

Optimality of Public Persuasion for Single-Good Allocation

Chen Chen

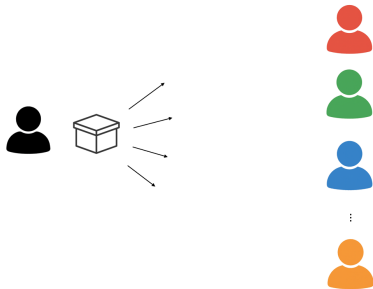
Xuyuanda Qi



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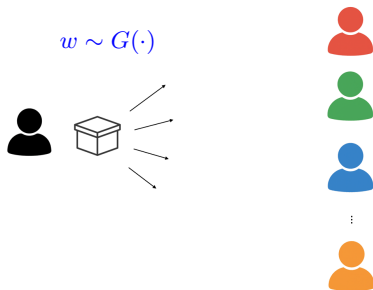
Model

- A sender allocates **an indivisible good** among n receivers
 - ▶ Sender can allocate to at most one receiver
 - ▶ Each receiver decides whether to accept based on self-interest



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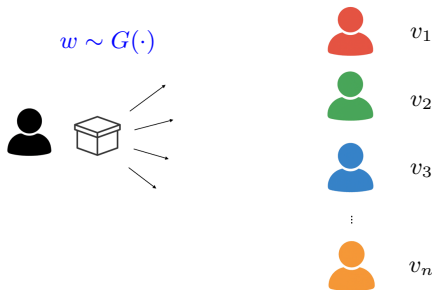
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- Sender strategically discloses good's characteristics $w \in \Omega$, with prior dist $G(\cdot)$

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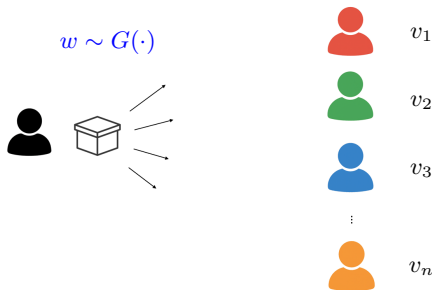
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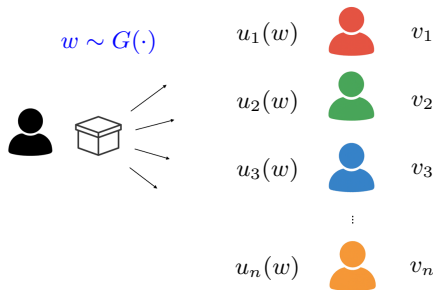


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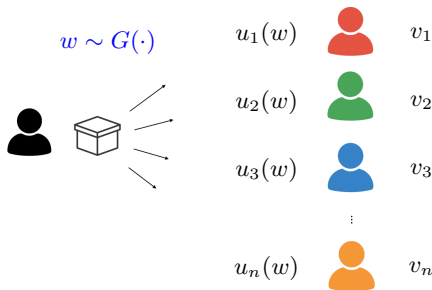
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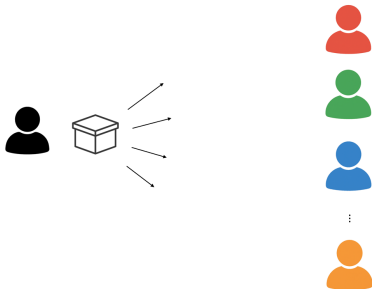
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- **Applications:** (i) school advisor promotes student for job positions, (ii) incubator pitches startup to VC investors

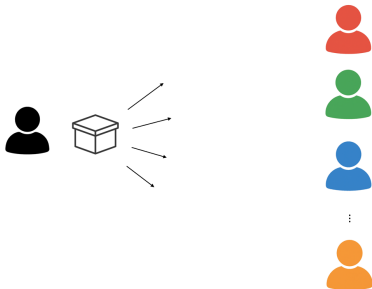
Public versus private persuasion

Sender commits to persuasion mechanism $f(s|w)$: joint dist of sending signals $\mathbf{s} = (s_1, s_2, \dots, s_n)$ conditional on w



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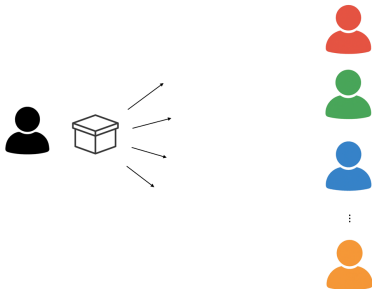
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- **Public persuasion:** $f(s|w) = 0$ if $s_i \neq s_j$ for some $i, j \in [n]$; that is receivers always receive the same signal
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- Model permits receivers to **communicate** after receiving signals
 - Receivers can communicate in an arbitrary way (including in self-interest)

Model (cont.)

The game proceeds as follows:

1. Sender commits to a persuasion mechanism $f(\cdot|w)$ and a signal space $\mathbf{S} = \bigotimes_{i=1}^n S_i$.
2. Sender observes the good's characteristics $w \sim G(w)$. A signal $\mathbf{s} = (s_i)_{i \in [n]} \sim f(\cdot|w)$ is generated and sent to the receivers.
3. Receivers communicate with one another in a certain way.
4. Each receiver i decides whether to accept the good based on the signal and communication.
5. Sender accepts the best offer (if she receives any).

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Main result: Public persuasion is **optimal** regardless of how receivers communicate

\implies Sender eliminates any communication for her own interest

The first-best relaxation


Proposition. The optimal value \bar{V} of:


$$\begin{aligned} \max_{q(i|w) \geq 0} \quad & \sum_{i=1}^n v_i \int_{w \in \Omega} q(i|w) dG(w) \\ \text{s.t.} \quad & \int_{w \in \Omega} u_i(w) q(i|w) dG(w) \geq 0, \forall i \in [n], \quad (\text{participation constr.}) \\ & \sum_{i \in [n]} q(i|w) \leq 1, \forall w \in \Omega. \end{aligned}$$

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Proof sketch: let $q(i|w)$ be the allocation probabilities under equilibrium.

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Theorem. Under the public persuasion: (i) it is an equilibrium for each receiver $i \in [n]$ to extend an offer only upon receiving signal $s = i$; (ii) the expected payoff of the mechanism equals \bar{V} .

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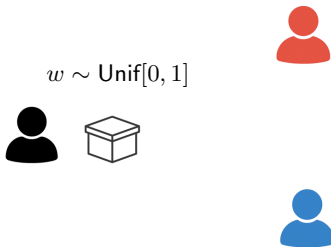
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Suboptimality of vanilla private persuasion

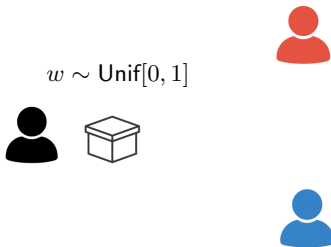
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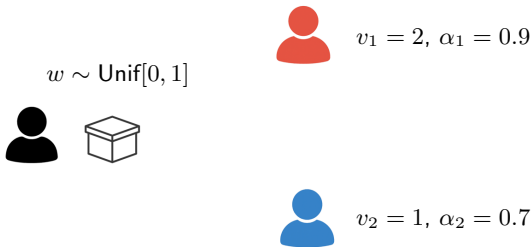
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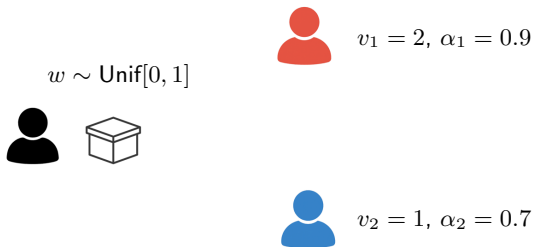
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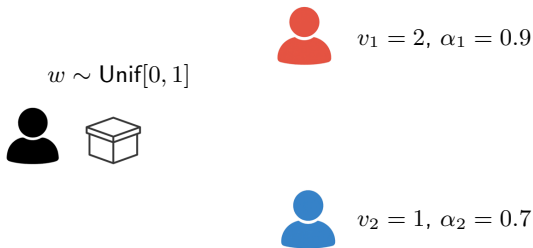


- **Vanilla private persuasion:** recommend receiver 1 to accept the good when $w \geq 0.8$, and recommend receiver 2 to accept the good when $w \geq 0.4$.

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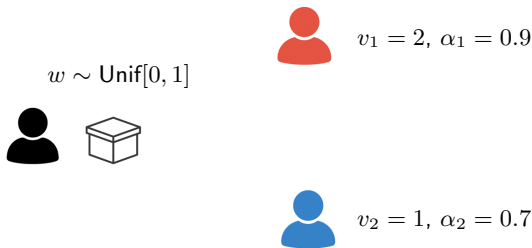


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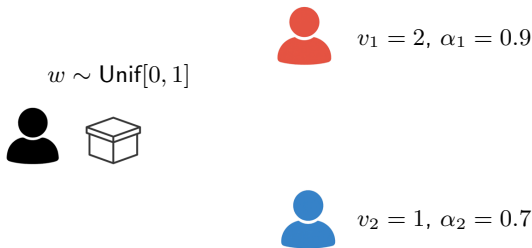


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- Suboptimal outcome: only goods with $w \in [0.8, 1]$ are allocated

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- **Key assumptions:** sender (i) allocates a **single good** and (ii) has a known **preference ranking** over receivers.

Special case: linear utilities

Utility function $u_i(w) = \kappa_i(w - \alpha_i)$ for each receiver i , where $w \in [0, 1]$

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Related literature

- Extreme-point approach to characterize an optimal persuasion mechanism: [\[Candogan, 2022\]](#), [\[Kleiner et al., 2021\]](#), [\[Arieli et al., 2023\]](#)
- Dual approach for optimality conditions: [\[Dworczak and Martini, 2019\]](#)

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- our approach
- ▶ [\[Dworczak and Martini, 2019\]](#) study a more general problem, interpreting dual price as Walrasian equilibrium price in a persuasion economy
 - ▶ We consider a special case in which sender's utility is piecewise constant and increasing in posterior mean (as in [\[Candogan, 2022\]](#)), but we explicitly characterize set of optimal persuasion mechanisms by dualizing different constraints

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First-best relaxation with linear utilities:

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dualize participation constrs
with dual variable $\mu_i \geq 0$



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Optimality conditions

Lagrangian dual problem:

$$V^{\text{LR}}(\boldsymbol{\mu}) = \int_0^1 \left(\max_{\substack{q(i|w) \geq 0, \\ \sum_{i \in [n]} q(i|w) \leq 1}} \sum_{i=1}^n \underbrace{\{v_i + \mu_i(w - \alpha_i)\}} \cdot q(i|w) \right) \cdot g(w) dw.$$

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$\boldsymbol{\mu}^* \triangleq \arg \min_{\boldsymbol{\mu} \in \mathbb{R}_+^n} V^{\text{LR}}(\boldsymbol{\mu})$: optimal dual variable

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$\implies h(w; \boldsymbol{\mu}^*)$ is convex, increasing, and piecewise linear

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$$V^{\text{LR}}(\boldsymbol{\mu}) = \int_0^1 \left(\max_{\substack{q(i|w) \geq 0, \\ \sum_{i \in [n]} q(i|w) \leq 1}} \sum_{i=1}^n \underbrace{\{v_i + \mu_i(w - \alpha_i)\}}_{\substack{\text{Receiver } i\text{'s line: passing through} \\ \text{point } (\alpha_i, v_i) \text{ with slope } \mu_i \geq 0}} \cdot q(i|w) \right) \cdot g(w) dw.$$

allocate to receiver with highest positive value for a given w

Receiver i 's line: passing through point (α_i, v_i) with slope $\mu_i \geq 0$

Upper envelope function: $h(w; \boldsymbol{\mu}^*) \triangleq \max_{i \in [n]} \left\{ v_i + \mu_i^*(w - \alpha_i) \right\} \vee 0$

$\boldsymbol{\mu}^* \triangleq \arg \min_{\boldsymbol{\mu} \in \mathbb{R}_+^n} V^{\text{LR}}(\boldsymbol{\mu})$: optimal dual variable

$\implies h(w; \boldsymbol{\mu}^*)$ is convex, increasing, and piecewise linear

Optimality conditions (informal). A (public) persuasion mechanism is **optimal** iff it satisfies:

1. For each linear segment of $h(w; \boldsymbol{\mu}^*)$: allocate w in this range exclusively to receivers whose point (α_i, v_i) lie on the segment
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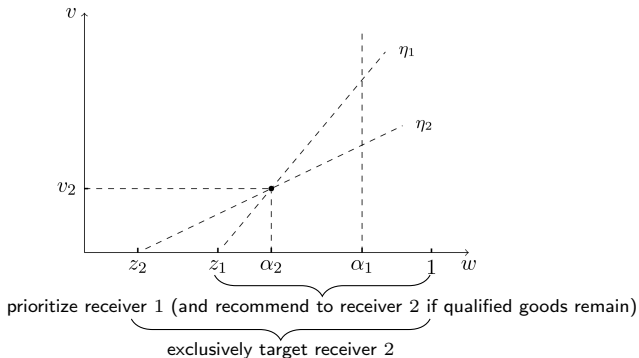
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\implies Problem **decouples** over segments of $h(w; \boldsymbol{\mu}^*)$

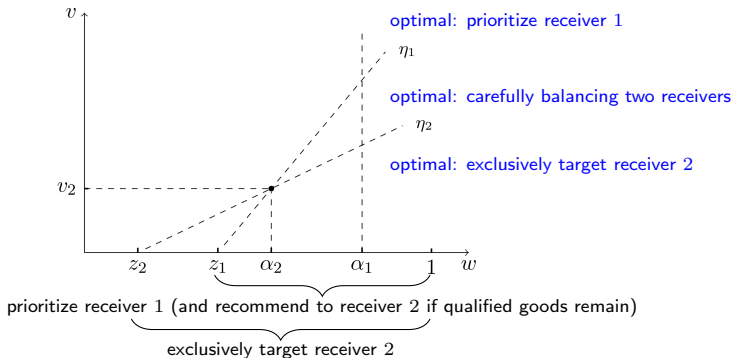
Optimal persuasion mechanisms

Two-receiver case: closed-form characterization



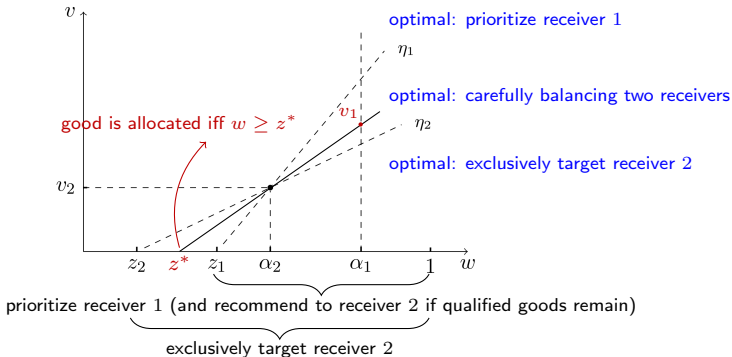
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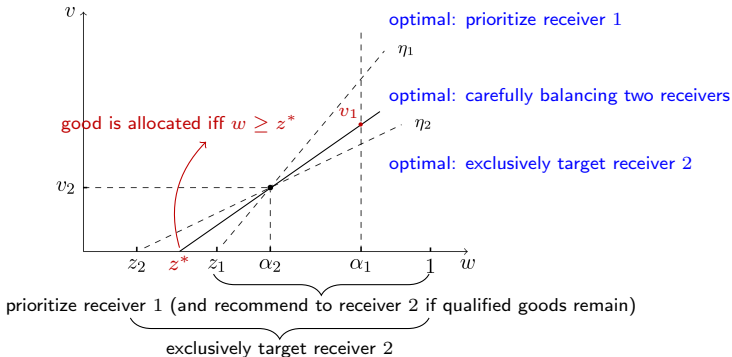


Any mechanism that satisfies the following is **optimal**:

1. Good is allocated if and only if $w \geq z^*$
2. Both receivers' participation constraints bind

Optimal persuasion mechanisms

Two-receiver case: closed-form characterization



General case:

- **Explicit** characterization of upper envelope function $h(w; \mu^*)$
- **Multiple ways** to construct optimal persuasion mechanisms

Summary

- We study **single-good resource allocation** in the **Bayesian persuasion** context, where the sender has known preferences over the receivers.
- Operational takeaway: **public persuasion** remains **optimal**, irrespective of how receivers communicate.
 - ▶ Analysis is based on **first-best relaxation**: public persuasion obtains first-best performance.

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Reference: C. Chen and X. Qi. 2024. Optimality of Public Persuasion for Single-Good Allocation. Major Revision at *OR*.

Appendix