PRACTICA Nro. 1 – TRANSFORMADAS INTEGRALES
SERIES DE FOURIER

1.- Encontrar el período de las funciones:

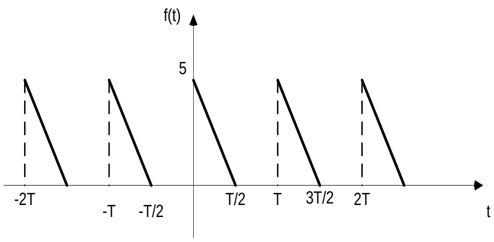
a) $f_{(t)} = sen\left(\frac{t}{3}\right) + sen\left(\frac{t}{5}\right)$ b) $f_{(t)} = \cos\left(\frac{3}{4}t\right) + sen\left(\frac{2}{5}t\right) + sen\left(\frac{t}{2}\right)$ c) $f_{(t)} = sen\left(\frac{3}{2}t\right) + sen\left(\frac{5}{4}t\right) + sen\left(\frac{t}{6}\right)$

d) $f_{(t)} = \left|\cos\left(\frac{4t}{3}\right)\right| + \left|\cos\left(\frac{3t}{5}\right)\right|$ e) $f_{(t)} = sen^2(5t) + \left|\cos\left(\frac{2}{5}t\right) + sen\left(\frac{4}{3}t\right)\right|$

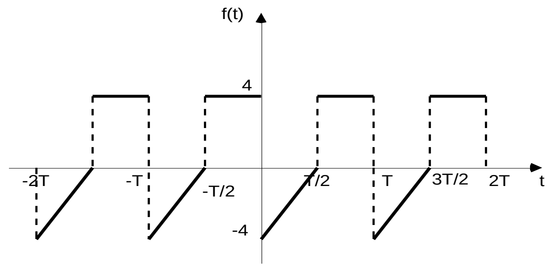
R.-a) T=30π; b) T=40π; c) 24π; d) T=15π; e) T=15π.

2.- Dadas las funciones periódicas, definirlas en un período y hallar las series de Fourier.

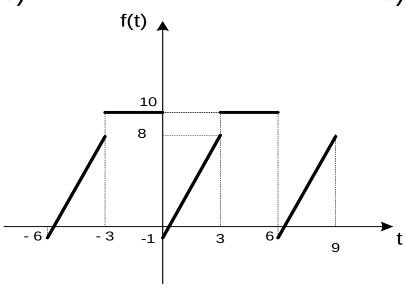
a)



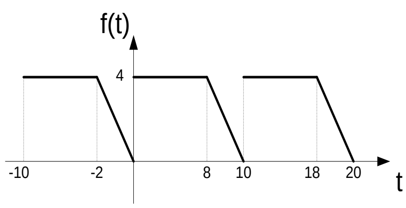
b)



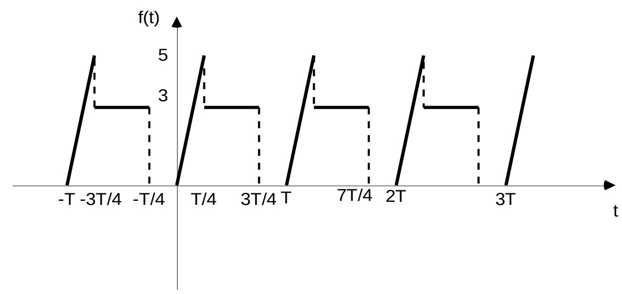
c)



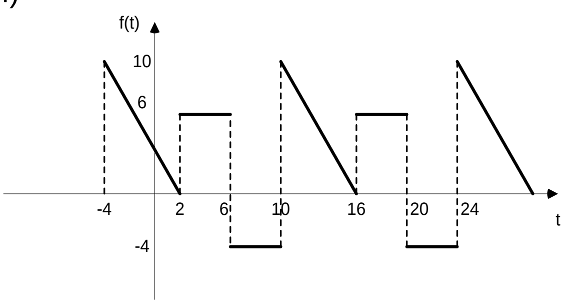
d)



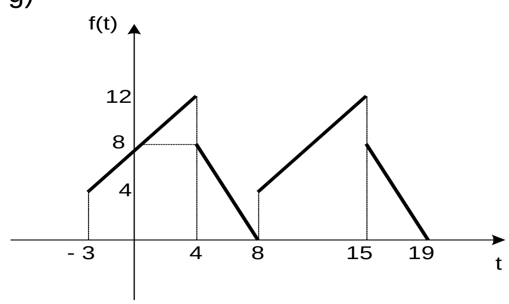
e)



f)



g)



R.- a) $f_{(t)} = \frac{5}{4} + \sum_{n=1}^{\infty} \frac{5}{n^2\pi^2}\left(1-(-1)^n\right)\cos\left(\frac{2\pi nt}{T}\right) + \frac{5}{\pi n} sen\left(\frac{2\pi nt}{T}\right)$

b) $f_{(t)} = 1 + \sum_{n=1}^{\infty}\left[\frac{4\left((-1)^n - 1\right)}{n^2\pi^2}\cos\left(\frac{2\pi nt}{T}\right) + \frac{4\left((-1)^n - 2\right)}{n\pi} sen\left(\frac{2\pi nt}{T}\right)\right]$

c) $f_{(t)} = \frac{27}{4} + \sum_{n=1}^{\infty}\left[\frac{9}{\pi^2 n^2}\left(\cos(\pi n) - 1\right)\cos\left(\frac{\pi t}{3}\right) + \left(\frac{2\cos(\pi n) - 11}{\pi n}\right) sen\left(\frac{\pi t}{3}\right)\right]$

d) $f_{(t)} = \frac{18}{5} + \sum_{n=1}^{\infty}\left[\frac{10}{\pi^2 n^2}\left(\cos\left(\frac{8\pi n}{5}\right) - 1\right)\cos\left(\frac{\pi n}{5}t\right) + \left(\frac{4}{\pi n} + \frac{10}{\pi^2 n^2} sen\left(\frac{8\pi n}{5}\right)\right) sen\left(\frac{\pi n}{5}t\right)\right]$

e) $f_{(t)} = \frac{17}{8} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi nt}{T}\right) + b_n sen\left(\frac{2\pi nt}{T}\right)\right]$ donde:

$$a_n = \frac{2sen\left(\frac{\pi n}{2}\right)+3sen\left(\frac{3\pi n}{2}\right)}{\pi n} + \frac{10\left(\cos\left(\frac{\pi n}{2}\right)-1\right)}{\pi^2 n^2}; \quad b_n = \frac{-2\cos\left(\frac{\pi n}{2}\right)-3\cos\left(\frac{3\pi n}{2}\right)}{\pi n} + \frac{10}{\pi^2 n^2} sen\left(\frac{\pi n}{2}\right)$$

f) $f_{(t)} = \frac{19}{7} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{\pi n t}{7}\right) + b_n sen\left(\frac{\pi n t}{7}\right)\right]$ donde:

$$a_n = \frac{-6sen\left(\frac{2\pi n}{7}\right)+10sen\left(\frac{4\pi n}{7}\right)+10sen\left(\frac{6\pi n}{7}\right)-4sen\left(\frac{10\pi n}{7}\right)}{\pi n} - \frac{35\left(\cos\left(\frac{2\pi n}{7}\right)-\cos\left(\frac{4\pi n}{7}\right)\right)}{3\pi^2 n^2}$$

$$b_n = \frac{6\cos\left(\frac{2\pi n}{7}\right)+10\cos\left(\frac{4\pi n}{7}\right)-10\cos\left(\frac{6\pi n}{7}\right)+4\cos\left(\frac{10\pi n}{7}\right)}{\pi n} - \frac{35\left(sen\left(\frac{2\pi n}{7}\right)+sen\left(\frac{4\pi n}{7}\right)\right)}{3\pi^2 n^2}$$

g) $f_{(t)} = \frac{144}{11} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{2\pi n t}{11}\right) + b_n sen\left(\frac{2\pi n t}{11}\right)\right]$ donde:

$$a_n = \frac{\frac{121}{7}\cos\left(\frac{8\pi n}{11}\right)-\frac{44}{7}\cos\left(\frac{6\pi n}{11}\right)-11\cos\left(\frac{16\pi n}{11}\right)}{\pi^2 n^2} + \frac{4}{\pi n}\left(sen\left(\frac{8\pi n}{11}\right)+sen\left(\frac{6\pi n}{11}\right)\right)$$

$$b_n = \frac{\frac{121}{7}sen\left(\frac{8\pi n}{11}\right)+\frac{44}{7}sen\left(\frac{6\pi n}{11}\right)-11sen\left(\frac{16\pi n}{11}\right)}{\pi^2 n^2} + \frac{4}{\pi n}\left(\cos\left(\frac{6\pi n}{11}\right)-\cos\left(\frac{8\pi n}{11}\right)\right)$$

3.- Dadas las funciones definidas en un período, graficarlas tres períodos y hallar sus series de Fourier

a) $f_{(t)} = \begin{cases} 4t-5 & 0<t<1 \\ 3 & 1<t<2 \end{cases} \quad T=2$

b) $f_{(t)} = \begin{cases} -3t+1 & -1<t<0 \\ 2t+3 & 0<t<1 \end{cases} \quad T=2$

c) $f_{(t)} = \begin{cases} 5 & -\frac{T}{4}<t<\frac{T}{3} \\ -\frac{15}{T}t+10 & \frac{T}{3}<t<\frac{2T}{3} \\ -5 & \frac{2T}{3}<t<\frac{3T}{4} \end{cases}$

d) $f_{(t)} = \begin{cases} -2t^2 & 0<t<4 \\ 0 & 4<t<8 \end{cases} \quad T=8$

e) $f_{(t)} = \begin{cases} -5 & 0<t<2 \\ 0 & 2<t<4 \quad T=8 \\ -\frac{t}{2}+4 & 4<t<8 \end{cases}$

f) $f_{(t)} = \begin{cases} 1-t^2 & 0<t<3 \\ 10 & 3<t<6 \end{cases} \quad T=6$

R.- a) $f_{(t)} = \sum_{n=1}^{\infty}\left[\frac{4}{n^2\pi^2}(-1+\cos(\pi n))\cos(\pi n t) + \frac{-8+4\cos(\pi n)}{\pi n} sen(\pi n t)\right];$

b) $f_{(t)} = \frac{13}{4} + \sum_{n=1}^{\infty}\left[\left(\frac{5(\cos(\pi n)-1)}{\pi^2 n^2}\right)\cos(\pi n t) + \left(\frac{2-\cos(\pi n)}{\pi n}\right)sen(\pi n t)\right];$

c) $f_{(t)} = \frac{10}{3} + \sum_{n=1}^{\infty}\left[a_n \cos(n\varpi_0 t) + b_n sen(n\varpi_0 t)\right];$

$$a_n = \frac{-10\cos(\pi n)sen\left(\frac{\pi}{2}n\right)+5sen\left(\frac{4\pi}{3}n\right)}{\pi n}; b_n = \frac{10\cos(\pi n)\cos\left(\frac{\pi}{2}n\right)-5\cos\left(\frac{4\pi}{3}n\right)}{\pi n} - \frac{15sen\left(\frac{\pi}{3}n\right)\cos(\pi n)}{\pi^2 n^2}$$

d) $f_{(t)} = -\frac{16}{3} + \sum_{n=1}^{\infty}\left[\frac{64}{\pi^2 n^2}\cos\left(\frac{\pi n t}{4}\right) + \left(\frac{32}{\pi n}\cos(\pi n) + \frac{64}{\pi^3 n^3}(1-\cos(\pi n))\right)sen\left(\frac{\pi n t}{4}\right)\right];$

e) $f_{(t)} = -\frac{3}{4} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{\pi}{4}nt\right) + b_n sen\left(\frac{\pi}{4}nt\right)\right]$ donde:

$$a_n = -\frac{5}{\pi n} sen\left(\frac{\pi}{2}n\right) + \frac{2}{\pi^2 n^2}\left(\cos(\pi n) - 1\right);\ b_n = \frac{5\left(\cos\left(\frac{\pi}{2}n\right) - 1\right) + 2\cos(\pi n)}{\pi n}$$

f) $f_{(t)} = 4 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{\pi n t}{3}\right) + b_n sen\left(\frac{\pi n t}{3}\right)\right]$ donde:

$$a_n = -\frac{18}{\pi^2 n^2}\cos(\pi n);\ b_n = \frac{-9 + 18\cos(\pi n)}{\pi n} + \frac{18\left(1 - \cos(\pi n)\right)}{\pi^3 n^3}$$