UMSS- FACULTAD DE CIENCIAS Y TECNOLOGIA
DEPARTAMENTO DE MATEMATICAS
SEMESTRE 1-2022 (4-05-2023)

PRIMER PARCIAL – TRANSFORMADAS INTEGRALES

APELLIDOS:………………………………… NOMBRES:……………………………
CARRERA:…………………………… CARNET DE IDENTIDAD:……………………

1.- Dada la función definida en un período T=5: $f_{(t)} = 2t \ldots\ldots 0 < t < 5$ a partir del teorema de Parseval, demostrar que: $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$ **(10 pts.)**

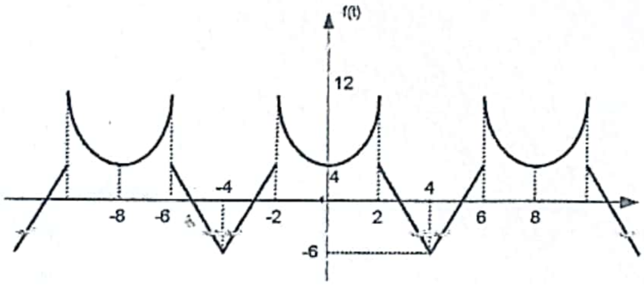
2.- Dada la siguiente función que se repite con un período T=6, hallar su serie de Fourier aplicando el método de diferenciación. **(25 pts.)**

$$f_{(t)} = \begin{cases} -\frac{1}{9}t^3 - 10 & 0 < t < 3 \\ 3t - 8 & 3 < t < 6 \end{cases}$$

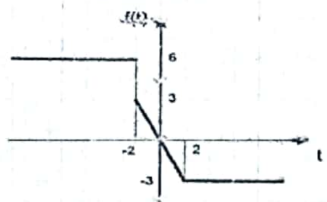
3.- Calcular las transformadas de Fourier: **(40 pts.)**

a) $\mathcal{F}\left\{\frac{(t+3)\cos(2t)}{(t-5)^3}\right\}$ b) $\mathcal{F}\left\{\frac{(t-1)^2 e^{-j5(t-1)}}{(t-1)^2 - 12(t-1) + 40}\right\}$

4.- Hallar la serie de Fourier de la función (las curvas son parábolas) **(~~20~~ pts.)** (30 pts)



5.- Calcular la transformada de Fourier de la función: **(15 pts.)**



1er P. Transformadas

1.-

$f(t) = 2t \quad 0<t<5 \quad . \quad T=5 = \omega_0 = \frac{2\pi}{5} \qquad C_0 = \frac{1}{5}\int_0^5 2t\,dt = \frac{2}{5}\cdot\frac{t^2}{2}\Big|_0^5 \Rightarrow \boxed{C_0 = 5}$

$C_n = \frac{1}{5}\int_0^5 2t e^{-j2\pi/5 nt}\,dt = \frac{2}{5}\cdot\left[\frac{te^{-j\frac{2\pi}{5}nt}}{-j\frac{2\pi}{5}n} - \frac{e^{-j\frac{2\pi}{5}nt}}{(-j\frac{2\pi}{5}n)^2}\right]\Big|_0^5 = \frac{2}{5}\cdot\left[-\frac{25}{j2\pi n}\cancel{e^{-j2\pi n}}^{1} + \underbrace{\frac{25}{4\pi^2 n^2}\cancel{e^{-j2\pi n}}^{1} - \frac{25}{4\pi^2 n^2}}_{0}\right]$

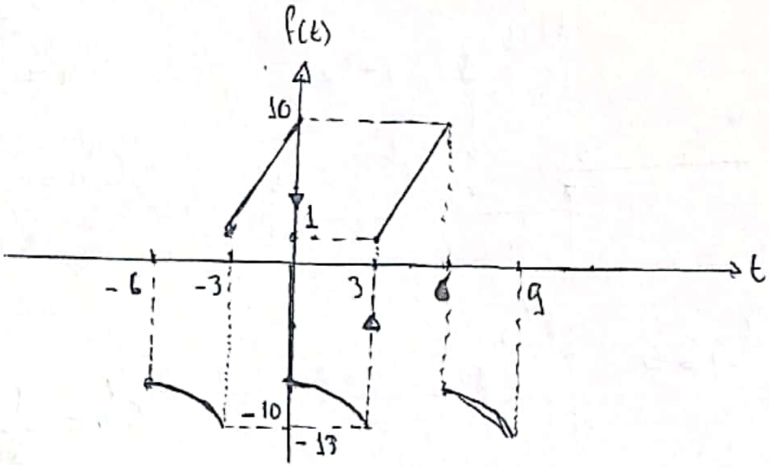
$C_n = j\frac{5}{\pi n} \Rightarrow \underline{b_n = -\frac{10}{\pi n}}$

$\frac{1}{5}\int_0^5 4t^2\,dt = 25 + \frac{1}{2}\sum_{n=1}^{\infty}\frac{100}{\pi^2 n^2} \Rightarrow \frac{4}{5}\cdot\frac{t^3}{3}\Big|_0^5 = 25 + \frac{50}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2} \Rightarrow \frac{25}{3} = \frac{50}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2} \cdot \frac{\pi^2}{50}$

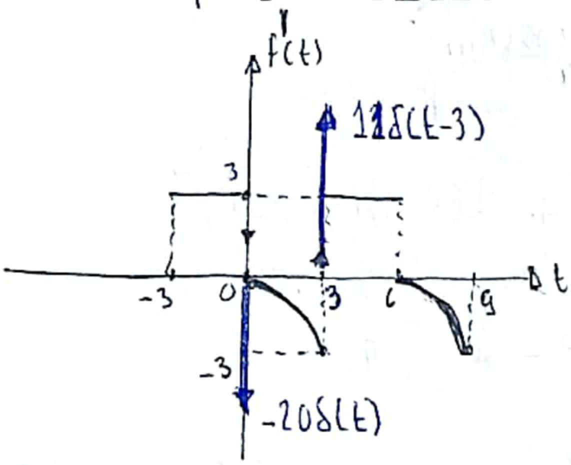
$\therefore \quad \boxed{\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}}$

2.- $T=6 \rightarrow \omega_0 = \pi/3$

$$f(t) = \begin{cases} -\frac{1}{9}t^3 - 10 & 0<t<3 \\ 3t-8 & 3<t<6 \end{cases}$$



$$f'(t) = \begin{cases} -\frac{1}{3}t^2 & 0<t<3 \\ 3 & 3<t<6 \end{cases}$$



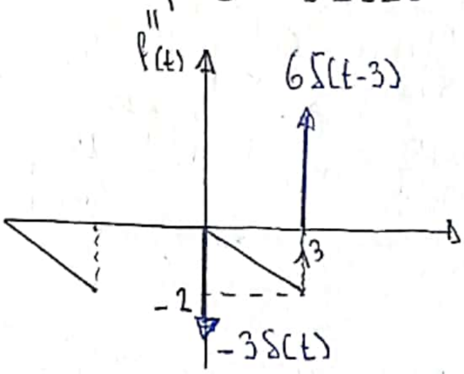
$\gamma'_n = \frac{1}{6}\int_0^6 (-20\delta(t) + 14\delta(t-3))e^{-j\frac{\pi}{3}nt}\,dt$

$\gamma'_n = -\frac{10}{3} + \frac{7}{3}\cos(\pi n)$

$C'_n = \frac{\gamma'_n}{j\frac{\pi}{3}n} = \underline{\frac{j}{\pi n}(-10 + 7\cos(\pi n))}$

Escaneado con CamScanner

$$f''(t) = \begin{cases} -\frac{2}{3}t & 0<t<3 \\ 0 & 3<t<6 \end{cases}$$



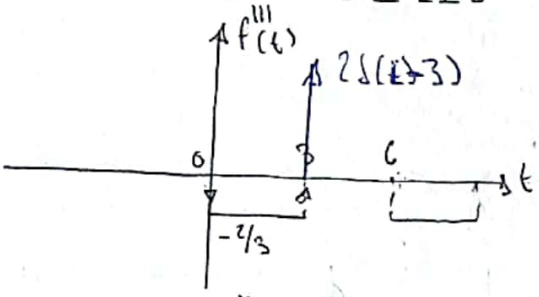
$$\gamma''_n = \frac{1}{6}\cdot\int_0^6 (-3\delta(t)+6\delta(t-3))\, e^{-j\frac{\pi}{3}nt}\, dt$$

$$\gamma''_n = -\frac{1}{2} + \cos(\pi n)$$

$$C''_n = \frac{\gamma''_n}{(j\frac{\pi}{3}n)^2} = -\frac{9}{\pi^2 n^2}\left(-\frac{1}{2}+\cos(\pi n)\right)$$

$$\boxed{C''_n = \frac{9/2 - 9\cos(\pi n)}{\pi^2 n^2}}$$

$$f'''(t) = \begin{cases} -2/3 & 0<t<3 \\ 0 & 3<t<6 \end{cases}$$



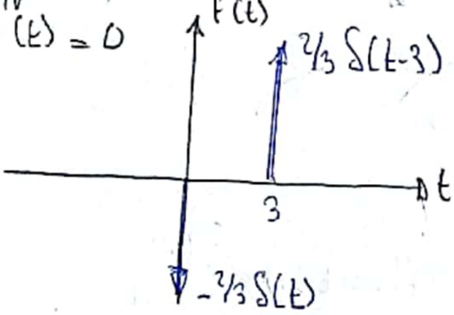
$$\gamma'''_n = \frac{1}{6}\int_0^6 2\delta(t-3)\, e^{-j\frac{\pi}{3}nt}\, dt$$

$$\gamma'''_n = \frac{1}{3}\cos(\pi n)$$

$$C'''_n = \frac{\gamma'''_n}{(j\frac{\pi}{3}n)^3} = j\frac{27}{\pi^3 n^3}\cdot\frac{1}{3}\cos(\pi n)$$

$$\boxed{C'''_n = j\,\frac{9\cos(\pi n)}{\pi^3 n^3}}$$

$$f^{IV}(t) = 0$$



$$\gamma^{IV}_n = \frac{1}{6}\int_0^6 \left(-\frac{2}{3}\delta(t) + \frac{2}{3}\delta(t-3)\right) e^{-j\frac{\pi}{3}nt}\, dt$$

$$\gamma^{IV}_n = -\frac{1}{9} + \frac{1}{9}\cos(\pi n)$$

$$C^{IV}_n = \frac{\gamma^{IV}_n}{(j\frac{\pi}{3}n)^4} = \boxed{\frac{-9+9\cos(\pi n)}{\pi^4 n^4}}$$

$$a_n = 2\,\mathrm{Re}\{C_n\} \qquad b_n = -2\,\mathrm{Im}\{C_n\} \rightarrow$$

$$\boxed{a_n = \frac{9-18\cos(\pi n)}{\pi^2 n^2} + \frac{18(\cos(\pi n)-1)}{\pi^4 n^4}}$$

$$\boxed{b_n = \frac{-20+14\cos(\pi n)}{\pi n} - \frac{18\cos(\pi n)}{\pi^3 n^3}}$$

$$C_0 = \frac{1}{6}\cdot\left[\int_0^3 \left(-\frac{1}{9}t^3 - 10\right)dt + \int_3^6 (3t-8)\,dt\right] = \frac{1}{6}\cdot\left[\left(-\frac{1}{9}\cdot\frac{t^4}{4} - 10t\right)\Big|_0^3 + \left(\frac{3t^2}{2} - 8t\right)\Big|_3^6\right]$$

$$C_0 = \frac{1}{6}\cdot\left[-\frac{129}{4} + 6 + \frac{21}{2}\right] \rightarrow \boxed{C_0 = -\frac{21}{8}} \rightarrow \boxed{f(t) = -\frac{21}{8} + \sum_{n=1}^{\infty}\left(a_n\cos\left(\frac{\pi}{3}nt\right) + b_n \sin\left(\frac{\pi}{3}nt\right)\right)}$$

Escaneado con CamScanner

3.- a) $\mathcal{F}\left\{\frac{(t+3)\cos(2t)}{(t-5)^3}\right\} \Rightarrow F(\omega)$ $\qquad \cos(2t) \nearrow \omega: \omega-2$, $\searrow \omega+2$

$$F_1(\omega) = \mathcal{F}\left\{\frac{t}{(t-5)^3} + \frac{3}{(t-5)^3}\right\}$$

$$\mathcal{F}\left\{\frac{1}{t}\right\} = -j\pi\,\text{sgn}(\omega) \qquad f(t) = 1/t \to f'(t) = -1/t^2 \to f''(t) = \frac{2}{t^3}$$

$$\mathcal{F}\left\{\frac{2}{t^3}\right\} = \mathcal{F}\{f''(t)\} = (j\omega)^2 \cdot \mathcal{F}\left\{\frac{1}{t}\right\} = -\omega^2 \cdot -j\pi\,\text{sgn}(\omega) \to \mathcal{F}\left\{\frac{1}{t^3}\right\} = j\frac{\pi}{2}\omega^2\,\text{sgn}(\omega)$$

$$\mathcal{F}\left\{\frac{1}{(t-5)^3}\right\} = j\frac{\pi}{2}\omega^2 e^{-j5\omega}\,\text{sgn}(\omega) \Rightarrow \mathcal{F}\left\{\frac{t}{(t-5)^3}\right\} = j\frac{d}{d\omega}\left[j\frac{\pi}{2}\omega^2 e^{-j5\omega}\,\text{sgn}(\omega)\right]$$

$$\mathcal{F}\left\{\frac{t}{(t-5)^3}\right\} = -\frac{\pi}{2}\cdot\left[2\omega e^{-j5\omega}\,\text{sgn}(\omega) + \omega^2 e^{-j5\omega}\cdot(-j5)\,\text{sgn}(\omega) + \underbrace{\omega^2 e^{-j5\omega}\cdot 2\delta(\omega)}_{0}\right]$$

$$\mathcal{F}\left\{\frac{t}{(t-5)^3}\right\} = -\frac{\pi}{2}\cdot\omega e^{-j5\omega}\,\text{sgn}(\omega)\cdot(2-j5\omega)$$

$$F_1(\omega) = -\frac{\pi}{2}\omega e^{-j5\omega}\,\text{sgn}(\omega)(2-j5\omega) + j\frac{3\pi}{2}\omega^2 e^{-j5\omega}\,\text{sgn}(\omega)$$

$$F_1(\omega) = \frac{\pi}{2}\omega e^{-j5\omega}\,\text{sgn}(\omega)\cdot\left[-2+j5\omega+j3\omega\right] = \underline{\frac{\pi}{2}\omega e^{-j5\omega}\,\text{sgn}(\omega)\cdot(-2+j8\omega)}$$

$$\boxed{F(\omega) = \frac{\pi}{4}\cdot\left[(\omega-2)e^{-j5(\omega-2)}\,\text{sgn}(\omega-2)(-2+j8(\omega-2)) + (\omega+2)e^{-j5(\omega+2)}\,\text{sgn}(\omega+2)(-2+j8(\omega+2))\right]}$$

b) $\mathcal{F}\left\{\frac{(t-1)^2 e^{-j5(t-1)}}{(t-1)^2-12(t-1)+40}\right\} = F(\omega) \qquad F_1(\omega) = \mathcal{F}\left\{\frac{t^2 e^{-j5t}}{t^2-12t+40}\right\}$

$$\mathcal{F}\left\{\frac{1}{\underset{+6^2-6^2}{t^2-12t+40}}\right\} = \mathcal{F}\left\{\frac{1}{(t-6)^2+4}\right\} = \frac{\pi}{2}e^{-j6\omega}\cdot e^{-2|\omega|} = \frac{\pi}{2}e^{-2|\omega|-j6\omega}$$

$$\mathcal{F}\left\{\frac{t}{t^2-12t+40}\right\} = j\frac{\pi}{2}e^{-2|\omega|-j6\omega}\cdot(-2\,\text{sgn}(\omega)-j6)$$

$$\mathcal{F}\left\{\frac{t^2}{t^2-12t+40}\right\} = -\frac{\pi}{2}\cdot e^{-2|\omega|-j6\omega}\cdot(-2\,\text{sgn}(\omega)-j6)^2 + j\frac{\pi}{2}\cancel{e^{-2|\omega|-j6\omega}}\cdot(-4\delta(\omega))$$ $\quad 1 \quad \omega=0$

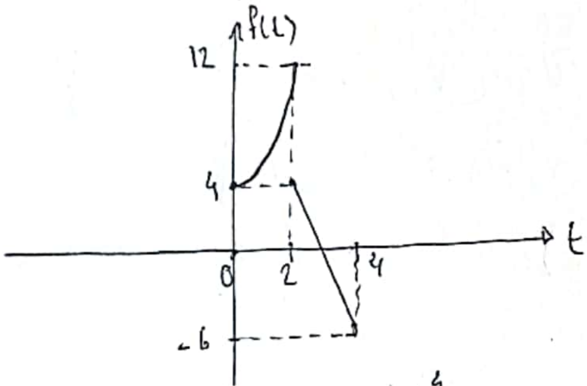
$$\mathcal{F}\left\{\frac{t^2}{t^2-12t+40}\right\} = -\frac{\pi}{2}e^{-2|\omega|-j6\omega}\cdot(4\cancel{\text{sgn}^2(\omega)}+j24\,\text{sgn}(\omega)-6) - j2\pi\delta(\omega)$$

$$\mathcal{F}\left\{\frac{t^2}{t^2-12t+40}\right\} = \frac{\pi}{2}e^{-2|\omega|-j6\omega}\cdot(2-j24\,\text{sgn}(\omega)) - j2\pi\delta(\omega)$$

$$\mathcal{F}\left\{\frac{t^2 e^{-j5t}}{t^2-12t+40}\right\} = \frac{\pi}{\cancel{2}}\, e^{-2|\omega+5|-j6(\omega+5)} \cdot \left(\cancel{2}^{\,1} - j\cancel{24}^{\,12}\,\text{sgn}(\omega+5)\right) - j2\pi\,\delta(\omega+5)$$

$$\boxed{F(\omega) = \pi e^{-j\omega} \cdot \left[e^{-2|\omega+5|-j6(\omega+5)} \cdot \left(1 - j12\,\text{sgn}(\omega+5)\right) - j2\pi\,\delta(\omega+5)\right]}$$

4.- Simetría par: $T = 8 \rightarrow \omega_0 = \pi/4$



$y = At^2 + 4$

$12 = 4A + 4 \rightarrow A = 2$

$$f(t) = \begin{cases} 2t^2 + 4 & 0 < t < 2 \\ -5t + 14 & 2 < t < 4 \end{cases}$$

$$C_0 = \frac{2}{8} \cdot \left[\int_0^2 (2t^2+4)\,dt + \int_2^4 (-5t+14)\,dt \right] = \frac{1}{4} \cdot \left[\left(\frac{2t^3}{3} + 4t\right)\Big|_0^2 + \left(-\frac{5t^2}{2} + 14t\right)\Big|_2^4 \right]$$

$$C_0 = \frac{1}{4} \cdot \left[\frac{40}{3} + 16 - 18 \right] \Rightarrow \boxed{C_0 = \frac{17}{6}}$$

$$a_n = \frac{4}{8} \cdot \left[\int_0^2 (2t^2+4)\cos\left(\frac{\pi}{4}nt\right)dt + \int_2^4 (-5t+14)\cos\left(\frac{\pi}{4}nt\right) dt \right]$$

$u = t^2 \rightarrow du = 2t\,dt$

$\int dv = \int \cos\left(\frac{\pi}{4}nt\right)dt \rightarrow v = \dfrac{\text{sen}\left(\frac{\pi}{4}nt\right)}{\frac{\pi}{4}n}$

$$a_n = \frac{1}{2} \cdot \Bigg[2 \cdot \left(\frac{t^2\,\text{sen}\left(\frac{\pi}{4}nt\right)}{\frac{\pi}{4}n}\Bigg|_0^2 - \int_0^2 \frac{\text{sen}\left(\frac{\pi}{4}nt\right)}{\frac{\pi}{4}n} \cdot 2t\,dt \right) + \frac{4\,\text{sen}\left(\frac{\pi}{4}nt\right)}{\frac{\pi}{4}n}\Bigg|_0^2$$

$$- 5 \cdot \left(\frac{t\,\text{sen}\left(\frac{\pi}{4}nt\right)}{\frac{\pi}{4}n} + \frac{\cos\left(\frac{\pi}{4}nt\right)}{\left(\frac{\pi}{4}n\right)^2} \right)\Bigg|_2^4 + \frac{14\,\text{sen}\left(\frac{\pi}{4}nt\right)}{\frac{\pi}{4}n}\Bigg|_2^4 \Bigg]$$

$$a_n = \frac{1}{2} \cdot \Bigg[2 \cdot \left(\frac{16}{\pi n}\,\text{sen}\left(\frac{\pi}{2}n\right) - \frac{8}{\pi n} \cdot \left(-\frac{t\cos\left(\frac{\pi}{4}nt\right)}{\frac{\pi}{4}n} + \frac{\text{sen}\left(\frac{\pi}{4}nt\right)}{\left(\frac{\pi}{4}n\right)^2} \right)\Bigg|_0^2 \right) + \frac{16}{\pi n}\,\text{sen}\left(\frac{\pi}{2}n\right)$$

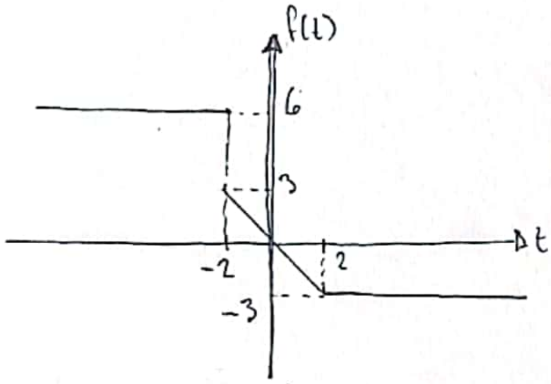
$$- 5 \cdot \left(-\frac{8}{\pi n}\,\text{sen}\left(\frac{\pi}{2}n\right) + \frac{16}{\pi^2 n^2}\cos(\pi n) - \frac{16}{\pi^2 n^2}\cos\left(\frac{\pi}{2}n\right) \right) - \frac{56}{\pi n}\,\text{sen}\left(\frac{\pi}{2}n\right) \Bigg]$$

$$a_n = \frac{1}{2}\cdot\left[\frac{32}{\pi n}\,\text{sen}\left(\frac{\pi}{2}n\right) - \frac{16}{\pi n}\cdot\left(-\frac{8}{\pi n}\cos\left(\frac{\pi}{2}n\right) + \frac{16}{\pi^2 n^2}\,\text{sen}\left(\frac{\pi}{2}n\right)\right) - \frac{80}{\pi^2 n^2}\cos(\pi n) + \frac{80}{\pi^2 n^2}\cos\left(\frac{\pi}{2}n\right)\right]$$

$$\boxed{a_n = \frac{16}{\pi n}\,\text{sen}\left(\frac{\pi}{2}n\right) + \frac{104\cos\left(\frac{\pi}{2}n\right) - 40\cos(\pi n)}{\pi^2 n^2} - \frac{128}{\pi^3 n^3}\,\text{sen}\left(\frac{\pi}{2}n\right)}$$

$$\boxed{f(t) = \frac{17}{6} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{\pi}{4}nt\right)}$$

5.-



$$f(t) = \begin{cases} 6 & t < -2 \\ -\frac{3}{2}t & -2 < t < 2 \\ -3 & t > 2 \end{cases}$$

$$f(t) = 6 - 6u(t+2) - \frac{3}{2}t\left(u(t+2) - u(t-2)\right) - 3u(t-2)$$

$$F(\omega) = 12\pi\delta(\omega) - 6e^{j2\omega}\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) - j\frac{3}{2}\frac{d}{d\omega}\cdot\left[e^{j2\omega}\left(\frac{1}{j\omega} + \pi\delta(\omega)\right) - e^{-j2\omega}\left(\frac{1}{j\omega} + \pi\delta(\omega)\right)\right] - 3e^{-j2\omega}\left(\frac{1}{j\omega} + \pi\delta(\omega)\right)$$

$$F(\omega) = 12\pi\delta(\omega) - \frac{6e^{j2\omega}}{j\omega} - 6\pi\delta(\omega) - j\frac{3}{2}\frac{d}{d\omega}\cdot\left[\frac{\overbrace{e^{j2\omega} - e^{-j2\omega}}^{2j\,\text{sen}(2\omega)}}{\cancel{j}\omega} + \cancel{\pi\delta(\omega)} - \cancel{\pi\delta(\omega)}\right] - \frac{3e^{-j2\omega}}{j\omega} + -3\pi\delta(\omega)$$

$$F(\omega) = 3\pi\delta(\omega) - \frac{6e^{j2\omega} + 3e^{-j2\omega}}{j\omega} - j\frac{3}{\cancel{2}}\frac{d}{d\omega}\cdot\left[\frac{\cancel{2}\,\text{sen}(2\omega)}{\omega}\right]$$

$$\boxed{F(\omega) = 3\pi\delta(\omega) - \frac{6e^{j2\omega} + 3e^{-j2\omega}}{j\omega} - j3\cdot\left(\frac{2\omega\cos(2\omega) - \text{sen}(2\omega)}{\omega^2}\right)}$$