Optimality of Public Persuasion for Single-Good Allocation

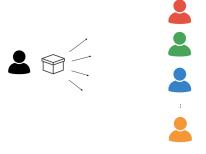
Chen Chen

Xuyuanda Qi

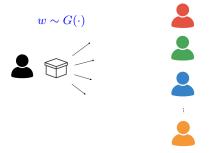


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 - ► Sender can allocate to at most one receiver
 - ► Each receiver decides whether to accept based on self-interest

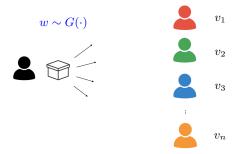


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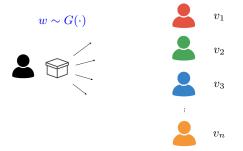
■ Sender strategically discloses good's characteristics $w \in \Omega$, with prior dist $G(\cdot)$

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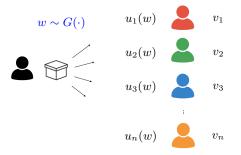
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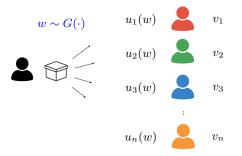
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- **Receiver** i's utility: $u_i(w)$ from receiving good of chars w; zero otherwise

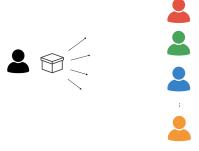
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- Applications: (i) school advisor promotes student for job positions, (ii) incubator pitches startup to VC investors

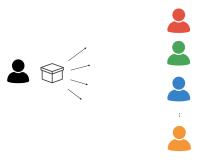
Public versus private persuasion

Sender commits to persuasion mechanism f(s|w): joint dist of sending signals $s=(s_1,s_2,\cdots,s_n)$ conditional on w



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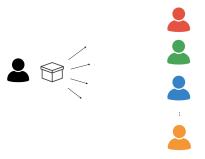
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- Public persuasion: f(s|w) = 0 if $s_i \neq s_j$ for some $i, j \in [n]$; that is receivers always receive the same signal
- Private persuasion: otherwise

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- Public persuasion: f(s|w) = 0 if $s_i \neq s_j$ for some $i, j \in [n]$; that is receivers always receive the same signal
- Private persuasion: otherwise
- Model permits receives to **communicate** after receiving signals
 - ► Receivers can communicate in an arbitrary way (including in self-interest)

Model (cont.)

The game proceeds as follows:

- 1. Sender commits to a persuasion mechanism $f(\cdot|w)$ and a signal space $\mathbf{S} = \bigotimes_{i=1}^{n} S_{i}$.
- 2. Sender observes the good's characteristics $w \sim G(w)$. A signal $\mathbf{s} = (s_i)_{i \in [n]} \sim f(\cdot|w)$ is generated and sent to the receivers.
- 3. Receivers communicate with one another in a certain way.
- 4. Each receiver i decides whether to accept the good based on the signal and communication.
- 5. Sender accepts the best offer (if she receives any).

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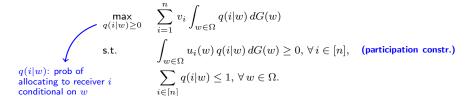
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Main result: Public persuasion is optimal regardless of how receivers communicate

⇒ Sender eliminates any communication for her own interest

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$$\max_{q(i|w)\geq 0} \quad \sum_{i=1}^n v_i \int_{w\in\Omega} q(i|w)\,dG(w)$$
 s.t.
$$\int_{w\in\Omega} u_i(w)\,q(i|w)\,dG(w)\geq 0,\,\forall\,i\in[n],\quad\text{(participation constr.)}$$

$$q(i|w)\text{: prob of allocating to receiver }i$$

$$\sum_{i\in[n]} q(i|w)\leq 1,\,\forall\,w\in\Omega.$$

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Proof sketch: let q(i|w) be the allocation probabilities under equilibrium.

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Public persuasion: Sender broadcasts s=i with prob $q^*(i|w)$

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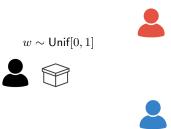
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⇒ Public persuasion is optimal

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- Receiver 2, aware of presence of receiver 1, will never extend an offer:
 - if extending an offer: only goods with $w \in [0.4, 0.8]$ will accept
- Suboptimal outcome: only goods with $w \in [0.8, 1]$ are allocated

Extension

- Weak preference: sender's utility satisfies $0 \le v_n \le \cdots \le v_2 \le v_1$ Public persuasion is optimal? ✓
- Multiple actions: each receiver selects from multiple actions regarding the good Public persuasion is optimal? ✓
- Uncertain Preference: sender's offer values $\{v_i\}$ are uncertain and possibly correlated with the good's characteristics w
 - ► Ordinal ranking over receivers remains fixed: ✓
 - ► Arbitrary correlation: ×
- Multiple goods: sender has multiple goods to allocate

Public persuasion is optimal? in general X

Counter-example: two identical goods to allocate to two receivers \longrightarrow externalities between receivers vanish \longrightarrow problem decouples over receivers

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Key assumptions: sender (i) allocates a **single good** and (ii) has a known **preference ranking** over receivers.

Utility function $u_i(w) = \kappa_i(w - \alpha_i)$ for each receiver i, where $w \in [0, 1]$

- lacktriangle Receivers care only about posterior mean: accept iff it exceeds $lpha_i$
- $\,\blacksquare\,$ Sender equivalently optimizes dist of posterior means of w (we use an alternative approach)

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Related literature

- Extreme-point approach to characterize an optimal persuasion mechanism: [Candogan, 2022], [Kleiner et al., 2021], [Arieli et al., 2023]
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- [Dworczak and Martini, 2019] study a more general problem, interpreting dual price as Walrasian equilibrium price in a persuasion economy
 - ► We consider a special case in which sender's utility is piecewise constant and increasing in posterior mean (as in [Candogan, 2022]), but we explicitly characterize set of optimal persuasion mechanisms by dualizing different constraints

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First-best relaxation with linear utilities:

$$\begin{split} \max_{q(i|w)\geq 0} \quad & \sum_{i=1}^n \, v_i \cdot \int_0^1 \, q(i|w) \, g(w) \, dw \\ \text{s.t.} \quad & \int_0^1 w \cdot q(i|w) \, g(w) \, dw \geq \alpha_i \int_0^1 \, q(i|w) \, g(w) \, dw, \, \forall \, i \in [n], \\ & \sum_{i \in [n]} q(i|w) \leq 1, \, \forall \, w \in [0,1]. \end{split}$$

Assumption (WLOG): receivers' hiring thresholds satisfy $0 < \alpha_n < \cdots < \alpha_2 < \alpha_1$

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$$\max_{\substack{q(i|w)\geq 0}} \quad \sum_{i=1}^n v_i \cdot \int_0^1 q(i|w) \, g(w) \, dw \qquad \qquad \text{dualize participation constrs.}$$

$$\text{s.t.} \qquad \int_0^1 w \cdot q(i|w) \, g(w) \, dw \geq \alpha_i \int_0^1 q(i|w) \, g(w) \, dw, \ \forall \, i \in [n],$$

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Lagrangian dual problem:

$$V^{\mathrm{LR}}(\boldsymbol{\mu}) = \int_0^1 \left(\max_{\substack{q(i|w) \geq 0, \\ \sum_{i \in [n]} q(i|w) \leq 1}} \sum_{i=1}^n \underbrace{\left\{ v_i + \mu_i \big(w - \alpha_i \big) \right\}} \cdot q(i|w) \right) \cdot g(w) \, dw.$$

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allocate to receiver with highest

point (α_i, v_i) with slope $\mu_i > 0$

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$$\underline{ \text{Upper envelope function:}} \ h \big(w; \pmb{\mu}^* \big) \triangleq \underset{i \in [n]}{\operatorname{arg min}} \underbrace{ V^{\operatorname{LR}}(\pmb{\mu}) : \text{ optimal dual variable} }_{\boldsymbol{\mu}^* (w - \alpha_i)} \Big\} \vee 0$$

Optimality conditions

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 $\implies h(w; \mu^*)$ is convex, increasing, and piecewise linear

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Optimality conditions (informal). A (public) persuasion mechanism is optimal iff it satisfies:

- 1. For each linear segment of $h(w; \mu^*)$: allocate w in this range exclusively to receivers whose point (α_i, v_i) lie on the segment
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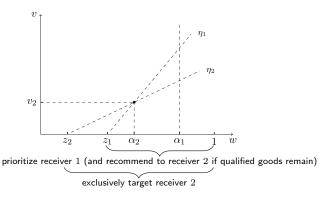
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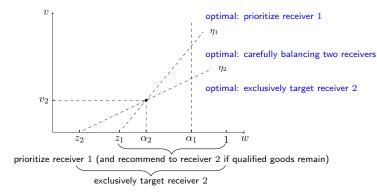
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$$\implies$$
 Problem **decouples** over segments of $h(w; \mu^*)$

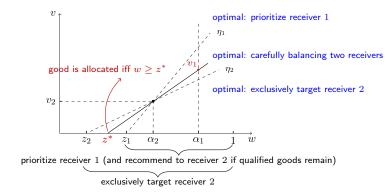
Two-receiver case: closed-form characterization



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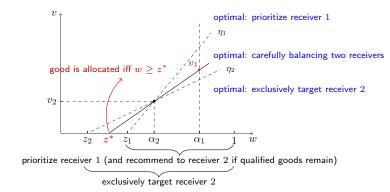
Two-receiver case: closed-form characterization



Any mechanism that satisfies the following is optimal:

- 1. Good is allocated if and only if $w \geq z^*$
- 2. Both receivers' participation constraints bind

Two-receiver case: closed-form characterization



General case:

- **Explicit** characterization of upper envelope function $h(w; \mu^*)$
- Multiple ways to construct optimal persuasion mechanisms

- We study single-good resource allocation in the Bayesian persuasion context, where the sender has known preferences over the receivers.
- Operational takeaway: public persuasion remains optimal, irrespective of how receivers communicate.
 - Analysis is based on first-best relaxation: public persuasion obtains first-best performance.

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Reference: C. Chen and X. Qi. 2024. Optimality of Public Persuasion for Single-Good Allocation. Major Revision at *OR*.

