Actuators

Actuator based on induction motor drive

Lecture 8

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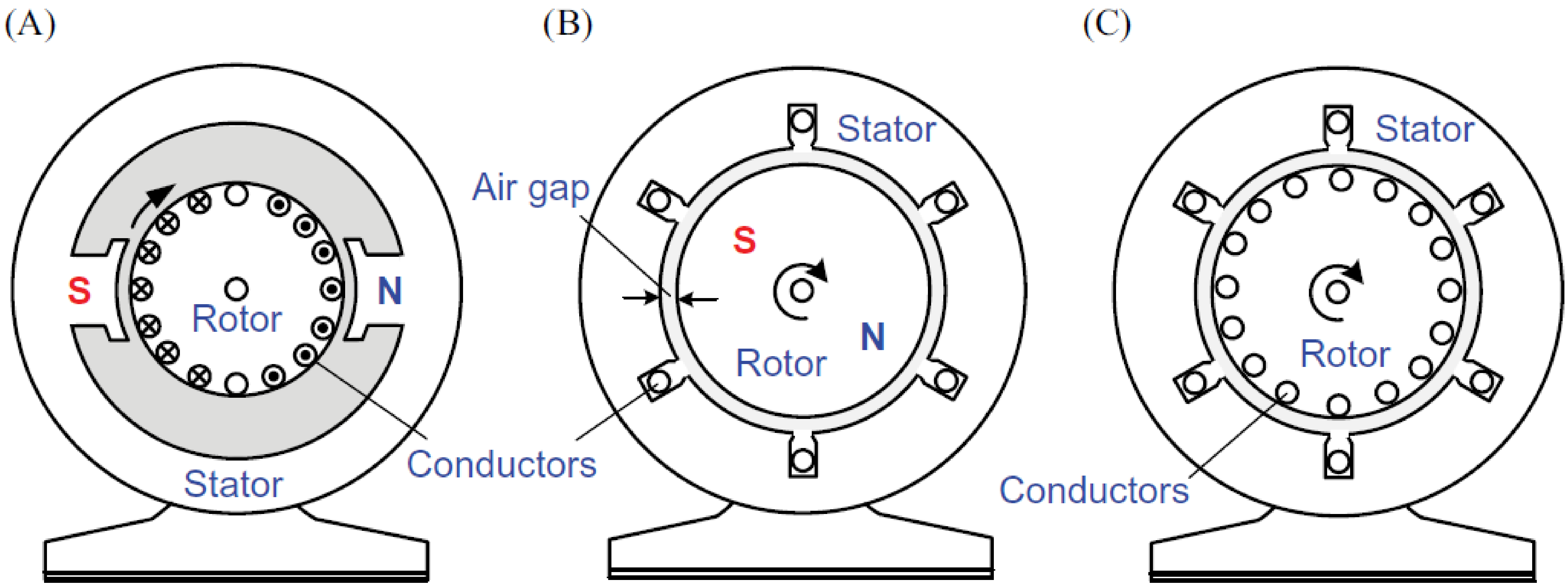
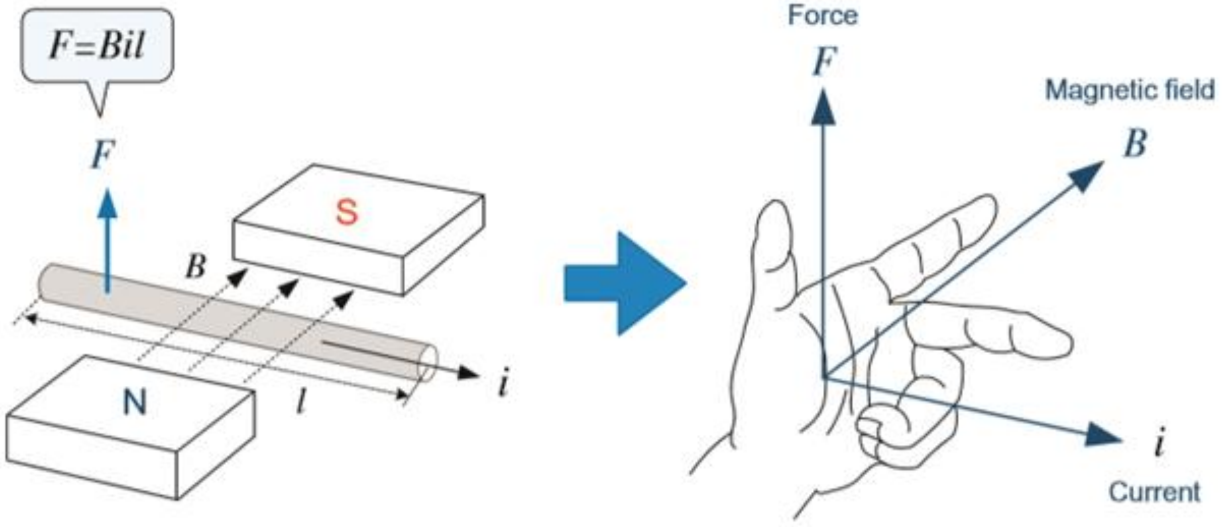
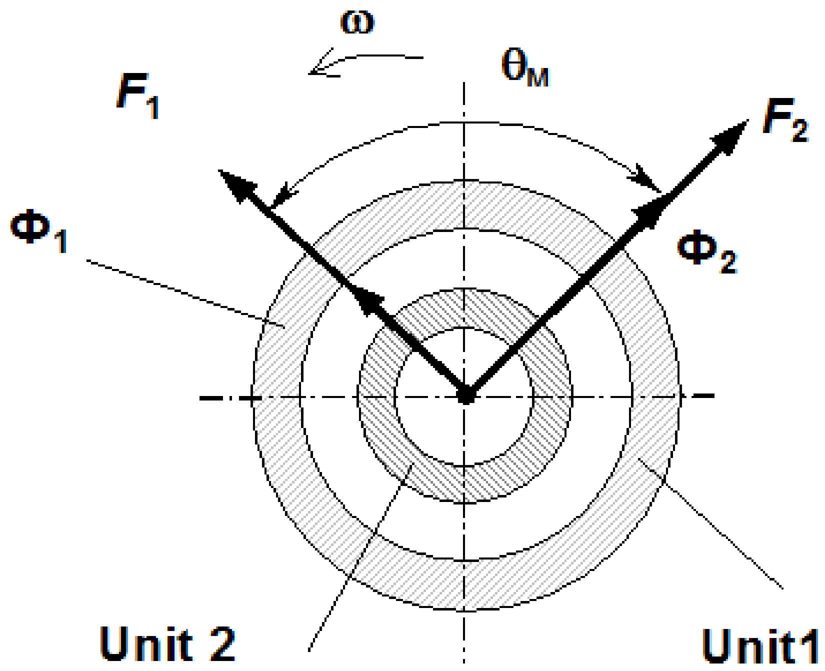
Configuration of electric motors



Configuration of electric motors. (A) DC motor, (B) AC synchronous motor, and (C) AC induction motor.

Two forms of Amper’s law





$$T = F_1 \times \Phi_2 \sin\theta_M$$

Amper’s law **<u>the 1st Form</u>** (Force for a current carrying conductor)

Generalized form of the Ampere’s Law **<u>the 2nd Form (</u>**The torque tends to align vectors)

Conditions for instantaneous torque control of motors

Construction with commutator => Always 90° between I_a and Φ_f => T_{max}

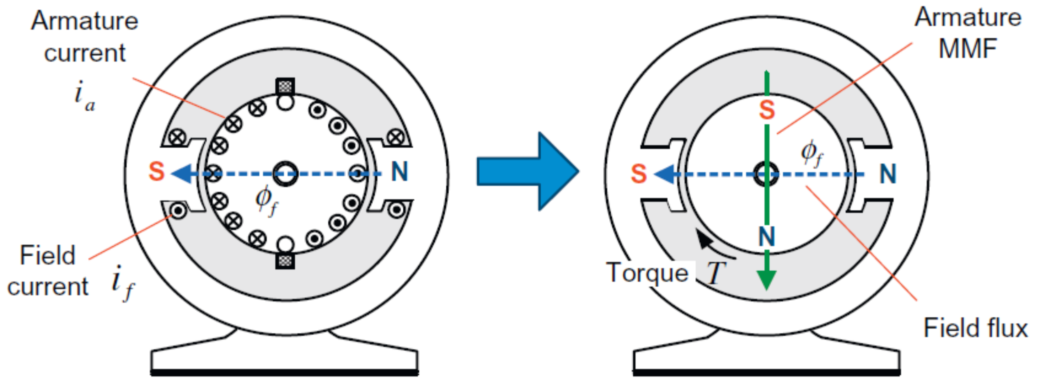


Figure Separately excited DC motor

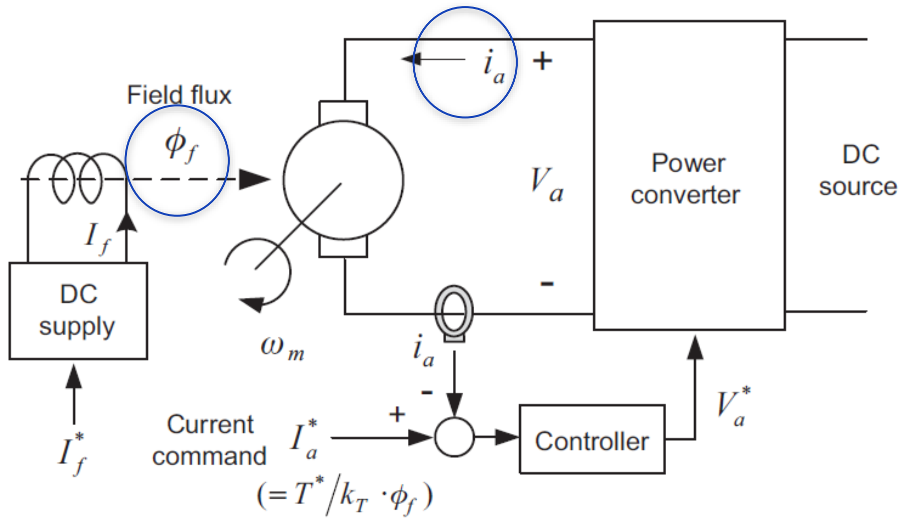


Figure Torque control system of a DC motor

Since the space angle between the armature current and the field flux always remains at 90 electrical degrees without using any particular control technique, the developed torque can be maximized under a given flux and current.

$$T = k|\phi_f||i_a|$$

$$T = k'|i_a|, \qquad k' = k|\phi_f|$$

This implies that it is possible to control the instantaneous torque of a DC motor by controlling only the magnitude of the armature current $|i_a|$.

Conditions for instantaneous torque control of motors

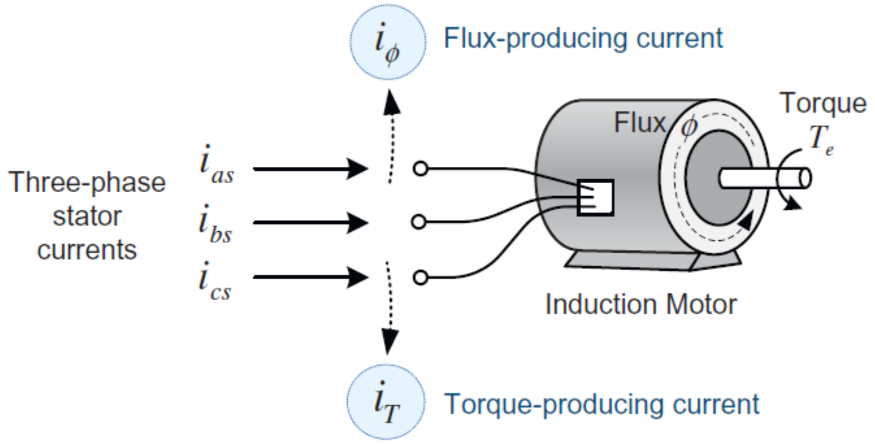
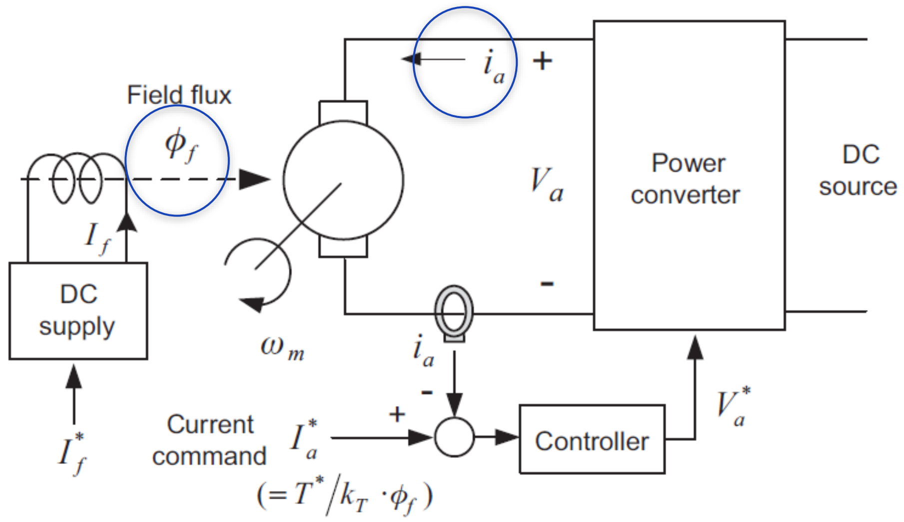


Figure   Currents of an induction motor



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Complex vector

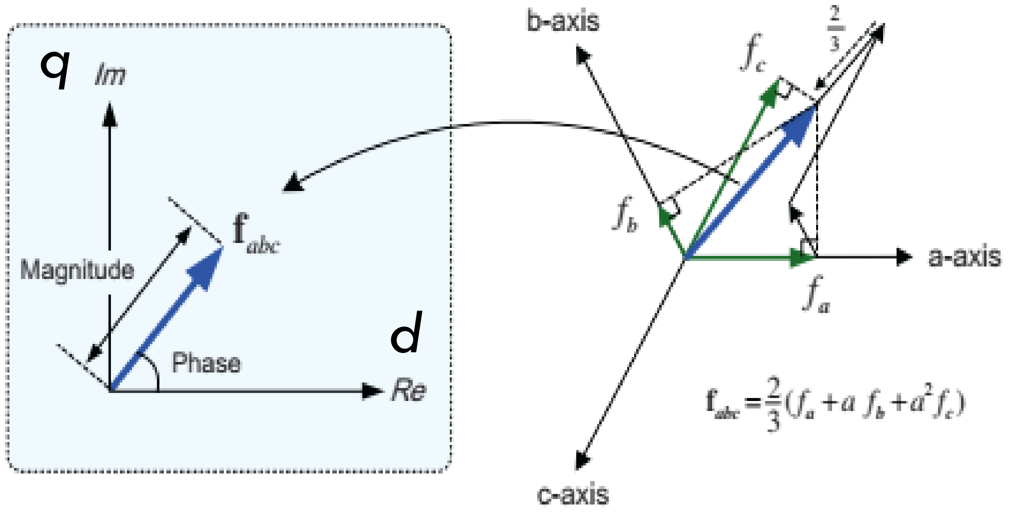


Figure Complex space vector

A complex space vector is defined as

$$f_{abc} \equiv \frac{2}{3}(f_a + af_b + a^2 f_c) \qquad a \equiv e^{j(2\pi/3)},\ a^2 = e^{j(4\pi/3)}$$

Complex vector

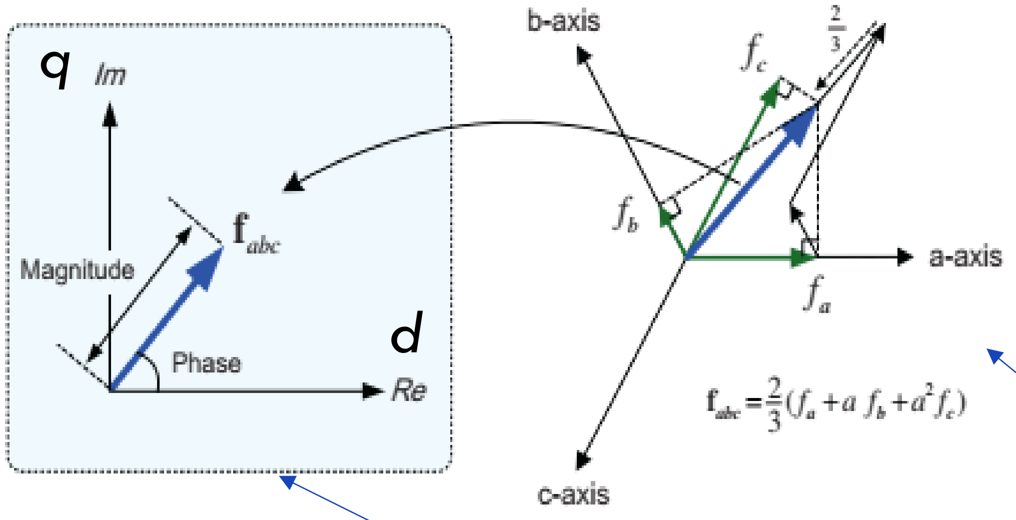


Figure Complex space vector

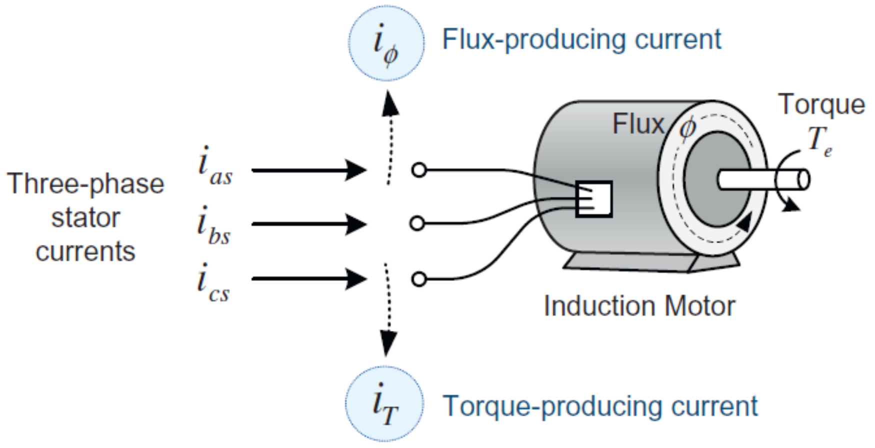


Figure Currents of an induction motor

Representation and description of f_{abc} in three-axis frame *abc*

Representation and description of f_{abc} in two-axis frame *Re-Im* or *dq*

A complex space vector is defined as

$$f_{abc} \equiv \frac{2}{3}(f_a + a f_b + a^2 f_c) \qquad a \equiv e^{j(2\pi/3)},\ a^2 = e^{j(4\pi/3)}$$

$$f_{dq}^s = f_d^s + j f_q^s \qquad \text{or} \qquad f_{dq}^e = f_d^e + j f_q^e$$

Complex vector

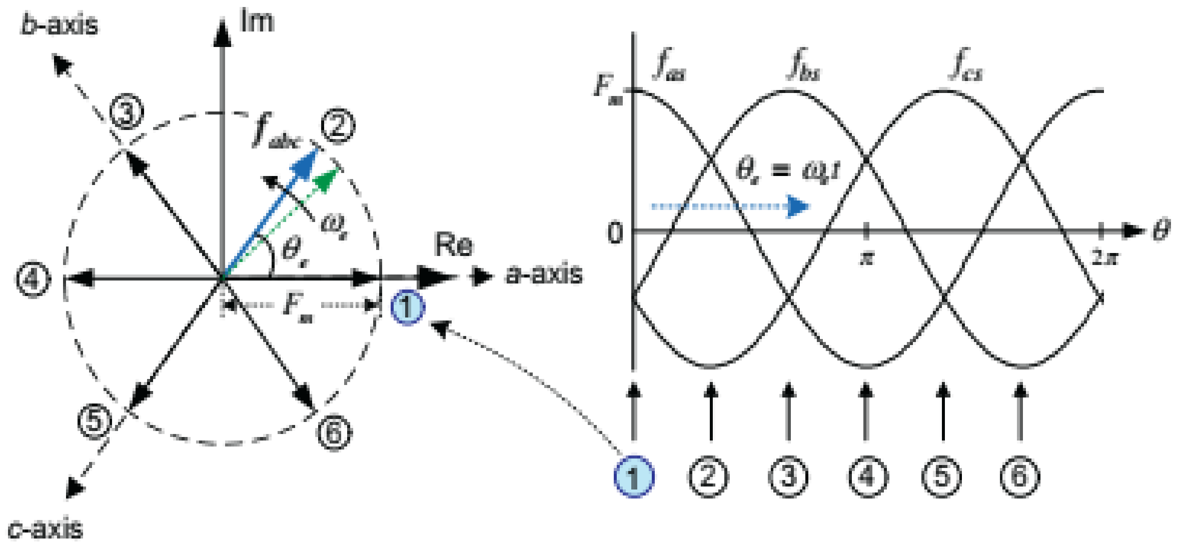
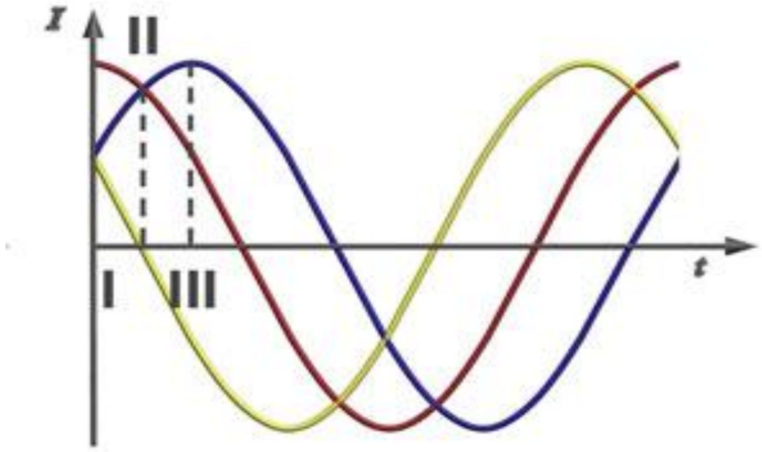
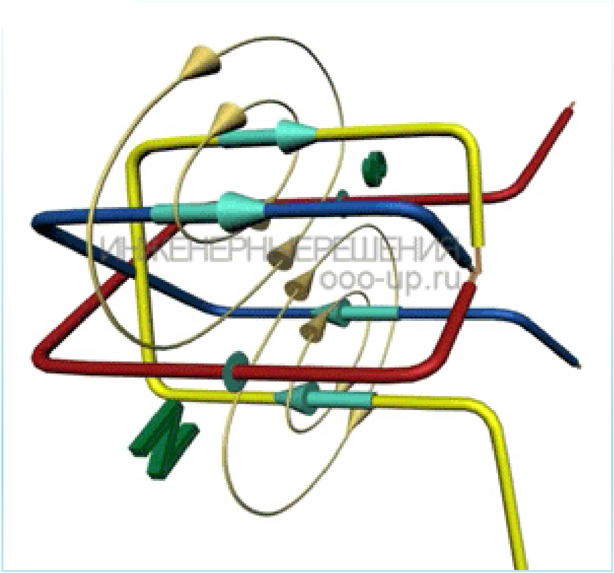


Figure Three-phase quantities and complex space vector



Firgure. Three-phase currents



Firgure. Rotating magnetic field can be represented as space vector too

AC motors

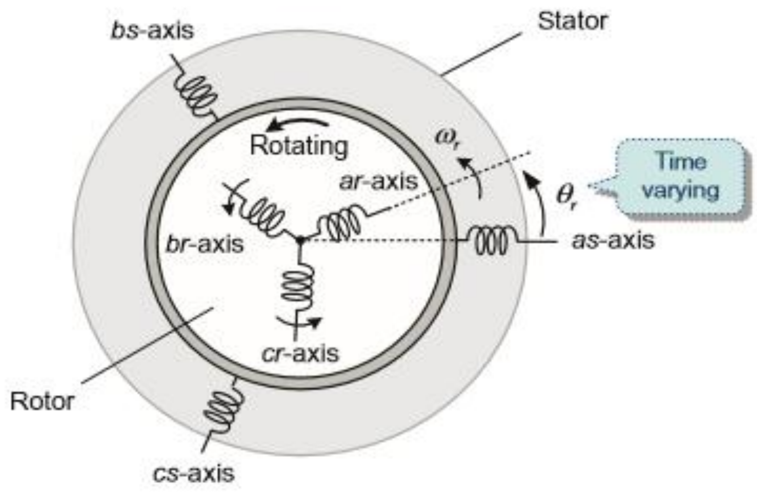


Fig. Angular position between stator and rotor windings

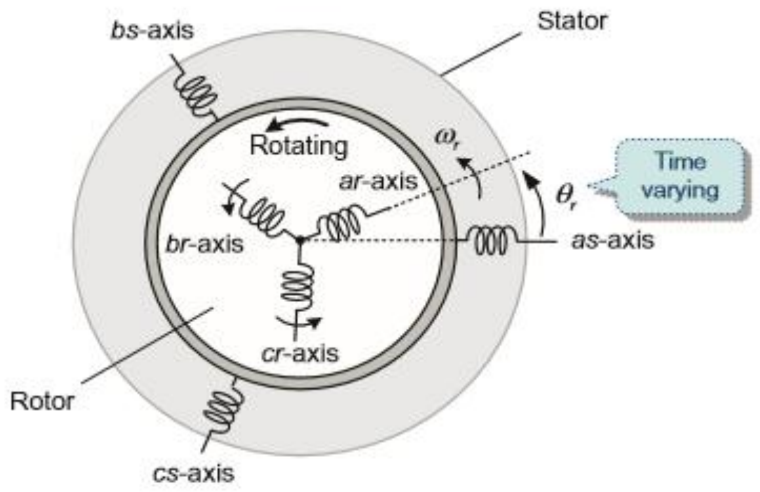
$$v(t) = Ri(t) + \frac{d\lambda(t)}{dt} = Ri(t) + \frac{d\,L(\theta_r)i(t)}{dt}$$

$\theta_r = \omega_r t$

λ - flux linkage

λ depends on the mutual-inductance which implies the amount of flux linking between the two windings (stator and rotor) - a time-varying parameter

AC motors



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$\theta_r = \omega_r t$

λ - flux linkage

- for each winding
- time-varying variables

COMPLEX

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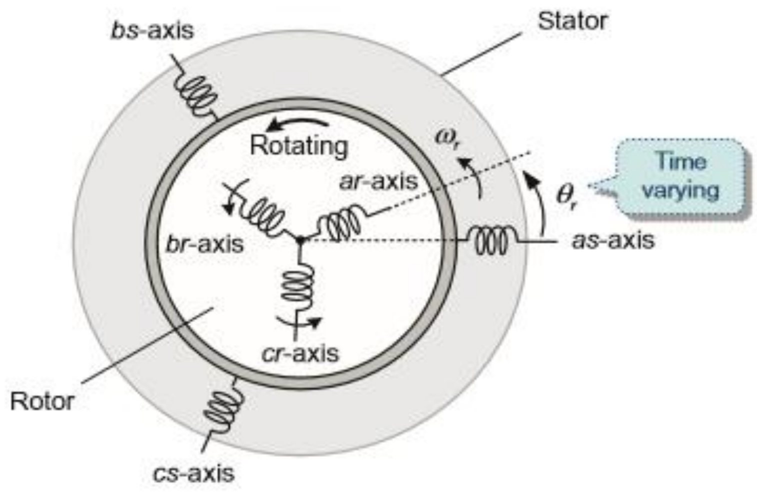
AC motors



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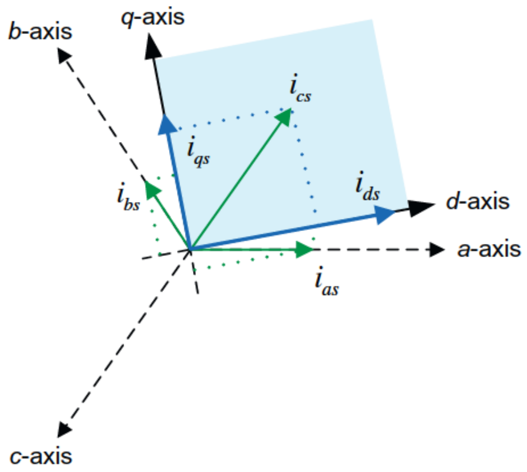

Figure Reference frame transformation for currents

$$\boxed{v(t) = Ri(t) + \frac{d\lambda(t)}{dt} = Ri(t) + \frac{d\,L(\theta_r)i(t)}{dt}}$$

$\theta_r = \omega_r t$

λ - flux linkage

- for each winding
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COMPLEX

λ depends on the mutual-inductance which implies the amount of flux linking between the two windings (stator and rotor) - a time-varying parameter

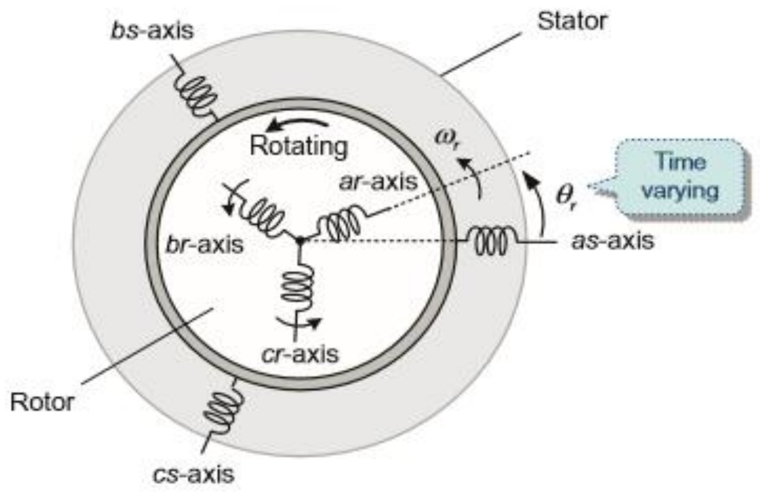
$$f_{dq}^{s} = f_d^{s} + j f_q^{s} \Rightarrow$$

$$u_s^s = u_{ds}^s + j u_{qs}^s$$

$$i_s^s = i_{ds}^s + j i_{qs}^s$$

$$\lambda_s^s = \lambda_{ds}^s + j \lambda_{qs}^s$$

AC motors



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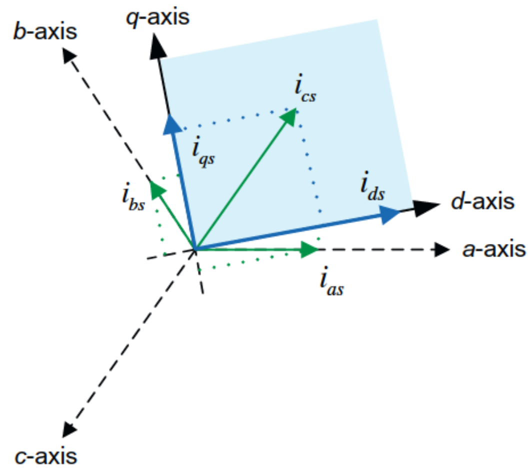

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λ - flux linkage

- for each winding
- time-varying variables

COMPLEX

λ depends on the mutual-inductance which implies the amount of flux linking between the two windings (stator and rotor) - a time-varying parameter

$$f_{dq}^s = f_d^s + jf_q^s \Rightarrow$$

$$u_s^s = u_{ds}^s + ju_{qs}^s$$

$$i_s^s = i_{ds}^s + ji_{qs}^s$$

$$\lambda_s^s = \lambda_{ds}^s + j\lambda_{qs}^s$$

$$\boxed{\begin{aligned} v_{ds}^s &= R_s i_{ds}^s + s\lambda_{ds}^s \\ v_{qs}^s &= R_s i_{qs}^s + s\lambda_{qs}^s \end{aligned}}$$

MORE SIMPLE

d-q axes model of an induction motor

Voltage equation in the *d-q* axes

- The voltage equations for the stator

$$v_{abcs} = R_s i_{abcs} + \frac{d\lambda_{abcs}}{dt} \rightarrow \qquad v_{ds}^{\omega} = R_s i_{ds}^{\omega} + \frac{d\lambda_{ds}^{\omega}}{dt} - \omega\lambda_{qs}^{\omega}$$

$$v_{qs}^{\omega} = R_s i_{qs}^{\omega} + \frac{d\lambda_{qs}^{\omega}}{dt} + \omega\lambda_{ds}^{\omega}$$

λ - Flux linkage

ω- rotational speed of the frame

- The voltage equations for the rotor

$$v_{abcr} = R_r i_{abcr} + \frac{d\lambda_{abcr}}{dt} \rightarrow \qquad v_{dr}^{\omega} = R_r i_{dr}^{\omega} + \frac{d\lambda_{dr}^{\omega}}{dt} - (\omega - \omega_r)\lambda_{qr}^{\omega}$$

$$v_{qr}^{\omega} = R_r i_{qr}^{\omega} + \frac{d\lambda_{qr}^{\omega}}{dt} + (\omega - \omega_r)\lambda_{dr}^{\omega}$$

ω- rotational speed of the frame

ω_r- rotational speed of the rotor

For squirrel-cage rotor induction motors, since the rotor bars are short-circuited by the end rings, the rotor voltage is zero, and thus, $v_{dr}^{\omega} = 0, v_{qr}^{\omega} = 0$.

<u>Flux linkage equations in the *d*-*q* axes</u>

- Stator flux linkage

$$\lambda_{ds}^{\omega} = L_{ls} i_{ds}^{\omega} + L_m (i_{ds}^{\omega} + i_{dr}^{\omega}) = L_s i_{ds}^{\omega} + L_m i_{dr}^{\omega}$$

$$\lambda_{qs}^{\omega} = L_{ls} i_{qs}^{\omega} + L_m \left(i_{qs}^{\omega} + i_{qr}^{\omega}\right) = L_s i_{qs}^{\omega} + L_m i_{qr}^{\omega}$$

$$L_m = \frac{3}{2} L_{ms}, \qquad L_s = L_{ls} + L_m$$

- Rotor flux linkage

$$\lambda_{dr}^{\omega} = L_{lr} i_{dr}^{\omega} + L_m (i_{dr}^{\omega} + i_{ds}^{\omega}) = L_r i_{dr}^{\omega} + L_m i_{ds}^{\omega}$$

$$\lambda_{qr}^{\omega} = L_{lr} i_{qr}^{\omega} + L_m \left(i_{qr}^{\omega} + i_{qs}^{\omega}\right) = L_r i_{qr}^{\omega} + L_m i_{qs}^{\omega}$$

d-q axes model of an induction motor

- Stationary reference frame ($\omega = 0$)

$$v_{ds}^{s} = R_s i_{ds}^{s} + s\lambda_{ds}^{s} \qquad \lambda_{ds}^{s} = L_s i_{ds}^{s} + L_m i_{dr}^{s}$$
$$v_{qs}^{s} = R_s i_{qs}^{s} + s\lambda_{qs}^{s} \qquad \lambda_{qs}^{s} = L_s i_{qs}^{s} + L_m i_{qr}^{s}$$
$$0 = R_r i_{dr}^{s} + s\lambda_{dr}^{s} + \omega_r \lambda_{qr}^{s} \qquad \lambda_{dr}^{s} = L_r i_{dr}^{s} + L_m i_{ds}^{s}$$
$$0 = R_r i_{qr}^{s} + s\lambda_{qr}^{s} - \omega_r \lambda_{dr}^{s} \qquad \lambda_{qr}^{s} = L_r i_{qr}^{s} + L_m i_{qs}^{s}$$

- Synchronously rotating reference frame ($\omega = \omega_e$)

$$v_{ds}^{e} = R_s i_{ds}^{e} + s\lambda_{ds}^{e} - \omega_e \lambda_{qs}^{e} \qquad \lambda_{ds}^{e} = L_s i_{ds}^{e} + L_m i_{dr}^{e}$$
$$v_{qs}^{e} = R_s i_{qs}^{e} + s\lambda_{qs}^{e} + \omega_e \lambda_{ds}^{e} \qquad \lambda_{qs}^{e} = L_s i_{qs}^{e} + L_m i_{dr}^{e}$$
$$0 = R_r i_{dr}^{e} + s\lambda_{dr}^{e} - (\omega_e - \omega_r)\lambda_{qr}^{e} \qquad \lambda_{qs}^{e} = L_r i_{dr}^{e} + L_m i_{ds}^{e}$$
$$0 = R_r i_{qr}^{e} + s\lambda_{qr}^{e} + (\omega_e - \omega_r)\lambda_{qr}^{e} \qquad \lambda_{qr}^{e} = L_r i_{qr}^{e} + L_m i_{qs}^{e}$$

Here, *s* denotes a differential operator.

Torque equation in the *d-q* axes

$$T_e = \frac{3}{2}\frac{P}{2} L_m \left(i_{qs}^{\omega} i_{dr}^{\omega} - i_{ds}^{\omega} i_{qr}^{\omega} \right) \qquad L_m = \frac{3}{2} L_{ms}$$

$$T_e = \frac{3}{2}\frac{P}{2} L_m Im\left[i_{dqr}^{*} i_{dqs} \right] =$$
$$= \frac{3}{2}\frac{P}{2} L_m \left(i_{qs} i_{dr} - i_{ds} i_{qr} \right) =$$
$$= \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\left[\lambda_{dqr}^{*} i_{dqs} \right] =$$
$$= \frac{3}{2}\frac{L_m}{L_r} \left(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds} \right) = \ldots$$

d-q axes model of an induction motor

- Stationary reference frame ($\omega = 0$)

$$v_{ds}^{s} = R_s i_{ds}^{s} + s\lambda_{ds}^{s} \qquad \lambda_{ds}^{s} = L_s i_{ds}^{s} + L_m i_{dr}^{s}$$
$$v_{qs}^{s} = R_s i_{qs}^{s} + s\lambda_{qs}^{s} \quad (*) \qquad \lambda_{qs}^{s} = L_s i_{qs}^{s} + L_m i_{qr}^{s} \quad (***)$$
$$0 = R_r i_{dr}^{s} + s\lambda_{dr}^{s} + \omega_r \lambda_{qr}^{s} \qquad \lambda_{dr}^{s} = L_r i_{dr}^{s} + L_m i_{ds}^{s}$$
$$0 = R_r i_{qr}^{s} + s\lambda_{qr}^{s} - \omega_r \lambda_{dr}^{s} \quad (**) \qquad \lambda_{qr}^{s} = L_r i_{qr}^{s} + L_m i_{qs}^{s} \quad (****)$$

- Synchronously rotating reference frame ($\omega = \omega_e$)

$$v_{ds}^{e} = R_s i_{ds}^{e} + s\lambda_{ds}^{e} - \omega_e \lambda_{qs}^{e} \qquad \lambda_{ds}^{e} = L_s i_{ds}^{e} + L_m i_{dr}^{e}$$
$$v_{qs}^{e} = R_s i_{qs}^{e} + s\lambda_{qs}^{e} + \omega_e \lambda_{ds}^{e} \qquad \lambda_{qs}^{e} = L_s i_{qs}^{e} + L_m i_{dr}^{e}$$
$$0 = R_r i_{dr}^{e} + s\lambda_{dr}^{e} - (\omega_e - \omega_r)\lambda_{qr}^{e} \qquad \lambda_{qs}^{e} = L_r i_{dr}^{e} + L_m i_{ds}^{e}$$
$$0 = R_r i_{qr}^{e} + s\lambda_{qr}^{e} + (\omega_e - \omega_r)\lambda_{qr}^{e} \qquad \lambda_{qr}^{e} = L_r i_{qr}^{e} + L_m i_{qs}^{e}$$

Here, s denotes a differential operator.

Torque equation in the d-q axes

$$T_e = \frac{3}{2}\frac{P}{2} L_m \left(i_{qs}^{\omega} i_{dr}^{\omega} - i_{ds}^{\omega} i_{qr}^{\omega}\right) \qquad L_m = \frac{3}{2} L_{ms}$$

$$T_e = \frac{3}{2}\frac{P}{2} L_m Im\left[i_{dqr}^{*} i_{dqs}\right] =$$
$$\boxed{= \frac{3}{2}\frac{P}{2} L_m \left(i_{qs} i_{dr} - i_{ds} i_{qr}\right) =} \quad (*****)$$
$$= \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\left[\lambda_{dqr}^{*} i_{dqs}\right] =$$
$$= \frac{3}{2}\frac{L_m}{L_r}\left(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}\right) = \ldots$$

model of IM in stationary reference frame

- Induction motor *d-q* equations in the stationary reference frame

$$v_{ds}^{s} = R_s i_{ds}^{s} + s\lambda_{ds}^{s} \rightarrow \lambda_{ds}^{s} = \int (v_{ds}^{s} - R_s i_{ds}^{s})dt$$

(*)

$$v_{qs}^{s} = R_s i_{qs}^{s} + s\lambda_{qs}^{s} \rightarrow \lambda_{qs}^{s} = \int \left(v_{qs}^{s} - R_s i_{qs}^{s}\right)dt$$

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$$0 = R_r i_{dr}^{s} + s\lambda_{dr}^{s} + \omega_r \lambda_{qr}^{s} \rightarrow \lambda_{dr}^{s} = \int (-R_r i_{dr}^{s} - \omega_r \lambda_{qr}^{s})dt$$

(**)

$$0 = R_r i_{qr}^{s} + s\lambda_{qr}^{s} - \omega_r \lambda_{dr}^{s} \rightarrow \lambda_{qr}^{s} = \int (-R_r i_{qr}^{s} + \omega_r \lambda_{dr}^{s})dt$$

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(**)

$$0 = R_r i_{qr}^{s} + s\lambda_{qr}^{s} - \omega_r \lambda_{dr}^{s} \rightarrow \lambda_{qr}^{s} = \int\left(-R_r i_{qr}^{s} + \omega_r \lambda_{dr}^{s}\right)dt$$

$$i_{ds}^{s} = \frac{L_r \lambda_{ds}^{s} - L_m \lambda_{dr}^{s}}{L_s L_r - L_m^2}, i_{qs}^{s} = \frac{L_r \lambda_{qs}^{s} - L_m \lambda_{qr}^{s}}{L_s L_r - L_m^2} \qquad (***)$$

$$i_{dr}^{s} = \frac{L_s \lambda_{dr}^{s} - L_m \lambda_{ds}^{s}}{L_s L_r - L_m^2}, i_{qr}^{s} = \frac{L_s \lambda_{qr}^{s} - L_m \lambda_{qs}^{s}}{L_s L_r - L_m^2} \qquad (****)$$

model of IM in stationary reference frame

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(**)

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(***)

$$i_{dr}^s = \frac{L_s \lambda_{dr}^s - L_m \lambda_{ds}^s}{L_s L_r - L_m^2}, i_{qr}^s = \frac{L_s \lambda_{qr}^s - L_m \lambda_{qs}^s}{L_s L_r - L_m^2}$$

(****)

$$T_e = \frac{3}{2}\frac{P}{2} L_m \left(i_{qs}^s i_{dr}^s - i_{ds}^s i_{qr}^s\right)$$

(*****)

model of IM in stationary reference frame

- Induction motor *d-q* equations in the stationary reference frame

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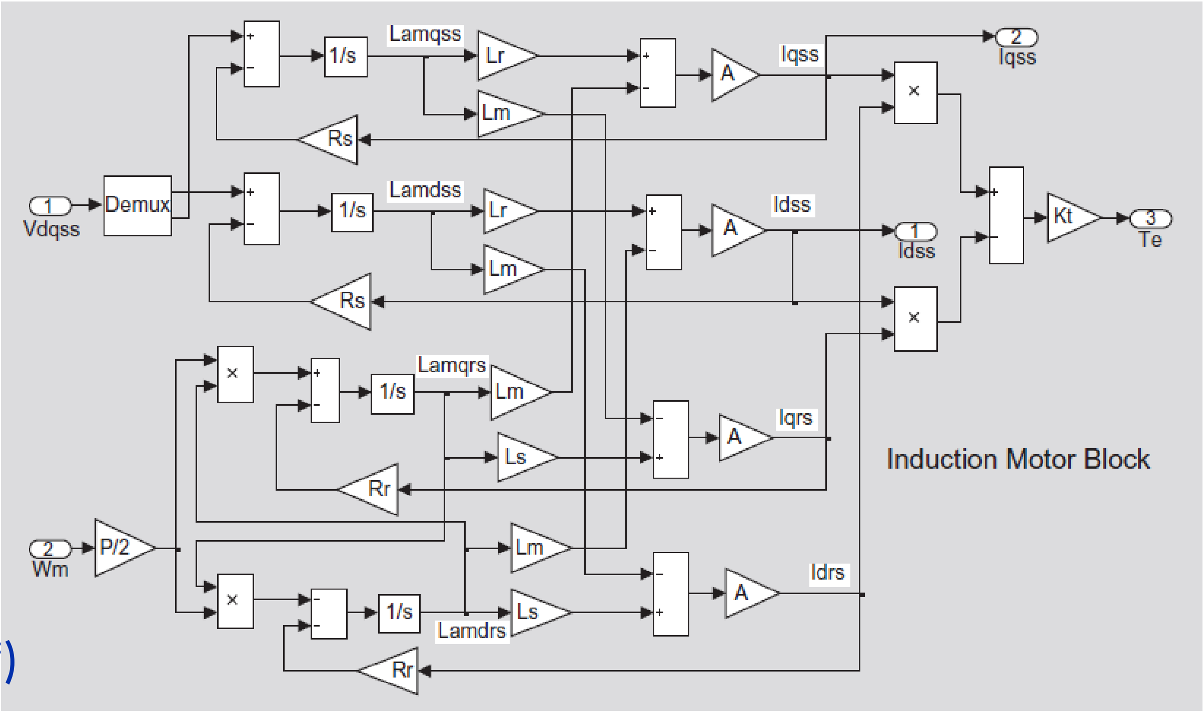
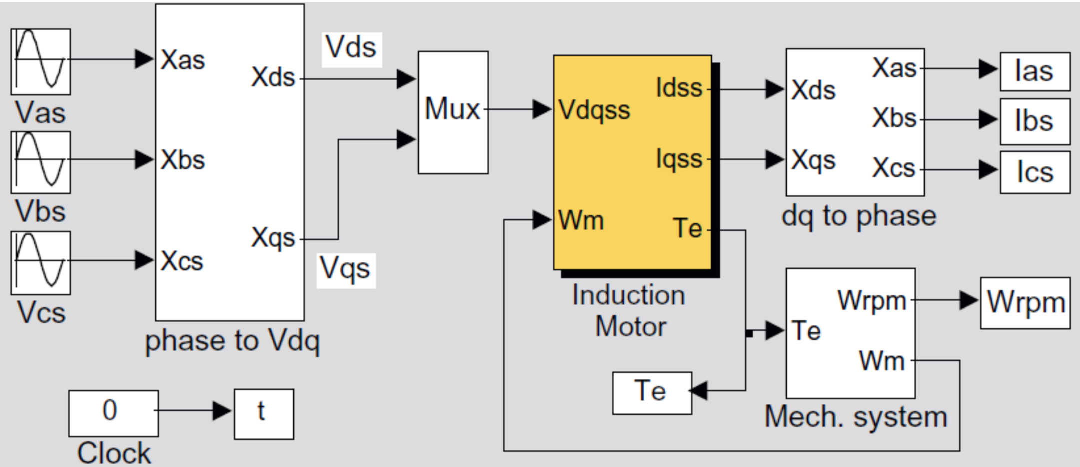
$$i_{dr}^s = \frac{L_s \lambda_{dr}^s - L_m \lambda_{ds}^s}{L_s L_r - L_m^2}, i_{qr}^s = \frac{L_s \lambda_{qr}^s - L_m \lambda_{qs}^s}{L_s L_r - L_m^2} \qquad (****)$$

$$T_e = \frac{3}{2}\frac{P}{2} L_m (i_{qs}^s i_{dr}^s - i_{ds}^s i_{qr}^s) \qquad (*****)$$



Simulink math model of Induction (Induction Motor Block) in stationary reference frame

model of IM in stationary reference frame



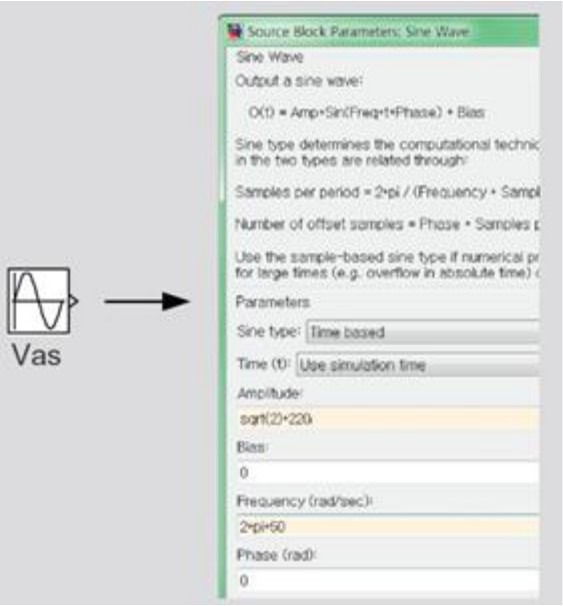
- Applied voltages

$$v_{as} = V_m cos\omega_e t$$

$$v_{bs} = V_m cos(\omega_e t - 120^o)$$

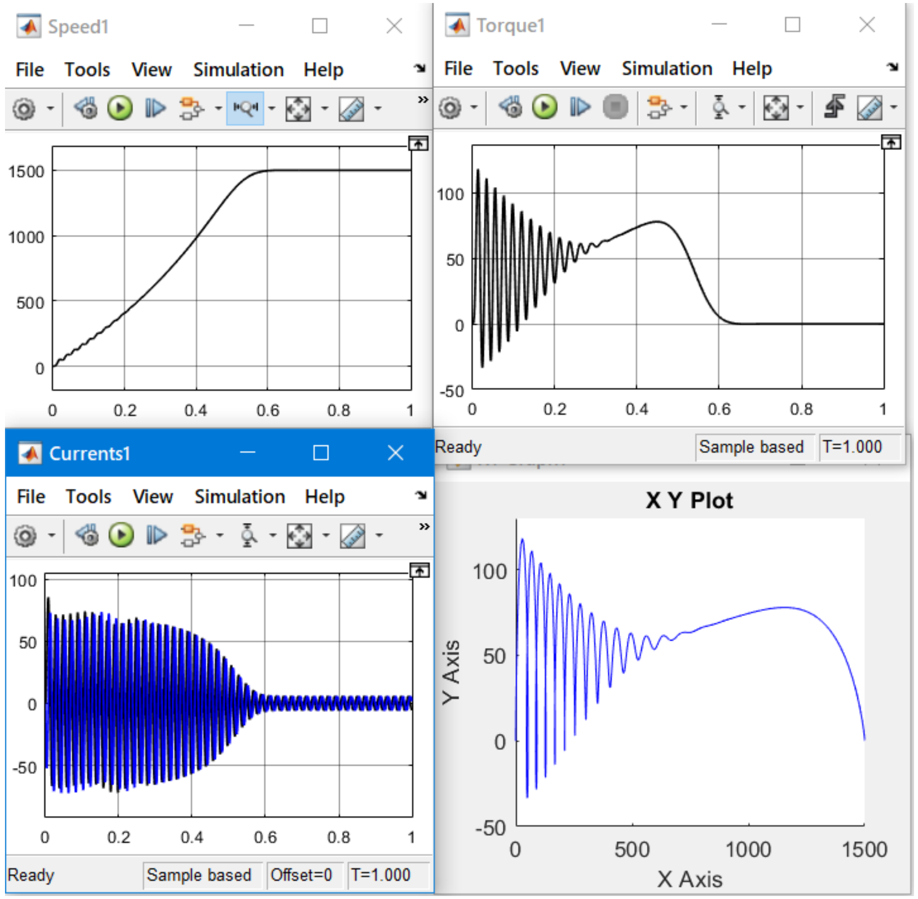
$$v_{cs} = V_m cos(\omega_e t - 240^o)$$

$$(V_m = \sqrt{2} * 220, \omega_e = 2\pi 50)$$



Speed [rpm]

Torque [N*m]



Currents [A]

Torque-speed characteristic

d-q axes model of an induction motor

- Stationary reference frame ($\omega = 0$)

$$v_{ds}^s = R_s i_{ds}^s + s\lambda_{ds}^s \qquad \lambda_{ds}^s = L_s i_{ds}^s + L_m i_{dr}^s$$
$$v_{qs}^s = R_s i_{qs}^s + s\lambda_{qs}^s \qquad \lambda_{qs}^s = L_s i_{qs}^s + L_m i_{qr}^s$$
$$0 = R_r i_{dr}^s + s\lambda_{dr}^s + \omega_r \lambda_{qr}^s \qquad \lambda_{dr}^s = L_r i_{dr}^s + L_m i_{ds}^s$$
$$0 = R_r i_{qr}^s + s\lambda_{qr}^s - \omega_r \lambda_{dr}^s \qquad \lambda_{qr}^s = L_r i_{qr}^s + L_m i_{qs}^s$$

- Synchronously rotating reference frame ($\omega = \omega_e$)

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e \quad (*) \qquad \lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \quad (***)$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e \qquad \lambda_{qs}^e = L_s i_{qs}^e + L_m i_{dr}^e$$
$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e \quad (**) \qquad \lambda_{qs}^e = L_r i_{dr}^e + L_m i_{ds}^e \quad (****)$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{qr}^e \qquad \lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e$$

Here, s denotes a differential operator.

<u>Torque equation in the *d-q* axes</u>

$$T_e = \frac{3}{2}\frac{P}{2} L_m \left(i_{qs}^\omega i_{dr}^\omega - i_{ds}^\omega i_{qr}^\omega\right) \qquad L_m = \frac{3}{2} L_{ms}$$

$$T_e = \frac{3}{2}\frac{P}{2} L_m Im\left[i_{dqr}^* i_{dqs}\right] =$$
$$= \frac{3}{2}\frac{P}{2} L_m \left(i_{qs} i_{dr} - i_{ds} i_{qr}\right) =$$
$$= \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\left[\lambda_{dqr}^* i_{dqs}\right] =$$
$$\boxed{= \frac{3}{2}\frac{L_m}{L_r}\left(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}\right) = \ldots} \quad (*****)$$

Model of IM in synchronous reference frame 1

- Induction motor *d-q* equations in the synchronous reference frame

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e \qquad (*)$$

$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \qquad (**)$$

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow$$
$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow \qquad (***)$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow$$
$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \qquad (****)$$

s is the time derivative operator *d/dt*

$\sigma = (1 - L_m^2/L_s L_r)$ is a total leakage factor

Model of IM in synchronous reference frame 1

- Induction motor *d-q* equations in the synchronous reference frame

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$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e \qquad (*)$$
$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e \qquad (**)$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow \qquad \lambda_{ds}^e = L_s i_{ds}^e + L_m\left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right)$$
$$(***)$$
$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow \qquad \lambda_{qs}^e = L_s i_{qs}^e + L_m\left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right)$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow \qquad i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$
$$(****)$$
$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \qquad i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

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Model of IM in synchronous reference frame 1

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$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e \qquad (**)$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$

Replace λ_{ds}^e, λ_{qs}^e and i_{dr}^e, i_{qr}^e in stator and rotor voltage equation through λ_{dr}^e, λ_{qr}^e and i_{ds}^e, i_{qs}^e

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow \qquad \lambda_{ds}^e = L_s i_{ds}^e + L_m\left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right)$$
$$(***)$$
$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow \qquad \lambda_{qs}^e = L_s i_{qs}^e + L_m\left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right)$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow \qquad i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$
$$(****)$$
$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \qquad i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

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$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e \quad (**)$$
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Replace λ_{ds}^e, λ_{qs}^e and i_{dr}^e, i_{qr}^e in stator and rotor voltage equation through λ_{dr}^e, λ_{qr}^e and i_{ds}^e, i_{qs}^e

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow \quad \lambda_{ds}^e = L_s i_{ds}^e + L_m\left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right)$$
$$(***)$$
$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow \quad \lambda_{qs}^e = L_s i_{qs}^e + L_m\left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right)$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow \quad i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$
$$(****)$$
$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \quad i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

$$v_{ds}^e = R_{\sigma s}(1 + sT'_s)i_{ds}^e - \omega_e L_{\sigma s} i_{qs}^e - \frac{k_2}{T_r}\lambda_{dr}^e - \omega_r k_2 \lambda_{qr}^e$$
$$v_{qs}^e = R_{\sigma s}(1 + sT'_s)i_{qs}^e + \omega_e L_{\sigma s} i_{ds}^e - \frac{k_2}{T_r}\lambda_{qr}^e + \omega_r k_2 \lambda_{dr}^e$$
$$0 = -R_r k_2 i_{ds}^e + \frac{1}{T_r}(1 + sT_r)\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$
$$0 = -R_r k_2 i_{qs}^e + \frac{1}{T_r}(1 + sT_r)\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$

$$R_{\sigma s} = (R_s + R_r k_2^2)$$
$$L_{\sigma s} = (L_s - L_m^2/L_r) = L_s\sigma$$
$$k_2 = \frac{L_m}{L_r}$$
$$T'_s = \frac{L_{\sigma s}}{R_{\sigma s}}$$

s is the time derivative operator d/dt

$\sigma = (1 - L_m^2/L_s L_r)$ is a total leakage factor

$$T = \frac{3}{2}\frac{P}{2}k_2(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e)$$

Model of IM in synchronous reference frame 1

- Induction motor *d-q* equations in the synchronous reference frame

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e \qquad (*)$$
$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e \qquad (**)$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$

Replace λ_{ds}^e, λ_{qs}^e and i_{dr}^e, i_{qr}^e in stator and rotor voltage equation through λ_{dr}^e, λ_{qr}^e and i_{ds}^e, i_{qs}^e

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow \qquad \lambda_{ds}^e = L_s i_{ds}^e + L_m\left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right)$$
$$(***)$$
$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow \qquad \lambda_{qs}^e = L_s i_{qs}^e + L_m\left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right)$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow \qquad i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$
$$(****)$$
$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \qquad i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

$$v_{ds}^e = R_{\sigma s}(1 + sT'_s)i_{ds}^e - \omega_e L_{\sigma s} i_{qs}^e - \frac{k_2}{T_r}\lambda_{dr}^e - \omega_r k_2 \lambda_{qr}^e$$
$$v_{qs}^e = R_{\sigma s}(1 + sT'_s)i_{qs}^e + \omega_e L_{\sigma s} i_{ds}^e - \frac{k_2}{T_r}\lambda_{qr}^e + \omega_r k_2 \lambda_{dr}^e$$
$$0 = -R_r k_2 i_{ds}^e + \frac{1}{T_r}(1 + sT_r)\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$
$$0 = -R_r k_2 i_{qs}^e + \frac{1}{T_r}(1 + sT_r)\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$

$$R_{\sigma s} = (R_s + R_r k_2^2)$$
$$L_{\sigma s} = (L_s - L_m^2/L_r) = L_s\sigma$$
$$k_2 = \frac{L_m}{L_r}$$
$$T'_s = \frac{L_{\sigma s}}{R_{\sigma s}}$$

s is the time derivative operator *d/dt*

$\sigma = (1 - L_m^2/L_s L_r)$ is a total leakage factor

$$T = \frac{3}{2}\frac{P}{2}k_2(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e) \qquad (*****)$$

Model of IM in synchronous reference frame 1

- Induction motor *d*-*q* equations in the synchronous reference frame

$$v_{ds}^{e} = R'_{s}(1 + sT'_{s})i_{ds}^{e} - \omega_{e}L'_{s}i_{qs}^{e} - \frac{k_{2}}{T_{r}}\lambda_{dr}^{e} - \omega_{r}k_{2}\lambda_{qr}^{e}$$

$$v_{qs}^{e} = R'_{s}(1 + sT'_{s})i_{qs}^{e} + \omega_{e}L'_{s}i_{ds}^{e} - \frac{k_{2}}{T_{r}}\lambda_{qr}^{e} + \omega_{r}k_{2}\lambda_{dr}^{e}$$

$$0 = -R_{r}k_{2}i_{ds}^{e} + \frac{1}{T_{2}}(1 + sT_{r})\lambda_{dr}^{e} - (\omega_{e} - \omega_{r})\lambda_{qr}^{e}$$

$$0 = -R_{r}k_{2}i_{qs}^{e} + \frac{1}{T_{2}}(1 + sT_{r})\lambda_{qr}^{e} + (\omega_{e} - \omega_{r})\lambda_{dr}^{e}$$

$$T = \frac{3}{2}\frac{P}{2}k_{2}(\lambda_{dr}^{e}i_{qs}^{e} - \lambda_{qr}^{e}i_{ds}^{e})$$

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- Induction motor *d-q* equations in the synchronous reference frame

$$v_{ds}^e = R'_s(1+sT'_s)i_{ds}^e - \omega_e L'_s i_{qs}^e - \frac{k_2}{T_r}\lambda_{dr}^e - \omega_r k_2 \lambda_{qr}^e \Rightarrow i_{ds}^e = \frac{1}{R_{\sigma s}(1+sT'_s)}\left(v_{ds}^e + \omega_e L_{\sigma s} i_{qs}^e + \frac{k_2}{T_r}\lambda_{dr}^e + \omega_r k_2 \lambda_{qr}^e\right.$$

$$v_{qs}^e = R'_s(1+sT'_s)i_{qs}^e + \omega_e L'_s i_{ds}^e - \frac{k_2}{T_r}\lambda_{qr}^e + \omega_r k_2 \lambda_{dr}^e \Rightarrow i_{qs}^e = \frac{1}{R_{\sigma s}(1+sT'_s)}\left(v_{qs}^e - \omega_e L_{\sigma s} i_{ds}^e + \frac{k_2}{T_r}\lambda_{qr}^e - \omega_r k_2 \lambda_{dr}^e\right.$$

$$0 = -R_r k_2 i_{ds}^e + \frac{1}{T_2}(1+sT_r)\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$

$$0 = -R_r k_2 i_{qs}^e + \frac{1}{T_2}(1+sT_r)\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$

$$\Rightarrow \lambda_{dr}^e = \frac{T_r}{(1+sT_r)}\left(R_r k_2 i_{ds}^e + (\omega_e - \omega_r)\lambda_{qr}^e\right)$$

$$\Rightarrow \lambda_{qr}^e = \frac{T_r}{(1+sT_r)}\left(R_r k_2 i_{qs}^e + (\omega_e - \omega_r)\lambda_{dr}^e\right)$$

$$T = \frac{3}{2}\frac{P}{2}k_2(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e)$$

$$T = \frac{3}{2}\frac{P}{2}k_2(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e)$$

Model of IM in synchronous reference frame 1

- Induction motor *d*-*q* equations in the synchronous reference frame

$$i_{ds}^{e} = \frac{1}{R'_s(1+sT'_s)}(v_{ds}^{e} + \omega_e L'_s i_{qs}^{e} + \frac{k_2}{T_r}\lambda_{dr}^{e} + \omega_r k_2 \lambda_{qr}^{e} \longrightarrow$$

$$i_{qs}^{e} = \frac{1}{R'_s(1+sT'_s)}(v_{qs}^{e} - \omega_e L'_s i_{ds}^{e} + \frac{k_2}{T_r}\lambda_{qr}^{e} - \omega_r k_2 \lambda_{dr}^{e} \longrightarrow$$

$$\lambda_{dr}^{e} = \frac{T_r}{(1+sT_r)}(R_r k_2 i_{ds}^{e} + (\omega_e - \omega_r)\lambda_{qr}^{e}) \qquad \longrightarrow$$

$$\lambda_{qr}^{e} = \frac{T_r}{(1+sT_r)}(R_r k_2 i_{qs}^{e} + (\omega_e - \omega_r)\lambda_{dr}^{e}) \qquad \longrightarrow$$

$$T = \frac{3}{2}\frac{P}{2}k_2(\lambda_{dr}^{e} i_{qs}^{e} - \lambda_{qr}^{e} i_{ds}^{e}) \qquad \longrightarrow$$

Model of IM in synchronous reference frame 1

- Induction motor *d-q* equations in the synchronous reference frame

$$\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0$$

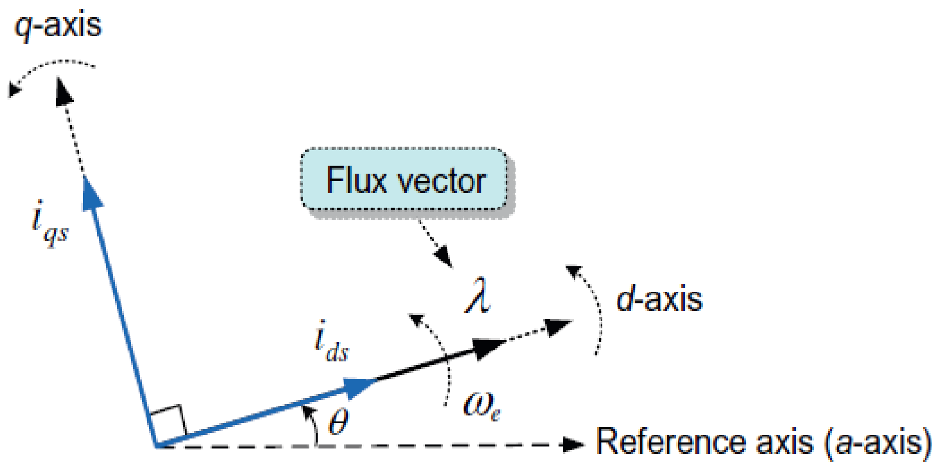
$$i_{ds}^e = \frac{1}{R'_s(1+sT'_s)}\left(v_{ds}^e + \omega_e L'_s i_{qs}^e + \frac{k_2}{T_r}\lambda_{dr}^e + \omega_r k_2 \lambda_{qr}^e\right. \rightarrow$$

$$i_{qs}^e = \frac{1}{R'_s(1+sT'_s)}\left(v_{qs}^e - \omega_e L'_s i_{ds}^e + \frac{k_2}{T_r}\lambda_{qr}^e - \omega_r k_2 \lambda_{dr}^e\right. \rightarrow$$

$$\lambda_{dr}^e = \frac{T_r}{(1+sT_r)}\left(R_r k_2 i_{ds}^e + (\omega_e - \omega_r)\lambda_{qr}^e\right) \qquad \rightarrow$$

$$\lambda_{qr}^e = \frac{T_r}{(1+sT_r)}\left(R_r k_2 i_{qs}^e + (\omega_e - \omega_r)\lambda_{dr}^e\right) \qquad \rightarrow$$

$$T = \frac{3}{2}\frac{P}{2}k_2\left(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e\right) \qquad \rightarrow$$



Model of IM in synchronous reference frame 1

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$$\boxed{\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0}$$

$$i_{ds}^e = \frac{1}{R'_s(1+sT'_s)}\left(v_{ds}^e + \omega_e L'_s i_{qs}^e + \frac{k_2}{T_r}\lambda_{dr}^e + \omega_r k_2 \lambda_{qr}^e\right) \longrightarrow \quad i_{ds}^e = \frac{1}{R_{\sigma s}(1+sT'_s)}\left(v_{ds}^e + \omega_e L_{\sigma s} i_{qs}^e + \frac{k_2}{T_r}\lambda_{dr}^e\right)$$

$$i_{qs}^e = \frac{1}{R'_s(1+sT'_s)}\left(v_{qs}^e - \omega_e L'_s i_{ds}^e + \frac{k_2}{T_r}\lambda_{qr}^e - \omega_r k_2 \lambda_{dr}^e\right) \longrightarrow \quad i_{qs}^e = \frac{1}{R_{\sigma s}(1+sT'_s)}\left(v_{qs}^e - \omega_e L_{\sigma s} i_{ds}^e - \omega_r k_2 \lambda_{dr}^e\right)$$

$$\lambda_{dr}^e = \frac{T_r}{(1+sT_r)}\left(R_r k_2 i_{ds}^e + (\omega_e - \omega_r)\lambda_{qr}^e\right) \longrightarrow \quad \lambda_{dr}^e = \frac{R_r k_2 T_r}{(1+sT_r)} i_{ds}^e = \frac{L_m}{(1+pT_r)} i_{ds}^e$$

$$\lambda_{qr}^e = \frac{T_r}{(1+sT_r)}\left(R_r k_2 i_{qs}^e + (\omega_e - \omega_r)\lambda_{dr}^e\right) \longrightarrow \quad \omega_e - \omega_r = \omega_{sl} = \frac{k_2 R_r}{\lambda_{dr}^e} i_{qs}^e = \frac{L_m R_r}{L_r}\frac{i_{qs}^e}{\lambda_{dr}^e}$$

$$T = \frac{3}{2}\frac{P}{2} k_2\left(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e\right) \longrightarrow \quad T = \frac{3}{2}\frac{P}{2} k_2 \lambda_{dr}^e i_{qs}^e$$

Model of IM in synchronous reference frame 1

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$$\lambda_{dr}^e = \frac{T_r}{(1+sT_r)}\left(R_r k_2 i_{ds}^e + (\omega_e - \omega_r)\lambda_{qr}^e\right) \longrightarrow \quad \lambda_{dr}^e = \frac{R_r k_2 T_r}{(1+sT_r)} i_{ds}^e = \frac{L_m}{(1+pT_r)} i_{ds}^e$$

$$\lambda_{qr}^e = \frac{T_r}{(1+sT_r)}\left(R_r k_2 i_{qs}^e + (\omega_e - \omega_r)\lambda_{dr}^e\right) \longrightarrow \quad \omega_e - \omega_r = \omega_{\mathrm{sl}} = \frac{k_2 R_r}{\lambda_{dr}^e} i_{qs}^e = \frac{L_m R_r}{L_r}\frac{i_{qs}^e}{\lambda_{dr}^e}$$

$$T = \frac{3}{2}\frac{P}{2} k_2\left(\lambda_{dr}^e i_{qs}^e - \lambda_{qr}^e i_{ds}^e\right) \longrightarrow \quad T = \frac{3}{2}\frac{P}{2} k_2 \lambda_{dr}^e i_{qs}^e$$

- i_{ds}^e - flux-producing current
- i_{qs}^e - torque-producing current.

Model of IM in synchronous reference frame 1

- Induction motor *d-q* equations in the synchronous reference frame

$$\lambda_{dr}^{e} = |\lambda|, \qquad \lambda_{qr}^{e} = 0$$

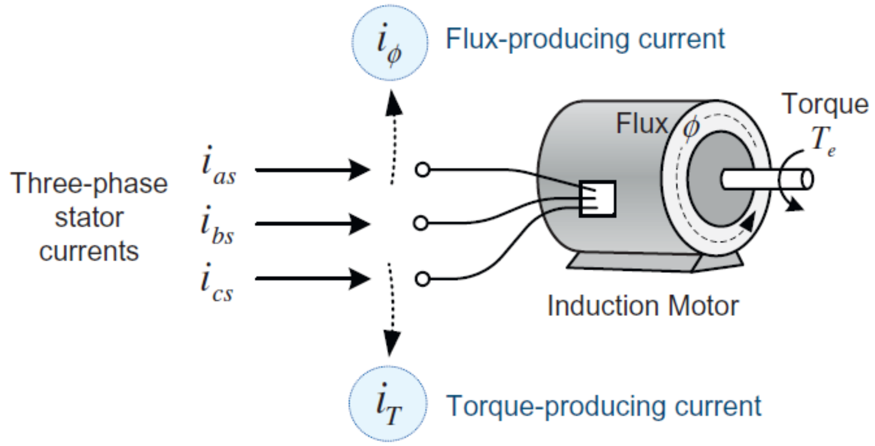


Figure Currents of an induction motor

$$i_{ds}^{e} = \frac{1}{R_{\sigma s}(1+sT'_s)}\left(v_{ds}^{e} + \omega_e L_{\sigma s} i_{qs}^{e} + \frac{k_2}{T_r}\lambda_{dr}^{e}\right)$$

$$i_{qs}^{e} = \frac{1}{R_{\sigma s}(1+sT'_s)}\left(v_{qs}^{e} - \omega_e L_{\sigma s} i_{ds}^{e} - \omega_r k_2 \lambda_{dr}^{e}\right)$$

$$\lambda_{dr}^{e} = \frac{R_r k_2 T_r}{(1+sT_r)} i_{ds}^{e} = \frac{L_m}{(1+pT_r)} i_{ds}^{e}$$

$$\omega_e - \omega_r = \omega_{sl} = \frac{k_2 R_r}{\lambda_{dr}^{e}} i_{qs}^{e} = \frac{L_m R_r}{L_r} \frac{i_{qs}^{e}}{\lambda_{dr}^{e}}$$

$$T = \frac{3}{2}\frac{P}{2} k_2 \lambda_{dr}^{e} i_{qs}^{e}$$

- i_{ds}^{e} - flux-producing current
- i_{qs}^{e} - torque-producing current.

d-q axes model of an induction motor

- Stationary reference frame ($\omega = 0$)

$$v_{ds}^s = R_s i_{ds}^s + s\lambda_{ds}^s \qquad \lambda_{ds}^s = L_s i_{ds}^s + L_m i_{dr}^s$$
$$v_{qs}^s = R_s i_{qs}^s + s\lambda_{qs}^s \qquad \lambda_{qs}^s = L_s i_{qs}^s + L_m i_{qr}^s$$
$$0 = R_r i_{dr}^s + s\lambda_{dr}^s + \omega_r \lambda_{qr}^s \qquad \lambda_{dr}^s = L_r i_{dr}^s + L_m i_{ds}^s$$
$$0 = R_r i_{qr}^s + s\lambda_{qr}^s - \omega_r \lambda_{dr}^s \qquad \lambda_{qr}^s = L_r i_{qr}^s + L_m i_{qs}^s$$

- Synchronously rotating reference frame ($\omega = \omega_e$)

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e \qquad (*) \qquad \lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e \qquad \lambda_{qs}^e = L_s i_{qs}^e + L_m i_{dr}^e \qquad (***)$$
$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e \qquad (**) \qquad \lambda_{qs}^e = L_r i_{dr}^e + L_m i_{ds}^e$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{qr}^e \qquad \lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \qquad (****)$$

Here, *s* denotes a differential operator.

<u>Torque equation in the *d-q* axes</u>

$$T_e = \frac{3}{2}\frac{P}{2} L_m\left(i_{qs}^{\omega} i_{dr}^{\omega} - i_{ds}^{\omega} i_{qr}^{\omega}\right) \qquad L_m = \frac{3}{2} L_{ms}$$

$$T_e = \frac{3}{2}\frac{P}{2} L_m Im\left[i_{dqr}^* i_{dqs}\right] =$$
$$= \frac{3}{2}\frac{P}{2} L_m\left(i_{qs} i_{dr} - i_{ds} i_{qr}\right) =$$
$$= \frac{3}{2}\frac{P}{2}\frac{L_m}{L_r} Im\left[\lambda_{dqr}^* i_{dqs}\right] =$$
$$\boxed{= \frac{3}{2}\frac{L_m}{L_r}\left(\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds}\right) = \ldots} \qquad (*****)$$

Model of IM in synchronous reference frame 2

- Induction motor *d*-*q* equations in the synchronous reference frame

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e$$

(*)

$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e$$

$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$

$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$

(**)

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow$$

(***)

$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow$$

(****)

$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow$$

Model of IM in synchronous reference frame 2

- Induction motor *d-q* equations in the synchronous reference frame

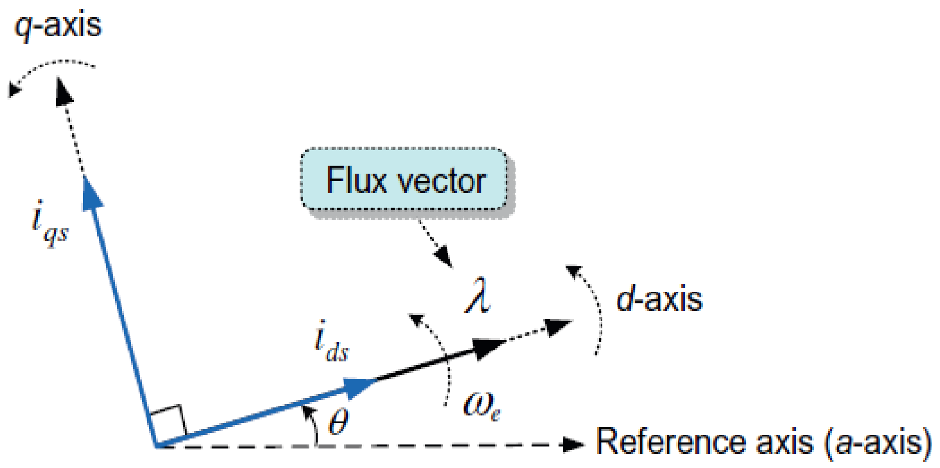
$$\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0$$

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e$$
(*)

$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$
(**)

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow$$
(***)
$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow$$
$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow$$
(****)



Model of IM in synchronous reference frame 2

- Induction motor *d-q* equations in the synchronous reference frame

$$\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0$$

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e$$
(*)

$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e$$
(**)

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow \qquad \lambda_{ds}^e = L_s i_{ds}^e + L_m\left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right) = \sigma L_s i_{ds}^e + \lambda_{dr}^e k_2$$
(***)

$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow \qquad \lambda_{qs}^e = L_s i_{qs}^e + L_m\left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right) = \sigma L_s i_{qs}^e + \lambda_{qr}^e k_2$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow \qquad i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$
(****)

$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \qquad i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

$\sigma = (1 - L_m^2/L_s L_r)$ is a total leakage factor

$T_{\sigma s} = L_s\sigma/R_s = L_{\sigma s}/R_s$

Model of IM in synchronous reference frame 2

$$\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0$$

- Induction motor *d-q* equations in the synchronous reference frame

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e \qquad (*)$$

$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e$$

$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$

$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \qquad (**)$$

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow$$

$$(***)$$

$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow$$

$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \qquad (****)$$

Replace λ_{ds}^e, λ_{qs}^e and i_{dr}^e, i_{qr}^e in stator and rotor voltage equation through λ_{dr}^e, λ_{qr}^e and i_{ds}^e, i_{qs}^e

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m \left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right) =$$

$$= \sigma L_s i_{ds}^e + \lambda_{dr}^e k_2$$

$$\lambda_{qs}^e = L_s i_{qs}^e + L_m \left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right) =$$

$$= \sigma L_s i_{qs}^e + \lambda_{qr}^e k_2$$

$$i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$

$$i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

$\sigma = (1 - L_m^2 / L_s L_r)$ is a total leakage factor

$T_{\sigma s} = L_s \sigma / R_s = L_{\sigma s} / R_s$

Model of IM in synchronous reference frame 2

- Induction motor *d*-*q* equations in the synchronous reference frame

$$\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0$$

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e \qquad (*)$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e$$
$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \quad (**)$$

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow$$
$$(***)$$
$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow$$
$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \quad (****)$$

Replace λ_{ds}^e, λ_{qs}^e and i_{dr}^e, i_{qr}^e in stator and rotor voltage equation through λ_{dr}^e, λ_{qr}^e and i_{ds}^e, i_{qs}^e

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m\left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right) =$$
$$= \sigma L_s i_{ds}^e + \lambda_{dr}^e k_2$$

$$\lambda_{qs}^e = L_s i_{qs}^e + L_m\left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right) =$$
$$= \sigma L_s i_{qs}^e + \lambda_{qr}^e k_2$$

$$i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$

$$i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

$$v_{ds}^e = R_s(1 + sT_{\sigma s})i_{ds}^e - \omega_e L_{\sigma s} i_{qs}^e + s\lambda_{dr}^e k_2$$
$$v_{qs}^e = R_s(1 + sT_{\sigma s})i_{qs}^e + \omega_e L_{\sigma s} i_{ds}^e + \lambda_{dr}^e k_2\, \omega_e$$
$$i_{ds}^e = \frac{\lambda_{dr}^e}{L_m}\,\frac{(1+sT_r)}{L_m}$$
$$i_{qs}^e = \frac{\lambda_{dr}^e}{L_m}\,\omega_{\text{sl}}\, T_r$$

$\sigma = (1 - L_m^2/L_s L_r)$ is a total leakage factor

$T_{\sigma s} = L_s\sigma/R_s = L_{\sigma s}/R_s$

$\omega_e - \omega_r = \omega_{\text{sl}} \qquad z_p = \frac{P}{2}$

Model of IM in synchronous reference frame 2

$$\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0$$

- Induction motor *d*-*q* equations in the synchronous reference frame

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e \quad (*)$$
$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \quad (**)$$

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \rightarrow \qquad \lambda_{ds}^e = L_s i_{ds}^e + L_m\left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right) = \sigma L_s i_{ds}^e + \lambda_{dr}^e k_2$$

$$(***)$$

$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \rightarrow \qquad \lambda_{qs}^e = L_s i_{qs}^e + L_m\left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right) = \sigma L_s i_{qs}^e + \lambda_{qr}^e k_2$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \rightarrow \qquad i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$

$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \rightarrow \quad (****) \quad i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

Replace λ_{ds}^e, λ_{qs}^e and i_{dr}^e, i_{qr}^e in stator and rotor voltage equation through λ_{dr}^e, λ_{qr}^e and i_{ds}^e, i_{qs}^e

$$v_{ds}^e = R_s(1 + sT_{\sigma s})i_{ds}^e - \omega_e L_{\sigma s} i_{qs}^e + s\lambda_{dr}^e k_2$$
$$v_{qs}^e = R_s(1 + sT_{\sigma s})i_{qs}^e + \omega_e L_{\sigma s} i_{ds}^e + \lambda_{dr}^e k_2\, \omega_e$$
$$i_{ds}^e = \frac{\lambda_{dr}^e}{L_m}\frac{(1+sT_r)}{L_m}$$
$$i_{qs}^e = \frac{\lambda_{dr}^e}{L_m}\omega_{sl} T_r$$
$$T = \frac{3}{2}\frac{P}{2} k_2 \lambda_{dr}^e i_{qs}^e = \frac{3}{2}\frac{P}{2}\frac{\lambda_{dr}^{e2}}{R_r}\omega_{sl} = \frac{3}{2} z_p \frac{\lambda_{dr}^{e2}}{R_r}\omega_{sl} =$$

$\sigma = (1 - L_m^2/L_s L_r)$ is a total leakage factor

$T_{\sigma s} = L_s\sigma/R_s = L_{\sigma s}/R_s$

$\omega_e - \omega_r = \omega_{sl}$ $\qquad z_p = \frac{P}{2}$

Model of IM in synchronous reference frame 2

- Induction motor *d*-*q* equations in the synchronous reference frame

$$\lambda_{dr}^e = |\lambda|, \qquad \lambda_{qr}^e = 0$$

$$v_{ds}^e = R_s i_{ds}^e + s\lambda_{ds}^e - \omega_e \lambda_{qs}^e$$
$$v_{qs}^e = R_s i_{qs}^e + s\lambda_{qs}^e + \omega_e \lambda_{ds}^e \quad (*)$$
$$0 = R_r i_{dr}^e + s\lambda_{dr}^e - (\omega_e - \omega_r)\lambda_{qr}^e$$
$$0 = R_r i_{qr}^e + s\lambda_{qr}^e + (\omega_e - \omega_r)\lambda_{dr}^e \quad (**)$$

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m i_{dr}^e \longrightarrow$$

$$(***)$$

$$\lambda_{qs}^e = L_s i_{qs}^e + L_m i_{qr}^e \longrightarrow$$

$$\lambda_{dr}^e = L_r i_{dr}^e + L_m i_{ds}^e \longrightarrow$$

$$\lambda_{qr}^e = L_r i_{qr}^e + L_m i_{qs}^e \longrightarrow \quad (****)$$

Replace λ_{ds}^e, λ_{qs}^e and i_{dr}^e, i_{qr}^e in stator and rotor voltage equation through λ_{dr}^e, λ_{qr}^e and i_{ds}^e, i_{qs}^e

$$\lambda_{ds}^e = L_s i_{ds}^e + L_m\left(\frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}\right) =$$
$$= \sigma L_s i_{ds}^e + \lambda_{dr}^e k_2$$

$$\lambda_{qs}^e = L_s i_{qs}^e + L_m\left(\frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}\right) =$$
$$= \sigma L_s i_{qs}^e + \lambda_{qr}^e k_2$$

$$i_{dr}^e = \frac{\lambda_{dr}^e - L_m i_{ds}^e}{L_r}$$

$$i_{qr}^e = \frac{\lambda_{qr}^e - L_m i_{qs}^e}{L_r}$$

$$v_{ds}^e = R_s(1 + sT_{\sigma s})i_{ds}^e - \omega_e L_{\sigma s} i_{qs}^e + s\lambda_{dr}^e k_2$$
$$v_{qs}^e = R_s(1 + sT_{\sigma s})i_{qs}^e + \omega_e L_{\sigma s} i_{ds}^e + \lambda_{dr}^e k_2\, \omega_e$$
$$i_{ds}^e = \frac{\lambda_{dr}^e}{L_m}\frac{(1+sT_r)}{L_m}$$
$$i_{qs}^e = \frac{\lambda_{dr}^e}{L_m}\omega_{sl}\, T_r$$

$$T = \frac{3}{2}\frac{P}{2} k_2 \lambda_{dr}^e i_{qs}^e = \frac{3}{2}\frac{P}{2}\frac{\lambda_{dr}^{e2}}{R_r}\omega_{sl} = \frac{3}{2} z_p \frac{\lambda_{dr}^{e2}}{R_r}\omega_{sl} =$$
$$= \frac{3}{2} z_p \frac{L_m}{L_r}\lambda_{dr}^e i_{qs}^e = \frac{3}{2} z_p \frac{L_m}{T_r R_r}\lambda_{dr}^e\, i_{qs}^e$$

$\sigma = (1 - L_m^2/L_s L_r)$ is a total leakage factor

$T_{\sigma s} = L_s\sigma/R_s = L_{\sigma s}/R_s$

$\omega_e - \omega_r = \omega_{sl}$ $\qquad z_p = \frac{P}{2}$

Model of IM in synchronous reference frame 2

$$i_{ds}^e = \frac{1}{R_s(1+sT_{\sigma s})}\left(v_{ds}^e + \omega_e L_{\sigma s} i_{qs}^e - s\lambda_{dr}^e k_2\right)$$

$$i_{qs}^e = \frac{1}{R_s(1+sT_{\sigma s})}\left(v_{qs}^e - \omega_e L_{\sigma s} i_{ds}^e - \lambda_{dr}^e k_2\, \omega_e\right)$$

$$i_{ds}^e = \frac{\lambda_{dr}^e}{L_m}\frac{(1+sT_r)}{L_m}$$

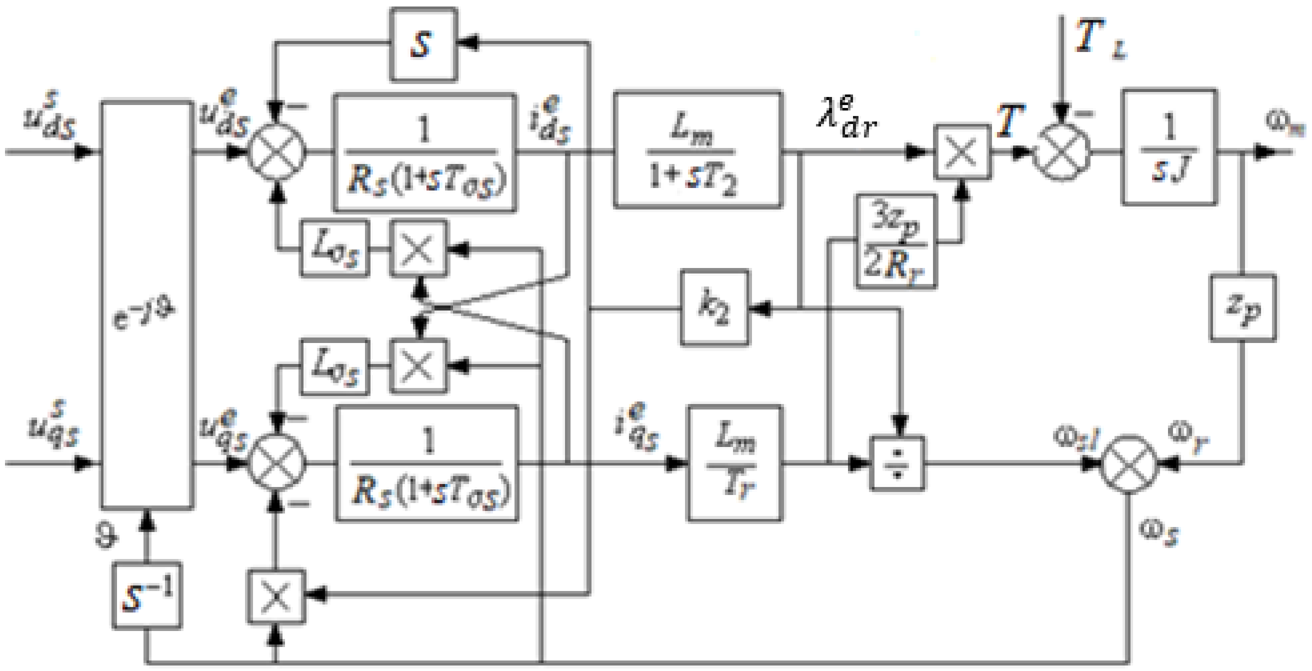
$$i_{qs}^e = \frac{\lambda_{dr}^e}{L_m}\omega_{\mathrm{sl}}\, T_r$$

$$i_s^e = sqrt({i_{ds}^e}^2 + {i_{qs}^e}^2)$$

$$T = \frac{3}{2} z_p \frac{L_m}{T_r R_r}\lambda_{dr}^e i_{qs}^e$$

$$\lambda_{dr}^e = i_s^e L_m / sqrt(1 + (\omega_{\mathrm{sl}}\, T_r)\text{^}2)$$

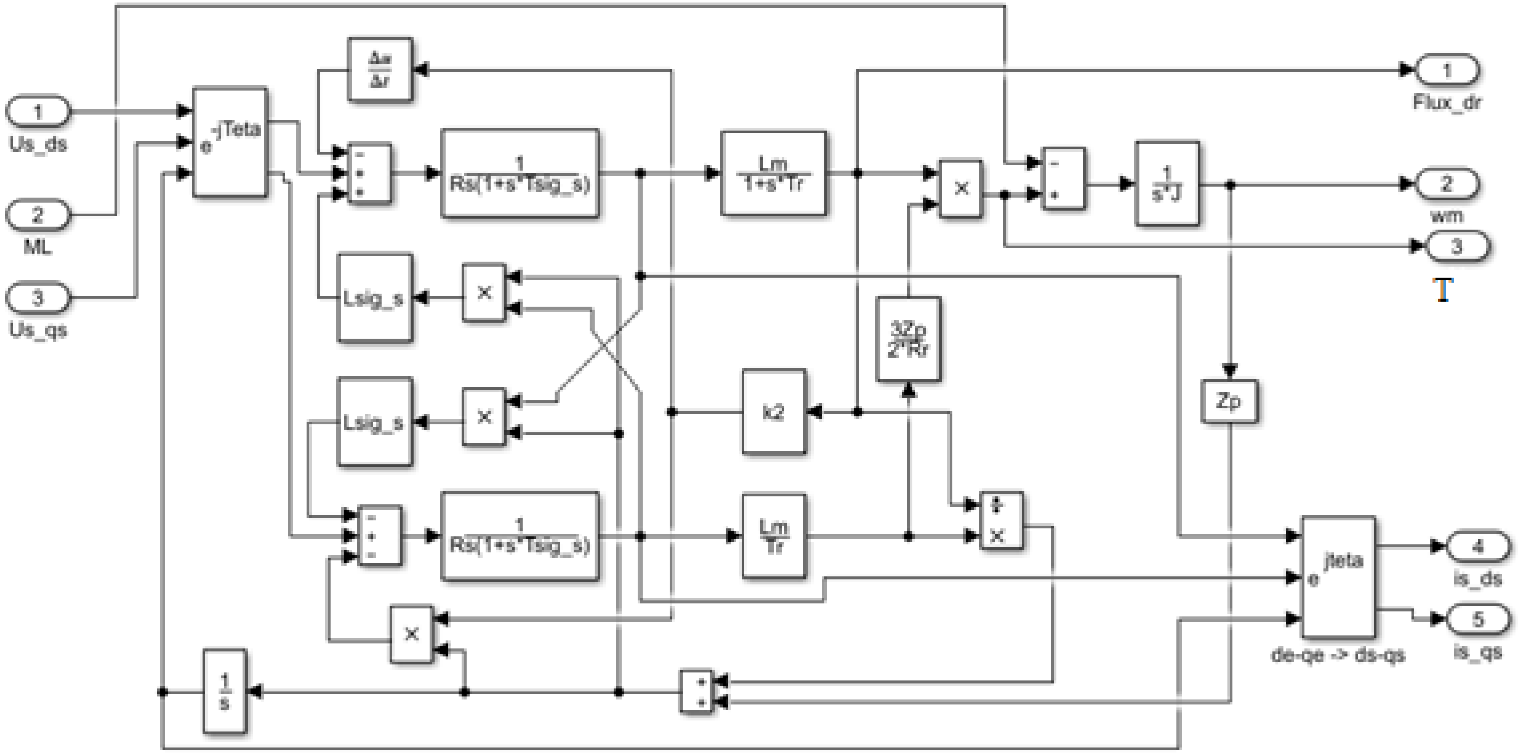
$$\omega_e - \omega_r = \omega_{\mathrm{sl}} = s\, \omega_e$$



Block diagram of Induction motor (Induction Motor Block) in synchronous reference frame

Model of IM in synchronous reference frame 2

Simulink math model Induction motor in synchronous reference frame



$$i_{ds}^{e} = \frac{1}{R_s(1+sT_{\sigma s})}\left(v_{ds}^{e} + \omega_e L_{\sigma s} i_{qs}^{e} - s\lambda_{dr}^{e} k_2\right)$$

$$i_{qs}^{e} = \frac{1}{R_s(1+sT_{\sigma s})}\left(v_{qs}^{e} - \omega_e L_{\sigma s} i_{ds}^{e} - \lambda_{dr}^{e} k_2\, \omega_e\right)$$

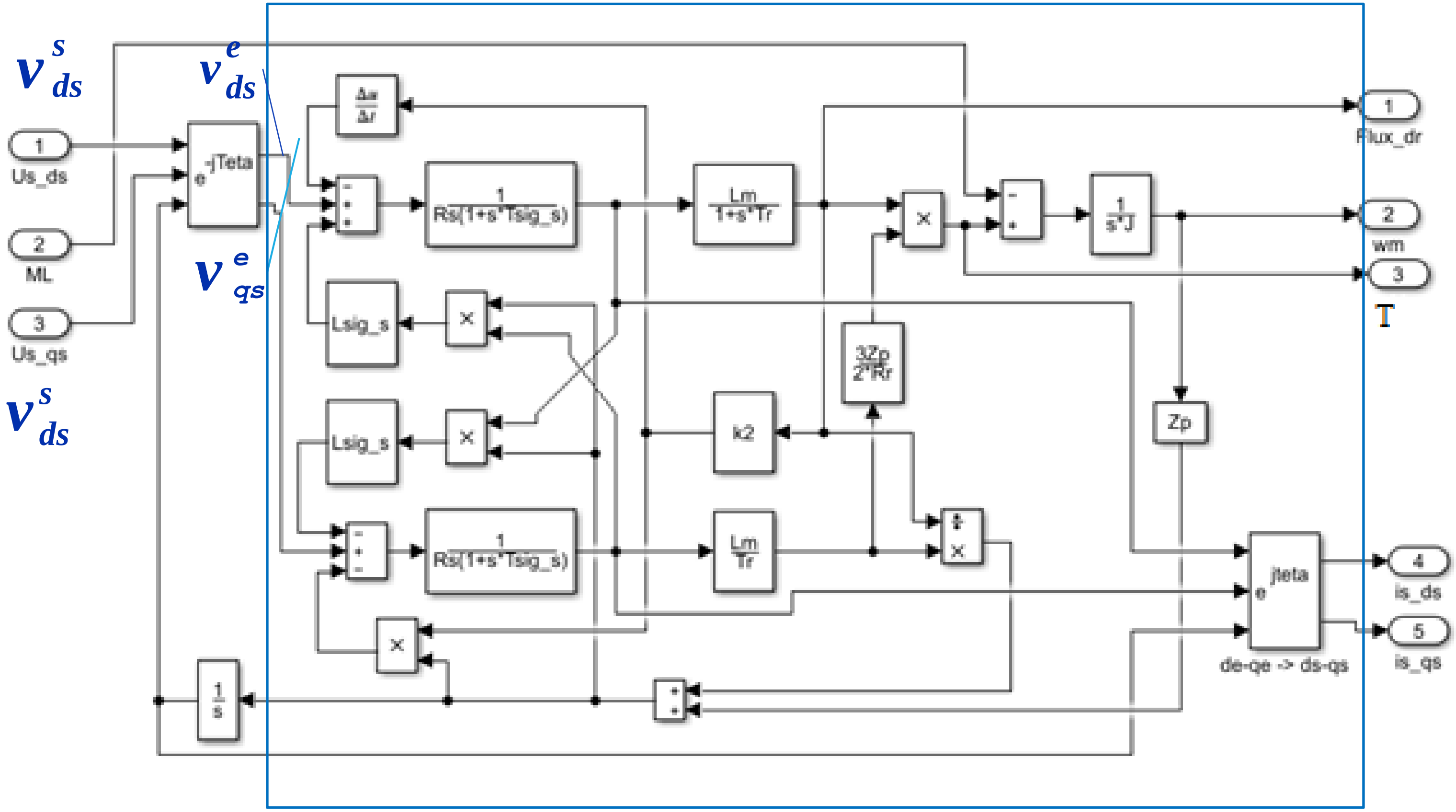
$$i_{ds}^{e} = \frac{\lambda_{dr}^{e}}{L_m}\, \frac{(1+sT_r)}{L_m}$$

$$i_{qs}^{e} = \frac{\lambda_{dr}^{e}}{L_m}\, \omega_{\mathrm{sl}}\, T_r$$

$$T = \frac{3}{2} z_p \frac{L_m}{T_r R_r} \lambda_{dr}^{e} i_{qs}^{e} \qquad z_p = \frac{P}{2}$$

Model of IM in synchronous reference frame 2

Simulink math model Induction motor in synchronous reference frame



$$i_{ds}^{e} = \frac{1}{R_s(1+sT_{\sigma s})}\left(v_{ds}^{e} + \omega_e L_{\sigma s} i_{qs}^{e} - s\lambda_{dr}^{e} k_2\right)$$

$$i_{qs}^{e} = \frac{1}{R_s(1+sT_{\sigma s})}\left(v_{qs}^{e} - \omega_e L_{\sigma s} i_{ds}^{e} - \lambda_{dr}^{e} k_2\, \omega_e\right)$$

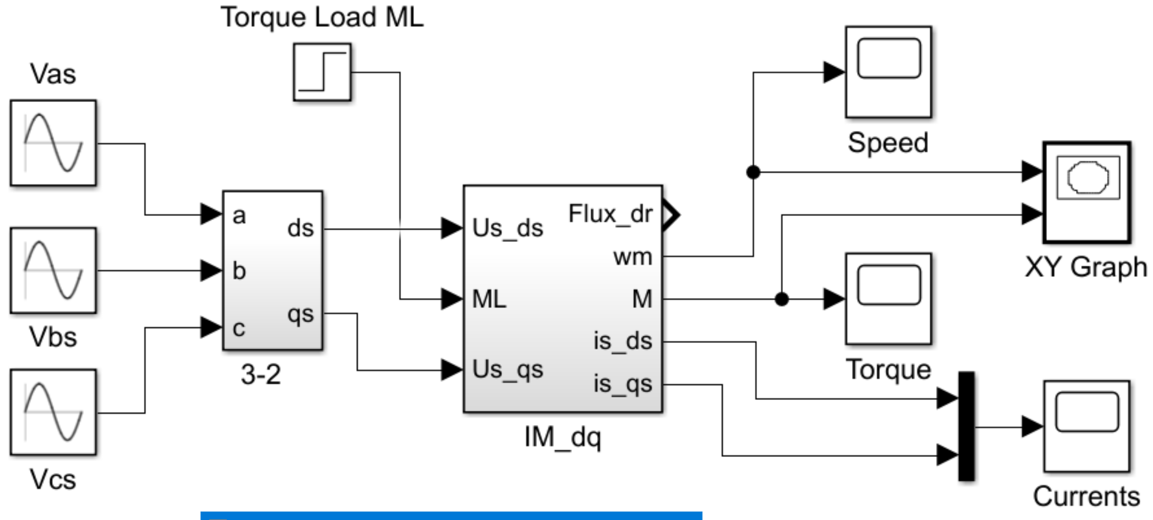
$$i_{ds}^{e} = \frac{\lambda_{dr}^{e}}{L_m}\,\frac{(1+sT_r)}{L_m}$$

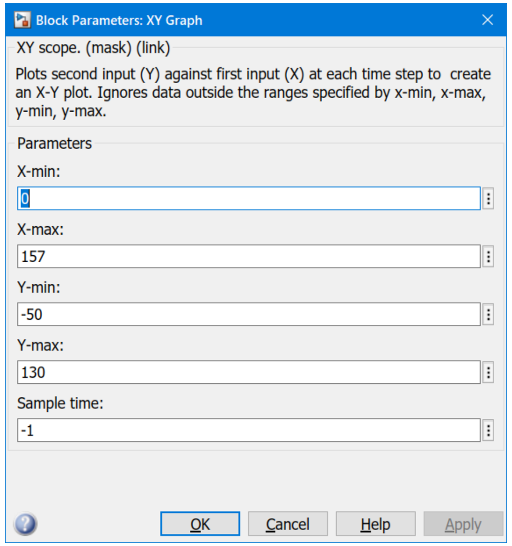
$$i_{qs}^{e} = \frac{\lambda_{dr}^{e}}{L_m}\,\omega_{\text{sl}}\, T_r$$

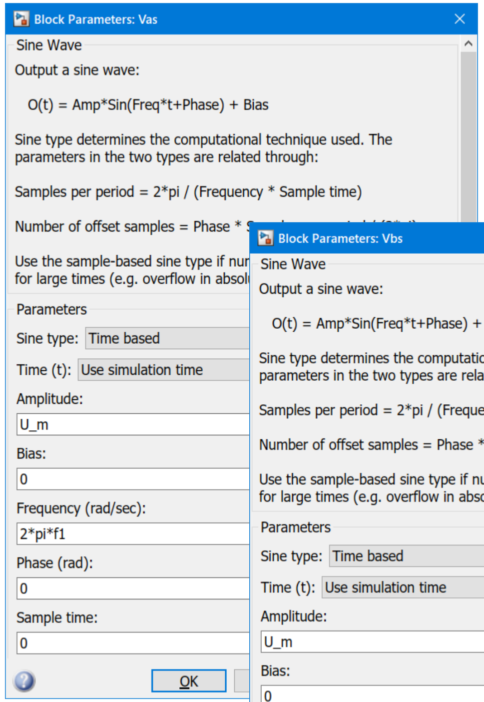
$$T = \frac{3}{2} z_p \frac{L_m}{T_r R_r} \lambda_{dr}^{e} i_{qs}^{e} \qquad z_p = \frac{P}{2}$$

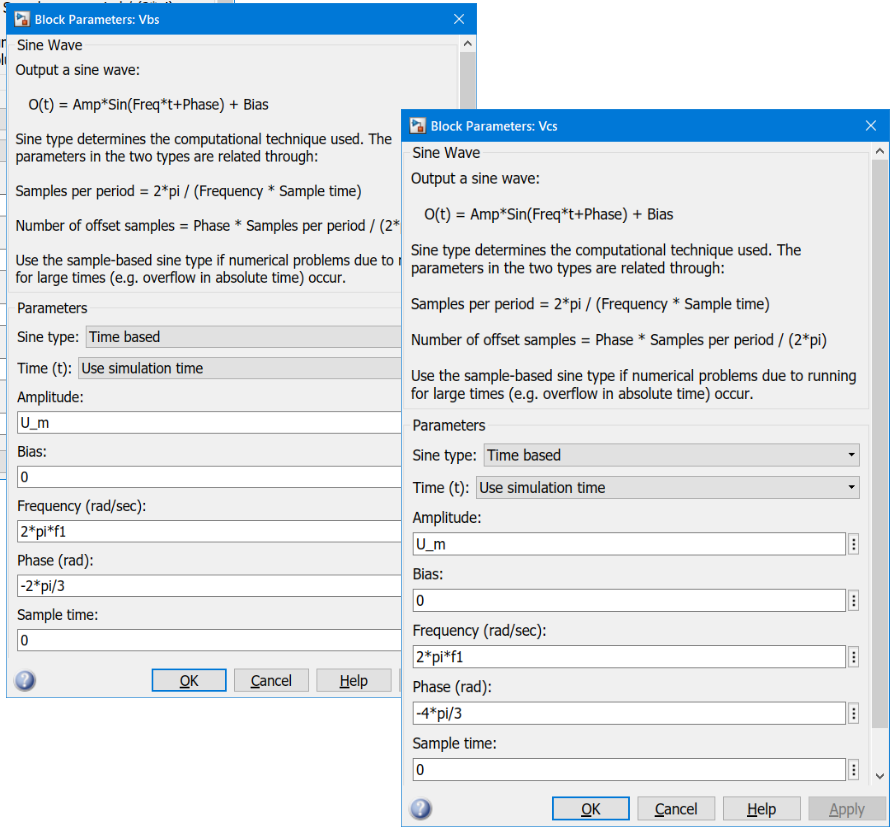
- Induction motor math model in synchronous reference frame with input signals of voltage

Model of IM in synchronous reference frame 2









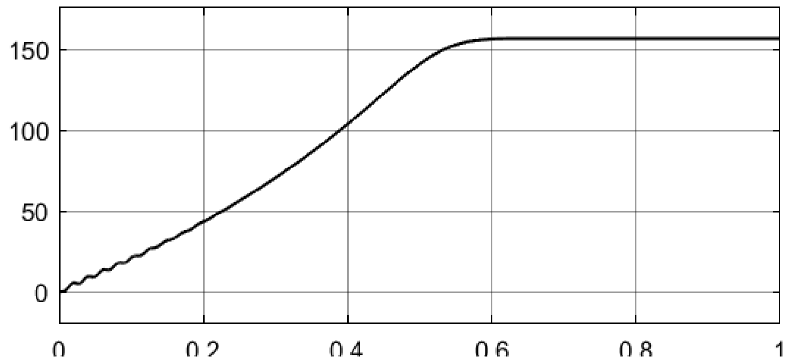
$$U_m = \sqrt{2} * 220$$

$$f_1 = f_s = 50$$

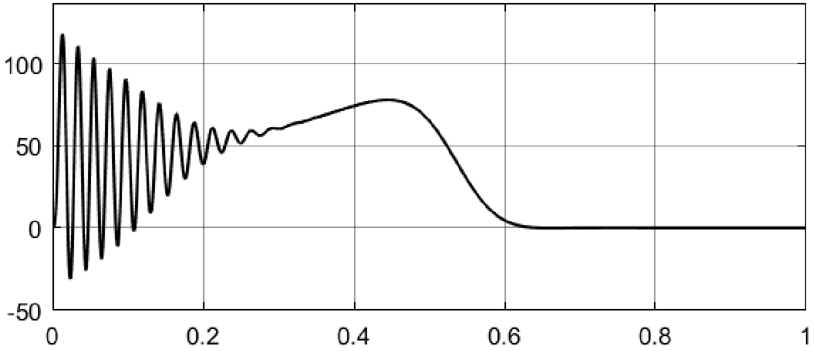
Model of IM in synchronous reference frame 2

The same results as in simulation IM with stationary reference frame!

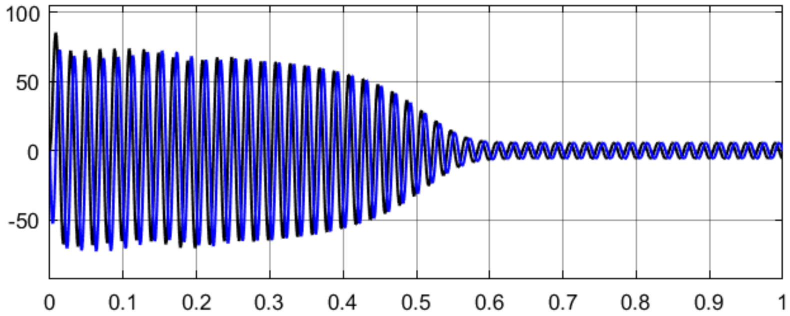
Speed [rad/s]



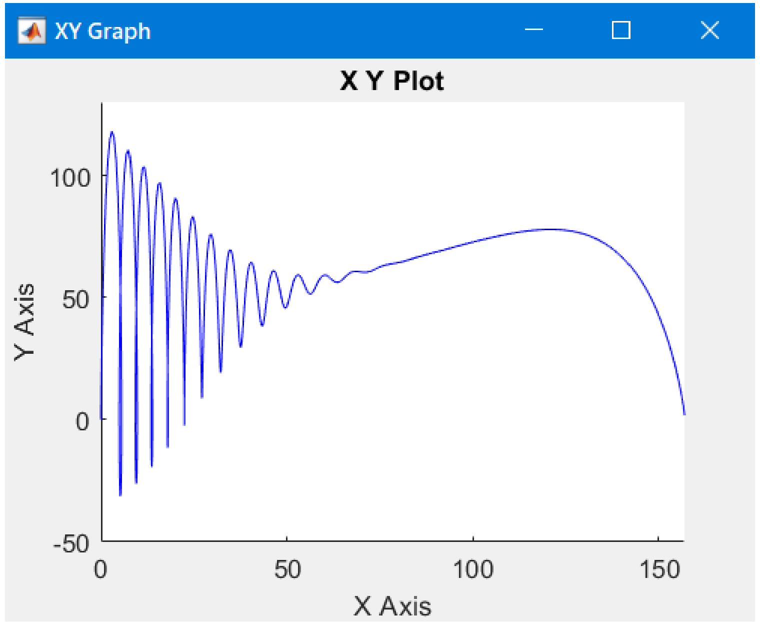
Torque [N*m]



Currents [A]

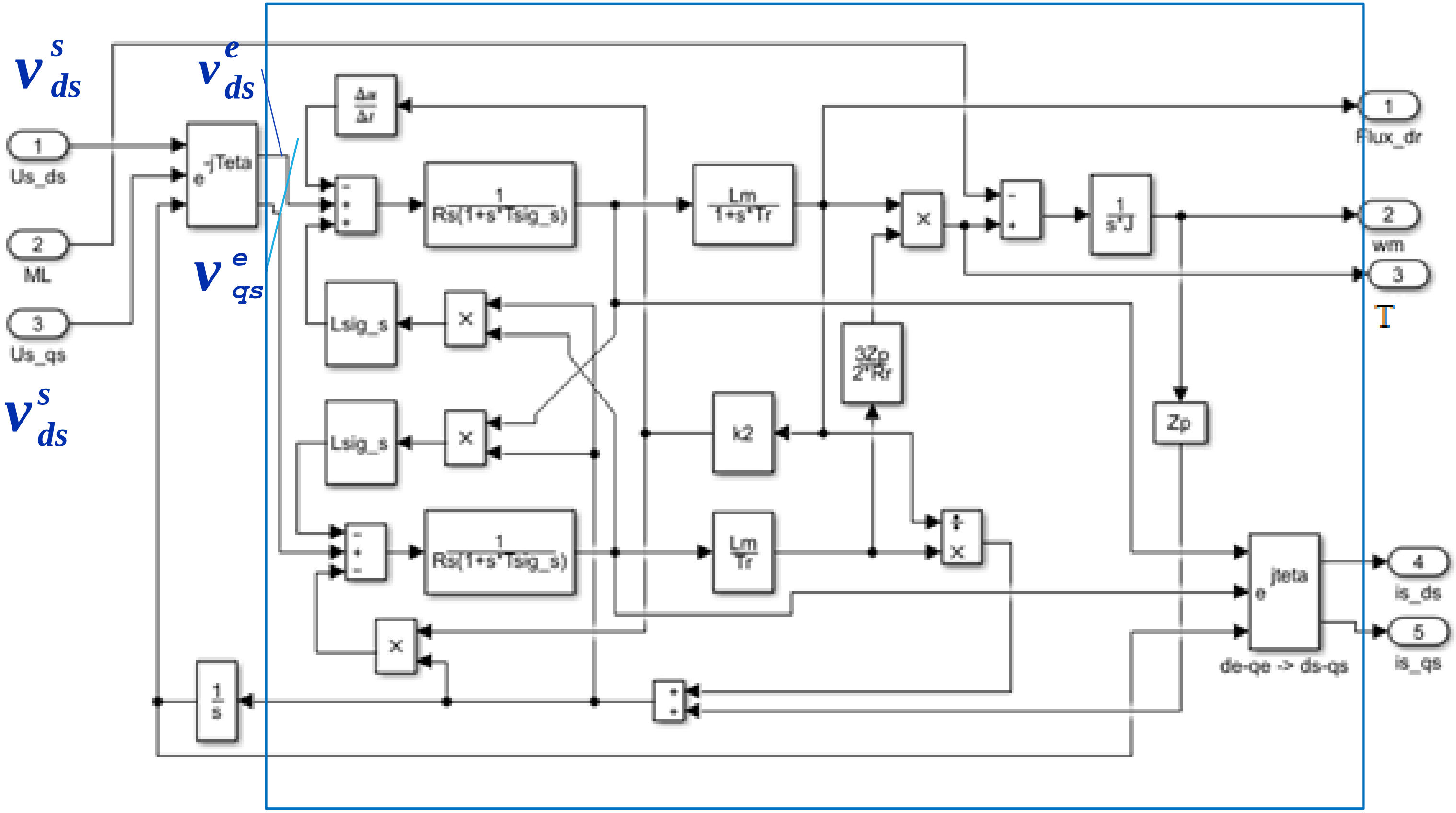


Torque-speed characteristic



Model of IM in synchronous reference frame 2

Simulink math model Induction motor in synchronous reference frame



$$i^e_{ds} = \frac{1}{R_s(1+sT_{\sigma s})}\left(v^e_{ds} + \omega_e L_{\sigma s} i^e_{qs} - s\lambda^e_{dr} k_2\right)$$

$$i^e_{qs} = \frac{1}{R_s(1+sT_{\sigma s})}\left(v^e_{qs} - \omega_e L_{\sigma s} i^e_{ds} - \lambda^e_{dr} k_2\, \omega_e\right)$$

$$i^e_{ds} = \frac{\lambda^e_{dr}}{L_m}\frac{(1+sT_r)}{L_m}$$

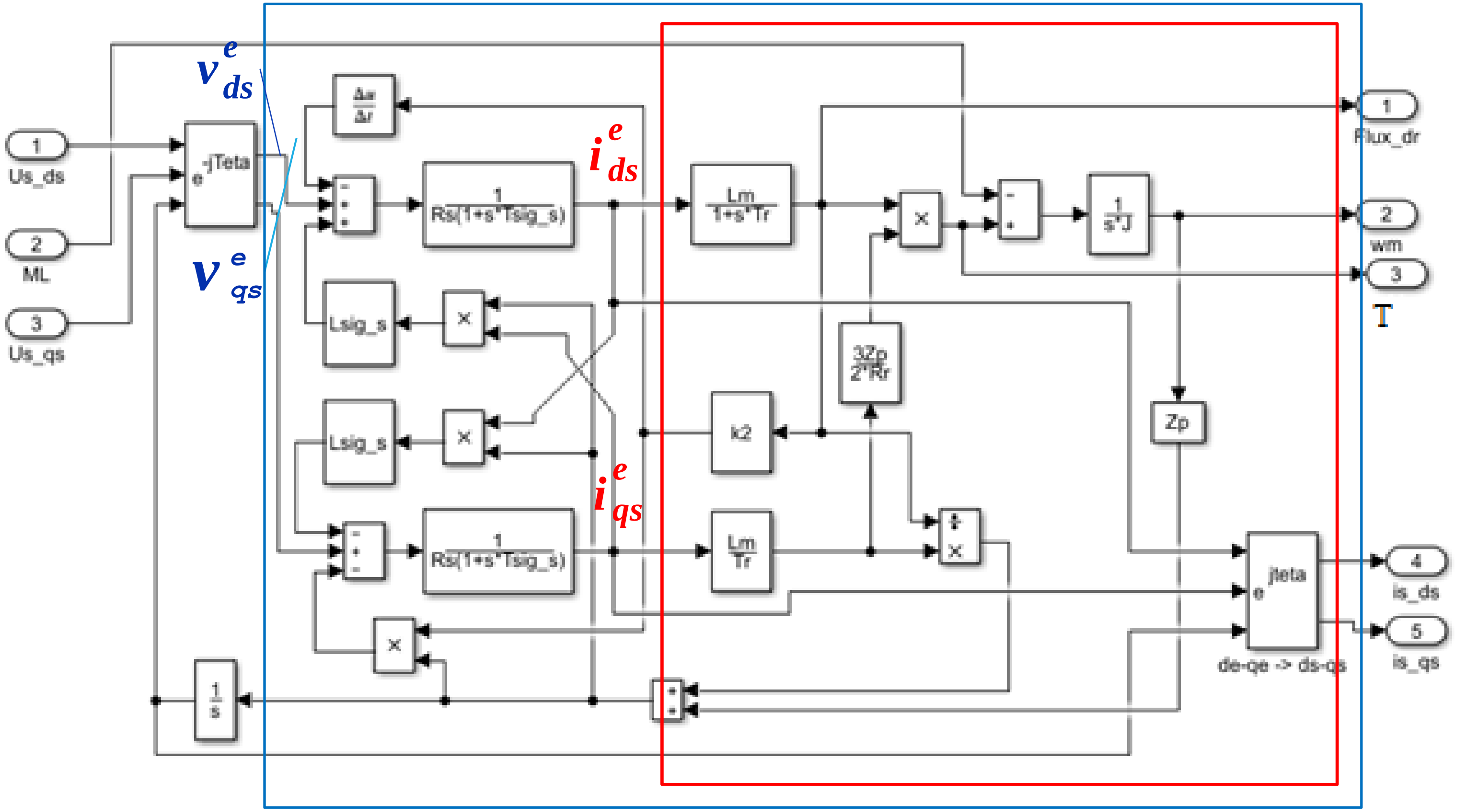
$$i^e_{qs} = \frac{\lambda^e_{dr}}{L_m}\omega_{\text{sl}}\, T_r$$

$$T = \frac{3}{2} z_p \frac{L_m}{T_r R_r}\lambda^e_{dr} i^e_{qs} \qquad z_p = \frac{P}{2}$$

- Induction motor math model in synchronous reference frame with input signals of voltage

Model of IM in synchronous reference frame 2

Simulink math model Induction motor in synchronous reference frame



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- Induction motor math model in synchronous reference frame with input signals of voltage
- Induction motor math model in synchronous reference frame with input signals of current

Model of IM in synchronous reference frame 2

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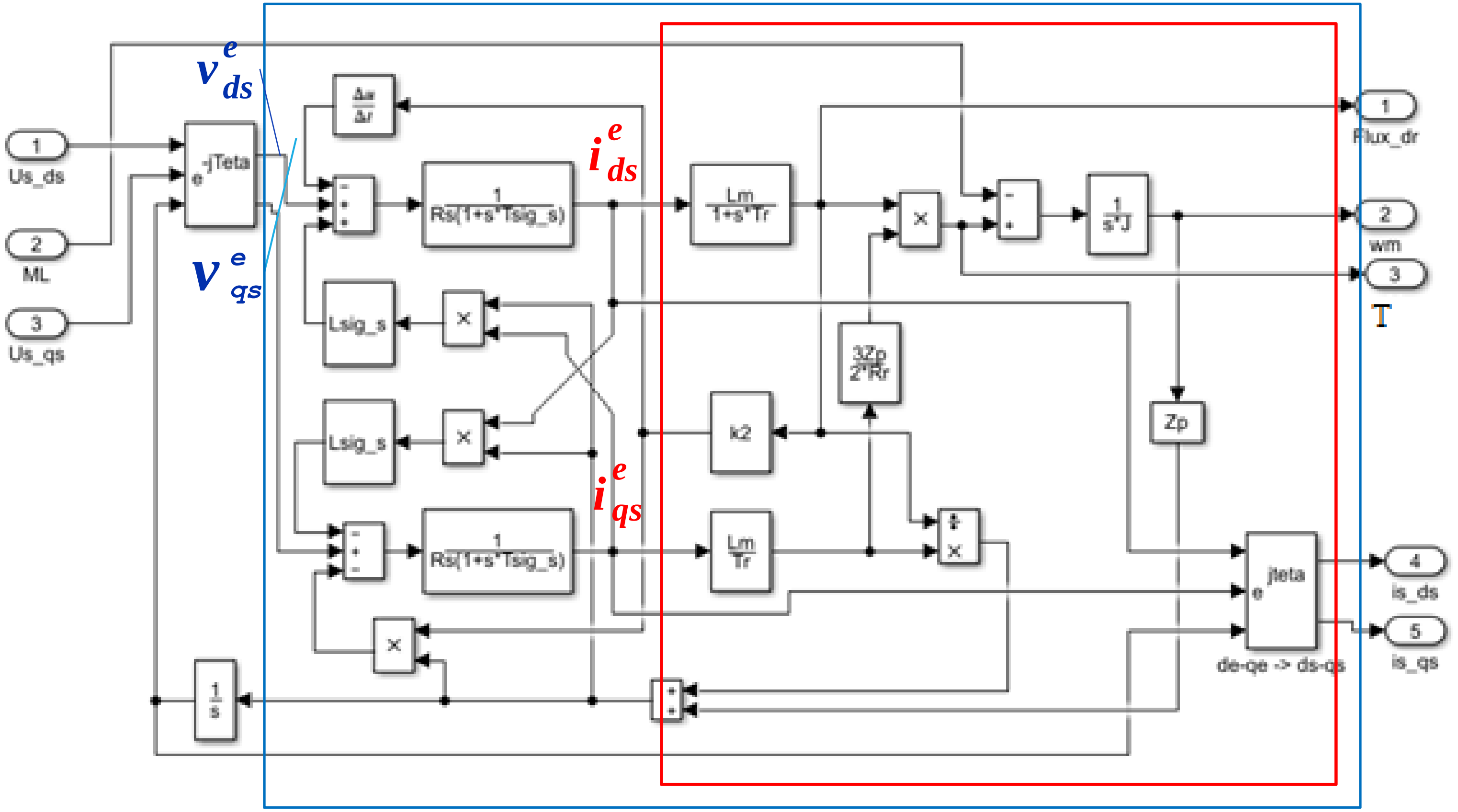

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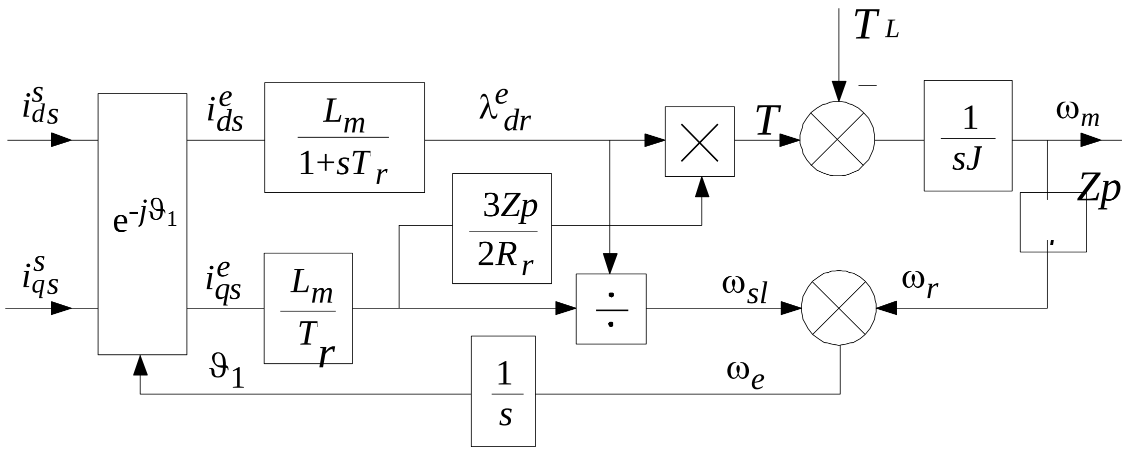
Currents of winding of IM can be formed directly by control system

- Induction motor math model in synchronous reference frame with input signals of voltage
- Induction motor math model in synchronous reference frame with input signals of current

Current model of IM in synchronous reference frame 2

$$\lambda_{dr}^{e} = \frac{L_m}{1+T_r s} i_{ds}^{e} \quad \left(T_r = \frac{L_r}{R_r}\right) \qquad \omega_{sl} = \frac{L_m R_r}{L_r} \frac{i_{qs}^{e}}{\lambda_{dr}^{e}}$$

$$T = \frac{3}{2} z_p \frac{L_m}{T_r R_r} \lambda_{dr}^{e} i_{qs}^{e} = \frac{3}{2} z_p \frac{\lambda_{dr}^{e2}}{R_r} \omega_{sl}$$



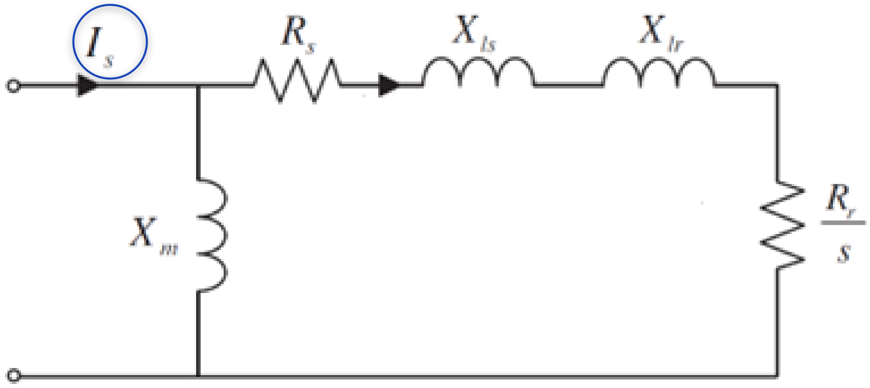


Figure Complete per phase equivalent circuit of a three-phase induction motor powered from a current source

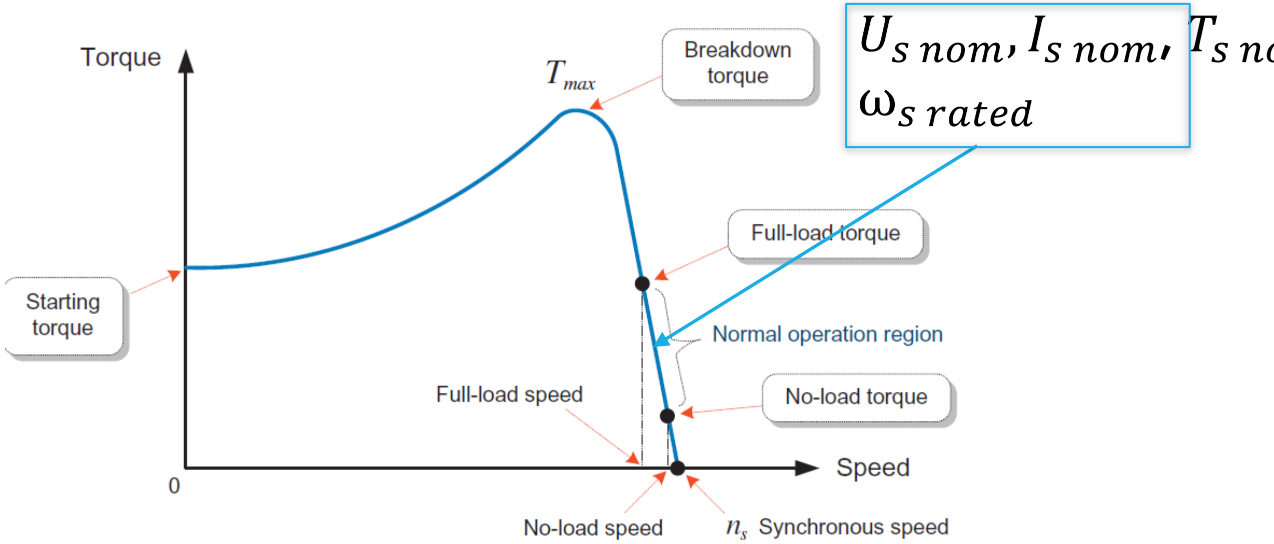


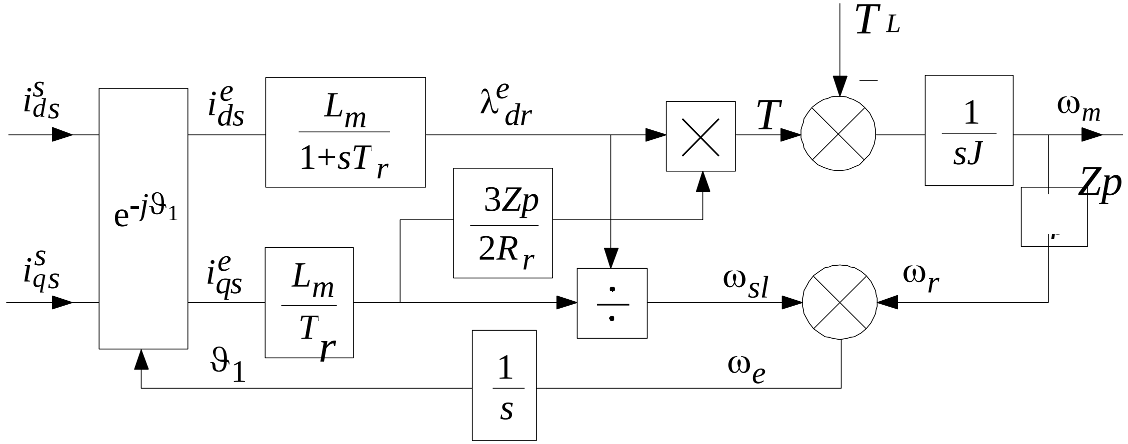
Figure. Speed versus torque curve for an induction motor powered from a voltage source

- Induction motor math model in synchronous reference frame with input signals of current

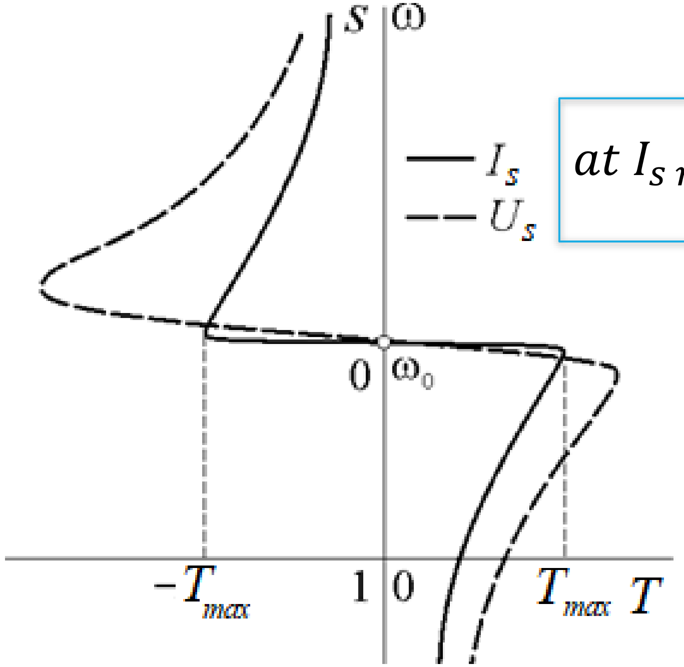
Current model of IM in synchronous reference frame 2

$$\lambda_{dr}^{e} = \frac{L_m}{1+T_r s} i_{ds}^{e} \quad \left(T_r = \frac{L_r}{R_r}\right) \qquad \omega_{sl} = \frac{L_m R_r}{L_r} \frac{i_{qs}^{e}}{\lambda_{dr}^{e}}$$

$$T = \frac{3}{2} z_p \frac{L_m}{T_r R_r} \lambda_{dr}^{e} i_{qs}^{e} = \frac{3}{2} z_p \frac{\lambda_{dr}^{e2}}{R_r} \omega_{sl}$$



- Induction motor math model in synchronous reference frame with input signals of current



at $I_{s\,nom}(I_{s\,rated})$

$$\frac{T_{maxU}}{T_{maxI}} = 3 \ldots 1 \quad at\ I_{nom}(I_{rated}) \qquad \frac{s_{maxU}}{s_{maxI}} = 3 \ldots 20$$

$$\frac{T_{maxI}}{T_{rated}} = 1{,}3 \ldots 4{,}5 \qquad > \qquad \frac{T_{maxU}}{T_{rated}} = 1{,}3 \ldots 3$$

Hysteresis current controller

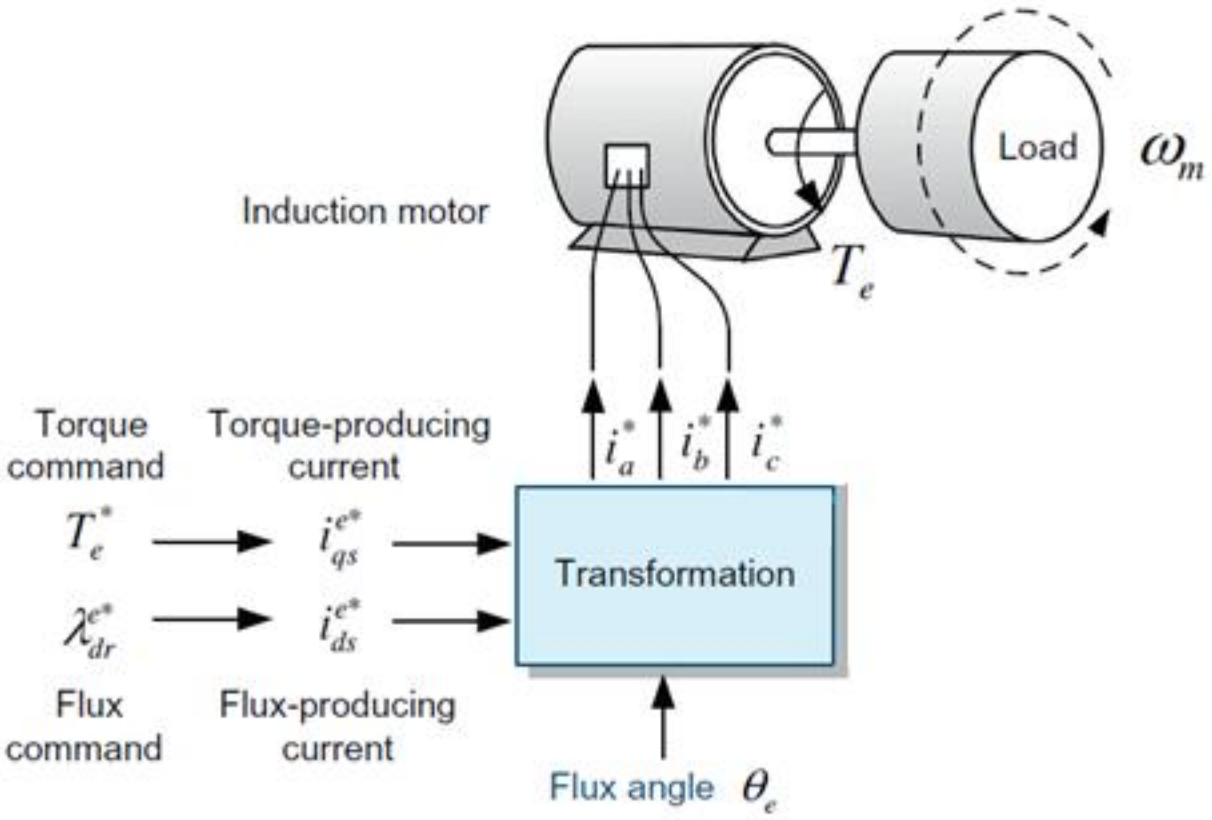


Figure Instantaneous torque control method of an induction motor

Hysteresis current controller

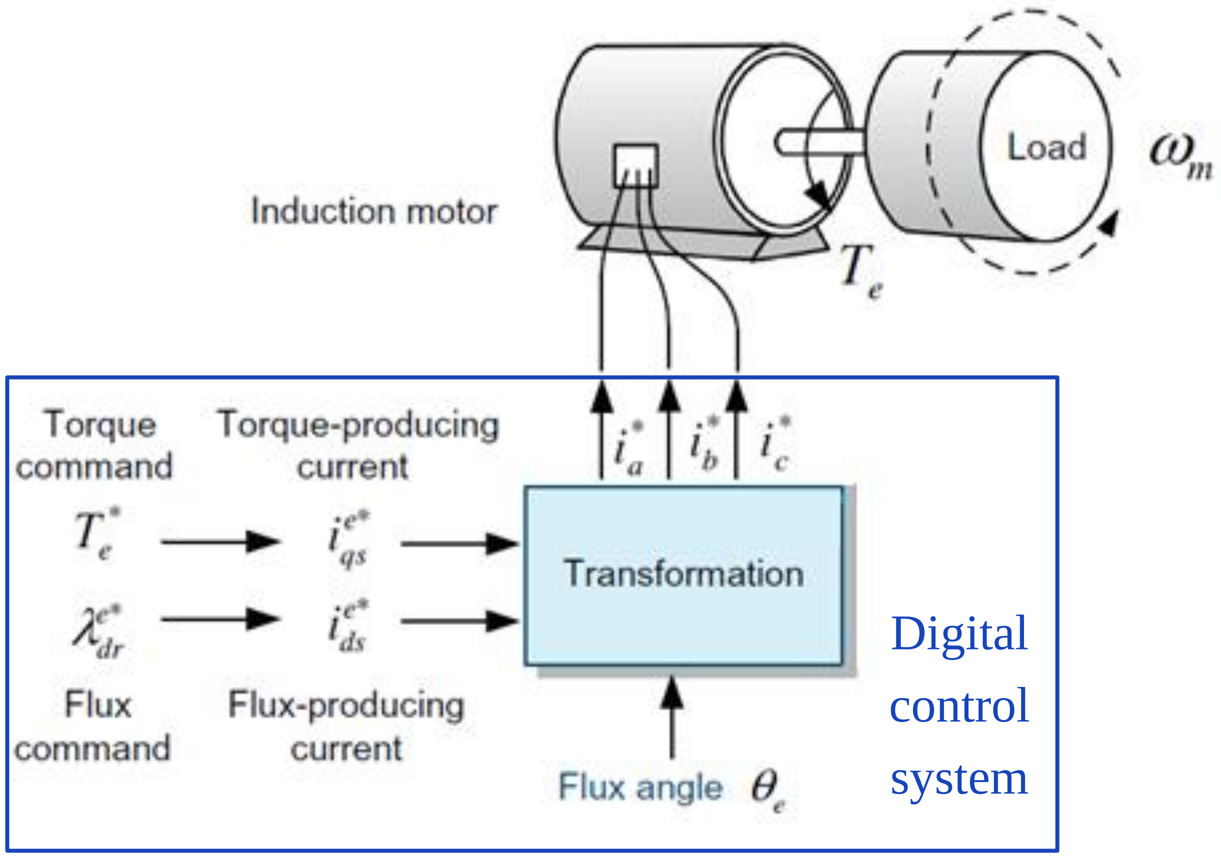


Figure Instantaneous torque control method of an induction motor

Hysteresis current controller

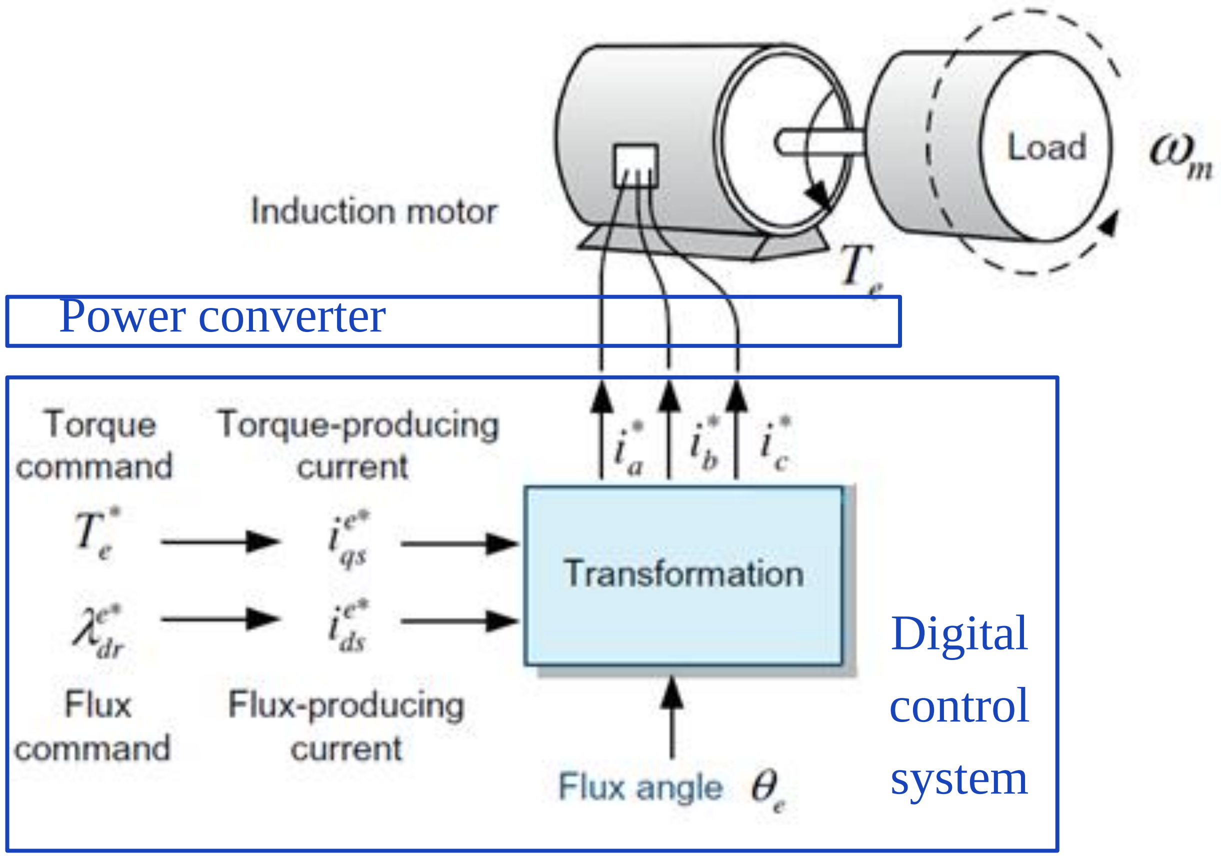


Figure Instantaneous torque control method of an induction motor

Control system of an induction motor

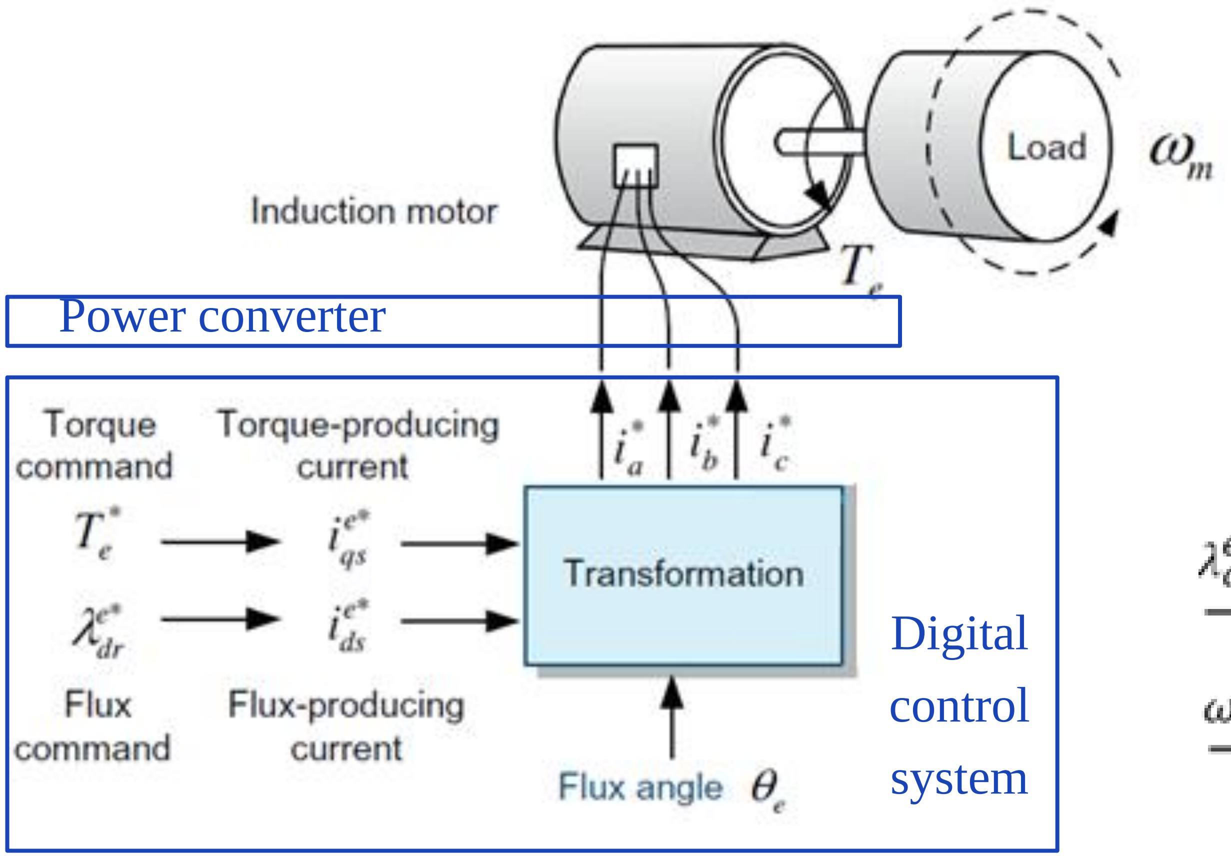


Figure Control system of an induction motor

Thus currents of winding of IM are formed by control system

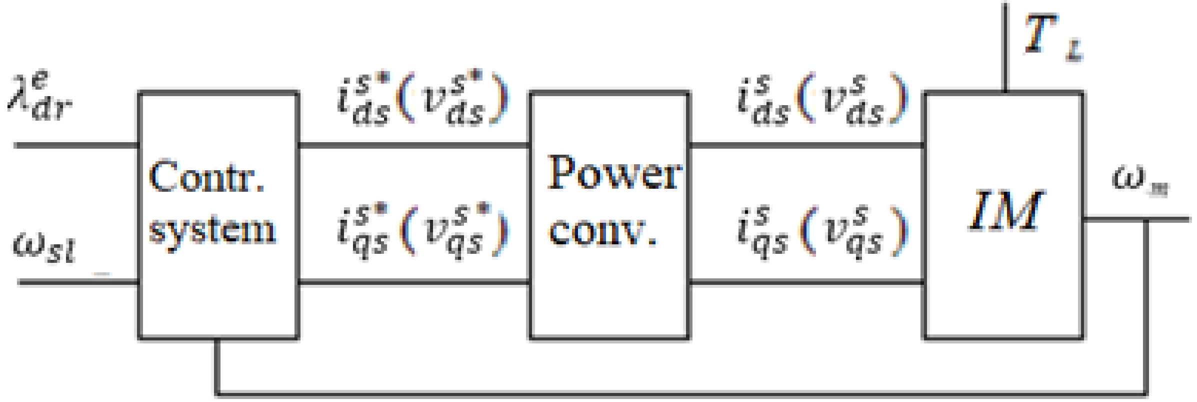


Figure Control system of an induction motor

Hysteresis current regulators as elements of AC motor control system

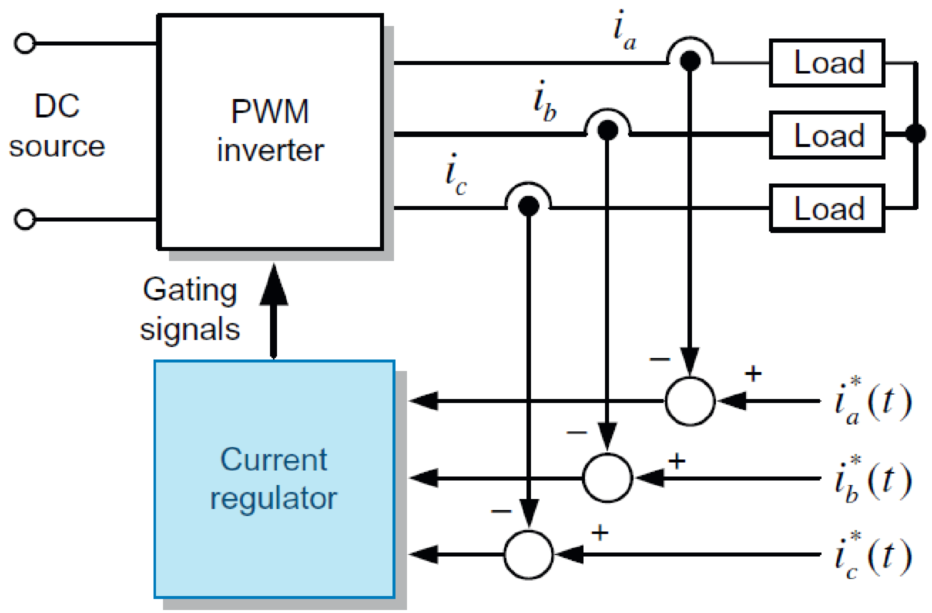


Figure Current control of a three-phase load

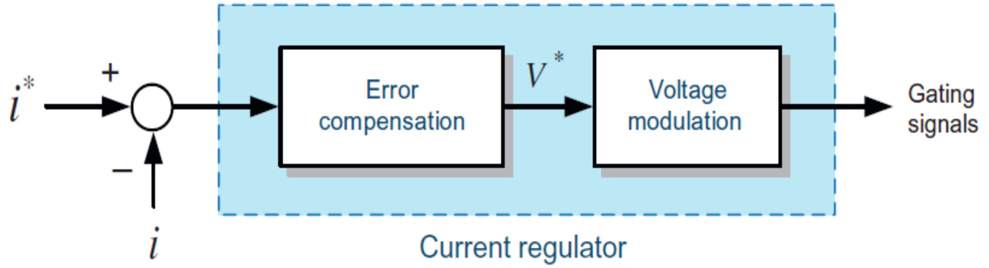


Figure Configuration of a current regulator

The current regulator plays a role in generating gating signals for the switching devices of a pulse width modulation (PWM) inverter

A current regulator consists of an error compensation part and a voltage modulation part

Hysteresis current regulators as elements of AC motor control system

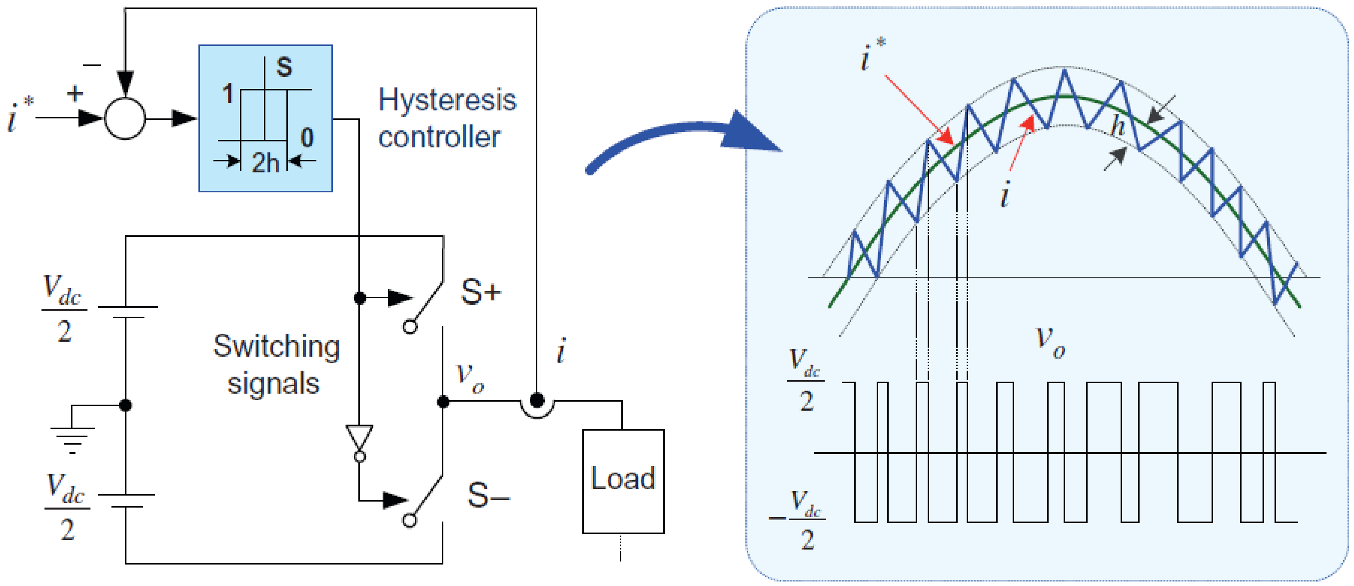


Figure Operation principle of a hysteresis regulator

h - hysteresis band

- $i^* - i \leq -h$: lower switch S- is turned on to decrease the load current by producing a negative voltage $(-\frac{1}{2}V_{dc})$
- $i^* - i \geq h$: lower switch S+ is turned on to decrease the load current by producing a negative voltage $(\frac{1}{2}V_{dc})$

Hysteresis current regulators have been widely used for small systems because of their simple implementation and an excellent dynamic performance.

A power converter with the hysteresis regulators can be considered a non-inertia element with the gain is equal one, if the number of switching for the period of the current is at least 20-30, and the hysteresis band 2h is not more than 5-7% of the maximum current value

Thank you for your attention