PRACTICA Nro. 7 – TRANSFORMADAS INTEGRALES
TRANSFORMADA INVERSA DE LAPLACE

1.- Calcular la transformada inversa de Laplace aplicando las respectivas propiedades:

a) $\mathcal{L}^{-1}\left\{\dfrac{1}{s+4}-\dfrac{2}{3s+1}+\dfrac{4}{s^2+9}\right\}$

b) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^3}+\dfrac{4}{s^5}-\dfrac{3}{\sqrt{s^5}}\right\}$

c) $\mathcal{L}^{-1}\left\{\dfrac{s+4}{s^2+6s+13}\right\}$

d) $\mathcal{L}^{-1}\left\{\dfrac{6}{3s^2+1}-\dfrac{s}{2s^2+4}+\dfrac{2}{(4s-3)^3}\right\}$

e) $\mathcal{L}^{-1}\left\{\dfrac{2s-6}{s^2+6s+25}\right\}$

f) $\mathcal{L}^{-1}\left\{\dfrac{2s-3}{9s^2-6s+2}\right\}$

g) $\mathcal{L}^{-1}\left\{\dfrac{1}{\sqrt{3s-5}}-\sqrt{(s-4)^7}\,e^{-7s}\right\}$

h) $\mathcal{L}^{-1}\left\{\ln\left(\dfrac{2s+1}{4s+5}\right)\right\}$

i) $\mathcal{L}^{-1}\left\{e^{-4s}\ln\left(1+\dfrac{4}{(s+4)^2}\right)\right\}$

j) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^2(s^2+5)}\right\}$

k) $\mathcal{L}^{-1}\left\{\dfrac{1}{s^3(3s+2)}\right\}$

l) $\mathcal{L}^{-1}\left\{\dfrac{e^{-s}}{(s+2)^3(s^2+4s+13)}\right\}$

RESPUESTAS:

a) $f_{(t)}=e^{-4t}-\dfrac{2}{3}e^{-\frac{t}{3}}+\dfrac{4}{3}sen(3t)$; b) $f_{(t)}=\dfrac{t^2}{2}+\dfrac{t^4}{6}-4\sqrt{\dfrac{t^3}{\pi}}$; c) $f_{(t)}=e^{-3t}\left[\cos(2t)+\tfrac{1}{2}sen(2t)\right]$

d) $f_{(t)}=2\sqrt{3}sen\left(\dfrac{\sqrt{3}}{3}t\right)-\dfrac{\cos\left(\sqrt{2}t\right)}{2}+\dfrac{t^2e^{\frac{3}{4}t}}{64}$; e) $f_{(t)}=e^{-3t}\left[2\cos(4t)-3sen(4t)\right]$; f) $f_{(t)}=\dfrac{1}{9}e^{\frac{t}{3}}\left(2\cos\left(\dfrac{t}{3}\right)-7sen\left(\dfrac{t}{3}\right)\right)$

g) $f_{(t)}=\dfrac{t^{-1/2}e^{\frac{5t}{3}}}{\sqrt{3\pi}}u(t)-\dfrac{105}{16\sqrt{\pi}}e^{4(t-7)}(t-7)^{-9/2}u(t-7)$; h) $f_{(t)}=\dfrac{e^{-\frac{5}{4}t}-e^{-\frac{t}{2}}}{t}$; i) $f_{(t)}=\dfrac{2e^{-4(t-4)}\left(1-\cos(2t-8)\right)}{t-4}u(t-4)$

j) $f_{(t)}=-\dfrac{sen\left(\sqrt{5}t\right)}{5\sqrt{5}}+\dfrac{t}{5}$; k) $f_{(t)}=-\dfrac{27}{8}e^{-\frac{2}{3}t}+\dfrac{3}{4}t^2-\dfrac{9}{4}t+\dfrac{27}{8}$; l) $f_{(t)}=e^{-2(t-1)}\left(\dfrac{\cos(3t-3)}{81}-\dfrac{1}{81}+\dfrac{(t-1)^2}{18}\right)u(t-1)$

2.-Calcular las transformadas inversas descomponiendo en fracciones parciales:

a) $\mathcal{L}^{-1}\left\{\dfrac{s-4}{(s+5)(s^2-3s+2)}\right\}$

b) $\mathcal{L}^{-1}\left\{\dfrac{s^2-2s+1}{2s^3+7s^2-4s}\right\}$

c) $\mathcal{L}^{-1}\left\{\dfrac{s-6}{s^3-s^2-5s-3}\right\}$

d) $\mathcal{L}^{-1}\left\{\dfrac{s+3}{(s^2+2s+1)(s^2-4)}\right\}$

e) $\mathcal{L}^{-1}\left\{\dfrac{-s^2-40s-76}{(s-2)(s^2+8s+20)}\right\}$

f) $\mathcal{L}^{-1}\left\{\dfrac{s^2+3}{(s^3+8)(s+2)}\right\}$

g) $\mathcal{L}^{-1}\left\{\dfrac{2s^3+4s+10}{(s^2+10s+25)(s^2+2s+20)}\right\}$

h) $\mathcal{L}^{-1}\left\{\dfrac{s^2+3}{(s-1)^2(s^2-4s+3)}\right\}$

i) $\mathcal{L}^{-1}\left\{\dfrac{s^2+8s+4}{(s^2-3s+2)(s^2+6s+10)}\right\}$

j) $\mathcal{L}^{-1}\left\{\dfrac{4s^2+2s+10}{(s^2-4s+10)(s^2+4)}\right\}$

k) $\mathcal{L}^{-1}\left\{\dfrac{2s^2+1}{(s^2+3)(s-3)^2}\right\}$

l) $\mathcal{L}^{-1}\left\{\dfrac{s^2+2s+1}{(s^2+5s+15)(s^2+4)}\right\}$

RESPUESTAS:

a) $f_{(t)}=-\dfrac{3}{14}e^{-5t}-\dfrac{2}{7}e^{2t}+\dfrac{1}{2}e^{t}$; b) $f_{(t)}=-\dfrac{1}{4}+\dfrac{1}{18}e^{\frac{t}{2}}+\dfrac{25}{36}e^{-4t}$; c) $f_{(t)}=\dfrac{7}{4}te^{-t}+\dfrac{3}{16}e^{-t}-\dfrac{3}{16}e^{3t}$

d) $f_{(t)}=\frac{1}{9}e^{-t}-\frac{2te^{-t}}{3}+\frac{5}{36}e^{2t}-\frac{1}{4}e^{-2t}$; e) $f_{(t)}=-4e^{2t}+3e^{-4t}\cos(2t)-7e^{-4t}sen(2t)$;

f) $f_{(t)}=-\frac{1}{24}e^{-2t}+\frac{7}{12}te^{-2t}+\frac{e^{t}}{24}\left(\cos\left(\sqrt{3}t\right)+\frac{7}{\sqrt{3}}sen\left(\sqrt{3}t\right)\right)$;

g) $f_{(t)}=\frac{662}{245}e^{-5t}-\frac{52}{7}te^{-5t}+e^{-t}\left(-\frac{172}{245}\cos\left(\sqrt{19}t\right)-\frac{922}{245\sqrt{19}}sen\left(\sqrt{19}t\right)\right)$; h) $f_{(t)}=e^{t}\left(-t^{2}-2t-\frac{3}{2}\right)+\frac{3}{2}e^{3t}$;

i) $f_{(t)}=\frac{12}{13}e^{2t}-\frac{13}{17}e^{t}+e^{-3t}\left(-\frac{35}{221}\cos t-\frac{123}{221}sent\right)$; j) $f_{(t)}=-\frac{3}{25}\cos(2t)-\frac{17}{50}sen(2t)+e^{2t}\left[\frac{3}{25}\cos\left(\sqrt{6}t\right)+\frac{111}{25\sqrt{6}}sen\left(\sqrt{6}t\right)\right]$;

k) $f_{(t)}=-\frac{5}{24}\cos\left(\sqrt{3}t\right)-\frac{5}{24\sqrt{3}}sen\left(\sqrt{3}t\right)+\frac{5}{24}e^{3t}+\frac{19}{12}te^{3t}$;

l) $f_{(t)}=e^{-\frac{5}{2}t}\left(-\frac{37}{221}\cos\left(\frac{\sqrt{35}}{2}t\right)+\frac{243}{221\sqrt{35}}sen\left(\frac{\sqrt{35}}{2}t\right)\right)+\frac{37}{221}\cos(2t)+\frac{7}{442}sen(2t)$

3.- Calcular las transformadas inversas aplicando la integral de convolución y las propiedades que correspondan:

a) $\mathcal{L}^{-1}\left\{\frac{s}{\left(s^{2}+7\right)^{2}}\right\}$ b) $\mathcal{L}^{-1}\left\{\frac{s}{(s+5)\left(s^{2}+4\right)}\right\}$ c) $\mathcal{L}^{-1}\left\{\frac{s}{\left(s^{2}+4\right)\left(s^{2}+9\right)}\right\}$

d) $\mathcal{L}^{-1}\left\{\frac{1}{\left(s^{2}+2\right)^{2}}\right\}$ e) $\mathcal{L}^{-1}\left\{\frac{s^{2}}{\left(s^{2}+5\right)^{2}}\right\}$ f) $\mathcal{L}^{-1}\left\{\frac{1}{s^{2}\left(s^{2}+4\right)^{2}}\right\}$

g) $\mathcal{L}^{-1}\left\{\frac{1}{(s+3)^{2}\left(s^{2}+4\right)}\right\}$ h) $\mathcal{L}^{-1}\left\{\frac{s}{(s+5)^{2}\left(s^{2}+1\right)}\right\}$ i) $\mathcal{L}^{-1}\left\{\frac{s}{\left(s^{2}+9\right)^{3}}\right\}$

j) $\mathcal{L}^{-1}\left\{\frac{e^{-6s}}{\left(s^{2}+9\right)^{3}}\right\}$ k) $\mathcal{L}^{-1}\left\{\frac{s+4}{\left(s^{2}+8s+20\right)^{3}}\right\}$ l) $\mathcal{L}^{-1}\left\{\frac{(s+1)^{2}e^{-3s}}{\left(s^{2}+2s+5\right)^{3}}\right\}$

RESPUESTAS

a) $f_{(t)}=\frac{1}{2\sqrt{7}}tsen\left(\sqrt{7}t\right)$; b) $f_{(t)}=\frac{1}{29}(5\cos(2t)+2sen(2t))-\frac{5}{29}e^{-5t}$; c) $f_{(t)}=-\frac{1}{5}\cos(3t)+\frac{1}{5}\cos(2t)$;

d) $f_{(t)}=\frac{sen\left(\sqrt{2}t\right)}{4\sqrt{2}}-\frac{t\cos\left(\sqrt{2}t\right)}{4}$; e) $f_{(t)}=\frac{t\cos\left(\sqrt{5}t\right)}{2}+\frac{sen\left(\sqrt{5}t\right)}{2\sqrt{5}}$; f) $f_{(t)}=\frac{t}{16}+\frac{t\cos(2t)}{32}-\frac{3}{64}sen(2t)$

g) $f_{(t)}=\frac{te^{-3t}}{13}+\frac{6}{169}e^{-3t}+\frac{5}{338}sen(2t)-\frac{6}{169}\cos(2t)$; h) $f_{(t)}=-\frac{5}{26}te^{-5t}-\frac{6}{169}e^{-5t}+\frac{6}{169}\cos t+\frac{5}{338}sent$

i) $f_{(t)}=\frac{tsen(3t)}{216}-\frac{t^{2}\cos(3t)}{72}$; j) $f_{(t)}=\left(-\frac{(t-6)^{2}sen(3t-18)}{216}-\frac{(t-6)\cos(3t-18)}{216}+\frac{sen(3t-18)}{648}\right)u(t-6)$

k) $f_{(t)}=e^{-4t}\left(\frac{tsen(2t)}{64}-\frac{t^{2}\cos(2t)}{32}\right)$; l) $f_{(t)}=e^{-(t-3)}\left(\frac{(t-3)^{2}sen(2t-6)}{16}-\frac{(t-3)\cos(2t-6)}{32}+\frac{sen(2t-6)}{64}\right)u(t-3)$