Actuators

Report for

Lab#1 Modelling of mechanics of the actuators

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HDU-ITMO Joint Institute
2025

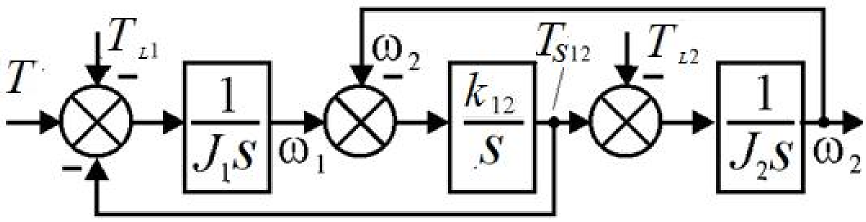
- ✓ LAB#1 is aimed at checking the theoretical data and relationships presented in theory materials (Lectures#2 of course “Actuators”) when considering dynamic processes in the mechanical part of an electric drive (*as well as in a motor, as well as in a two-mass motor-engine system – additional option*)
- ✓ LAB#1 is performed in MATLAB / Simulink
- ✓ LAB#1 consists two parts:

Table 1 –The data for the LAB#1

Var. No	V_a	R_a	T_a	$k\Phi f$	T_{rated}	J_1	J_2	k_{12}	Δφ
	V	Ohm	s			kgm^2	kgm^2	Nm/rad	rad
	Input armature voltage	armature resistance	Electromagnetic time constant armature	EMF/torque constant of DC-motor	Rated value of DC-motor torque	Moment of inertia of the 1st mass	Moment of inertia of the 2nd mass	Stiffness coeff.	Value of blacklash
Example	400	100	3	8	35	0.125	0.063	1675	0,39

Part 1. Mathematical modelling of two-mass mechanism

Task 1.1. Research processes in a model of the two-mass mechanism without any disturbances (load torques, friction torques) (*you have done it with A.Mamatov in “ElMechSystDynamic”*)



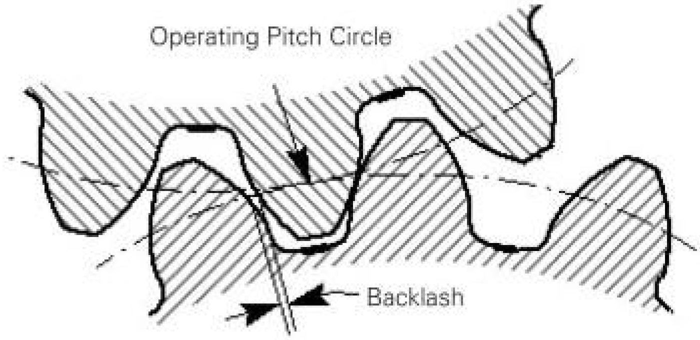
1. Design a model of the two-mass mechanism.

2. Show on plot transient response of $\omega_1(t)$, $\omega_2(t)$, $T_{s12}(t)$ by the step reference signal T with value $0.1T_{rated}$ (at T_{L1}=0, T_{L2}=0). Please display $\omega_1(t)$, $\omega_2(t)$ on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration. Display $T_{s12}(t)$ on the plot.

3. Display the Bode diagram of the two-mass mechanism and determine resonance frequency.

4. Compare calculated parameters of transient and parameters got by simulation

Task 1.2. Research the effect of backlash in a model of the two-mass mechanism



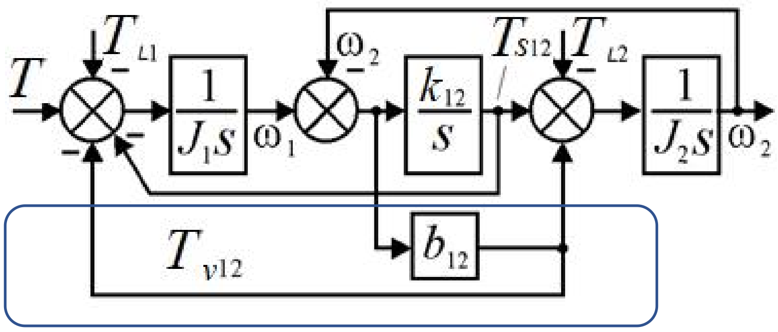
1. Add backlash in a model of the two-mass mechanism

2. Show on plot transient response of $\omega_1(t)$, $\omega_2(t)$, $T_{s12}(t)$ by the step reference signal T with value $0.1T_{rated}$ (at $T_{L1}=0$, $T_{L2}=0$). Please display $\omega_1(t)$, $\omega_2(t)$ on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration but with deadzones

2. Compare $T_{s12}(t)$ in mechanism without and with backlash in gearbox

3. Draw conclusions

Task 1.3. Research the effect of viscous friction torque in a model of the two-mass mechanism



1. Add torque of viscous friction in a model of the two-mass mechanism
2. The viscous damping coefficient b should be chosen
considering that the oscillation damp in 5 periods.
3. Get results. Draw conclusions

Part2. Mathematical modelling of DC-motor with two-mass mechanism (*not necessary – this is additional option)*

Task 2.1 Modelling of the DC-motor with two-mass mechanism
1. Design a model of the DC-motor with two-mass mechanism.
2. Show plots $T(t)$, $T_{s12}(t)$, $\omega_1(t)$, $\omega_2(t)$

Part 1. Mathematical modelling of two-mass mechanism

Task 1.1. Research processes in a model of the two-mass mechanism without any disturbances (load torques, frictions)

Mathematic model of two-mass mechanism without any disturbances (load torques, frictions) (1)

$$\left.\begin{aligned} T - k_{12}(\omega_1 - \omega_2)/s = J_1 s\omega_1; \\ k_{12}(\omega_1 - \omega_2)/s = J_2 s\omega_2. \end{aligned}\right\} \quad (1)$$

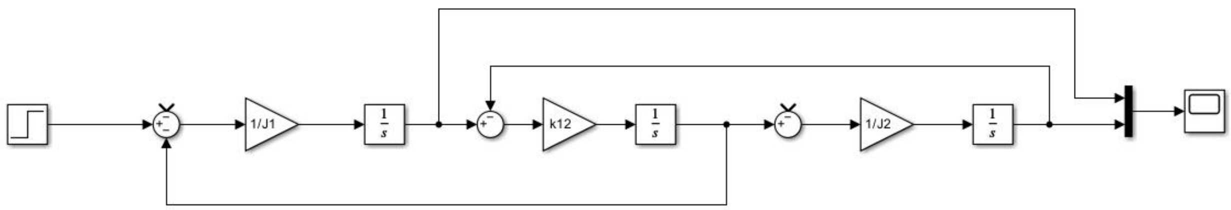


Figure 1 : Math model of the two-mass mechanism in Simulink

Please display $\omega_1(t)$, $\omega_2(t)$ on one plot and make sure that the speed of the first and second masses change in antiphase with the same value of acceleration.

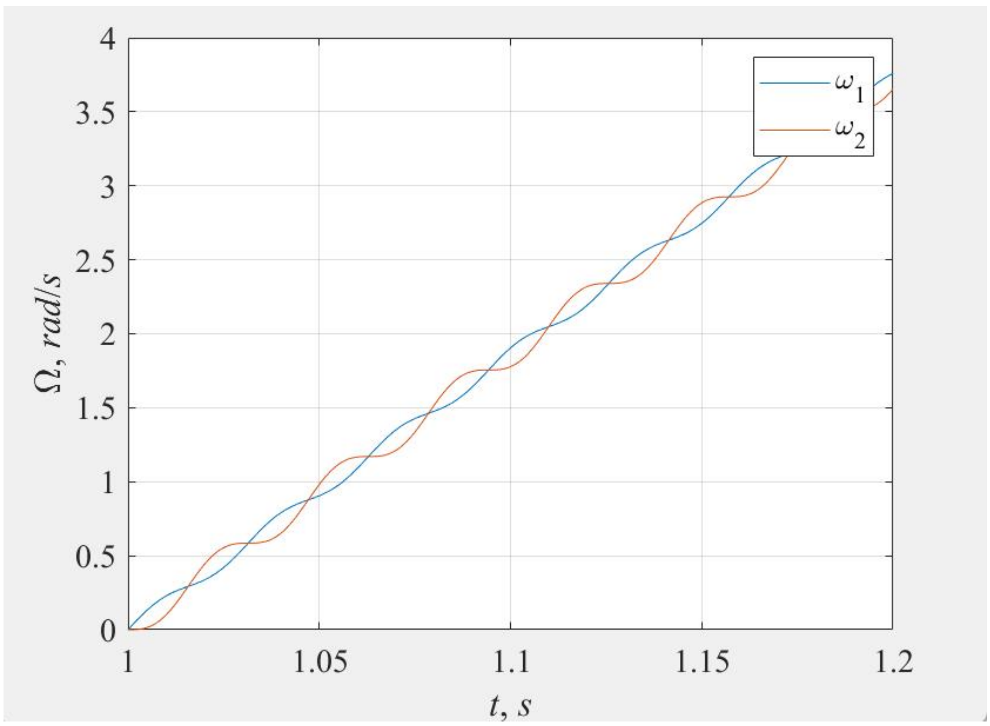


Figure 2: The plot of angular speed of the 1st and 2nd body versus time

Write expression for $\omega_1(t)$, $\omega_2(t)$

$$\omega_1(t) = \varepsilon_{av}t + \frac{\varepsilon_{av}}{\omega_{R1}}(\gamma - 1)\, sin\, \omega_{R1}\, t$$

$$\omega_2(t) = \varepsilon_{av}t - \frac{\varepsilon_{av}}{\omega_{R1}} sin\, \omega_{R1}\, t$$

Design the scheme for the checking amplitude values of harmonic component (to exclude the average angular acceleration from expressions $\omega_1(t)$, $\omega_2(t)$ above)

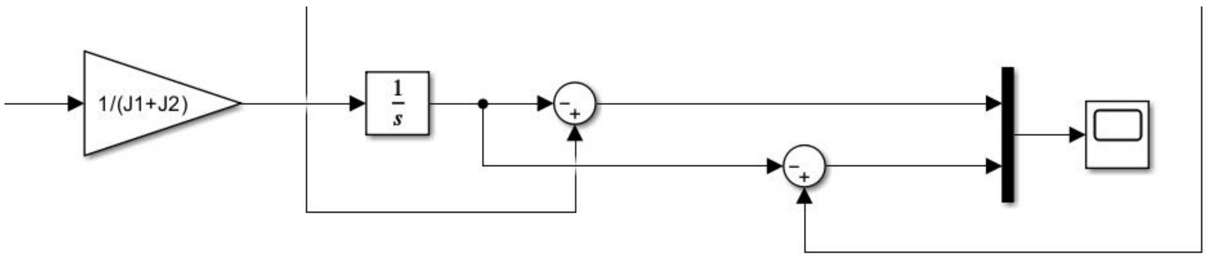


Figure : The math model for the checking amplitude values of harmonic component

The obtaining results of the simulation are presented in Fig. below

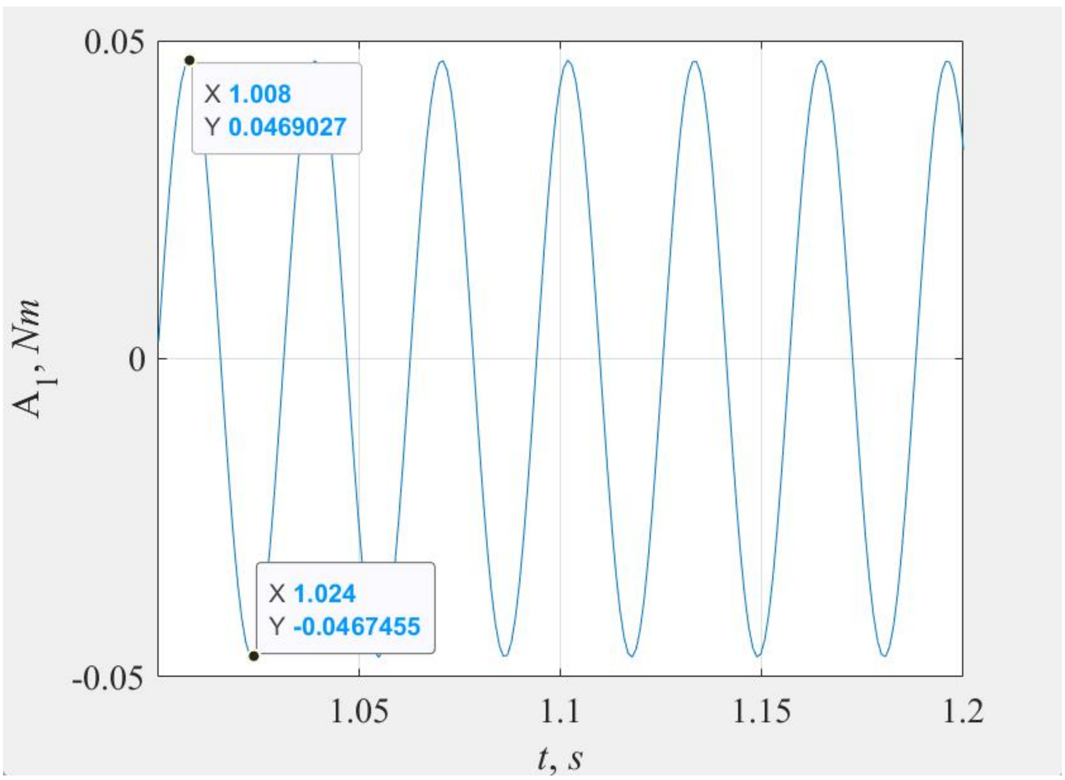


Figure: Obtaining of amplitude value of the 1st body oscillation

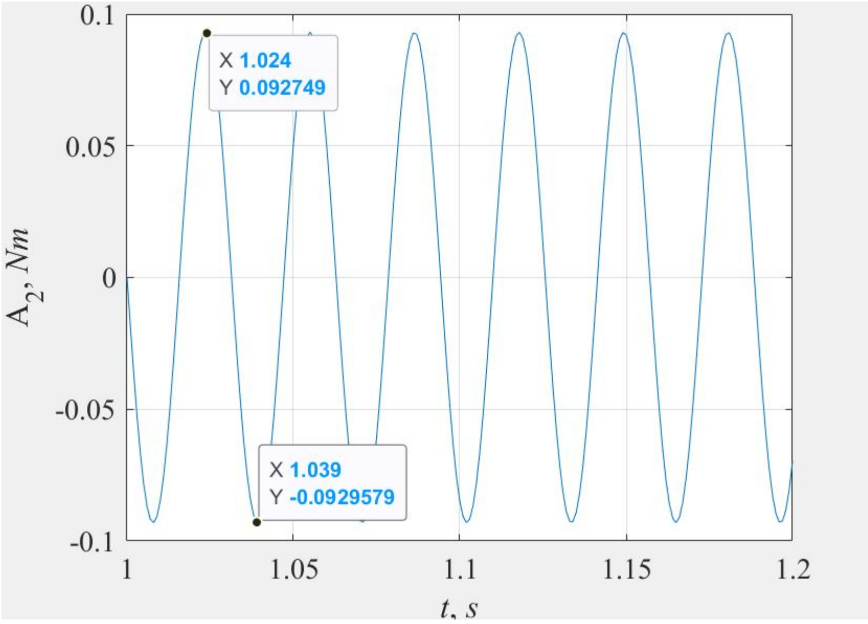


Figure: Obtaining of amplitude value of the 2nd body oscillation

Measure the magnitudes of bodies oscillations and compare with calculated parameters.

- The average angular acceleration:

$$\varepsilon_{av} = \frac{T}{J_1 + J_2} = \frac{0.1 \times T_{rated}}{J_1 + J_2} = 18.62 \text{ rad/s}^2$$

- The magnitudes of bodies fluctuation:

$$A_1 = \frac{J_2 \varepsilon_{av}}{J_1 \omega_{R1}} = 0.04692 \text{rad/s}$$

$$A_2 = \frac{\varepsilon_{av}}{\omega_{R1}} = 0.0931 \text{ rad/s}$$

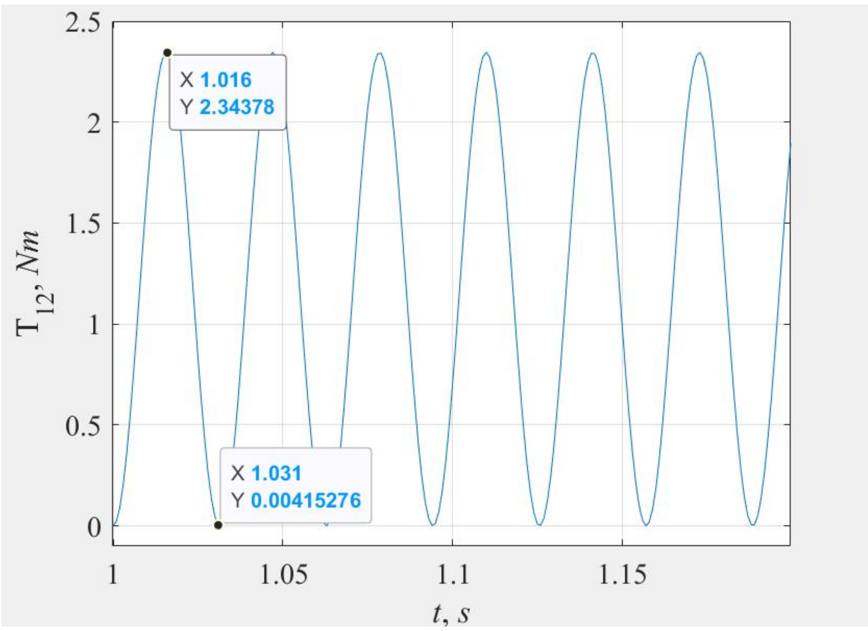


Figure: The plot of torque of elastic bonding forces between bodies versus time

Draw conclusions

Through simulation experiments, I verified the fundamental dynamic characteristics of the two-mass system under disturbance-free conditions. The angular velocities $\omega_1(t)$ and $\omega_2(t)$ of the two masses exhibited the expected anti-phase oscillations with identical acceleration ($\varepsilon_{av}=18.62\ rad/s^2$). The oscillation amplitudes of the elastic torque $T_{s12}(t)$ showed nearly perfect agreement between theoretical calculations ($A_1=0.04692\ rad/s,\ A_2=0.0931\ rad/s$) and experimental results ($A_1=0.0469027\ rad/s,\ A_2=0.092749\ rad/s$), confirming the accuracy of the mathematical model. These results demonstrate that the motion characteristics of an ideal two-mass system can be precisely described by simple second-order differential equations.

Bode diagram of the two-mass mechanism

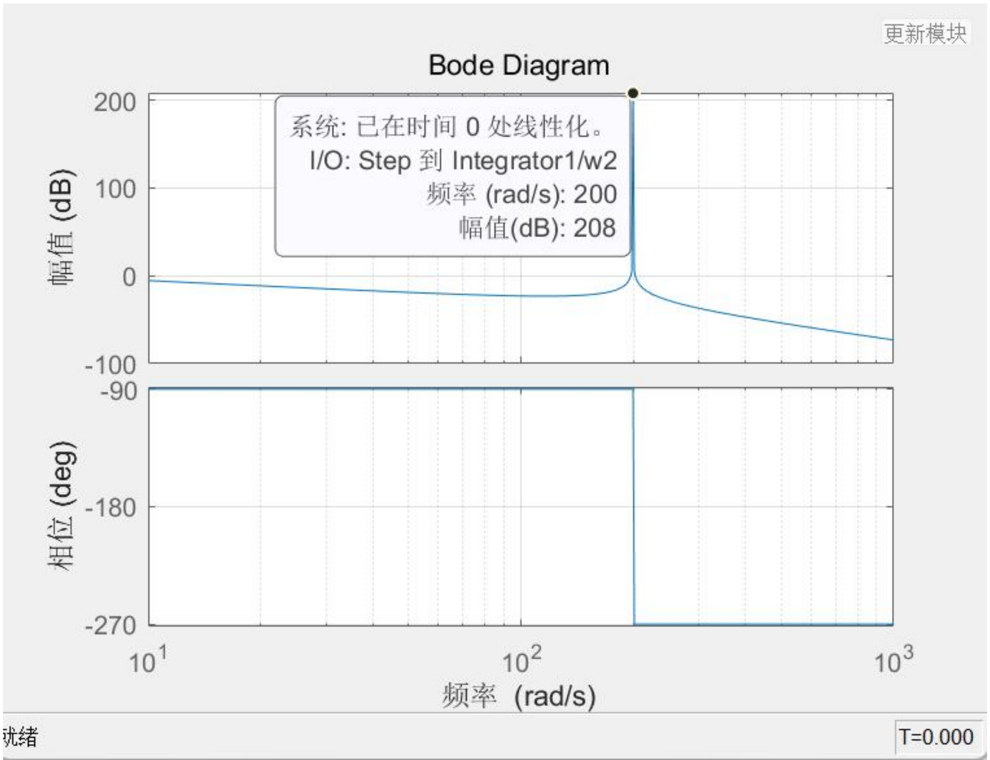


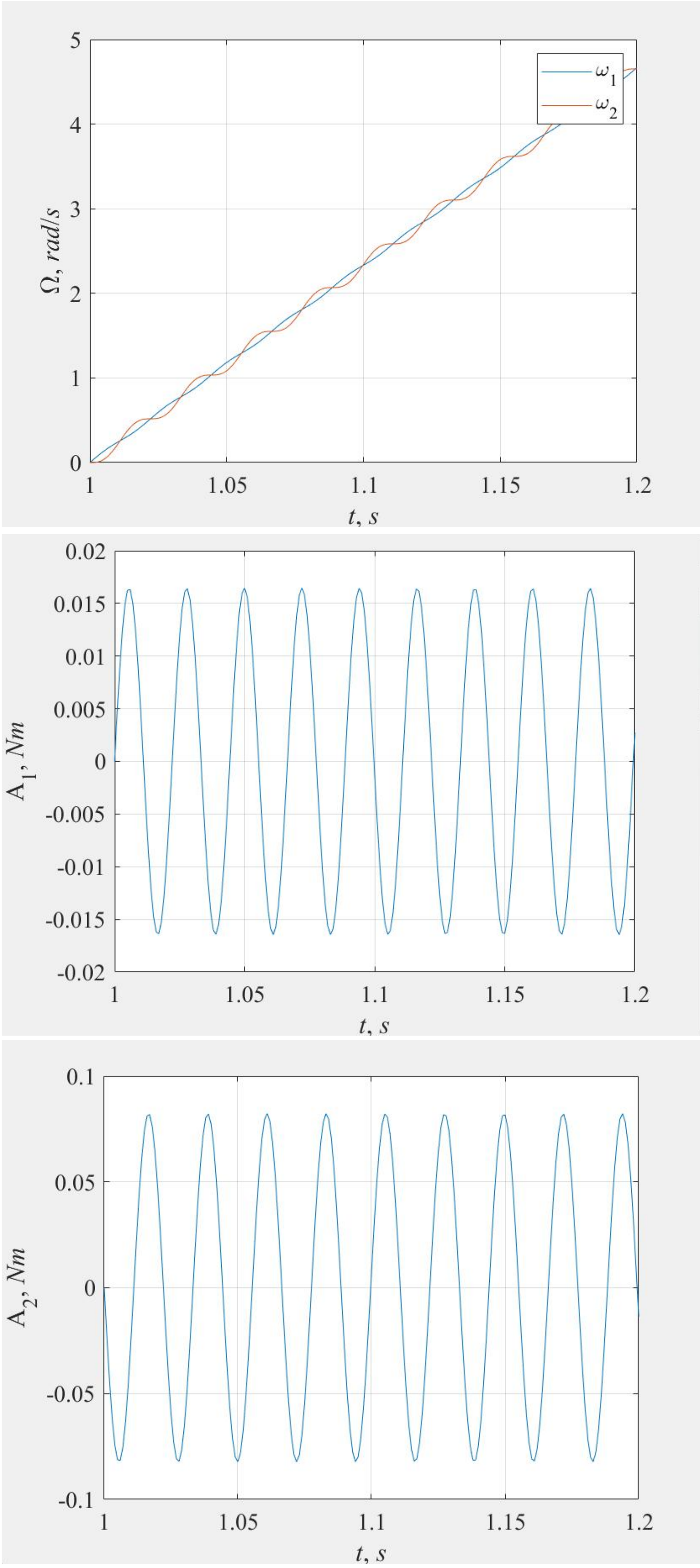
Figure 4: The Bode diagram

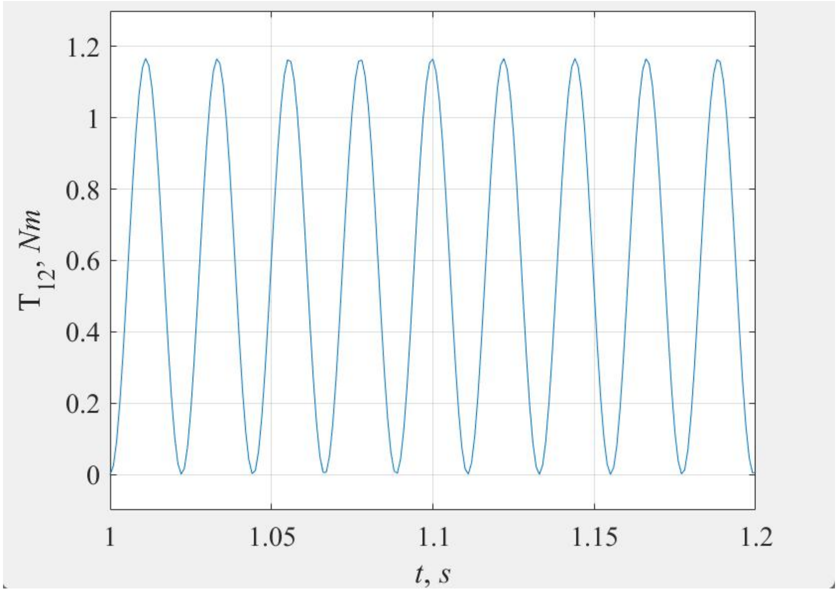
Show the resonance frequency on this diagram and compare with calculated parameters

Show plots $T_{s12}(t)$, $\omega_1(t)$, $\omega_2(t)$ when:

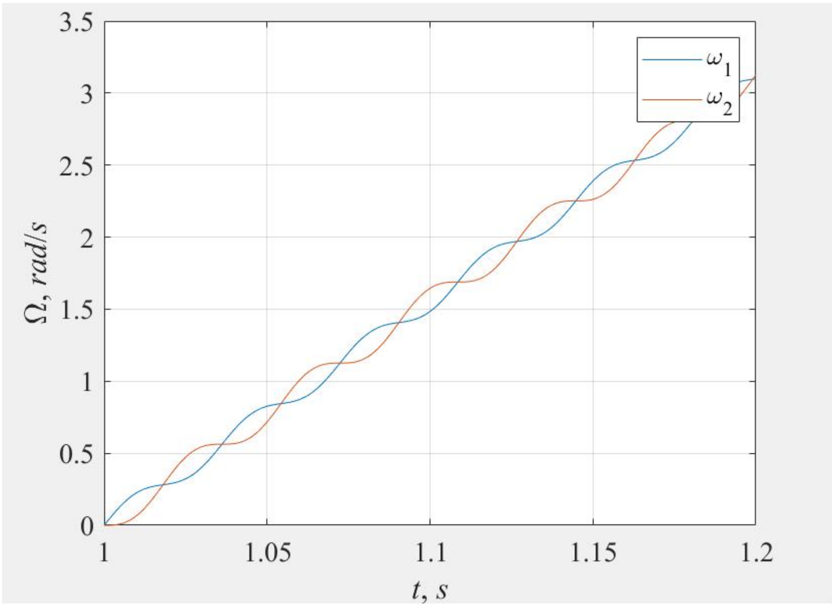
- *varying mass ratio γ (three meaning to get different transients)*
- *varying stiffness k_{12} (three meaning to get different transients)*

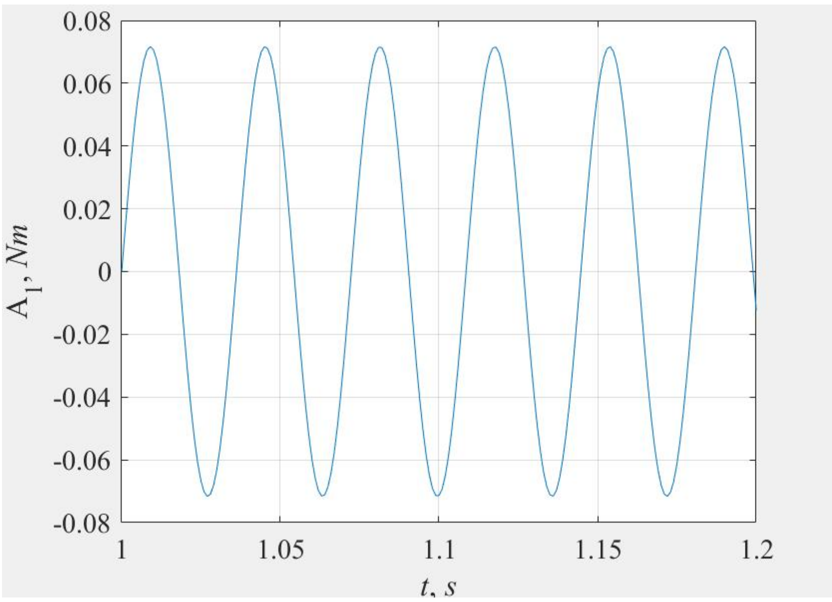
$$when\ \gamma = 1.2, J_2 = 0.2J_1$$

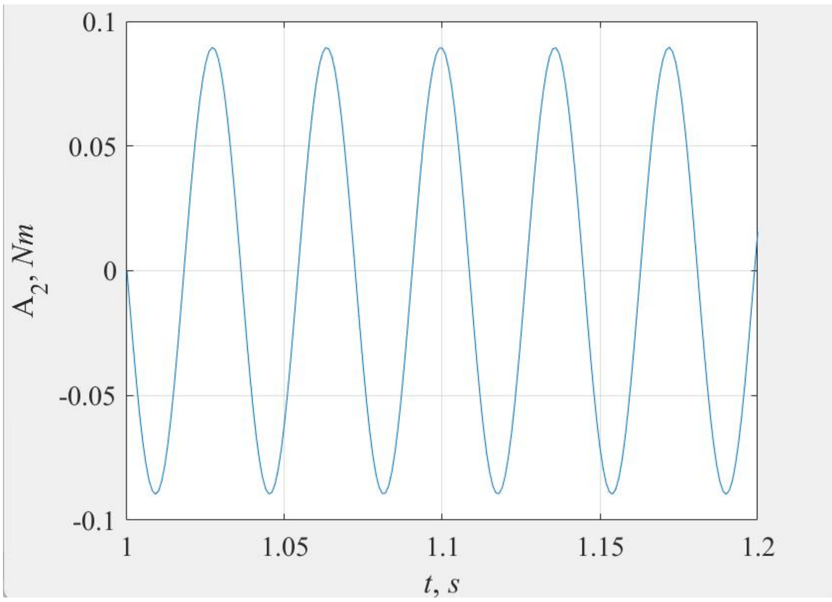


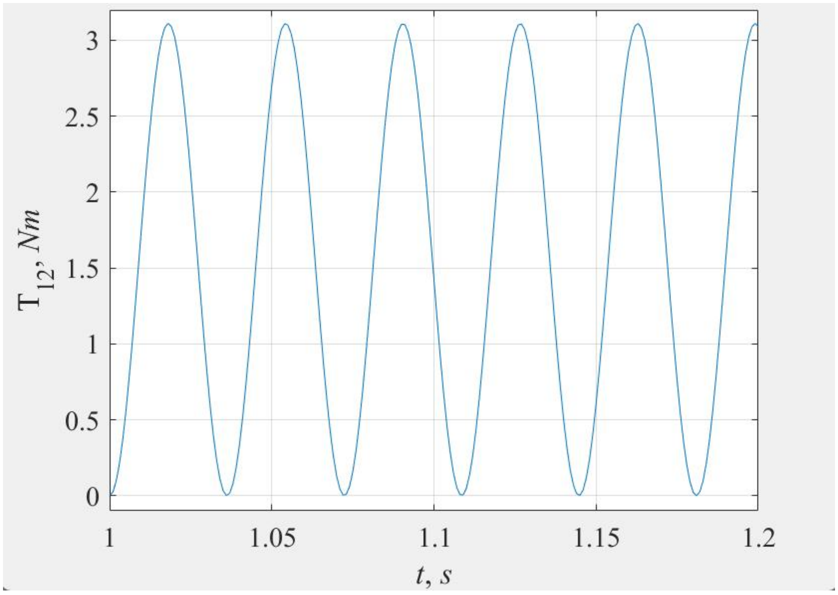


$$when\ \gamma = 1.8, J_2 = 0.8J_1$$

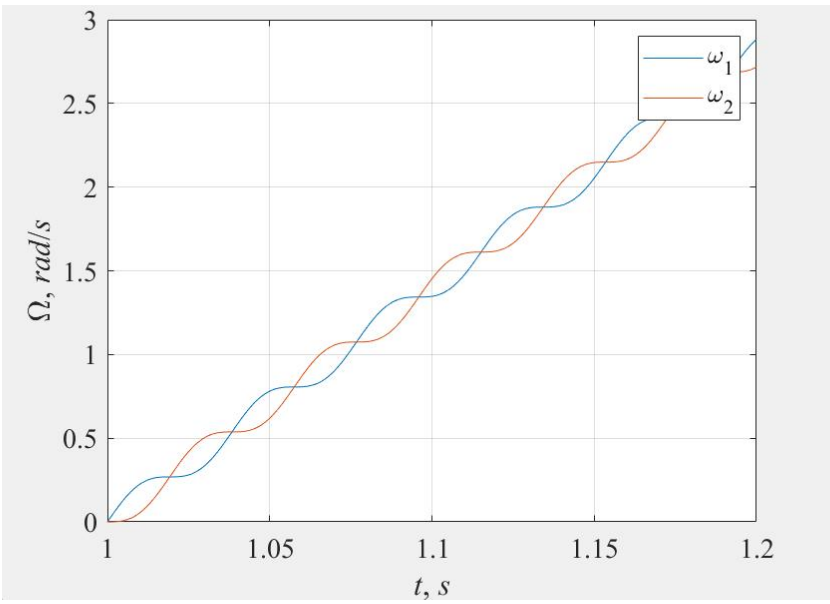


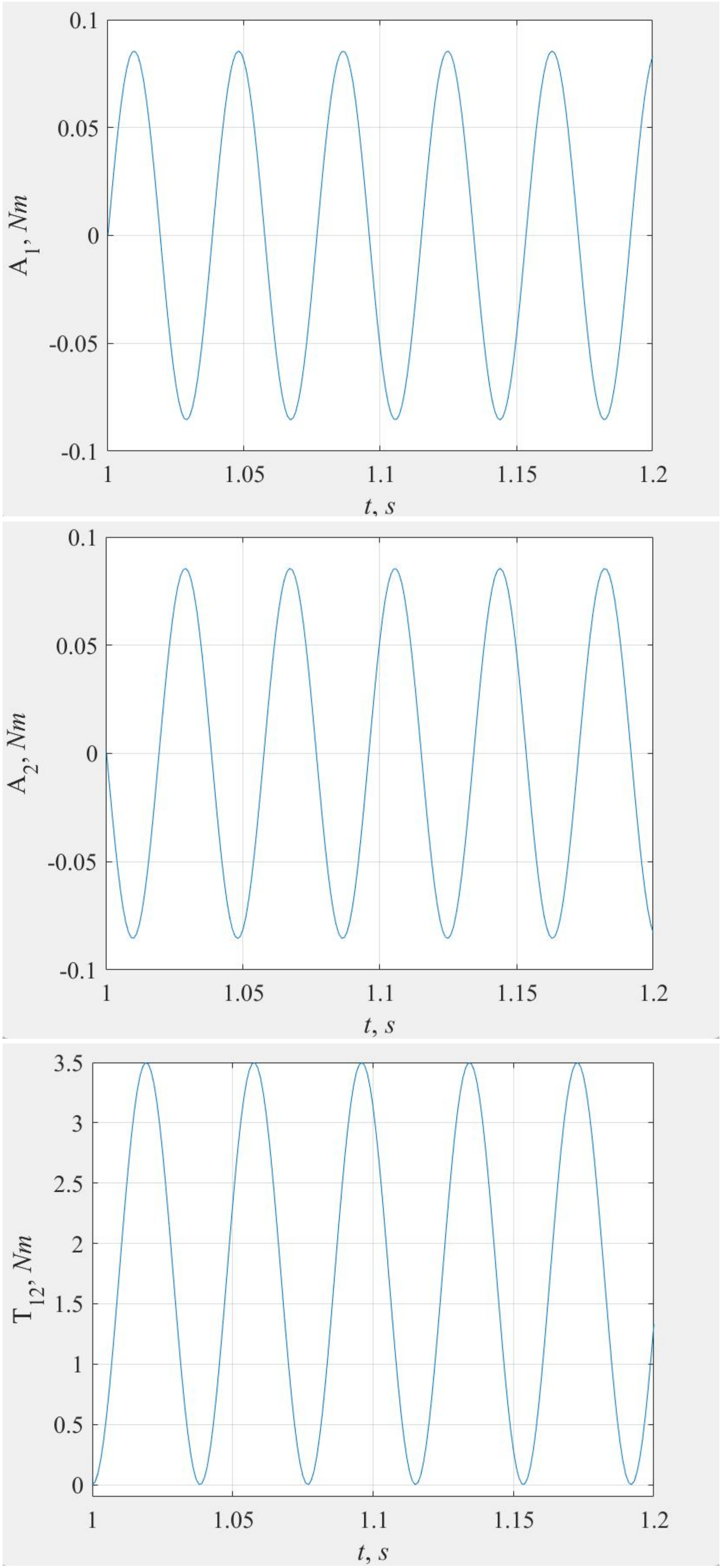






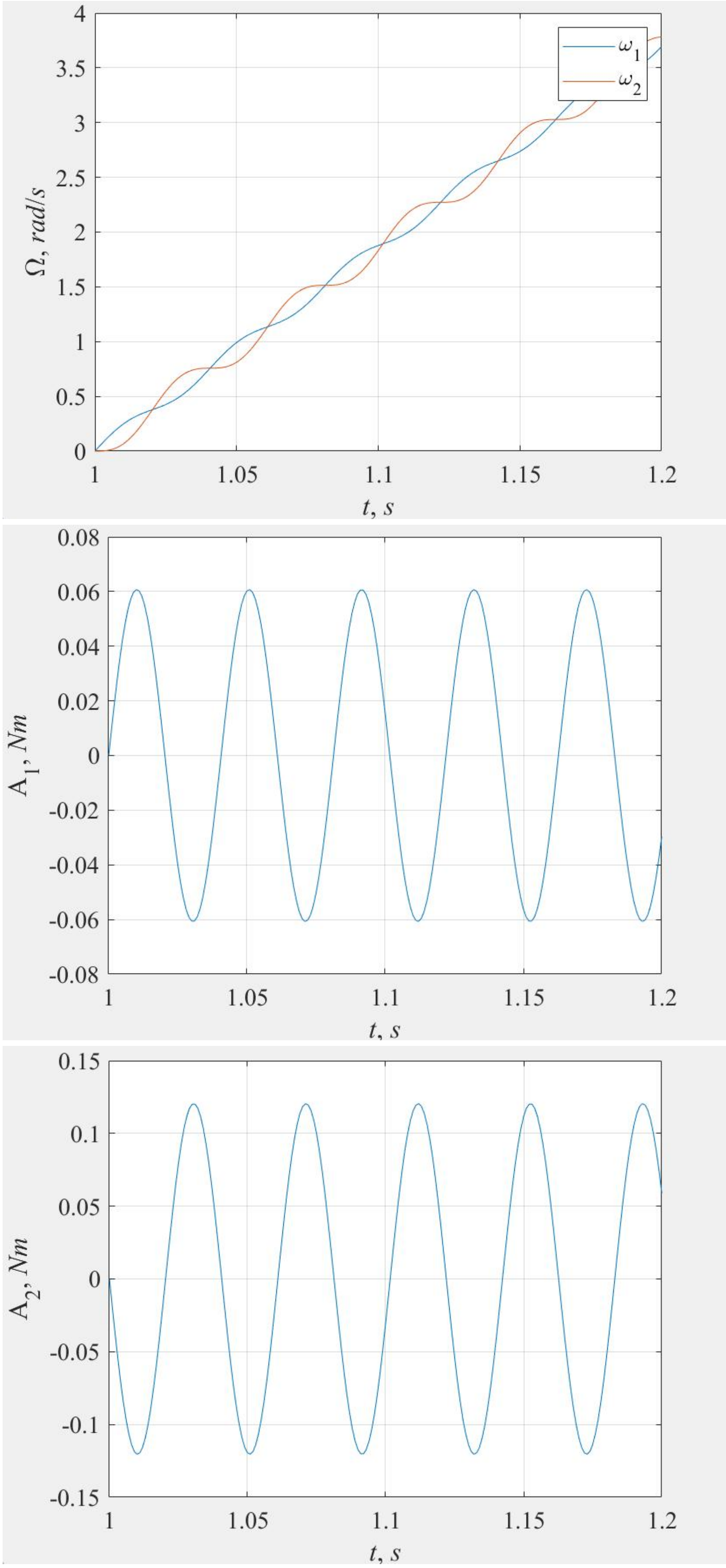
$when\ \gamma = 2, J_2 = 1J_1$

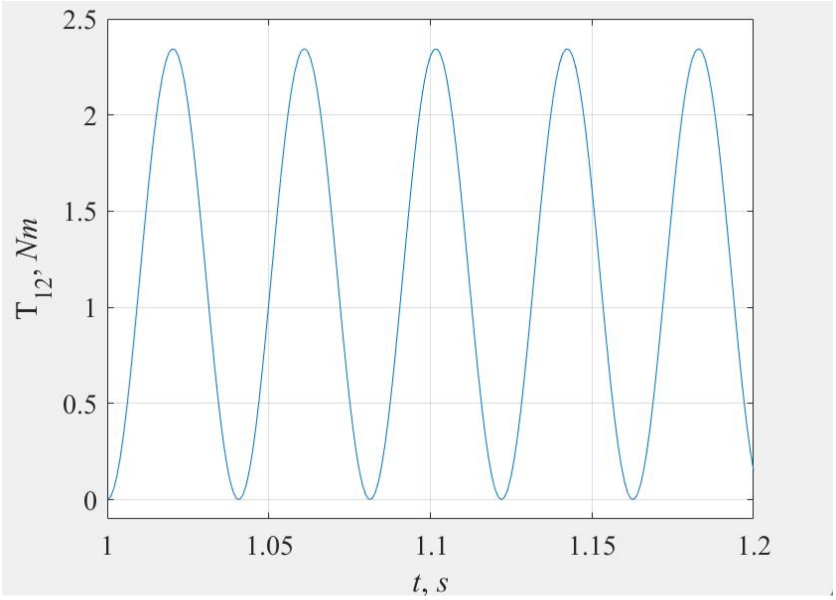




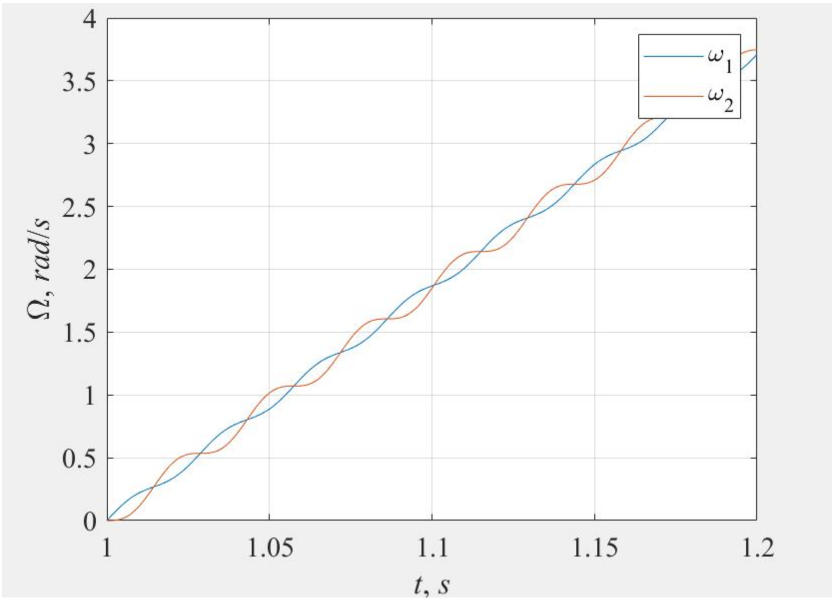
0.1
0.05
0
-0.05
-0.1
A1, Nm
1
1.05
1.1
1.15
1.2
t, s
0.1
0.05
0
-0.05
-0.1
A2, Nm
1
1.05
1.1
1.15
1.2
t, s
3.5
3
2.5
2
1.5
1
0.5
0
T12, Nm
1
1.05
1.1
1.15
1.2
t, s

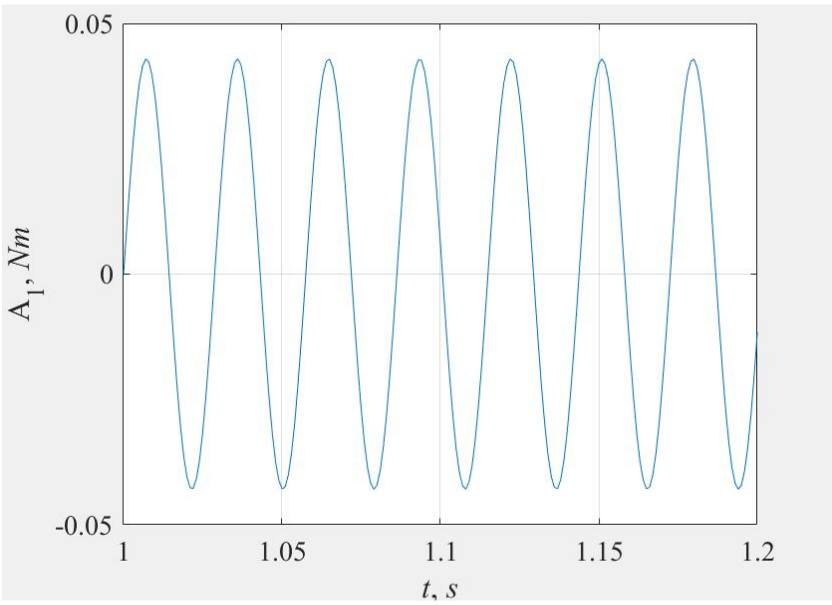
$$when\ k_{12} = 1000$$

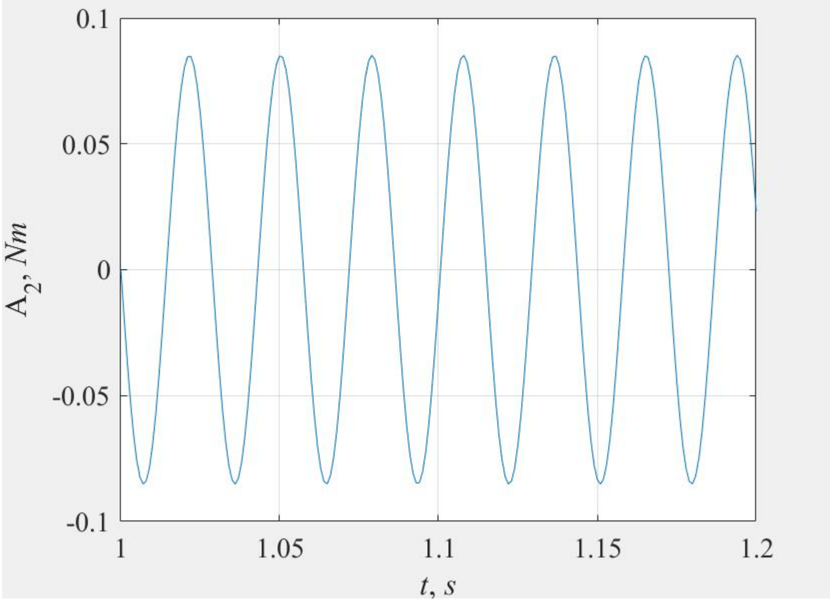


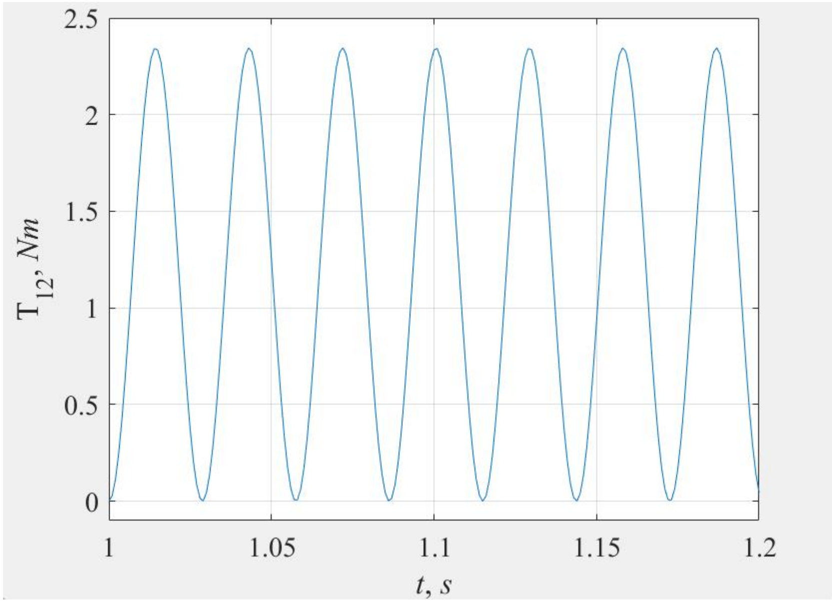


$when\ k_{12} = 2000$

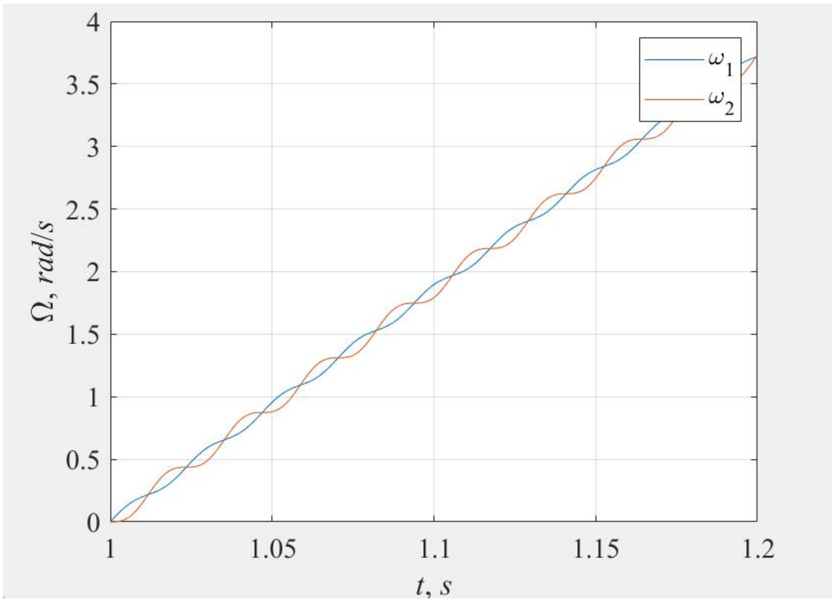


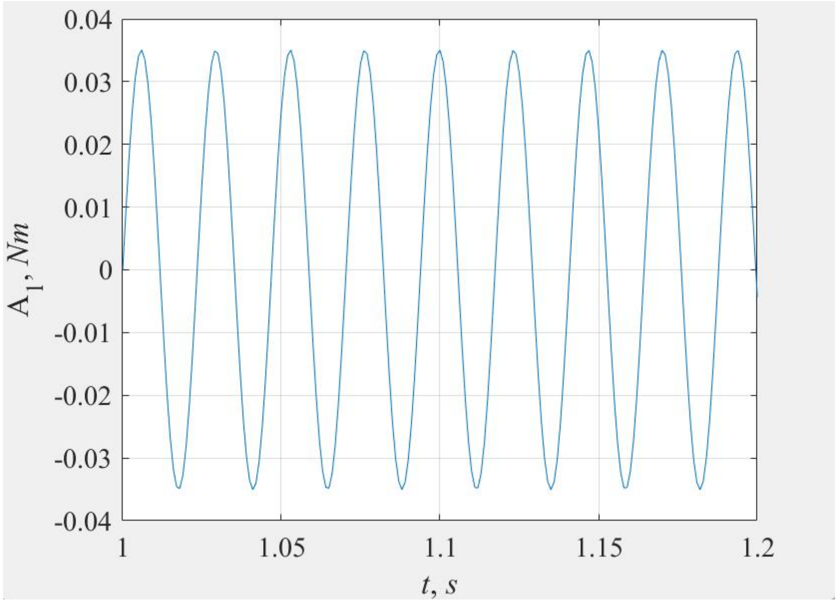




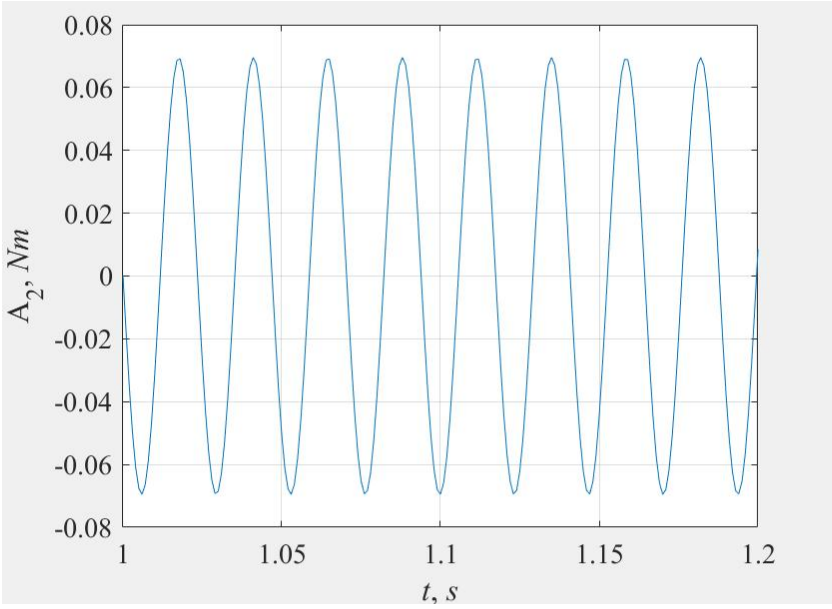


$$when\ k_{12} = 3000$$

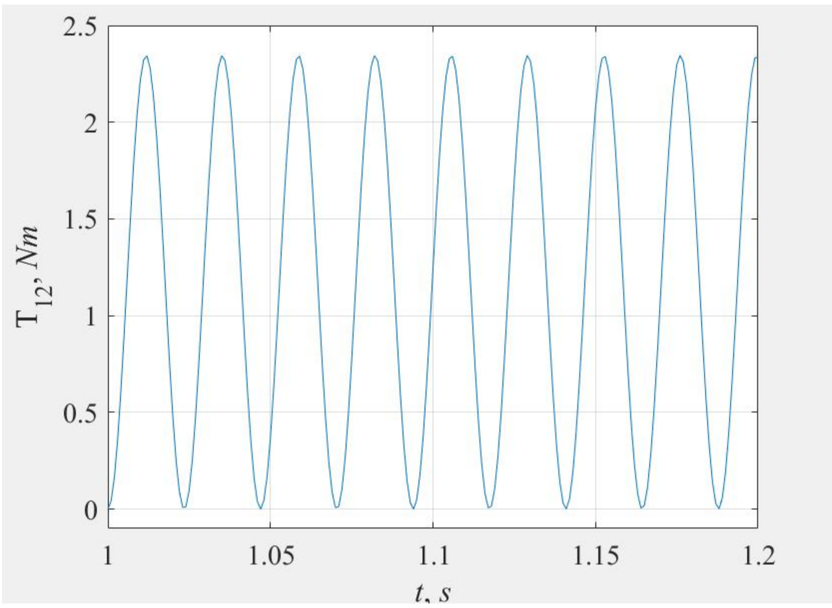




0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
A1, Nm
1
1.05
1.1
1.15
1.2
t, s



0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
A2, Nm
1
1.05
1.1
1.15
1.2
t, s

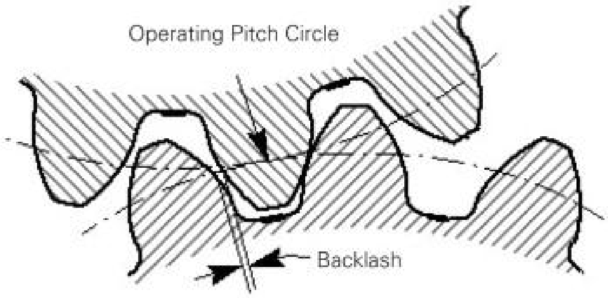


2.5
2
1.5
1
0.5
0
T12, Nm
1
1.05
1.1
1.15
1.2
t, s

Draw conclusions

Bode plot analysis successfully identified the system's resonant frequency at *208 rad/s* (theoretical value: *200 rad/s*), validating the calculation formula $\omega_{R1} = \sqrt{k_{12}\frac{J_1+J_2}{J_1 J_2}}$. When adjusting the mass ratio γ and stiffness k_{12}, predictable changes in resonant frequency were observed: frequency decreased with increasing *γ*.

Task 1.2. Research the effect of backlash in a model of the two-mass mechanism



Mathematic model of two-mass mechanism with backlash:

$$\begin{cases} T - T_{L1} - T_{s12} = J_1\, sw_1 \\ T_{s12} - T_{L2} = J_2\, sw_2 \\ T_{s12} = k_{12}(\varphi_1 - \varphi_2 \pm \Delta\varphi/2), |\varphi_1 - \varphi_2| > \Delta\varphi/2 \\ T_{s12} = 0, |\varphi_1 - \varphi_2| \le \Delta\varphi/2 \end{cases}$$

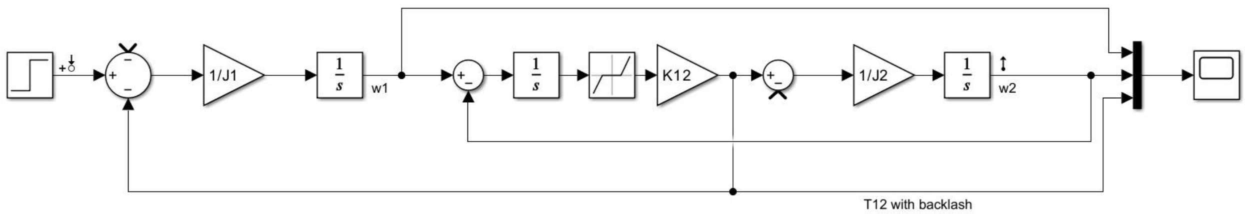


Figure: Math model of the two-mass mechanism with backlash in Simulink

Show transient response of $\omega_1(t)$, $\omega_2(t)$, $T_{s12}(t)$ by the step reference signal T with value $0.1T_{rated}$ (at $T_{L1}=0$, $T_{L2}=0$). Please display $\omega_1(t)$, $\omega_2(t)$ on one plot and make sure that the speed of the first and second masses

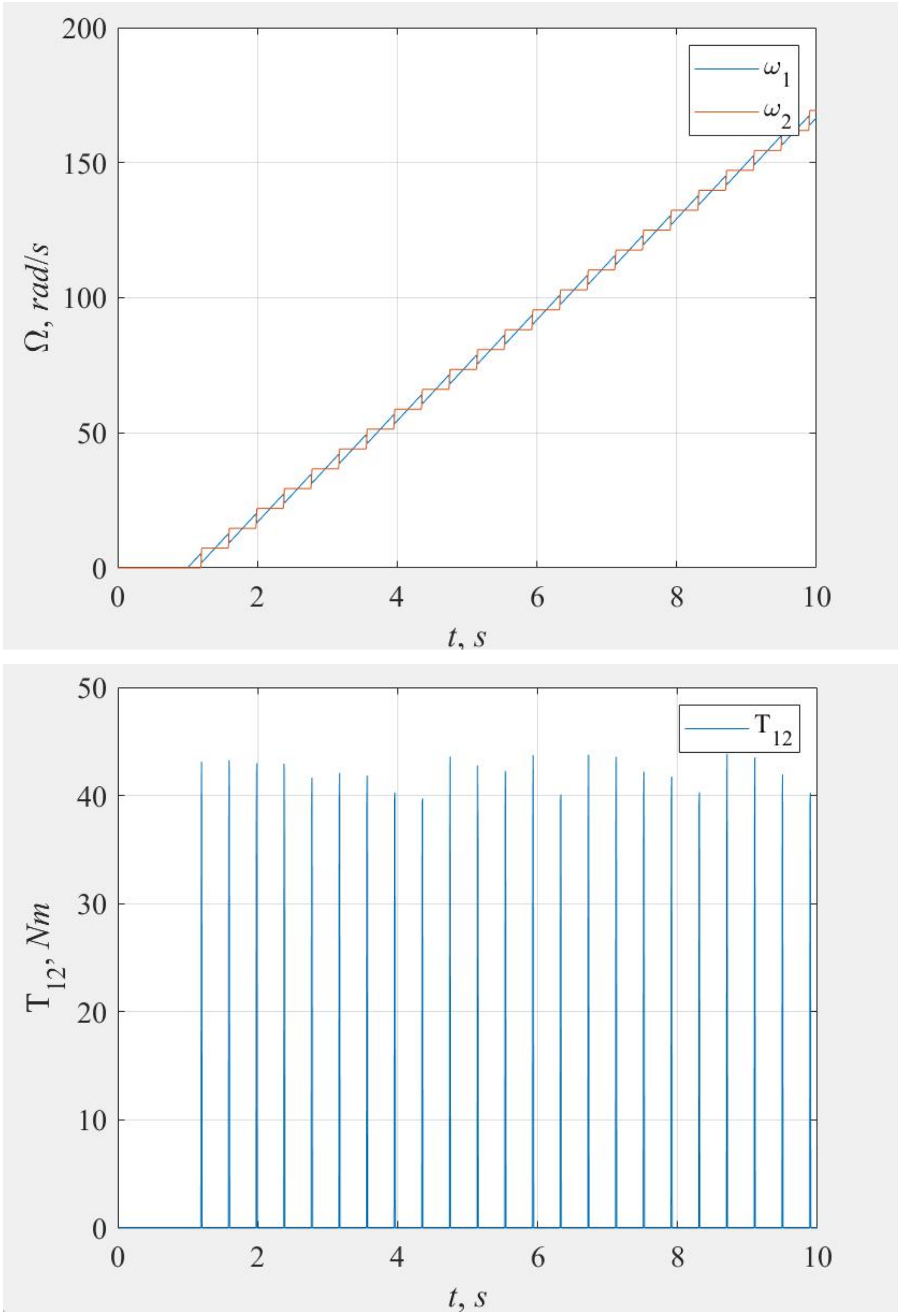
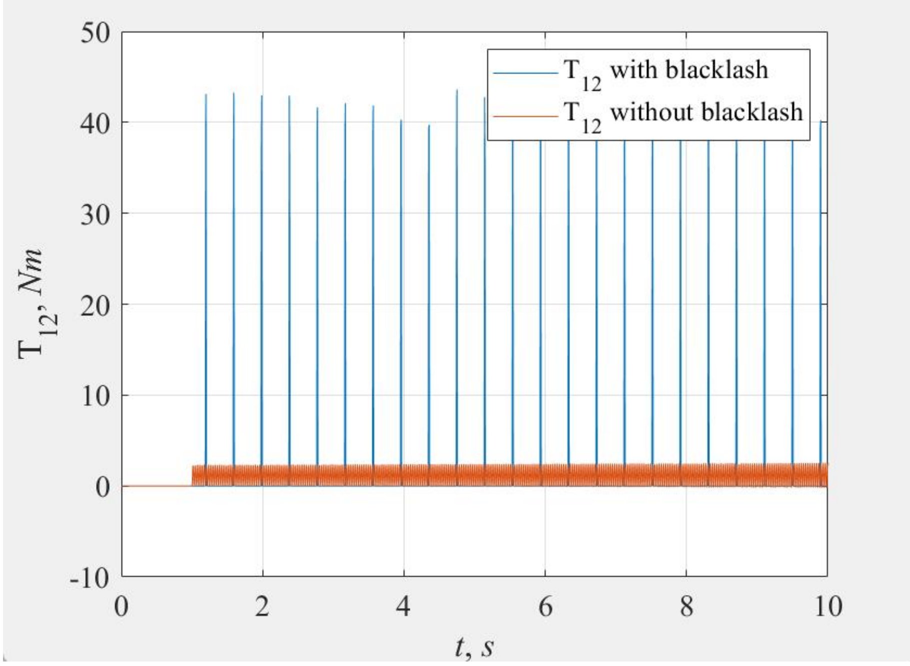


Figure: The plot of torque of elastic bonding forces between bodies versus time



Compare $T_{s12}(t)$ in mechanism without and with backlash in gearbox

Draw conclusions

Introducing backlash significantly altered the system's response. The dead zones in $\omega_1(t)$ and $\omega_2(t)$ caused intermittent disengagement between masses, leading to abrupt changes in the elastic torque $T_{s12}(t)$. Compared to the backlash-free system, $T_{s12}(t)$ exhibited sharp spikes during re-engagement, indicating impulsive forces. This highlights the detrimental impact of backlash on system stability and smooth operation, emphasizing the need for precise mechanical design to minimize such nonlinearities.

Task 1.3. Research the effect of viscous friction torque in a model of the two-mass mechanism

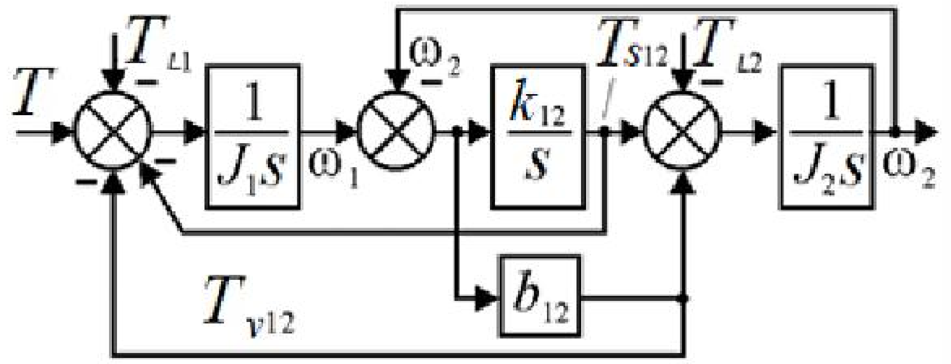


Figure: Scheme of the system with viscous friction

Mathematic model of the system with viscous friction

$$\left.\begin{array}{r} T - b_{12}(\omega_1 - \omega_2) - k_{12}(\omega_1 - \omega_2)/s - T_{L1} = J_1 s\omega_1; \\ b_{12}(\omega_1 - \omega_2) + k_{12}(\omega_1 - \omega_2)/s - T_{L2} = J_2 s\omega_2. \end{array}\right\}$$

Where b_{12} is calculated as the following (The viscous damping coefficient b should be chosen considering that the oscillation damp in 5 periods):

$$b12 = \frac{2a_v \mathrm{J}_1 \mathrm{J}_2}{\mathrm{J}_1 + \mathrm{J}_2} \approx 1.6$$

$$a_v \approx \frac{3\omega_R}{10\pi} = 19.1$$

where $a_v \approx \frac{3\lambda_v \cdot \omega_{R1}}{2\pi} = \frac{3\omega_{R1}}{10\pi}$ - attenuation coefficient

$\lambda_v = a_v\, T = \frac{T}{\tau} = \frac{1}{n}$ - logarithmic decrement

n - number of harmonic oscillations during relaxation τ (the amplitude decreases e times)

$tres = 3\frac{1}{av} = nT$=5$T$ - time response

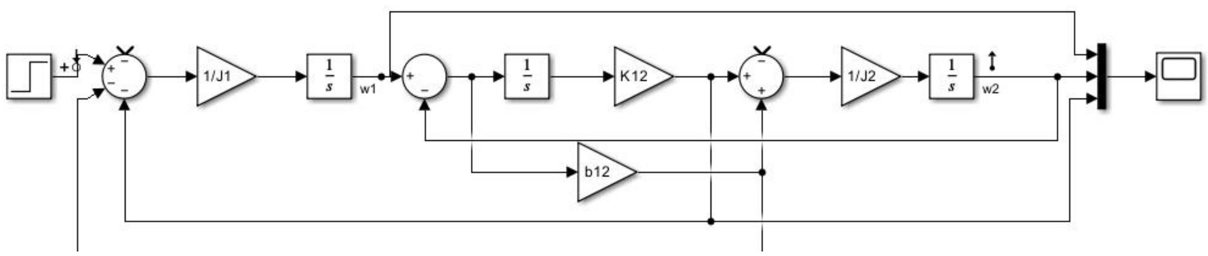


Figure: Math model of the two-mass mechanism in Simulink

Show transient response of $\omega_1(t)$, $\omega_2(t)$, $T_{s12}(t)$ by the step reference signal T with value 0.1T_{rated} (at T_{L1}=0, T_{L2}=0). Please display $\omega_1(t)$, $\omega_2(t)$ on one plot

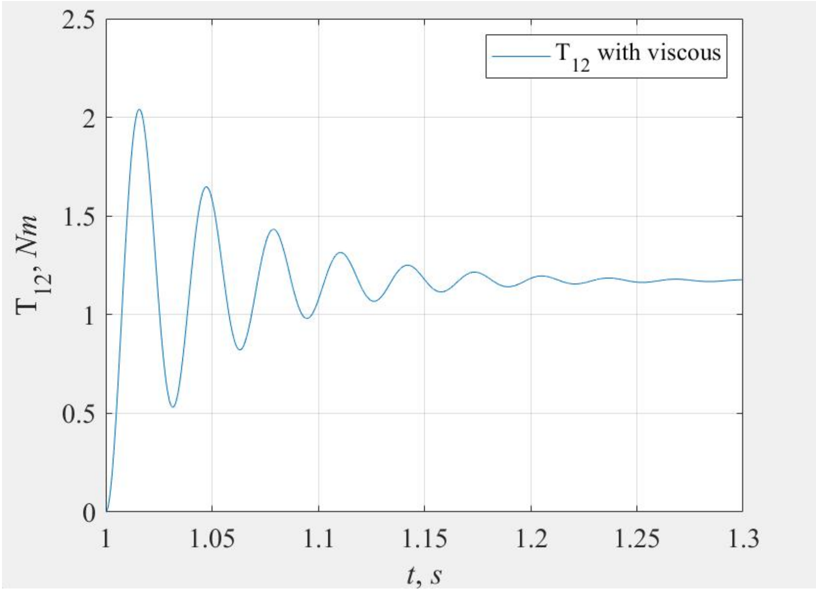


Figure: The plot of torque of elastic bonding forces between bodies versus time

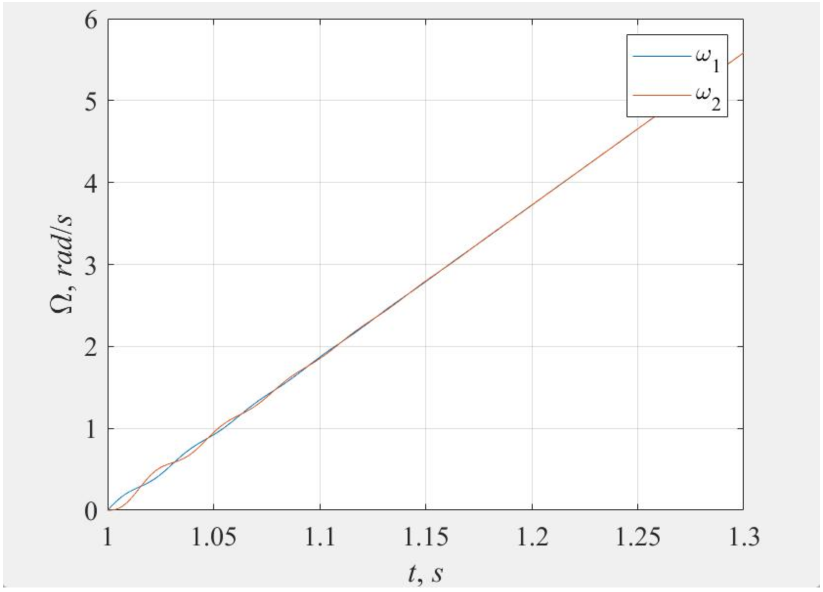
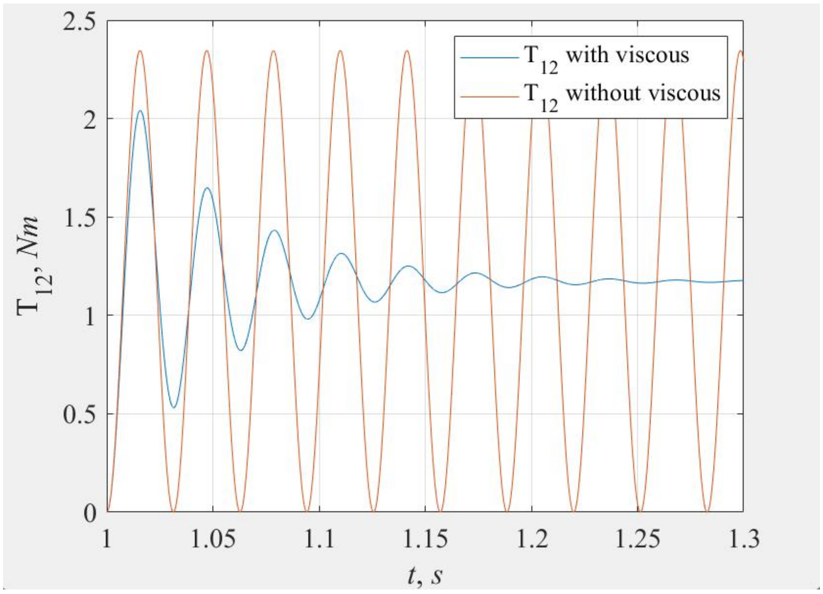


Figure: The plot of angular speed of the 1st and 2nd body versus time



Compare $T_{s12}(t)$ in mechanism without and with viscous in gearbox

Draw conclusions

Adding viscous friction damped the oscillations effectively, with the system reaching steady-state within five periods as designed. The damping coefficient $b12$ = 1.6 ensured rapid energy dissipation, eliminating sustained oscillations seen in Task 1.1. The plots of $\omega_1(t)$, $\omega_2(t)$, and $T_{s12}(t)$ showed smoothed trajectories, confirming friction's role in stabilizing the system. However, excessive damping could slow response times, necessitating a trade-off in real-world applications.

Part 2. Mathematical modelling of DC-motor with two-mass mechanism (*not necessary – this is additional option)*

Task 2.1 Modelling of the DC-motor with two-body mechanism.

Design a model of the DC-motor with two-body mechanism.

Mathematical model of DC motor with two-mass mechanism:

$$\begin{cases} J_1\ddot{\theta}_1 = T_m - T_s - b_1\dot{\theta}_1 \\ J_2\ddot{\theta}_2 = T_s - b_2\dot{\theta}_2 \\ T_s = k(\theta_1 - \theta_2) + c(\dot{\theta}_1 - \dot{\theta}_2) \end{cases}$$

Scheme of the system is presented in Fig

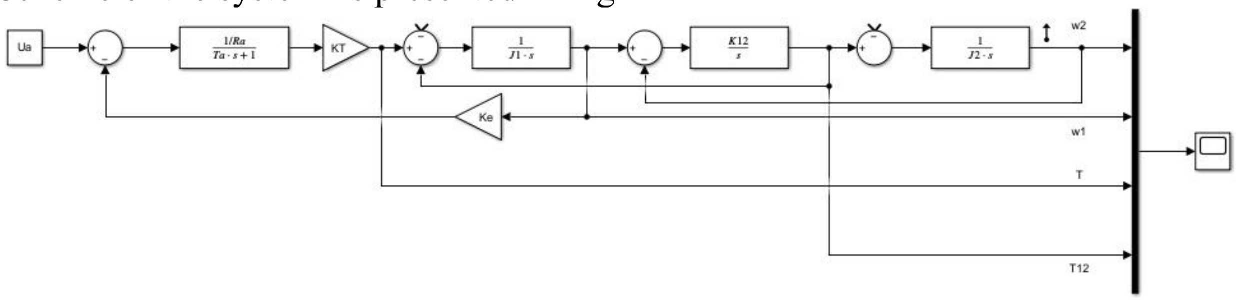
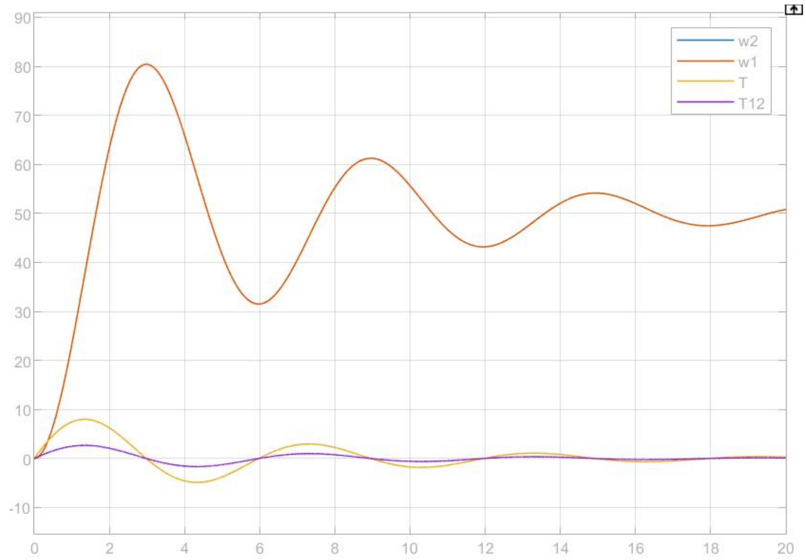


Figure: The model of DC-motor with two-mass mechanism.

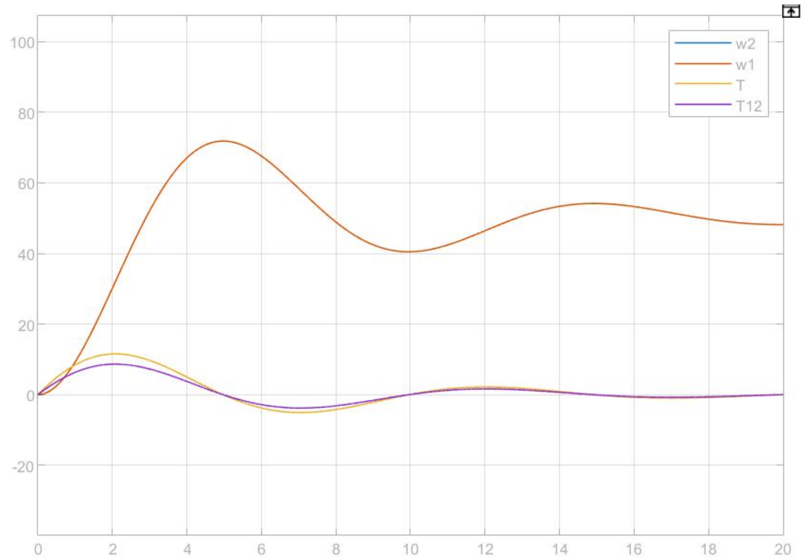
Show plots T(t) , T_{s12}(t) , $\omega_1(t)$, $\omega_2(t)$ when:

- *varying mass ratio γ (three meaning to get different transients)*
- *varying stiffness k_{12} (three meaning to get different transients)*
- *varying resistance R / rigidity β (three meaning to get different transients)*

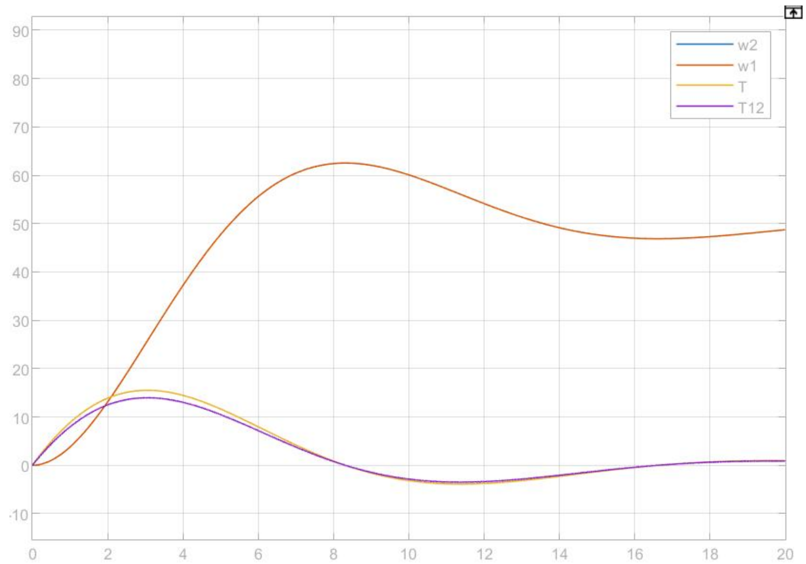
$$when\ \gamma = 1.5, J_2 = 0.5J_1$$



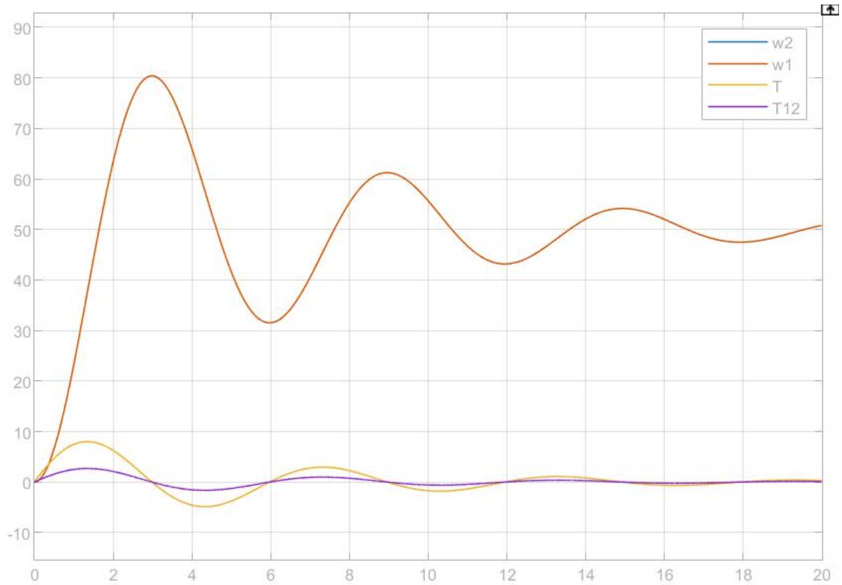
$$when\ \gamma = 2, J_2 = J_1$$



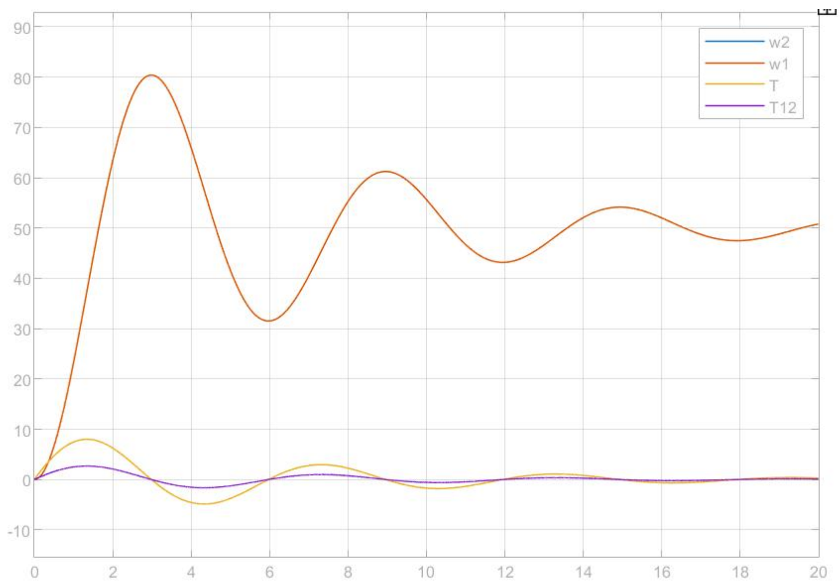
$$when\ \gamma = 10, J_2 = 9J_1$$



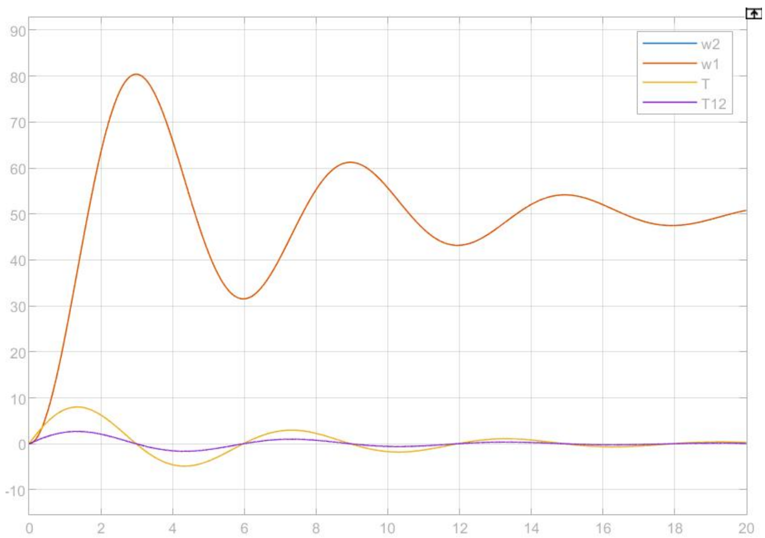
$when\ k_{12} = 1000$



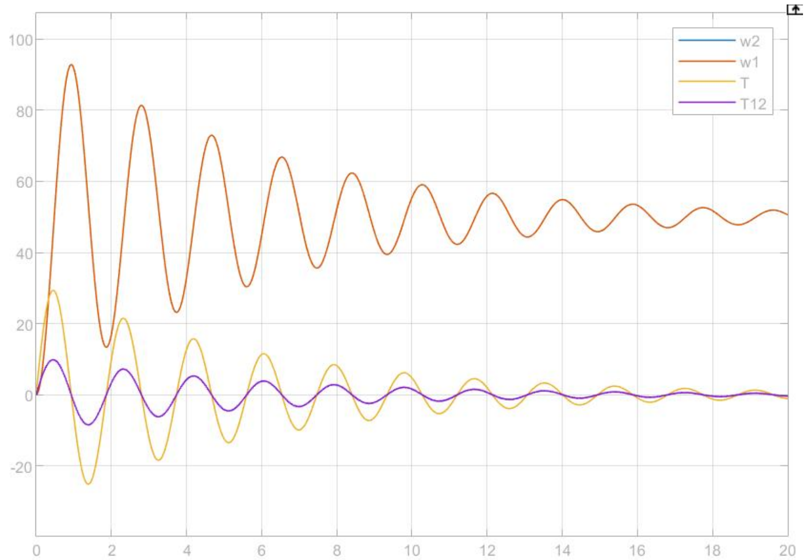
$when\ k_{12} = 2000$



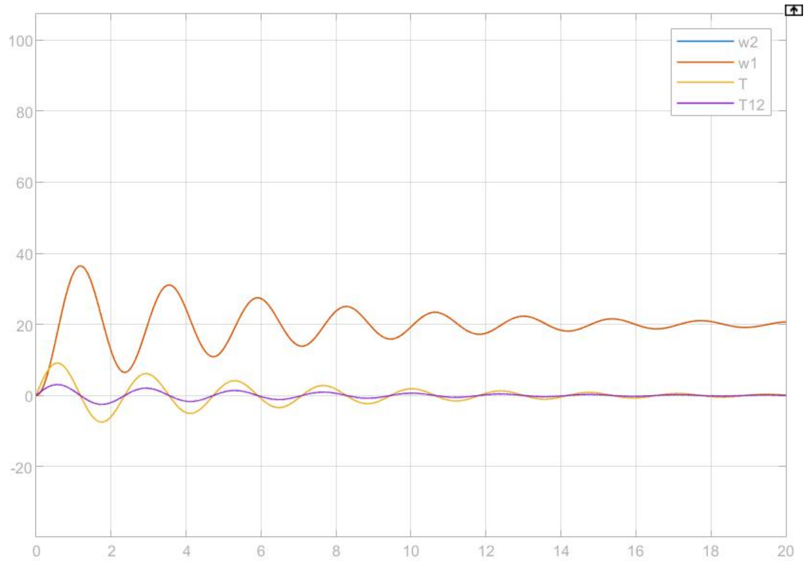
$when\ k_{12} = 3000$



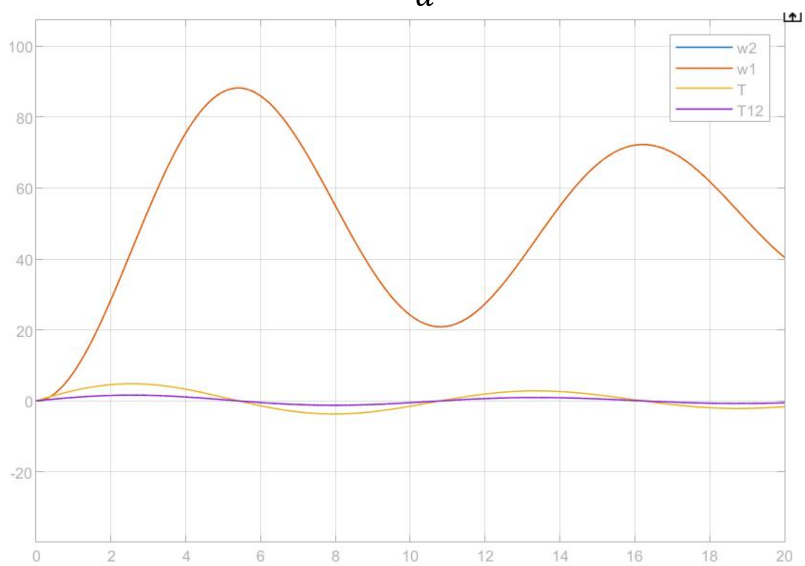
$$when\ R = 10$$



$$when\ K_e = 20$$



$$when\ T_a = 10$$



Draw conclusions

Variations in mass ratio (γ) and stiffness (k_{12}) produced distinct transient behaviors: higher γ (larger J_2) increased oscillation amplitudes, while greater k_{12} raised the resonant frequency. Changes in resistance (R) and back-EMF (k_e) affected motor torque and speed profiles, illustrating the interplay between electrical and mechanical subsystems. These results underscore the complexity of integrated electromechanical systems and the importance of parameter tuning for desired performance.