Optimality of Public Persuasion for Single-Good Allocation

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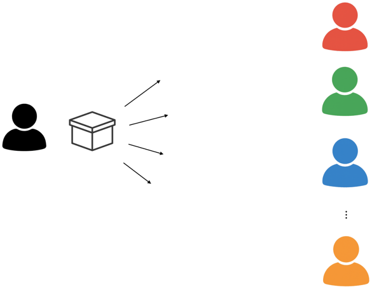
Model

- A sender allocates **an indivisible good** among n receivers
  - Sender can allocate to at most one receiver
  - Each receiver decides whether to accept based on self-interest



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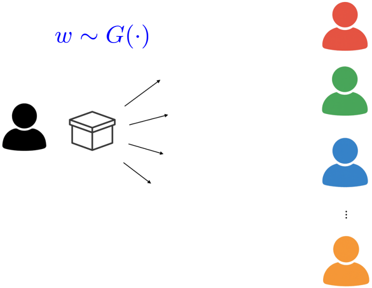

- Sender strategically discloses good's characteristics $w \in \Omega$, with prior dist $G(\cdot)$

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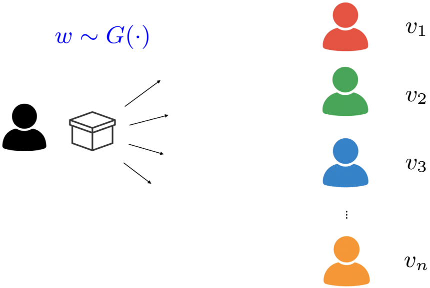

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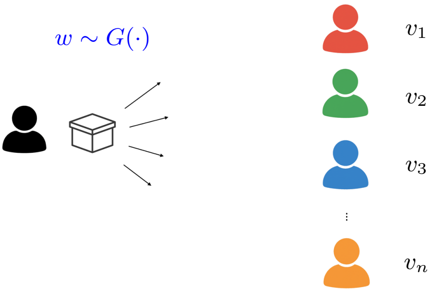

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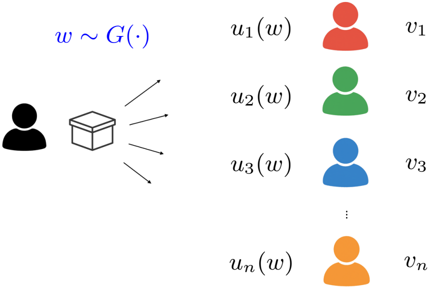

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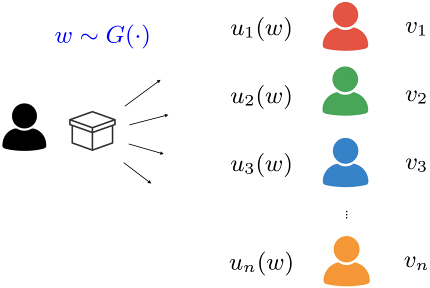

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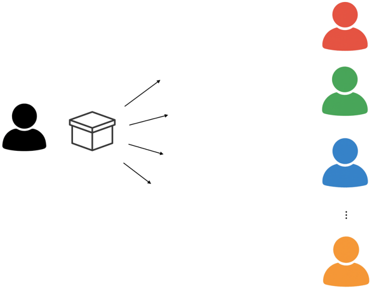
- **Applications**: (i) school advisor promotes student for job positions, (ii) incubator pitches startup to VC investors

Public versus private persuasion

Sender commits to persuasion mechanism $f(\boldsymbol{s}|w)$: joint dist of sending signals $\boldsymbol{s} = (s_1, s_2, \cdots, s_n)$ conditional on w



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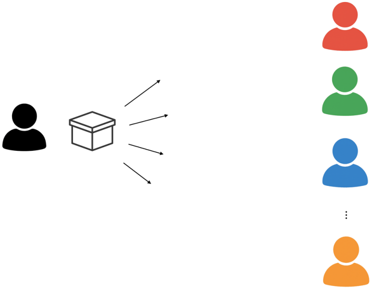

- **Public persuasion:** $f(\boldsymbol{s}|w) = 0$ if $s_i \neq s_j$ for some $i, j \in [n]$; that is receivers always receive the same signal
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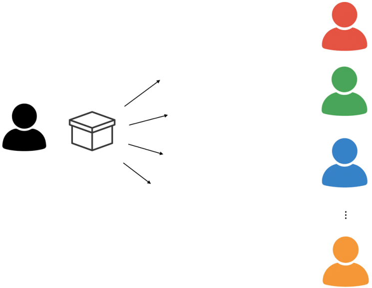

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- Model permits receives to **communicate** after receiving signals
  - Receivers can communicate in an arbitrary way (including in self-interest)

Model (cont.)

The game proceeds as follows:

1. Sender commits to a persuasion mechanism $f(\cdot|w)$ and a signal space $\mathbf{S} = \bigotimes_{i=1}^{n} S_i$.
2. Sender observes the good's characteristics $w \sim G(w)$. A signal $\mathbf{s} = (s_i)_{i \in [n]} \sim f(\cdot|w)$ is generated and sent to the receivers.
3. Receivers communicate with one another in a certain way.
4. Each receiver i decides whether to accept the good based on the signal and communication.
5. Sender accepts the best offer (if she receives any).

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Main result: Public persuasion is optimal regardless of how receivers communicate

$\Longrightarrow$ Sender eliminates any communication for her own interest

The first-best relaxation

Proposition. The optimal value $\bar{V}$ of:

$$\begin{aligned}
\max_{q(i|w)\geq 0} \quad & \sum_{i=1}^{n} v_i \int_{w\in\Omega} q(i|w)\, dG(w) \\
\text{s.t.} \quad & \int_{w\in\Omega} u_i(w)\, q(i|w)\, dG(w) \geq 0,\ \forall\, i \in [n], \quad \textbf{(participation constr.)} \\
& \sum_{i\in[n]} q(i|w) \leq 1,\ \forall\, w \in \Omega.
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Proof sketch: let $q(i|w)$ be the allocation probabilities under equilibrium.

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Public persuasion: Sender broadcasts $s = i$ with prob $q^*(i|w)$

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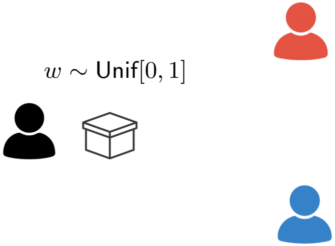
$\Longrightarrow$ Public persuasion is optimal

Suboptimality of vanilla private persuasion

Suppose the receivers cannot communicate.



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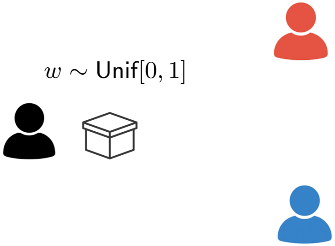

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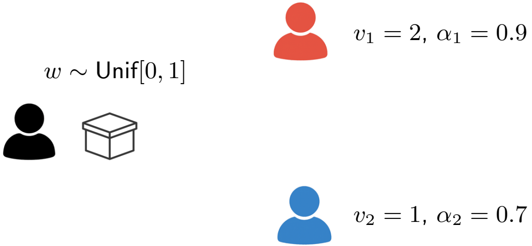

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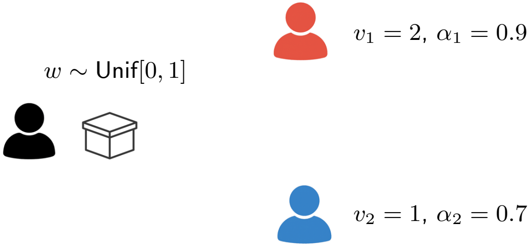

- **Vanilla private persuasion:** recommend receiver 1 to accept the good when $w \geq 0.8$, and recommend receiver 2 to accept the good when $w \geq 0.4$.

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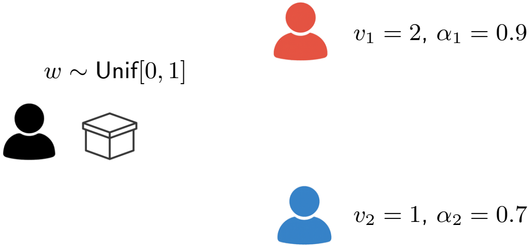

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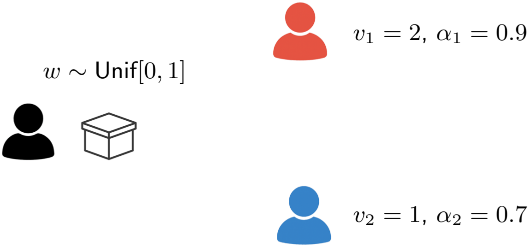

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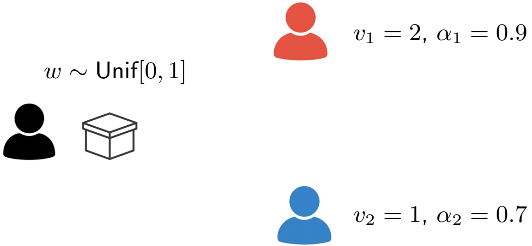

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- Suboptimal outcome: only goods with $w \in [0.8, 1]$ are allocated

Extension

- **Weak preference:** sender's utility satisfies $0 \leq v_n \leq \cdots \leq v_2 \leq v_1$

  Public persuasion is optimal? ✓

- **Multiple actions:** each receiver selects from multiple actions regarding the good

  Public persuasion is optimal? ✓

- **Uncertain Preference:** sender's offer values $\{v_i\}$ are uncertain and possibly correlated with the good's characteristics w
  - Ordinal ranking over receivers remains fixed: ✓
  - Arbitrary correlation: ×

- **Multiple goods:** sender has multiple goods to allocate

  Public persuasion is optimal? in general ×

  Counter-example: two identical goods to allocate to two receivers $\longrightarrow$ externalities between receivers vanish $\longrightarrow$ problem decouples over receivers

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- **Key assumptions:** sender (i) allocates a **single good** and (ii) has a known **preference ranking** over receivers.

Special case: linear utilities

Utility function $u_i(w) = \kappa_i(w - \alpha_i)$ for each receiver i, where $w \in [0, 1]$

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- Sender equivalently optimizes dist of posterior means (we use an alternative approach)

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Related literature

- Extreme-point approach to characterize an optimal persuasion mechanism: [Candogan, 2022], [Kleiner et al., 2021], [Arieli et al., 2023]
- Dual approach for optimality conditions: [Dworczak and Martini, 2019]

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- **Dual approach** for optimality conditions: [Dworczak and Martini, 2019]
  - [Dworczak and Martini, 2019] study a more general problem, interpreting dual price as Walrasian equilibrium price in a persuasion economy
  - We consider a special case in which sender's utility is piecewise constant and increasing in posterior mean (as in [Candogan, 2022]), but we explicitly characterize set of optimal persuasion mechanisms by dualizing different constraints

our approach

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First-best relaxation with linear utilities:

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Assumption (WLOG): receivers' hiring thresholds satisfy $0 < \alpha_n < \cdots < \alpha_2 < \alpha_1$

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First-best relaxation with linear utilities:

dualize participation constrs. with dual variable $\mu_i \geq 0$

$$\max_{q(i|w)\geq 0} \quad \sum_{i=1}^{n} v_i \cdot \int_0^1 q(i|w)\, g(w)\, dw$$

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Optimality conditions

Lagrangian dual problem:

$$V^{\mathrm{LR}}(\boldsymbol{\mu}) = \int_0^1 \left(\max_{\substack{q(i|w)\geq 0,\\ \sum_{i\in[n]} q(i|w)\leq 1}} \sum_{i=1}^{n} \underbrace{\Big\{ v_i + \mu_i \big(w - \alpha_i\big) \Big\}} \cdot q(i|w) \right) \cdot g(w)\, dw.$$

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$\boldsymbol{\mu}^* \triangleq \arg\min_{\boldsymbol{\mu}\in\mathbb{R}^n_+} V^{\mathrm{LR}}(\boldsymbol{\mu})$: optimal dual variable

Upper envelope function: $h\big(w;\boldsymbol{\mu}^*\big) \triangleq \max_{i\in[n]} \Big\{v_i + \mu_i^*\big(w-\alpha_i\big)\Big\} \vee 0$

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$\implies h\big(w;\boldsymbol{\mu}^*\big)$ is convex, increasing, and piecewise linear

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Receiver i's line: passing through point (α_i, v_i) with slope $\mu_i \geq 0$

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Optimality conditions (informal). A (public) persuasion mechanism is optimal iff it satisfies:

1. For each linear segment of $h\left(w;\boldsymbol{\mu}^*\right)$: allocate w in this range exclusively to receivers whose point (α_i, v_i) lie on the segment
2. Receivers' participation constrs bind (for all segments with positive slopes)

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<u>Upper envelope function:</u> $h(w; \boldsymbol{\mu}^*) \triangleq \max_{i\in[n]} \left\{ v_i + \mu_i^* (w - \alpha_i) \right\} \vee 0$

$\Longrightarrow h(w; \boldsymbol{\mu}^*)$ is convex, increasing, and piecewise linear

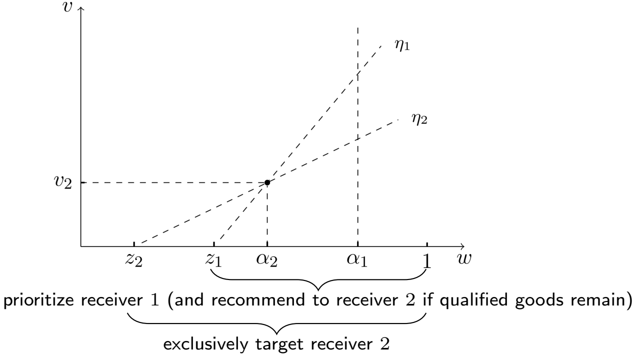
Optimality conditions (informal). A (public) persuasion mechanism is optimal iff it satisfies:

1. For each linear segment of $h(w; \boldsymbol{\mu}^*)$: allocate w in this range exclusively to receivers whose point (α_i, v_i) lie on the segment
2. Receivers' participation constrs bind (for all segments with positive slopes)

$\Longrightarrow$ Problem **decouples** over segments of $h(w; \boldsymbol{\mu}^*)$

Optimal persuasion mechanisms

Two-receiver case: closed-form characterization



Optimal persuasion mechanisms

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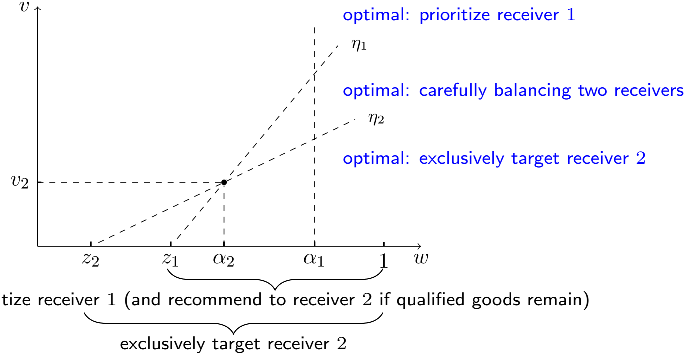
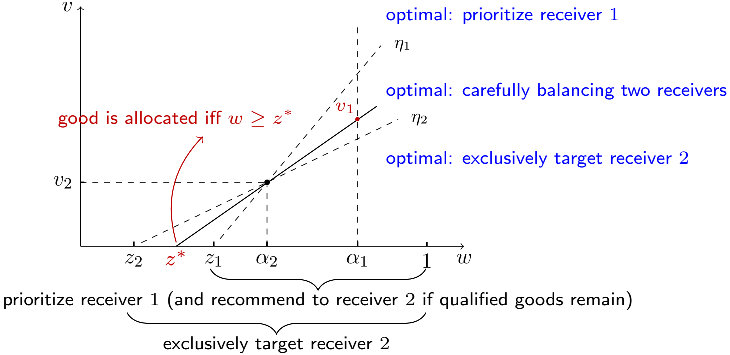


Optimal persuasion mechanisms

Two-receiver case: closed-form characterization

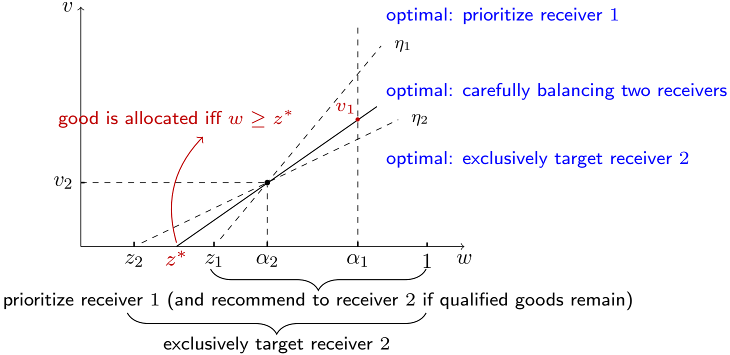


Any mechanism that satisfies the following is optimal:

1. Good is allocated if and only if $w \geq z^*$
2. Both receivers' participation constraints bind

Optimal persuasion mechanisms

Two-receiver case: closed-form characterization



General case:

- **Explicit** characterization of upper envelope function $h(w; \boldsymbol{\mu}^*)$
- **Multiple ways** to construct optimal persuasion mechanisms

Summary

- We study single-good resource allocation in the Bayesian persuasion context, where the sender has known preferences over the receivers.
- Operational takeaway: **public persuasion** remains optimal, irrespective of how receivers communicate.
  - Analysis is based on first-best relaxation: public persuasion obtains first-best performance.

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Reference: C. Chen and X. Qi. 2024. Optimality of Public Persuasion for Single-Good Allocation. Major Revision at *OR*.

Appendix