CAP VIII.- TENSIONES COMBINADAS

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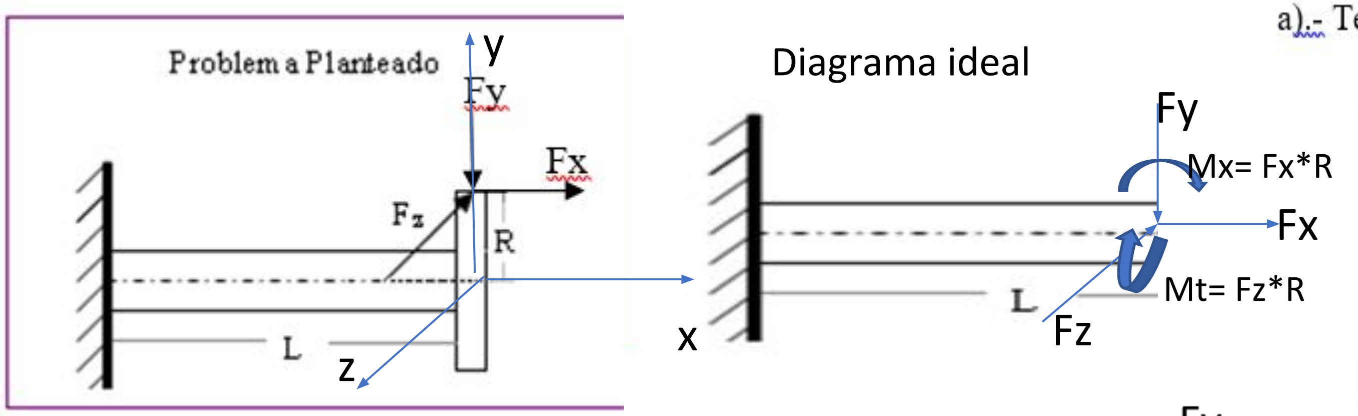
Dpto de: Ingeniería Mecánica

FCyT- UMSS

FLEXO – TRACSO – TORSION EN EL ESPACIO EN VIGAS DE SECCION CIRCULAR

EJERCICIO 1.-



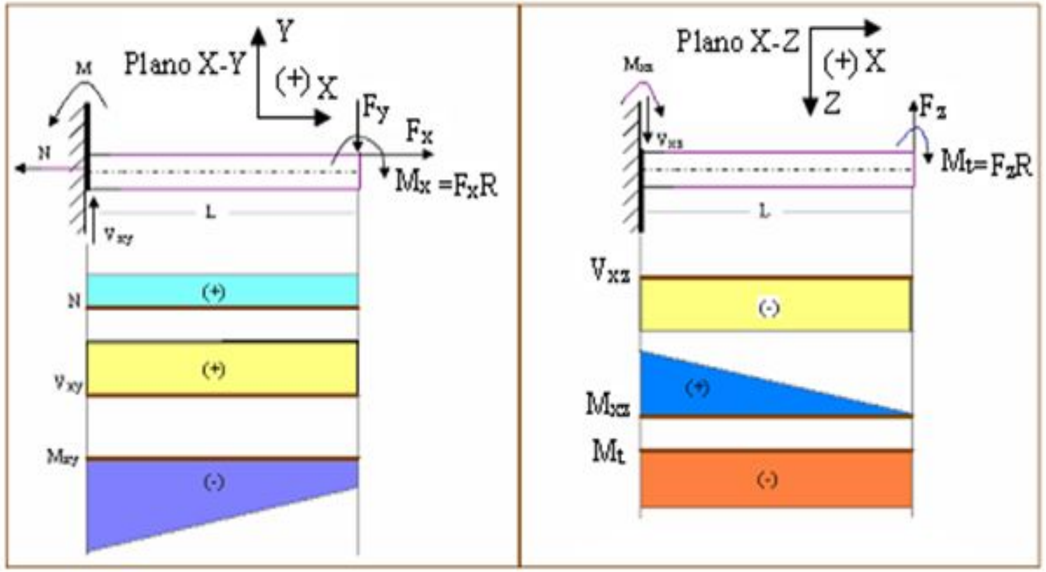
a).- Tensiones resultantes debido a la flexión:

Fy

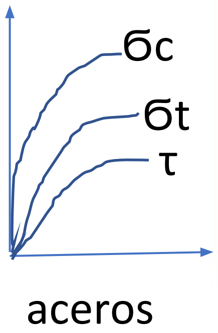
Diagramas de esfuerzos normales – cortantes, momentos flectores y torsores en cada plano:

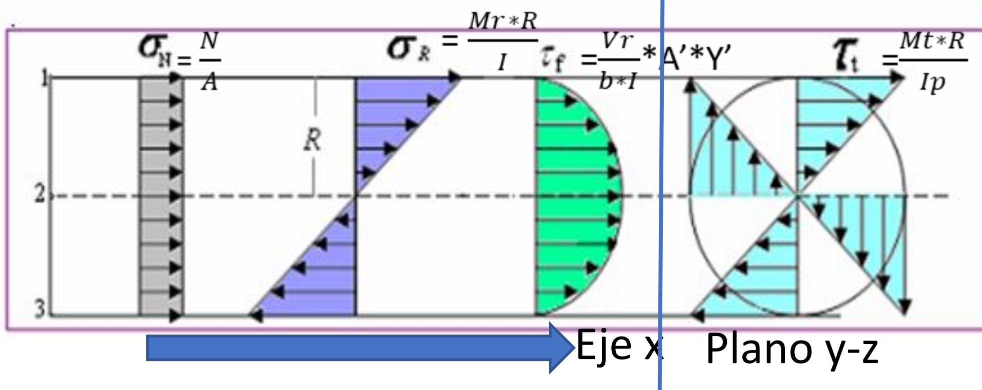
Como para **secciones circulares** **Iyy=Izz=Irr, entonces**

$$\boxed{M_R = \sqrt{M_{XY}^2 + M_{XZ}^2}} \Rightarrow \boxed{\sigma_R = \frac{M_R * R}{I}}$$



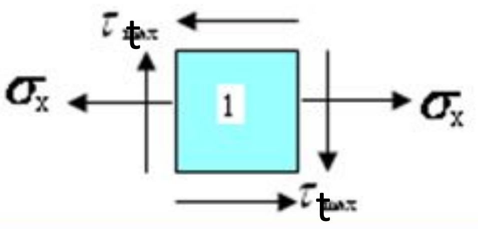
b).- Diagrama de tensiones combinadas en la sección crítica: $Vr = \sqrt{Vxy^2 + Vxz^2}$





FLEXO – TRACSO – TORSION EN EL ESPACIO EN VIGAS DE SECCION CIRCULAR

c).- Análisis de tensiones el punto crítico, que en este caso es el punto (1 – 1).

Elemento con estado de tensiones	Ecuaciones para el calculo	
 	$\sigma_x = \frac{N}{A} + \frac{M_R * R}{I}$ $\tau_{max} = \frac{M_t * R}{I_p}$	$I = \frac{\pi * \phi^4}{64}$ $I_p = \frac{\pi * \phi^4}{32}$

d).- Ecuaciones para su dimensionamiento (Criterio de Mohr):

Ecuación para la tensión máxima	Ecuación para la cortante máxima
$\sigma_{max} = \left(\frac{\sigma_x}{2}\right) + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \leq \bar{\sigma}$	$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \leq \bar{\tau}$

Ejercicio 1.-

Hallar el diámetro del eje para:

Fx= 1000 Kg   Fy= 1500 Kg   Fz= 2500 kg   R= 15cm   SAE 1045

Longitud del eje= 40 cm

σf= 4200 Kg/cm2

Tf=0,5 σf

n=2

FLEXO – TORSO – COMPRESION EN EL ESPACIO

EJERCICIO 2

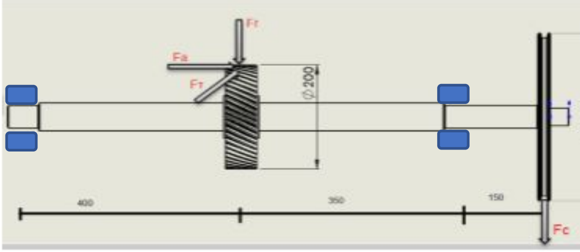
ESPECIFICACIONES DEL MOTOR Y DATOS

$Pot = 4Hp$

$n = 1500\ rpm$

Par de polos=2

Alimentación 220 V monofásico



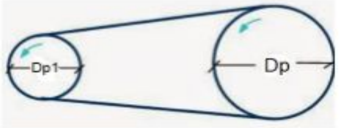
POLEA

Dp=0.3m

Dp1=0.1m

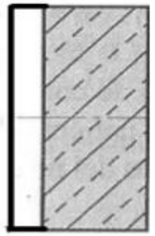
$i = \frac{1}{3}$

$n = 500\ rpm$



ENGRANAJE

De=0.2m



SAE 1045

n=3

$$\sigma_r = 630\ Mpa = 6424\ kg/cm^2$$
$$\sigma_f = 530\ Mpa = 5400\ kg/cm^2$$

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CALCULO DE LAS FUERZAS DEL ENGRANAJE

$$Pot = \frac{M_t * w}{75}$$

LOS FENIX

$$W = \frac{2*\pi*n}{60} = \frac{2*\pi*500}{60} = 52.35\ rad/s$$

$$M_t = \frac{4*75}{52.35} = 5.73\ kg.m$$

$$M_t = F_t * r_e$$

$$F_t = \frac{5.73}{0.1} = 57.306\ kg$$

Angulo de presion α=20˚

Angulo de helice ψ=30˚

$F_r = tag(\alpha) * F_t$= 20.859 kg

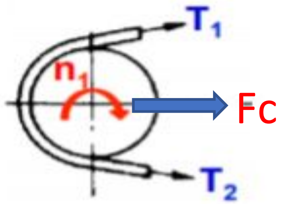
$F_a = tag(\psi) * F_t$= 33.06 kg

CALCULO DE LA CORREA

μ = Coeficiente de rosamiento (0.4-0.6)

θ= angulo de abrazamiento (π rad)

$$T_1 = T_2 e^{\theta\mu}$$



$$Mt_p = (T_1 + T_2)\frac{D_p}{2}$$

$$T_1 = 3.51 T_2$$

$$T_1 + T_2 = 38.2$$

$$T_1 = 53.52\ kg$$

$$T_2 = 15.21\ kg$$

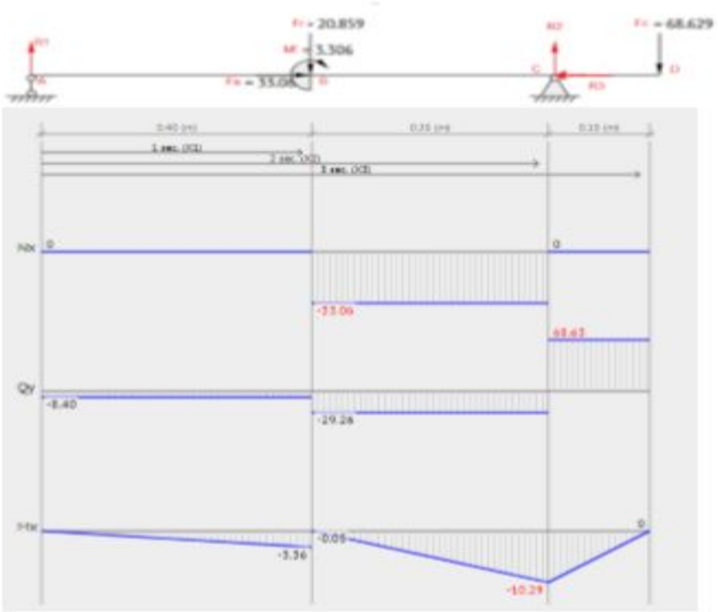
$$F_c = T_1 + T_2$$

$$F_c = 68.629\ kg$$

FLEXO – TORSO – COMPRESION EN EL ESPACIO

LOS FENIX

PLANO X-Y



$$\sum F_x = 0$$

$$F_a = R_3 = 33.06\ kg$$

$$\sum F_y = 0$$

$$R_1 + R_2 = F_r + F_c$$

$$R_1 + R_2 = 89.488$$

LOS FENIX

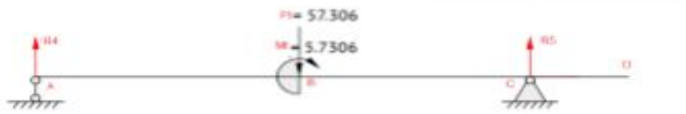
$$\sum M_A = 0$$

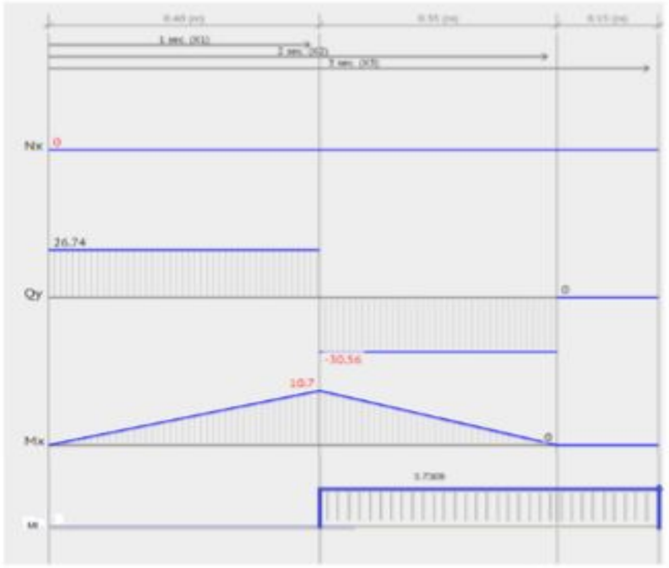
$$M_F + 0.4F_r + 0.9F_c = 0.75R_2$$

$$R_2 = 97.888\ kg$$

$$R_1 = -8.399\ kg$$

PLANO X-Z





FLEXO – TORSO – COMPRESION EN EL ESPACIO

LOS FENIX

$$\sum F_z = 0$$

$$R_4 + R_5 = 57.306$$

$$\sum M_A = 0$$

$$0.4F_T = 0.75R_5$$

$$R_5 = 30.56\ kg$$

$$R_1 = 26.74\ kg$$

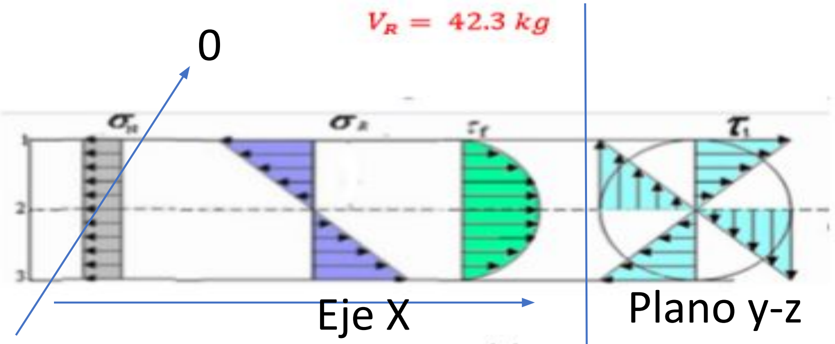
$$M_T = 5.7306\ kg.m$$

SECCION CRITICA B-B

$$M_R = 11.21\ kg.m$$

$$M_T = 5.7306\ kg.m$$

$$V_R = 42.3\ kg$$



$$\sigma_N = \frac{N}{A} \qquad \sigma_R = \frac{M_R * Y_{MAX}}{I} \qquad \tau_f = \frac{Va'Y'}{I} \qquad \tau_T = \frac{M_T * Y_{MAX}}{I_P}$$

FLEXION

$$\sigma_R = \frac{M_R * Y_{MAX}}{I} \qquad \frac{Y_{MAX}}{I} = z = \frac{\pi \varnothing^3}{32}$$

$$\sigma_R = \frac{M_R}{z} \qquad \sigma_R = \frac{11.21x100}{\frac{\pi \varnothing^3}{32}} = \frac{11418.41}{\varnothing^3}\ (\mathrm{kg}/cm^2)$$

LOS FENIX

TORSION

$$\tau_T = \frac{M_T * Y_{MAX}}{I_P} \qquad \frac{Y_{MAX}}{I_P} = z = \frac{\pi \varnothing^3}{16}$$

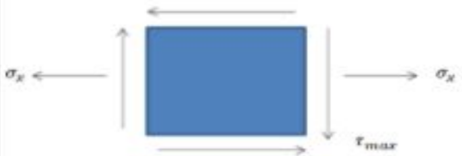
$$\tau_T = \frac{M_T}{z} \qquad \tau_T = \frac{5.7306 * 100}{\frac{\pi \varnothing^3}{16}} = \frac{2918.57}{\varnothing^3}\ (\mathrm{kg}/cm^2)$$

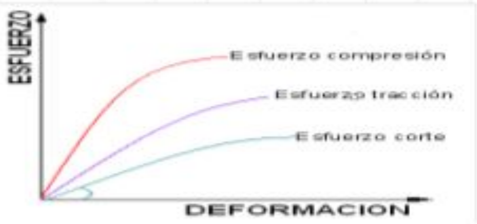
$$\tau_T = \tau_{xy}$$

ESFUERZO NORMAL

$$\sigma_N = \frac{N}{A} = \frac{33.06}{\frac{\pi \varnothing^2}{4}} = \frac{42.09}{\varnothing^2}\ (\mathrm{kg}/cm^2)$$

PARA EL PUNTO CRÍTICO





$$\sigma_x = -\sigma_N + \sigma_R$$

$$\sigma_N = 0$$

$$\sigma_x = \sigma_R$$

$$\sigma = \frac{\sigma_f}{n} = \frac{5400}{3} = 1800 (\mathrm{kg}/cm^2)$$

FLEXO – TORSO – COMPRESION EN EL ESPACIO

LOS FENIX

$$\tau = \frac{0.5\sigma_f}{n} = \frac{0.5 * 5400}{3} = 900(\mathrm{kg}/cm^2)$$

$$\sigma_{\frac{max}{min}} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}{}^2} \quad ; \quad \tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}{}^2}$$

$$\sigma_{\frac{max}{min}} = \frac{\frac{11418.41}{\text{Ø}^3}}{2} \pm \sqrt{\left(\frac{\frac{11418.41}{\text{Ø}^3}}{2}\right)^2 + \left(\frac{2918.57}{\text{Ø}^3}\right)^2} < 1800$$

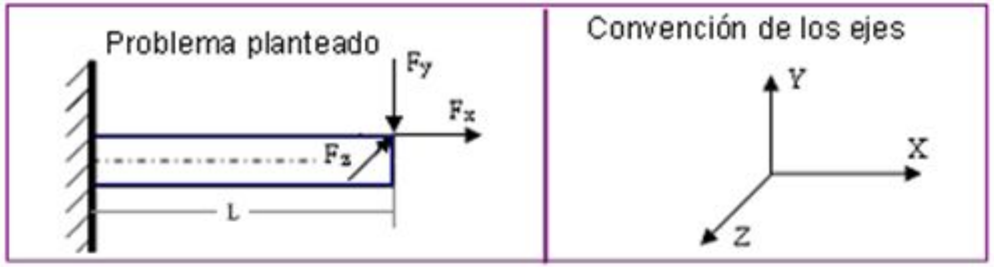
$$\text{Ø} = 1.888\ cm$$

$$\tau_{max} = \sqrt{\left(\frac{\frac{11418.41}{\text{Ø}^3}}{2}\right)^2 + \left(\frac{2918.57}{\text{Ø}^3}\right)^2} < 900$$

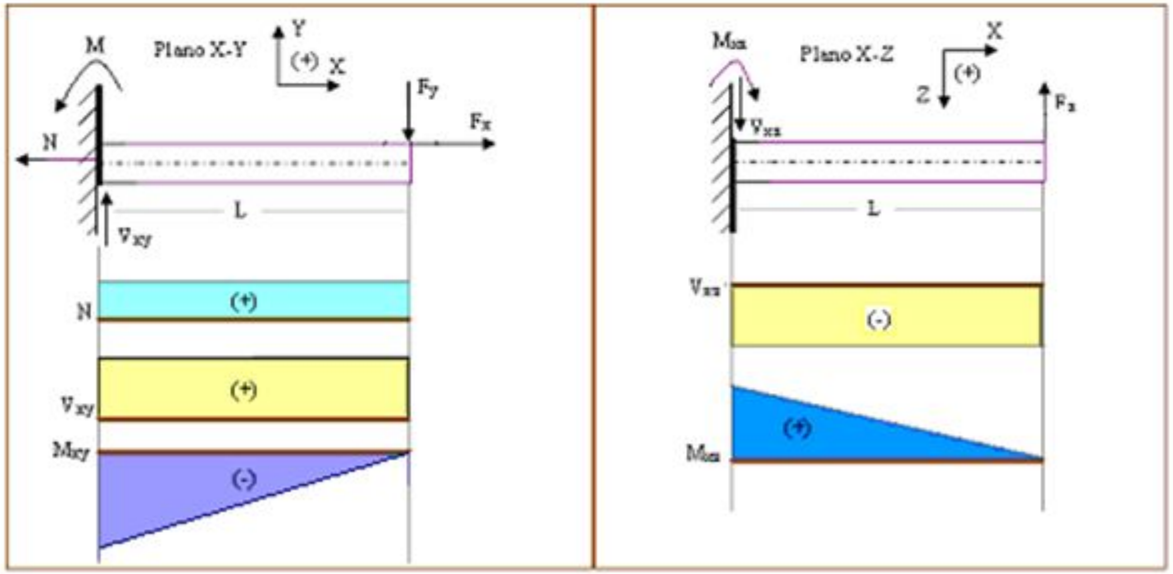
$$\text{Ø} = 1.92\ cm$$

Normalizano a 1pulg = 2.54 cm

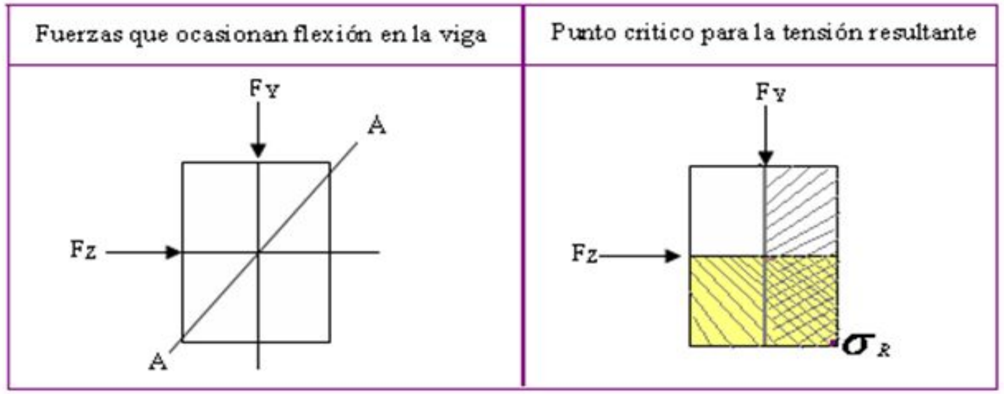
FLEXO – TRACSION EN EL ESPACIO EN VIGAS DE SECCION NO CIR



iagrama de esfuerzos normales – cortantes y momentos flectores en cada plano:



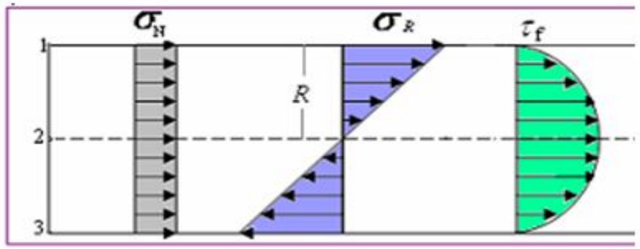
a) Tensiones resultantes debido a la flexión:



Tensión resultante debido a la flexión en el punto crítico de la sección rectangular

$$\sigma_R = \frac{M_{XY} * Y_{max}}{I_{ZZ}} + \frac{M_{XZ} * Z_{max}}{I_{YY}}$$

b) Análisis de tensiones en la

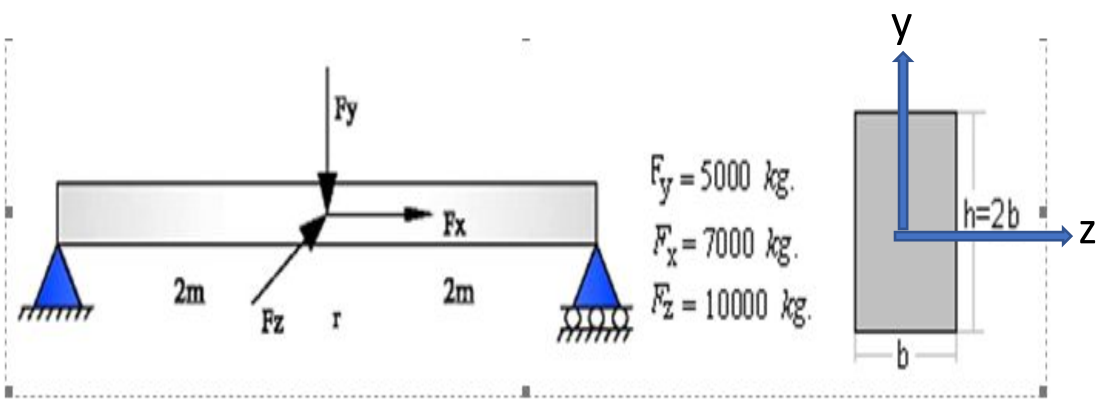


c) Ecuaciones de dimensionamiento en el punto más crítico

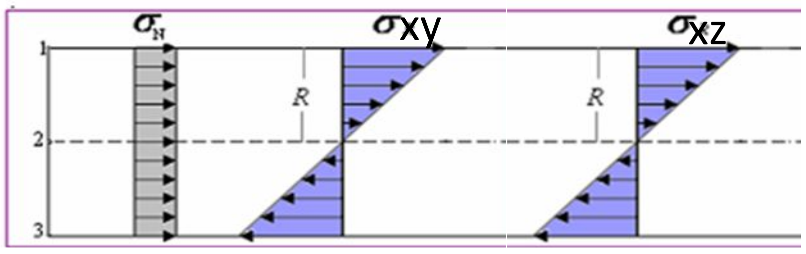
Ecuaciones para el calculo
$\sigma_x = \frac{N}{A} + \frac{M_{XY} * Y_{max}}{I_{ZZ}} + \frac{M_{XZ} * Z_{max}}{I_{YY}} \leq \bar{\sigma}$

EJERCICIOS

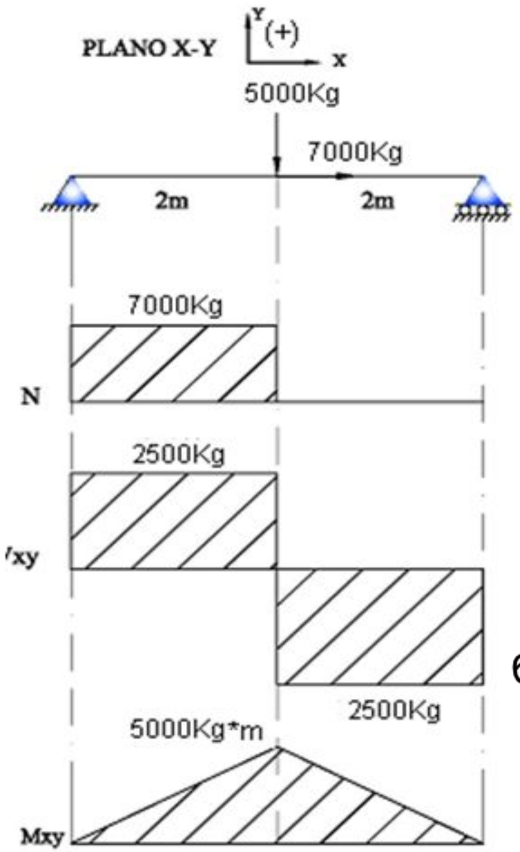
Ejercicio 3.- Calcular las dimensiones de una viga rectangular de h=2b, a partir de la estructura siguiente en base a los siguientes datos: σ_f=2100Kg/cm^2, n=3

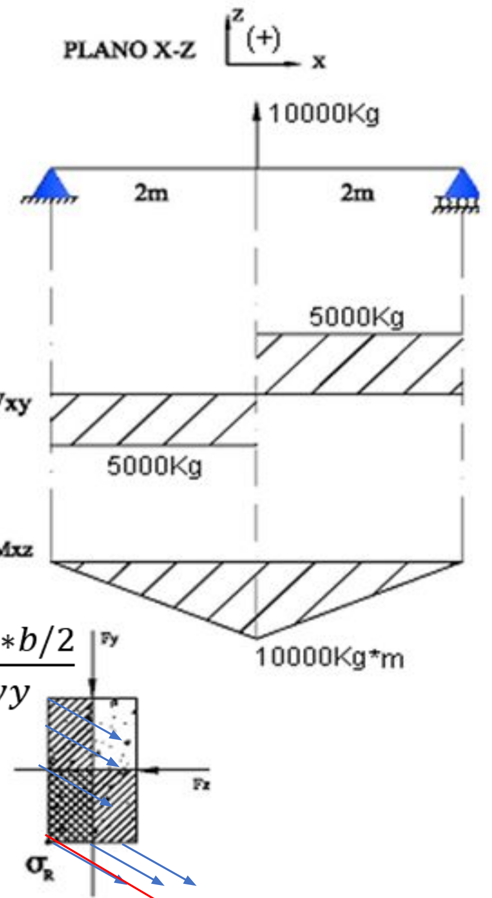


b) Análisis de tensiones en la



Solución





$$\sigma_{xz} = \frac{Mxz * b/2}{Iyy}$$

$$\sigma_{xy} = \frac{Mxy * h/2}{Izz}$$

$$\sigma_R = \frac{N}{A} + \left|\frac{M_{MAX(xy)} * Y_{MAX}}{I_{zz}}\right| + \left|\frac{M_{MAX(xz)} * Z_{MAX}}{I_{YY}}\right| \leq \bar{\sigma}$$

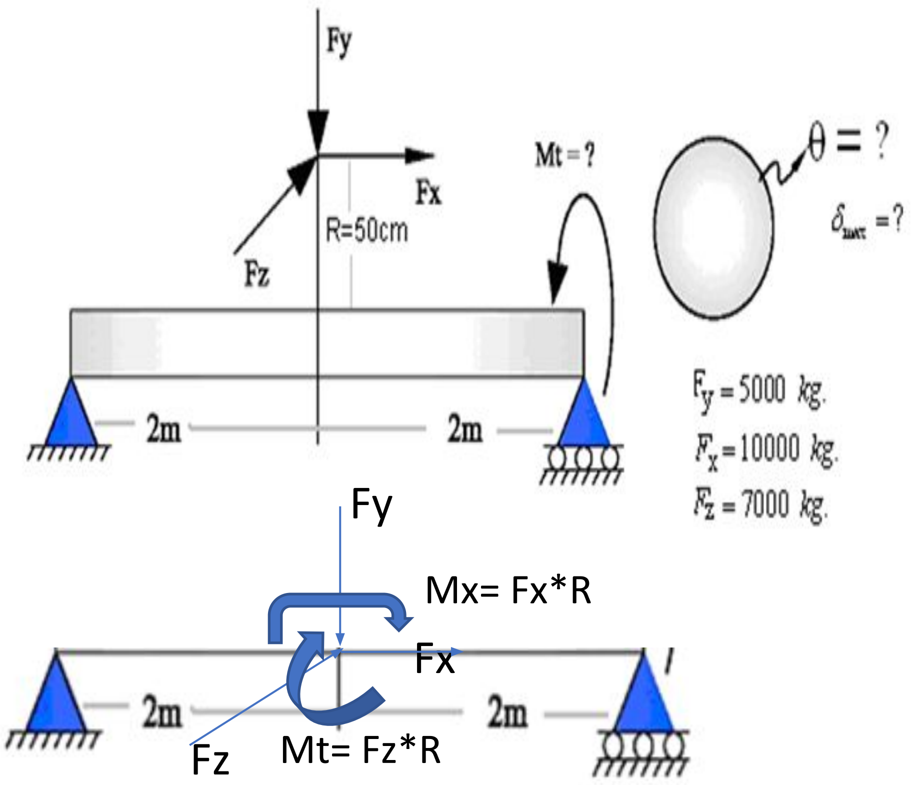
$$\frac{10000}{(b)(2b)} + \frac{(500000)(b)(12)}{b(2b)^3} + \frac{(1000000)(b/2)(12)}{(2b)(b^3)} \leq 700$$

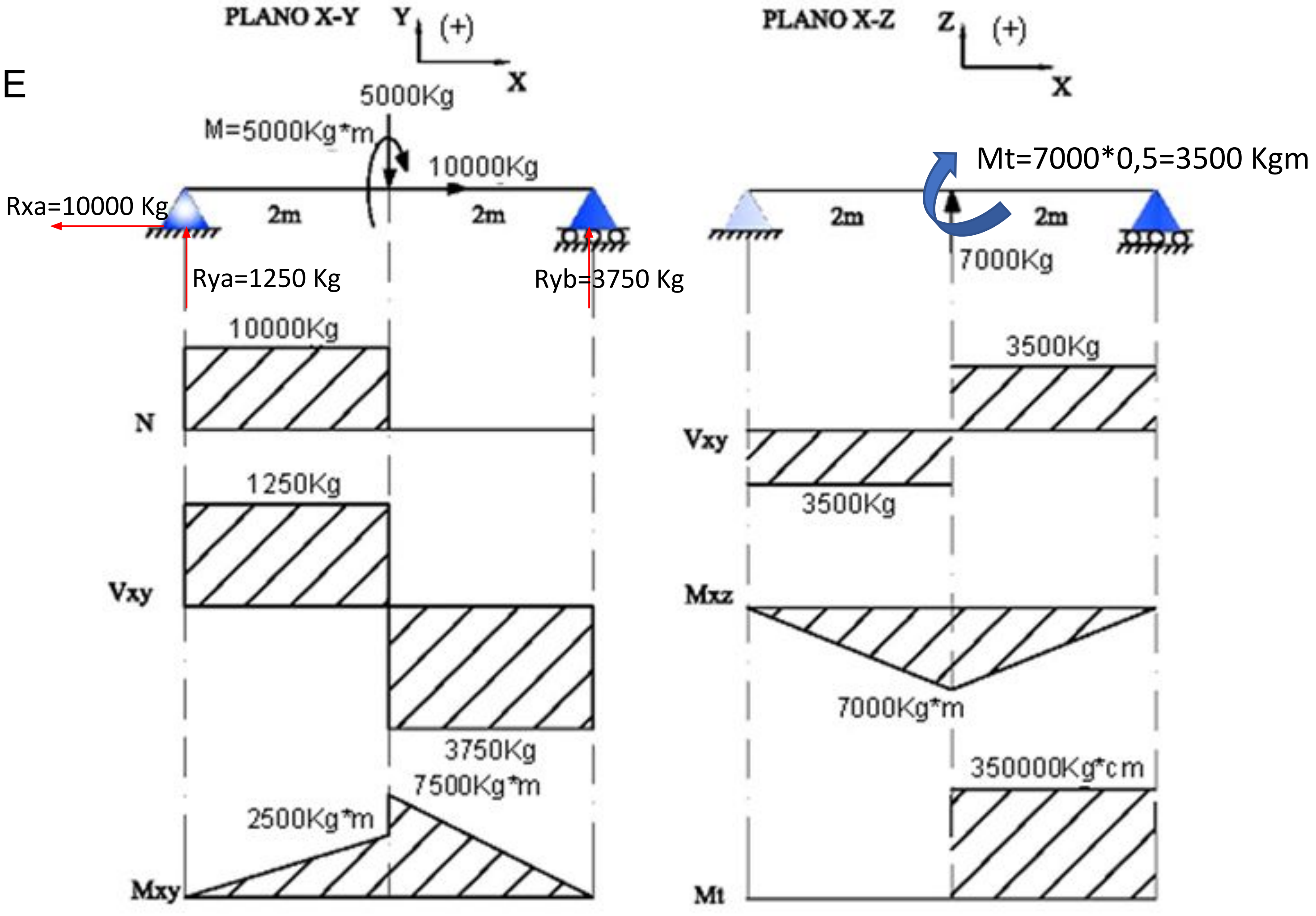
$b \geq 17.49$ cm.

$b = 18$ cm.

$h = 36$ cm.

Ejercicio 4.- Calcular el diámetro de una viga circular que cumpla las condiciones:
σ_f=2100Kg/cm^2, τ_f=0.5σ_f, n=3, μ = 0.2, θ=0.25^0 y E =2.1x10^{6}Kg/cm^2



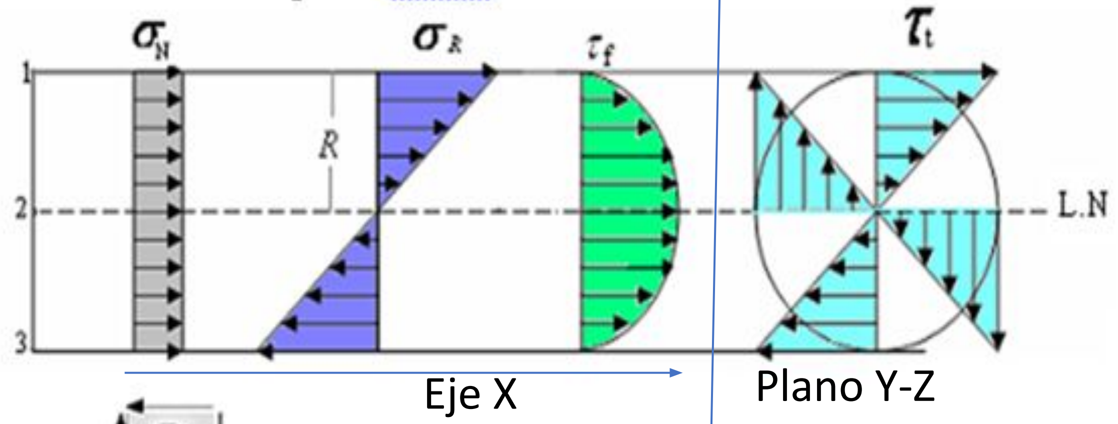


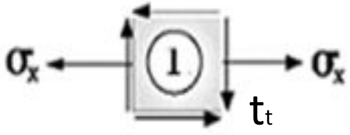
$$M_R = \sqrt{M_{XY}^{\;2} + M_{XZ}^{\;2}} \Rightarrow M_R = \sqrt{7500^2 + 7000^2} \Rightarrow$$

$$M_R = 10259.14 Kg * m \Rightarrow$$

$$M_R = 1025914 Kg * cm$$

Determinación del punto critico.





$$\sigma_R = \frac{M_R * Y_{MAX}}{I}, \qquad \sigma_N = \frac{N}{A}, \qquad \tau_t = \frac{M_t * R}{I_p}$$

$$\sigma_X = \sigma_n + \sigma_R \quad \Rightarrow \sigma_X = \frac{10000}{\frac{\pi}{4}\O^4} + \frac{(1025914)(\O/2)}{\frac{\pi}{64}\O^4} \Rightarrow \frac{\sigma_X}{2} = \frac{20000}{\pi\O^2} + \frac{16414624}{\pi\O^3}$$

$$\tau_{XY} = \tau_t = \frac{M_t * R}{I_p} \Rightarrow \tau_{XY} = \frac{(350000)(\O/2)}{\frac{\pi}{32}\O^4} \Rightarrow \tau_{XY} = \frac{5600000}{\pi\phi^3}$$

$$\sigma_{MAX} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \leq 700$$

$$\therefore \frac{20000}{\pi\O^2} + \frac{16414624}{\pi\O^3} + \sqrt{\left(\frac{20000}{\pi\O^2} + \frac{16414624}{\pi\O^3}\right)^2 + \left(\frac{5600000}{\pi\O^3}\right)^2} \leq 700$$

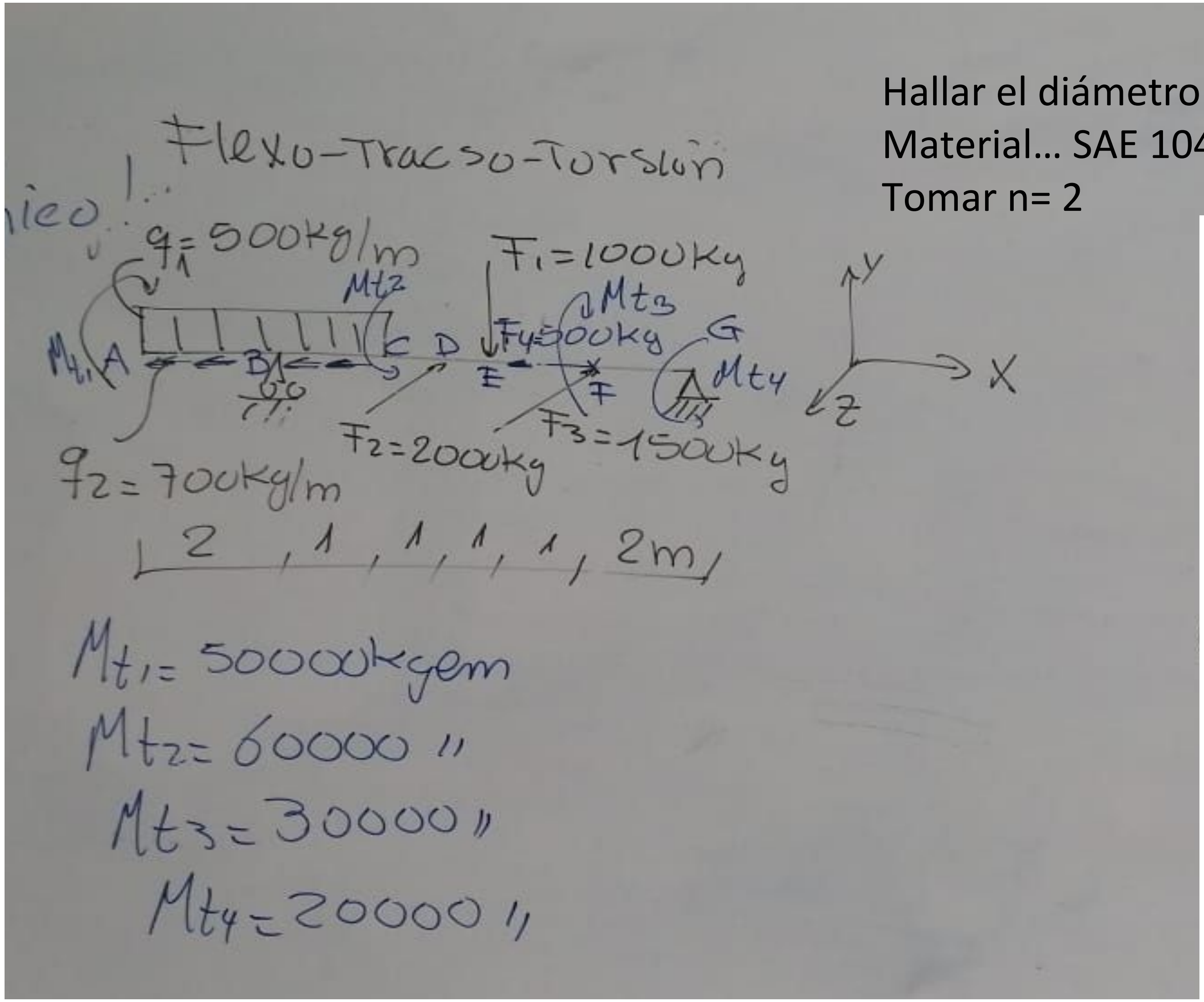
$\therefore \O \geq$ 25,08cm › $\O =$ 26 cm

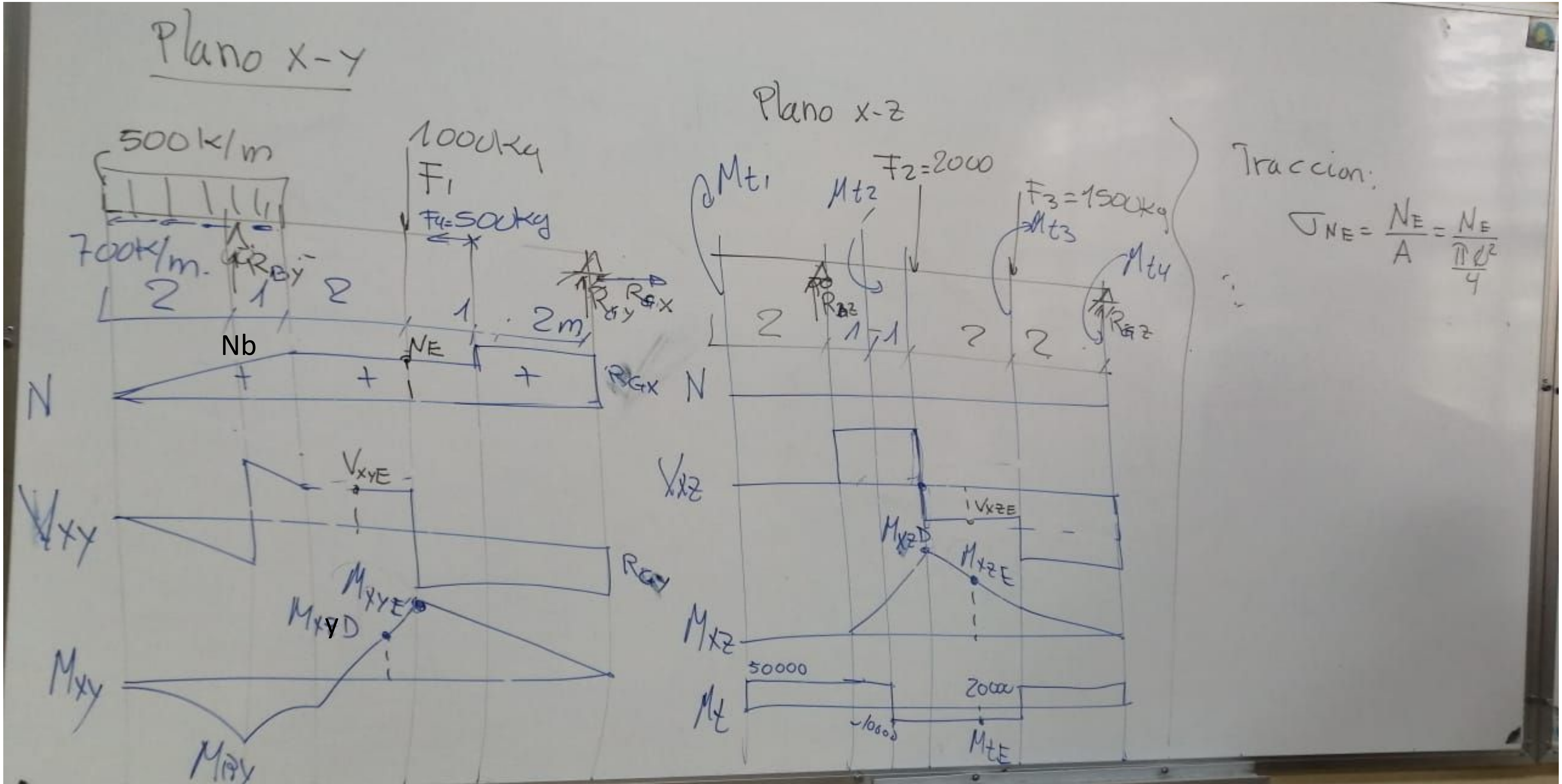
Ecuación para la cortante máxima
$\tau_{max} = R = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \leq \bar{\tau}$

Solución es $\O =$ 26 cm

Ejercicio 5.-

Hallar el diámetro del eje
Material… SAE 1045
Tomar n= 2





Plano x-y
500k/m
700k/m.
1000kg
F1
F4=500kg
RBy
RGy
RGx
2
1
2
1
2m
Nb
NE
N
+
VxyE
Vxy
MxyE
MxyD
Mxy
MBy
Plano x-z
Mt1
Mt2
F2=2000
F3=1500kg
Mt3
Mt4
RBz
RGz
VxzE
Vxz
MxzD
MxzE
Mxz
50000
20000
-10000
Mt
MtE
Traccion:
$\sigma_{NE} = \frac{N_E}{A} = \frac{N_E}{\frac{\pi d^2}{4}}$

Secciones criticas.

(Buscar Flexion maxima $\overline{x-y}$ y $\overline{xz}$

Seccion EE:

Flexion:

$M_{max_{xy}} = M_{xyE} =$

$M_{xz} = M_{xzE} =$

$M_{R_E} = \sqrt{M_{xy_E}^2 + M_{xz_E}^2} =$

$\sigma_{RE} = \frac{M_{RE} * \phi/2}{I} = \frac{M_{RE}}{\frac{\pi\phi^3}{32}}$

Torsion:

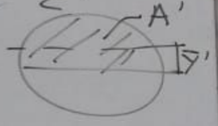
$M_t = M_{tE}$

$\tau = \frac{M_{tE}\,\phi/2}{I_p} = \frac{M_t}{\frac{\pi\phi^3}{16}}$

$V_{RE} = \sqrt{V_{xy_E}^2 + V_{xz_E}^2} \Rightarrow V_R$

$\tau_{RE} = \frac{V_{RE}}{I\,b} A'\bar{y}'$

$A' = \frac{A}{2}; \; \bar{y} = Tab$

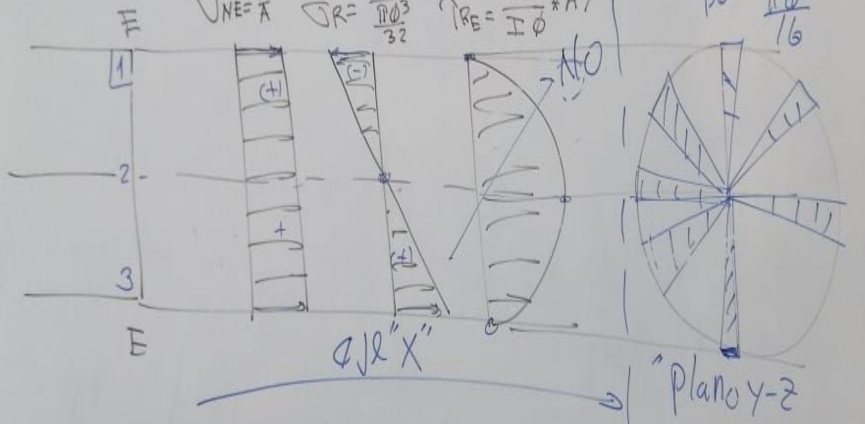


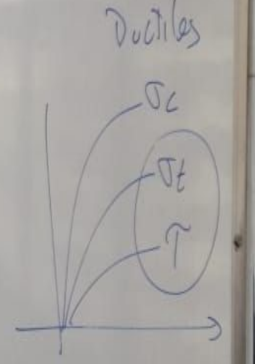
Tracción:

$\sigma_N = \frac{N}{A}$

Diagrama de tensiones

$\sigma_{NE} = \frac{N}{A}$ $\quad \sigma_R = \frac{M_{RE}}{\frac{\pi\phi^3}{32}}$ $\quad \tau_{RE} = \frac{V_{RE}}{I\phi} * A'\bar{y}'$ $\quad \tau_t = \frac{M_t}{\frac{\pi\phi^3}{16}}$





$\sigma_x = \sigma_N + \sigma_R$ $\quad \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_t^2} \le \bar{\sigma}_1 \Rightarrow \phi_1 \ge \ldots$

$\sigma_{max} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_t^2}$

$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_t^2} \le \bar{\tau} \Rightarrow \phi_2 \ge \ldots$

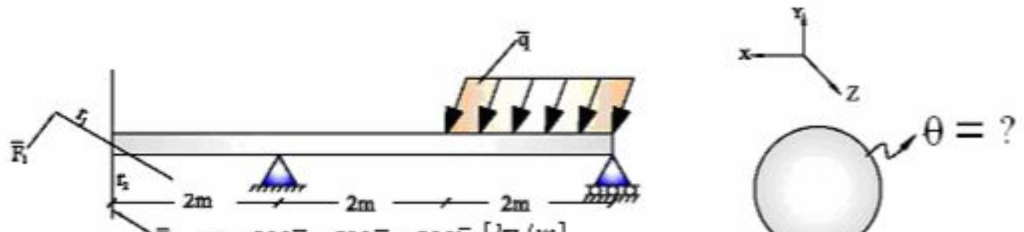
$\phi_{final} = \text{Mayor}\begin{cases}\phi_1 \\ \phi_2\end{cases}$ M_{xy}...

"Lo mismo para Seccion DD'" $\Rightarrow \phi$

Lo mismo para secciones BB y EE

Ejercicio 6.-

PROBLEMA 10.4.- Calcular el diámetro y el ángulo de torsión máximo que puede soportar la viga, con los siguientes datos: $\sigma_f=4200 Kg/cm^2$, $\tau_f=0.5\sigma_f$, $n=3$, $\mu = 0.3$ y $E = 2.1x10^6 Kg/cm^2$.



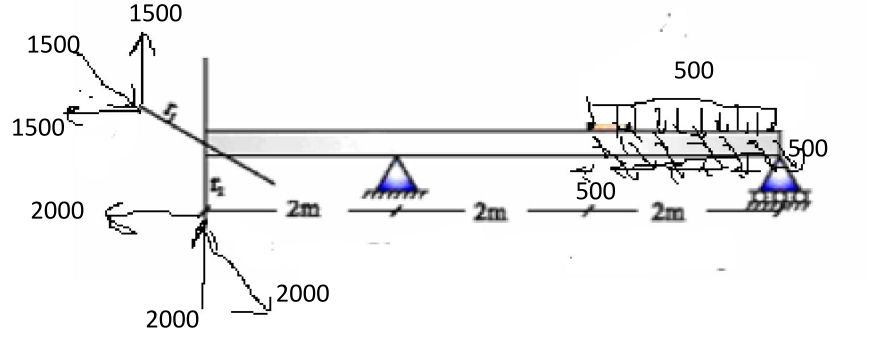
$$\bar{q} = 500\bar{i} - 500\bar{j} + 500\bar{k} \ [kg/m]$$

$$\bar{F}_1 = 1500\bar{i} + 1500\bar{j} + 1500\bar{k} \ [kg]$$

$$\bar{F}_2 = 2000\bar{i} + 2000\bar{j} + 2000\bar{k} \ [kg]$$

$$r_1 = r_2 = 20 \ cm$$

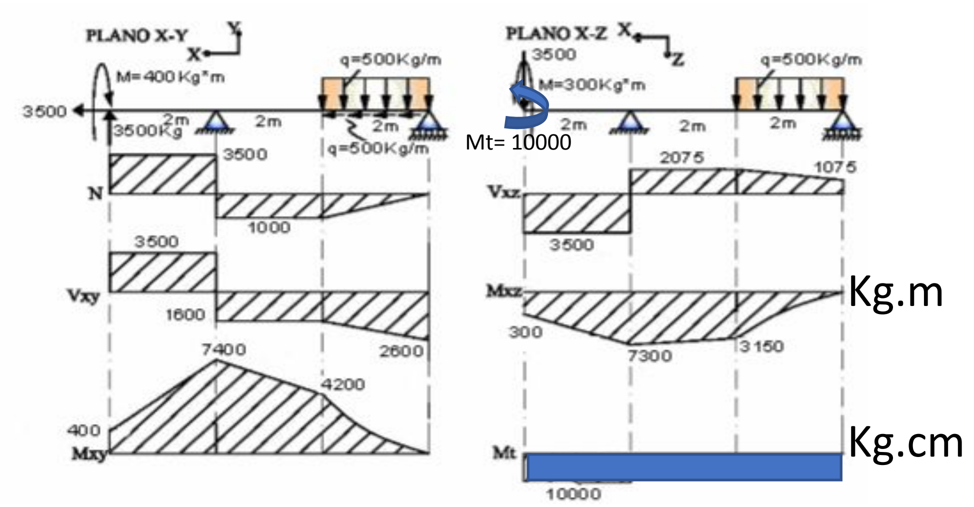
Solución:



Para F1:
Fy= 1500 Kg.
Fz= 1500 Kg
Fx= 1500 Kg
Mt= 1500 *20 = 30000 Kg.cm
Mxz= -1500*20 = - 30000

Para F2:
Fy= 2000 Kg.
Fz= 2000 Kg
Fx= 2000 Kg
Mt= -2000 *20 = - 40000 Kg.cm

Para q:
qy= - 500 Kg.m
qz= 500 Kg.m
qx= 500 Kg.m

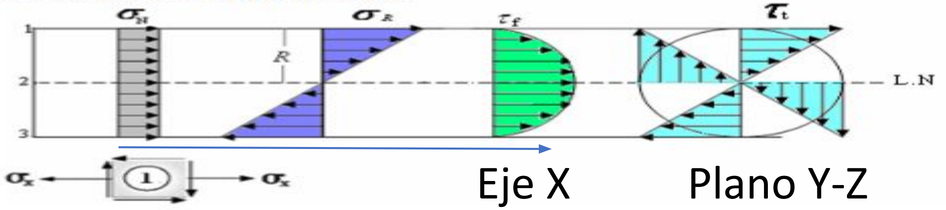


$$M_R = \sqrt{M_{XY}^2 + M_{XZ}^2} \Rightarrow M_R = \sqrt{7400^2 + 7300^2} \Rightarrow$$

$$M_R = 10394.71 Kg*m \Rightarrow$$

$$M_R = 1039471 Kg*cm$$

Determinación del punto critico



$$\sigma_X = \sigma_N + \frac{M_R * Y_{MAX}}{I} \Rightarrow$$

$$\sigma_X = \frac{3500}{\frac{\pi}{4}\varnothing^2} + \frac{(1039471)(\varnothing/2)}{\frac{\pi}{64}\varnothing^4} \Rightarrow \sigma_X = \frac{14000}{\pi\phi^2} + \frac{33263072}{\pi\phi^3}$$

$$\tau = \frac{M_t * R}{I_P} \Rightarrow \tau = \frac{(10000)(\varnothing/2)}{\frac{\pi}{32}\varnothing^4} \Rightarrow \tau = \frac{160000}{\pi\phi^3}$$

$$\sigma_{MAX} = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \leq \bar{\sigma}$$

$$\sigma_{MAX} = \frac{7000}{\pi\varnothing^2} + \frac{16631536}{\pi\varnothing^3} + \sqrt{\left(\frac{7000}{\pi\varnothing^2} + \frac{16631536}{\pi\varnothing^3}\right)^2 + \left(\frac{160000}{\pi\varnothing^3}\right)^2} \leq 1400$$

$$\Rightarrow \varnothing \geq 19.63 \ cm \Rightarrow \varnothing = 20 \ cm.$$

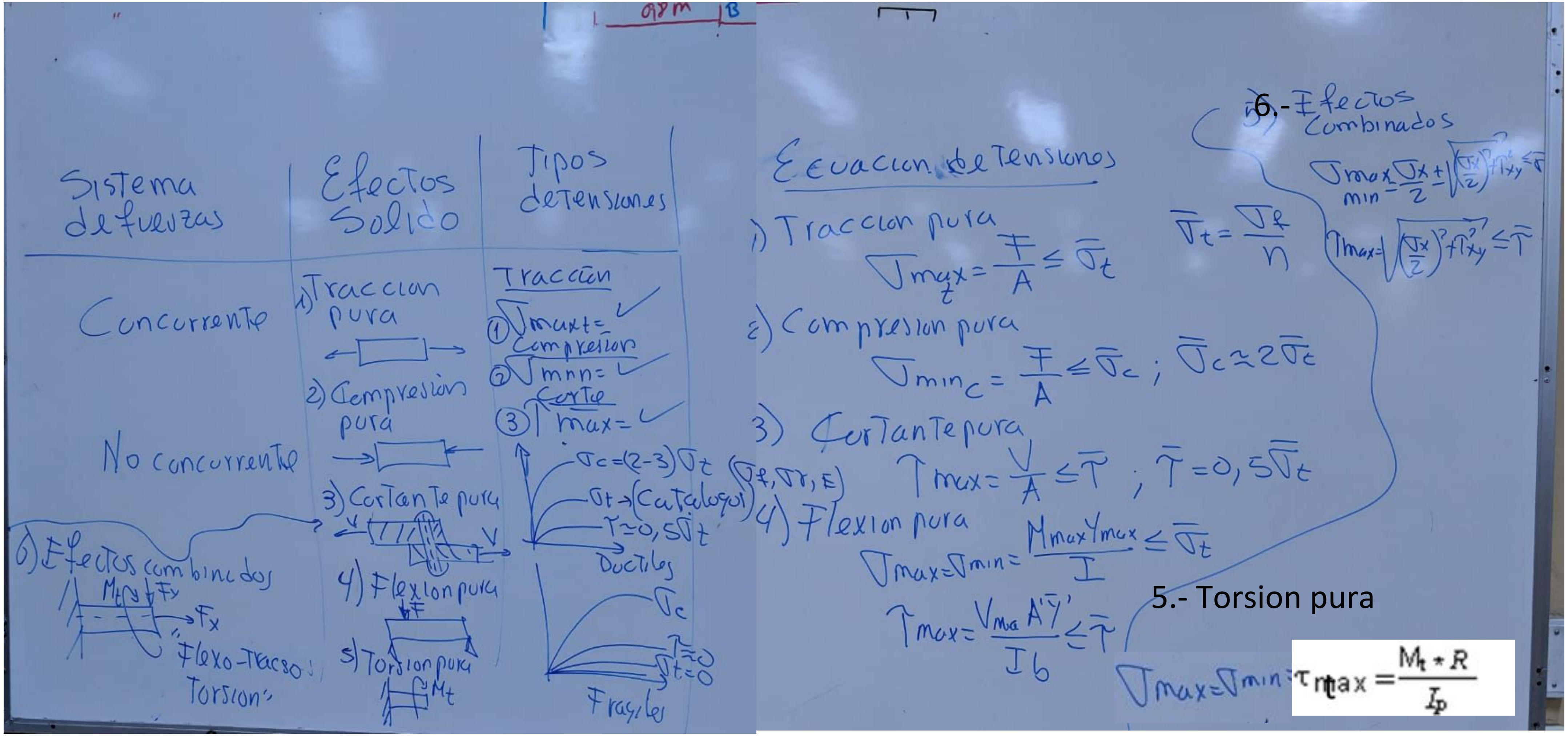
$$\tau_{MAX} = \frac{\sigma_{max} - \sigma_{min}}{2} \Rightarrow \tau_{MAX} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau)^2} \leq \bar{\tau}$$

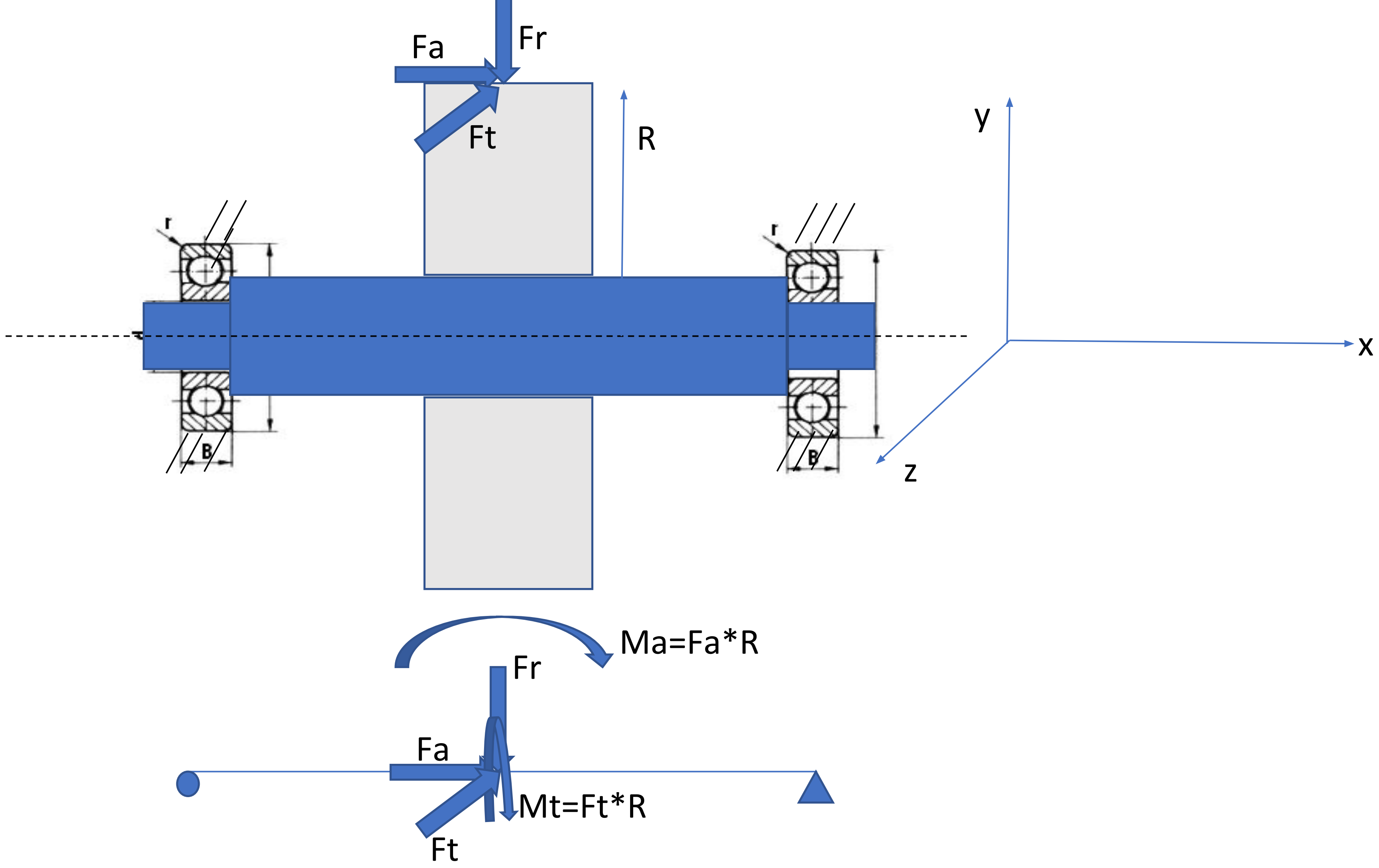
$$\tau_{MAX} = \sqrt{\left(\frac{7000}{\pi\varnothing^2} + \frac{16631536}{\pi\varnothing^3}\right)^2 + \left(\frac{160000}{\pi\varnothing^3}\right)^2} \leq 700 \quad \Rightarrow \varnothing \geq 19.63 \ cm \Rightarrow \varnothing = 20 \ cm.$$

$$\theta_{MAX} = \Sigma\theta_i = \frac{M_t * L}{G * I_P} \Rightarrow \theta = \frac{10000*200*2.6*32}{2.1x10^6 * \pi * 20^4} \Rightarrow \theta = 1.57639x10^{-4}$$

$$\theta_{MAX} = 1.57639x10^{-4} * 57.3 \Rightarrow \theta_{MAX} = 0.00903°$$

RESUMEN CONTENIDO RESISTENCIA DE MATERIALES I




Fa
Fr
Ft
R
y
x
z
r
B
r
B
Ma=Fa*R
Fr
Fa
Mt=Ft*R
Ft

GRACIAS.........