrature JPM-OS scheme (JPM-OS-Q)

Gaussian quadrature is a well-known technique for approximation of integrals of the form $I=\int f(x)p(x)\,dx$, where $f(x)$ is often a probability density function (i.e., it is positive and it integrates to unity) and $p(x)$ is an function belonging to a particular family of functions. The quadrature approximates the integral as a finite weighted summation of the form $I\approx\sum_i w_i p(x_i)$, where the nodes x_i and the corresponding weights w_i are selected in a manner that maximizes the accuracy of the approximation, while keeping the number of nodes small. In the context of the JPM-OS-Q method and Eq. (2), the integral extends over multiple dimensions, $f(x)$ corresponds to the joint distribution of storm characteristics $f_{\underline{X}}(\underline{x})$, $p(x)$ corresponds to the conditional surge-exceedence probability $P[\eta(\underline{x})>\eta]$, and each node can be thought of as one synthetic storm.

In Gaussian quadrature, the number of nodes, the nodal locations, and the weights are selected so that the summation will evaluate the integral exactly if $p(x)$ is a polynomial of a certain degree and $f(x)$ is a particular function (e.g., a standard normal probability density). This technique is used frequently in one dimension. Miller and Rice (1983) provide implementation details and results for a variety of commonly used probability distributions. The improvement in calculation efficiency (number of function evaluations needed for a specified accuracy) can be very great compared to simpler methods.

Unfortunately, extension of these so-called zero-error rules to more than one dimension is problematic. A conceptually straightforward, but computationally impractical, extension (the so-called product rules) consists of applying a one-dimensional quadrature separately in each dimension, resulting in a very large number of nodes, many of which have very low weights (this product rule was, in fact, used to construct the JPM reference case to be described in the next section). Some of the more sophisticated multi-dimensional approaches often lead to some weights being negative (see Genz and Keister, 1996). Negative weights create stability problems and make it impossible to interpret the weights in terms of the occurrence rates of synthetic storms.

In contrast to Gaussian quadrature, Bayesian quadrature (also termed Gaussian-process quadrature) defines the family of functions $p(x)$ as all possible realizations of a random process having a certain auto-covariance function, and seeks to minimize the integration error in a mean-squared sense. According to Diaconis (1988), this approach dates back to the work of Poincare in 1896. It is also closely related to Kriging (e.g., Journel and Huijbregts, 1978), and to least-squares regression.

Conceptually, it is straightforward to extend Bayesian quadrature to multiple dimensions, although if the number of dimensions is greater than five or six, the calculation of the nodal locations and weights may exceed one day of computer time on one processor of a modern personal computer; however, this may be a small expenditure compared to the savings in the number of storm simulations.

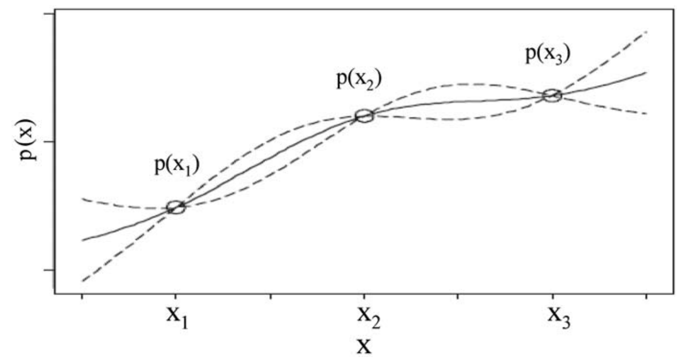

Fig. 5. Illustration of the conditional distribution of a random function $p(x)$ at intermediate points between sampling nodes.

Fig. 5 illustrates the concept of Bayesian quadrature in one dimension and its probabilistic nature. The illustrated function $p(x)$ has been sampled at 3 nodes x_1, x_2, x_3 so that the values of $p(x)$ are known at those points. For other values of x, $p(x)$ is only known in a approximate, probabilistic sense. That is, its conditional mean and standard deviation (given the values at x_1, x_2, x_3) are known, through use of the auto-covariance function of $p(x)$. The solid and dashed lines in Fig. 5 indicate the conditional mean and the one standard deviation range around the mean, respectively. The shape of the mean curve and the width of the standard deviation range depend on the distance to the nodes and on the auto-covariance function $k(x,y)=E[p(x)p(y)]$, where $E[\,]$ denotes expected value. The auto-covariance function contains information about the degree of continuity or smoothness of realizations of $p(x)$, at both small and large scales (see Vanmarcke, 1983).

When considering multi-dimensional integrals, the quantities x and y above are vectors and the multi-dimensional auto-covariance function $k(x,y)$ decays faster as a function of $|x-y|$ in the directions associated with the most influential quantities. A convenient measure of the reciprocal of this rate of decay is the correlation distance or scale of fluctuation (see Vanmarcke, 1983).

Returning to Fig. 5, the quantity to be evaluated is the integral of the product of $p(x)$, which we know only in a probabilistic sense, and the known function $f(x)$. Bayesian quadrature uses this probabilistic information to determine the optimal placement of the nodes and the associated weights. This is achieved using two nested optimization loops. The inner loop determines the weights that minimize the mean-squared error of the approximation to the integral, for given nodal locations, under the constraint that the weights add to unity. The mathematical details of this optimization are very similar to Kriging and involve the closed-form evaluation of multiple integrals, followed by matrix operations. The outer loop determines the optimal nodal locations and involves numerical optimization in multiple dimensions. The resulting algorithm tends to space the nodes more closely in the regions where $f(x)$ is highest and it tends to space them more regularly along the dimensions associated with the more influential quantities. Further details on the mathematics are provided in Toro et al. (2010).

In the present implementation of the quadrature JPM-OS method, $f(x)$ was taken to be a standard multi-normal probability distribution, and the auto-covariance function was defined in terms of the associated dimensionless x (known as standard normal space). The nodal locations are also determined in standard normal space, and are then mapped into the physical space of ΔP, R_p, etc., using a distribution transformation that takes into account the joint cumulative distributions of the physical quantities (this transformation is sometimes called the Rosenblatt transformation; see Madsen et al., 1986). The correlation distances, which serve as the parameters of the auto-covariance function $k(x,y)$, are determined using expert judgment, guided by the results of sensitivity tests (obtained in this study using SLOSH to perform numerical sensitivity experiments—see Niedoroda et al., 2010).

In the results reported here, application of the quadrature approach to define a minimal set of synthetic storms and their rates involved the following steps: first, the distribution of ΔP was discretized into three broad slices, roughly corresponding to hurricane intensity categories 3, 4, and 5. Second, for each ΔP slice, the joint-probability distribution of ΔP (within the slice), R_p, V_f, and θ was discretized using Bayesian quadrature. The result is a set of ΔP_i, R_{p_i}, $V_{f,i}$, θ_i, and their associated probabilities. Finally, the distribution of landfall location was discretized by constructing a set of tracks for each of the synthetic storms defined in the previous two steps. The tracks are offset from one another by the distance R_p measured perpendicular to themselves. The probability assigned to each artificial storm is easily computed as the product of the probabilities resulting from these three steps. The corresponding rates of occurrence are obtained after a further multiplication by the storm density and the track spacing (because each track represents all possible tracks half way to its adjacent tracks).

Because the specification of the correlation distances was in part dependent on expert judgment, numerical experiments were used to help define the final simulation set. This was done using results derived from a JPM reference case discussed in the next section. Four JPM-OS-Q candidate sets were evaluated having 303, 193, 158 and 147 parameter combinations (synthetic storms). Results for the 100- and 500-year coastal flood elevations at 65 points distributed across coastal Mississippi showed that the first three cases all adequately approximated the JPM reference case, but that the last case showed some unacceptable departures. The JPM-OS-Q set having 158 synthetic storms (denoted as OS6; see Fig. 6 for a description of the parameter set) was selected for the subsequent analysis, although other sets of comparable or greater size would have been equally acceptable.

Table 1 shows the way that the pressure deficit was divided into discrete slices for the OS6 set, and Table 2 shows the adopted correlation distances for each parameter. Note that although the central pressure is the most important parameter overall, it does not have the smallest correlation distance since these distances apply only within the slices for which the pressure variation is limited.

The optimal parameter selection for OS6 is shown graphically in Fig. 6. In this figure, the off-diagonal graphical elements show the pairwise parameter combinations used. The center of each circle corresponds to a specific parameter pair, while the radius of each circle indicates the weight assigned to that pair. This figure can be compared with the JPM-OS-RS simulation set illustrated in Fig. 4. Note that the locations of the OS-Q storms do not display the relative regularity evident in the OS-RS case. Instead, the choices are quite complex, having been selected by a numerical optimization procedure.

4.3. JPM reference case (JPM-REF)

In order to establish a benchmark against which the OS calculations could be tested and compared, a highly detailed JPM analysis (JPM-REF), was also performed. The reference case parameters were selected using one-dimensional Gaussian quadratures as described above, resulting in a much larger storm simulation set than used for either of the two OS approaches. The JPM-REF set consisted of 2967 simulations, including

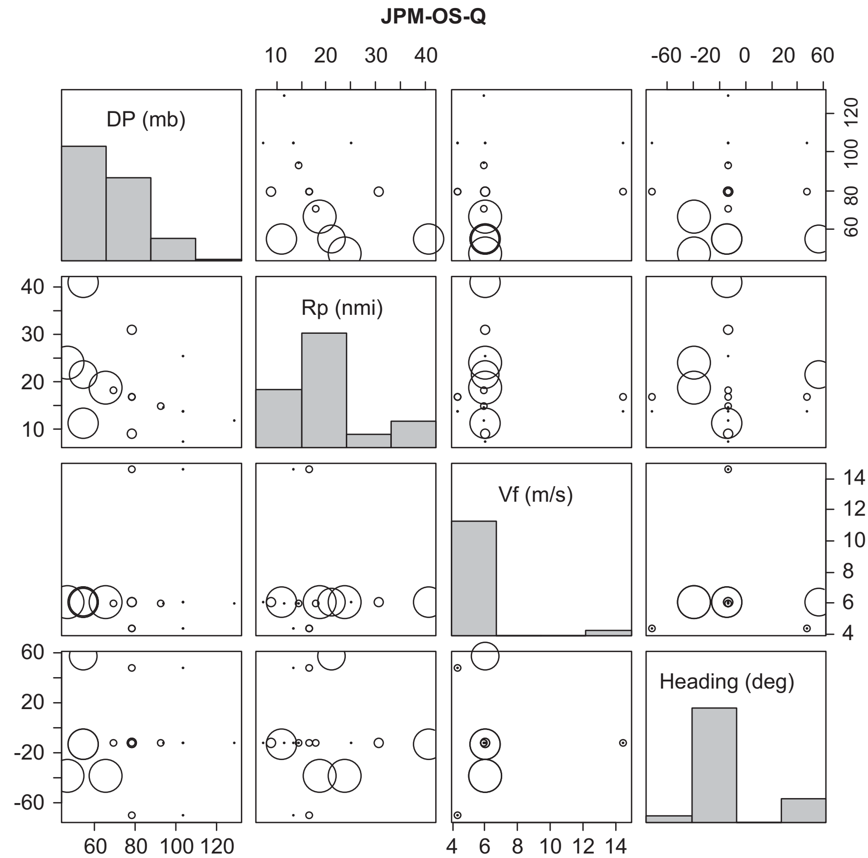

Fig. 6. Graphical representation of the disposition of parameters for the 158 synthetic storms of the JPM-Q-OS6 simulation set. Circle sizes indicate the assigned probability weights.

Table 1
Discretization of ΔP in the Bayesian-quadrature JPM-OS6 set.

Slice	Cat. 3	Cat.4	Cat.5
ΔP range (mb)	45–70	70–95	95–135
Probability	0.657	0.261	0.062
# of nodes in Bayesian quadrature	5	7	7

Table 2
Correlation distances in the Bayesian-quadrature JPM-OS6 set.

Parameter	ΔP	R_p	V_f	θ
Correlation distance (std. normal units)	4	2.5	6	5

approximately seven tracks per storm. It should be noted that although this is a large number of storms, it is less than would be required for a brute-force JPM set of the same accuracy, since the one-dimensional Gaussian quadrature method introduces a gain in efficiency.

5. Findings and discussion

Fig. 7 shows a comparison of the 100-year surge elevations from all three analyses at 65 shoreline points ordered according to longitude. Figs. 8 and 9 show plots of the differences between JPM-REF and JPM-OS6-Q and JPM-REF and JPM-OS-RS, respectively, while Fig. 10 shows the differences between the Q and RS methods. It is seen that in each comparison the differences are small and not notably biased, with mean differences of near zero, and standard deviations of about 0.5 ft. Similar findings at other recurrence intervals and for other simulation sets can be found in the companion documents referenced earlier.

The first conclusion to be drawn is that both OS schemes do, in fact, yield a very large reduction of computational effort with very little loss of accuracy. Whereas, the JPM-REF determinations required nearly 3000 storm simulations, both OS variants succeeded in reproducing those results with good accuracy using only about 5% as many simulations.

The second noteworthy point is that the methods give nearly identical results, despite very great differences in approach. This may be an important fact for the eventual development of a more or less automated approach that might incorporate ideas from both the RS and the Q methods. For each approach, the work reported here required some degree of expert judgment that might be eliminated or minimized in a future synthesis incorporating both quadrature and response-surface aspects. Perhaps a hybrid approach would be possible, although it is difficult to imagine that the simulation effort could be reduced much below the already very low level demonstrated here.

It must be kept in mind that there remain major uncertainties in surge estimation methods that will only be reduced through additional refinement of models and by an increase in parameter dimensionality, both driving computational costs upward. For example, hurricanes are not fully characterized by the five simple parameters considered here and in most past studies, but instead display complex variability which can only be captured by consideration of still more parameters (e.g., Holland's B affects the shape of the wind profile and might be considered as an additional time-varying JPM parameter), as might deviations in

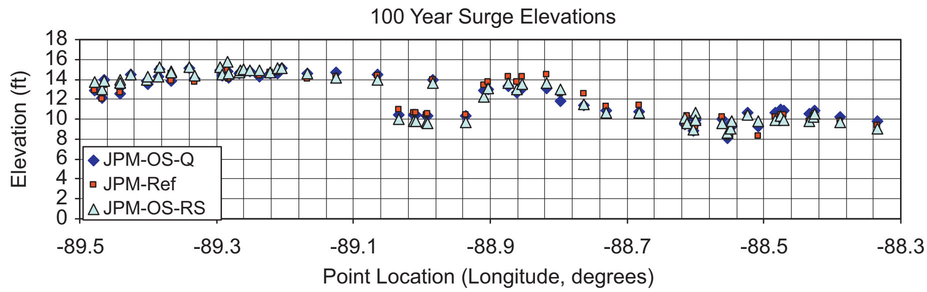

Fig. 7. 100-year surge elevations at selected points from the JPM-OS6-Q, JPM-OS-RS, and JPM-REF analyses.

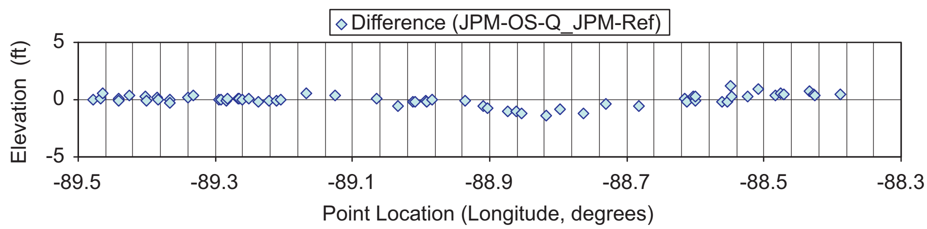

Fig. 8. Differences between JPM-OS6-Q and JPM-REF.

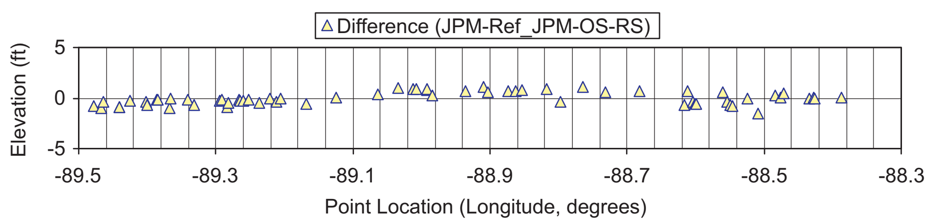

Fig. 9. Differences between JPM-OS-RS and JPM-REF.

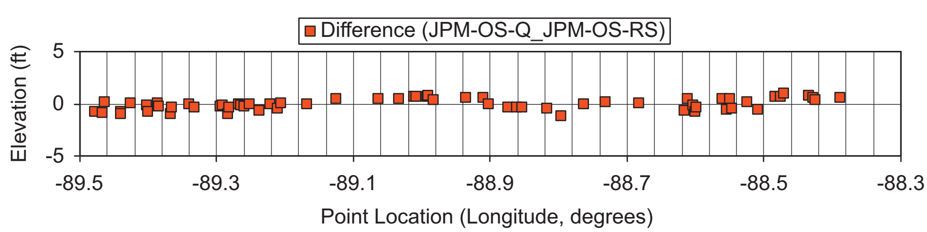

Fig. 10. Differences between JPM-OS-RS and JPM-OS-Q.

the evolution of storms as they approach the coast. Similarly, astronomic tide can interact with the development of the surge, so that tide amplitude and phase also represent additional parameter dimensions. For the specific case of the FEMA Mississippi study, tide was sufficiently small to allow an approximate treatment without enlarging the JPM space, but more complex methods will be required in many other regions. Other physical processes that are missing or only crudely accounted for in most work include coastal landform erosion, despite the fact that it can greatly modify the very shape of the computational basin, and rainfall which can play an important role in semi-confined waterways. The additional effects associated with wind waves are most commonly treated by ad hoc add-on methods. Future improvements in hazard evaluation, then, will only increase the already great computational burden, and so will require commensurate improvements in the optimization of the statistical simulation methods in order to remain practical.

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