PRACTICA Nro. 6 – TRANSFORMADAS INTEGRALES
TRANSFORMADAS DE LAPLACE

1.- Calcular la transformada de las funciones dadas:

a) $\mathcal{L}\left\{\left(3t^3-5t\right)^2\right\}$ b) $\mathcal{L}\left\{\left(t^3-2t\right)^3\right\}$ c) $\mathcal{L}\left\{\left(e^{3t}+2e^{-t}\right)^2\right\}$ d) $\mathcal{L}\left\{\left(1-e^{-2t}\right)^3\right\}$

e) $\mathcal{L}\left\{3\cos(5t)-6sen(3t)\right\}$ f) $\mathcal{L}\left\{4sen^2\left(\frac{\pi}{3}t\right)\right\}$ g) $\mathcal{L}\left\{\cos^4(5t)\right\}$ h) $\mathcal{L}\left\{sen^3(2t)\right\}$

i) $\mathcal{L}\left\{e^{3t}\cos\sqrt{10}t\right\}$ j) $\mathcal{L}\left\{\left(e^{-8t}-2e^{3t}\right)\left(3t^2-2t^5\right)^2\right\}$ k) $\mathcal{L}\left\{\left(t^2-te^{-4t}\right)^3\right\}$ l) $\mathcal{L}\left\{\left(e^{-4t}+3e^{5t}\right)^2\cos^2(2t)\right\}$

RESPUESTAS

a) $F_{(s)}=\frac{6480}{s^7}-\frac{720}{s^5}+\frac{50}{s^3}$; b) $F(s)=\frac{362880}{s^{10}}-\frac{30240}{s^8}+\frac{1440}{s^6}-\frac{48}{s^4}$; c) $F_{(s)}=\frac{6s^2-24s-40}{(s-6)(s-2)(s+2)}$

d) $F(s)=\frac{1}{s}-\frac{3}{s+2}+\frac{3}{s+4}-\frac{1}{s+6}$; e) $F_{(s)}=\frac{3s}{s^2+25}-\frac{18}{s^2+9}$

f) $F(s)=\frac{2}{s}-\frac{18s}{9s^2+4\pi^2}$; g) $F(s)=\frac{s}{8s^2+3200}+\frac{s}{2s^2+200}+\frac{3}{8s}$; h) $F(s)=-\frac{3}{2\left(s^2+36\right)}+\frac{3}{2\left(s^2+4\right)}$;

i) $F(s)=\frac{s-3}{10+(s-3)^2}$; j) $F(s)=\frac{216}{(s+8)^5}-\frac{60480}{(s+8)^8}+\frac{1415200}{(s+8)^{11}}-\frac{432}{(s+3)^5}+\frac{120460}{(s-3)^8}-\frac{29030400}{(s-3)^{11}}$

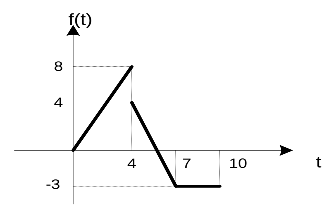
k) $F(s)=\frac{720}{s^7}-\frac{360}{(s+4)^6}+\frac{72}{(s+8)^5}-\frac{6}{(s+12)^4}$;

l) $F(s)=\frac{1}{2s+16}+\frac{3}{s-1}+\frac{9}{2s-20}+\frac{s+8}{2(s+8)^2+32}+\frac{3s-3}{(s-1)^2+16}+\frac{9s-90}{2(s-10)^2+32}$

2.- Expresar en términos u(t) y calcule sus transformadas aplicando desplazamiento en "t":

a) $f_{(t)}=\begin{cases}3t.......0<t<2\\4..........t>2\end{cases}$ b) $f_{(t)}=\begin{cases}-t+1.......0<t<4\\t^2..........t>4\end{cases}$ c) $f_{(t)}=\begin{cases}3t+1 & 0<t<1\\4-t^2 & 1<t<2\end{cases}$

d) $f_{(t)}=\begin{cases}e^{-3t}+4 & 0<t<2\\t^2+2 & 2<t<4\end{cases}$ e) $f_{(t)}=\begin{cases}sen(\pi t) & 0<t<0.5\\1 & 0.5<t<1\\2-t & 1<t<2\end{cases}$ f)



RESPUESTAS

a) $F(s)=\frac{3}{s^2}\left(1-e^{-2s}\right)-\frac{2}{s}e^{-2s}$ b) $F(s)=-\frac{1}{s^2}+\frac{1}{s}+e^{-4s}\left(\frac{2}{s^3}+\frac{9}{s^2}+\frac{19}{s}\right)$

c) $F(s)=\frac{3}{s^2}+\frac{1}{s}+e^{-s}\left(-\frac{2}{s^3}-\frac{5}{s^2}-\frac{1}{s}\right)+e^{-2s}\left(\frac{2}{s^3}+\frac{4}{s^2}\right)$;

d) $F(s)=\frac{1}{s+3}+\frac{4}{s}+e^{-2s}\left(-\frac{e^{-6}}{s+3}+\frac{2}{s^3}+\frac{4}{s^2}+\frac{2}{s}\right)+e^{-4s}\left(-\frac{2}{s^3}-\frac{8}{s^2}-\frac{18}{s}\right)$

e) $F(s)=\frac{\pi}{s^2+\pi^2}+e^{-0.5s}\left(-\frac{s}{s^2+\pi^2}+\frac{1}{s}\right)-\frac{e^{-s}}{s^2}+\frac{e^{-2s}}{s^2}$

f) $F(s)=\dfrac{2}{s^2}+e^{-4s}\left(-\dfrac{13}{3s^2}-\dfrac{4}{s}\right)+\dfrac{7e^{-7s}}{3s^2}+\dfrac{3e^{-10s}}{s}$

3.- Calcular utilizando la transformada de la derivada:

a) Si $\mathcal{L}\{f'_{(t)}\}=\dfrac{2s^3+3s^2}{s^2-4}$, hallar $\mathcal{L}\{f_{(t)}\}$ sabiendo que $f_{(0)}=-3$

b) Si $\mathcal{L}\{sen\sqrt{t}\}=\dfrac{\sqrt{\pi}}{2s^{3/2}}e^{-\frac{1}{4s}}$, hallar $\mathcal{L}\left\{\dfrac{\cos\sqrt{t}}{\sqrt{t}}\right\}$

c) Si $\mathcal{L}\{f'''_{(t)}\}=\dfrac{2}{s^2-4}$; $f_{(0)}=f'_{(0)}=f''_{(0)}=-2$, hallar: $\mathcal{L}\{f_{(t)}\}$

RESPUESTAS:

a) $F_{(s)}=\dfrac{2s^3+12}{s(s^s-4)}$; b) $F_{(s)}=\sqrt{\dfrac{\pi}{s}}e^{-\frac{1}{4s}}$; c) $F_{(s)}=\dfrac{-2s^4-2s^3+6s^2+8s+10}{s^3(s^2-4)}$

4.- Calcular las transformadas utilizando las propiedades correspondientes:

a) $\mathcal{L}\{t^2e^{-2t}u(t-2)\}$ b) $\mathcal{L}\{t\cosh t\}$ c) $\mathcal{L}\{t^2e^{-\frac{t}{3}}sen(4t)\}$

d) $\mathcal{L}\{te^{-2t}sen(4t)u(t-\frac{\pi}{3})\}$ e) $\mathcal{L}\{t^2e^{-4t}\cosh t\}$ f) $\mathcal{L}\left\{\int_0^t(t^3e^{-t}-2)^2dt\right\}$

g) $\mathcal{L}\left\{\int_0^t t^3\cos t\,dt\right\}$ h) $\mathcal{L}\left\{\int_0^t(e^{3t}+te^{-5t})^3dt\right\}$ i) $\mathcal{L}\left\{\dfrac{e^{-2t}-e^{-4t}}{t}\right\}$

j) $\mathcal{L}\left\{\int_0^t\dfrac{1-e^{-2t}}{t}dt\right\}$ k) $\mathcal{L}\left\{\int_0^t t^2e^{-3t}senh(2t)dt\right\}$ l) $\mathcal{L}\left\{\dfrac{sen^2(\frac{2}{3}t)}{t}\right\}$

m) $\mathcal{L}\left\{\int_0^t\dfrac{e^{-3t}sen^2(5t)}{t}dt\right\}$ n) $\mathcal{L}\left\{t\int_0^t\dfrac{e^{-2t}sen(4t)}{t}dt\right\}$ o) $\mathcal{L}\left\{te^{-4t}\int_0^t\dfrac{e^{-t}-e^{-2t}}{t}dt\right\}$

p) $\mathcal{L}\left\{\int_0^t\dfrac{e^{-t}\cos(3t)-e^{-3t}\cos(5t)}{t}dt\right\}$ q) $\mathcal{L}\left\{\left(\sqrt{t^3}e^{-2t}+2t\right)^3\right\}$ r) $\mathcal{L}\left\{\int_0^t\left(t^{-5/2}+t^{-2}e^{3t}\right)\left(e^{-6t}-t^{-3/2}\right)dt\right\}$

RESPUESTAS

a) $F(s)=e^{-2(s+2)}\left(\dfrac{2}{(s+2)^3}+\dfrac{4}{(s+2)^2}+\dfrac{4}{(s+2)}\right)$; b) $F(s)=\dfrac{s^2+25}{(s^2-25)^2}$; c) $F(s)=\dfrac{24(s+1/3)^2-128}{[(s+1/3)^2+16]^3}$;

d) $F(s)=e^{-\frac{\pi}{3}(s+2)}\left[\dfrac{\pi}{3}\left(-\dfrac{2}{(s+2)^2+16}-\dfrac{\sqrt{3}(s+2)}{2(s+2)^2+32}\right)-\dfrac{4(s+2)}{((s+2)^2+16)^2}+\dfrac{-2\sqrt{3}(s+2)^2+32\sqrt{3}}{(2(s+2)^2+32)^2}\right]$

e) $F(s)=\dfrac{1}{(s+3)^3}+\dfrac{1}{(s+5)^3}$; f) $F(s)=\dfrac{720}{s(s+2)^7}-\dfrac{24}{s(s+1)^4}+\dfrac{4}{s^2}$; g) $F_{(s)}=\dfrac{6(s^4-6s^2+1)}{s(s^2+1)^4}$

h) $F_{(s)}=\dfrac{1}{s^2-9s}+\dfrac{3}{s(s-1)^2}+\dfrac{6}{s(s+7)^3}+\dfrac{6}{s(s+15)^4}$; i) $F_{(s)}=\ln\left(\dfrac{s+4}{s+2}\right)$; j) $F_{(s)}=\dfrac{1}{s}\ln\left(\dfrac{s+2}{s}\right)$

k) $F_{(s)}=\dfrac{12(s+3)^2+16}{s[s^2+6s+5]^3}$; l) $F_{(s)}=\dfrac{1}{2}\ln\left(\dfrac{\sqrt{s^2+16/9}}{s}\right)$; m) $F_{(s)}=\dfrac{1}{2s}\ln\left(\dfrac{\sqrt{s^2+6s+109}}{s+3}\right)$;

n) $F_{(s)} = \frac{1}{s^2}\arctan\left(\frac{4}{s+2}\right) + \frac{4}{s\left((s+2)^2+16\right)}$; o) $F_{(s)} = \frac{1}{(s+4)^2}\ln\left(\frac{s+6}{s+5}\right) - \frac{1}{(s+4)(s+6)} + \frac{1}{(s+4)(s+5)}$

p) $F_{(s)} = \frac{1}{2s}\ln\left(\frac{(s+3)^2+25}{(s+1)^2+9}\right)$; q) $F_{(s)} = \frac{945\sqrt{\pi}}{32(s+6)^{\frac{11}{2}}} + \frac{144}{(s+4)^5} + \frac{315\sqrt{\pi}}{4(s+2)^{\frac{9}{2}}} + \frac{48}{s^4}$;

r) $F_{(s)} = \frac{1}{s}\left(\frac{4\sqrt{\pi}}{3(s+6)^{-\frac{3}{2}}}\right) - \frac{\Gamma_{(-3)}}{s^{-3}} + \frac{\Gamma_{(-1)}}{(s+3)^{-1}} + \frac{8\sqrt{\pi}}{15(s-3)^{-\frac{5}{2}}}$

5.- Evaluar las integrales aplicando la transformada de Laplace:

a) $\int_0^\infty e^{-3t}\,\frac{sen^2(2t)}{t}dt$ b) $\int_0^\infty t^2 e^{-4t} sen(3t)dt$ c) $\int_0^\infty \frac{e^{-4t}-e^{-3t}}{t}dt$

d) $\int_0^\infty \frac{\cos(3t)-\cos(5t)}{t}dt$ e) $\int_o^\infty t^{-5/2}e^{-3t}dt$ f) $\int_0^\infty \frac{e^{-t}\cos 3t - e^{-2t}\cos 5t}{t}dt$

g) $\int_0^\infty \frac{e^{-3\sqrt{3}t}sen3t}{t}dt$ h) $\int_0^\infty \frac{\left(e^{-4t}-e^{-2t}\right)senht}{t}dt$ i) $\int_0^\infty \frac{e^{-\sqrt{2}t}senht.sent}{t}dt$

j) $\int_0^\infty t^{-5/2}e^{-4t}\cosh(2t)dt$ k) $\int_0^\infty \frac{\cosh(2t)sen(3t)}{t}dt$ l) $\int_0^\infty\int_0^t \frac{e^{-2t}sen(2u)}{u}dudt$

m) $\int_0^\infty\int_0^t \frac{e^{-2t-3u}-e^{-2t-u}}{u}dudt$ n) $\int_0^\infty\int_0^t x^{3/2}e^{-\frac{t}{4}-2x}dxdt$ o) $\int_0^\infty\int_0^t \frac{e^{-4t}\cos x - e^{-4t}\cos(3x)}{x}dxdt$

p) $\int_0^\infty\int_0^t \frac{sen^2(2x)}{x}e^{-\frac{1}{2}t}dxdt$ q) $\int_0^\infty\int_0^t \frac{e^{-4t}sen^3(2x)}{x}dxdt$ r) $\int_0^\infty\int_0^t t\,xe^{-3t}\cos(2x)dxdt$

s) $\int_0^\infty\int_0^t \frac{te^{-5t-4u}sen(2u)}{u}dudt$ t) $\int_0^\infty\int_0^t \frac{te^{-\frac{1}{3}t-u}sen^2(3u)}{u}dudt$ u) $\int_0^\infty\int_0^t \frac{1-\cos x}{x^2}e^{-2t}dxdt$

RESPUESTAS

a) $\frac{1}{2}\ln\left(\frac{5}{3}\right)$; b) $\frac{234}{15625}$; c) $\ln\left(\frac{3}{4}\right)$; d) $\ln\left(\frac{5}{3}\right)$; e) $4\sqrt{3\pi}$; f) $\frac{1}{2}\ln\left(\frac{29}{10}\right)$; g) $\frac{\pi}{6}$; h) $\frac{1}{2}\ln\left(\frac{5}{9}\right)$; i) $\frac{\pi}{8}$; j) 20.71; k) 0;

l) $\frac{\pi}{8}$; m) $\frac{1}{2}\ln\left(\frac{3}{5}\right)$; n) $\frac{32}{81}\sqrt{\pi}$; o) $\frac{1}{8}\ln\left(\frac{25}{17}\right)$;p) $\frac{1}{2}\ln 65$; q) 0.0255; r) $\frac{11}{19773}$; s) 0.0135; t) 7.949; u) 0.12025