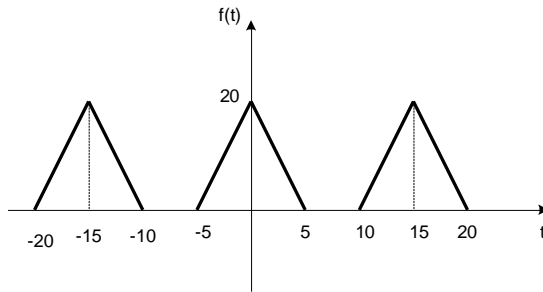


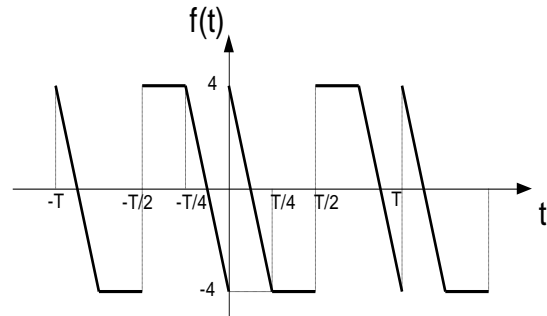
PRACTICA Nro. 2 – TRANSFORMADAS INTEGRALES
ANALISIS DE FORMAS DE ONDA PERIODICAS

1.- Determine las series de Fourier de las funciones, considerando si tienen simetría par o impar.

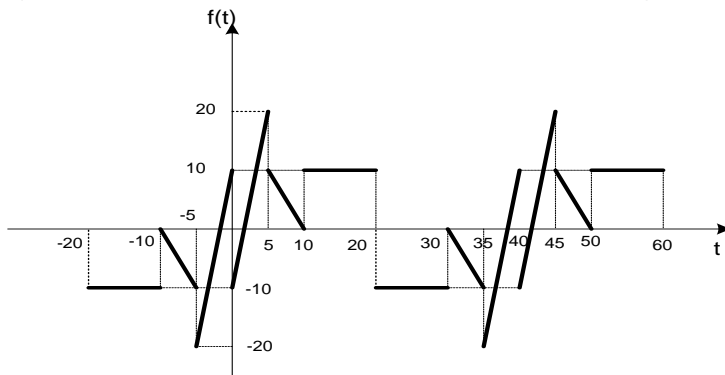
a)



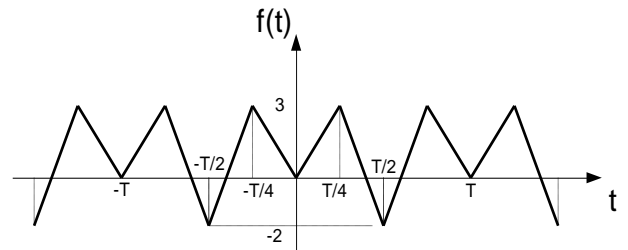
b)



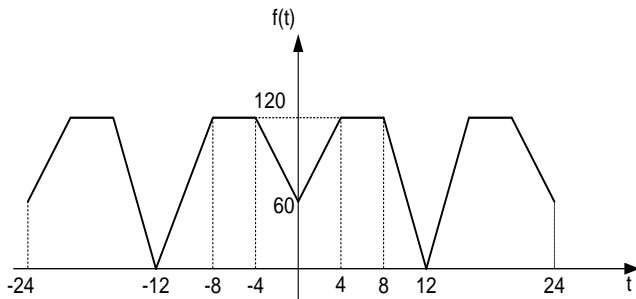
c)



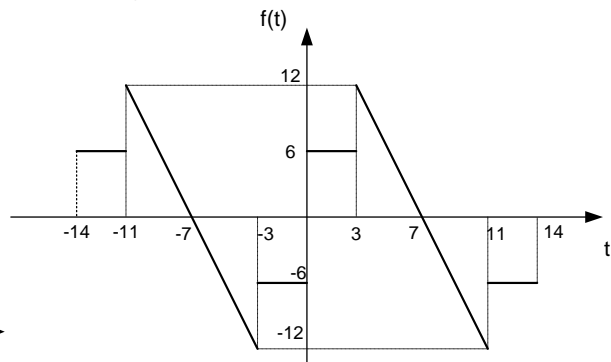
d)



e)



f)



RESPUESTAS

a) Simetría par:
$$f(t) = \frac{20}{3} + \sum_{n=1}^{\infty} \frac{60}{n^2 \pi^2} \left[1 - \cos\left(\frac{2\pi n}{3}\right) \right] \cos\left(\frac{2\pi n t}{15}\right)$$

b) Simetría impar:
$$f(t) = \sum_{n=1}^{\infty} \left[\frac{8}{\pi n} (1 + (-1)^n) - \frac{32}{n^2 \pi^2} \operatorname{sen}\left(\frac{\pi n}{2}\right) \right] \operatorname{sen}\left(\frac{2\pi n t}{T}\right)$$

c) Simetría impar: $f(t) = \sum_{n=1}^{\infty} \left[b_n \operatorname{sen}\left(\frac{\pi n t}{20}\right) \right];$

$$b_n = \frac{-20 \cos\left(\frac{\pi}{4}\right) - 20 + 20 \cos\left(\frac{\pi}{2}\right) - 20 \cos(\pi)}{\pi n} + \frac{320 \operatorname{sen}\left(\frac{\pi}{4}\right) - 80 \operatorname{sen}\left(\frac{\pi}{2}\right)}{\pi^2 n^2}$$

d) Simetría par: $f(t) = 1 + \sum_{n=1}^{\infty} \left[\frac{32 \cos\left(\frac{\pi}{2}\right) - 12 - 20 \cos(\pi)}{\pi^2 n^2} \right] \cos\left(\frac{2 \pi n t}{T}\right)$

e) simetría par

$$f(t) = 90 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi n t}{12}\right); a_n = \frac{360(\cos\left(\frac{\pi}{3}\right) - 1) + 720(\cos\left(\frac{2\pi}{3}\right) - \cos(\pi))}{\pi^2 n^2}$$

f) Simetría impar:

$$f(t) = \sum_{n=1}^{\infty} \left[\frac{12}{\pi n} \left(\cos\left(\frac{3\pi n}{7}\right) + 1 \right) + \frac{42}{\pi^2 n^2} \operatorname{sen}\left(\frac{3\pi n}{7}\right) \right] \operatorname{sen}\left(\frac{\pi n t}{7}\right)$$

2.- Dadas las funciones definidas en un período, gráfíquelas y determine sus series de Fourier considerando si tienen simetría par o impar.

a) $f(t) = \begin{cases} 1 & -4 < t < -2 \\ t^2 - 4 & -2 < t < 2 \\ 1 & 2 < t < 4 \end{cases}$

b) $f(t) = \begin{cases} 3 & 0 < t < \frac{T}{4} \\ -\frac{12}{T}t + 6 & \frac{T}{4} < t < \frac{3T}{4} \\ -3 & \frac{3T}{4} < t < T \end{cases}$

c) $f(t) = \begin{cases} -2 & -\frac{T}{2} < t < -\frac{T}{4} \\ \frac{16}{T}t - 2 & -\frac{T}{4} < t < 0 \\ \frac{16}{T}t + 2 & 0 < t < \frac{T}{4} \\ 2 & \frac{T}{4} < t < \frac{T}{2} \end{cases}$

d) $f(t) = \begin{cases} \frac{3}{2}t + \frac{27}{2} & -6 < t < -3 \\ t^2 & -3 < t < 3 \\ -\frac{3}{2}t + \frac{27}{2} & 3 < t < 6 \end{cases}$

RESPUESTAS

a) Simetría par

$$f(t) = -\frac{5}{6} + \sum_{n=1}^{\infty} \left[\frac{32}{\pi^2 n^2} \cos\left(\frac{\pi n}{2}\right) - \frac{64}{\pi^3 n^3} \operatorname{sen}\left(\frac{\pi n}{2}\right) - \frac{2}{\pi n} \operatorname{sen}\left(\frac{\pi n}{2}\right) \right] \cos\left(\frac{\pi n t}{4}\right)$$

b) Simetría impar

$$f(t) = \sum_{n=1}^{\infty} \left[\left[\frac{1}{n \pi} + \frac{12 \operatorname{sen}(\pi n / 2)}{\pi^2 n^2} \right] \operatorname{sen}\left(\frac{2 \pi n t}{T}\right) \right]$$

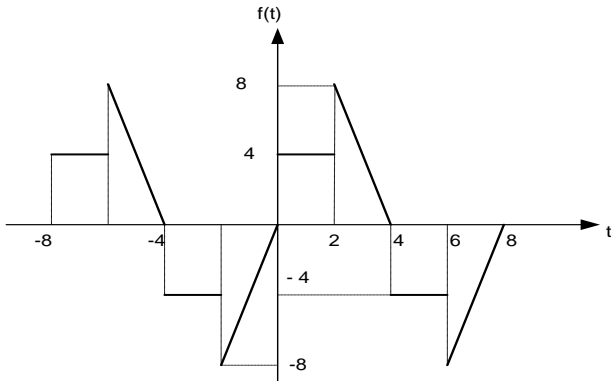
c) Simetría impar: $f(t) = \sum_{n=1}^{\infty} \left[\frac{-8 \cos\left(\frac{\pi}{2}\right) + 4 - 4 \cos(\pi)}{\pi n} + \frac{16}{\pi^2 n^2} \operatorname{sen}\left(\frac{\pi n}{2}\right) \right] \operatorname{sen}\left(\frac{2 \pi n t}{T}\right)$

d) Simetría par

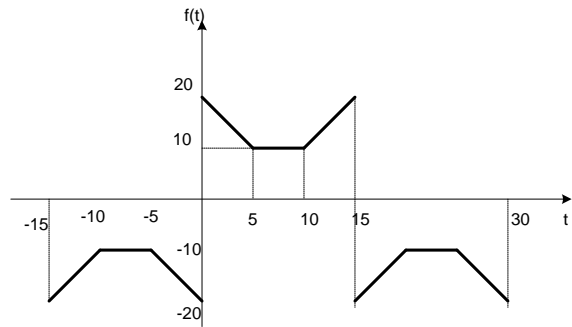
$$f(t) = \frac{39}{8} + \sum_{n=1}^{\infty} \left[\left[\frac{90 \cos(\pi n / 2)}{n \pi} - \frac{144 \operatorname{sen}(\pi n / 2)}{\pi^3 n^3} - \frac{18 \cos(\pi n)}{\pi^2 n^2} \right] \cos\left(\frac{\pi n t}{6}\right) \right]$$

3.- Hallar la serie de Fourier de las funciones considerando simetrías de media y de cuarto de onda:

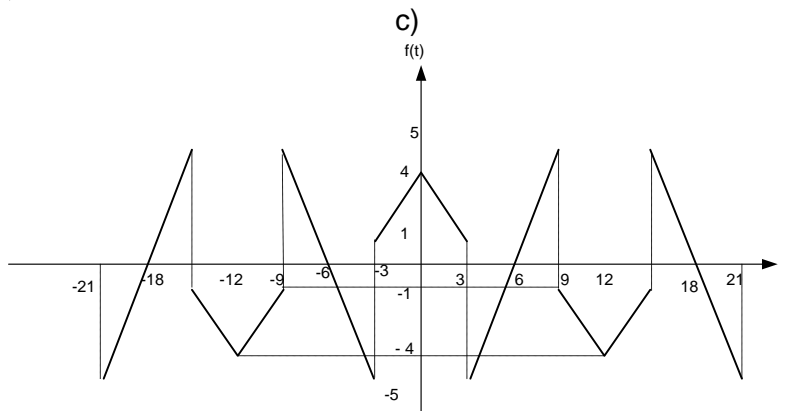
a)



b)



c)



RESPUESTAS:

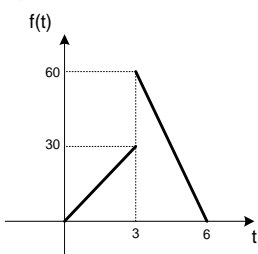
$$a) f(t) = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[\left(\frac{-8}{\pi n} \operatorname{sen}\left(\frac{\pi n}{2}\right) + \frac{32}{\pi^2 n^2} \right) \cos\left(\frac{\pi n t}{4}\right) + \left(\frac{8}{\pi n} + \frac{32}{\pi^2 n^2} \operatorname{sen}\left(\frac{\pi n}{2}\right) \right) \operatorname{sen}\left(\frac{\pi n t}{4}\right) \right]$$

$$b) f(t) = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[-\frac{120}{\pi^2 n^2} \operatorname{sen}\left(\frac{\pi n}{3}\right) + \frac{80}{\pi n} \right] \operatorname{sen}\left(\frac{\pi n t}{15}\right)$$

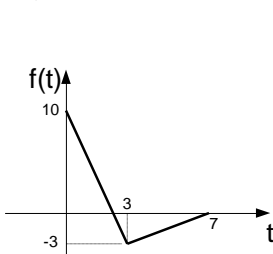
$$c) f(t) = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[\frac{24}{\pi n} \operatorname{sen}\left(\frac{\pi n}{4}\right) + \frac{-128 \cos\left(\frac{\pi n}{4}\right) + 48}{\pi^2 n^2} \right] \cos\left(\frac{\pi n t}{12}\right)$$

4.- Dadas las funciones definidas en un intervalo finito hallar sus series de Fourier y expandirlas periódicamente para simetría par e impar.

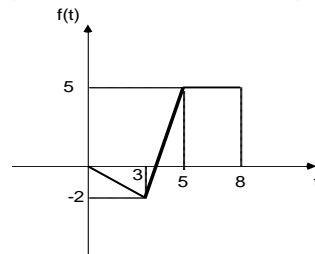
a)



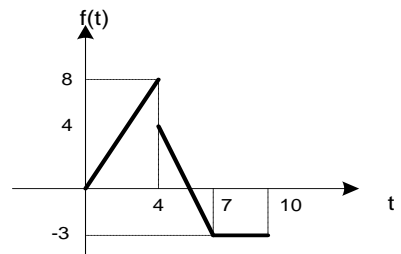
b)



c)

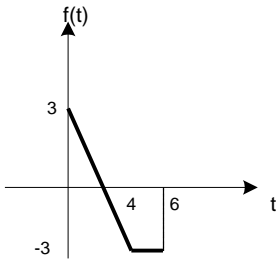


d)

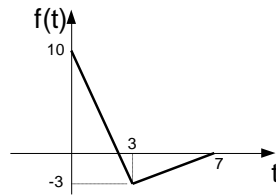


5.- Expandir las funciones con simetría de media onda grafique tres períodos y determine su serie de Fourier con 3 armónicos diferentes de cero:

a)



b)

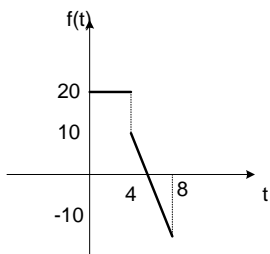


$$R.- a) f(t) = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[\frac{18}{\pi^2 n^2} \left(1 - \cos\left(\frac{2\pi n}{3}\right) \right) \cos\left(\frac{\pi n t}{6}\right) - \frac{18}{\pi^2 n^2} \operatorname{sen}\left(\frac{2\pi n}{3}\right) \operatorname{sen}\left(\frac{\pi n t}{6}\right) \right]$$

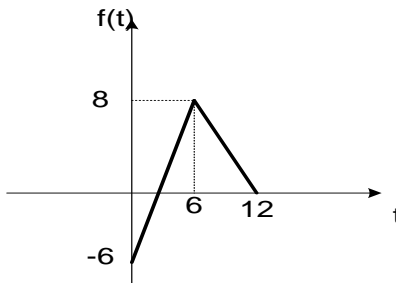
$$f(t) = 2.74 \cos\left(\frac{\pi t}{6}\right) - 1.58 \operatorname{sen}\left(\frac{\pi t}{6}\right) + 0.109 \cos\left(\frac{5\pi t}{6}\right) + 0.0631 \operatorname{sen}\left(\frac{5\pi t}{6}\right) + 0.056 \cos\left(\frac{7\pi t}{6}\right) - 0.032 \operatorname{sen}\left(\frac{7\pi t}{6}\right) + \dots$$

6.- Expandir las funciones con simetría de cuarto de onda par e impar, grafique dos períodos y determinar su serie de Fourier con 3 armónicos distintos de cero.

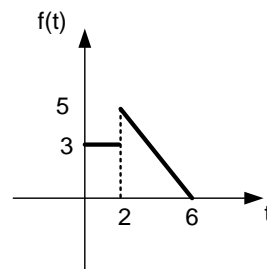
a)



b)



c)



RESPUESTAS:

a) Para SCO par:

$$f(t) = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[\frac{40}{\pi n} \left(\operatorname{sen}\left(\frac{\pi n}{4}\right) - \operatorname{sen}\left(\frac{\pi n}{2}\right) \right) + \frac{320}{\pi^2 n^2} \cos\left(\frac{\pi n}{4}\right) \right] \cos\left(\frac{\pi n t}{16}\right)$$

$$f(t) = 19.197 \cos\left(\frac{\pi t}{16}\right) + 4.698 \cos\left(\frac{3\pi t}{16}\right) - 5.264 \cos\left(\frac{5\pi t}{16}\right) + \dots$$

Para SCO impar:

$$f(t) = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[\frac{-40 \cos\left(\frac{\pi n}{4}\right) + 80}{\pi n} + \frac{320 \left(\operatorname{sen}\left(\frac{\pi n}{4}\right) - \operatorname{sen}\left(\frac{\pi n}{2}\right) \right)}{\pi^2 n^2} \right] \operatorname{sen}\left(\frac{\pi n t}{16}\right)$$

$$f(t) = 6.965 \operatorname{sen}\left(\frac{\pi t}{16}\right) + 17.639 \operatorname{sen}\left(\frac{3\pi t}{16}\right) + 4.68 \operatorname{sen}\left(\frac{5\pi t}{16}\right) + \dots$$

b) Para SCO par: $f_{(t)} = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[\frac{352\cos\left(\frac{\pi n}{4}\right) - 244}{\pi^2 n^2} \right] \cos\left(\frac{\pi n t}{24}\right)$

Para SCO impar: $f_{(t)} = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[\frac{352\text{sen}\left(\frac{\pi n}{4}\right) - 128}{\pi^2 n^2} - \frac{24}{\pi n} \right] \text{sen}\left(\frac{\pi n t}{24}\right)$

c) Para SCO par: $f_{(t)} = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[-\frac{8}{\pi n} \text{sen}\left(\frac{\pi n}{6}\right) + \frac{60}{\pi^2 n^2} \cos\left(\frac{\pi n}{6}\right) \right] \cos\left(\frac{\pi n t}{12}\right)$

$$f_{(t)} = 3.992\cos\left(\frac{\pi t}{12}\right) - 0.849\cos\left(\frac{\pi t}{4}\right) - 0.465\cos\left(\frac{5\pi t}{12}\right) + 0.074\cos\left(\frac{7\pi t}{12}\right) + \dots$$

Para SCO impar: $f_{(t)} = \sum_{\substack{n=1 \\ n:\text{impar}}}^{\infty} \left[\frac{8\cos\left(\frac{\pi n}{6}\right) + 12}{\pi n} - \frac{60}{\pi^2 n^2} \left(\text{sen}\left(\frac{\pi n}{2}\right) - \text{sen}\left(\frac{\pi n}{6}\right) \right) \right] \text{sen}\left(\frac{\pi n t}{12}\right)$