3.

2枚硬幣 C_0, C_1

3組試驗 $\{HHH, HHT, TTT\}$

$$\text{Initial value} \begin{cases} k = 0.5 & , P(C_0) \\ P_0 = 0.6 & , P(H|C_0) \\ P_1 = 0.1 & , P(H|C_1) \end{cases}$$

E-step

$P(HHH|C_0) = P_0^3 = 0.6^3 = 0.216$

$P(HHH|C_1) = P_1^3 = 0.1^3 = 0.001$

$P(HHH, C_0) = 0.216 \times 0.5 = 0.108$

$P(HHH, C_1) = 0.001 \times 0.5 = 0.0005$

出現HHH時，來自 C_0 的機率 $= \dfrac{P(HHH, C_0)}{P(HHH, C_0) + P(HHH, C_1)} = \dfrac{0.108}{0.1085} = 0.9954$

〃 C_1 $= 1 - 0.9954 = 0.1046$

$P(HHT|C_0) = P_0^2(1-P_0) = 0.6^2 \times 0.4 = 0.144$

$P(HHT|C_1) = P_1^2(1-P_1) = 0.1^2 \times 0.9 = 0.009$

出現 HHT 時，來自 C_0 的機率 $= \dfrac{0.144 \times 0.5}{0.144 \times 0.5 + 0.009 \times 0.5} = 0.9411$

〃 C_1 $= 0.0589$

$P(TTT|C_0) = (1-P_0)^3 = 0.4^3 = 0.064$

$P(TTT|C_1) = 0.9^3 = 0.729$

出現TTT時，來自 C_0 的機率 $= \dfrac{0.064\times0.5}{0.064\times0.5+0.729\times0.5} = 0.0805$

" C_1 $= 0.9195$

M-Step:

$$k^{(1)} = \frac{0.9954+0.9411+0.0805}{3} = 0.6723$$

$$p_0^{(1)} = \frac{3\times0.9954+2\times0.9411+0.0805}{3(0.9954+0.9411+0.0805)} = 0.881$$

$$p_1^{(1)} = \frac{3\times0.0046+2\times0.0589+0.9196}{3(0.9954+0.0589+0.9195)} = 0.0435$$