

Actuators

Mechanics of the actuators

Lecture

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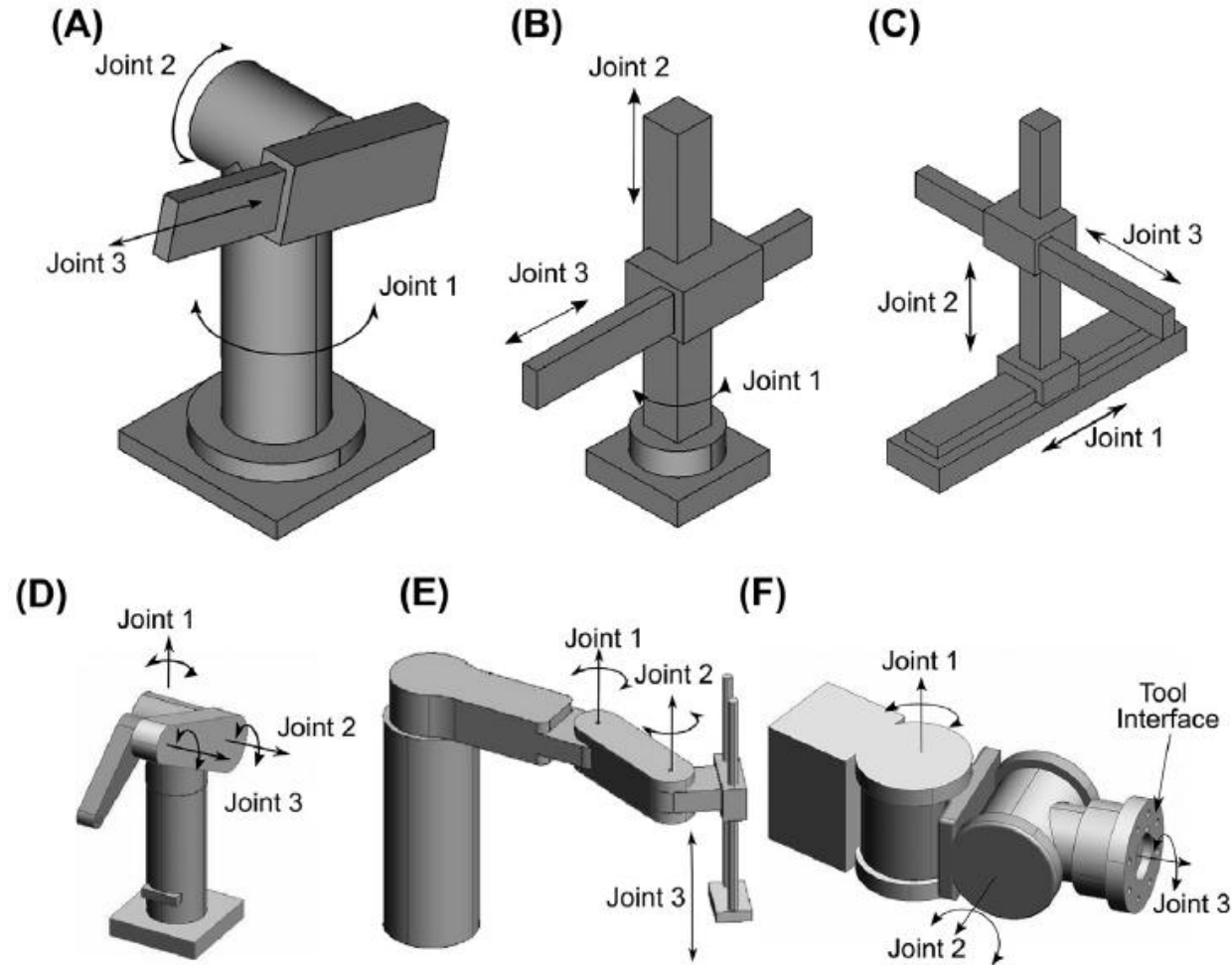
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Introduction

Example of application rotary and linear actuators



☐ Rotary motion

☐ Linear motion

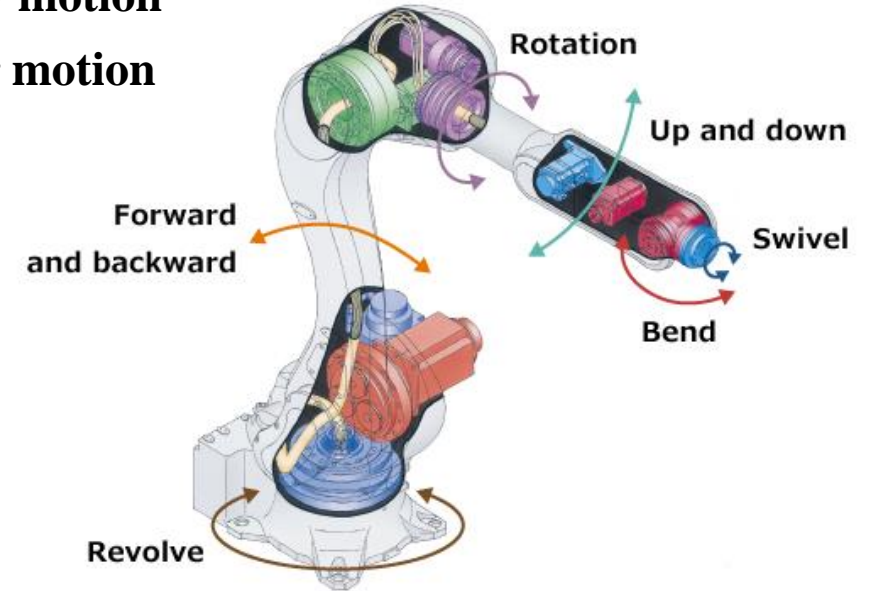


Figure. The standard configurations of industrial manipulators (A) to (E) and wrist (F) using only three joints: (A) Polar. (B) Cylindrical. (C) Cartesian and Gantry. (D) Jointed Arm. (E) SCARA. (F) Wrist.

In engineering, **actuating devices** are converters that convert an input signal (electrical, optical, mechanical, pneumatic, etc.) into an output signal (usually in motion), acting on the control object (working body).

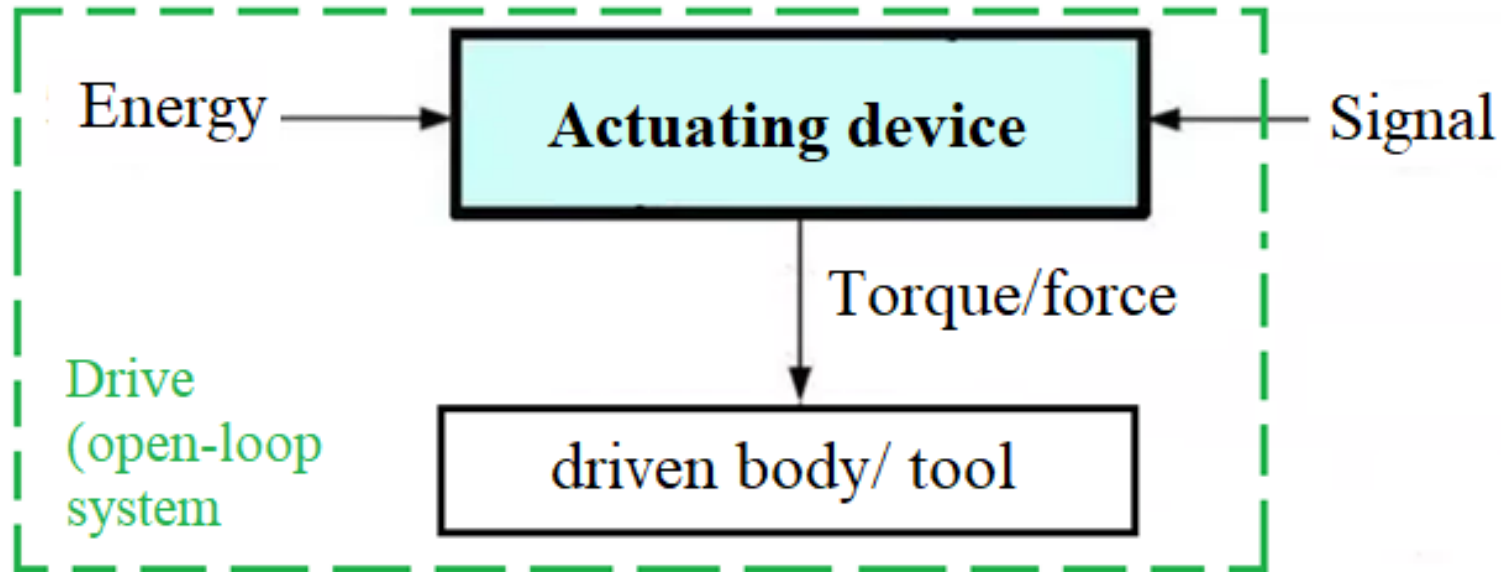
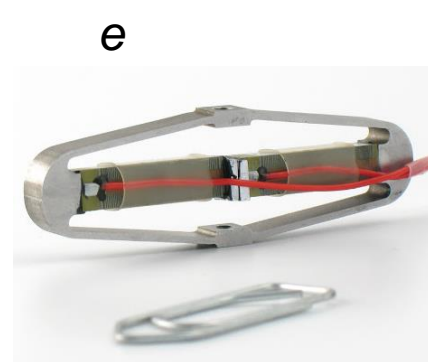
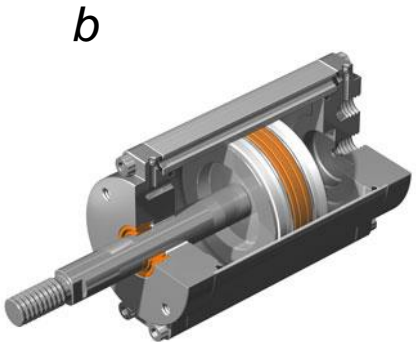
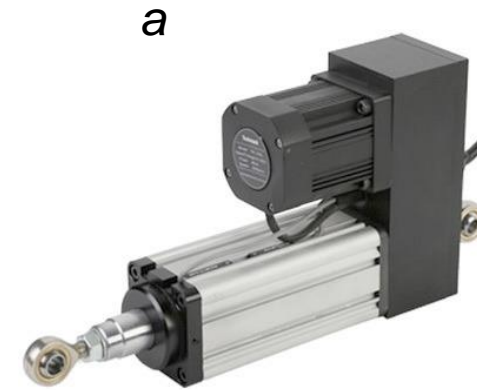


Figure. Open loop drive system

- ☐ Rotary motion
- ☐ Linear motion

Devices of this type include:

- electromechanical devices (motor + gearbox);
- pneumatic and hydraulic devices;
- piezo devices;
- electromagnetic (solenoid) devices, etc.



☐ Rotary motion

☐ Linear motion

Figure. Electromechanical device (a), Pneumatically and hydraulic powered actuators (b,c), electromagnetic device (d), piezo device (e)

Actuator – what is it?

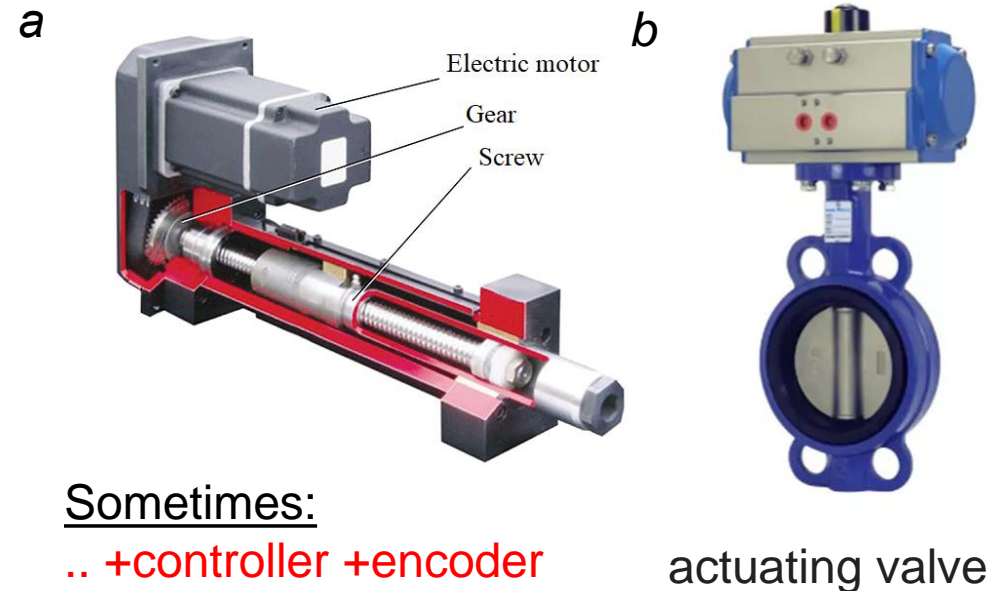
An actuator (actuator) is an actuating device that allows you to perform rotary or linear movements, acting on the control object, by changing the flow of energy or materials.

In common case: Actuator = Motor + Gearbox + Tool

Additional features:

- generally, the term "actuator" refers to devices that provide linear motion (linear actuator);
- The "actuator" is not intended for constant movement / rotation of the shaft, but for precise positioning;
- most rotating "actuators" install "slowly" and do not rotate more than 90 degrees;
- some "actuators" are designed to be in only two positions (open-closed)

Widely used in power generation, machine tools, home appliances, industry and other industries (actuating valves, pumps, motors and switches).



Sometimes:

.. +controller +encoder

Figure. Examples of: actuator (a) and actuator-based device (b)

Dexterous artificial hand



Figure - Model
Smart Motor
Hand (C6M)

Dexterous artificial hands have been developed using *electric*, *pneumatic* or *hydraulic* actuators.

Actuator requirements:

- compact size
- simplicity
- good **stiffness** and bandwidth
- cost-effective



using of
**electrically
powered
actuators**

The Hand is driven by 20 Smart Motor units mounted below the wrist which provide compliant movements. Each Smart Motor is coupled to the corresponding joint of the Hand.

Integrated electronics in the Smart Motor unit drives a high efficiency rare-earth motor and also manages corresponding force sensors.

The Hand system (hand, sensors, and all motors) has a total weight of 4 kg.

Electrically powered actuators



Figure. Electrically powered actuators *a* – actuators with ball screw, *b* – linear actuators, *c* - linear motor representation *d* - electromagnetic device

PROS:

- **Fast:** Electric actuators are directly driven.
- **Precise:** Electric actuators are precise devices.
- **Clean:** Electricity is a clean energy source

CONS

- **Low power to weight/ volume ratio**
- May **need mechanical brakes** to help hold load
- DC motors require **brush maintenance**



Pneumatically powered actuators

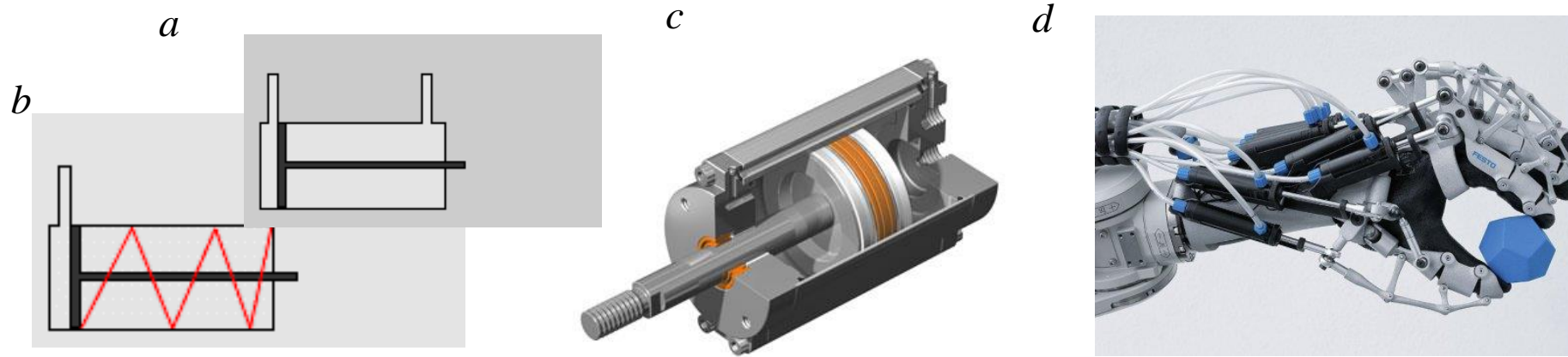
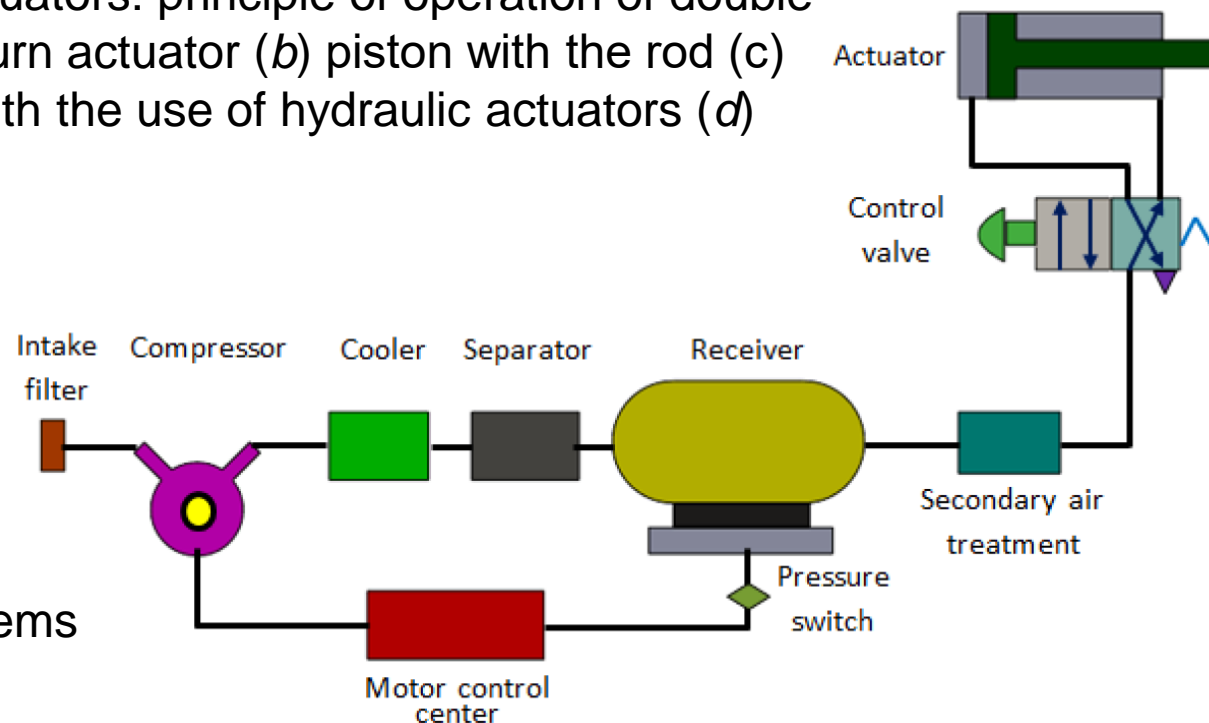


Figure. Pneumatically powered actuators: principle of operation of double acting actuator (a) and spring return actuator (b) piston with the rod (c) and dexterous artificial hands with the use of hydraulic actuators (d)



Figure. fully pneumatic systems



- ☐ **Compressor / compressed gas cylinder**
- ☐ **Pneumatic motor (pneumatic cylinder)**

+

- air preparation (purification) unit
- compressed air distribution unit;
- compressed air transmission system in cylinder.

Pneumatically powered actuators

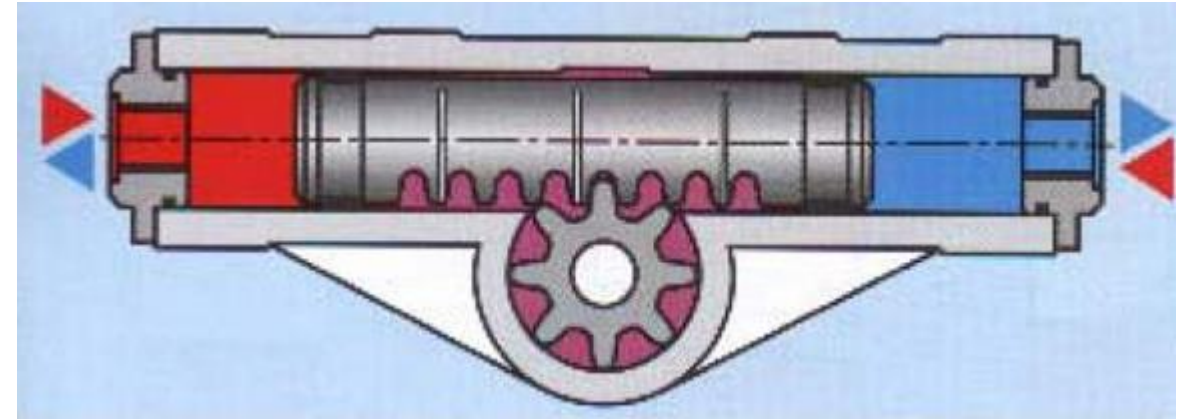
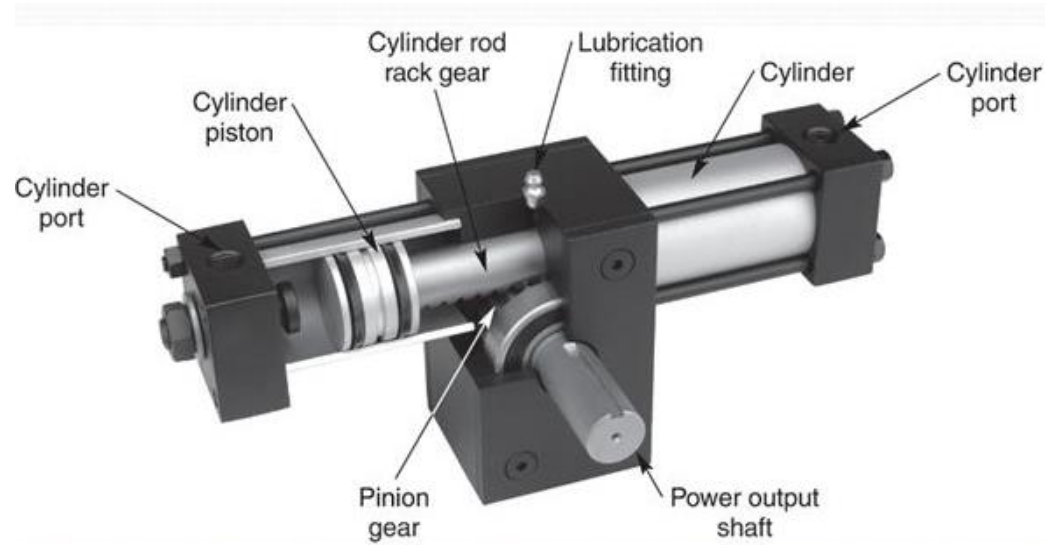


Figure. Rack-and-pinion limited rotation actuator

PROS:

- **Fast:** Compressed air allows for high-speed motion
- **Economical:** On average, pneumatic actuators aren't expensive
- **Simple :** they're quite basic in design

CONS

- **Limited power** (air is more compressible than oil in hydraulic system)
- **Shorter life cycle** (powered by compressed air)
- **Strong temperature** dependence

- Here maximum angle may be **larger than 360°**

Hydraulically powered actuators

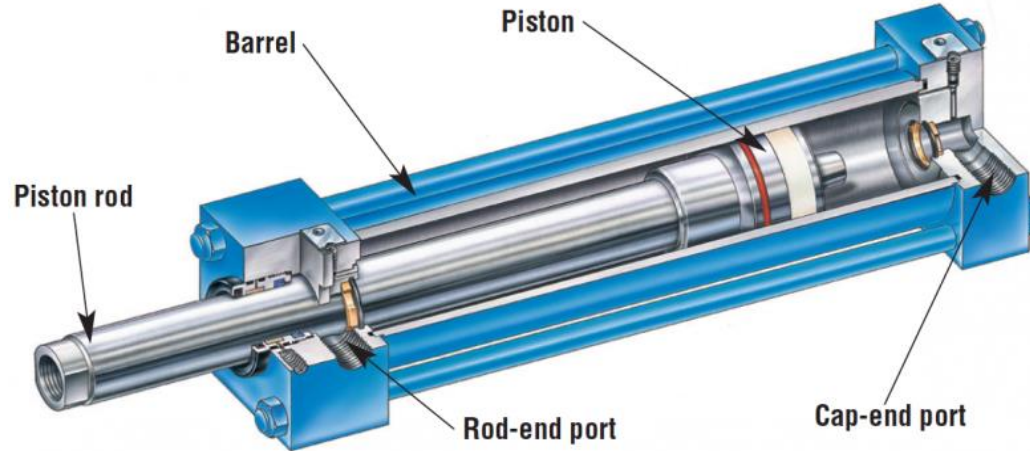
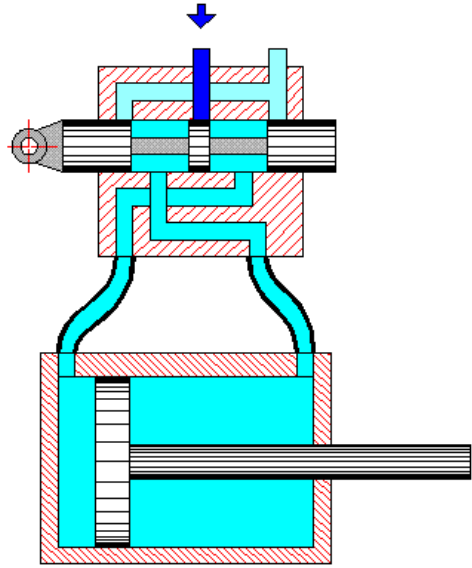


Figure. Hydraulically powered actuators

PROS:

- **Force:** high horsepower-to-weight ratio
- **Safety:** design has been long-proven to be safe and secure
- **Mobility:** They are self-contained and portable

CONS

- Large **initial investment**
- Required **Maintenance**
- **Leakage** of hydraulic oil

□ Pump

□ Hydraulic motor

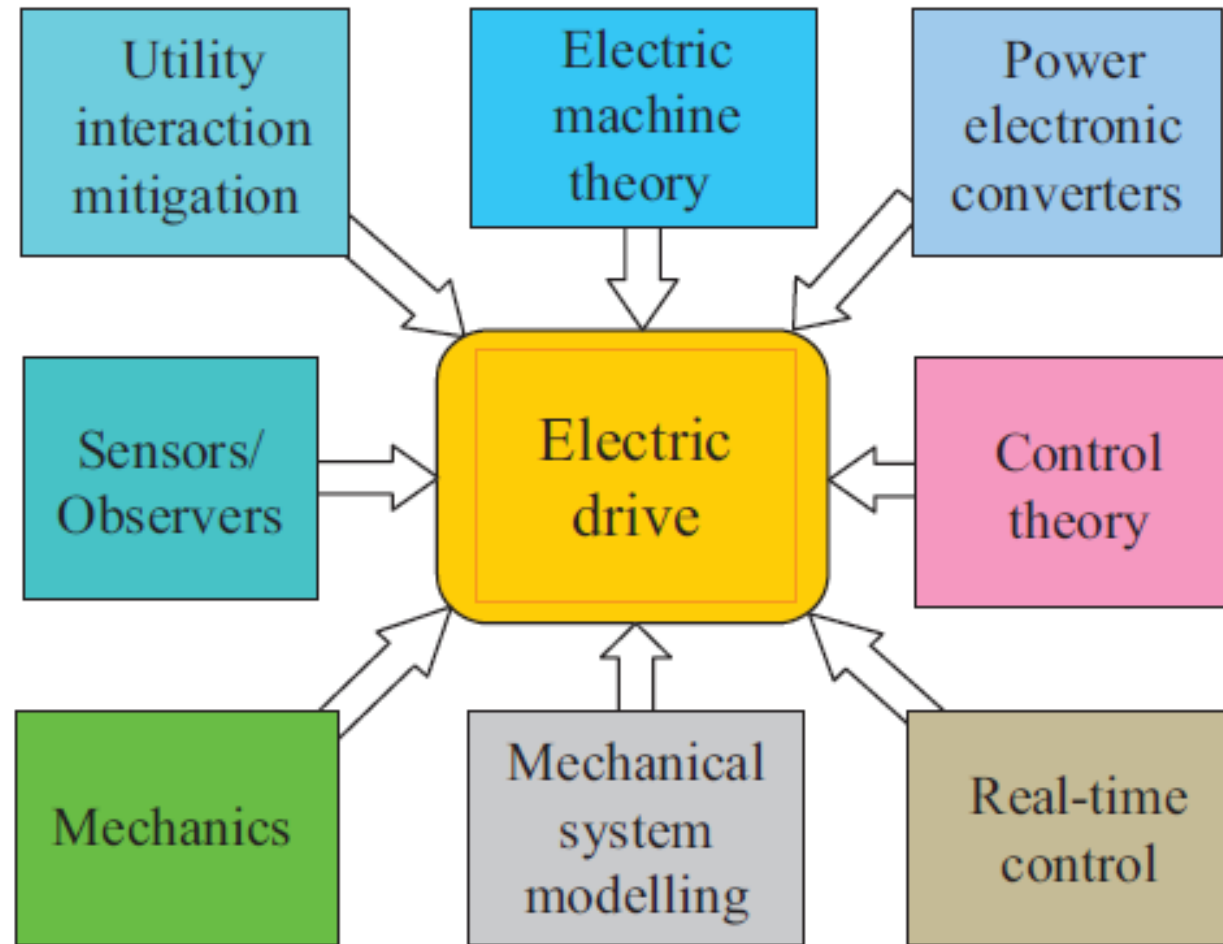
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- fluid reservoir;
- hydraulic lines, including a pressure line (pipes, hoses);
- hydraulic control devices such as a throttle or hydraulic valve;
- filtering devices;
- temperature control systems (heat exchangers).

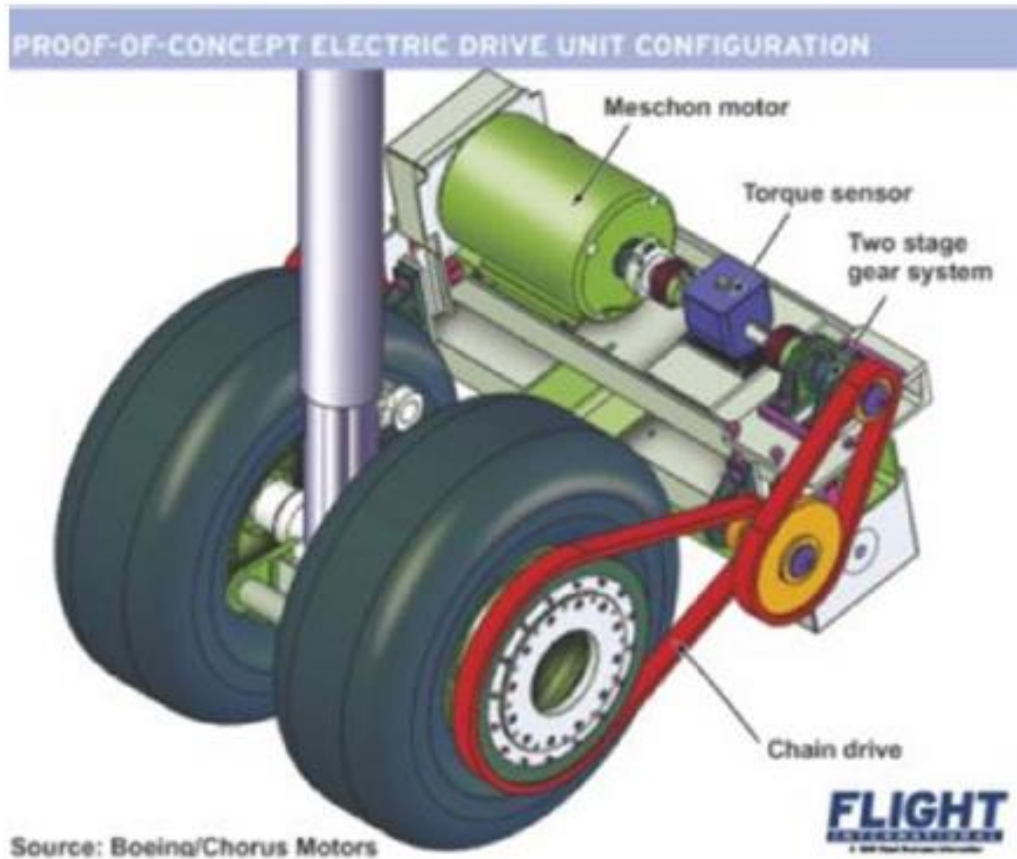
Comparison of various types of actuators

electric actuators	hydraulic actuators	pneumatic actuators
<u>PROS:</u>		
<ul style="list-style-type: none">- Fast: directly driven- Precise: no backlash and flex- Clean: use of electricity	<ul style="list-style-type: none">- Force: high horsepower-to-weight ratio- Safety: design has been long-proven to be safe and secure- Mobility: They are self-contained and portable	<ul style="list-style-type: none">- Fast: compressed air allows for high-speed motion- Economical: cheap- Simple: quite basic in design
<u>CONS</u>		
<ul style="list-style-type: none">- Low power to weight/ volume ratio- May need mechanical brakes to help hold load- DC motors require brush maintenance	<ul style="list-style-type: none">- Large initial investment- Required Maintenance- Leakage of hydraulic oil	<ul style="list-style-type: none">- Limited power (use air)- Shorter life cycle (using pressure)- Strong temperature dependence

The concept of electric drive



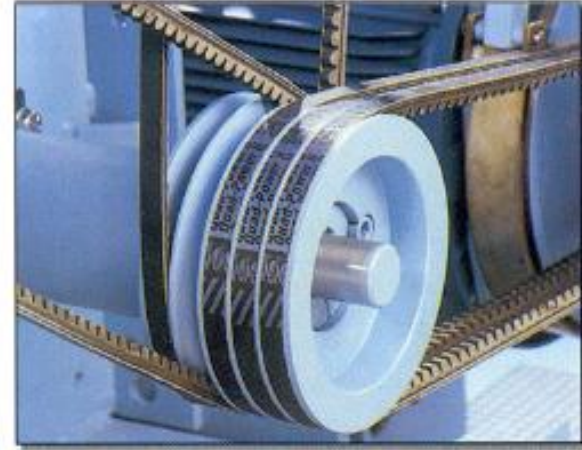
Application of electric drive: Transport



Drive Motor at the **wheel** of an airplane



Electric bike



Precise drilling and Belt Conveyor

Application of electric drive: In robotics

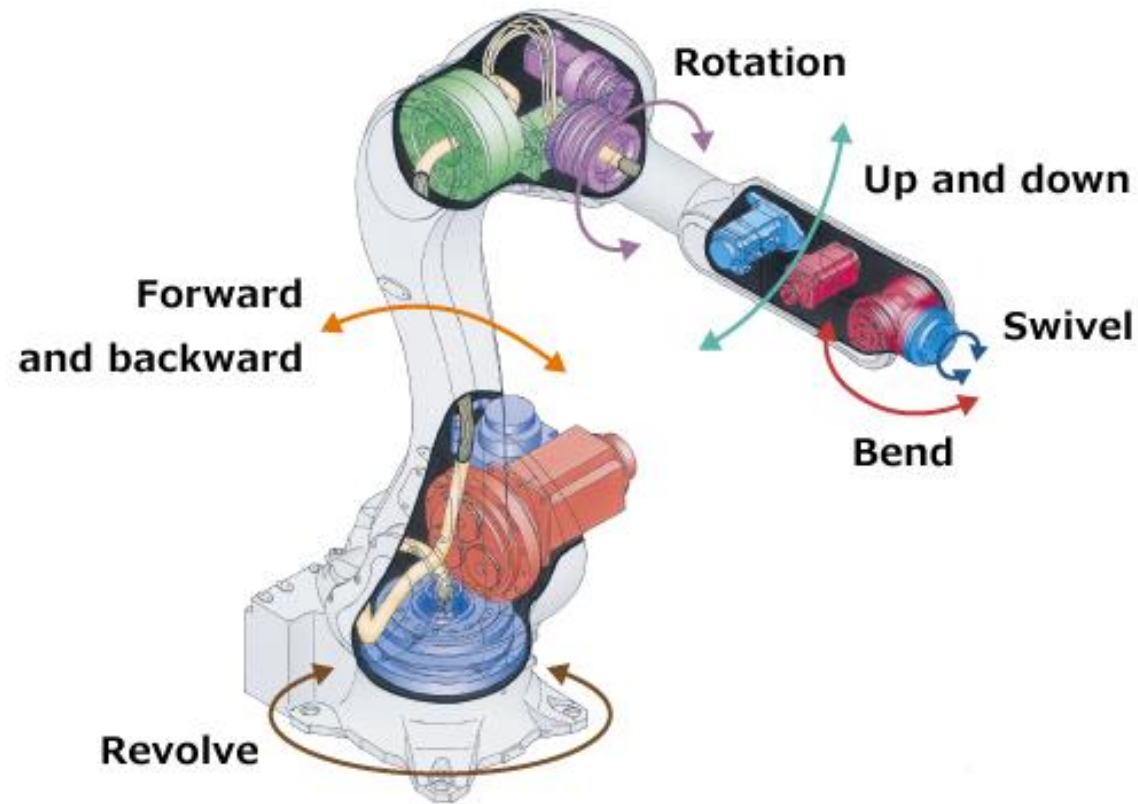


Figure. Articulated robots use at least three rotary joints, and as the name suggests, 6-axis articulated robots use six such joints to provide six degrees of movement, from the base to the wrist.

Image credit: [Kawasaki](#)



Figure. In this application, placing an articulated robot on a 7th axis allows the robot to transfer parts from a machining center to another, stationary robot.

Application of electric drive: In robotics

Drives convert any type of energy into mechanical movement and include a motor and a device for controlling it, and it may also include mechanisms for converting and transmitting motion (gearboxes, converters of rotational motion into linear motion and vice versa), a brake and a clutch.

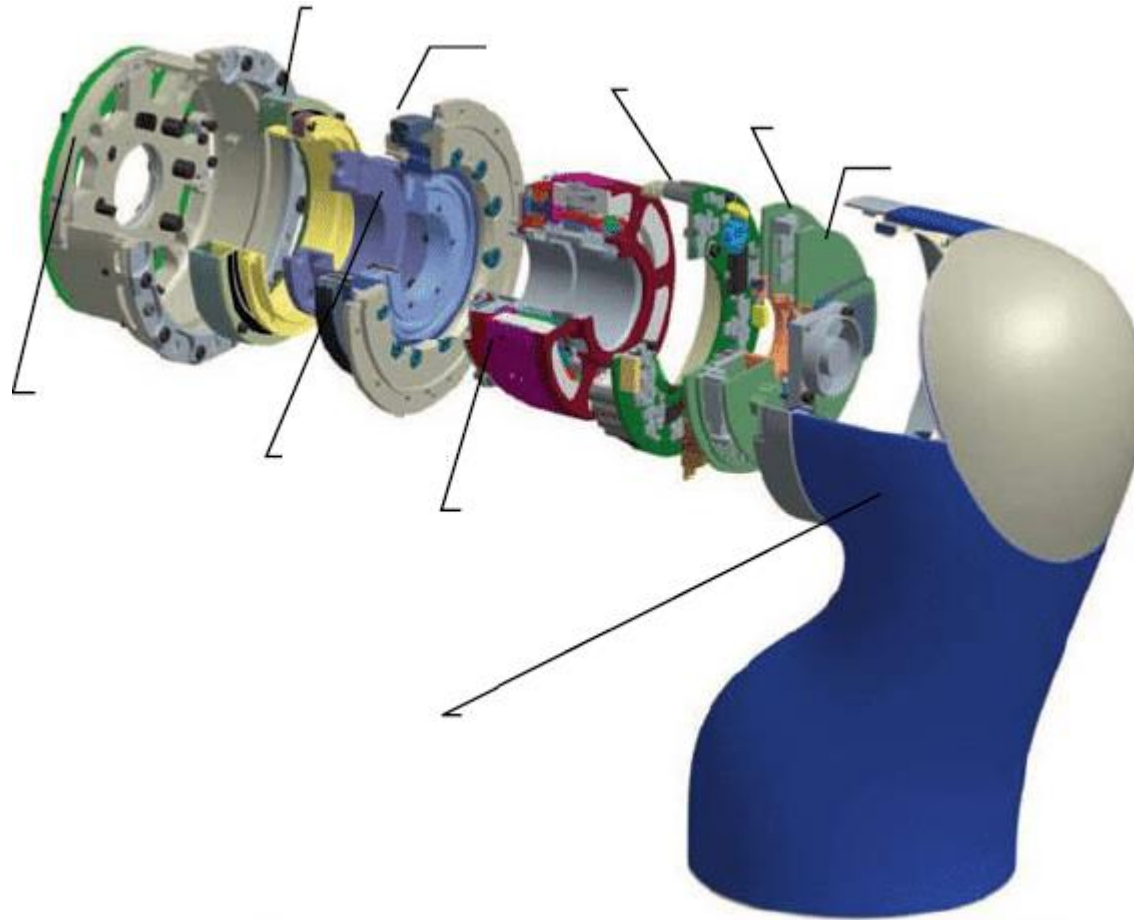


Figure. The mechatronic joint design

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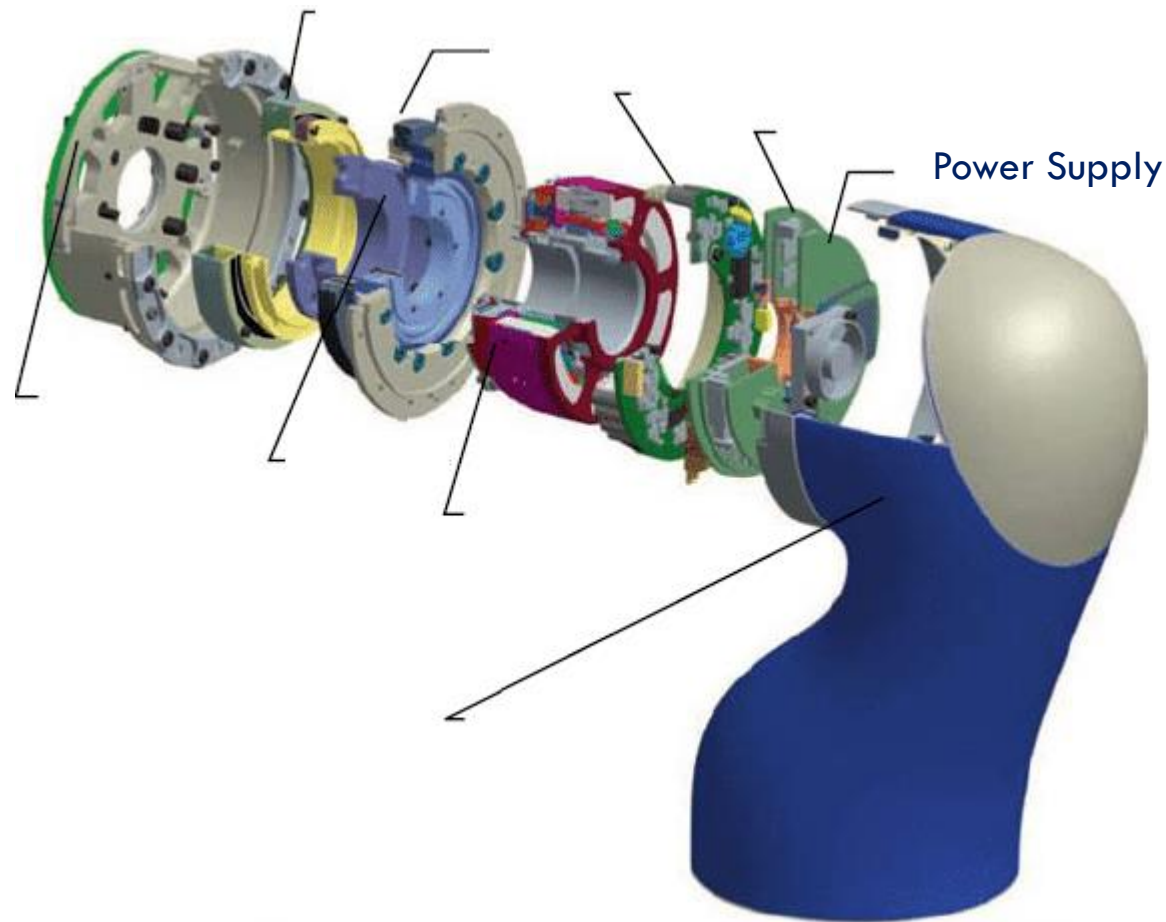


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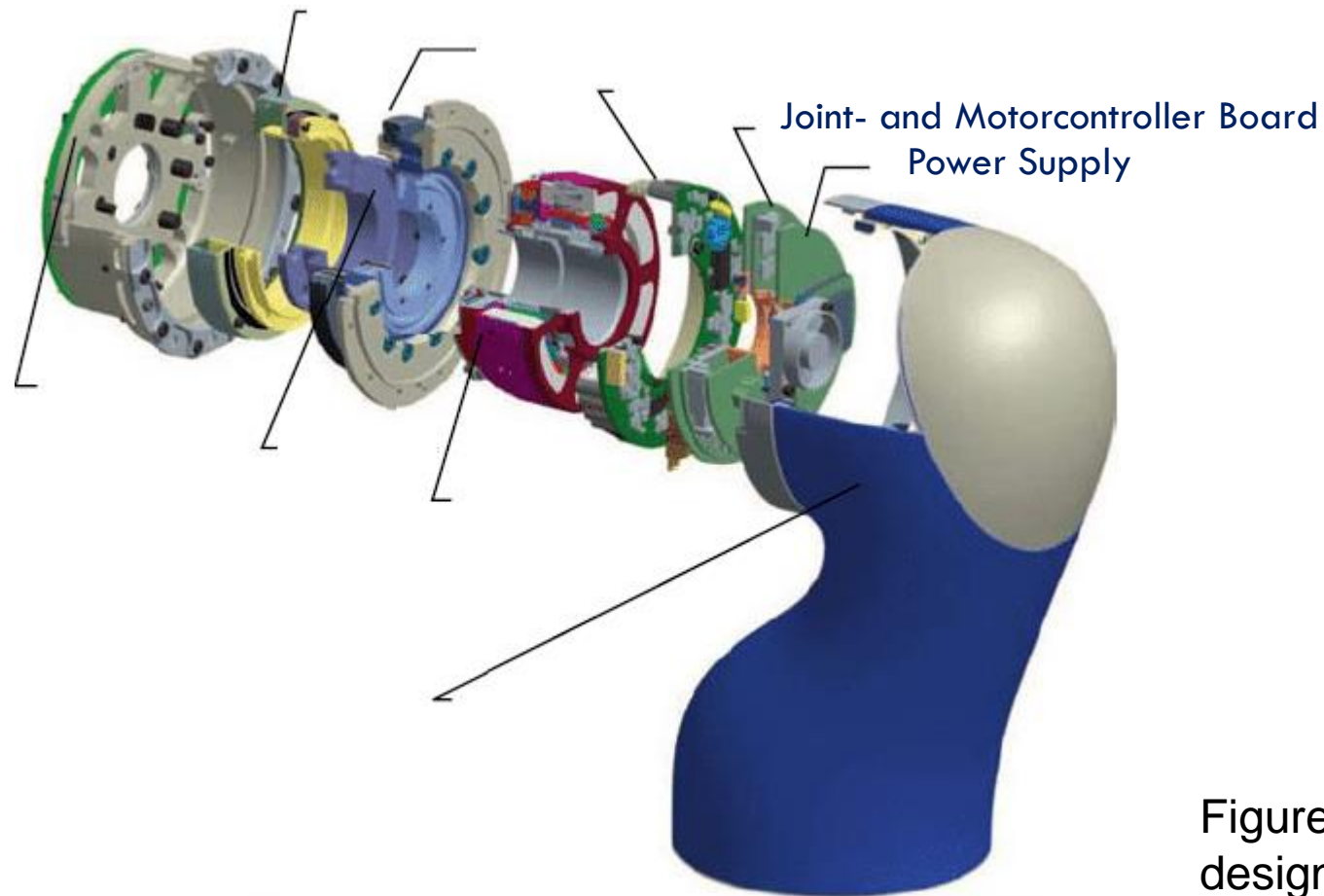


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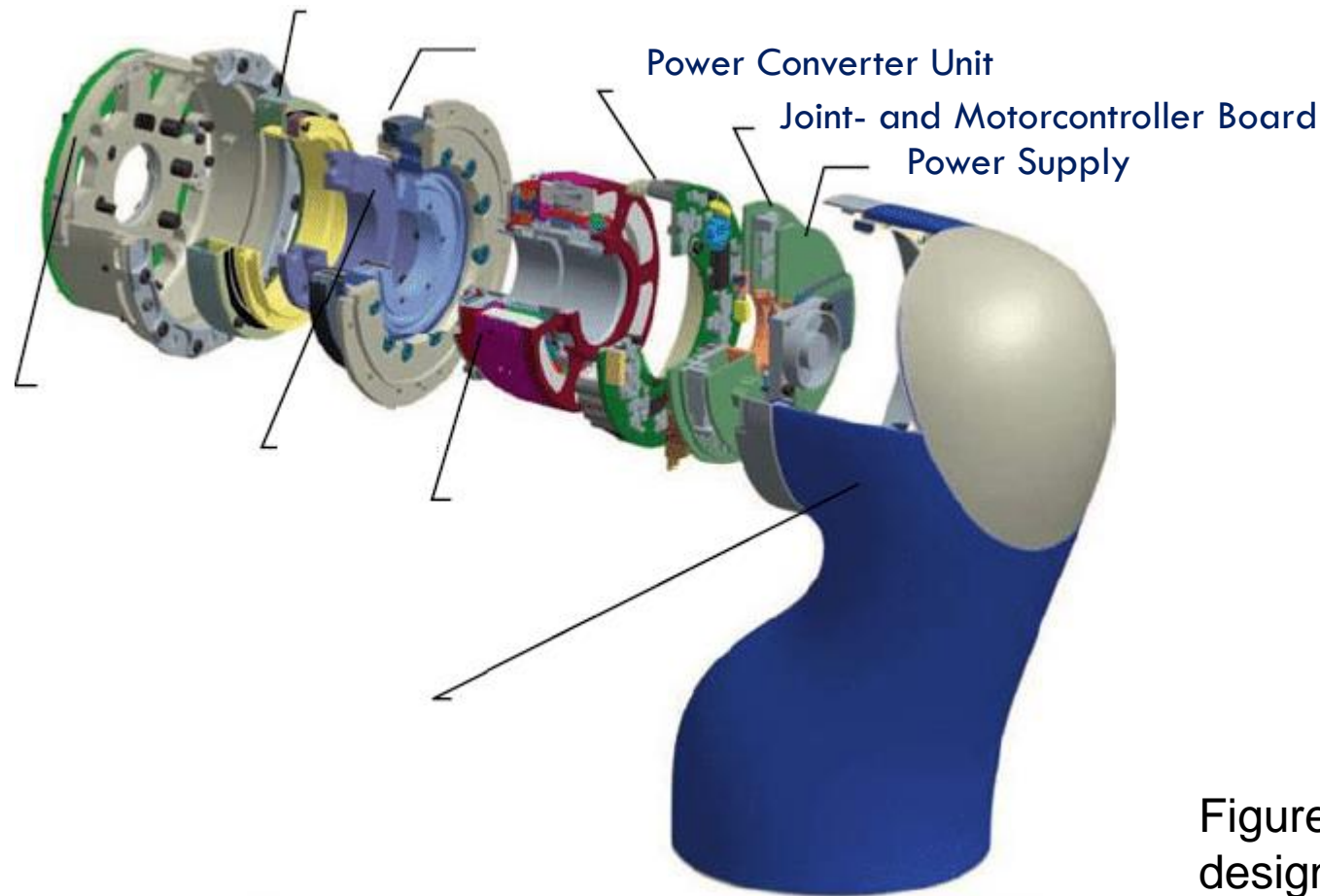


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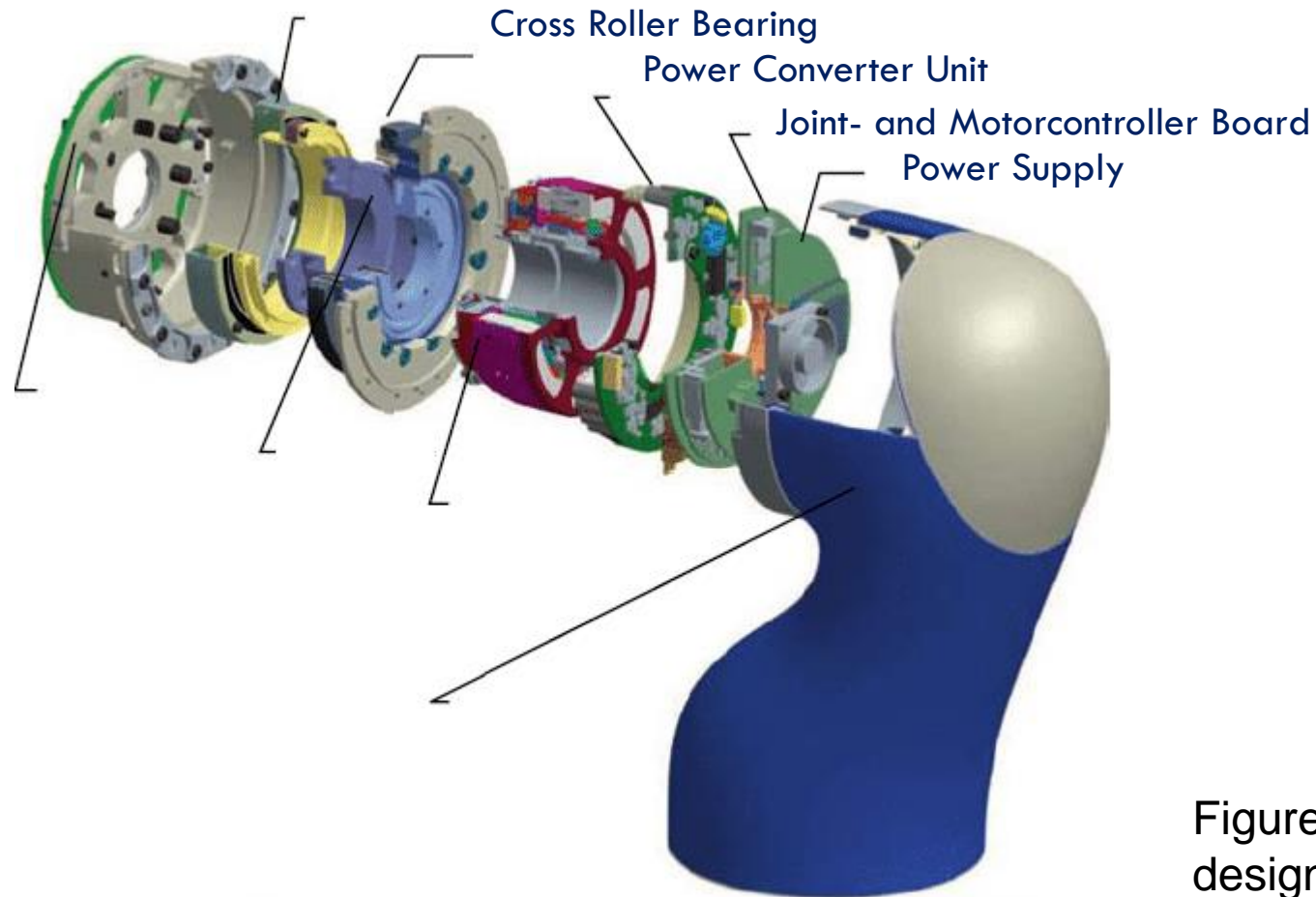


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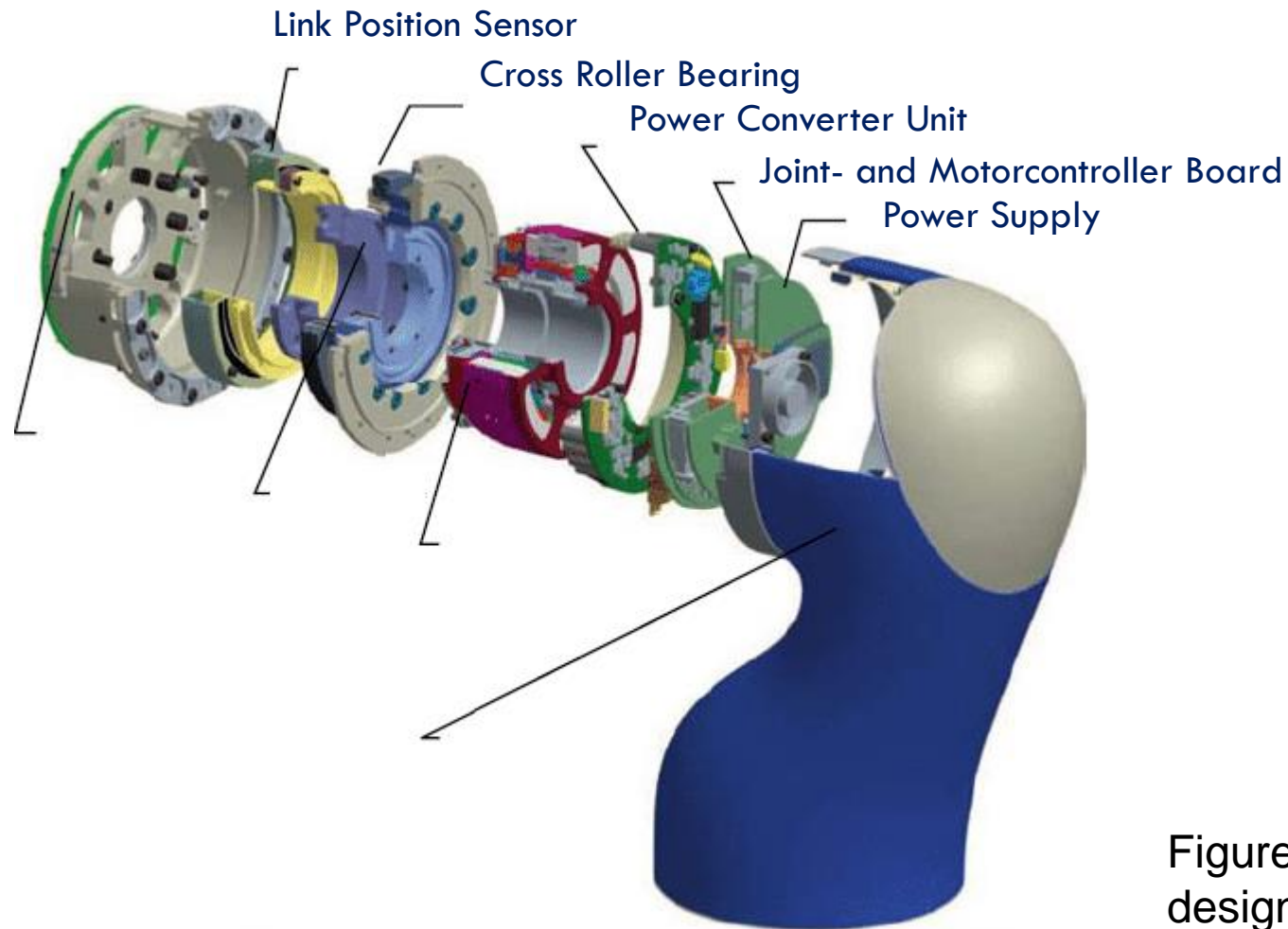


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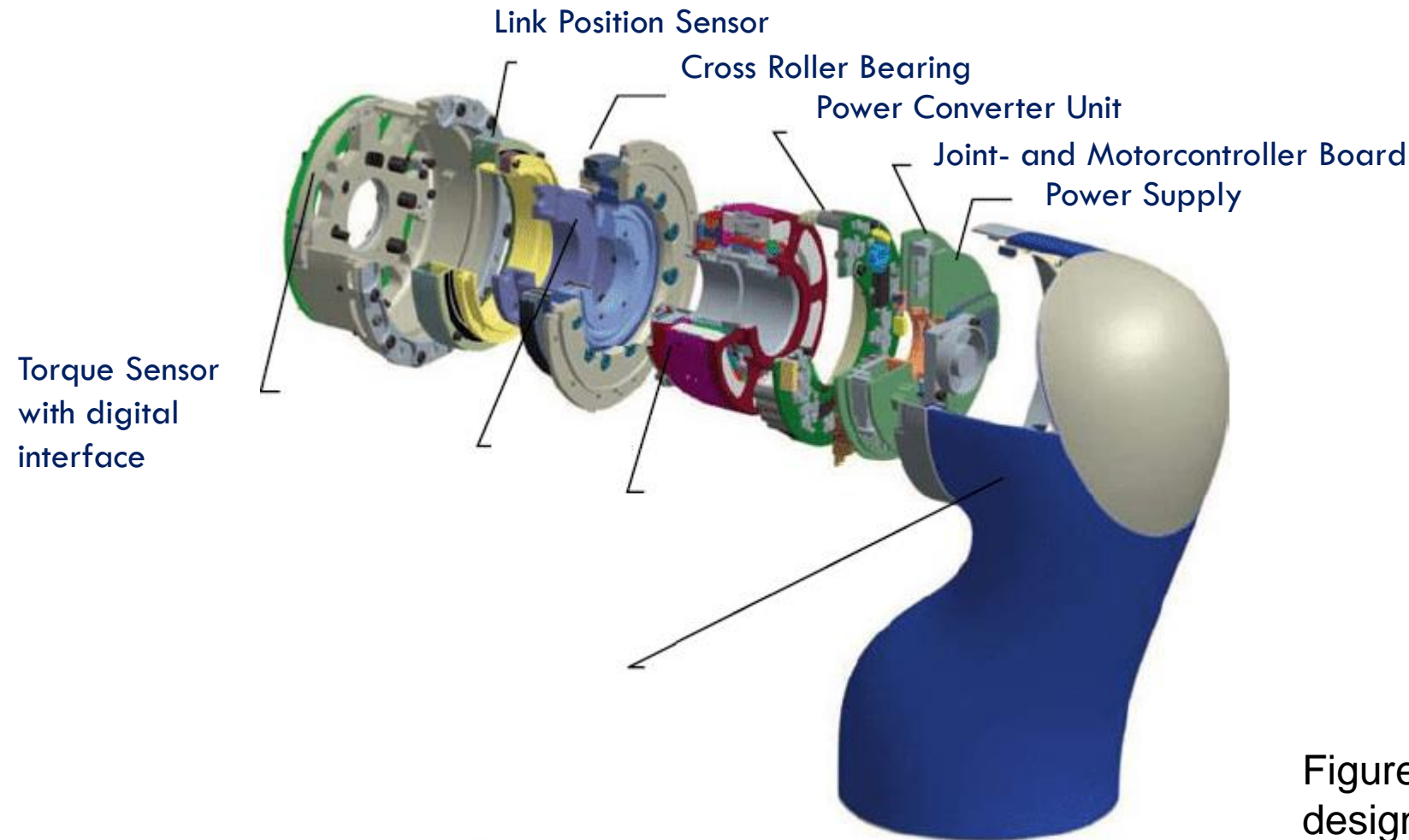


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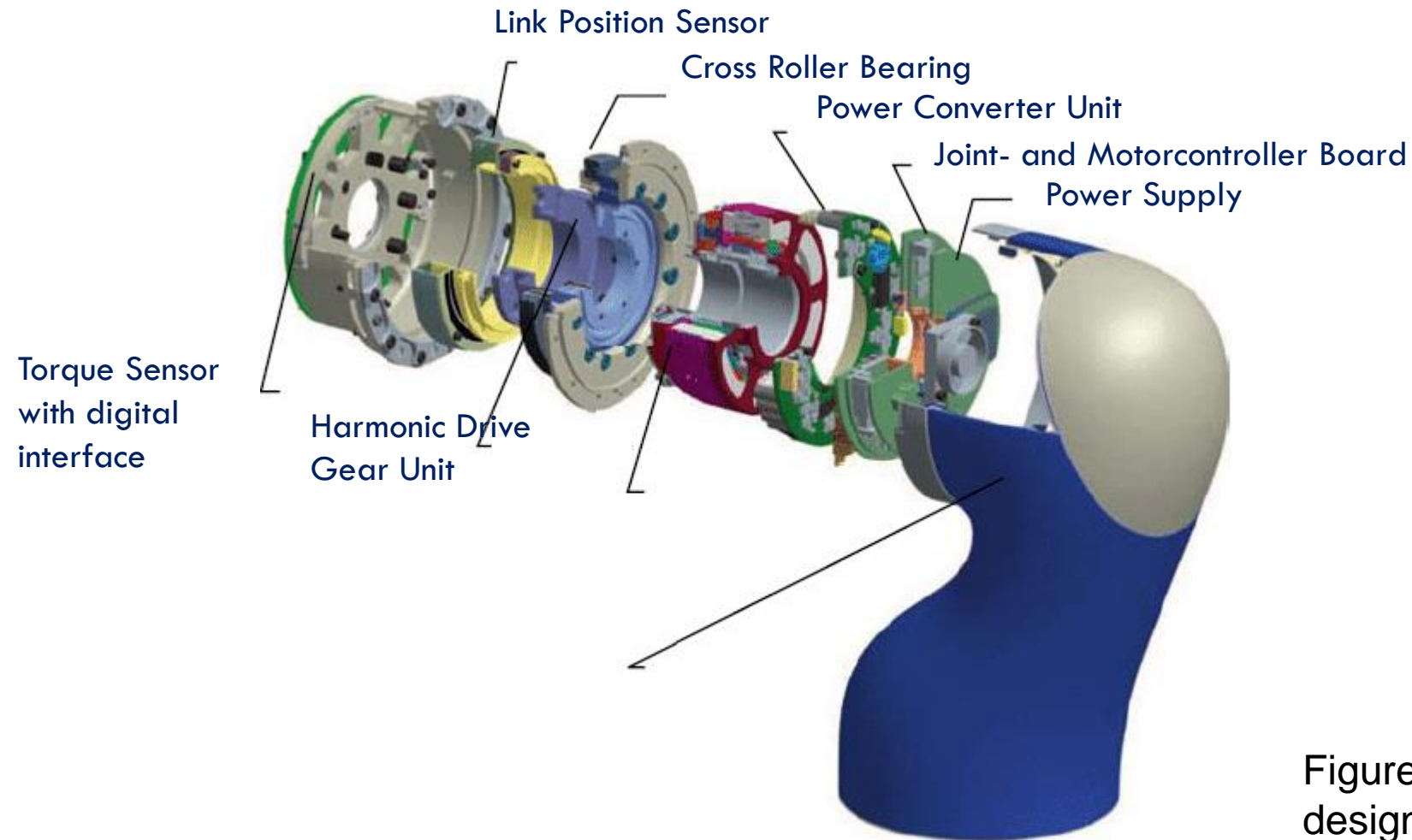


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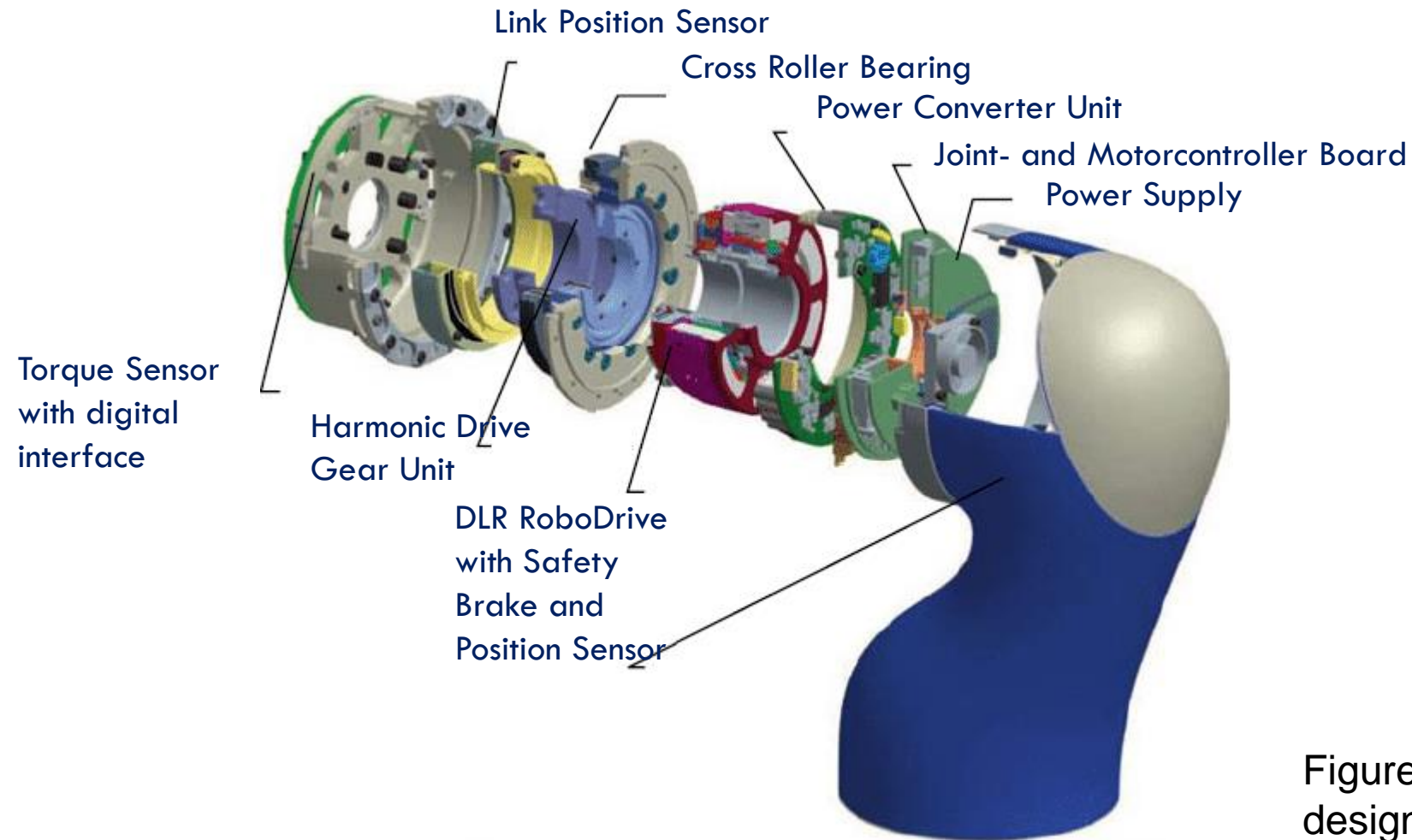


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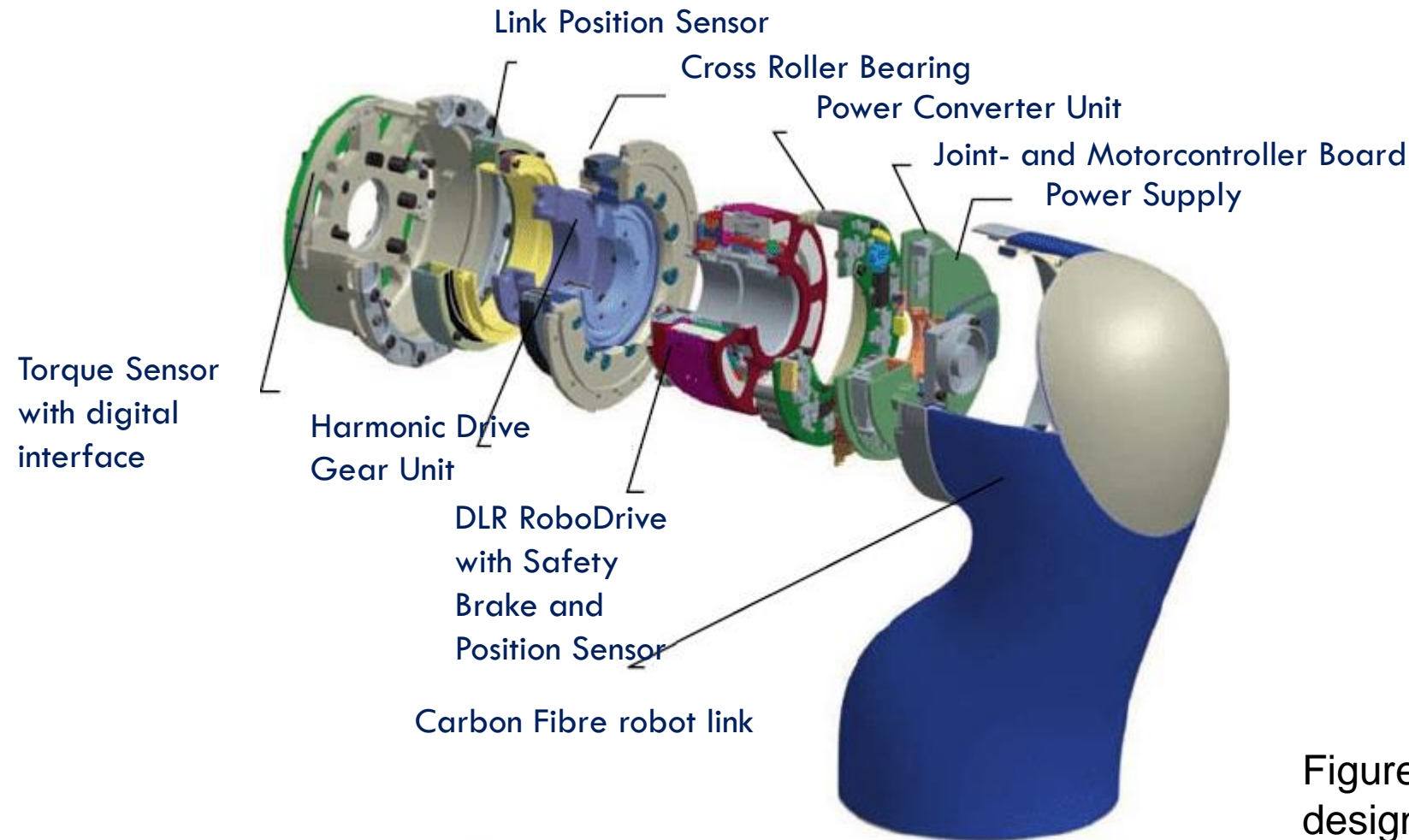
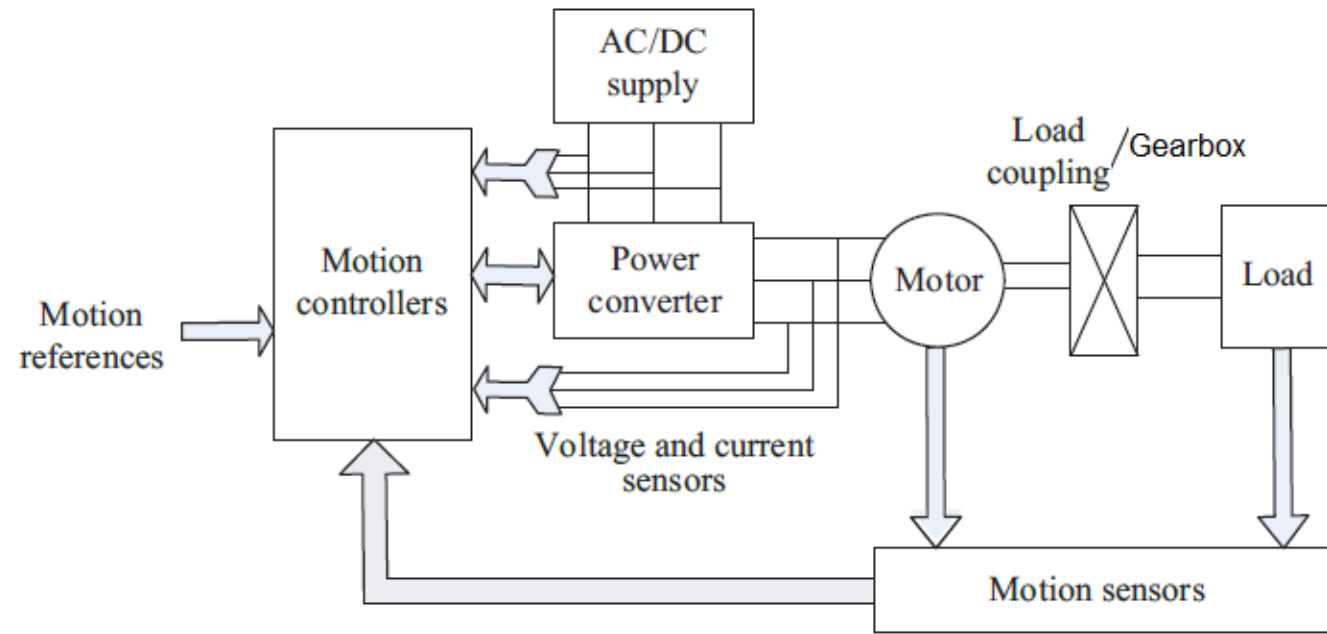
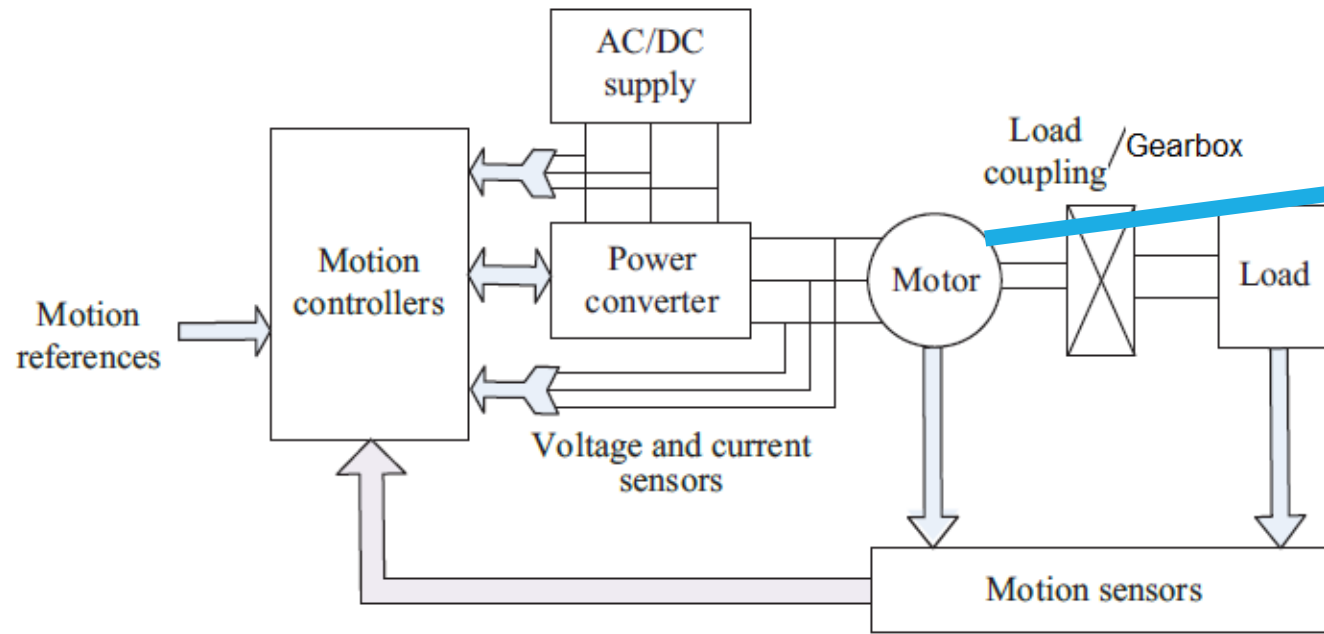


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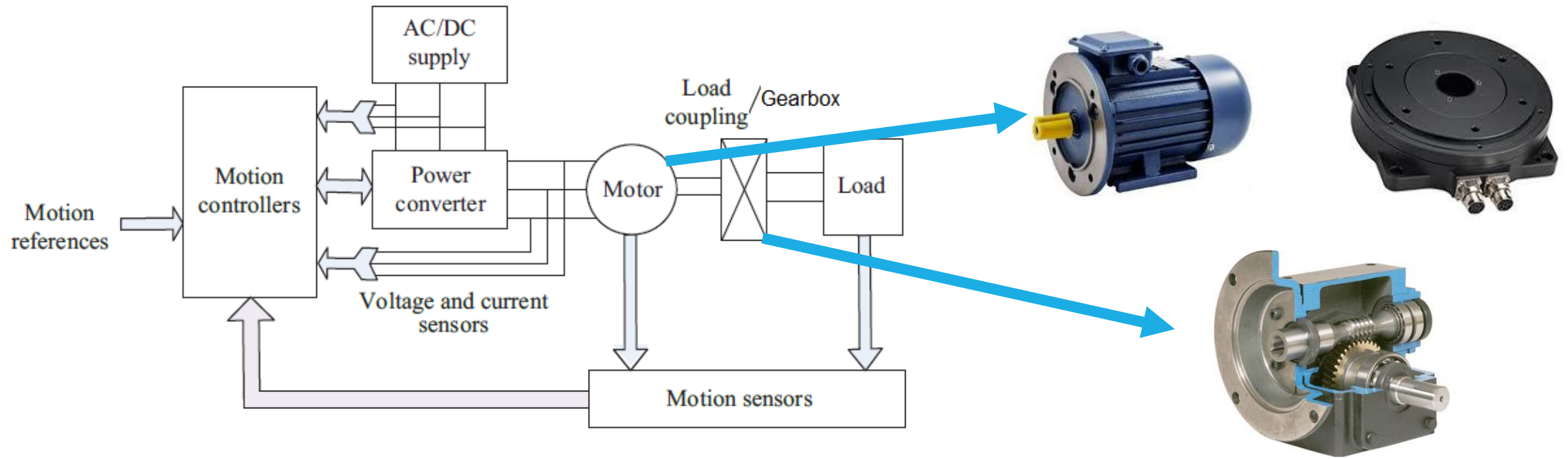
Structure of actuator system based on an electric drive



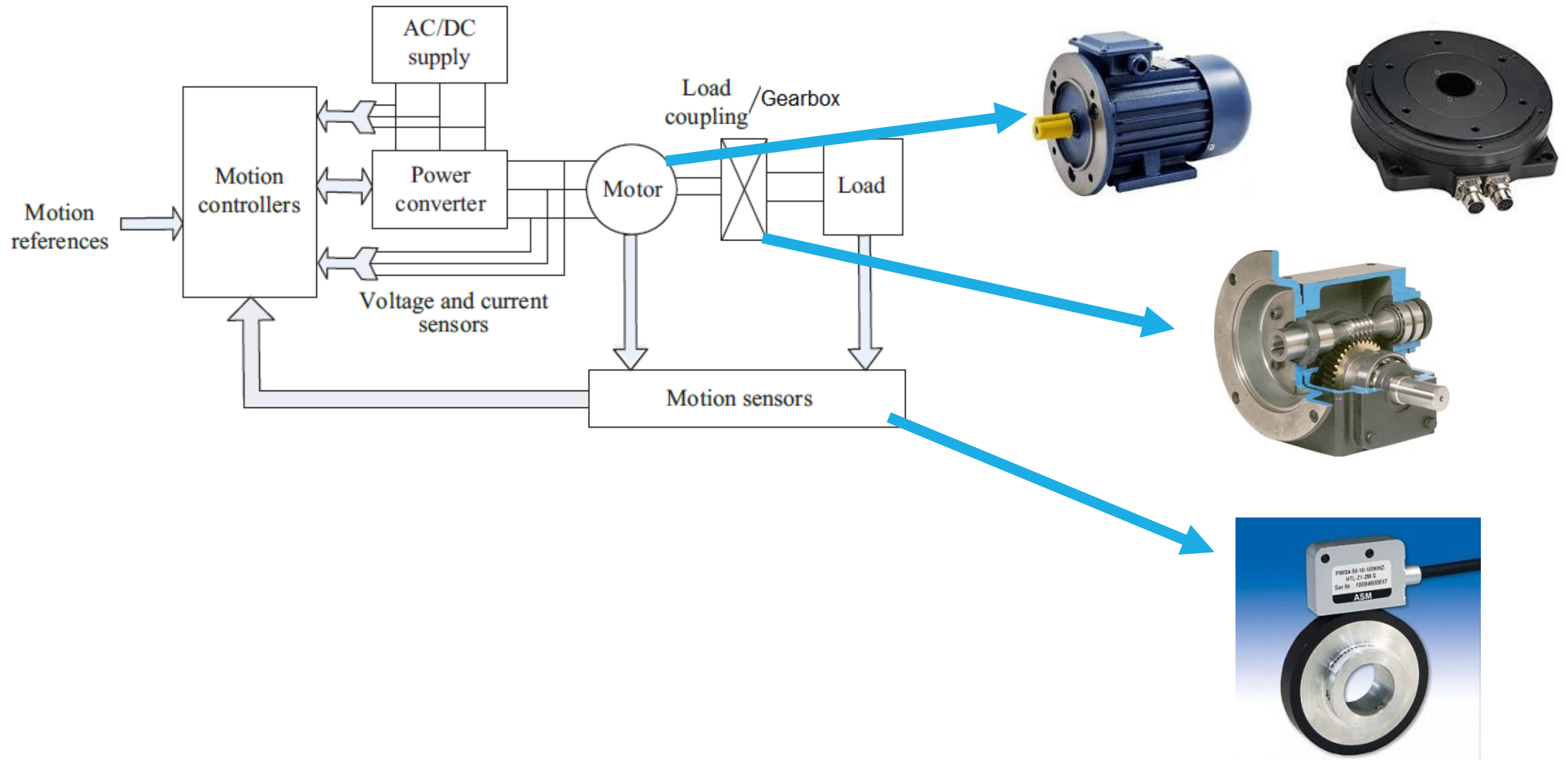
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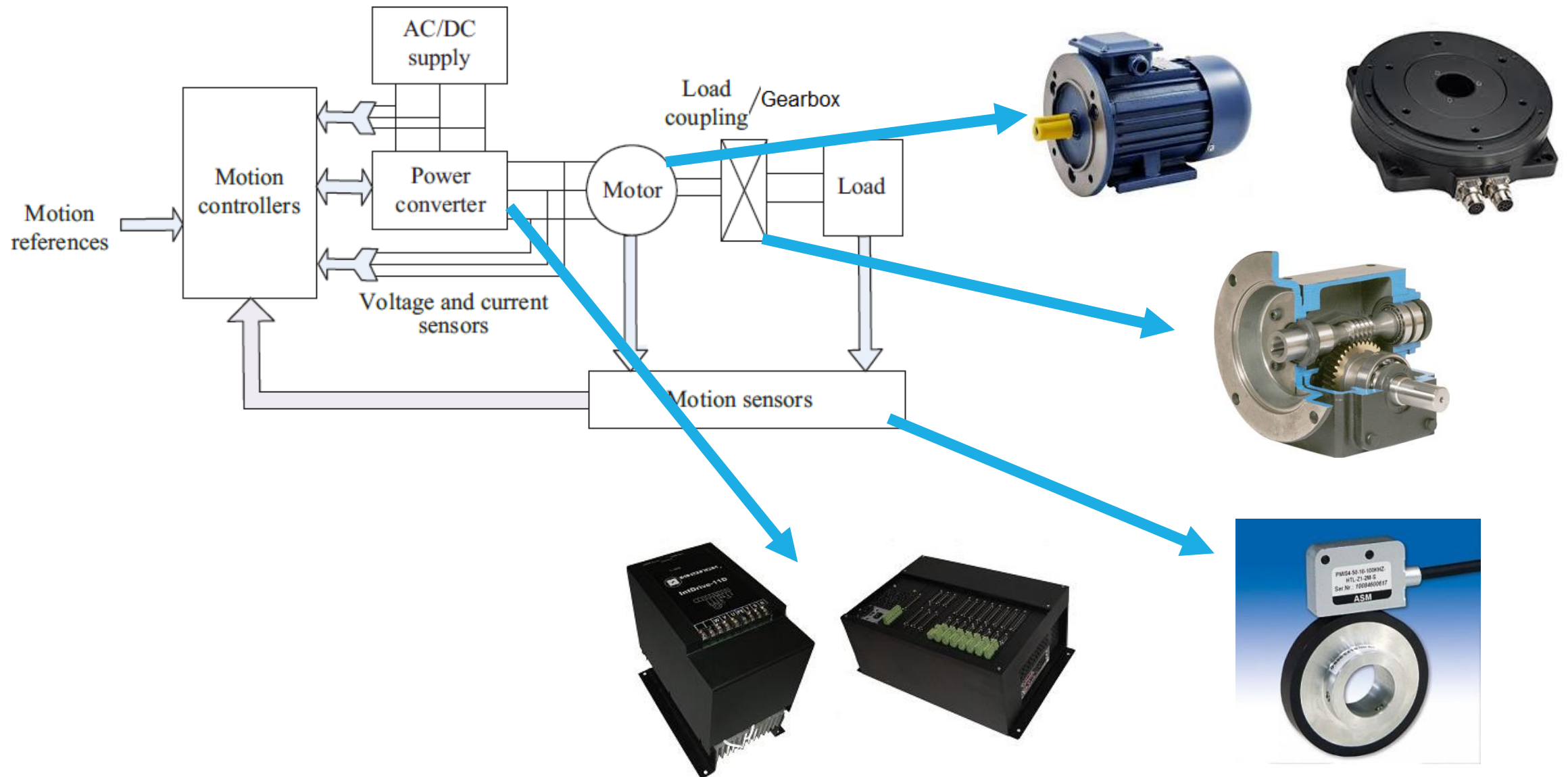
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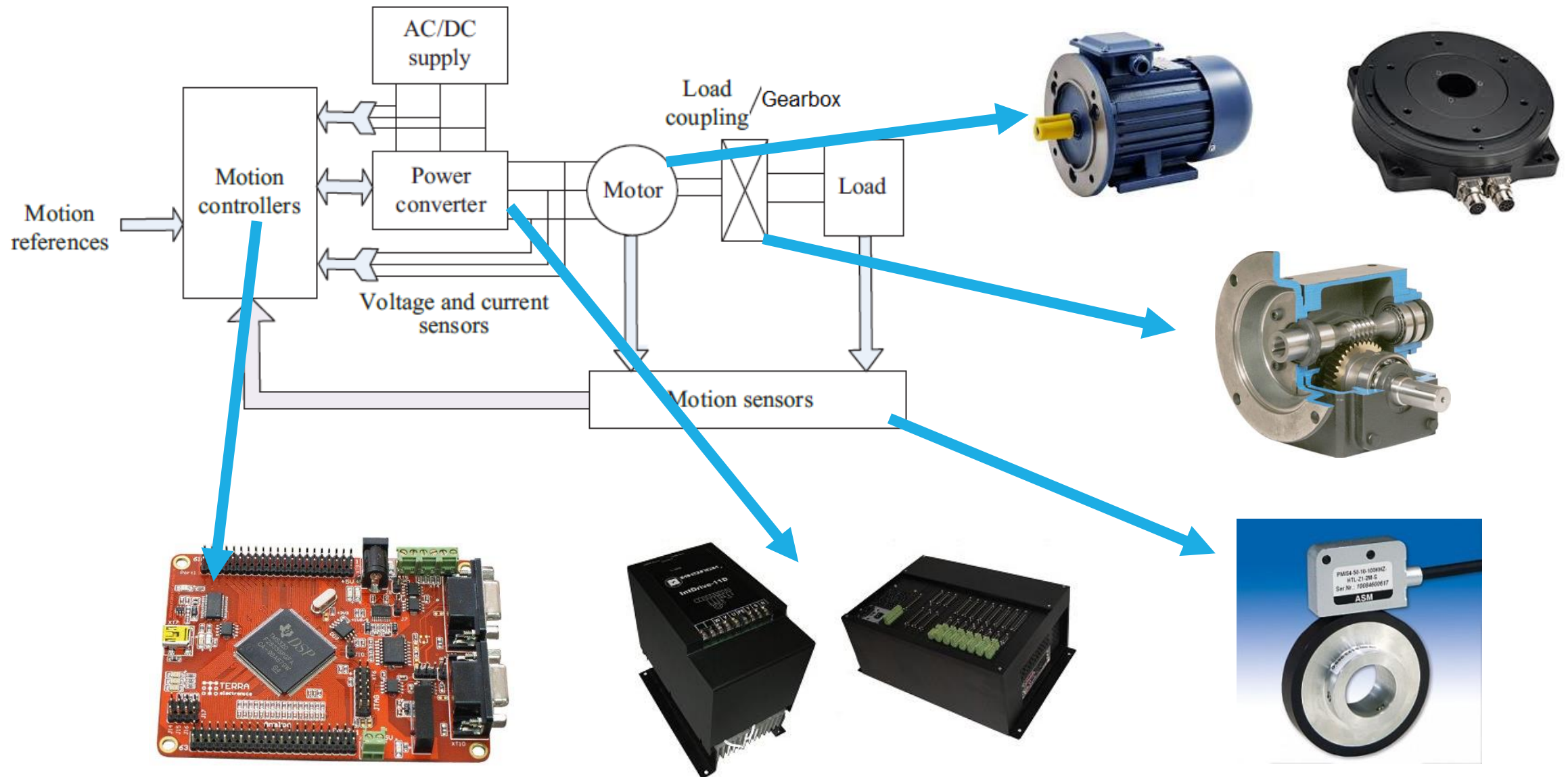
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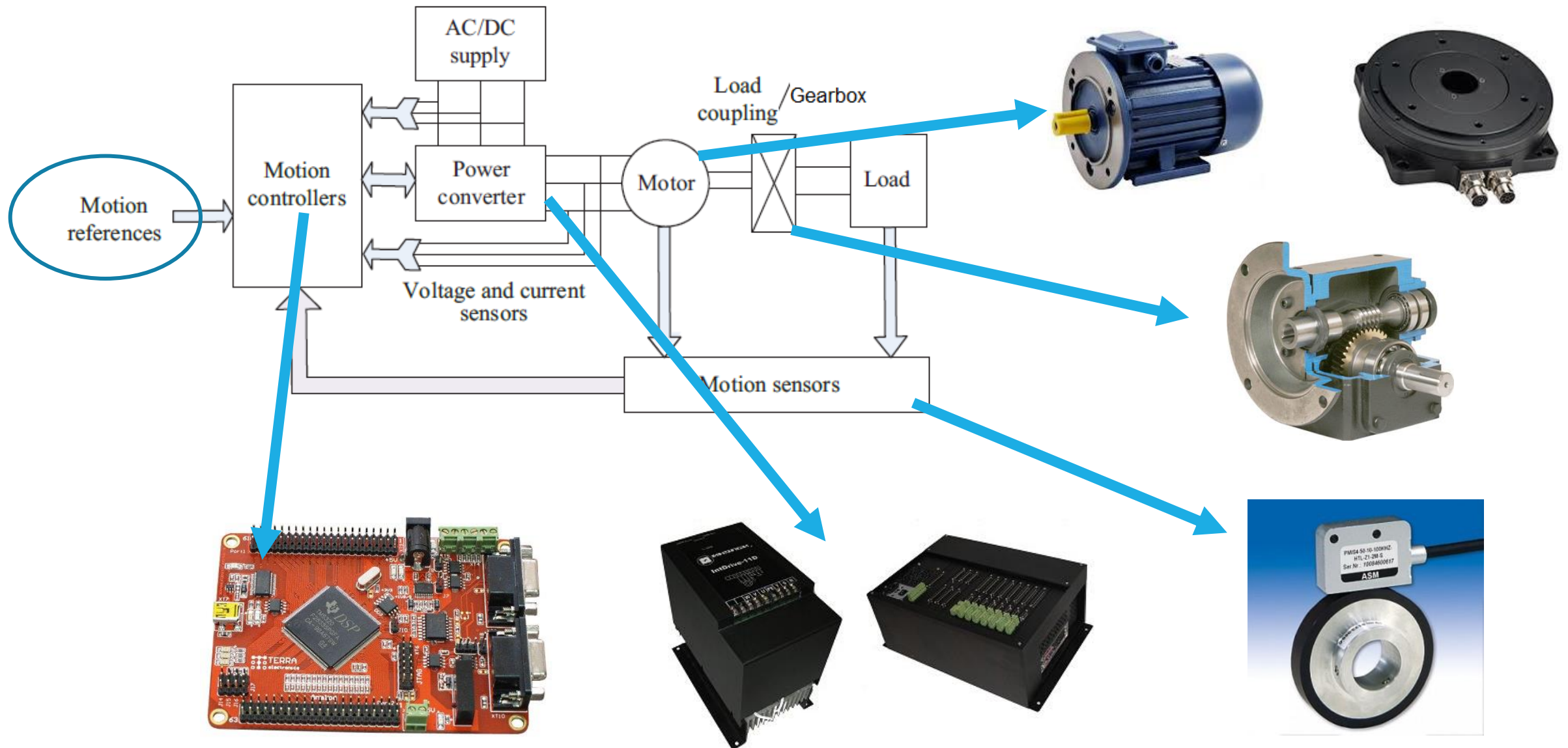
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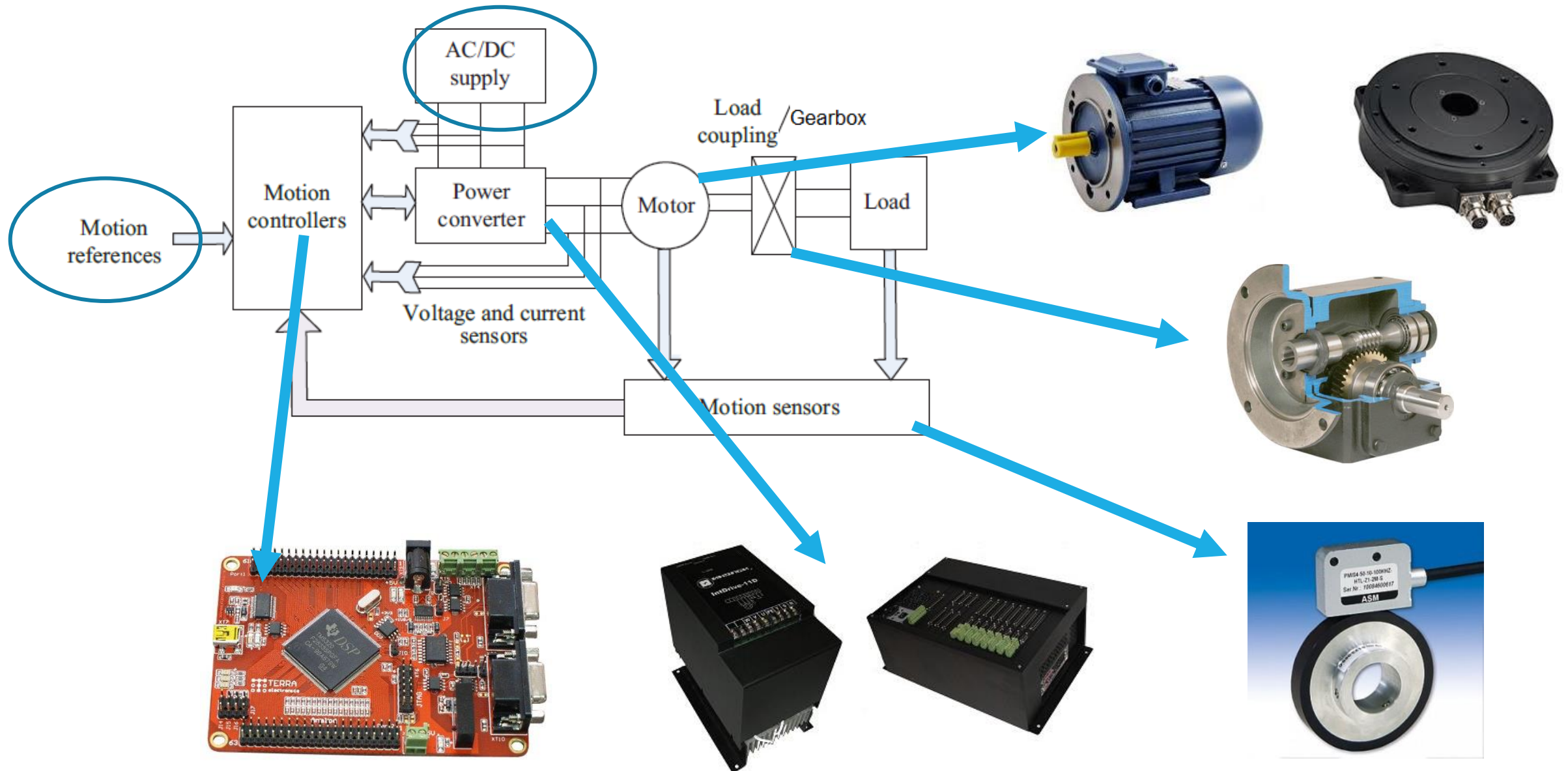
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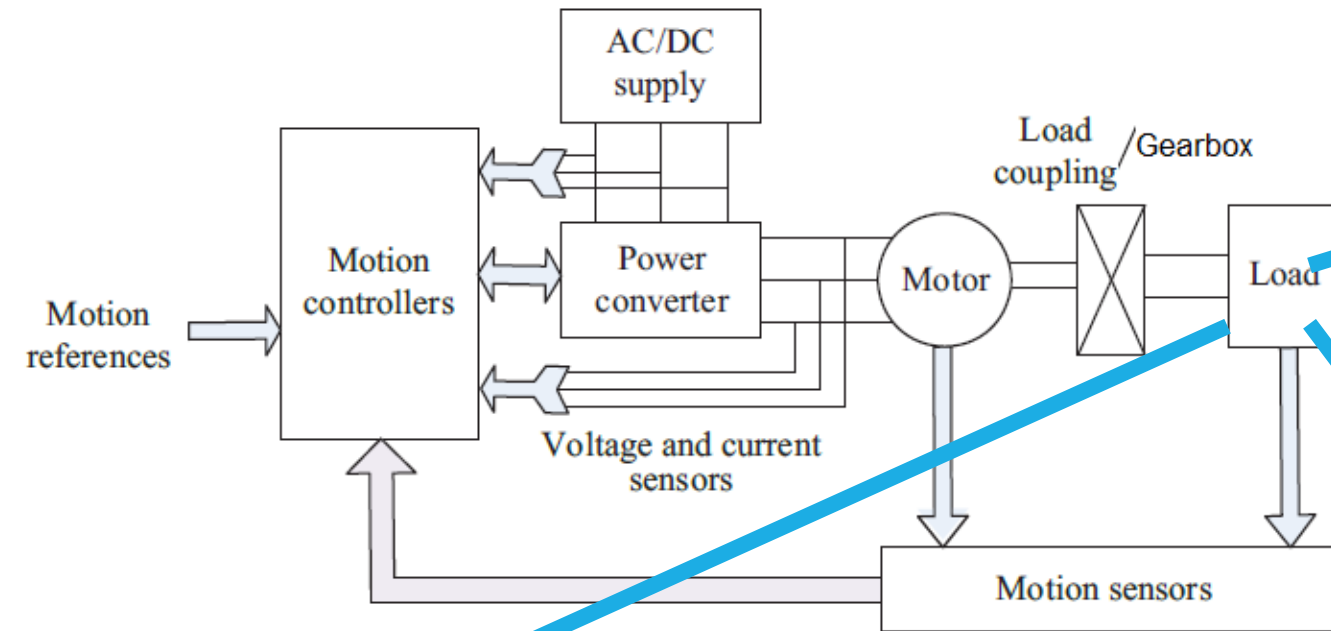
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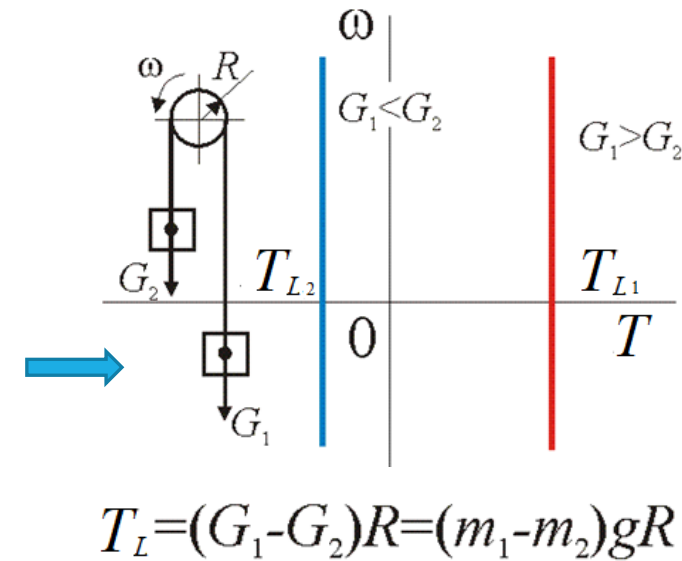
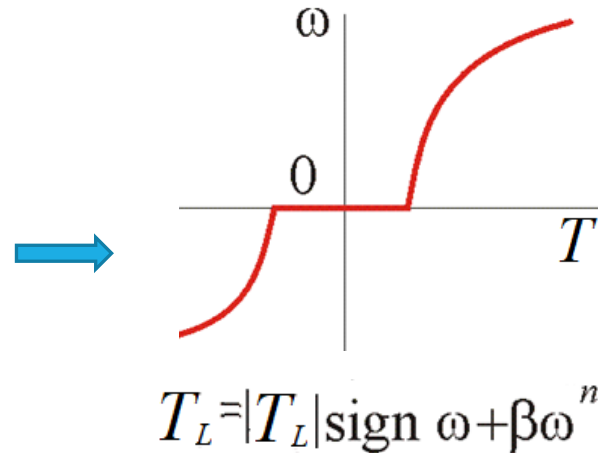
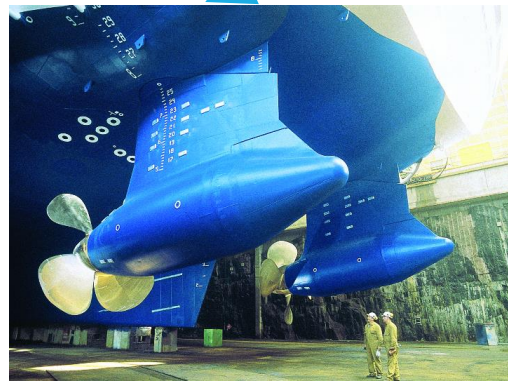
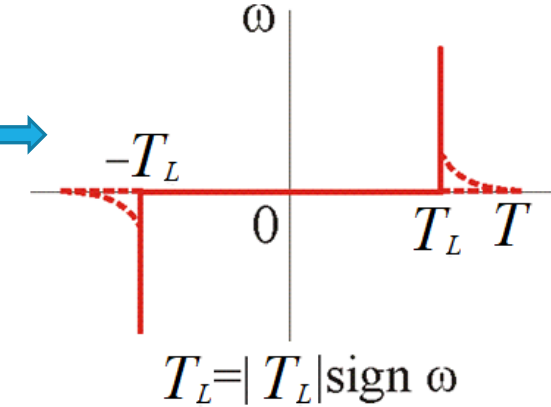
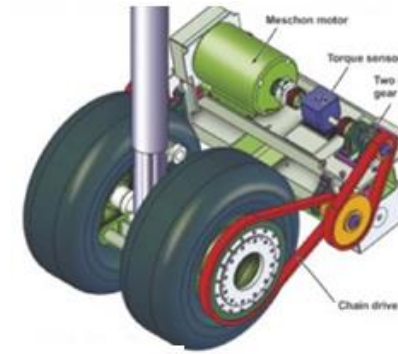
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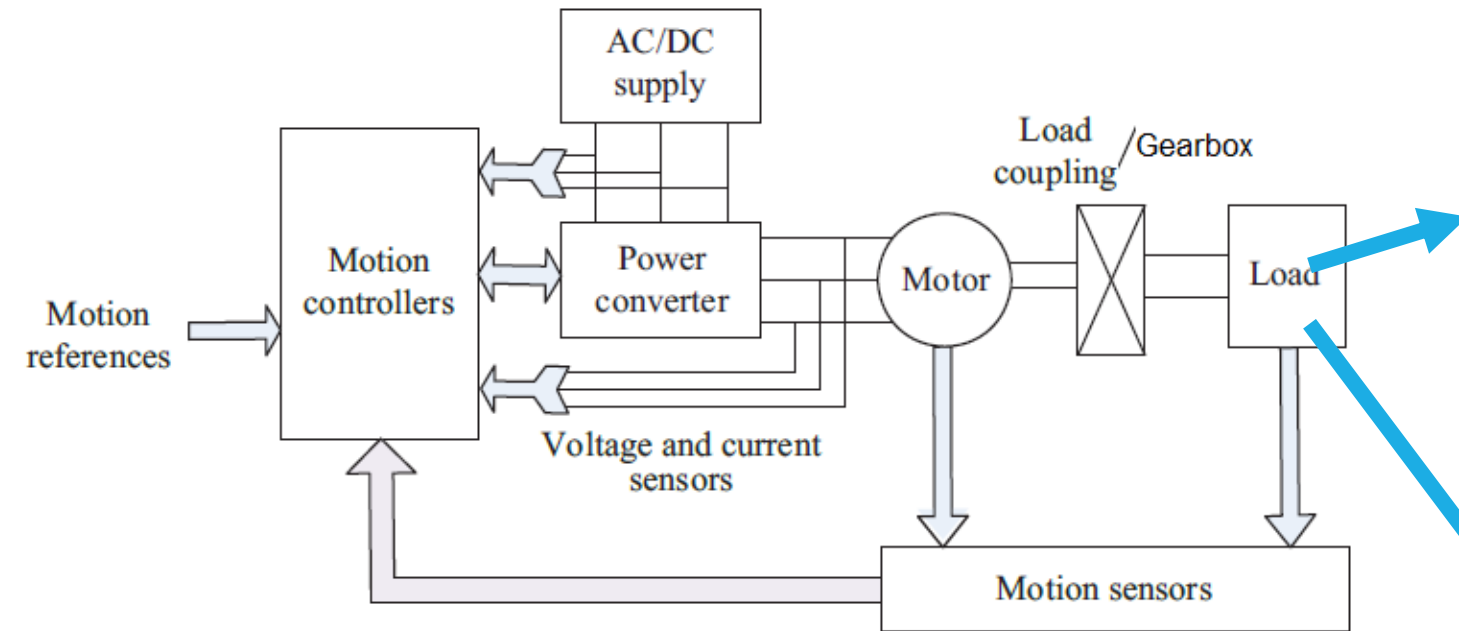
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Structure of actuator system



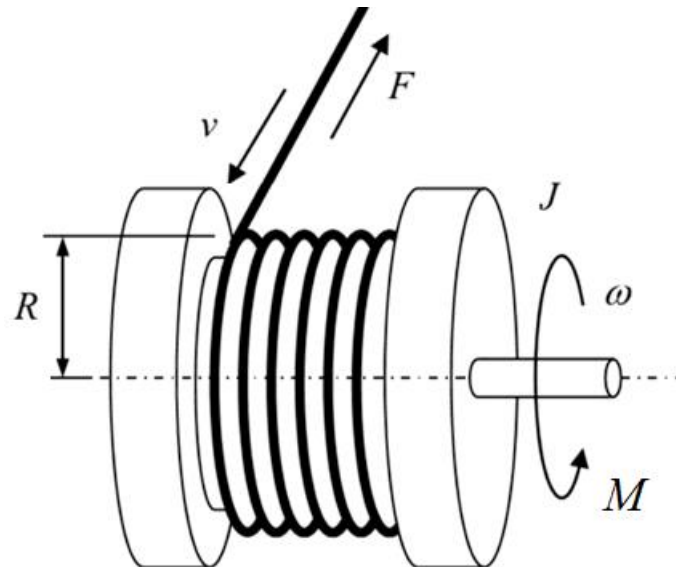
Структура электропривода



A simplified structure of a fully controlled drive



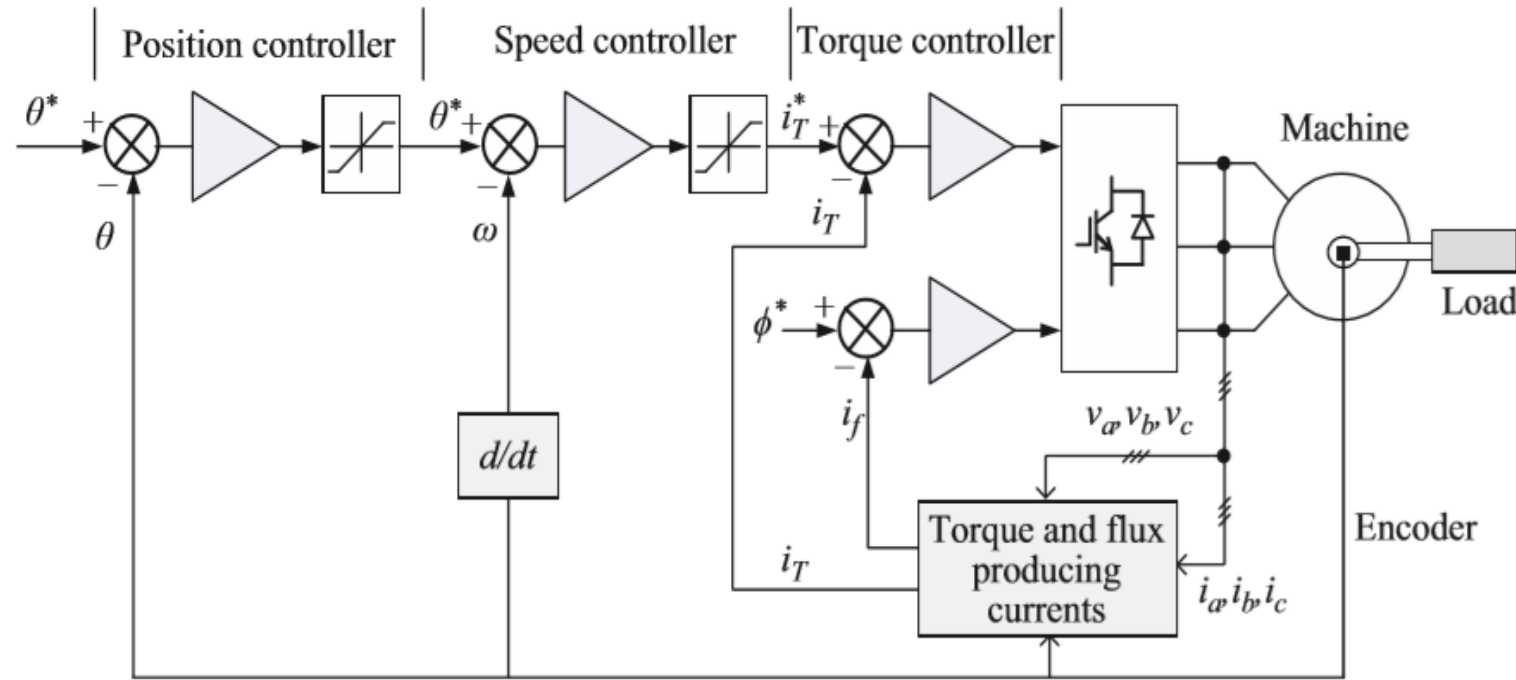
rolling mill



✓ change in moment of inertia



Control system of a variable-speed-electric drive



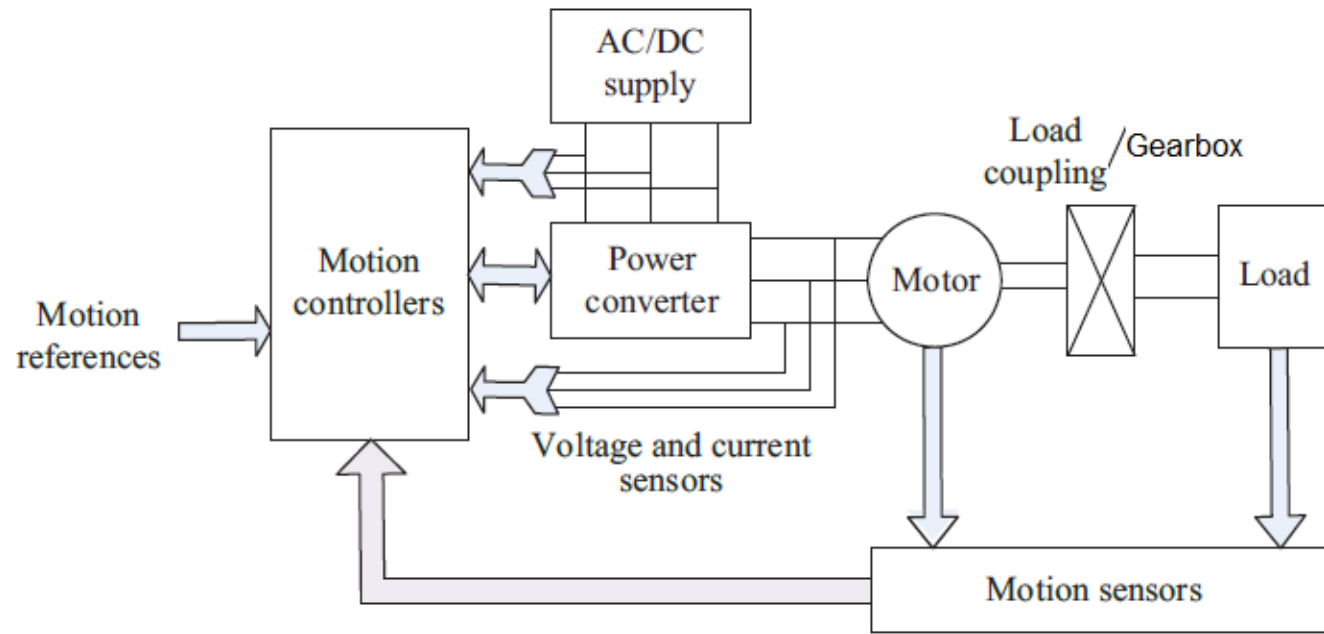
The traditional structure of controller arrangements follows the hierarchical (**Cascade Control Strategy**) structure: The innermost controller is for torque, the reference for which is the output of the speed controller. The reference for the speed controller is the output from the position controller

ATTENDANCE

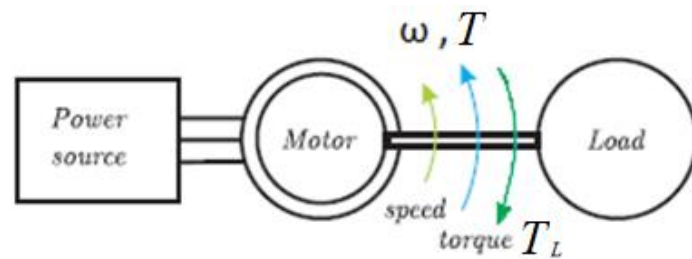


Static characteristics and stability of electric drive

Structure of actuator system based on an electric drive



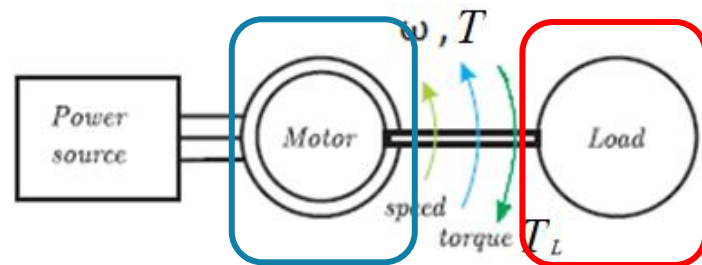
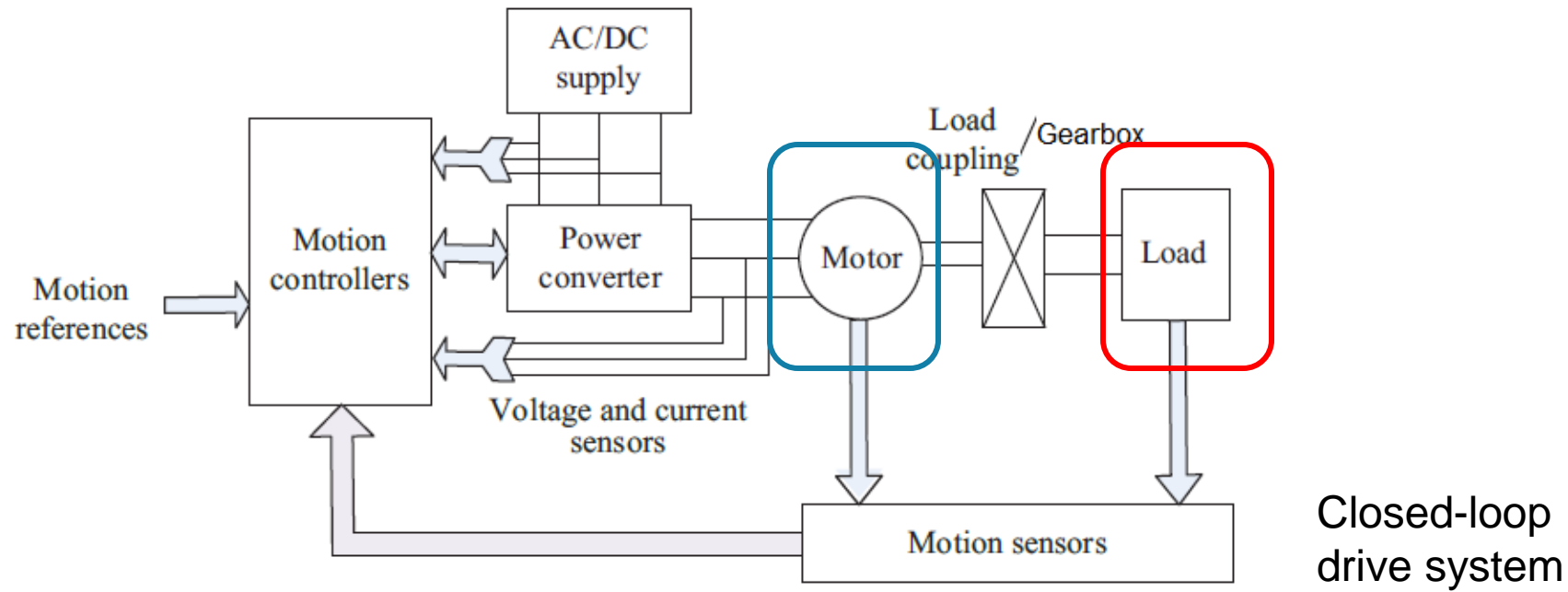
Closed-loop drive system



T – Motor torque
 T_L – Load torque

Open-loop drive system

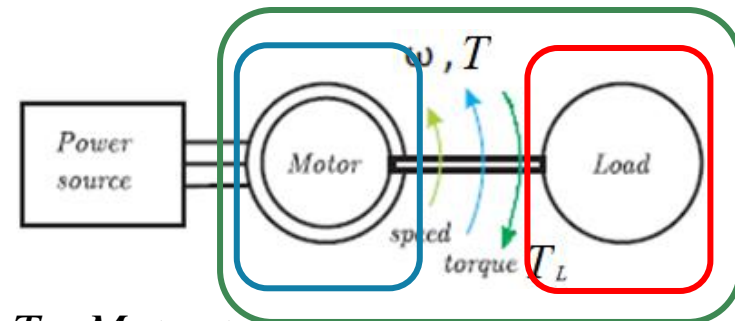
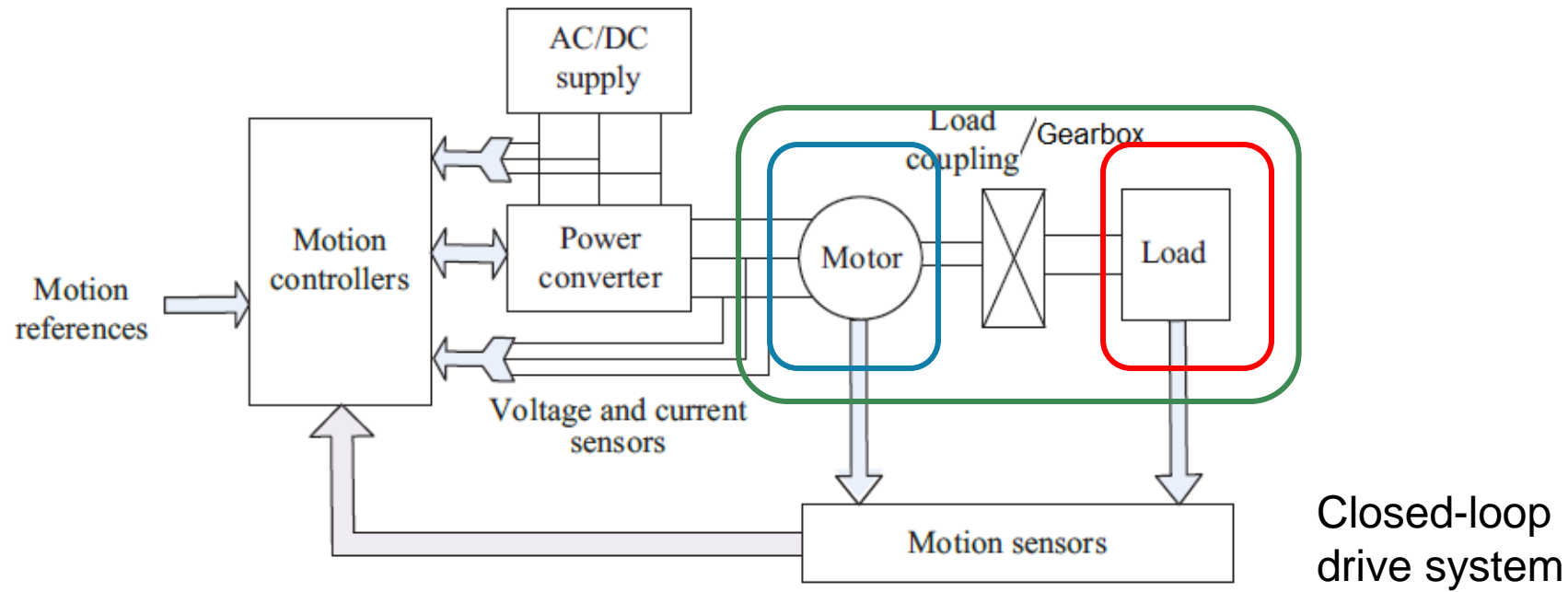
Structure of actuator system based on an electric drive



T – Motor torque
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Open-loop
drive system

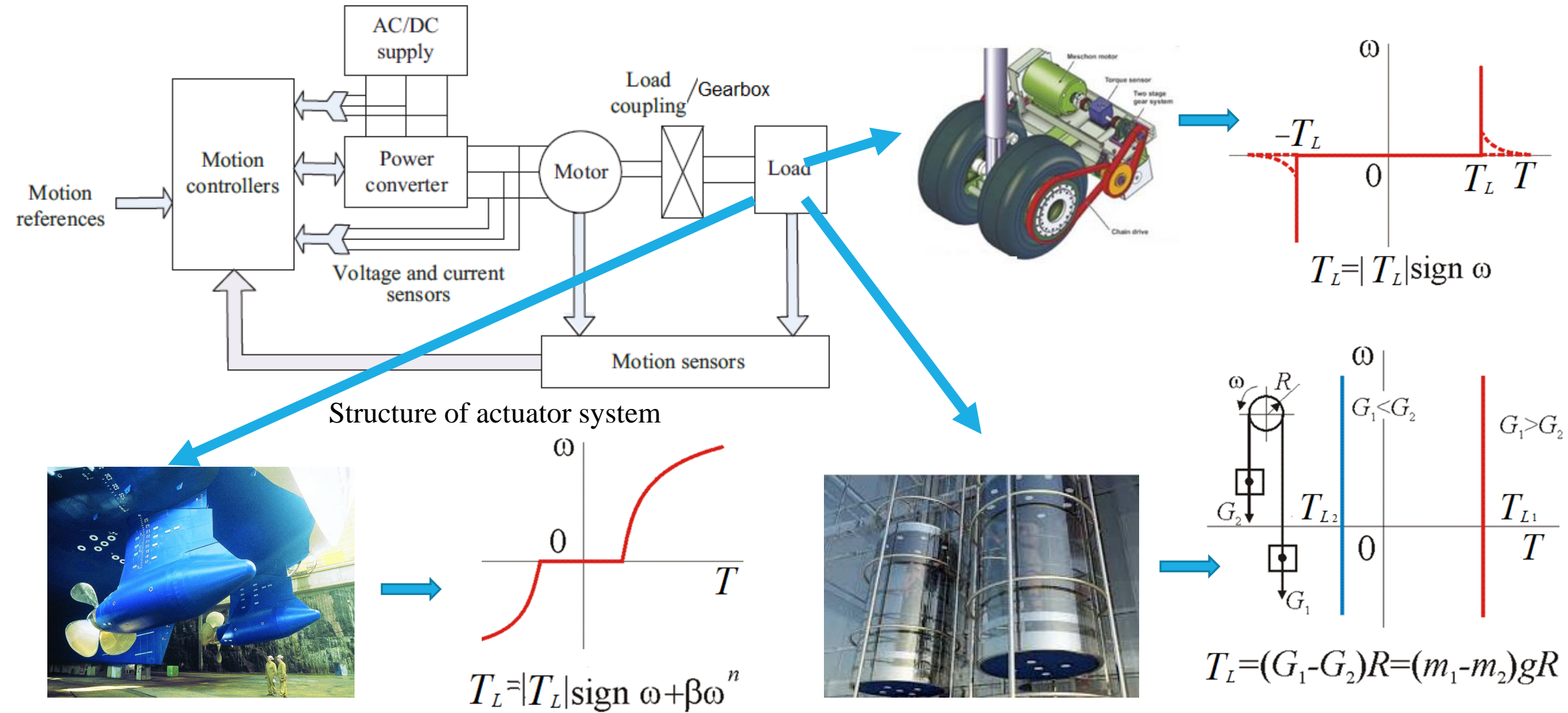
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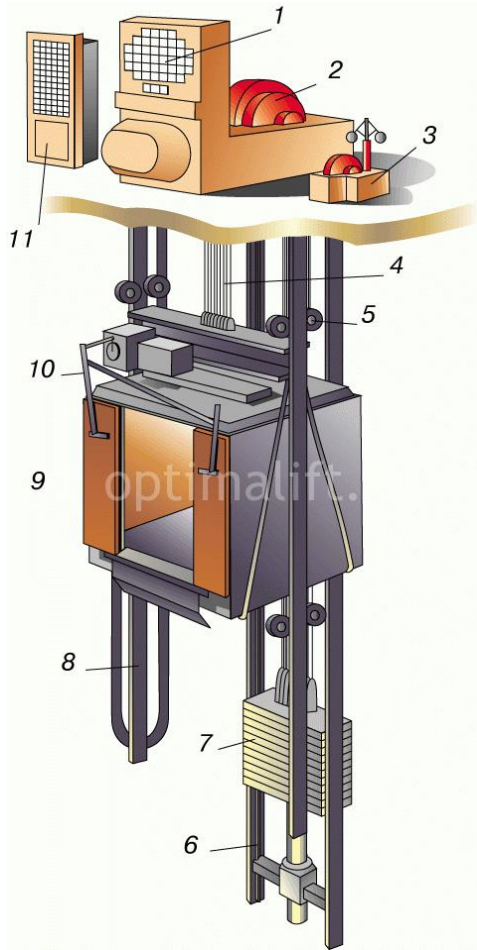
T – Motor torque
 T_L – Load torque

Open-loop
drive system

Structure of actuator system based on an electric drive



Types of working mechanisms



Mechanical or static characteristics determine properties of electric motor and drives (speed/ torque characteristics)

- Rotation movements: $T = f(\omega)$, $T = f(\alpha)$
- Linear movements: $F = f(v)$, $F = f(s)$

- | | |
|----|---------------------------|
| 1) | $T_L = \text{const}$ |
| 2) | $T_L = F(\omega)$ |
| 3) | $T_L = F(\alpha)$ |
| 4) | $T_L = F(\omega, \alpha)$ |
| 5) | $T_L = F(t)$ |

T_L - static torque or load torque

T – motor torque

(we need to develop such value by motor to provide a given motion or force in static mode $T = T_L$)

Figure. Automatic passenger elevator (scheme): 1 - a computer that controls the operation of the elevator; 2 - motor; 3 - elevator control system; 4 - cab cables; 5 - rollers; 6 - rails of the counterweight; 7 - counterweight; 8 - rails of the elevator car; 9 - a cabin; 10 - a mechanism for opening cabin doors; 11 - memory bank floor data.

Types of working mechanisms: 1st type ($T_L = \text{const}$)

1st type working mechanisms - static load (or torque) $T_L = \text{const}$

Forces and torques created by external sources of energy relative to the drive and independent of its movement are called *active*.

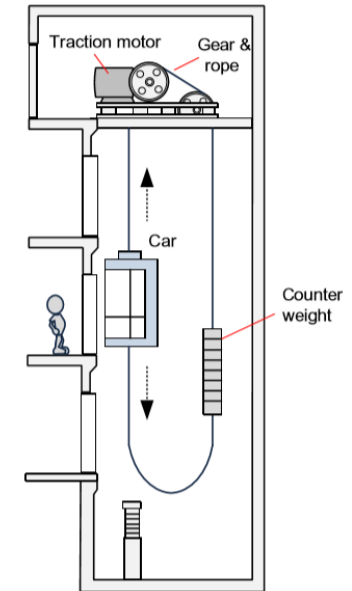
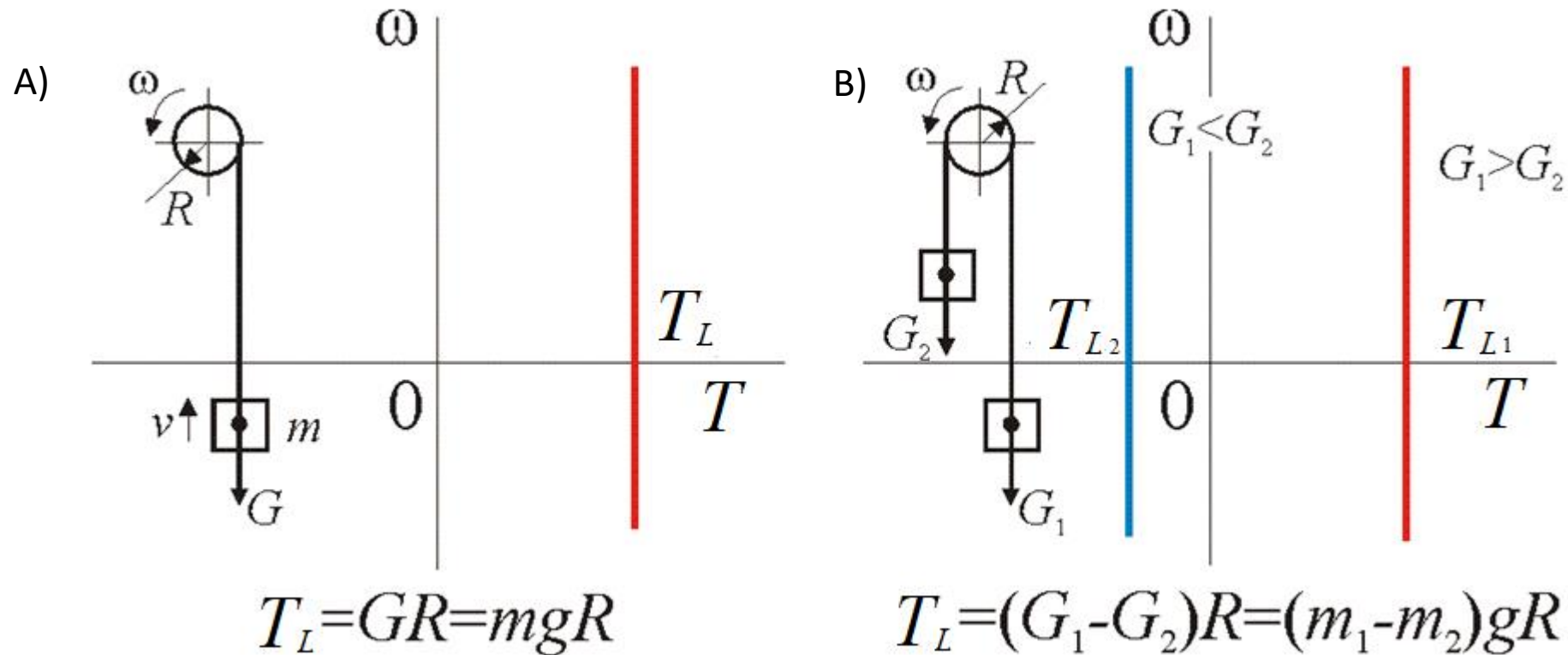


Figure . Examples of active loads created by an unbalanced (a) and balanced (b) lifting mechanism

Types of working mechanisms: 1st type ($T_L = \text{const}$)

Reactive are forces and torques that arise as a reaction to active action. These loads always act in the opposite direction to the movement of the electric drive, i.e. change their direction when the sign of speed changes $M_L = |M_L| \text{sign } \omega$

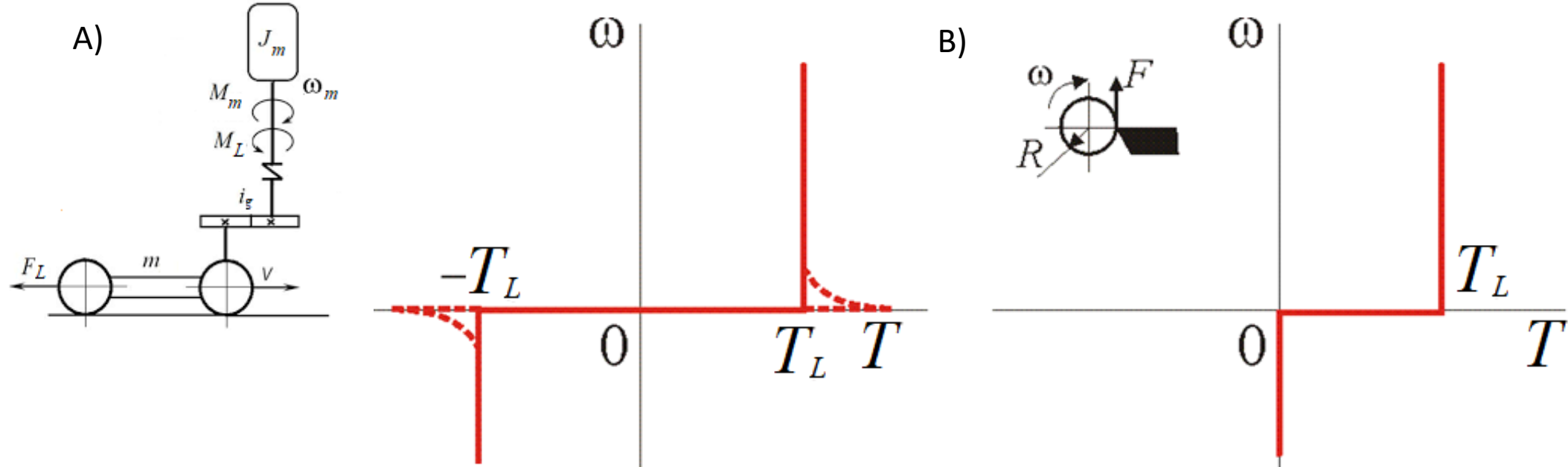


Figure 3. Examples of reactive loads created by dry friction (a) and created on the spindle shaft of a lathe when machining a part with a cutter (b)

Types of working mechanisms: 2st type ($T_L = f(\omega)$)

2st type working mechanisms - static torque depends on speed $T_L = f(\omega)$

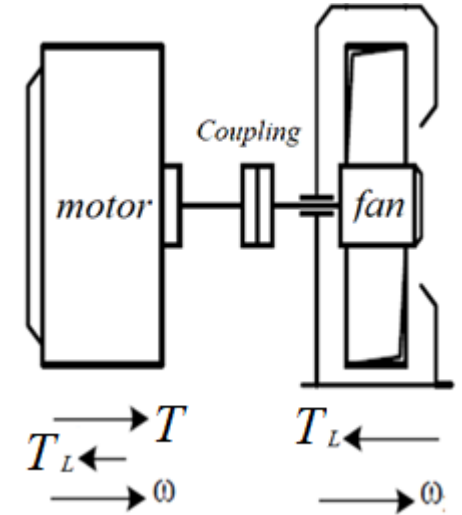
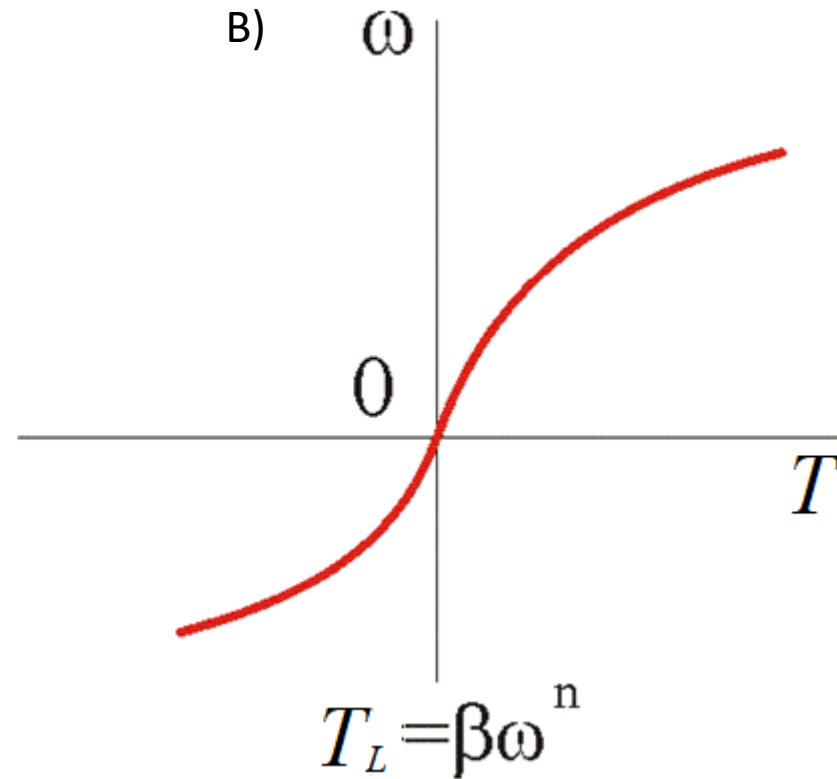
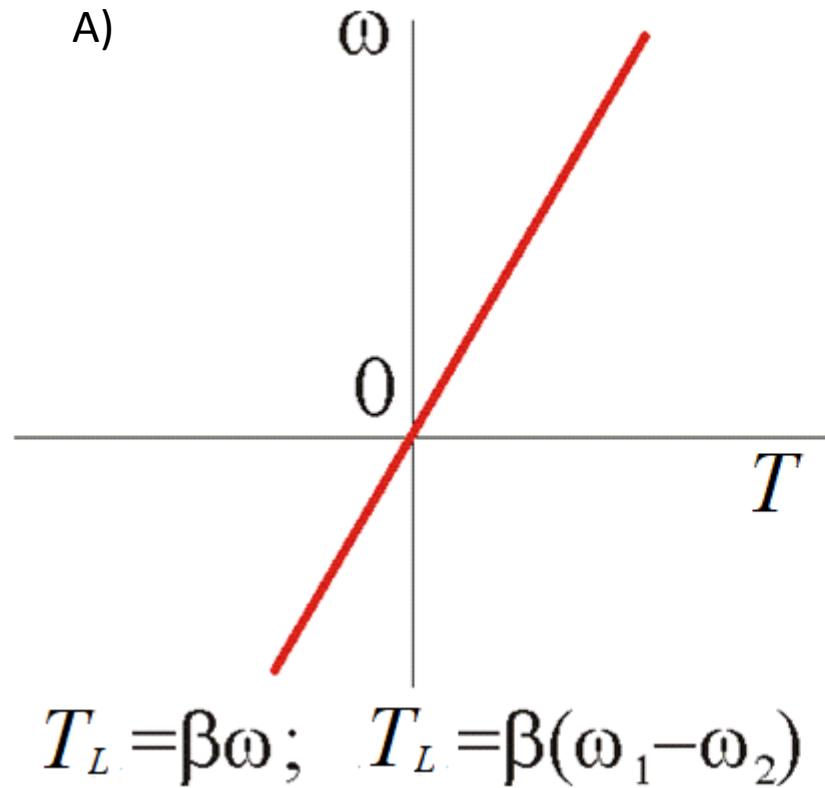


Figure. Reactive «viscous friction» torque (a) and torque load of fan (b)

Types of working mechanisms: : 2st type ($T_L = f(\omega)$)

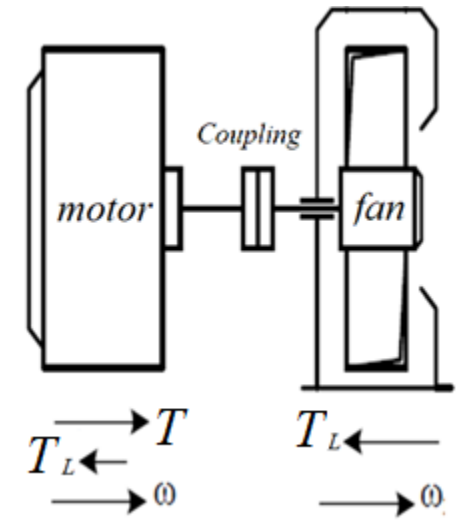
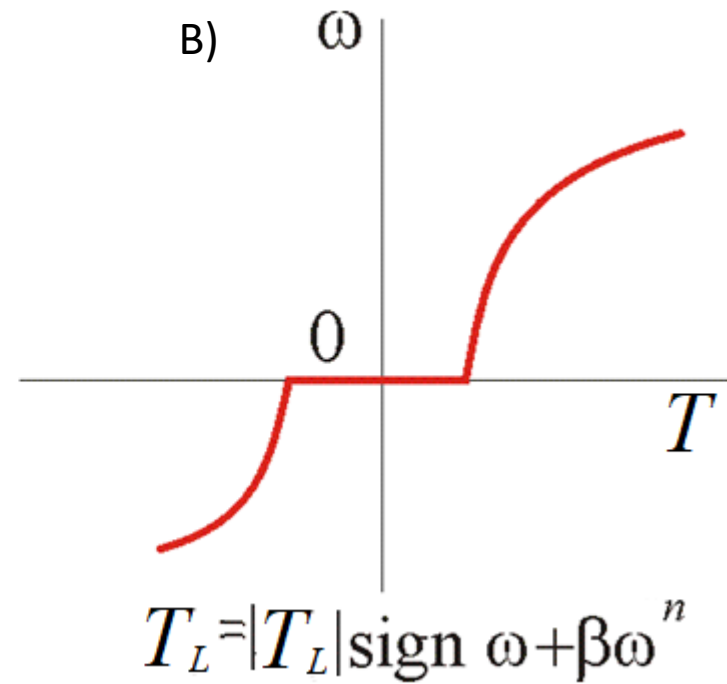
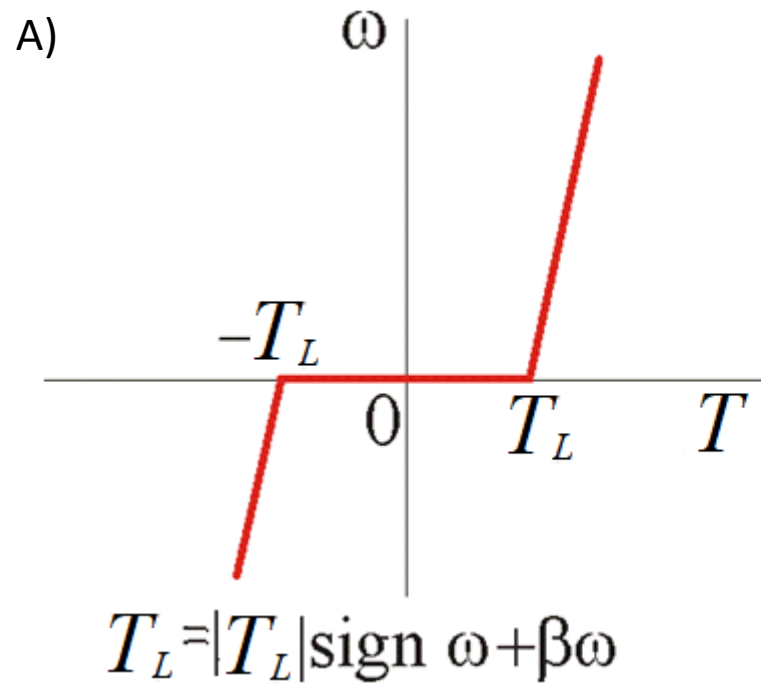


Figure. Reactive «viscous friction» torque (a) and torque load of fan (b) with nonlinearity

Types of working mechanisms: 3^d type ($T_L = f(\varphi)$) and 4th type ($T_L = f(\omega, \varphi)$)

3^d type working mechanisms - static torque depends on the path, i.e. $T_L = f(\varphi)$.

4th type working mechanisms - static torque depends on both on speed and on the path

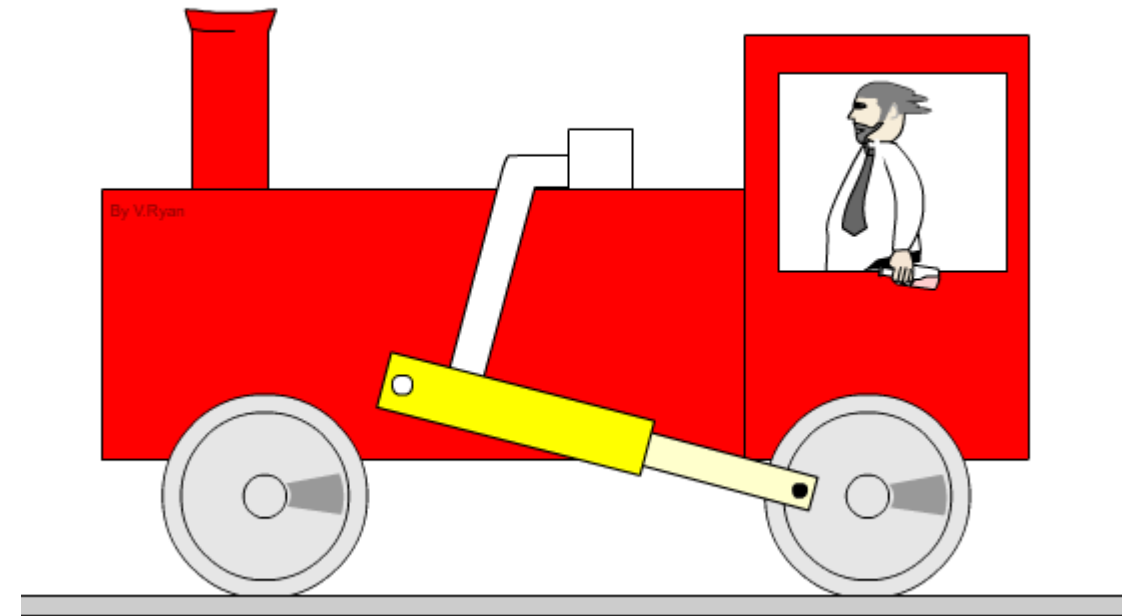
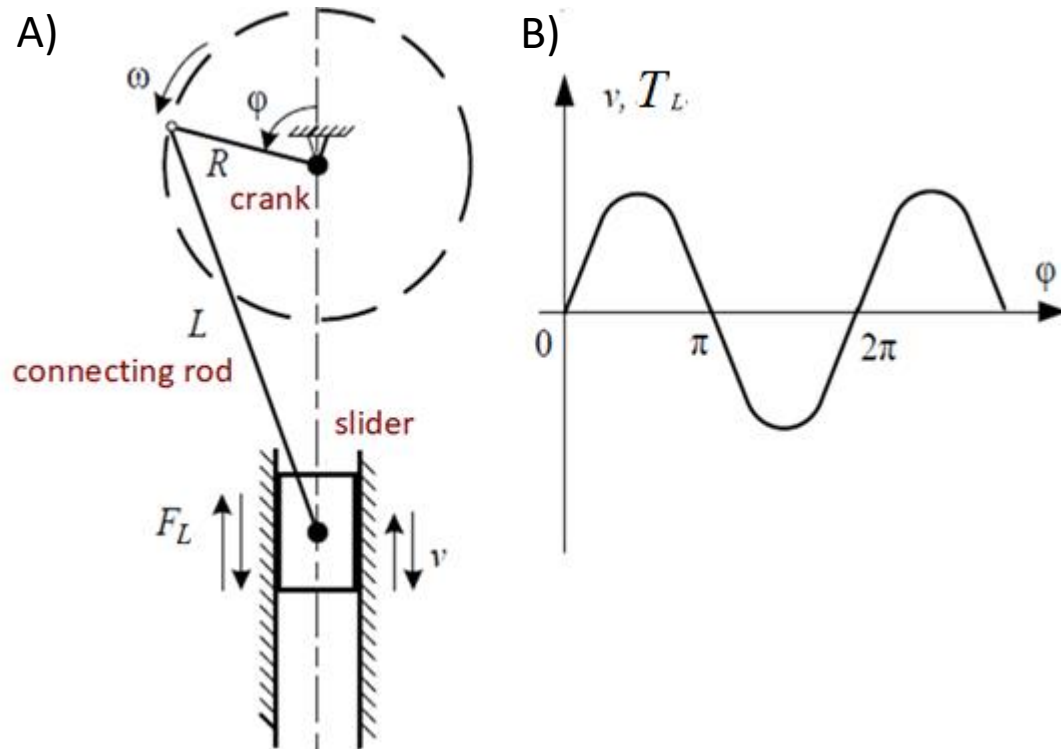


Figure. Kinematic diagram of the crank mechanism (a) the corresponding dependence $T_L = f(\varphi)$ (b)

Types of working mechanisms: 5^d type ($T_L = f(t)$)

5th type working mechanisms - static torque is a function of time $T_L = f(t)$, as well as mechanisms in which the load is random in nature.

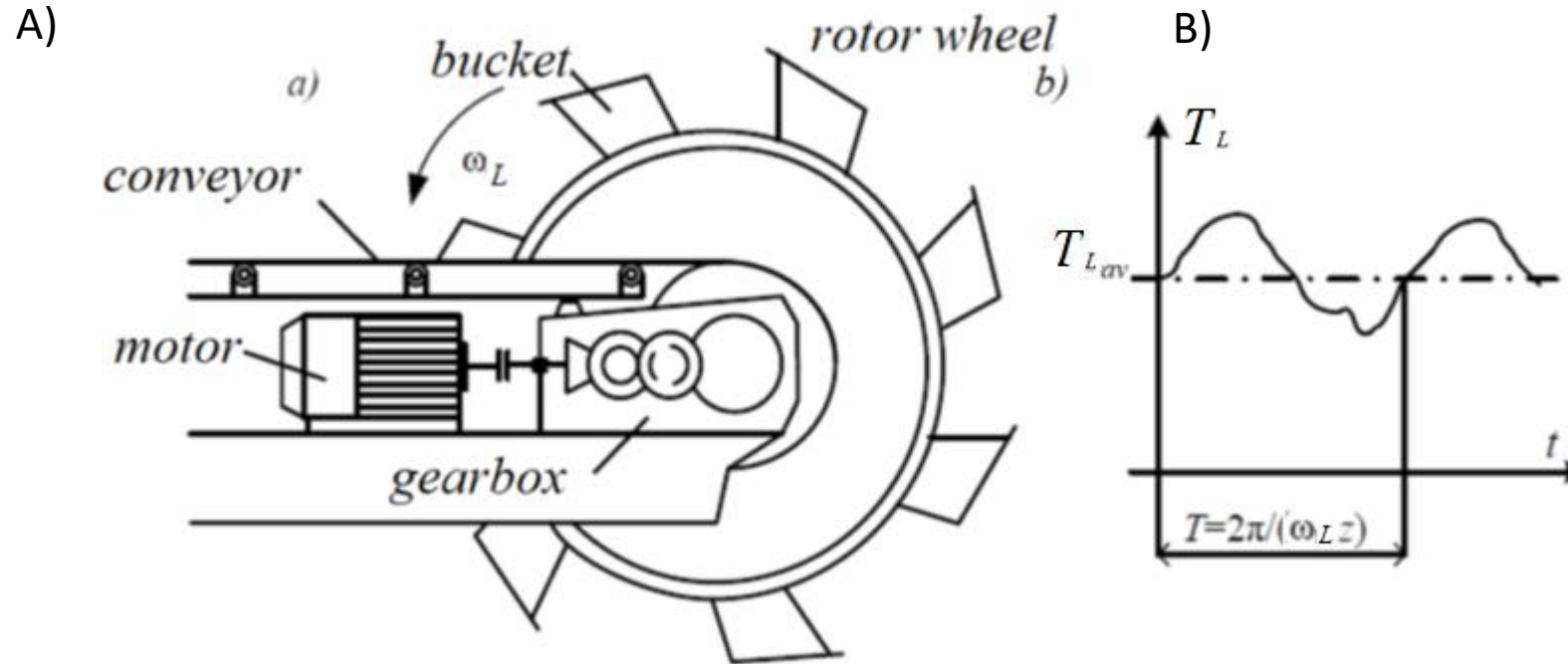
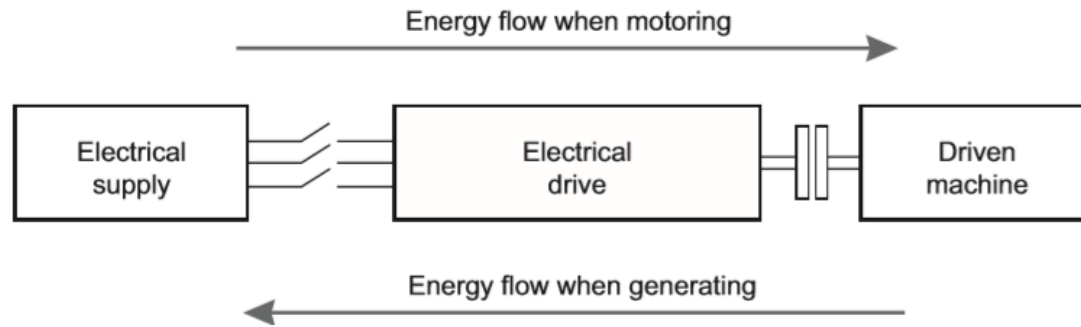
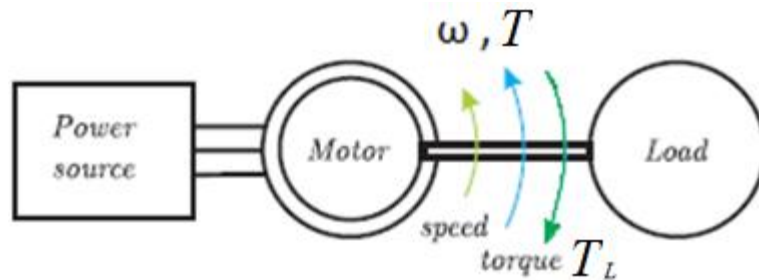


Figure 7. Kinematic diagram of the electric drive of the excavator's rotor wheel (a) and the corresponding dependence $T_L = f(t)$ (b) and bucket wheel excavators used on open cast mines (c)

Modes of operation

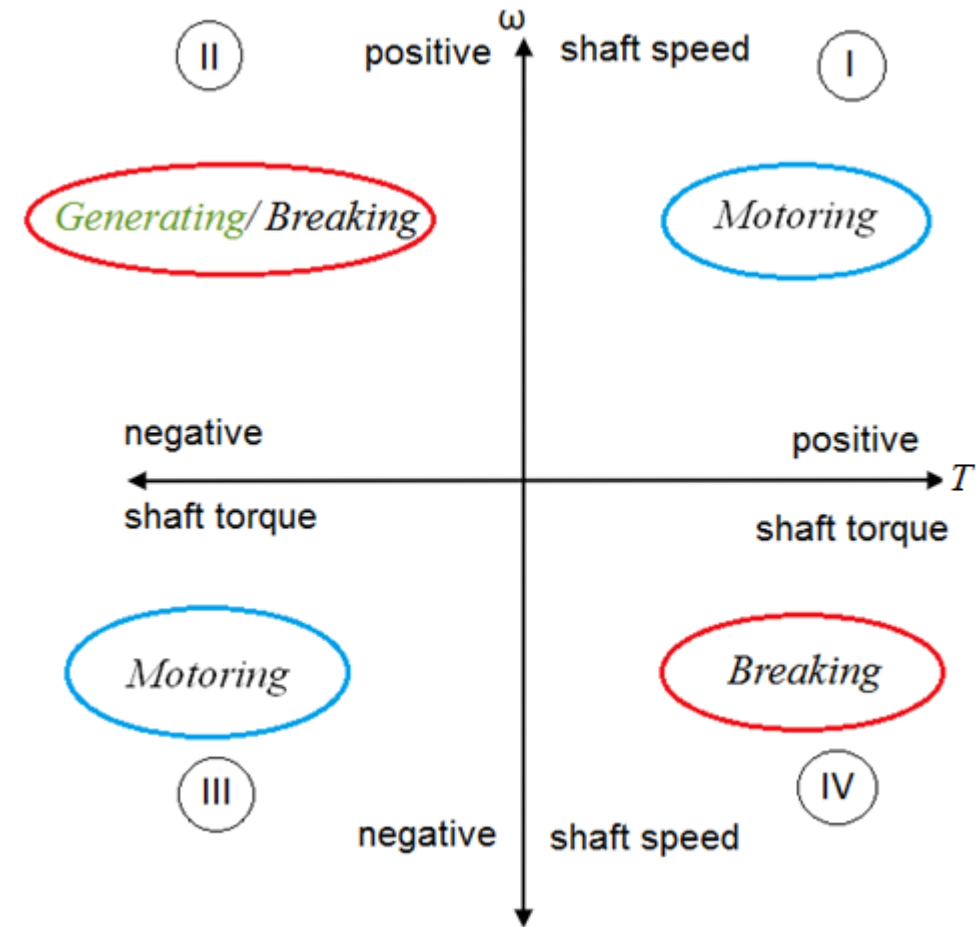


(a) Energy flows



T – Motor torque
 T_L – Load torque

(b) Motor with power supply



(c) Operating modes

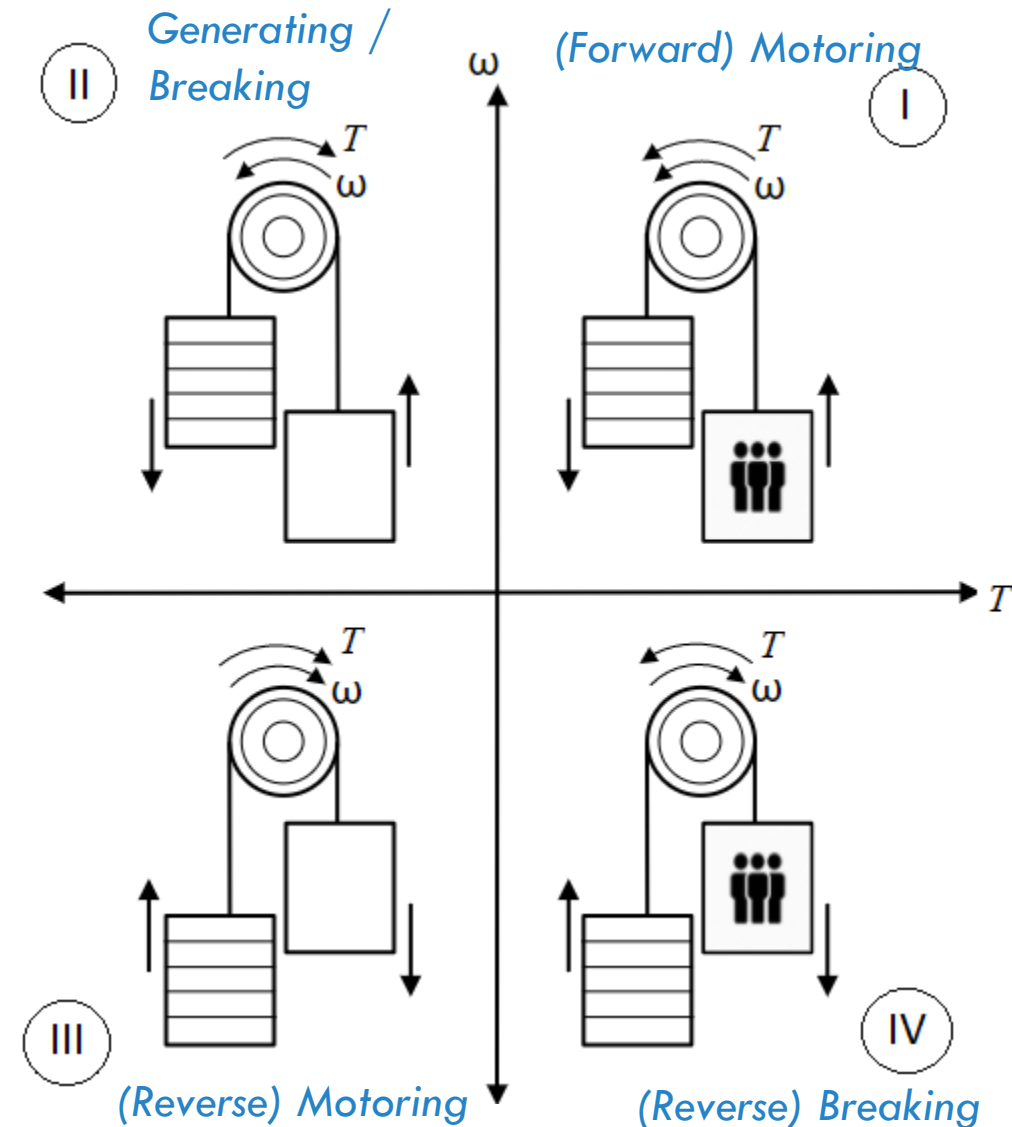
Modes of operation

Depending on the design of the controller, the electrical drive operates in the I/III quadrant only (motoring mode), e.g. a **pump**, or in all four quadrants, e.g. a **hoist**.

Regenerative braking of motor (in II quadrant)

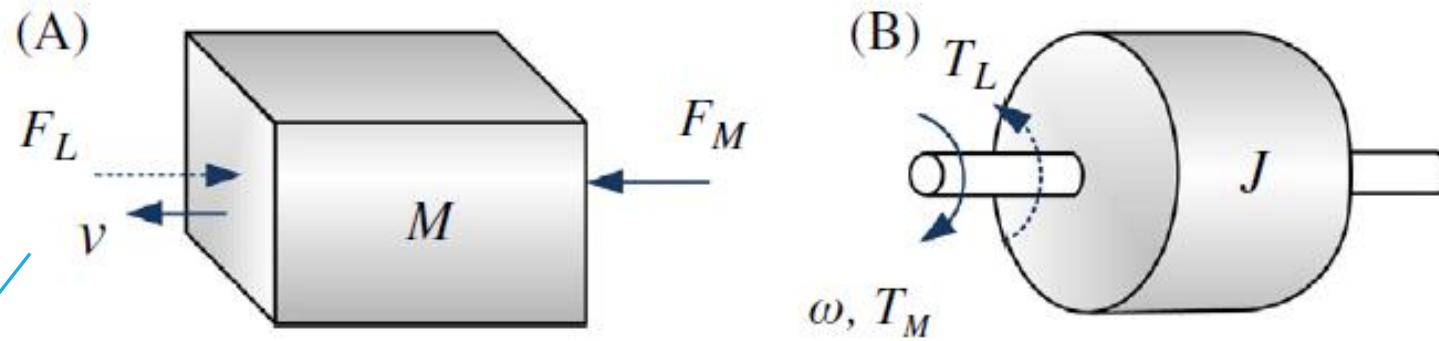
- most of the mechanical energy from the load is returned to the power source. Attention is drawn to the fact that some power sources are not able to accept any (or only a limited amount (batteries)) generated energy.
- ‘*dissipative*’ braking operation where most of the mechanical load energy is burned up in an external brake-chopper resistor. Brake choppers come in all sizes, in off-shore cranes and locomotives, power levels of several megawatts are common practice.

Reverse braking mode of motor (in IV quadrant) is one where mechanical power is completely returned to the motor at the same time some or none electrical power is delivered, i.e. both mechanical and electrical input power are dissipated in the motor.



Drive's operating modes

The equation of motion



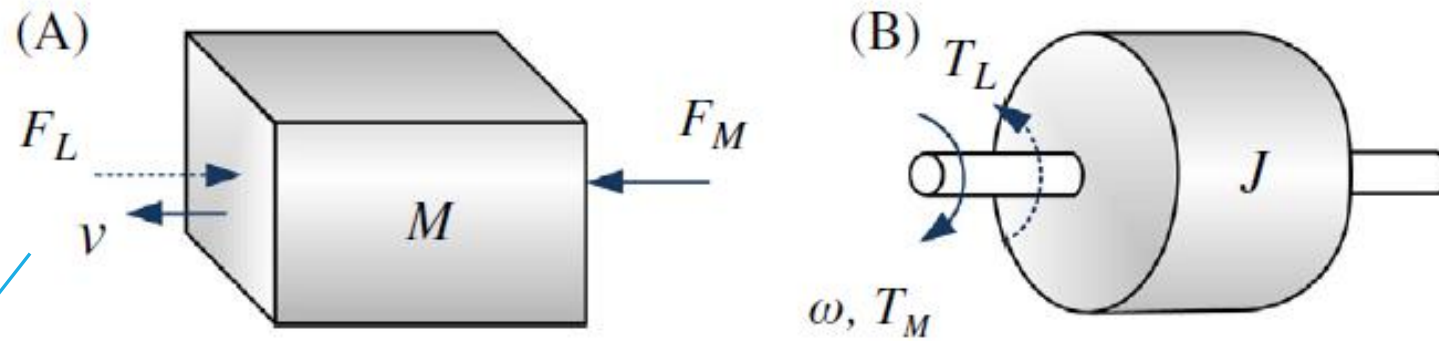
(A) Linear motion and (B) rotational motion objects.

$$F_M - F_L = \frac{d}{dt}(Mv) = M \frac{dv}{dt} + v \frac{dM}{dt}$$

$$F_M - F_L = M \frac{dv}{dt} = Ma \quad \text{Newton's second law}$$

$a(=dv/dt)$ object's acceleration

The equation of motion



(A) Linear motion and (B) rotational motion objects.

$$F_M - F_L = \frac{d}{dt}(Mv) = M \frac{dv}{dt} + v \frac{dM}{dt}$$

$$F_M - F_L = M \frac{dv}{dt} = Ma$$

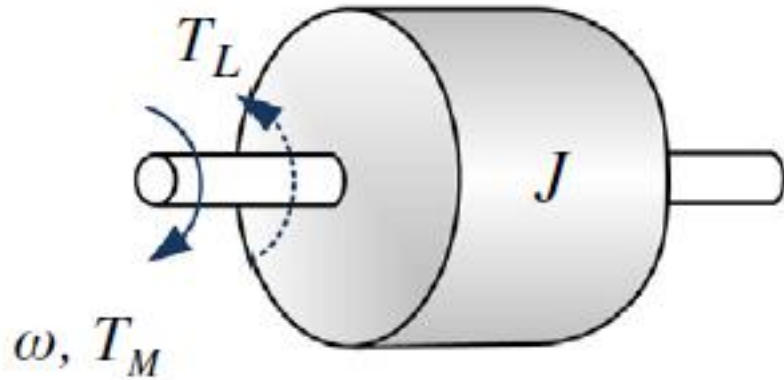
$a (= dv/dt)$ object's acceleration

Newton's second law

$$T_M - T_L = \frac{d}{dt}(J\omega) = J \frac{d\omega}{dt} + \omega \frac{dJ}{dt}$$

T_M and T_L - driving and load torque,
 ω - angular speed of the rotating object
 J - the moment of inertia

The equation of motion



Newton's second law

$$T_M - T_L = J \frac{d\omega}{dt} = J\alpha$$

$\alpha (= d\omega/dt)$ - object's angular acceleration

$T_M > T_L$ the object will accelerate

$T_M < T_L$ it will deaccelerate

$T_M = T_L$ the speed will not be changed

The equation of motion

The movement of the mechanical part of the electric drive is described by equation which is called *fundamental equations* of motion.

$$T - T_L = J_{\Sigma} \frac{d\omega}{dt}$$

where T – motor torque; T_L – total static (load) torque acting on all elements of the system; J_{Σ} – total moment of inertia of moving masses.

This equation allows us to analyze the *dynamic* modes of motion of the electric drive, which can be *transient* or *steady*.

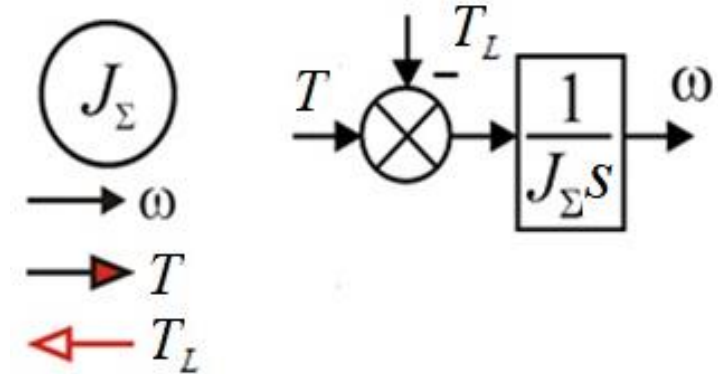
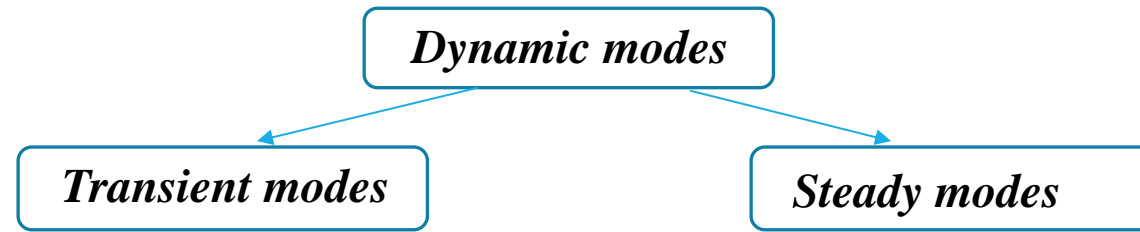


Figure. Model of one-mass mechanical system

The modes of operation



Transient dynamic modes (transient processes) in an electric drive are associated with its transition from one steady state to another under the influence of a change in T or T_L .

Steady-state dynamic modes (stationary) of an electric drive occur when T , T_L or J_Σ changes periodically (with a variable reduction radius \tilde{r}), when there is a mode of steady-state forced oscillations of **the coordinates of the electric drive (torque, speed, etc.)**.

A particular case of steady-state dynamic mode is the **steady-state static mode** of operation of the electric drive, when the right-hand side of equation (1), which is a dynamic torque, is zero:

$$T - T_L = J_\Sigma \frac{d\omega}{dt} = T_{DYN} = 0$$

and correspondingly, $d\omega/dt = 0$, $T = T_L$, T_{DYN} - dynamic torque

The modes of operation

$$T - T_L = J_{\Sigma} \frac{d\omega}{dt}$$

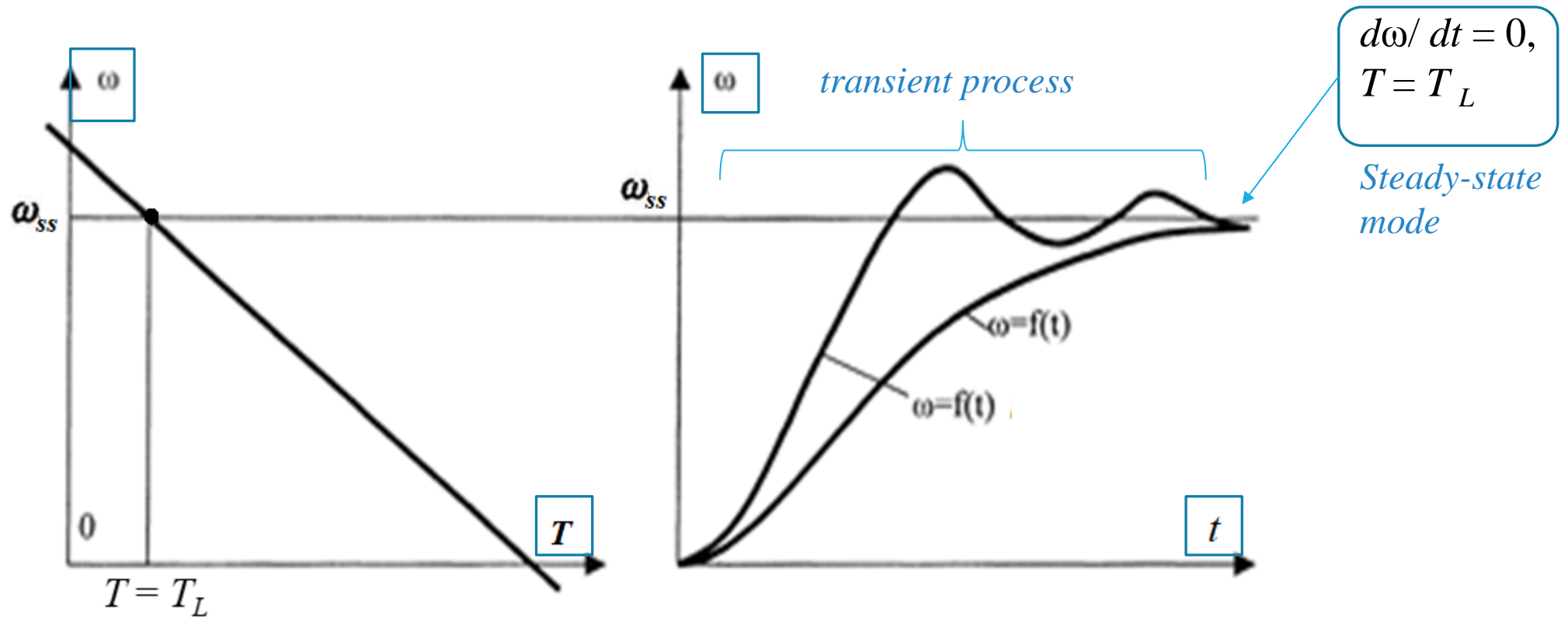
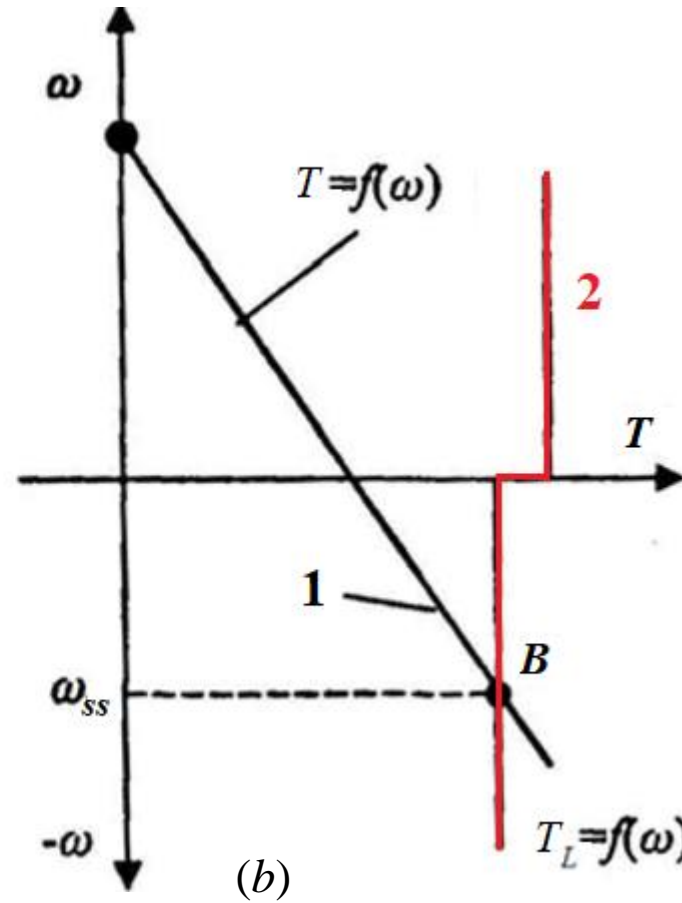
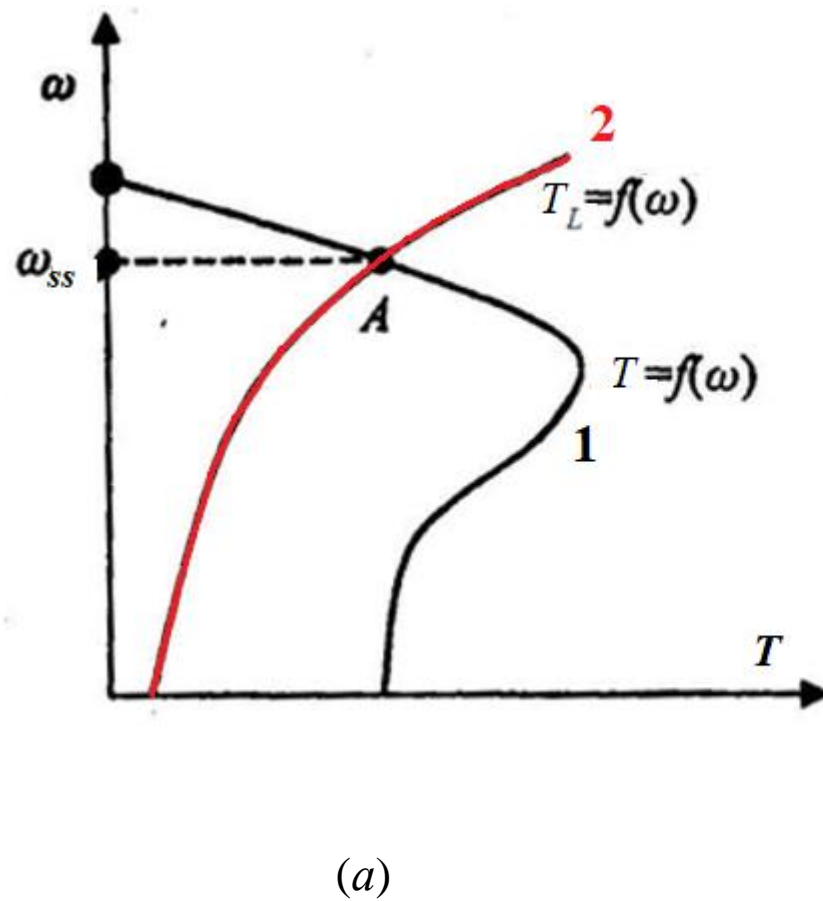


Figure. Static speed-to-torque curve (a) and transient process in the drive (b)

The modes of operation

The point (equilibrium point) of the steady-state in the coordinate system T, ω is obtained as the intersection point of the mechanical characteristics $\omega = f(T)$ and $\omega = f(T_L)$, in which with $T = T_L$.



A, B - equilibrium points

ω_{ss} - steady-state speed

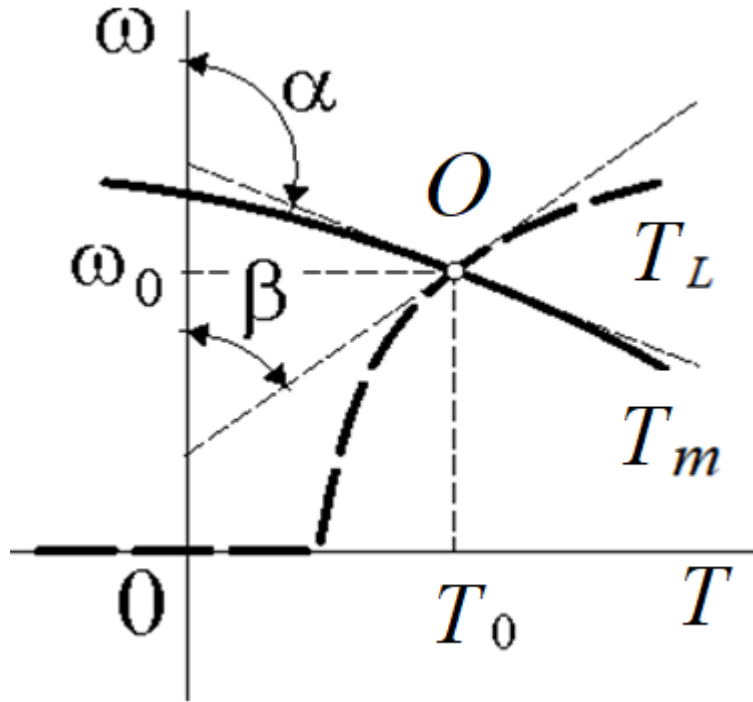
$\omega = f(T)$ - mechanical speed-torque characteristic of the motor (1)

$\omega = f(T_L)$ - mechanical speed-torque characteristic of the mechanism (2)

Figure. Graphical determination of the speed of steady-state operation of the drive

Static stability of electric drive

To ensure the static stability of the drive, *it is necessary that at the point under consideration the slope of the mechanical characteristics of the motor is less than the slope of the mechanical characteristics of the load.*



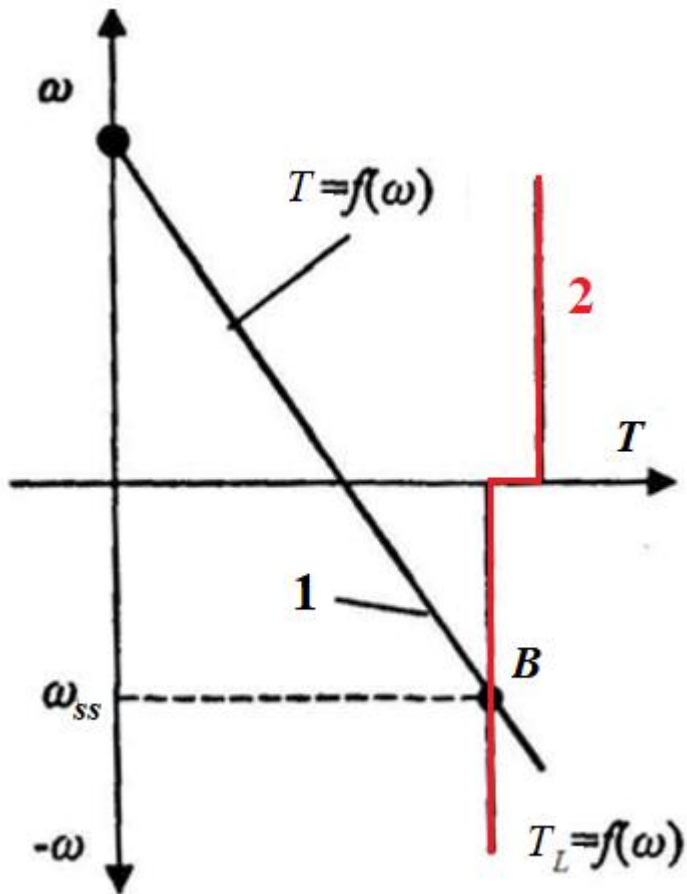
$$\left. \frac{dT_m}{d\omega} \right|_{\omega=\omega_0} < \left. \frac{dT_L}{d\omega} \right|_{\omega=\omega_0} \quad (8)$$

T – motor torque; T_L – total static (load) torque
 O – equilibrium point

Figure. Static speed-torque curve of motor and load

Static stability of electric drive

It follows from condition (8) that stable operation of the drive at point with zero torque ($T_L = 0$) or under a static load of the type $T_L = \text{const}$ is possible only if there is negative slope $\left. \frac{dT_m}{d\omega} \right|_{\omega=\omega_0} < 0$

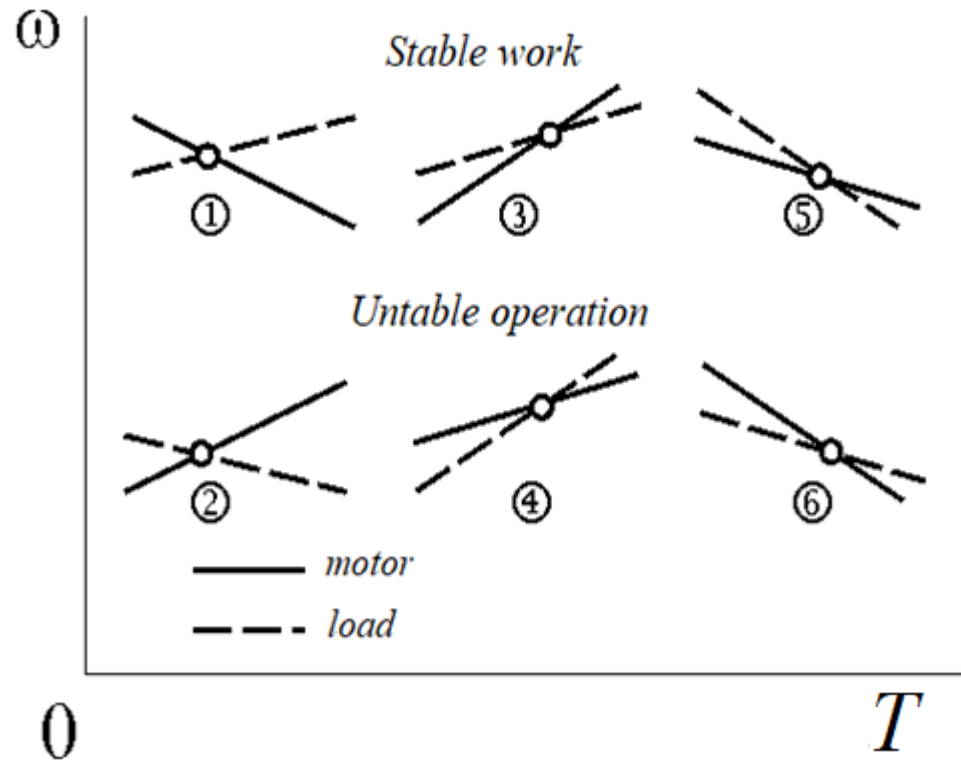


$$\left. \frac{dT_m}{d\omega} \right|_{\omega=\omega_0} < \left. \frac{dT_L}{d\omega} \right|_{\omega=\omega_0} \quad (8)$$

Figure. Static speed-torque characteristics (curve) of motor and load

Static stability of electric drive

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Figure. Static speed-torque characteristics (curve) of motor and load

Four main categories of mechanical characteristics

$$\beta_h = \frac{T_2 - T_1}{\omega_2 - \omega_1} = \frac{\Delta T}{\Delta \omega}.$$

– rigidity factor of speed-torque curve

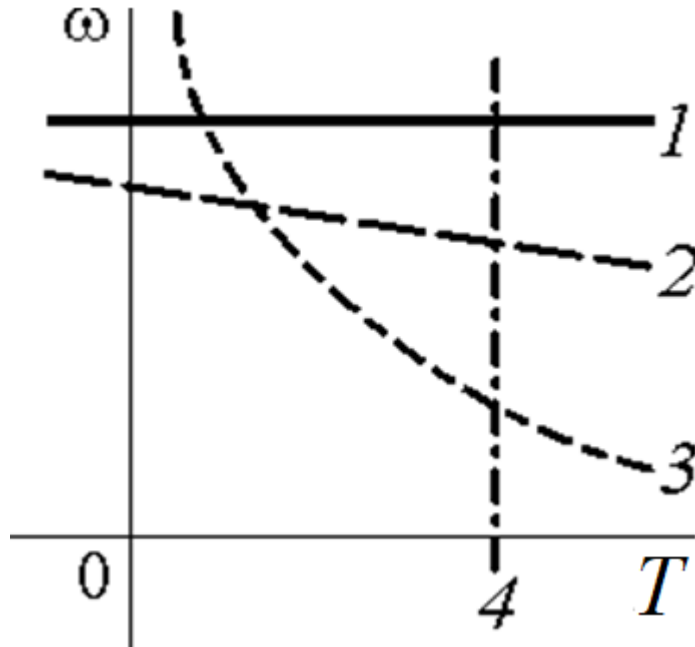
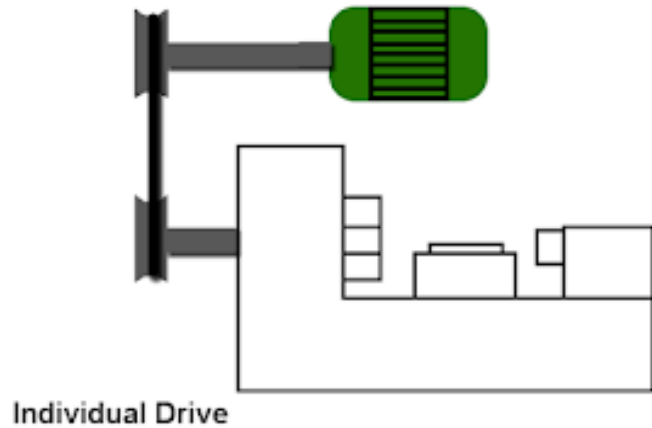


Figure. Static speed-torque characteristics of motor and load

1. *Absolutely “rigid” characteristic* – $\beta_h = \infty$ motor speed remains constant independently of the torque. Synchronous motors, for example, have this characteristic 1 (1, Fig. 14).
2. *“Rigid” characteristic* – $0 \ll \beta_h < \infty$ motor speed changes slightly with changing torque. Separately excited DC motors and asynchronous motors on the working area of the mechanical characteristic have such characteristic (2, Fig. 14).
3. *“Soft” characteristic* $\beta_h > 0$ - the motor speed changes significantly with a change in torque. DC series motors have this characteristic. Moreover, the slope of torque versus ω of their characteristics is different at different points (3, Fig. 14).
4. *Absolutely “soft” characteristic* $\beta_h = 0$ – motor torque remains constant independently of rotation speed. This characteristic is, for example, separately excited DC motors when feeding the armature from a current source (4, Fig. 14).

Design schemes of the mechanical part of the electric drive



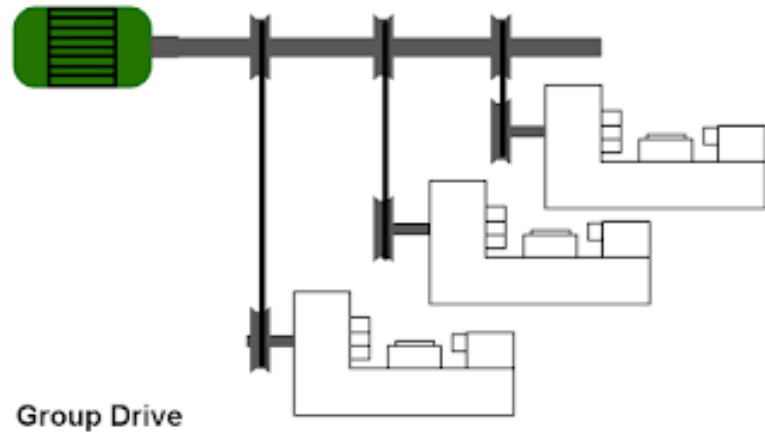
In the **Individual Drive** system, each machine tool has its own electric motor which drives the machine through belt, chain, gearing or by direct coupling. The system is also called as a self-contained drive.

Advantages:

- The machines can be installed at any desired position
- If there is fault in one motor other machines will not be affected
- Speed of each machine can be controlled
- The absence of belts and line shafts reduces the risk of accidents to operator

Disadvantages:

- Initial high cost
- For a single machine such as lathe, one single motor is used to control various mechanisms by means of mechanical parts like gears and mechanism so power loss will take place



The **Group Drive system** uses a high powered motor which drives the group of machines through belting and pulleys serve to vary their speed.

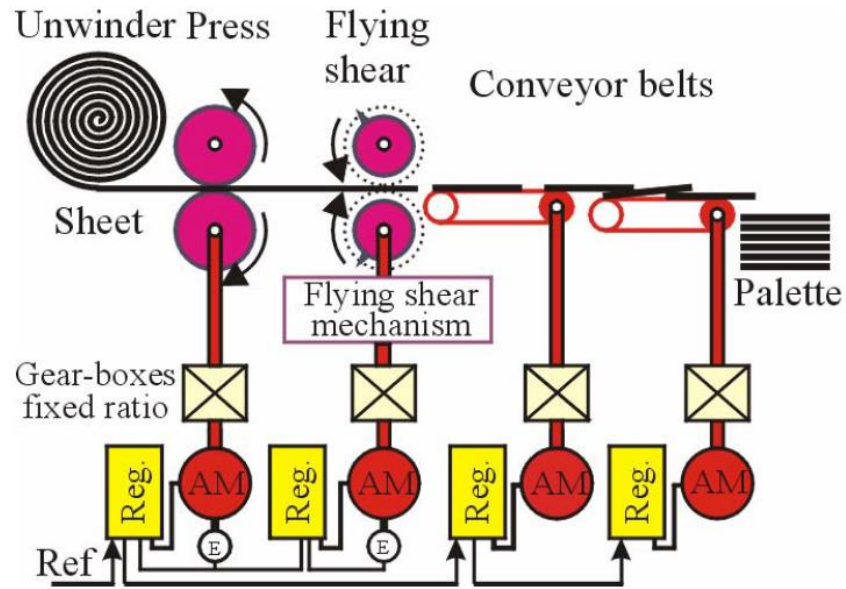
Advantages:

- Installation cost and cost of one large motor will be much less than a number of small motor
- The efficiency and power factor of a large group motor will be higher
- In group motor drive operations can be stopped simultaneously

Disadvantages:

- The breakdown of large motor causes all operations to be stopped
- If the most of the machines are idle then the main motor will operate on load with less efficiency
- Noise level at the work site is quite high
- Speed control of individual machines are not possible

Multi-motor Drives



The **Multi-motor** drive consist of several individual motor which serve to one of many motions or mechanism in some production unit. For example, in travelling crane, there are three motors used. One for hoisting, other for long travel motion and third for cross travel motion.

Features:

- In multi motor drives, separate motors are used for operating different parts of same mechanism
- Each motor is used to drive only one of the many working mechanisms in a machine
- Examples are metal cutting machine tools, paper making machines and rolling mills , etc.,



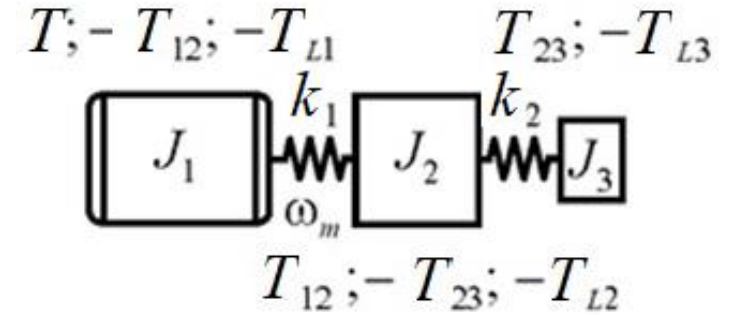
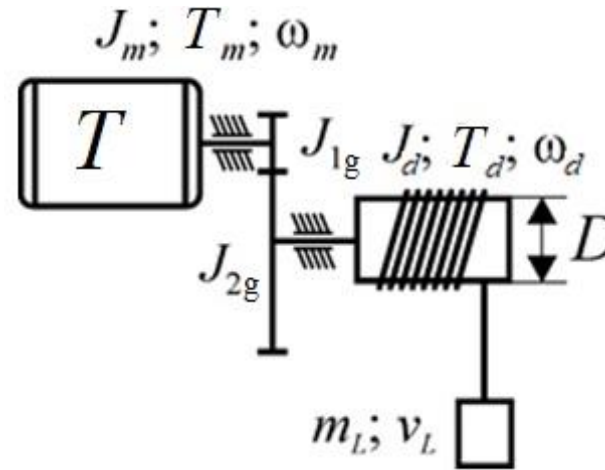
Types of electric drives

INDIVIDUAL DRIVE	GROUP DRIVE
It is suitable for small size workshop where machines may be moved frequently and machines are scattered over large area	It is suitable for medium and large size workshop where machines are not scattered over large area
Speed of a machine can be controlled separately	Cone pulleys required to obtain a wide range of speed
Machine shaft can be rotated in any direction	Difficult to change the direction of main shaft
Individual machine does not affect other machines when the failure of a motor occurs.	Failure of the main motor will stop the entire group of machines.
Less power is wasted if less machines are working.	More power is wasted if less machines are running but more economical when all machines are working in full load.
High initial capital investment.	Less initial capital investment.

Conversion of kinematic schemes



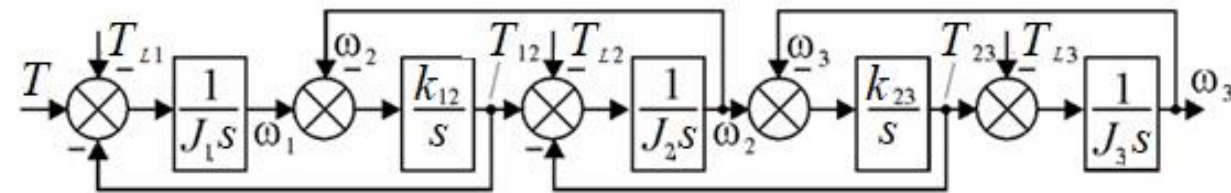
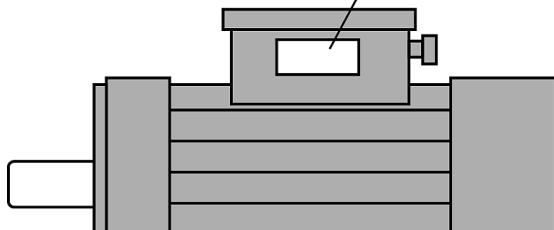
Real mechanism



kinematic schemes

1	Motor	3 ~	50 Hz	IEC 34-6	8
2				No.	7
3		15 kW		2910 r/min	6
			Cl. F	cos φ 0,90	
		400 V		230 V	4
		27,5 A		48,7 A	
	Cat. No.		IP 54	kg	

- 1 – Motor type and rated frequency
- 2 – Rated power
- 3 – Rated current and voltage in case of Star connection
- 4 – Rated current and voltage in case of Delta connection
- 5 – IP protection class
- 6 – power factor
- 7 – rotor speed
- 8 – cooling method

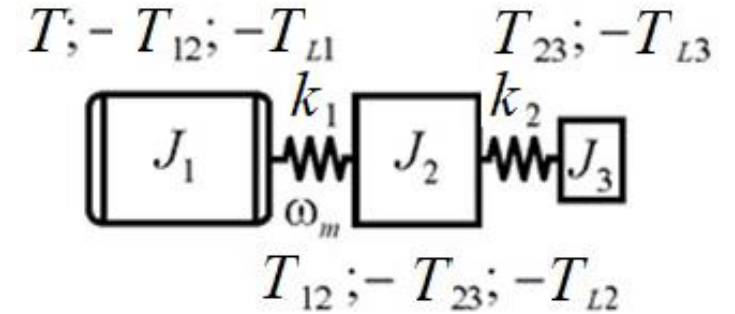
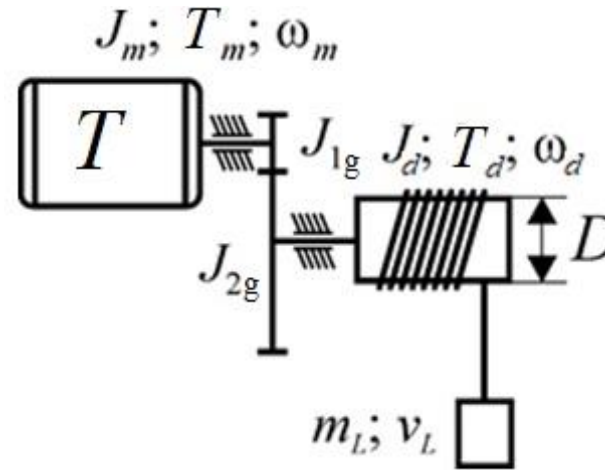


Mathematical model of three-mass mechanism (load)

Conversion of kinematic schemes

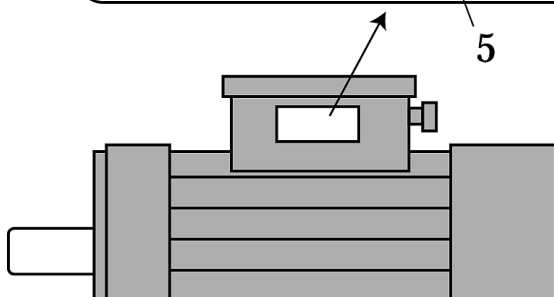


Real mechanism



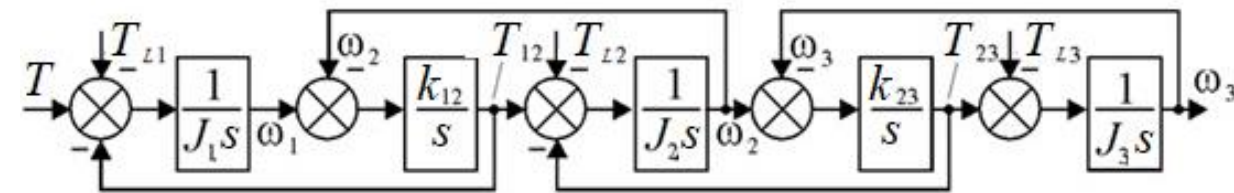
kinematic schemes

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				Cl. F	
				cos φ 0,90	
	400 V			230 V	4
	27,5 A			48,7 A	
	Cat. No.			IP 54	5
				kg	

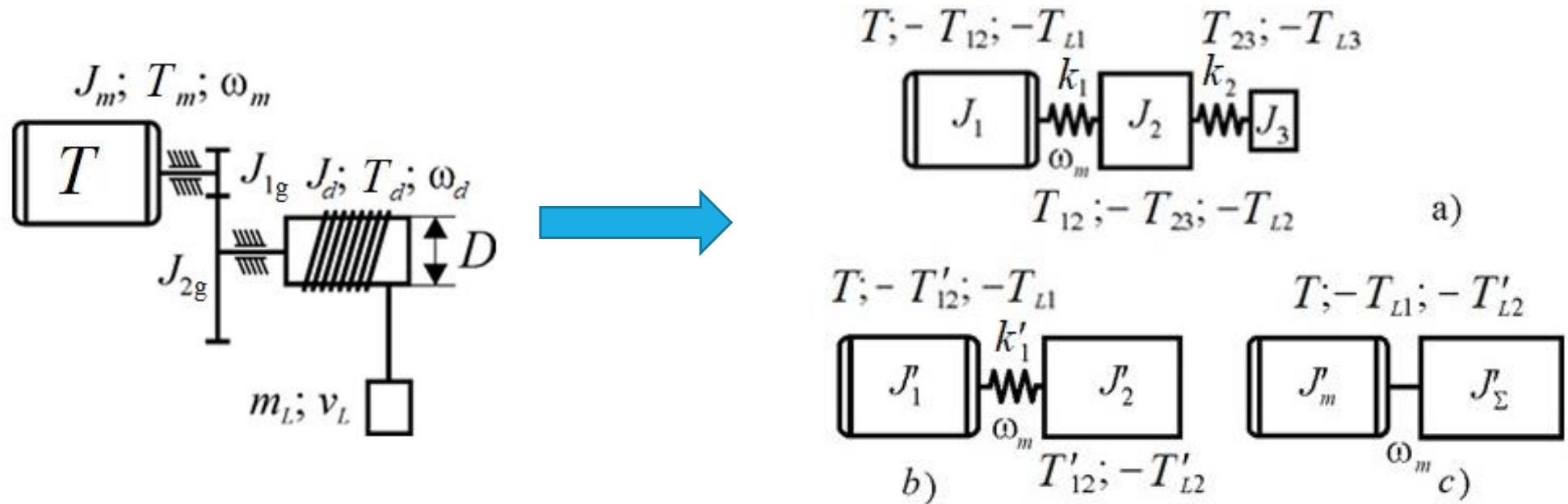


- 1 – Motor type and rated frequency
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- 5 – IP protection class
- 6 – power factor
- 7 – rotor speed
- 8 – cooling method

?

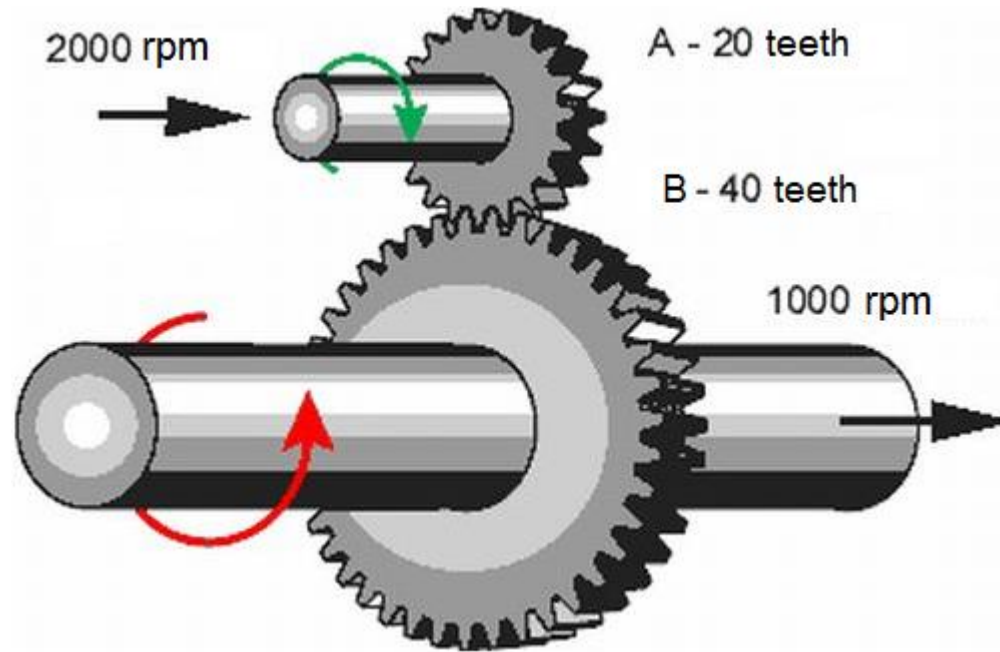


Mathematical model



- Conversion of static torques and forces
- Conversion of moments of inertia
- Link Stiffness Conversion

Gearbox. Basic relationships, principle of operation



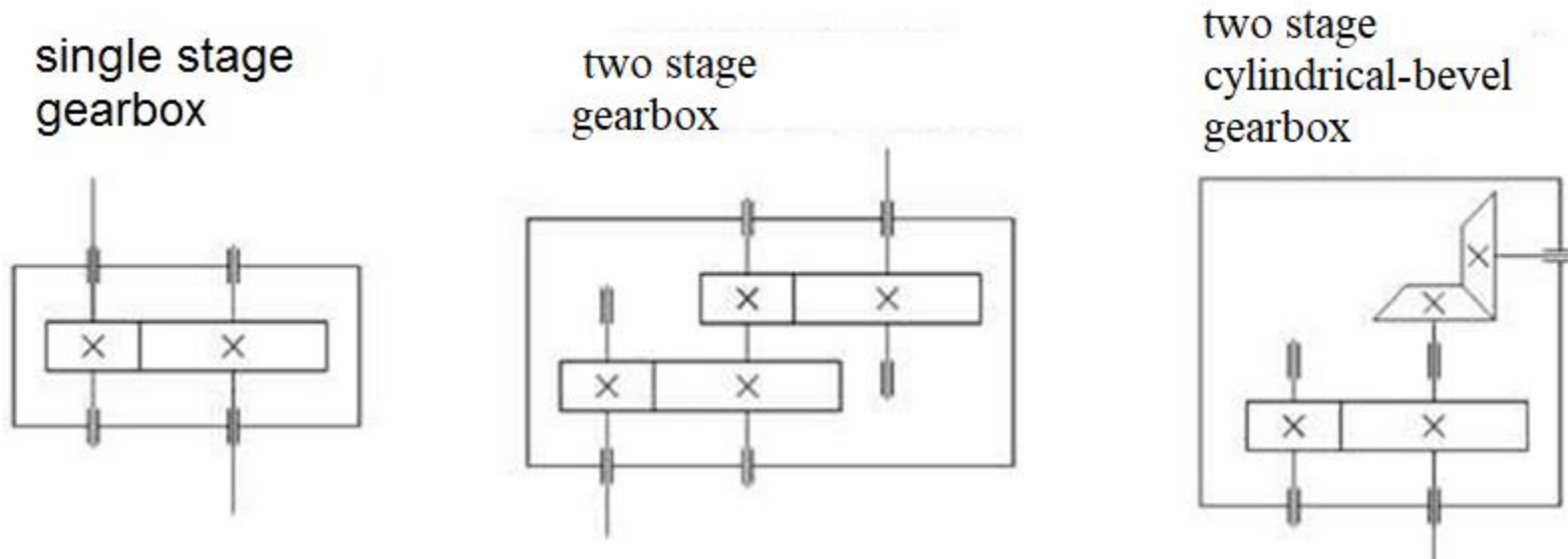
The gear ratio $j=j_{AB}$ is determined when the drive wheel A

$$j = j_{AB} = \frac{\omega_A}{\omega_B} = \frac{z_B}{z_A}$$

j - this is the ratio of the angular speed of the drive gear (A) to the angular speed of the driven gear (B).

j - this is the ratio of the number of teeth of the driven gear (B) to the number of teeth of the drive wheel (A)

Gearbox. Classification of cylindrical gears by the number of stages

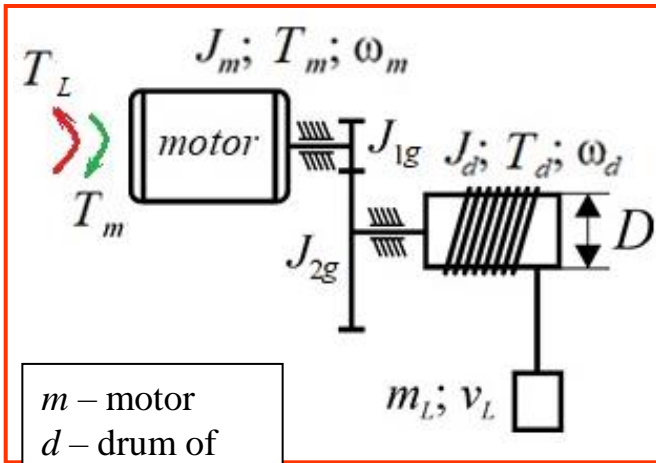


The number of gear stages is selected depending on the total gear ratio.
Gears have the following number of stages, depending on values:

- single-stage - $j = 1.6 \dots 6.3$;
- two-stage - $j = 8 \dots 40$;
- three-stage - $j = 25 \dots 60$.

- Conversion of static torques and forces
- Conversion of moments of inertia
- Link Stiffness Conversion

Conversion of static torques and forces to the motor shaft in the presence of n -transmission gear in the drive



m – motor
 d – drum of the winch
 n -th transmission gear

$j_k = \frac{\omega_{k-1}}{\omega_k}$ – gear ratio from $(k-1)$ -th to k -th shaft

η_k – efficiency of k -th transmission

Conversion of static torques for the axes of rotation

$$T_d \omega_d = T'_d \omega_m = T_L \omega_m,$$

$$\text{where } T_d = \frac{G_L D}{2} = \frac{m_L g D}{2} -$$

The power on the drum shaft is equal to the power on the motor shaft (T'_d - converted meaning of drum torque to the motor shaft)

static torque created on the drum shaft by a load mass m_L (m_L [kg], G_L - weight or load gravity [N]);

$$T_L = T_d \frac{\omega_d}{\omega_m} = T_d \frac{1}{j}; \quad j = \omega_m / \omega_d$$

– gear ratio from drum to motor

$$T_L \omega_m = T_1 \omega_1 = T_2 \omega_2 = \dots = T_n \omega_n$$

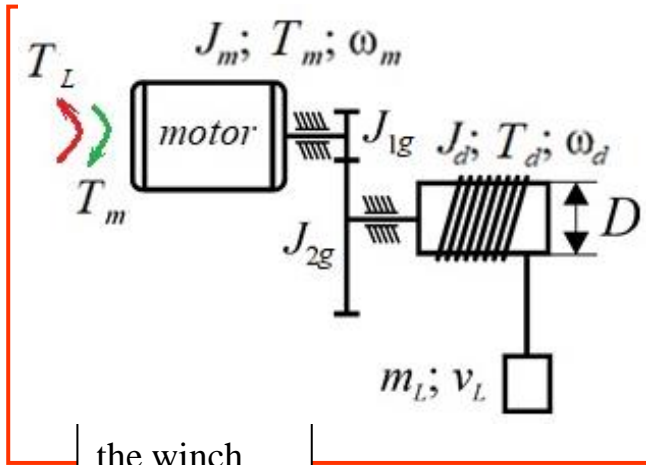
$$T_L \omega_m = \frac{T_1 \omega_1}{\eta_1} = \frac{T_2 \omega_2}{\eta_1 \eta_2} = \dots = \frac{T_n \omega_n}{\eta_1 \eta_2 \dots \eta_n} \Rightarrow T_L = T'_n = T_n \frac{\omega_n}{\omega_m (\eta_1 \eta_2 \dots \eta_n)} = \frac{T_n}{j_{1n} \eta_{1n}}$$

– conversation of torque T_n of n -th transmission gear to the motor shaft side

$$j_{1n} = j_1 j_2 \dots j_n = \prod_{k=1}^n j_k; \quad \eta_{1n} = \eta_1 \eta_2 \dots \eta_n = \prod_{k=1}^n \eta_k$$

– full gear ratio and efficiency of transmission from n -th shaft to motor.

Conversion of static torques and forces to the motor shaft in the presence of n -transmission gear in the drive



the winch
 n -th
transmission
gear

Conversion linear motion (lifting load) to static torque T_L

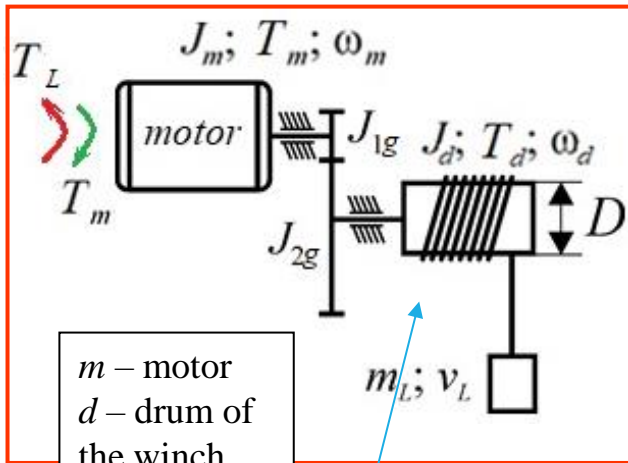
$$T_{LG}\omega_m = F_{ms}v_{ms} \Rightarrow T_{LG} = F_{ms} \frac{v_{ms}}{\omega_m} = F_{ms}\tilde{r};$$

$$T_{LG} = F_n \tilde{r}_n \frac{1}{\eta_{1n}}; \quad \tilde{r} = \frac{v_{ms}}{\omega_m} \text{ - radius of transformation of } n\text{-th component moving to the motor speed } \omega_m.$$

$$T_L = \sum_{k=1}^p \frac{T_k}{j_{1k}\eta_{1k}} + \sum_{i=1}^q \frac{F_i \tilde{r}_i}{\eta_{1i}}$$

The final formula of the static torque - the sum of all the torques converted to the motor side

Conversion of static torques and forces to the motor shaft in the presence of n -transmission gear in the drive



m – motor
 d – drum of the winch
 n -th transmission gear

Conversion linear motion (lifting load) to static torque T_L

$$T_{LG}\omega_m = F_{ms}v_{ms} \Rightarrow T_{LG} = F_{ms} \frac{v_{ms}}{\omega_m} = F_{ms}\tilde{r};$$

$$T_{LG} = F_n \tilde{r}_n \frac{1}{\eta_{1n}}; \quad \tilde{r} = \frac{v_{ms}}{\omega_m} \text{ - radius of transformation of } n\text{-th component moving to the motor speed } \omega_m.$$

$$T_L = \sum_{k=1}^p \frac{T_k}{j_{1k}\eta_{1k}} + \sum_{i=1}^q \frac{F_i \tilde{r}_i}{\eta_{1i}}$$

The final formula of the static torque - the sum of all the torques converted to the speed of the motor shaft





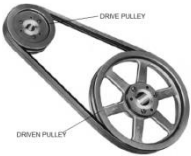



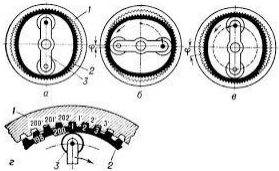
Rotating motion

Linear motion

As an example, let's determine the static load torque on the motor shaft (converted values) T_{LG} , that created by mass m_L , lifted by winch with speed v_L .

$$T_{LG} = T_d \frac{\omega_d}{\omega_m \eta_{12}} = \frac{m_L g D}{2\eta_d} \cdot \frac{\omega_d}{\omega_m \eta_{12}} = \frac{m_L g D}{2\eta_d} \cdot \frac{2v_L}{\omega_m \eta_{12} D} = m_L g \frac{v_L}{\omega_m \eta_d \eta_{12}}$$

Kinematic pairs efficiency

Type of pair (transmission)	η	Type of pair (transmission)	η
Friction 	0,90...0,95	Chain 	0,96...0,98
Flat belt 	0,97...0,98	Screw nut 	$\leq 0,7$
V-belt 	0,92...0,97	Ball screw 	$\approx 0,9$
Gear			
cylindrical (spur) gear 	0,90...0,98		
worm gear 	0,70...0,92		
wave 	0,70...0,90		

➤ Conversion of static torques and forces

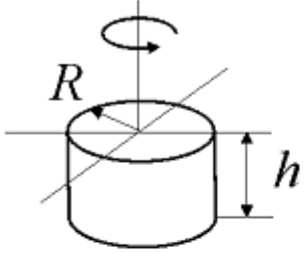
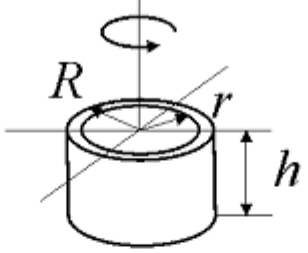
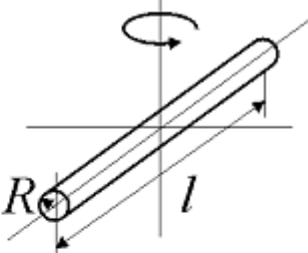
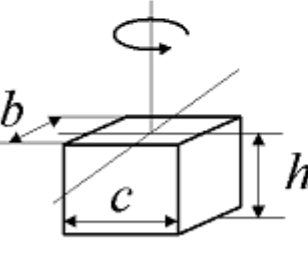
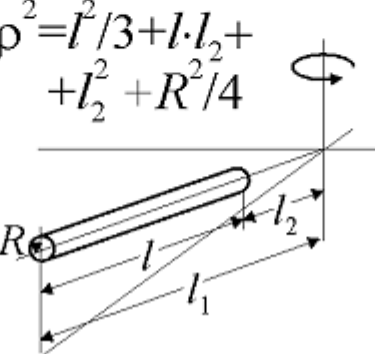

➤ Conversion of moments of inertia

➤ Link Stiffness Conversion

$$J = mr^2; \quad J = \sum_{k=1}^n m_k r_k^2; \quad J = \int_V r^2 dm \Rightarrow J = m\rho^2$$

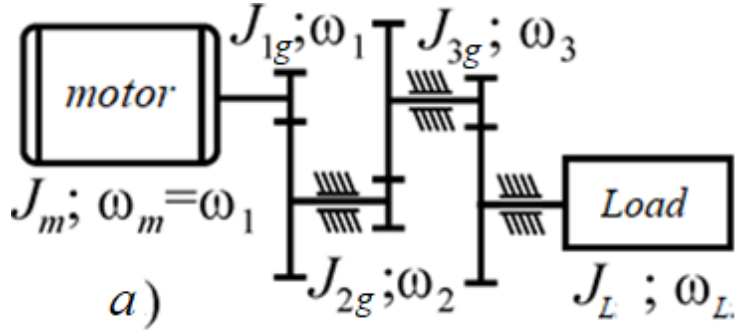
ρ - radius of inertia

Radius of inertia of simple geometric bodies

 $\rho^2 = R^2/2$	 $\rho^2 = (R^2 + r^2)/2$	 $\rho^2 = (l^2 + 3R^2)/12$
 $\rho^2 = (b^2 + c^2)/12$	 $\rho^2 = l^2/3 + l \cdot l_2 + l_2^2 + R^2/4$	 $\rho^2 = \rho_0^2 + a^2$

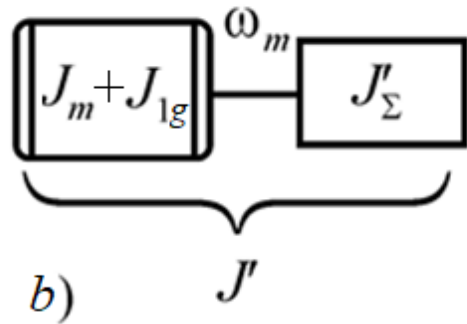
ρ_0 - radius of inertia of an arbitrary body obtained relative to an axis located at the center of gravity

Conversion of rotating masses to the motor shaft



The total kinetic energy of the flywheel masses of the motor, gearbox and load is equal:

$$\frac{J'_m \omega_m^2}{2} = \frac{J_{1g} \omega_1^2}{2} + \frac{J_{2g} \omega_2^2}{2} + \frac{J_{3g} \omega_3^2}{2} + \frac{J_L \omega_L^2}{2}$$



$$\begin{aligned} J' &= (J_m + J_{1g}) + J_{2g} \frac{\omega_2^2}{\omega_1^2} + J_{3g} \frac{\omega_3^2}{\omega_1^2} + \dots + J_{ng} \frac{\omega_n^2}{\omega_1^2} + J_L \frac{\omega_L^2}{\omega_1^2} = \\ &= (J_m + J_{1g}) + J_{2g} \frac{1}{j_{12}^2} + J_{3g} \frac{1}{j_{13}^2} + \dots + J_{ng} \frac{1}{j_{1n}^2} + J_L \frac{1}{j_{1L}^2} = \\ &= (J_m + J_{1g}) + J'_2 + J'_3 + \dots + J'_n + J'_L = (J_m + J_{1g}) + J'_\Sigma \end{aligned}$$

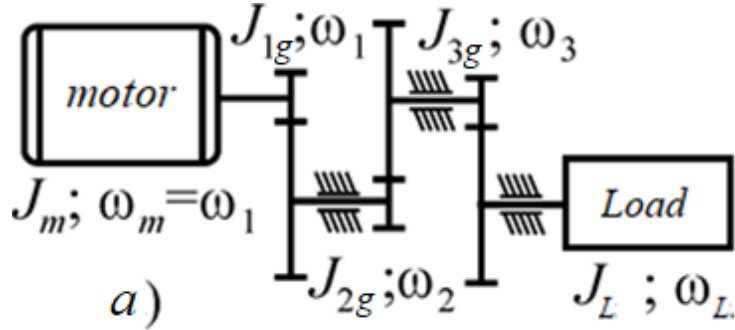
Rotating motion

where: $j_{12}, j_{13} \dots j_{1n}, j_{1L}$ – gear ratios between motor shaft and the shafts corresponding to the index numbers;

$J'_2 = J_2 / j_{12}^2, J'_3 = J_3 / j_{13}^2 \dots J'_L = J_L / j_{1L}^2$ – the moments of inertia of the elements converted to the motor shaft (therefore elements rotate at engine speed);

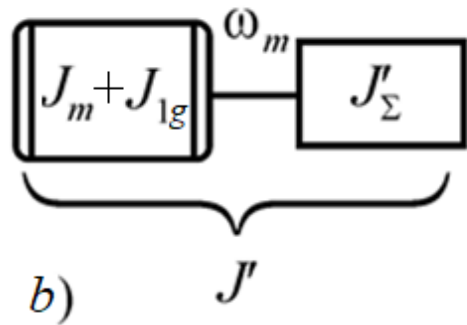
$J'_\Sigma = J'_2 + J'_3 + \dots + J'_n + J'_L$ – the total converted moment of inertia of the elements rotating at speeds different from the motor speed .

Conversion of rotating linearly moving masses to the motor shaft



$$\frac{J'_L \omega_m^2}{2} = \frac{mv_L^2}{2} \Rightarrow J'_L = m \left(\frac{v_L}{\omega_m} \right)^2 = m\tilde{r}^2$$

Linear
motion



$$J' = (J_m + J_{1g}) + \frac{J_{2g}}{j_{12}^2} + \frac{J_{3g}}{j_{13}^2} + \dots + \frac{J_{pg}}{j_{1p}^2} + m_1 \left(\frac{v_1}{\omega_m} \right)^2 + m_2 \left(\frac{v_2}{\omega_m} \right)^2 + \dots$$

$$\dots + m_q (v_q / \omega_m)^2$$

$$J' = (J_m + J_{1g}) + \sum_{k=2}^p J_{kg} / j_{1k}^2 + \sum_{i=1}^q m_i \tilde{r}_i^2$$

The total moment of inertia (the sum of the moments of inertia converted to the motor side)

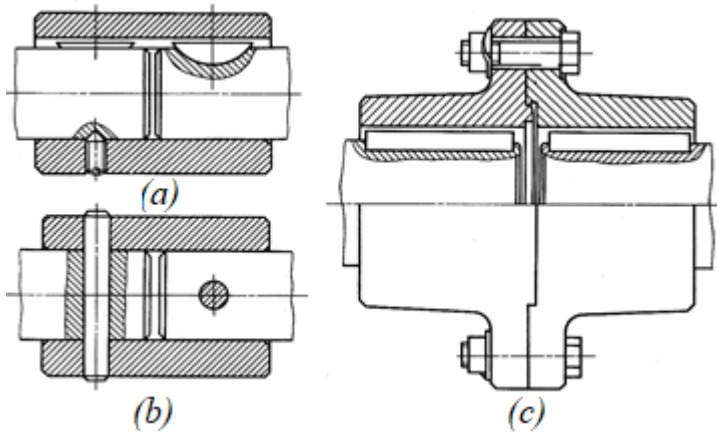
Inertia of motor+first gear

Rotating motion

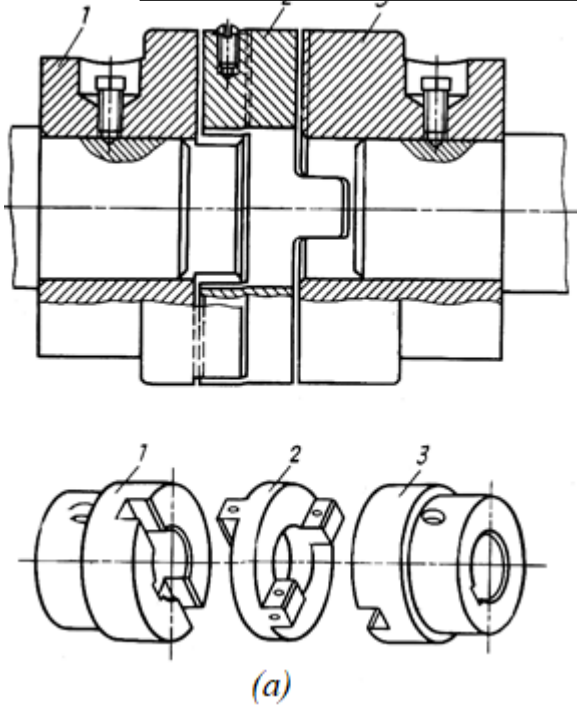
Linear motion

Couplings

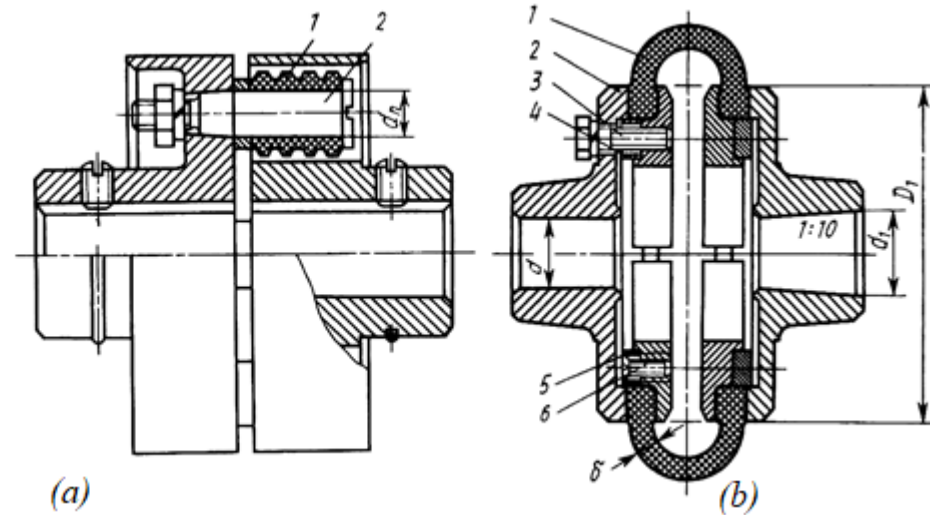
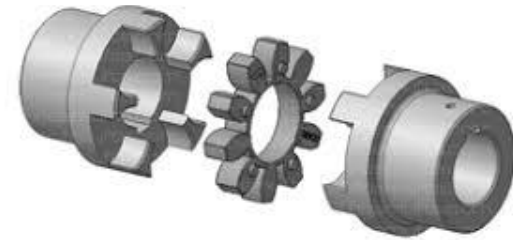
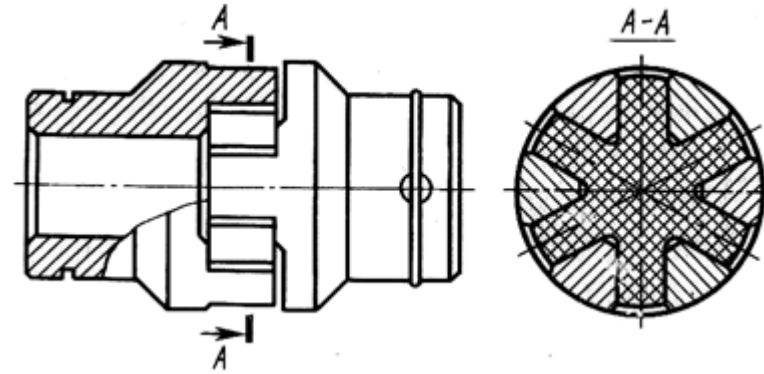
Rigid shaft couplings



Compensating coupling



Elastic coupling



- Conversion of static torques and forces
- Conversion of moments of inertia
- Link Stiffness Conversion

Rotating motion

$$j_i = \frac{\omega_1}{\omega_i} = \frac{d\varphi_1}{d\varphi_i} \implies d\varphi_1 = d\varphi'_i = d\varphi_i j_i \implies \varphi'_i = j_{1i} \varphi_i \quad \text{-- angular movement } \varphi_i \text{ converted to the motor side, based on the ratio } j$$

stiffness of the i -th rotating element $k_i = \frac{T_i}{\Delta\varphi_i}$, where T_i and $\Delta\varphi_i$ – torque and corresponding deformation of the i -th element

When stiffness is converted, the potential energy must be equal before and after conversion

$$\frac{k_i \Delta\varphi_i^2}{2} = \frac{k'_i \Delta\varphi'^2_i}{2} \implies$$

$$k'_i = \frac{k_i}{j_{1i}^2}$$

– stiffness of the i -th rotating element converted to the motor side

Linear motion

$$\tilde{r} = \frac{v_i}{\omega_1} = \frac{dl_j/dt}{d\varphi_1/dt} \Rightarrow \varphi'_j = l_j/\tilde{r}_{1j} \quad \text{linear } l_j \text{ movements converted to the motor side}$$

When stiffness is converted, the potential energy must be equal before and after conversion

$$\frac{k_i \Delta l_j^2}{2} = \frac{k'_i \Delta l_i'^2}{2} \Rightarrow$$

$$k'_j = k_j \tilde{r}_{1j}^2$$

– stiffness of the j -th rotating element converted to the motor side

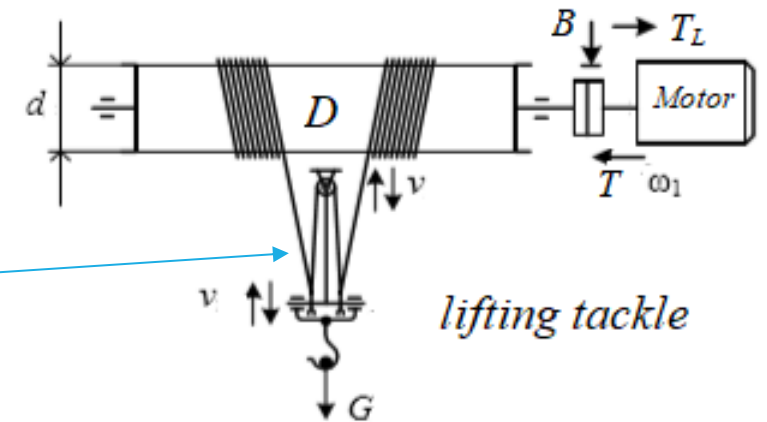
- when connected in series

$$1/k_{sum} = 1/k_1 + 1/k_2 + 1/k_3 + \dots$$

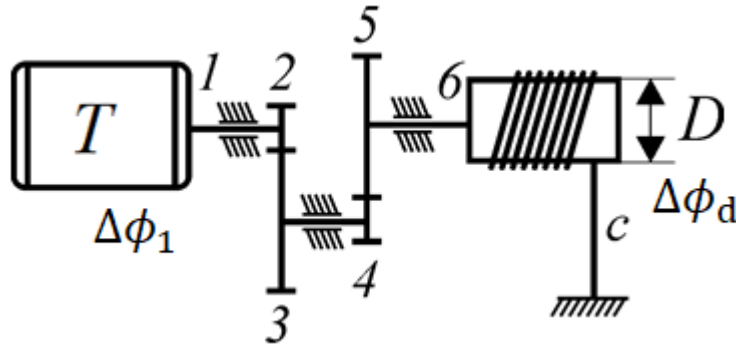
- with parallel connection of elastic elements

$$k_{sum} = k_1 + k_2 + k_3 + \dots$$

Total sum. stiffness



Stiffness conversion. Example: drum with an elastic cable



Δl – extension of cable

T_d – torque on drum shaft

T – motor torque

Example: drum with an elastic cable

$$k = \frac{F}{\Delta l} \quad \text{– definition of stiffness}$$

$$T_d = T \cdot j_{23} j_{45} = T \cdot j_{15} = F \cdot D/2 = \Delta l \cdot k D/2; \quad (1)$$

$$\Delta \phi_d = \frac{2 \Delta l}{D};$$

$$\Delta \phi_1 = \Delta \phi_d j_{15} = 2 \Delta l \cdot \frac{j_{15}}{D} = \frac{T}{k'}; \quad \begin{array}{l} \text{– drum shaft angle of rotation} \\ \text{– motor shaft angle of rotation} \end{array}$$

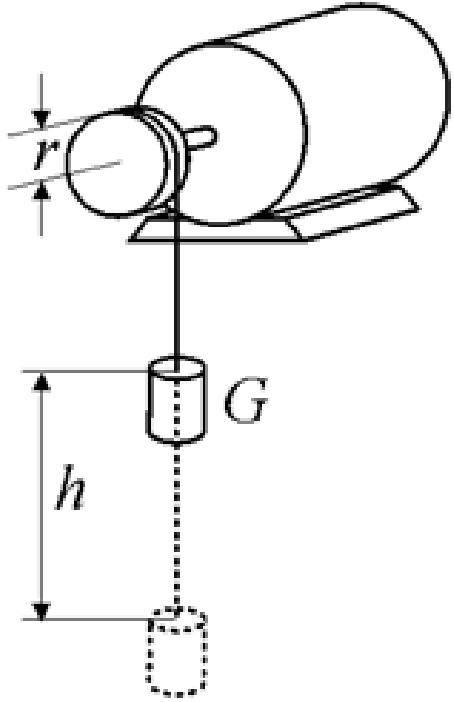
$$\Delta \phi_1 = \frac{4T \cdot j_{15}^2}{k D^2} = \frac{T}{k'}$$

⇓

$$k' = k \left(\frac{D}{2j_{15}} \right)^2 = k \tilde{r}^2; \quad \tilde{r} = \frac{D}{2j_{15}}$$

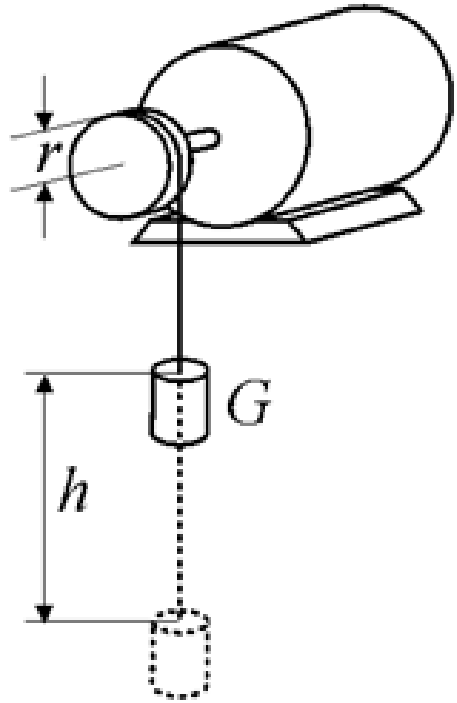
– stiffness of cable converted to the motor side

Experimental obtaining moment of inertia



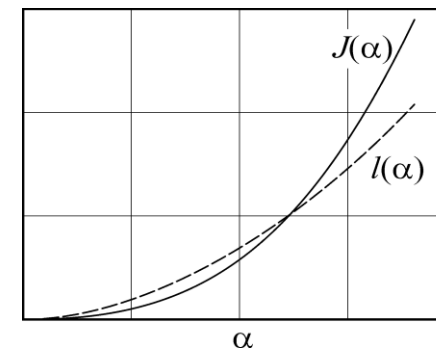
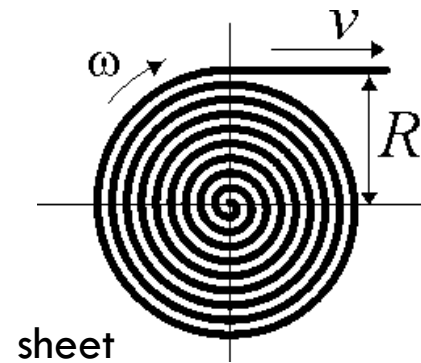
$$J = Gr^2 \left(\frac{t^2}{2h} - \frac{1}{g} \right)$$

Experimental obtaining moment of inertia

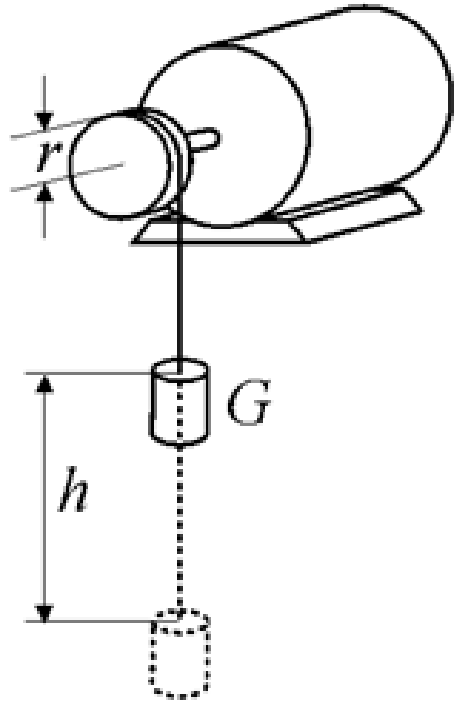


$$J = Gr^2 \left(\frac{t^2}{2h} - \frac{1}{g} \right)$$

Mechanisms with variable static moments and inertial properties (spiral winding mechanism)

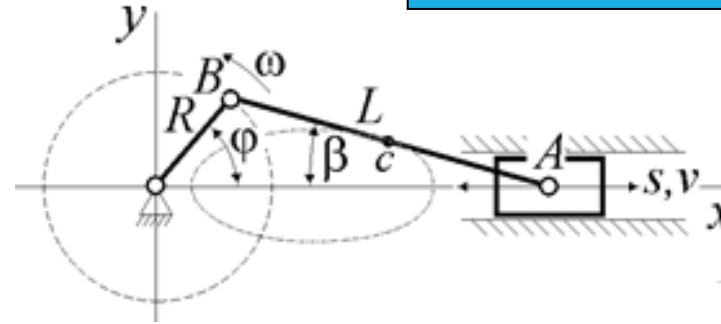


Experimental obtaining moment of inertia



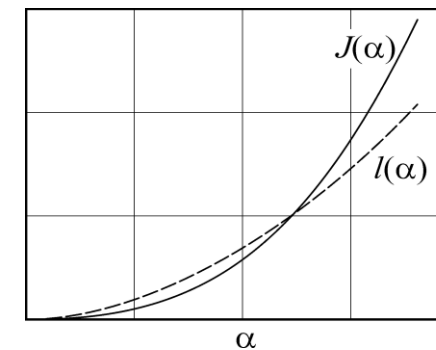
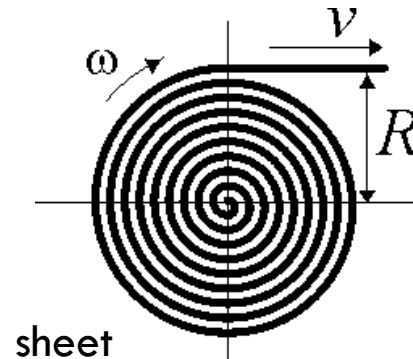
$$J = Gr^2 \left(\frac{t^2}{2h} - \frac{1}{g} \right)$$

Mechanisms with variable static moments and inertial properties (crank mechanism)



$$\tilde{r}(\phi) = \frac{v(\phi)}{\omega} = R \left(\sin \phi + \frac{\lambda}{2} \sin 2 \phi \right) \xrightarrow{L \rightarrow \infty \Rightarrow \lambda \rightarrow 0} R \sin \phi$$

Mechanisms with variable static moments and inertial properties (spiral winding mechanism)



Thank you for your attention