

PRACTICA Nro. 7 – TRANSFORMADAS INTEGRALES
TRANSFORMADA INVERSA DE LAPLACE

1.- Calcular la transformada inversa de Laplace aplicando las respectivas propiedades:

$$\begin{array}{lll}
 \text{a) } \mathcal{L}^{-1}\left\{\frac{1}{s+4} - \frac{2}{3s+1} + \frac{4}{s^2+9}\right\} & \text{b) } \mathcal{L}^{-1}\left\{\frac{1}{s^3} + \frac{4}{s^5} - \frac{3}{\sqrt{s^5}}\right\} & \text{c) } \mathcal{L}^{-1}\left\{\frac{s+4}{s^2+6s+13}\right\} \\
 \text{d) } \mathcal{L}^{-1}\left\{\frac{6}{3s^2+1} - \frac{s}{2s^2+4} + \frac{2}{(4s-3)^3}\right\} & \text{e) } \mathcal{L}^{-1}\left\{\frac{2s-6}{s^2+6s+25}\right\} & \text{f) } \mathcal{L}^{-1}\left\{\frac{2s-3}{9s^2-6s+2}\right\} \\
 \text{g) } \mathcal{L}^{-1}\left\{\frac{1}{\sqrt{3s-5}} - \sqrt{(s-4)^7} e^{-7s}\right\} & \text{h) } \mathcal{L}^{-1}\left\{\ln\left(\frac{2s+1}{4s+5}\right)\right\} & \text{i) } \mathcal{L}^{-1}\left\{e^{-4s} \ln\left(1 + \frac{4}{(s+4)^2}\right)\right\} \\
 \text{j) } \mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+5)}\right\} & \text{k) } \mathcal{L}^{-1}\left\{\frac{1}{s^3(3s+2)}\right\} & \text{l) } \mathcal{L}^{-1}\left\{\frac{e^{-s}}{(s+2)^3(s^2+4s+13)}\right\}
 \end{array}$$

RESPUESTAS:

$$\begin{array}{lll}
 \text{a) } f_{(t)} = e^{-4t} - \frac{2}{3}e^{-\frac{t}{3}} + \frac{4}{3}\sin(3t); & \text{b) } f_{(t)} = \frac{t^2}{2} + \frac{t^4}{6} - 4\sqrt{\frac{t^3}{\pi}}; & \text{c) } f_{(t)} = e^{-3t}[\cos(2t) + \frac{1}{2}\sin(2t)] \\
 \text{d) } f_{(t)} = 2\sqrt{3}\sin\left(\frac{\sqrt{3}}{3}t\right) - \frac{\cos(\sqrt{2}t)}{2} + \frac{t^2 e^{\frac{3}{4}t}}{64}; & \text{e) } f_{(t)} = e^{-3t}[2\cos(4t) - 3\sin(4t)]; & \text{f) } f_{(t)} = \frac{1}{9}e^{\frac{t}{3}}\left(2\cos\left(\frac{t}{3}\right) - 7\sin\left(\frac{t}{3}\right)\right) \\
 \text{g) } f_{(t)} = \frac{t^{-\frac{1}{2}}e^{\frac{5t}{3}}}{\sqrt{3\pi}}u(t) - \frac{105}{16\sqrt{\pi}}e^{4(t-7)}(t-7)^{-\frac{5}{2}}u(t-7); & \text{h) } f_{(t)} = \frac{e^{-\frac{5}{4}t} - e^{-\frac{t}{2}}}{t}; & \text{i) } f_{(t)} = \frac{2e^{-4(t-4)}(1 - \cos(2t-8))}{t-4}u(t-4) \\
 \text{j) } f_{(t)} = -\frac{\sin(\sqrt{5}t)}{5\sqrt{5}} + \frac{t}{5}; & \text{k) } f_{(t)} = -\frac{27}{8}e^{-\frac{2}{3}t} + \frac{3}{4}t^2 - \frac{9}{4}t + \frac{27}{8}; & \text{l) } f_{(t)} = e^{-2(t-1)}\left(\frac{\cos(3t-3)}{81} - \frac{1}{81} + \frac{(t-1)^2}{18}\right)u(t-1)
 \end{array}$$

2.-Calcular las transformadas inversas descomponiendo en fracciones parciales:

$$\begin{array}{lll}
 \text{a) } \mathcal{L}^{-1}\left\{\frac{s-4}{(s+5)(s^2-3s+2)}\right\} & \text{b) } \mathcal{L}^{-1}\left\{\frac{s^2-2s+1}{2s^3+7s^2-4s}\right\} & \text{c) } \mathcal{L}^{-1}\left\{\frac{s-6}{s^3-s^2-5s-3}\right\} \\
 \text{d) } \mathcal{L}^{-1}\left\{\frac{s+3}{(s^2+2s+1)(s^2-4)}\right\} & \text{e) } \mathcal{L}^{-1}\left\{\frac{-s^2-40s-76}{(s-2)(s^2+8s+20)}\right\} & \text{f) } \mathcal{L}^{-1}\left\{\frac{s^2+3}{(s^3+8)(s+2)}\right\} \\
 \text{g) } \mathcal{L}^{-1}\left\{\frac{2s^3+4s+10}{(s^2+10s+25)(s^2+2s+20)}\right\} & \text{h) } \mathcal{L}^{-1}\left\{\frac{s^2+3}{(s-1)^2(s^2-4s+3)}\right\} & \text{i) } \mathcal{L}^{-1}\left\{\frac{s^2+8s+4}{(s^2-3s+2)(s^2+6s+10)}\right\} \\
 \text{j) } \mathcal{L}^{-1}\left\{\frac{4s^2+2s+10}{(s^2-4s+10)(s^2+4)}\right\} & \text{k) } \mathcal{L}^{-1}\left\{\frac{2s^2+1}{(s^2+3)(s-3)^2}\right\} & \text{l) } \mathcal{L}^{-1}\left\{\frac{s^2+2s+1}{(s^2+5s+15)(s^2+4)}\right\}
 \end{array}$$

RESPUESTAS:

$$\begin{array}{lll}
 \text{a) } f_{(t)} = -\frac{3}{14}e^{-5t} - \frac{2}{7}e^{2t} + \frac{1}{2}e^t; & \text{b) } f_{(t)} = -\frac{1}{4} + \frac{1}{18}e^{\frac{t}{2}} + \frac{25}{36}e^{-4t}; & \text{c) } f_{(t)} = \frac{7}{4}te^{-t} + \frac{3}{16}e^{-t} - \frac{3}{16}e^{3t}
 \end{array}$$

- d) $f(t) = \frac{1}{9}e^{-t} - \frac{2te^{-t}}{3} + \frac{5}{36}e^{2t} - \frac{1}{4}e^{-2t}$; e) $f(t) = -4e^{2t} + 3e^{-4t} \cos(2t) - 7e^{-4t} \sin(2t)$;
- f) $f(t) = -\frac{1}{24}e^{-2t} + \frac{7}{12}te^{-2t} + \frac{e^t}{24} \left(\cos(\sqrt{3}t) + \frac{7}{\sqrt{3}} \sin(\sqrt{3}t) \right)$;
- g) $f(t) = \frac{662}{245}e^{-5t} - \frac{52}{7}te^{-5t} + e^{-t} \left(-\frac{172}{245} \cos(\sqrt{19}t) - \frac{922}{245\sqrt{19}} \sin(\sqrt{19}t) \right)$; h) $f(t) = e^t \left(-t^2 - 2t - \frac{3}{2} \right) + \frac{3}{2}e^{3t}$;
- i) $f(t) = \frac{12}{13}e^{2t} - \frac{13}{17}e^t + e^{-3t} \left(-\frac{35}{221} \cos t - \frac{123}{221} \sin t \right)$; j) $f(t) = -\frac{3}{25} \cos(2t) - \frac{17}{50} \sin(2t) + e^{2t} \left[\frac{3}{25} \cos(\sqrt{6}t) + \frac{111}{25\sqrt{6}} \sin(\sqrt{6}t) \right]$;
- k) $f(t) = -\frac{5}{24} \cos(\sqrt{3}t) - \frac{5}{24\sqrt{3}} \sin(\sqrt{3}t) + \frac{5}{24}e^{3t} + \frac{19}{12}te^{3t}$;
- l) $f(t) = e^{-\frac{5}{2}t} \left(-\frac{37}{221} \cos\left(\frac{\sqrt{35}}{2}t\right) + \frac{243}{221\sqrt{35}} \sin\left(\frac{\sqrt{35}}{2}t\right) \right) + \frac{37}{221} \cos(2t) + \frac{7}{442} \sin(2t)$

3.- Calcular las transformadas inversas aplicando la integral de convolución y las propiedades que correspondan:

- a) $\mathcal{L}^{-1} \left\{ \frac{s}{(s^2 + 7)^2} \right\}$ b) $\mathcal{L}^{-1} \left\{ \frac{s}{(s+5)(s^2 + 4)} \right\}$ c) $\mathcal{L}^{-1} \left\{ \frac{s}{(s^2 + 4)(s^2 + 9)} \right\}$
- d) $\mathcal{L}^{-1} \left\{ \frac{1}{(s^2 + 2)^2} \right\}$ e) $\mathcal{L}^{-1} \left\{ \frac{s^2}{(s^2 + 5)^2} \right\}$ f) $\mathcal{L}^{-1} \left\{ \frac{1}{s^2(s^2 + 4)^2} \right\}$
- g) $\mathcal{L}^{-1} \left\{ \frac{1}{(s+3)^2(s^2 + 4)} \right\}$ h) $\mathcal{L}^{-1} \left\{ \frac{s}{(s+5)^2(s^2 + 1)} \right\}$ i) $\mathcal{L}^{-1} \left\{ \frac{s}{(s^2 + 9)^3} \right\}$
- j) $\mathcal{L}^{-1} \left\{ \frac{e^{-6s}}{(s^2 + 9)^3} \right\}$ k) $\mathcal{L}^{-1} \left\{ \frac{s+4}{(s^2 + 8s + 20)^3} \right\}$ l) $\mathcal{L}^{-1} \left\{ \frac{(s+1)^2 e^{-3s}}{(s^2 + 2s + 5)^3} \right\}$

RESPUESTAS

- a) $f(t) = \frac{1}{2\sqrt{7}} t \sin(\sqrt{7}t)$; b) $f(t) = \frac{1}{29} (5 \cos(2t) + 2 \sin(2t)) - \frac{5}{29} e^{-5t}$; c) $f(t) = -\frac{1}{5} \cos(3t) + \frac{1}{5} \cos(2t)$;
- d) $f(t) = \frac{\sin(\sqrt{2}t)}{4\sqrt{2}} - \frac{t \cos(\sqrt{2}t)}{4}$; e) $f(t) = \frac{t \cos(\sqrt{5}t)}{2} + \frac{\sin(\sqrt{5}t)}{2\sqrt{5}}$; f) $f(t) = \frac{t}{16} + \frac{t \cos(2t)}{32} - \frac{3}{64} \sin(2t)$
- g) $f(t) = \frac{te^{-3t}}{13} + \frac{6}{169} e^{-3t} + \frac{5}{338} \sin(2t) - \frac{6}{169} \cos(2t)$; h) $f(t) = -\frac{5}{26} te^{-5t} - \frac{6}{169} e^{-5t} + \frac{6}{169} \cos t + \frac{5}{338} \sin t$
- i) $f(t) = \frac{t \sin(3t)}{216} - \frac{t^2 \cos(3t)}{72}$; j) $f(t) = \left(-\frac{(t-6)^2 \sin(3t-18)}{216} - \frac{(t-6) \cos(3t-18)}{216} + \frac{\sin(3t-18)}{648} \right) u(t-6)$
- k) $f(t) = e^{-4t} \left(\frac{t \sin(2t)}{64} - \frac{t^2 \cos(2t)}{32} \right)$; l) $f(t) = e^{-(t-3)} \left(\frac{(t-3)^2 \sin(2t-6)}{16} - \frac{(t-3) \cos(2t-6)}{32} + \frac{\sin(2t-6)}{64} \right) u(t-3)$