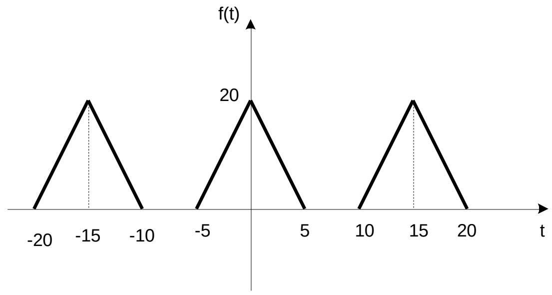
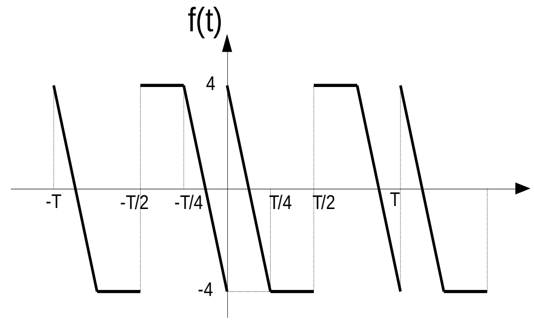
PRACTICA Nro. 2 – TRANSFORMADAS INTEGRALES
ANALISIS DE FORMAS DE ONDA PERIODICAS

1.- Determine las series de Fourier de las funciones, considerando si tienen simetría par o impar.

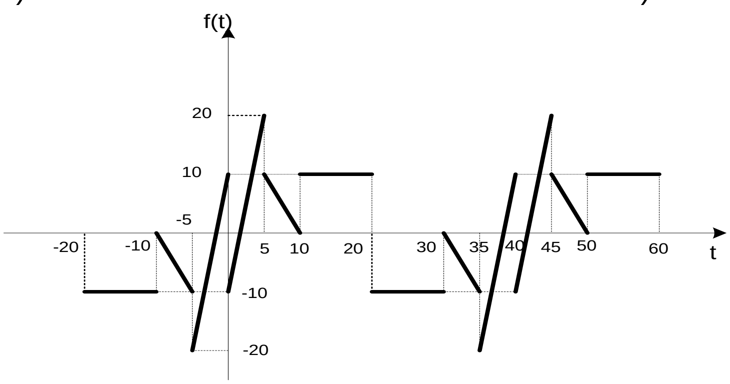
a)



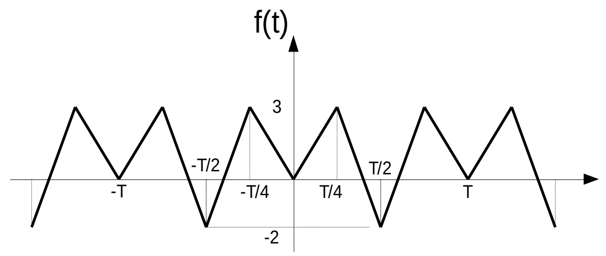
b)



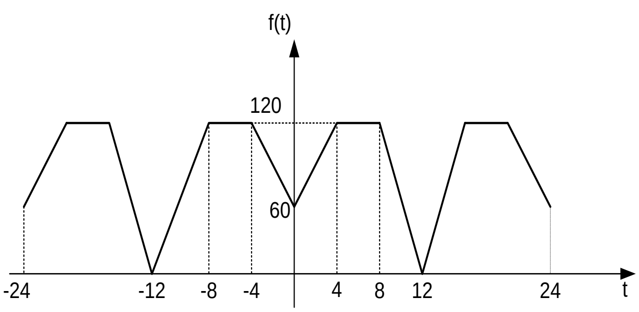
c)



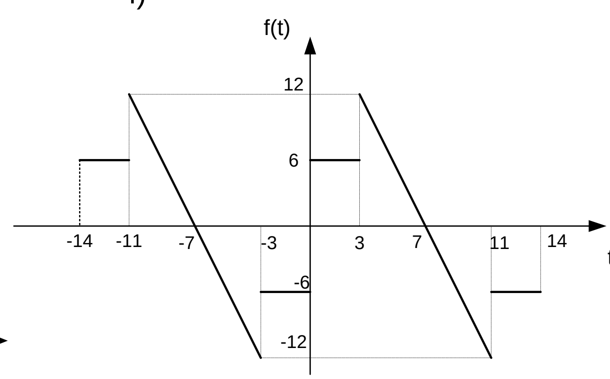
d)



e)



f)



RESPUESTAS

a) Simetría par: $f_{(t)} = \frac{20}{3} + \sum_{n=1}^{\infty} \frac{60}{n^2\pi^2}\left[1 - \cos\left(\frac{2\pi n}{3}\right)\right]\cos\left(\frac{2\pi n t}{15}\right)$

b) Simetría impar: $f_{(t)} = \sum_{n=1}^{\infty}\left[\frac{8}{\pi n}(1+(-1)^n) - \frac{32}{n^2\pi^2}\, sen\left(\frac{\pi n}{2}\right)\right] sen\left(\frac{2\pi n t}{T}\right)$

c) Simetría impar: $f_{(t)} = \sum_{n=1}^{\infty} \left[b_n sen\left(\frac{\pi n t}{20}\right)\right];$

$$b_n = \frac{-20\cos\left(\frac{\pi n}{4}\right) - 20 + 20\cos\left(\frac{\pi n}{2}\right) - 20\cos(\pi n)}{\pi n} + \frac{320 sen\left(\frac{\pi n}{4}\right) - 80 sen\left(\frac{\pi n}{2}\right)}{\pi^2 n^2}$$

d) Simetría par: $f_{(t)} = 1 + \sum_{n=1}^{\infty}\left[-\frac{30}{\pi n} sen\left(\frac{\pi n}{2}\right) + \frac{32\cos\left(\frac{\pi n}{2}\right) - 12 - 20\cos(\pi n)}{\pi^2 n^2}\right]\cos\left(\frac{2\pi n t}{T}\right)$

e) simetría par

$$f_{(t)} = 90 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi n t}{12}\right);\ a_n = \frac{360\left(\cos\left(\frac{\pi n}{3}\right) - 1\right) + 720\left(\cos\left(\frac{2\pi n}{3}\right) - \cos(\pi n)\right)}{\pi^2 n^2}$$

f) Simetría impar:

$$f_{(t)} = \sum_{n=1}^{\infty}\left[\frac{12}{\pi n}\left(\cos\left(\frac{3\pi n}{7}\right) + 1\right) + \frac{42}{\pi^2 n^2} sen\left(\frac{3\pi n}{7}\right)\right] sen\left(\frac{\pi n t}{7}\right)$$

2.- Dadas las funciones definidas en un período, grafíquelas y determine sus series de Fourier considerando si tienen simetría par o impar.

a) $f_{(t)} = \begin{cases} 1 & -4 < t < -2 \\ t^2 - 4 & -2 < t < 2 \\ 1 & 2 < t < 4 \end{cases}$

b) $f_{(t)} = \begin{cases} 3 & 0 < t < \frac{T}{4} \\ -\frac{12}{T} t + 6 & \frac{T}{4} < t < \frac{3T}{4} \\ -3 & \frac{3T}{4} < t < T \end{cases}$

c) $f_{(t)} = \begin{cases} -2 & -\frac{T}{2} < t < \frac{-T}{4} \\ \frac{16}{T} t - 2 & -\frac{T}{4} < t < 0 \\ \frac{16}{T} t + 2 & 0 < t < \frac{T}{4} \\ 2 & \frac{T}{4} < t < \frac{T}{2} \end{cases}$

d) $f_{(t)} = \begin{cases} \frac{3}{2} t + \frac{27}{2} & -6 < t < -3 \\ t^2 & -3 < t < 3 \\ -\frac{3}{2} t + \frac{27}{2} & 3 < t < 6 \end{cases}$

RESPUESTAS

a) Simetría par

$$f_{(t)} = -\frac{5}{6} + \sum_{n=1}^{\infty}\left[\frac{32}{\pi^3 n^3}\cos\left(\frac{\pi n}{2}\right) - \frac{64}{\pi^2 n^2} sen\left(\frac{\pi n}{2}\right) - \frac{2}{\pi n} sen\left(\frac{\pi n}{2}\right)\right]\cos\left(\frac{\pi n t}{4}\right)$$

b) Simetría impar

$$f_{(t)} = \sum_{n=1}^{\infty}\left[\left[\frac{1}{n\ \pi} + \frac{12 sen(\pi n/\ 2)}{\pi^2 n^2}\right] sen\left(\frac{2\pi n t}{T}\right)\right]$$

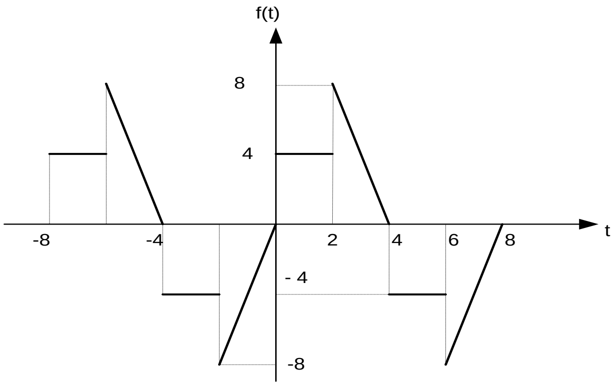
c) Simetría impar: $f_{(t)} = \sum_{n=1}^{\infty}\left[\frac{-8\cos\left(\frac{\pi n}{2}\right) + 4 - 4\cos(\pi n)}{\pi n} + \frac{16}{\pi^2 n^2} sen\left(\frac{\pi n}{2}\right)\right] sen\left(\frac{2\pi n t}{T}\right)$

d) Simetría par

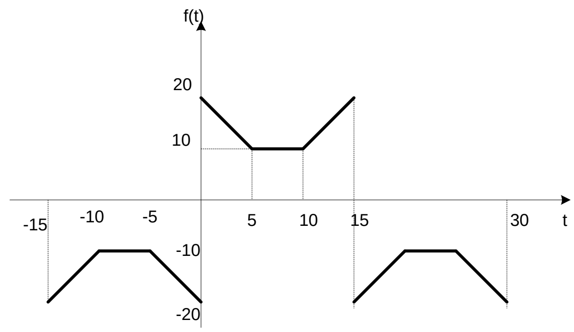
$$f_{(t)} = \frac{39}{8} + \sum_{n=1}^{\infty}\left[\left[\frac{90\cos(\pi n/\ 2)}{n\ \pi} - \frac{144 sen(\pi n/\ 2)}{\pi^3 n^3} - \frac{18\cos(\pi n)}{\pi^2 n^2}\right]\cos\left(\frac{\pi n t}{6}\right)\right]$$

3.- Hallar la serie de Fourier de las funciones considerando simetrías de media y de cuarto de onda:

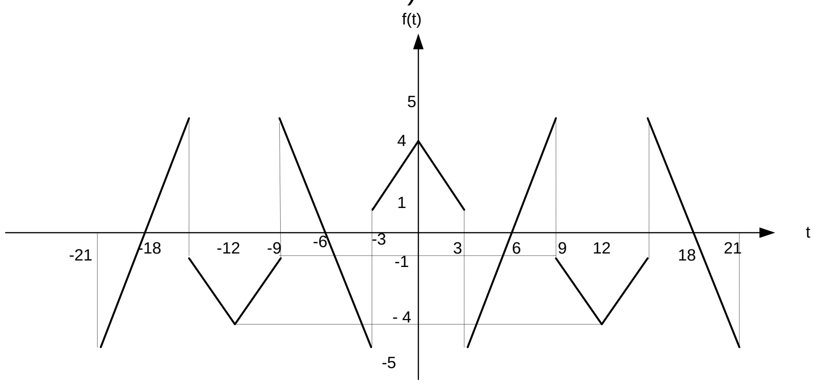
a)



b)



c)



RESPUESTAS:

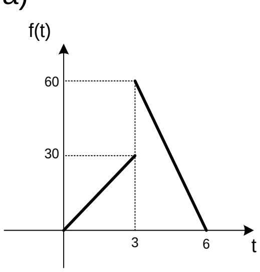
a) $$f_{(t)}=\sum_{\substack{n=1\\ n:impar}}^{\infty}\left[\left(\frac{-8}{\pi n}sen\left(\frac{\pi n}{2}\right)+\frac{32}{\pi^2 n^2}\right)\cos\left(\frac{\pi n t}{4}\right)+\left(\frac{8}{\pi n}+\frac{32}{\pi^2 n^2}sen\left(\frac{\pi n}{2}\right)\right)sen\left(\frac{\pi n t}{4}\right)\right]$$

b) $$f_{(t)}=\sum_{\substack{n=1\\ n:impar}}^{\infty}\left[-\frac{120}{\pi^2 n^2}sen\left(\frac{\pi n}{3}\right)+\frac{80}{\pi n}\right]sen\left(\frac{\pi n t}{15}\right)$$

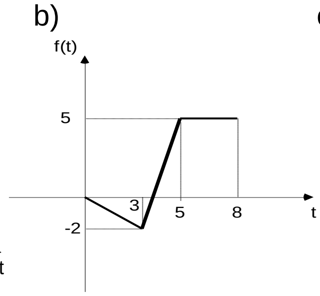
c) $$f_{(t)}=\sum_{\substack{n=1\\ n:impar}}^{\infty}\left[\frac{24}{\pi n}sen\left(\frac{\pi n}{4}\right)+\frac{-128\cos\left(\frac{\pi n}{4}\right)+48}{\pi^2 n^2}\right]\cos\left(\frac{\pi n t}{12}\right)$$

4.- Dadas las funciones definidas en un intervalo finito hallar sus series de Fourier y expandirlas periódicamente para simetría par los incisos a) y c) y para simetría impar los incisos b) y d).

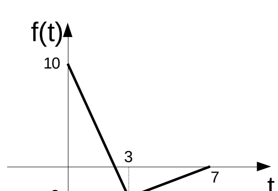
a)



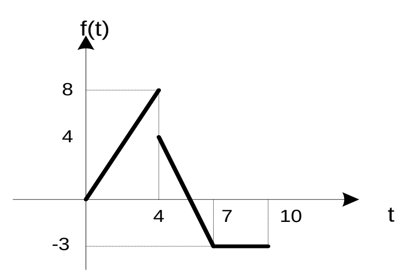
b)



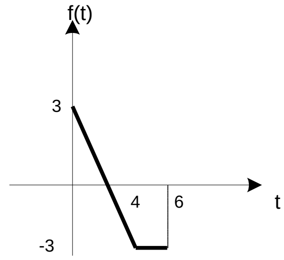
c)



d)



5.- Expandir la función con simetría de media onda grafique en el intervalo: -24<t<24 y determine su serie de Fourier con 3 armónicos diferentes de cero:



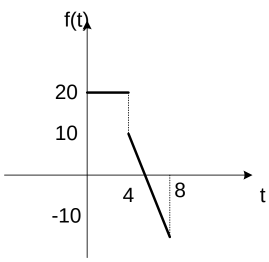
R.- $f_{(t)} = \sum_{\substack{n=1 \\ n:impar}}^{\infty} \left[\frac{18}{\pi^2 n^2}\left(1-\cos\left(\frac{2\pi n}{3}\right)\right)\cos\left(\frac{\pi n t}{6}\right) - \frac{18}{\pi^2 n^2} sen\left(\frac{2\pi n}{3}\right) sen\left(\frac{\pi n t}{6}\right)\right]$

Desarrollando 3 armónicos:

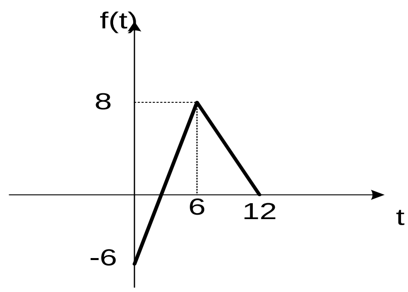
$$f_{(t)} = 2.74\cos\left(\frac{\pi t}{6}\right) - 1.58 sen\left(\frac{\pi t}{6}\right) + 0.109\cos\left(\frac{5\pi t}{6}\right) + 0.0631 sen\left(\frac{5\pi t}{6}\right) + 0.056\cos\left(\frac{7\pi t}{6}\right) - 0.032 sen\left(\frac{7\pi t}{6}\right) + \ldots$$

6.- Expandir las funciones con simetría de cuarto de onda par e impar, grafique dos períodos y determinar su serie de Fourier con 3 armónicos distintos de cero.

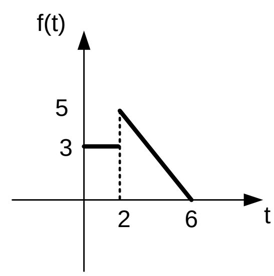
a)



b)



c)



RESPUESTAS:

a) Para SCO par:

$$f_{(t)} = \sum_{\substack{n=1 \\ n:impar}}^{\infty} \left[\frac{40}{\pi n}\left(sen\left(\frac{\pi n}{4}\right) - sen\left(\frac{\pi n}{2}\right)\right) + \frac{320}{\pi^2 n^2}\cos\left(\frac{\pi n}{4}\right)\right]\cos\left(\frac{\pi n t}{16}\right)$$

$$f_{(t)} = 19.197\cos\left(\frac{\pi t}{16}\right) + 4.698\cos\left(\frac{3\pi t}{16}\right) - 5.264\cos\left(\frac{5\pi t}{16}\right) + \ldots$$

Para SCO impar:

$$f_{(t)} = \sum_{\substack{n=1 \\ n:impar}}^{\infty} \left[\frac{-40\cos\left(\frac{\pi n}{4}\right) + 80}{\pi n} + \frac{320\left(sen\left(\frac{\pi n}{4}\right) - sen\left(\frac{\pi n}{2}\right)\right)}{\pi^2 n^2}\right] sen\left(\frac{\pi n t}{16}\right)$$

$$f_{(t)} = 6.965 sen\left(\frac{\pi t}{16}\right) + 17.639 sen\left(\frac{3\pi t}{16}\right) + 4.68 sen\left(\frac{5\pi t}{16}\right) + \ldots$$

b) Para SCO par: $f_{(t)} = \sum_{\substack{n=1 \\ n:impar}}^{\infty} \left[\frac{352\cos\left(\frac{\pi n}{4}\right) - 244}{\pi^2 n^2} \right] \cos\left(\frac{\pi n t}{24}\right)$

Para SCO impar: $f_{(t)} = \sum_{\substack{n=1 \\ n:impar}}^{\infty} \left[\frac{352\,sen\left(\frac{\pi n}{4}\right) - 128}{\pi^2 n^2} - \frac{24}{\pi n} \right] sen\left(\frac{\pi n t}{24}\right)$

c) Para SCO par: $f_{(t)} = \sum_{\substack{n=1 \\ n:impar}}^{\infty} \left[-\frac{8}{\pi n} sen\left(\frac{\pi n}{6}\right) + \frac{60}{\pi^2 n^2} \cos\left(\frac{\pi n}{6}\right) \right] \cos\left(\frac{\pi n t}{12}\right)$

$$f_{(t)} = 3.992\cos\left(\frac{\pi t}{12}\right) - 0.849\cos\left(\frac{\pi t}{4}\right) - 0.465\cos\left(\frac{5\pi t}{12}\right) + 0.074\cos\left(\frac{7\pi t}{12}\right) + \ldots$$

Para SCO impar: $f_{(t)} = \sum_{\substack{n=1 \\ n:impar}}^{\infty} \left[\frac{8\cos\left(\frac{\pi n}{6}\right) + 12}{\pi n} - \frac{60}{\pi^2 n^2} \left(sen\left(\frac{\pi n}{2}\right) - sen\left(\frac{\pi n}{6}\right) \right) \right] sen\left(\frac{\pi n t}{12}\right)$