Electrical Machines

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Transformers

Part II

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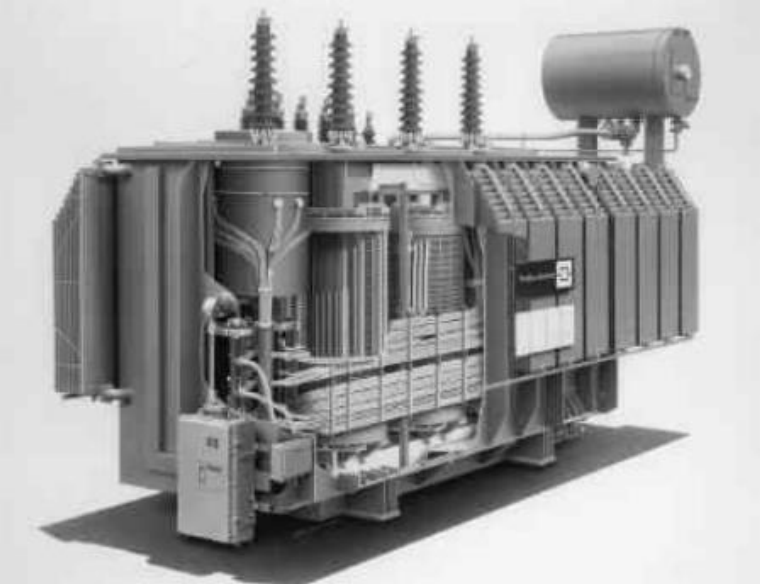
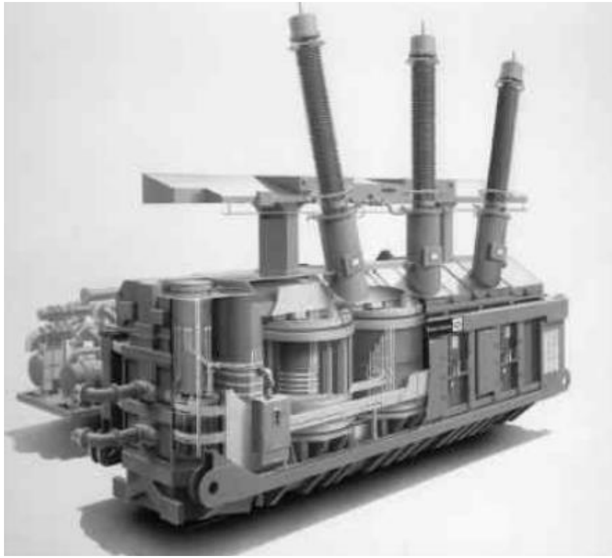
Three-phase transformers

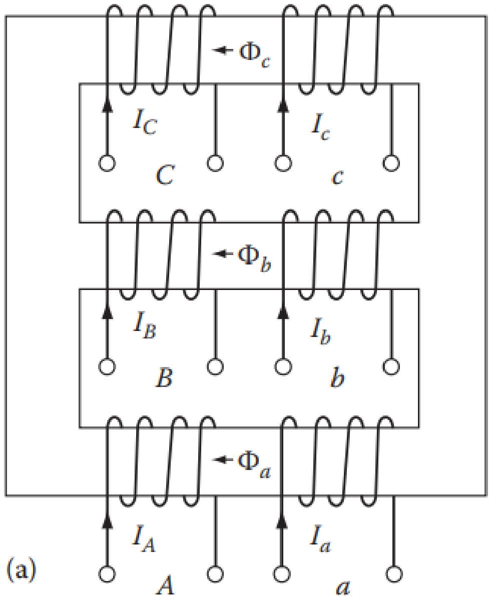


A 40MVA, 110 kV ± 16%/21 kV, three-phase, core-type transformer, 5.2m high, 9.4m long, 3m wide, weighing 80 tons. (Courtesy of Siemens AG, Munich, Germany.)

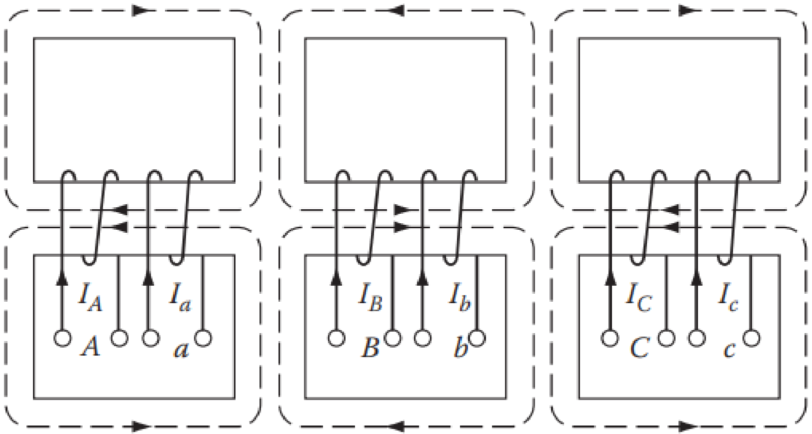


A 850/950/1100XWA, 415 kV ± 11%/27 kV, three-phase, shell-type transformer, 11.3 in high, 14 in long, 5.7 in wide, weighing (without cooling oil) 552 tons. (Courtesy of Siemens AG, Munich, Germany.)

Three-phase transformers



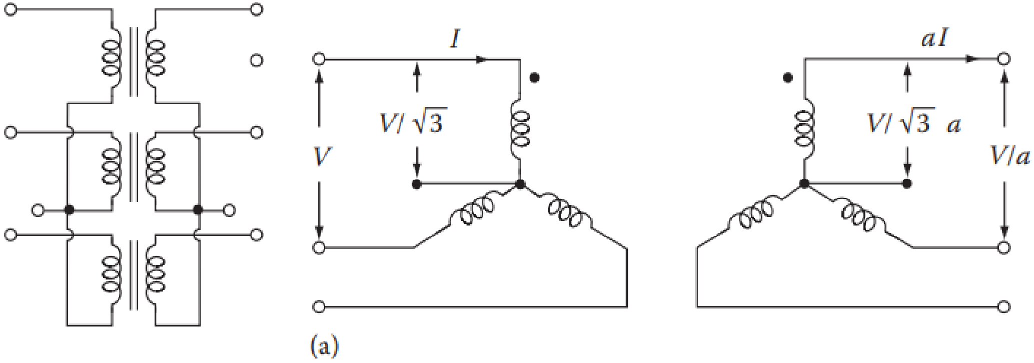
Core type three-phase transformer



Shell type three-phase transformer

Three-phase transformers

The wye–wye connection has two very serious problems:
1. If loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.
2. Third-harmonic voltages can be large.



wye–wye connection

Relations between line values and winding values of currents and voltages:

$$V_{L1} = \sqrt{3} \cdot V_1$$
$$V_{L2} = \sqrt{3} \cdot V_2$$
$$I_{L1} = I_1$$
$$I_{L2} = I_2$$

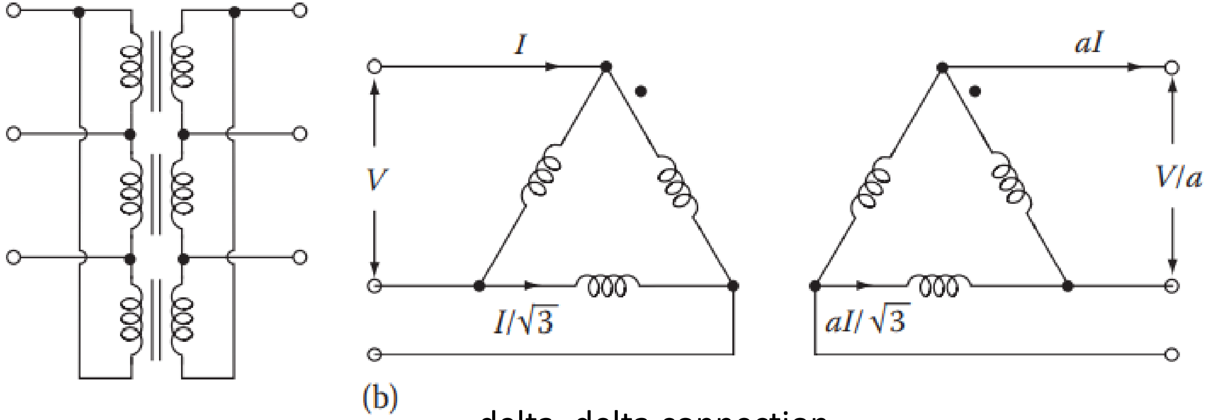
Three-phase transformers

Both the unbalance problem and the third-harmonic problem can be solved using one of two techniques:

1. ***Solidly ground the neutrals of the transformers***, especially the primary winding's neutral. This connection permits the additive third-harmonic components to cause a current flow in the neutral instead of building up large voltages. The neutral also provides a return path for any current imbalances in the load.

2. Add a ***third (tertiary) winding*** connected in Δ to the transformer bank. If a third Δ-connected winding is added to the transformer then the third-harmonic components of voltage in the Δ will add up, causing a circulating current flow within the winding. This suppresses the third-harmonic components of voltage in the same manner as grounding the transformer neutrals.

Three-phase transformers



delta–delta connection

Relations between line values and winding values of currents and voltages:

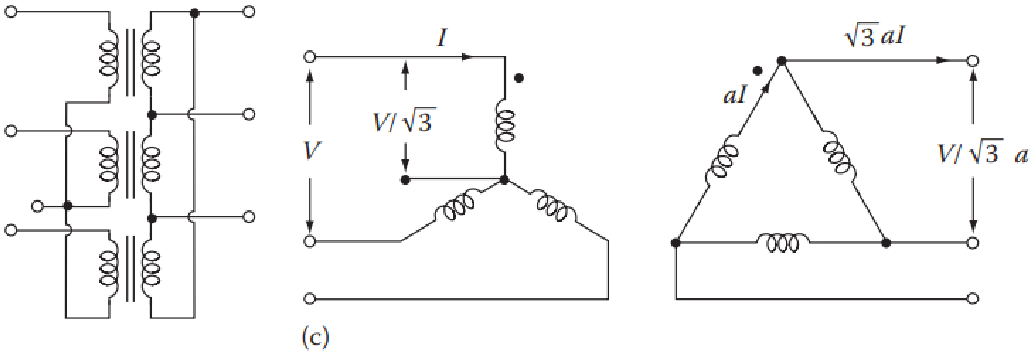
$$V_{L1} = V_1$$
$$V_{L2} = V_2$$
$$I_{L1} = \sqrt{3} \cdot I_1$$
$$I_{L2} = \sqrt{3} \cdot I_2$$

Three-phase transformers

In the wye–delta connection, there is no problem with third-harmonic components in its voltages, since they are absorbed in a circulating current on the delta side.

This connection can be used with unbalanced loads. In high-voltage transmission systems, the high-voltage side is connected in delta and the low-voltage side is connected in wye.

Due to the delta connection, the secondary voltage is shifted 30° with respect to the primary voltage.



wye–delta connection

Relations between line values and winding values of currents and voltages:

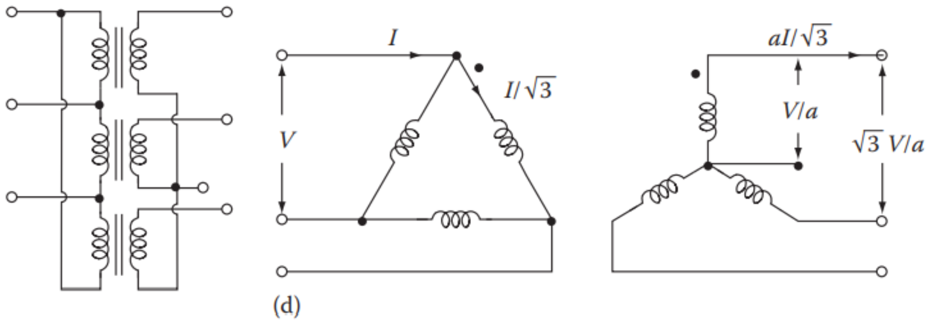
$$V_{L1} = \sqrt{3} \cdot V_1$$
$$V_{L2} = V_2$$
$$I_{L1} = I_1$$
$$I_{L2} = \sqrt{3} \cdot I_2$$

Three-phase transformers

In the delta–wye connection, there is also no problem with third-harmonic components in its voltages. It has the same advantages and the same phase shift as the wye–delta connection.

The secondary voltage lags the primary voltage by 30°, as is the case for the wye–delta connection. This connection is basically used to step up a low voltage to a high voltage.

In general, when a wye–delta or delta–wye connection is used, the wye is preferably on the high-voltage side, and the neutral is grounded.



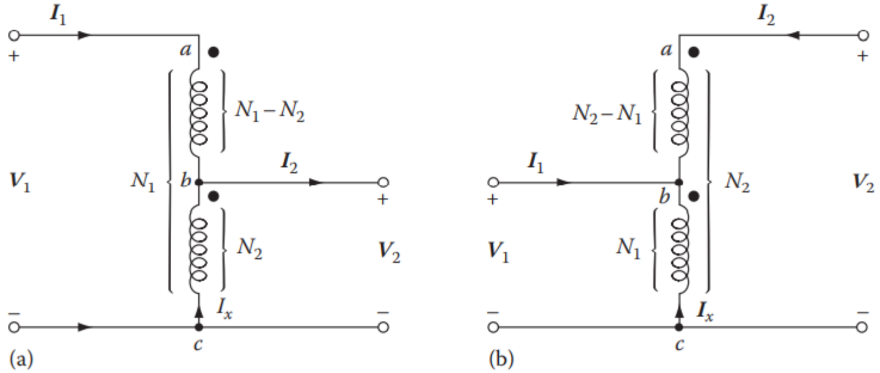
delta-wye connection

Relations between line values and winding values of currents and voltages:

$$V_{L1} = V_1$$
$$V_{L2} = \sqrt{3} \cdot V_2$$
$$I_{L1} = \sqrt{3} \cdot I_1$$
$$I_{L2} = I_2$$

Autotransformers

- An autotransformer has a single winding, part of which is common to both the primary and the secondary simultaneously.
- In an autotransformer, there is no electrical isolation between the input side and the output side.
- The power is transferred from the primary to the secondary through both induction and conduction. An autotransformer can be used as a step-down or step-up transformer.



step-down autotransformer

step-up autotransformer

Autotransformers

For a step-down autotransformer when a > 1, since the excitation current is neglected, then I_1 and I_2 are in phase, and the current in the common section of the winding as:

$$I_x = I_2 - I_1$$

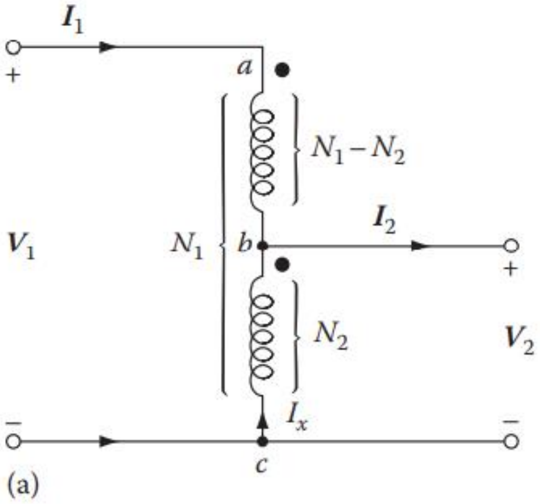
The mmfs of the two windings are equal.

Thus, according to the mmf balance equation:

$$N_2 I_x = (N_1 - N_2) I_1$$

$$\Downarrow$$

$$I_x = I_2 - I_1 = aI_1 - I_1 = (a-1)I_1 = \frac{N_1 - N_2}{N_2} I_1$$



step-down autotransformer

Autotransformers

The turn ratio ***a*** for step-down autotransformer can be evaluated as:

$$a = \frac{N_1}{N_2} = \frac{N_c + N_s}{N_c}$$

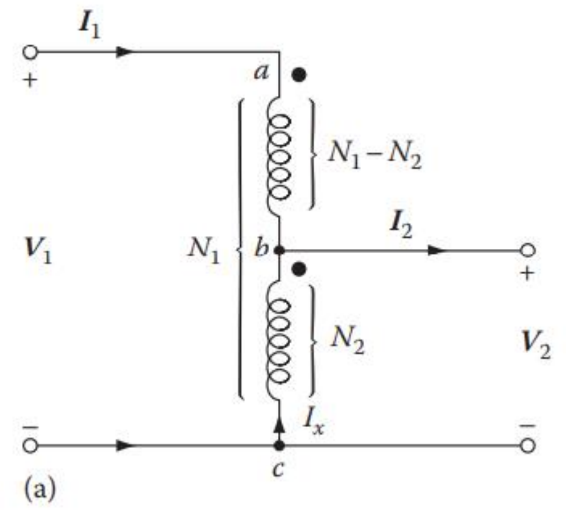
then:

$$\frac{I_2}{I_1} = \frac{N_c + N_s}{N_c}$$

where:

N_c is the number of turns in common winding = N_2

N_s is the number of turns in series winding = $N_1 - N_2$



step-down autotransformer

Autotransformers

Similarly, it can be shown that:

$$\frac{V_2}{V_1} = \frac{N_c}{N_c + N_s}$$

The apparent power delivered to the load is S_{out} and can be expressed as:

$$S_{out} = V_2 I_2 = V_2 I_1 + V_2 (I_2 - I_1) = S_{cond} + S_{ind}$$

where:

S_{cond} is the conductively transferred power to the load through N_2 winding

S_{ind} is the inductively transferred power to the load through $N_1 - N_2$ winding

The S_{cond} and S_{ind} are related to S_{out} by:

$$\frac{S_{cond}}{S_{out}} = \frac{I_1}{I_2} = \frac{1}{a} = \frac{N_2}{N_1} \qquad \frac{S_{ind}}{S_{out}} = \frac{I_2 - I_1}{I_2} = \frac{a-1}{a} = \frac{N_s}{N_c + N_s} = \frac{N_1 - N_2}{N_1}$$

Autotransformers

Similarly for a step-up autotransformer when a < 1, since the excitation current is neglected, then I_1 and I_2 are in phase, and the current in the common section of the winding as:

$$I_x = I_2 - I_1$$

The apparent power delivered to the load is S_{out} and can be expressed as:

$$S_{out} = V_2 I_2 = V_1 I_1 = V_1 I_2 + V_1 (I_1 - I_2) = S_{cond} + S_{ind}$$

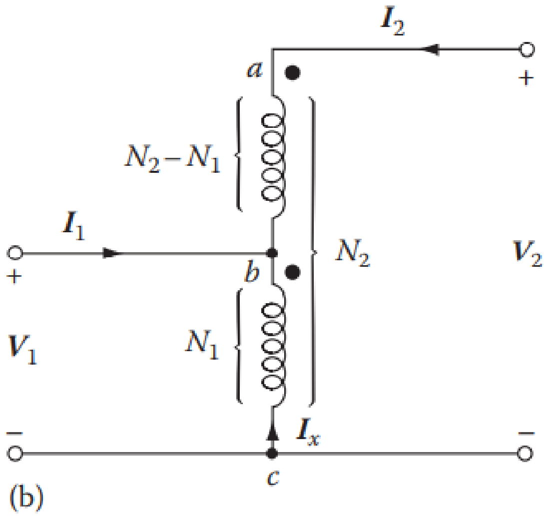
where,

S_{cond} is the conductively transferred power to the load through N_2 winding

S_{ind} is the inductively transferred power to the load through $N_2 - N_1$ winding

The S_{cond} and S_{ind} are related to S_{out} by:

$$\frac{S_{cond}}{S_{out}} = \frac{I_2}{I_1} = a \qquad \frac{S_{ind}}{S_{out}} = \frac{I_1 - I_2}{I_1} = 1 - a \qquad a = \frac{N_c}{N_c + N_s}$$



step-up autotransformer

Autotransformers

The advantages:

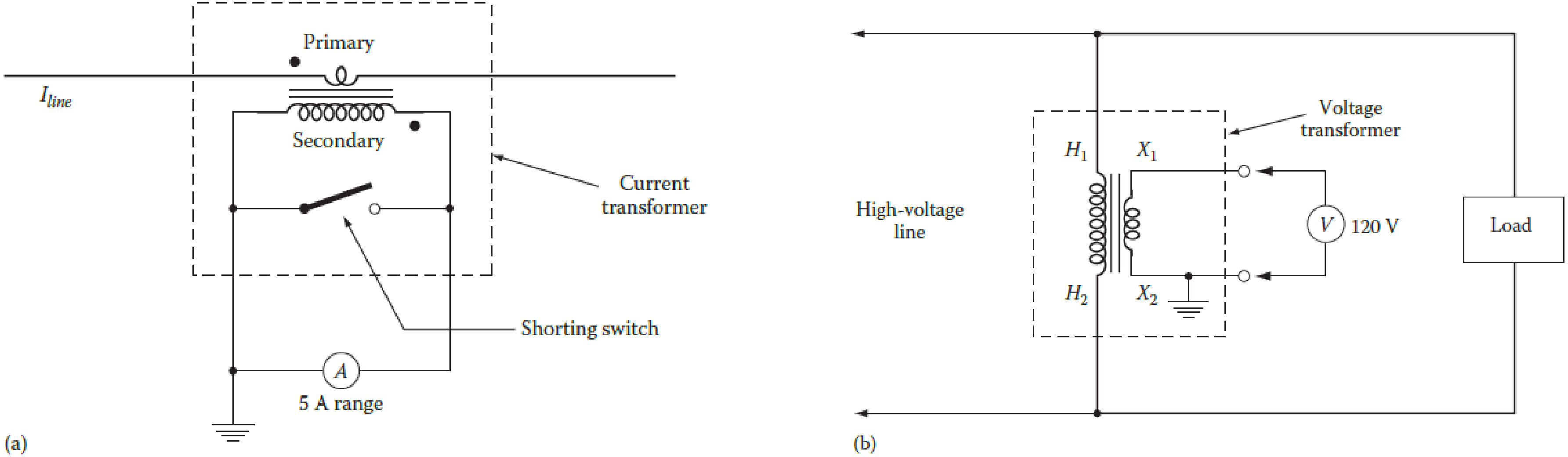
- lower leakage reactances
- lower losses
- smaller excitation current requirements
- an autotransformer is cheaper than the equivalent two-winding transformer (especially when the voltage ratio does not vary too greatly from 1 to 1).

The disadvantages:

- no electrical isolation between the primary and secondary
- a greater short-circuit current than for the two-winding transformer.

Three-phase autotransformer banks generally have wye-connected main windings with the neutral normally connected solidly to ground. In addition, it is common practice to include a third winding connected in delta, called the tertiary winding.

Instrument transformers



Instrument transformer connections: (a) current transformer connection and (b) voltage transformer connection.

Inrush current

Occasionally, upon energizing a power transformer, a **transient phenomenon** (due to magnetizing current characteristics) takes place even if there is no load connected to its secondary.

As a result, **its magnetizing current peak may be several times** (about 8–10 times) the rated transformer current, or it may be practically unnoticeable.

Because of losses in the excited winding and magnetic circuit, this current ultimately decreases to the normal value of the excitation current (i.e., to about 5% or less of the rated transformer current).

Such a transient event is known as the **inrush current** phenomenon.

It may cause **(1) a momentary dip in the voltage if the impedance of the excitation source is significant**, **(2) undue stress in the transformer windings**, or **(3) improper operation of protective devices**

Inrush current

The magnitude of such an inrush current depends on the magnitude, polarity, and rate of change in applied voltage at the time of switching. For example, assume that the applied voltage, at t = 0, happens to be:

$$v(t) = \sqrt{2}V_1 \sin \omega t = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi}{dt}$$

The resultant flux is:

$$\Phi = \frac{\sqrt{2}V_1}{N_1} \int_0^t \sin \omega t dt + \Phi_0$$

where $\Phi(0) = \Phi_r$ (i.e., the residual flux). Therefore,

$$\Phi = \frac{\sqrt{2}V_1}{\omega N_1}\left(1 - \cos \omega t\right) + \Phi_r = -\Phi_m \cos \omega t + \Phi_m + \Phi_r$$

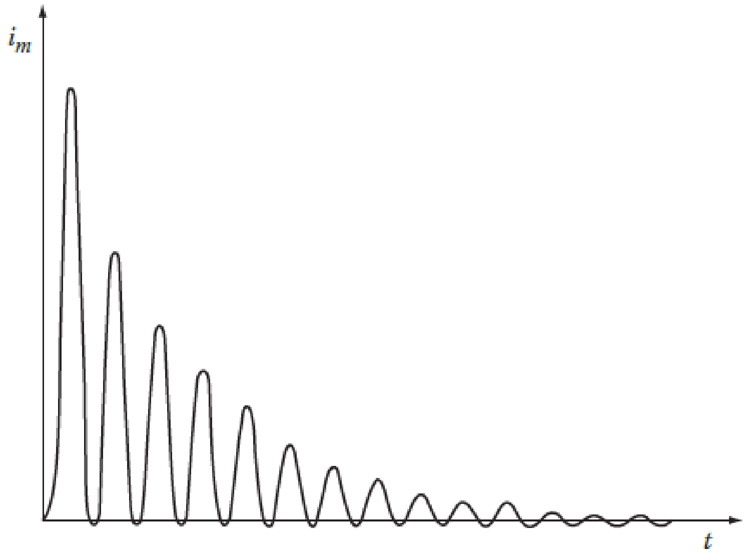
Inrush current

If the dc component flux $\Phi_m + \Phi_r$ is constant, at $\omega t = \pi$, the instantaneous flux is:

$$\Phi = 2\Phi_m + \Phi_r$$

That is, the maximum value of the flux may be more than twice the maximum of the normal flux, since there is often residual magnetism in the core when it is initially energized.

Such doubling of the maximum flux in the core causes a tremendously large magnetization current.



Inrush current phenomenon in a power transformer

Laboratory task

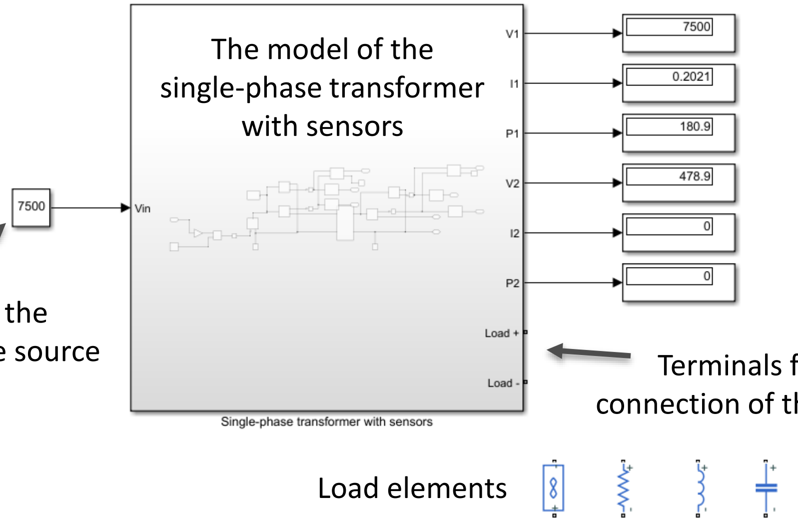
The research of the single-phase two winding transformer

Exercises:

- ❑ Determination of the equivalent circuit parameters
- ❑ Determination of the performance characteristics

Description of the Simulink model

RMS values of voltages, currents and active powers



RMS value of the sinusoidal voltage source

Terminals for connection of the load

Load elements

Laboratory task

Determination of the equivalent circuit parameters. Part 1.

1.1 Open the m-file which will have been sent to you with the model:

Single_phase_transformer.slx
transformer_lab_sc.m

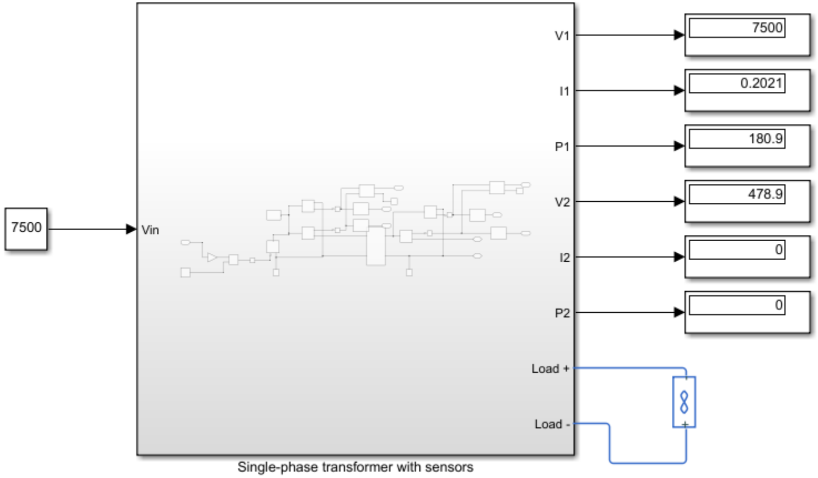
1.2 Complete m-file with rated parameters of the transformer using nameplate information:

```
% Transformer nameplate information: 15 kVA, 7500/480 V, 60 Hz
% Add following parameters to m-file: rated primary voltage, rated secondary voltage,
% rated primary current, rated secondary current.
```

Laboratory task

Determination of the equivalent circuit parameters. Part 2.

2.1 Perform open-circuit test (use rated primary voltage):



2.2 Complete m-file with data of the open-circuit test and write a script for calculation of the magnetization branch parameters (R_c, X_m):

```
%% Open-circuit test
% Simulate the open-circuit test and then add following parameters to m-file: Voc, Ioc, Poc


% Write script for determination of the magnetization branch parameters (Rc, Xm)
```

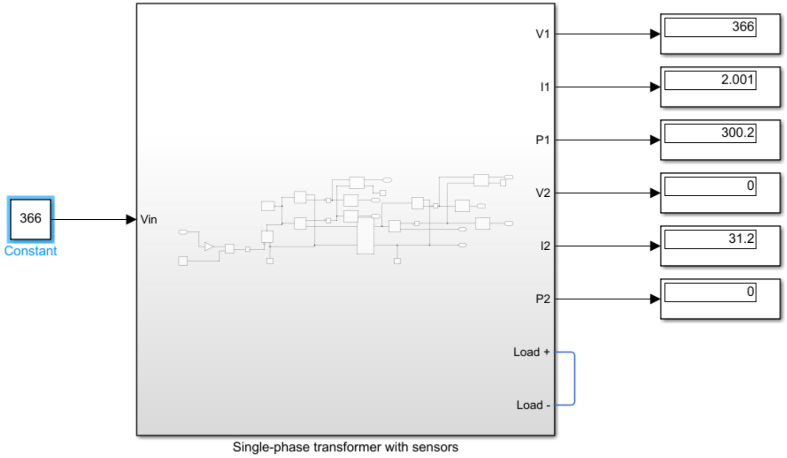
2.3 Draw phasor diagram for open-circuit test.

ITMO

Laboratory task

Determination of the equivalent circuit parameters. Part 3.

3.1 Make short-circuit test (you must find the primary voltage value when there are rated current in primary winding):



3.2 Complete m-file with data of the short-circuit test and write a script for calculation of the leakage resistances (R_1, R_2) and reactances (X_{l1}, X_{l2}) :

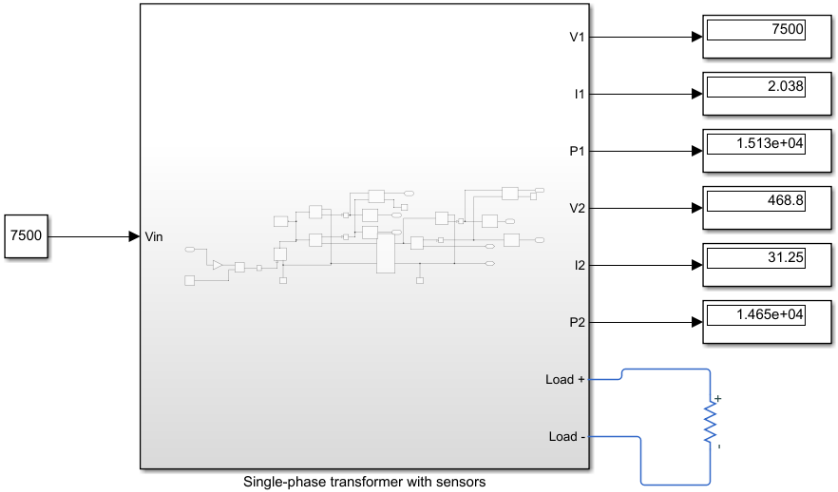
```
%% Short - circuit test
% Simulate the short-circuit test and then add following parameters to m-file: Vsc, Isc, Psc


% Write script for determination of the leakage resistances (R1, R2) and reactances (Xl1, Xl2)
```

3.3 Draw phasor diagram for short-circuit test.

Laboratory task

Determination of the performance characteristics. Part 1.

1.1 Make 10 tests with R-loads (values from m-file):



1.2 Complete m-file with data of R-load test.

Laboratory task

Determination of the performance characteristics. Part 1.

1.3 Calculate load ratio, load power factor, transformer efficiency and voltage regulation:

$$\beta = \frac{I_2}{I_{2\_rated}} \qquad \cos\theta_{load} = \frac{P_2}{V_2 \cdot I_2} \qquad \eta = \frac{P_2}{P_2 + \beta^2 P_{cu} + P_{core}}$$

$$\Delta V\%_{2\_\mathrm{mod}} = \frac{\left(V_{2\_no\_load} - V_{2\_load}\right)\times 100}{V_{2\_load}}$$

$$\Delta V\%_{2\_calc} = \frac{\beta \frac{V_{sc}}{a}(\cos\theta_{sc}\cos\theta_{load} + \sin\theta_{sc}\sin\theta_{load})\times 100}{V_{2\_load}}$$

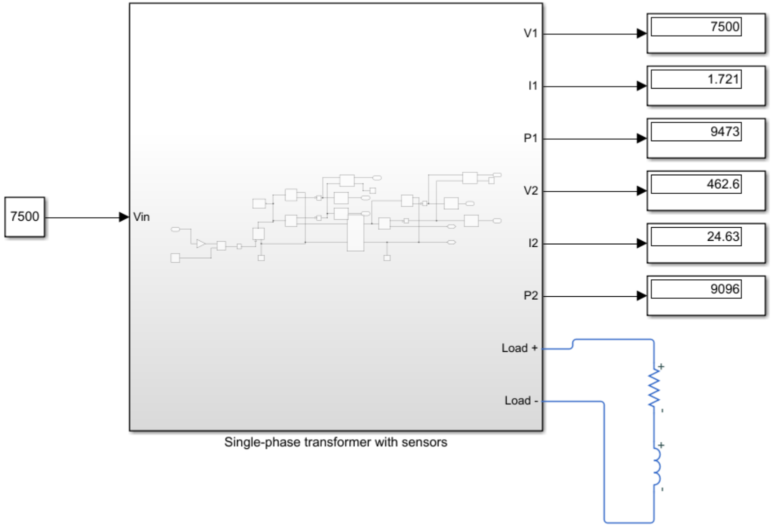
1.4 Draw following diagrams: $P_2(\beta)$, $PF_{load}(\beta)$, $\eta(\beta)$, $\Delta V\%_{2\_calc}(\beta)$, $\Delta V\%_{2\_mod}(\beta)$

1.5 Draw phasor diagram for one of the tests.

Laboratory task

Determination of the performance characteristics. Part 2.

2.1 Make 10 tests with RL-loads (values from m-file):



2.2 Complete m-file with data of RL-load test.

Laboratory task

Determination of the performance characteristics. Part 2.

2.3 Calculate load ratio, load power factor, transformer efficiency and voltage regulation:

$$\beta = \frac{I_2}{I_{2\_rated}} \qquad \cos\theta_{load} = \frac{P_2}{V_2 \cdot I_2} \qquad \eta = \frac{P_2}{P_2 + \beta^2 P_{cu} + P_{core}}$$

$$\Delta V\%_{2\_\text{mod}} = \frac{\left(V_{2\_no\_load} - V_{2\_load}\right) \times 100}{V_{2\_load}}$$

$$\Delta V\%_{2\_calc} = \frac{\beta \frac{V_{sc}}{a} (\cos\theta_{sc} \cos\theta_{load} + \sin\theta_{sc} \sin\theta_{load}) \times 100}{V_{2\_load}}$$

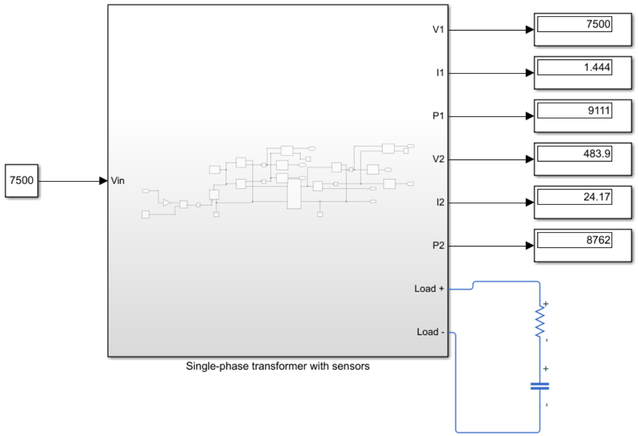
2.4 Draw following diagrams: $P_2(\beta)$, $PF_{load}(\beta)$, $\eta(\beta)$, $\Delta V\%_{2\_calc}(\beta)$, $\Delta V\%_{2\_mod}(\beta)$

2.5 Draw phasor diagram for one of the tests.

Laboratory task

Determination of the performance characteristics. Part 3.

3.1 Make 10 tests with RC-loads (values from m-file):



3.2 Complete m-file with data of RC-load test.

Laboratory task

Determination of the performance characteristics. Part 3.

3.3 Calculate load ratio, load power factor, transformer efficiency and voltage regulation:

$$\beta = \frac{I_2}{I_{2\_rated}} \qquad \cos\theta_{load} = \frac{P_2}{V_2 \cdot I_2} \qquad \eta = \frac{P_2}{P_2 + \beta^2 P_{cu} + P_{core}}$$

$$\Delta V\%_{2\_\mathrm{mod}} = \frac{\left(V_{2\_no\_load} - V_{2\_load}\right)\times 100}{V_{2\_load}}$$

$$\Delta V\%_{2\_calc} = \frac{\beta \frac{V_{sc}}{a}(\cos\theta_{sc}\cos\theta_{load} + \sin\theta_{sc}\sin\theta_{load})\times 100}{V_{2\_load}}$$

3.4 Draw following diagrams: $P_2(\beta)$, $PF_{load}(\beta)$, $\eta(\beta)$, $\Delta V\%_{2\_calc}(\beta)$, $\Delta V\%_{2\_mod}(\beta)$

3.5 Draw phasor diagram for one of the tests.

Laboratory task

Report requirements

- your name in English, your HDU and ITMO numbers, your photo.
- good resolution screenshots of the transformer models with results of open-circuit and short-circuit tests.
- values of calculated parameters of equivalent circuit
- equations that you’ve used for calculation
- good resolution figures of the performance characteristics (white background, legend, unit of measurements).
- listing of your m-file with calculations.
- conclusions

Laboratory task

10 points - MAX

Deadline

2024 / 10 / 04

Penalties

- inaccurate figures: – 1 points
- wrong calculations in any part of the lab – 2 points
- no conclusions or inadequate conclusion: – 1 points
- skipping the deadline: – 3 points

Typical tasks of the test

1. Consider a 15 kVA, 7500/480 V, 60 Hz distribution transformer. Assume that the open-circuit and short-circuit tests were performed on the primary side of the transformer and that the following data were obtained: Voc = 7500 V, Ioc = 0.2 A, Poc = 180 W, Vsc = 366 V, Isc = 2 A , Psc = 300 W.

Determine the impedance of the approximate equivalent circuit referred to the primary side.

The power factor during the open-circuit test is:

$$PF_{oc} = \cos\theta_{oc} = \frac{P_{oc}}{V_{oc} I_{oc}} = \frac{180}{7500 \cdot 0.2} = 0.12$$

The excitation admittance is:

$$\mathbf{Y}_e = \mathbf{Y}_{oc} = \frac{I_{oc}}{V_{oc}} \angle -\arccos(PF_{oc}) = \frac{0.2}{7500} \angle -83.11^\circ =$$

$$= 0.0000267 \cdot \cos(-83.11^\circ) + j0.0000267 \cdot \sin(-83.11^\circ) \approx$$

$$\approx 0.0000032 - j0.0000265 = \frac{1}{R_c} - j\frac{1}{X_m}$$

$$R_c \approx \frac{1}{0.0000032} = 312500\,\Omega = 312.50\,k\Omega$$

$$X_m \approx \frac{1}{0.0000265} = 377730\,\Omega = 377.73\,k\Omega$$

Typical tasks of the test

The power factor during the short-circuit test is:

$$PF_{sc} = \cos\theta_{sc} = \frac{P_{sc}}{V_{sc} I_{sc}} = \frac{300}{366 \cdot 2} = 0.41$$

The series (i.e., the equivalent) impedance is:

$$\mathbf{Z}_{eq1} = \mathbf{Z}_{sc} = \frac{V_{sc}}{I_{sc}} \angle \arccos(PF_{sc}) = \frac{366}{2} \angle 65.81^\circ =$$

$$= 183 \cdot \cos(65.81^\circ) + j183 \cdot \sin(65.81^\circ) = 75 + j166.93\Omega = R_{eq1} + jX_{eq1}$$

$$R_{eq1} = 75\,\Omega, \quad X_{eq1} = 166.93\,\Omega,$$

Typical tasks of the test

2. A 75 kVA, 2400/240 V, 60 Hz distribution transformer has equivalent resistance and reactance of 0.009318 Ω and 0.058462 Ω, respectively, which are both referred to its secondary side. Use the exact equation for V_1 and determine the full-load voltage regulation:

(a) At 0.85 lagging power factor.

(b) At unity power factor.

(c) At 0.85 leading power factor.

At 0.85 lagging power factor:

$$I_2 = \frac{S}{V_2} = \frac{75000}{240} = 312.5\ A$$

$$\theta = \arccos(0.85) = 31.79°$$

$$\mathbf{I}_2 = 312.5\angle -31.79°$$

Typical tasks of the test

Secondary voltage under no load condition:

$$\frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2 R_{eq2} + j\mathbf{I}_2 X_{eq2} =$$

$$= 240\angle 0° + \left(312.5\angle -31.79°\right)(0.009318) + j\left(312.5\angle -31.79°\right)(0.058462) =$$

$$= 252.4873\angle 3.18°$$

Voltage regulation:

$$\%V_{reg} = \frac{V_1 / a - V_{2,FL}}{V_{2,FL}} \times 100 = \frac{252.48 - 240}{240} \times 100 = 5.2\%$$

Typical tasks of the test

At unity power factor:

$$\theta = \arccos(1) = 0°$$

$$\mathbf{I}_2 = 312.5\angle 0°$$

Secondary voltage under no load condition:

$$\frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2 R_{eq2} + j\mathbf{I}_2 X_{eq2} =$$

$$= 240\angle 0° + \left(312.5\angle 0°\right)(0.009318) + j\left(312.5\angle 0°\right)(0.058462) =$$

$$= 243.598\angle 4.3°$$

Voltage regulation:

$$\%V_{reg} = \frac{V_1 / a - V_{2,FL}}{V_{2,FL}} \times 100 = \frac{243.598 - 240}{240} \times 100 = 1.5\%$$

Typical tasks of the test

At 0.85 leading power factor :

$$\theta = \arccos(0.85) = 31.79^\circ$$
$$\mathbf{I}_2 = 312.5\angle 31.79^\circ$$

Secondary voltage under no load condition:

$$\frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2 R_{eq2} + j\mathbf{I}_2 X_{eq2} =$$
$$= 240\angle 0^\circ + \left(312.5\angle 31.79^\circ\right)(0.009318) + j\left(312.5\angle 31.79^\circ\right)(0.058462) =$$
$$= 233.4755\angle 4.19^\circ$$

Voltage regulation:

$$\%V_{reg} = \frac{V_1 / a - V_{2,FL}}{V_{2,FL}} \times 100 = \frac{233.4755 - 240}{240} \times 100 = -2.72\%$$

Typical tasks of the test

3. Consider a three-phase, 15 MVA, 138/13.8 kV distribution substation transformer that is being used as a step-down transformer. Determine the ratings and turn ratios of the transformer, if it is connected in:

(a) Wye–delta

(b) Delta–wye

(c) Delta–delta

(d) Wye–wye

Typical tasks of the test

The rated primary line current is:

$$I_{L1} = \frac{S_{3f}}{\sqrt{3}V_{L1}} = \frac{15\times10^6}{\sqrt{3}\times138\times10^3} = 62.7555A$$

The rated secondary line current is:

$$I_{L2} = \frac{S_{3f}}{\sqrt{3}V_{L2}} = \frac{15\times10^6}{\sqrt{3}\times13.8\times10^3} = 627.555A$$

The rated VA per phase:

$$S_{1f} = \frac{S_{3f}}{3} = \frac{15\times10^6}{3} = 5\times10^6 VA$$

The rated line voltages:

$$V_{L1} = 138\,kV$$

$$V_{L2} = 13.8\,kV$$

Typical tasks of the test

(a) wye–delta connection

$$I_1 = I_{L1} = 62.755A$$

$$I_2 = \frac{I_{L2}}{\sqrt{3}} = 362.32\ A$$

$$V_1 = \frac{V_{L1}}{\sqrt{3}} = \frac{138\times10^3}{\sqrt{3}} = 79.674\times10^3\ V$$

$$V_2 = V_{L2} = 13.8\times10^3\ V$$

$$a = \frac{V_1}{V_2} = \frac{79.674\times10^3}{13.8\times10^3} = 5.7735$$

(b) delta–wye connection

$$I_1 = \frac{I_{L1}}{\sqrt{3}} = 36.232$$

$$I_2 = I_{L2} = 627.555A$$

$$V_1 = V_{L1} = 138\times10^3\ V$$

$$V_2 = \frac{V_{L2}}{\sqrt{3}} = 7.967\times10^3\ V$$

$$a = \frac{V_1}{V_2} = \frac{138\times10^3}{7.967\times10^3} = 17.32$$

Typical tasks of the test

(c) delta–delta connection

$$I_1 = \frac{I_{L1}}{\sqrt{3}} = 36.232\ A$$

$$I_2 = \frac{I_{L2}}{\sqrt{3}} = 362.32\ A$$

$$V_1 = V_{L1} = 138 \times 10^3\ V$$

$$V_2 = V_{L2} = 13.8 \times 10^3\ V$$

$$a = \frac{V_1}{V_2} = \frac{138 \times 10^3}{13.8 \times 10^3} = 10$$

(d) wye–wye connection

$$I_1 = I_{L1} = 62.755A$$

$$I_2 = I_{L2} = 627.555A$$

$$V_1 = \frac{V_{L1}}{\sqrt{3}} = 79.67 \times 10^3\ V$$

$$V_2 = \frac{V_{L2}}{\sqrt{3}} = 7.967 \times 10^3\ V$$

$$a = \frac{V_1}{V_2} = \frac{79.67 \times 10^3}{7.967 \times 10^3} = 10$$

Test

- You have 1 attempt for the test
- For each new attempt, you will receive a one-point penalty
- For each anonymous response (without a name and number), all students of the course will receive a half-point penalty

QR-code for the test

https://forms.yandex.ru/u/66e3aff4f47e7347b42f7e4b/

Thank you!

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