UMSS- FACULTAD DE CIENCIAS Y TECNOLOGIA
DEPARTAMENTO DE MATEMATICAS
SEMESTRE 2-2023 (31-10-2023)

PRIMER PARCIAL – TRANSFORMADAS INTEGRALES

APELLIDOS:.. NOMBRES:.................................
CARRERA:................................... CARNET DE IDENTIDAD:.......................

1.- (30 pts.) Dada la siguiente función que se repite con un período T=6, hallar su serie de Fourier aplicando el método de diferenciación y determine los primeros 3 armónicos diferentes de cero

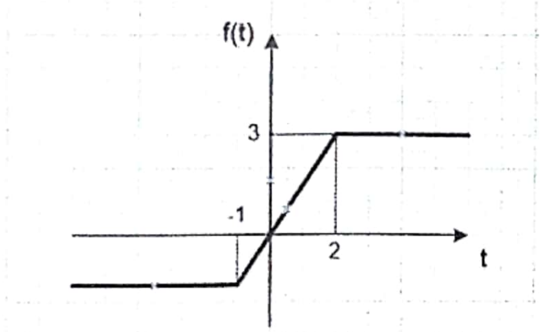
$$f_{(t)} = \begin{cases} \frac{1}{9}t^3 + 5 & 0 < t < 3 \\ -\frac{1}{3}t^2 + 2t + 5 & 3 < t < 6 \end{cases}$$

2.- (40 pts.) Calcular las transformadas de Fourier:

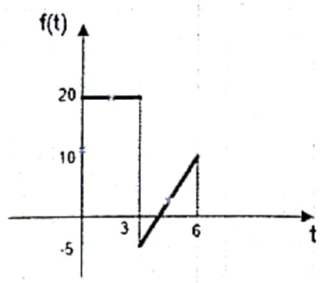
a) $\mathcal{F}\left\{\dfrac{\cos(2t)}{12 - jt + 6t^2}\right\}$

b) $\mathcal{F}\left\{\dfrac{(t-4)^2 e^{-j2(t-4)}}{(t-4)^2 + 6(t-4) + 40}\right\}$

3.- (15 pts.) Calcular la transformada de Fourier de la función:

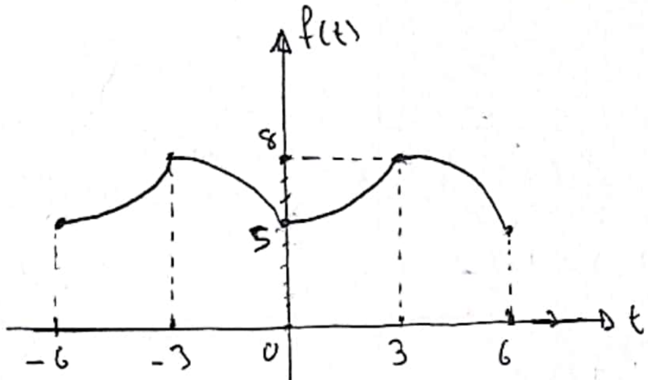


4.- (20 pts.) La siguiente función, expandir con una simetria de cuarto de onda impar graficando en el intervalo -36<t<36 y determine su serie de Fourier com 3 armónicos diferentes de cero



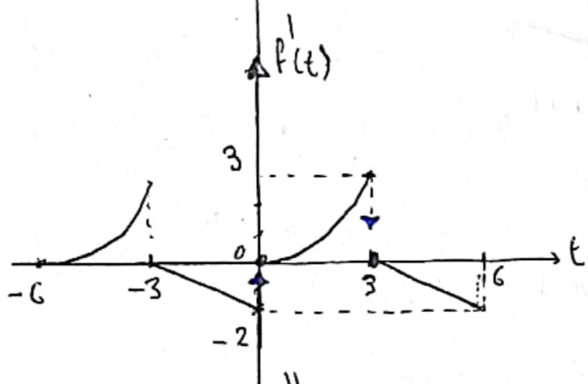
1er Parcial_ Transformadas

1. $T = 6 \rightarrow \omega_0 = \frac{\pi}{3}$



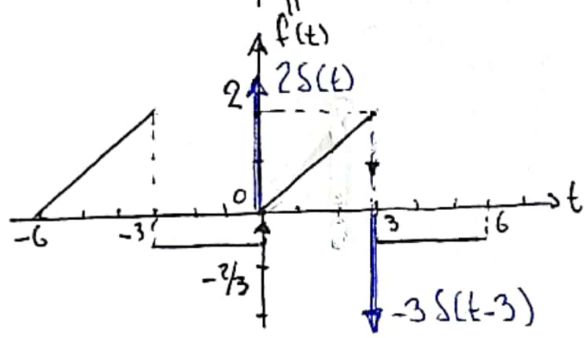
$$f(t) = \begin{cases} \frac{1}{9}t^3 + 5 & 0 < t < 3 \\ -\frac{1}{3}t^2 + 2t + 5 & 3 < t < 6 \end{cases}$$

$$f'(t) = \begin{cases} \frac{1}{3}t^2 & 0 < t < 3 \\ -\frac{2}{3}t + 2 & 3 < t < 6 \end{cases}$$



$$\boxed{\gamma'_n = 0 \rightarrow C'_n = 0}$$

$$f''(t) = \begin{cases} \frac{2}{3}t & 0 < t < 3 \\ -\frac{2}{3} & 3 < t < 6 \end{cases}$$



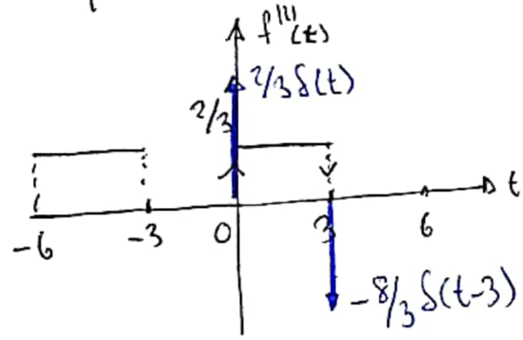
$$\gamma''_n = \frac{1}{6}\int_0^6 (2\delta(t) - 3\delta(t-3))e^{-j\frac{\pi}{3}nt}\,dt$$

$$\gamma''_n = \frac{1}{3} - \frac{1}{2}\cos(\pi n)$$

$$C''_n = \frac{\gamma''_n}{(j\frac{\pi}{3}n)^2} = -\frac{9}{\pi^2 n^2}\cdot\gamma''_n$$

$$\boxed{C''_n = -\frac{3 - \frac{9}{2}\cos(\pi n)}{\pi^2 n^2}}$$

$$f'''(t) = \begin{cases} \frac{2}{3} & 0 < t < 3 \\ 0 & 3 < t < 6 \end{cases}$$



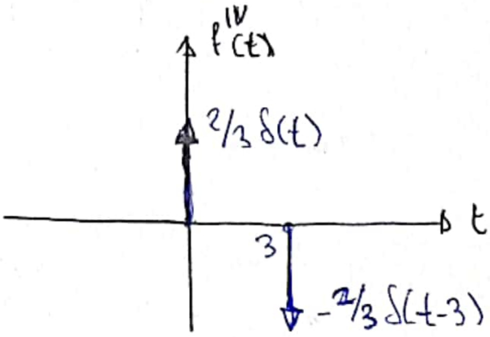
$$\gamma'''_n = \frac{1}{6}\int_0^6 \left(\frac{2}{3}\delta(t) - \frac{8}{3}\delta(t-3)\right)e^{-j\frac{\pi}{3}nt}\,dt$$

$$\gamma'''_n = \frac{1}{9} - \frac{4}{9}\cos(\pi n)$$

$$C'''_n = \frac{\gamma'''_n}{(j\frac{\pi}{3}n)^3} = j\frac{27}{\pi^3 n^3}\gamma'''_n$$

$$\boxed{C'''_n = j\left(\frac{3 - 12\cos(\pi n)}{\pi^3 n^3}\right)}$$

$f^{IV}_{(t)} = 0$



$$\gamma^{IV}_{(n)} = \frac{1}{6}\cdot\int_0^6 \left(\frac{2}{3}\delta(t) - \frac{2}{3}\delta(t-3)\right) e^{-j\frac{\pi}{3}nt}\,dt$$

$$\gamma^{IV}_n = \frac{1}{9} - \frac{1}{9}\cos(\pi n)$$

$$C^{IV}_n = \frac{\gamma^{IV}_n}{(j\frac{\pi}{3}n)^4} = \frac{81}{\pi^4 n^4}\cdot\gamma^{IV}_n \rightarrow \boxed{C^{IV}_n = \frac{9(1-\cos(\pi n))}{\pi^4 n^4}}$$

$$a_n = 2\,\mathrm{Re}\{C_n\} \rightarrow \boxed{a_n = \frac{-6+9\cos(\pi n)}{\pi^2 n^2} + \frac{18(1-\cos(\pi n))}{\pi^4 n^4}}$$

$$b_n = -2\,\mathrm{Im}\{C_n\} \rightarrow \boxed{b_n = \frac{-6+24\cos(\pi n)}{\pi^3 n^3}}$$

$$C_0 = \frac{1}{6}\cdot\left[\int_0^3\left(\frac{1}{9}t^3+5\right)dt + \int_3^6\left(-\frac{1}{3}t^2+2t+5\right)dt\right]$$

$$C_0 = \frac{1}{6}\cdot\left[\left(\frac{1}{9}\cdot\frac{t^4}{4}+5t\right)\Big|_0^3 + \left(-\frac{t^3}{9}+t^2+5t\right)\Big|_3^6\right]$$

$$C_0 = \frac{1}{6}\cdot\left[\frac{69}{4} + 42 - 21\right] \Rightarrow \boxed{C_0 = \frac{51}{8}}$$

$$\boxed{f(t) = \frac{51}{8} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{\pi}{3}nt\right) + b_n \operatorname{sen}\left(\frac{\pi}{3}nt\right)\right]}$$

n	a_n	b_n
1	-1,15	-0,968
2	0,076	0,073
3	-0,164	-0,036

$$\boxed{\begin{aligned} f(t) = \frac{51}{8} &- 1{,}15\cos\left(\frac{\pi}{3}t\right) - 0{,}968\operatorname{sen}\left(\frac{\pi}{3}t\right) \\ &+ 0{,}076\cos\left(\frac{2\pi}{3}t\right) + 0{,}073\operatorname{sen}\left(\frac{2\pi}{3}t\right) \\ &- 0{,}164\cos(\pi t) - 0{,}036\operatorname{sen}(\pi t)\end{aligned}}$$

2.- a) $\mathcal{F}\left\{\frac{\cos(2t)}{12-jt+6t^2}\right\} \rightarrow \frac{e^{j2t}+e^{-j2t}}{2} \Rightarrow \omega : \omega-2 \; ; \; \omega : \omega+2$

$$F_1(\omega) = \mathcal{F}\left\{\frac{1}{12-jt+6t^2}\right\} = \mathcal{F}\left\{\frac{1}{12-jt-6(jt)^2}\right\} = \mathcal{F}\left\{\frac{1}{(4-3jt)(3+2jt)}\right\}$$

$4 \quad -3jt$

$3 \quad 2jt$

$$\frac{1}{(4-3jt)(3+2jt)} = \frac{A}{4-3jt} + \frac{B}{3+2jt}$$

$$1 = A(3+2jt) + B(4-3jt)$$

$$jt = -3/2: \quad 1 = B(4-3(-3/2)) \rightarrow \boxed{B = \frac{2}{17}}$$

$$jt = 4/3: \quad 1 = A(3+2(4/3)) \rightarrow \boxed{A = \frac{3}{17}}$$

$$F_1(\omega) = \mathcal{F}\left\{\frac{3/17}{4-3jt} + \frac{2/17}{3+2jt}\right\} = \mathcal{F}\left\{\frac{1/17}{\frac{4}{3}-jt} + \frac{1/17}{\frac{3}{2}+jt}\right\}$$

$$\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a+j\omega} \rightarrow \mathcal{F}\left\{\frac{1}{a+jt}\right\} = 2\pi e^{a\omega}u(-\omega)$$

$$\mathcal{F}\{e^{at}u(t)\} = \frac{1}{a-j\omega} \rightarrow \mathcal{F}\left\{\frac{1}{a-jt}\right\} = 2\pi e^{-a\omega}u(\omega)$$

$$F_1(\omega) = \frac{2\pi}{17}\left[e^{-4/3\omega}u(\omega) + e^{3/2\omega}u(-\omega)\right]$$

$$\boxed{F(\omega) = \frac{\pi}{17}\cdot\left[e^{-4/3(\omega-2)}u(\omega-2) + e^{\frac{3}{2}(\omega-2)}u(-\omega+2) + e^{-4/3(\omega+2)}u(\omega+2) + e^{\frac{3}{2}(\omega+2)}u(-\omega-2)\right]}$$

b) $$\mathcal{F}\left\{\frac{(t-4)^2 e^{-j2(t-4)}}{(t-4)^2+6(t-4)+40}\right\} = F(\omega) \quad t-4 \rightarrow \cdot e^{-j4\omega}$$

$$F_1(\omega) = \mathcal{F}\left\{\frac{t^2 e^{-j2t}}{t^2+6t+40}\right\} \rightarrow \omega : \omega+2$$

$$F_2(\omega) = \mathcal{F}\left\{\frac{1}{\underbrace{t^2+6t+40}_{+3^2-3^2}}\right\} = \mathcal{F}\left\{\frac{1}{(t+3)^2+31}\right\} = \frac{\pi}{\sqrt{31}}e^{-\sqrt{31}|\omega|}\cdot e^{j3\omega} = \frac{\pi}{\sqrt{31}}e^{-\sqrt{31}|\omega|+j3\omega}$$

$$\mathcal{F}\left\{\frac{t}{t^2+6t+40}\right\} = j\frac{\pi}{\sqrt{31}}\cdot e^{-\sqrt{31}|\omega|+j3\omega}\cdot(-\sqrt{31}\,\mathrm{sgn}(\omega)+j3) =$$

$$\mathcal{F}\left\{\frac{t^2}{t^2+6t+40}\right\} = j^2\frac{\pi}{\sqrt{31}}\cdot\left[e^{-\sqrt{31}|\omega|+j3\omega}\cdot(-\sqrt{31}\,\mathrm{sgn}(\omega)+j3)^2 + \underbrace{e^{-\sqrt{31}|\omega|+j3\omega}}_{\omega=0}\cdot(-\sqrt{31}\cdot 2\delta(\omega))\right]$$

$$\mathcal{F}\left\{\frac{t^2}{t^2+6t+40}\right\} = -\frac{\pi}{\sqrt{31}}\cdot\left[e^{-\sqrt{31}|\omega|+j3\omega}\cdot(31 - j6\sqrt{31}\,\mathrm{sgn}(\omega) - 3) - 2\sqrt{31}\,\delta(\omega)\right]$$

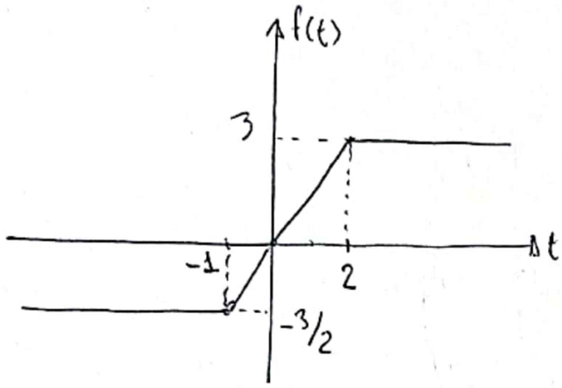
$$\mathcal{F}\left\{\frac{t^2}{t^2+6t+40}\right\} = -\pi\cdot\left[e^{-\sqrt{31}|\omega|+j3\omega}\cdot\left(\frac{28}{\sqrt{31}} - j6\,\mathrm{sgn}(\omega)\right) - 2\delta(\omega)\right]$$

Escaneado con CamScanner

$$F_1(\omega) = -\pi \cdot \left[e^{-\sqrt{31}\,|\omega+2| + j3(\omega+2)} \cdot \left(\frac{28}{\sqrt{31}} - j6\,\mathrm{sgn}(\omega+2) \right) - 2\,\delta(\omega+2) \right]$$

$$\boxed{F(\omega) = -\pi \cdot e^{-j4\omega} \left[e^{-\sqrt{31}\,|\omega+2| + j3(\omega+2)} \cdot \left(\frac{28}{\sqrt{31}} - j6\,\mathrm{sgn}(\omega+2) \right) - 2\,\delta(\omega+2) \right]}$$

3.-



$f(t) = \frac{3}{2} t$

$t = -1 ; \ f(-1) = -\frac{3}{2}$

$$f(t) = -\frac{3}{2} + \frac{3}{2} u(t+1) + \frac{3}{2} t \left(u(t+1) - u(t-2) \right) + 3u(t-2)$$

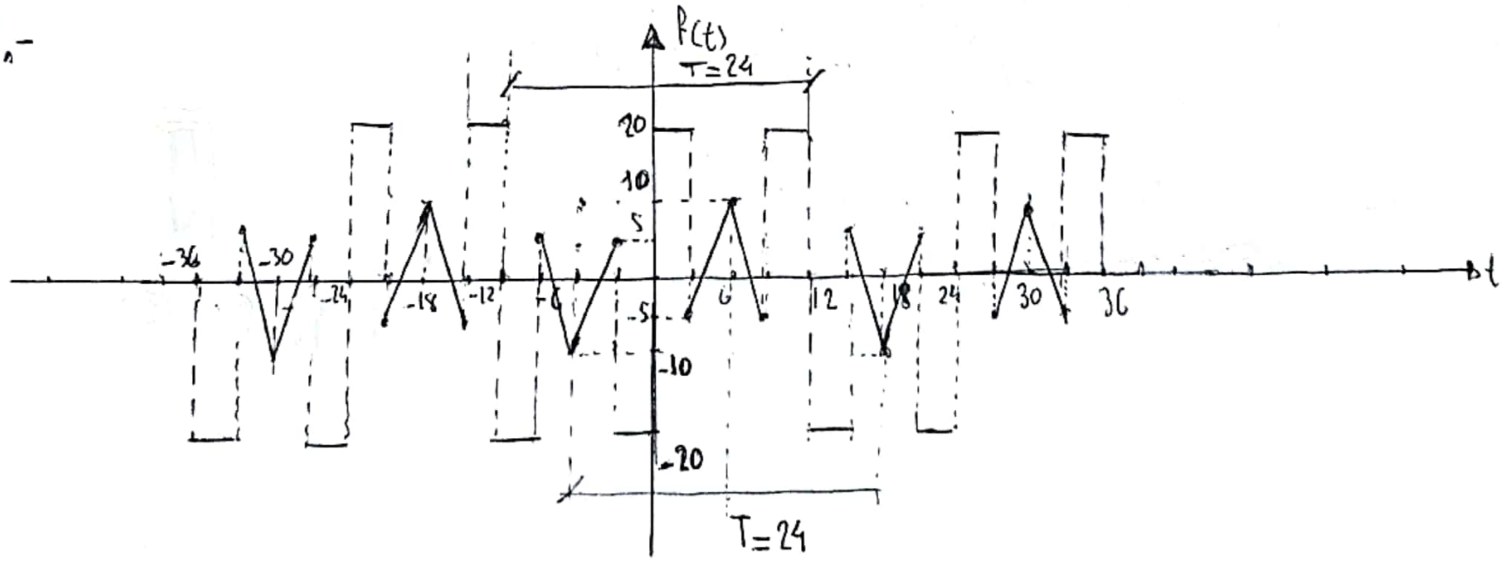
$$F(\omega) = \cancel{2}\pi\left(-\frac{3}{\cancel{2}}\right)\delta(\omega) + \frac{3}{2} e^{j\omega} \cdot \left(\frac{1}{j\omega} + \pi\delta(\omega) \right) + \frac{3}{2} j \frac{d}{d\omega} \left[e^{j\omega}\left(\frac{1}{j\omega} + \pi\delta(\omega) \right) - e^{-j2\omega} \cdot \left(\frac{1}{j\omega} + \pi\delta(\omega) \right) \right] + 3e^{-j2\omega} \cdot \left(\frac{1}{j\omega} + \pi\delta(\omega) \right)$$

$$F(\omega) = -3\cancel{\pi\delta(\omega)} + \frac{3e^{j\omega}}{2j\omega} + \frac{3}{2}\pi\delta(\omega) + \frac{3}{2}\cancel{j} \frac{d}{d\omega} \left[\frac{e^{j\omega} - e^{-j2\omega}}{\cancel{j}\omega} + \cancel{\pi\delta(\omega)} - \cancel{\pi\delta(\omega)} \right] + \frac{3e^{-j2\omega}}{j\omega} \cancel{+3\pi\delta(\omega)}$$

$$F(\omega) = \frac{\frac{3}{2} e^{j\omega} + 3e^{-j2\omega}}{j\omega} + \frac{3}{2}\pi\delta(\omega) + \frac{3}{2} \cdot \left[\frac{(je^{j\omega} + j2e^{-j2\omega})\omega + e^{j\omega} - e^{-j2\omega}}{\omega^2} \right]$$

$$\boxed{F(\omega) = \frac{\frac{3}{2} e^{j\omega} + 3e^{-j2\omega}}{j\omega} + \frac{3}{2}\pi\delta(\omega) + \frac{3}{2} \cdot \left[\frac{e^{j\omega}(1+j\omega) + e^{-j2\omega}(-1+j2\omega)}{\omega^2} \right]}$$

4.-



$$T = 24 \rightarrow \omega_0 = \frac{\pi}{12} \qquad f(t) = \begin{cases} 20 & 0 < t < 3 \\ 5t - 20 & 3 < t < 6 \end{cases}$$

$$b_n = \frac{8}{24}\left[\int_0^3 20 \operatorname{sen}\left(\frac{\pi}{12} nt\right) dt + \int_3^6 (5t-20) \operatorname{sen}\left(\frac{\pi}{12} nt\right) dt\right]$$

$$b_n = \frac{1}{3} \cdot \left[-\frac{20\cos(\pi/12\, nt)}{\frac{\pi}{12} n}\Bigg|_0^3 + 5\left(-\frac{t\cos(\pi/12\, nt)}{\frac{\pi}{12} n}\Bigg|_3^6 + \frac{\operatorname{sen}(\frac{\pi}{12} nt)}{(\frac{\pi}{12} n)^2}\Bigg|_3^6 \right) + \frac{20\cos(\pi/12\, nt)}{\frac{\pi}{12} n}\Bigg|_3^6 \right]$$

$$b_n = \frac{1}{3} \cdot \left[-\frac{240}{\pi n}\cos\left(\frac{\pi}{4} n\right) + \frac{240}{\pi n} + 5 \cdot \left(\cancel{-\frac{72}{\pi n}\cos\left(\frac{\pi}{2} n\right)}^{\,0} + \frac{36}{\pi n}\cos\left(\frac{\pi}{4} n\right) + \frac{144}{\pi^2 n^2}\operatorname{sen}\left(\frac{\pi}{2} n\right) - \frac{144}{\pi^2 n^2}\operatorname{sen}\left(\frac{\pi}{4} n\right)\right) + \cancel{\frac{240}{\pi n}\cos\left(\frac{\pi}{2} n\right)}^{\,0} - \frac{240}{\pi n}\cos\left(\frac{\pi}{4} n\right) \right]$$

$$b_n = \frac{1}{3} \cdot \left[-\frac{300}{\pi n}\cos\left(\frac{\pi}{4} n\right) + \frac{240}{\pi n} + \frac{720}{\pi^2 n^2}\left(\operatorname{sen}\left(\frac{\pi}{2} n\right) - \operatorname{sen}\left(\frac{\pi}{4} n\right)\right)\right]$$

$$\boxed{b_n = \frac{-100\cos(\pi/4\, n) + 80}{\pi n} + \frac{240}{\pi^2 n^2} \cdot \left(\operatorname{sen}\left(\frac{\pi}{2} n\right) - \operatorname{sen}\left(\frac{\pi}{4} n\right)\right)} \quad n: \text{impar}$$

n	b_n
1	10,079
3	11,378
5	11,255

$$\boxed{f(t) = 10{,}079 \operatorname{sen}\left(\frac{\pi}{12} t\right) + 11{,}378 \operatorname{sen}\left(\frac{\pi}{4} t\right) + 11{,}255 \operatorname{sen}\left(\frac{5\pi}{12} t\right)}$$