

**PRACTICA Nro. 3 – TRANSFORMADAS INTEGRALES  
SERIE COMPLEJA DE FOURIER Y ESPECTROS DISCRETOS**

1.- Graficar las funciones periódicas y determinar la serie compleja y a partir de ella la serie trigonométrica de Fourier.

$$a) f(t) = \begin{cases} t+4 & 0 < t < 2 \\ -2t+10 & 2 < t < 4 \end{cases} \quad T=4 \quad b) f(t) = \begin{cases} t^2-4 & 0 < t < 3 \\ 5 & 3 < t < 6 \end{cases} \quad T=6$$

$$c) f(t) = e^{-2t} \quad -2\pi < t < 2\pi \quad T=4\pi \quad d) f(t) = \begin{cases} e^{3t} & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases} \quad T=2$$

RESPUESTAS:

$$a) f(t) = \frac{9}{2} + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \left[ \left( \frac{3}{\pi^2 n^2} (\cos(\pi n) - 1) - \frac{j}{\pi n} \right) e^{\frac{j\pi n t}{2}} \right]; f(t) = \frac{9}{2} + \sum_{n=1}^{\infty} \left[ \left( \frac{6}{\pi^2 n^2} (\cos(\pi n) - 1) \cos\left(\frac{\pi n t}{2}\right) + \frac{2}{n\pi} \sin\left(\frac{\pi n t}{2}\right) \right) \right]$$

$$b) f(t) = 2 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \left[ \frac{9 \cos(\pi n)}{\pi^2 n^2} + j \left( \frac{9}{2\pi n} + \frac{9(1 - \cos(\pi n))}{\pi^3 n^3} \right) \right] e^{j \frac{\pi n t}{3}}$$

$$f(t) = 2 + \sum_{n=1}^{\infty} \left[ \frac{18 \cos(\pi n)}{\pi^2 n^2} \cos\left(\frac{\pi n t}{3}\right) + \left( \frac{18(\cos(\pi n) - 1)}{\pi^3 n^3} - \frac{9}{\pi n} \right) \sin\left(\frac{\pi n t}{3}\right) \right]$$

$$c) f(t) = \frac{e^{4\pi} - e^{-4\pi}}{8\pi} + \frac{e^{4\pi} - e^{-4\pi}}{\pi} \sum_{n=1}^{\infty} \left[ \frac{4 \cos(\pi n)}{n^2 + 16} \cos\left(\frac{nt}{2}\right) + \frac{n \cos(\pi n)}{n^2 + 16} \sin\left(\frac{nt}{2}\right) \right]$$

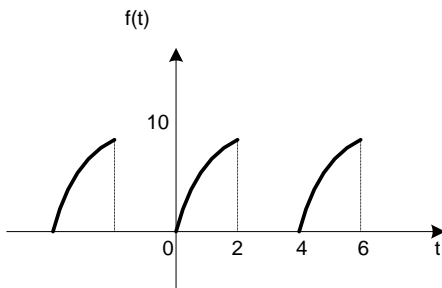
$$d) f(t) = \frac{e^3 - 1}{6} + \sum_{n=1}^{\infty} \left[ \left( \frac{3e^3 \cos(\pi n) - 3}{9 + \pi^2 n^2} \right) \cos(\pi n t) + \left( \frac{\pi(1 - e^3 \cos(\pi n))}{9 + \pi^2 n^2} \right) \sin(\pi n t) \right]$$

2.- Determinar la serie compleja y a partir de ella la serie trigonométrica de Fourier de las siguientes ondas senoidales rectificadas.

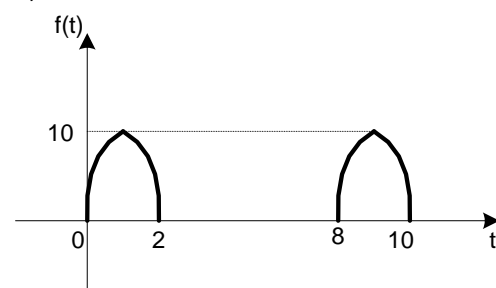
$$a) f(t) = \begin{cases} \sin\left(\frac{\pi t}{2}\right) & 0 < t < 2 \\ 0 & 2 < t < 4 \end{cases} \quad T=4$$

$$b) f(t) = |10 \sin(4t)|$$

c)



d)



## RESPUESTAS

$$a) f(t) = \frac{1}{\pi} + \frac{1}{2} \operatorname{sen}\left(\frac{\pi}{2}t\right) + \sum_{n=2}^{\infty} \left( \frac{1 + \cos(\pi n)}{\pi(1-n^2)} \right) \cos\left(\frac{\pi n t}{2}\right); b) f(t) = \frac{20}{\pi} + \sum_{n=1}^{\infty} \frac{40}{\pi(1-4n^2)} \cos(8nt)$$

$$c) f(t) = \frac{10}{\pi} + \sum_{n=1}^{\infty} \left[ \frac{20}{\pi(1-4n^2)} \cos\left(\frac{\pi n t}{2}\right) + \frac{40n \cos(\pi n)}{\pi(1-4n^2)} \operatorname{sen}\left(\frac{\pi n t}{2}\right) \right]$$

$$d) f(t) = \frac{5}{\pi} + \frac{5}{2} \operatorname{sen}\left(\frac{\pi}{2}t\right) + \sum_{\substack{n=1 \\ n \neq 2}}^{\infty} \left[ \frac{20(\cos(\frac{\pi n}{2})+1)}{\pi(4-n^2)} \cos\left(\frac{\pi n t}{4}\right) + \frac{20 \operatorname{sen}(\frac{\pi n}{2})}{\pi(1-4n^2)} \operatorname{sen}\left(\frac{\pi n t}{4}\right) \right]$$

3.- Expresar las siguientes funciones en términos de la función escalón unitario  $u(t)$ :

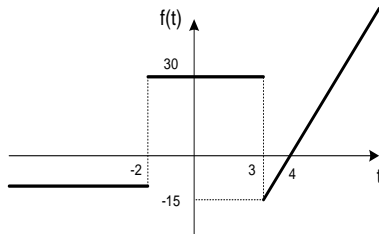
$$a) f(t) = \begin{cases} -4 & -7 < t < -2 \\ 4 & -2 < t < 1 \\ 6 & 1 < t < 5 \end{cases}$$

$$b) f(t) = \begin{cases} -5t & -5 < t < -2 \\ 4t+1 & -2 < t < 0 \\ -3t+6 & 0 < t < 2 \end{cases}$$

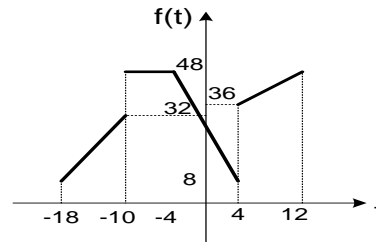
$$c) f(t) = \begin{cases} -4 & t < -2 \\ 5 & -2 < t < 4 \\ -10 & t > 4 \end{cases}$$

$$d) f(t) = \begin{cases} 2-t & t < 3 \\ t-4 & 3 < t < 10 \\ 0 & 10 < t < 12 \\ 2t-10 & t > 12 \end{cases}$$

e)



f)



## RESPUESTAS

$$a) f(t) = -4u(t+7) + 8u(t+2) + 2u(t-1) - 6u(t-5);$$

$$b) f(t) = -5tu(t+5) + (9t+1)u(t+2) + (-7t+5)u(t) + (3t-6)u(t-2); c) f(t) = -4 + 9u(t+2) - 15u(t-5);$$

$$d) f(t) = 2 - t + (2t-6)u(t-3) + (-t+4)u(t-10) + (2t-10)u(t-12)$$

$$e) f(t) = -15 + 45u(t+2) + (15t-90)u(t-3);$$

$$f) f(t) = (3t+62)u(t+18) + (-3t-14)u(t+10) + (-5t-20)u(t+4) + \left(\frac{13}{2}t+2\right)u(t-4) + \left(-\frac{3}{2}t-30\right)u(t-12)$$

4.- Evaluar las integrales con función impulso:

$$a) \int_{-\infty}^{\infty} \left( \frac{t^2-1}{2t+5} \right) \delta(t+4) dt$$

$$b) \int_{-\infty}^{\infty} t \cos(2t) \delta(t - \frac{\pi}{6}) dt$$

$$c) \int_{-\infty}^{\infty} (2t-1) \delta(3t-4) dt$$

$$d) \int_{-\infty}^{\infty} \delta'(t-2) e^{2t} \ln t dt$$

$$e) \int_{-\infty}^{\infty} \delta'(t-1) t^2 e^{-3t} \cos(4t) dt$$

$$f) \int_{-\infty}^{\infty} \delta''(t - \frac{1}{2}) t^3 \ln(2t) dt$$

RESPUESTAS

a)  $-5$ ; b)  $\frac{\pi}{12}$ ; c)  $\frac{5}{9}$ ; d)  $102.99$ ; e)  $e^{-3}(\cos 4 + 4\operatorname{sen} 4)$ ; f)  $\frac{5}{2}$

5.- Aplicando diferenciación, determine las series de Fourier de las funciones definidas en un período y graficarlas:

$$\begin{aligned} \text{a) } f(t) &= \begin{cases} -3t+1 & -1 < t < 0 \\ 2t+3 & 0 < t < 1 \end{cases} \quad T=2 & \text{b) } f(t) &= \begin{cases} 2 & 0 < t < \frac{T}{2} \\ \frac{3}{T}t + \frac{3}{2} & \frac{T}{2} < t < T \end{cases} \\ \text{c) } f(t) &= \begin{cases} \frac{t^2}{5} + 1 & 0 < t < 5 \\ -4 & 5 < t < 10 \end{cases} \quad T=10 & \text{d) } f(t) &= \begin{cases} -5t+10 & 0 < t < 2 \\ \frac{t^2}{4} + 4 & 2 < t < 4 \end{cases} \quad T=4 \\ \text{e) } f(t) &= \begin{cases} t^2 & 0 < t < 2 \\ -2t+8 & 2 < t < 6 \\ -1 & 6 < t < 8 \end{cases} \quad T=8 & \text{f) } f(t) &= \begin{cases} 1-t^2 & 0 < t < 1 \\ 5t-2 & 1 < t < 2 \\ 3 & 2 < t < 4 \end{cases} \quad T=4 \\ \text{g) } f(t) &= \begin{cases} t^3+1 & 0 < t < 2 \\ 2t-8 & 2 < t < 4 \end{cases} \quad T=4 & \text{h) } f(t) &= \begin{cases} t^3+1 & 0 < t < 2 \\ 12-t^2 & 2 < t < 4 \end{cases} \quad T=4 \\ \text{i) } f(t) &= \begin{cases} \frac{t^3}{5} + 3 & 0 < t < 5 \\ 2 & 5 < t < 8 \end{cases} \quad T=8 \end{aligned}$$

RESPUESTAS

$$\begin{aligned} \text{a) } f(t) &= \frac{13}{4} + \sum_{n=1}^{\infty} \left[ \left( \frac{5(\cos(\pi n) - 1)}{\pi^2 n^2} \right) \cos(\pi n t) + \left( \frac{2 - \cos(\pi n)}{\pi n} \right) \operatorname{sen}(\pi n t) \right] \\ \text{b) } f(t) &= \frac{23}{8} + \sum_{n=1}^{\infty} \left[ \left( \frac{6(1 - \cos(\pi n))}{\pi^2 n^2} \right) \cos\left(\frac{2\pi n}{T} t\right) + \left( \frac{-5 + 2\cos(\pi n)}{\pi n} \right) \operatorname{sen}\left(\frac{2\pi n}{T} t\right) \right] \\ \text{c) } f(t) &= -\frac{2}{3} + \sum_{n=1}^{\infty} \left[ \frac{10}{\pi^2 n^2} \cos(\pi n) \cos\left(\frac{\pi n t}{5}\right) + \left( \frac{5 - 10\cos(\pi n)}{\pi n} + \frac{10(\cos(\pi n) - 1)}{\pi^3 n^3} \right) \operatorname{sen}\left(\frac{\pi n t}{5}\right) \right] \\ \text{d) } f(t) &= \frac{17}{3} + \sum_{n=1}^{\infty} \left[ \frac{14 - 12\cos(\pi n)}{\pi^2 n^2} \cos\left(\frac{\pi n t}{2}\right) + \left( \frac{2 + 5\cos(\pi n)}{\pi n} - \frac{2 - 2\cos(\pi n)}{\pi^3 n^3} \right) \operatorname{sen}\left(\frac{\pi n t}{2}\right) \right] \\ \text{e) } f(t) &= \frac{1}{12} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{\pi n t}{4}\right) + b_n \operatorname{sen}\left(\frac{\pi n t}{4}\right) \right] \text{ donde:} \\ a_n &= -\frac{3}{\pi n} \operatorname{sen}\left(\frac{3\pi n}{2}\right) + \frac{24\cos(\frac{\pi n}{2}) - 8\cos(\frac{3\pi n}{2})}{\pi^2 n^2} - \frac{32}{\pi^3 n^3} \operatorname{sen}\left(\frac{\pi n}{2}\right) \\ b_n &= \frac{1 + 3\cos(\frac{\pi n}{2})}{\pi n} + \frac{24\operatorname{sen}(\frac{\pi n}{2}) - 8\operatorname{sen}(\frac{3\pi n}{2})}{\pi^2 n^2} - \frac{32}{\pi^3 n^3} \left( 1 - \cos\left(\frac{\pi n}{2}\right) \right) \end{aligned}$$

f)  $f(t) = \frac{73}{24} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{\pi n t}{2}\right) + b_n \operatorname{sen}\left(\frac{\pi n t}{2}\right) \right]$  donde:

$$a_n = -\frac{3}{\pi n} \operatorname{sen}\left(\frac{\pi n}{2}\right) + \frac{-14 \cos\left(\frac{\pi n}{2}\right) + 10 \cos(\pi n)}{\pi^2 n^2} + \frac{8}{\pi^3 n^3} \operatorname{sen}\left(\frac{\pi n}{2}\right)$$

$$b_n = -\frac{2}{\pi n} + \frac{3}{\pi n} \cos\left(\frac{\pi n}{2}\right) - \frac{5}{\pi n} \cos(\pi n) - \frac{14}{\pi^2 n^2} \operatorname{sen}\left(\frac{\pi n}{2}\right) + \frac{8}{\pi^3 n^3} \left(1 - \cos\left(\frac{\pi n}{2}\right)\right)$$

g)  $f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{\pi n t}{2}\right) + b_n \operatorname{sen}\left(\frac{\pi n t}{2}\right) \right]$  donde:

$$a_n = \frac{4 + 20 \cos(\pi n)}{\pi^2 n^2} + \frac{48(1 - \cos(\pi n))}{\pi^4 n^4}; \quad b_n = \frac{1 - 13 \cos(\pi n)}{\pi n} + \frac{48 \cos(\pi n)}{\pi^3 n^3}$$

h)  $f(t) = \frac{17}{6} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{\pi n t}{4}\right) + b_n \operatorname{sen}\left(\frac{\pi n t}{4}\right) \right]$ , donde:

$$a_n = \frac{-16 + 32 \cos(\pi n)}{\pi^2 n^2} + \frac{48 - 48 \cos(\pi n)}{\pi^4 n^4}; \quad b_n = \frac{5 - \cos(\pi n)}{\pi n} + \frac{-8 + 56 \cos(\pi n)}{\pi^3 n^3}$$

i)  $f(t) = \frac{209}{32} + \sum_{n=1}^{\infty} \left[ a_n \cos\left(\frac{\pi n t}{4}\right) + b_n \operatorname{sen}\left(\frac{\pi n t}{4}\right) \right]$ , donde:

$$a_n = \frac{26}{\pi n} \operatorname{sen}\left(\frac{5\pi n}{4}\right) + \frac{60}{\pi^2 n^2} \cos\left(\frac{5\pi n}{4}\right) - \frac{96}{\pi^3 n^3} \operatorname{sen}\left(\frac{5\pi n}{4}\right) + \frac{384}{5\pi^4 n^4} \left(1 - \cos\left(\frac{5\pi n}{4}\right)\right)$$

$$b_n = \frac{2}{\pi n} \left(1 - 13 \cos\left(\frac{5\pi n}{4}\right)\right) + \frac{60}{\pi^2 n^2} \operatorname{sen}\left(\frac{5\pi n}{4}\right) + \frac{96}{\pi^3 n^3} \cos\left(\frac{5\pi n}{4}\right) - \frac{384}{5\pi^4 n^4} \operatorname{sen}\left(\frac{5\pi n}{4}\right);$$

6.- Graficar los espectros de amplitud y fase de las funciones periódicas:

a)  $f(t) = 5t \quad 0 < t < 3; T=3$

b)  $f(t) = t + 1 \quad 0 < t < 5; T=5$

7.- Dada la función definida en un período:  $f(t) = \frac{1}{2} - \frac{t}{T} \dots \dots 0 < t < T$  a partir del teorema de

Parseval, demostrar que:  $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$

8.- Dada la función definida en un período:  $f(t) = \begin{cases} -1 & -\frac{T}{2} < t < 0 \\ 1 & 0 < t < \frac{T}{2} \end{cases}$  a partir del teorema de Parseval

demostrar:  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

9.- Dada la función definida en un período  $T=2\pi$ :  $f(t) = t^2 \dots -\pi < t < \pi$  aplicando el teorema de

Parseval demostrar:  $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

10.- Demostrar a partir de la función definida en un período:  $f_{(t)} = \begin{cases} -\frac{2}{T}t & -\frac{T}{2} < t < 0 \\ \frac{2}{T}t & 0 < t < \frac{T}{2} \end{cases}$  mediante el

teorema de Parseval:  $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$