Electrical Machines

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Permanent magnet synchronous motors

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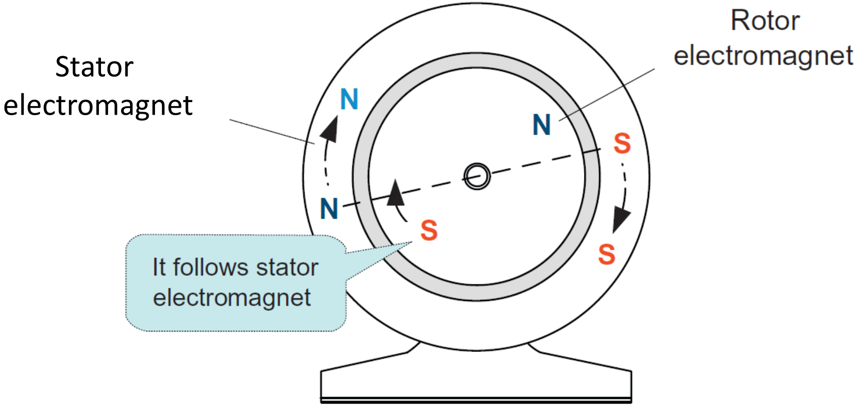
Synchronous machines

The magnetic field on the rotor is generated either by a **permanent magnet** or by a **field winding powered by a DC power supply** separated from the stator AC power source.

In this machine, **the rotor magnetic field is stationary relative to the rotor**. To produce a torque, the rotor should rotate at the same speed as the stator rotating magnetic field. This speed is called **the synchronous speed**.



$\omega_e = \frac{p}{2}\omega_m$ electrical angular frequency

ω_m mechanical angular frequency

$\theta_e = \frac{p}{2}\theta_m$ electrical angle

θ_m mechanical angle

p number of poles

Synchronous machines

Almost **all three-phase power** is generated by three-phase synchronous machines operated as generators.

Synchronous generators are also called **alternators** and are normally large machines producing electrical power at hydro, nuclear, or thermal power plants.

4 pole generators (ALSTROM power)

2 pole generators for gas and steam turbines (SIEMENS)

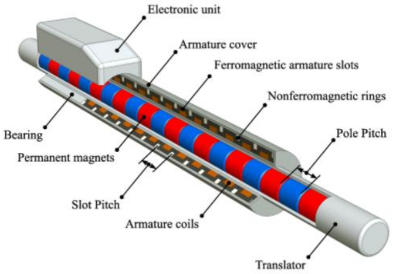
Gearless synchronous generator for wind power plants (SIEMENS)

Synchronous machines

Permanent magnet synchronous motors (PMSM)

Brushless DC-motor (BLDC)

Stepper motor

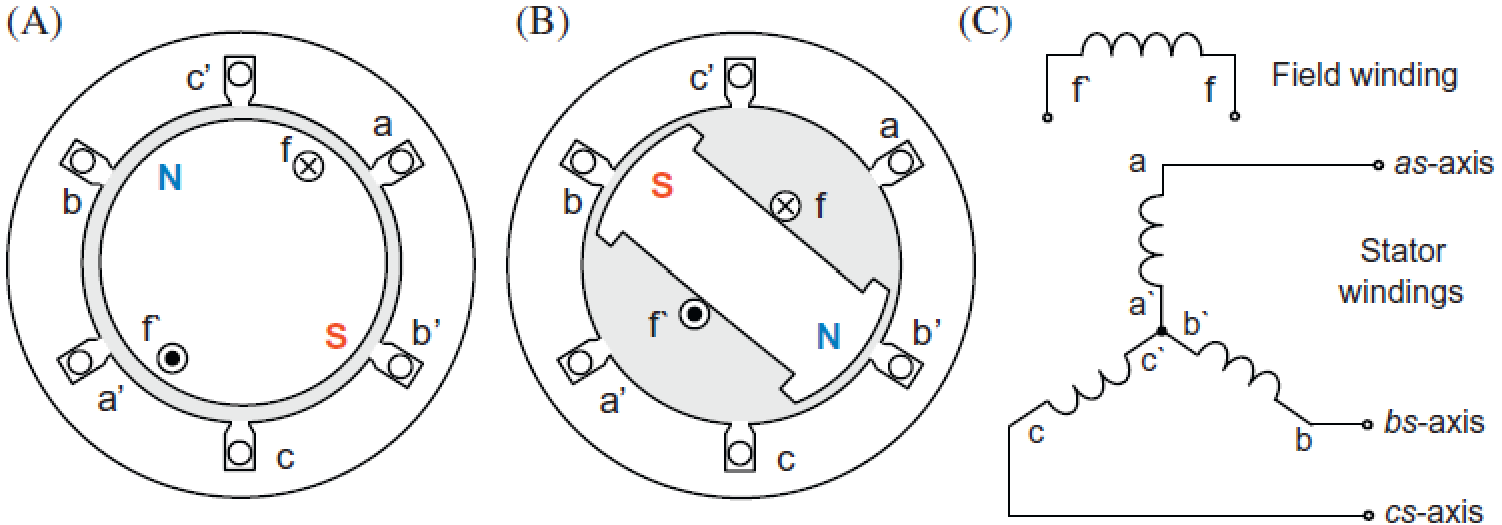


Concept of linear PMSM

Hysteresis motor

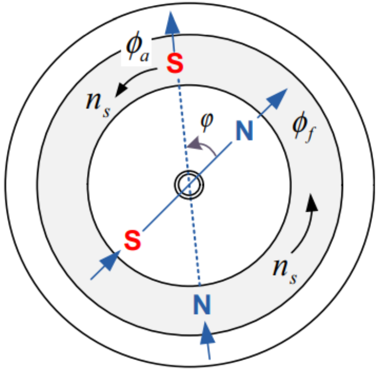
Relustance motor

Synchronous machines

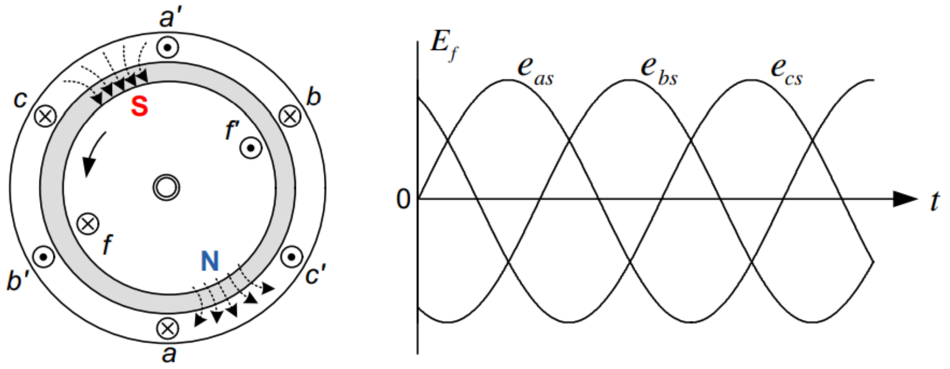


Stator and rotor structure of a synchronous machine.
(A) Cylindrical type, (B) salient pole type, and (C) stator windings.

Cylindrical rotor synchronous motor



Rotating field flux of stator and the field flux of rotor



Excitation voltage induced by the field flux.

The stator voltage equation:

$$V_s = R_s I_s + \frac{d\lambda_s}{dt}$$

The rotating field flux generated by I_s in the stator:

$$\phi_a = \phi_{ar} + \phi_{al}$$

The flux in the air gap

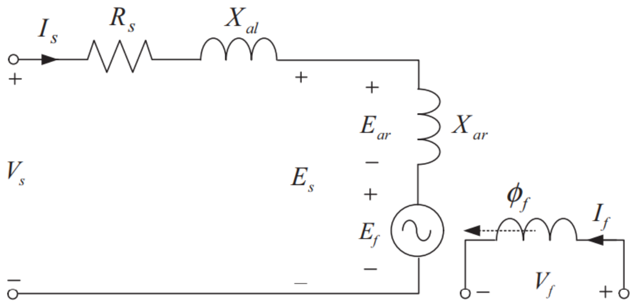
$$\phi_s = \phi_f + \phi_{ar}$$

ϕ_f the field flux generated in the rotor

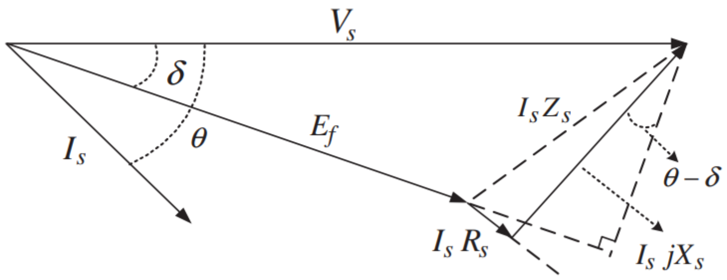
The flux linkage of the stator winding

$$\lambda_s = N_s\left(\phi_f + \phi_{ar}\right)$$

Cylindrical rotor synchronous motor



Per phase equivalent circuit of a cylindrical rotor synchronous motor.



Phasor diagram for a synchronous motor.

The total induced voltage in the stator winding

$$E_s = E_f + E_{ar}$$

The armature reaction voltage

$$E_{ar} = jX_{ar} I_a$$

The stator voltage equation

$$\begin{aligned} V_s &= I_s R_s + jX_{al} I_s + E_s \\ &= I_s R_s + jX_{al} I_s + jX_{ar} I_s + E_f \\ &= I_s Z_s + E_f \end{aligned}$$

Cylindrical rotor synchronous motor

The input power of a synchronous motor:

$$P = 3V_s I_s \cos\theta$$

From the phasor diagram:

$$V_s \cos\delta = E_f + I_s X_s \sin(\theta-\delta) + I_s R_s \cos(\theta-\delta)$$

$$V_s \sin\delta = I_s X_s \cos(\theta-\delta) - I_s R_s \sin(\theta-\delta)$$

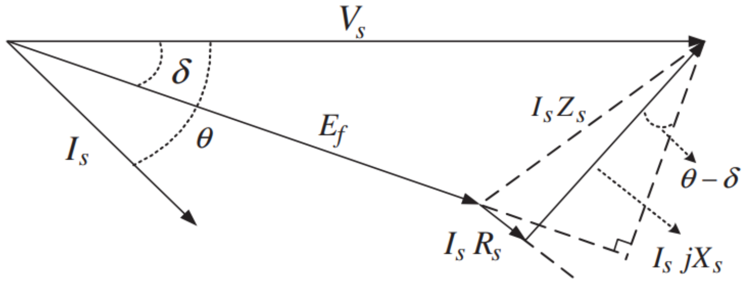
If the stator winding resistance is neglected:

$$I_s \cos\theta = \frac{E_f \sin\delta}{X_s}$$

$$P = 3\frac{V_s E_f}{X_s}\sin\delta = P_{\max}\sin\delta$$

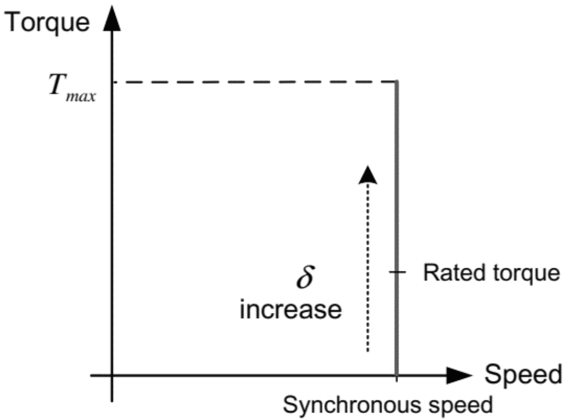
$$Q = 3\frac{V_s E_f \cos\delta - V_s}{X_s}$$

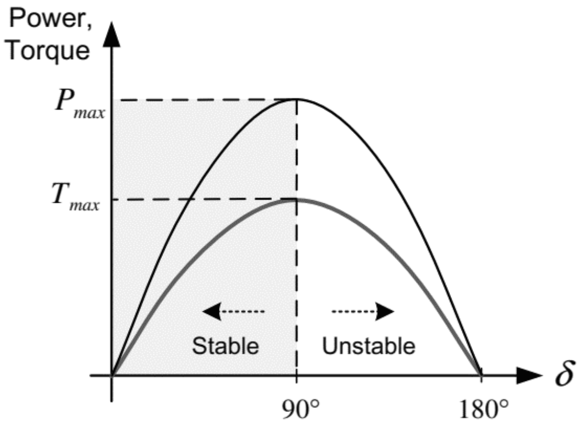
$$T = \frac{P}{\omega_m} = 3\frac{V_s E_f}{\omega_m X_s}\sin\delta = T_{\max}\sin\delta$$



Phasor diagram for a synchronous motor.

Cylindrical rotor synchronous motor





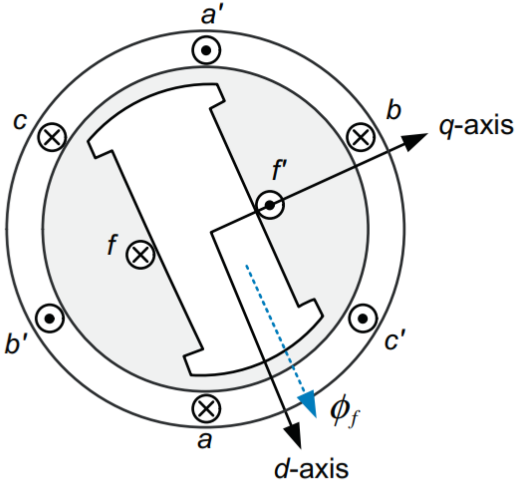
$$P = 3\frac{V_s E_f}{X_s}\sin\delta = P_{\max}\sin\delta$$

$$Q = 3\frac{V_s E_f\cos\delta - V_s}{X_s}$$

$$T = \frac{P}{\omega_m} = 3\frac{V_s E_f}{\omega_m X_s}\sin\delta = T_{\max}\sin\delta$$

Output power and torque for a cylindrical rotor synchronous motor.

Salient pole rotor synchronous motor



Salient pole rotor synchronous motor.

To consider the saliency of the rotor in a synchronous motor model, we need to define the d and q axes.

The d-axis is defined as the axis along the poles.

The q-axis is defined as the axis between the poles, which is also the direction of the excitation voltage.

$$L_d > L_q$$

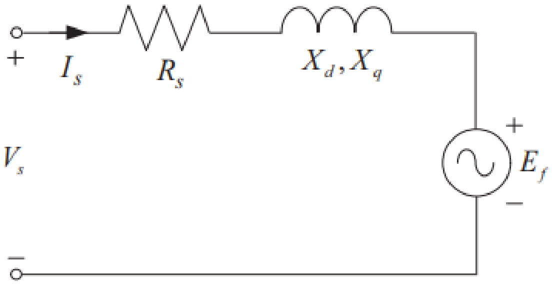
Thus the stator current

$$I_s = I_q - jI_d$$

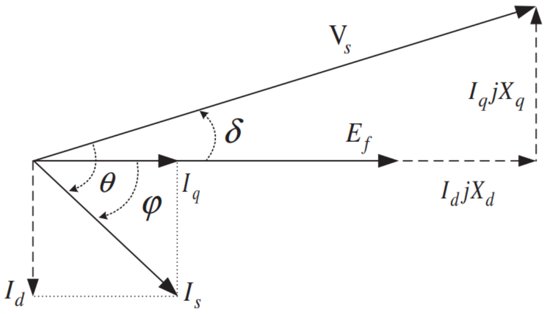
$$I_q = I_s \cos\varphi \qquad I_d = I_s \sin\varphi$$

φ the phase angle between I_q and I_d

Salient pole rotor synchronous motor



Equivalent circuit of a salient pole rotor synchronous motor.



Phasor diagram of a salient pole synchronous motor.

The d and q axes synchronous reactances:

$$X_d = X_{ad} + X_{al}$$

$$X_q = X_{aq} + X_{al}$$

X_{al} the armature leakage reactance

$$X_q = (0.5 \sim 0.8) X_d$$

The voltage equation of the salient pole rotor synchronous motor:

$$V_s = I_s R_s + jI_d X_d + jI_q X_q + E_f$$

Salient pole rotor synchronous motor

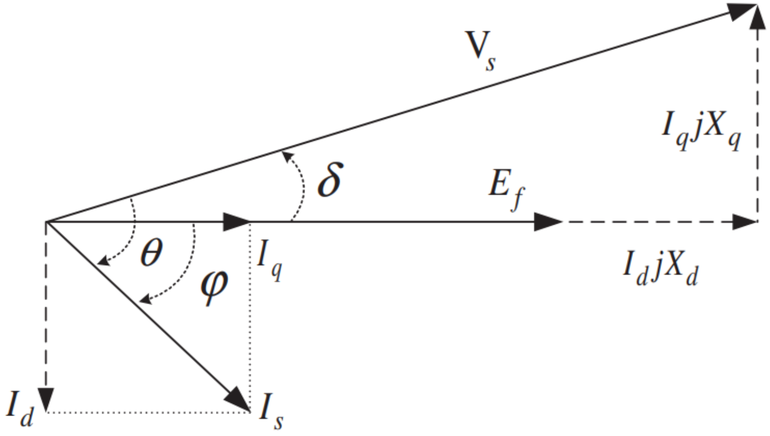
The input power in terms of d-q axes currents:

$$P = 3V_s I_s \cos\theta = 3V_s I_s \cos(\varphi + \delta)$$
$$= 3V_s I_s (\cos\varphi\cos\delta - \sin\varphi\sin\delta)$$
$$= 3V_s (I_q \cos\delta - I_d \sin\delta)$$

From the phasor diagram:

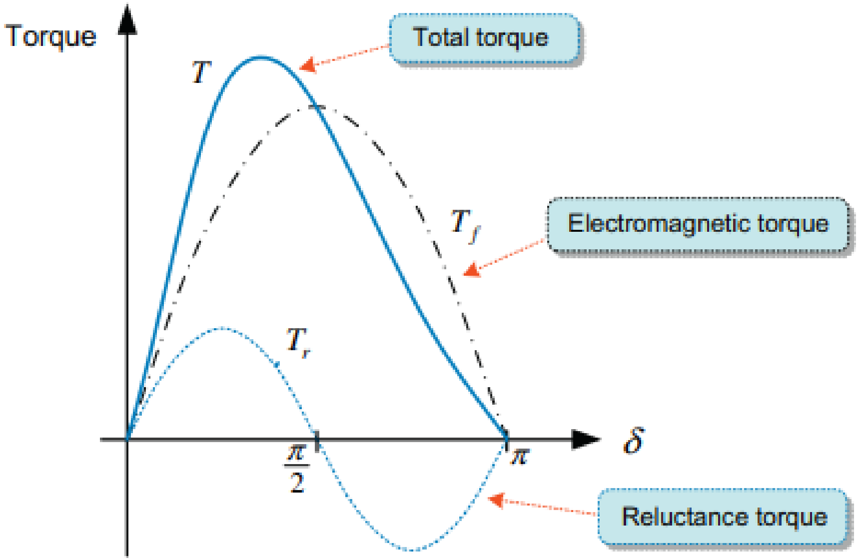
$$V_s \cos\delta = E_f + I_d X_d$$

$$V_s \sin\delta = I_q X_q$$



Phasor diagram of a salient pole synchronous motor.

Salient pole rotor synchronous motor



Torque of a salient pole synchronous motor.

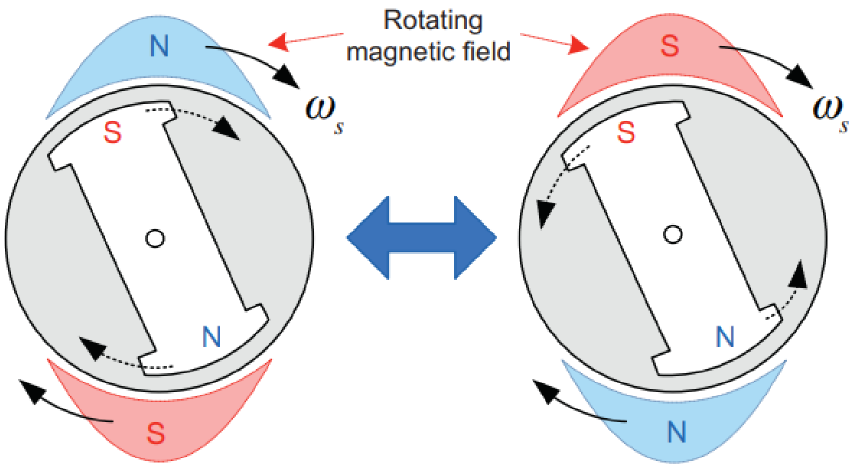
$$P = 3\frac{V_s E_f}{X_d}\sin\delta + 3\frac{V_s^2\left(X_d - X_q\right)}{2X_d X_q}\sin 2\delta$$

$$T = \frac{P}{\omega_m} = 3\frac{V_s E_f}{\omega_m X_d}\sin\delta + 3\frac{V_s^2\left(X_d - X_q\right)}{2\omega_m X_d X_q}\sin 2\delta = T_f + T_r$$

T_f the electromagnetic torque

T_r the reluctance torque

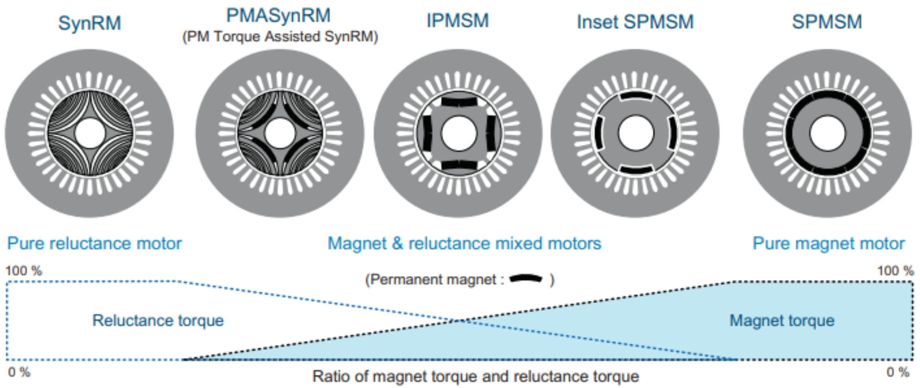
Starting of synchronous motors



Synchronization in a synchronous motor.

1. Separate motor is used to drive the rotor to run close to the synchronous speed.
2. Starting the synchronous motor as an induction motor. In this case, the rotor has a special damping winding for the purpose of starting.
3. Starting the motor slowly at a reduced frequency by using a PWM inverter

Permanent magnet synchronous motors

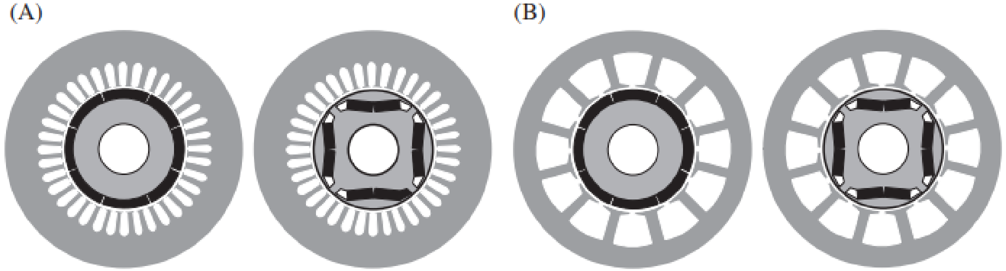


Several categories of synchronous motors.

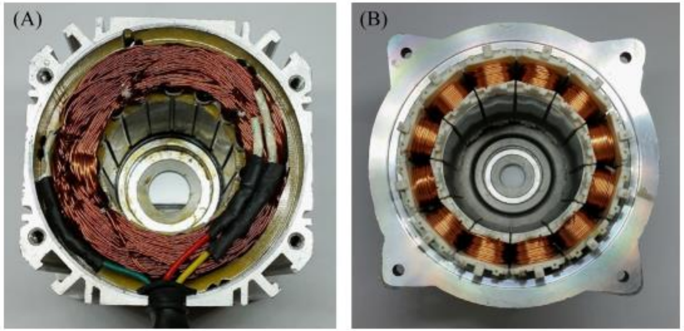
Compared to DC excitation of synchronous machines, the advantages of permanent magnet (PM) excitation are the following:

- Speeds can be comparable to those of cage induction machines (of the same dimensions)
- Joule losses in the rotor are eliminated, which is particularly advantageous for lower power ratings, where efficiency is generally low.
- For lower power ratings, permanent magnet excitation results in a lower volume of the required material, i.e. a more compact machine.

Permanent magnet synchronous motors

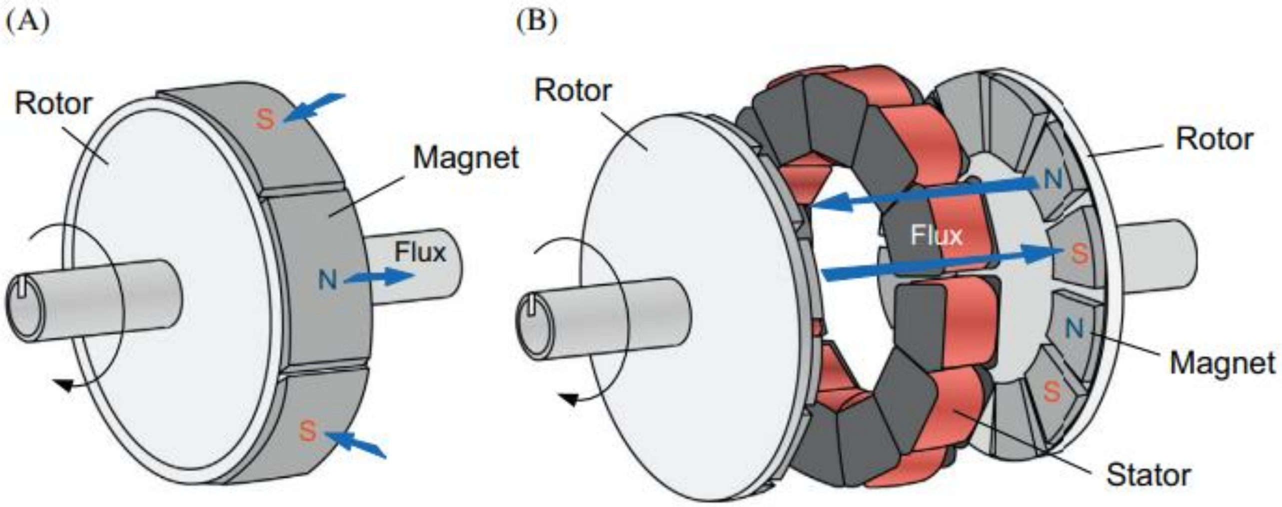


Different structures for the stator and rotor of PMSMs. (A) Distributed winding and (B) concentrated winding.



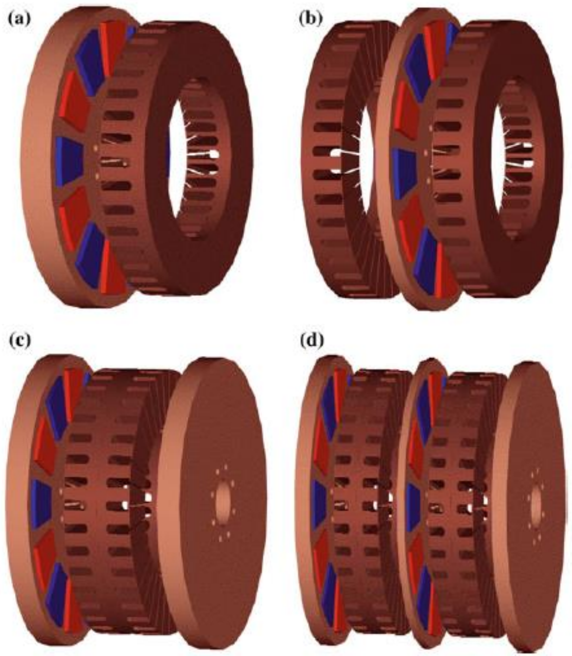
Comparison of distributed and concentrated windings. (A) Distributed winding and (B) concentrated winding.

Permanent magnet synchronous motors

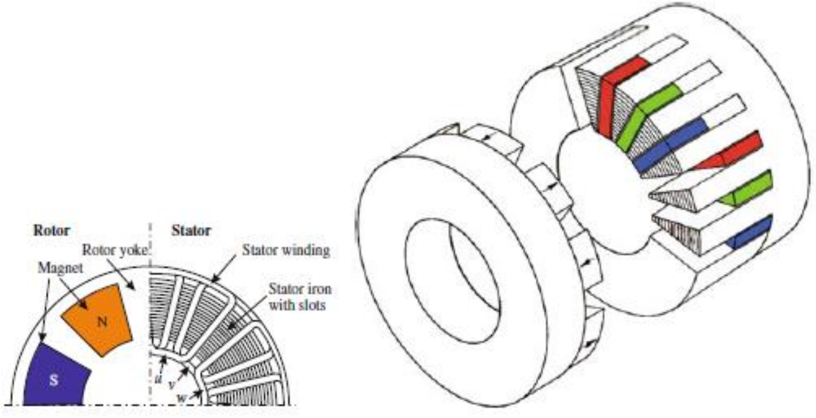


PMSM configurations. (A) Radial-flux type and (B) axial-flux type.

Permanent magnet synchronous motors

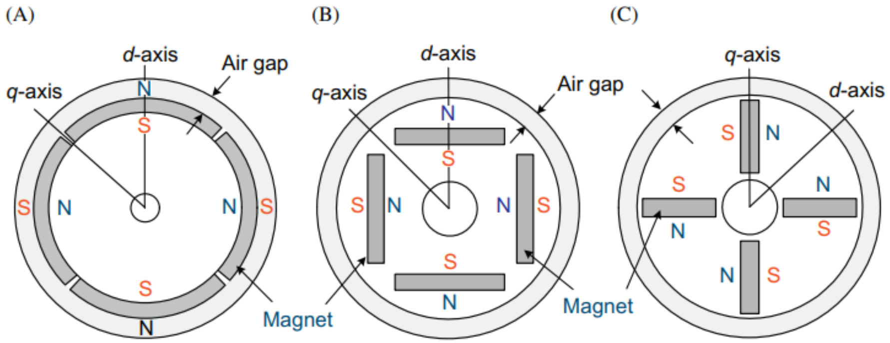


Axial flux machines; **a** single stator and rotor; **b** double stator and single rotor; **c** single stator and double rotor; **d** multi-stage with two stator modules and three rotor modules

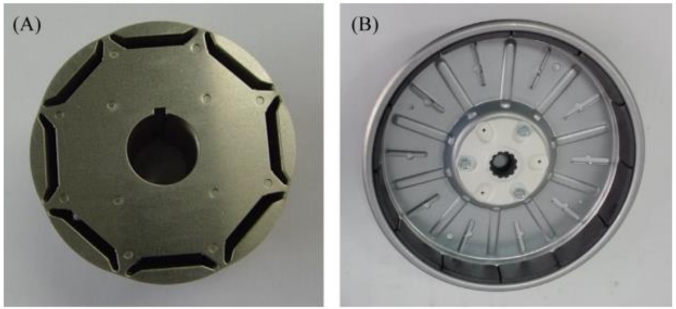


Single stator - single rotor axial flux PM machine

Permanent magnet synchronous motors



Rotor topologies of PMSMs (four-pole). (A) Surface mounted, (B) interior (parallel), and (C) interior (perpendicular).



Rotor configurations of IPMSMs. (A) Inner rotor type and
(B) outer rotor type.

Permanent magnet synchronous motors

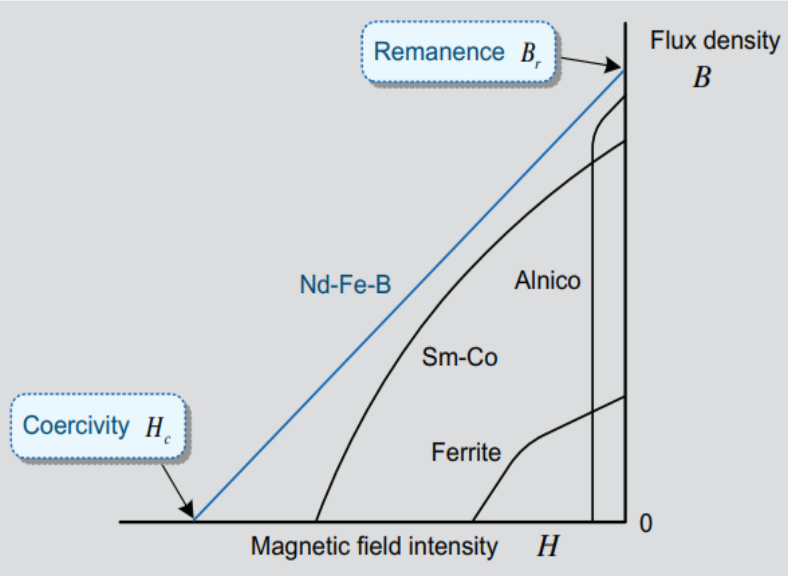
The most important permanent magnet materials:

- cast materials (e.g. Alnico, Ticonal),
- ceramic materials (e.g. Barium Ferrite),
- rare earth materials (e.g. SmCo5), and
- amorphous materials (e.g. NdFeB).

The main properties of a hard magnetic material:

- the intersection points of the demagnetising characteristic with the y-axis and the x-axis, i.e. the remanent induction B_r (for $H = 0$) and the coercive force H_c (for $B = 0$)
- the temperature coefficients of B_r and H_c
- the shape and specifically the slopes of the minor loops that determine the repetitive demagnetising and magnetising; these minor loops can be approximated by the recoil lines, i.e. straight lines with a slope almost equal to the slope of the demagnetising curve in B_r .

Permanent magnet synchronous motors



B/H characteristic curves of the magnetic materials

- cast materials (Alnico, Ticonal) yield a high induction but are rather susceptible to demagnetisation; the temperature coefficients are quite low (−2*10−4 for B_r and (−7 . . . + 3)*10−4 for H_c)
- ceramic materials (Sr- and Ba-ferrites) are less susceptible to demagnetisation but offer much lower induction values. Moreover, their temperature coefficients are rather high ($-2*10^{-3}$ for B_r and (+2 . . . + 5)$*10^{-3}$ for H_c)
- rare earth (SmCo5) and amorphous (NdFeB) permanent magnet materials show demagnetising characteristics that are relatively close to the ideal demagnetizing line with B_r >≈ 1 and dB/dH ≈ μ_o, but they are quite expensive. Their temperature coefficients are quite low (−5*10−4 for B_r and −3*10−4 for H_c).

Model of permanent magnet synchronous motors

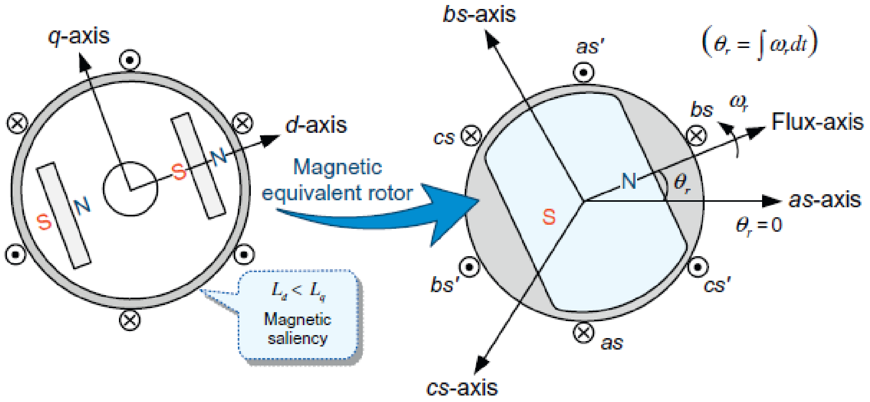
The stator voltage equations:

$$v_{abcs} = R_s i_{abcs} + \frac{d\lambda_{abcs}}{dt}$$

$$v_{abcs} = \begin{bmatrix} v_{as} & v_{bs} & v_{cs} \end{bmatrix}^T$$

$$i_{abcs} = \begin{bmatrix} i_{as} & i_{bs} & i_{cs} \end{bmatrix}^T$$

$$\lambda_{abcs} = \begin{bmatrix} \lambda_{as} & \lambda_{bs} & \lambda_{cs} \end{bmatrix}^T$$



Rotor and its equivalent of an IPMSM.

The flux linkage of an IPMSM has the following four components: the flux linkages due to the stator currents and the flux linkage due to the permanent magnet as:

$$\lambda_{as} = \lambda_{asas} + \lambda_{asbs} + \lambda_{ascs} + \phi_{asf}$$

$$\lambda_{bs} = \lambda_{bsas} + \lambda_{bsbs} + \lambda_{bscs} + \phi_{bsf}$$

$$\lambda_{cs} = \lambda_{csas} + \lambda_{csbs} + \lambda_{cscs} + \phi_{csf}$$

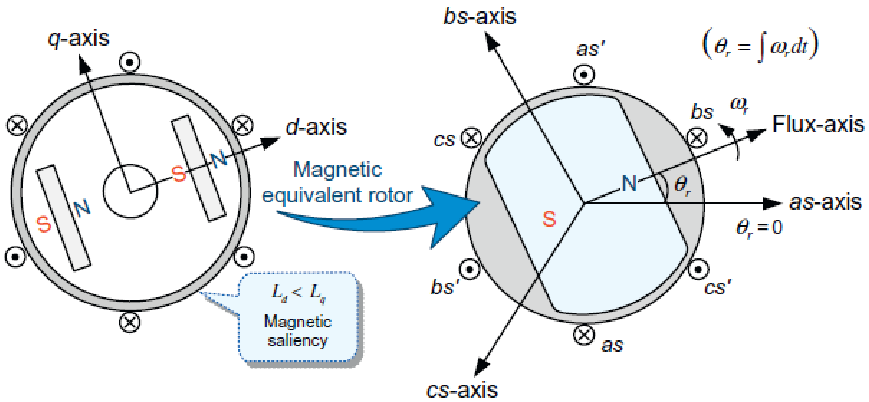
Model of permanent magnet synchronous motors

Flux linkages can be expressed as the product of the related current and inductance as:

$$\lambda_{as} = L_{asas} i_{as} + L_{asbs} i_{bs} + L_{ascs} i_{cs} + L_{asf} I_f$$
$$\lambda_{bs} = L_{bsas} i_{as} + L_{bsbs} i_{bs} + L_{bscs} i_{cs} + L_{bsf} I_f$$
$$\lambda_{cs} = L_{csas} i_{as} + L_{csbs} i_{bs} + L_{cscs} i_{cs} + L_{csf} I_f$$

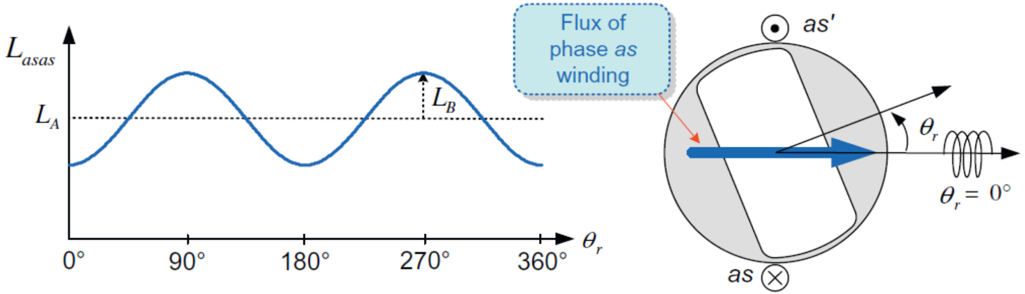
Determine the stator inductances of:

$$L_s = \begin{bmatrix} L_{asas} & L_{asbs} & L_{ascs} \\ L_{bsas} & L_{bsbs} & L_{bscs} \\ L_{csas} & L_{csbs} & L_{cscs} \end{bmatrix}$$



Rotor and its equivalent of an IPMSM.

Model of permanent magnet synchronous motors



Self-inductance of phase as winding with respect to rotor positions.

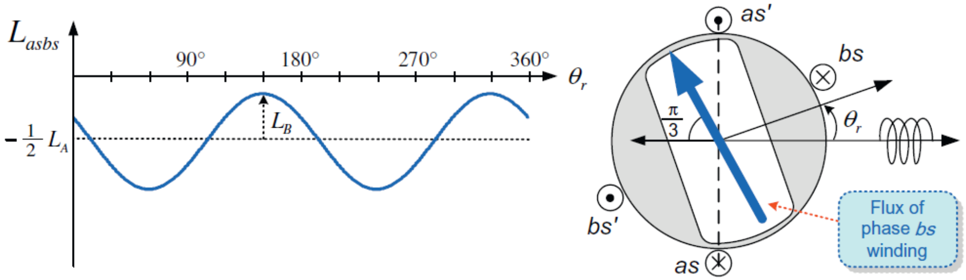
The self-inductances vary sinusoidally with respect to the rotor angle θ_r and can be expressed as:

$$L_{asas} = L_{ls} + L_A - L_B \cos 2\theta_r$$

$$L_{bsbs} = L_{ls} + L_A - L_B \cos 2\left(\theta_r - \frac{2\pi}{3}\right)$$

$$L_{cscs} = L_{ls} + L_A - L_B \cos 2\left(\theta_r + \frac{2\pi}{3}\right)$$

Model of permanent magnet synchronous motors



Mutual-inductance between the phase as and bs stator windings.

The mutual-inductances between the stator windings also vary sinusoidally with respect to the angle θ_r:

$$L_{asbs} = L_{bsas} = -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right)$$

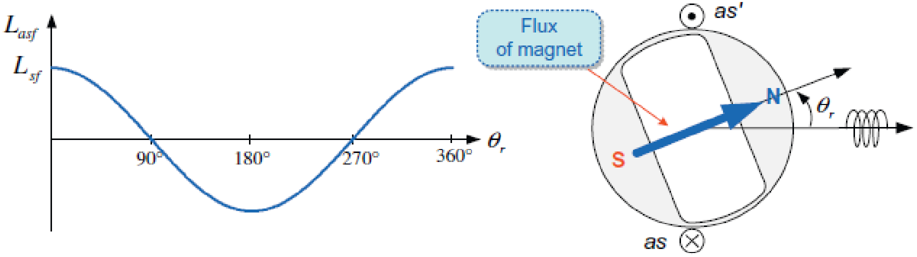
$$L_{ascs} = L_{csas} = -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right)$$

$$L_{bscs} = L_{csbs} = -\frac{1}{2}L_A - L_B \cos 2\theta_r$$

Model of permanent magnet synchronous motors

With these inductances, the stator inductance is given by:

$$L_s = \begin{bmatrix} L_{ls} + L_A - L_B \cos 2\theta_r & -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & L_{ls} + L_A - L_B \cos 2\left(\theta_r - \frac{2\pi}{3}\right) & -\frac{1}{2}L_A - L_B \cos 2\theta_r \\ -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) & -\frac{1}{2}L_A - L_B \cos 2\theta_r & L_{ls} + L_A - L_B \cos 2\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$



Mutual-inductance between stator as winding and magnet.

Unlike self-inductances, the mutual-inductances vary by $\cos\theta_r$ per one mechanical revolution of the rotor:

$$L_{asf} = L_{sf} \cos\theta_r$$

$$L_{bsf} = L_{sf} \cos\left(\theta_r - \frac{2\pi}{3}\right)$$

$$L_{csf} = L_{sf} \cos\left(\theta_r + \frac{2\pi}{3}\right)$$

Model of permanent magnet synchronous motors

From these inductances, the total flux linkages of an IPMSM are given by:

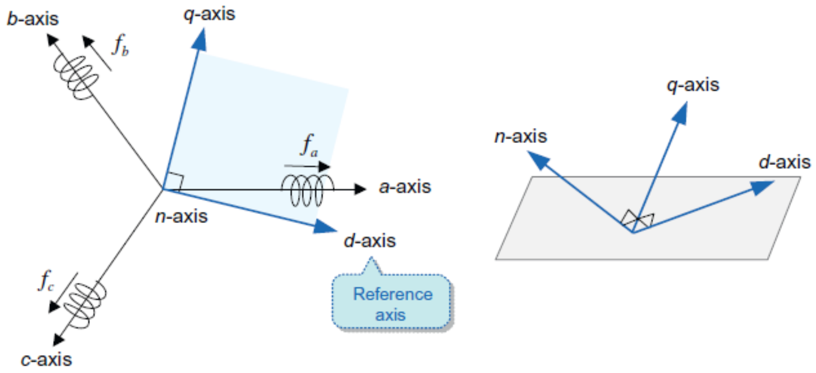
$$\lambda_{abcs} = L_s i_{abcs} + L_f I_f =$$

$$= \begin{bmatrix} L_{ls} + L_A - L_B \cos 2\theta_r & -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r - \frac{\pi}{3}\right) & L_{ls} + L_A - L_B \cos 2\left(\theta_r - \frac{2\pi}{3}\right) & -\frac{1}{2}L_A - L_B \cos 2\theta_r \\ -\frac{1}{2}L_A - L_B \cos 2\left(\theta_r + \frac{\pi}{3}\right) & -\frac{1}{2}L_A - L_B \cos 2\theta_r & L_{ls} + L_A - L_B \cos 2\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} +$$

$$+L_{sf} \begin{bmatrix} \cos\theta_r \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix} I_f$$

All the inductances in the flux linkages of an IPMSM are time varying except for at a standstill of the motor. Therefore, the time-varying coefficients will appear in the voltage equations of an IPMSM. In the case of an SPMSM, we can easily obtain the flux linkages by letting L_B=0:

Reference frame transformation



abc coordinate and d/q axes coordinate.

d-axis (direct axis)

The direction of the d variable, which is called the d-axis, is normally chosen as the direction of the magnetic flux in the AC motor. In the vector control for AC motors, the d-axis is regarded as the reference axis, and the flux-producing component of motor current is aligned along the d-axis.

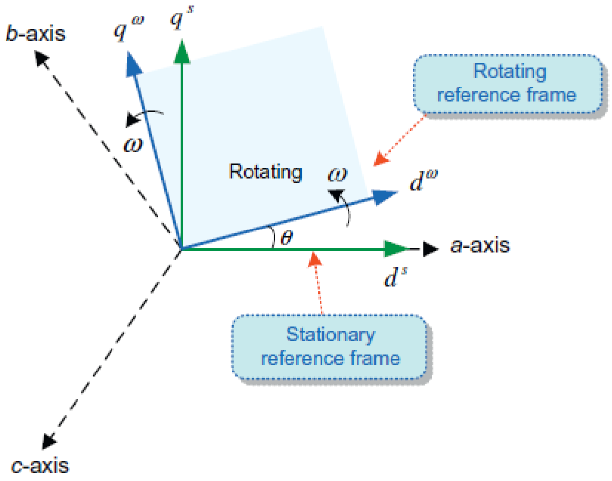
q-axis (quadrature axis)

The direction of the q variable, which is called the q-axis, is defined as the direction 90 ahead of the d-axis. In the vector control for AC motors, the torque-producing component of motor current or the back-EMF is aligned along the q-axis.

n-axis (neutral axis)

The direction of the n variable, which is called the n-axis, is defined as the direction that is orthogonal to both the d- and q-axes. The n-axis has nothing to do with the mechanical output power of the AC motor but is related to the losses.

Reference frame transformation



d/q axes stationary reference frame and rotating reference frame.

Stationary reference frame

This frame of reference remains stationary. In other words, in the stationary reference frame, the d/q coordinate system does not rotate.

This reference will be denoted by d^s/q^s axes. Normally, in AC motor drives, d^s-axis is chosen as the axis of phase *as*. This reference frame is also called stator reference frame.

Rotating reference frame

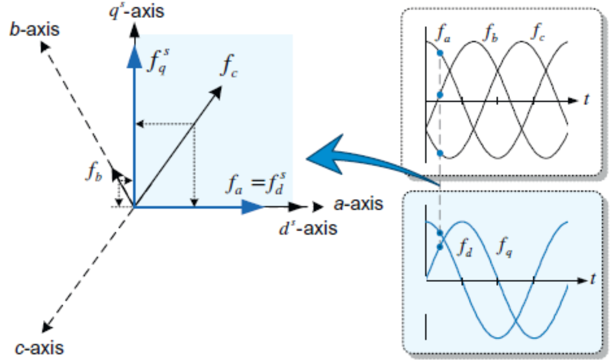
This frame of reference rotates at an angular speed ω. In other words, in the rotating reference frame, the d/q coordinate system rotates at a speed ω. The speed of rotation can be chosen arbitrarily.

This reference frame will be denoted by d^{ω}/q^{ω} axes.

The angle between the rotating reference frame and the stationary reference frame may vary over time. This angle θ is given by an integral of the angular velocity ω of the rotating reference frame as:

$$\theta = \int \omega(\tau)d\tau + \theta(0)$$

Reference frame transformation



d/q axes stationary reference frame and rotating reference frame.

The reference frame transformation can be simply considered as the orthogonal projection of the three-phase *abc* variables onto the d/q axis in the stationary reference frame:

$$f_d^s = k\left[f_a \cos(0) + f_b \cos(-\frac{2\pi}{3}) + f_c \cos(\frac{2\pi}{3}) \right]$$

$$f_q^s = k\left[f_a \cos(0) + f_b \sin(-\frac{2\pi}{3}) + f_c \sin(\frac{2\pi}{3}) \right]$$

The frame of reference can rotate at an angular velocity ω. So, the transformation of the three-phase stationary *abc* variables into *dqn* variables in the arbitrary reference frame rotating at a speed ω can be generally formulated as:

$$f_{dqn}^{\omega} = T(\theta) f_{abc}$$

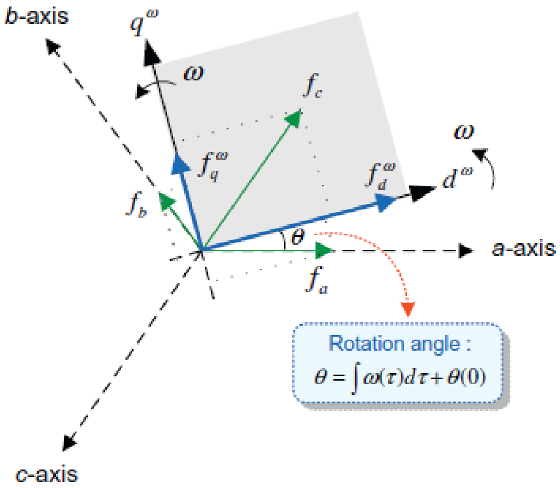
$$f_{dqn} = \begin{bmatrix} f_d & f_q & f_n \end{bmatrix}^T$$

$$f_{abc} = \begin{bmatrix} f_a & f_b & f_c \end{bmatrix}^T$$

The transformation matrix:

$$T(\theta) = \begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin\theta & -\sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Reference frame transformation



Transformation into the arbitrary rotating reference frame.

The coefficient k can be chosen arbitrarily. In the transformation matrix T(θ), the coefficient k is chosen as 2/3. In this case the magnitude of the *dq* variables is exactly identical to that of the *abc* variables, so, this transformation is called ***magnitude invariance transformation***.

However, the power and the torque evaluated in the *dqn* variables become 2/3 less than those evaluated in the *abc* variables, which will be shown later. On the other hand, when using the coefficient of √(2/3), the power remains the same value in the two reference frames, i.e., $P_{dqn}=P_{abc}$. So, this transformation is called ***power invariance transformation***.

However, the magnitude of the *dq* variables is not equal to the magnitude of the *abc* variables. The coefficient of 2/3 is normally used when applying the transformation to motor variables.

Reference frame transformation

From setting θ=0 in, the transformation of the three-phase *abc* variables into *dqn* variables in the stationary reference frame is given by:

$$f_{dqn}^{s}=T\left(0\right)f_{abc}=\frac{2}{3}\begin{bmatrix}1 & -\frac{1}{2} & -\frac{1}{2}\\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2}\\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}$$

This is known as **Clark's transformation.**

$$f_d^s=\frac{2f_a-f_b-f_c}{3}$$

$$f_q^s=\frac{1}{\sqrt{3}}\left(f_b-f_c\right)$$

$$f_n^s=\frac{\sqrt{2}\left(f_a+f_b+f_c\right)}{3}$$

In the stationary reference frame, the *dqn* variables are arithmetically related to the *abc* variables. In particular, if the sum of the *abc* variables in a balanced three-phase system with no neutral connection is zero, i.e., $f_a+f_b+f_c=0$, then the n-axis variable is zero, i.e., $f_s^n=0$. Thus, $f_s^d=f_a$, i.e., the d^s-axis variable is always equal to the phase *a*-axis variable. In this case the *dq* variables are reduced as following:

$$f_d^s=f_a$$

$$f_q^s=\frac{1}{\sqrt{3}}\left(f_b-f_c\right)$$

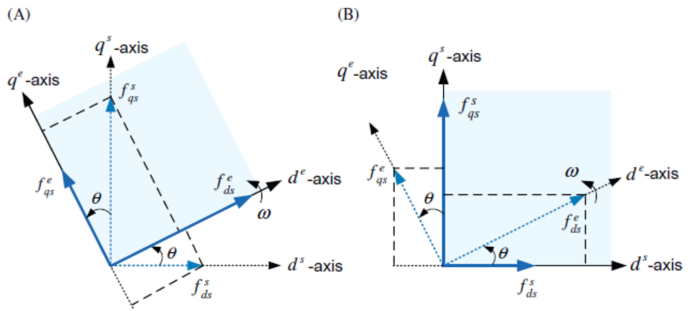
Inverse **Clark's transformation:**

$$f_a=f_d^s$$

$$f_b=-\frac{1}{2}f_d^s+\frac{\sqrt{3}}{2}f_q^s$$

$$f_c=-\frac{1}{2}f_d^s-\frac{\sqrt{3}}{2}f_q^s$$

Reference frame transformation



Transformation between reference frames. (A) Stationary into rotating frame and (B) rotating into stationary frame.

The transformation of the stationary reference frame into the rotating reference frame can be formulated as:

$$f_{dqn}^e = R(\theta)\, f_{dqn}^s = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f_d^s \\ f_q^s \\ f_n^s \end{bmatrix}$$

This is known as Park's transformation.

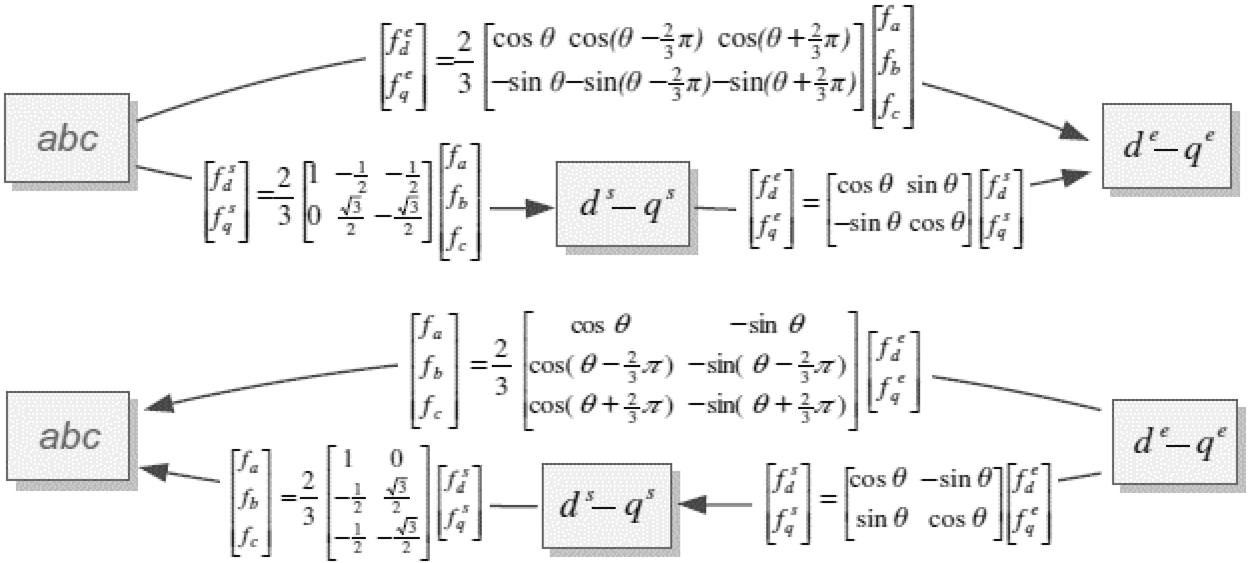
Transformation of stationary reference frame into rotating reference frame:

$$f_d^e = f_d^s \cos(\theta) + f_q^s \sin(\theta)$$
$$f_q^e = -f_d^s \sin(\theta) + f_q^s \cos(\theta)$$

Inverse transformation of stationary reference frame into rotating reference frame:

$$f_d^s = f_d^e \cos(\theta) - f_q^e \sin(\theta)$$
$$f_q^s = f_d^e \sin(\theta) + f_q^e \cos(\theta)$$

Reference frame transformation



Reference frame transformations.

Reference frame transformation

The instantaneous power may be expressed in *abc* variables as:

$$P = v_a i_a + v_b i_b + v_c i_c = V_{abc}^T I_{abc}$$

By transforming the above equation into the arbitrary reference frame rotating at any angular velocity ω, the instantaneous power is given by:

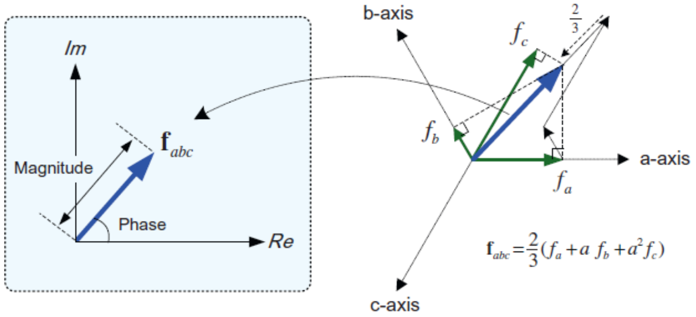
$$P = V_{abc}^T I_{abc} = \left[T^{-1}(\theta) V_{dqn}^{\omega}\right]^T \left[T^{-1}(\theta) I_{dqn}^{\omega}\right] =$$

$$= \frac{3}{2}\left(V_{dqn}^{\omega}\right)^T I_{dqn}^{\omega} = \frac{3}{2}\left(v_d^{\omega} i_d^{\omega} + v_q^{\omega} i_q^{\omega} + v_n^{\omega} i_n^{\omega}\right)$$

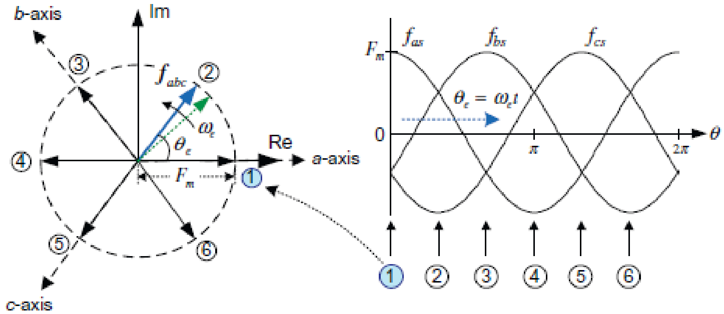
$$T^{-1}(\theta) = \frac{3}{2} T^T(\theta)$$

Although the power has a 3/2 factor due to the choice of the constant used in the transformation matrix T(θ), it is expressed as the sum of the product of the voltage and the current in each axis like the power expression in the three-phase system. The waveforms of the voltage, current, and flux linkage vary according to the angular velocity of the reference frame, whereas the waveform of the power remains unchanged.

Reference frame transformation



Complex space vector.



Three-phase quantities and complex space vector.

This vector is called the complex space vector, or in short, space vector. The complex vector representation allows us to analyze the three-phase system as a whole instead of looking at each phase individually. A space vector is different from a phasor, which is a representation of the amplitude and phase for a sine wave in the steady-state condition:

$$f_{abc} \equiv \frac{2}{3}\left(f_a + af_b + a^2 f_c \right)$$

$$a = e^{j(2\pi/3)}, \quad a^2 = e^{j(4\pi/3)}$$

Inversely, the quantities of a three-phase system from the space vector f_{abc} are given by:

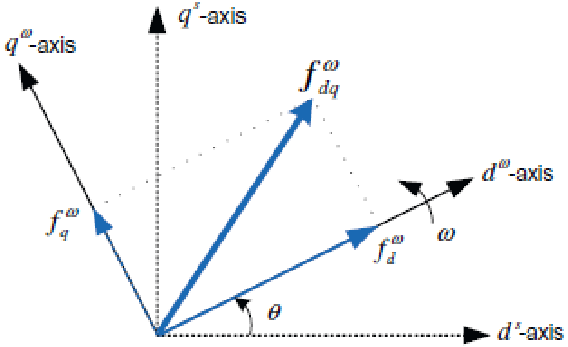
$$f_a = \mathrm{Re}\left[f_{abc} \right] + f_n^{\omega} =$$

$$= \mathrm{Re}\left[\frac{2}{3}\left(f_a + af_b + a^2 f_c \right) \right] + \frac{1}{3}\left(f_a + f_b + f_c \right)$$

$$f_b = \mathrm{Re}\left[a^2 f_{abc} \right] + f_n^{\omega}$$

$$f_c = \mathrm{Re}\left[a \; f_{abc} \right] + f_n^{\omega}$$

Reference frame transformation



Vector f^ω_{dq} in the arbitrary reference frame rotating at an angular velocity.

Thus, we can consider that the space vector f_{abc} equals to the vector f^s_{dq} in the stationary reference frame, which has the d- and q-axes components:

$$f_{dq}^s = f_d^s + jf_q^s$$

A vector f^ω_{dq} in the arbitrary reference frame rotating at a speed ω can be expressed as:

$$f_{dq}^\omega = f_d^\omega + jf_q^\omega$$

The d- and q-axes components of this vector are given by:

$$f_d^\omega = \frac{2}{3}\left[f_a \cos\theta + f_b \cos\left(\theta - \frac{2}{3}\pi\right) + f_b \cos\left(\theta - \frac{4}{3}\pi\right)\right]$$

$$f_d^\omega = \frac{2}{3}\left[f_a \sin\theta + f_b \sin\left(\theta - \frac{2}{3}\pi\right) + f_b \sin\left(\theta - \frac{4}{3}\pi\right)\right]$$

Reference frame transformation

Using Euler formula

$$f_{dq}^{\omega} = \frac{2}{3}\left[\begin{array}{l} f_a\left(\cos\theta - j\sin\theta\right) + f_b\left(\cos\left(\theta - \frac{2\pi}{3}\right) - j\sin\left(\theta - \frac{2\pi}{3}\right)\right) + \\ + f_c\left(\cos\left(\theta - \frac{4\pi}{3}\right) - j\sin\left(\theta - \frac{4\pi}{3}\right)\right) \end{array}\right] =$$

$$= \frac{2}{3}\left[f_a e^{-j\theta} + f_a e^{-j\left(\theta - \frac{2\pi}{3}\right)} + f_a e^{-j\left(\theta - \frac{4\pi}{3}\right)} \right] = \frac{2}{3}\left[f_a + a f_a + a^2 f_a \right] e^{-j\theta} =$$

$$= f_{abc} e^{-j\theta}$$

By letting θ=0, we can identify that the space vector f_{abc} equals to the vector f^{s}_{dq} in the dq axis of the stationary reference frame as:

$$f_{dq}^{s} = f_{abc} e^{-j0} = f_{abc}$$

Vector f^{ω}_{dq} in the rotating reference frame with the angular velocity $\omega=\omega_e$ can be expressed as

$$f_{dq}^{e} = f_{abc} e^{-j\theta_e} = f_{dq}^{s} e^{-j\theta_e}$$

The instantaneous power is expressed in terms of the complex vector as:

$$P = \frac{3}{2}\mathrm{Re}\left[V_{abc} I_{abc}^{*}\right] = \frac{3}{2}\mathrm{Re}\left[V_{dq}^{\omega} I_{dq}^{\omega *}\right]$$

where I^{*}_{abc}; I^{*}_{dq} are the complex conjugates of I_{abc}, I_{dq}, respectively.

d-q axes model of a permanent magnet synchronous motor

The dq axes stator voltage equations

$$v_{ds}^{\omega} = R_s i_{ds}^{\omega} + \frac{d\lambda_{ds}^{\omega}}{dt} - \omega\lambda_{qs}^{\omega}$$

$$v_{qs}^{\omega} = R_s i_{qs}^{\omega} + \frac{d\lambda_{qs}^{\omega}}{dt} + \omega\lambda_{ds}^{\omega}$$

$$v_{ns}^{\omega} = R_s i_{ns}^{\omega} + \frac{d\lambda_{ns}^{\omega}}{dt}$$

The transformation of the stator flux linkage of an IMPSM into the dq axes rotating at an arbitrary speed ω is as follows:

$$\lambda_{abcs} = L_s i_{abcs} + L_f i_f$$

$$T(\theta)\lambda_{abcs} = T(\theta) L_s i_{abcs} + T(\theta) L_f i_f$$

$$\lambda_{dqns}^{\omega} = T(\theta) L_s \left(T^{-1}(\theta) i_{dqns}^{\omega}\right) + T(\theta) L_f i_f$$

d-q axes model of a permanent magnet synchronous motor

Transformations of inductances needed by the above equation are as follows.

From the following stator inductance:

$$L_s = \begin{bmatrix} L_{ls}+L_A & -\frac{1}{2}L_A & -\frac{1}{2}L_A \\ -\frac{1}{2}L_A & L_{ls}+L_A & -\frac{1}{2}L_A \\ -\frac{1}{2}L_A & -\frac{1}{2}L_A & L_{ls}+L_A \end{bmatrix} - L_B \begin{bmatrix} \cos 2\theta_r & \cos 2\left(\theta_r - \frac{\pi}{3}\right) & \cos 2\left(\theta_r + \frac{\pi}{3}\right) \\ \cos 2\left(\theta_r - \frac{\pi}{3}\right) & \cos 2\left(\theta_r - \frac{2\pi}{3}\right) & \cos 2\theta_r \\ \cos 2\left(\theta_r + \frac{\pi}{3}\right) & \cos 2\theta_r & \cos 2\left(\theta_r - \frac{4\pi}{3}\right) \end{bmatrix}$$

$$\Downarrow$$

$$T(\theta) L_s T^{-1}(\theta) = \begin{bmatrix} L_{ls}+\frac{3}{2}L_A & 0 & 0 \\ 0 & L_{ls}+\frac{3}{2}L_A & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} + \frac{3}{2}L_B \begin{bmatrix} -\cos 2(\theta-\theta_r) & \sin 2(\theta-\theta_r) & 0 \\ \sin 2(\theta-\theta_r) & \cos 2(\theta-\theta_r) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From the inductance:

$$L_f = L_{sf} \begin{bmatrix} \cos\theta_r \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{4\pi}{3}\right) \end{bmatrix}$$

$$\Downarrow$$

$$T(\theta) L_f = L_{sf} \begin{bmatrix} \cos(\theta-\theta_r) \\ -\sin(\theta-\theta_r) \\ 0 \end{bmatrix}$$

d-q axes model of a permanent magnet synchronous motor

The stator flux linkage in the dq axes rotating at an arbitrary speed ω is given as

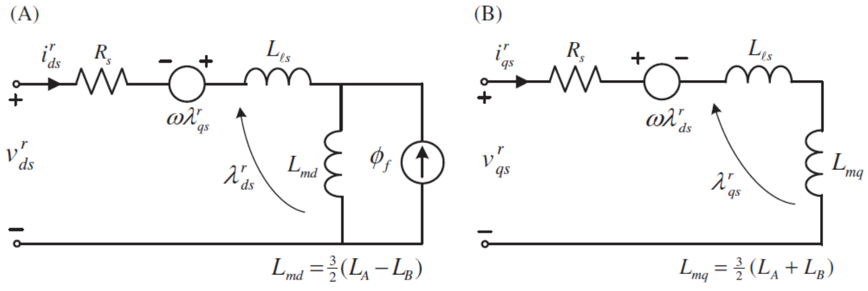
$$\lambda_{dqs}^{\omega} = \begin{bmatrix} L_{ls} + \frac{3}{2}\left(L_A - L_B \cos 2\left(\theta - \theta_r\right)\right) & \frac{3}{2}\sin 2\left(\theta - \theta_r\right) & 0 \\ \frac{3}{2}\sin 2\left(\theta - \theta_r\right) & L_{ls} + \frac{3}{2}\left(L_A + L_B \cos 2\left(\theta - \theta_r\right)\right) & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} i_{dqs}^{\omega} + \begin{bmatrix} \cos\left(\theta - \theta_r\right) \\ -\sin\left(\theta - \theta_r\right) \\ 0 \end{bmatrix} \phi_f$$

$$\phi_f = L_{sf} i_f$$

$$L_{ds} = L_{ls} + \frac{3}{2}\left(L_A - L_B\right), \quad L_{qs} = L_{ls} + \frac{3}{2}\left(L_A + L_B\right)$$

$$\lambda_{dqs}^{\omega} = \begin{bmatrix} \frac{L_{ds} + L_{qs}}{2} + \frac{L_{ds} - L_{qs}}{2}\cos 2\left(\theta - \theta_r\right) & \frac{L_{ds} - L_{qs}}{2}\sin 2\left(\theta - \theta_r\right) & 0 \\ \frac{L_{ds} - L_{qs}}{2}\sin 2\left(\theta - \theta_r\right) & \frac{L_{ds} + L_{qs}}{2} - \frac{L_{ds} - L_{qs}}{2}\cos 2\left(\theta - \theta_r\right) & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} i_{dqs}^{\omega} + \begin{bmatrix} \cos\left(\theta - \theta_r\right) \\ -\sin\left(\theta - \theta_r\right) \\ 0 \end{bmatrix} \phi_f$$

d-q axes model of a permanent magnet synchronous motor



d^r/q^r axes equivalent circuit of an IPMSM. (A) d^r-axis and (B) q^r-axis.

The stator flux linkages with a constant inductance in the rotor reference frame:

$$\lambda_{dqn}^r=\begin{bmatrix} L_{ds} & 0 & 0 \\ 0 & L_{qs} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}\begin{bmatrix} i_{ds}^r \\ i_{qs}^r \\ i_{ns}^r \end{bmatrix}+\begin{bmatrix} \phi_f \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda_{ds}^r=L_{ds}^r i_{ds}^r+\phi_f$$

$$\lambda_{qs}^r=L_{qs}^r i_{qs}^r$$

$$\lambda_{ns}^r=L_{ls}^r i_{ns}^r$$

By letting $L_{ds}=L_{qs}$ in the above equations, we can obtain the flux linkages of an SPMSM with a cylindrical rotor.

This stator flux linkage can be expressed in the stationary reference frame (θ=0) as

$$\lambda_{dqs}^s=\begin{bmatrix} \frac{L_{ds}+L_{qs}}{2}+\frac{L_{ds}-L_{qs}}{2}\sin 2\theta_r & \frac{L_{ds}-L_{qs}}{2}\sin 2\theta_r & 0 \\ \frac{L_{ds}-L_{qs}}{2}\sin 2\theta_r & \frac{L_{ds}+L_{qs}}{2}-\frac{L_{ds}-L_{qs}}{2}\sin 2\theta_r & 0 \\ 0 & 0 & L_{ls} \end{bmatrix} i_{dqs}^s+\begin{bmatrix} \cos\theta_r \\ -\sin\theta_r \\ 0 \end{bmatrix}\phi_f$$

d-q axes model of a permanent magnet synchronous motor

The input power may be expressed in the dq axes as:

$$P_{in} = \frac{3}{2}\left[v_{ds}^r i_{ds}^r + v_{qs}^r i_{qs}^r \right]$$

$$P_{in} = \frac{3}{2}\left(\left(R_s i_{ds}^r + \frac{d\lambda_{ds}^r}{dt} - \omega_r \lambda_{qs}^r\right) i_{ds}^r - \left(R_s i_{qs}^r + \frac{d\lambda_{qs}^r}{dt} + \omega_r \lambda_{qs}^r\right) i_{qs}^r\right) =$$

$$= \frac{3}{2}\left(R_s \left(i_{ds}^{r2} + i_{qs}^{r2}\right) + i_{ds}^r \frac{d\lambda_{ds}^r}{dt} + i_{qs}^r \frac{d\lambda_{qs}^r}{dt} + \omega_r \phi_f i_{qs}^r + \omega_r \left(L_{ds} - L_{qs}\right) i_{ds}^r i_{qs}^r\right)$$

The output torque is obtained by dividing the output power by the rotor speed ω_r as

$$T_e = \frac{p}{2}\frac{3}{2}\left[\phi_f i_{qs}^r + \left(L_{ds} - L_{qs}\right) i_{ds}^r i_{qs}^r \right]$$

For the torque of an SPMSM with a cylindrical rotor, by letting $L_{ds} = L_{qs} = L_s$, we can obtain:

$$T_e = \frac{p}{2}\frac{3}{2}\phi_f i_{qs}^r$$

d-q axes model of a permanent magnet synchronous motor

Voltage and flux linkage equations of an IPMSM in the d^r /q^r axes

$$v_{ds}^r = R_s i_{ds}^r + \frac{d\lambda_{ds}^r}{dt} - \omega\lambda_{qs}^r$$

$$v_{qs}^r = R_s i_{qs}^r + \frac{d\lambda_{qs}^r}{dt} + \omega\lambda_{ds}^r$$

$$\lambda_{ds}^r = L_{ds} i_{ds}^r + \phi_f$$

$$\lambda_{qs}^r = L_{qs} i_{qs}^r$$

$$T_e = \frac{p}{2}\frac{3}{2}\left[\phi_f i_{qs}^r + \left(L_{ds} - L_{qs}\right) i_{ds}^r i_{qs}^r\right]$$

Thank you!

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