

$$3. D = \{(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)\}$$

$$\Phi = \begin{bmatrix} \varphi_1(x_1) & \varphi_2(x_2) & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

$$y = Xw + \varepsilon, \text{ 其中 } \varepsilon \sim N(0, \sigma^2 I), \text{ 假设 } w \sim N(0, b^{-1} I)$$

$$y | X, w \sim N(Xw, \sigma^2 I)$$

$$p(y | X, w) = \frac{1}{(2\pi\sigma^2)^{n/2}} \exp \left\{ -\frac{1}{2\sigma^2} (y - Xw)^T (y - Xw) \right\}$$

$$p(w) = \frac{1}{(2\pi/b)^{n/2}} \exp \left\{ -\frac{b}{2} w^T w \right\}$$

$$\hat{a} = \frac{1}{\sigma^2}$$

$$p(w | X, y) \propto p(y | X, w) p(w)$$

$$\propto e^{-\frac{a}{2} (y - Xw)^T (y - Xw) - \frac{b}{2} w^T w}$$

$$\propto e^{a(y - Xw)^T (y - Xw) + b w^T w}$$

$$\text{其中, } a(y - Xw)^T (y - Xw) + b w^T w$$

$$= a(y^T y - 2y^T Xw + w^T X^T X w) + b w^T w$$

$$\hat{A} = aX^T X + bI$$

$$= a y^T y - 2a y^T X w + w^T (a X^T X + b I) w$$

$$-2w^T \Lambda \mu = -2w^T a X^T y$$

$$\Lambda \mu = a X^T y$$

$$\mu = a^{-1} X^T y$$

$\frac{1}{b} \propto p(\omega)$  prior of  $\omega$  mean =  $m$ , Cov matrix of  $\omega$  inverse =  $S$

$$p(\omega | X, y) \propto p(y | X, \omega) p(\omega)$$

$$\propto e^{-\frac{a}{2} \sum_i (y_i - x_i^T \omega)^2 - \frac{1}{2} (\omega - m)^T S (\omega - m)}$$

$$= -\frac{a}{2} \sum_i (y_i - x_i^T \omega)^2 - \frac{1}{2} (\omega - m)^T S (\omega - m)$$

$$= -\frac{a}{2} \|X\omega - y\|^2 - \frac{1}{2} (\omega - m)^T S (\omega - m)$$

$$= -\frac{a}{2} \left( \|X\omega - y\|^2 + \frac{1}{a} (\omega - m)^T S (\omega - m) \right)$$

$$= a \|X\omega - y\|^2 + (\omega - m)^T S (\omega - m)$$

$$= a (X\omega - y)^T (X\omega - y) + (\omega - m)^T S (\omega - m)$$

$$= a (\omega^T X^T X \omega - 2\omega^T X^T y + y^T y) + (\omega^T S \omega - 2\omega^T S m + m^T S m)$$

$$= \omega^T (a X^T X + S) \omega - 2\omega^T (a X^T y + S m) + a y^T y + m^T S m$$

$$\text{Compare with } (\omega - \mu)^T \Lambda (\omega - \mu) = \omega^T \Lambda \omega - 2\omega^T \Lambda \mu + \mu^T \mu$$

$$\Lambda = a X^T X + S$$

$$\mu = \Lambda^{-1} (a X^T y + S m)$$