Incentivizing Resource Pooling

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Joint work with:
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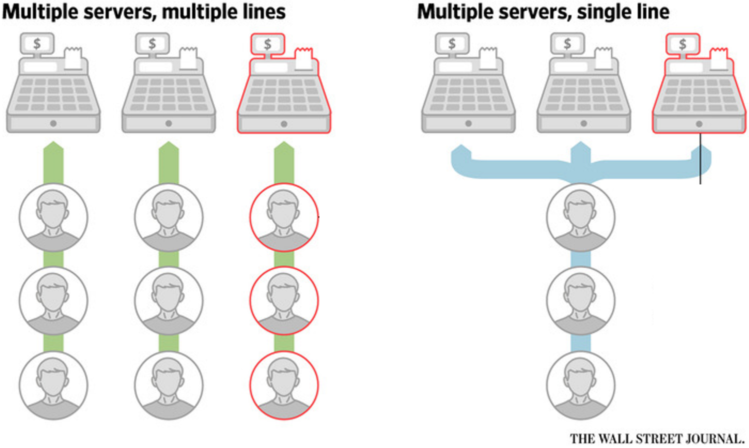
The Line Dance

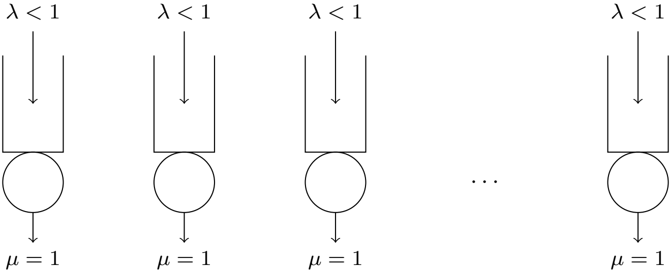
Queuing theory, the mathematical study of lines, helps businesses, call centers, computer networks and others figure out how to keep things moving.



Resource pooling significantly improves service

Resource pooling: known fact

N servers: job arrival rate $\lambda < 1$, server processing rate $\mu = 1$



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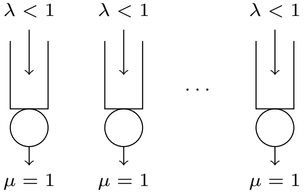
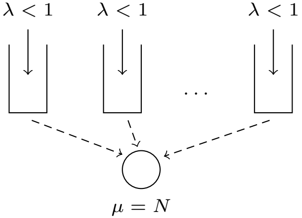


$vs.$

With resource pooling:



 # jobs in system: $N \cdot \frac{\lambda}{1-\lambda}$ (linear) vs. $\frac{\lambda}{1-\lambda}$ (constant)

	Without resource pooling	With resource pooling
# jobs in system:	$N \cdot \frac{\lambda}{1-\lambda}$	$\frac{\lambda}{1-\lambda}$
	linear	constant

Can resource pooling be achieved in **decentralized** systems?

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Decentralization boosts security, privacy, and scalability

Motivation

- **Goal:** design mechanism to incentivize resource pooling in a decentralized setting.
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Golem Network
Market cap: $170M

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Golem Network Market cap: \$170M	Akash Network Market cap: \$320M	iExec Market cap: \$75M

- Essential aspects of the problem:
  - Large-scale system: number of servers N is large
  - Servers have limited information about one another

A marketplace design problem:

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Incentivize resource pooling, in private information setting

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for queueing, matching, and general stochastic systems

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A talk on a different day...

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  - Provides tight theoretical guarantees

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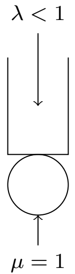
- Propose an approximation-based analytical framework
  - Simplifies the design and analysis of token-based mechanisms
  - Provides tight theoretical guarantees
  - Can be applied to more general settings

Model setup

- N strategic servers
- Jobs arrive with Poisson(λ) where $\lambda < 1$; capacity units arrive with Poisson(1)



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  1. Holding cost: each waiting job costs one per unit of time
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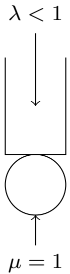

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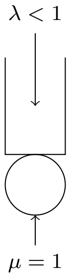

- **Servers' objective:** minimizing own long-run average total cost
- Limited information:
  (a) A server's arrivals and actions are private information
  (b) Precise knowledge of number of servers N not required (except knowing that it is relative large)

Related Literature

Resource pooling:

- Power of resource pooling: [Tsitsiklis and Xu, 2013]
- Decentralized setup with two servers: [Hu and Caldentey, 2023]

Mean-field equilibrium:

- Analysis of complex operational problems: [Iyer et al., 2014], [Balseiro et al., 2015], [Kanoria and Saban, 2021], [Arnosti et al., 2021]
- Fluid mean-field equilibrium similar in spirit to [Balseiro et al., 2015]

Scrip system:

- Analysis of scrip system: [Kash et al., 2007], [Kash et al., 2015], [Johnson et al., 2014], [Bo et al., 2018]

Other related work:

- Cooperative game model: [Anily and Haviv, 2010], [Anily and Haviv, 2014], [Karsten et al., 2015]
- Supermarket game: [Xu and Hajek, 2013], [Yang et al., 2019]

Outline

- **Motivation, research question, and literature review**
- **Token-based mechanism**
  - Solution concept: Fluid mean-field equilibrium (FMFE)
  - Characterization of FMFE
  - Designing key element of mechanism
- **FMFE strategy as near-optimal best response**
  - Asymptotic analysis for large markets
  - Numerical analysis for small markets
- **Extension to heterogeneous servers**
- **Takeaway**

Token-based mechanism

In the mechanism, a server can:

- Request help from others without recall at any time.

- When a capacity unit arrives, either: (i) serve its job, (ii) help others, or (iii) be idle and waste the unit without recall.

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  - The *oldest* job in shared pool is served (if pool is non-empty)

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- The value of ϕ is critical to system performance

Equilibrium concept: Fluid mean-field equilibrium

Approximation methodology similar to (Balseiro et al. 2015)

- **Mean-field approximation:** each server optimizes by assuming state of shared pool is fixed at long-run average $\implies$
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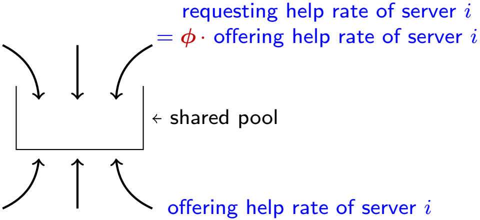

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  $\implies$ Closed-form characterization (next slide)
- Fluid mean-field equilibrium (FMFE):

$$\boxed{w \in H(w)}$$

conjectured waiting time

induced waiting time when all servers follow best responses to w

Server's best response

Closed-form solution: threshold policy w.r.t. queue length:

- Request help only when queue length exceeds a threshold k (which depends on ϕ and w)
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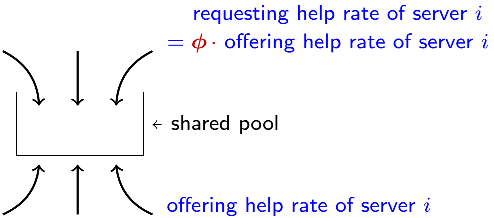

Best response when $w < 1$

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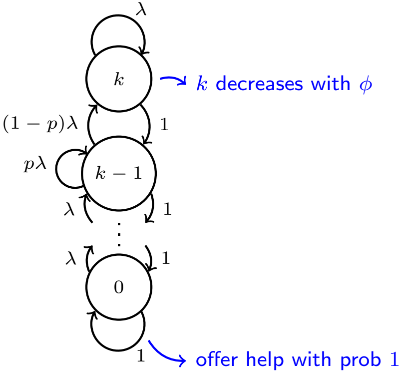
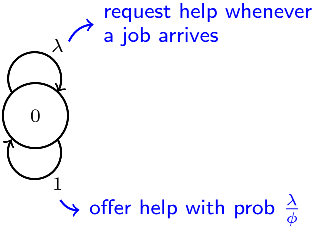


$\boldsymbol{\phi \leq \lambda}$



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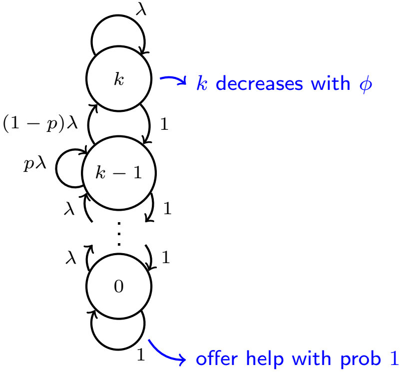

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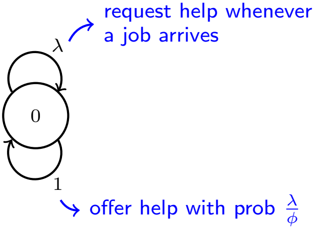

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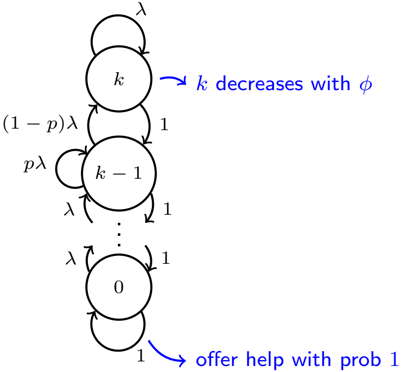

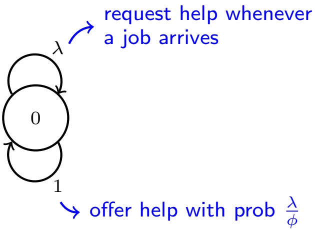

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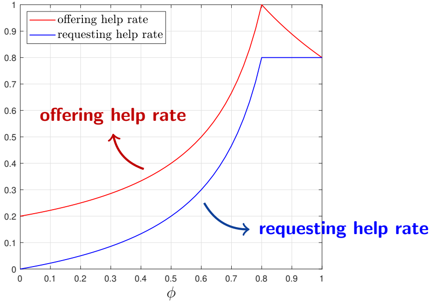
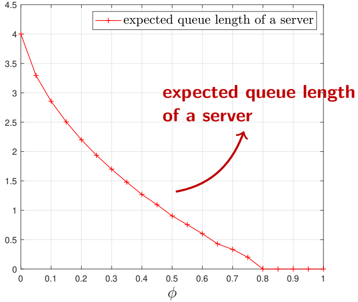
Optimal value of ϕ





Proposition. The expected total number of jobs in system, denoted by $Q_\Sigma(\phi)$, satisfies:

1. When $\phi < \lambda$: $\lim_{N\to\infty} Q_\Sigma(\phi)/N = q(\phi) > 0$
2. When $\phi \geq \lambda$: $Q_\Sigma(\phi) = \frac{\phi}{1-\phi}$

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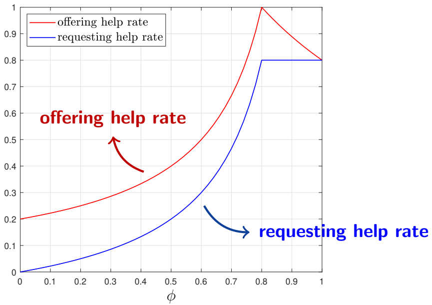

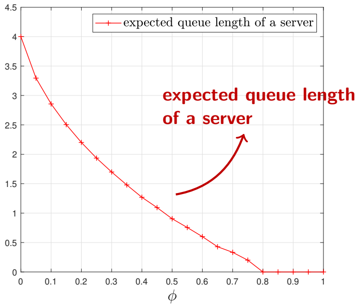

Main result. The optimal value is $\boldsymbol{\phi = \lambda}$. Moreover, this induces complete resource pooling: it is each server's best strategy to (i) request help whenever a job arrives, (ii) offer help when queue is empty.

$\Longrightarrow$ System's dynamics and performance match those under centralized control

FMFE as **good approximation** of servers' strategies

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2. A coupling argument and a drift analysis to show:
   (a) shared pool's queue length transitions to stationary distribution quickly as $N \to \infty$
   (b) in stationary distribution, shared pool is non-empty with probability $\phi - \frac{c(\lambda,\phi,\delta)}{N^{1-\delta}}$

Analysis for small market

- Mechanism uses $\phi = \lambda$.
- Consider the fluid setup: tokens can go negative but expected rates of earning and spending tokens are equal.
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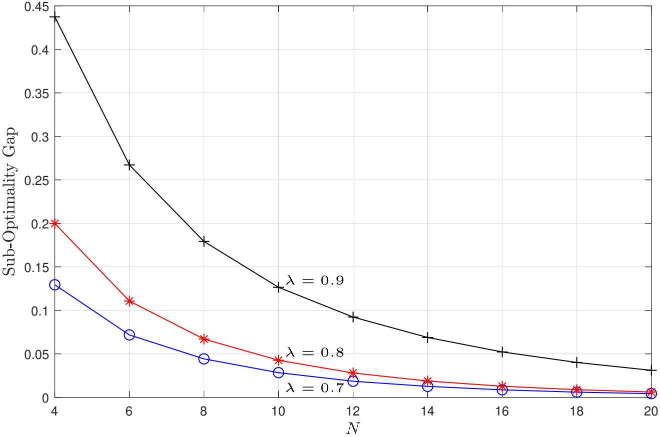
$\Rightarrow$ Optimal strategy depends only on two states: q_1 (own queue length) and q_0

$\Longrightarrow$ Tractable optimization problem!

Numerical results

(a) job processing cost $c = 1$; (b) job arrival rate $\lambda \in \{0.7, 0.8, 0.9\}$

Sub-optimality gap $= \frac{\text{Cost of complete resource pooling} - \text{Cost of optimal strategy}}{\text{Cost of optimal strategy}}$



- The value of playing strategically is small even with few servers (and when server one can perfectly monitor the shared pool)

Extension: heterogeneous servers

- For each server i: job arrival rate λ_i and processing rate μ_i; let $\rho_i = \frac{\lambda_i}{\mu_i}$
- Assume $0 < \underline{\rho} \leq \rho_i \leq \bar{\rho} < 1$ and $0 < \underline{\lambda} \leq \lambda_i \leq \bar{\lambda}$ for all servers

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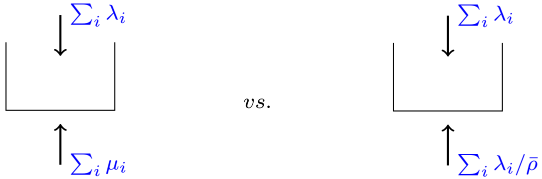

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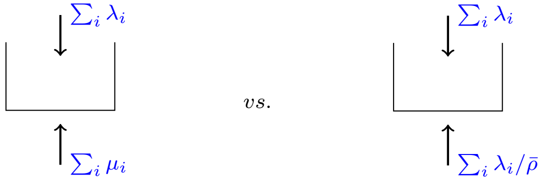

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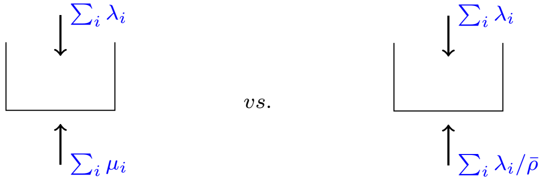

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  $\Rightarrow$ Costs allocated <u>fairly</u> in our mechanism!

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- We study incentivizing resource pooling in a decentralized setting, where servers have limited information about others
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  - Analysis based on fluid mean-field equilibrium
  - Numerical results show that benefit from unilateral deviation is small even with only a few servers

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Reference: C. Chen, Y. Chen, and P. Qian. 2023. Incentivizing Resource Pooling. Under review.

Working paper available at `https://papers.ssrn.com/abstract=4586771`

Appendix

Minimum number of servers to sustain FMFE

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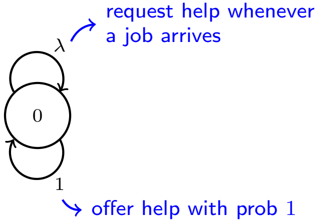

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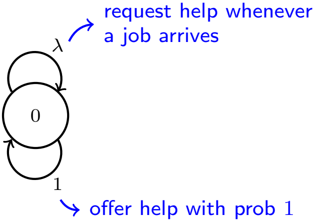
  - $w \leq 1 \Rightarrow N \geq \left\lceil \frac{1}{1-\lambda} \right\rceil$



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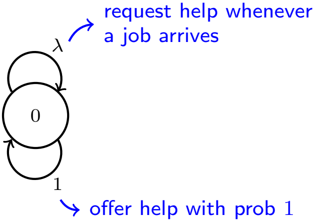

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