Electrical Machines

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Magnetic circuits

DC motor

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Transformers

Synchronous Machines

Servomotors

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AC Machinery

Brushless DC motor


Galina Demidova

DC motor hometask


Dmitry Lukichev

AC hometask


Alexander Mamatov

Transformer hometask

SM hometask

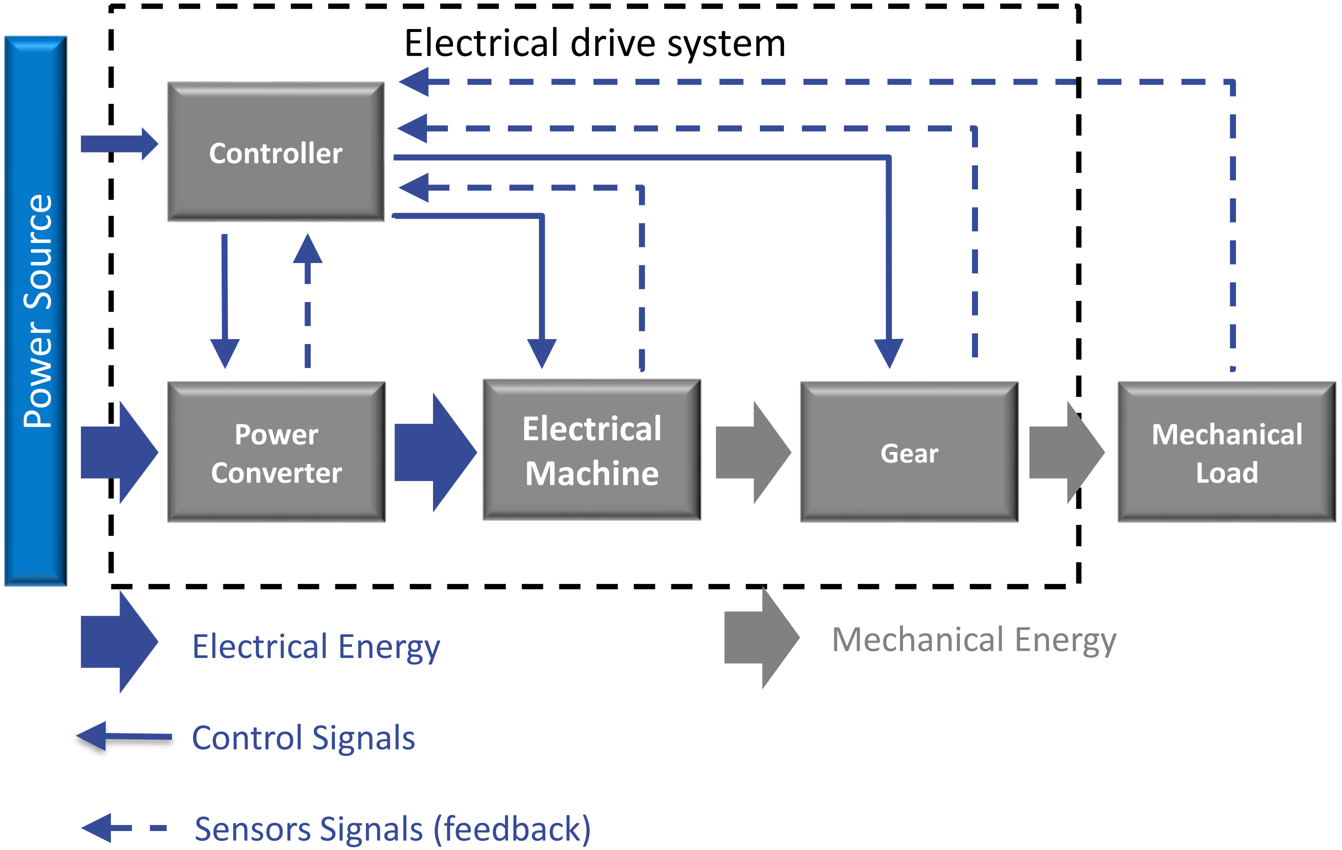
An introduction to electric machines.
Simple magnetic circuits.

COURSE AIMS

- To give an overview about Electrical Machines and their applications in industry, service and everyday life.
- To give basic knowledge in the basic composition of Electrical Machines, components used in these systems and application principles of these systems.
- To give basic knowledge for designing and exploitation of Electrical Machines in the respective field.

LEARNING OUTCOMES

- Knows and orients in main structures of electrical machines and knows respective application areas.
- Knows the main type of electrical machines, their components and orients in respective basic applications.
- Knows and is able to evaluate the characteristics of electrical machines and the role of different knowledge in the design and integration of the components into a whole operational system.


Electrical drive system
Power Source
Controller
Power Converter
Electrical Machine
Gear
Mechanical Load
Electrical Energy
Mechanical Energy
Control Signals
Sensors Signals (feedback)

Electrical Machines

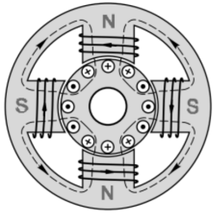
Power type

D.C. (direct current)

A.C. (Alternating current)

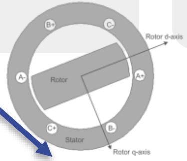
Electric machine type

Direct Current Commutator Machine

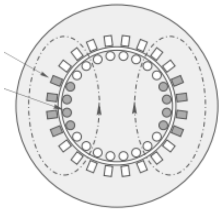


BLDC (Brushless DC motor)

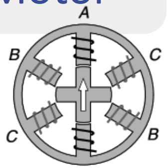
Synchronous Machine



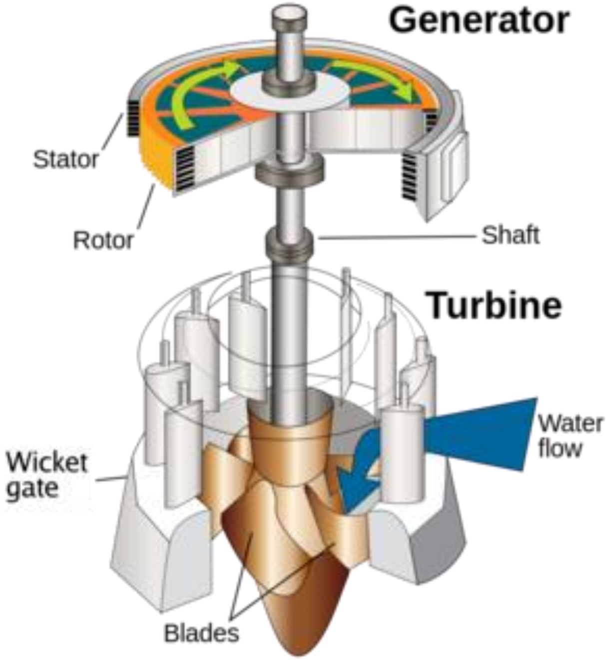
Induction Machine



Stepper Motor



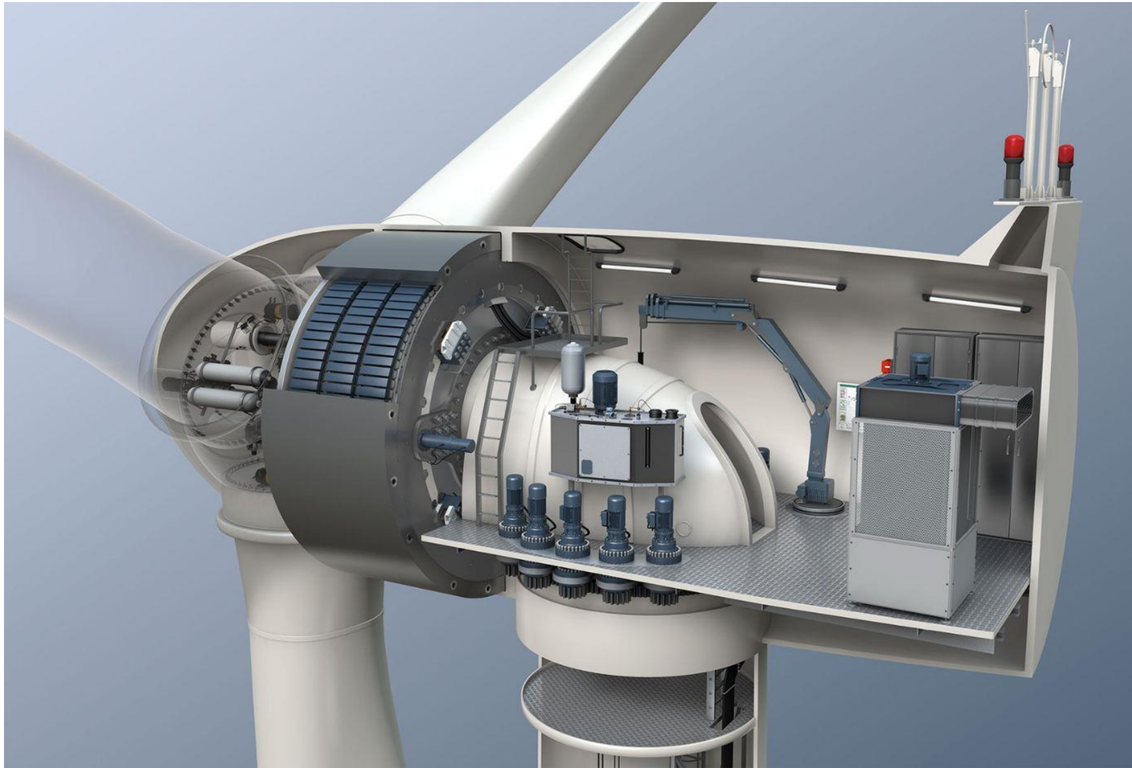
PMSM (Permanent Magnet Synchronous Motor)


Generator
Stator
Rotor
Shaft
Turbine
Water flow
Wicket gate
Blades

AC Electrical Machines

Generators

- ❑ Utility generators
- ❑ Backup generators
- ❑ Wind turbines



AC Electrical Machines

Motors

- Single phase induction
  - Washing machines
  - Compressor
  - Dryers

  Household use

- 3-phase induction
  - Cranes
  - Elevators
  - Fire pumps

  Industry use

- Synchronous Motors
  - Servomotors
  - Clocks
  - Synchronous condenser

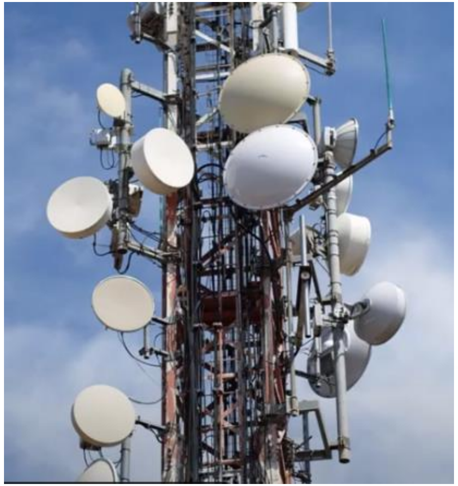





DC Electrical Machines

Generators

- ❑ Early power systems
- ❑ Standalone systems (cellphone towels)
- ❑ Lift, cranes, trains



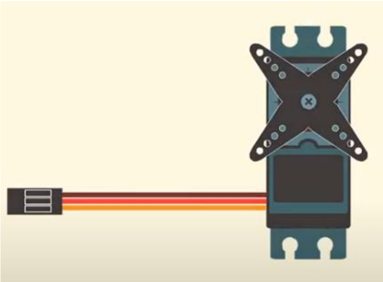
DC Electrical Machines

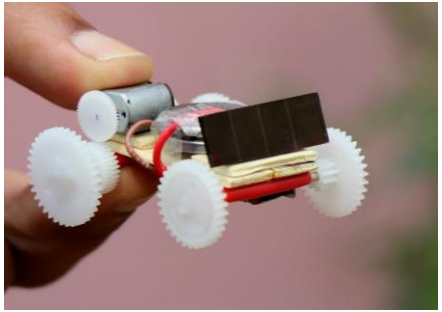
Motors

Toys

Medical equipment

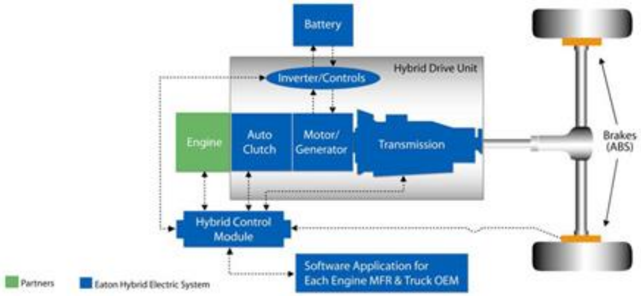
Electric vehicles



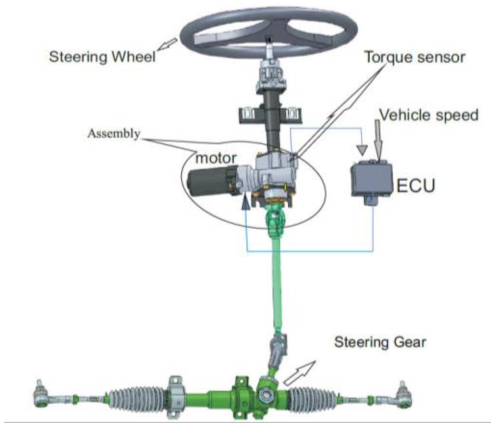


Electric vehicles

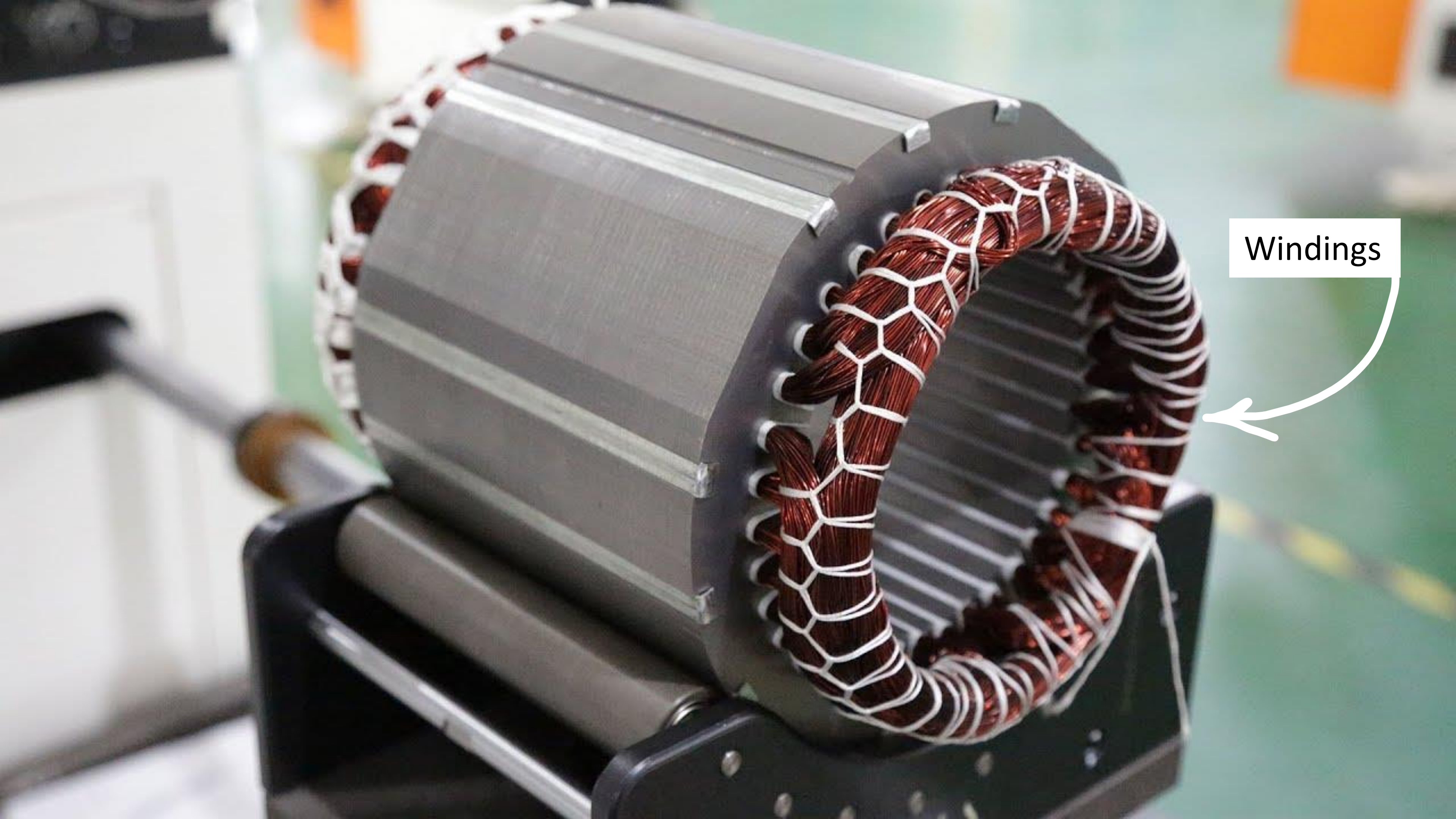
Hybrid electric vehicle drivetrain



Electronic power steering systems



HVAC (Heating, Ventilation And Air conditioning) system


Windings

Windings

copper

aluminum

copper

Advantages

- Stronger than aluminum
- Higher Current carrying capacity
- Transformer with copper winding less expensive
- No Galvanic corrosion
- Smaller winding size
- Easy to repair broken wire connection

Disadvantages

- Expensive
- Less Flexible
- Lesser resources available

Advantages

- Less Cost
- Corrosion resistive
- Conductivity
- More Flexible
- Lower eddy losses

aluminum

Disadvantages

- Susceptible to oxidation at Joints
- Higher Resistivity

Resistivity of Copper is 1.68×10^{-8} Ohm

Resistivity of Aluminum 2.65×10^{-8} Ohm

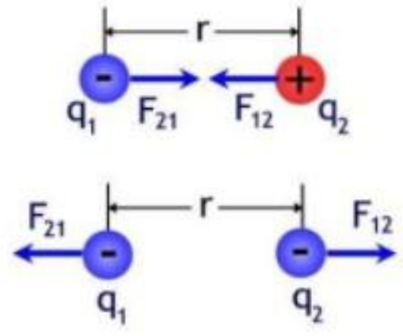
Aluminum/Copper = $(2.65 \times 10^{-8}) / (1.68 \times 10^{-8})$ = **1.6**

- Conductivity

- Magnetic materials are those materials in which a state of magnetization can be induced.
- Such materials when magnetized create a magnetic field in the surrounding space.
  - Magnetic materials include the elements
    iron
    nickel
    cobalt

alloys containing some of these such as **steel** and some of their compounds.

Magnetic Field



The force of attraction / repulsion between two magnetic poles is directly proportional to the strength of the poles and inversely proportional to the square of the distance between them

$$F_E = \frac{kq_1q_2}{r^2}$$

Electric field:

1) A distribution of electric charge at rest creates an electric field E in the surrounding space.
2) The electric field exerts a force $\bar{F}_E = q\bar{E}$ on any other charges in presence of that field.

Magnetic field:

1) A moving charge or current creates a magnetic field in the surrounding space (in addition to $\bar{E}$).
2) The magnetic field exerts a force $\bar{F}_m$ on any other moving charge or current present in that field.

The magnetic field is a vector field vector quantity associated with each point in space.

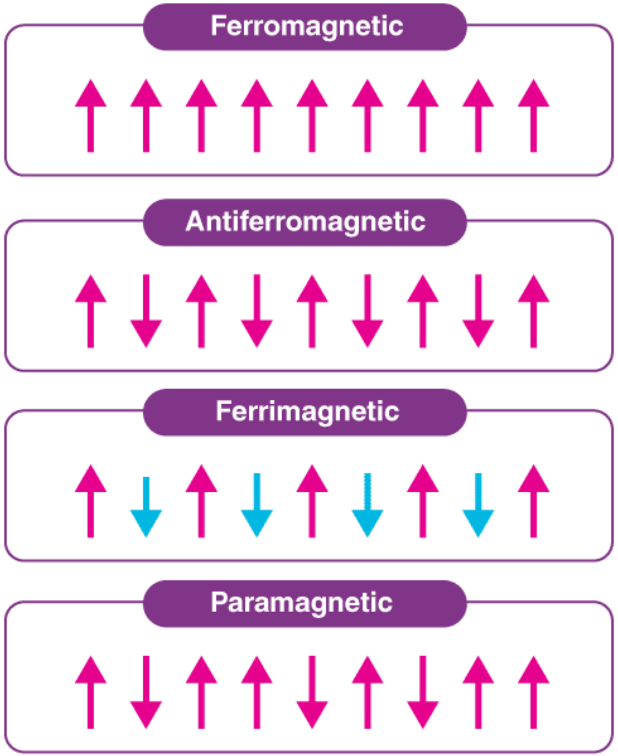
$$\bar{F}_m = |q| v_\perp B = |q| vB \sin\varphi$$

$$\boxed{\bar{F}_m = q\bar{v} \times \bar{B}}$$

$\bar{F}_m$ is always perpendicular to $\bar{B}$ and $\bar{v}$

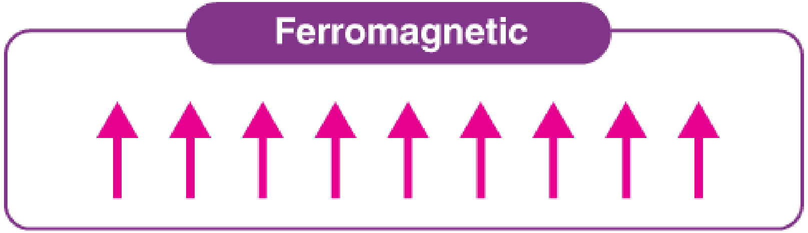
Classification

- Ferromagnetic
- Paramagnetic
- Diamagnetic
- Magnetically Soft Material
- Magnetically Hard Material

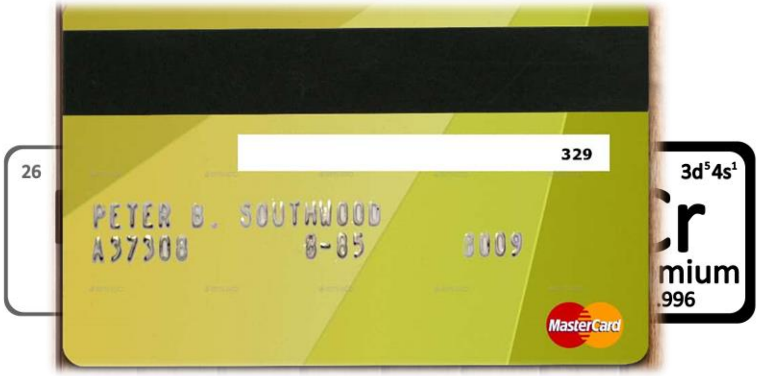


Ferromagnetic

- ❑ A type of material that is highly attracted to magnets and can become permanently magnetized is called a ferromagnetic.
- ❑ The relative permeability is much greater than unity and are dependent on the field strength
- ❑ These have hight susceptibility

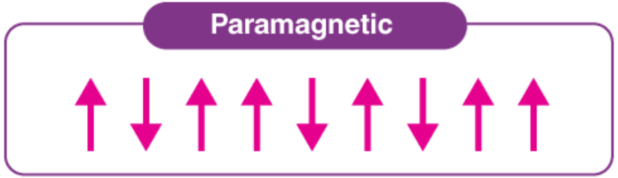


Spins are aligned parallel in magnetic domains

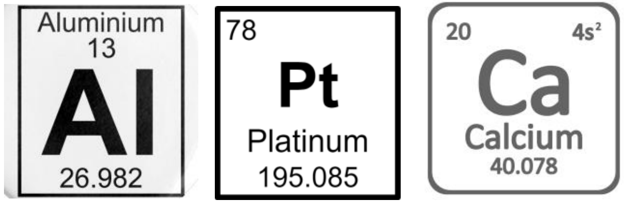


Paramagnetic

- ❑ It is a substance or body which very weakly attracted by the poles of a magnet, but not retaining any permanent magnetism.
- ❑ These have relative permeability slightly greather than unity and are magnetized slightly.
- ❑ They attract the lines of forces weakly.



Spins are randomly oriented



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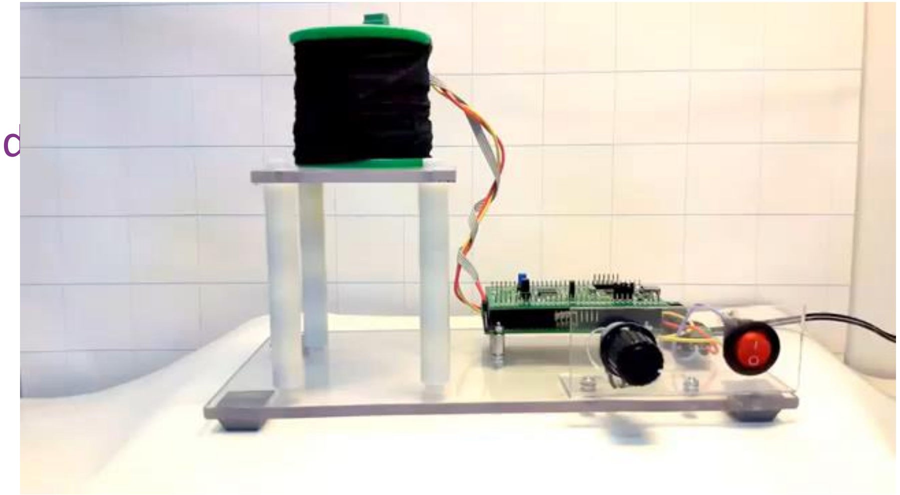
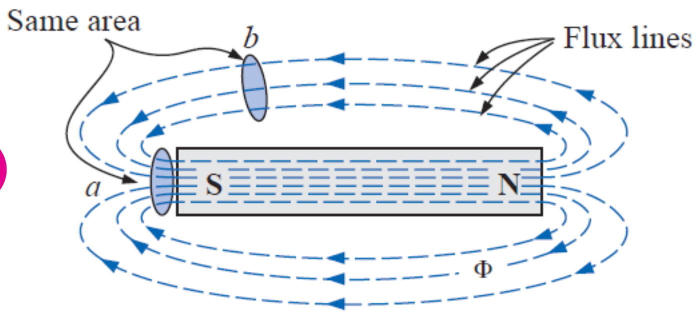
Diamagnetic

- It is substance which create a magnetic field in opposite to an externally applied field.
- Susceptibility is negative.
- These have relative permeability slightly less than unity.
- They reppel the lines of force slightly.

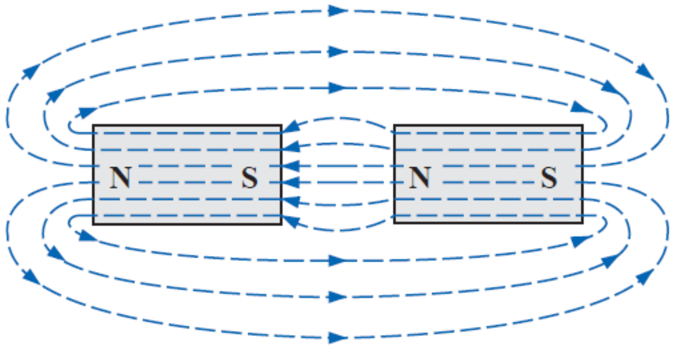


1



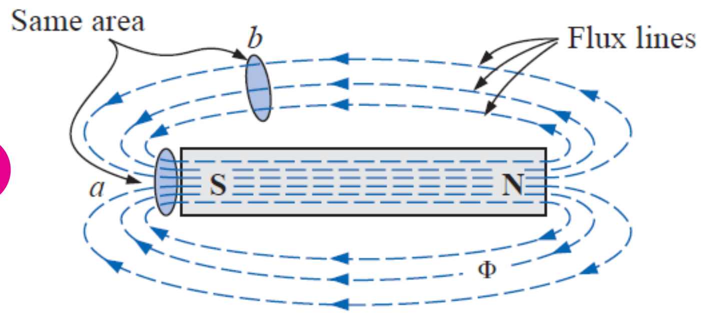
Flux distribution for a permanent magnet

2



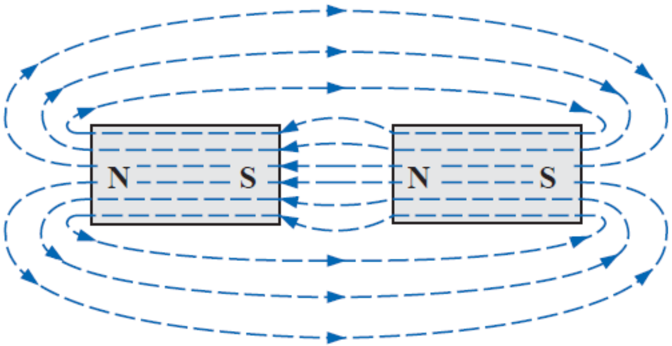
Flux distribution for two adjacent, opposite poles

1



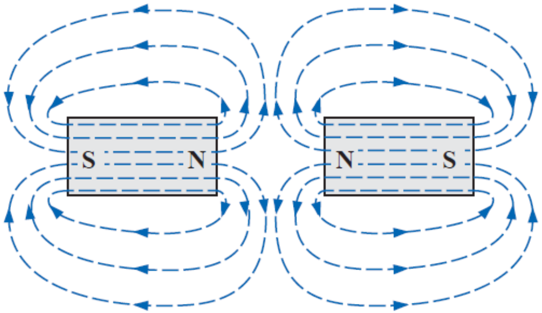
Flux distribution for a permanent magnet

2



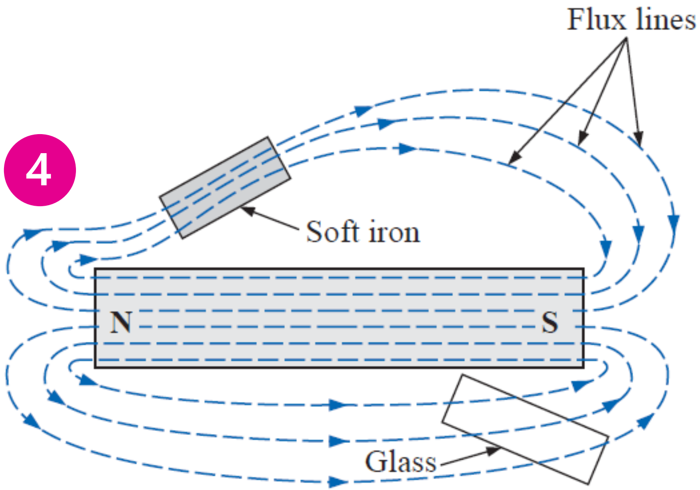
Flux distribution for two adjacent, opposite poles

3



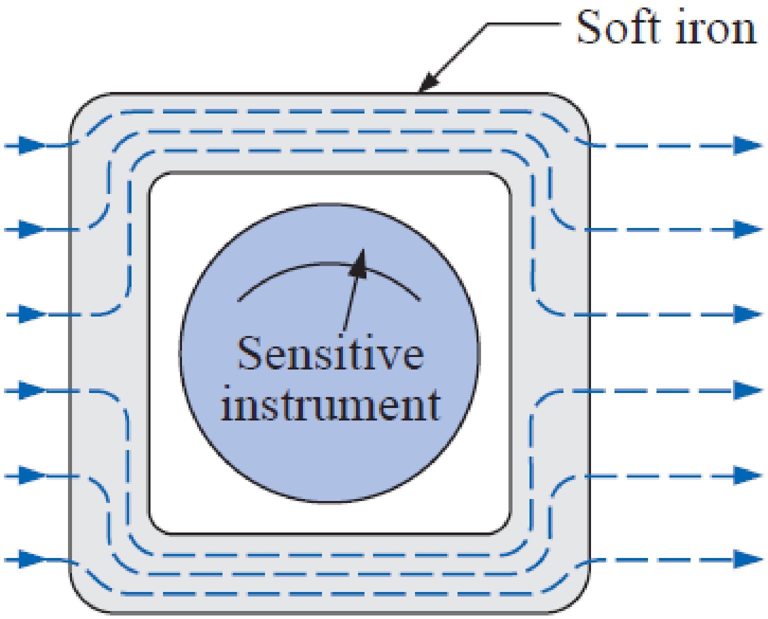
Flux distribution for two adjacent, like poles

4



Effect of a ferromagnetic sample on the flux distribution of a permanent magnet

5



Effect of a magnetic shield on the flux distribution

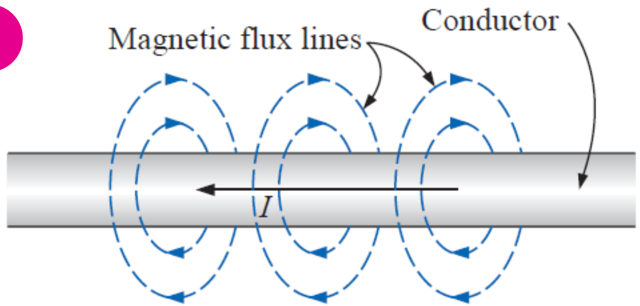
Flux distributio
permanent ma

adjacent, like poles

Flux distributi
adjacent, opposite poles

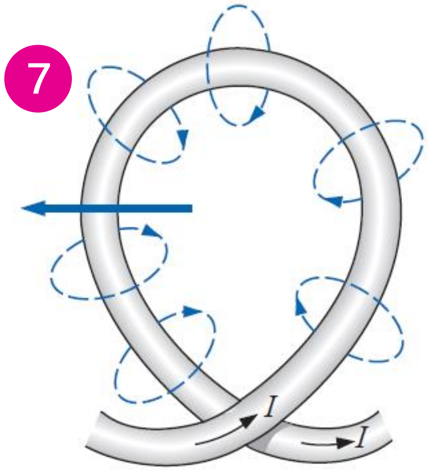
c sample on the flux
distribution of a permanent magnet

6



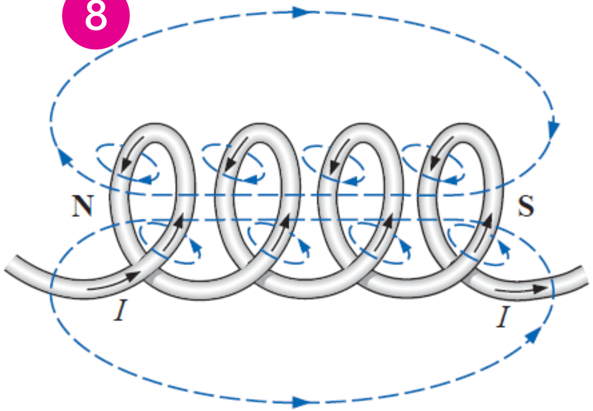
Magnetic flux lines around a current-carrying conductor

7



Flux distribution of a single-turn coil

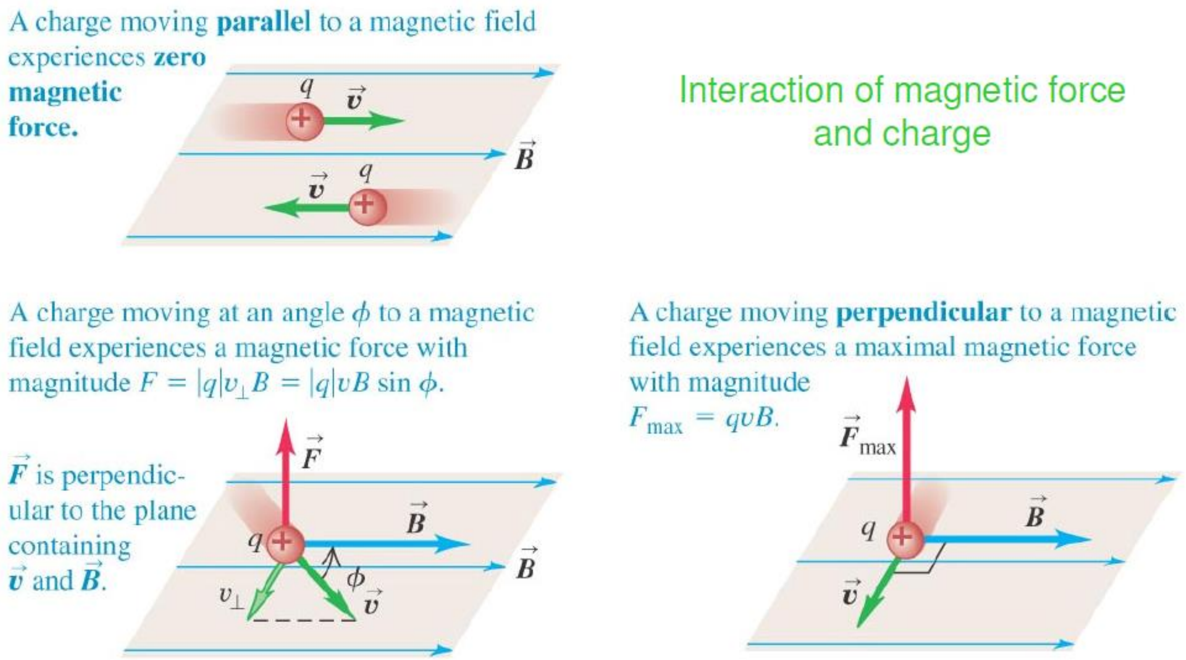
8



Flux distribution of a current-carrying coil

Magnetic Field

The moving charge interacts with the fixed magnet. The force between them is at a maximum when the velocity of the charge is perpendicular to the magnetic field.

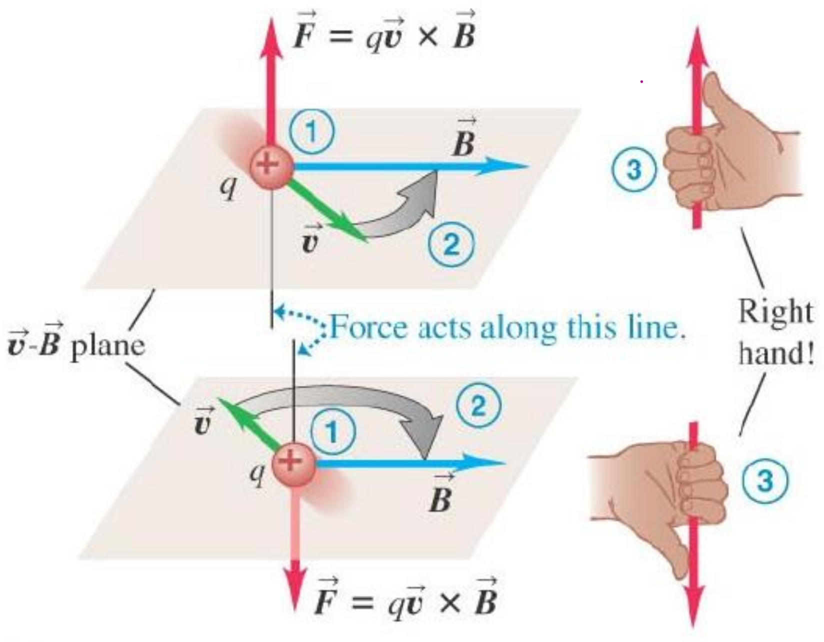


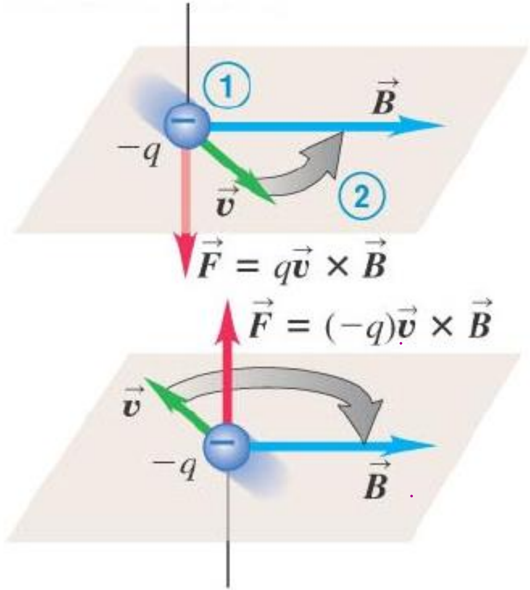
Magnetic Field

Right Hand Rule

Positive charge moving in magnetic field direction of force follows right hand rule

Negative charge F direction contrary to right hand rule.





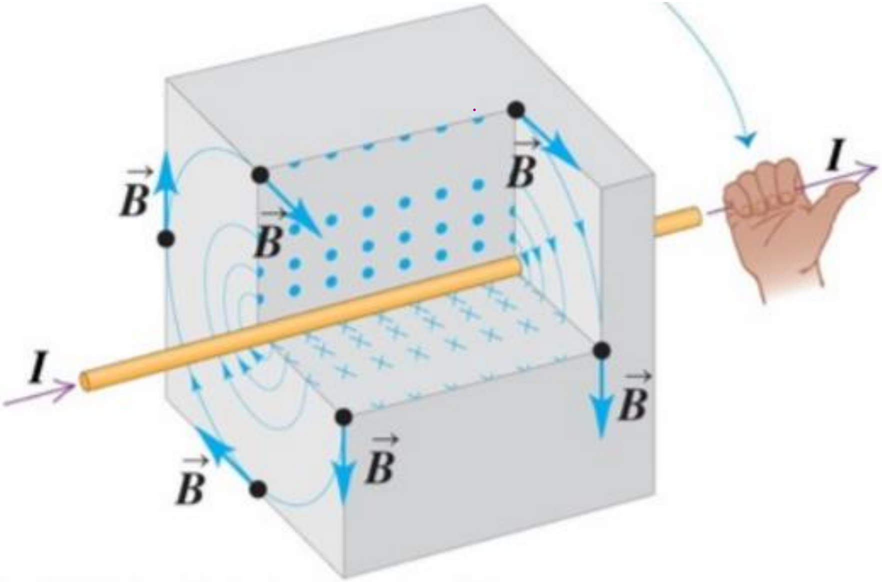
$$\bar{F}_m = |q| v B_\perp$$

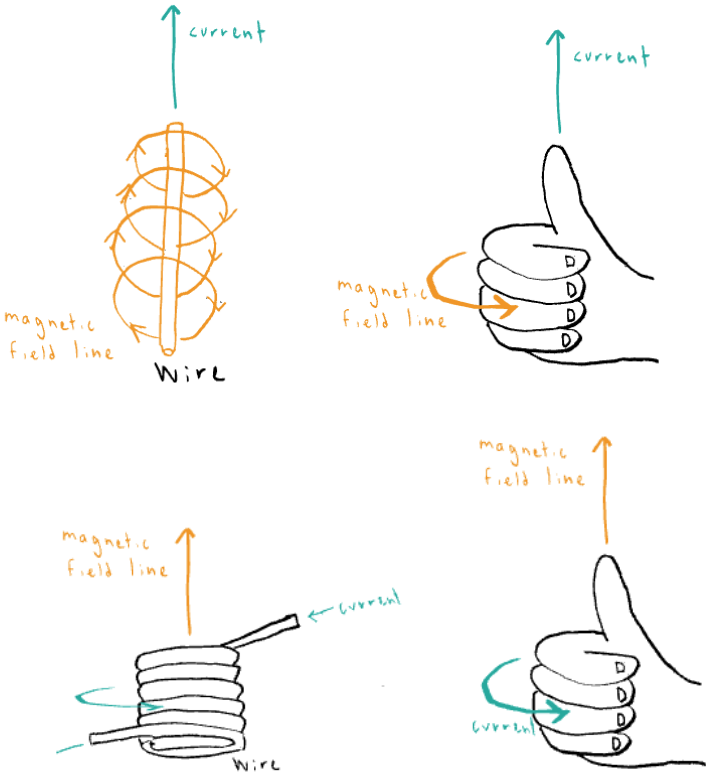
Units:

1 Tesla = 1 N s / C m = 1 N/A m

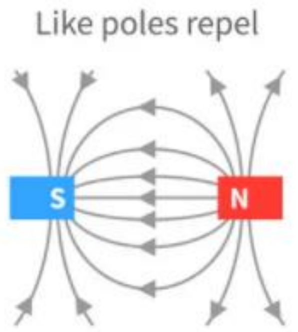
1 Gauss = 10-4 T

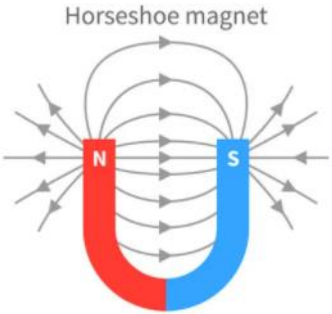
Right Hand Rule

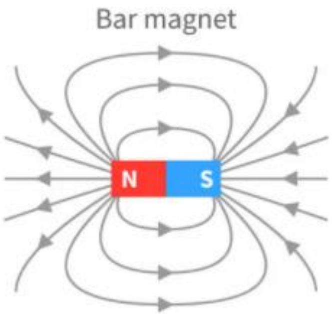


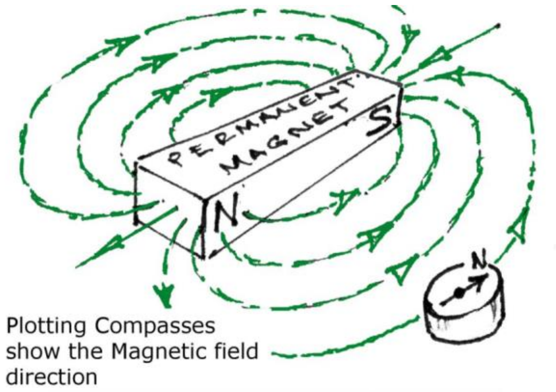


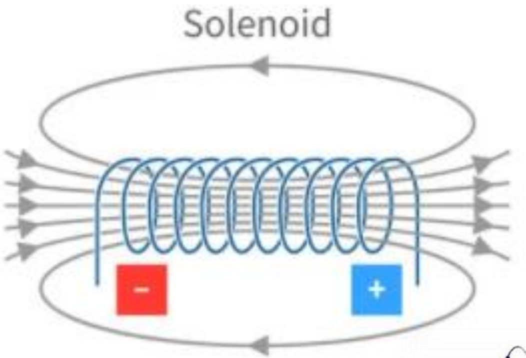
Magnetic field

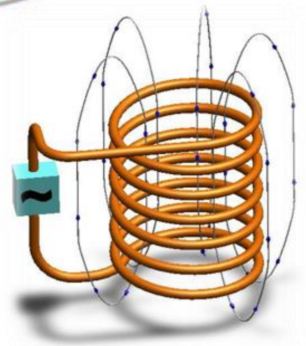


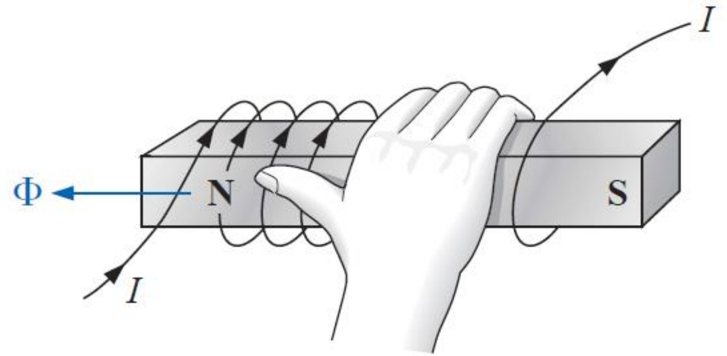








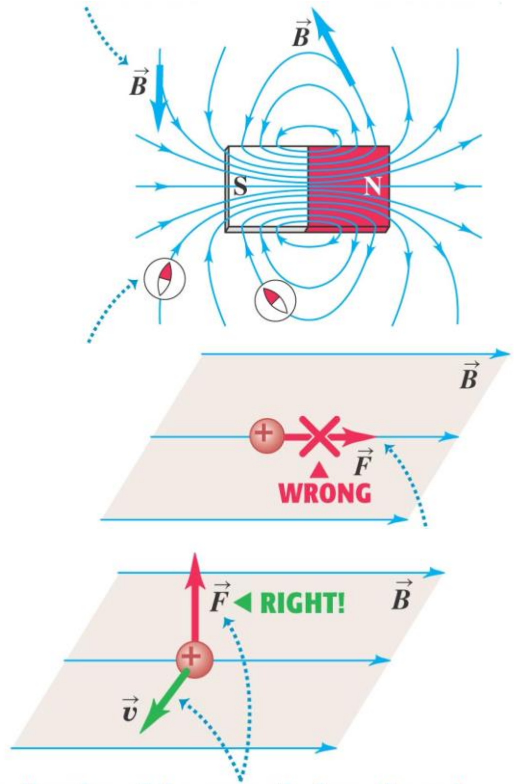




Determining the direction of flux for an electromagnet

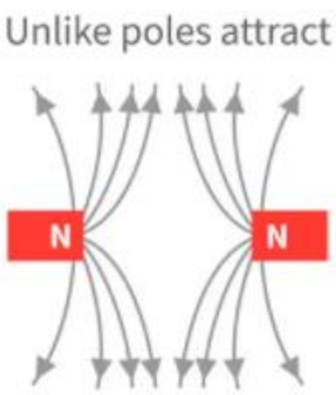
Magnetic Field Lines and Magnetic Flux

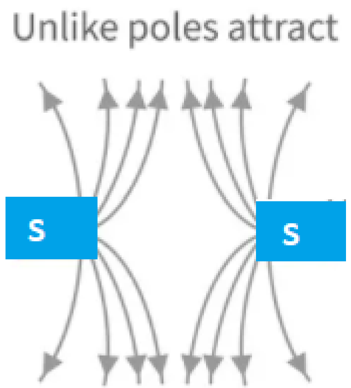
- Magnetic field lines may be traced from N toward S (analogous to the electric field lines).
    - At each point they are tangent to magnetic field vector.
- The more densely packed the field lines, the stronger the field at a point.
    - Field lines never intersect.
- The field lines point in the same direction as a compass (from N toward S).
    - Magnetic field lines are not “lines of force”.



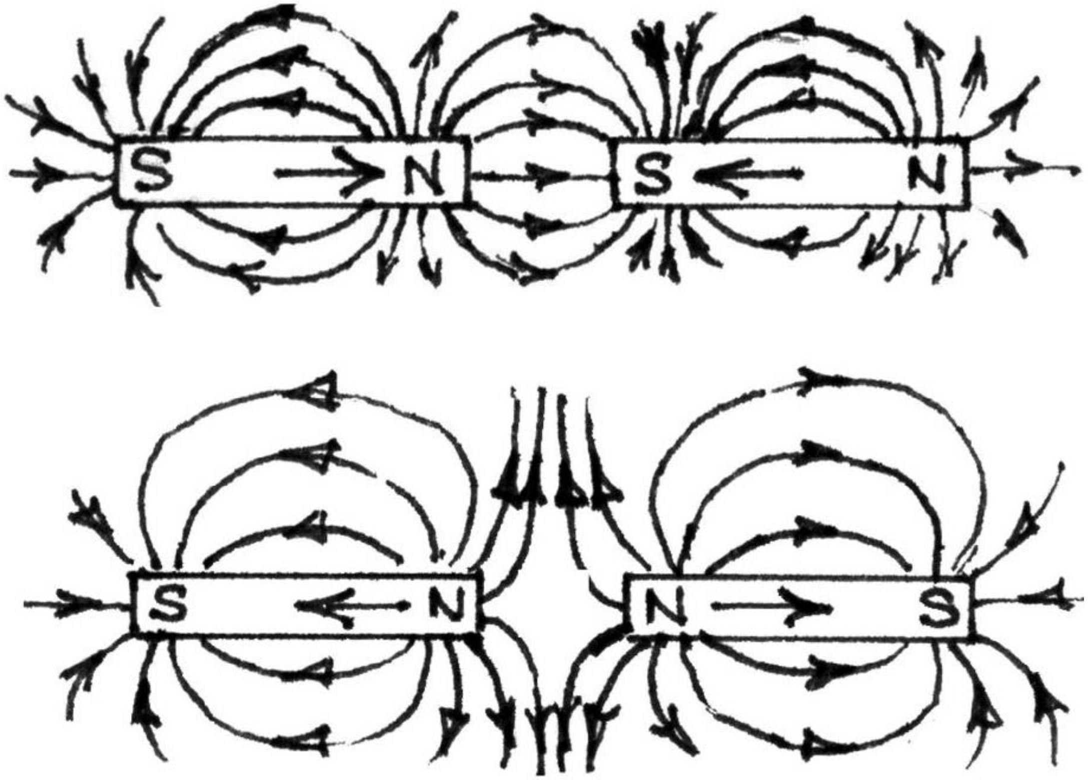
The direction of the magnetic force depends on the velocity $\vec{v}$, as expressed by the magnetic force law $\vec{F} = q\vec{v} \times \vec{B}$.

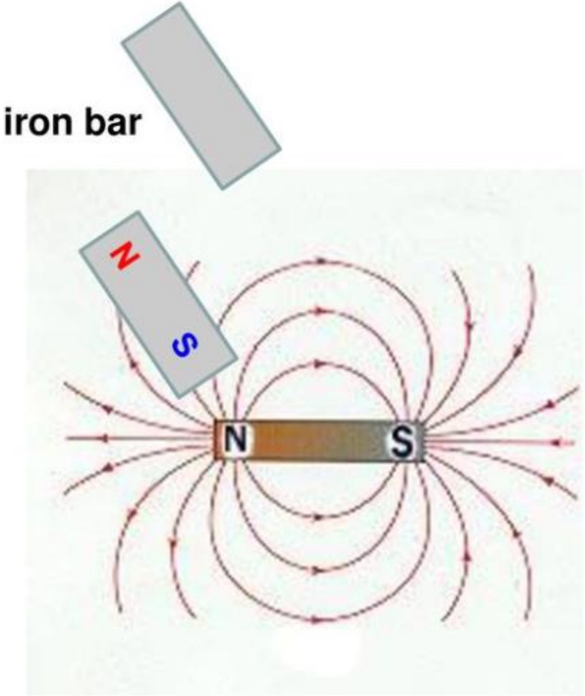
Magnetic fields between two bar magnets





Practical part





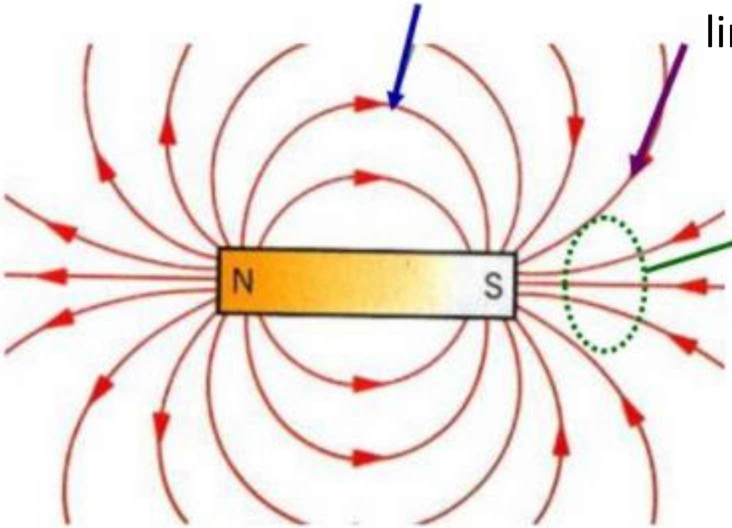
If the material is magnetically hard it will retain its magnetism once removed from the field.

Magnetic Field

B - SI unit
T - tesla

Magnetic field lines

B is tanget to the field lines at any point

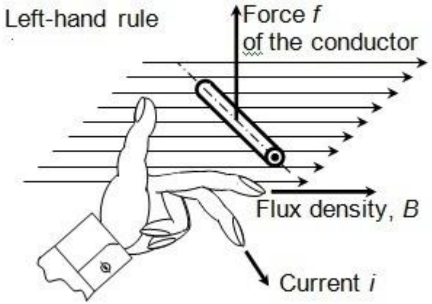


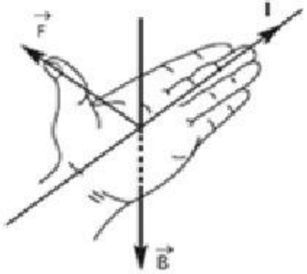
The stronger the magnetic field the denser the magnetic field lines

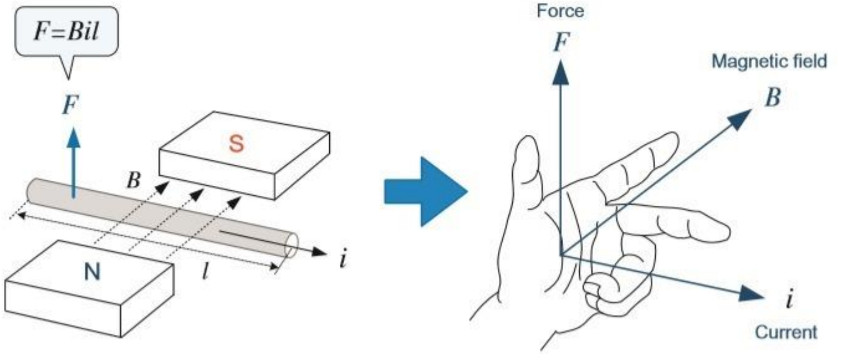
Magnetic Field

Fleming's left-hand rule
(for electric motors)

Direction of the force: (Fleming's Left-hand rule)

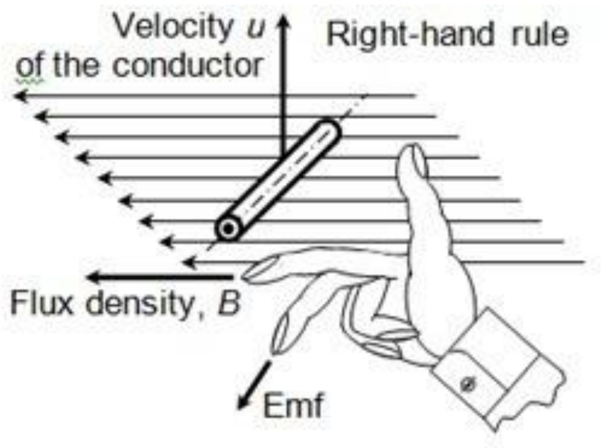


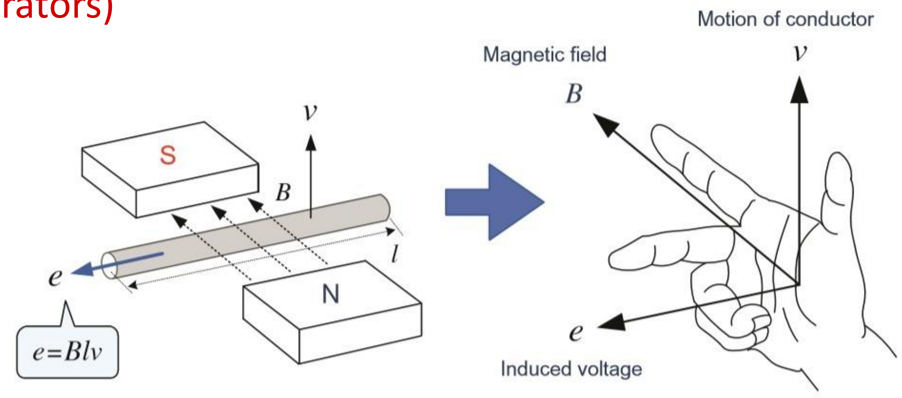




Magnetic Field

Fleming's right-hand rule
(for generators)



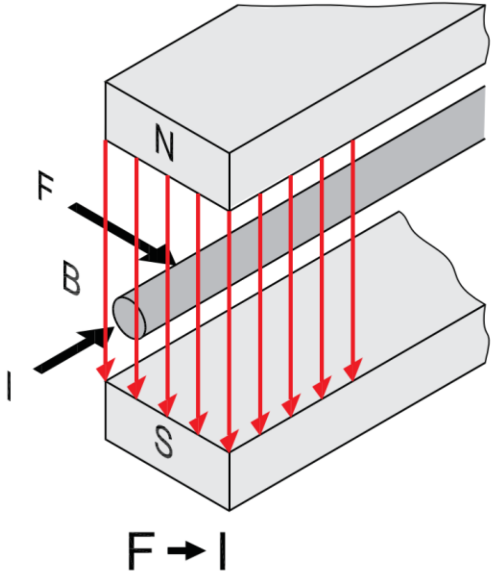


If charged particle moves in region where both, *E* and *B* are present:

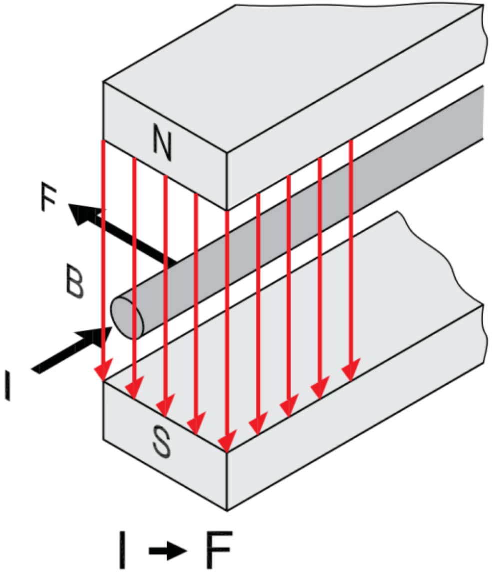
$$\bar{F}_m = |q|\left(\bar{E} + \bar{v} \times \bar{B}\right)$$

Principle for electromagnetic induction

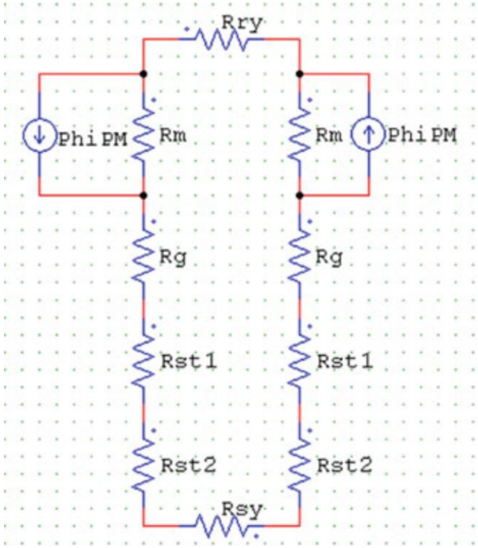
Fleming's Right-hand Rule

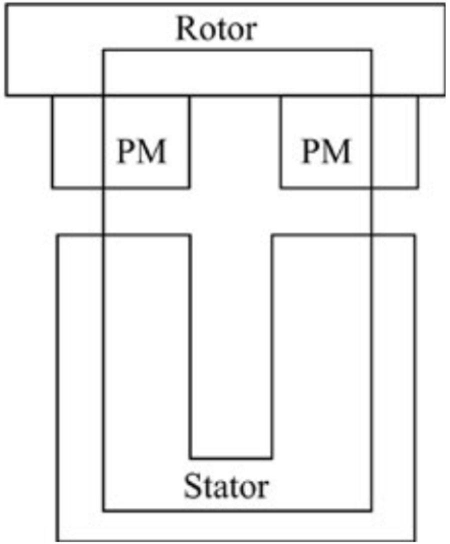


Fleming’s Left-hand Rule



- The **magnetic circuit model** acts as a uniform principle in descriptive magnetostatics, and as an approximate computational aid in electrical machine design.
- The model uses the conception of magnetic reluctance to establish an equivalent circuit for approximate analysis of static magnetic field in electrical machines.



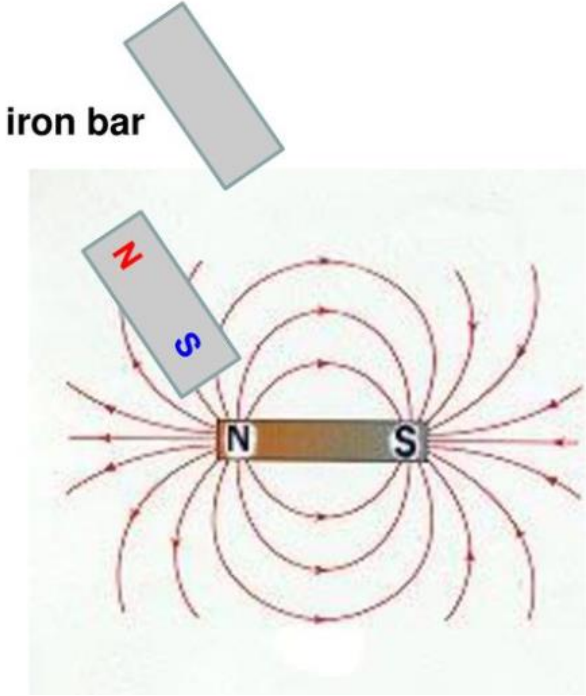


Comparison of Electric and Magnetic Force

- ❑ Electric force vector along direction of electric field
- ❑ Electric force acts on charged particle regardless of whether particle is moving
- ❑ Electric force does work in displacing a charged particle

- ❑ Magnetic force vector perpendicular to magnetic field
- ❑ Magnetic force acts on charged particle only when particle is in motion
- ❑ Magnetic force associated with steady magnetic field does no work when a particle is displaced → force perpendicular to displacement of its point of application

B(H) Characteristics



$$B_{ind} = \mu_0 M$$

$$\mu_0 = 4\pi \cdot 10^{-7} [Wb / Am]$$

$$B = \mu_0 M + \mu_0 H$$

$$M = \chi H$$

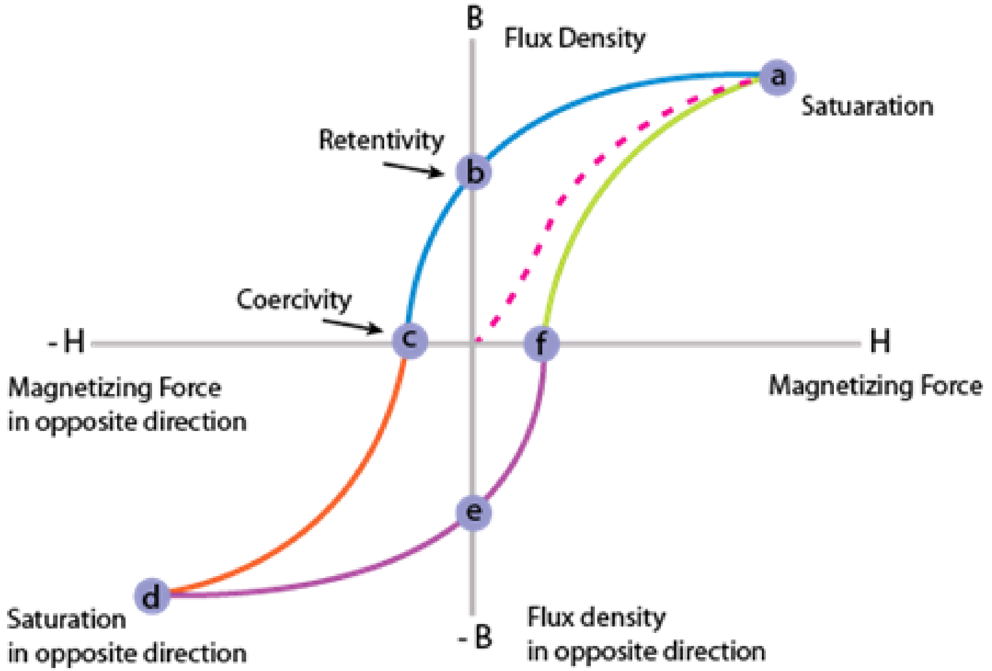
magnetic susceptibility of the material

$$B = \mu_0 \underbrace{(\chi + 1)}_{\mu_r} H = \mu H$$

μ_r relative permeability

Simple magnetic circuits

B(H) Characteristics

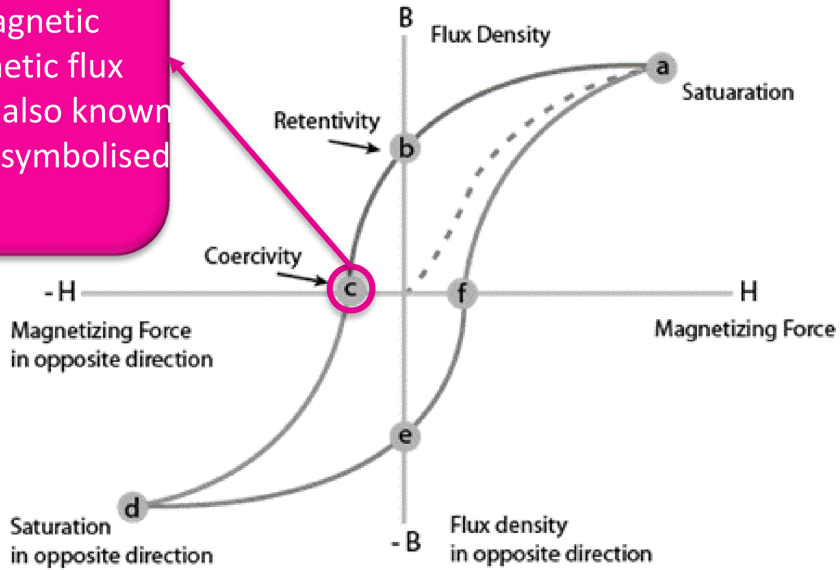


Typical B-H loop of a ferromagnetic material

Chukwuchekwa, Nkwachukwu 2011. *Investigation of magnetic properties and Barkhausen noise of electrical steel.* PhD Thesis, Cardiff University.

B(H) Characteristics

This is the amount of reverse magneti[c]
field that is applied to a magnetic
material to make the magnetic flux
density return to zero. It is also known
as the coercive field and is symbolised
as H_C.



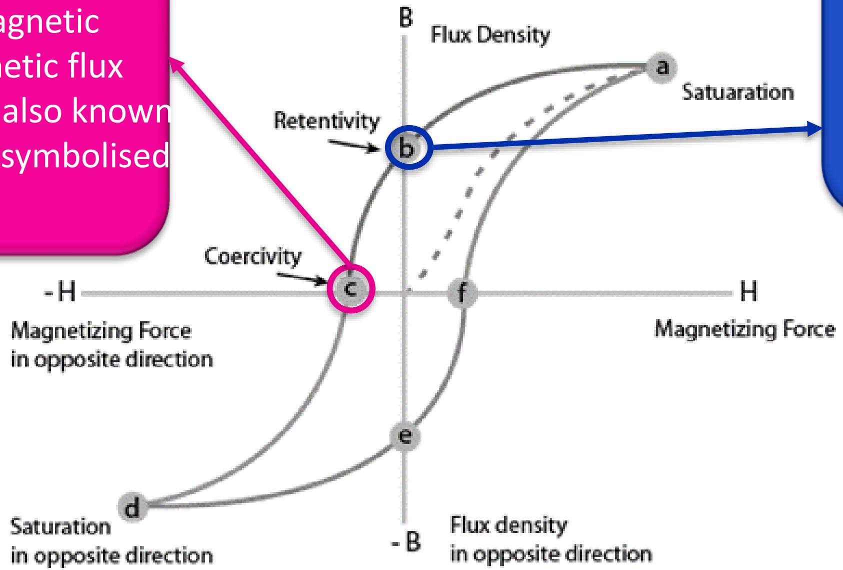
Typical B-H loop of a ferromagnetic material

Chukwuchekwa, Nkwachukwu 2011. *Investigation of magnetic properties and Barkhausen noise of electrical steel.* PhD Thesis, Cardiff University.

B(H) Characteristics

This is the amount of reverse magneti field that is applied to a magnetic material to make the magnetic flux density return to zero. It is also known as the coercive field and is symbolised as H_C.

This is the magnetic flux density that remains in a material when the magnetic field is zero. It can be represented with the symbol BR.



Typical B-H loop of a ferromagnetic material

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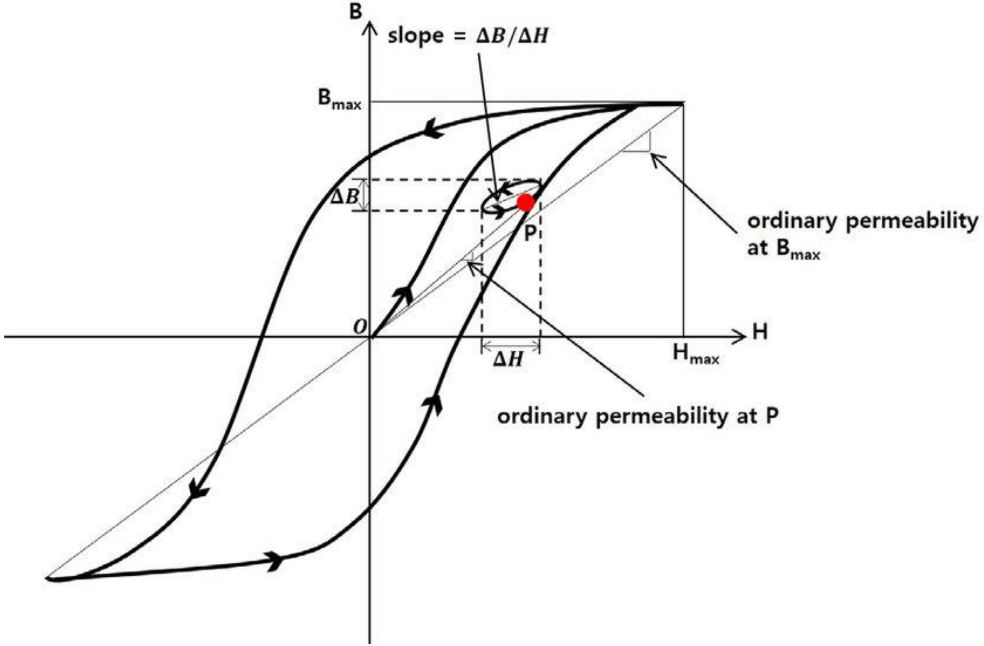


B-H Curve of a typical ferromagnetic material

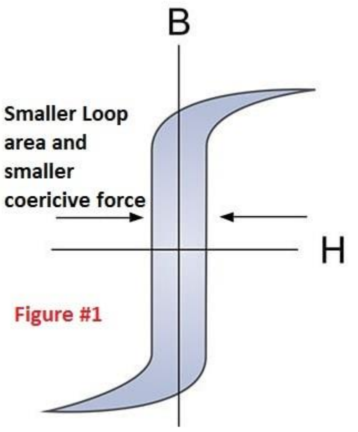
Total loss = Static hysteresis Loss + Classical eddy current loss + Excess (anomalous) loss

Park, Jooyoung;Kim, Junkyeong;Zhang, Aoqi;Lee, Hwanwoo;Park, Seunghee "Embedded EM Sensor for Tensile Force Estimation of PS tendon of PSC Girder" Journal of the Computational Structural Engineering Institute of Korea. 2015. Dec, 28(6): 691-697

Magnetically Soft material

Characteristics:

- ❑ They have hight permeability
- ❑ The magnetic energy stored is not high
- ❑ They have negligible coercive force
- ❑ They have low remanence
- ❑ Hystersis loop is narrow



Soft Magnetic Material

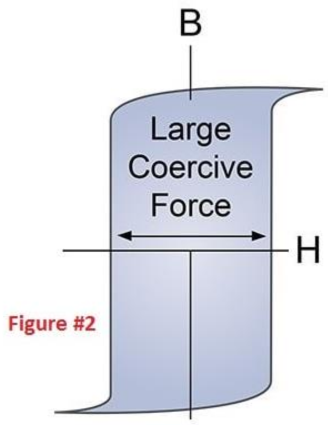
Examples:

pure or ingot iron
cast iron
carbon steel
manganese and nickel steel

Magnetically Hard Material

Characteristics:

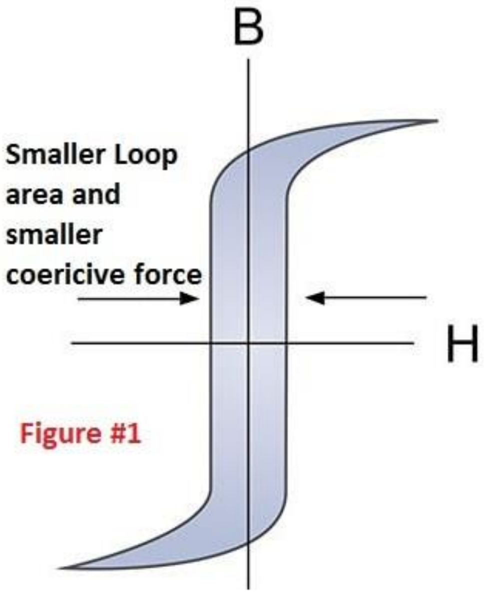
- They posses hight value of BH product
- High retentivity
- High coercitivity
- Strong magnetic reluctance
- Hysteresis loop is more rectangular in shape



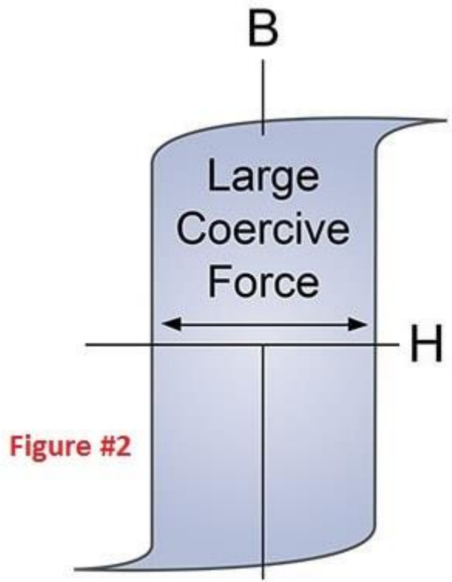
Hard Magnetic Material

Examples:

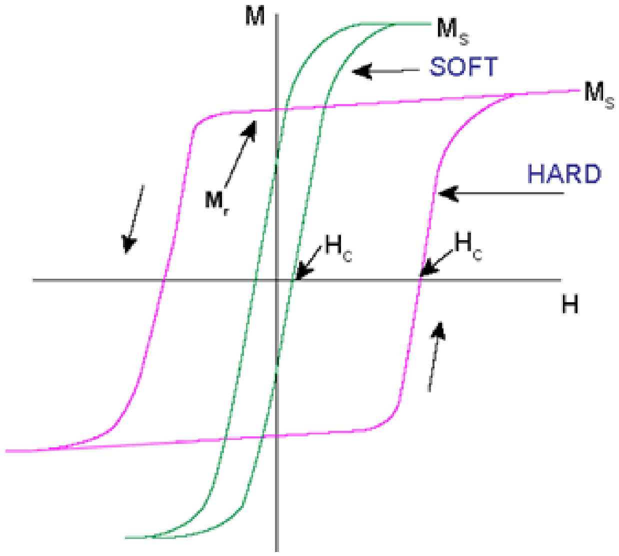
Tungsten steel
Cobalt steel
Chromium steel



Soft Magnetic Material

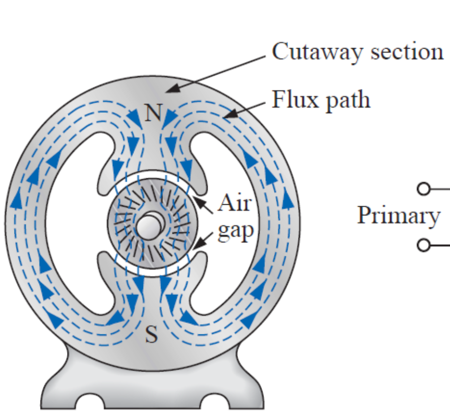


Hard Magnetic Material

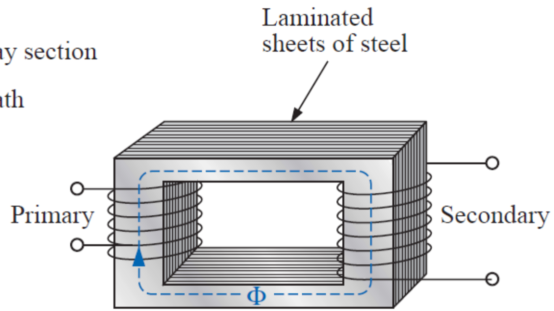


Terms connected with magnetic materials

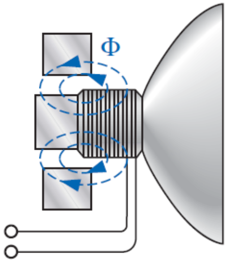
- Magnetic force
- Magnetic flux density
- Magnetic field strength
- Relative permeability
- Absolute permeability
- Remanence
- Retentivity
- Curie temperature (T_c)
- Hysteresis loop and losses



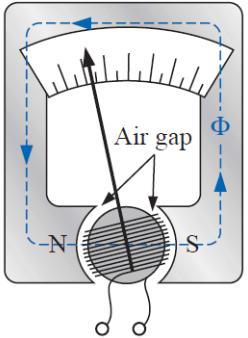
Generator



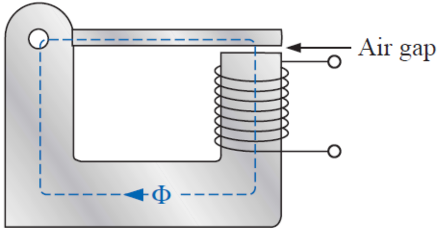
Transformer



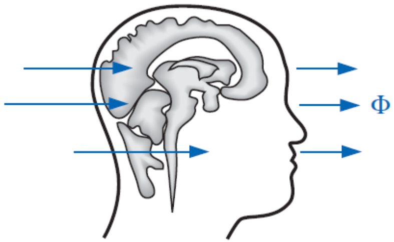
Loudspeaker



Meter movement



Relay

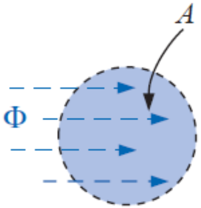


Medical Applications: Magnetic resonance imaging.

Magnetic Circuit Properties

$$\boxed{B = \frac{\Phi}{A}}$$

B = teslas (T)
Φ = webers (Wb)
A = square meters (m^2)



$$\boxed{B = \mu_r \mu_0 H}$$

B is the **magnetic flux density** in teslas

$\mu_0 = 4\pi \cdot 10^{-7} [H / m]$ for air, we can use the permeability of free space

$$\boxed{\mu_r = \frac{\mu}{\mu_o}}$$

μ_r is the permeability of the material in henries/meter

H - is the magnetizing force in amp-turns/meter

Antiferromagnetic	$\mu_r \approx 1,0$	$\mathbf{B} \approx \mu_0 \mathbf{H}$
Ferromagnetic	$\mu_r \gg 1,0$	

Magnetizing force **H**

Given the characteristics of the coil and the path length of the magnetic circuit, the magnetic flux gives rise to a magnetizing force

$$\oint_l \mathbf{H}\, d\mathbf{l} = \sum \pm I$$

magnetizing force [A/m]

coil current in amps [A]

l is the length of the magnetic path in meters

$$\sum_{k=1}^{n} H_k l_k = \sum \pm I$$

for *m* coils with the number of turns *w* and current *I*

$$\sum_{k=1}^{n} H_k l_k = \sum_{p=1}^{m} \pm I_p w_p = \sum_{p=1}^{m} \pm F_p$$

magneto-motive force **F** in amp-turns for p-th turns $$F_p = I_p w_p$$

Magnetic flux **Φ:**

$$\Phi = \int_S \mathbf{B}\, d\mathbf{S} = \int_S B \cos\alpha \, dS \qquad [Wb]=[T][m^2]$$

dS – Area *S*

α – the angle between the direction of magnetic flux density vector and the perpendicular to the surface *dS*

For a perpendicular surface: $\Phi = BS$

$$F_{mk} = H_k l_k$$ magneto-motive force

magnetizing force $H_k = B_k / (\mu_k \mu_0)$

$$F_{mk} = B_k l_k / (\mu_k \mu_0)$$

if S_k is perpendicular to l_k $\Longrightarrow$ $F_{\text{м}k} = \Phi_k \dfrac{l_k}{\mu_k \mu_0 S_k} = \Phi_k R_{\text{м}k}$

$$R_{\text{м}k} = \frac{l_k}{\mu_k \mu_0 S_k}$$ – reluctance *k*-th circuit unit [amp-turns/weber]

Nomenclature

B is the flux density in teslas

μ is the permeability of the material in henries/meter

H is the magnetizing force in amp-turns/meter

w is the number of turns or loops in the coil

I is the coil current in amps

l is the length of the magnetic path in meters

R is the reluctance in amp-turns/weber

S is the cross sectional area of the material in square meters

F is the magneto-motive force (MMF) in amp-turns

Φ is the magnetic flux in webers

Example

A magnetic flux of $6 \cdot 10^{-5}$ Wb exists in a core whose cross section has dimensions of 1 centimeter by 1 centimeters. Determine the flux density in teslas.

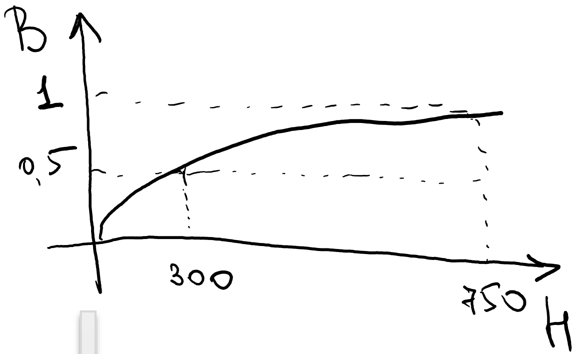
convert the dimensions and find the area

$$S = 0.01[m] \cdot 0.01[m] = 1 \cdot 10^{-4}[m^2]$$

$$B = \frac{\Phi}{S} = \frac{6 \cdot 10^{-5}[Wb]}{1 \cdot 10^{-4}[m^2]} = 0.6[T]$$

Example

The toroid is made of cast steel, has a 500 turn coil, a cross section of 1 cm by 1 cm, and an average path length of 50 cm. Determine the flux in webers if a current of 0.5 amps feeds the coil.



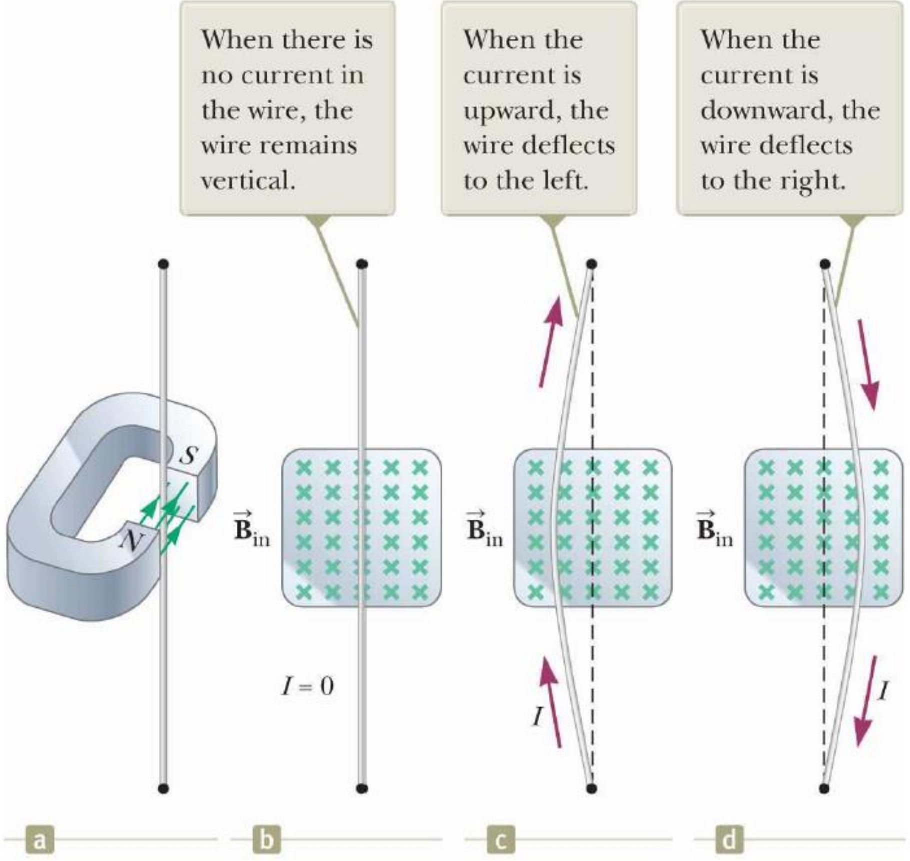
$$H = \frac{w \cdot I}{l} = \frac{500[turns] \cdot 0.5[A]}{0.5[m]} = 500[At / m]$$

$$S = 0.01[m] \cdot 0.01[m] = 1 \cdot 10^{-4}[m^2]$$

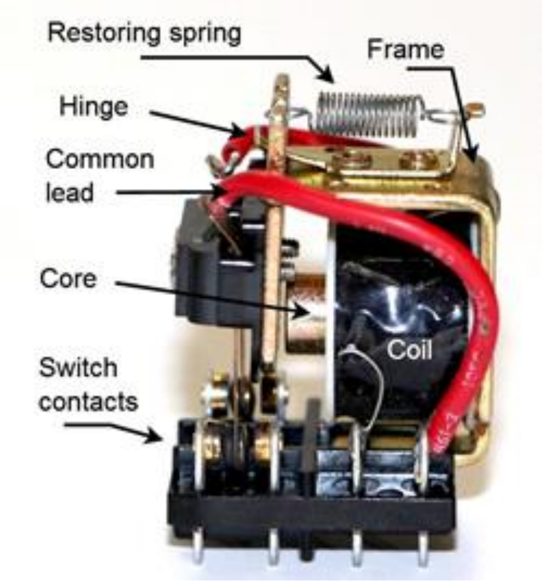
find the magnetizing force from the BH curve $B = 0.75[T]$

$$\Phi = B \cdot \mathrm{S} = 0.5[\mathrm{T}] \cdot 1 \cdot 10^{-4}[m^2] = 0.75 \cdot 10^{-4}[Wb]$$

Magnetic Force Acting on a Current-Carrying Conductor



$$q\vec{\mathbf{v}}_e \times \vec{\mathbf{B}} = -e\vec{\mathbf{v}}_e \times \vec{\mathbf{B}}$$

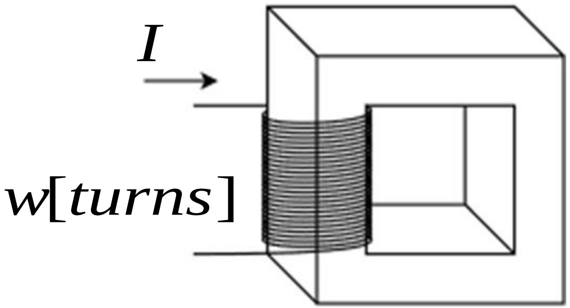


Magnetic circuits analysis

Table

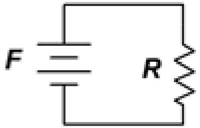
Section	Flux Φ (Wb)	Area S (m2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	"Drop" Hl (At)

Example



Magnetic system

Assume the core is made of sheet steel, has a 200 turn coil, a cross section of 1 cm by 1 cm, and an average length of 12 cm.
Determine tpathhe coil current required to achieve a flux of $1{\cdot}10^{-4}$ webers.



electrical circuit analogy

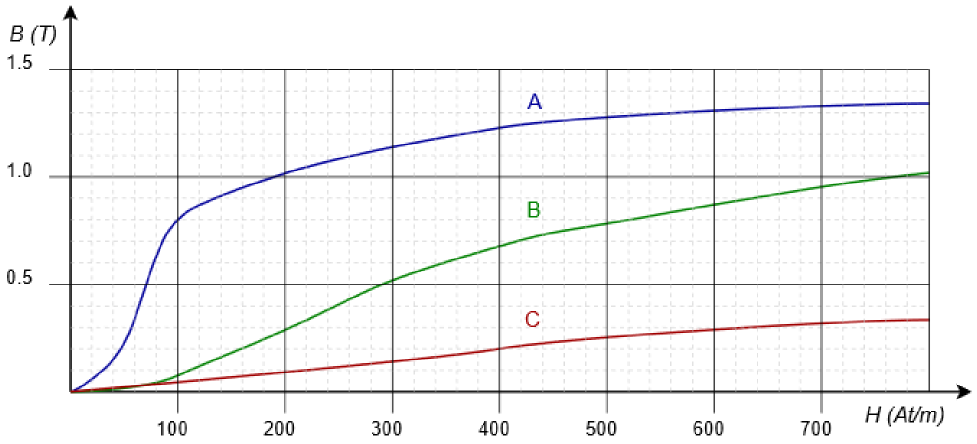
$$wI = H_{sheet} l_{sheet}$$

Section	Flux Φ (Wb)	Area S(m2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	“Drop” Hl (At)
Sheet Steel	$1{\cdot}10^{-4}$	$1{\cdot}10^{-4}$			0.12	

$$B = \frac{\Phi}{S} = \frac{1 \cdot 10^{-4}[Wb]}{1 \cdot 10^{-4}[m^2]} = 1[T]$$

Section	Flux Φ (Wb)	Area S(m2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	"Drop" Hl (At)
Sheet Steel	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$	1		0.12	

ITMO



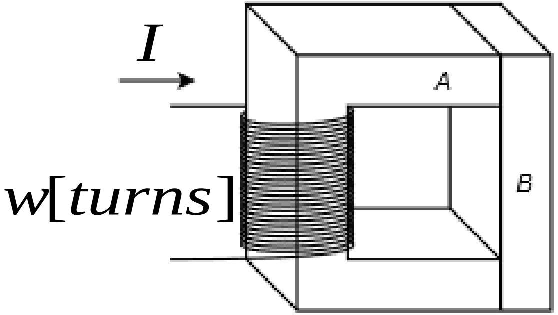
Section	Flux Φ (Wb)	Area S(m2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	"Drop" Hl (At)
Sheet Steel	$1 \cdot 10^{-4}$	$1 \cdot 10^{-4}$	1	190	0.12	

$$H \cdot l = 190 \cdot 0.12 = 22.8[At]$$

Section	Flux Φ (Wb)	Area S(m2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	“Drop” Hl (At)
Sheet Steel	$1{\cdot}10^{-4}$	$1{\cdot}10^{-4}$	1	190	0.12	22.8

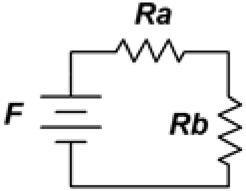
$$I = \frac{H \cdot l}{w} = \frac{22.8[At]}{200[t]} = 114[mA]$$

Example



Magnetic system

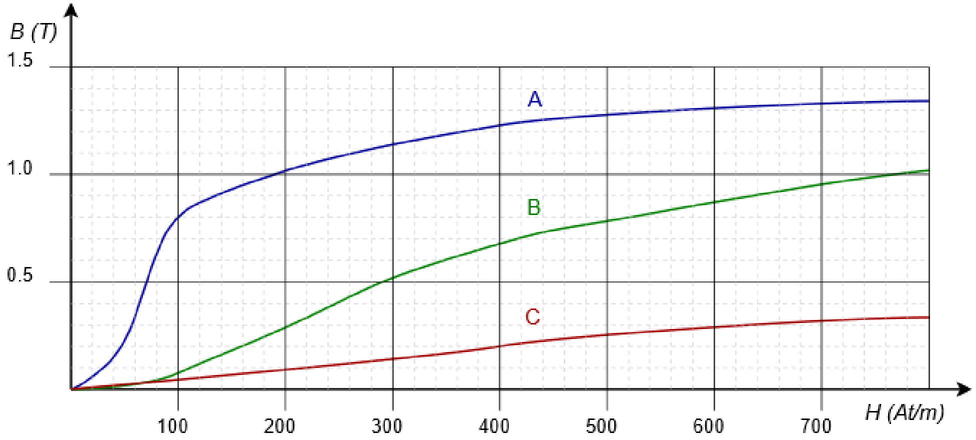
Given the magnetic system assume section A is made of sheet steel and section B is made of cast steel. Each part has a cross section of 2 cm by 2 cm. The path length of A is 12 cm and the path length of B is 4 cm.
If the coil has 50 turns, determine the coil current required to achieve a flux of $2 \cdot 10^{-4}$ webers.



electrical circuit analogy

$$wI = H_{sheet} l_{sheet} + H_{cast} l_{cast}$$

Section	Flux Φ (Wb)	Area S (m^2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	“Drop” Hl (At)
Sheet Steel	$2 \cdot 10^{-4}$	$4 \cdot 10^{-4}$	0.5		0.12	
Cast Steel	$2 \cdot 10^{-4}$	$4 \cdot 10^{-4}$	0.5		0.04	

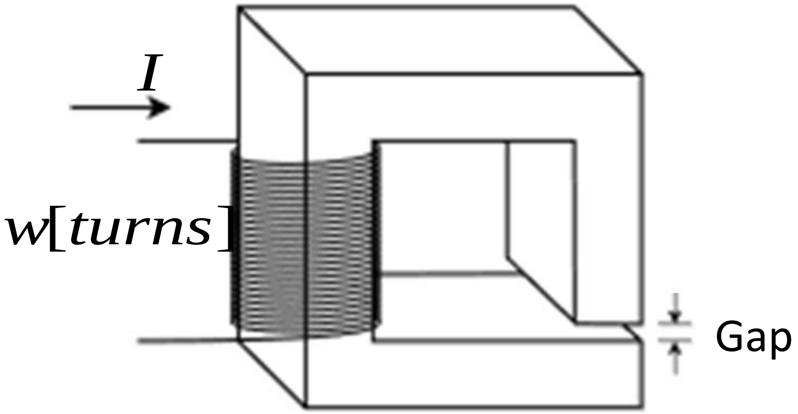


Section	Flux Φ (Wb)	Area S (m^2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	"Drop" Hl (At)
Sheet Steel	$2 \cdot 10^{-4}$	$4 \cdot 10^{-4}$	0.5	70	0.12	
Cast Steel	$2 \cdot 10^{-4}$	$4 \cdot 10^{-4}$	0.5	290	0.04	

Section	Flux Φ (Wb)	Area S (m^2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	“Drop” Hl (At)
Sheet Steel	$2 \cdot 10^{-4}$	$4 \cdot 10^{-4}$	0.5	70	0.12	8.4
Cast Steel	$2 \cdot 10^{-4}$	$4 \cdot 10^{-4}$	0.5	290	0.04	11.6

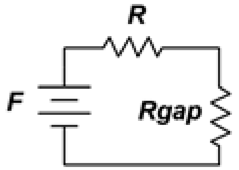
$$I = \frac{H \cdot l}{w} = \frac{20[At]}{50[t]} = 400[mA]$$

Example



Magnetic system

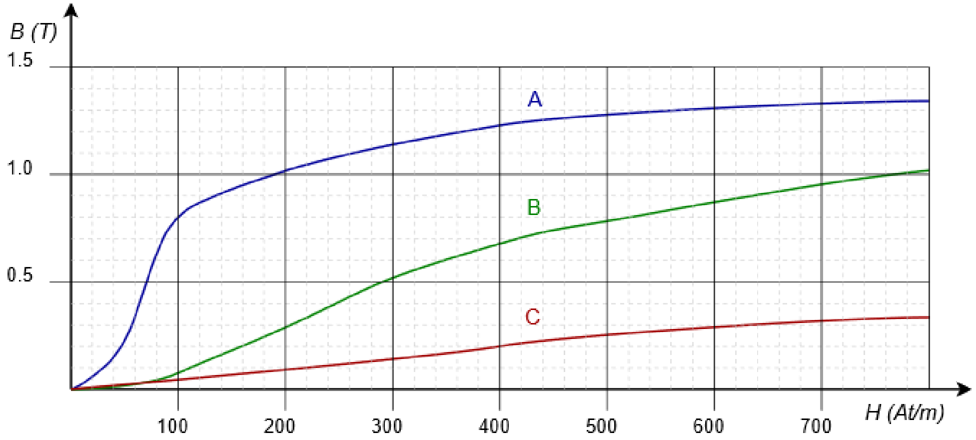
Given the magnetic system assume the core is made of sheet steel. The cross section throughout is $3 \cdot 10^{-4}\ m^2$. The path length of the main core is 8 cm and the path length of the gap is 1 mm. How many turns will the coil need in order for a coil current of 400 mA to achieve a flux of $1.2 \cdot 10^{-4}$ webers?



electrical circuit analogy

$$wI = H_{sheet} l_{sheet} + H_{gap} l_{gap}$$

Section	Flux Φ (Wb)	Area S (m^2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	"Drop" Hl (At)
Sheet Steel	$1.2 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	0.4		$8 \cdot 10^{-2}$	
Gap	$1.2 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	0.4		$1 \cdot 10^{-3}$	



Section	Flux Φ (Wb)	Area S (m^2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	"Drop" Hl (At)
Sheet Steel	$1.2 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	0.4	63	$8 \cdot 10^{-2}$	
Gap	$1.2 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	0.4	$3.183 \cdot 10^{-5}$	$1 \cdot 10^{-3}$	

Sheet Steel $$H \cdot l = 8 \cdot 10^{-2} \cdot 63 = 5[At]$$

Gap $$H \cdot l = 1 \cdot 10^{-3} \cdot 3.183 \cdot 10^{5} = 318.3[At]$$

Section	Flux Φ (Wb)	Area S (m^2)	Flux Density B (T)	Magnetizing Force H (At/m)	Length l (m)	"Drop" Hl (At)
Sheet Steel	$1.2 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	0.4	63	$8 \cdot 10^{-2}$	5.0
Gap	$1.2 \cdot 10^{-4}$	$3 \cdot 10^{-4}$	0.4	$3.183 \cdot 10^{5}$	$1 \cdot 10^{-3}$	318.3

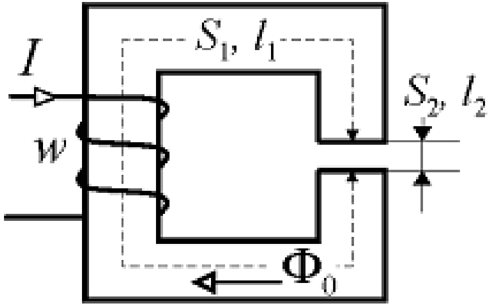
$$w = \frac{H \cdot l}{I} = \frac{323.3[At]}{400[mA]} = 808[turns]$$

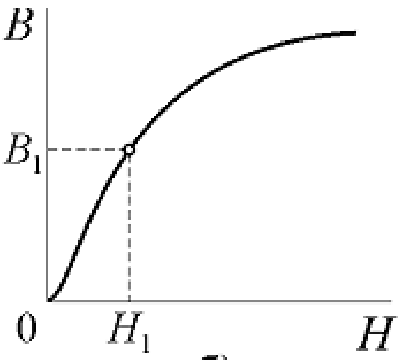
Simple electro-magnetic circuits

An unbranched magnetic circuit is a circuit, through all the elements of which the same magnetic flux

Direct way

consists in determining the MMF at a given magnetic flux Φ





The magnetic flux in the magnetic circuit and in the gap is the same

$$\Phi = B_1 S_1 = B_2 S_2$$

magnetizing force in the gap

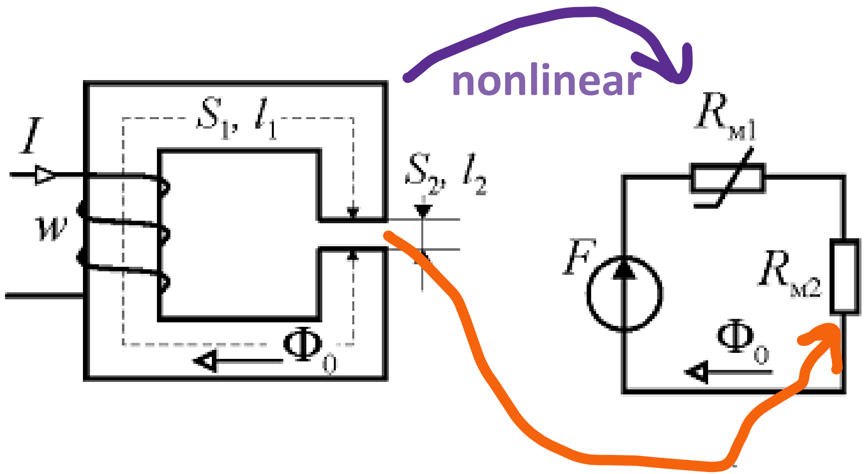
$$H_2 = B_2 / \mu_0 = \Phi / (S_2 \mu_0)$$

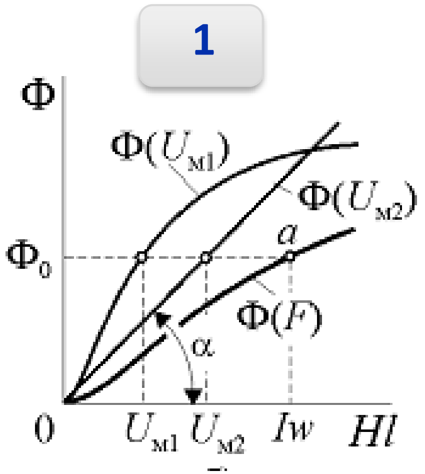
H_1 determined by the magnetization curve B(H) in accordance with the value of the flux density $B_1 = \Phi / S_1$

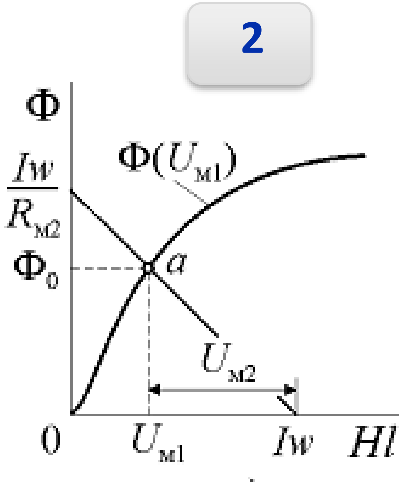
MMF $F = H_1 l_1 + H_2 l_2$

Opposite way

consists in determining magnetic flux Φ at a given MMF



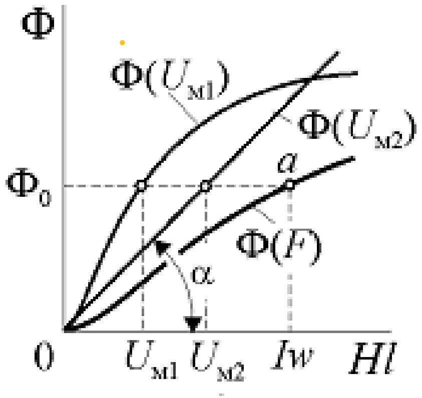




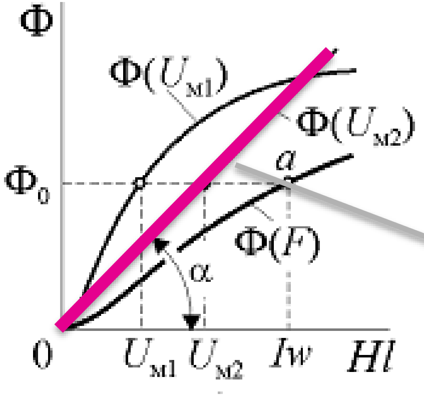
$$wI = H_1 l_1 + H_2 l_2$$

$$\Phi_0 \frac{l_1}{\mu_1(U_{M1})\mu_0 S_1} + \Phi_0 \frac{l_2}{\mu_0 S_2} = \Phi_0 \left[R_{M1}(U_{M1}) + R_{M2} \right] = Iw = F$$

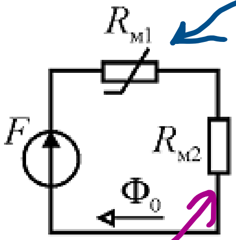
Graphical method

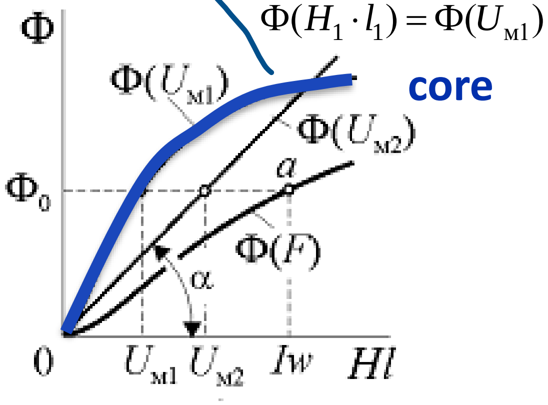


$$\alpha = \operatorname{arcctg}\left(R_{м2} m_\Phi / m_U\right)$$

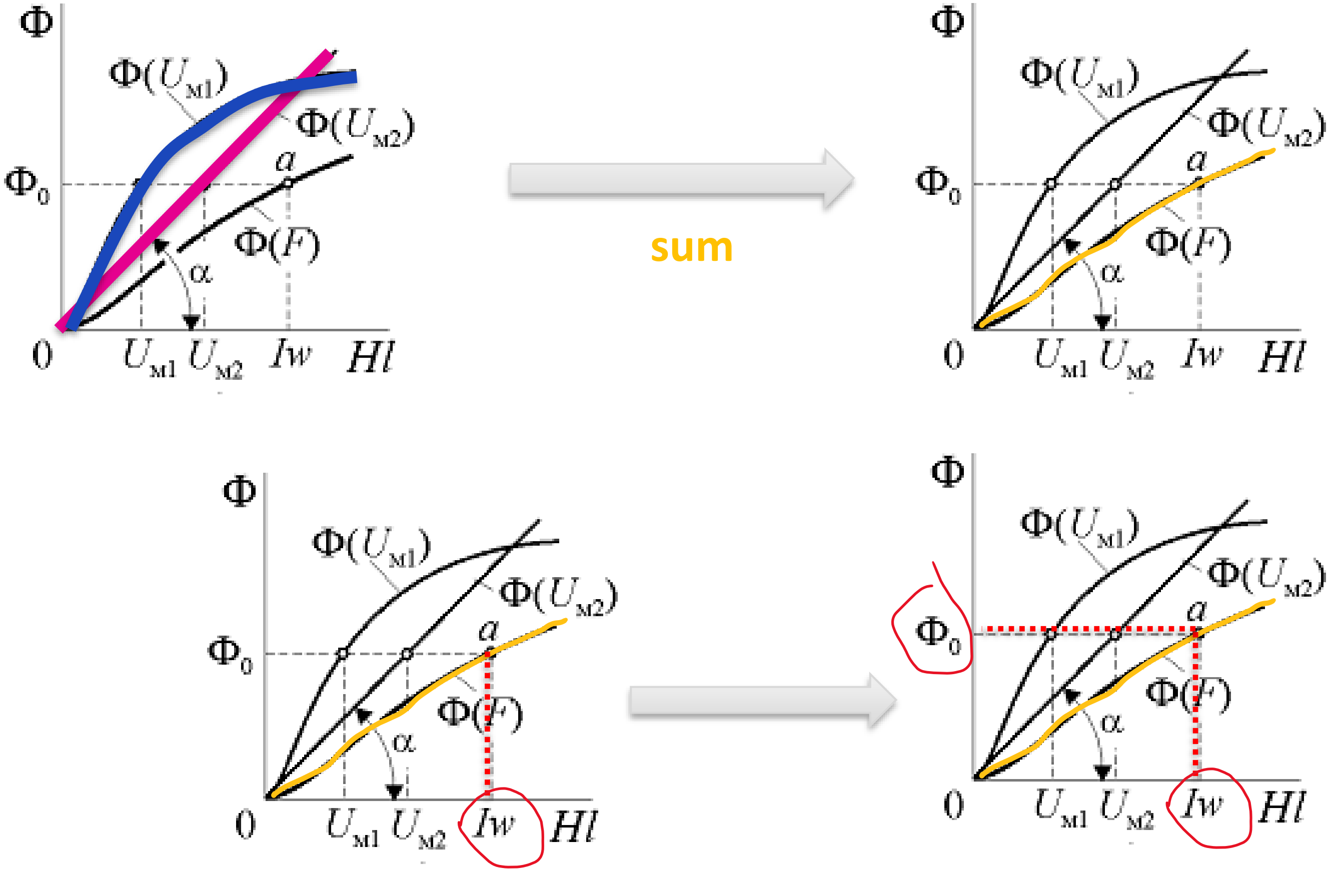


air gap

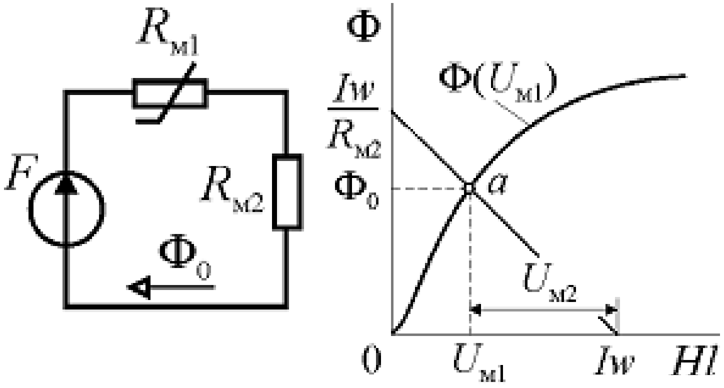




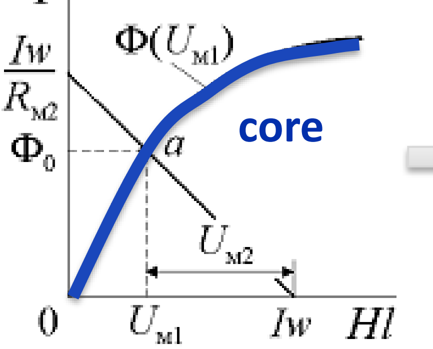
Graphical method

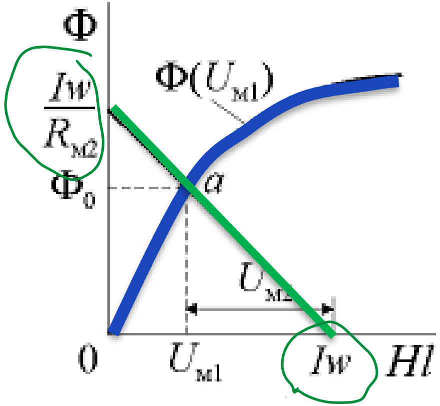


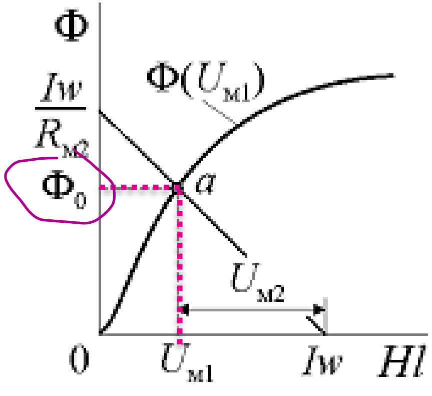
2 Full-load characteristic method



$$\Phi(H_1 \cdot l_1) = \Phi(U_{м1})$$

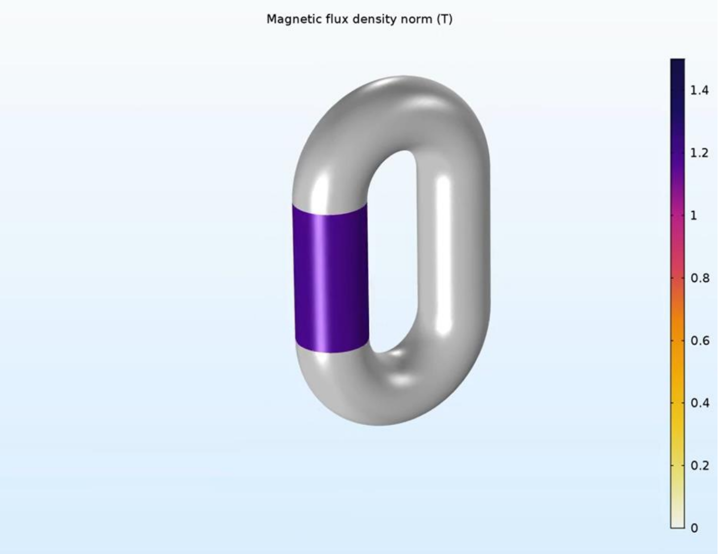






- ❑ Weakly magnetic materials
  paramagnetic and diamagnetic materials
- ❑ Soft irons
- ❑ Hard irons
  permanent magnets

ferromagnetic

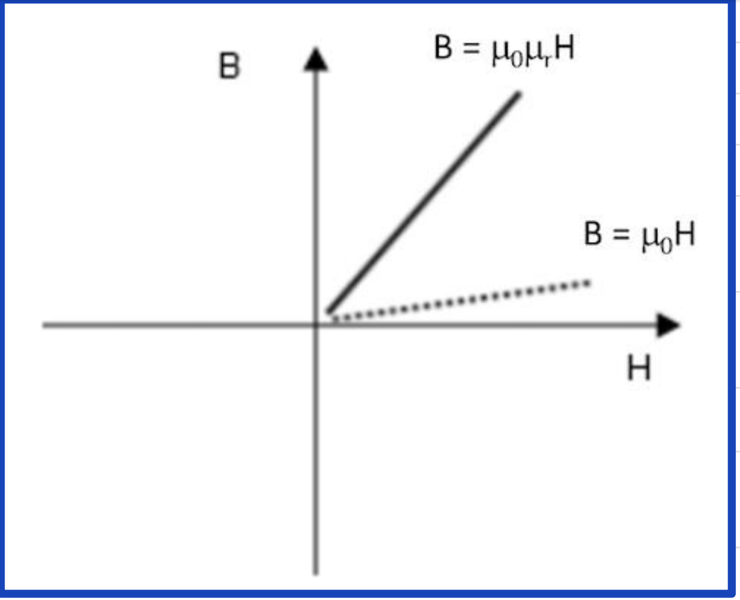


Constitutive Relations	Soft Iron, (Fully Time-dependent)	Soft Iron (AC Feeding)	Permanent Magnets (Fully Time-Dependent)	Required Information
Relative Permeability	✓	✓		1 scalar (or tensor)
Magnetic Losses		✓		2 scalars (or tensors)
B-H Curve	✓			1 function
Effective B-H Curve		✓		1 function
Magnetization			✓	1 vector
Remanent Flux Density			✓	1 scalar (or tensor) and 1 vector
BH Nonlinear Permanent Magnet			✓	Function and a direction
Hysteresis Jiles-Atherton Model	✓		✓	5 scalars (or tensors)
External Magnetic Material	✓	✓	✓	Externally compiled code

Laws for Soft Irons

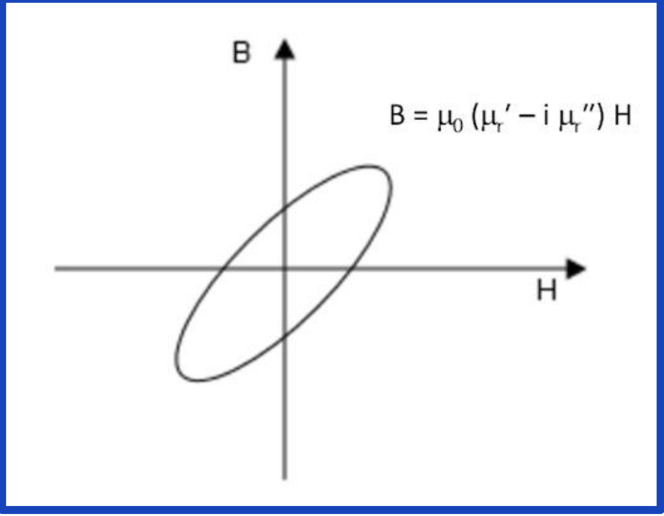
Laws for Permanent Magnets

Constitutive Relations	Soft Iron, (Fully Time-dependent)	Soft Iron (AC Feeding)	Permanent Magnets (Fully Time-Dependent)	Required Information
Relative Permeability	✓	✓		1 scalar (or tensor)
Magnetic Losses				rs)
B-H Curve				
Effective B-H Cur				
Magnetization				
Remanent Flux D				
BH Nonlinear Pe				
Hysteresis Jiles-A				
External Magnet				



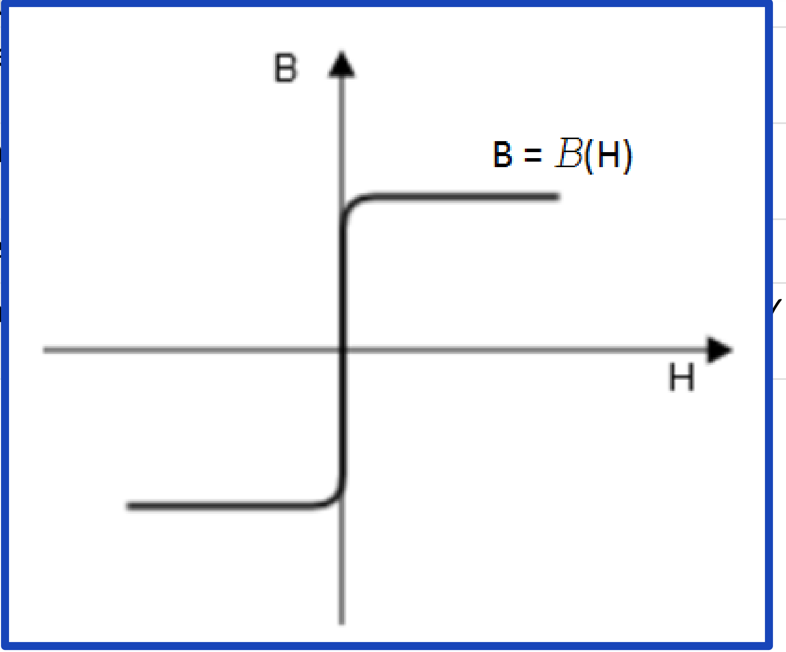
- May be used in soft irons if the fields are small
- Laminated iron in the column of a power transformer under short-circuit feeding is typically very well described by this law

Constitutive Relations	Soft Iron, (Fully Time-dependent)	Soft Iron (AC Feeding)	Permanent Magnets (Fully Time-Dependent)	Required Information
Relative Permeability	✓	✓		1 scalar (or tensor)
Magnetic Losses		✓		2 scalars (or tensors)
B-H Curve	✓			1 function
Effective B-H (		✓		1 function
Magnetizatior				
Remanent Flu				r)
BH Nonlinear				
Hysteresis Jile				rs)
External Magı		✓		ed



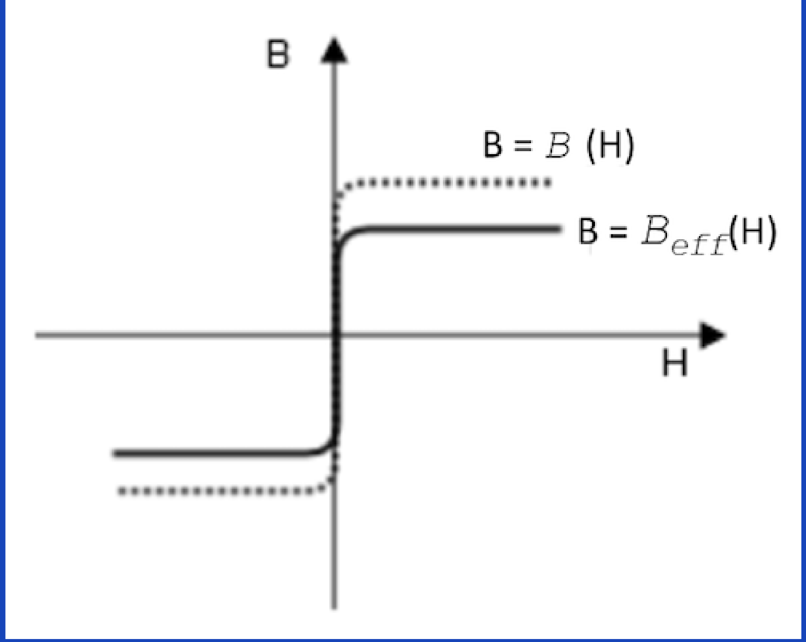
- Typical of all ferromagnetic materials at higher frequencies
- Ferrite materials employed in high-frequency inductors, transformers, or flux concentrators typically offer such data in their data sheets

Constitutive Relations	Soft Iron, (Fully Time-dependent)	Soft Iron (AC Feeding)	Permanent Magnets (Fully Time-Dependent)	Required Information
Relative Permeability	✓	✓		1 scalar (or tensor)
Magnetic Losses		✓		2 scalars (or tensors)
B-H Curve	✓			1 function
Effective B-H Curve		✓		1 function
Magnetization			✓	1 vector
Remane				r)
BH Non				
Hystere				rs)
Externa		✓		d



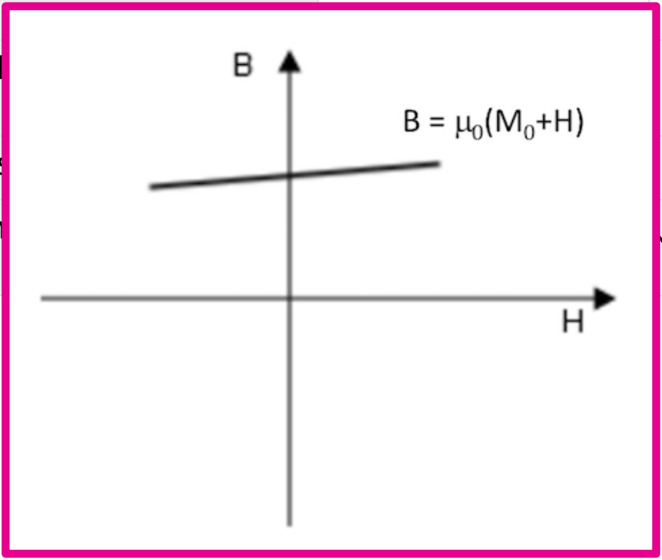
- Most common description for magnetic soft iron steels
- Includes the effect of magnetic saturation
- Used for moving magnetic circuits such as motors and generators (i.e., a circuit with varying reluctance)
- Behaves as the *Relative Permeability* constitutive relation for small fields

Constitutive Relations	Soft Iron, (Fully Time-dependent)	Soft Iron (AC Feeding)	Permanent Magnets (Fully Time-Dependent)	Required Information
Relative Permeability	√	√		1 scalar (or tensor)
Magnetic Losses		√		2 scalars (or tensors)
B-H Curve	√			1 function
Effective B-H Curve		√		1 function
Magnetization			√	1 vector
Remanent I				r)
BH Nonline				
Hysteresis J				)
External Ma				



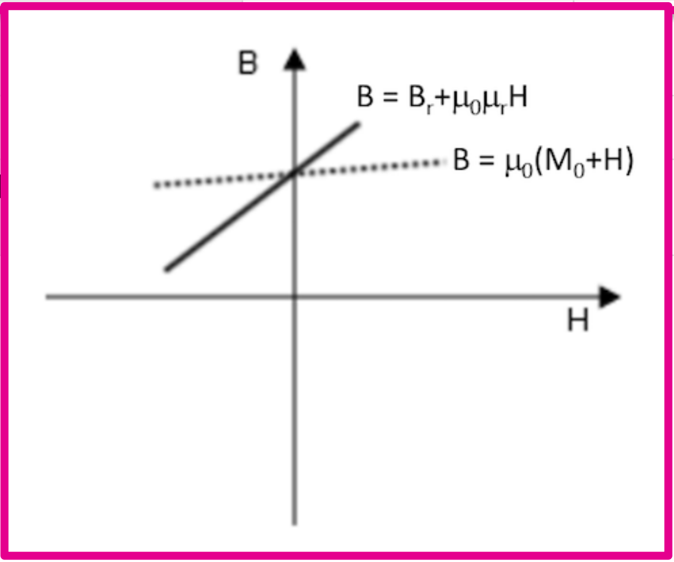
- Generalization of the *B-H Curve* constitutive relation specific for AC feeding
- Works for static circuits or for cases where the geometrical configuration is slowly changing with respect to the AC external magnetic field
- Ferromagnetic parts in an induction heating apparatus or a transformer core under open circuit may use this condition
- Behaves as the *Relative Permeability* constitutive relation for small fields

Constitutive Relations	Soft Iron, (Fully Time-dependent)	Soft Iron (AC Feeding)	Permanent Magnets (Fully Time-Dependent)	Required Information
Relative Permeability	√	√		1 scalar (or tensor)
Magnetic Losses		√		2 scalars (or tensors)
B-H Curve	√			1 function
Effective B-H Curve		√		1 function
Magnetization			√	1 vector
Remanent Flux Density			√	1 scalar (or tensor)
BH Nonlinear I				
Hysteresis Jiles				ors)
External Magn		√		led

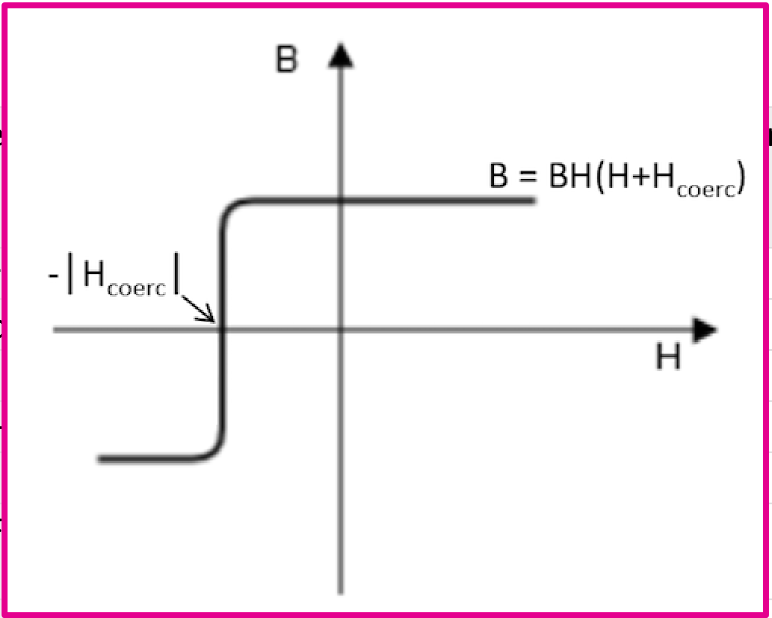


- Typical description for rare earth permanent magnets
  - Used in modern motors, generators, and sensors

Constitutive Relations	Soft Iron, (Fully Time-dependent)	Soft Iron (AC Feeding)	Permanent Magnets (Fully Time-Dependent)	Required Information
Relative Permeability	✓	✓		1 scalar (or tensor)
Magnetic Losses		✓		2 scalars (or tensors)
B-H Curve	✓			1 function
Effective B-H Curve		✓		1 function
Magnetization			✓	1 vector
Remanent Flux Density			✓	1 scalar (or tensor)
BH Nonlinear Perm				
Hysteresis Jiles-Ath				ors)
External Magnetic I				led



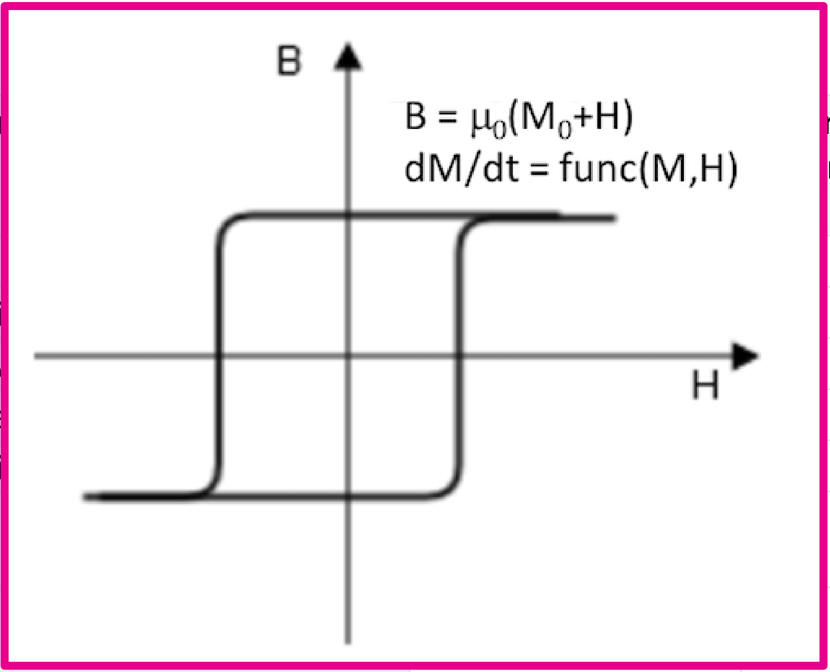
- Generalization of the *Magnetization* constitutive relation
- Gives the option to better include the effect of demagnetizing due to an externally applied field in the opposite direction to the current magnetization
- May be useful for AlNiCo-like materials under small changes of the applied field



Constitutive		ro ng		
Relative Per				
Magnetic Lo				
B-H Curve				
Effective B-H				
Magnetizati				
Remanent F				
BH Nonlinear Permanent Magnet				direction
Hysteresis Jiles-Atherton Model	✓		✓	5 scalars (or tensors)
External Magnetic Material	✓	✓	✓	Externally compiled code

- Specific for modeling the demagnetization of permanent magnets when only uniaxial data is provided from the magnet manufacturer
- Common for AlNiCo and rare earth magnets under high temperature conditions
- Mix of *B-H Curve* and *Magnetization* constitutive relations, as it uses a description similar to the *B-H Curve* constitutive relation, but shifts the curve in the B-H plane

Constitu		ro ng		
Relative				
Magneti				
B-H Curv				
Effective				
Magneti				
Remane				
BH Nonl				
Hysteresis Jiles-Atherton Model	✓		✓	5 scalars (or tensors)
External Magnetic Material	✓	✓	✓	Externally compiled code



- Very flexible in modeling different materials, as it contains many different parameters
- Can be used for fine-tuning loss computation in motors and other electrical machines (even though its applicability is sometimes limited by the difficulty in retrieving material parameters)