

# On Subdivision and Meaning of Graph

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## Abstract

In this paper, we formulate subdivision, observe their iteration, and hence reveal what graph really is.

## Introduction

Different graphs may represent the same shape, it's because graphs can be divided. This brings us *subdivision*. However, even in classical theory there's no rigorous definition, which is we aim for.

## Set Viewpoint

Since it's said on set footing, we should change our view from element to set:

$$G \rightarrow \mathcal{G} := \langle G \rangle \simeq \text{P}(G)$$

Hence the link on sets is:  $\forall A \in \mathcal{G}, \forall B \in \mathcal{G}$ ,

$$A \sqsupseteq B : \exists x \in A, \exists y \in B \text{ s.t. } x \sqsupseteq y$$

Now we can formulate subdivision:

**Definition.** Graphs  $f^* : \mathcal{H} \rightarrow \mathcal{G}$  satisfy

$$\begin{aligned} f^*(\bigcap_{A \in \mathcal{A}} A) &= \bigcap_{A \in \mathcal{A}} f^*(A) \\ \bullet \quad \forall \mathcal{A}, \quad f^*(\bigcup_{A \in \mathcal{A}} A) &= \bigcup_{A \in \mathcal{A}} f^*(A) \end{aligned}$$

- $\forall A, f^*(A^c) = f^*(A)^c$
- $\forall A, \forall B, f^*(A) \sqsupseteq f^*(B) \Rightarrow A \sqsupseteq B$

is subdivision. In application, we usually add like faithfulness and (co)dimension preserving:

- $\forall s \in H, \begin{cases} s \text{ is point} \Rightarrow f^*(s) \text{ is partible} \\ s \text{ is not point} \Rightarrow f^*(s) \text{ is partible} \end{cases}$
- $\forall A, \begin{cases} \dim f^*(A) = \dim A \\ \text{codim } f^*(A) = \text{codim } A \end{cases}$

# Element Viewpoint

As element view and set view are equivalent, we can rephrase it in elementary saying:

**Definition.** Graphs  $f : G \rightarrow H$  satisfy

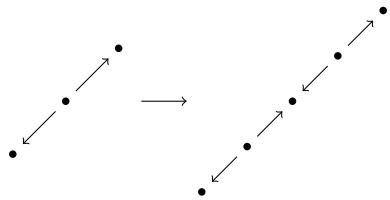
- $x \sqsupseteq y \Rightarrow f(x) \sqsupseteq f(y)$

is subdivision. The corresponding addons are:

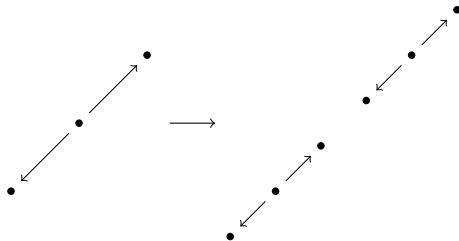
- $\forall y \in H, \exists x \in G \text{ s.t. } y = f(x) \text{ and } \begin{cases} y \text{ is point} \Rightarrow \exists x' \text{ s.t. } f(x) = f(x') \\ y \text{ is not point} \Rightarrow \exists x' \text{ s.t. } f(x) = f(x') \end{cases}$
- $\forall x, \begin{pmatrix} \text{codim } x \geq \text{codim } f(x) \\ \dim x \leq \dim f(x) \end{pmatrix}$

## What is Graph

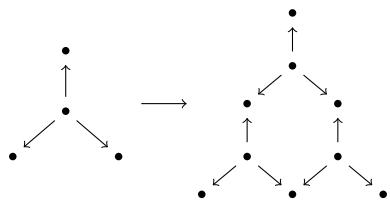
If we repeat the subdivision of  $\mathbb{R}$ :



it seems that points are getting dense and close to edge itself. This make us think if “edge” is actually its subdivision, and  $\mathbb{R}$  is actually a *fractal*. The other choice is



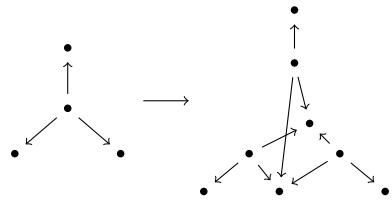
which produces Cantor Set! If we do for 3-edge:



this produces Sierpiński Triangle! It also gives an explanation of  $n$ -edge. In summary,

“Vertex, edge, face are not real, only subdivision reveals them.”

We end with an example never seen:



## Reference

- [1] Jerry Chen “General Theory of Graph”
- [2] Jerry Chen “Pseudo-Set Theory” (on printing)