3. $Y \sim Bin(n, \theta)$

$\theta \sim Beta(\alpha, \beta)$

$$f(y|\theta) = C_y^n \theta^y (1-\theta)^{n-y}$$

$$\pi(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}$$

$$\pi(\theta|y) \propto f(y|\theta) \cdot \pi(\theta)$$

$$\propto \theta^{\alpha+y-1} (1-\theta)^{n-y+\beta-1}$$

$$\therefore \pi(\theta|y) = c\,\theta^{\alpha+y-1} (1-\theta)^{n-y+\beta-1} \sim Beta(\underset{Y=1}{\underline{\underline{\alpha+y}}}, \underset{Y=0}{\underline{\underline{n-y+\beta}}})$$

4. Poisson pdf: $f(y) = \dfrac{e^{-\theta}\theta^y}{y!}$

$$f(\underset{\sim}{y}|\theta) = \prod_{i=1}^{n} \frac{e^{-\theta}\theta^{y_i}}{y_i!} = \frac{e^{-n\theta}\theta^{\sum_{i=1}^{n} y_i}}{\prod_{i=1}^{n} y_i!}$$

$$\pi(\theta) = \frac{\theta^{\alpha-1}}{\beta^\alpha \Gamma(\alpha)} e^{-\frac{\theta}{\beta}}$$

$$\pi(\theta|\underset{\sim}{y}) \propto e^{-n\theta}\theta^{\sum_{i=1}^{n} y_i} \cdot \theta^{\alpha-1} e^{-\frac{\theta}{\beta}}$$

$$\propto \theta^{\sum_{i=1}^{n} y_i + \alpha - 1} e^{-(n+\frac{1}{\beta})\theta}$$

$$\therefore \pi(\theta|\underset{\sim}{y}) \sim \Gamma\left(\sum_{i=1}^{n} y_i + \alpha, n+\frac{1}{\beta}\right)$$

$$l(\theta) = -n\theta + \sum_{i=1}^{n} y_i \ln\theta - \sum_{i=1}^{n} \ln y_i$$

$$\frac{\partial}{\partial\theta} l(\theta) = -n + \frac{\sum_{i=1}^{n} y_i}{\theta} = 0$$

$$\hat{\theta} = \frac{\sum_{i=1}^{n} y_i}{n} = \bar{Y}$$