ITMO UNIVERSITY

Operating modes of the electric drives

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- Nominal operating modes of motors
- Calculation of motor power in continuous operation mode
- Calculation of motor power for short-term operation mode
- Optimal control of the position drives

Load diagrams of the electric drive

Load diagrams of the electric drive

The load diagrams of the electric drive are the dependences of static and dynamic loads on time

The load diagram of the actuator is the dependence of the static load torque $T_{dist}(t)$ on time. It is usually supplemented by a diagram of the given rotation speeds $\omega^*(t)$.

The motor load diagram is the dependence of the motor torque (corresponding to the drive tachogram) over time.

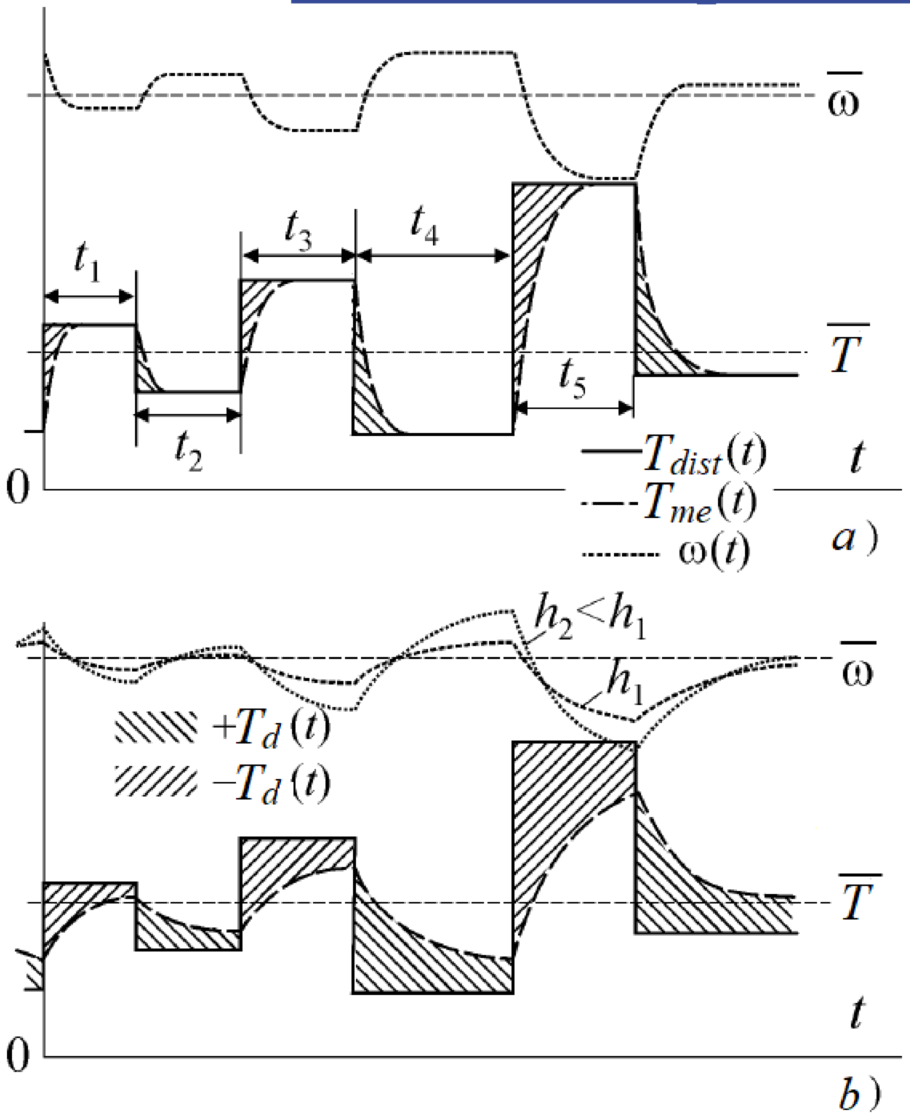
Load diagrams of the electric drive

All production mechanisms can be divided into two large groups: **continuous mechanisms** and **cyclic mechanisms**.

Continuous action mechanisms work during a work shift or even several days.

A feature of the **mechanisms of cyclic action** is the presence in the cycle of one or more starts, reverses, brakings.

Load diagram of a mechanism operating in continuous operation with variable load



$T_{me}(t)$ – magnetoelectric torque

$T_{dist}(t)$ – disturbance torque

$T_d(t)$ – dynamic torque

$T_m = J/h$ – electromechanical time constant

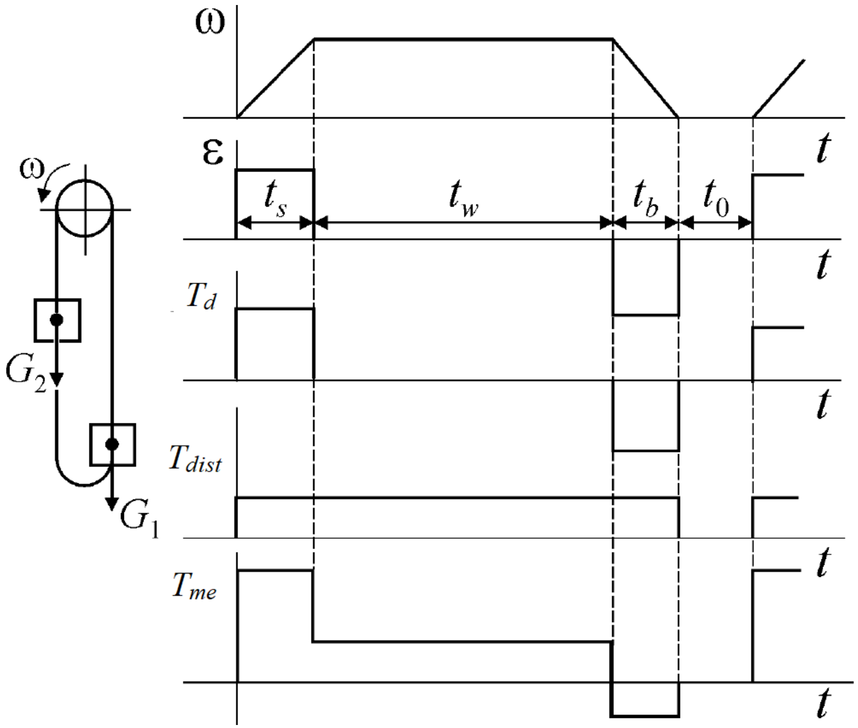
J – inertia

h – rigidity of the mechanical characteristic of the motor

$t_q > 3T_m$ – case of *a)*

$t_q < 3T_m$ – case of *b)*

Load diagram of a cyclic action mechanism



$T_{me}(t)$ – magnetoelectric torque

$T_{dist}(t)$ – disturbance torque

$T_d(t)$ – dynamic torque

t_s – interval of starting

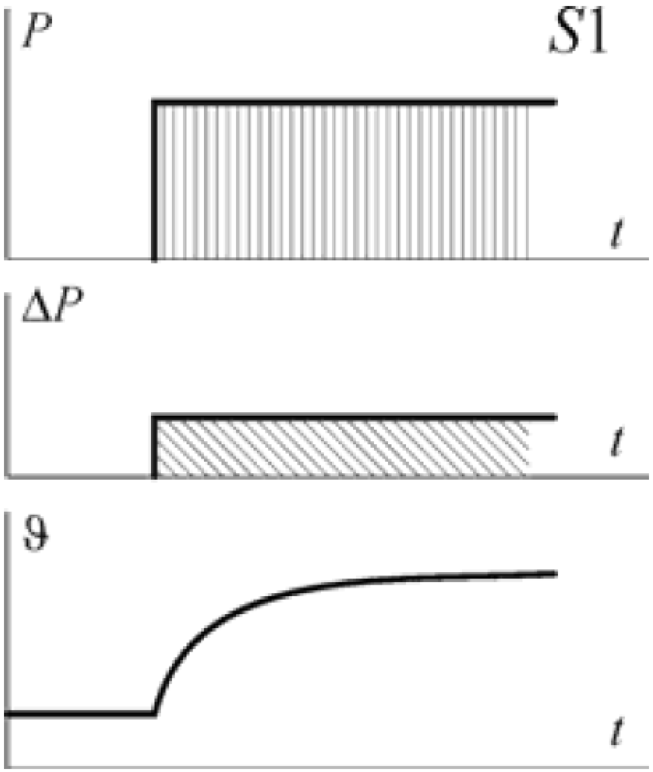
t_w – interval of operation with a constant rated load

t_b – interval of braking

t_c – interval of cycle

Nominal operating modes of motors

Long-term nominal operation mode of a motor

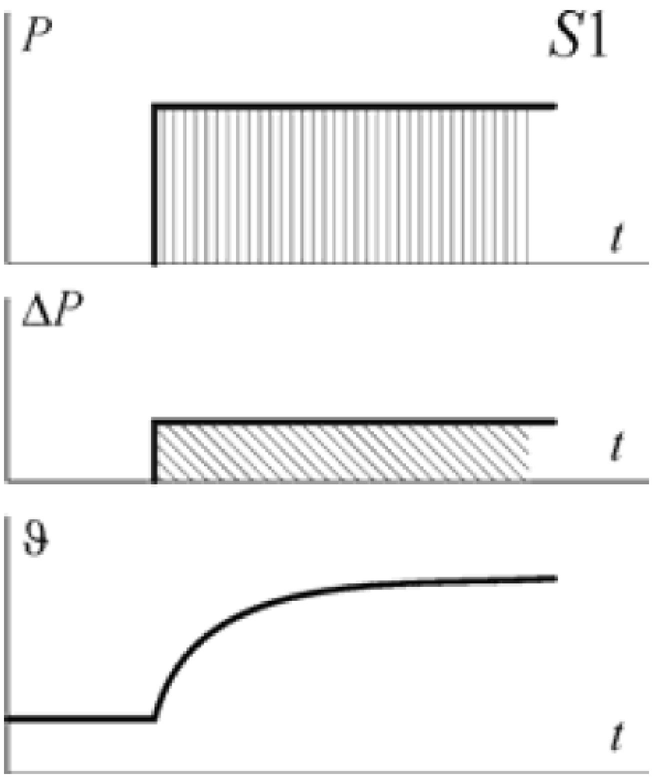


P – net mechanical power

ΔP – power loss

θ – motor temperature

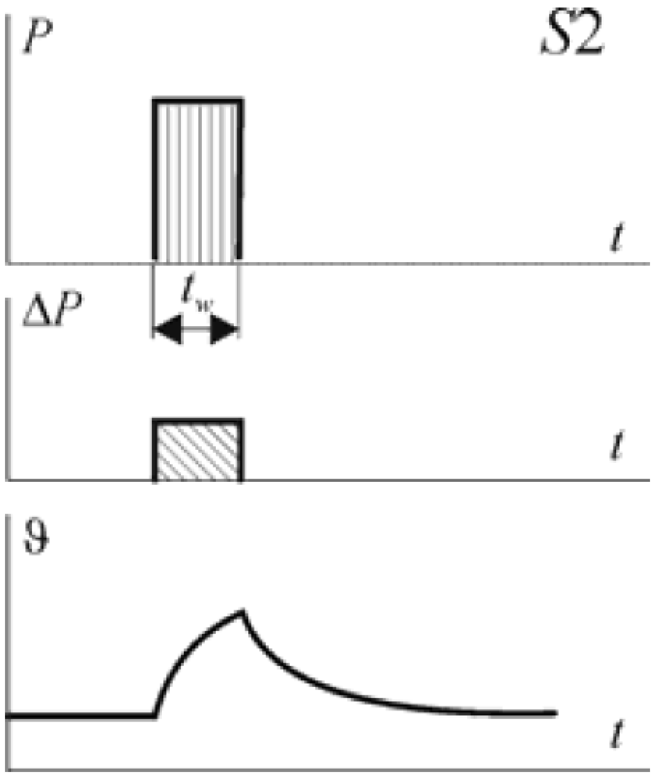
Long-term nominal operation mode of a motor



Features of S1:

- the motor have time to heat up to a steady temperature

Short-term nominal operation mode of a motor



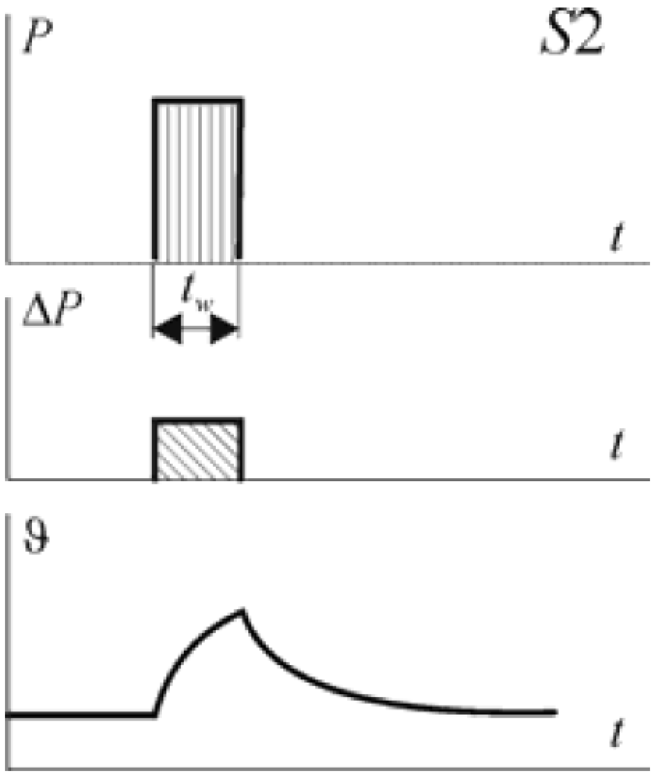
P – net mechanical power

ΔP – power loss

θ – motor temperature

t_w = 10, 30, 60, 90 min

Short-term nominal operation mode of a motor

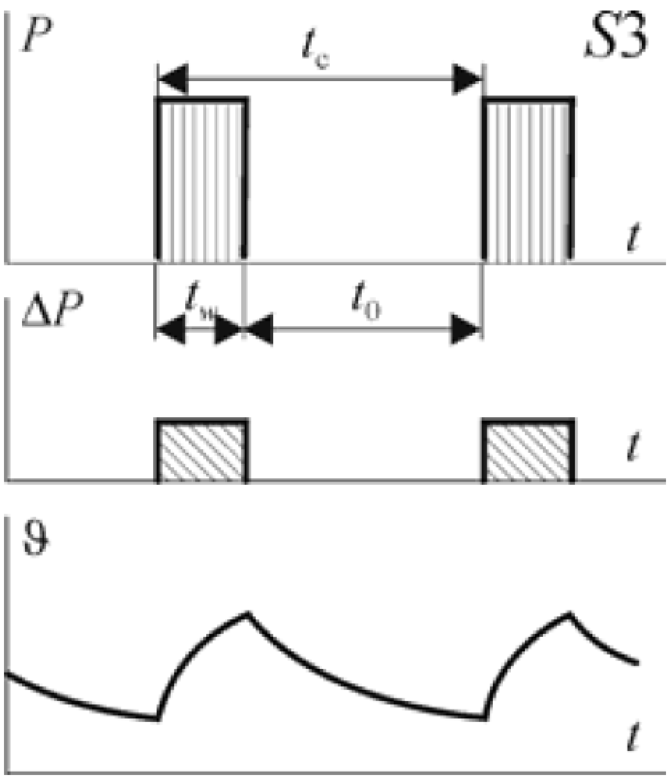


Features of S2:

- during operation motor does not have time to heat up to a steady temperature

- during the shutdown state motor cools down to ambient temperature

Intermittent-short-term nominal operation mode of a motor



t_w – interval of operation with a constant rated load

t_0 – interval of disconnection from power supply

t_c – interval of cycle

CD – duration of switching on

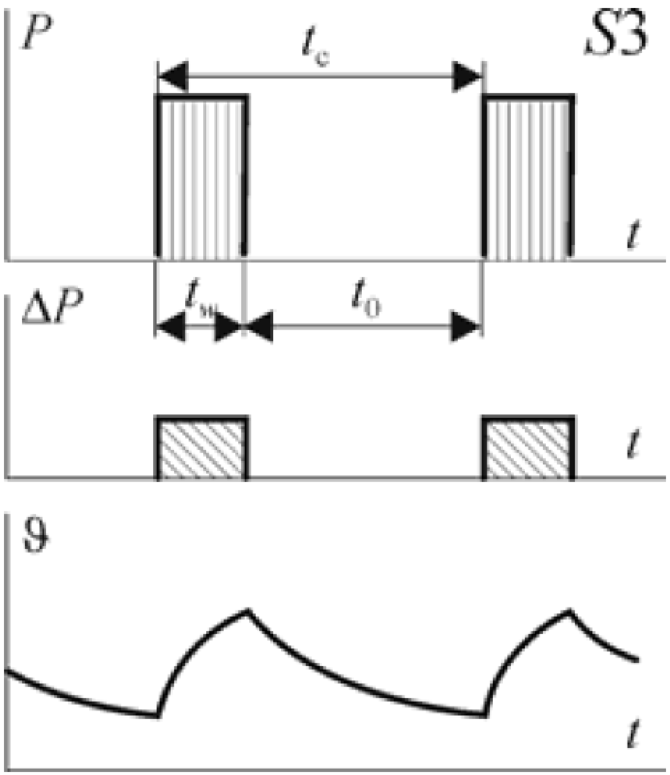
$$CD = \frac{t_w}{t_w + t_0} \cdot 100\% = \frac{t_w}{t_c} \cdot 100\%$$

CD = 15%, 25%, 40%, 60%

CD = 25%

CD = 40%

Intermittent-short-term nominal operation mode of a motor



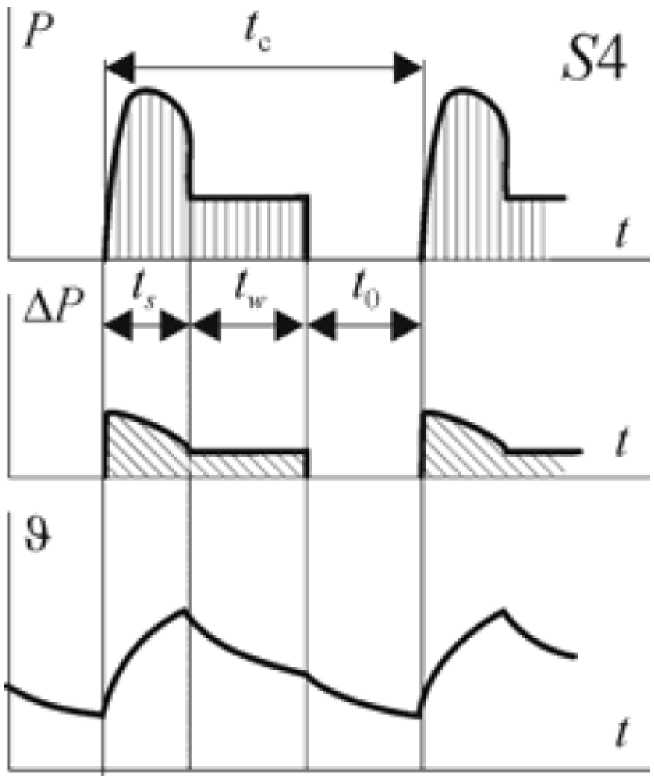
Features of S3:

- short-term operation with a rated load tw (working intervals) periodically alternate with intervals of the off state

- the duration of all intervals is insufficient to achieve steady temperature values

- the average value of the motor temperature over the period is higher than the ambient temperature

Intermittent-short-term nominal operation mode of a motor with frequent starts



t_s – interval of starting

t_w – interval of operation with a constant rated load

t_0 – interval of disconnection from power supply

t_c – interval of cycle

CD – duration of switching on

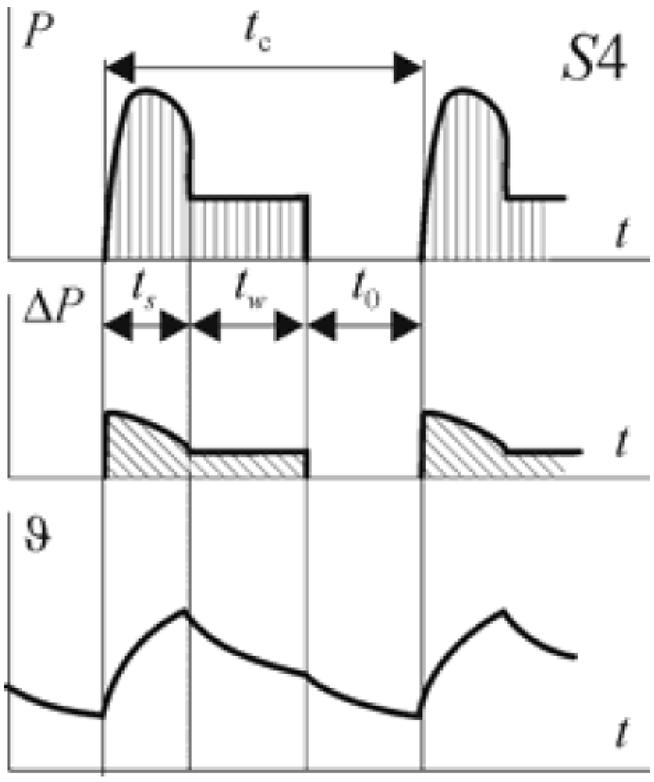
$$CD = \frac{t_s + t_w}{t_s + t_w + t_0} \cdot 100\% = \frac{t_s + t_w}{t_c} \cdot 100\%$$

CD = 15%, 25%, 40%, 60%

$$FI = \frac{J_\Sigma}{J_m}$$

FI = 1.2; 1.6; 3.5; 4; 6.3 and 10

Intermittent-short-term nominal operation mode of a motor with frequent starts



Features of S4:

- sufficient starting intervals

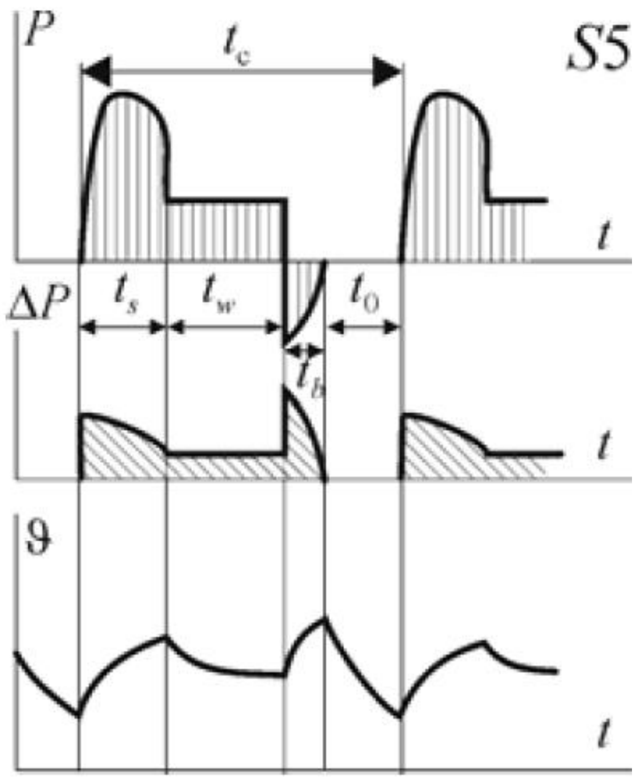
- the inertia coefficient FI characterizes the starting conditions

- starting losses significantly affect the thermal regime of the motor

- the duration of all intervals is insufficient to achieve steady temperature values

- the average value of the motor temperature over the period is higher than the ambient temperature

Intermittent-short-term nominal operation mode of a motor with frequent starts and electrical braking



t_s – interval of starting

t_w – interval of operation with a constant rated load

t_0 – interval of disconnection from power supply

t_b – interval of braking

t_c – interval of cycle

CD – duration of switching on

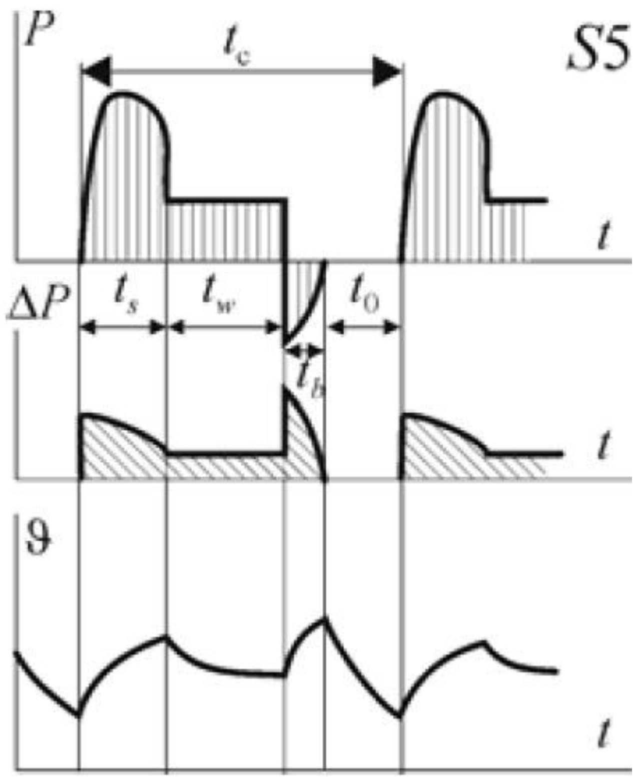
$$CD = \frac{t_s + t_w + t_b}{t_s + t_w + t_b + t_0} \cdot 100\% = \frac{t_s + t_w + t_b}{t_c} \cdot 100\%$$

CD = 15%, 25%, 40%, 60%

$$FI = \frac{J_\Sigma}{J_m}$$

FI = 1.2; 1.6; 2; 3.5; 4

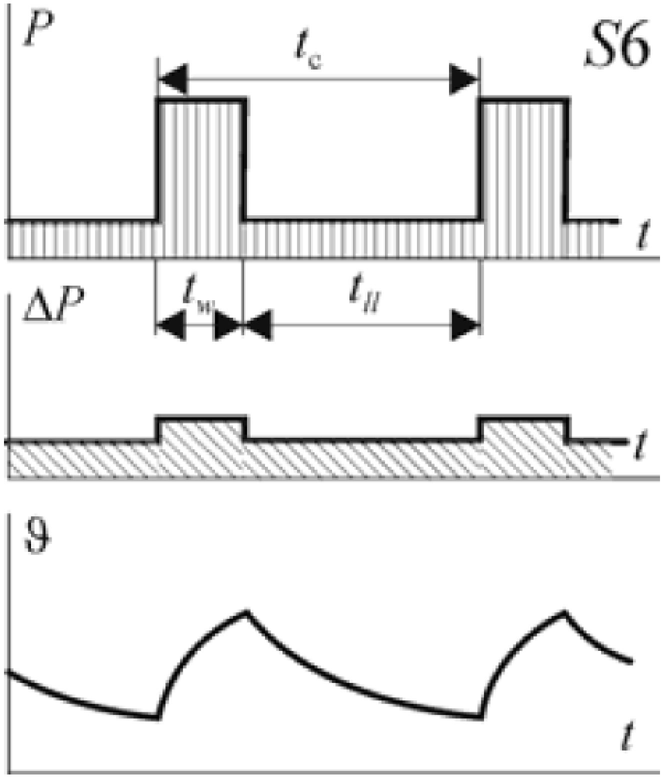
Intermittent-short-term nominal operation mode of a motor with frequent starts and electrical braking



Features of S5:

- sufficient starting intervals
- sufficient braking intervals
- the inertia coefficient FI characterizes the starting conditions
- the duration of all intervals is insufficient to achieve steady temperature values
- all mode characteristics and normalized values are practically the same as for S4
- the average value of the motor temperature over the period is higher than the ambient temperature

Intermittent nominal operation mode of a motor

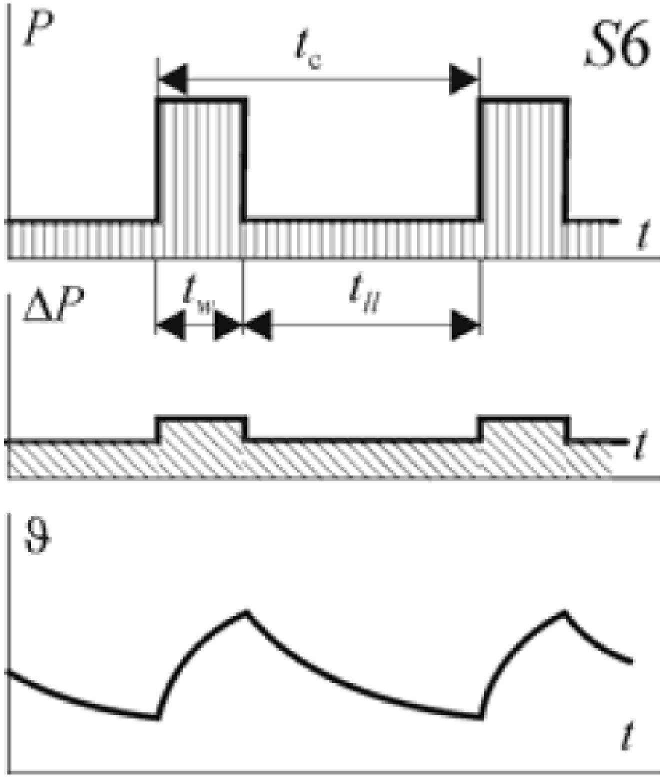


t_w – interval of operation with a constant rated load

t_{II} – interval of ideal idle

t_c – interval of cycle

Intermittent nominal operation mode of a motor



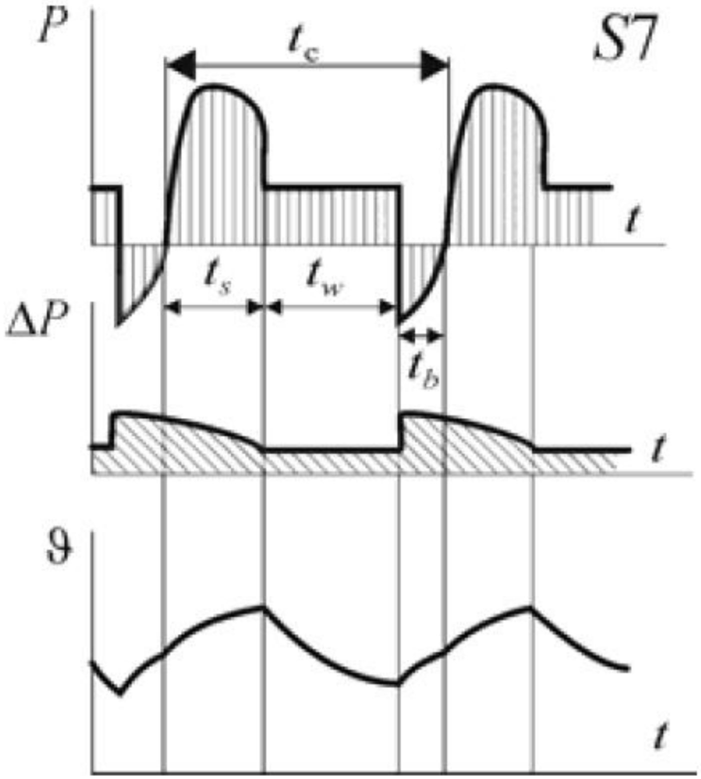
Features of S6:

- short-term operation with a rated load tw (working intervals) periodically alternate with intervals of the <u>ideal idle</u>

- the duration of all intervals is insufficient to achieve steady temperature values

- the average value of the motor temperature over the period is higher than the ambient temperature

Intermittent nominal operation of a motor with frequent reverses



t_s – interval of starting

t_w – interval of operation with a constant rated load

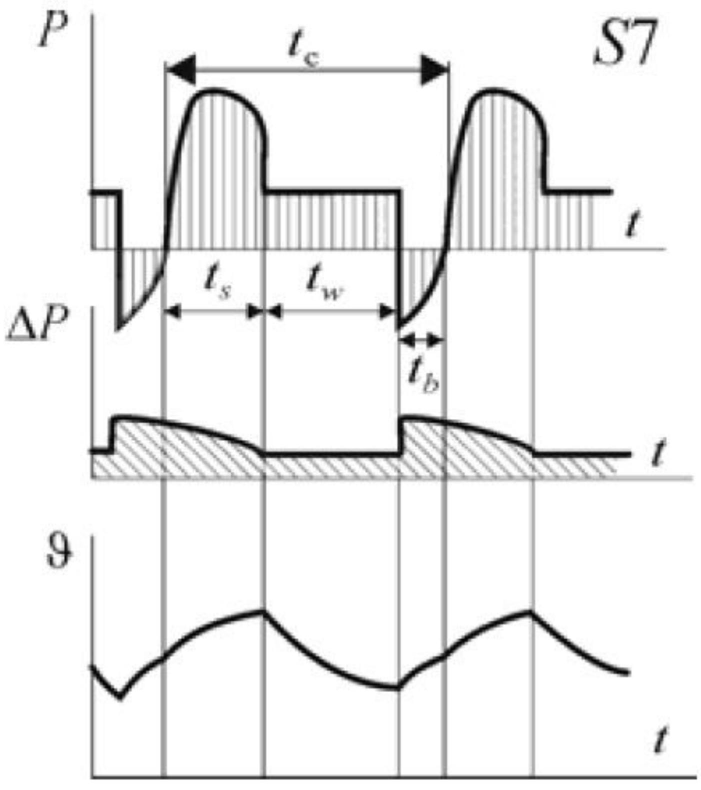
t_b – interval of braking

t_c – interval of cycle

$$FI = \frac{J_\Sigma}{J_m}$$

FI = 1.2; 1.6; 2; 3.5; 4

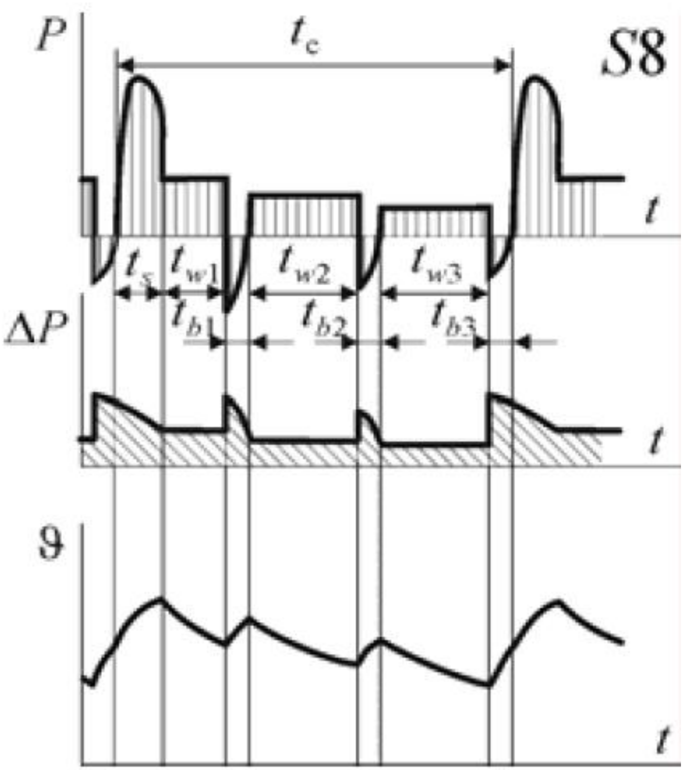
Intermittent nominal operation of a motor with frequent reverses



Features of S7:

- sufficient starting intervals
- sufficient braking intervals
- there are no pauses
- the inertia coefficient *FI* characterizes the starting conditions
- the duration of all intervals is insufficient to achieve steady temperature values
- all mode characteristics and normalized values are practically the same as for S5
- the average value of the motor temperature over the period is higher than the ambient temperature
- reverse modes significantly affect the temperature regime

Intermittent nominal operation of a motor with two or more values of angular speeds



t_s – interval of starting

$t_{w.q}$ – interval of operation with a constant rated load at the qth stage

$t_{b.q}$ – interval of braking at the qth stage

t_c – interval of cycle

$t_{p.q}$ – t_s or $t_{b.q}$

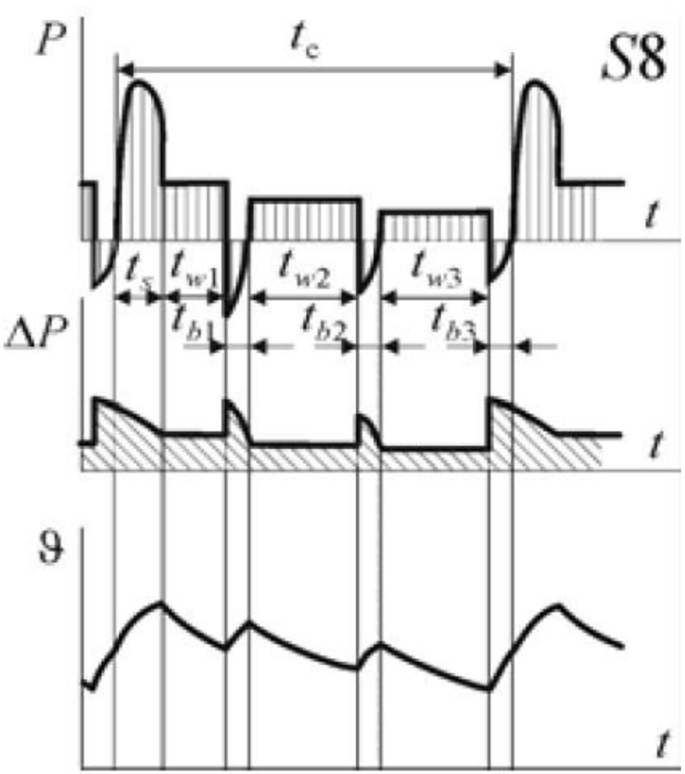
CD_q – duration of switching on during *q*-th stage

$$CD_q = \frac{t_{p.q} + t_{w.q}}{t_c} \cdot 100\%$$

$$FI = \frac{J_\Sigma}{J_m}$$

FI = 1.2; 1.6; 2; 3.5; 4

Intermittent nominal operation of a motor with two or more values of angular speeds



Features of S7:

- sufficient starting intervals
- sufficient braking intervals
- there are no pauses
- the inertia coefficient *FI* characterizes the starting conditions
- the duration of all intervals is insufficient to achieve steady temperature values
- all mode characteristics and normalized values are practically the same as for S5
- the average value of the motor temperature over the period is higher than the ambient temperature
- reverse modes significantly affect the temperature regime
- there are several accelerations and decelerations during duty cycle

Nominal operating modes of motors

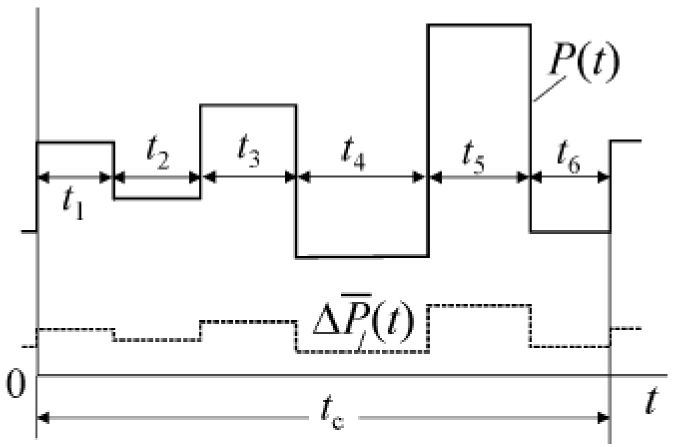
S1, S2 and S3 are the main ones in solving practical problems

Modes S4 and S5 can be reduced by equivalent transformations to mode S3

Modes S6 – S8 can be reduced by equivalent transformations to mode S1

Calculation of motor power in continuous operation mode

Calculation of motor power in continuous operation mode



Motor power selection formula by the average loss method:

$$\Delta\overline{P} = \frac{\sum_{i=1}^{n} \Delta\overline{P}_i t_i}{\sum_{i=1}^{n} \beta_i t_i} \leq \Delta P_N \,.$$

β_i - coefficient of heat transfer deterioration on the i-th cycle interval

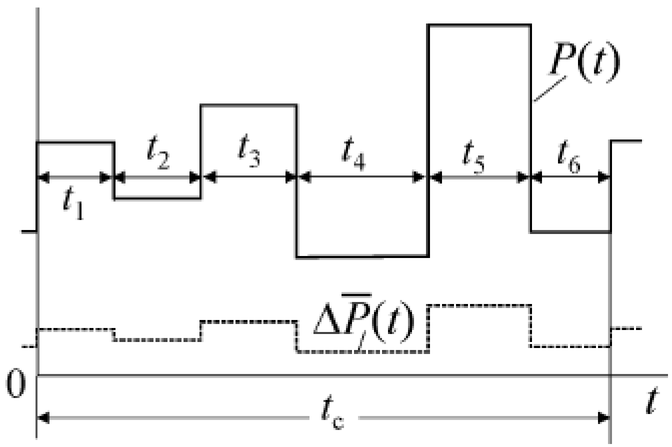
β_0 - coefficient of deterioration of heat transfer in a stationary state

If at all intervals the motor is running at a speed close to the nominal speed or if the engine has independent ventilation:

$$\Delta\overline{P} = \frac{1}{t_c} \sum_{i=1}^{n} \Delta\overline{P}_i t_i \leq \Delta P_N \,,$$

Calculation of motor power in continuous operation mode

Motor power selection by the average loss method:



1. According to the load diagram of the mechanism, the average power on the motor shaft is determined.

$$\overline{P} = \frac{1}{t_c}\sum_{i=1}^{n} P_i t_i \,.$$ for const speed

$$\overline{P} = \frac{\sum_{i=1}^{n} P_i \frac{\omega_N}{\omega_i} t_i}{\sum_{i=1}^{n} \beta_i t_i}.$$ for various speed

2. Based on the result of calculating the average power, a motor with a rated power is selected from the catalog

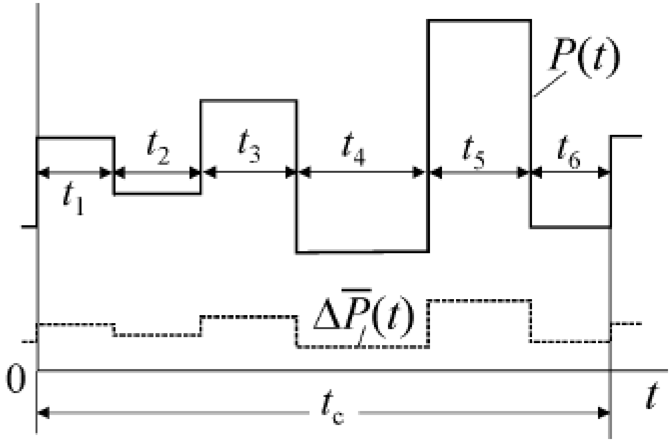
$$P_N = k\overline{P}, \qquad k = 1.1 \dots 1.3$$

3. For each interval of constant load, the power losses are determined from the efficiency curves or from the reference table of the catalog

4. Determine the average losses in the motor and compare them with the nominal

$$\Delta\overline{P} \le \Delta P_N = P_N(1-\eta_N)/\eta_N$$

Calculation of motor power in continuous operation mode



Equivalent current method

$$I_{eq} = \sqrt{\frac{\sum_{i=1}^{n} I_i^2 t_i}{\sum_{i=1}^{n} \beta_i t_i}} \leq I_N \; .$$

Equivalent torque method

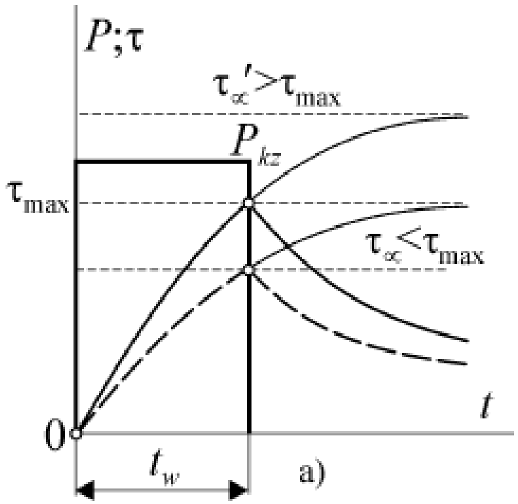
$$M_{eq} = \sqrt{\frac{\sum_{i=1}^{n} M_i^2 t_i}{\sum_{i=1}^{n} \beta_i t_i}} \leq M_N$$

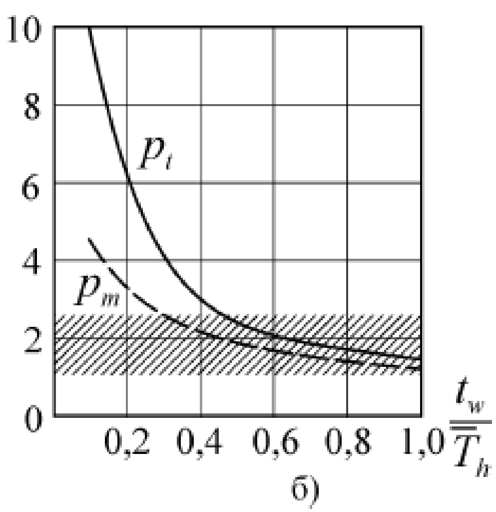
Equivalent power method

$$P_{eq} = \sqrt{\frac{\sum_{i=1}^{n} \left(P_i \frac{\omega_N}{\omega_i} \right)^2 t_i}{\sum_{i=1}^{n} \beta_i t_i}} \; .$$

Calculation of motor power for short-term operation mode

Calculation of motor power for short-term operation mode





τ_∞ - steady-state temperature rise in long-term.

τ'_∞ - steady-state temperature rise in short-term.

$$\tau'_\infty\left(1-e^{-t_w/\overline{T}_h}\right)=\tau_{\max}=\tau_\infty$$

$$\overline{T}_h=\left(T_{h0}+T_{he}\right)/2$$

$$\tau_\infty=\Delta P_N/A \qquad \tau'_\infty=\Delta P_{kz}/A$$

Thermal overload factor:

$$p_t=\frac{\tau'_\infty}{\tau_\infty}=\frac{\Delta P_{kz}}{\Delta P_N}=\frac{1}{1-e^{-t_w/\overline{T}_h}};$$

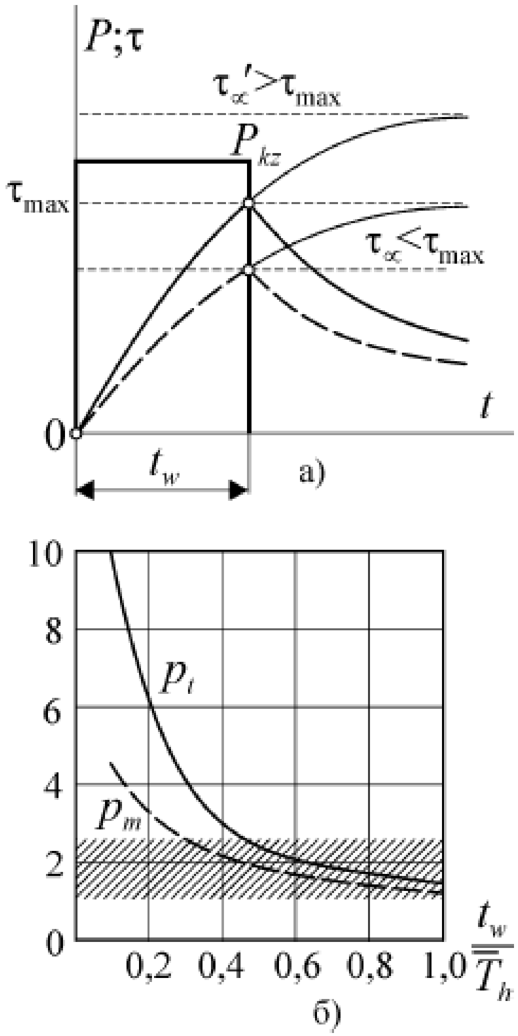
$$p_t=\frac{\Delta P_{cN}+\Delta P_{vN}\left(P_{kz}/P_N\right)^2}{\Delta P_{cN}+\Delta P_{vN}}=\frac{a+p_m^2}{a+1}\Rightarrow p_m=\sqrt{(1+a)p_t-a}$$

Mechanical overload factor:

$$p_m=P_{kz}/P_N \qquad a=\Delta P_{cN}/\Delta P_{vN}$$

$$p_m=\sqrt{\frac{1+a}{1-e^{-t_w/\overline{T}_h}}-a}$$

Calculation of motor power for short-term operation mode



Temperature rise in long-term

$$\tau_{\max} = \frac{\Delta P_{kz}}{A}\left(1 - e^{-t_w/\overline{T}_h}\right) = \frac{\Delta P_{kz\,N}}{A}\left(1 - e^{-t_{wN}/\overline{T}_h}\right).$$

Thermal overload factor:

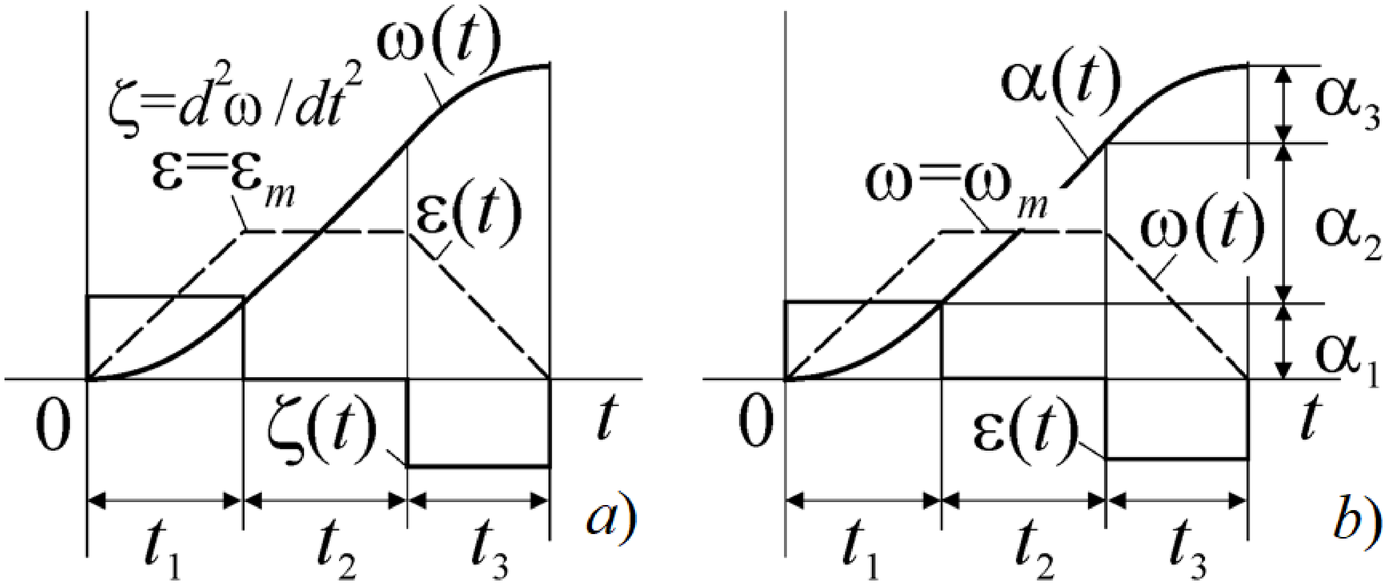
$$p_t = \frac{\Delta P_{kz}}{\Delta P_{kz\,N}} = \frac{1 - e^{-t_{wN}/\overline{T}_h}}{1 - e^{-t_w/\overline{T}_h}} = \frac{a + \left(P_{kz}/P_{kz\,N}\right)^2}{1 + a}$$

Mechanical overload factor:

$$P_{kz} = P_{kz\,N}\sqrt{(1+a)\frac{1 - e^{-t_{wN}/\overline{T}_h}}{1 - e^{-t_w/\overline{T}_h}} - a} \xrightarrow{a \to 0} P_{kz\,N}\sqrt{p_t}$$

Optimal control of the position drives

Optimal control of the position drives

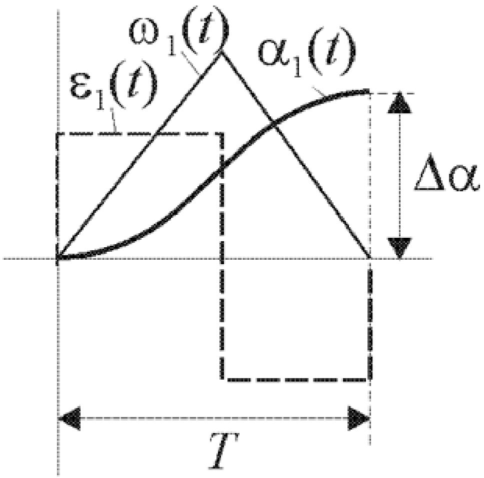


$$\underline{T=\min;\ Q=\min;\ \zeta=d\varepsilon/dt=\min}$$

$$M(t)=J\cdot\varepsilon(t);\ dQ=M(t)\cdot d\alpha$$

$$Q=C\int_0^{\Delta\alpha/2}\varepsilon(t)d\alpha=C\int_0^{T/2}\varepsilon(t)\cdot\omega(t)dt$$

1. Optimal control of the position drives



$$\varepsilon(t) = \varepsilon_{1m} \cdot sg(t)$$

$$\omega(t) = \varepsilon_{1m} t$$

$$\alpha(t) = \varepsilon_{1m} t^2 / 2$$

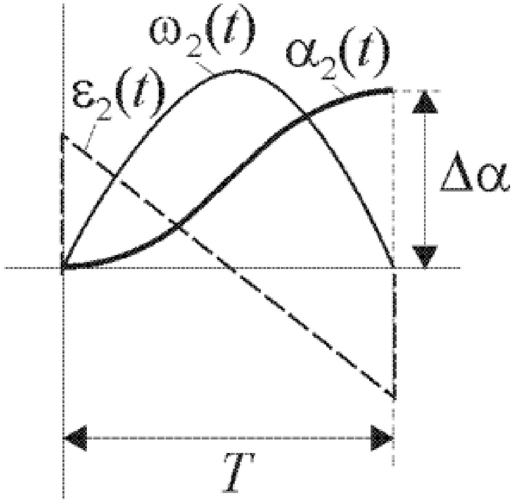
	1
ε	1
q	1
ω	1

$$\omega_{1m} = \frac{\varepsilon_{1m} T}{2}$$

$$Q_1 = J\varepsilon_{1m}^2 \int_0^{T/2} t\,dt = \frac{J\varepsilon_{1m}^2 T^2}{8}$$

$$\frac{d\varepsilon}{dt}_{\max} = \pm\infty$$

2. Optimal control of the position drives



$$\varepsilon(t) = \varepsilon_{2m}\frac{T-2t}{T}$$

$$\omega(t) = \varepsilon_{2m}\left(t - \frac{t^2}{T}\right)$$

$$\alpha(t) = \varepsilon_{2m}\left(\frac{t^2}{2} - \frac{t^3}{3T}\right)$$

	1	2
ε	1	1,5
q	1	0,56
ω	1	0,75

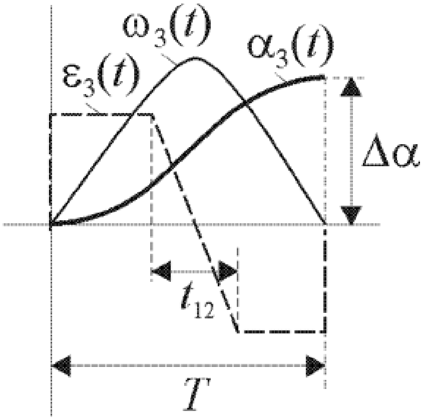
$$\frac{\alpha_1}{2} = \frac{\varepsilon_{1m}t^2}{2}\bigg|_{t=T/2} = \frac{\varepsilon_{1m}T^2}{8} = \frac{\alpha_2}{2} = \varepsilon_{2m}t^2\left(\frac{1}{2} - \frac{t}{3T}\right)\bigg|_{t=T/2} = \frac{\varepsilon_{2m}T^2}{12} \Rightarrow \boxed{\frac{\varepsilon_{2m}}{\varepsilon_{1m}} = \frac{3}{2}}$$

$$\omega_{2m} = \varepsilon_{2m}t(1 - t/T)\big|_{t=T/2} = \frac{\varepsilon_{2m}T}{4} = \frac{3}{2}\varepsilon_{1m}\frac{T}{4} \Rightarrow \boxed{\frac{\omega_{2m}}{\omega_{1m}} = \frac{3}{4}}$$

$$Q_2 = J\varepsilon_2^2\int_0^{T/2}\left(\frac{T-2t}{T}\right)\left(t - \frac{t^2}{T}\right)dt = \frac{J\varepsilon_2^2}{2T^2}\left(Tt^2 - 2Tt^3 + t^4\right)\Big|_0^{T/2} = \frac{J\varepsilon_2^2T^2}{32} = \frac{J9\varepsilon_1^2T^2}{4\cdot 32}$$

$$\boxed{\frac{Q_2}{Q_1} = \frac{18}{32} = 0{,}56 \qquad \frac{d\varepsilon}{dt}_{\max} = \pm\infty}$$

3. Optimal control of the position drives



	1	2	3
ε	1	1,5	1,05
q	1	0,56	0,62
ω	1	0,75	0,85

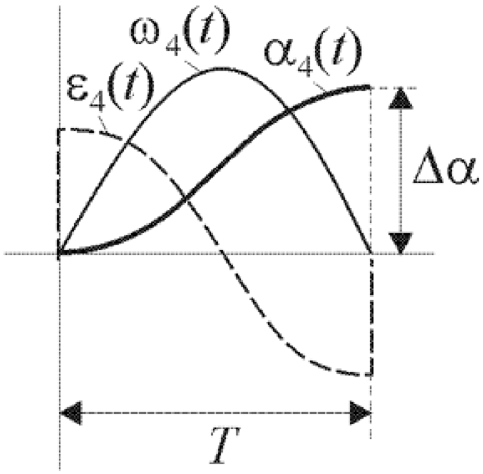
$$1,0\Big|_{t_{12}=0} \le \frac{\varepsilon_{3m}}{\varepsilon_{1m}} \le 1,5\Big|_{t_{12}=T}$$

$$1,0\Big|_{t_{12}=0} \ge \frac{\omega_{3m}}{\omega_{1m}} \ge 0,75\Big|_{t_{12}=T}$$

$$1,0\Big|_{t_{12}=0} \ge \frac{Q_3}{Q_1} \ge 0,56\Big|_{t_{12}=T}$$

$$\frac{d\varepsilon}{dt}_{\max} = \pm\infty$$

4. Optimal control of the position drives



$$\varepsilon(t) = \varepsilon_{4m} \cos \pi \frac{t}{T}$$

$$\omega(t) = \varepsilon_{4m} \frac{T}{\pi} \sin \pi \frac{t}{T}$$

$$\alpha(t) = \varepsilon_{4m} \frac{T^2}{\pi^2} \left(1 - \cos \pi \frac{t}{T}\right)$$

	1	2	3	4
ε	1	1,5	1,05	1,23
q	1	0,56	0,62	0,61
ω	1	0,75	0,85	0,78

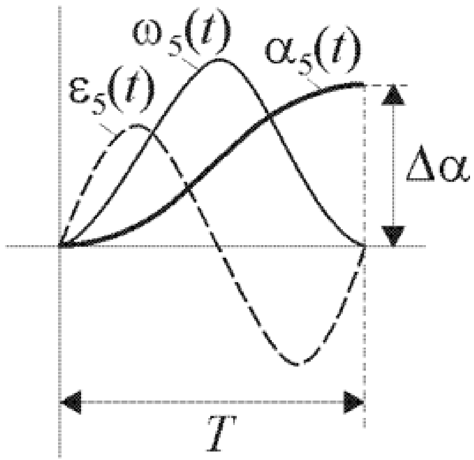
$$\frac{\alpha_1}{2} = \frac{\varepsilon_{1m} T^2}{8} = \frac{\alpha_4}{2} = \varepsilon_{4m} \frac{T^2}{\pi^2} \left(1 - \cos \frac{\pi t}{T}\right)\Bigg|_{t=T/2} = \frac{\varepsilon_{4m} T^2}{\pi^2} \Rightarrow \boxed{\frac{\varepsilon_{4m}}{\varepsilon_{1m}} = \frac{\pi^2}{8} = 1,23}$$

$$\omega_3 = \varepsilon_{4m} \frac{T}{\pi} \sin \frac{\pi t}{T}\Bigg|_{t=T/2} = \frac{\varepsilon_{4m} T}{\pi} = \varepsilon_{1m} \frac{\pi T}{8} \Rightarrow \boxed{\frac{\omega_{4m}}{\omega_{1m}} = \frac{\pi}{4} = 0,78}$$

$$Q_4 = \frac{J \varepsilon_{4m}^2 T}{\pi} \int_0^{T/2} \cos \frac{\pi t}{T} \cdot \sin \frac{\pi t}{T} \, dt = \frac{J \varepsilon_{4m}^2 T^2}{2\pi^2} \left[1 - \cos^2 \frac{\pi t}{T}\right]\Bigg|_0^{T/2} = \frac{J \varepsilon_{4m}^2 T^2}{2\pi^2} = \frac{J \pi^2 \varepsilon_{1m}^2 T^2}{128}$$

$$\boxed{\begin{array}{l} \dfrac{Q_4}{Q_1} = \dfrac{\pi^2 8}{128} = 0,61 \\ \dfrac{d\varepsilon}{dt}_{\max} = \pm\infty \end{array}}$$

5. Optimal control of the position drives



$$\varepsilon(t) = \varepsilon_{5m} \sin 2\pi \frac{t}{T}$$

$$\omega(t) = \varepsilon_{5m} \frac{T}{2\pi}\left(1 - \cos 2\pi \frac{t}{T}\right)$$

$$\alpha(t) = \varepsilon_{5m} \frac{T}{2\pi}\left(t - \frac{T}{2\pi} \sin 2\pi \frac{t}{T}\right)$$

	1	2	3	4	5
ε	1	1,5	1,05	1,23	1,57
q	1	0,56	0,62	0,61	1
ω	1	0,75	0,85	0,78	1

$$\frac{\alpha_1}{2} = \frac{\varepsilon_{1m}T^2}{8} = \frac{\alpha_5}{2} = \varepsilon_{5m}\frac{T}{2\pi}\left(t - \frac{T}{2\pi}\sin\frac{2\pi t}{T}\right)\Bigg|_{t=T/2} = \frac{\varepsilon_{5m}T^2}{4\pi} \Rightarrow \boxed{\frac{\varepsilon_{5m}}{\varepsilon_{1m}} = \frac{\pi}{2} = 1{,}57}$$

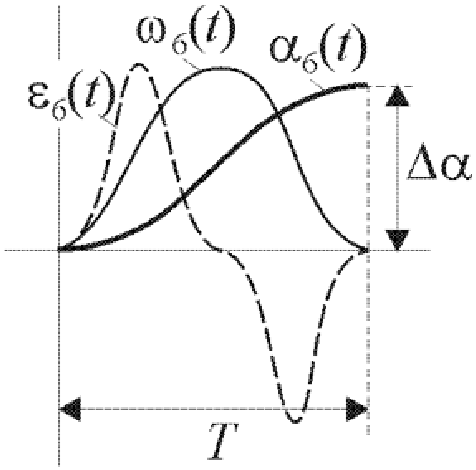
$$\omega_{5m} = \varepsilon_{5m}\frac{T}{2\pi}\left(1 - \cos\frac{2\pi t}{T}\right)\Bigg|_{t=T/2} = \frac{\varepsilon_{5m}T}{\pi} = \frac{\pi\varepsilon_{1m}T}{2\pi} = \omega_{1m} \Rightarrow \boxed{\frac{\omega_{5m}}{\omega_{1m}} = 1}$$

$$Q_5 = \frac{J\varepsilon_{5m}^2 T}{2\pi}\int_0^{T/2} \sin\frac{2\pi t}{T}\cdot\left(1 - \cos\frac{2\pi t}{T}\right)dt =$$

$$= \frac{J\varepsilon_{5m}^2 T^2}{8\pi^2}\left[\cos\frac{2\pi t}{T}\left(\cos\frac{2\pi t}{T} - 2\right) + 1\right]\Bigg|_0^{T/2} = \frac{J\varepsilon_{5m}^2 T^2}{2\pi^2} = \frac{J\pi^2\varepsilon_{1m}^2 T^2}{4\cdot 2\pi^2} = \frac{J\varepsilon_{1m}^2 T^2}{8} = Q_1$$

$$\boxed{\begin{array}{l} \dfrac{Q_5}{Q_1} = 1 \\ \dfrac{d\varepsilon}{dt}_{\max} = \pm\varepsilon_{5m}\dfrac{2\pi}{T} \end{array}}$$

6. Optimal control of the position drives



$$\varepsilon(t)=\frac{\varepsilon_{6m}}{2}\left(1-\cos 4\pi\frac{t}{T}\right)sg(t)$$

$$\omega(t)=\frac{\varepsilon_{6m}}{2}\left[\left(t-\frac{T}{4\pi}\sin 4\pi\frac{t}{T}\right)sg(t)+\frac{1-sg(t)}{2}\right]$$

$$\alpha(t)=\frac{\varepsilon_{6m}}{4}\left[t^2+\frac{T^2}{8\pi^2}\left(\cos 4\pi\frac{t}{T}-1\right)\right]$$

	1	2	3	4	5	6
ε	1	1,5	1,05	1,23	1,57	2
q	1	0,56	0,62	0,61	1	1
ω	1	0,75	0,85	0,78	1	1

$$\frac{\alpha_1}{2}=\frac{\varepsilon_{1m}T^2}{8}=\frac{\alpha_6}{2}=\frac{\varepsilon_{6m}}{4}\left[t^2+\frac{T^2}{8\pi^2}\left(\cos\frac{4\pi t}{T}-1\right)\right]\Bigg|_{t=T/2}=\frac{\varepsilon_{6m}T^2}{16}\Rightarrow\boxed{\frac{\varepsilon_{6m}}{\varepsilon_{1m}}=2}$$

$$\omega_{6m}=\frac{\varepsilon_{6m}}{2}\left(t-\frac{T}{4\pi}\sin\frac{4\pi t}{T}\right)\Bigg|_{t=T/2}=\frac{\varepsilon_{6m}T}{4}=\frac{2\varepsilon_{1m}T}{4}=\omega_{1m}\Rightarrow\boxed{\frac{\omega_{6m}}{\omega_{1m}}=1}$$

$$Q_6=\frac{J\varepsilon_{6m}^2}{4}\int_0^{T/2}\left(1-\cos\frac{4\pi t}{T}\right)\left(t-\frac{T}{4\pi}\sin\frac{4\pi t}{T}\right)dt=$$

$$=\frac{J\varepsilon_{6m}^2T^2}{128\pi^2}\left[\frac{16\pi^2t^2}{T^2}-\frac{8\pi t}{T}\sin\frac{4\pi t}{T}-\cos^2\frac{4\pi t}{T}+1\right]\Bigg|_0^{T/2}=\frac{J\varepsilon_{6m}^2T^2}{32}=\frac{J\varepsilon_{1m}^2T^2}{8}=Q_1$$

$$\boxed{\begin{gathered}\frac{Q_6}{Q_1}=1\\ \frac{d\varepsilon}{dt}_{\max}=\pm\varepsilon_{6m}\frac{2\pi}{T}\end{gathered}}$$

Optimal control of the position drives