PRACTICA Nro. 5 – TRANSFORMADAS INTEGRALES
TRANSFORMADA INVERSA DE FOURIER

1.- Calcular la transformada inversa de Fourier de las funciones por fracciones parciales:

a) $F_{(\omega)} = \dfrac{3}{(4+j\varpi)(3+j2\varpi)}$ b) $F_{(\omega)} = \dfrac{6}{(3-j2\omega)(1+j4\omega)}$ c) $F_{(\omega)} = \dfrac{4+j\varpi}{5-j9\varpi+2\varpi^2}$

d) $F_{(\omega)} = \dfrac{3+j\varpi}{12+j8\varpi-\varpi^2}$ e) $F_{(\omega)} = \dfrac{2-j\varpi}{(2+j\varpi)(25+\varpi^2)}$ f) $F_{(\omega)} = \dfrac{6}{(4+\varpi^2)(1+\omega^2)}$

g) $F_{(\omega)} = \dfrac{2j}{3\varpi+j2\varpi^2}$ h) $F_{(\omega)} = \dfrac{2+j\varpi}{(6\varpi^2-j7\varpi+3)(4+j\varpi)}$

RESPUESTAS

a) $f_{(t)} = \frac{3}{5}\left(e^{-\frac{3}{2}t} - e^{-4t}\right)u(t)]$; b) $f_{(t)} = \frac{3}{7}\left(e^{\frac{3}{2}t}u(-t) - e^{-\frac{1}{4}t}u(t)\right)$; c) $f_{(t)} = \frac{9}{22}e^{\frac{1}{2}t}u(-t) - \frac{1}{11}e^{-5t}u(t)$; d) $f_{(t)} = \frac{9}{4}e^{-6t}u(t) + \frac{1}{4}e^{-2t}u(t)$

e) $f_{(t)} = \left(\frac{4}{21}e^{-2t} - \frac{7}{30}e^{-5t}\right)u(t) - \frac{3}{70}e^{5t}u(-t)$; f) $f_{(t)} = -\frac{1}{2}e^{-2|t|} + e^{-|t|}$; g) $f_{(t)} = -\frac{\operatorname{sgn}(t)}{3} + \frac{2}{3}e^{-\frac{3}{2}t}u(t)$;

h) $f_{(t)} = \left(\frac{1}{55}e^{-\dots\frac{3}{2}t} + \frac{2}{65}e^{-4t}\right)u(t) + \frac{7}{143}e^{\frac{1}{3}t}u(-t)$

2.- Hallar la convolución entre las funciones:

a) $f_{1(t)} = t^2u_{(t)}$; $f_{2(t)} = 4u_{(t)} - 4u_{(t-4)}$ b) $f_{1(t)} = \begin{cases} 2t & 0<t<3 \\ 6 & 3<t<5 \end{cases}$; $f_{2(t)} = e^{-3t}u_{(t)}$ c) $f_{1(t)} = \begin{cases} 2t & 0<t<5 \\ 10 & 5<t<8 \end{cases}$; $f_{2(t)} = 3e^{-2t}u_{(t)}$

d) $f_{1(t)} = sen\left(\frac{\pi}{2}t\right)\left(u_{(t)} - u_{(t-2)}\right)$; $f_{2(t)} = 10e^{-t}u_{(t)}$ e) $f_{1(t)} = \begin{cases} 2t & 0<t<2 \\ -2t+8 & 2<t<4 \end{cases}$; $f_{2(t)} = 3e^{5t}u_{(-t)}$

RESPUESTAS

a) $f_{(t)} = \begin{cases} \frac{4}{3}t^3 & 0<t<4 \\ 16t^2 - 64t + \frac{256}{3} & t>4 \end{cases}$; b) $f_{(t)} = \begin{cases} \frac{2}{3}t - \frac{2}{9} + \frac{2}{9}e^{-3t} & 0<t<3 \\ -\frac{2}{9}e^{-3t+9} + \frac{2}{9}e^{-3t} + 2 & 3<t<5 \\ -\frac{2}{9}e^{-3t+9} + \frac{2}{9}e^{-3t} + 2e^{-3t+15} & t>5 \end{cases}$ c) $f_{(t)} = \begin{cases} 3t - \frac{3}{2} + \frac{3}{2}e^{-2t} & 0<t<5 \\ -\frac{3}{2}e^{-2t+10} + \frac{3}{2}e^{-2t} + 15 & 5<t<8 \\ -\frac{3}{2}e^{-2t+10} + \frac{3}{2}e^{-2t} + 15e^{-2t+16} & t>8 \end{cases}$

d) $f_{(t)} = \begin{cases} \frac{40}{4+\pi^2}\left[sen\left(\frac{\pi t}{2}\right) - \frac{\pi}{2}\cos\left(\frac{\pi t}{2}\right) + \frac{\pi}{2}e^{-t}\right] & 0<t<2 \\ \frac{20\pi}{4+\pi^2}\left(e^{-t+2} + e^{-t}\right) & t>2 \end{cases}$ e) $f_{(t)} = \begin{cases} \frac{6}{25}e^{5t-20} - \frac{12}{25}e^{5t-10} + \frac{6}{25}e^{5t} & t<0 \\ -\frac{12}{25}e^{5t-10} + \frac{6}{25}e^{5t-20} + \frac{6}{5}t + \frac{6}{25} & 0<t<2 \\ \frac{6}{25}e^{5t-20} - \frac{6}{5}t + \frac{114}{25} & 2<t<4 \end{cases}$

3.- Calcular la transformada inversa de Fourier aplicando la integral de convolución:

a) $F_{(\omega)} = \dfrac{1}{(2+j\varpi)(6+j\varpi)^2}$ b) $F_{(\omega)} = \dfrac{1}{(3-j\varpi)(2+j\varpi)^2}$ c) $F_{(\omega)} = \dfrac{10}{(4+\omega^2)(4-j5\omega)}$

d) $F_{(\omega)} = \dfrac{3}{(1+\omega^2)(2+j4\omega)}$ e) $F_{(\omega)} = \dfrac{sen(2\omega)}{\omega(2+j\omega)}$ f) $F_{(\omega)} = \dfrac{sen(3\omega)}{\omega(2-j4\omega)}$

g) $F_{(\omega)} = \dfrac{sen(2\omega)}{\omega(3+j\omega)^2}$ h) $F_{(\omega)} = \dfrac{6\pi\delta''_{(\omega)}}{4+j3\omega}$

RESPUESTAS

a) $f_{(t)} = \left(-\frac{t}{4}e^{-6t} - \frac{e^{-6t}}{16} + \frac{e^{-2t}}{16}\right)u(t)$; b) $f_{(t)} = \frac{e^{3t}}{25}u(-t) + \left[\frac{te^{-2t}}{5} + \frac{e^{-2t}}{25}\right]u(t)$ c) $f_{(t)} = \left(\frac{25}{42}e^{\frac{4}{5}t} - \frac{5}{12}e^{2t}\right)u(-t) + \frac{5}{28}e^{-2t}u(t)$;

d) $f_{(t)} = \frac{1}{4}e^{t}u(-t) + \left(-\frac{3}{4}e^{-t} + e^{-\frac{t}{2}}\right)u(t)$; e) $f_{(t)} = \left(\frac{1-e^{-2t-4}}{4}\right)u(t+2) + \left(\frac{e^{-2t+4}-1}{4}\right)u(t-2)$;

f) $f_{(t)} = \begin{cases} \frac{1}{4}\left(e^{\frac{1}{2}(t+3)} - e^{\frac{1}{2}(t-3)}\right) & t < -3 \\ \frac{1}{4}\left(1 - e^{\frac{1}{2}(t-3)}\right) & -3 < t < 3 \end{cases}$;g) $f_{(t)} = \left(-\frac{t}{6}e^{-3t-6} - \frac{7}{18}e^{-3t-6} + \frac{1}{18}\right)u(t+2) + \left(-\frac{1}{18} + \frac{1}{6}te^{-3t+6} - \frac{5}{18}e^{-3t+6}\right)u(t-2)$;

h) $f_{(t)} = -\frac{3}{4}t^2 + \frac{9}{8}t - \frac{27}{32}$

4.- Calcule la transformada inversa aplicando propiedades y lo que corresponda:

a) $F_{(\omega)} = \frac{e^{j(2\varpi-4)}}{3 - j(2-\varpi)}$ b) $F_{(\omega)} = \frac{4e^{-j(\varpi-3)}}{\varpi^2 - 6\varpi + 10}$ c) $F_{(\omega)} = \frac{e^{j(2\varpi+4)}sen(5\omega+10)}{\omega+2}$

d) $F_{(\omega)} = \frac{3\cos(2\varpi)}{\varpi^2 + 8\varpi + 20}$ e) $F_{(\omega)} = \frac{1}{j4(\varpi-2) - 3(\varpi-2)^2}$ f) $F_{(\omega)} = \frac{e^{-j3\varpi}}{(2+j3\varpi)(1-j2\varpi)}$

RESPUESTAS:

a) $f_{(t)} = e^{(-3+j2)t-6}u(t+2)$; b) $f_{(t)} = 2e^{-|t-1|+j3t}$; c) $f_{(t)} = \frac{1}{2}e^{-j2t}\left(u(t+7) - u(t-3)\right)$; d) $f_{(t)} = \frac{3}{8}\left(e^{-2|t+2|-j4(t+2)} + e^{-2|t-2|-j4(t-2)}\right)$;

e) $f_{(t)} = -\frac{1}{8}e^{j2t}u(-t) + \left(-\frac{1}{4}e^{\left(-\frac{4}{3}+j2\right)t} + \frac{1}{8}e^{j2t}\right)u(t)$; f) $f_{(t)} = \frac{1}{7}e^{\frac{t-3}{2}}u(-t+3) + \frac{1}{7}e^{-\frac{2}{3}(t-3)}u(t-3)$

5.- Hallar una solución particular para las siguientes ecuaciones diferenciales ordinarias:

a) $x'_{(t)} + 3x_{(t)} = t^2u(t)$ b) $x''_{(t)} + 8x'_{(t)} + 12x_{(t)} = \delta(t-3)$

c) $x''_{(t)} + 3x'_{(t)} + 2x_{(t)} = tu(t)$ d) $x''_{(t)} + 3x'_{(t)} = \delta(t)$

e) $x''_{(t)} - 2x'_{(t)} - 8x_{(t)} = \delta(t+6)$ f) $x''_{(t)} + 2x'_{(t)} + x_{(t)} = e^{-2t}u(t)$

g) $x''_{(t)} + 6x'_{(t)} + 5x_{(t)} = t^2u(t)$ h) $x''_{(t)} + 4x'_{(t)} + 4x_{(t)} = 2e^{3t}u(-t)$

RESPUESTAS

a) $x_{(t)} = \left(\frac{t^2}{3} - \frac{2t}{9} + \frac{2}{27} - \frac{2e^{-3t}}{27}\right)u(t)$; b) $x_{(t)} = \frac{e^{-2(t-3)}}{4} - \frac{e^{-6(t-3)}}{4}u(t-3)$; c) $x_{(t)} = \left(\frac{t}{2} - \frac{3}{4} + e^{-t} - \frac{1}{4}e^{-2t}\right)u(t)$;

d) $x_{(t)} = \frac{1}{6}\text{sgn}(t) - \frac{1}{3}e^{-3t}u(t)$; e) $x_{(t)} = -\frac{e^{4(t+6)}}{6}u(-t-6) - \frac{e^{-2(t+6)}}{6}u(t+6)$; f) $x_{(t)} = \left(te^{-t} - e^{-t} + e^{-2t}\right)u(t)$;

g) $x_{(t)} = \left(\frac{t^2}{5} - \frac{12}{25}t + \frac{62}{125} + \frac{e^{-5t}}{250} - \frac{e^{-t}}{2}\right)u(t)$;h) $x_{(t)} = \frac{2}{25}u(-t) + \left(\frac{2}{5}te^{-2t} + \frac{2}{25}e^{-2t}\right)u(t)$