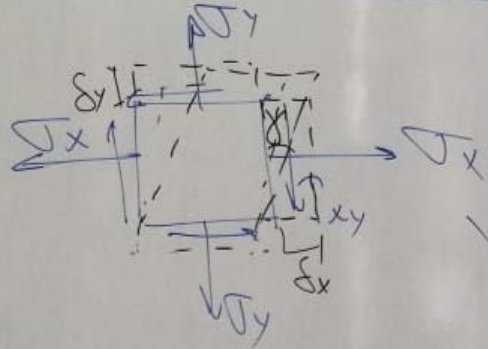


CAP V.- DEFORMACIONES EN EL PLANO Y CILINDROS DE PARED DELGADA

Profesor: Ing. Guido Gomez U.
Dpto de: Ingeniería Mecánica
FCyT- UMSS

DEFORMACIONES EN EL PLANO

CAP: Deformaciones Combinadas



Tens. combi.

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \rightarrow \text{Normales}$$

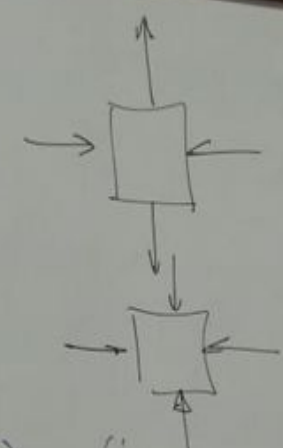
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Deformaciones Combinadas

→ Cortantes

→ Longitudinales → $(\sigma) \Rightarrow$ Tema de Resist I

→ Angulares → (τ) } Tema de Resist II

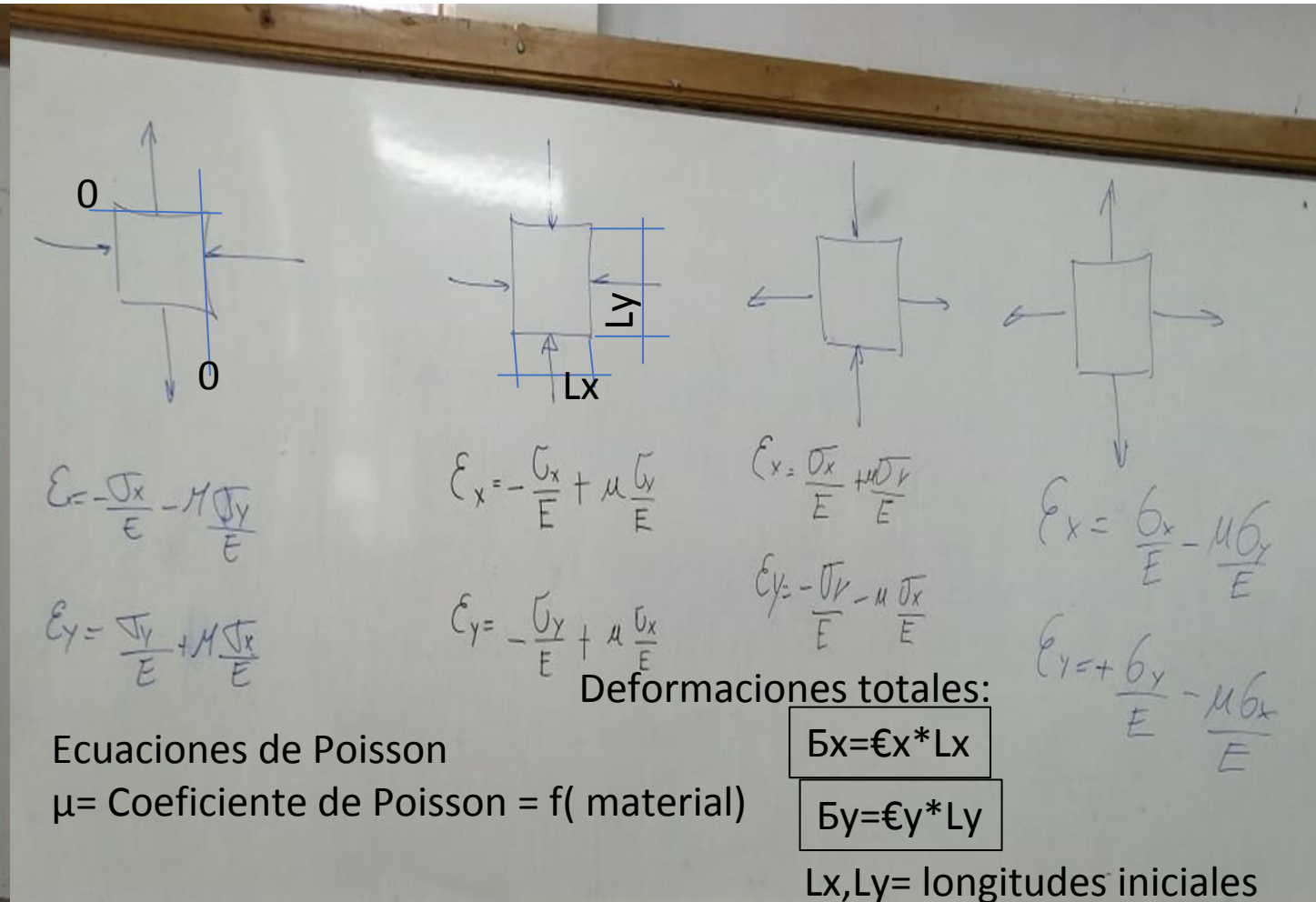
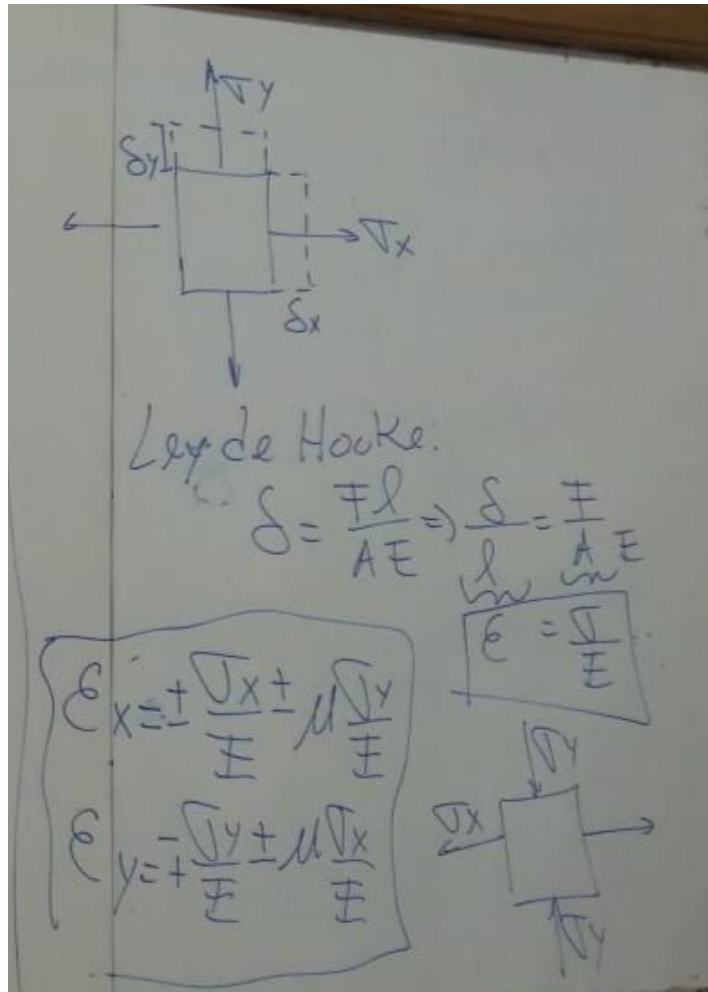


Objetivos:

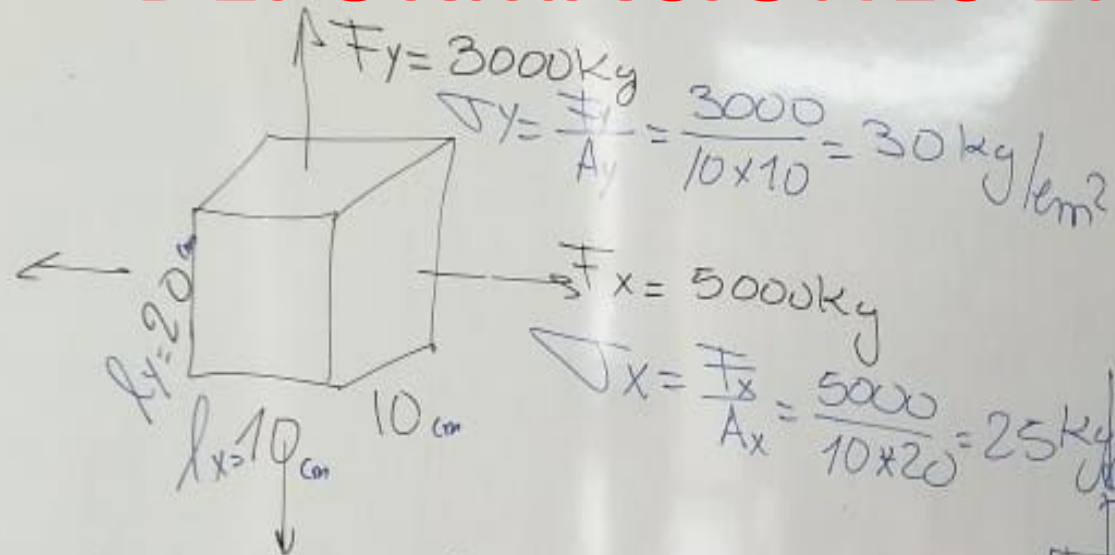
- 1) Hallar ecuaciones y nos permitan determinar el valor de las deform. longitudinales en el plano
- 2) Dimensionar la rigidez

3.- Aplicacion en cilindros de pared delgada

DEFORMACIONES EN EL PLANO



DEFORMACIONES EN EL PLANO

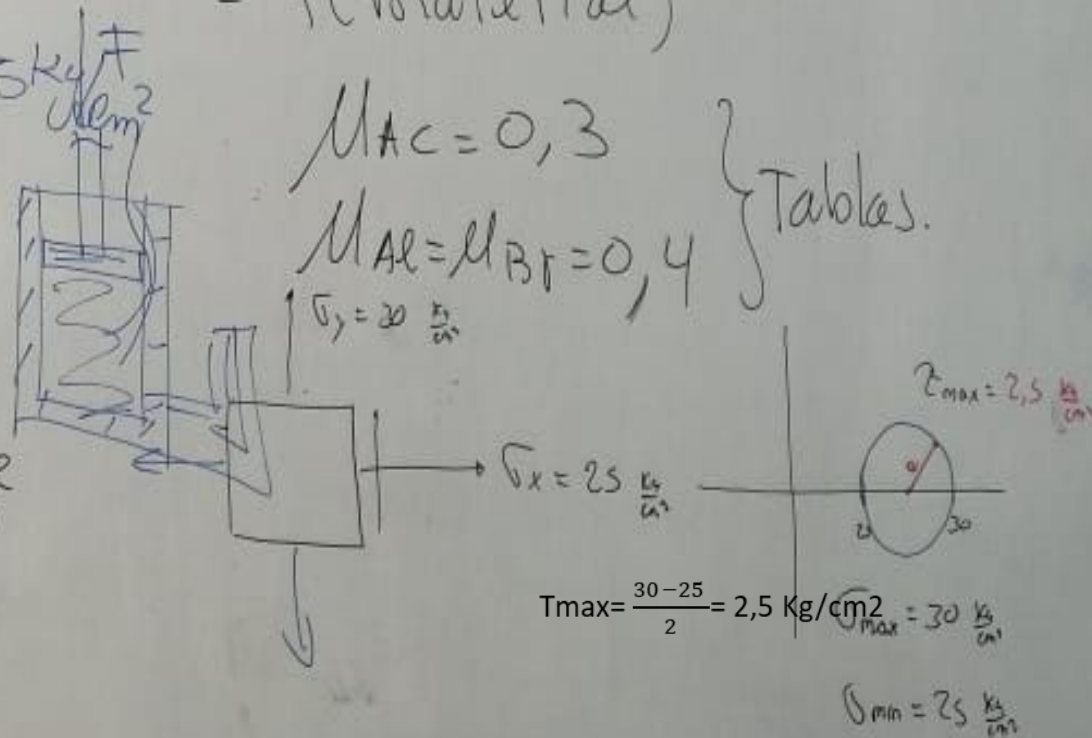


$\mu = \text{Coeficiente de Poisson}$
 $= f(\text{material})$

$\mu_{Ac} = 0,3$
 $\mu_{Al} = \mu_{Br} = 0,4$ } Tablas.

a) $\sigma_{\max} = ?$
 $\sigma_{\min} = ?$
 $\tau_{\max} = ?$

b) $\epsilon_x = ? \rightarrow \delta_x = ?$
 $\epsilon_y = ? \rightarrow \delta_y = ?$
 $E = 2,1 \times 10^6 \text{ kg/cm}^2$
 $\mu = 0,3$



$$\epsilon_x = \frac{\sigma_x}{E} - \mu \frac{\sigma_y}{E}$$

$$\epsilon_x = \frac{25}{2,1 \times 10^6} - 0,3 \frac{30}{2,1 \times 10^6}$$

$$\epsilon_x = 7,62 \times 10^{-6}$$

$$\delta_x = \epsilon_x \cdot l_x = 7,62 \times 10^{-6} \times 10 \text{ cm}$$

$$\delta_x = 7,62 \times 10^{-5} \text{ cm}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \mu \frac{\sigma_x}{E}$$

$$\epsilon_y = \frac{30}{2,1 \times 10^6} - 0,3 \frac{25}{2,1 \times 10^6}$$

$$\delta_y = \epsilon_y \cdot l_y = 1,02 \times 10^{-5} \text{ cm}$$

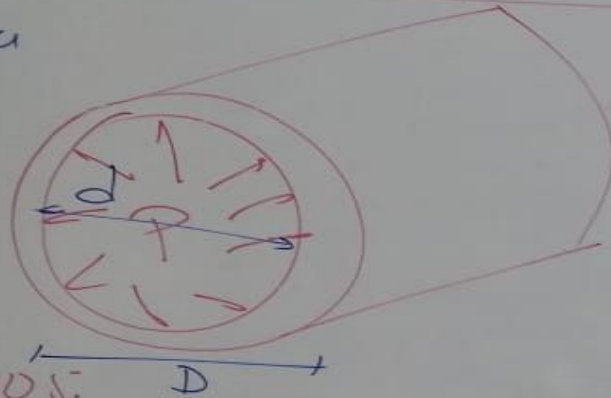
$$\delta_y = 2,14 \times 10^{-5} \text{ cm}$$

CILINDROS DE PARED DELGADA

= 5 cm y $e = 2$ cm ..pared gruesa
= 200 cm $e = 2$ cm.. pared delgada

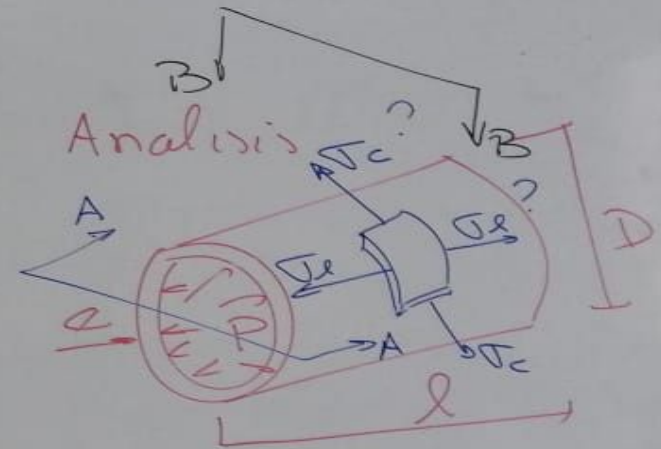
CAP: Cilindros de pared delgada

Pared delgada
 $D \approx d$



Objetivos:

- 1) Que tipo de tensiones y deformaciones producen?
- 2) Como se distribuyen?
- 3) Como se dimensionan?

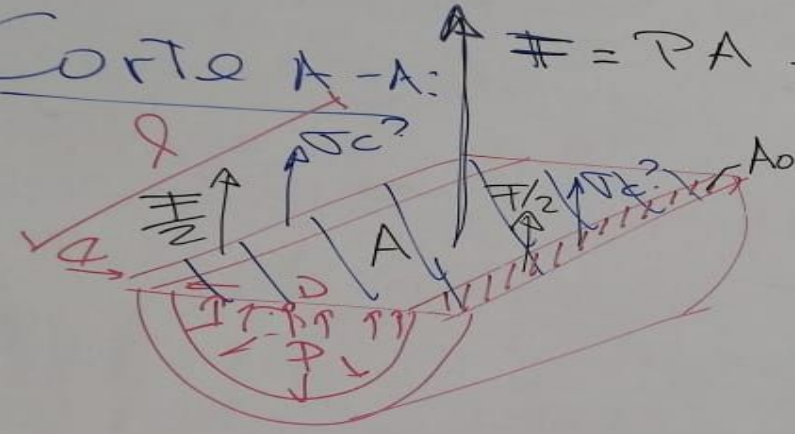


Objetivo

$\sigma_{max} = ?$
$\sigma_{min} = ?$
$\tau_{max} = ?$

CILINDROS DE PARED DELGADA

Corte A-A:

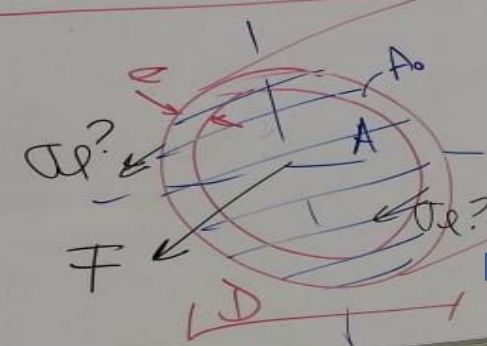


$$F = PA = P \cdot \pi D \ell$$

$$\sigma_c = \frac{F/2}{A_0} = \frac{F/2}{2\ell D}$$

$$\sigma_c = \frac{P D \ell}{2\ell D} \Rightarrow \sigma_c = \frac{PD}{2\ell}$$

Corte B-B:



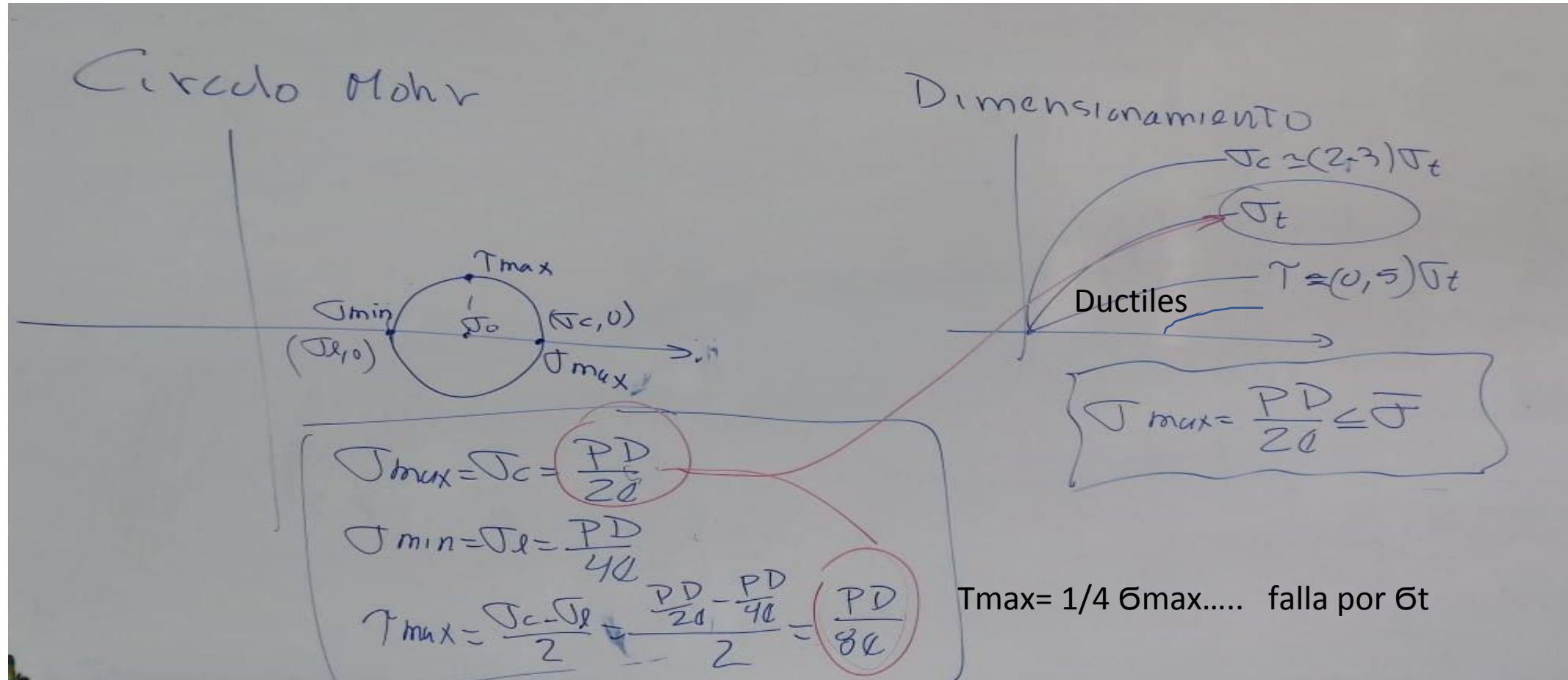
$$F = PA = P \pi D^2 \ell / 4$$

$$\sigma_l = \frac{F}{A_0} = \frac{F}{\pi D \ell} = \frac{P \pi D^2 \ell / 4}{\pi D \ell} \Rightarrow \sigma_l = \frac{PD}{4\ell}$$

$\sigma_c \approx 2\sigma_l$

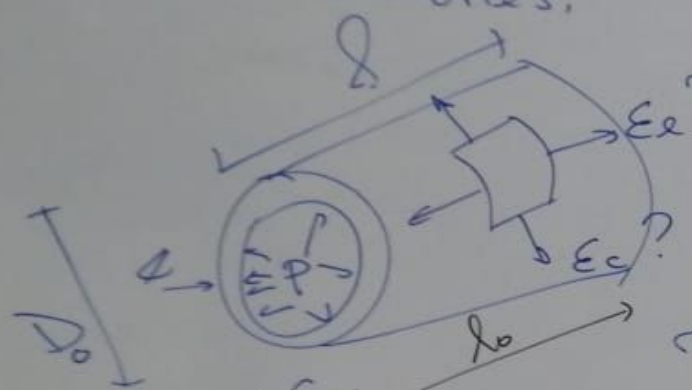
πD e

CILINDROS DE PARED DELGADA



CILINDROS DE PARED DELGADA

Deformaciones:



ϵ_l , ϵ_c , D_0 , l_0

Objetivos:

$D_{final} = ?$
 $l_{final} = ?$

Poisson:

$$\epsilon_c = \frac{\sigma_c}{E} - \mu \frac{\sigma_l}{E}$$

$$\epsilon_l = \frac{\sigma_l}{E} - \mu \frac{\sigma_c}{E}$$

Deformación diametral:

$$\epsilon_c = \frac{\delta_c}{l_0} \Rightarrow \delta_c = \epsilon_c l_0$$

$$\delta_c = \epsilon_c * \underbrace{\pi D_0}_{\text{Perim. inicial}}$$

πD_0 , δ_c

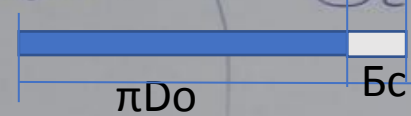
Perímetro final

$$l_f = l_0 + \delta_c$$

$$\pi D_f = \pi D_0 + \epsilon_c \pi D_0$$

$$D_f = D_0 + \epsilon_c D_0$$

Partes de un Cilindro Hidráulico



Ojo del Vástago, Vástago, Sellos Hidráulicos, Barril, Pistón, Puerto de Salida, Puerto de Entrada



La deformación no afecta a la funcionalidad

La deformación circunferencial afecta a la funcionalidad

CILINDROS DE PARED DELGADA

Deformación long:

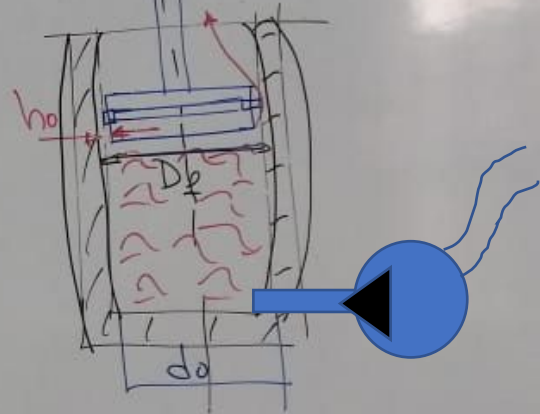
$$\epsilon_l = \frac{\delta_l}{l_0} \Rightarrow \delta_l = \epsilon_l l_0$$

$$l_f = l_0 + \delta_l$$

$$l_f = l_0 + \epsilon_l l_0$$

$$l_f = l_0 (1 + \epsilon_l)$$

Holgura:



holgura Máxima admisible } $\bar{h}_0 = f(\text{lubricante}, T)$

$$\bar{h}_0 = 0,02 \rightarrow 0,003 \text{ mm}$$

Calculo holgura

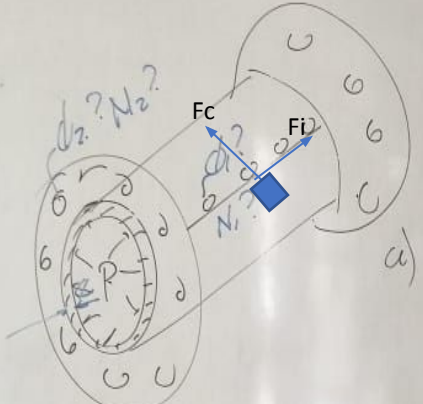
$$h_f = \frac{D_f - D_0}{2}$$

dimensionamiento a la rigidez

$$\frac{D_f - D_0}{2} \leq \bar{h}_0$$

EJERCICIOS RESUELTOS

a)



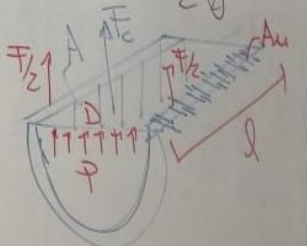
$P = 5 \text{ kg/cm}^2$
 $D = 30 \text{ cm}$
 $l = 100 \text{ cm}$

$\sigma_{\max} = \frac{PD}{2t} \leq \bar{\sigma}$

SAE 1020 $\rightarrow \sigma_f = 2100 \text{ kg/cm}^2$
 $n = 2$

$\langle \rangle \frac{PD}{2t} \Rightarrow \langle \rangle \text{--- Normalizar a ---}$

b)



$F_c = PA = PDl$

$\frac{F_c}{2} \leq \bar{\sigma} \Rightarrow \frac{PDl}{2N_1 \pi \frac{\phi_1^2}{4}} \leq \bar{\sigma} \Rightarrow$

$\Rightarrow N_1 \phi_1 \geq \dots \checkmark$

realidad $l_1 = 50$

$l_1 = 50 \text{ cm} \Rightarrow N_1 = 21 =$

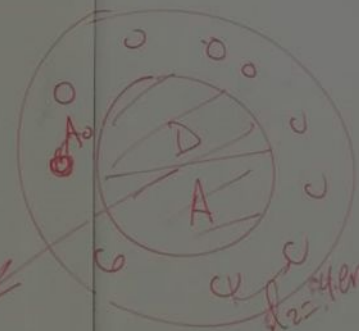
100 cm

125

100 cm

125

c)



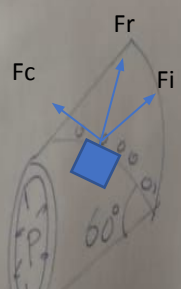
$F_l = PA = \frac{P \pi D^2}{4}$

$\frac{F_l}{N_2 A_0} \leq \bar{\sigma} \Rightarrow \frac{P \pi D^2 / 4}{N_2 \pi \frac{\phi_2^2}{4}} \leq \bar{\sigma} \Rightarrow$

$N_2 \phi_2 \geq \dots \checkmark \Rightarrow \phi_2 = \checkmark$


Permetro $l_2 = \pi D = \pi \cdot 30 = 94 \Rightarrow N_2 = \frac{94}{4} + 1 = 24,5 \Rightarrow 25$

EJERCICIOS RESUELTOS




$P = 5 \text{ kg/cm}^2$
 $D = 70 \text{ cm}$
 $l = 150 \text{ cm}$
 $\sigma_f = 2200 \text{ kg/cm}^2$
 $n = 2$

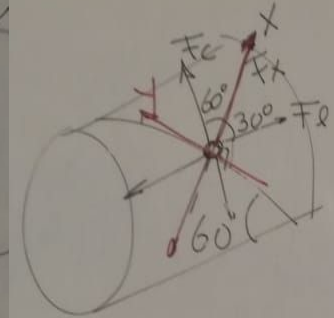
$\frac{PD}{2l} \leq \bar{\tau}$
 $\Rightarrow \frac{PD}{2\bar{\tau}} = \frac{5 \times 70}{2 \times 2200}$
 $l \geq 0,159 \text{ cm}$



$F_c = P \times A = P \times lD$
 $F_c = 5 \times 150 \times 70 = 52500 \text{ kg}$



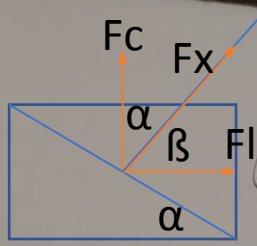
$F_f = P A_1 = 5 \times \frac{\pi D^2}{4} = 19242 \text{ kg}$



$F_x = F_c \cos 60 + F_f \cos 30$
 $F_x = 52500 \cos 60 + 19242 \cos 30$
 $F_x = 42914 \text{ kg}$

$\frac{F_x}{N A_0} \leq \bar{\tau} \Rightarrow$

$\sigma_f = 4500 \text{ kg/cm}^2 \Rightarrow \bar{\tau} = 0,5 \frac{\sigma_f}{n}$
 $n = 2$
 $\bar{\tau} = 0,5 \times \frac{4500}{2} = 1125$

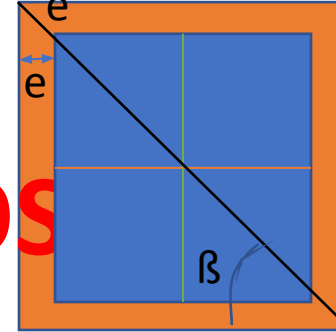


$\phi = \frac{3''}{4} = 1,9 \text{ cm} \Rightarrow A_0 = \frac{\pi 1,9^2}{4} = 2,83 \text{ cm}^2$

$N \geq \frac{F_x}{\bar{\tau} A_0} = \frac{42914}{1125 \times 2,83}$

$N \geq 13,48 \Rightarrow \boxed{N = 14}$

EJERCICIOS PROPUESTOS



$$S = (n-2) \cdot 180^\circ$$

$$S = (8-2) \cdot 180$$

$$S = 1080^\circ$$

$$\Theta = 1080/8 = 135^\circ$$

1.-

$\beta = 90 - \Theta/2 = 22,5^\circ$

$\alpha = 45^\circ$

$L = 50 \text{ cm}$

$P = 5 \text{ kg/cm}^2$

a) $d = ?$

$SAE1020 \rightarrow n=2$

b) $\phi_{\text{remachos}} = ?$

$SAE1045 \rightarrow n=3$

$N=20 \text{ remachos}$

Tension longitudinal:

$\sigma_c = \frac{P \cdot L}{2e} \leq \sigma_{ad}$

$\sigma_l = \frac{P \cdot A_l}{A_o} = \frac{P \cdot A_l}{8 \cdot 50 \cdot e} \leq \sigma_{ad}$

$L = 50 + 2 \cdot 50 \cos 45 = 120,7$

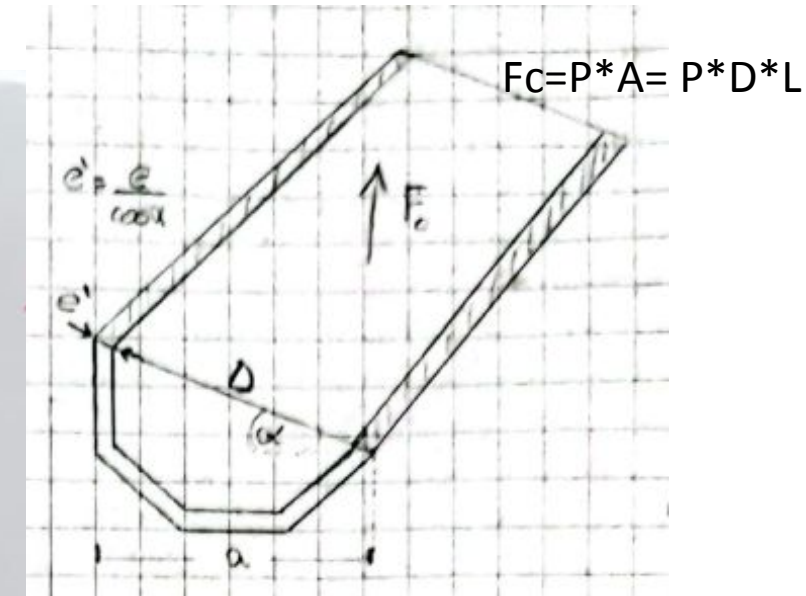
$Tg \beta = 50/L$

$D = L / \cos \beta = 130,64$

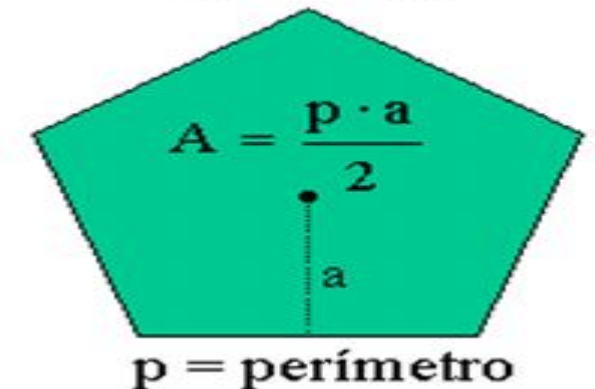
$a = D/2 \cdot \cos \beta = 60,37$

$p = 8 \cdot 50 = 400$

$\sigma_c = \frac{PD}{2e} \leq \sigma_{ad} \Rightarrow \frac{PD}{2e} \leq \sigma_{ad} \Rightarrow \frac{PD}{2e} \leq \sigma_{ad}$

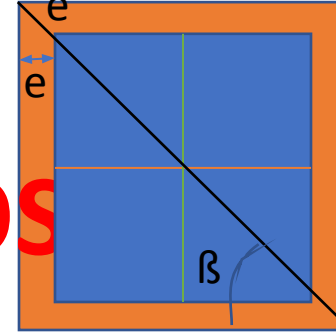


Polígono regular



A diagram showing a large orange square divided into four smaller squares by a horizontal and vertical green line. The top-left small square is shaded blue. A diagonal black line runs from the top-left corner of the large square to the bottom-right corner. The label β is placed near the bottom-right corner of the large square, indicating the ratio of the width of the blue square to the height of the orange square.

$$\Theta = 1080/8 = 135^\circ$$



1.-

$P = 5 \text{ kg/cm}^2$

$\alpha = 45^\circ$

β

$\theta/2$

D

L

F_c

F_f

F_r

45°

$\phi?$

50cm

50cm

$P = 5 \text{ kg/cm}^2$

b) Remaches:


$F_c = P \cdot D \cdot l$

$F_f = P \cdot A_l$

$F_r = F_c \cdot \cos 45^\circ + F_f \cdot \sin 45^\circ$

$N \cdot \frac{F_r}{A_r} \leq T_{ad}$

$A_r = \frac{\pi \cdot d^2}{4}$




$A = \frac{p \cdot a}{2}$

$p = \text{perímetro}$

EJERCICIOS PROPUESTOS

2.-

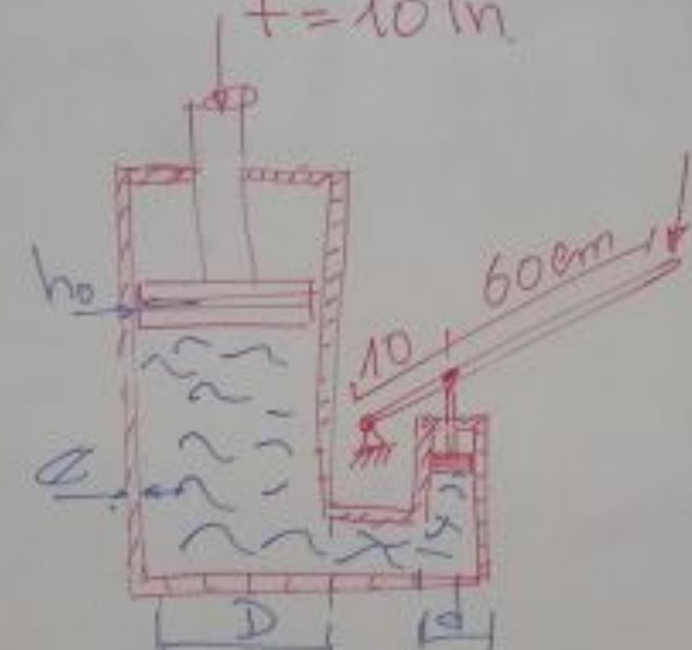


$l = 200 \text{ cm}$
 $D = 70 \text{ cm}$
 $p = 2 \text{ kg/cm}^2$

a) $\alpha = ?$
 SAE 1010 $\rightarrow n = 2$

b) $\phi_{\text{Resacas}} = 1/2$
 SAE 1030 $\rightarrow n = 2$
 $N = ?$

3.-



$F = 107 \text{ N}$
 60 cm
 10
 $D = 10 \text{ cm}$
 $d = 2 \text{ cm}$

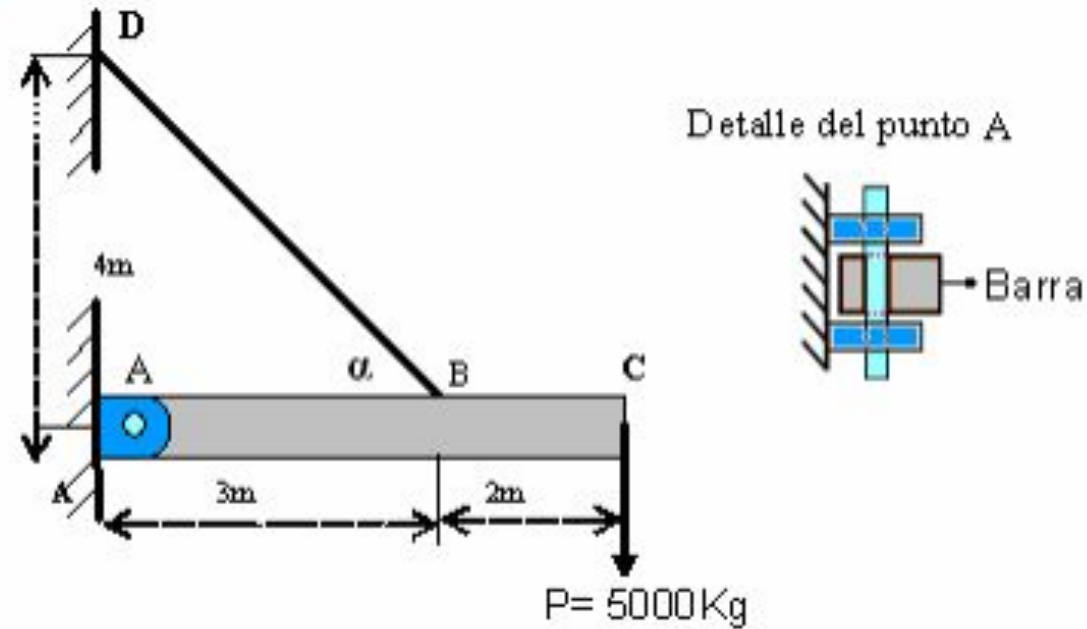
a) $f = ?$

b) $\alpha = ?$
 SAE 1020 $\rightarrow n = 2$
 $h_0 = 0,008 \text{ mm}$
 $\mu = 0,3$

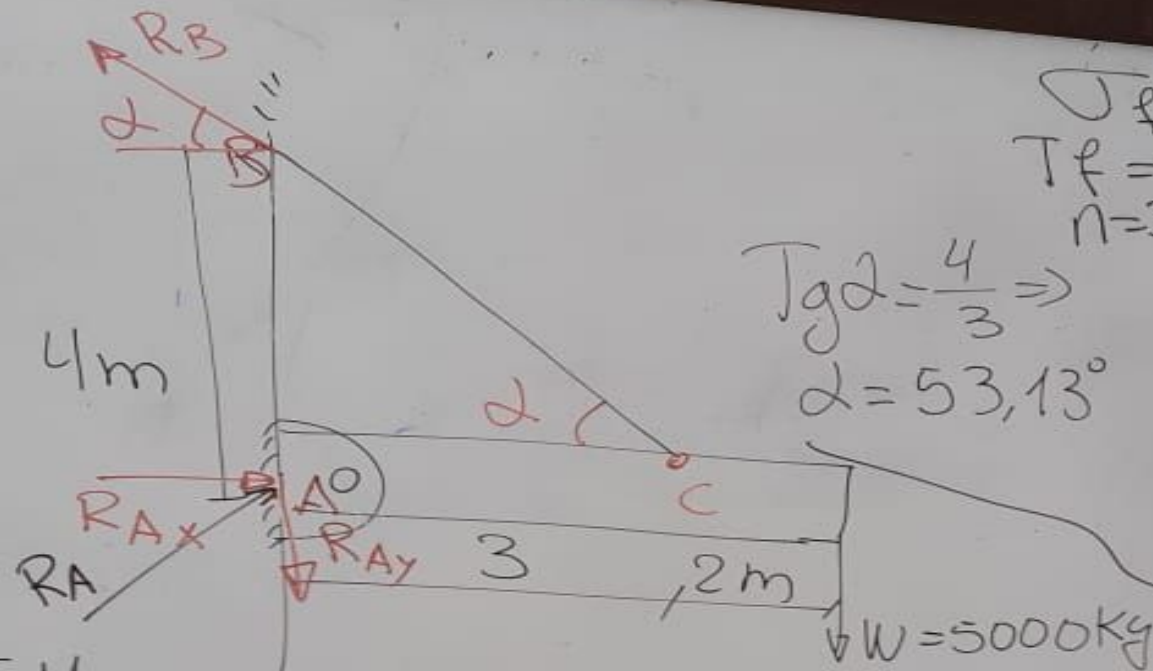
RESISTENCIA DE MATERIALES
Ingenieria Civil
1er PARCIAL

PREGUNTA 1.-

2.5.- A partir de la figura presentada. Calcular el diámetro del pasador A que soporta la barra AC a cortante simple, si $\sigma_f = 2100 \text{ kg/cm}^2$ y un $\tau_f = 0.5\sigma_f$ con un factor de seguridad de 3.



2. -



$$\sigma_f = 2100 \text{ kg/cm}^2$$

$$\tau_f = 0,5 \sigma_f$$

$$n = 3$$

$$\tan \alpha = \frac{4}{3} \Rightarrow$$

$$\alpha = 53,13^\circ$$

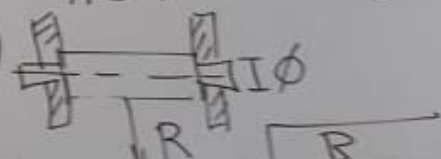
$$\sum F_y = 0$$

$$10416 \text{ send} - 3333 - 5000 = 0$$

$$0 = 0 \Rightarrow 0 \text{ K}$$

$$R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$$

$$R_A = 7083 \text{ Kg}$$



$$\frac{R}{2A} \leq \bar{\tau} \Rightarrow A \geq \sqrt{\frac{R}{2\bar{\tau}}}$$

$$\phi \geq \sqrt{\frac{4 \times 7083}{2 \times 350 \times 700}}$$

$$\boxed{\phi \geq 3,59 \text{ cm}}$$

$$\sum M_A = 0$$

$$- R_B \cos \alpha \times 4 + W \times 5 = 0 \Rightarrow R_B = \frac{5000 \times 5}{4 \cos \alpha}$$

$$R_B = 10416 \text{ Kg}$$

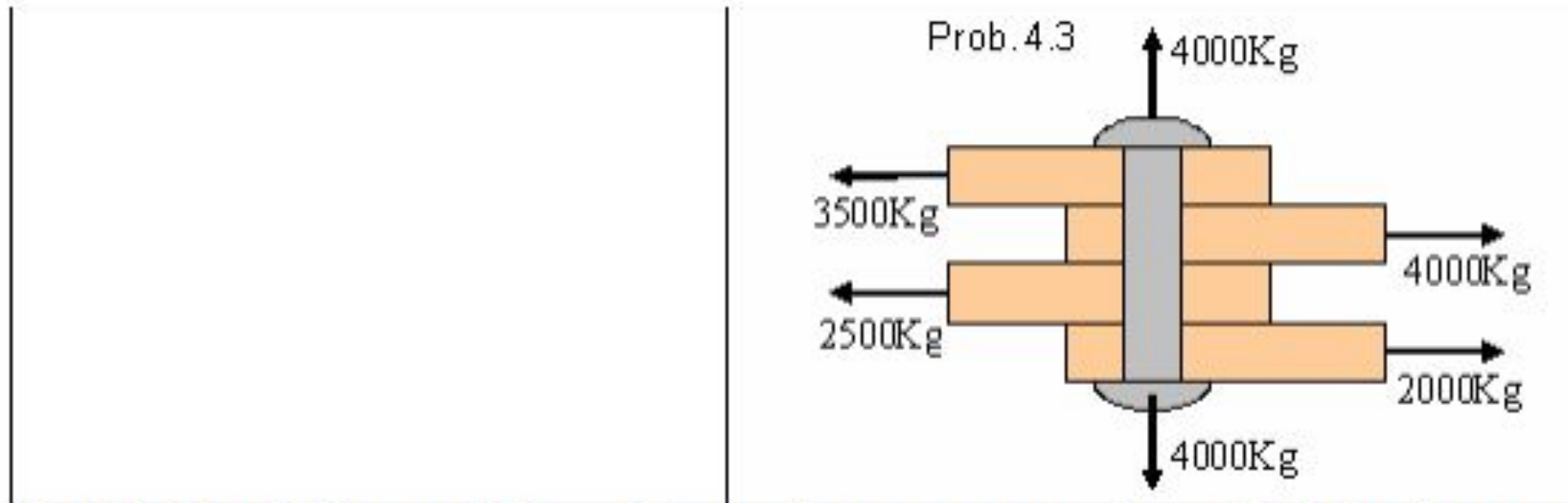
$$\sum M_B = 0$$

$$- 4 R_{AX} + 5W = 0 \Rightarrow R_{AX} = \frac{5}{4} \times 5000 = 6250 \text{ Kg}$$

$$\sum M_C = 0$$

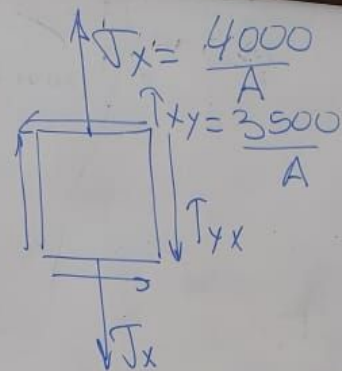
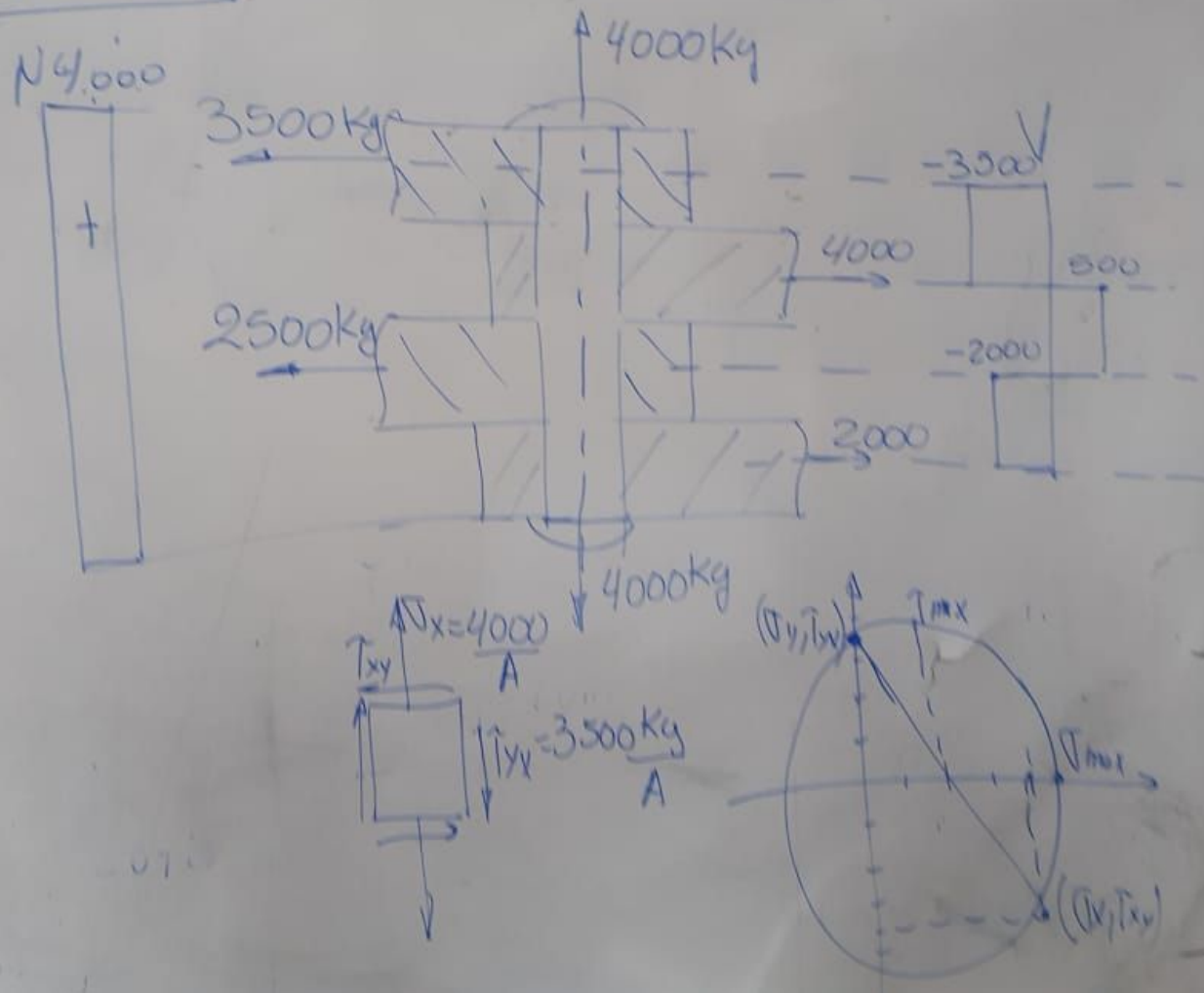
$$- 3 R_{AY} + 2W = 0 \Rightarrow R_{AY} = \frac{2}{3} \times 5000 = 3333$$

PREGUNTA 1.-



1.3.- Calcular el diámetro del remache que tiene que soportar la acción de las fuerzas axiales que se presentan en la figura adjunta, para cuyo efecto se tiene los esfuerzos de $\sigma_f = 4200 \text{ kg/cm}^2$ y un $\tau_f = 0.5\sigma_f$, con un factor de seguridad 2.

Problema 1 -



$$\sigma_f = 4200 \text{ kg/cm}^2$$

$$\tau_f = 0, \sigma_f$$

$$n = 2$$

$$\sigma_{max} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \leq \bar{\sigma}$$

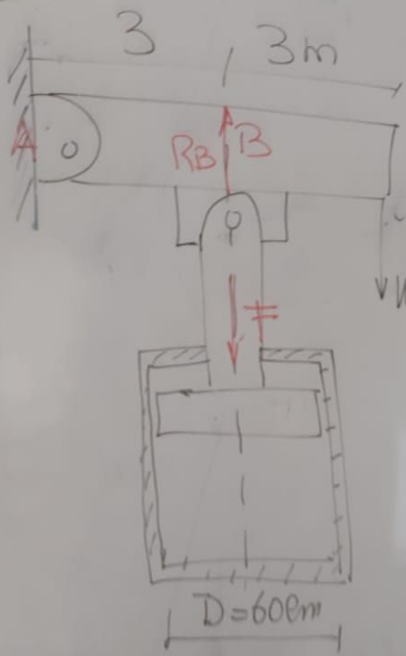
$$\sigma_{max} = \frac{4000}{2A} \pm \sqrt{\left(\frac{4000}{2A}\right)^2 + \left(\frac{3500}{A}\right)^2} \leq \bar{\sigma}$$

$$\sigma_{max} = \frac{2000}{A} \pm \frac{4031}{A} = 2100 \Rightarrow A \geq \sqrt{\frac{6031}{2100}} \Rightarrow \phi \geq \sqrt{\frac{4 \cdot 6031}{\pi \cdot 2100}}$$

$$\phi \geq 1,91 \text{ cm}$$

$$\sigma_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} \leq \bar{\sigma} \Rightarrow \frac{2657}{A} \leq 10500 \Rightarrow \phi \geq \sqrt{\frac{4 \cdot 4031}{\pi \cdot 10500}}$$

$$\phi \geq 2,21 \text{ cm}$$



$$\sum M_A = 0$$

$$-3R_B + 6 \times W = 0$$

$$R_B = \frac{6 \times 8000}{3}$$

$$R_B = 16000 \text{ kg}$$

$$F = R_B = 16000 \text{ kg}$$

$$\sigma_c = \frac{PD}{2l} \leq \bar{\sigma}$$

$$P = \frac{F}{A} = \frac{16000}{\pi \times 60^2} = 5,65 \frac{\text{kg}}{\text{cm}^2}$$

$$\Delta \sigma_c = \frac{PD}{2l} = \frac{5,65 \times 60}{2 \times 2100} = 0,08 \text{ cm} = 0,8 \text{ mm}$$

$$\text{Sea } d = 1 \text{ mm}$$

Comprobamos halgora

$$\sigma_c = \frac{PD}{2d} = \frac{5,65 \times 60}{2 \times 0,1} = 1695 \frac{\text{kg}}{\text{cm}^2}$$

$$\sigma_1 = \frac{PD}{4d} = 847 \frac{\text{kg}}{\text{cm}^2}$$

$$\epsilon_c = \frac{1}{E} (\sigma_c - \mu \sigma_1) = \frac{1}{2,1 \times 10^6} (1695 - 0,3 \times 847) = 6,86 \times 10^{-4}$$

$$6,86 \times 10^{-4} = 0,13$$

$$\delta_c = l_0 \epsilon_c = \pi \times 60 \times 0,13 = 188,62$$

$$l_f = l_0 + \delta_c = \pi \times 60 + 188,62 = 377,62$$

$$\sigma_f = 4200 \text{ Kg/cm}^2$$

$$\mu_f = 0,5$$

$$E = 2,1 \times 10^6 \text{ kg/cm}^2$$

$$\mu = 0,3$$

$$h_0 = 0,15 \text{ mm}$$

$$l_f = \pi D_f \Rightarrow D_f = \frac{l_f}{\pi}$$

$$D_f = \frac{377,62}{\pi} = 60,04 \text{ cm}$$

$$h_0 = \frac{D_f - D_0}{2} = \frac{60,04 - 60}{2} = 0,02 \text{ cm}$$

$$h_0 = 0,02 \text{ mm} \leq h_0 = 0,15 \text{ mm} \text{ No cumple}$$

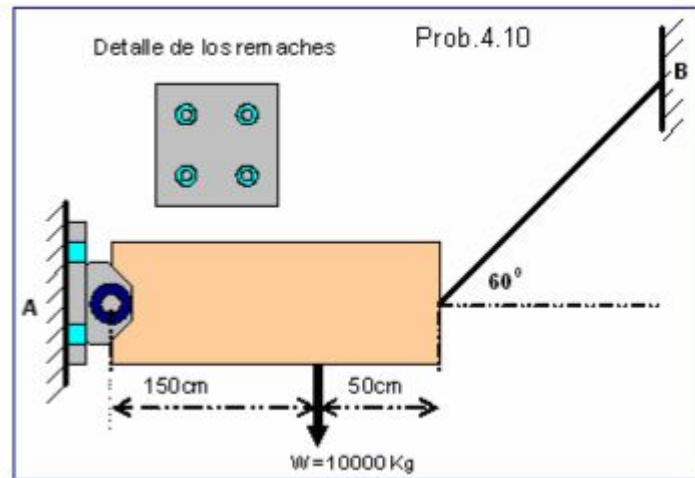
$$\text{Sea } e = 1,5 \text{ mm}$$

$$h_0 = 0,13 \text{ cumple}$$

PREGUNTA 1.-

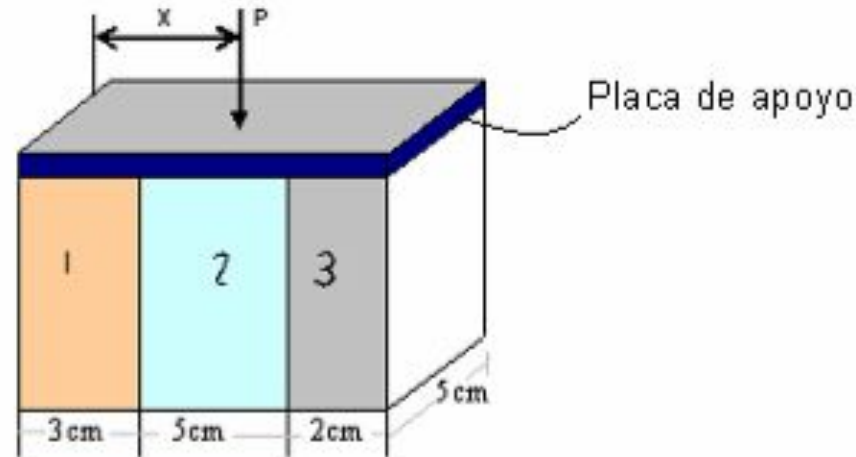
4.10.- Para la figura representada se pide dimensionar:

- a).- El diámetro del perno del pasador A. $\sigma_f=2100\text{kg/cm}^2$, $\tau_f=0.5 \sigma_f$ y $n=3$
- b).- El diámetro del cable con $\sigma_f=2800\text{kg/cm}^2$, $\tau_f=0.5 \sigma_f$ y un $n=2$
- c).- El diámetro de los remaches la unión de la figura para un $\sigma_f=4200\text{kg/cm}^2$ y un $\tau_f=0.5\sigma_f$ con un factor de seguridad de 3.



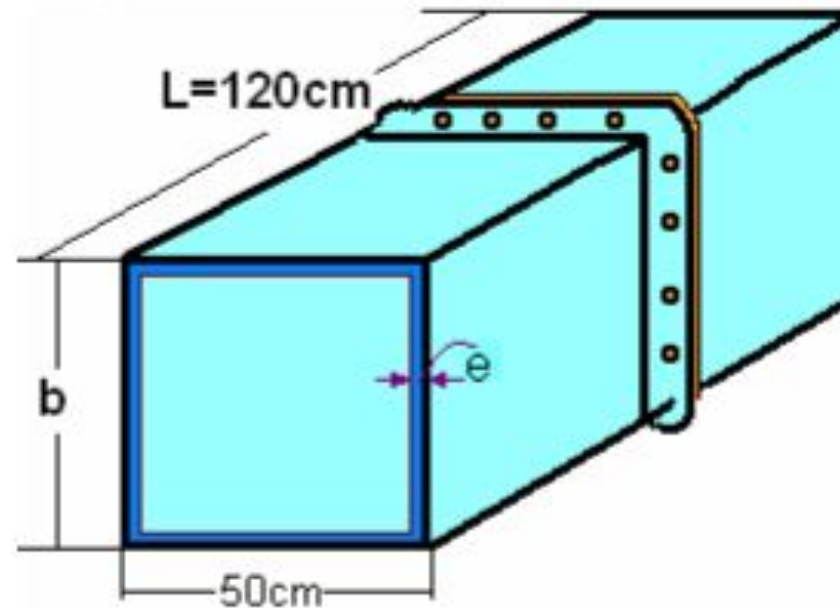
PREGUNTA 2.-

3.19.- Un miembro compuesto de tres bloques prismáticos es comprimido por una carga **P** a cierta distancia **X**. se pide calcular el valor de dicha carga **P** y la distancia **X** con los siguientes datos: $E_1=2.1 \cdot 10^6 \text{ kg/cm}^2$, $E_2=7 \cdot 10^5 \text{ kg/cm}^2$ y $E_3=1.4 \cdot 10^6 \text{ kg/cm}^2$ y los esfuerzos admisibles son: $\sigma_1=2100 \text{ kg/cm}^2$, $\sigma_3=700 \text{ kg/cm}^2$ y $\sigma_2=1050 \text{ kg/cm}^2$.



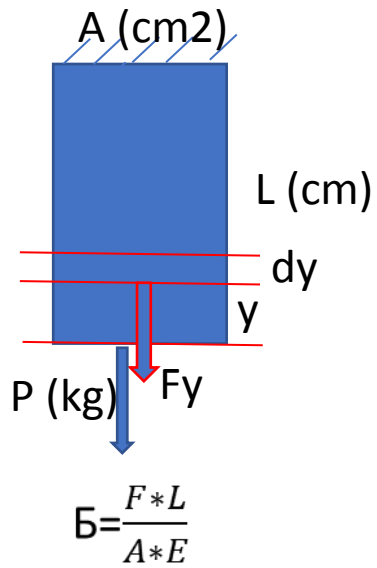
PREGUNTA 3.-

5.12.- Determinar el espesor de la plancha, las longitudes finales y el numero de remaches, para cuyo efecto se tiene los esfuerzos de $\sigma_f = 2100 \text{ kg/cm}^2$, $\tau_f = 0.5 \sigma_f$, $n = 2$. Si la presión interna es de 5 kg/cm^2 , $E = 2.1 \cdot 10^6 \text{ kg/cm}^2$, $b = 80 \text{ cm}$, $\mu = 0.3$.



Diametro remaches = $\frac{1}{2}''$

Pregunta 1.- Hallar la deformación total



$$F_y = \gamma \cdot \text{Vol} = \gamma \cdot A \cdot y$$

$$d\delta = \frac{F_y \cdot dy}{A \cdot E}$$

$$\delta_\gamma = \int \frac{F_y \cdot dy}{A \cdot E} = \int \frac{\gamma \cdot A \cdot y \cdot dy}{A \cdot E} = \frac{\gamma \cdot l^2}{2E}$$

$$\delta_p = \frac{P \cdot L}{A \cdot E}$$

$$\delta_t = \delta_\gamma + \delta_p$$

$$\gamma = 7000 \text{ (kg/m}^3\text{)}$$

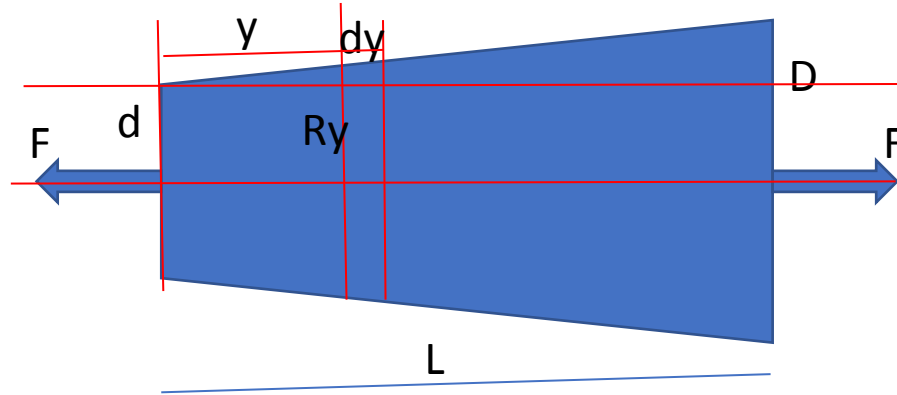
$$E = 2,1 \text{ E}6 \text{ (kg/cm}^2\text{)}$$

$$L = 50 \text{ cm}$$

$$P = 2000 \text{ Kg}$$

$$A = 2 \text{ cm}^2$$

Pregunta 4.- Hallar la deformación total



$$E = 2,1 \text{ E6 (kg/cm}^2\text{)}$$

$$L = 50 \text{ cm}$$

$$P = 2000 \text{ Kg}$$

$$d = 2 \text{ cm}$$

$$D = 6 \text{ cm}$$

$$\delta p = \frac{P \cdot L}{A \cdot E}$$

$$d\delta = \frac{F \cdot dy}{A_y \cdot E}$$

$$\frac{\frac{D}{2} - d/2}{L} = \frac{R_y - d/2}{y}$$

$$R_y = \left(\frac{D-d}{2L}\right)y + d/2$$

$$A_y = \frac{\pi \cdot R_y^2}{4}$$

$$\delta = \int \frac{F \cdot dy}{A_y \cdot E} = \frac{F \cdot dy}{\left(\pi \left(\left(\frac{D-d}{2L}\right)y + \frac{d}{2}\right)^2 \cdot E\right)}$$



GRACIAS.....