Often beams are not perfectly Gaussian, so the focusability and waist size are measured in-situ. The M^2 value can then be used to characterise the difference of a beam from the ideal Gaussian, and modifies the beam propagation formula as

$$w_M^2 = w_{0_M}^2 + \left(\frac{z\lambda_0 M^2}{\pi w_{0_M}}\right)^2 , \tag{2.1.14}$$

where w_M is the beam width and w_{0_M} is the measured beam waist. Importantly, M^2 is not measured from a single focal spot, but from multiple points within its Rayleigh range [131]. At the Gemini laser facility, $M^2 = 4.1$ have been measured for an $F = 40$ focusing geometry [129], while $M^2 = 3.6$ have been measured for an $F = 2$ focusing geometry[1].

2.1.3 General Beam Propagation

Rearranging (2.1.8) we find

$$\frac{\partial \mathbf{E}_0}{\partial z} = \left(\frac{i}{2k}\nabla_\perp^2 + \frac{ik}{2}(\eta^2 - 1)\right)\mathbf{E}_0 . \tag{2.1.15}$$

This describes the change in an electric field as it propagates, given its current values. Without the need to assume any symmetry, this equation can be reaarranged and solved using a finite-difference scheme as

$$\mathbf{E}_0(x, y, z + \delta z) = \exp\left(\delta z \frac{i}{2k}\nabla_\perp^2\right)\exp\left(\delta z \frac{ik}{2}(\eta^2 - 1)\right)\mathbf{E}_0(x, y, z) , \tag{2.1.16}$$

$$\mathbf{E}_0(x, y, z + \delta z) = \exp(\delta z A)\exp(\delta z B)\,\mathbf{E}_0(x, y, z) , \tag{2.1.17}$$

where δz is a small step in the propagation axis, and the operators A and B have been defined. This is known as a beam propagation method (BPM) [132]. It can be shown [133] that the operators are commutable, and the solution has accuracy $\sim (\delta z)^3$ when implemented in a split-step scheme, like

$$\mathbf{E}_0(x, y, z + \delta z) = \exp\left(\frac{\delta z}{2}A\right)\exp(\delta z B)\exp\left(\frac{\delta z}{2}A\right)\mathbf{E}_0(x, y, z) . \tag{2.1.18}$$

The operator, B, is a straight-forward, complex multiplicative factor. However, the derivatives in the operator, A, are more involved. Instead, it can be shown that this operator is equivalent to

[1] Analysis performed by E. Los, Imperial College London

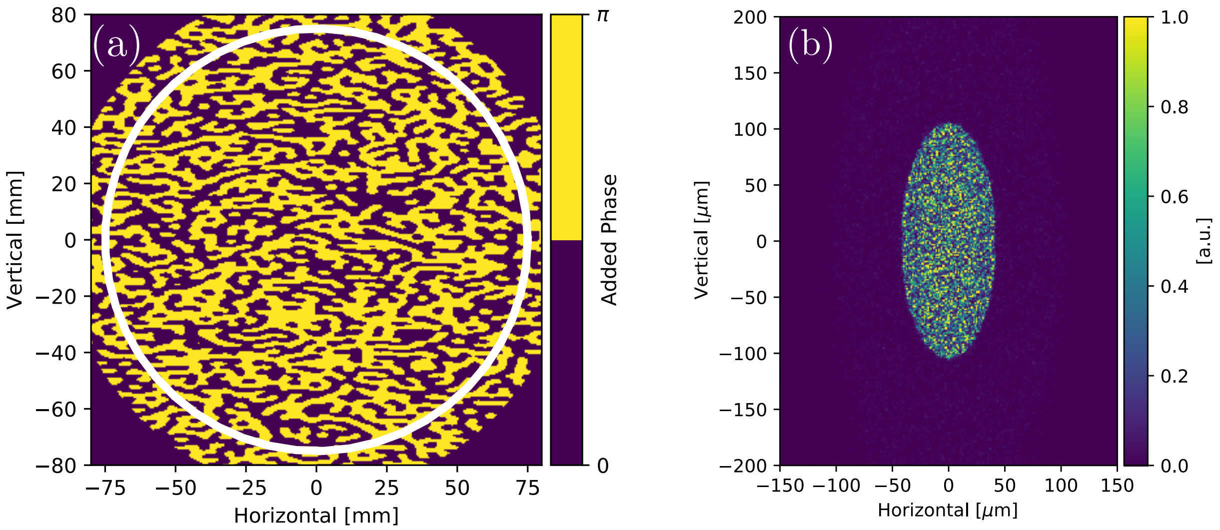

Figure 2.2: Beam Propagation through a Distributive Phase Mask. (a) Ideal, binary, distributive phase mask applied to a collimated, flat-top beam of width 75 mm (white). (b) Simulated $f/2$ focal spot of the beam given the phase mask. The mask was designed to create an 80 μm $\times$ 200 μm uniform focal spot.

$$\exp\left(\frac{\delta z}{2}A\right) = \mathcal{F}^{-1}\exp\left(\frac{i\delta z}{4k}k_{\perp}^{2}\right)\mathcal{F}\ , \tag{2.1.19}$$

where $\mathcal{F}$ is the Fourier transform. The propagation of the pulse can then be implemented in a Fourier-transform based beam propagation method (FFT-BPM) [133]. A code[2] employing this method was used to simulate the propagation of electromagnetic waves through a distributed phase mask, as shown in Figure 2.2.

In vacuum, the operator $B = 0$, resulting in many of the intermediate Fourier transforms steps cancelling each other out. Propagation of the beam to any point in space, Δz, can then be performed in one step, following

$$\mathbf{E}_0(x, y, z + \Delta z) = \mathcal{F}^{-1}\exp\left(\frac{i\Delta z}{2k}k_{\perp}^{2}\right)\mathcal{F}\ \mathbf{E}_0(x, y, z)\ . \tag{2.1.20}$$

This highlights how the vacuum focal spot of a general laser beam is dependent on both its spatial and spectral Fourier components [134].

2.1.4 Flat-top Beams

In order to extract the most energy from amplifiers and ensure uniformity across the beam, laser beam profiles are often pseudo flat-top as opposed to Gaussian. The Gemini laser pulse is typically an 150 mm diameter pseudo flat-top beam, with inhomoegeneities caused by the Ti:sapphire crystal amplifier [135]. From its Fourier transform, the focal spot of a laser with

[2] Original code developed by R. Shalloo, DESY