Actuators

Vector representation of AC electric drive and Actuator based on AC motor drive

Lecture 3

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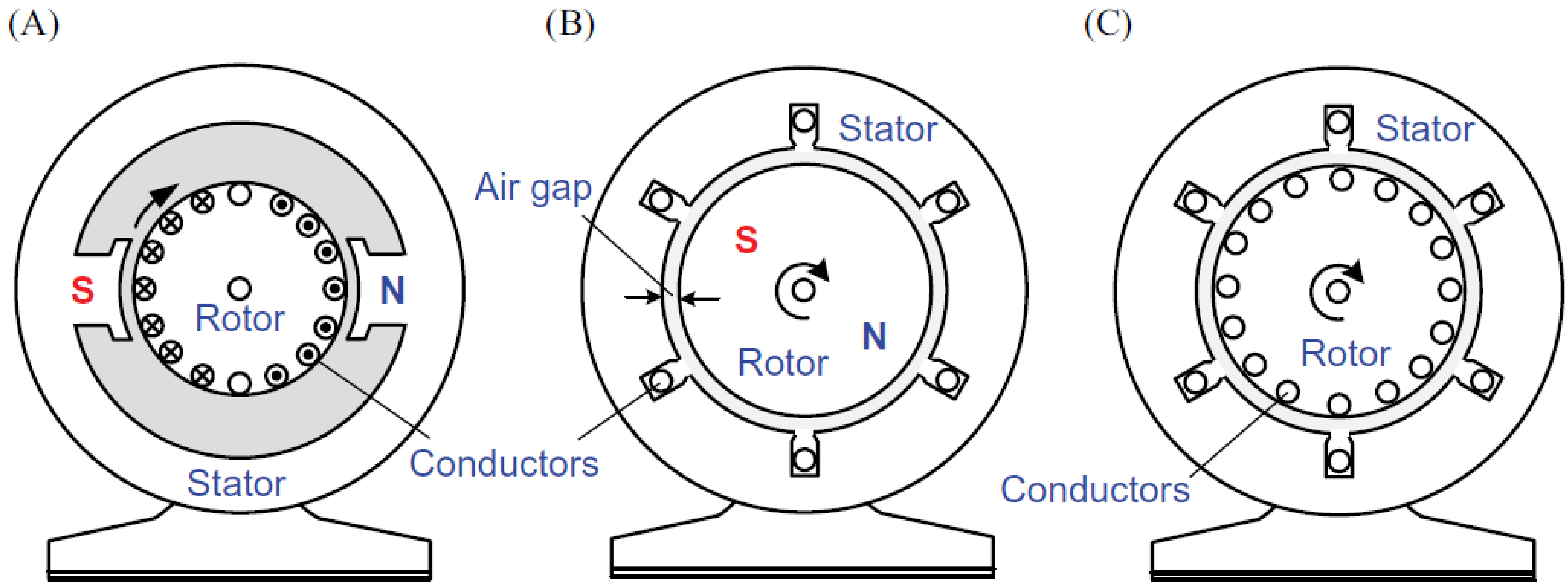
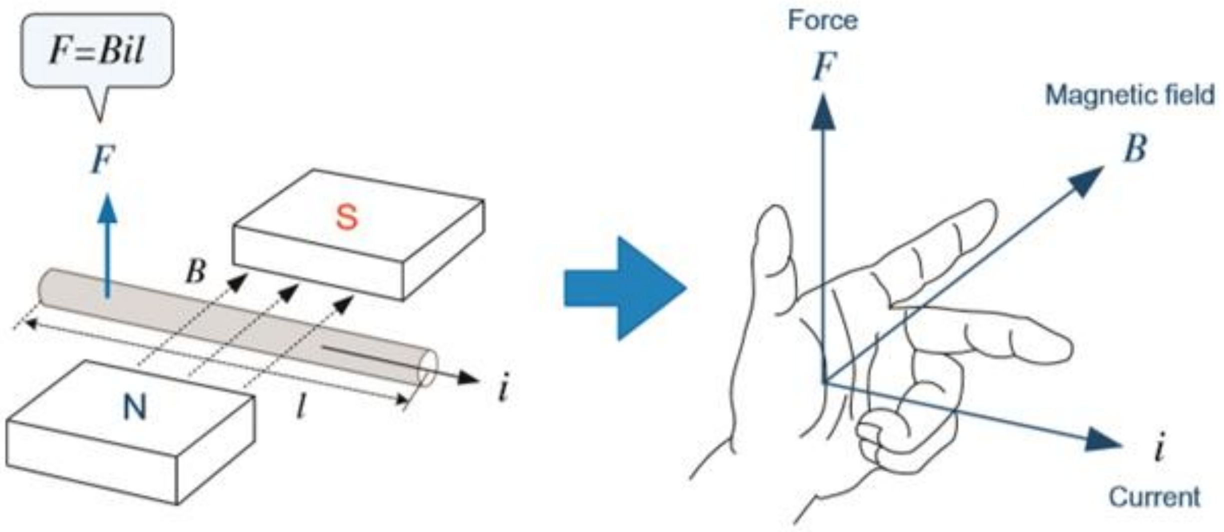
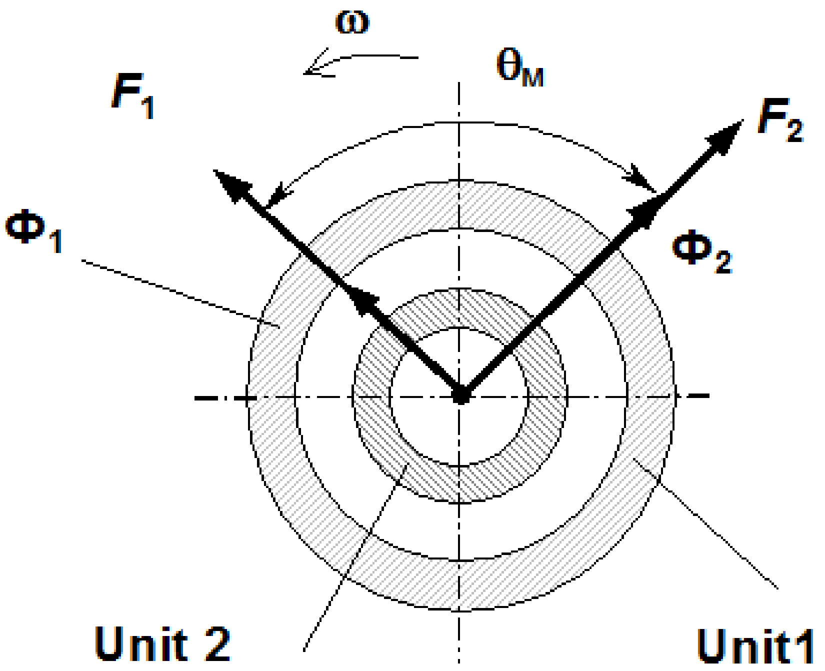
Configuration of electric motors



Configuration of electric motors. (A) DC motor, (B) AC synchronous motor, and (C) AC induction motor.

Two forms of Amper’s law





$$T = F_1 \times \Phi_2 \sin\theta_M$$

Amper’s law **<u>the 1st Form</u>** (Force for a current carrying conductor)

Generalized form of the Ampere’s Law **<u>the 2nd Form (</u>**The torque tends to align vectors)

Conditions for instantaneous torque control of motors

Construction with commutator => Always 90° between I_a and Φ_f => T_{max}

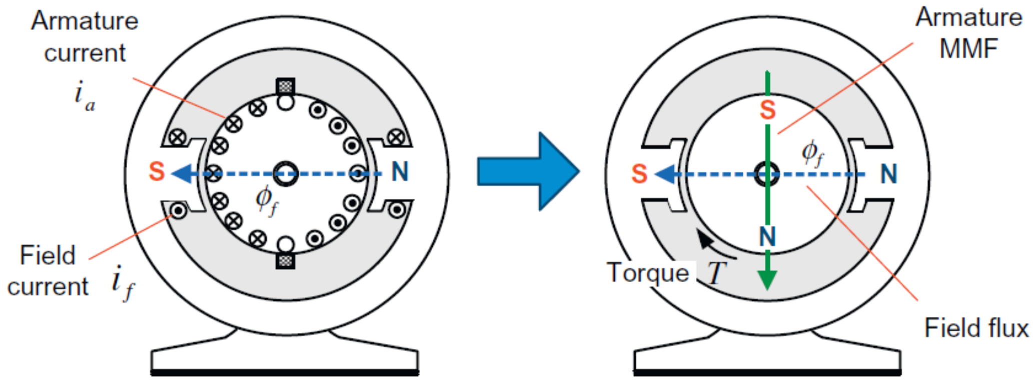


Figure Separately excited DC motor

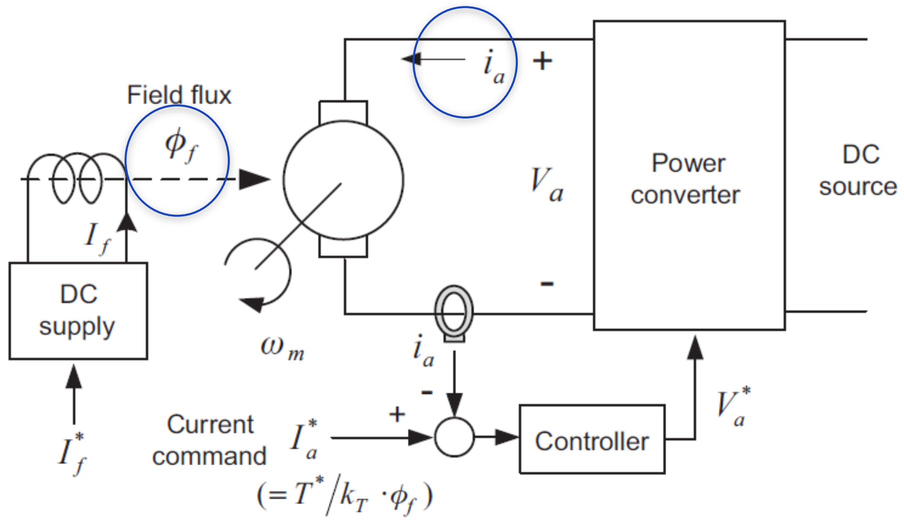


Figure Torque control system of a DC motor

Since the space angle between the armature current and the field flux always remains at 90 electrical degrees without using any particular control technique, the developed torque can be maximized under a given flux and current.

$$T = k|\phi_f||i_a|$$

$$T = k'|i_a|, \qquad k' = k|\phi_f|$$

This implies that it is possible to control the instantaneous torque of a DC motor by controlling only the magnitude of the armature current $|i_a|$.

Conditions for instantaneous torque control of motors

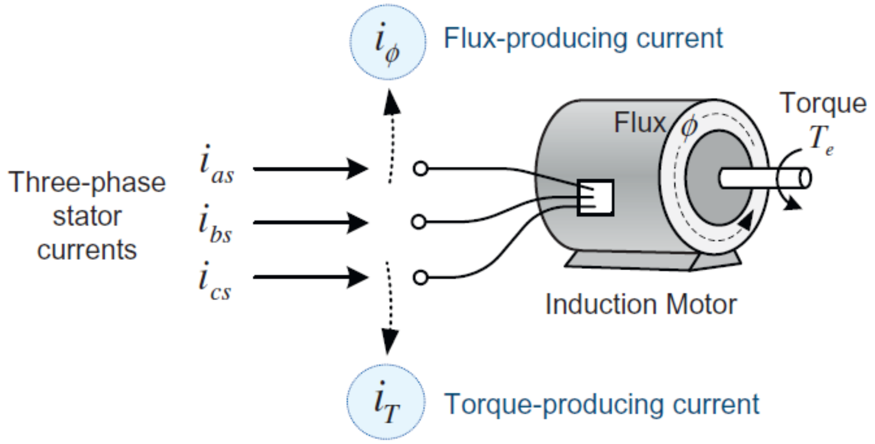
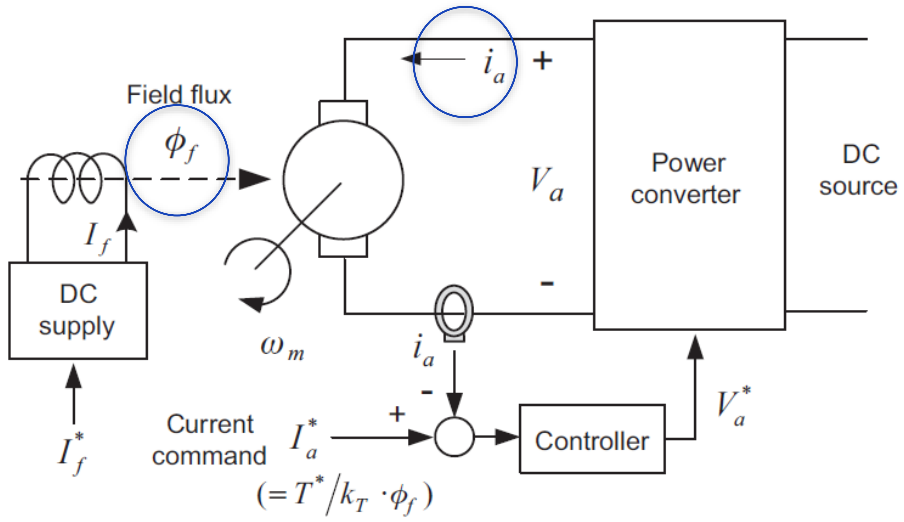


Figure Currents of an induction motor



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Complex vector

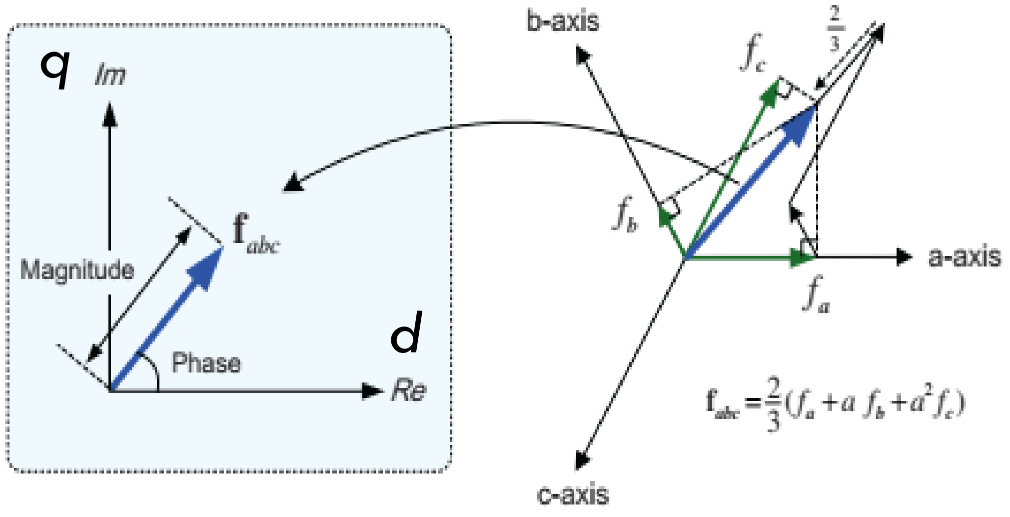
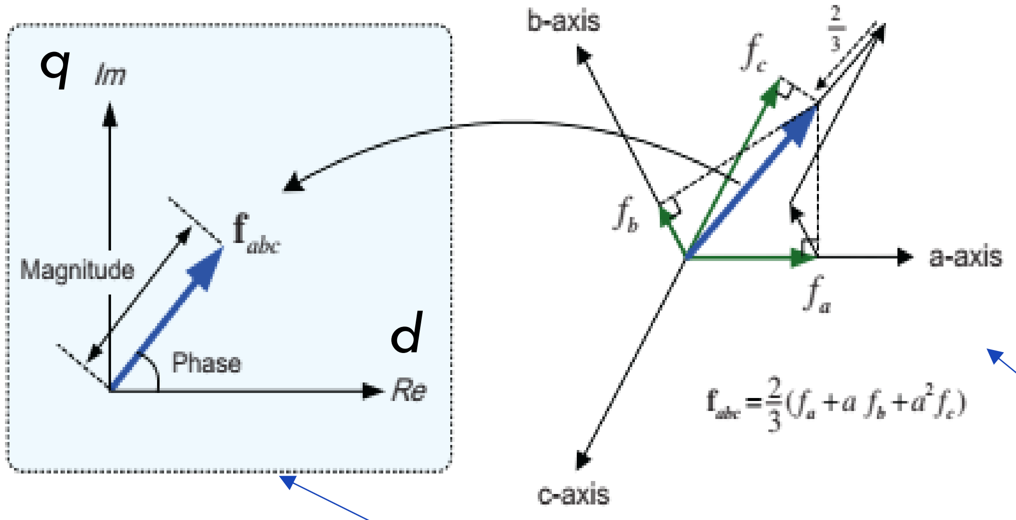


Figure Complex space vector

A complex space vector is defined as

$$f_{abc} \equiv \frac{2}{3}(f_a + af_b + a^2 f_c) \qquad a \equiv e^{j(2\pi/3)},\ a^2 = e^{j(4\pi/3)}$$

Complex vector



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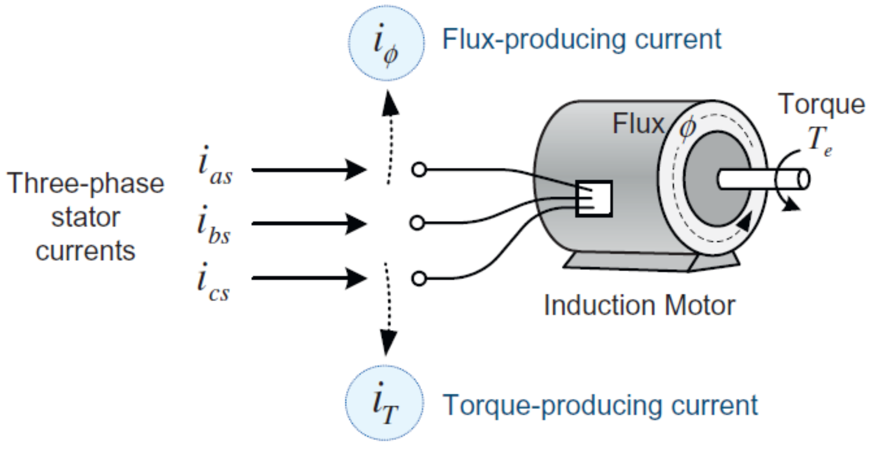

Figure Currents of an induction motor

Representation and description of f_{abc} in three-axis frame *abc*

Representation and description of f_{abc} in two-axis frame *Re-Im* or *dq*

A complex space vector is defined as

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$$f_{dq}^s = f_d^s + jf_q^s \qquad \text{or} \qquad f_{dq}^e = f_d^e + jf_q^e$$

Complex vector

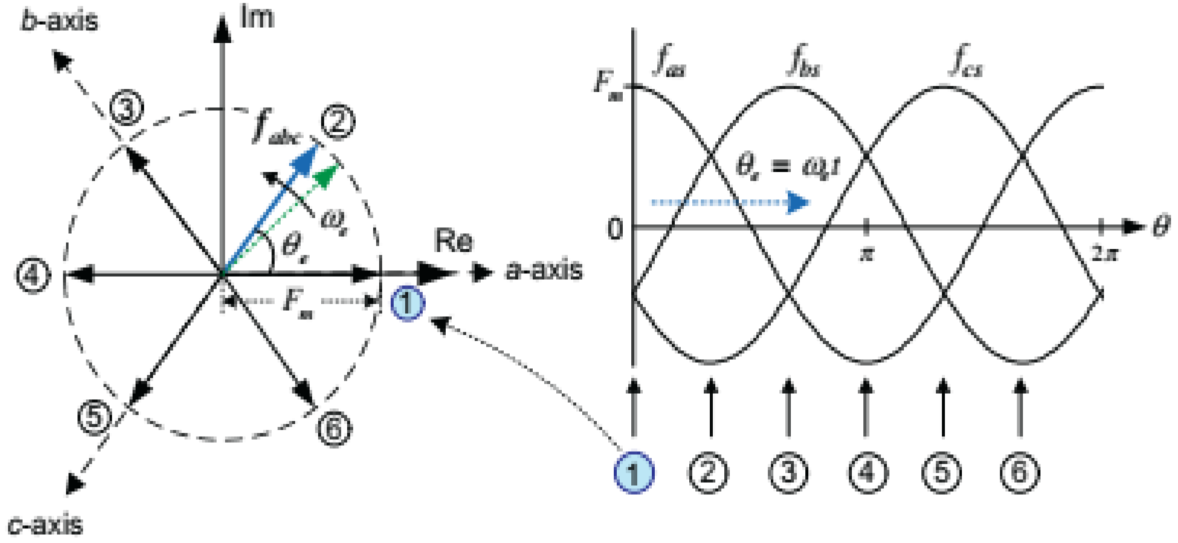
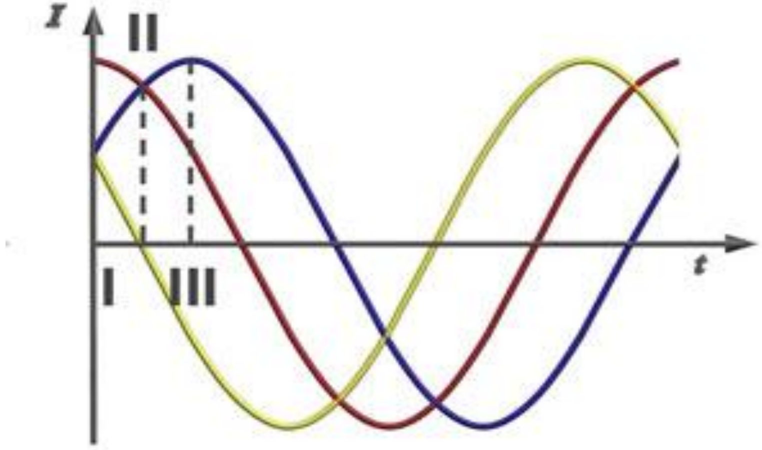
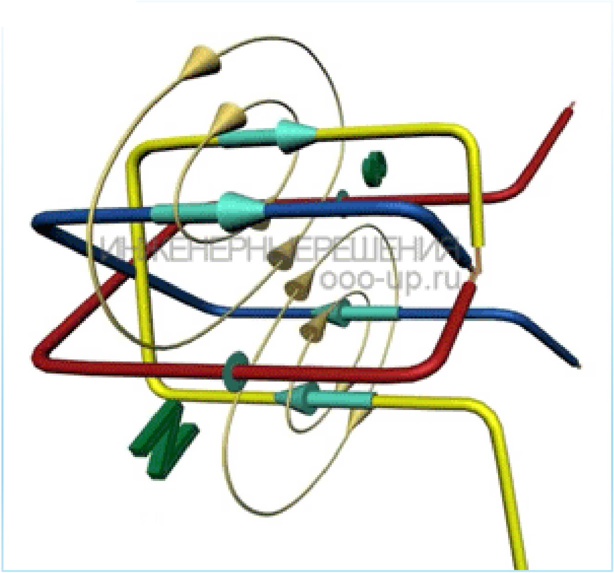


Figure Three-phase quantities and complex space vector





Firgure. Three-phase currents

Firgure. Rotating magnetic field can be represented as space vector too

AC motors

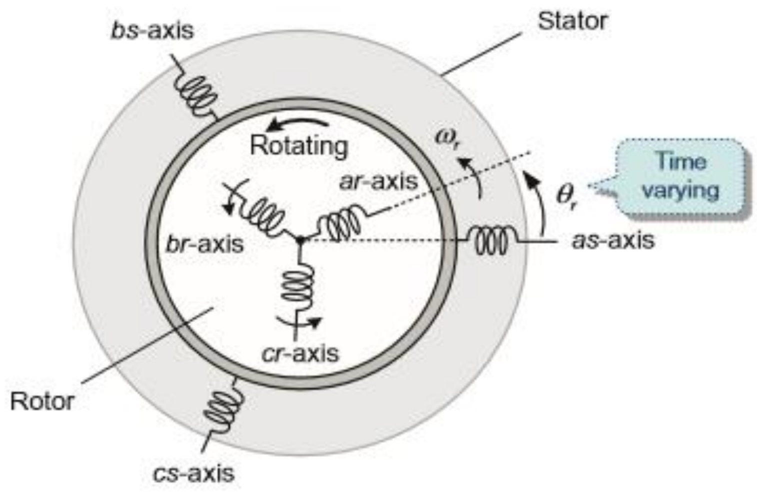


Fig. Angular position between stator and rotor windings

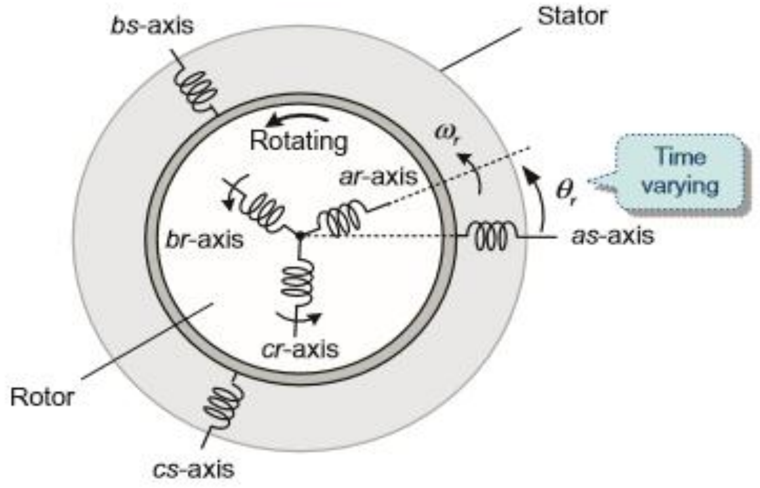
$$v(t) = Ri(t) + \frac{d\lambda(t)}{dt} = Ri(t) + \frac{d\,L(\theta_r)i(t)}{dt}$$

$\theta_r = \omega_r t$

λ - flux linkage

λ depends on the mutual-inductance which implies the amount of flux linking between the two windings (stator and rotor) - a time-varying parameter

AC motors



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- for each winding
- time-varying variables

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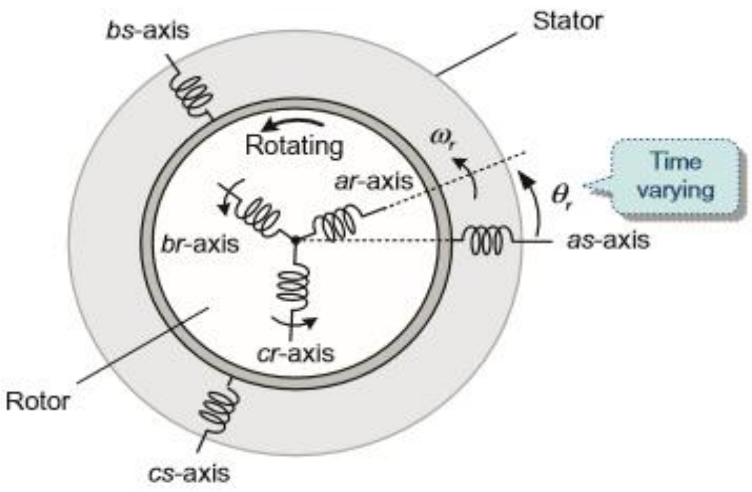
AC motors



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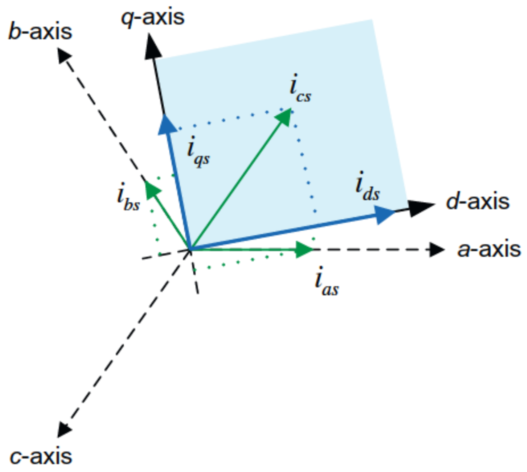

Figure   Reference frame transformation for currents

$$\boxed{v(t) = Ri(t) + \frac{d\lambda(t)}{dt} = Ri(t) + \frac{d\,L(\theta_r)i(t)}{dt}}$$

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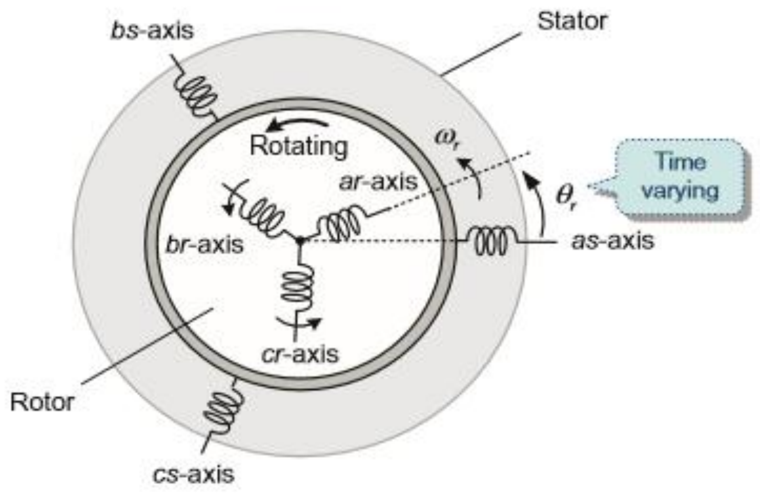
$$f_{dq}^s = f_d^s + jf_q^s \Rightarrow \begin{aligned} u_s^s &= u_{ds}^s + ju_{qs}^s \\ i_s^s &= i_{ds}^s + ji_{qs}^s \\ \lambda_s^s &= \lambda_{ds}^s + j\lambda_{qs}^s \end{aligned}$$

AC motors



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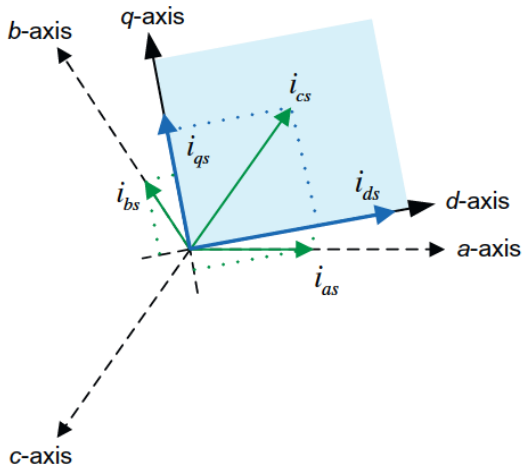

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- for each winding
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COMPLEX

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$$f_{dq}^s = f_d^s + jf_q^s \Rightarrow$$

$$u_s^s = u_{ds}^s + ju_{qs}^s$$

$$i_s^s = i_{ds}^s + ji_{qs}^s$$

$$\lambda_s^s = \lambda_{ds}^s + j\lambda_{qs}^s$$

$$\boxed{\begin{aligned} v_{ds}^s &= R_s i_{ds}^s + s\lambda_{ds}^s \\ v_{qs}^s &= R_s i_{qs}^s + s\lambda_{qs}^s \end{aligned}}$$

MORE SIMPLE

Complex vector

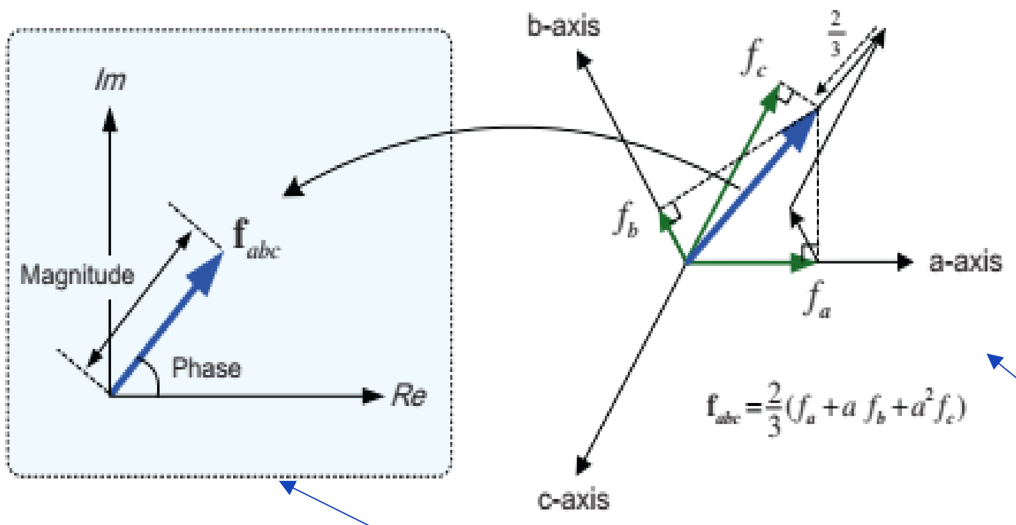


Figure Complex space vector

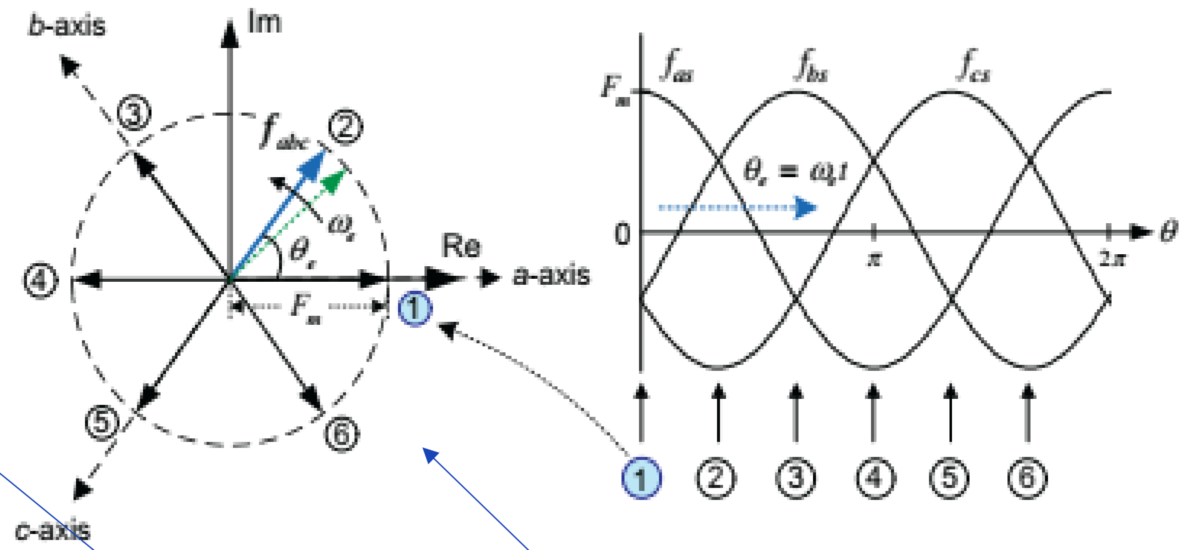


Figure Three-phase quantities and complex space vector

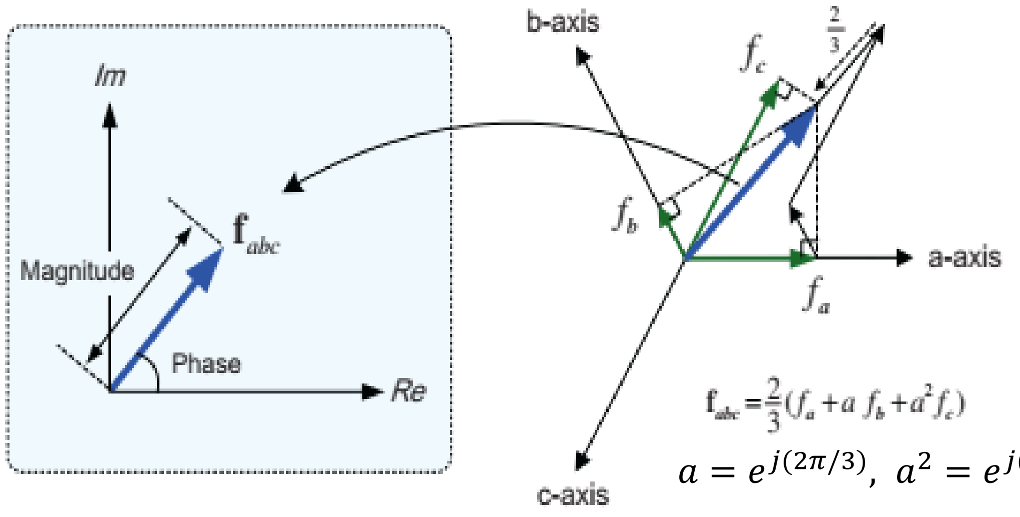
A complex space vector is defined as

$$f_{abc} \equiv \frac{2}{3}(f_a + af_b + a^2 f_c) \qquad a \equiv e^{j(2\pi/3)},\ a^2 = e^{j(4\pi/3)}$$

Representation and description of f_{abc} in three-axis frame *abc*

Representation and description of f_{abc} in two-axis frame *Re-Im* or *dq*

Reference frame transformation by complex vector



$a = e^{j(2\pi/3)},\ a^2 = e^{j(4\pi/3)}$

Figure Complex space vector

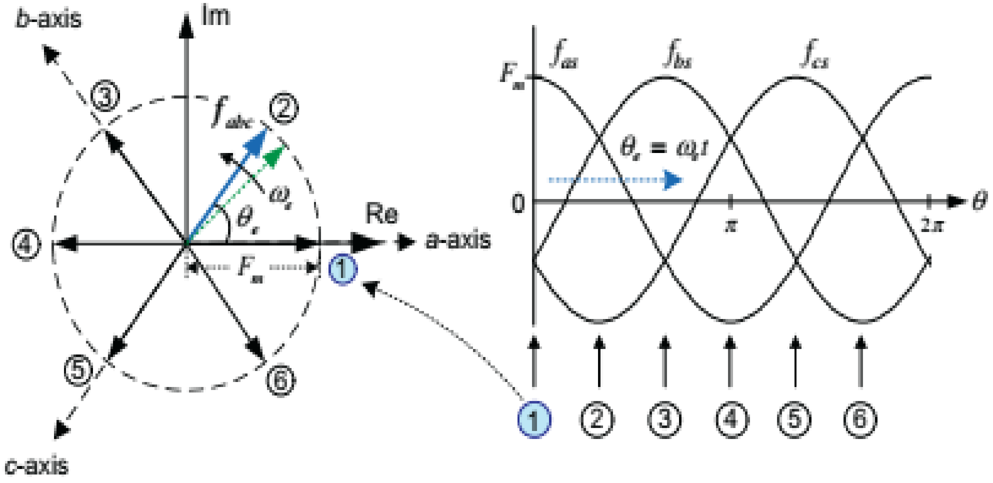


Figure Three-phase quantities and complex space vector

space vector f_{abc} equals to the vector f_{dq}^s

$$f_{dq}^s = f_d^s + jf_q^s$$

$$f_{dq}^\omega = f_d^\omega + jf_q^\omega \longrightarrow$$

$$f_d^\omega = \frac{2}{3}\left[f_a \cos\theta + f_b \cos\left(\theta - \frac{2}{3}\pi\right) + f_c \cos\left(\theta - \frac{4}{3}\pi\right)\right]$$

$$f_q^\omega = -\frac{2}{3}\left[f_a \sin\theta + f_b \sin\left(\theta - \frac{2}{3}\pi\right) + f_c \sin\left(\theta - \frac{4}{3}\pi\right)\right]$$

Reference frame transformation by complex vector

$$f_{dq}^{\omega} = f_d^{\omega} + jf_q^{\omega}$$ therefore

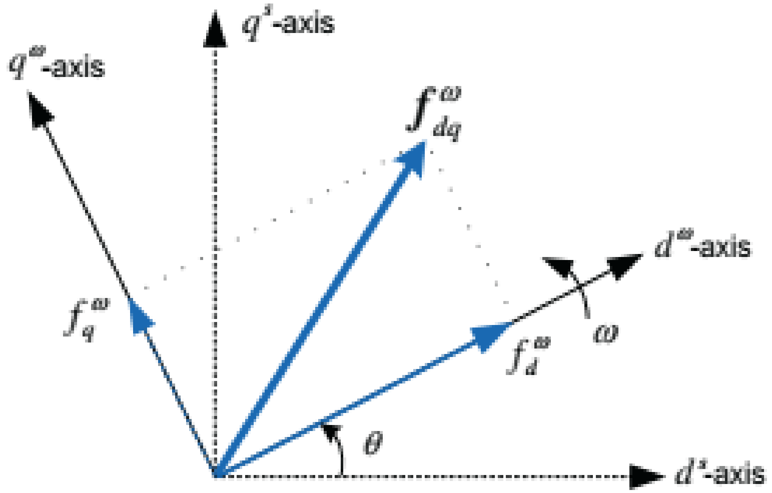


Figure Vector f_{dq}^{ω} in the arbitrary reference frame rotating at an angular velocity

$$f_{dq}^{\omega} = \frac{2}{3}\left[f_a(\cos\theta - j\sin\theta) + f_b\left(\cos\left(\theta - \frac{2}{3}\pi\right) - j\sin\left(\theta - \frac{2}{3}\pi\right)\right) + \right.$$

$$\left. + f_e\left(\cos\left(\theta - \frac{4}{3}\pi\right) - j\sin\left(\theta - \frac{4}{3}\pi\right)\right)\right] =$$

$$= \frac{2}{3}\left[f_a e^{-j\theta} + f_a e^{-j\left(\theta - \frac{2}{3}\pi\right)} + f_a e^{-j\left(\theta - \frac{4}{3}\pi\right)}\right] = \frac{2}{3}[f_a + af_a + a^2 f_a]e^{-j\theta} =$$

$$= f_{abc}e^{-j\theta}$$

A vector f_{dq}^{ω} in the rotating reference frame with the angular velocity $\omega = \omega_e$ can be expressed as

$$\boxed{f_{dq}^{e} = f_{abc}e^{-j\theta_e} = f_{dq}^{s}e^{-j\theta_e}} \quad \left(\theta_e = \int \omega_e(t)dt + \theta(0)\right)$$

And

$$\boxed{f_{dq}^{s} = f_{dq}^{e}e^{j\theta_e}}$$

Types of the d-q reference frame

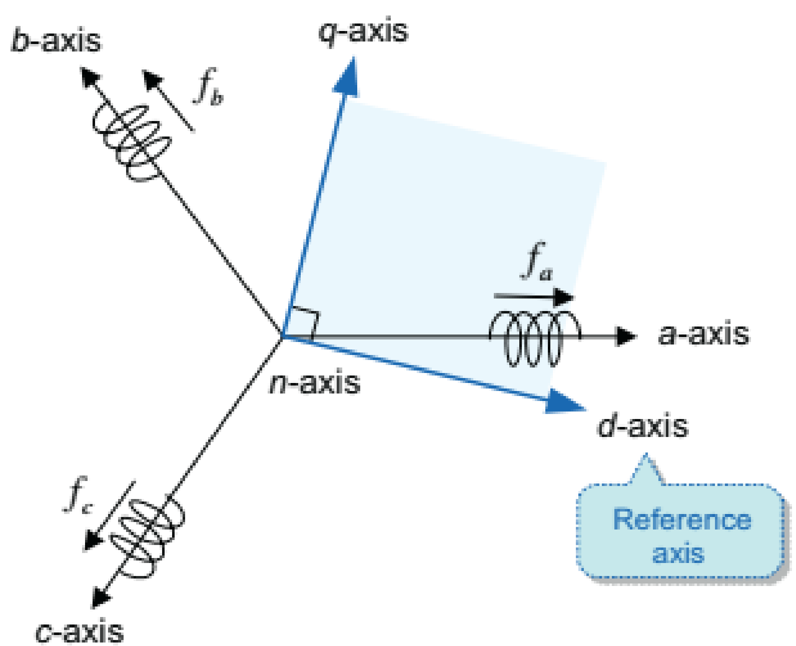
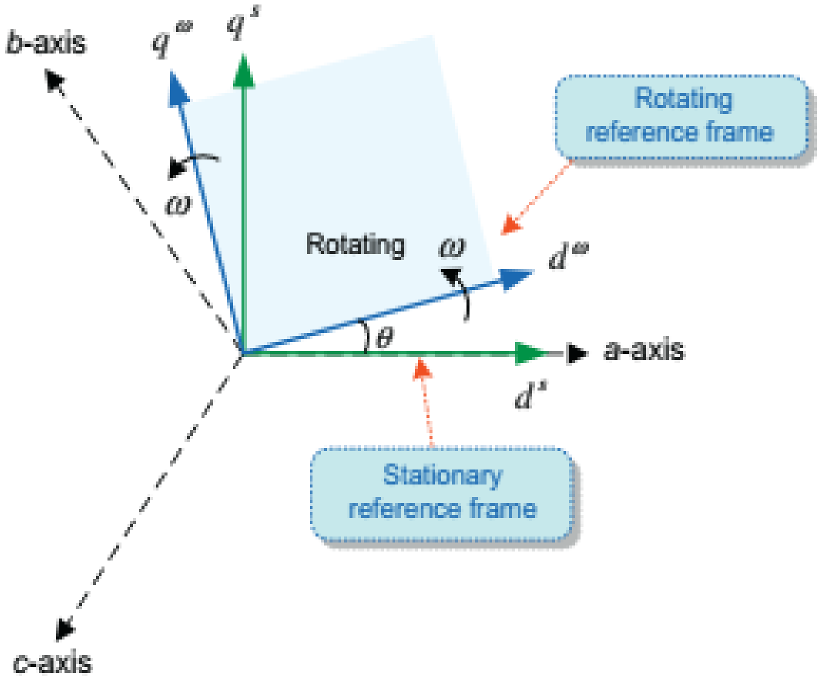


Figure *abc* coordinate and *dq* axes coordinate



$$\theta = \int \omega(\tau) d\tau + \theta(0)$$

Figure *d-q* axes stationary reference frame and rotating reference frame

- d-axis (direct axis)
- q-axis (quadrature axis)
- n-axis (neutral axis)

- ❑ *d-q* coordinate system rotates at a speed ω. ⟹ $d^{\omega} - q^{\omega}$ axes
- ❑ *Stationary frame* ω=0. ⟹ $d^{s} - q^{s}$ axes
- ❑ synchronously rotating reference frame or *synchronous reference* frame $\omega = \omega_e$ ⟹ $d^{e} - q^{e}$ axes
- ❑ *rotor reference* frame $\omega = \omega_r$ ⟹ $d^{r} - q^{r}$ axes

Reference frame transformation by matrix equation

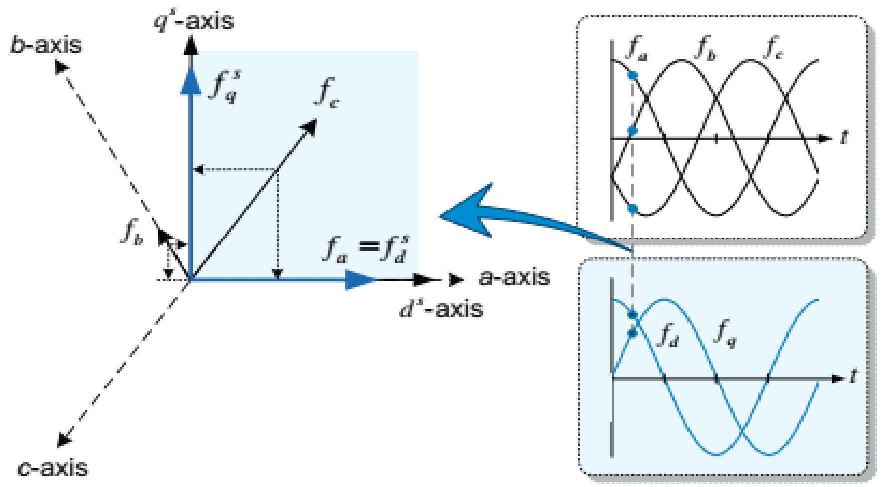


Figure Transformation into the stationary reference frame

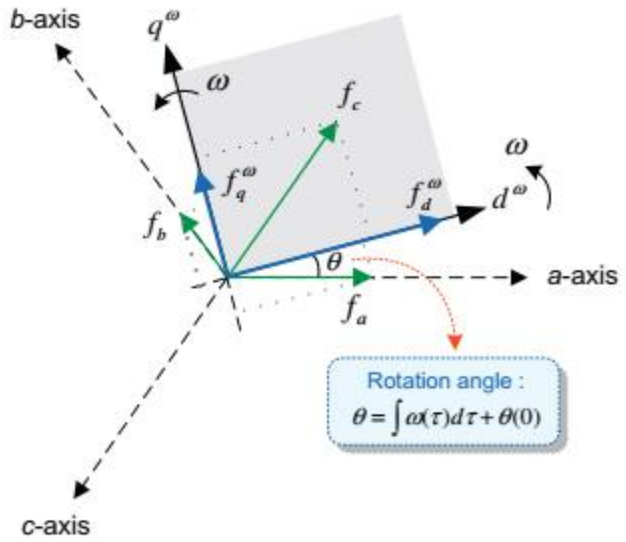


Figure Transformation into the arbitrary rotating reference frame

Transformation of *abc* variables into *dq(n)* variables in the stationary reference frame

$$f_d^s = \frac{2f_a - f_b - f_c}{3}$$

$$f_q^s = \frac{1}{\sqrt{3}}(f_b - f_c)$$

$$f_n^s = \frac{2(f_a + f_b + f_c)}{3}$$

Clarke's transformation

Inverse transformation

$$f_a = f_d^s$$

$$f_b = -\frac{1}{2}f_d^s + \frac{\sqrt{3}}{2}f_q^s$$

$$f_c = -\frac{1}{2}f_d^s - \frac{\sqrt{3}}{2}f_q^s$$

Inverse Clarke's transformation

Reference frame transformation by matrix equation

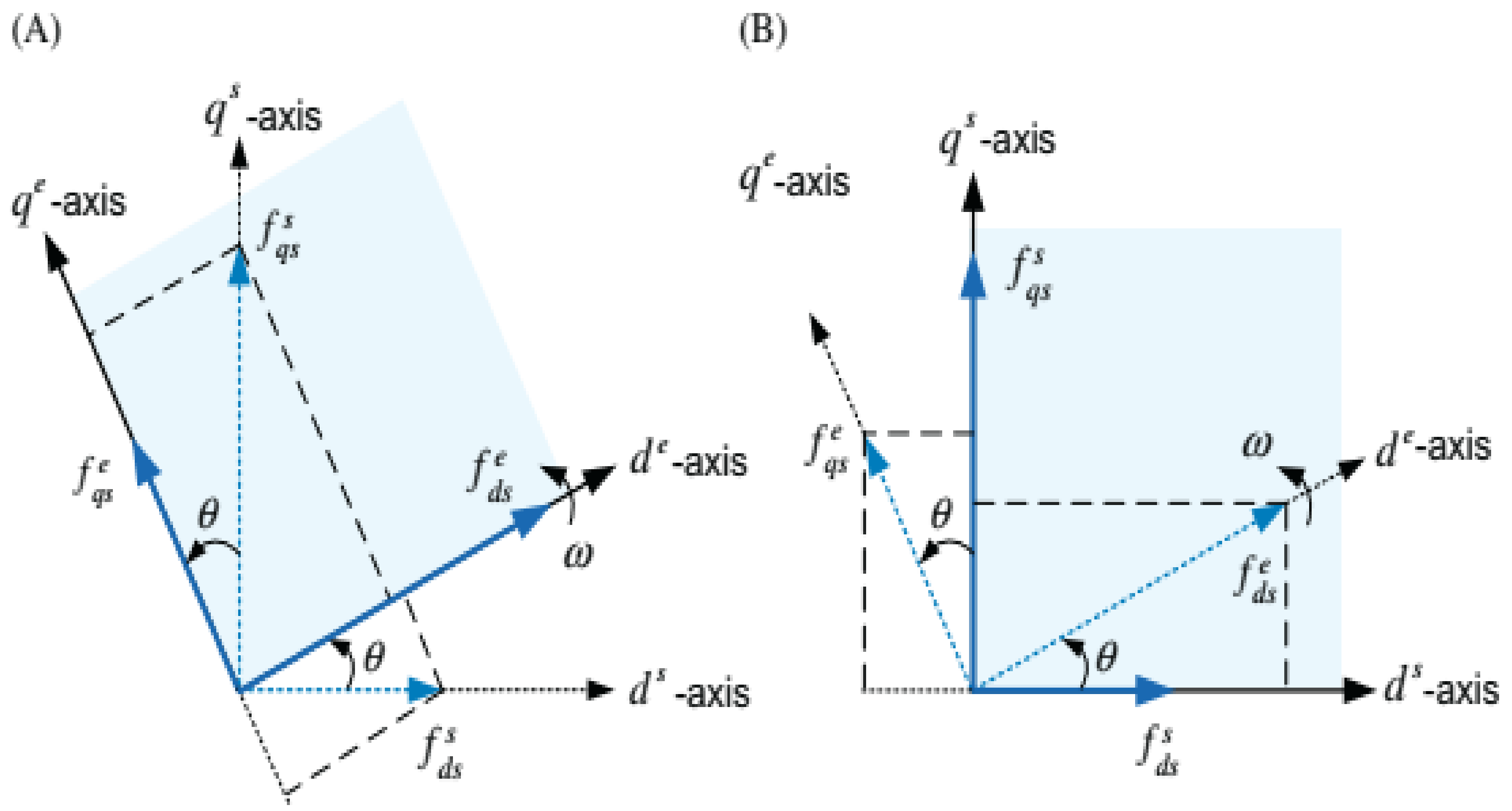


Figure Transformation between reference frames. (A) Stationary into rotating frame and (B) rotating into stationary frame

➢ Transformation of stationary reference frame into rotating reference frame

$$f_d^e = f_d^s \cos\theta + f_q^s \sin\theta$$

$$f_q^e = -f_d^s \sin\theta + f_q^s \cos\theta$$

Park's transformation

➢ Inverse transformation of rotating reference frame into stationary reference frame

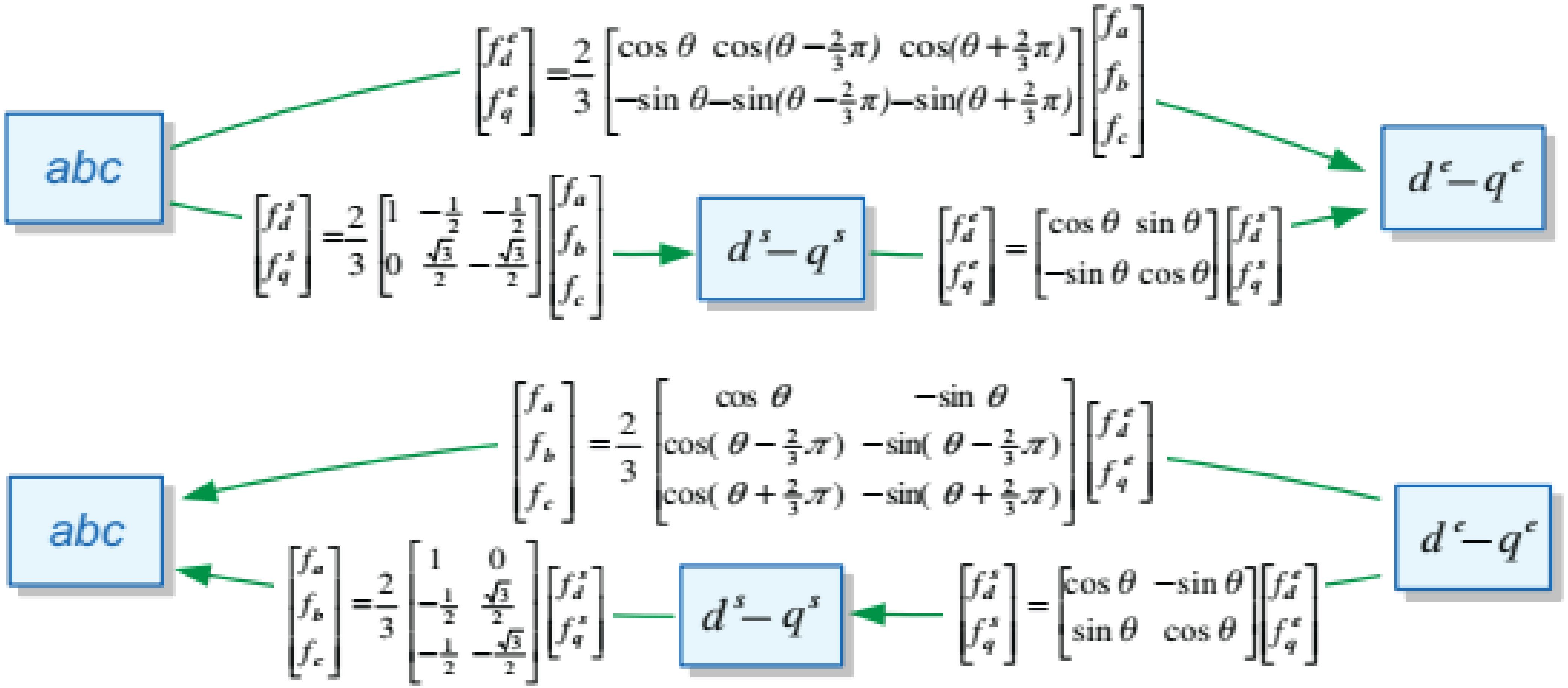
$$f_d^s = f_d^e \cos\theta - f_q^e \sin\theta$$

$$f_q^s = f_d^e \sin\theta + f_q^e \cos\theta$$

Inverse Park's transformation

Reference frame transformation by matrix equation

Attendance test #9



$$\begin{bmatrix} f_d^e \\ f_q^e \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos(\theta-\frac{2}{3}\pi) & \cos(\theta+\frac{2}{3}\pi) \\ -\sin\theta & -\sin(\theta-\frac{2}{3}\pi) & -\sin(\theta+\frac{2}{3}\pi) \end{bmatrix}\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

$$\begin{bmatrix} f_d^s \\ f_q^s \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix}$$

$$\begin{bmatrix} f_d^e \\ f_q^e \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} f_d^s \\ f_q^s \end{bmatrix}$$

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \cos\theta & -\sin\theta \\ \cos(\theta-\frac{2}{3}\pi) & -\sin(\theta-\frac{2}{3}\pi) \\ \cos(\theta+\frac{2}{3}\pi) & -\sin(\theta+\frac{2}{3}\pi) \end{bmatrix}\begin{bmatrix} f_d^e \\ f_q^e \end{bmatrix}$$

$$\begin{bmatrix} f_a \\ f_b \\ f_c \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} f_d^s \\ f_q^s \end{bmatrix}$$

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Figure Reference frame transformations

AC motors

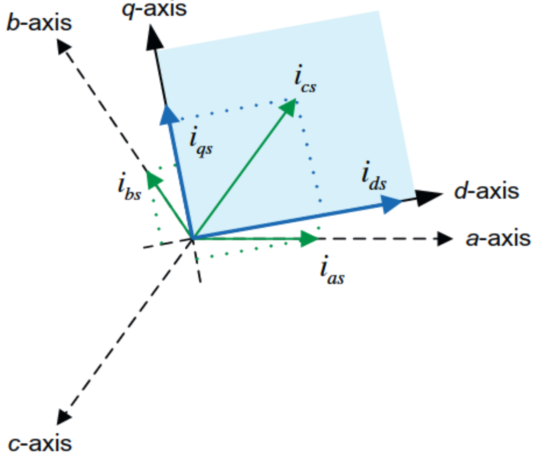


Figure Reference frame transformation for currents

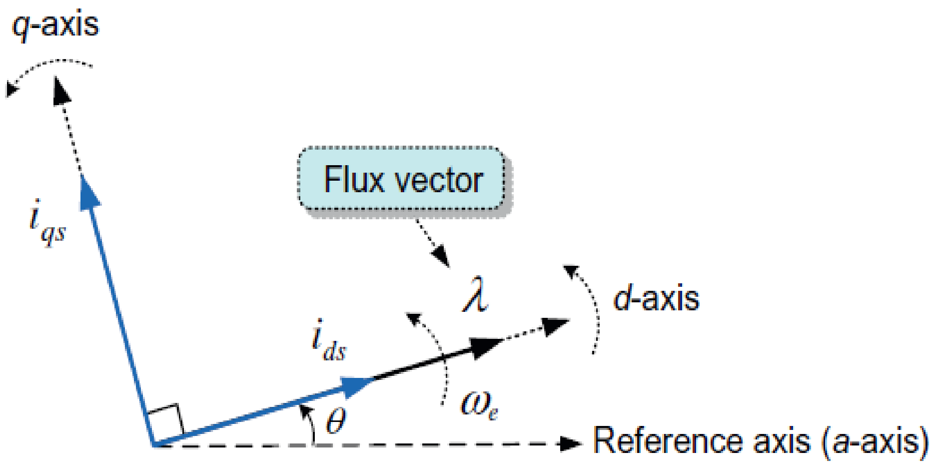
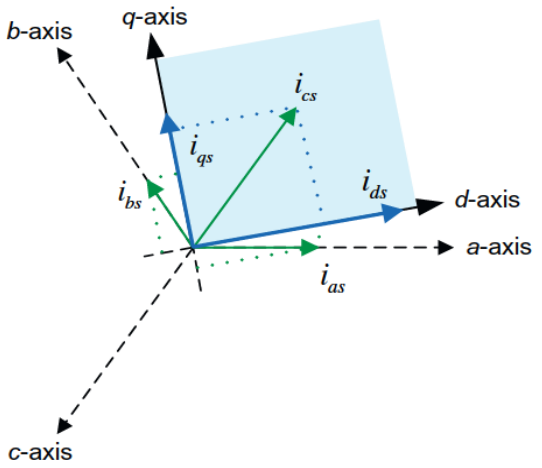


Figure . Assignment of d-q axes based on the flux vector λ

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\left[\lambda^*_{dqr} i_{dqs}\right]$$

AC motors



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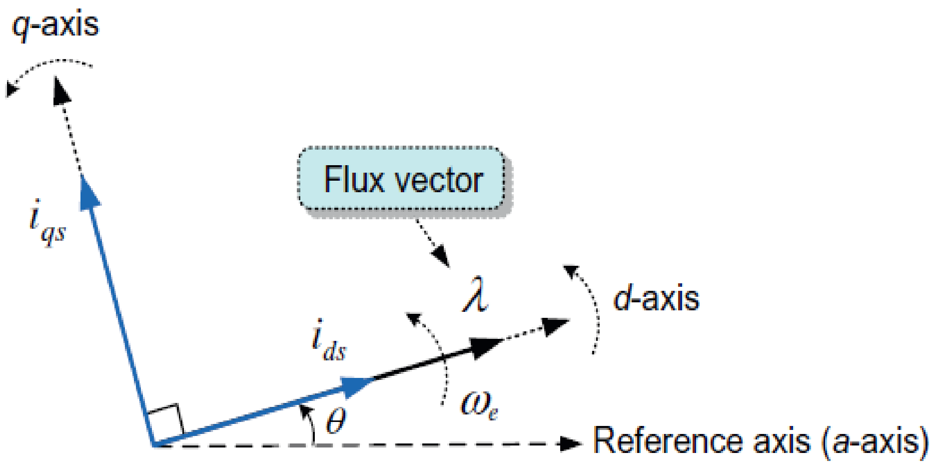
$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im[\lambda^*_{dqr} i_{dqs}]$$

$$f^e_{dq} = f^e_d + jf^e_q \Rightarrow \begin{array}{l} i^e_s = i^e_{ds} + ji^e_{qs} \\ \lambda^e_s = \lambda^e_{ds} + j\lambda^e_{qs} \end{array}$$



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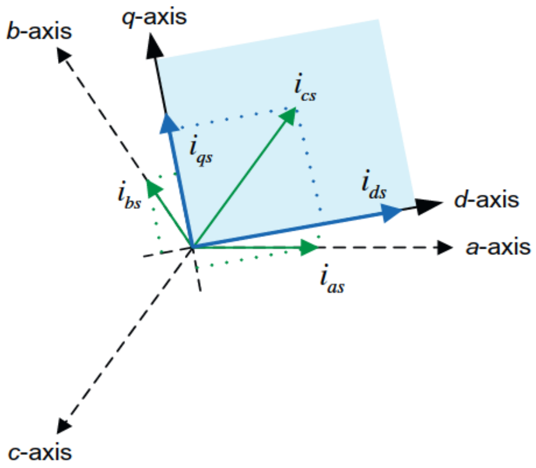

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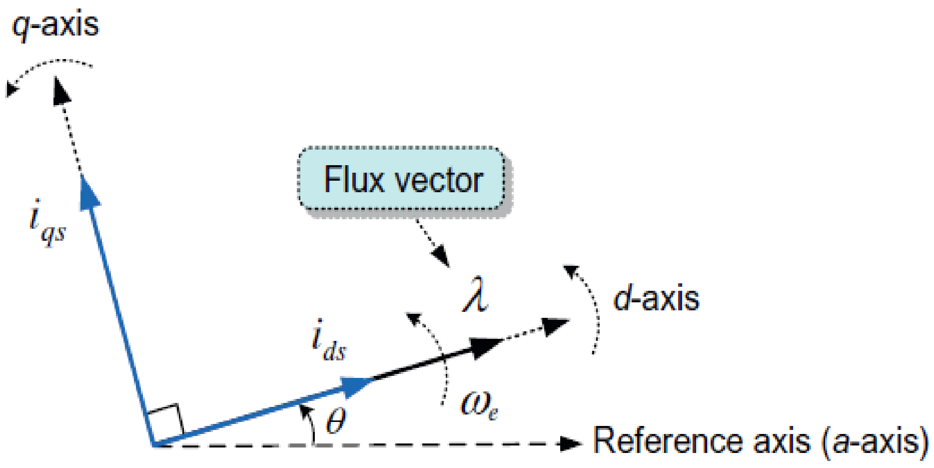

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$$\boxed{T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\left[\lambda_{dqr}^{*} i_{dqs}\right]}$$

$$f_{dq}^e = f_d^e + jf_q^e \quad \Rightarrow \quad \begin{aligned} i_s^e &= i_{ds}^e + ji_{qs}^e \\ \lambda_s^e &= \lambda_{ds}^e + j\lambda_{qs}^e \end{aligned}$$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}\left(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e\right) = k|\lambda| i_{qs}^e$$

$$\left(\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0, k = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}\right)$$

$$T_e = k' i_{qs}^e \;\; (k' = k|\lambda|)$$

AC motors

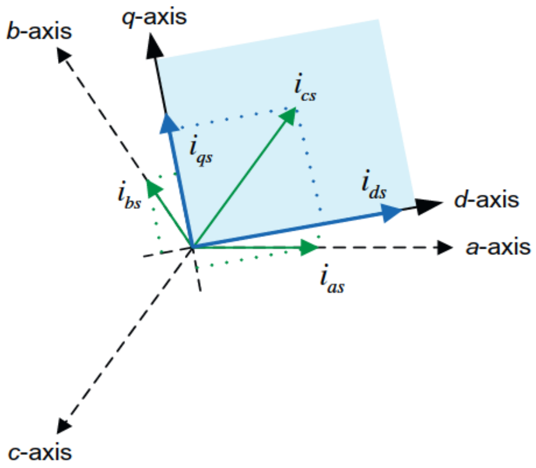


Figure   Reference frame transformation for currents

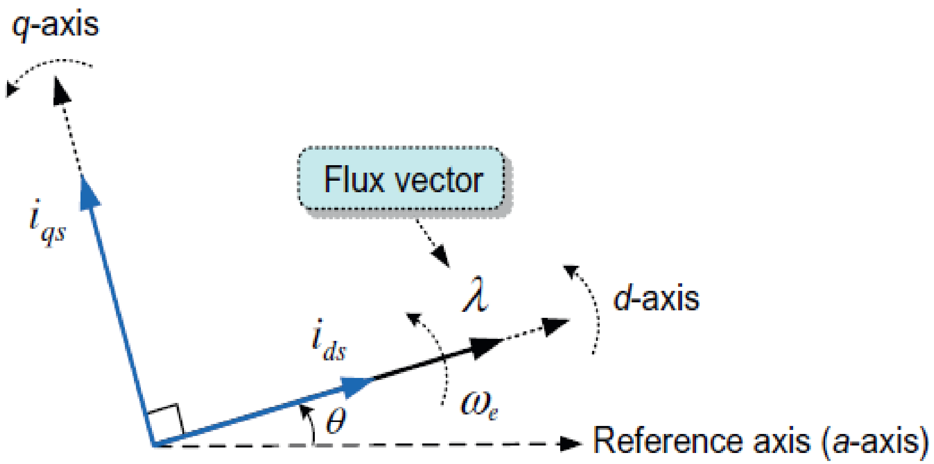


Figure . Assignment of d-q axes based on the flux vector λ

$$\boxed{T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im[\lambda^*_{dqr} i_{dqs}]}$$

$$f^e_{dq} = f^e_d + jf^e_q \Rightarrow \begin{array}{l} i^e_s = i^e_{ds} + ji^e_{qs} \\ \lambda^e_s = \lambda^e_{ds} + j\lambda^e_{qs} \end{array}$$

$$T_e = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}(\lambda^e_{dr} i^e_{qs} - \lambda^e_{qr} i^e_{ds}) = k|\lambda| i^e_{qs}$$

$$\left(\lambda^e_{dr} = |\lambda|, \quad \lambda^e_{qr} = 0, k = \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r}\right)$$

$$T_e = k' i^e_{qs} \ \ (k' = k|\lambda|)$$

- the *d*-axis stator current is assigned as the flux-producing current
- the *q*-axis stator current is assigned as the torque-producing current.

Instantaneous torque control of an induction motor

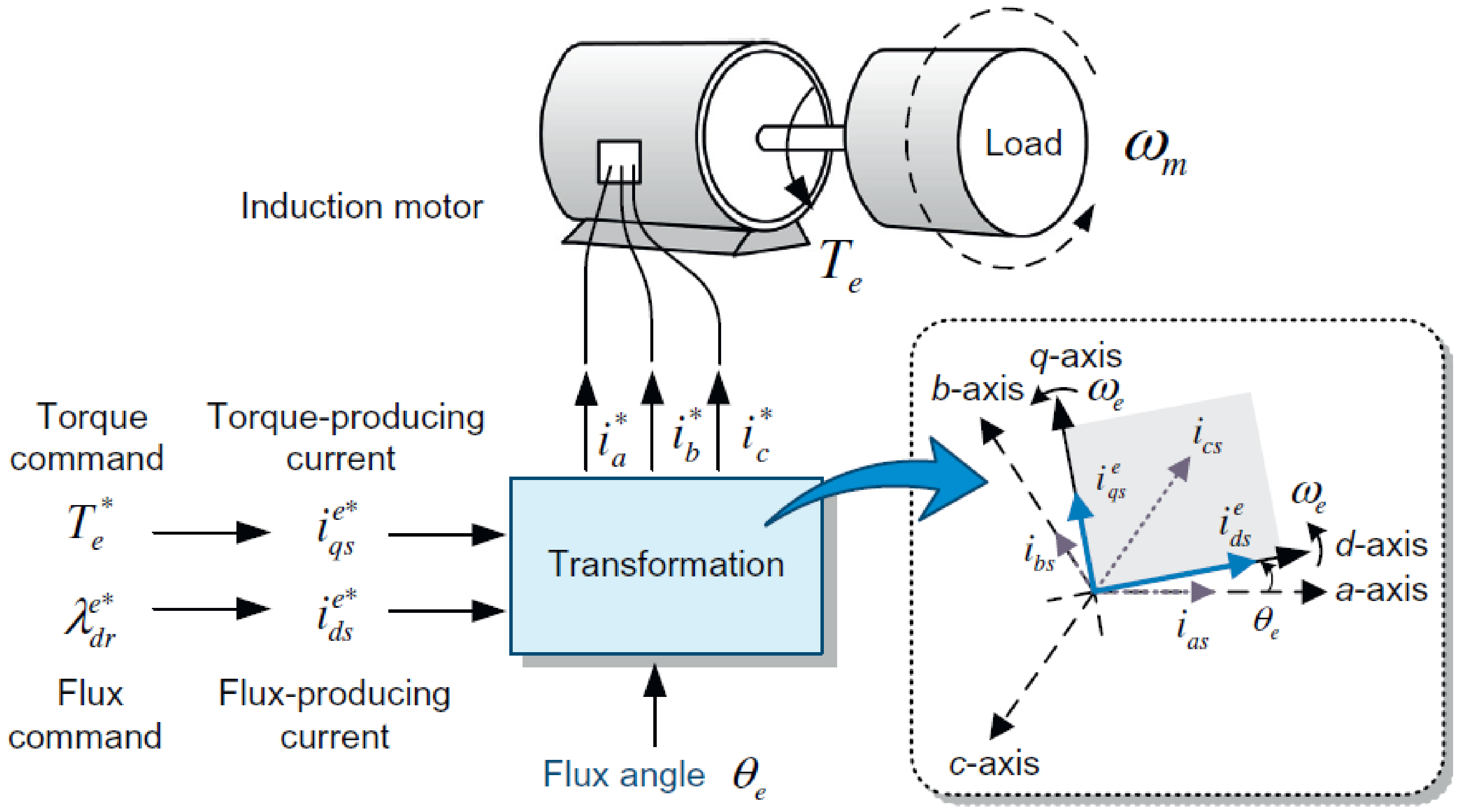


Figure Instantaneous torque control method of an induction motor

- Based on the rated flux command λ_{ds}^{e*}, the required flux-producing current command i_{ds}^{e*} is calculated.
- On the other hand, the torque-producing current command i_{qs}^{e*} is calculated based on the output torque command T_e^* required for driving a given load.
- These command currents i_{ds}^{e*} and i_{qs}^{e*} are given in the synchronous rotating reference frame. Thus we need to obtain the three-phase stator current commands through the inverse transformation into these d— q current commands

Here, it should be noted that this transformation requires the knowledge of the angular position of the flux vector, so called the *flux angle*.

Two-pole, three-phase, wye-connected symmetrical induction motor

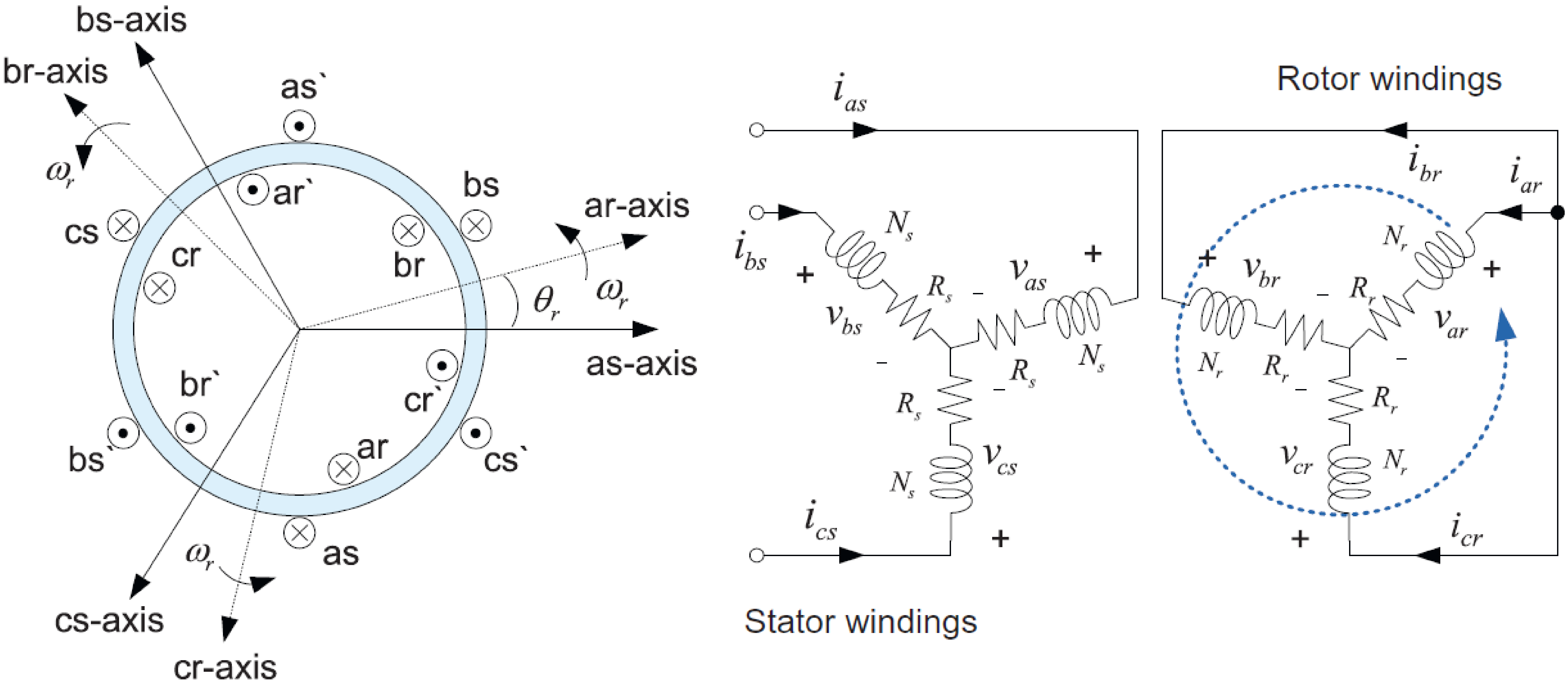


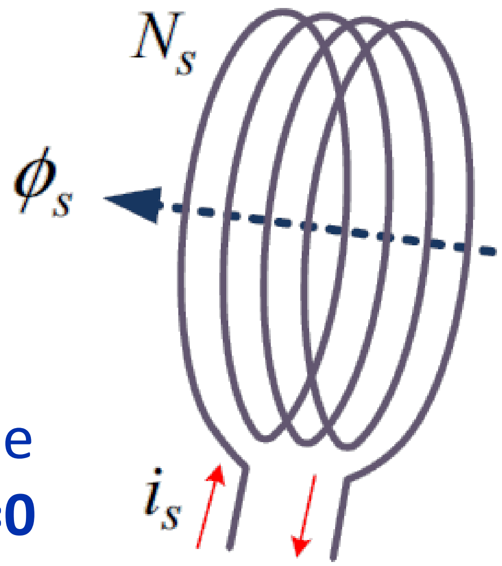
Figure Two-pole, three-phase, wye-connected symmetrical induction motor

Rotating machine: Self-inductance and mutual inductance

The inductance L of a coil is defined as the flux linkage per ampere of current in the coil

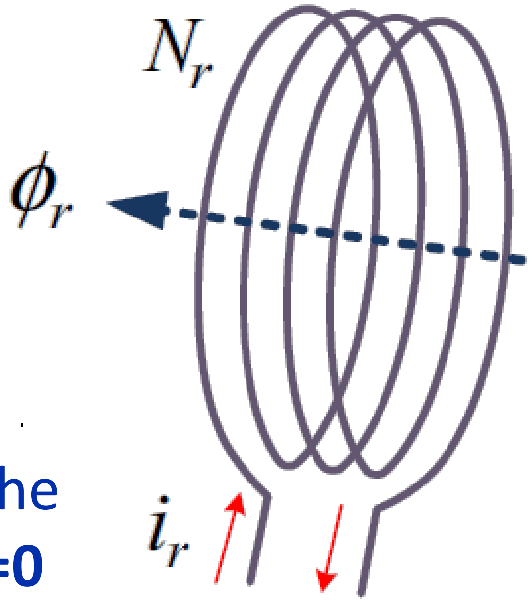
- Self-inductance: Self-inductance is the flux linkage produced in the winding by the current in that same winding divided by that current.

$$L_{ss} = \frac{\lambda_s}{i_s} = \frac{N_s\,\phi_s}{i_s}$$



Self-inductance of the stator's winding at $\boldsymbol{i_r}$**=0**

$$L_{rr} = \frac{N_r\,\phi_r}{i_r}$$



Self-inductance of the rotor's winding at $\boldsymbol{i_s}$**=0**

ϕ is the *magnetizing flux*

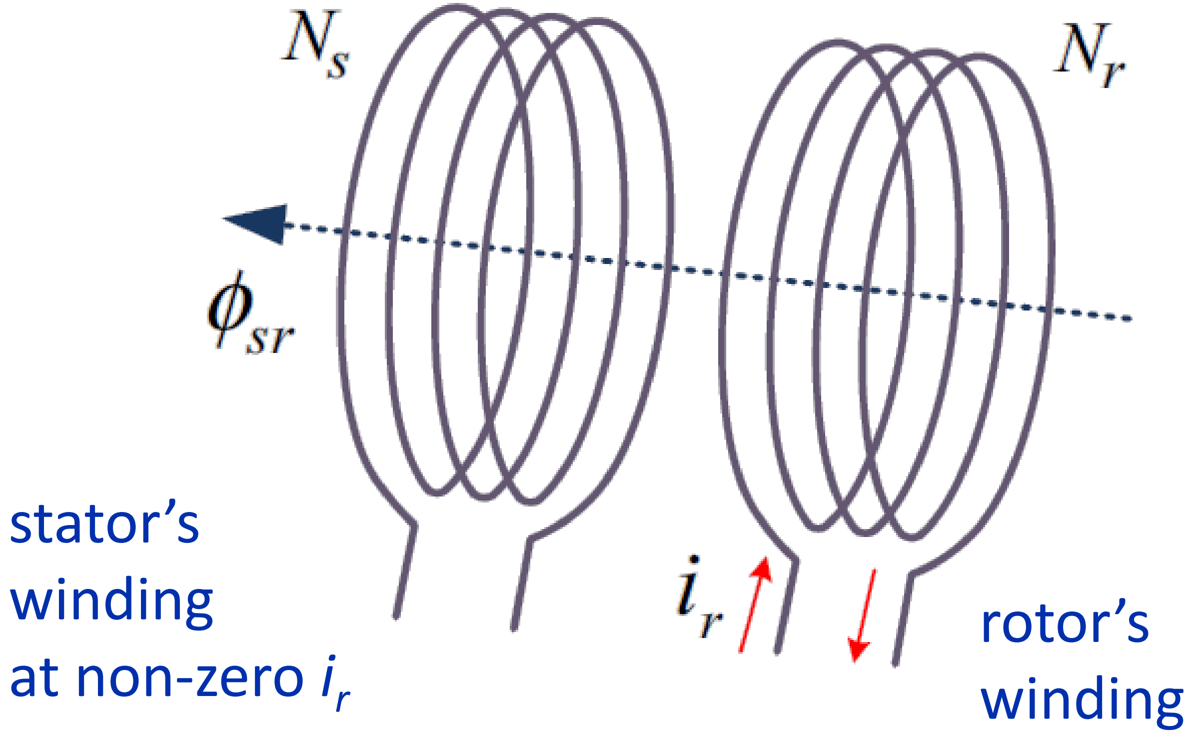
N_S, N_S - number of coils' turns

Rotating machine: Self-inductance and mutual inductance

Mutual-inductance: Mutual-inductance is the flux linkage produced in one winding by the current in the other winding divided by that current

$$L_{sr} = \frac{N_s \phi_{sr}}{i_r}, \qquad L_{rs} = \frac{N_r \phi_{rs}}{i_s}$$

$$L_{sr} = L_{rs} = L_m$$



winding axes match

AC motors

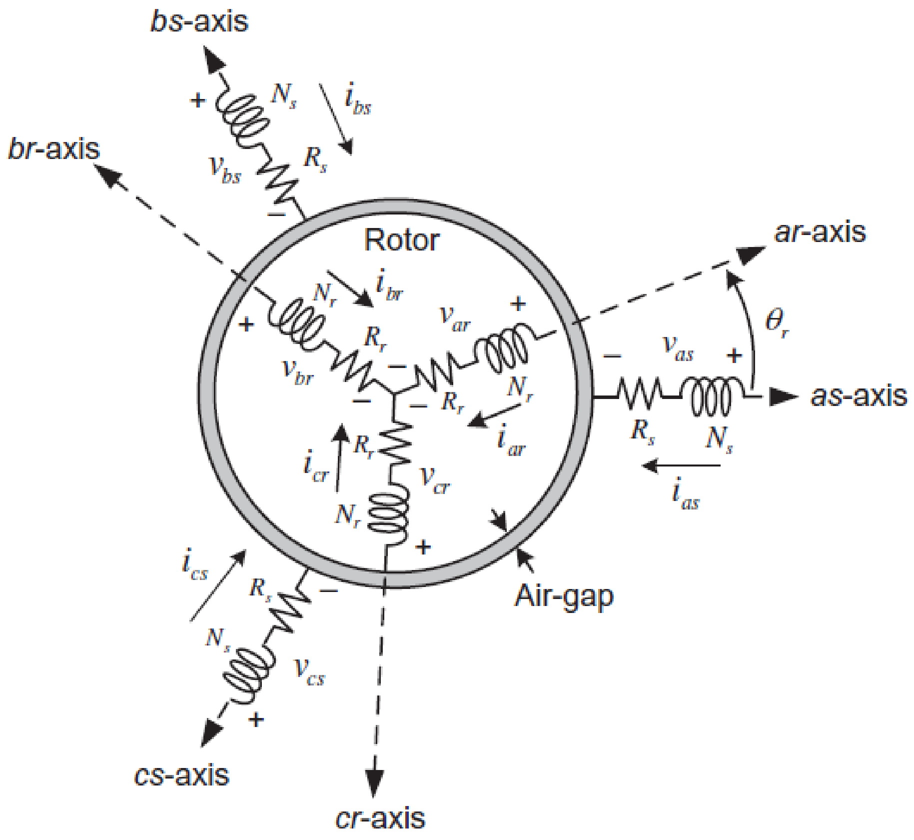


Figure. Stator and rotor windings of an induction motor

$$v_{as} = R_s i_{as} + \frac{d\lambda_{as}}{dt} \qquad v'_{ar} = R' i'_{ar} + \frac{d\lambda'_{ar}}{dt}$$

$$v_{bs} = R_s i_{bs} + \frac{d\lambda_{bs}}{dt} \qquad v'_{br} = R' i'_{br} + \frac{d\lambda'_{br}}{dt}$$

$$v_{cs} = R_s i_{cs} + \frac{d\lambda_{cs}}{dt} \qquad v'_{cr} = R' i'_{cr} + \frac{d\lambda'_{cr}}{dt}$$

v_{as}, v_{bs}, v_{cs} - the stator voltages

i_{as}, i_{bs}, i_{cs} - the stator currents

$v'_{ar}, v'_{br}, v'_{cr}$ - the rotor voltages

$i'_{ar}, i'_{br}, i'_{cr}$ - the rotor currents

$\lambda_{as}, \lambda_{bs}, \lambda_{cs}$ - the stator flux linkages

$\lambda_{ar}, \lambda_{br}, \lambda_{cr}$ - the rotor flux linkages

AC motors

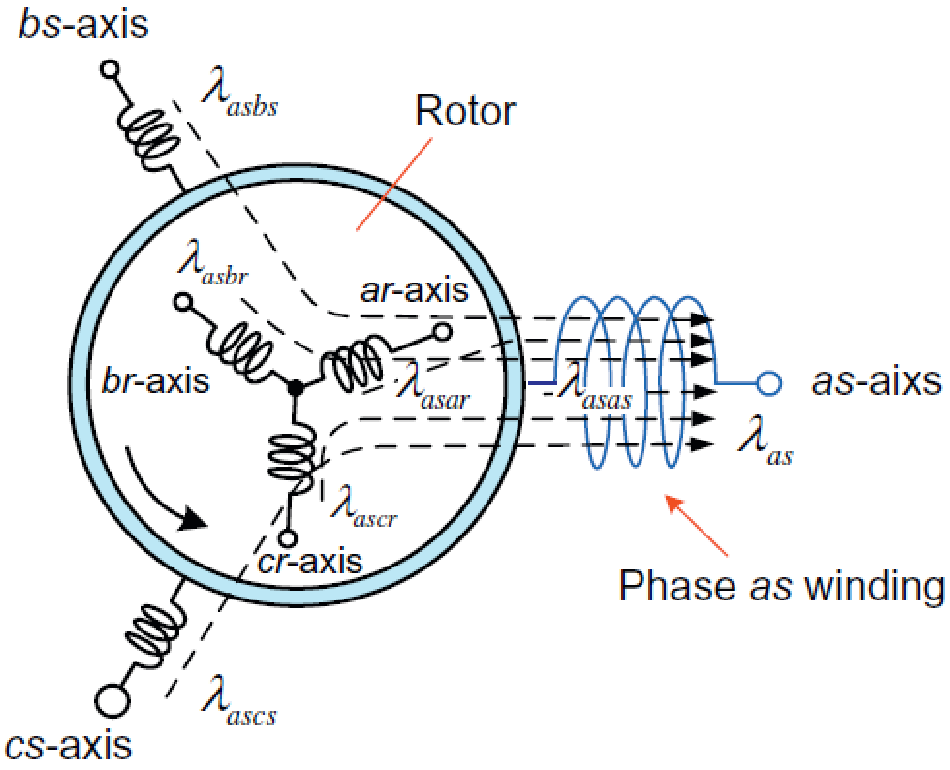


Figure. Flux linkage λ_{as} of the phase as winding

For example, the total flux linkage λ_{as} of the phase *as* windin

$$\lambda_{as} = \lambda_{asas} + \lambda_{asbs} + \lambda_{ascs} + \lambda_{asar} + \lambda_{asbr} + \lambda_{ascr} =$$

$$= L_{asas}\, i_{as} + L_{asbs} i_{bs} + L_{ascs} i_{cs} + L_{asar} i_{ar} + L_{asbr} i_{br} + L_{ascr} i_{cr}$$

We can express all flux linkages and all currents of the phases using common vectors as

$$\begin{bmatrix} \lambda_{abcs} \\ \lambda_{abcr} \end{bmatrix} = \begin{bmatrix} L_s & L_{sr} \\ L_{rs} & L_r \end{bmatrix} \begin{bmatrix} i_{abcs} \\ i_{abcr} \end{bmatrix}$$

Stator windings

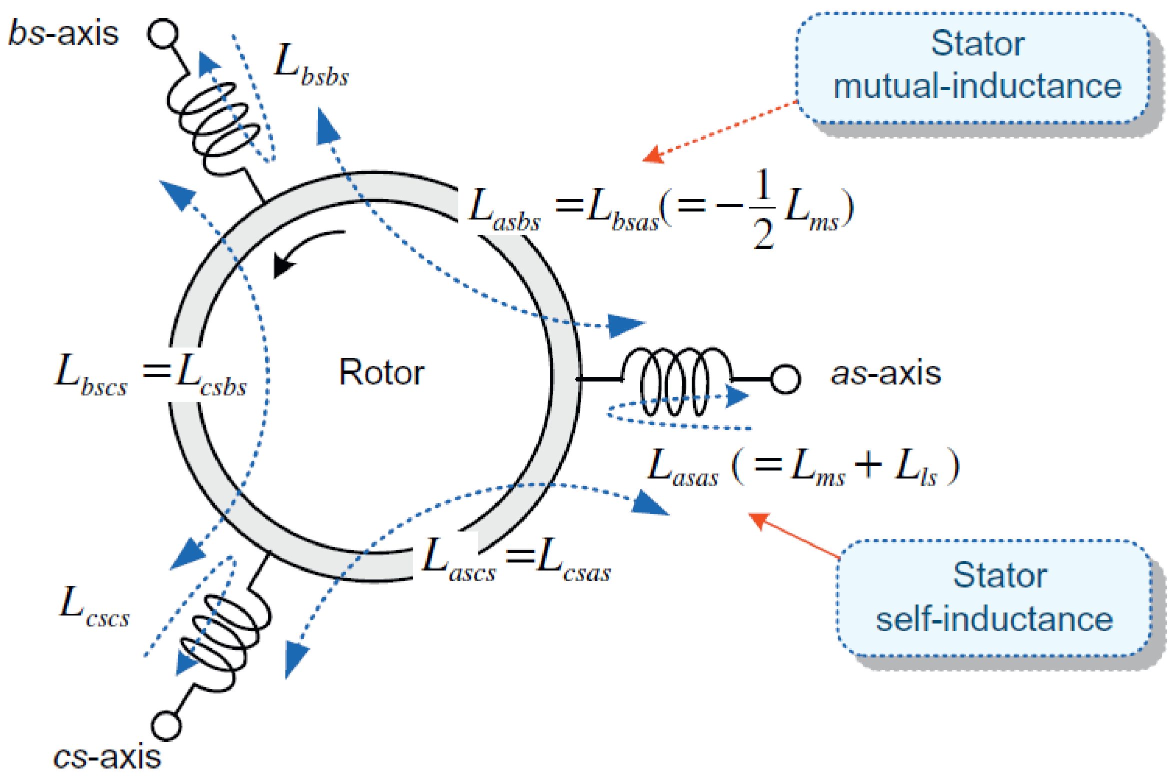


Figure. Stator inductances

the stator self-inductances $L_{asas}, L_{bsbs}, L_{cscs}$ consist of the leakage inductance L_{ls} and the magnetizing inductance L_{ms} as

$$L_{asas} = L_{bsbs} = L_{cscs} = L_{ls} + L_{ms}$$

where $L_{ms} = \mu_o N_s^2 (rl/g)(\pi/4)$, μ_o is the permeability of air, l is the axial length of the air gap and r is the radius to the mean of the air gap

The mutual-inductances $L_{asbs}, L_{ascs}, L_{bsas}, L_{bscs}, L_{csas}, L_{csbs}$ between the two stator windings, which are displaced from each other by 120°, are all the same and are related to the magnetizing inductance as

$$L_{asbs} = L_{ascs} = L_{bsas} = L_{bscs} = L_{csas} = L_{csbs} = L_{ms} \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} L_{ms}$$

Rotor windings

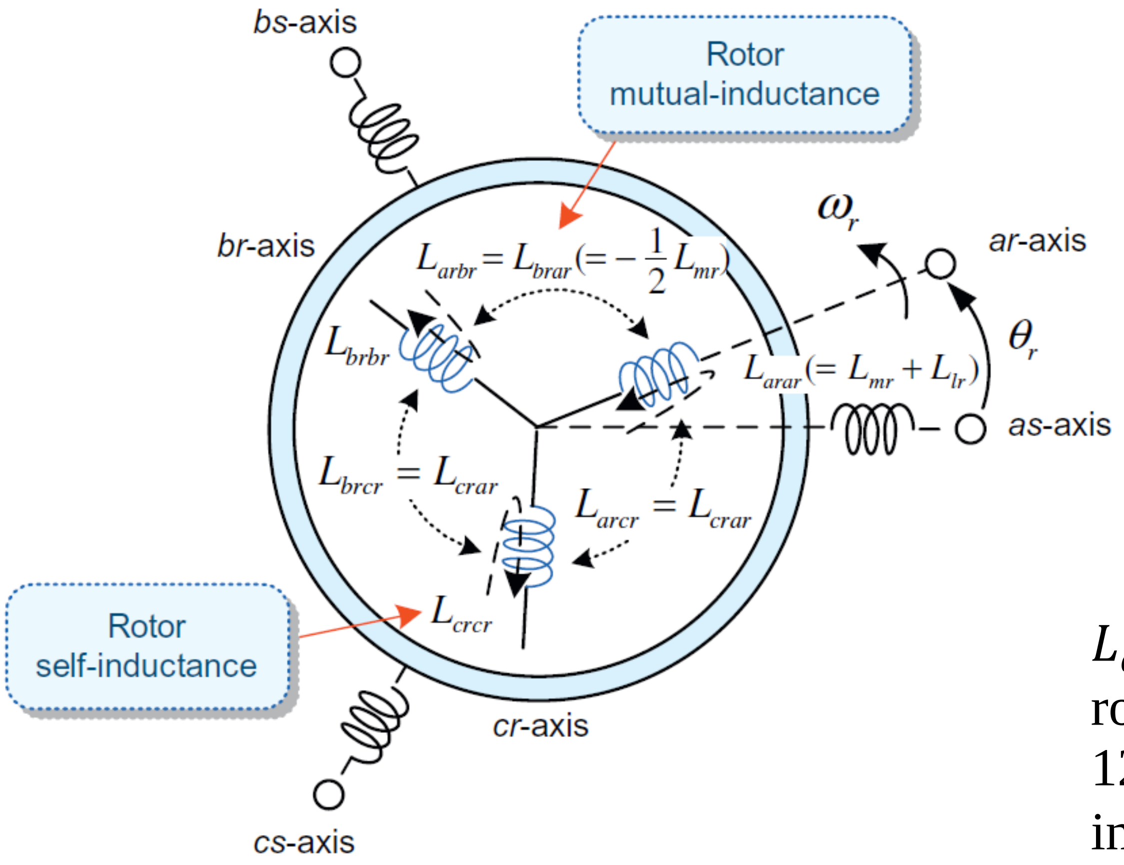


Figure. Inductances of the rotor windings

the rotor self-inductances $L_{arar}, L_{brbr}, L_{crcr}$ consist of the leakage inductance L_{lr} and the magnetizing inductance L_{mr}

$$L_{arar} = L_{brbr} = L_{crcr} = L_{lr} + L_{mr}$$

where $L_{mr} = \mu_o N_s^2 \left(\frac{rl}{g}\right)\left(\frac{\pi}{4}\right) = (\frac{N_r}{N_s})^2 L_{ms} = (n)^2 L_{ms}$

The mutual-inductances $L_{arbr}, L_{arcr}, L_{brar}, L_{brcr}, L_{crar}, L_{crbr}$ between the two rotor windings, which are displaced from each other by 120°, are all the same and related to the magnetizing inductance as

$$L_{arbr} = L_{arcr} = L_{brar} = L_{brcr} = L_{crar} = L_{crbr}$$

$$= L_{mr} \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} L_{mr} = -\frac{1}{2}\left(\frac{N_r}{N_s}\right)^2 L_{ms}$$

Inductance between the stator and rotor windings

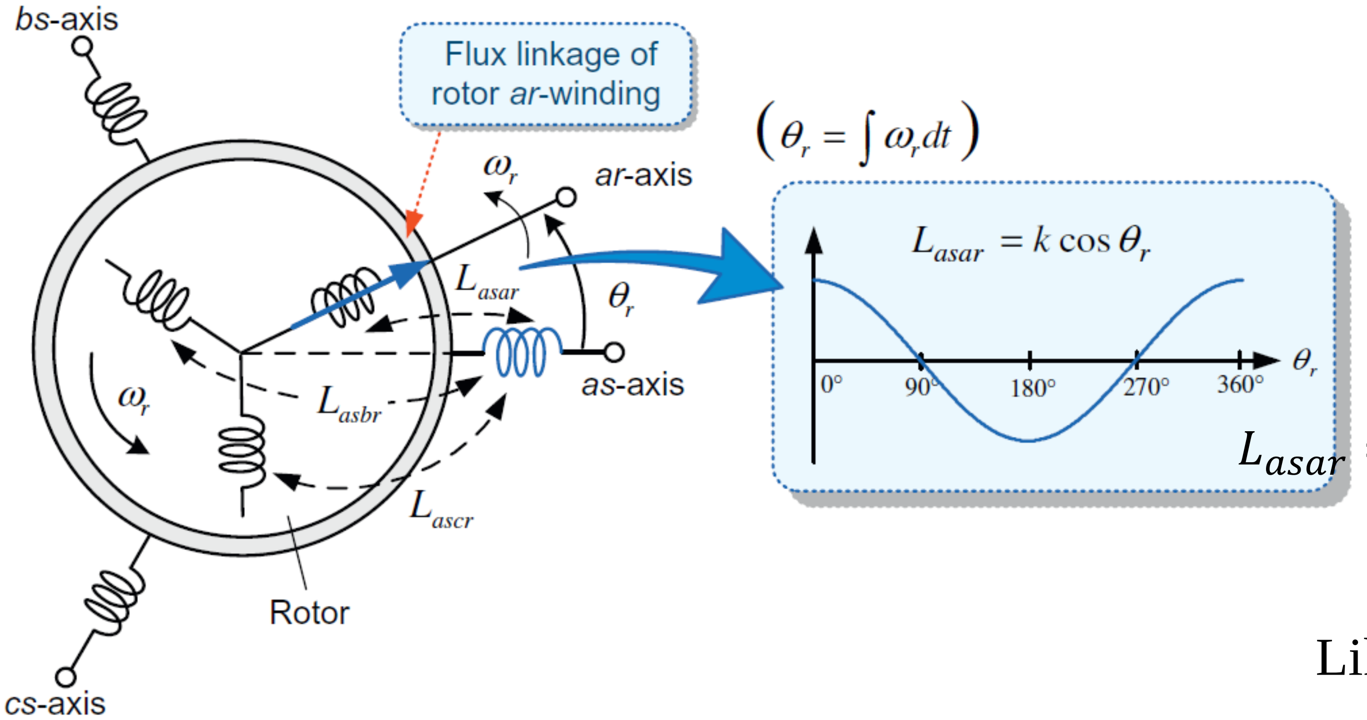


Figure. Mutual-inductance between the stator as winding and the rotor ar winding

As an example, examine the mutual-inductance L_{asar}, which represents the ratio of the flux linking in the stator *as* winding to the rotor *ar* winding current generating the flux

$$L_{asar} = L_{mr}\left(\frac{N_s}{N_r}\right)cos\theta_r = L_{ms}\left(\frac{N_r}{N_s}\right)cos\theta_r \quad \left(\theta_r = \int \omega_r dt\right)$$

Likewise, the other mutual-inductances are given by

$$L_{asar} = L_{bsbr} = L_{cscr} = \left(\frac{N_r}{N_s}\right)L_{ms}cos\theta_r$$

$$L_{asbr} = L_{bscr} = L_{csar} = \left(\frac{N_r}{N_s}\right)L_{ms}cos(\theta_r + \frac{2\pi}{3})$$

$$L_{ascr} = L_{bsar} = L_{csbr} = \left(\frac{N_r}{N_s}\right)L_{ms}cos(\theta_r - \frac{2\pi}{3})$$

flux linkage of the stator windings

The flux linkage of the stator windings is given by

$$\lambda_{abcs} = L_s i_{abcs} + L_{sr} i_{abcr} = \begin{bmatrix} L_{ls} + L_{ms} & -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & L_{ls} + L_{ms} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} & L_{ls} + L_{ms} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

$$+ nL_{ms} \begin{bmatrix} cos\theta_r & cos\left(\theta_r + \frac{2\pi}{3}\right) & cos\left(\theta_r - \frac{2\pi}{3}\right) \\ cos\left(\theta_r - \frac{2\pi}{3}\right) & cos\theta_r & cos\left(\theta_r + \frac{2\pi}{3}\right) \\ cos\left(\theta_r + \frac{2\pi}{3}\right) & cos\left(\theta_r - \frac{2\pi}{3}\right) & cos\theta_r \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

flux linkage of the stator windings

The flux linkage of the stator windings is given by

$$\lambda_{abcs} = L_s i_{abcs} + L_{sr} i_{abcr} = \begin{bmatrix} L_{ls}+L_{ms} & -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & L_{ls}+L_{ms} & -\frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & -\frac{L_{ms}}{2} & L_{ls}+L_{ms} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

$$+nL_{ms} \begin{bmatrix} cos\theta_r & cos(\theta_r+\frac{2\pi}{3}) & cos(\theta_r-\frac{2\pi}{3}) \\ cos(\theta_r-\frac{2\pi}{3}) & cos\theta_r & cos(\theta_r+\frac{2\pi}{3}) \\ cos(\theta_r+\frac{2\pi}{3}) & cos(\theta_r-\frac{2\pi}{3}) & cos\theta_r \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

Expression for common vectors of stator flux linkage and common vectors of stator and rotor currents

SIMPLE

Expression for common vectors of stator flux linkage and the stator and rotor currents <u>of separate phase windings</u>

COMPLEX

flux linkage of the stator windings

The flux linkage of the rotor windings is given by

$$\lambda_{abcr} = L_r i_{abcr} + L_{rs} i_{abcs} = \begin{bmatrix} L_{ls} + n^2 L_{ms} & -n^2 \frac{L_{ms}}{2} & -n^2 \frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & L_{ls} + n^2 L_{ms} & -n^2 \frac{L_{ms}}{2} \\ -n^2 \frac{L_{ms}}{2} & -n^2 \frac{L_{ms}}{2} & L_{ls} + n^2 L_{ms} \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

$$+ n L_{ms} \begin{bmatrix} cos\theta_r & cos\left(\theta_r - \frac{2\pi}{3}\right) & cos\left(\theta_r + \frac{2\pi}{3}\right) \\ cos\left(\theta_r + \frac{2\pi}{3}\right) & cos\theta_r & cos\left(\theta_r - \frac{2\pi}{3}\right) \\ cos\left(\theta_r - \frac{2\pi}{3}\right) & cos\left(\theta_r + \frac{2\pi}{3}\right) & cos\theta_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

flux linkage of the stator windings

The flux linkage of the rotor windings is given by

$$\lambda_{abcr} = L_r i_{abcr} + L_{rs} i_{abcs} = \begin{bmatrix} L_{ls} + n^2 L_{ms} & -n^2 \frac{L_{ms}}{2} & -n^2 \frac{L_{ms}}{2} \\ -\frac{L_{ms}}{2} & L_{ls} + n^2 L_{ms} & -n^2 \frac{L_{ms}}{2} \\ -n^2 \frac{L_{ms}}{2} & -n^2 \frac{L_{ms}}{2} & L_{ls} + n^2 L_{ms} \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

$$+ n L_{ms} \begin{bmatrix} cos\theta_r & cos(\theta_r - \frac{2\pi}{3}) & cos(\theta_r + \frac{2\pi}{3}) \\ cos(\theta_r + \frac{2\pi}{3}) & cos\theta_r & cos(\theta_r - \frac{2\pi}{3}) \\ cos(\theta_r - \frac{2\pi}{3}) & cos(\theta_r + \frac{2\pi}{3}) & cos\theta_r \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix}$$

Expression for common vectors of rotor flux linkage and common vectors of stator and rotor currents

SIMPLE

Expression for common vectors of rotor flux linkage and the stator and rotor currents of separate phase windings

COMPLEX

Thank you for your attention