UMSS- FACULTAD DE CIENCIAS Y TECNOLOGIA
DEPARTAMENTO DE MATEMATICAS
SEMESTRE 2-2023 (21-12-2023)

SEGUNDO PARCIAL – TRANSFORMADAS INTEGRALES

APELLIDOS:... NOMBRES:.................................
CARRERA:................................... CARNET DE IDENTIDAD:.......................

1.- Un circuito RLC tiene como componentes: R=60 [Ω]; L=20 [H]; C=5 [mF] dadas las condiciones iniciales: $V_{C(0)} = 40[V]$, $i_{L(0)} = 2.5[A]$.Determinar la corriente en función del tiempo si se aplica la fuente de voltaje: $v_{(t)} = 100\cos(2t)[V]$.

2.- Dado el sistema: hallar solamente x(t)

$$\begin{cases} \frac{dx}{dt} = 3x - 2y + 5e^{-4t} \\ \frac{dy}{dt} = 2x - y + +2e^{-3t} \end{cases} ; \quad \begin{array}{l} x_{(0)} = -4 \\ y_{(0)} = -5 \end{array}$$

3.- Evaluar la integral: $\displaystyle\int_0^{\infty}\int_0^{t} \frac{te^{-\frac{1}{2}t-4x} sen^2(2x)}{x} dxdt$

4.- Resolver dadas las condiciones iniciales:

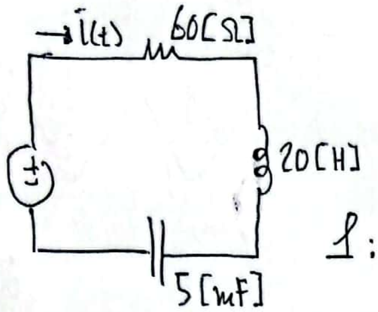
$$y''' + 4y' = 4sen(2t) \; ; \; y_{(0)} = 1; y'_{(0)} = 2; y''_{(0)} = -1$$

5.- Hallar la transformada inversa de Fourier de la función:

$$F_{(\omega)} = \frac{1}{(4+\varpi^2)(2+j\varpi)^2}$$

2^{do} Parcial _ Transformadas

1.-



$$\frac{1}{5\times10^{-3}}\int_0^t i\,dt + 40 + 20\frac{di}{dt} + 60i = 100\cos(2t)$$

$$\mathcal{L}:\ \frac{200\,I(s)}{s} + \frac{40}{s} + 20(s\,I(s) - 2{,}5) + 60\,I(s) = \frac{100s}{s^2+4}$$

$$I(s)\left(\frac{200}{s} + 20s + 60\right) = \frac{100s}{s^2+4} - \frac{40}{s} + 50$$

$$I(s) = \frac{\dfrac{100s^2 - 40(s^2+4) + 50s(s^2+4)}{s(s^2+4)}}{\dfrac{200 + 20s^2 + 60s}{s}} \Rightarrow I(s) = \frac{+50s^3 + 60s^2 + 200s - 160}{(20s^2+60s+200)(s^2+4)} \quad \%\,20$$

$$I(s) = \frac{+2{,}5s^3 + 3s^2 + 10s - 8}{(s^2+3s+10)(s^2+4)} = \frac{As+B}{s^2+3s+10} + \frac{Cs+D}{s^2+4} \quad \cdot (s^2+4)(s^2+3s+10)$$

$$+(3/2)^2 - (3/2)^2$$

$$+2{,}5s^3 + 3s^2 + 10s - 8 = (As+B)(s^2+4) + (Cs+D)(s^2+3s+10)$$

s^3: $2{,}5 = A + C$

s^2: $3 = B + 3C + D$

s: $10 = 4A + 10C + 3D$

cte: $-8 = 4B + 10D$

$\Rightarrow$ $A = \frac{5}{3}$, $B = \frac{13}{6}$, $C = \frac{5}{6}$, $D = -\frac{5}{3}$

$$i(t) = \mathcal{L}^{-1}\left\{\frac{\frac{5}{3}s + \frac{13}{6}}{(s+3/2)^2 + 31/4} + \frac{5/6}{s^2+4} - \frac{5/3}{s^2+4}\right\}$$

$$i(t) = \mathcal{L}^{-1}\left\{\frac{5/3(s+3/2)}{(s+3/2)^2+31/4} + \frac{-1/3}{(s+3/2)^2+31/4}\right\} + \frac{5}{6}\cos(2t) - \frac{5}{6}\operatorname{sen}(2t)$$

$$\boxed{i(t) = e^{-3/2\,t}\cdot\left(\frac{5}{3}\cos\left(\frac{\sqrt{31}}{2}t\right) + \frac{-2}{3\sqrt{31}}\operatorname{sen}\left(\frac{\sqrt{31}}{2}t\right)\right) + \frac{5}{6}\cos(2t) - \frac{5}{6}\operatorname{sen}(2t)\ [A]}$$

2.-

$$\begin{cases} \dfrac{dx}{dt} = 3x - 2y + 5e^{-4t} & x(0) = -4 \\[2ex] \dfrac{dy}{dt} = 2x - y + 2e^{-3t} & y(0) = -5 \end{cases}$$

$$\mathcal{L}:\quad sX(s) + 4 = 3X(s) - 2Y(s) + \frac{5}{s+4}$$

$$sY(s) + 5 = 2X(s) - Y(s) + \frac{2}{s+3}$$

$$\begin{cases} (s-3)X(s) + 2Y(s) = \dfrac{5}{s+4} - 4 \\ -2X(s) + (s+1)Y(s) = \dfrac{2}{s+3} - 5 \end{cases}$$

$$X(s) = \frac{\begin{vmatrix} \frac{5}{s+4} - 4 & 2 \\ \frac{2}{s+3} - 5 & s+1 \end{vmatrix}}{\begin{vmatrix} s-3 & 2 \\ -2 & s+1 \end{vmatrix}} = \frac{\frac{5s+5}{s+4} - 4s - 4 - \frac{4}{s+3} + 10}{(s-3)(s+1) + 4}$$

$$X(s) = \frac{\frac{(5s+5)(s+3) - 4(s+4) + (-4s+6)(s^2+7s+12)}{(s+4)(s+3)}}{s^2 - 2s + 1}$$

$$X(s) = \frac{5s^2 + 20s + 15 - 4s - 16 - 4s^3 - 22s^2 - 6s + 72}{(s+4)(s+3)(s-1)^2}$$

$$X(s) = \frac{-4s^3 - 17s^2 + 10s + 71}{(s+4)(s+3)(s-1)^2} = \frac{A}{s+4} + \frac{B}{s+3} + \frac{C}{s-1} + \frac{D}{(s-1)^2}$$

$$\cdot (s+4)(s+3)(s-1)^2$$

$$-4s^3 - 17s^2 + 10s + 71 = A(s+3)(s-1)^2 + B(s+4)(s-1)^2 + C(s+4)(s+3)(s-1) + D(s+4)(s+3)$$

$s=1$: $\quad 60 = D(5)(4) \rightarrow \boxed{D=3}$

$s=-3$: $\quad -4 = B(1)(-4)^2 \rightarrow \boxed{B = -1/4}$

$s=-4$: $\quad 15 = A(-1)(-5)^2 \rightarrow \boxed{A = -3/5}$

$s=0$: $\quad 71 = 3A + 4B - 12C + 12D \rightarrow \boxed{C = -\frac{63}{20}}$

$$x(t) = \mathcal{L}^{-1}\left\{ \frac{-3/5}{s+4} - \frac{1/4}{s+3} - \frac{63/20}{s-1} + \frac{3}{(s-1)^2} \right\}$$

$$\boxed{x(t) = -\frac{3}{5}e^{-4t} - \frac{1}{4}e^{-3t} - \frac{63}{20}e^{t} + 3te^{t}}$$

3.- $I = \int_0^\infty \int_0^t \frac{te^{-\frac{1}{2}t-4x}\operatorname{sen}^2(2x)}{x}\,dx\,dt = \int_0^\infty te^{-\frac{1}{2}t}dt \cdot \int_0^t \frac{e^{-4x}\operatorname{sen}^2(2x)}{x}dx$

$\mathcal{L}\left\{\frac{\operatorname{sen}^2(2x)}{x}\right\} = \mathcal{L}\left\{\frac{1-\cos(4x)}{2x}\right\} = \frac{1}{2}\int_s^\infty \left(\frac{1}{s} - \frac{s}{s^2+16}\right)ds$

$\mathcal{L}\left\{\frac{\operatorname{sen}^2(2x)}{x}\right\} = \frac{1}{2}\cdot\left[\ln s - \frac{1}{2}\ln(s^2+16)\right]\Big|_s^\infty = \frac{1}{2}\ln\left(\frac{s}{(s^2+16)^{1/2}}\right)\Big|_s^\infty$

$\mathcal{L}\left\{\frac{\operatorname{sen}^2(2x)}{x}\right\} = \frac{1}{2}\ln\left[\frac{(s^2+16)^{1/2}}{s}\right]$

$\mathcal{L}\left\{\frac{e^{-4x}\operatorname{sen}^2(2x)}{x}\right\} = \frac{1}{2}\ln\left[\frac{((s+4)^2+16)^{1/2}}{s+4}\right] = \frac{1}{2}\ln\left[\frac{(s^2+8s+32)^{1/2}}{s+4}\right]$

$\mathcal{L}\left\{\int_0^t \frac{e^{-4x}\operatorname{sen}^2(2x)}{x}dx\right\} = \frac{1}{2s}\ln\left[\frac{(s^2+8s+32)^{1/2}}{s+4}\right] = \frac{1}{2s}\cdot\left[\frac{1}{2}\ln(s^2+8s+32) - \ln(s+4)\right]$

$\mathcal{L}\left\{t\int_0^t \frac{e^{-4x}\operatorname{sen}^2(2x)}{x}dx\right\} = +\frac{1}{2s^2}\cdot\ln\left[\frac{(s^2+8s+32)^{1/2}}{s+4}\right] - \frac{1}{2s}\cdot\left[\frac{\cancel{2}s+\cancel{8}^{4}}{\cancel{2}(s^2+8s+32)} - \frac{1}{s+4}\right]$

$\mathcal{L}\left\{t\int_0^t \frac{e^{-4x}\operatorname{sen}^2(2x)}{x}dx\right\} = \frac{1}{2s^2}\cdot\ln\left[\frac{(s^2+8s+32)^{1/2}}{s+4}\right] - \frac{s+4}{2s^3+16s^2+64s} + \frac{1}{2s^2+8s} = F(s)$

$\int_0^\infty \left[t\int_0^t \frac{e^{-4x}\operatorname{sen}^2(2x)}{x}dx\right]e^{-st}dt = F(s)$

$s = 1/2 \quad I = \int_0^\infty\int_0^t \frac{te^{-4x-\frac{1}{2}t}\operatorname{sen}^2(2x)}{x}dx\,dt = F(1/2)$

$I = \frac{2}{2(1/4)}\cdot\ln\left[\frac{\sqrt{145}/\cancel{2}}{9/\cancel{2}}\right] - \frac{9/2}{145/4} + \frac{1}{9/2} \Rightarrow \boxed{I = 0{,}68037}$

4.- $y''' + 4y' = 4\operatorname{sen}(2t) \qquad y(0) = 1;\ y'(0) = 2;\ y''(0) = -1$

$\mathcal{L}: \quad s^3Y(s) - s^2 - 2s + 1 + 4(sY(s) - 1) = \frac{8}{s^2+4}$

$Y(s)(s^3+4s) = \frac{8}{s^2+4} + s^2 + 2s + 3 \qquad \div\ s^3+4s$

$Y(s) = \frac{8}{s(s^2+4)^2} + \frac{s^2}{s(s^2+4)} + \frac{2s}{s(s^2+4)} + \frac{3}{s(s^2+4)}$

$$Y(s) = \frac{8}{s(s^2+4)^2} + \frac{s}{s^2+4} + \frac{2}{s^2+4} + \frac{3}{s(s^2+4)}$$

$$y_1(t) = \mathcal{L}^{-1}\left\{\frac{8}{(s^2+4)^2}\right\} = \mathcal{L}^{-1}\left\{8 \cdot \frac{1}{s^2+4} \cdot \frac{1}{s^2+4}\right\} = \overset{4}{\cancel{8}} \cdot \frac{\operatorname{sen}(2t)}{\cancel{2}} * \frac{\operatorname{sen}(2t)}{\cancel{2}}$$

$$y_1(t) = 4\int_0^t \operatorname{sen}(2\tau)\operatorname{sen}(2t-2\tau)\,d\tau = 2\int_0^t \left(\cos(4\tau-2t) - \cos(2t)\right)d\tau$$

$$y_1(t) = 2\left[\frac{\operatorname{sen}(4\tau-2t)}{4} - \tau\cos(2t)\right]\Big|_0^t = 2\cdot\left[\frac{\operatorname{sen}(2t)+\operatorname{sen}(+2t)}{4} - t\cos(2t)\right]$$

$$y_1(t) = \operatorname{sen}(2t) - 2t\cos(2t)$$

$$y_2(t) = \mathcal{L}^{-1}\left\{\frac{8}{s(s^2+4)^2}\right\} = \int_0^t \left[\operatorname{sen}(2t) - 2t\cos(2t)\right]dt$$

$$y_2(t) = \left[-\frac{\cos(2t)}{2} - 2\left(\frac{t\operatorname{sen}(2t)}{2} + \frac{\cos(2t)}{4}\right)\right]\Big|_0^t$$

$$y_2(t) = -\frac{\cos(2t)}{2} + \frac{1}{2} - 2\cdot\left(\frac{t\operatorname{sen}(2t)}{2} + \frac{\cos(2t)}{4} - \frac{1}{4}\right)$$

$$\underline{y_2(t) = -\cos(2t) + 1 - t\operatorname{sen}(2t)}$$

$$y_3(t) = \mathcal{L}^{-1}\left\{\frac{3}{s(s^2+4)}\right\} = \int_0^t 3\frac{\operatorname{sen}(2t)}{2}\,dt = \frac{3}{2}\cdot -\frac{\cos(2t)}{2}\Big|_0^t$$

$$\underline{y_3(t) = -\frac{3}{4}\cos(2t) + \frac{3}{4}}$$

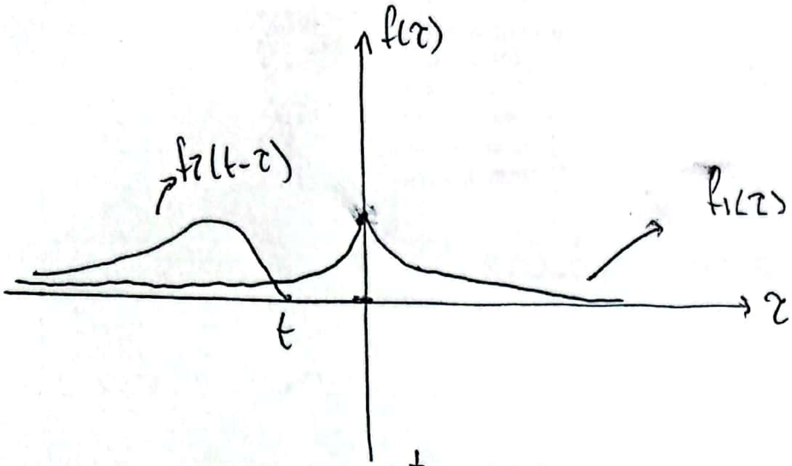
$$y(t) = -\cancel{\cos(2t)} + 1 - t\operatorname{sen}(2t) + \cancel{\cos(2t)} + \operatorname{sen}(2t) - \frac{3}{4}\cos(2t) + \frac{3}{4}$$

$$\boxed{y(t) = -t\operatorname{sen}(2t) + \frac{7}{4} + \operatorname{sen}(2t) - \frac{3}{4}\cos(2t)}$$

5.- $$\mathcal{F}^{-1}\left\{\frac{1}{(4+\omega^2)(2+j\omega)^2}\right\} = \mathcal{F}^{-1}\left\{\frac{e^{-2|t|}}{4} * te^{-2t}u(t)\right.$$

$$f_1(t) = \frac{e^{-2|t|}}{4} = \begin{cases}\frac{e^{+2t}}{4} & t<0\\ \frac{e^{-2t}}{4} & t>0\end{cases} \Rightarrow f_1(\tau) = \begin{cases}\frac{e^{2\tau}}{4} & \tau<0\\ \frac{e^{-2\tau}}{4} & \tau>0\end{cases}$$

$$f_2(t) = t e^{-2t} u(t) \Rightarrow f_2(t-\tau) = (t-\tau) e^{-2(t-\tau)} u(t-\tau)$$



$$t<0: \quad f(t) = \int_{-\infty}^{t} \frac{e^{2\tau}}{4} \cdot (t-\tau) e^{-2(t-\tau)} d\tau = \frac{1}{4} e^{-2t} \int_{-\infty}^{t} \left(t e^{4\tau} - \tau e^{4\tau}\right) d\tau$$

$$f(t) = \frac{1}{4} e^{-2t} \cdot \left(\frac{t e^{4\tau}}{4}\Big|_{-\infty}^{t} - \left(\frac{\tau e^{4\tau}}{4} - \frac{e^{4\tau}}{16}\right)\Big|_{-\infty}^{t}\right)$$

$$f(t) = \frac{1}{4} e^{-2t} \cdot \left(\cancel{\frac{t e^{4t}}{4}} - \left(\cancel{\frac{t e^{4t}}{4}} - \frac{e^{4t}}{16}\right)\right) = \frac{1}{4} e^{-2t} \cdot \frac{e^{4t}}{16}$$

$$\boxed{f(t) = \frac{e^{2t}}{64}}$$

$$t>0: \quad f(t) = \int_{-\infty}^{0} \frac{e^{2\tau}}{4} \cdot (t-\tau) e^{-2(t-\tau)} d\tau + \int_{0}^{t} \frac{e^{-2\tau}}{4} \cdot (t-\tau) e^{-2(t-\tau)} d\tau$$

$$f(t) = \frac{1}{4} e^{-2t} \cdot \left(\cancelto{0}{\frac{t e^{4\tau}}{4}}\Big|_{-\infty}^{0} - \left(\cancelto{0}{\frac{\tau e^{4\tau}}{4}} - \frac{e^{4\tau}}{16}\right)\Big|_{-\infty}^{0}\right) + \frac{1}{4} e^{-2t} \cdot \int_{0}^{t} (t-\tau) d\tau$$

$$f(t) = \frac{1}{4} e^{-2t} \cdot \left(\frac{1}{16}\right) + \frac{1}{4} e^{-2t} \cdot \left(t\tau - \frac{\tau^2}{2}\right)\Big|_{0}^{t}$$

$$f(t) = \frac{e^{-2t}}{64} + \frac{1}{4} e^{-2t} \cdot \left(t^2 - \frac{t^2}{2}\right) = \frac{e^{-2t}}{64} + \frac{1}{4} e^{-2t} \cdot \frac{t^2}{2}.$$

$$f(t) = \frac{e^{-2t}}{64} + \frac{t^2 e^{-2t}}{8} \Rightarrow \boxed{f(t) = \frac{e^{2t}}{64} u(-t) + \left(\frac{e^{-2t}}{64} + \frac{t^2 e^{-2t}}{8}\right) u(t)}$$