**UMSS- FACULTAD DE CIENCIAS Y TECNOLOGIA**
**DEPARTAMENTO DE MATEMATICAS**
**GESTIÓN 2-2021 (18-11-2021)**

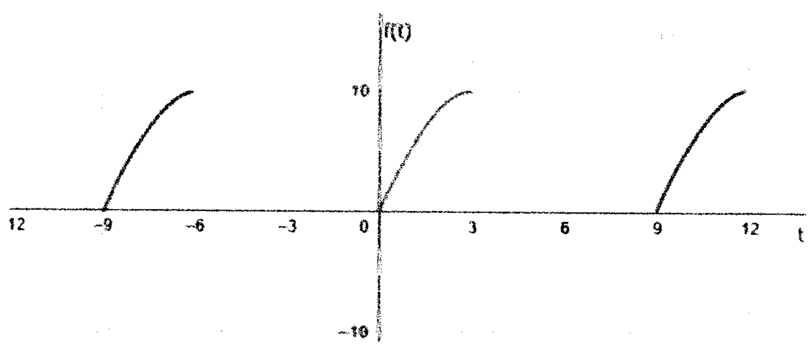
## PRIMER PARCIAL – TRANSFORMADAS INTEGRALES

APELLIDOS:.......................................... NOMBRES:..............................
CARRERA:................................... CARNET DE IDENTIDAD:....................

1.- Dada la siguiente función que se repite con un período T=8, hallar su serie de Fourier aplicando el método de diferenciación. Luego determine el valor del tercer armónico **30 pts.**

$$f_{(t)}=\begin{cases}\frac{1}{8}t^3-2t+5 & 0<t<4\\ -2t+8 & 4<t<8\end{cases}$$

2.- Hallar la serie de Fourier de la porción de onda senoidal de la figura y determine los primeros 3 armónicos diferentes de cero **20 pts.**




3.- Determine el período de la siguiente función: **10 pts.**

$$f_{(t)}=sen\left(\frac{2}{3}t\right)+\cos\left(\frac{5}{4}t\right)+\left|sen(6t)\right|$$

4.- Calcular las transformadas de Fourier de las funciones: **40 pts.**

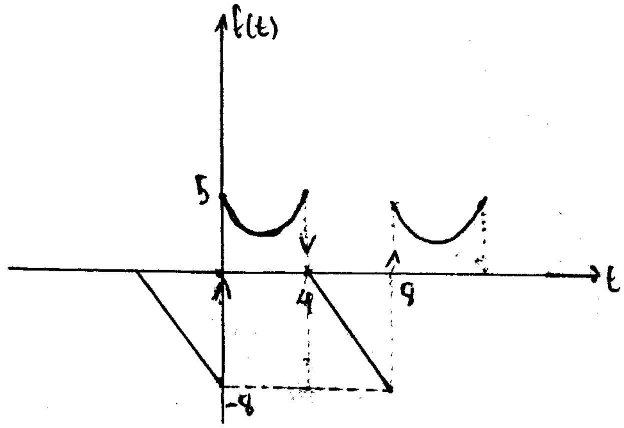
a) $\mathcal{F}\left\{t\cos\left(\frac{\pi}{4}t\right)\operatorname{sgn}(t-4)\right\}$ b) $\mathcal{F}\left\{\frac{(2jt-1)e^{-j4t}}{2-5jt+3t^2}\right\}$

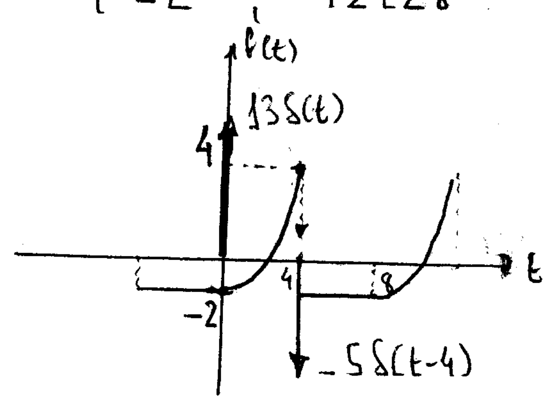
# 1er Parcial _ Transformadas

1.- $f(t) = \begin{cases} \frac{1}{8}t^3 - 2t + 5 & 0 < t < 4 \\ -2t + 8 & 4 < t < 8 \end{cases}$

$T = 8 \rightarrow \omega_0 = \frac{\pi}{4}$

$f'(t) = \begin{cases} \frac{3}{8}t^2 - 2 & 0 < t < 4 \\ -2 & 4 < t < 8 \end{cases}$



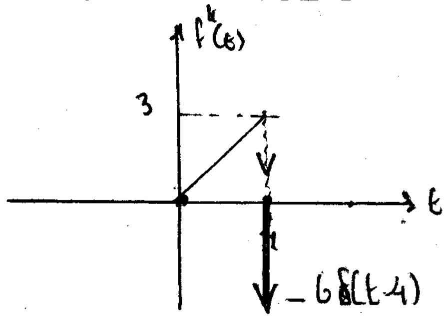

$$\gamma'_n = \frac{1}{8}\int_0^8 \left(13\delta(t) - 5\delta(t-4)\right) e^{-j\frac{\pi}{4}nt}\,dt = \frac{13}{8} - \frac{5}{8}\cos(\pi n)$$

$$c'_n = \frac{\gamma'_n}{j\frac{\pi}{4}n} = -\frac{j}{\pi n}\left(\frac{13}{2} - \frac{5}{2}\cos(\pi n)\right)$$

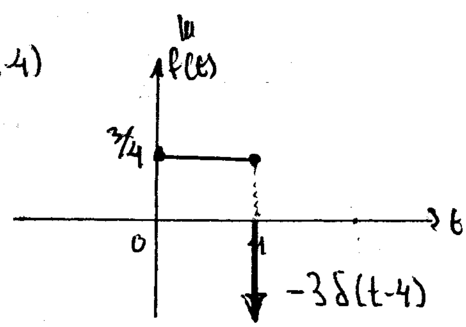
$f''(t) = \begin{cases} \frac{3}{4}t & 0 < t < 4 \\ 0 & 4 < t < 8 \end{cases}$

$$\gamma''_n = \frac{1}{8}\int_0^8 -6\delta(t-4) e^{-j\frac{\pi}{4}nt}\,dt = -\frac{3}{4}\cos(\pi n)$$

$$c''_n = \frac{\gamma''_n}{(j\frac{\pi}{4}n)^2} = -\frac{16}{\pi^2 n^2}\gamma''_n \rightarrow c''_n = \frac{12}{\pi^2 n^2}\cos(\pi n)$$




$f'''(t) = \begin{cases} \frac{3}{4} & 0 < t < 4 \\ 0 & 4 < t < 8 \end{cases}$



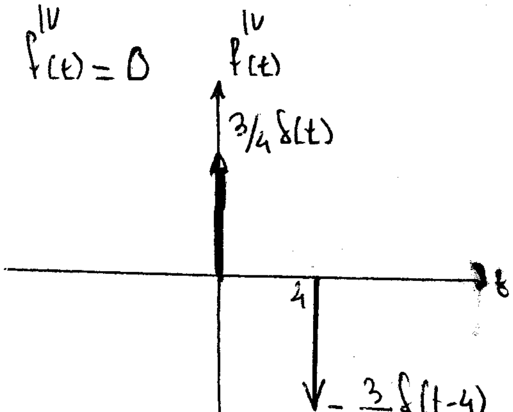

$$\gamma'''_n = \frac{1}{8}\int_0^8 -3\delta(t-4) e^{-j\frac{\pi}{4}nt}\,dt$$

$$\gamma'''_n = -\frac{3}{8}\cos(\pi n)$$

$$c'''_n = \frac{\gamma'''_n}{(j\frac{\pi}{4}n)^3} = j\frac{64}{\pi^3 n^3}\cdot -\frac{3}{8}\cos(\pi n)$$

$$c'''_n = -j\frac{24}{\pi^3 n^3}\cos(\pi n)$$

$f^{IV}(t) = 0$




$$\gamma^{IV}_n = \frac{1}{8}\int_0^8 \left(\frac{3}{4}\delta(t) - \frac{3}{4}\delta(t-4)\right) e^{-j\frac{\pi}{4}nt}\,dt$$

$$\gamma^{IV}_n = \frac{3}{32} - \frac{3}{32}\cos(\pi n) \qquad c^{IV}_n = \frac{\gamma^{IV}_n}{(j\frac{\pi}{4}n)^4} = \frac{256}{\pi^4 n^4}\gamma^{IV}_n$$

$$c^{IV}_n = \frac{24}{\pi^4 n^4}\left(1 - \cos(\pi n)\right)$$

$$a_n = 2\,\mathrm{Re}\{C_n\} \rightarrow \boxed{a_n = \frac{24}{\pi^2 n^2}\cos(\pi n) + \frac{48(1-\cos(\pi n))}{\pi^4 n^4}}$$

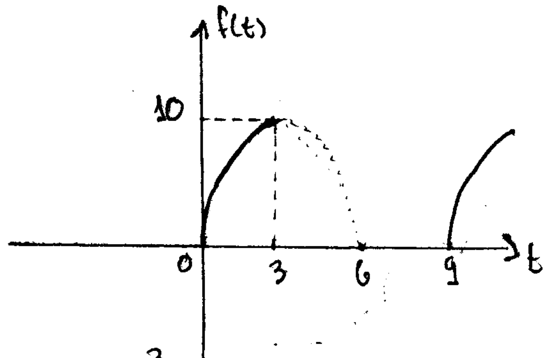
$$b_n = -2\,\mathrm{Im}\{C_n\} \rightarrow \boxed{b_n = \frac{13-5\cos(\pi n)}{\pi n} + \frac{48\cos(\pi n)}{\pi^3 n^3}}$$

$$C_0 = \frac{1}{8}\left[\int_0^4 \left(\frac{t^3}{8} - 2t + 5\right)dt + \int_4^8 (-2t+8)\,dt\right] = \frac{1}{8}\left[12 - 16\right] \rightarrow \boxed{C_0 = -\frac{1}{2}}$$

$$\boxed{f(t) = -\frac{1}{2} + \sum_{n=0}^{\infty}\left[a_n\cos\left(\frac{\pi}{4}nt\right) + b_n \,\mathrm{sen}\left(\frac{\pi}{4}nt\right)\right]}$$

3er armónico: $n = 3$ $\left.\begin{array}{l} a_n = -0.258 \\ b_n = 1.853 \end{array}\right\}$ $\boxed{f_3(t) = -0.258\cos\left(\frac{3\pi}{4}t\right) + 1.853\,\mathrm{sen}\left(\frac{3\pi}{4}t\right)}$

2.-




$$T = 9 \rightarrow \omega_0 = \frac{2\pi}{9}$$

$$T' = 12 \rightarrow \omega' = \frac{\pi}{6} \rightarrow f(t) = 10\,\mathrm{sen}\left(\frac{\pi}{6}t\right) \quad 0 < t < 3$$

$$C_n = \frac{1}{9}\int_0^3 10\,\mathrm{sen}\left(\frac{\pi}{6}t\right) e^{-j\frac{2\pi}{9}nt}\,dt$$

$$C_n = \frac{10}{9}\int_0^3 \left(\frac{e^{j\frac{\pi}{6}t} - e^{-j\frac{\pi}{6}t}}{2j}\right) e^{-j\frac{2\pi}{9}nt}\,dt = \frac{5}{9j}\int_0^3 \left(e^{j\pi\left(\frac{1}{6}-\frac{2}{9}n\right)t} - e^{-j\pi\left(\frac{1}{6}+\frac{2}{9}n\right)t}\right)dt$$

$$C_n = \frac{5}{9j}\left[\frac{e^{j\pi\left(\frac{1}{6}-\frac{2}{9}n\right)t}}{j\pi\left(\frac{1}{6}-\frac{2}{9}n\right)} + \frac{e^{-j\pi\left(\frac{1}{6}+\frac{2}{9}n\right)t}}{+j\pi\left(\frac{1}{6}+\frac{2}{9}n\right)}\right]\Bigg|_0^3 = -\frac{5}{\pi}\cdot\left[\frac{e^{j\frac{\pi}{2}}\cdot e^{-j\frac{2\pi n}{3}} - 1}{1.5 - 2n} + \frac{e^{-j\frac{\pi}{2}}\cdot e^{-j\frac{2\pi n}{3}} - 1}{1.5 + 2n}\right]$$

$$e^{\pm j\pi/2} = \cancel{\cos(\pi/2)}^{\,0} \pm j\,\mathrm{sen}(\pi/2) = \pm j \rightarrow C_n = -\frac{5}{\pi}\cdot\left[\frac{je^{-j\frac{2\pi n}{3}} - 1}{1.5-2n} + \frac{-je^{-j\frac{2\pi n}{3}} - 1}{1.5+2n}\right]$$

$$C_n = -\frac{5}{\pi}\cdot\left[\frac{\left(je^{-j\frac{2\pi n}{3}} - 1\right)(1.5+2n) + \left(-je^{-j\frac{2\pi n}{3}} - 1\right)(1.5-2n)}{(1.5-2n)(1.5+2n)}\right]$$

$$C_n = -\frac{5}{\pi}\cdot\left[\frac{\cancel{1.5je^{-j\frac{2\pi n}{3}}} - 1.5 + 2jne^{-j\frac{2\pi n}{3}} - \cancel{2n} - \cancel{1.5je^{-j\frac{2\pi n}{3}}} - 1.5 + 2jne^{-j\frac{2\pi n}{3}} + \cancel{2n}}{2.25 - 4n^2}\right]$$

$$C_n = -\frac{5}{\pi}\cdot\left[\frac{4jn\left(\cos\left(\frac{2\pi}{3}n\right) - j\,\mathrm{sen}\left(\frac{2\pi}{3}n\right)\right) - 3}{2.25 - 4n^2}\right] = -\frac{5}{\pi}\cdot\left[\frac{4n\,\mathrm{sen}\left(\frac{2\pi}{3}n\right) - 3 + j4n\cos\left(\frac{2\pi}{3}n\right)}{2.25-4n^2}\right]$$

$$n = 0:\quad C_0 = -\frac{5}{\pi}\cdot\frac{-3}{2.25} \rightarrow \boxed{C_0 = \frac{20}{3\pi}}$$

$$\boxed{a_n = -\frac{10}{\pi}\left(\frac{4n\,\mathrm{sen}\left(\frac{2\pi}{3}n\right) - 3}{2.25 - 4n^2}\right)}$$

$$\boxed{f(t) = \frac{20}{3\pi} + \sum_{n=1}^{\infty}\left[a_n\cos\left(\frac{2\pi}{9}nt\right) + b_n\,\mathrm{sen}\left(\frac{2\pi}{9}nt\right)\right]}$$

$$\boxed{b_n = \frac{40n\cos\left(\frac{2\pi}{3}n\right)}{\pi(2.25 - 4n^2)}}$$

(Armónicos al final)

3.- $f(t) = \sin\left(\frac{2}{3}t\right) + \cos\left(\frac{5}{4}t\right) + |\sin(6t)|$

$$T_1 = \frac{2\pi}{2/3} = 3\pi \;;\quad T_2 = \frac{2\pi}{5/4} = \frac{8\pi}{5} \;;\quad T_3 = \frac{\pi}{6}$$

$$T = 3\pi a = \frac{8\pi}{5} b = \frac{\pi}{6} c \rightarrow 3a = \frac{8}{5} b = \frac{1}{6} c \quad \times 30$$

$$90a = 48b = 5c \qquad a;\, b;\, c: \text{Mínimos} \in \mathbb{N}$$

MCM(90,48,5):

$$\left.\begin{array}{l} 90 = 18\times 5 = 3\times 6\times 5 = 2\times 3^2\times 5 \\ 48 = 3\times 16 = 2^4\times 3 \\ 5 = 5 \end{array}\right\} MCM = 2^4 . 3^2 . 5 = 720 \Rightarrow \begin{array}{l} a = 8 \\ b = 15 \\ c = 144 \end{array}$$

$\therefore \boxed{T = 24\pi}$ (Puede también hacerse como se hizo en clase)

4.- a) $\mathcal{F}\left\{t\cos\left(\frac{\pi}{4}t\right)\mathrm{sgn}(t-4)\right\}$

$$\mathcal{F}\{\mathrm{sgn}(t)\} = \frac{2}{j\omega} \rightarrow \mathcal{F}\{\mathrm{sgn}(t-4)\} = \frac{2e^{-j4\omega}}{j\omega} \rightarrow \mathcal{F}\left\{\cos\left(\frac{\pi}{4}t\right)\mathrm{sgn}(t-4)\right\} = \frac{e^{-j4(\omega+\pi/4)}}{j(\omega+\pi/4)} + \frac{e^{-j4(\omega-\pi/4)}}{j(\omega-\pi/4)}$$

$$F_s(\omega) = \frac{e^{-j4\omega}\cdot \cancel{e^{-j\pi}}^{-1}}{j(\omega+\pi/4)} + \frac{e^{-j4\omega}\cdot \cancel{e^{j\pi}}^{-1}}{j(\omega-\pi/4)} = je^{-j4\omega}\cdot\left(\frac{\omega - \cancel{\pi/4} + \omega + \cancel{\pi/4}}{\omega^2 - \frac{\pi^2}{16}}\right) = \frac{j2\omega e^{-j4\omega}}{\omega^2 - \frac{\pi^2}{16}}$$

$$F(\omega) = \mathcal{F}\left\{t\cos\left(\frac{\pi}{4}t\right)\mathrm{sgn}(t-4)\right\} = j\frac{dF_s(\omega)}{d\omega} = -\frac{(2e^{-j4\omega} + 2\omega\cdot(-j4e^{-j4\omega}))(\omega^2 - \pi^2/16) - 2\omega(2\omega e^{-j4\omega})}{\left(\omega^2 - \frac{\pi^2}{16}\right)^2}$$

$$\boxed{F(\omega) = -\frac{e^{-j4\omega}\left[(2 - j8\omega)(\omega^2 - \pi^2/16) - 4\omega^2\right]}{\left(\omega^2 - \frac{\pi^2}{16}\right)^2}}$$

(1ª forma)

b) $\mathcal{F}\left\{\frac{(2jt-1)e^{-j4t}}{2 - 5jt + 3t^2}\right\}$

$$2 - 5jt - 3(jt)^2 = (2+jt)(1-3jt)$$

$$\begin{array}{ll} 2 & jt \\ 1 & -3jt \end{array}$$

$$F(\omega) = \mathcal{F}\left\{\frac{(2jt-1)e^{-j4t}}{(2+jt)(1-3jt)}\right\} \Rightarrow \frac{2jt-1}{(2+jt)(1-3jt)} = \frac{A}{2+jt} + \frac{B}{1-3jt}$$

$$2jt - 1 = A(1-3jt) + B(2+jt)$$

$jt = -2$: $\quad -5 = 7A \rightarrow \underline{A = -5/7}$

$jt = 1/3$: $\quad 2/3 - 1 = B(2 + 1/3) \rightarrow \underline{B = -1/7}$

$$F(\omega) = \mathcal{F}\left\{ \frac{-5/7}{2+jt} - \frac{1/7}{1-3jt} \right\}$$

$$\mathcal{F}\{e^{-at}u(t)\} = \frac{1}{a+j\omega} \rightarrow \mathcal{F}\left\{\frac{1}{a+jt}\right\} = 2\pi e^{a\omega}u(-\omega)$$

$$\mathcal{F}\{e^{at}u(-t)\} = \frac{1}{a-j\omega} \rightarrow \mathcal{F}\left\{\frac{1}{a-jt}\right\} = 2\pi e^{-a\omega}u(\omega)$$

$$F(\omega) = -\frac{10}{7}\pi e^{2\omega}u(-\omega) - \frac{2}{21}\pi e^{-\frac{1}{3}\omega}u(\omega)$$

$$e^{-j4t} \rightarrow \omega : \omega+4 \Rightarrow \boxed{F(\omega) = -\frac{10}{7}\pi e^{2(\omega+4)}u(-\omega-4) - \frac{2}{21}\pi e^{-\frac{1}{3}(\omega+4)}u(\omega+4)}$$

Armónicos del (2)

$$\boxed{f(t) = \frac{20}{3\pi} + 0.844\cos\left(\frac{2\pi}{9}t\right) + 3.638\,\text{sen}\left(\frac{2\pi}{9}t\right) + -2.298\cos\left(\frac{4\pi}{9}t\right) + 0.926\,\text{sen}\left(\frac{4\pi}{9}t\right) - 0.283\cos\left(\frac{2\pi}{3}t\right) - 1.132\,\text{sen}\left(\frac{2\pi}{3}t\right)}$$

(*) 1a) Otra manera: $\mathcal{F}\{t\cos(\pi/4\, t)\,\text{sgn}(t-4)\}$

$$\mathcal{F}\{\text{sgn}(t-4)\} = \frac{2e^{-j4\omega}}{j\omega} \rightarrow \mathcal{F}\{t\,\text{sgn}(t-4)\} = j\left[\frac{-j8e^{-j4\omega}\cdot j\omega - j2e^{-j4\omega}}{-\omega^2}\right] = \frac{-j8\omega e^{-j4\omega} - 2e^{-j4\omega}}{\omega^2}$$

$$F(\omega) = \frac{1}{2}\left[\frac{-j8(\omega-\pi/4)e^{-j4(\omega-\pi/4)} - 2e^{-j4(\omega-\pi/4)}}{(\omega-\pi/4)^2} + \frac{-j8(\omega+\pi/4)e^{-j4(\omega+\pi/4)} - 2e^{-j4(\omega+\pi/4)}}{(\omega+\pi/4)^2}\right]$$

$$F(\omega) = \frac{-j4(\omega-\pi/4)e^{-j4\omega}\cancel{e^{j\pi}}^{-1} - e^{-j4\omega}\cancel{e^{j\pi}}^{-1}}{(\omega-\pi/4)^2} + \frac{-j4(\omega+\pi/4)e^{-j4\omega}\cancel{e^{j\pi}}^{-1} - e^{-j4\omega}\cancel{e^{j\pi}}^{-1}}{(\omega+\pi/4)^2}$$

$$\boxed{F(\omega) = e^{-j4\omega}\left[\frac{j(4\omega-\pi)+1}{(\omega-\pi/4)^2} + \frac{+j(4\omega+\pi)+1}{(\omega+\pi/4)^2}\right]}$$