SERIE TRIGONOMÉTRICA DE *FOURIER*:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(n\omega_0 t) + b_n \operatorname{sen}(n\omega_0 t)]$$

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt$$

$$b_n = \frac{2}{T}\int_0^T f(t)\operatorname{sen}(n\omega_0 t)dt$$

FORMULAS ÚTILES:

$$\operatorname{sen}(\pi n) = 0; \quad n \in \mathbb{N}$$

$$\cos(\pi n) = (-1)^n; \quad n \in \mathbb{N}$$

$$\operatorname{sen}(2\pi n) = 0; \quad n \in \mathbb{N}$$

$$\cos(2\pi n) = 1; \quad n \in \mathbb{N}$$

$$\int e^{at}dt = \frac{1}{a}e^{at}$$

$$\int te^{at}dt = \frac{t}{a}e^{at} - \frac{1}{a^2}e^{at}$$

$$\int \operatorname{sen}(at)dt = -\frac{\cos(at)}{a}$$

$$\int \cos(at)dt = \frac{\operatorname{sen}(at)}{a}$$

$$\int t\operatorname{sen}(at)dt = -\frac{t}{a}\cos(at) + \frac{1}{a^2}\operatorname{sen}(at)$$

$$\int t\cos(at)dt = \frac{t}{a}\operatorname{sen}(at) + \frac{1}{a^2}\cos(at)$$

SIMETRÍAS DE ONDA:

	a_0	a_n	b_n
PAR	$\frac{4}{T}\int_0^{T/2} f(t)dt$	$\frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)dt$	0
IMPAR	0	0	$\frac{4}{T}\int_0^{T/2} f(t)\operatorname{sen}(n\omega_0 t)dt$
S.M.O.	0	$\begin{cases} \text{p:} & a_n = 0 \\ \text{i:} & a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)dt \end{cases}$	$\begin{cases} \text{p:} & b_n = 0 \\ \text{i:} & b_n = \frac{4}{T}\int_0^{T/2} f(t)\operatorname{sen}(n\omega_0 t)dt \end{cases}$
S.C.O. PAR	0	$\begin{cases} \text{p:} & a_n = 0 \\ \text{i:} & a_n = \frac{8}{T}\int_0^{T/4} f(t)\cos(n\omega_0 t)dt \end{cases}$	0
S.C.O. IMPAR	0	0	$\begin{cases} \text{p:} & b_n = 0 \\ \text{i:} & b_n = \frac{8}{T}\int_0^{T/4} f(t)\operatorname{sen}(n\omega_0 t)dt \end{cases}$

SERIE COMPLEJA DE *FOURIER*:

$$e^{j\theta} = \cos(\theta) + j\operatorname{sen}(\theta)$$

$$e^{-j\theta} = \cos(\theta) - j\,\mathrm{sen}(\theta)$$

$$e^{\pm j2\pi n} = 1$$

$$e^{\pm j\pi} = -1$$

$$e^{\pm j\pi n} = \cos(\pi n)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\mathrm{sen}(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_0 t}dt$$

$$c_0 = \frac{1}{T}\int_0^T f(t)dt$$

$$a_n = 2\,\mathbb{R}e\{c_n\}$$

$$b_n = -2\,\mathbb{I}m\{c_n\}$$

FUNCIÓN IMPULSO:

$$\int_{-\infty}^{\infty} \delta(t-t_0)dt = 1$$

$$\int_{-\infty}^{\infty} \phi(t)\delta(t-t_0)dt = \phi(t_0)$$

$$\int_{-\infty}^{\infty} \phi(t)\delta^{(n)}(t-t_0)dt = (-1)^n\phi^{(n)}(t_0)$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$t^n\delta(t) = 0; n \in \mathbb{N}$$

$$u'(t-t_0) = \delta(t-t_0)$$

$$\phi(t)\delta(t) = \phi(0)\delta(t)$$

$$\delta(-t) = \delta(t)$$

SERIE DE *FOURIER* POR DIFERENCIACIÓN:

$$c_n = c_n' + c_n'' + \cdots + c_n^{(k)}$$

$$\gamma_n' = \frac{1}{T}\int_0^T f'(t)e^{-jn\omega_0 t}dt$$

$$c_n' = \frac{\gamma_n'}{jn\omega_0}$$

$$\gamma_n'' = \frac{1}{T}\int_0^T f''(t)e^{-jn\omega_0 t}dt$$

$$c_n'' = \frac{\gamma_n''}{(jn\omega_0)^2}$$

$$\gamma_n^{(n)} = \frac{1}{T}\int_0^T f^{(k)}(t)e^{-jn\omega_0 t}dt$$

$$c_n^{(k)} = \frac{\gamma_n^{(k)}}{(jn\omega_0)^k}$$

TEOREMA DE LA MULTIPLICACIÓN:

$$\frac{1}{T}\int_0^T f_1(t)f_2(t)dt = \sum_{n=-\infty}^{\infty} c_1(n)c_2(-n) = \sum_{n=-\infty}^{\infty} c_1(-n)c_2(n)$$

TEOREMA DE *PARSEVAL*:

$$\frac{1}{T}\int_0^T f^2(t)dt = c_0^2 + \frac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)$$

TRANSFORMADA DE *FOURIER*:

$$\mathcal{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$F(\omega) = R(\omega) + jX(\omega)$$

$$|F(\omega)| = \sqrt{R^2(\omega) + X^2(\omega)}$$

$$\Theta(\omega) = \arctan\left(\frac{X(\omega)}{R(\omega)}\right)$$

PROPIEDADES DE LA TRANSFORMADA DE *FOURIER*:

1	Linealidad	$\mathcal{F}\{a_1f_1(t) + a_2f_2(t)\} = a_1F_1(\omega) + a_2F_2(\omega)$
2	Cambio de escala	$\mathcal{F}\{f(at)\} = \frac{1}{\lvert a\rvert}F\left(\frac{\omega}{a}\right)$
3	Desplazamiento en ω	$\mathcal{F}\{f(t)e^{jat}\} = F(\omega - a)$
4	Desplazamiento en t	$\mathcal{F}\{f(t-a)\} = F(\omega)e^{-ja\omega}$
5	Simetría	$\mathcal{F}\{F(t)\} = 2\pi f(-\omega)$
6	Multiplicación	$\mathcal{F}\{t^n f(t)\} = j^n \frac{d^{(n)}F(\omega)}{d\omega^n}; \quad n \in \mathbb{N}$
7	Derivada	$\mathcal{F}\{f^{(n)}(t)\} = (j\omega)^n F(\omega); \quad n \in \mathbb{N}$

FUNCIÓN SIGNO:

$$\operatorname{sgn}'(t) = 2\delta(t)$$

$$|t|' = \operatorname{sgn}(t)$$

$$\operatorname{sgn}^2(t) = 1$$

TABLA DE TRANSFORMADAS DE *FOURIER*:

	$f(t)$	$F(\omega) = \mathcal{F}\{f(t)\}$
1	$u(t+a) - u(t-a)$	$\dfrac{2\,\mathrm{sen}(a\omega)}{\omega}$
2	$\dfrac{\mathrm{sen}(at)}{t}$	$\pi[u(\omega+a) - u(\omega-a)]$
3	$e^{-at}u(t) \quad a>0$	$\dfrac{1}{a+j\omega}$
4	$e^{at}u(-t) \quad a>0$	$\dfrac{1}{a-j\omega}$
5	$e^{-a\|t\|} \quad a>0$	$\dfrac{2a}{a^2+\omega^2}$
6	$\dfrac{1}{t^2+a^2}$	$\dfrac{\pi}{a}e^{-a\|\omega\|}$
7	$\delta(t-a)$	$e^{-ja\omega}$
8	e^{jat}	$2\pi\delta(\omega-a)$
9	k	$2\pi k\delta(\omega)$
10	$\mathrm{sen}(at)$	$j\omega[\delta(\omega+a) - \delta(\omega-a)]$
11	$\cos(at)$	$\pi[\delta(\omega+a) + \delta(\omega-a)]$
12	$t^n e^{-at}u(t)$	$\dfrac{n!}{(a+j\omega)^{n+1}} \quad n \in \mathbb{N}$
13	$u(t)$	$\dfrac{1}{j\omega} + \pi\delta(\omega)$
14	$\mathrm{sgn}(t)$	$\dfrac{2}{j\omega}$
15	$\|t\|$	$-\dfrac{2}{\omega^2}$
16	$\dfrac{1}{t}$	$-j\pi\,\mathrm{sgn}(\omega)$
17	$\dfrac{1}{t^n}$	$\dfrac{j^n\pi\omega^{n-1}\,\mathrm{sgn}(\omega)}{(-1)^n(n-1)!}$

FUNCIONES TRIGONOMÉTRICAS DE ARCO DOBLE:

$$\mathrm{sen}(2x) = 2\,\mathrm{sen}(x)\cos(x)$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\cos(2x) = 1 - 2\,\mathrm{sen}^2(x)$$

FUNCIONES TRIGONOMÉTRICAS DE ARCO TRIPLE:

$$\mathrm{sen}(3x) = 3\,\mathrm{sen}(x) - 4\,\mathrm{sen}^3(x)$$

$$\cos(3x) = 4\cos^3(a) - 3\cos(x)$$

TRANSFORMADA INVERSA DE *FOURIER*:

$$\mathcal{F}\{f(t)\} = F(\omega) \rightarrow \mathcal{F}^{-1}\{F(\omega)\} = f(t)$$

TABLA DE TRANSFORMADAS INVERSAS DE *FOURIER*:

	$F(\omega)$	$f(t) = \mathcal{F}^{-1}\{F(\omega)\}$
1	$\dfrac{1}{a+j\omega}$	$e^{-at}u(t) \quad a>0$
2	$\dfrac{1}{a-j\omega}$	$e^{at}u(-t) \quad a>0$
3	$\dfrac{2a}{a^2+\omega^2}$	$e^{-a\lvert t\rvert} \quad a>0$
4	$\frac{1}{\omega}\operatorname{sen}(a\omega)$	$\frac{1}{2}[u(t+a)-u(t-a)]$
5	k	$k\delta(t)$
6	$\dfrac{1}{\omega}$	$\frac{1}{2}j\operatorname{sgn}(t)$

PROPIEDADES DE LA TRANSFORMADA INVERSA DE *FOURIER*:

1	Linealidad	$\mathcal{F}^{-1}\{a_1F_1(\omega)+a_2F_2(\omega)\} = a_1f_1(t)+a_2f_2(t)$
2	Desplazamiento en ω	$\mathcal{F}^{-1}\{F(\omega-a)\} = f(t)e^{jat}$
3	Desplazamiento en t	$\mathcal{F}^{-1}\{F(\omega)e^{-ja\omega}\} = f(t-a)$

CONVOLUCIÓN:

$$f(t) = f_1(t)*f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)d\tau$$

PROPIEDADES DE LA CONVOLUCIÓN:

1	Conmutatividad	$f_1(t)*f_2(t) = f_2(t)*f_1(t)$
2	Asociatividad	$f_1(t)*[f_2(t)*f_3(t)] = [f_1(t)*f_2(t)]*f_3(t)$
3	Distributividad	$f_1(t)*[f_2(t)+f_3(t)] = f_1(t)*f_2(t)+f_1(t)*f_3(t)$
4	Función impulso	$f_1(t)*\delta(t-t_0) = f_1(t-t_0)$
5	Función escalón unitario	$[f_1(t)u(t)]*[f_2(t)u(t)] = \int_0^t f_1(\tau)f_2(t-\tau)d\tau$

TRANSFORMADA DE *FOURIER* Y CONVOLUCIÓN:

$$\mathcal{F}\{f_1(t)*f_2(t)\} = F_1(\omega)F_2(\omega)$$

$$\mathcal{F}^{-1}\{F_1(\omega)F_2(\omega)\} = f_1(t)*f_2(t)$$

ECUACIONES DIFERENCIALES ORDINARIAS:

$$\mathcal{F}\{f'(t)\} = j\omega F(\omega)$$

$$\mathcal{F}\{f''(t)\} = (j\omega)^2F(\omega)$$

$$\mathcal{F}\{f^{(n)}(t)\} = (j\omega)^nF(\omega)$$

TRANSFORMADA DE *LAPLACE*:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} f(t)e^{-st}dt$$

PROPIEDADES DE LA TRANSFORMADA DE *LAPLACE*:

1	Linealidad	$\mathcal{L}\{a_1 f_1(t) + a_2 f_2(t)\} = a_1 F_1(s) + a_2 F_2(s)$
2	Desplazamiento en s	$\mathcal{L}\{f(t)e^{at}\} = F(s-a)$
3	Desplazamiento en t	$\mathcal{L}\{f(t-a)u(t-a)\} = F(s)e^{-as}$
		$\mathcal{L}\{f(t)u(t-a)\} = e^{-as}\mathcal{L}\{f(t+a)\}$
4	Multiplicación	$\mathcal{L}\{tf(t)\} = -\frac{dF(s)}{ds}$
		$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^{(n)}F(s)}{ds^n}$
5	División	$\mathcal{L}\left\{\frac{1}{t}f(t)\right\} = \int_s^\infty F(s)ds$
6	Derivadas	$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$
		$\mathcal{L}\{f''(t)\} = s^2F(s) - f(0)s - f'(0)$
		$\mathcal{L}\{f'''(t)\} = s^3F(s) - f(0)s^2 - f'(0)s - f''(0)$
7	Integrales	$\mathcal{L}\left\{\int_0^t f(t)dt\right\} = \frac{1}{s}F(s)$

TABLA DE TRANSFORMADAS DE *LAPLACE*:

	$f(t)$	$F(s) = \mathcal{L}\{f(t)\}$
1	k	$\dfrac{k}{s}$
2	t^n	$\dfrac{\Gamma(n+1)}{s^{n+1}}$
		$\dfrac{n!}{s^{n+1}}; \quad n \in \mathbb{N}$
3	e^{at}	$\dfrac{1}{s-a}$
4	$\operatorname{sen}(at)$	$\dfrac{a}{s^2+a^2}$
5	$\cos(at)$	$\dfrac{s}{s^2+a^2}$
6	$\operatorname{senh}(at)$	$\dfrac{a}{s^2-a^2}$
7	$\cosh(at)$	$\dfrac{s}{s^2-a^2}$
8	$u(t-a)$	$\dfrac{1}{s}e^{-as}$
9	$\delta(t-a)$	e^{-at}
10	$\dfrac{1}{t}\operatorname{sen}(at)$	$\arctan\left(\dfrac{a}{s}\right)$

FUNCIÓN *GAMMA*:

$$\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$$

PROPIEDADES DE LA FUNCIÓN *GAMMA*:

1	Propiedad 1	$\Gamma(n) = (n-1)\Gamma(n-1)$
		$\Gamma(n) = (n-1)(n-2)(n-3)\ldots(n-r)\Gamma(n-r)$
2	Propiedad 2	$\Gamma(n) = \frac{\Gamma(n+1)}{n}$
3	Propiedad 3	$\Gamma(n) = (n-1)!$
		$0! = 1$
4	Propiedad 4	$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
		$\Gamma\left(-\frac{1}{2}\right) = -2\sqrt{\pi}$

TEOREMA DEL VALOR INICIAL Y FINAL:

$$f(0) = \lim_{t\to 0} f(t) = \lim_{t\to\infty} sF(s)$$

$$f(\infty) = \lim_{t\to\infty} f(t) = \lim_{t\to 0} sF(s)$$

TRANSFORMADA INVERSA DE *LAPLACE*:

$$\mathcal{L}^{-1}\{F(s)\} = f(t); \quad t > 0$$

TABLA DE TRANSFORMADAS INVERSAS DE *LAPLACE*:

	$F(s)$	$f(t) = \mathcal{L}^{-1}\{F(s)\}; t > 0$
1	$\dfrac{k}{s}$	k
2	$\dfrac{1}{s^n}$	$\dfrac{t^{n-1}}{\Gamma(n)}$
		$\dfrac{t^{n-1}}{(n-1)!}; \quad n \in \mathbb{N}$
3	$\dfrac{1}{s-a}$	e^{at}
4	$\dfrac{1}{s^2+a^2}$	$\dfrac{1}{a}\operatorname{sen}(at)$
5	$\dfrac{s}{s^2+a^2}$	$\cos(at)$
6	$\dfrac{1}{s^2-a^2}$	$\dfrac{1}{a}\operatorname{senh}(at)$
7	$\dfrac{s}{s^2-a^2}$	$\cosh(at)$
8	$\arctan\left(\dfrac{a}{s}\right)$	$\dfrac{1}{t}\operatorname{sen}(at)$
9	k	$k\delta(t)$
10	e^{-as}	$\delta(t-a)$

PROPIEDADES DE LA TRANSFORMADA INVERSA DE *LAPLACE*:

1	Linealidad	$\mathcal{L}^{-1}\{a_1F_1(s)+a_2F_2(s)\}=a_1f_1(t)+a_2f_2(t)$
2	Desplazamiento en s	$\mathcal{L}^{-1}\{F(s-a)\}=f(t)e^{at}$
3	Desplazamiento en t	$\mathcal{L}^{-1}\{F(s)e^{-as}\}=f(t-a)u(t-a)$
4	División por s	$\mathcal{L}^{-1}\left\{\frac{F(s)}{s}\right\}=\int_0^t f(t)dt$
		$\mathcal{L}^{-1}\left\{\frac{F(s)}{s^n}\right\}=\int_0^t\int_0^t\cdots\int_0^t f(t)dt\dots dtdt$
5	Derivada	$\mathcal{L}^{-1}\{F'(s)\}=-tf(t)$
		$\mathcal{L}^{-1}\{F^{(n)}(s)\}=(-1)^n t^n f(t)$

DESCOMPOSICIÓN EN FRACCIONES PARCIALES:

$$\frac{P(s)}{(s-a_1)(s-a_2)\dots(s-a_n)}=\frac{A_1}{s-a_1}+\frac{A_2}{s-a_2}+\cdots+\frac{A_n}{s-a_n}$$

$$\frac{P(s)}{(s-a)^m(s-b)^n}=\frac{A_1}{s-a}+\frac{A_2}{(s-a)^2}+\cdots+\frac{A_m}{(s-a)^m}+\frac{B_1}{(s-b)}+\frac{B_2}{(s-a)^2}+\cdots+\frac{B_n}{(s-b)^n}$$

$$\frac{P(s)}{(s^2+a_1s+b_1)(s^2+a_2s+b_2)}=\frac{A_1s+B_1}{s^2+a_1s+b_1}+\frac{A_2s+B_2}{s^2+a_2s+b_2}$$

TRANSFORMADA DE *LAPLACE* Y CONVOLUCIÓN:

$$\mathcal{L}\{f_1(t)*f_2(t)\}=F_1(s)F_2(s)$$

$$\mathcal{L}^{-1}\{F_1(s)F_2(s)\}=f_1(t)*f_2(t)$$

APLICACIONES DE LA TRANSFORMADA DE *LAPLACE*:

$$\mathcal{L}\{f'(t)\}=sF(s)-f(0)$$

$$\mathcal{L}\{f''(t)\}=s^2F(s)-f(0)s-f'(0)$$

$$\mathcal{L}\{f'''(t)\}=s^3F(s)-f(0)s^2-f'(0)s-f''(0)$$