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Actuator based on DC drive

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Content

- General information about actuators
- Speed/torque characteristic
- Speed control of the DC motor
- Braking modes
- DC Motor Dynamics

General information about actuators

Equation of motion of the electric drive.

Newton's law in rotational motion

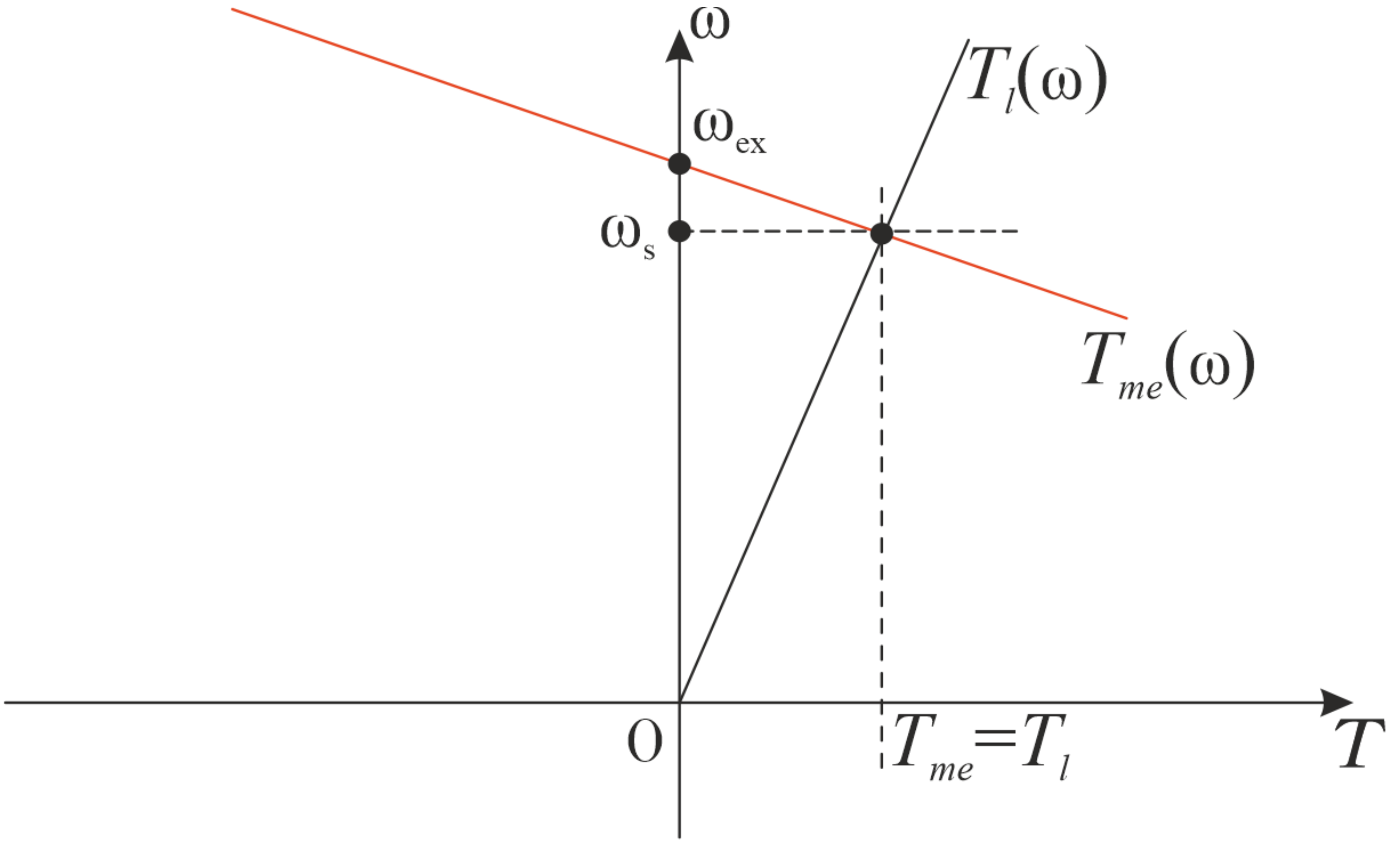
$$J\frac{d\omega}{dt} = T_{me} - T_{load}$$

In steady state mode:

$$\frac{d\omega}{dt} = 0$$

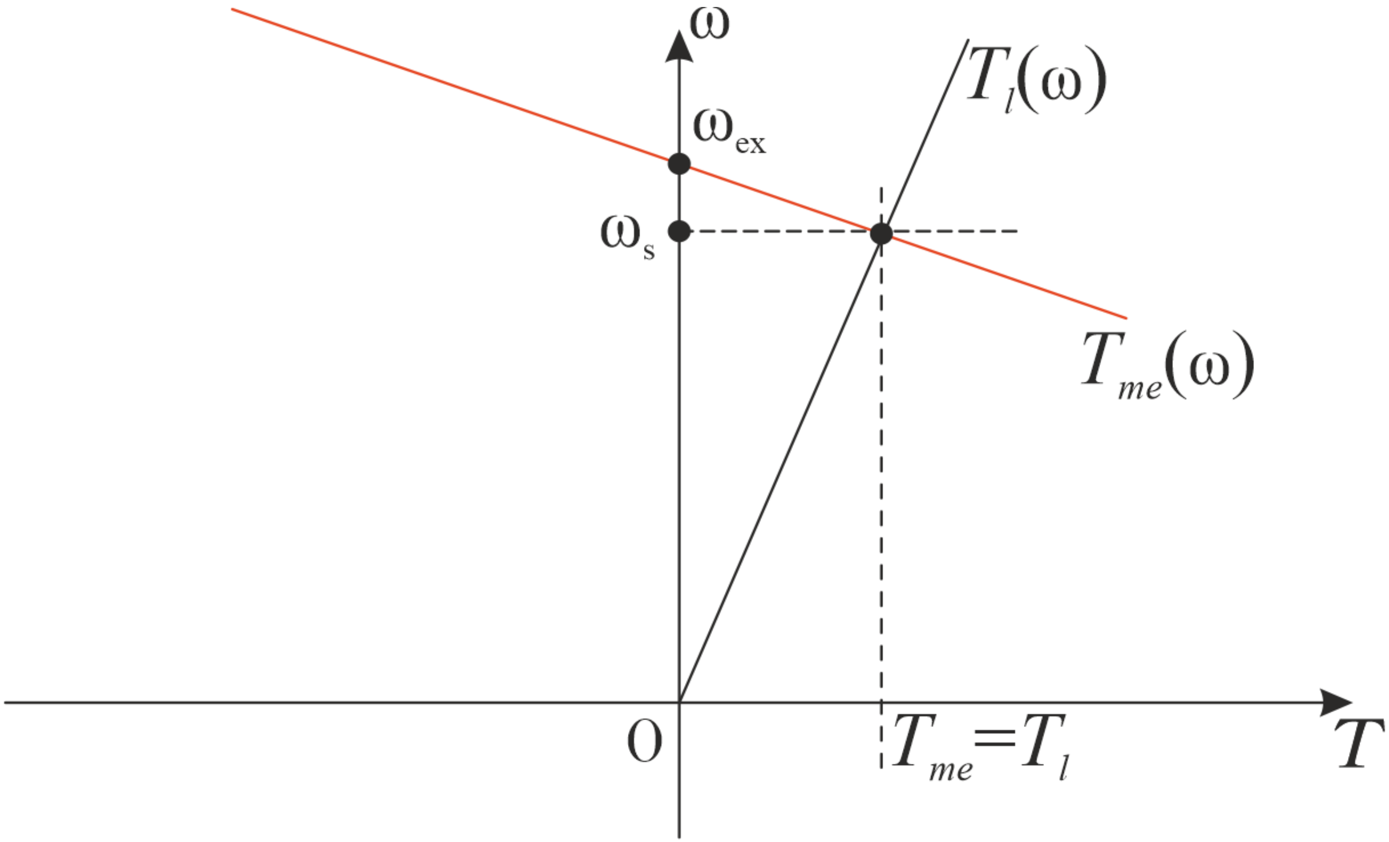
$$T_{me} = T_{load}$$

Static stability of the drive



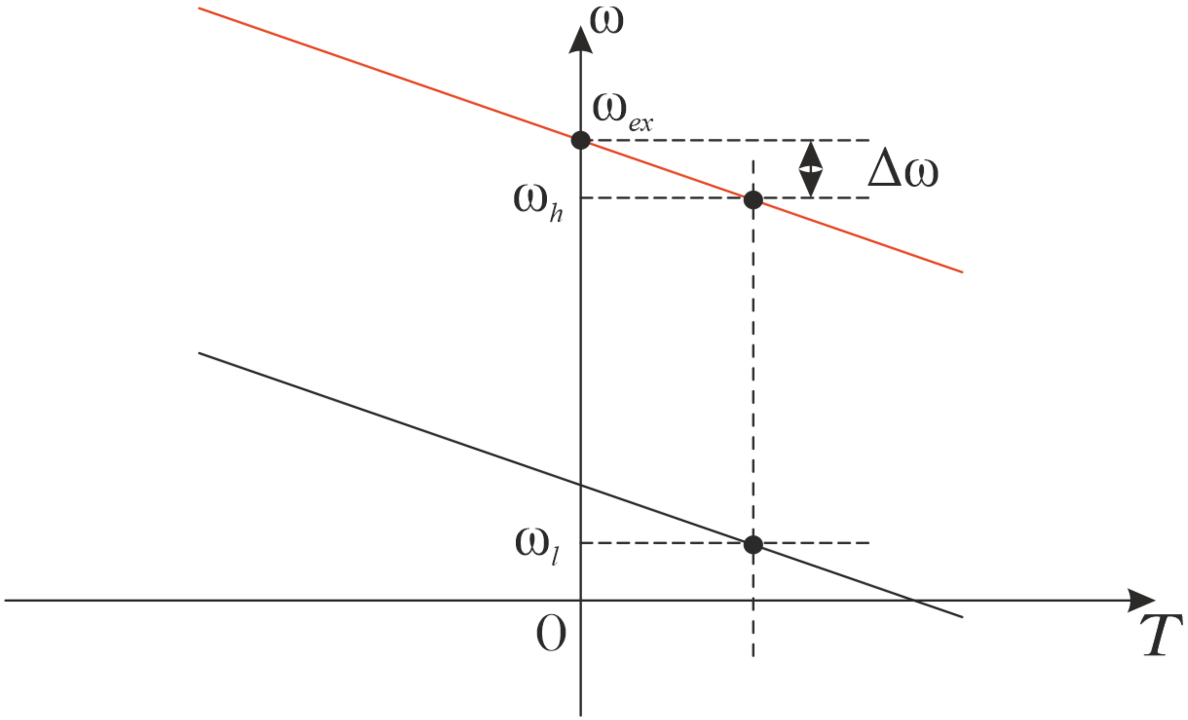
$$J\frac{d\omega}{dt} = T_{me} - T_{load} \lessgtr 0$$

Static stability of the drive



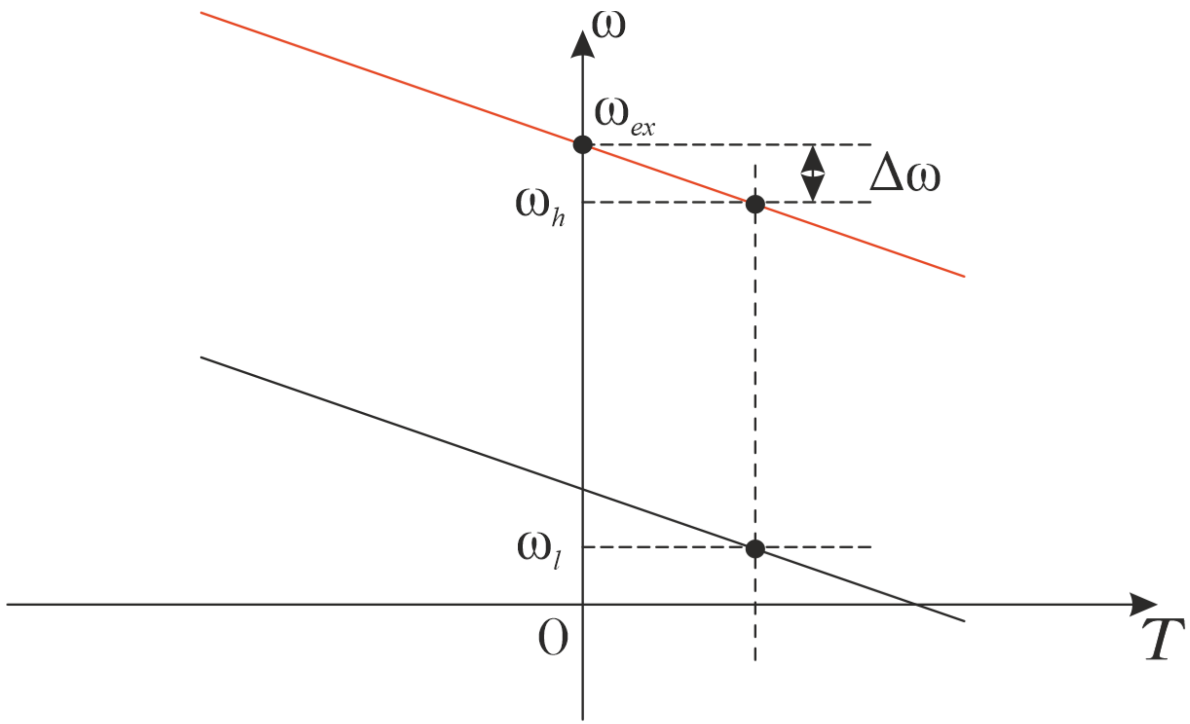
The condition of stability is a necessary condition for the performance of the electric drive. It should be noted that this is not always the case.

Speed control range



Speed control range: $D = \dfrac{\omega_h}{\omega_l}$

Speed control range



Δω - absolute static load error.

Δ - relative static load error.

$$\Delta = \frac{\Delta\omega}{\omega_{mean}}$$

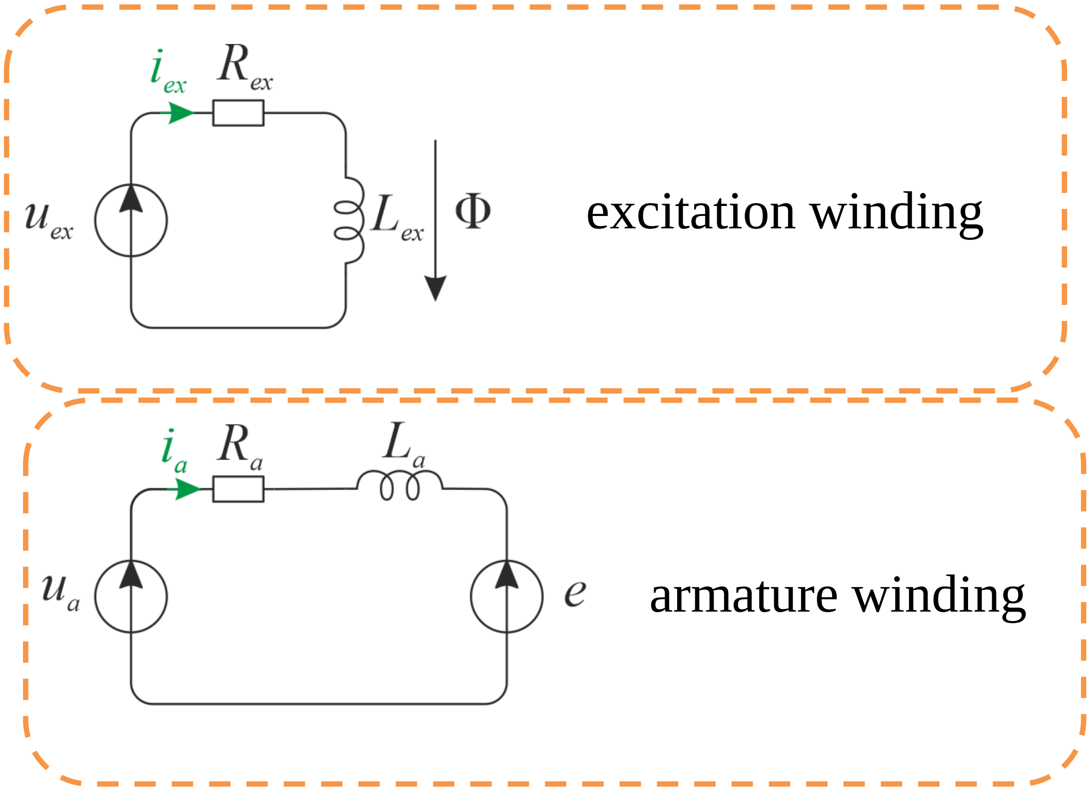
$$\Delta_h = \frac{\Delta\omega}{\omega_h}$$

$$\Delta_l = D \cdot \Delta_h$$

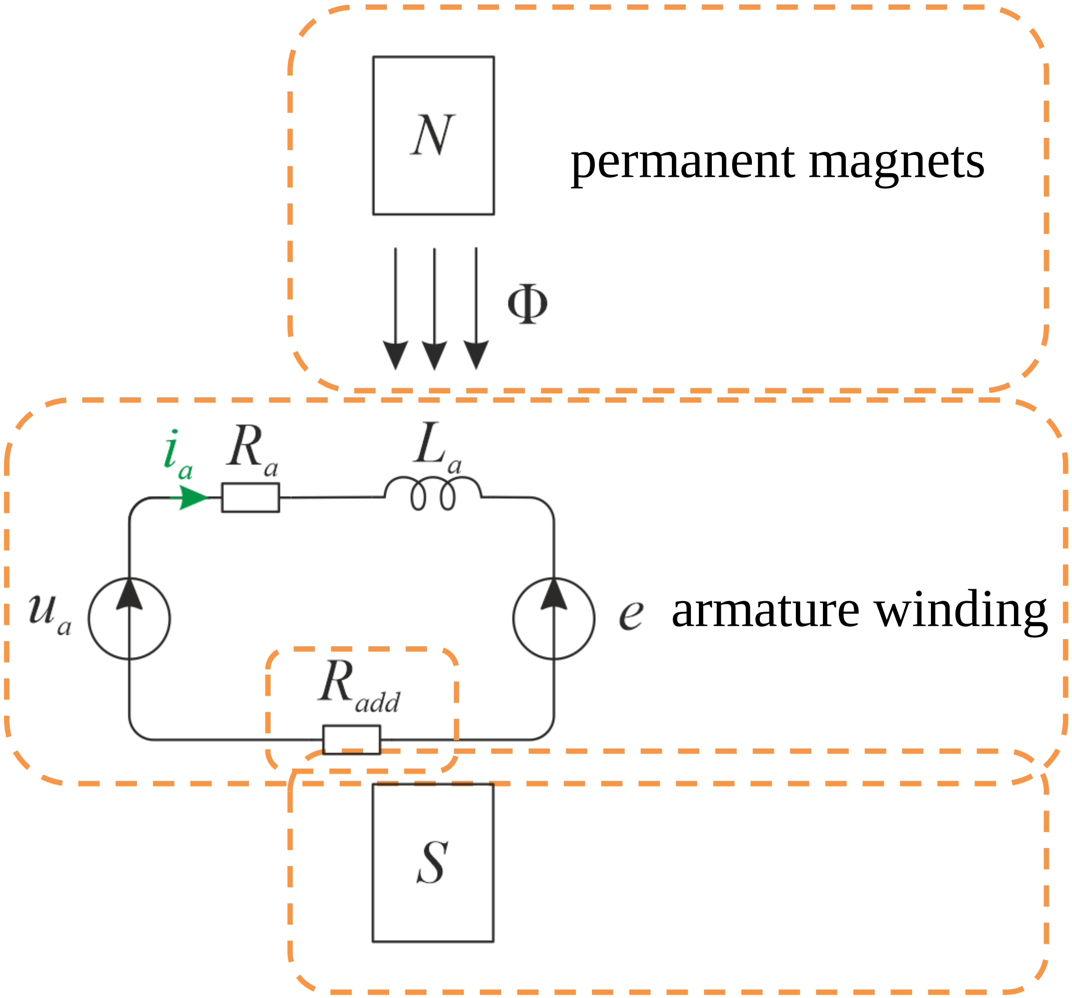
$$\Delta_l = \frac{\Delta\omega}{\omega_l}$$

Speed/torque characteristic

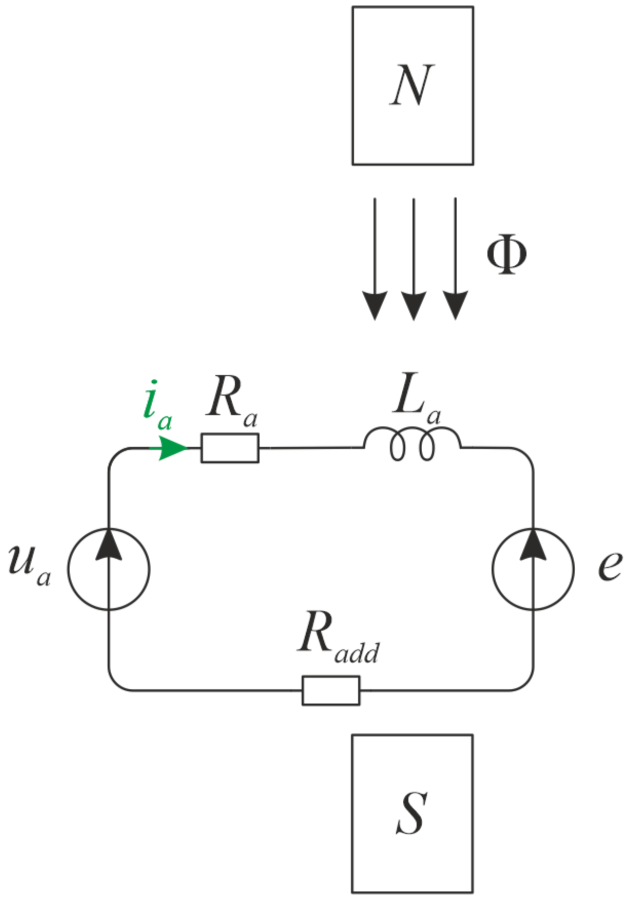
Separately excited DC motor



Separately excited DC motor



Separately excited DC motor



$$u_a = e + i_a(R_a + R_{add})$$

R_a - motor armature resistance

R_{add} – additional resistance

$$e = k \cdot \Phi \cdot \omega$$

e – back-EMF

$$k = \frac{p \cdot N}{2\pi \cdot a}$$

k – constructive coefficient

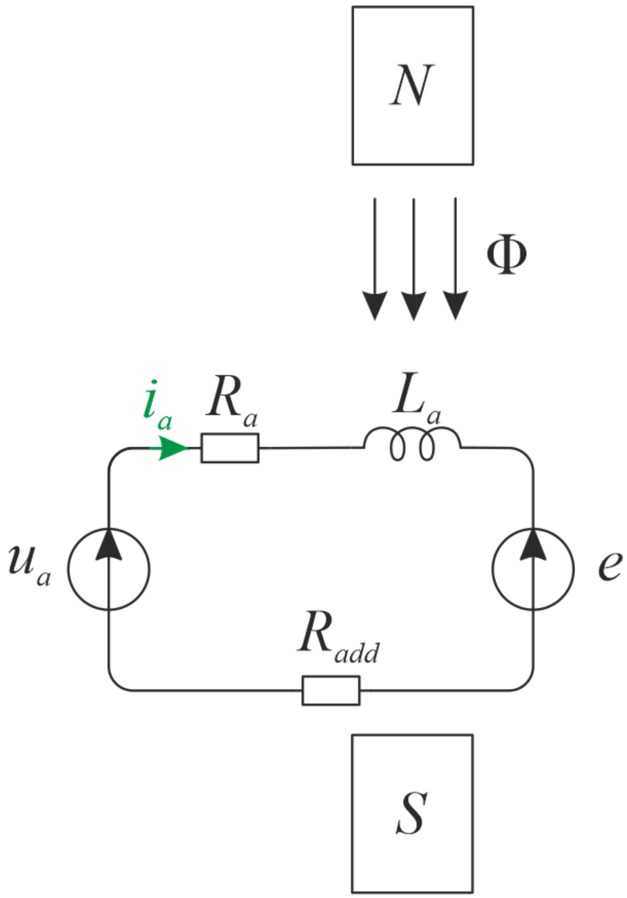
p – number of pole pairs

N – number of active armature winding conductors

a – number of pairs of parallel branches of the armature winding

Φ – magnetic flux

Separately excited DC motor



$$\omega = \boxed{\frac{u_a}{k\Phi}} - \boxed{\frac{i_a\left(R_a + R_{add}\right)}{k\Phi}}$$

Speed/current characteristic

$$T_{me} = k\Phi \cdot i_a$$

$$\omega = \frac{u_a}{k\Phi} - \frac{T_{me}\left(R_a + R_{add}\right)}{k^2\Phi^2}$$

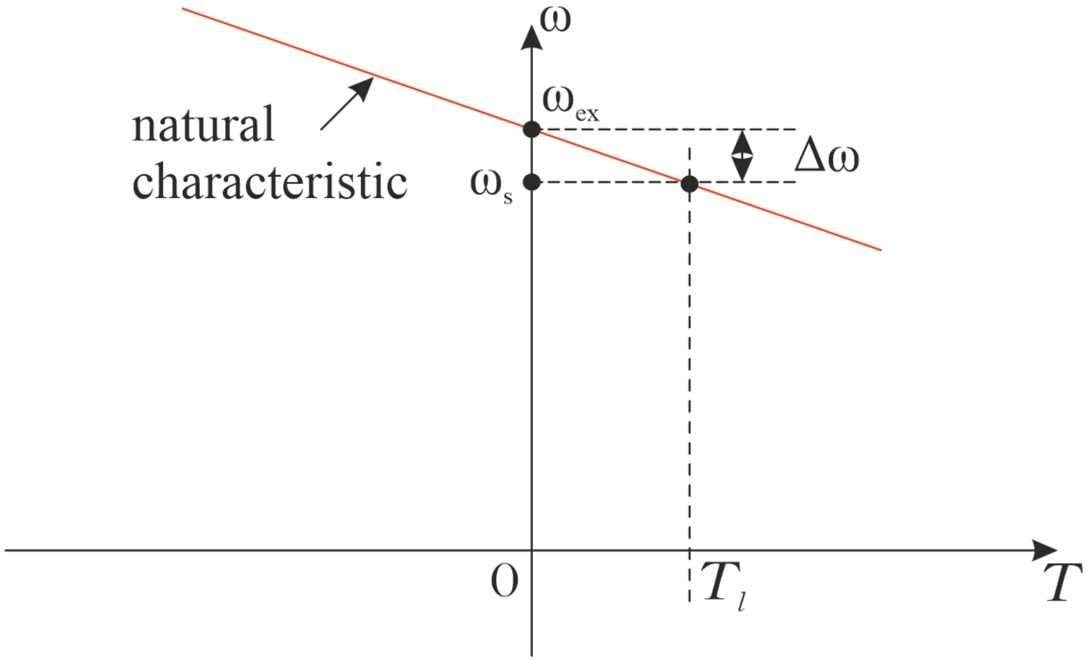
Speed/torque characteristic

$$\omega_0 = \frac{u_a}{k\Phi}$$ - angular speed of ideal idle

$$\Delta\omega = \frac{i_a\left(R_a + R_{add}\right)}{k\Phi} = \frac{T_{me}\left(R_a + R_{add}\right)}{k^2\Phi^2}$$

- static drop in angular speed

Separately excited DC motor

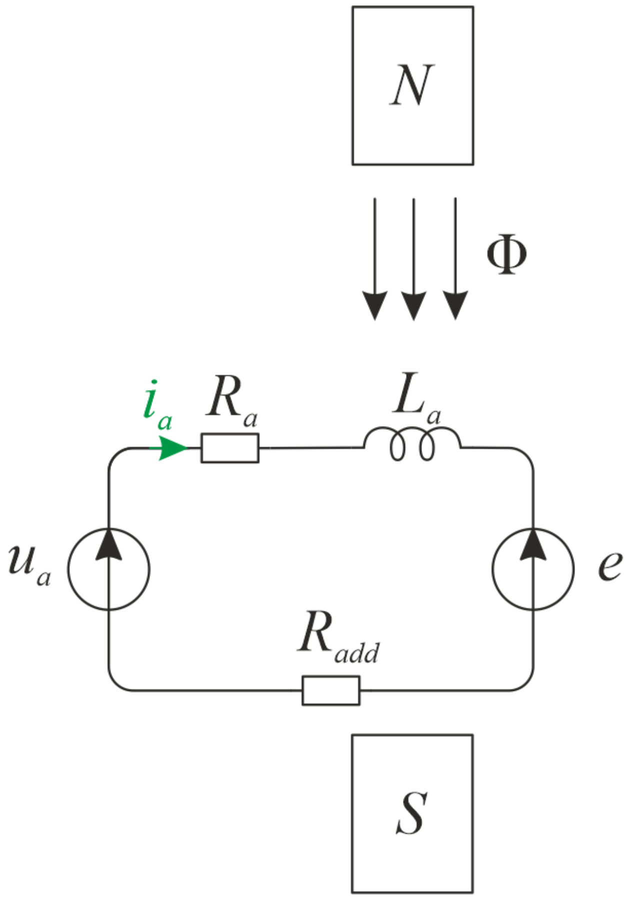


The characteristic obtained at the nominal value of the armature voltage u_{nom}, the nominal magnetic flux Φ_{nom} and the absence of external resistors in the armature circuit is called **natural.**

$$\Delta\omega = \frac{i_a R_a}{k\Phi} = \frac{T_{me} R_a}{k^2 \Phi^2}$$

$$\boxed{T_{me} = T_l}$$

Separately excited DC motor



$$\frac{1}{k\Phi} = k_{st} = \frac{\omega_{nom}}{u_{nom} - i_{nom} R_a}$$

k_{st} – static gain of the drive [rad/(s·V)]

$$\omega = k_{st} u_a - k_{st} i_a \left(R_a + R_{add}\right)$$

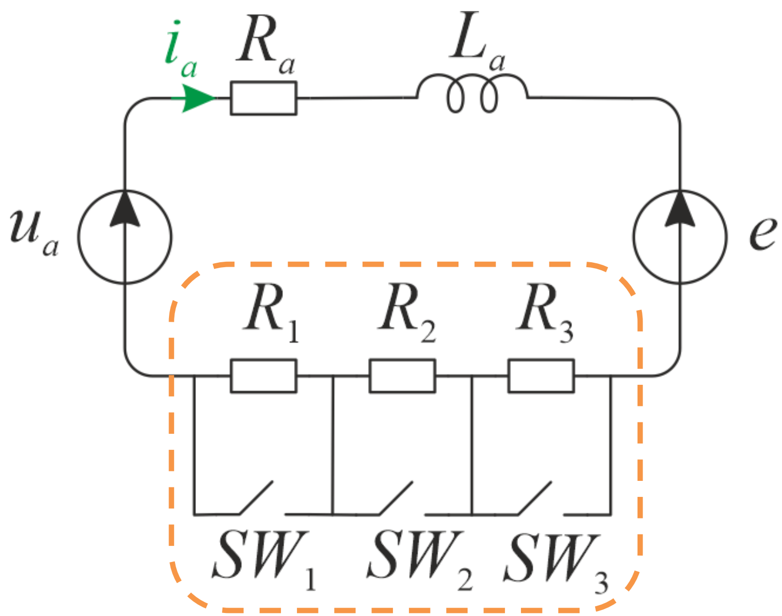
$$\omega = k_{st} u_a - k_{st}^{\,2} T_{me} \left(R_a + R_{add}\right)$$

$$\omega_0 = k_{st} u_a$$

$$\Delta\omega = k_{st} i_a \left(R_a + R_{add}\right) = k_{st}^{\,2} T_{me} \left(R_a + R_{add}\right)$$

Speed control of the DC motor

Speed control of the DC motor by adding resistances to armature circuit



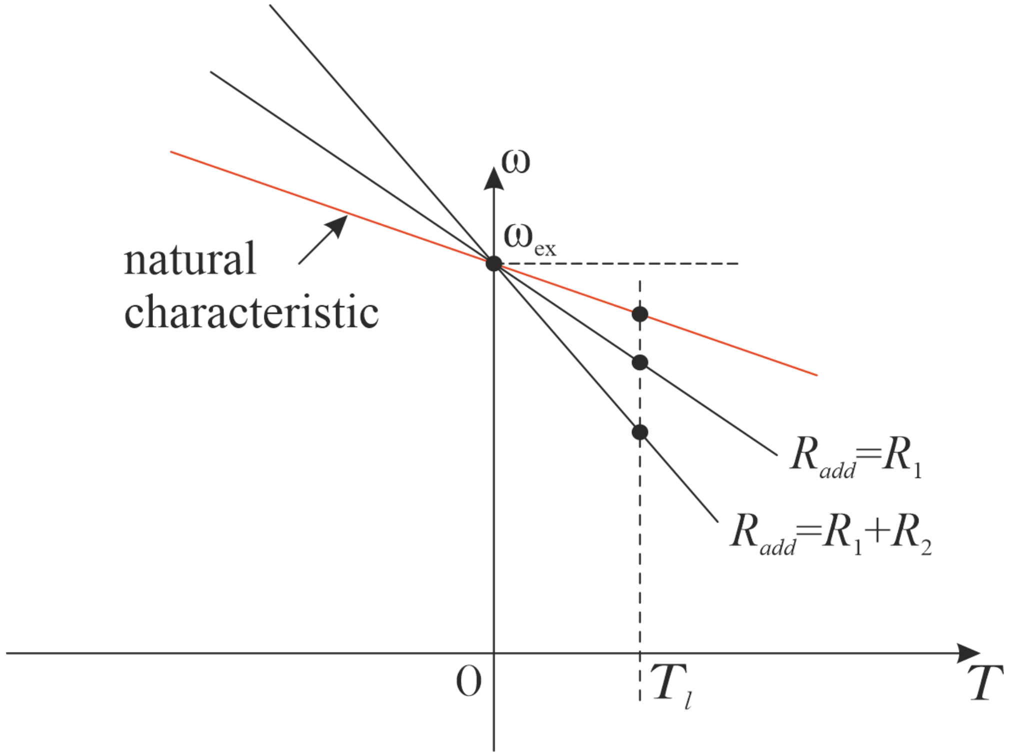
$$u_a = u_{nom}$$

$$\Phi_a = \Phi_{nom}$$

$$\omega = k_{st} u_a - k_{st} i_a \left(R_a + \boxed{R_{add}} \right)$$

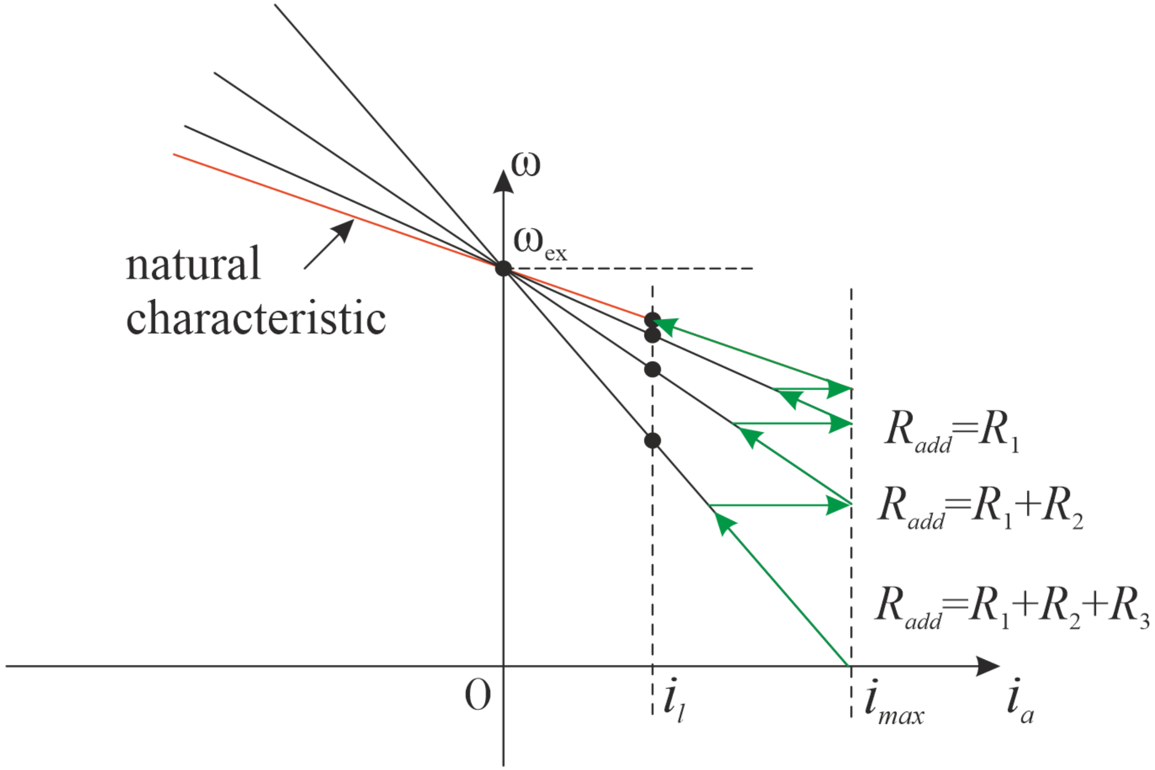
$$\omega = k_{st} u_a - k_{st}^{\,2} T_{me} \left(R_a + R_{add} \right)$$

Speed control of the DC motor by adding resistances to armature circuit



The characteristics obtained with the introduction of additional resistances are called **artificial - rheostatic.**

Speed control of the DC motor by adding resistances to armature circuit

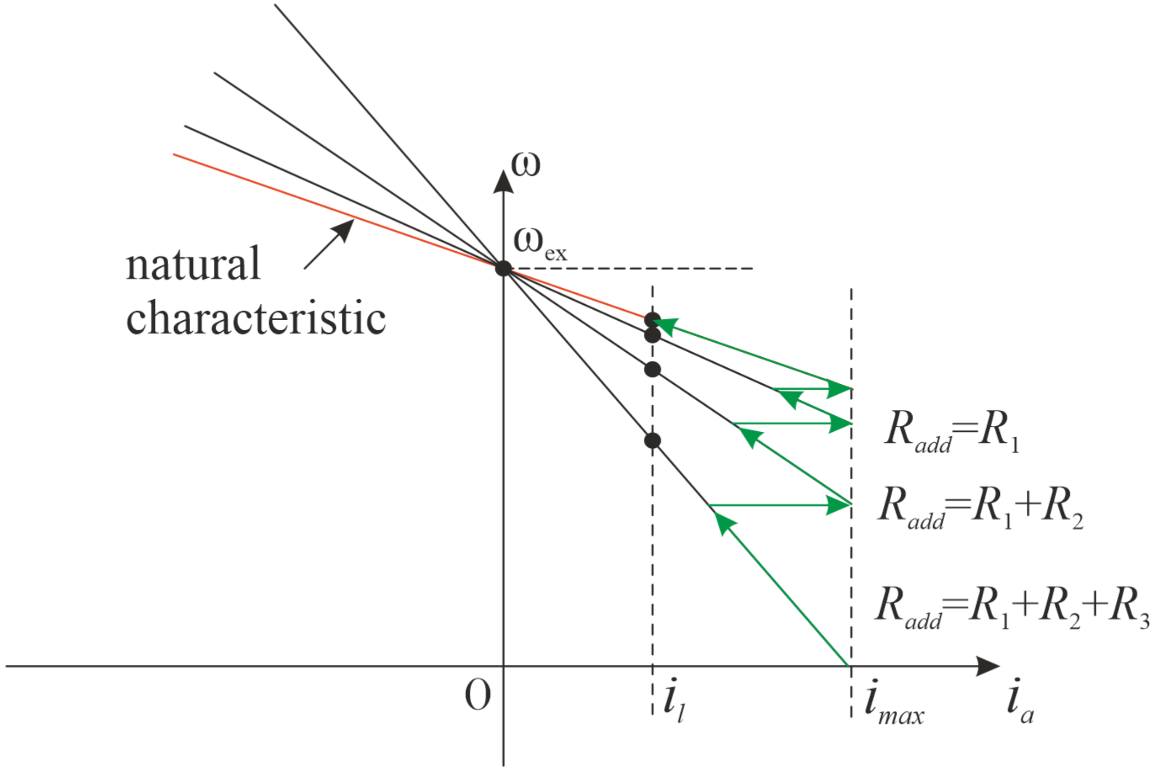


$$i_{inr} = \frac{u_{nom}}{R_a + R_{add}}$$

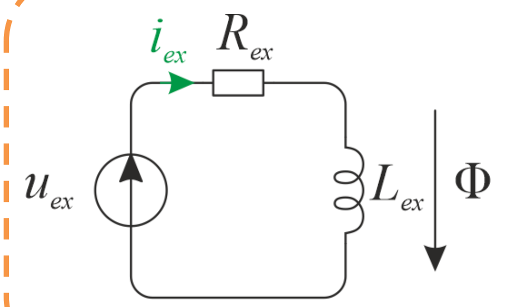
$$P > 0.5\ldots1.0 \text{ kW}$$

$$i_{inr} = \frac{u_{nom}}{R_a} > i_{\max}$$

Speed control of the DC motor by adding resistances to armature circuit

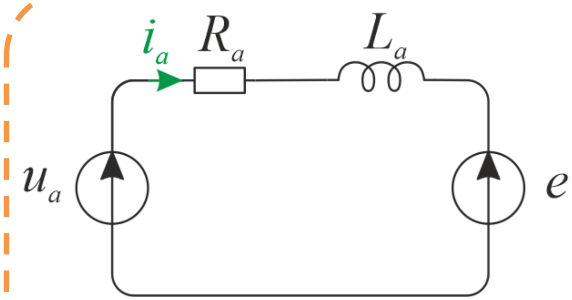


Speed control of the DC motor by changing armature voltage



excitation winding

$u_{ex} = \text{const}$



armature winding

$u_a = \text{var}$

$$\omega_s = \frac{u_a}{k\Phi} - \frac{i_a R_a}{k\Phi}$$

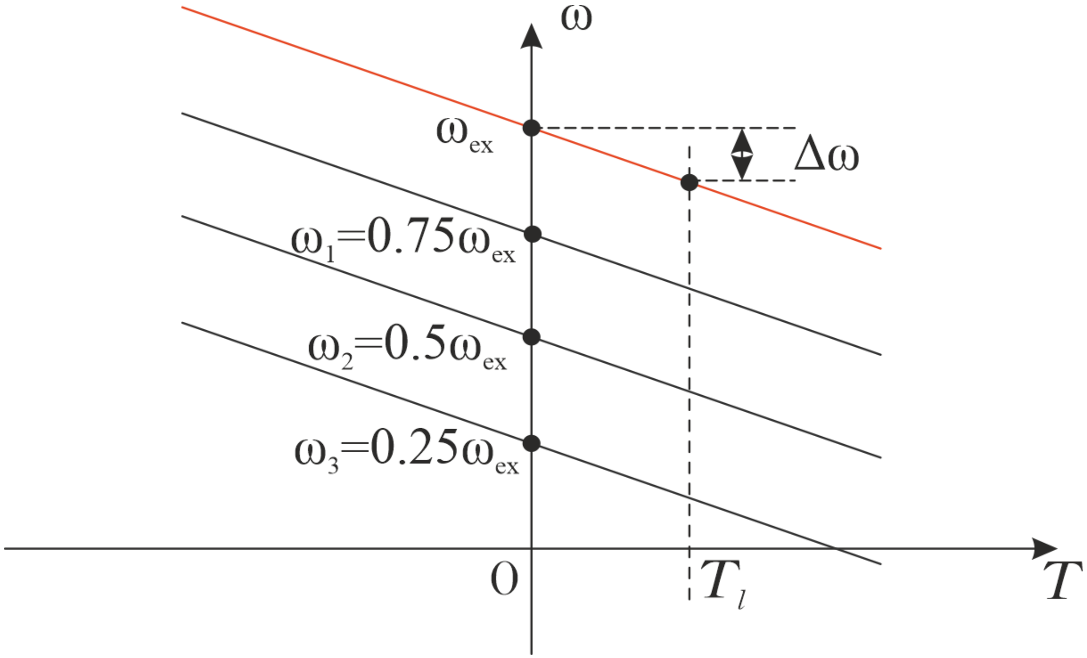
$$\omega_s = \boxed{\frac{u_a}{k\Phi}} - \boxed{\frac{T_{me} R_a}{k^2 \Phi^2}}$$

k - constructive coefficient

$$\omega_0 = \frac{u_a}{k\Phi} = \omega_{ex}, \omega_1, \omega_2 \ldots.$$

$$\Delta\omega = \frac{T_{me} R_a}{k^2 \Phi^2}$$

Speed control of the DC motor by changing armature voltage

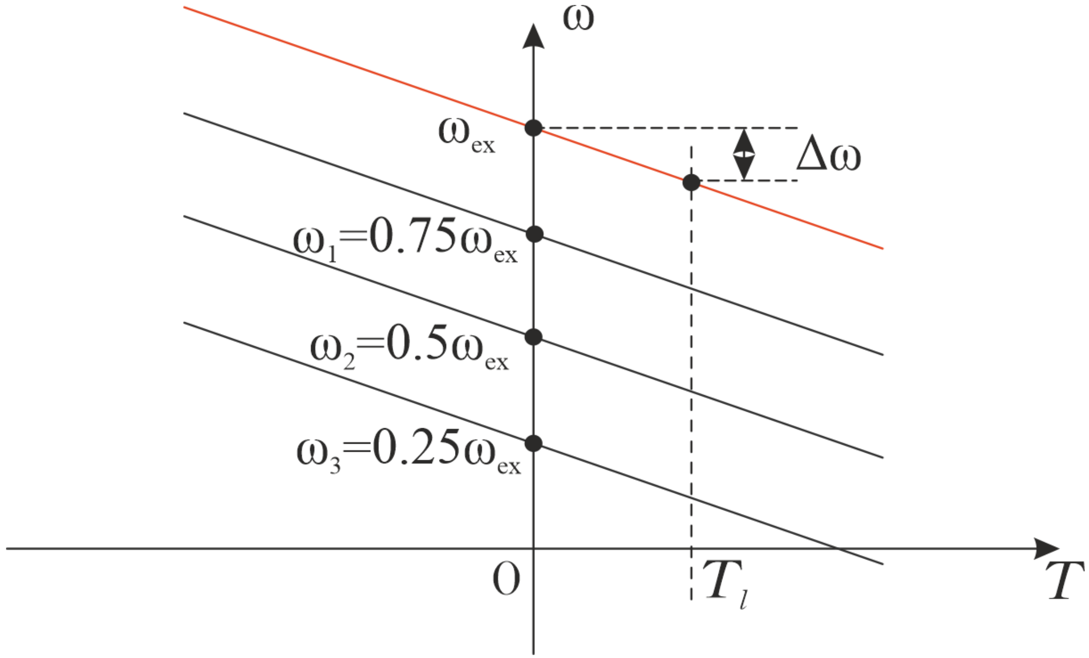


$$\omega_0 = \frac{u_a}{k\Phi} = \omega_{ex}, \omega_1, \omega_2 ....$$

$$\Delta\omega = \frac{T_{me} R_a}{k^2 \Phi^2}$$

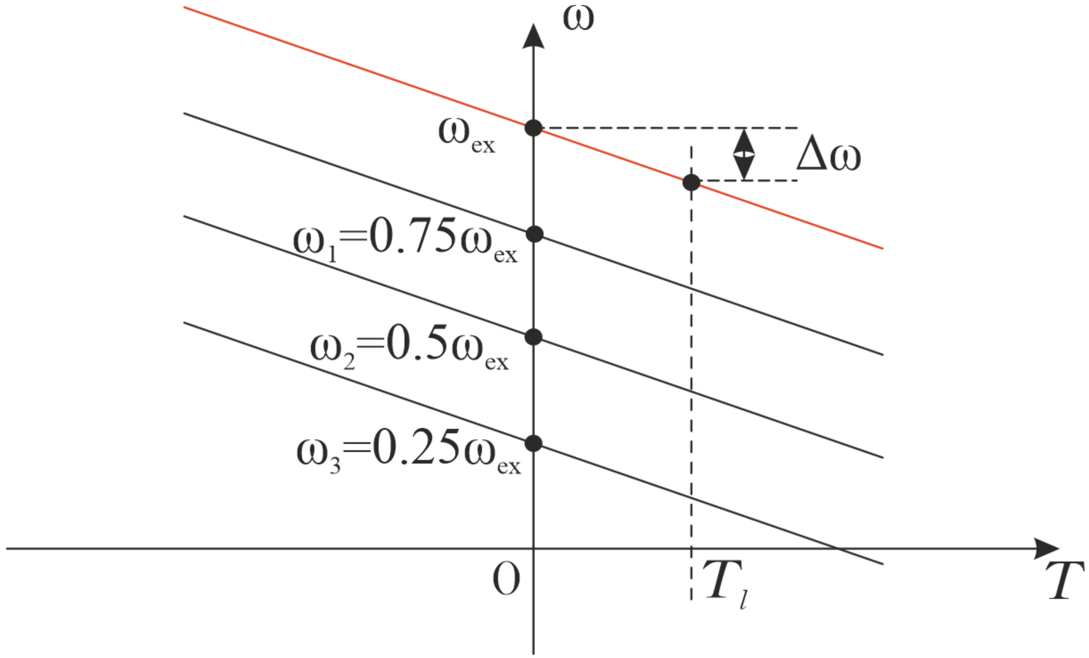
$$D = \frac{\omega_{ex}}{\omega_{\min}} = 1000 \div 10000$$

Speed control of the DC motor by changing armature voltage



Speed control by changing the armature voltage is the main method of speed control in wide-range drives.

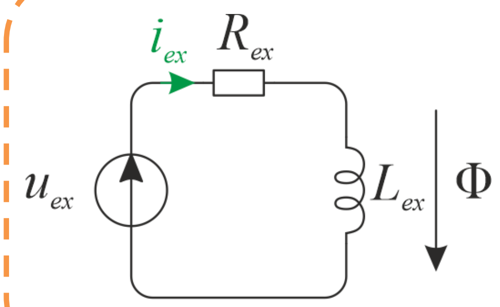
Speed control of the DC motor by changing armature voltage



$$T_{nom} = k\Phi i_{nom}$$

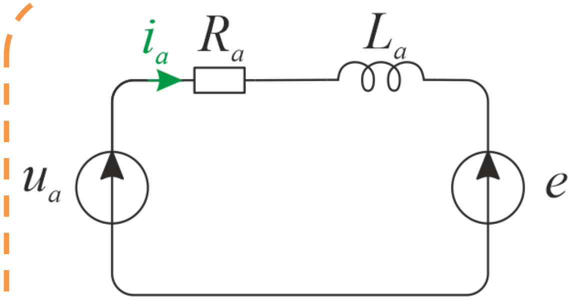
$$P = T_{nom}\omega_s$$

Speed control of the DC motor by changing the magnetic flux



excitation winding

u_{ex} = var



armature winding

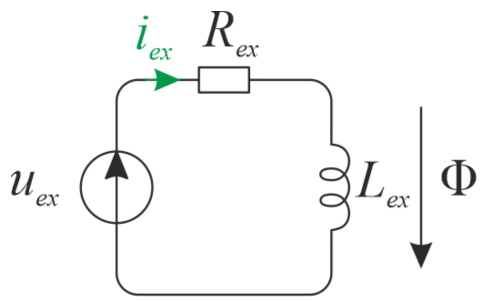
u_a = const

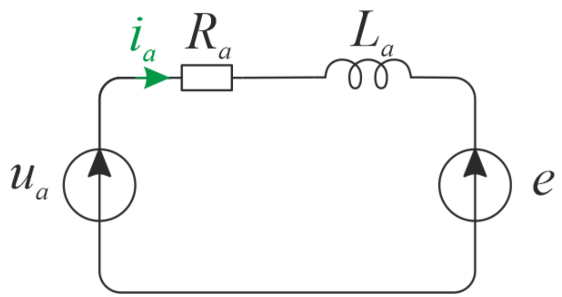
$$\omega = \frac{u_a}{k\Phi} - \frac{i_a R_a}{k\Phi}$$

$$\omega = \boxed{\frac{u_a}{k\Phi}} - \boxed{\frac{T_{me} R_a}{k^2 \Phi^2}}$$

k - constructive back-emf constant

Speed control of the DC motor by changing the magnetic flux



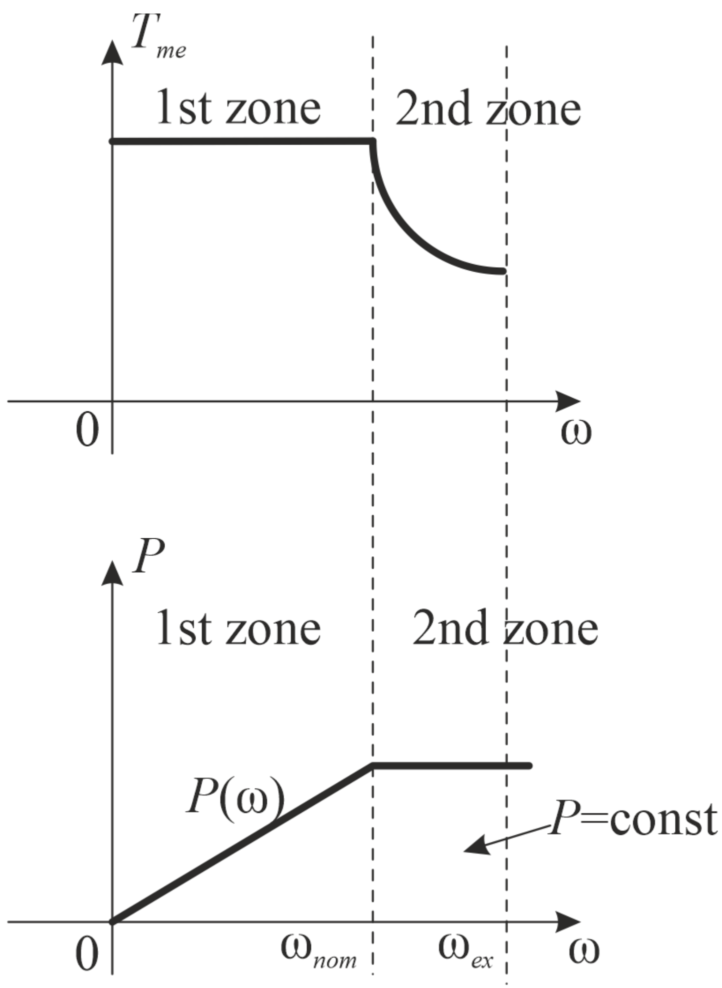


$$\omega_{nom} = \frac{u_{nom}}{k\Phi}$$

$$T_{nom} = k\Phi i_{nom} = k\Phi \frac{u_{nom}}{R_a}$$

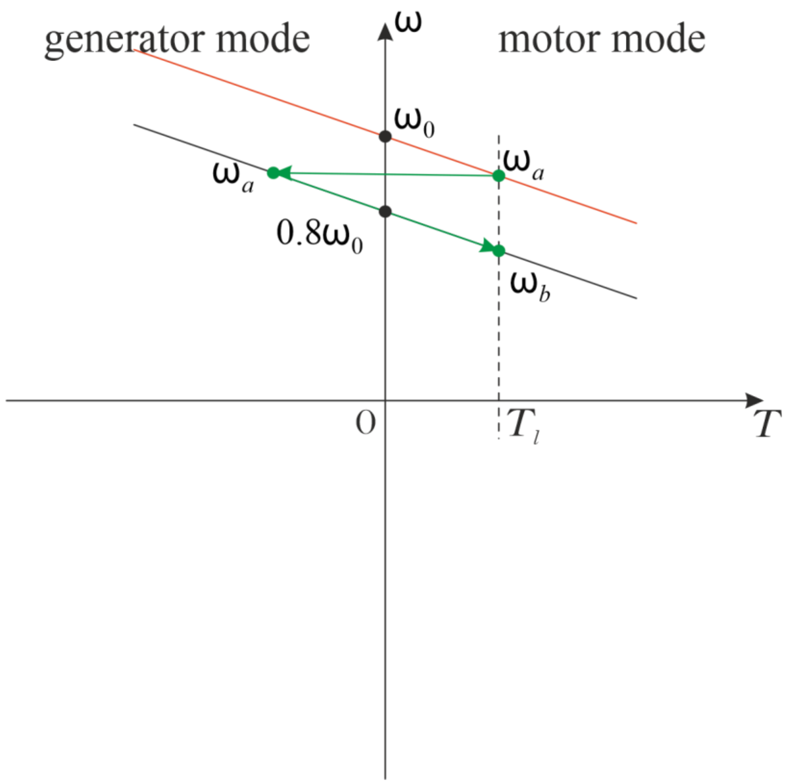
$$\Rightarrow \quad P_{nom} = \omega_{nom} T_{nom} = \frac{u_{nom}^{\;2}}{R_a} = const$$

Speed control of the DC motor by changing the magnetic flux



Braking modes

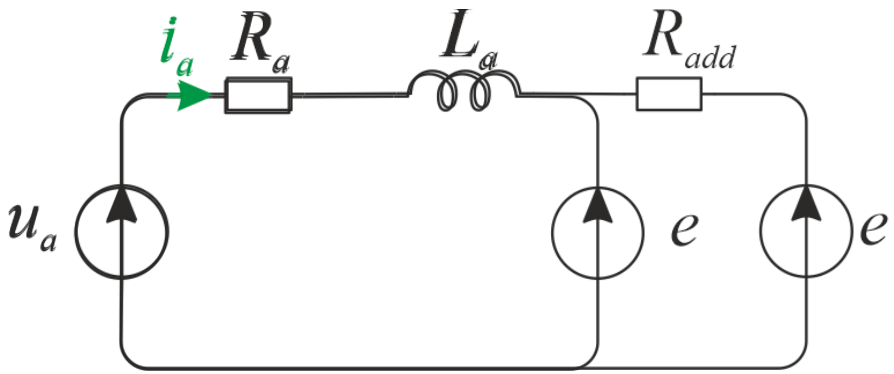
Regenerative braking with energy return to the power grid



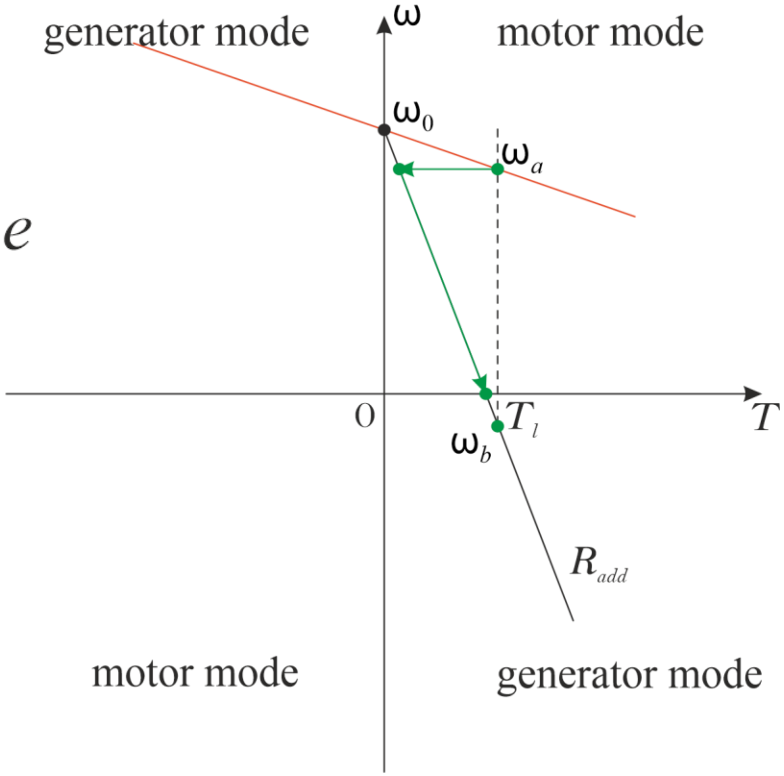
$U_0 = k_v \omega_0$ – nominal voltage $\Rightarrow$ $0.8U_0$

$$i_a = \frac{0.8U_0 - k_v \omega_0}{R_a} < 0 \Rightarrow T_{me} < 0 \Rightarrow \omega_a \to \omega_b$$

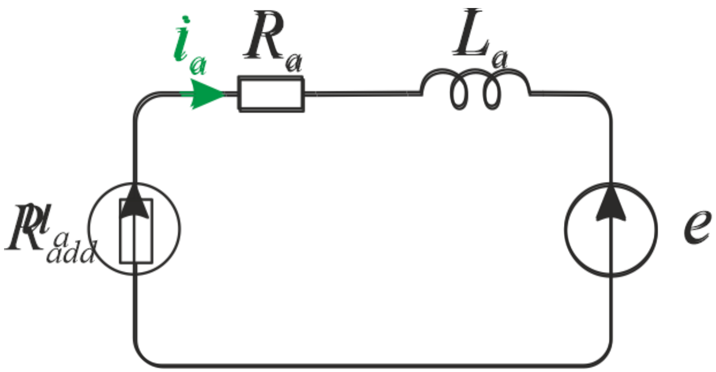
Reverse braking



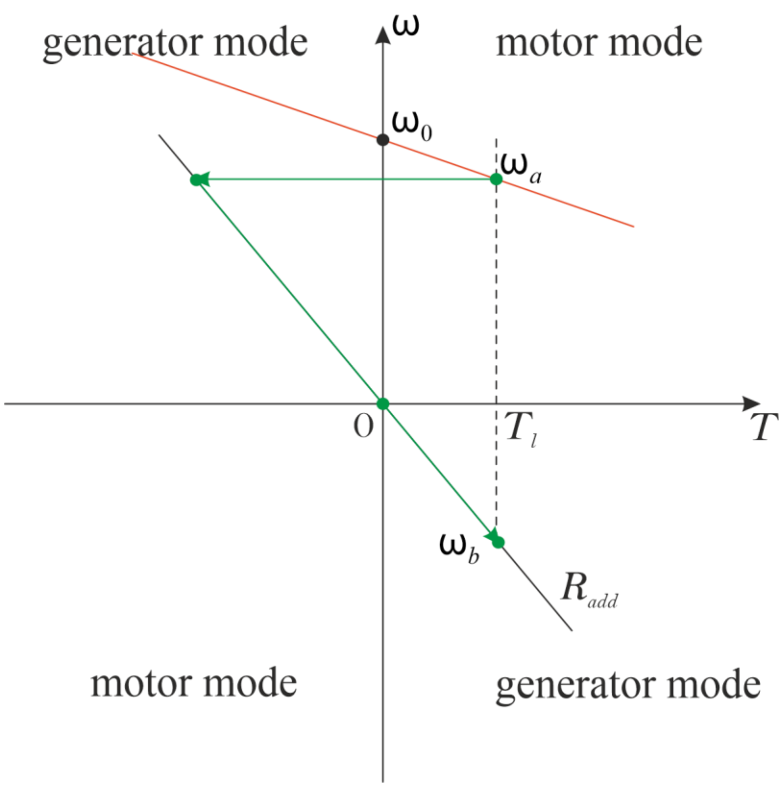
DC motor electrical part



Dynamic braking



DC motor electrical part



$$\omega = -\frac{T_{me}(R_a + R_{add})}{{k_v}^2}$$

$$T_{me} = -\frac{{k_v}^2\omega}{R_a + R_{add}}$$

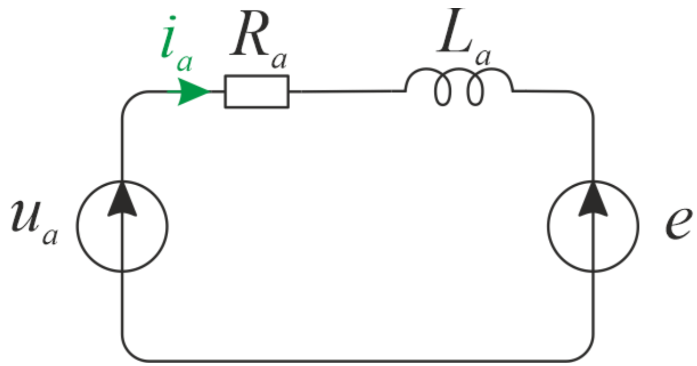
$k_v = k \cdot \Phi$ - back-emf constant

k - constructive back-emf constant

DC Motor Dynamics

Dynamic model of DC motor

Electrical part:



Time domain:

$$\frac{di_a(t)}{dt} = -\frac{R_a}{L_a} i_a(t) - \frac{1}{L_a} e(t) + \frac{1}{L_a} u_a(t)$$

$$\frac{di_a(t)}{dt} = -\frac{1}{T_e} i_a(t) - \frac{k \cdot \Phi}{R_a T_e} \omega(t) + \frac{1}{R_a T_e} u_a(t)$$

$$T_e = \frac{L_a}{R_a}$$

$$e_a(t) = k \cdot \Phi \cdot \omega(t)$$

T_e – electromagnetic time constant

e_a – back-EMF

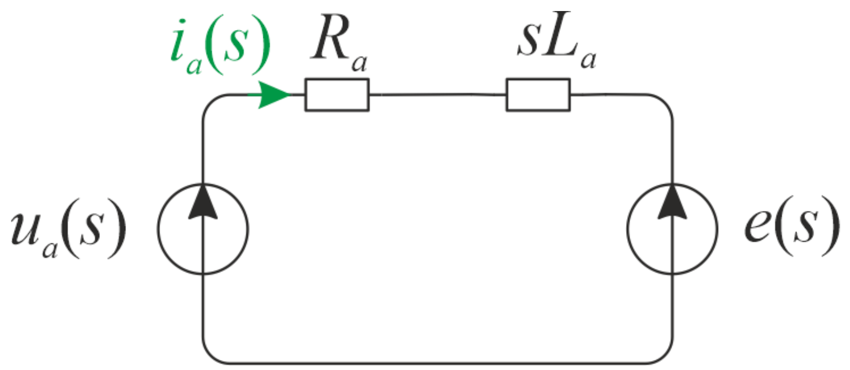
Φ – magnetic flux of field winding

ω – angular velocity

k – constructive coefficient

Dynamic model of DC motor

Electrical part:



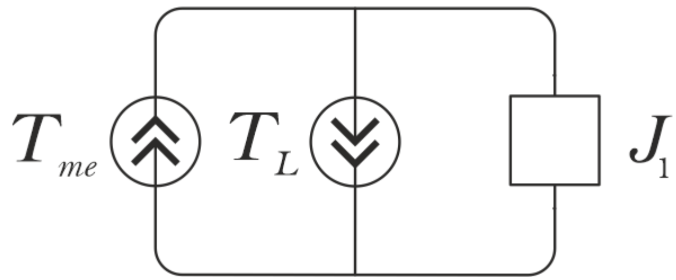
Laplace transform:

$$si_a(s) = -\frac{R_a}{L_a} i_a(s) - \frac{1}{L_a} e(s) + \frac{1}{L_a} u_a(s)$$

$$\boxed{si_a(s) = -\frac{1}{T_e} i_a(s) - \frac{k \cdot \Phi}{R_a T_e} \omega(s) + \frac{1}{R_a T_e} u_a(s)}$$

Dynamic model of DC motor

Mechanical part:



Time domain:

$$T_{dyn}(t) = T_{me}(t) - T_l(t)$$

$$T_{dyn}(t) = J\frac{d\omega(t)}{dt}$$

$$J\frac{d\omega(t)}{dt} = k \cdot \Phi \cdot i(t) - T_l(t)$$

$$T_{me}(t) = k \cdot \Phi \cdot i(t)$$

T_{me} – DC motor torque (magnetoelectric)

T_l – load torque

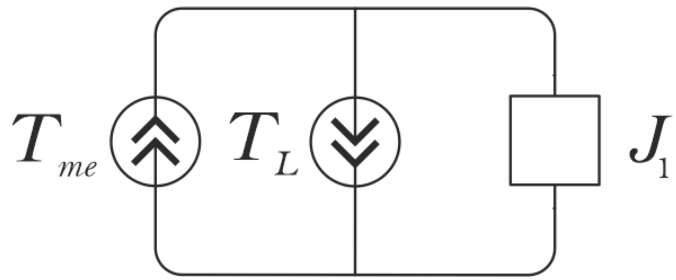
T_{dyn} – dynamic torque

ω – angular velocity

J – inertia

Dynamic model of DC motor

Mechanical part:



Laplace transform:

$$s\omega(s)=\frac{k\cdot\Phi}{J}i(s)-\frac{1}{J}T_l(s)$$

T_{me} – DC motor torque (magnetoelectric)

T_l – load torque

T_{dyn} – dynamic torque

ω – angular velocity

J – inertia

Dynamic model of DC motor

Time domain:

$$\begin{cases} \dfrac{di_a(t)}{dt} = -\dfrac{1}{T_e} i_a(t) - \dfrac{k \cdot \Phi}{R_a T_e} \omega(t) + \dfrac{1}{R_a T_e} u_a(t) \\ \dfrac{d\omega(t)}{dt} = \dfrac{k \cdot \Phi}{J} i(t) - \dfrac{1}{J} T_l(t) \end{cases}$$

Laplace domain:

$$\begin{cases} s i_a(s) = -\dfrac{1}{T_e} i_a(s) - \dfrac{k \cdot \Phi}{R_a T_e} \omega(s) + \dfrac{1}{R_a T_e} u_a(s) \\ s\omega(s) = \dfrac{k \cdot \Phi}{J} i(s) - \dfrac{1}{J} T_l(s) \end{cases}$$

Dynamic model of DC motor

Time domain:

$$\begin{cases} \dfrac{di_a(t)}{dt} = -\dfrac{1}{T_e} i_a(t) - \dfrac{1}{k_{st} R_a T_e}\omega(t) + \dfrac{1}{R_a T_e} u_a(t) \\ \dfrac{d\omega(t)}{dt} = \dfrac{k_{st} \cdot R_a}{T_m}\left(i(t) - i_l(t)\right) \end{cases}$$

$i_l(t)$ – equivalent disturbance current

$$i_l(t) = k_{st} T_l(t)$$

Laplace domain:

$$\begin{cases} s i_a(s) = -\dfrac{1}{T_e} i_a(s) - \dfrac{1}{k_{st} R_a T_e}\omega(s) + \dfrac{1}{R_a T_e} u_a(s) \\ s\omega(s) = \dfrac{k_{st} \cdot R_a}{T_m}\left(i(s) - i_l(s)\right) \end{cases}$$

k_v - back-EMF constant

k_{st} - static gain of the drive

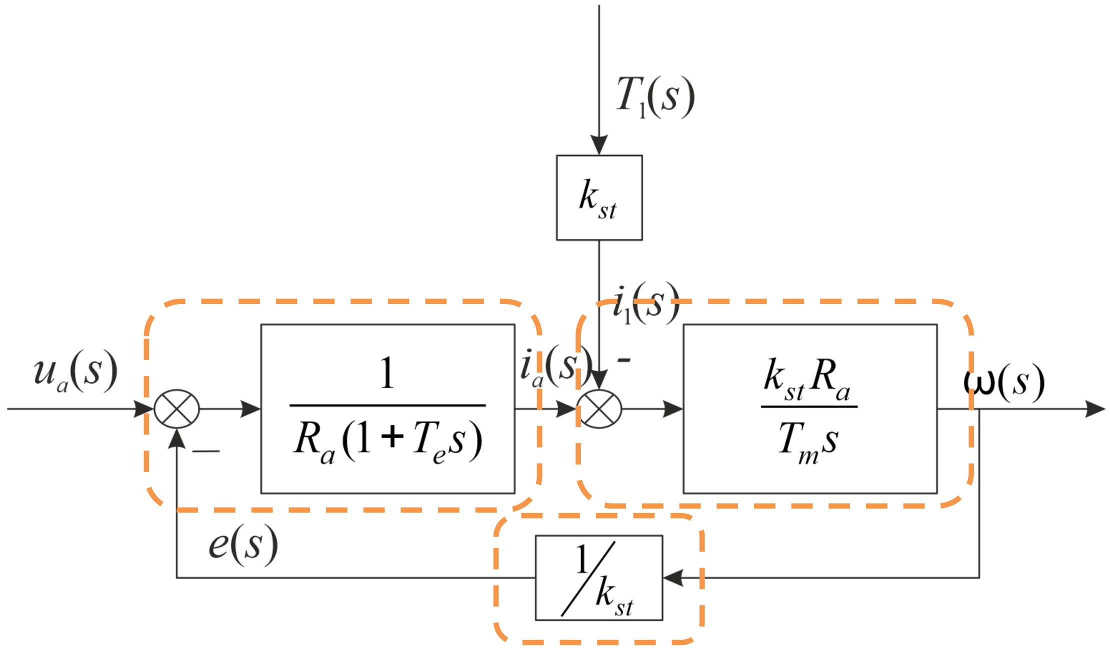
T_m - electromechanical constant

$$k_v = k \cdot \Phi$$

$$k_{st} = \frac{1}{k_v}$$

$$T_m = \frac{J \cdot R_a}{k_v^{\,2}}$$

Dynamic model of DC motor



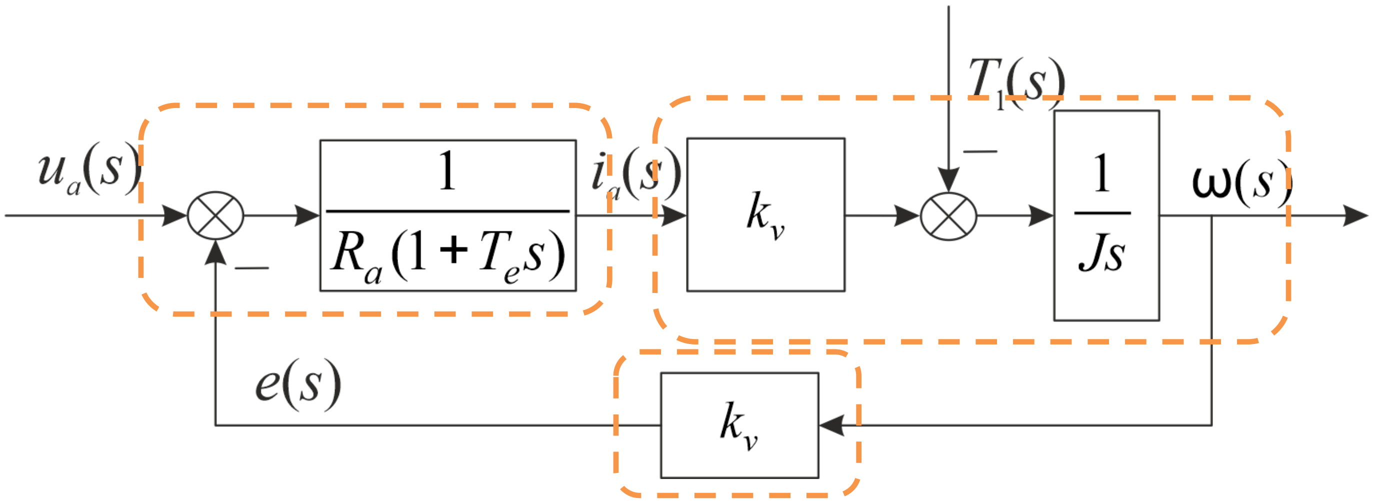
$$\begin{cases} s i_a(s) = -\frac{1}{T_e} i_a(s) - \frac{1}{k_{st} R_a T_e} \omega(s) + \frac{1}{R_a T_e} u_a(s) \\ s\omega(s) = \frac{k_{st} \cdot R_a}{T_m}\left(i(s) - i_l(s)\right) \end{cases}$$

$$k_v = k \cdot \Phi$$

$$k_{st} = \frac{1}{k_v}$$

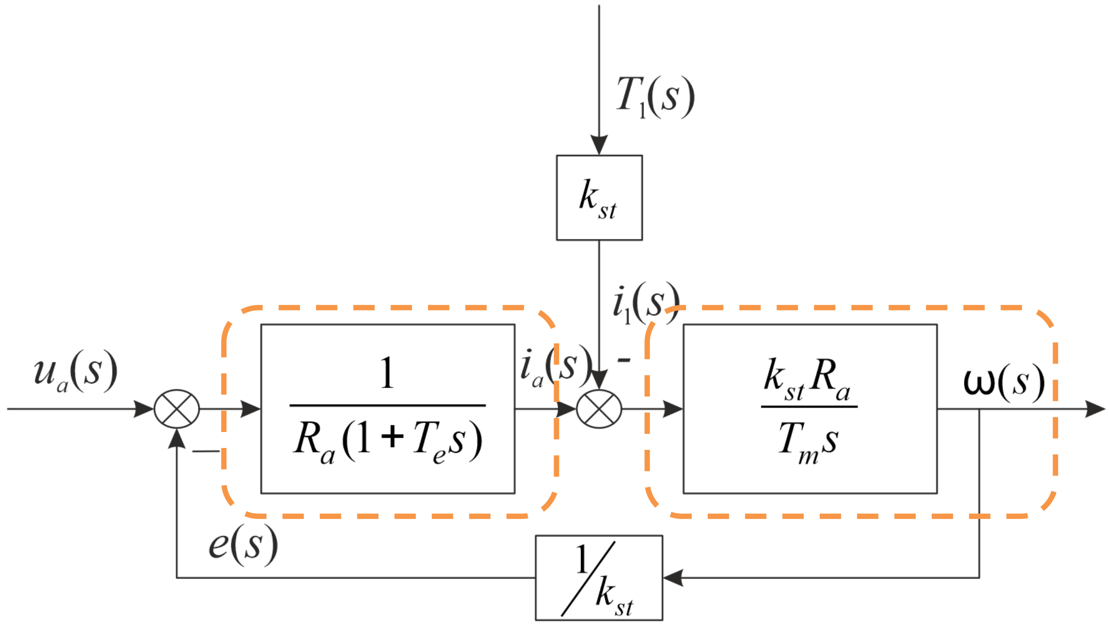
$$T_m = \frac{J \cdot R_a}{k_v{}^2}$$

Dynamic model of DC motor



$$
\begin{cases}
s i_a(s) = -\frac{1}{T_e} i_a(s) - \frac{k_v}{R_a T_e}\omega(s) + \frac{1}{R_a T_e} u_a(s) \\
s\omega(s) = \frac{1}{J}\left(k_v i(s) - T_l(s)\right)
\end{cases}
$$

Dynamic model of DC motor



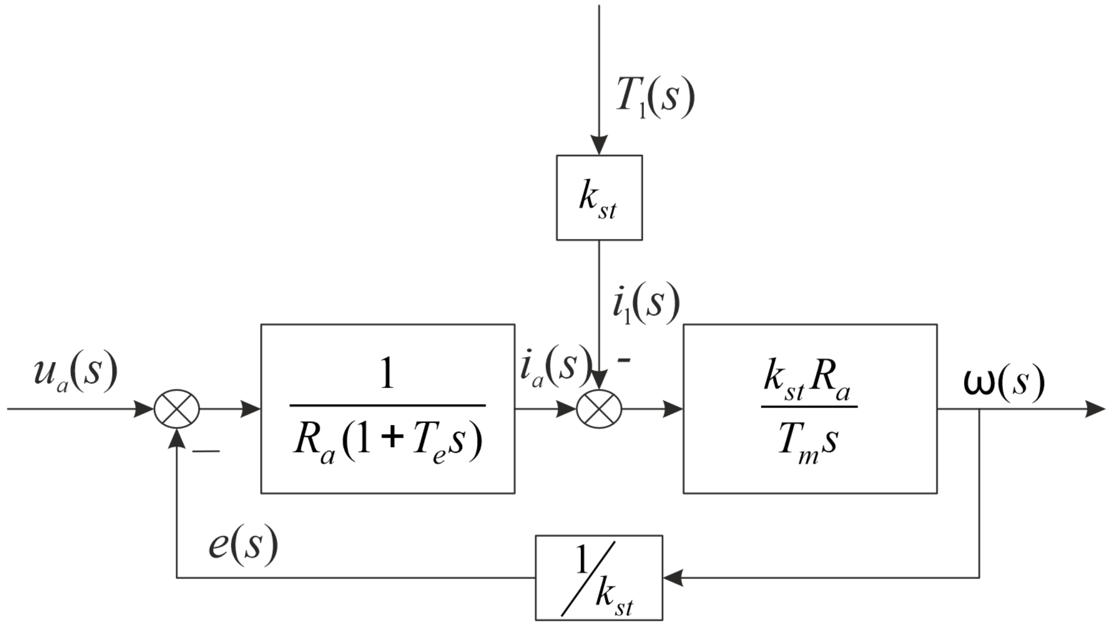
$$W_c(s) = \frac{\omega(s)}{u_a(s)} = \frac{W_e(s)W_m(s)}{1 + W_e(s)W_m(s)/k_{st}}$$

$$W_c(s) = \frac{k_{st}}{T_e T_m s^2 + T_m s + 1}$$

$$W_e(s) = \frac{i_a(s)}{u_a(s)} = \frac{1}{R_a(1+T_e s)}$$

$$W_m(s) = \frac{\omega(s)}{i_a(s)} = \frac{k_{st}R_a}{T_m s}$$

Dynamic model of DC motor



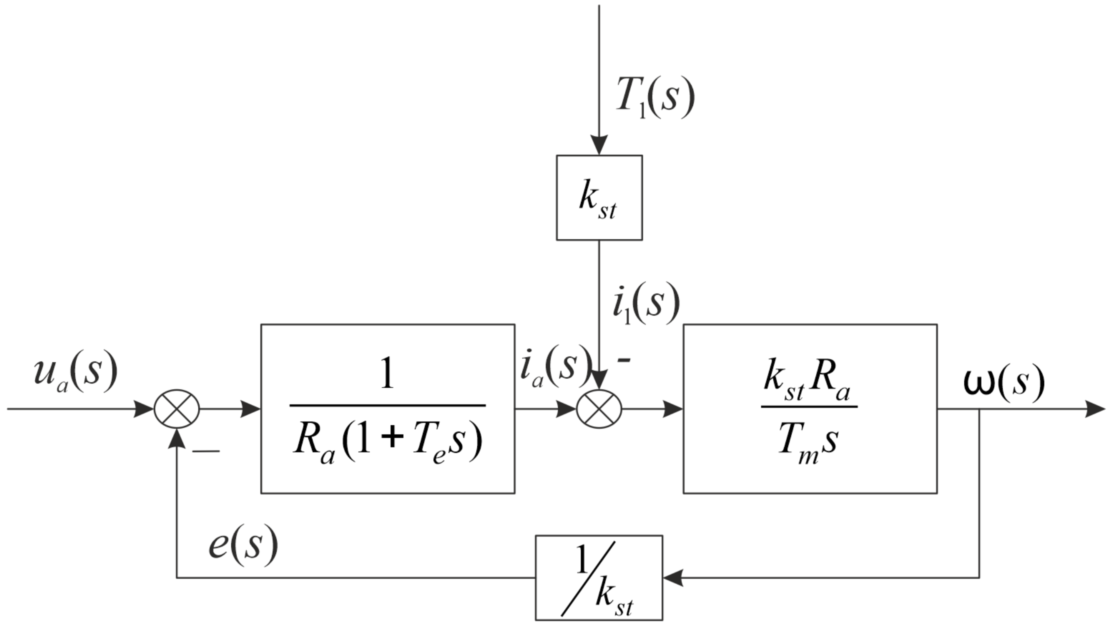
$$W_{dist}(s) = \frac{\omega(s)}{T_l(s)} = \frac{W_m(s)}{1 + W_c(s)W_m(s)/k_{st}}$$

$$W_e(s) = \frac{i_a(s)}{u_a(s)} = \frac{1}{R_a(1+T_e s)}$$

$$W_m(s) = \frac{\omega(s)}{i_a(s)} = \frac{k_{st}R_a}{T_m s}$$

$$W_{dist}(s) = \frac{k_{st}^{\ 2} \cdot R_a}{T_e T_m s^2 + T_m s + 1}$$

Dynamic model of DC motor



Two cases:

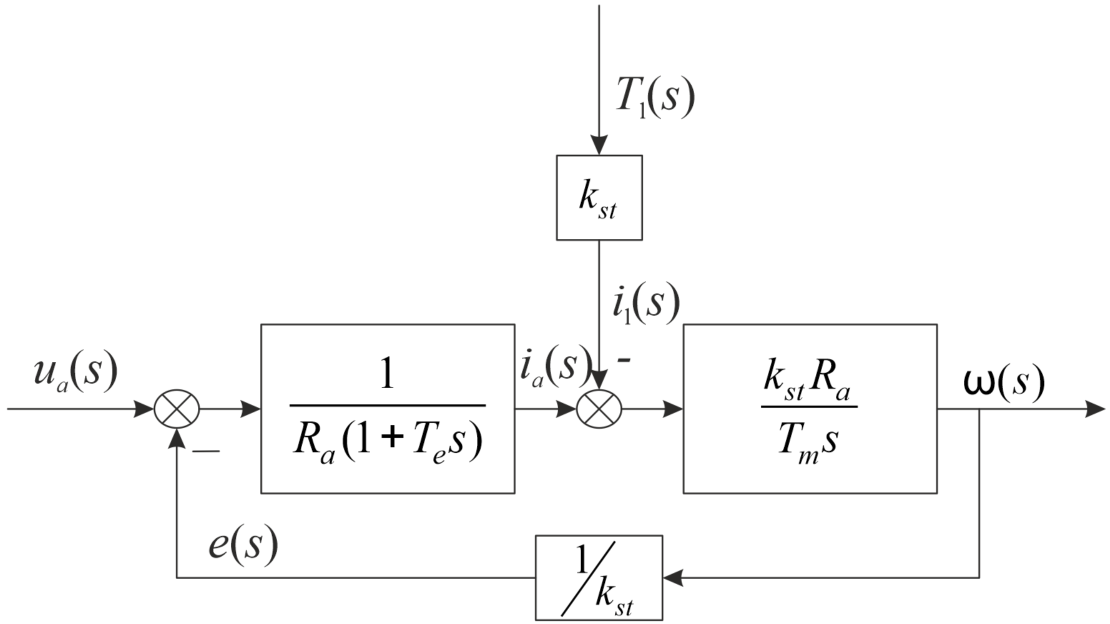
$$W_c(s) = \frac{k_{st}}{T_e T_m s^2 + T_m s + 1}$$

$$W_{dist}(s) = \frac{k_{st}^2 \cdot R_a}{T_e T_m s^2 + T_m s + 1}$$

$T_m \geq 4T_e$ real poles

$T_m < 4T_e$ complex poles

Dynamic model of DC motor



Case of real poles $T_m \geq 4T_e$

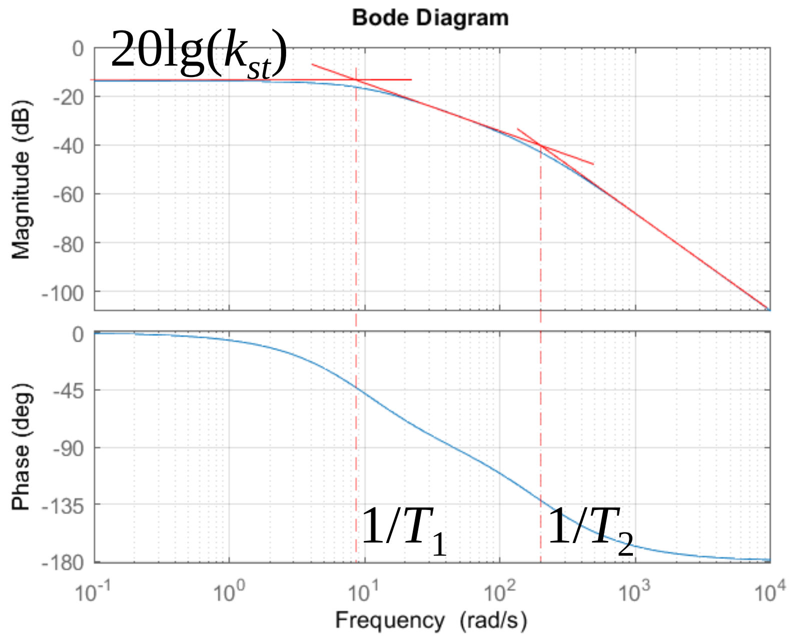
$$W_c(s) = \frac{k_{st}}{(T_1 s+1)(T_2 s+1)}$$

$$s_{1,2} = \frac{-T_m \pm \sqrt{T_m^{\ 2} - 4T_e T_m}}{2T_m T_e}$$

$$W_{dist}(s) = \frac{k_{st}^{\ 2} \cdot R_a}{(T_1 s+1)(T_2 s+1)}$$

$$T_1 = -\frac{1}{s_1},\ T_2 = -\frac{1}{s_2}$$

Dynamic model of DC motor



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$$W_c(s) = \frac{k_{st}}{(T_1 s + 1)(T_2 s + 1)}$$

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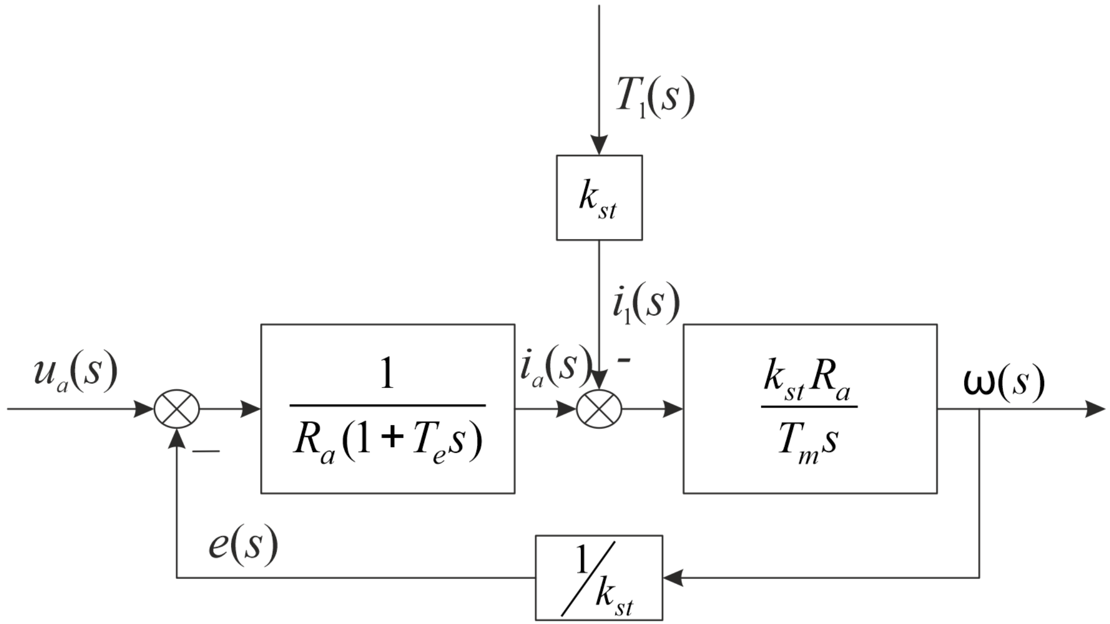
$$T_1 = -\frac{1}{s_1}, \; T_2 = -\frac{1}{s_2}$$

Dynamic model of DC motor

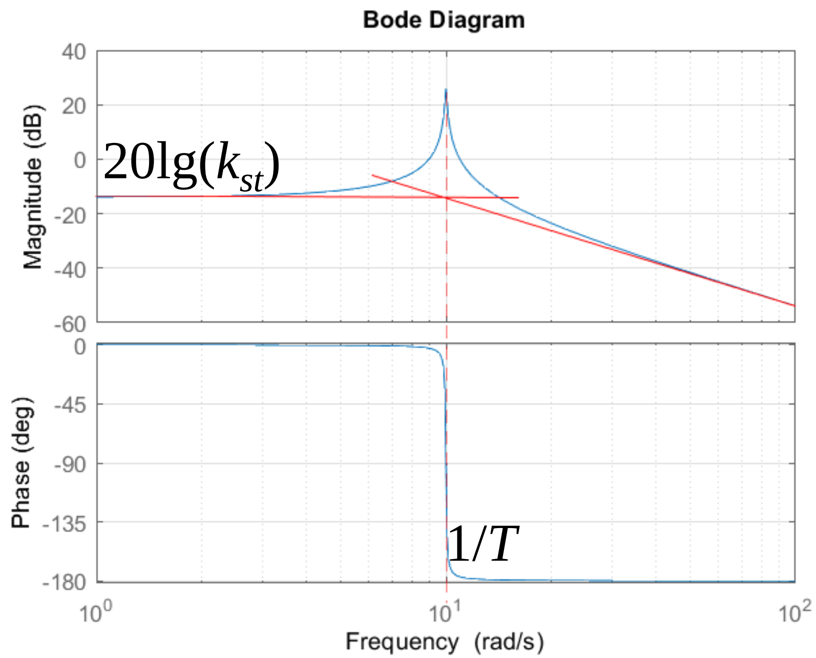


Case of imaginary poles $T_m < 4T_e$

$$W_c(s) = \frac{k_{st}}{T^2 s^2 + 2T\xi s + 1} \qquad T = \sqrt{T_e T_m}$$

$$W_{dist}(s) = \frac{k_{st}^{\ 2} \cdot R_a}{T^2 s^2 + 2T\xi s + 1} \qquad \xi = \frac{T_m}{2T}$$

Dynamic model of DC motor



Case of imaginary poles $T_m < 4T_e$

$$W_c(s) = \frac{k_{st}}{T^2 s^2 + 2T\xi s + 1} \qquad T = \sqrt{T_e T_m}$$

$$W_{dist}(s) = \frac{k_{st}{}^2 \cdot R_a}{T^2 s^2 + 2T\xi s + 1} \qquad \xi = \frac{T_m}{2T}$$