PRACTICA Nro. 3 – TRANSFORMADAS INTEGRALES
SERIE COMPLEJA DE FOURIER Y ESPECTROS DISCRETOS

1.- Graficar las funciones periódicas y determinar la serie compleja y a partir de ella la serie trigonométrica de Fourier.

a) $f_{(t)} = \begin{cases} t+4 & 0<t<2 \\ -2t+10 & 2<t<4 \end{cases}$ T=4 b) $f_{(t)} = \begin{cases} t^2-4 & 0<t<3 \\ 5 & 3<t<6 \end{cases}$ T=6

c) $f_{(t)} = e^{-2t} \quad -2\pi < t < 2\pi$ T=4π d) $f_{(t)} = \begin{cases} e^{3t} & 0<t<1 \\ 0 & 1<t<2 \end{cases}$ T=2

RESPUESTAS:

a) $f_{(t)} = \frac{9}{2} + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left[\left(\frac{3}{\pi^2 n^2}(\cos(\pi n)-1) - \frac{j}{\pi n} \right) \right] e^{\frac{j\pi n t}{2}}$; $f_{(t)} = \frac{9}{2} + \sum_{n=1}^{\infty} \left[\left(\frac{6}{\pi^2 n^2}(\cos(\pi n)-1)\cos\left(\frac{\pi n t}{2}\right) \right) + \frac{2}{n\pi} sen\left(\frac{\pi n t}{2}\right) \right]$

b) $f_{(t)} = 2 + \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left[\frac{9\cos(\pi n)}{\pi^2 n^2} + j\left(\frac{9}{2\pi n} + \frac{9(1-\cos(\pi n))}{\pi^3 n^3} \right) \right] e^{j\frac{\pi n t}{3}}$

$$f_{(t)} = 2 + \sum_{n=1}^{\infty} \left[\frac{18\cos(\pi n)}{\pi^2 n^2}\cos\left(\frac{\pi n t}{3}\right) + \left(\frac{18(\cos(\pi n)-1)}{\pi^3 n^3} - \frac{9}{\pi n} \right) sen\left(\frac{\pi n t}{3}\right) \right]$$

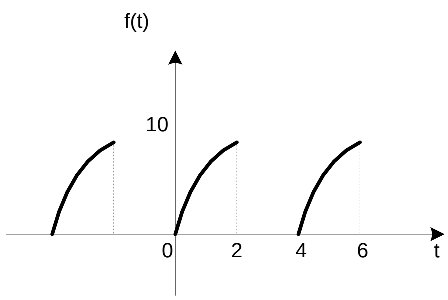
c) $f_{(t)} = \frac{e^{4\pi} - e^{-4\pi}}{8\pi} + \frac{e^{4\pi} - e^{-4\pi}}{\pi} \sum_{n=1}^{\infty} \left[\frac{4\cos(\pi n)}{n^2+16}\cos\left(\frac{nt}{2}\right) + \frac{n\cos(\pi n)}{n^2+16} sen\left(\frac{nt}{2}\right) \right]$

d) $f_{(t)} = \frac{e^3-1}{6} + \sum_{n=1}^{\infty} \left[\left(\frac{3e^3\cos(\pi n)-3}{9+\pi^2 n^2} \right)\cos(\pi n t) + \left(\frac{\pi n(1-e^3\cos(\pi n))}{9+\pi^2 n^2} \right) sen(\pi n t) \right]$

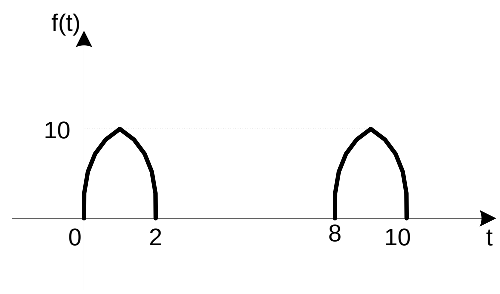
2.- Determinar la serie compleja y a partir de ella la serie trigonométrica de Fourier de las siguientes ondas senoidales rectificadas.

a) $f_{(t)} = \begin{cases} sen\left(\frac{\pi t}{2}\right) \ldots 0<t<2 \\ 0 \ldots\ldots\ldots\ldots 2<t<4 \end{cases}$ T=4 b) $f_{(t)} = |10sen(4t)|$

c)



d)



RESPUESTAS

a) $f_{(t)} = \frac{1}{\pi} + \frac{1}{2}sen\left(\frac{\pi}{2}t\right) + \sum_{n=2}^{\infty}\left(\frac{1+\cos(\pi n)}{\pi(1-n^2)}\right)\cos\left(\frac{\pi n t}{2}\right)$; b) $f_{(t)} = \frac{20}{\pi} + \sum_{n=1}^{\infty}\frac{40}{\pi(1-4n^2)}\cos(8nt)$

c) $f_{(t)} = \frac{10}{\pi} + \sum_{n=1}^{\infty}\left[\frac{20}{\pi(1-4n^2)}\cos\left(\frac{\pi n t}{2}\right) + \frac{40n\cos(\pi n)}{\pi(1-4n^2)}sen\left(\frac{\pi n t}{2}\right)\right]$

d) $f_{(t)} = \frac{5}{\pi} + \frac{5}{2}sen\left(\frac{\pi t}{2}\right) + \sum_{\substack{n=1\\ n\neq 2}}^{\infty}\left[\frac{20\left(\cos\left(\frac{\pi n}{2}\right)+1\right)}{\pi(4-n^2)}\cos\left(\frac{\pi n t}{4}\right) + \frac{20sen\left(\frac{\pi n}{2}\right)}{\pi(4-n^2)}sen\left(\frac{\pi n t}{4}\right)\right]$

3.- Expresar las siguientes funciones en términos de la función escalón unitario u(t):

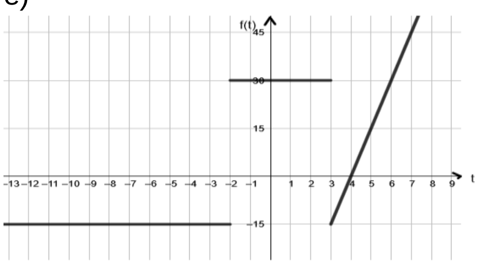
a) $f_{(t)} = \begin{cases} -4 & -7 < t < -2 \\ 4 & -2 < t < 1 \\ 6 & 1 < t < 5 \end{cases}$

b) $f_{(t)} = \begin{cases} -5t & -5 < t < -2 \\ 4t+1 & -2 < t < 0 \\ -3t+6 & 0 < t < 2 \end{cases}$

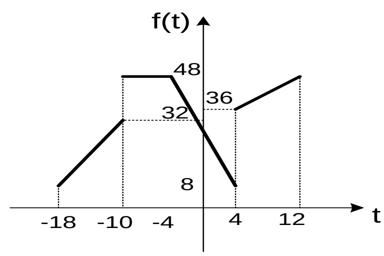
c) $f_{(t)} = \begin{cases} -4 & t < -2 \\ 5 & -2 < t < 4 \\ -10 & t > 4 \end{cases}$

d) $f_{(t)} = \begin{cases} 2-t & t < 3 \\ t-4 & 3 < t < 10 \\ 0 & 10 < t < 12 \\ 2t-10 & t > 12 \end{cases}$

e)



f)



RESPUESTAS

a) $f_{(t)} = -4u(t+7) + 8u(t+2) + 2u(t-1) - 6u(t-5)$;

b) $f_{(t)} = -5tu(t+5) + (9t+1)u(t+2) + (-7t+5)u(t) + (3t-6)u(t-2)$; c) $f_{(t)} = -4 + 9u(t+2) - 15u(t-5)$;

d) $f_{(t)} = 2 - t + (2t-6)u(t-3) + (-t+4)u(t-10) + (2t-10)u(t-12)$

e) $f_{(t)} = -15 + 45u(t+2) + (15t-90)u(t-3)$;

f) $f_{(t)} = (3t+62)u(t+18) + (-3t-14)u(t+10) + (-5t-20)u(t+4) + \left(\tfrac{13}{2}t+2\right)u(t-4) + \left(-\tfrac{3}{2}t-30\right)u(t-12)$

4.- Evaluar las integrales con función impulso:

a) $\int_{-\infty}^{\infty}\left(\frac{t^2-1}{2t+5}\right)\delta(t+4)dt$ b) $\int_{-\infty}^{\infty}t\cos(2t)\delta\left(t-\tfrac{\pi}{6}\right)dt$ c) $\int_{-\infty}^{\infty}(2t-1)\delta(3t-4)dt$

d) $\int_{-\infty}^{\infty}\delta'(t-2)e^{2t}\ln t\,dt$ e) $\int_{-\infty}^{\infty}\delta'(t-1)t^2e^{-3t}\cos(4t)dt$ f) $\int_{-\infty}^{\infty}\delta''\left(t-\tfrac{1}{2}\right)t^3\ln(2t)dt$

RESPUESTAS: a) -5; b) $\frac{\pi}{12}$; c) $\frac{5}{9}$; d)102.99; e) $e^{-3}(\cos 4 + 4sen4)$; f) $\frac{5}{2}$

5.- Aplicando diferenciación, determine las series de Fourier de las funciones definidas en un período y graficarlas:

a) $f_{(t)}=\begin{cases}-3t+1 & -1<t<0\\ 2t+3 & 0<t<1\end{cases}\quad T=2$

b) $f_{(t)}=\begin{cases}2 & 0<t<\frac{T}{2}\\ \dfrac{3}{T}t+\dfrac{3}{2} & \frac{T}{2}<t<T\end{cases}$

c) $f_{(t)}=\begin{cases}\dfrac{t^2}{5}+1 & 0<t<5\\ -4 & 5<t<10\end{cases}\quad T=10$

d) $f_{(t)}=\begin{cases}-5t+10 & 0<t<2\\ \dfrac{t^2}{4}+4 & 2<t<4\end{cases}\quad T=4$

e) $f_{(t)}=\begin{cases}t^2 & 0<t<2\\ -2t+8 & 2<t<6\\ -1 & 6<t<8\end{cases}\quad T=8$

f) $f_{(t)}=\begin{cases}1-t^2 & 0<t<1\\ 5t-2 & 1<t<2\\ 3 & 2<t<4\end{cases}\quad T=4$

g) $f_{(t)}=\begin{cases}t^3+1 & 0<t<2\\ 2t-8 & 2<t<4\end{cases}\quad T=4$

h) $f_{(t)}=\begin{cases}t^3+1 & 0<t<2\\ 12-t^2 & 2<t<4\end{cases}\quad T=4$

i) $f_{(t)}=\begin{cases}\dfrac{t^3}{5}+3 & 0<t<5\\ 2 & 5<t<8\end{cases}\quad T=8$

RESPUESTAS

a) $f_{(t)}=\dfrac{13}{4}+\sum_{n=1}^{\infty}\left[\left(\dfrac{5(\cos(\pi n)-1)}{\pi^2 n^2}\right)\cos(\pi nt)+\left(\dfrac{2-\cos(\pi n)}{\pi n}\right)sen(\pi nt)\right]$

b) $f_{(t)}=\dfrac{23}{8}+\sum_{n=1}^{\infty}\left[\dfrac{6(1-\cos(\pi n))}{\pi^2 n^2}\cos\left(\dfrac{2\pi n}{T}t\right)+\left(\dfrac{-5+2\cos(\pi n)}{\pi n}\right)sen\left(\dfrac{2\pi n}{T}t\right)\right]$

c) $f_{(t)}=-\dfrac{2}{3}+\sum_{n=1}^{\infty}\left[\dfrac{10}{\pi^2 n^2}\cos(\pi n)\cos\left(\dfrac{\pi nt}{5}\right)+\left(\dfrac{5-10\cos(\pi n)}{\pi n}+\dfrac{10(\cos(\pi n)-1)}{\pi^3 n^3}\right)sen\left(\dfrac{\pi nt}{5}\right)\right]$

d) $f_{(t)}=\dfrac{17}{3}+\sum_{n=1}^{\infty}\left[\dfrac{14-12\cos(\pi n)}{\pi^2 n^2}\cos\left(\dfrac{\pi nt}{2}\right)+\left(\dfrac{2+5\cos(\pi n)}{\pi n}+\dfrac{2-2\cos(\pi n)}{\pi^3 n^3}\right)sen\left(\dfrac{\pi nt}{2}\right)\right]$

e) $f_{(t)}=\dfrac{1}{12}+\sum_{n=1}^{\infty}\left[a_n\cos\left(\dfrac{\pi nt}{4}\right)+b_n sen\left(\dfrac{\pi nt}{4}\right)\right]$ donde:

$$a_n=-\frac{3}{\pi n}sen\left(\frac{3\pi n}{2}\right)+\frac{24\cos\left(\frac{\pi n}{2}\right)-8\cos\left(\frac{3\pi n}{2}\right)}{\pi^2 n^2}-\frac{32}{\pi^3 n^3}sen\left(\frac{\pi n}{2}\right)$$

$$b_n=\frac{1+3\cos\left(\frac{\pi n}{2}\right)}{\pi n}+\frac{24sen\left(\frac{\pi n}{2}\right)-8sen\left(\frac{3\pi n}{2}\right)}{\pi^2 n^2}-\frac{32}{\pi^3 n^3}\left(1-\cos\left(\frac{\pi n}{2}\right)\right)$$

f) $f_{(t)}=\dfrac{73}{24}+\sum_{n=1}^{\infty}\left[a_n\cos\left(\dfrac{\pi nt}{2}\right)+b_n sen\left(\dfrac{\pi nt}{2}\right)\right]$ donde:

$$a_n = -\frac{3}{\pi n} sen\left(\frac{\pi n}{2}\right) + \frac{-14\cos\left(\frac{\pi n}{2}\right) + 10\cos(\pi n)}{\pi^2 n^2} + \frac{8}{\pi^3 n^3} sen\left(\frac{\pi n}{2}\right)$$

$$b_n = -\frac{2}{\pi n} + \frac{3}{\pi n}\cos\left(\frac{\pi n}{2}\right) - \frac{5}{\pi n}\cos(\pi n) - \frac{14}{\pi^2 n^2} sen\left(\frac{\pi n}{2}\right) + \frac{8}{\pi^3 n^3}\left(1 - \cos\left(\frac{\pi n}{2}\right)\right)$$

g) $f_{(t)} = \frac{1}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{\pi n t}{2}\right) + b_n sen\left(\frac{\pi n t}{2}\right)\right]$ donde:

$$a_n = \frac{4 + 20\cos(\pi n)}{\pi^2 n^2} + \frac{48(1 - \cos(\pi n))}{\pi^4 n^4}\ ;\ b_n = \frac{1 - 13\cos(\pi n)}{\pi n} + \frac{48\cos(\pi n)}{\pi^3 n^3}$$

h) $f_{(t)} = \frac{17}{6} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{\pi n t}{4}\right) + b_n sen\left(\frac{\pi n t}{4}\right)\right]$, donde:

$$a_n = \frac{-16 + 32\cos(\pi n)}{\pi^2 n^2} + \frac{48 - 48\cos(\pi n)}{\pi^4 n^4}\ ;\ b_n = \frac{5 - \cos(\pi n)}{\pi n} + \frac{-8 + 56\cos(\pi n)}{\pi^3 n^3}$$

i) $f_{(t)} = \frac{209}{32} + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{\pi n t}{4}\right) + b_n sen\left(\frac{\pi n t}{4}\right)\right]$, donde:

$$a_n = \frac{26}{\pi n} sen\left(\frac{5\pi n}{4}\right) + \frac{60}{\pi^2 n^2}\cos\left(\frac{5\pi n}{4}\right) - \frac{96}{\pi^3 n^3} sen\left(\frac{5\pi n}{4}\right) + \frac{384}{5\pi^4 n^4}\left(1 - \cos\left(\frac{5\pi n}{4}\right)\right)$$

$$b_n = \frac{2}{\pi n}\left(\frac{1}{2} - 13\cos\left(\frac{5\pi n}{4}\right)\right) + \frac{60}{\pi^2 n^2} sen\left(\frac{5\pi n}{4}\right) + \frac{96}{\pi^3 n^3}\cos\left(\frac{5\pi n}{4}\right) - \frac{384}{5\pi^4 n^4} sen\left(\frac{5\pi n}{4}\right);$$

6.- Graficar los espectros de amplitud y fase de las funciones periódicas:

a) $f_{(t)} = 5t \quad 0 < t < 3$; T=3 b) $f_{(t)} = t + 1 \quad 0 < t < 5$; T=5

7.- Dada la función definida en un período T=5: $f_{(t)} = 2t \ldots\ldots 0 < t < 5$ a partir del teorema de Parseval, demostrar que: $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$

8.- Dada la función definida en un período: $f_{(t)} = \begin{cases} -1 & -\frac{T}{2} < t < 0 \\ 1 & 0 < t < \frac{T}{2} \end{cases}$ a partir del teorema de Parseval demostrar: $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$

9.- Dada la función definida en un período T=2π: $f_{(t)} = t^2 \ldots -\pi < t < \pi$ aplicando el teorema de Parseval demostrar: $\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$

10.- Demostrar a partir de la función definida en un período: $f_{(t)} = \begin{cases} -\frac{2}{T}t & -\frac{T}{2} < t < 0 \\ \frac{2}{T}t & 0 < t < \frac{T}{2} \end{cases}$ mediante el teorema de Parseval: $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$