

PRACTICA Nro. 1 – TRANSFORMADAS INTEGRALES SERIES DE FOURIER

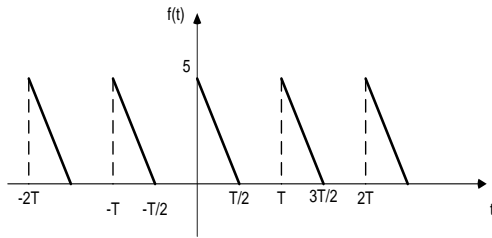
1.- Encontrar el período de las funciones:

a) $f(t) = \sin\left(\frac{t}{3}\right) + \sin\left(\frac{t}{5}\right)$ b) $f(t) = \cos\left(\frac{3}{4}t\right) + \sin\left(\frac{2}{5}t\right) + \sin\left(\frac{t}{2}\right)$ c) $f(t) = \sin\left(\frac{3}{2}t\right) + \sin\left(\frac{5}{4}t\right) + \sin\left(\frac{t}{6}\right)$
 d) $f(t) = \left|\cos\left(\frac{4t}{3}\right)\right| + \left|\cos\left(\frac{3t}{5}\right)\right|$ e) $f(t) = \sin^2(5t) + \left|\sin\left(\frac{4}{3}t\right)\right| + \cos\left(\frac{2}{5}t\right)$

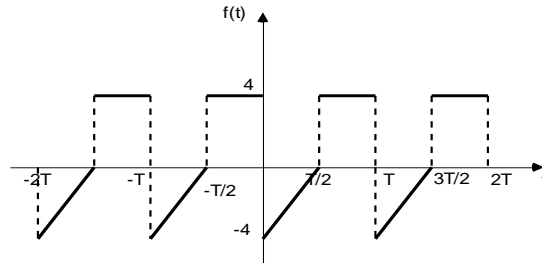
R.- a) $T=30\pi$; b) $T=40\pi$; c) 24π ; d) $T=15\pi$; e) $T=15\pi$.

2.- Dadas las funciones periódicas, definir las en un período y hallar las series de Fourier.

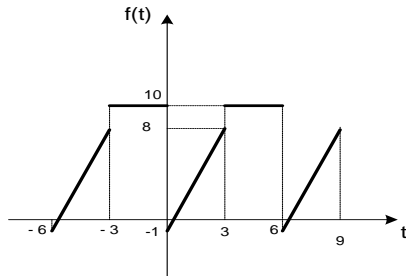
a)



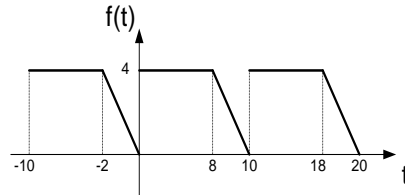
b)



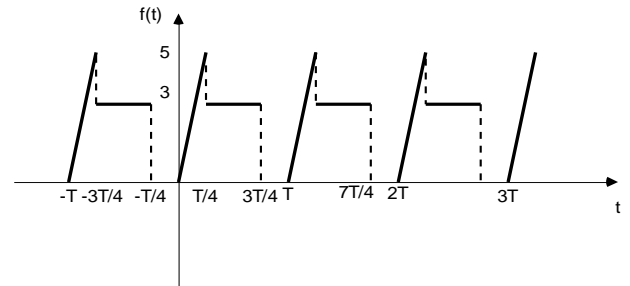
c)



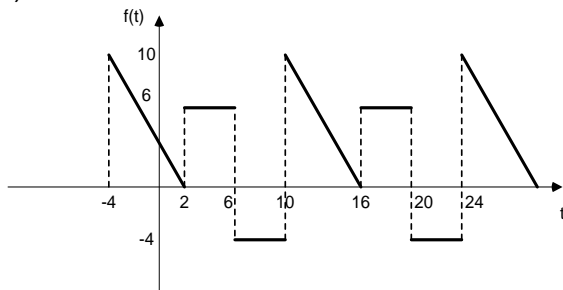
d)



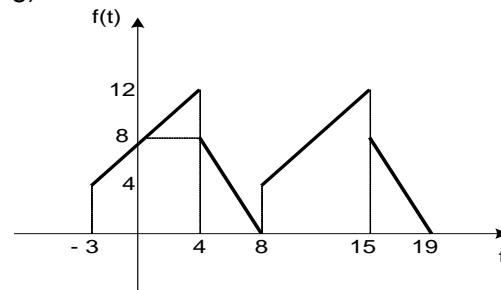
e)



f)



g)



R.- a) $f(t) = \frac{5}{4} + \sum_{n=1}^{\infty} \frac{5}{n^2 \pi^2} (1 - (-1)^n) \cos\left(\frac{2\pi n t}{T}\right) + \frac{5}{\pi n} \sin\left(\frac{2\pi n t}{T}\right)$

b) $f(t) = 1 + \sum_{n=1}^{\infty} \left[\frac{4((-1)^n - 1)}{n^2 \pi^2} \cos\left(\frac{2\pi n t}{T}\right) + \frac{4((-1)^n - 2)}{n \pi} \sin\left(\frac{2\pi n t}{T}\right) \right]$

c) $f(t) = \frac{27}{4} + \sum_{n=1}^{\infty} \left[\frac{9}{\pi^2 n^2} (\cos(\pi n) - 1) \cos\left(\frac{\pi n t}{3}\right) + \left(\frac{2 \cos(\pi n) - 11}{\pi n} \right) \sin\left(\frac{\pi n t}{3}\right) \right]$

d) $f(t) = \frac{18}{5} + \sum_{n=1}^{\infty} \left[\frac{10}{\pi^2 n^2} \left(\cos\left(\frac{8\pi n}{5}\right) - 1 \right) \cos\left(\frac{\pi n t}{5}\right) + \left(\frac{4}{\pi n} + \frac{10}{\pi^2 n^2} \sin\left(\frac{8\pi n}{5}\right) \right) \sin\left(\frac{\pi n t}{5}\right) \right]$

e) $f(t) = \frac{17}{8} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2\pi n t}{T}\right) + b_n \sin\left(\frac{2\pi n t}{T}\right) \right]$ donde:

$$a_n = \frac{2\operatorname{sen}\left(\frac{\pi}{2}\right) + 3\operatorname{sen}\left(\frac{3\pi}{2}\right)}{\pi n} + \frac{10(\cos\left(\frac{\pi}{2}\right) - 1)}{\pi^2 n^2}; \quad b_n = \frac{-2\cos\left(\frac{\pi}{2}\right) - 3\cos\left(\frac{3\pi}{2}\right)}{\pi n} + \frac{10}{\pi^2 n^2} \operatorname{sen}\left(\frac{\pi}{2}\right)$$

f) $f(t) = \frac{19}{7} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{\pi n t}{7}\right) + b_n \operatorname{sen}\left(\frac{\pi n t}{7}\right) \right]$ donde:

$$a_n = \frac{-6\operatorname{sen}\left(\frac{2\pi}{7}\right) + 10\operatorname{sen}\left(\frac{4\pi}{7}\right) + 10\operatorname{sen}\left(\frac{6\pi}{7}\right) - 4\operatorname{sen}\left(\frac{10\pi}{7}\right)}{\pi n} - \frac{35(\cos\left(\frac{2\pi}{7}\right) - \cos\left(\frac{4\pi}{7}\right))}{3\pi^2 n^2}$$

$$b_n = \frac{6\cos\left(\frac{2\pi}{7}\right) + 10\cos\left(\frac{4\pi}{7}\right) - 10\cos\left(\frac{6\pi}{7}\right) + 4\cos\left(\frac{10\pi}{7}\right)}{\pi n} - \frac{35(\operatorname{sen}\left(\frac{2\pi}{7}\right) + \operatorname{sen}\left(\frac{4\pi}{7}\right))}{3\pi^2 n^2}$$

g) $f(t) = \frac{144}{11} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2\pi n t}{11}\right) + b_n \operatorname{sen}\left(\frac{2\pi n t}{11}\right) \right]$ donde:

$$a_n = \frac{\frac{121}{7} \cos\left(\frac{8\pi}{11}\right) - \frac{44}{7} \cos\left(\frac{6\pi}{11}\right) - 11 \cos\left(\frac{16\pi}{11}\right)}{\pi^2 n^2} + \frac{4}{\pi n} \left(\operatorname{sen}\left(\frac{8\pi}{11}\right) + \operatorname{sen}\left(\frac{6\pi}{11}\right) \right)$$

$$b_n = \frac{\frac{121}{7} \operatorname{sen}\left(\frac{8\pi}{11}\right) + \frac{44}{7} \operatorname{sen}\left(\frac{6\pi}{11}\right) - 11 \operatorname{sen}\left(\frac{16\pi}{11}\right)}{\pi^2 n^2} + \frac{4}{\pi n} \left(\cos\left(\frac{6\pi}{11}\right) - \cos\left(\frac{8\pi}{11}\right) \right)$$

3.- Dadas las funciones definidas en un período, graficarlas tres períodos y hallar sus series de Fourier

a) $f(t) = \begin{cases} 4t-5 & 0 < t < 1 \\ 3 & 1 < t < 2 \end{cases} T=2$ b) $f(t) = \begin{cases} -3t+1 & -1 < t < 0 \\ 2t+3 & 0 < t < 1 \end{cases} T=2$ c) $f(t) = \begin{cases} 5 & -\frac{T}{4} < t < \frac{T}{3} \\ -\frac{15}{T}t + 10 & \frac{T}{3} < t < \frac{2T}{3} \\ -5 & \frac{2T}{3} < t < \frac{3T}{4} \end{cases}$

d) $f(t) = \begin{cases} -2t^2 & 0 < t < 4 \\ 0 & 4 < t < 8 \end{cases} T=8$ e) $f(t) = \begin{cases} -5 & 0 < t < 2 \\ 0 & 2 < t < 4 \\ -\frac{t}{2} + 4 & 4 < t < 8 \end{cases} T=8$ f) $f(t) = \begin{cases} 1-t^2 & 0 < t < 3 \\ 10 & 3 < t < 6 \end{cases} T=6$

R.- a) $f(t) = \sum_{n=1}^{\infty} \left[\frac{4}{n^2 \pi^2} (-1 + \cos(\pi n)) \cos(\pi n t) + \frac{-8 + 4\cos(\pi n)}{\pi n} \operatorname{sen}(\pi n t) \right];$

b) $f(t) = \frac{13}{4} + \sum_{n=1}^{\infty} \left[\left(\frac{5(\cos(\pi n) - 1)}{\pi^2 n^2} \right) \cos(\pi n t) + \left(\frac{2 - \cos(\pi n)}{\pi n} \right) \operatorname{sen}(\pi n t) \right];$

c) $f(t) = \frac{10}{3} + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 t) + b_n \operatorname{sen}(n\omega_0 t)];$

$$a_n = \frac{-10\cos(\pi n) \operatorname{sen}\left(\frac{\pi}{2} n\right) + 5\operatorname{sen}\left(\frac{4\pi}{3} n\right)}{\pi n}; \quad b_n = \frac{10\cos(\pi n) \cos\left(\frac{\pi}{2} n\right) - 5\cos\left(\frac{4\pi}{3} n\right)}{\pi n} - \frac{15\operatorname{sen}\left(\frac{\pi}{3} n\right) \cos(\pi n)}{\pi^2 n^2}$$

d) $f(t) = -\frac{16}{3} + \sum_{n=1}^{\infty} \left[\frac{64}{\pi^2 n^2} \cos\left(\frac{\pi n t}{4}\right) + \left(\frac{32}{\pi n} \cos(\pi n) + \frac{64}{\pi^3 n^3} (1 - \cos(\pi n)) \right) \operatorname{sen}\left(\frac{\pi n t}{4}\right) \right];$

e) $f(t) = -\frac{3}{4} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{\pi}{4} nt\right) + b_n \operatorname{sen}\left(\frac{\pi}{4} nt\right) \right]$ donde:

$$a_n = -\frac{5}{\pi n} \operatorname{sen}\left(\frac{\pi}{2} n\right) + \frac{2}{\pi^2 n^2} (\cos(\pi n) - 1); b_n = \frac{5(\cos(\frac{\pi}{2} n) - 1) + 2\cos(\pi n)}{\pi n}$$

f) $f(t) = 4 + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{\pi n t}{3}\right) + b_n \operatorname{sen}\left(\frac{\pi n t}{3}\right) \right]$ donde:

$$a_n = -\frac{18}{\pi^2 n^2} \cos(\pi n); b_n = \frac{-9 + 18\cos(\pi n)}{\pi n} + \frac{18(1 - \cos(\pi n))}{\pi^3 n^3}$$