

Actuators

Lection 9
H-bridge PWM algorithms

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Introduction

Switching methods

Symmetric

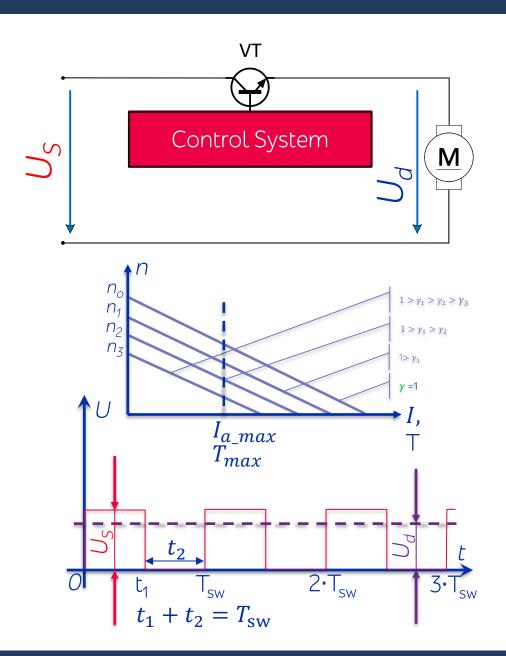
Asymmetric

Non-symmetric sequential switching

Energy efficiency of DC drive with PWM converter

Optimal switching frequency





[rpm] speed n $U_a$ [V] armature voltage [A]  $I_a$ - armature current resistance (armature windings) [Ohm]  $r_a$  additional resistance [Ohm]  $r_{cont}$  $[N \cdot m]$ torque [V/rpm] constructive constant - magnetic flux [Wb] switching frequency [Hz]  $T_{sw}$ - switching period [s] - duty cycle

$$f_{sw} = \frac{1}{T_{sw}}$$

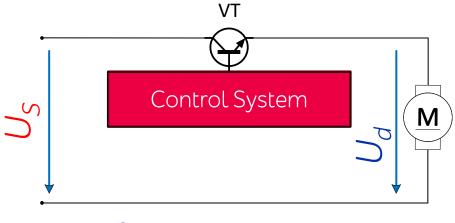
$$\gamma = \frac{t_1}{t_1 + t_2} = \frac{t_1}{T_{sw}} = t_1 \cdot f_{sw}$$

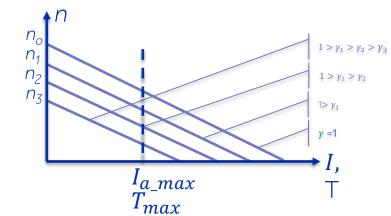
$$n = \frac{U_a - I_a \cdot r_a}{C_e \cdot \Phi} \quad \rightarrow \quad n = \frac{U_S \cdot \gamma}{C_e \cdot \Phi} - \frac{I_a \cdot r_a}{C_e \cdot \Phi}$$

$$U_d = \frac{1}{T_{sw}} \int_0^{t_1} u_S(t) dt$$

$$U_d = \frac{U_S \cdot t_1}{T_{sw}} = U_S \cdot f_{sw} \cdot t_1 = \frac{U_S \cdot t_1}{t_1 + t_2} = U_S \gamma$$







n	<ul><li>speed</li></ul>	[rpm]
$U_a$	<ul> <li>armature voltage</li> </ul>	[V]
$I_a$	<ul> <li>armature current</li> </ul>	[A]
$r_a$	<ul><li>resistance (armature windings)</li></ul>	[Ohm]
$r_{cont}$	<ul> <li>additional resistance</li> </ul>	[Ohm]
T	<ul><li>torque</li></ul>	$[N \cdot m]$
$C_e$	<ul> <li>constructive constant</li> </ul>	[V/rpm
Φ	<ul><li>magnetic flux</li></ul>	[Wb]
$f_{sw}$	<ul> <li>switching frequency</li> </ul>	[Hz]
$T_{\mathrm{sw}}$	<ul><li>– switching period</li></ul>	[s]
γ	<ul><li>duty cycle</li></ul>	

$$f_{SW} = \frac{1}{T_{SW}}$$

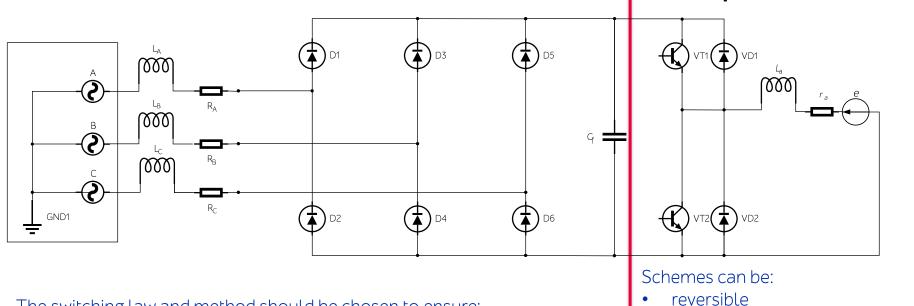
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The switching law and method should be chosen to ensure:

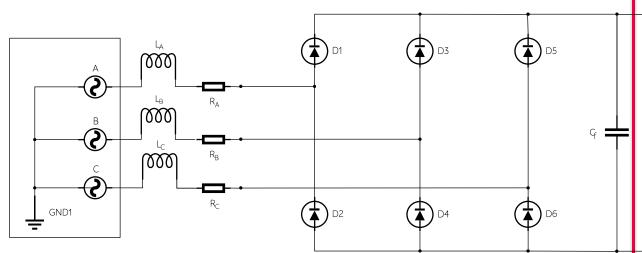
- 1. Equal load on elements of equipment.
- 2. The armature circuit of the DC machine should not break and should not significantly change its resistance at the time of current flow in the armature circuit.
- 3. Reversibility of electrical energy flows from the power source to the load and vice versa.

non-reversible

#### with

- symmetric switching law.
- asymmetrical switching law.





Transistors VT1 and VT2 are switched by sign-alternating pulses of reverse polarity, i.e.

when VT1 is switched on, VT2 is switched off and vice versa.

When VT1 is opened: the load circuit is connected to the power supply circuit.

When VT1 is turned off and VT2 is turned on, the DC drive armature circuit is disconnected from the power supply circuit and shortened in the circuits formed either by the open VT2 or by the VD2 shunting it.

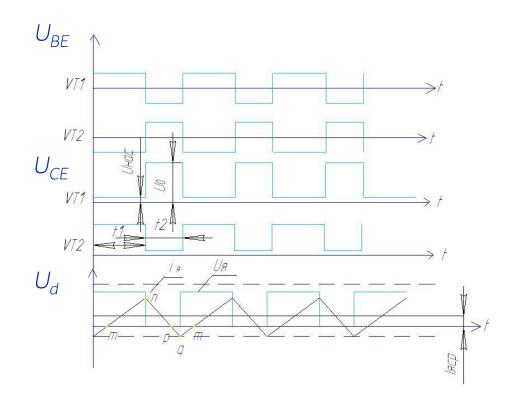
non-reversible circuitry providing

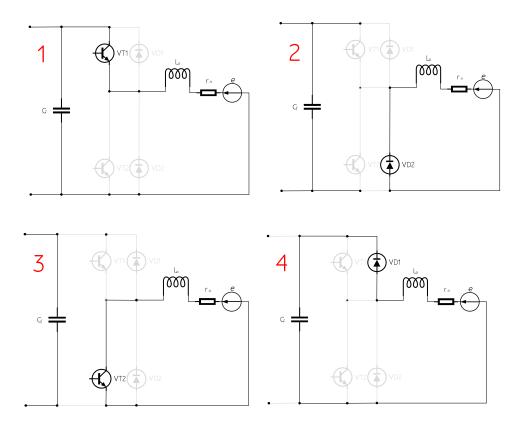
• dynamic braking mode

VT2 VD2

return of the energy of the rotating parts of the machine to the power grid.

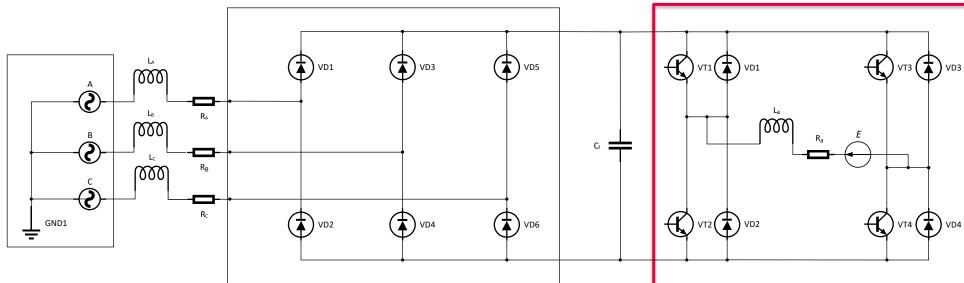






- **Mode 1.** Through the open VT1 at the time interval m-n, the energy of the power supply is consumed by the machine,  $I_a = I_s$ .
- **Mode 2.** When pulse  $t_1$  ends, the load is disconnected from the power source.
- Mode 3. At point p, the current changes direction under the action of the EMF of the armature.
- **Mode 4.** At the end of time, t<sub>2</sub> turns on VT1 and closes VT2.





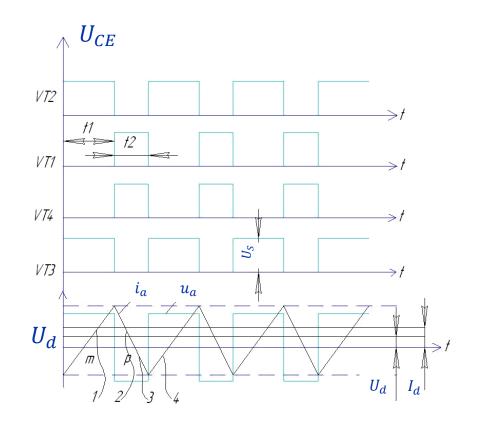
Reversible circuits can be used with 2 types of switching laws:

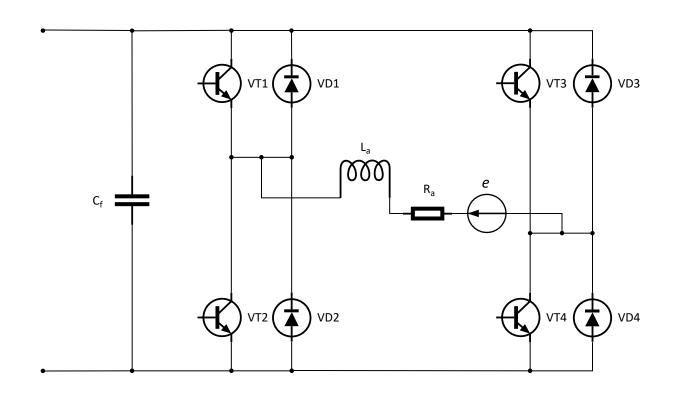
- Symmetric switching law
  - o («plus» and «minus» voltages alternates during one interval  $T_{
    m sw}$ ).
- Asymmetric switching law
  - o («plus» or «minus» voltage pulse and zero voltage during  $t_2$  time of  $T_{\rm sw}$ ).

reversible circuitry providing

- dynamic braking mode
- return of the energy of the rotating parts of the machine to the power grid.
- rotating of the drive in both directions



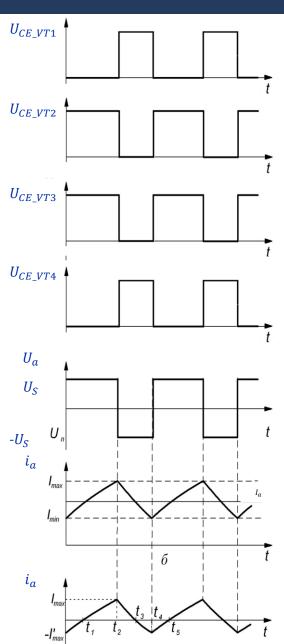


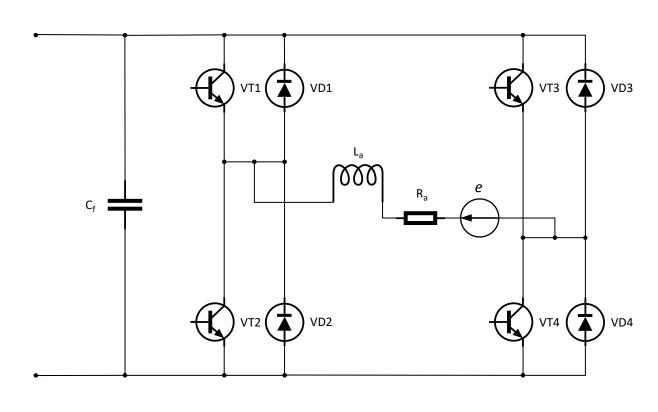


- At each switching interval, all 4 transistors of the bridge are switched
- Transistors of the same diagonal VT1-VT4 and VT2-VT3 could be controlled by the same PWM pulses.
- During the entire switching period, the armature circuit is connected to the circuit of the primary power supply.
- Increased current ripple due to the double amplitude of the supply voltage.
- Increased losses

Use for systems up to 20-50W.



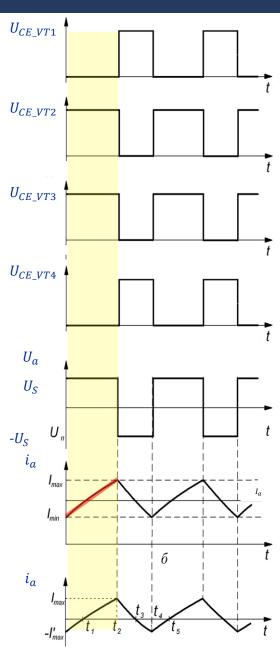


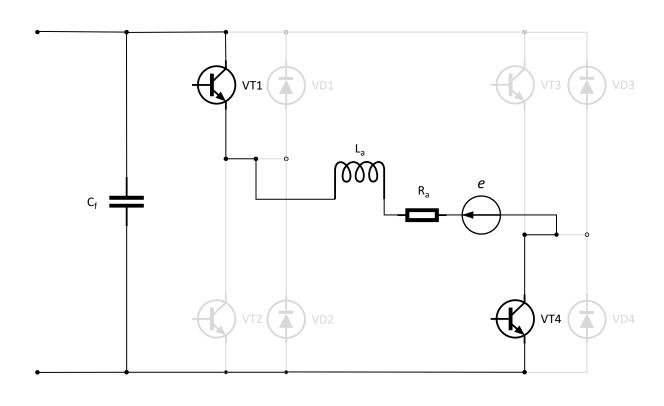


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For use in systems up to 20-50W.



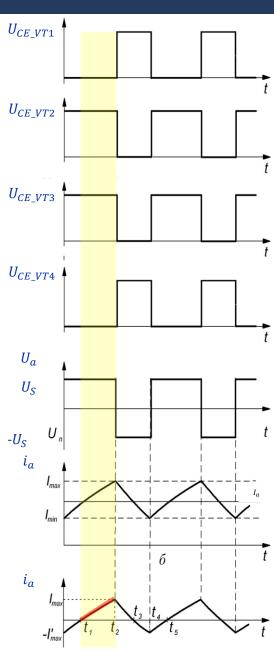


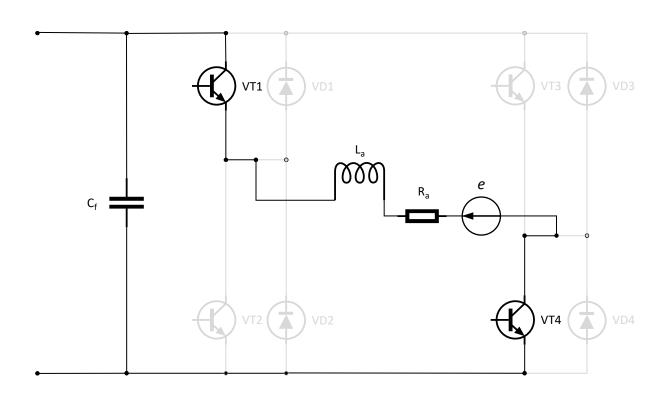


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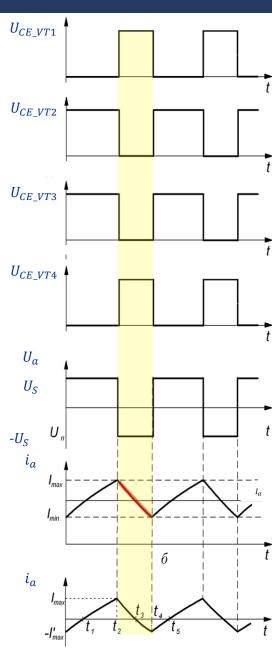


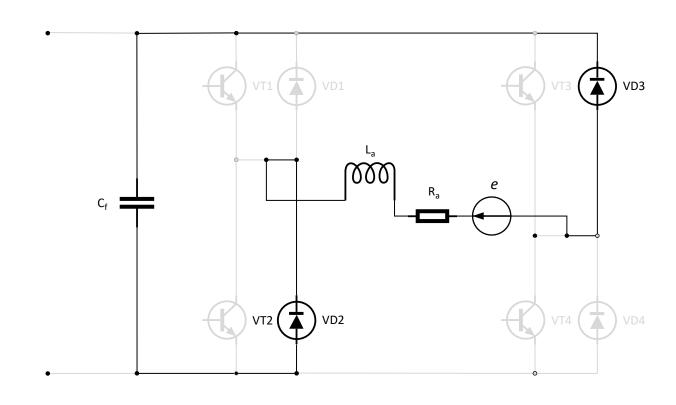


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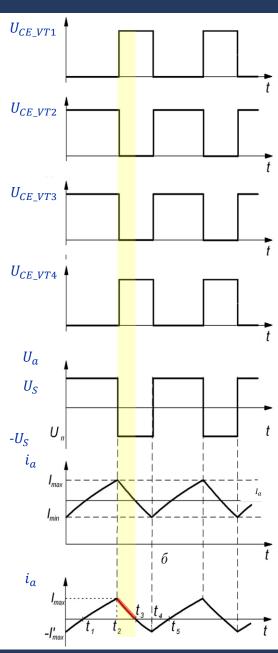


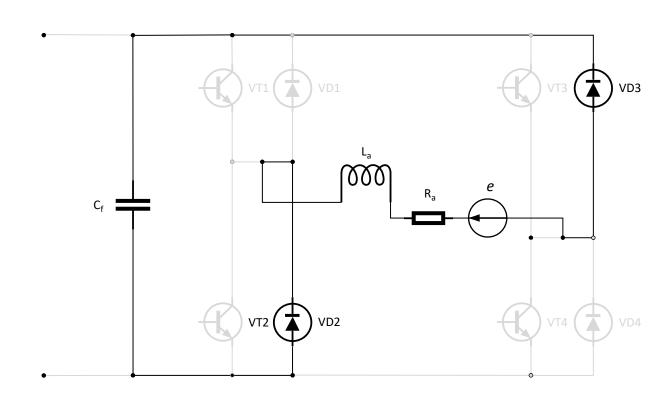


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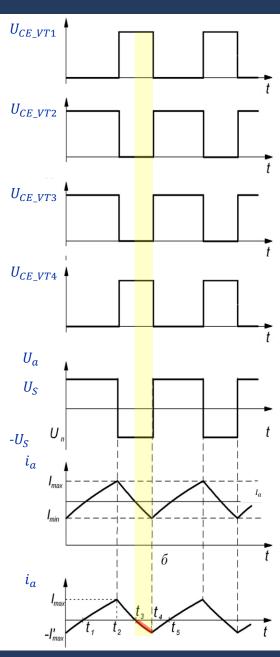


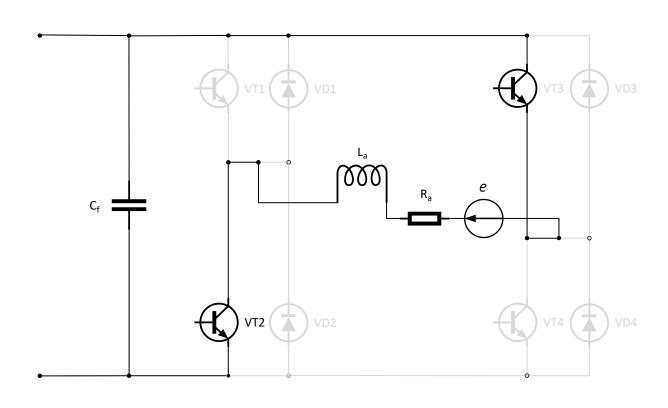


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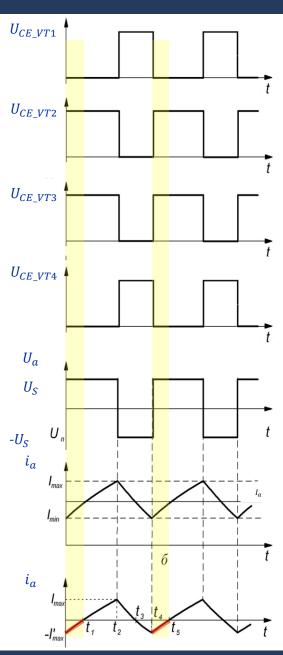


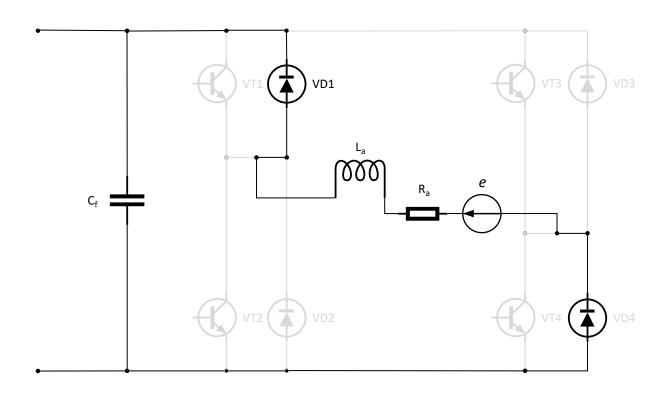


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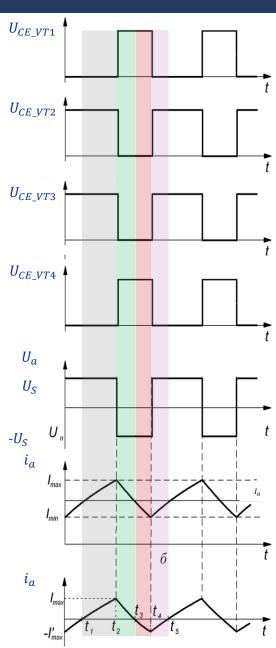


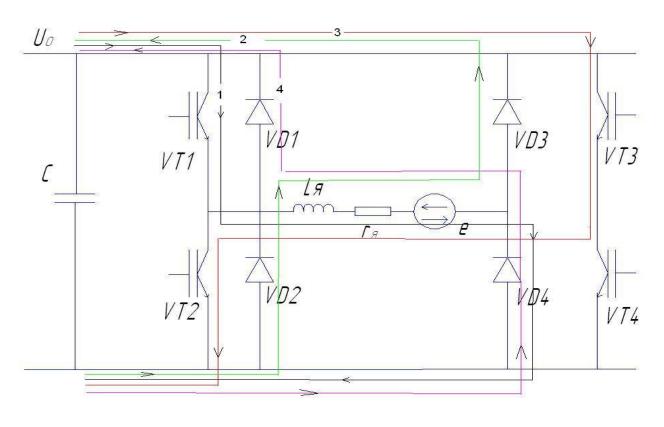


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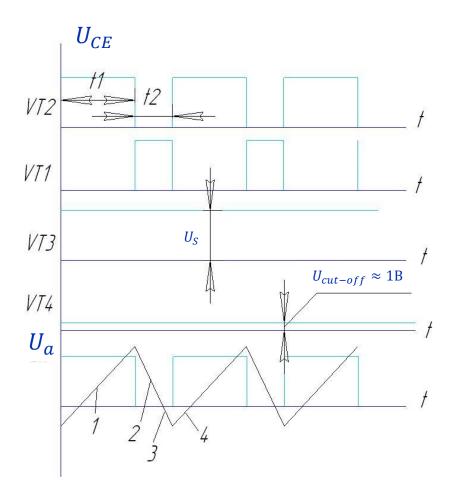


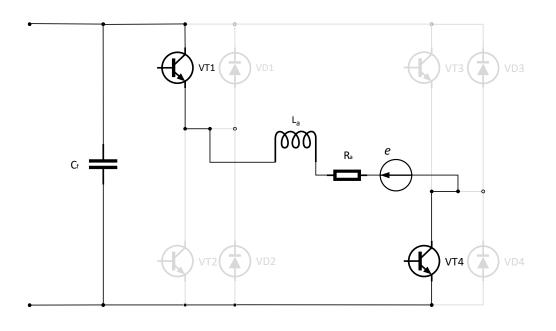


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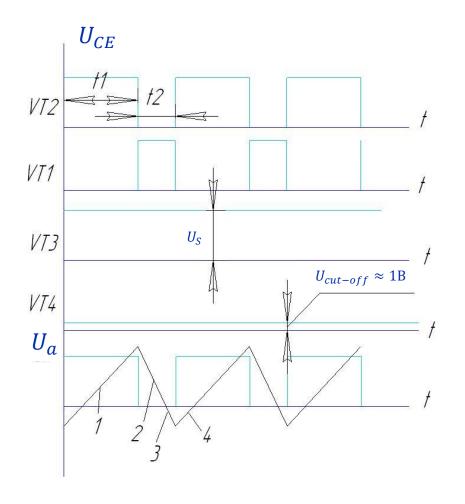


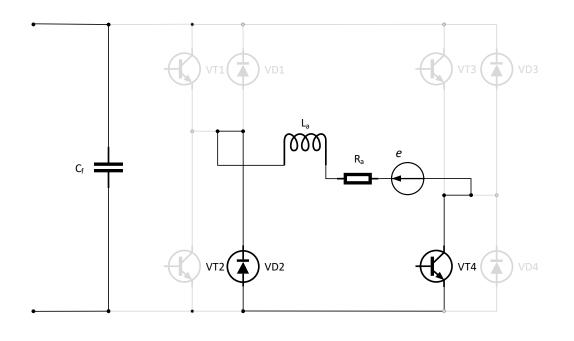




Method of asymmetric switching the power switches of the bridge No1:

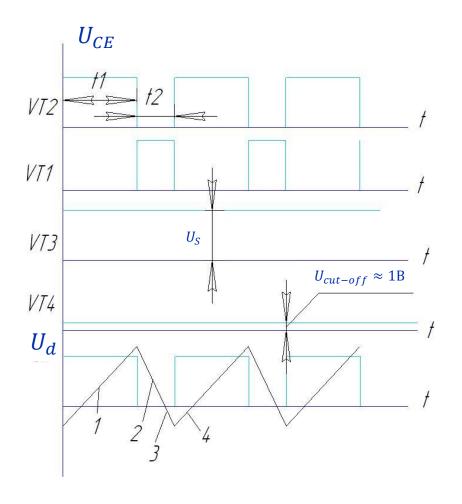


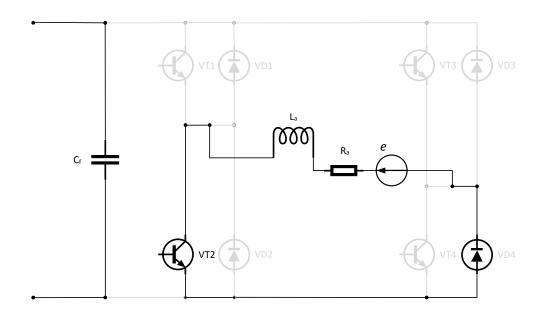




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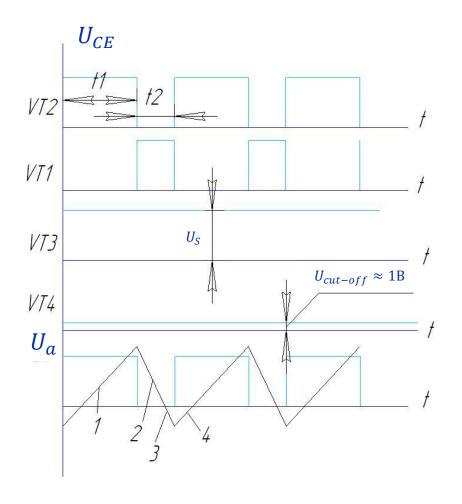


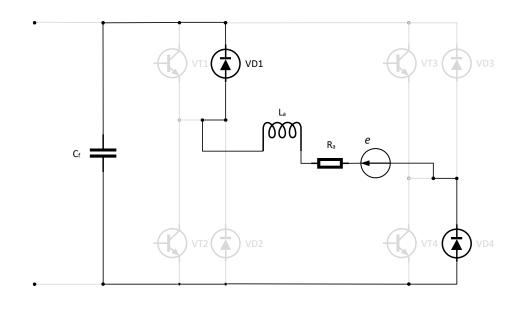




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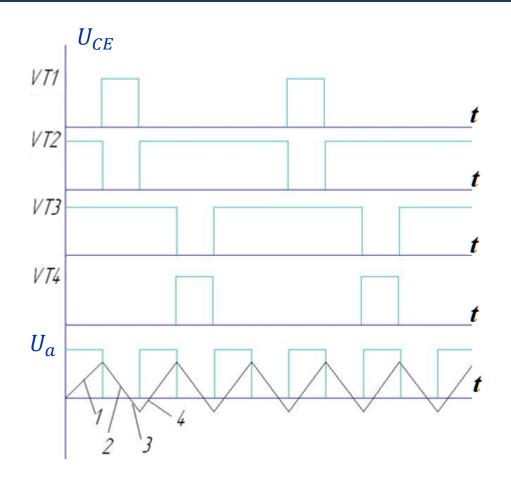


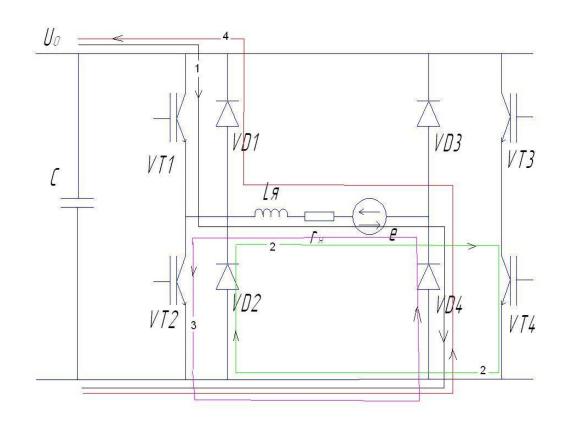




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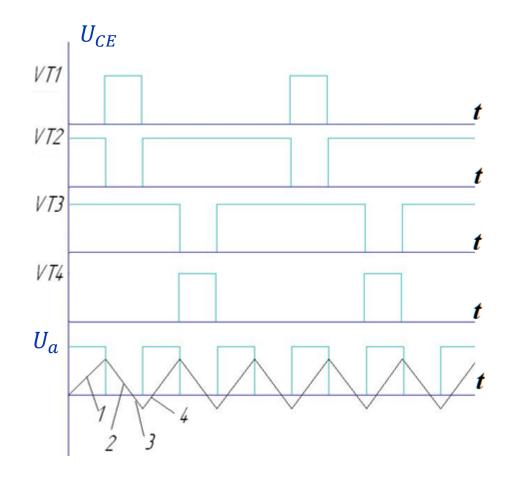


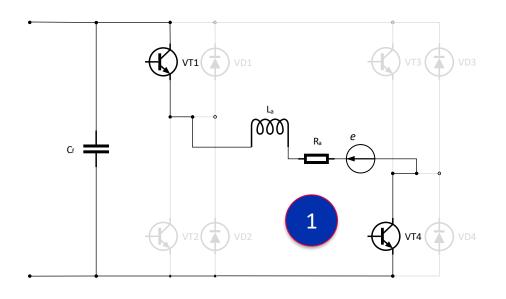




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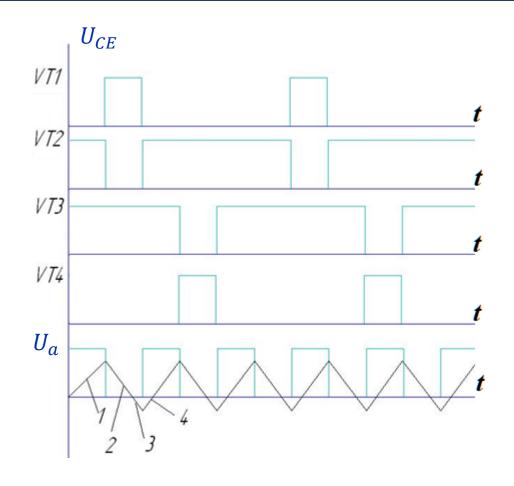


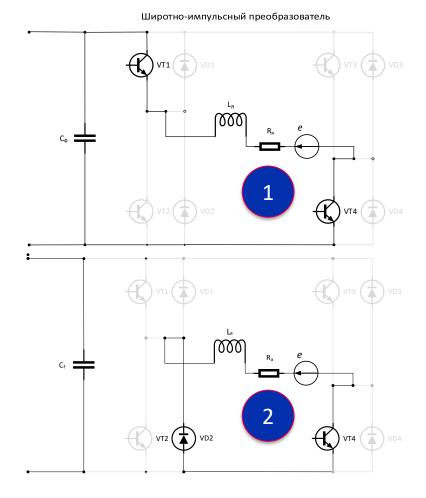




- alternate switching of the bridge power switches.
- Leads to the possibility of two times switching frequency reduction in relation to the switching frequency of Diagonal switching or symmetrical switching.

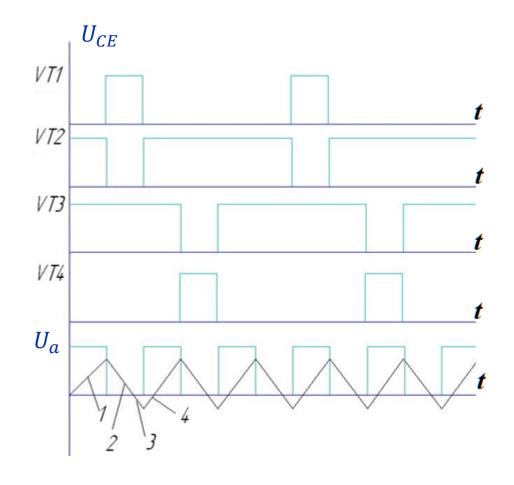


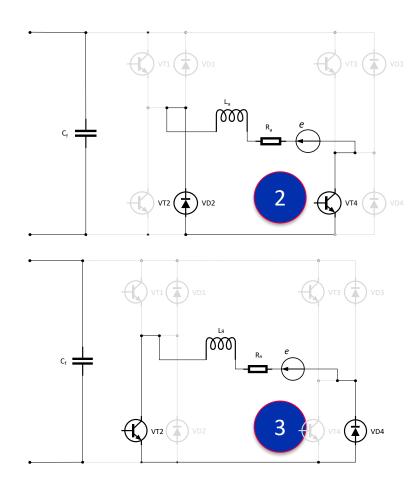




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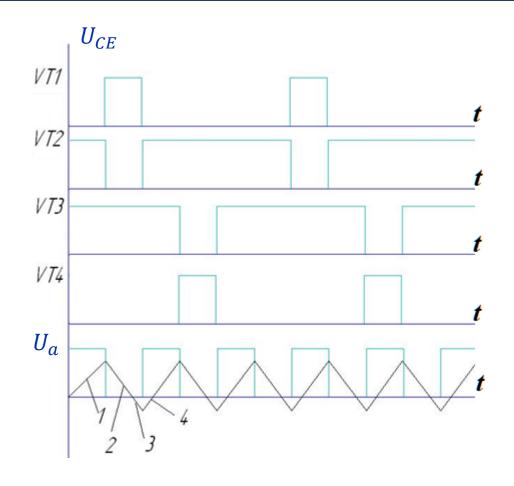


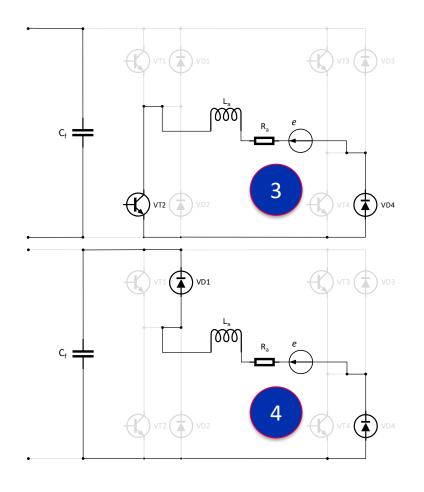




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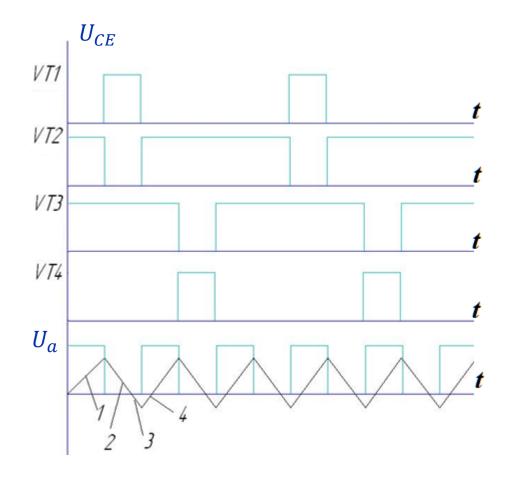


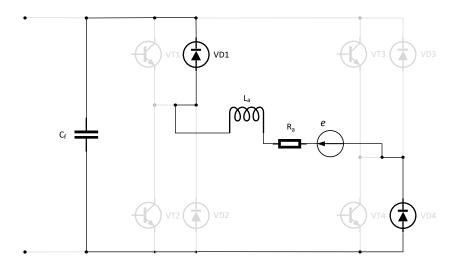




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Advantages:
Small current ripple
Even distribution of power losses between switches
Linear characteristics of invertor-DC drive system

Disadvantages:
Complicated control
Short circuit through transistors possibility



The scheme of the PWM converter stage, the type of switched devices, the law and the method of switching devices are determined by:

- The nature of the primary power supply (uncontrollable rectifier with filter, battery, generator)
- Load circuit parameters (R, RC, RL, R-L-e)

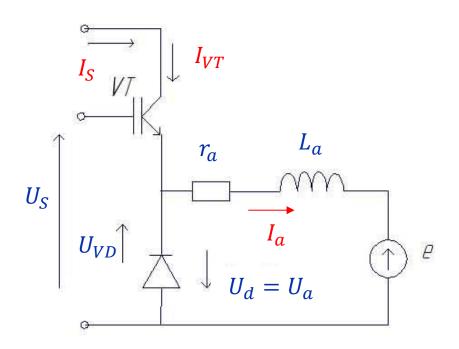
Requirements for the entire control system of PWM converter are distinguished by:

- According to the scheme of the PWM converter (reversible and non-reversing)
- By type of switching device (thyristors, transistors)
- According to the law and method of switching ( $f_{sw}$ =const,  $f_{sw}$ =var)

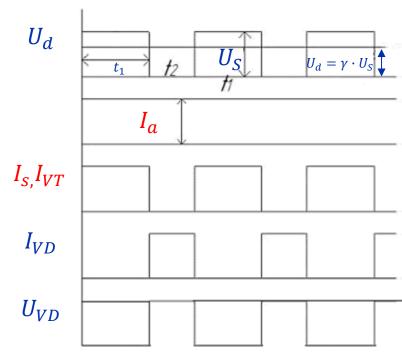
# Types of modulation:

- 1. Pulse Width Modulation (PWM) with Constant Switching Frequency  $(f_{SW} = const, T_{SW} = const, t_1 = var)$ .
- 2. Pulse-frequency modulation (PFM)  $(T_{sw} = var, t_1 = const)$
- 3. Pulse-frequency Width Modulation (PFWM)  $(t_1 = var, T_{sw} = var)$
- 4. Multi-zone pulse modulation (MZPM).



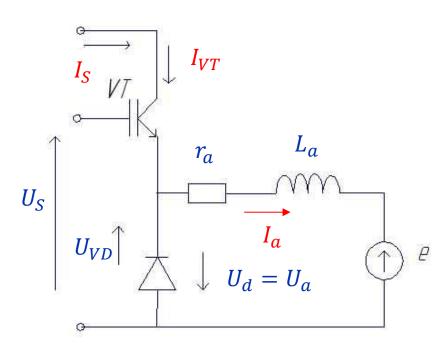


$$\begin{aligned} &U_d = \gamma \cdot U_S \\ &\gamma = \frac{t_1}{t_1 + t_2} = \frac{t_1}{T_{\text{SW}}} = t_1 \cdot f_{\text{SW}} \\ &t_{1min} = T_{\text{SW}} - t_{1max} \\ &U_{dmin} = \frac{t_{1min}}{T_{\text{SW}}} \cdot U_S = \gamma_{1min} \cdot U_S > 0 \\ &U_{dmax} = \frac{t_{1max}}{T_{\text{SW}}} \cdot U_S = \gamma_{1max} \cdot U_S \leq U_0 \end{aligned}$$



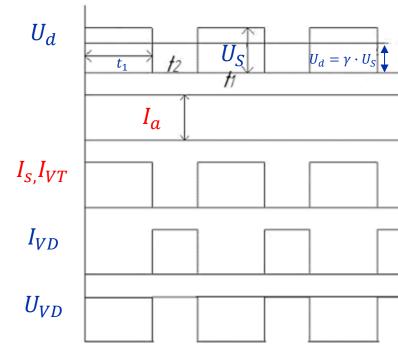
Power balance  $U_sI_s=U_aI_a$ 





$$\begin{split} &U_d = \gamma \cdot U_S \\ &\gamma = \frac{t_1}{t_1 + t_2} = \frac{t_1}{T_k} = t_1 \cdot f_k \\ &t_{1min} = T_{\text{SW}} - t_{1max} \\ &U_{dmin} = \frac{t_{1min}}{T_k} \cdot U_S = \gamma_{1min} \cdot U_S > 0 \\ &U_{dmax} = \frac{t_{1max}}{T_{\text{SW}}} \cdot U_S = \gamma_{1max} \cdot U_S \leq U_0 \end{split}$$

Power balance:  $U_sI_s=U_aI_a$ 



$$U_{d} = \gamma \cdot U_{S}$$

$$I_{d} = \gamma \cdot I_{a} = I_{VT}$$

$$t_{2} = (1 - \gamma) \cdot T_{SW}$$

$$U_{VTmax} = U_{VDmax} = U_{S}$$

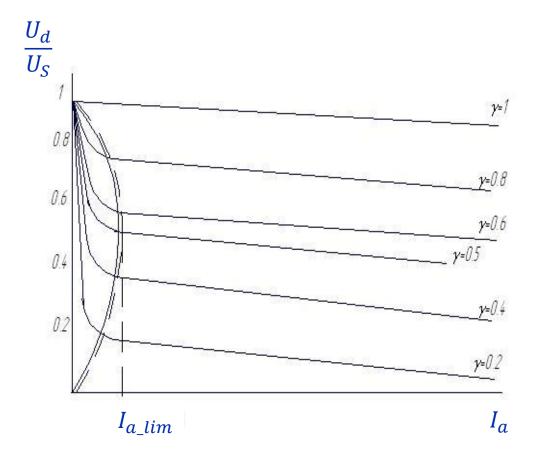
$$I_{amax} = \gamma_{max} \cdot I_{dmax}$$

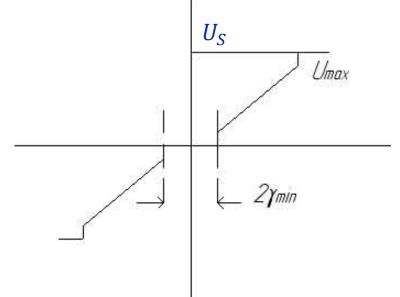
$$I_{dmax} = \frac{U_{S}}{r_{a}}$$

$$I_{VD} = (1 - \gamma) \cdot I_{amax}$$



Limits of  $U_d$ (min  $\mu$  max) are also limits for top switching frequency and distort the linear dependence  $U_d$ =f(Y)





$$U_a = f(I_a)$$

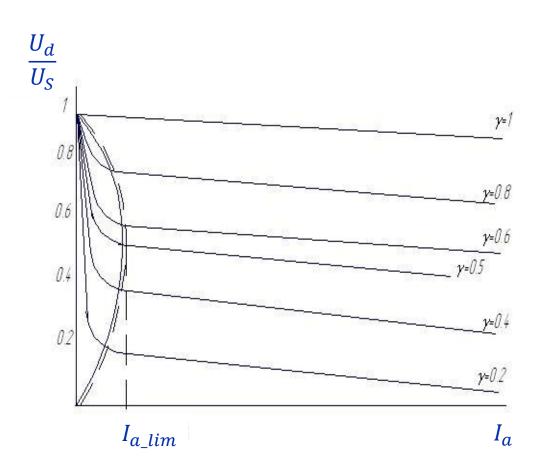
$$U_d = \gamma U_S$$

Taking into account the internal resistance of the power supply  $r_{in}$  and additional resistance in armature circuit:

$$U_d = \gamma U_S - r_{in} I_a$$

In relative units:

$$\overline{U}_d = \overline{U}_a = \gamma - \frac{r_{in}I_a}{U_S}$$



$$U_a = f(I_a)$$

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the boundary operation mode corresponds to the ratio

$$I_a = \frac{\Delta I_a}{2}$$

$$I_d = \gamma (1 - \gamma) \frac{U_S}{2L_a \cdot f_{sw}}$$

 $\Rightarrow$  Current  $I_{d\_lim}$  dependency of  $\gamma$  is determined by a multiplier  $\gamma(1-\gamma)$ 

 $\Rightarrow$  each of the external characteristics corresponds to its own value  $I_{d\_lim}$  at  $\gamma=0$  and  $\gamma=1$ 

 $\Rightarrow$ the maximum values are reached when  $\gamma = 0.5$ :

$$I_{d\_lim\_max} = 0.5(1 - 0.5) \frac{U_s}{2L_a \cdot f_{sw}} = \frac{0.125 \cdot U_s}{L_a \cdot f_{sw}}$$



Loss of power in the armature of the DC drive, which is the sum of the main component  $r_a I_a^2$ , corresponding to the DC machine supplied with DC voltage source, and additional component related with power supply from a PWM switching converter:

with a symmetric switching:

$$\Delta P_a = r_a I_a^2 + 4r_a I_a^2 [\gamma(1-\gamma) - \tau_{load}\beta] = r_a I_a^2 (1 + 4\alpha_K \alpha_P),$$

Non-symmetric sequential switching

$$\Delta P_a = r_a I_a^2 + r_a I_a^2 [\gamma(1-\gamma) - \tau_{load}\beta] = r_a I_a^2 (1 + \alpha_K \alpha_P),$$

$$\alpha_P = \gamma (1 - \gamma) - \tau_{load} \beta,$$
 $\alpha_K = \frac{I_K}{I_a},$ 
 $\tau_{load} = \frac{L_a}{r_a T_{sw}}$ 

$$\beta = \frac{(1-load)(1-e^{-(1-\gamma)\tau_{load}})}{1-e^{-\frac{1}{\tau_{load}}}} - \text{ armature current ripple coefficient}$$



Power loss in the DC drive armature ( $\Delta P_a$ ), which is the sum of the main component  $r_a I_a^2$ , corresponding to the DC machine supplied with DC voltage source, and additional ( $\Delta P_{a\_add}$ ) component related with power supply from a PWM switching converter:

Symmetric switching:

$$\Delta P_{a} = r_{a}I_{a}^{2} + 4r_{a}I_{a}^{2}[\gamma(1-\gamma) - \tau_{load}\beta]$$
  
=  $r_{a}I_{a}^{2}(1 + 4\alpha_{K}\alpha_{P})$ ,

additional losses per units (in p.u.)



Symmetric switching:

$$\Delta \bar{P}_{a\_add} = \frac{\Delta P_{a\_add}}{U_a I_a} = \frac{\Delta P_{a\_add}}{r_a I_a^2} = 4\alpha_K \alpha_P$$

Non-symmetric sequential switching

$$\Delta \bar{P}_{a\_add} = \frac{\Delta P_{a\_add}}{U_a I_a} = \frac{\Delta P_{a\_add}}{r_a I_a^2} = \alpha_K \alpha_P$$

Relative additional losses in the actuator system are a function of the coefficient  $\alpha_P$ , which in turn depends on  $\gamma$ ,  $\tau_{load}$ ,  $\beta$ 

$$\alpha_{P} = \gamma(1 - \gamma) - \tau_{load}\beta,$$

$$\alpha_{K} = \frac{I_{K}}{I_{a}},$$

$$\tau_{load} = \frac{L_{a}}{r_{a}T_{sw}}$$

$$\beta = \frac{(1 - e^{-\frac{\gamma}{\tau_{load}}})(1 - e^{-(1 - \gamma)\tau_{load}})}{1 - e^{-\frac{1}{\tau_{load}}}}$$



Power loss in the DC drive armature ( $\Delta P_a$ ), which is the sum of the main component  $r_a I_a^2$ , corresponding to the DC machine supplied with DC voltage source, and additional ( $\Delta P_{a\_add}$ ) component related with power supply from a PWM switching converter:

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Non-symmetric sequential switching

$$\begin{aligned} \Delta P_a &= r_a I_a^2 + r_a I_a^2 [\gamma (1 - \gamma) - \tau_{load} \beta] \\ &= r_a I_a^2 (1 + \alpha_K \alpha_P), \end{aligned}$$

additional losses per units (in p.u.)



Symmetric switching:

$$\Delta \bar{P}_{a\_add} = \frac{\Delta P_{a\_add}}{U_a I_a} = \frac{\Delta P_{a\_add}}{r_a I_a^2} = 4\alpha_K \alpha_P$$

Non-symmetric sequential switching

$$\Delta \bar{P}_{a\_add} = \frac{\Delta \bar{P}_{a\_add}}{U_a I_a} = \frac{\Delta \bar{P}_{a\_add}}{r_a I_a^2} = \alpha_K \alpha_P$$

It is known that the dependencies of additional losses of the function  $\gamma$  have a maximum at  $\gamma=0.5.$ 

In the area 
$$0 < \tau_{load}^{-1} < 3$$
 dependency  $(\alpha_P)_{\gamma=0.5} \approx \frac{0.0045}{\tau_{load}^2}$ . In the area  $\tau_{load}^{-1} > 3$   $(\alpha_P)_{\gamma=0.5} \approx \frac{0.0217}{\tau_{load}} - 0.0261$ 

$$\alpha_{P} = \gamma(1 - \gamma) - \tau_{load}\beta,$$

$$\alpha_{K} = \frac{I_{K}}{I_{a}},$$

$$\tau_{load} = \frac{L_{a}}{r_{a}T_{sw}}$$

$$\beta = \frac{(1 - e^{-\frac{\gamma}{\tau_{load}}})(1 - e^{-(1 - \gamma)\tau_{load}})}{1 - e^{-\frac{1}{\tau_{load}}}}$$



### Power losses

$$P_{full} = P_{sw} + P_{on} = K_a I_a U_a f_{sw} \frac{t_+ + t_-}{2} + I^2 r_{in}$$

Symmetric switching:

$$K_a = 1 + 2\alpha_K K_a'$$

Non-symmetric sequential switching:

$$K_a = 1 + \alpha_K K_a'$$

 $t_{+}$  - voltage drop time on the switch when opening the transistor (switch-on time of the transistor)

 $t_-$  - voltage build-up time on the key when opening the transistor (switch-off time of the transistor)

 $r_{in}$  - transistor resistance in conductive state

Value of  $K'_a$  is maximum at

$$\gamma = \gamma_{max} = \tau_{load} \ln \frac{1}{\tau_{load}(1 - e^{-\frac{1}{\tau_{load}}})}.$$

In the area  $0 < \tau_{load}^{-1} < 1$ :  $\gamma_{max} = 0.5$ .

In the area  $au_{load}^{-1}>1$  : maximum of  $K_a'=f( au_{load}^{-1})$  shifts to the area of smaller  $\gamma$ 

Symmetric switching:  $K_a \approx 1 + 0.232 \frac{\alpha_K}{\tau_{load}}$ 

Non-symmetric sequential switching:  $K_a \approx 1 + 0.116 \frac{\alpha_K}{\tau_{load}}$ 

$$\alpha_P = \gamma (1 - \gamma) - \tau_{load} \beta,$$

$$\alpha_K = \frac{I_K}{I_a},$$

$$\tau_{load} = \frac{L_a}{r_a T_{sw}}$$

$$\beta = \frac{\left(1 - e^{-\frac{\gamma}{\tau_{load}}}\right)\left(1 - e^{-(1 - \gamma)\tau_{load}}\right)}{1 - e^{-\frac{1}{\tau_{load}}}}$$



### Power losses

$$P_{full} = P_{sw} + P_{on} = K_a I_a U_a f_{sw} \frac{t_+ + t_-}{2} + I^2 r_{in}$$

Symmetric switching law:

$$K_a = 1 + 2\alpha_K K_a'$$

Non-symmetric sequential switching:

$$K_a = 1 + \alpha_K K_a'$$

 $t_{+}$  - voltage drop time on the switch when opening the transistor (switch-on time of the transistor)

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Symmetric switching:  $K_a \approx 1 + 0.232 \frac{\alpha_K}{\tau_{load}}$ 

Non-symmetric sequential switching:  $K_a \approx 1 + 0.116 \frac{\alpha_K}{\tau_{load}}$ 

Symmetric switching law:

$$\left(\Delta \bar{P}_{a\_add} + \bar{P}_{sw}\right) \approx \frac{\bar{0},018}{\tau_{load}^2} + \frac{\tau_+ + \tau_-}{2\tau_{load}} (\tau_{load} + 0.232\alpha_K)$$

Non-symmetric sequential switching:

$$\left(\Delta \bar{P}_{a\_add} + \bar{P}_{sw}\right) \approx \frac{0,0045}{\tau_{load}^2} + \frac{\tau_+ + \tau_-}{2\tau_{load}} (\tau_{load} + 0.116\alpha_K)$$



### Power losses

$$P_{full} = P_{sw} + P_{on} = K_a I_a U_a f_{sw} \frac{t_+ + t_-}{2} + I^2 r_{in}$$

Symmetric switching:

$$K_a = 1 + 2\alpha_K K_a'$$

Non-symmetric sequential switching:

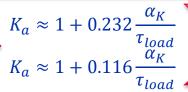
$$K_a = 1 + \alpha_K K_a'$$

In modern electric drives at switching frequencies above 1kHz:

- Always:  $au_{load}^{-1} > 1$
- the RMS current of the transistor is always determined by the Torque of the machine during starting modes
- the energy performance of DC drive with PWM converter is always determined by additional losses in the copper of the machine and switching losses in power transistors

Symmetric switching

Non-symmetric sequential switching:





$$\left( \Delta \bar{P}_{\text{a_add}} + \bar{P}_{\text{sw}} \right) \approx \frac{0.018}{\tau_{load}^2} + \frac{\tau_+ + \tau_-}{2\tau_{load}} (\tau_{load} + 0.232\alpha_K)$$

$$\left( \Delta \bar{P}_{\text{a_add}} + \bar{P}_{\text{sw}} \right) \approx \frac{0.0045}{\tau_{load}^2} + \frac{\tau_+ + \tau_-}{2\tau_{load}} (\tau_{load} + 0.116\alpha_K)$$

## Minimum losses condition

Minimum losses condition:

$$\frac{\delta \left(\Delta \bar{P}_{\text{a\_add}} + \bar{P}_{\text{sw}}\right)_{\gamma = 0.5}}{\delta (\tau_{load}^{-})} = 0$$

Symmetric switching: 
$$f_{sw\_opt} = 0.332 \sqrt[3]{\frac{\alpha_K r_a^2}{L_a^2 (t_+ + t_-)}}$$

Non-symmetric sequential switching: 
$$f_{sw\_opt} = 0.26 \sqrt[3]{\frac{\alpha_K r_a^2}{L_a^2(t_+ + t_-)}}$$

