

On Subdivision and Meaning of Graph

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Abstract

In this paper, we formulate subdivision, observe their iteration, and hence reveal what graph really is.

Introduction

Different graphs may represent the same shape, it's because graphs can be divided. This brings us *subdivision*. However, even in classical theory there's no rigorous definition, which is we aim for.

Set Viewpoint

Since it's said on set footing, we should change our view from element to set:

$$G \rightarrow \mathcal{G} := \langle G \rangle \simeq P(G)$$

Hence the link on sets is: $\forall A \in \mathcal{G}, \forall B \in \mathcal{G}$,

$$A \rightrightarrows B : \exists x \in A, \exists y \in B \text{ s.t. } x \rightrightarrows y$$

Now we can formulate subdivision:

Definition. Graphs $f^* : \mathcal{H} \rightarrow \mathcal{G}$ satisfy

- $f^*(\bigcap_{A \in \mathcal{A}} A) = \bigcap_{A \in \mathcal{A}} f^*(A)$
- $\bullet \forall \mathcal{A}, f^*(\bigcup_{A \in \mathcal{A}} A) = \bigcup_{A \in \mathcal{A}} f^*(A)$
- $\bullet \forall A, f^*(A^c) = f^*(A)^c$
- $\bullet \forall A, \forall B, f^*(A) \rightrightarrows f^*(B) \Rightarrow A \rightrightarrows B$

is subdivision. In application, we usually add like faithfulness and (co)dimension preserving:

- $\bullet \forall s \in H, \begin{cases} s \text{ is point} \Rightarrow f^*(s) \text{ is not partible} \\ s \text{ is not point} \Rightarrow f^*(s) \text{ is partible} \end{cases}$
- $\bullet \forall A, \begin{cases} \dim f^*(A) = \dim A \\ \text{codim } f^*(A) = \text{codim } A \end{cases}$

Element Viewpoint

As element view and set view are equivalent, we can rephrase it in elementary saying:

Definition. Graphs $f : G \rightarrow H$ satisfy

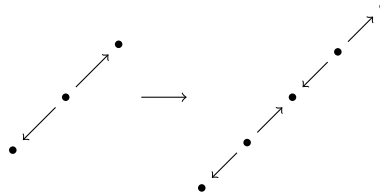
- $x \rightrightarrows y \Rightarrow f(x) \rightrightarrows f(y)$

is subdivision. The corresponding addons are:

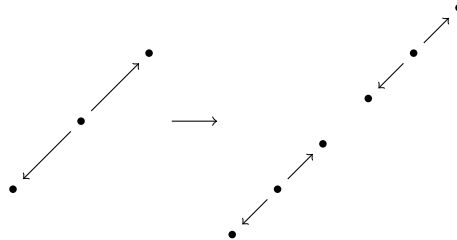
- $\forall y \in H, \exists x \in G \text{ s.t. } y = f(x) \text{ and } \begin{cases} y \text{ is point} \Rightarrow \nexists x' \text{ s.t. } f(x) = f(x') \\ y \text{ is not point} \Rightarrow \exists x' \text{ s.t. } f(x) = f(x') \end{cases}$
- $\forall x, \begin{pmatrix} \text{codim } x \geq \text{codim } f(x) \\ \text{dim } x \leq \text{dim } f(x) \end{pmatrix}$

What is Graph

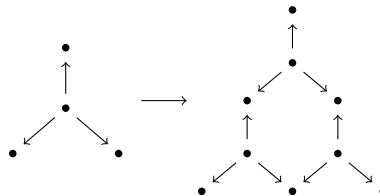
If we repeat the subdivision of \mathbb{R} :



it seems that points are getting dense and close to edge itself. This make us think if “edge” is actually its subdivision, and \mathbb{R} is actually a *fractal*. The other choice is



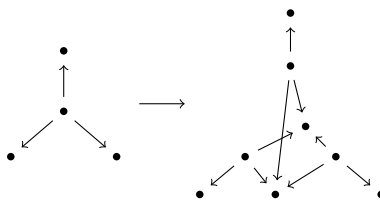
which produces Cantor Set! If we do for 3-edge:



this produces Sierpiński Triangle! It also gives an explanation of n -edge. In summary,

“Vertex, edge, face are not real, only subdivision reveals them.”

We end with an example never seen:



Reference

- [1] Jerry Chen “General Theory of Graph”
- [2] Jerry Chen “Pseudo-Set Theory” (on printing)