

TRANSFORMADA DE *FOURIER*:

$$\mathcal{F}\{f(t)\} = F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t} dt$$

$$F(\omega) = R(\omega) + jX(\omega)$$

$$|F(\omega)| = \sqrt{R^2(\omega) + X^2(\omega)}$$

$$\Theta(\omega) = \arctan\left(\frac{X(\omega)}{R(\omega)}\right)$$

TRANSFORMADA INVERSA DE *FOURIER*:

$$\mathcal{F}\{f(t)\} = F(\omega) \rightarrow \mathcal{F}^{-1}\{F(\omega)\} = f(t)$$

TABLA DE TRANSFORMADAS DE *FOURIER*:

| $f(t)$ | $F(\omega) = \mathcal{F}\{f(t)\}$ |
|--------------------------------------|---|
| 1 $u(t+a) - u(t-a)$ | $\frac{2 \operatorname{sen}(a\omega)}{\omega}$ |
| 2 $\frac{\operatorname{sen}(at)}{t}$ | $\pi[u(\omega+a) - u(\omega-a)]$ |
| 3 $e^{-at}u(t) \quad a > 0$ | $\frac{1}{a + j\omega}$ |
| 4 $e^{at}u(-t) \quad a > 0$ | $\frac{1}{a - j\omega}$ |
| 5 $e^{-a t } \quad a > 0$ | $\frac{a^2 + \omega^2}{\pi} e^{-a \omega }$ |
| 6 $\frac{1}{t^2 + a^2}$ | $\frac{\pi}{a} e^{-a \omega }$ |
| 7 $\delta(t-a)$ | $e^{-j\omega a}$ |
| 8 e^{jat} | $2\pi\delta(\omega-a)$ |
| 9 k | $2\pi k\delta(\omega)$ |
| 10 $\operatorname{sen}(at)$ | $j\omega[\delta(\omega+a) - \delta(\omega-a)]$ |
| 11 $\cos(at)$ | $\pi[\delta(\omega+a) + \delta(\omega-a)]$ |
| 12 $t^n e^{-at}u(t)$ | $\frac{n!}{(a + j\omega)^{n+1}} \quad n \in \mathbb{N}$ |
| 13 $u(t)$ | $\frac{1}{j\omega} + \pi\delta(\omega)$ |
| 14 $\operatorname{sgn}(t)$ | $\frac{2}{j\omega}$ |
| 15 $ t $ | $-\frac{2}{\omega^2}$ |
| 16 $\frac{1}{t}$ | $-j\pi \operatorname{sgn}(\omega)$ |
| 17 $\frac{1}{t^n}$ | $\frac{j^n \pi \omega^{n-1} \operatorname{sgn}(\omega)}{(-1)^n (n-1)!}$ |

PROPIEDADES DE LA TRANSFORMADA DE *FOURIER*:

| | | |
|---|----------------------------|---|
| 1 | Linealidad | $\mathcal{F}\{a_1 f_1(t) + a_2 f_2(t)\} = a_1 F_1(\omega) + a_2 F_2(\omega)$ |
| 2 | Cambio de escala | $\mathcal{F}\{f(at)\} = \frac{1}{ a } F\left(\frac{\omega}{a}\right)$ |
| 3 | Desplazamiento en ω | $\mathcal{F}\{f(t)e^{jat}\} = F(\omega - a)$ |
| 4 | Desplazamiento en t | $\mathcal{F}\{f(t-a)\} = F(\omega)e^{-j\omega a}$ |
| 5 | Simetría | $\mathcal{F}\{F(t)\} = 2\pi f(-\omega)$ |
| 6 | Multiplicación | $\mathcal{F}\{t^n f(t)\} = j^n \frac{d^{(n)} F(\omega)}{d\omega^n}; \quad n \in \mathbb{N}$ |
| 7 | Derivada | $\mathcal{F}\{f^{(n)}(t)\} = (j\omega)^n F(\omega); \quad n \in \mathbb{N}$ |

PROPIEDADES DE LA TRANSFORMADA INVERSA DE *FOURIER*

| | | |
|---|----------------------------|---|
| 1 | Linealidad | $\mathcal{F}^{-1}\{a_1 F_1(\omega) + a_2 F_2(\omega)\} = a_1 f_1(t) + a_2 f_2(t)$ |
| 2 | Desplazamiento en ω | $\mathcal{F}^{-1}\{F(\omega-a)\} = f(t)e^{jat}$ |
| 3 | Desplazamiento en t | $\mathcal{F}^{-1}\{F(\omega)e^{-j\omega a}\} = f(t-a)$ |

TABLA DE TRANSFORMADAS INVERSAS DE *FOURIER*:

| $F(\omega)$ | $f(t) = \mathcal{F}^{-1}\{F(\omega)\}$ |
|--|--|
| 1 $\frac{1}{a + j\omega}$ | $e^{-at}u(t) \quad a > 0$ |
| 2 $\frac{1}{a - j\omega}$ | $e^{at}u(-t) \quad a > 0$ |
| 3 $\frac{2a}{a^2 + \omega^2}$ | $e^{-a t } \quad a > 0$ |
| 4 $\frac{1}{\omega} \operatorname{sen}(a\omega)$ | $\frac{1}{2}[u(t+a) - u(t-a)]$ |
| 5 k | $k\delta(t)$ |
| 6 $\frac{1}{\omega}$ | $\frac{1}{2}j \operatorname{sgn}(t)$ |

FUNCIÓN SIGNO:

$$\operatorname{sgn}'(t) = 2\delta(t)$$

$$|t|' = \operatorname{sgn}(t)$$

$$\operatorname{sgn}^2(t) = 1$$

CONVOLUCIÓN:

$$f(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau)f_2(t-\tau)d\tau$$

TRANSFORMADA DE *FOURIER* Y CONVOLUCIÓN:

$$\mathcal{F}\{f_1(t) * f_2(t)\} = F_1(\omega)F_2(\omega)$$

$$\mathcal{F}^{-1}\{F_1(\omega)F_2(\omega)\} = f_1(t) * f_2(t)$$

PROPIEDADES DE LA CONVOLUCIÓN:

| | | |
|---|--------------------------|--|
| 1 | Conmutatividad | $f_1(t) * f_2(t) = f_2(t) * f_1(t)$ |
| 2 | Asociatividad | $f_1(t) * [f_2(t) * f_3(t)] = [f_1(t) * f_2(t)] * f_3(t)$ |
| 3 | Distributividad | $f_1(t) * [f_2(t) + f_3(t)] = f_1(t) * f_2(t) + f_1(t) * f_3(t)$ |
| 4 | Función impulso | $f_1(t) * \delta(t-t_0) = f_1(t-t_0)$ |
| 5 | Función escalón unitario | $[f_1(t)u(t)] * [f_2(t)u(t)] = \int_0^t f_1(\tau)f_2(t-\tau)d\tau$ |

FUNCIONES TRIGONOMÉTRICAS DE ARCO DOBLE:

$$\operatorname{sen}(2x) = 2 \operatorname{sen}(x) \cos(x)$$

$$\cos(2x) = 2 \cos^2(x) - 1$$

$$\cos(2x) = 1 - 2 \operatorname{sen}^2(x)$$

FUNCIONES TRIGONOMÉTRICAS DE ARCO TRIPLE:

$$\operatorname{sen}(3x) = 3 \operatorname{sen}(x) - 4 \operatorname{sen}^3(x)$$

$$\cos(3x) = 4 \cos^3(x) - 3 \cos(x)$$

ECUACIONES DIFERENCIALES ORDINARIAS:

$$\mathcal{F}\{f'(t)\} = j\omega F(\omega)$$

$$\mathcal{F}\{f''(t)\} = (j\omega)^2 F(\omega)$$

$$\mathcal{F}\{f^{(n)}(t)\} = (j\omega)^n F(\omega)$$