UMSS- FACULTAD DE CIENCIAS Y TECNOLOGIA
DEPARTAMENTO DE MATEMATICAS
SEMESTRE 1-2023 (29-06-2023)

SEGUNDO PARCIAL – TRANSFORMADAS INTEGRALES

APELLIDOS:…………………………………… NOMBRES:………………………………
CARRERA:……………………………… CARNET DE IDENTIDAD:……………………

1.- Un circuito RLC tiene como componentes: R=60 [Ω]; L=20 [H]; C=5 [mF] dadas las condiciones iniciales: $V_{C(0)} = 80[V]; i_{L(0)} = 1.5[A]$.Determinar la corriente en función del tiempo si se aplica la fuente de voltaje: $v_{(t)} = 100te^{-4t}[V]$.

2.- Dado el sistema: hallar solamente y(t)

$$\begin{cases} \frac{dx}{dt} = 3x - 2y + 5sen(3t) \\ \frac{dy}{dt} = 2x - y + +2\cos(3t) \end{cases}; \quad \begin{matrix} x_{(0)} = -5 \\ y_{(0)} = 3 \end{matrix}$$

3.- Resolver la ecuación diferencial:

$$y'' + 25y = f_{(t)};\ y_{(0)} = 5; \quad y'_{(0)} = -4 \text{ donde } f_{(t)} = \begin{cases} 3t^2 - 5 & 0 < t < 3 \\ 0 & t > 3 \end{cases}$$

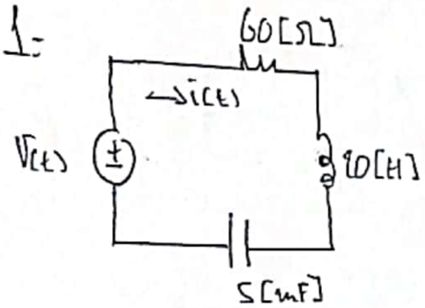
4.- Evaluar la integral: $\int_0^{\infty}\int_0^{t} \frac{te^{-\frac{1}{4}t-3x} sen^3(3x)}{x} dxdt$

5. Hallar una solución particular de la ecuación diferencial:

$$x''_{(t)} + 5x'_{(t)} + 6x_{(t)} = tu(-t)$$

2do Parcial _ Transformadas

1.-



$V_C(0) = 80\,[V]$

$i_L(0) = 1.5\,[A]$

$V(t) = 100\,t e^{-4t}\,[V]$

$$60\,i + 20\frac{di}{dt} + \frac{1}{5\times10^{-3}}\int_0^t i\,dt + 80 = 100\,t e^{-4t}$$

$$\mathcal{L}:\quad 60\,I(s) + 20\left(s\,I(s) - 1.5\right) + \frac{200\,I(s)}{s} + \frac{80}{s} = \frac{100}{(s+4)^2}$$

$$I(s)\left(60 + 20s + \frac{200}{s}\right) = \frac{100}{(s+4)^2} - \frac{80}{s} + 30$$

$$I(s) = \frac{\frac{100s - 80(s^2+8s+16) + 30s(s+4)^2}{s(s+4)^2}}{60 + 20s + \frac{200}{s}} = \frac{-80s^2 - 640s - 1280 + 30s(s^2+8s+16)}{\cancel{s}(s+4)^2\left(\frac{60s+20s^2+200}{\cancel{s}}\right)}$$

$$I(s) = \frac{30s^3 + 160s^2 - 60s - 1280}{(s+4)^2(20s^2+60s+200)} \quad \div 20 \Rightarrow I(s) = \frac{\frac{3}{2}s^3 + 8s^2 - 3s - 64}{(s+4)^2(s^2+3s+10)} = \frac{A}{s+4} + \frac{B}{(s+4)^2} + \frac{Cs+D}{s^2+3s+10}$$

$$\frac{3}{2}s^3 + 8s^2 - 3s - 64 = A(s+4)(s^2+3s+10) + B(s^2+3s+10) + (Cs+D)(s^2+8s+16) \quad \cdot (s+4)^2(s^2+3s+10)$$

$s^3:\quad \frac{3}{2} = A + C$

$s^2:\quad 8 = 7A + B + 8C + D$

$s:\quad -3 = 22A + 3B + 16C + 8D$

$\text{ctte}:\quad -64 = 40A + 10B + 16D$

$A = -\frac{15}{98}\qquad C = \frac{81}{49}$

$B = -\frac{10}{7}\qquad D = -\frac{267}{98}$

$$i(t) = \mathcal{L}^{-1}\left\{\frac{-15/98}{s+4} - \frac{10/7}{(s+4)^2} + \frac{\frac{81}{49}(s+3/2-3/2) - \frac{267}{98}}{(s+3/2)^2 + 31/4}\right\}$$

$$i(t) = \;.....\; + \mathcal{L}^{-1}\left\{\frac{\frac{81}{49}(s+3/2)}{(s+3/2)^2+31/4} - \frac{\frac{255}{49}}{(s+3/2)^2+31/4}\right\}$$

$$\boxed{i(t) = -\frac{15}{98}e^{-4t} - \frac{10}{7}t e^{-4t} + e^{-3/2\,t}\cdot\left(\frac{81}{49}\cos\left(\frac{\sqrt{31}}{2}t\right) - 1{,}869\,\text{sen}\left(\frac{\sqrt{31}}{2}t\right)\right)\,[A]}$$

2.- $\begin{cases} \dfrac{dx}{dt} = 3x - 2y + 5\,\text{sen}(3t) & x(0) = -5 \\ \dfrac{dy}{dt} = 2x - y + 2\cos(3t) & y(0) = 3 \end{cases} \xrightarrow{\mathcal{L}} \begin{cases} sX(s) + 5 = 3X(s) - 2Y(s) + \dfrac{15}{s^2+9} \\ sY(s) - 3 = 2X(s) - Y(s) + \dfrac{2s}{s^2+9} \end{cases}$

$$\begin{cases} (s-3)X(s) + 2Y(s) = \dfrac{15}{s^2+9} - 5 \\ -2X(s) + (s+1)Y(s) = \dfrac{2s}{s^2+9} + 3 \end{cases} \qquad Y(s) = \frac{\begin{vmatrix} s-3 & \frac{15}{s^2+9} - 5 \\ -2 & \frac{2s}{s^2+9} + 3 \end{vmatrix}}{\begin{vmatrix} s-3 & 2 \\ -2 & s+1 \end{vmatrix}}$$

$$Y(s) = \frac{(s-3)\left(\frac{2s}{s^2+9} + 3\right) + \frac{30}{s^2+9} - 10}{(s-3)(s+1) + 4}$$

$$Y(s) = \frac{\frac{2s^2 - 6s + 30}{s^2+9} + 3s - 19}{s^2 - 2s + 1} = \frac{\frac{2s^2 - 6s + 30 + (3s-19)(s^2+9)}{s^2+9}}{(s-1)^2}$$

$$\underline{Y(s) = \frac{3s^3 - 17s^2 + 21s - 141}{(s^2+9)(s-1)^2}} = \frac{As+B}{s^2+9} + \frac{C}{s-1} + \frac{D}{(s-1)^2} \quad \cdot (s-1)^2(s^2+9)$$

$$3s^3 - 17s^2 + 21s - 141 = (As+B)(s^2 - 2s + 1) + C(s-1)(s^2+9) + D(s^2+9)$$

s^3: $3 = A + C$

s^2: $-17 = -2A + B - C + D$

s: $21 = A - 2B + 9C$

ctte: $-141 = B - 9C + 9D$

$A = \frac{18}{25}$; $B = \frac{3}{25}$

$C = \frac{57}{25}$; $D = -\frac{67}{5}$

$$y(t) = \mathcal{L}^{-1}\left\{ \frac{\frac{18}{25}s}{s^2+9} + \frac{\frac{3}{25}}{s^2+9} + \frac{\frac{57}{25}}{s-1} - \frac{67/5}{(s-1)^2} \right\}.$$

$$\boxed{y(t) = \frac{18}{25}\cos(3t) + \frac{1}{25}\text{sen}(3t) + \frac{57}{25}e^t - \frac{67}{5}te^t}$$

3.- $f(t) = \begin{cases} 3t^2 - 5 & 0 < t < 3 \\ 0 & t > 3 \end{cases} \Rightarrow f(t) = (3t^2 - 5)u(t) - (3t^2 - 5)u(t-3)$

$$F(s) = \frac{3 \cdot 2}{s^3} - \frac{5}{s} - e^{-3s} \cdot \mathcal{L}\{3(t+3)^2 - 5\}$$

$3(t+3)^2 - 5 \longrightarrow 3(t^2 + 6t + 9) - 5$

$3t^2 + 18t + 22$

$$\underline{F(s) = \frac{6}{s^3} - \frac{5}{s} - e^{-3s}\left(\frac{6}{s^3} + \frac{18}{s^2} + \frac{22}{s}\right)}$$

$$y'' + 25y = f(t) \qquad y(0) = 5 \; ; \; y'(0) = -4$$

$$\mathcal{L}: \quad s^2Y(s) - 5s + 4 + 25Y(s) = F(s) \rightarrow Y(s)(s^2+25) = F(s) + 5s - 4$$

$$Y(s) = \frac{6}{s^3(s^2+25)} - \frac{5}{s(s^2+25)} - e^{-3s}\cdot\left(\frac{6}{s^3(s^2+25)} + \frac{18}{s^2(s^2+25)} + \frac{22}{s(s^2+25)}\right) + \frac{5s}{s^2+25} - \frac{4}{s^2+25}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+25)}\right\} = \int_0^t \frac{\operatorname{sen}(5t)}{5}dt = -\frac{\cos(5t)}{25}\Big|_0^t = -\frac{\cos(5t)}{25} + \frac{1}{25}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+25)}\right\} = \int_0^t \left(-\frac{\cos(5t)}{25} + \frac{1}{25}\right)dt = -\frac{\operatorname{sen}(5t)}{125} + \frac{t}{25}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^3(s^2+25)}\right\} = \int_0^t \left(-\frac{\operatorname{sen}(5t)}{125} + \frac{t}{25}\right)dt = -\frac{\cos(5t)}{625} + \frac{t^2}{50} + \frac{1}{625}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \left[-\frac{6}{625}\cos(5t) + \frac{3}{25}t^2 + \frac{6}{625} + \frac{\cos(5t)}{5} - \frac{1}{5} + 5\cos(5t) - \frac{4}{5}\operatorname{sen}(5t)\right]u(t)$$

$$-\left[-\frac{6}{625}\cos(5(t-3)) + \frac{3}{25}(t-3)^2 + \frac{6}{625} - \frac{18}{125}\operatorname{sen}(5(t-3)) + \frac{18}{25}(t-3) + -\frac{22}{25}\cos(5(t-3)) + \frac{22}{25}\right)\cdot u(t-3)$$

$$\boxed{y(t) = \left[\frac{3244}{625}\cos(5t) - \frac{4}{5}\operatorname{sen}(5t) + \frac{3}{25}t^2 - \frac{119}{625}\right]u(t) - \left[-\frac{556}{625}\cos(5t-15) - \frac{18}{125}\operatorname{sen}(5t-15) + \frac{3}{25}t^2 - \frac{119}{625}\right]u(t-3)}$$

4.-

$$I = \int_0^\infty \int_0^t \frac{te^{-\frac{1}{4}t-3x}\operatorname{sen}^3(3x)}{x}dx\,dt = \int_0^\infty te^{-\frac{1}{4}t}dt \cdot \int_0^t \frac{e^{-3x}\operatorname{sen}^3(3x)}{x}dx$$

$$\frac{\operatorname{sen}^3(3x)}{\cancel{x}} = \left(\frac{e^{j3x} - e^{-j3x}}{2j}\right)^3 = \frac{(e^{j3x})^3 - 3(e^{j3x})^2(e^{-j3x}) + 3(e^{j3x})(e^{-j3x})^2 - (e^{-j3x})^3}{-8j}$$

$$\operatorname{sen}^3(3x) = \frac{(e^{j9x} - e^{-j9x}) - 3(e^{j3x} - e^{-j3x})}{-8j} = \frac{2\cancel{j}\operatorname{sen}(9x) - 6\cancel{j}\operatorname{sen}(3x)}{-8\cancel{j}}$$

$$\operatorname{sen}^3(3x) = -\frac{1}{4}\operatorname{sen}(9x) + \frac{3}{4}\operatorname{sen}(3x)$$

Escaneado con CamScanner

$$\mathcal{L}\left\{\frac{\operatorname{sen}^3(3x)}{x}\right\} = \mathcal{L}\left\{-\frac{1}{4}\frac{\operatorname{sen}(9x)}{x} + \frac{3}{4}\frac{\operatorname{sen}(3x)}{x}\right\}$$

$$\mathcal{L}\left\{\frac{\operatorname{sen}^3(3x)}{x}\right\} = -\frac{1}{4}\arctan\left(\frac{9}{s}\right) + \frac{3}{4}\arctan\left(\frac{3}{s}\right)$$

$$\mathcal{L}\left\{\int_0^t \frac{e^{-3x}\operatorname{sen}^3(3x)}{x}dx\right\} = \frac{1}{s}\cdot\left(-\frac{1}{4}\arctan\left(\frac{9}{s+3}\right) + \frac{3}{4}\arctan\left(\frac{3}{s+3}\right)\right)$$

$$\mathcal{L}\left\{t\int_0^t \frac{e^{-3x}\operatorname{sen}^3(3x)}{x}dx\right\} = -\left[-\frac{1}{s^2}\left(-\frac{1}{4}\arctan\left(\frac{9}{s+3}\right) + \frac{3}{4}\arctan\left(\frac{3}{s+3}\right)\right)\right.$$

$$\left. + \frac{1}{s}\cdot\left[-\frac{1}{4}\cdot\frac{-\frac{9}{(s+3)^2}}{1+\frac{81}{(s+3)^2}} + \frac{3}{4}\cdot\frac{-\frac{3}{(s+3)^2}}{1+\frac{9}{(s+3)^2}}\right]\right]$$

$$\mathcal{L}\left\{t\int_0^t \frac{e^{-3x}\operatorname{sen}^3(3x)}{x}dx\right\} = \frac{1}{s^2}\left(-\frac{1}{4}\arctan\left(\frac{9}{s+3}\right) + \frac{3}{4}\arctan\left(\frac{3}{s+3}\right)\right) - \frac{1}{s}\cdot\left[\frac{9/4}{(s+3)^2+81} - \frac{9/4}{(s+3)^2+9}\right]$$

$$s = 1/4 \Rightarrow \quad \mathrm{II} = \int_0^\infty\int_0^t \frac{t e^{-3x-t/4}\operatorname{sen}^3(3x)}{x}dx\,dt = F(1/4)$$

$$\mathrm{II} = 16\cdot\left(-\frac{1}{4}\arctan\left(\frac{36}{13}\right) + \frac{3}{4}\arctan\left(\frac{12}{13}\right)\right) - 4\cdot\left[\frac{36}{1465} - \frac{36}{313}\right]$$

$$\boxed{\mathrm{II} \approx 4{,}4098}$$

5.- $x''(t) + 5x'(t) + 6x(t) = t u(-t)$

$\mathcal{F}:\ (j\omega)^2 X(\omega) + 5j\omega X(\omega) + 6X(\omega) = G(\omega)$

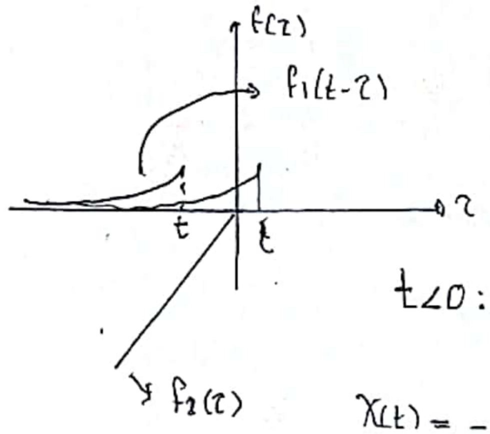
$X(\omega)\left(6 + 5j\omega + (j\omega)^2\right) = G(\omega) \rightarrow X(\omega) = \dfrac{G(\omega)}{(3+j\omega)(2+j\omega)}$

$$x(t) = \underbrace{e^{-3t}u(t) * e^{-2t}u(t)}_{} * t u(-t)$$

$$f_1(t) = \int_0^t e^{-3\tau}\cdot e^{-2(t-\tau)}d\tau = e^{-2t}\int_0^t e^{-\tau}d\tau = e^{-2t}\cdot -e^{-t}\Big|_0^t = -e^{-2t}\left(e^{-t}-1\right)$$

$$f_1(t) = \left(-e^{-3t} + e^{-2t}\right)u(t)$$

$$x(t) = f_1(t) * t\,u(-t) = \underbrace{(-e^{-3t}+e^{-2t})\,u(t)}_{f_1(t)} * \underbrace{t\,u(-t)}_{f_2(t)}$$



$$f_2(\tau) = \tau\,u(-\tau)$$

$$f_1(t-\tau) = -e^{-3(t-\tau)} + e^{-2(t-\tau)}\,u(t-\tau)$$

$t<0$: $$x(t) = \int_{-\infty}^{t} (-e^{-3(t-\tau)} + e^{-2(t-\tau)}) \cdot \tau\, d\tau$$

$$x(t) = -e^{-3t}\int_{-\infty}^{t} \tau e^{3\tau} d\tau + e^{-2t}\int_{-\infty}^{t} \tau e^{2\tau} d\tau$$

$$x(t) = -e^{-3t} \cdot \left(\frac{\tau e^{3\tau}}{3} - \frac{e^{3\tau}}{9} \right)\Bigg|_{-\infty}^{t} + e^{-2t} \cdot \left(\frac{\tau e^{2\tau}}{2} - \frac{e^{2\tau}}{4} \right)\Bigg|_{-\infty}^{t}$$

$$x(t) = -e^{-3t} \cdot \left(\frac{t e^{3t}}{3} - \frac{e^{3t}}{9} \right) + e^{-2t} \cdot \left(\frac{t e^{2t}}{2} - \frac{e^{2t}}{4} \right) = -\frac{t}{3} + \frac{1}{9} + \frac{t}{2} - \frac{1}{4}$$

$$\underline{x(t) = \frac{t}{6} - \frac{5}{36}}$$

$t>0$: $$x(t) = \int_{-\infty}^{0} (-e^{-3(t-\tau)} + e^{-2(t-\tau)})\,\tau\, d\tau$$

$$x(t) = -e^{-3t} \cdot \left(\frac{\tau e^{3\tau}}{3} - \frac{e^{3\tau}}{9} \right)\Bigg|_{-\infty}^{0} + e^{-2t} \cdot \left(\frac{\tau e^{2\tau}}{2} - \frac{e^{2\tau}}{4} \right)\Bigg|_{-\infty}^{0} = -e^{-3t} \cdot \left(-\frac{1}{9}\right) + e^{-2t} \cdot \left(-\frac{1}{4}\right)$$

$$\underline{x(t) = \frac{1}{9}e^{-3t} - \frac{1}{4}e^{-2t}}$$

Sol: $$\boxed{x(t) = \left(\frac{t}{6} - \frac{5}{36}\right)u(-t) + \left(\frac{1}{9}e^{-3t} - \frac{1}{4}e^{-2t}\right)u(t)}$$