Electrical Machines

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Transformers

Part I

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What is transformer?

A transformer is a static electromagnetic machine (no moving parts!).

- ❑ Changing the voltage and current levels in a given electrical system
- ❑ Establishing electrical isolation
- ❑ Impedance matching
- ❑ Measuring instruments

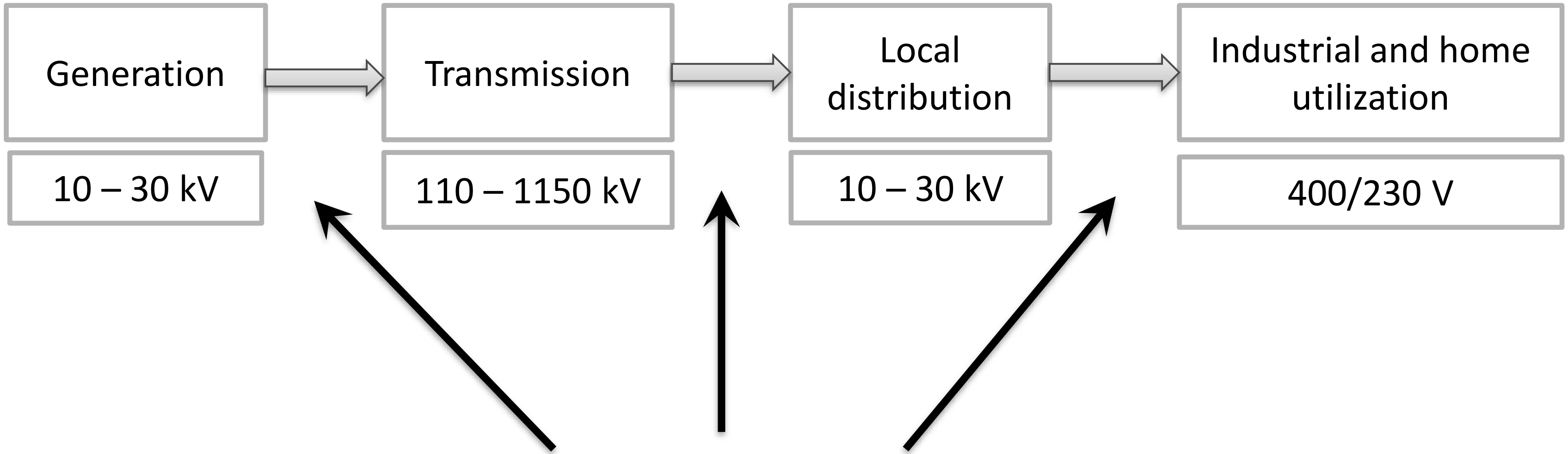
Single phase voltage transformer

High-voltage power transformer

https://the-rsgroup.com/low-loss-power-transformers-for-latvia/

Autotransformer

Where do we use transformers?



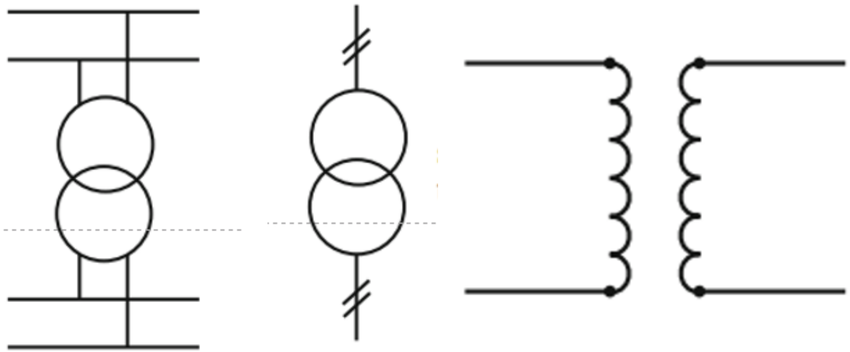
- Power and distribution transformers

Where do we use transformers?

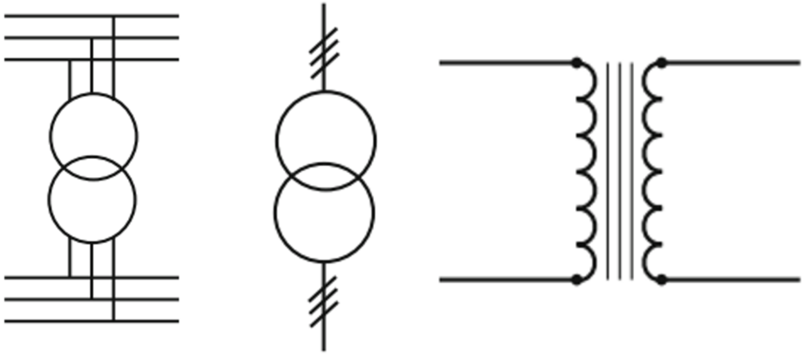
- **Isolating transformers:**
  - to electrically isolate electric circuits from each other or to block DC signals while maintaining AC continuity between the circuits
  - to eliminate electromagnetic noise in many types of circuits.
- **Various tasks in communication systems and control electrical circuits:** impedance matching, input transformers, output transformers, and insulation apparatus between electric circuits, and interstage transformers
- **Instrument transformers:**
  - to measure high voltages and large currents with standard small-range voltmeters (120V), ammeters (5 A), and wattmeters.
  - to transform voltages and currents to activate relays for control and protection.

Symbols of transformers

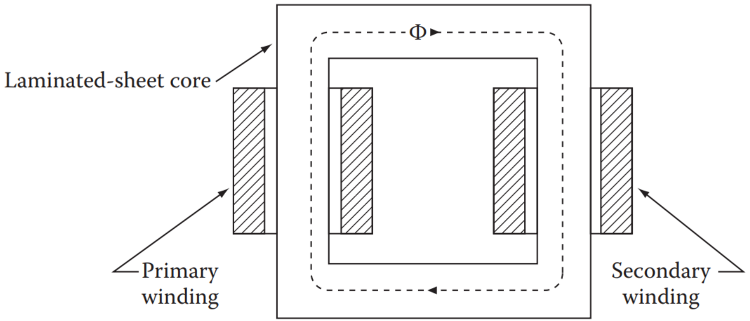
- Single-phase transformer



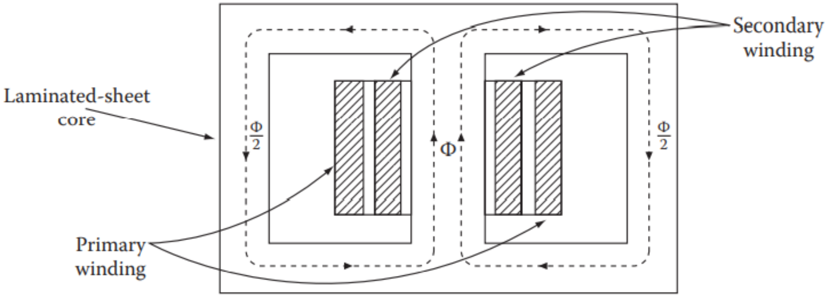
- Three-phase transformer



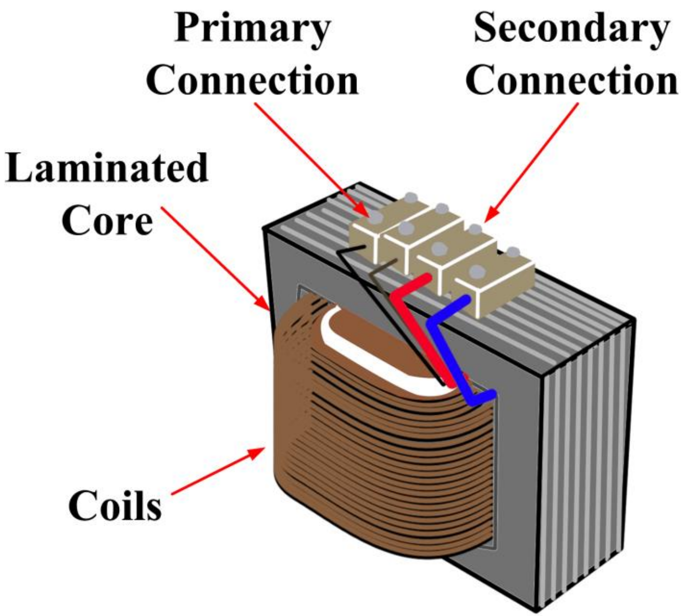
Transformer construction



Core type transformer



Shell type transformer



Faraday’s law of induction

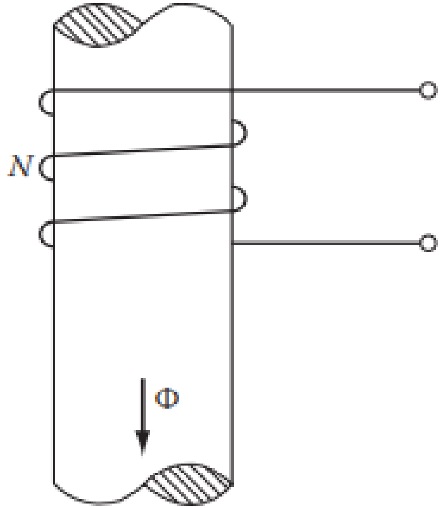
whenever a flux passes through a turn of a coil, a voltage (i.e., an electromotive force [emf]) is induced, in each turn of that coil, that is directly proportional to the rate of change in the flux with respect to time:

$$e_{ind} = \frac{d\Phi}{dt} \qquad (1)$$

where Φ is the flux that passes through the turn.

If such a coil has N turns and the same flux passes through all of them, the resulting induced voltage between the two terminals of the coil becomes:

$$e_{ind} = N\frac{d\Phi}{dt} \qquad (2)$$



Lenz’s law of induction

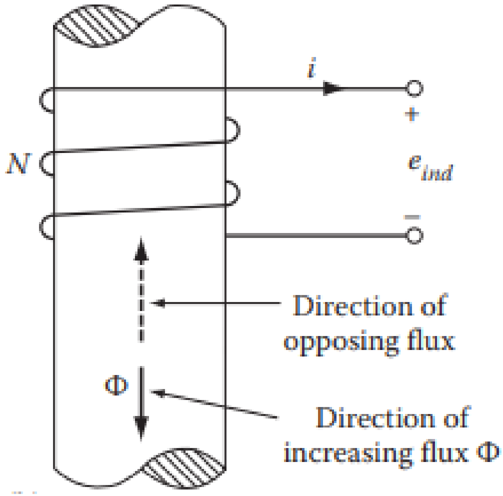
if the coil ends were connected together, the voltage built-up would produce a current that would create a new flux opposing the original flux change.

Equations (1), (2) can be re-expressed as:

$$e_{ind} = -\frac{d\Phi}{dt}$$

$$e_{ind} = -N\frac{d\Phi}{dt}$$

where the negative sign in the equations signifies that the polarity of the induced voltage opposes the change that produced it.



The magnitude of the induced voltage also can be determined using the flux linkage of a given coil:

$$e_{ind} = \frac{d\lambda}{dt} \qquad \lambda = \sum_{i=1}^{N} \Phi_i = Li$$

An applied **sinusoidal voltage** has to produce a **sinusoidally changing flux**, provided that the resistive voltage drop is negligible. The flux as a function of time is given as:

$$\Phi = \Phi_m \sin \omega t$$

where Φ_m is the maximum value of the flux, ω is 2πf, f is the frequency in Hz.

Then the induced voltage is given as:

$$e_{ind} = N\frac{d\Phi}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt} = \omega N \Phi_m \cos \omega t$$

the induced emf leads the flux by 90°.

The rms value of the induced emf is given as:

$$E = \frac{2\pi}{\sqrt{2}} fN\Phi_m = 4.44\, fN\Phi_m$$

If voltage drop due to the resistance of the winding is neglected, the counter-emf equals the applied voltage. Therefore:

$\boxed{V = E = 4.44\, fN\Phi_m}$ - **emf equation** of a transformer or **general transformer equation**

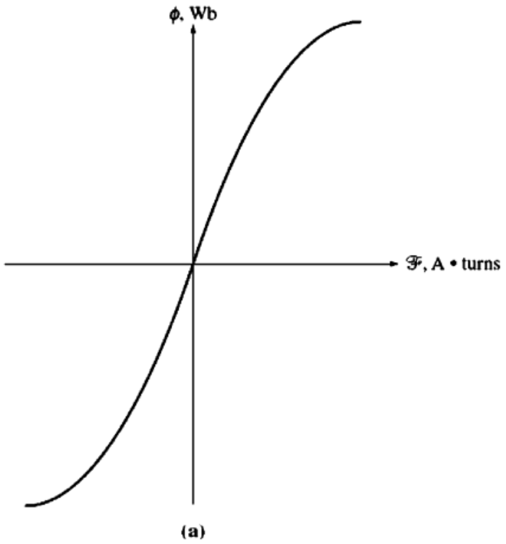
$$\Phi_m = \frac{V}{4.44\, fN}$$

The flux is determined by the applied voltage, the frequency of the applied voltage, and the number of turns in the winding.

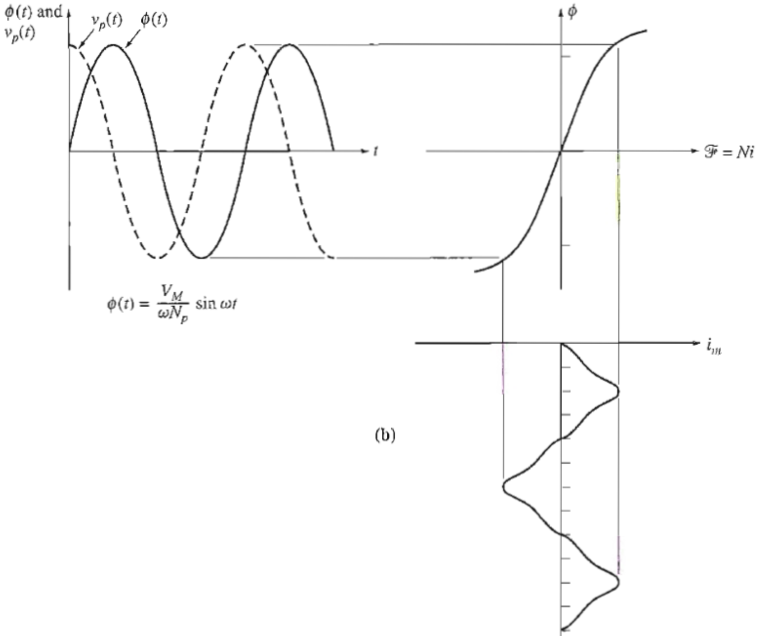
The excitation (or exciting) current adjusts itself to produce the maximum flux required.

If the maximum flux density takes place in a **saturated** core, the current has to increase disproportionately during each half period to provide this flux density.

For this reason, inductors with ferromagnetic cores end up having non-sinusoidal excitation currents.

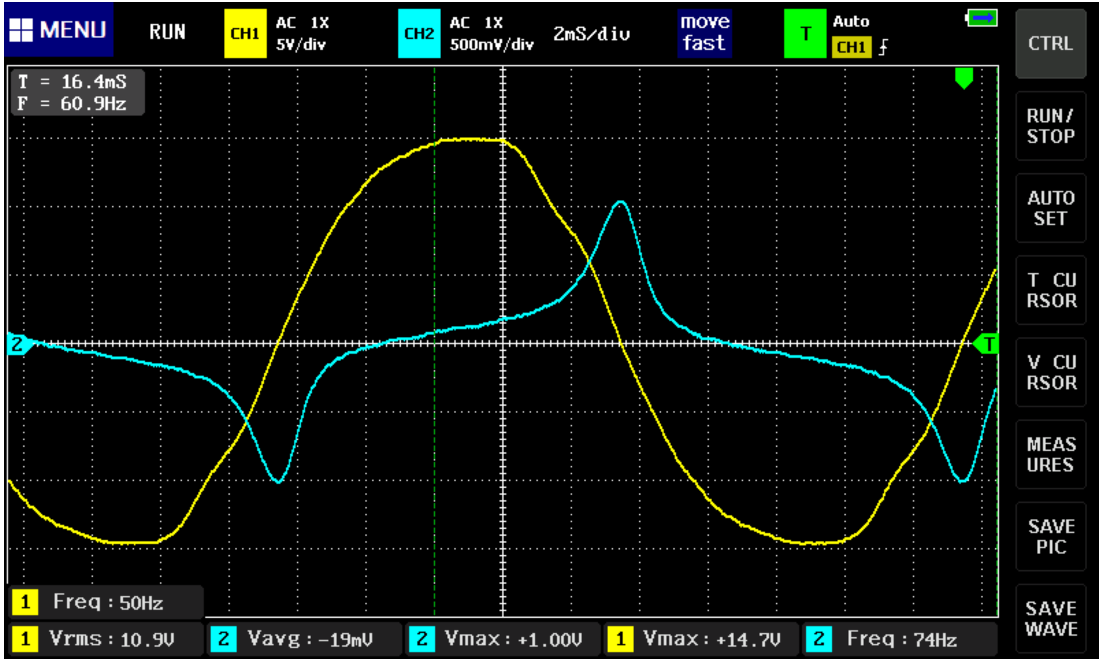


Magnetization curve of the transformer core



Magnetization current caused by the flux in the transformer core

Primary voltage and magnetizing current of the real transformer:



If the core is unsaturated and the resistance of the coil is negligible, the maximum value of the magnetizing current can be found from:

$$I_m = \frac{N\Phi_m}{L} = \frac{\sqrt{2}V}{\omega L}$$

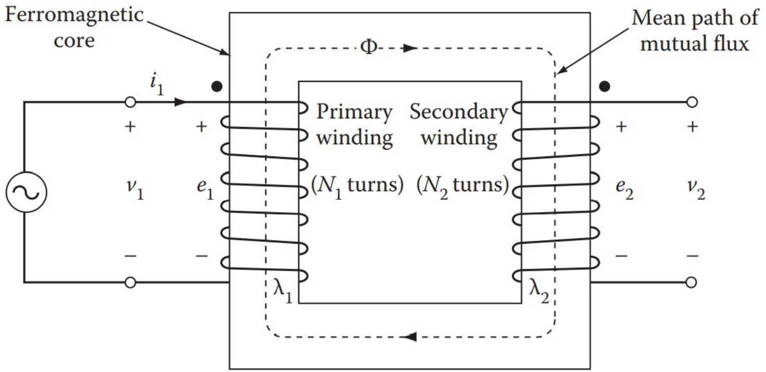
In the phasor form, the magnetizing current that produces the mutual flux is:

$$I_m = \frac{V}{j\omega L} = \frac{V}{jX_m}$$

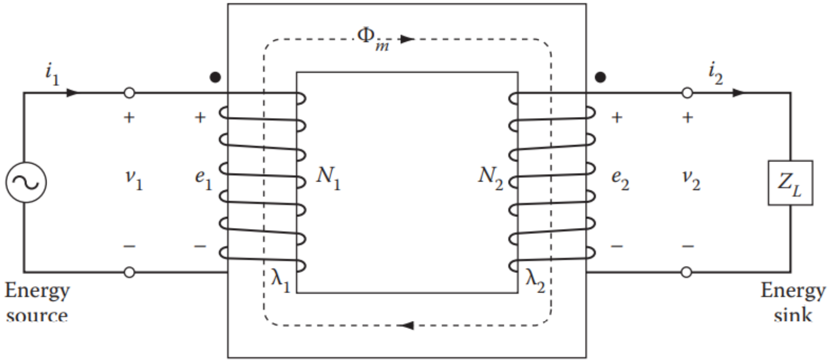
where X_m is the magnetizing reactance of the coil.

Ideal transformer

- The winding resistances are negligible.
- All magnetic flux is confined to the ferromagnetic core and links both windings, that is, leakage fluxes do not exist.
- The core losses are negligible.
- The excitation current required to establish flux in the core is negligible.
- The magnetic core material does not saturate.



«Ideal» transformer with no load



«Ideal» transformer with load

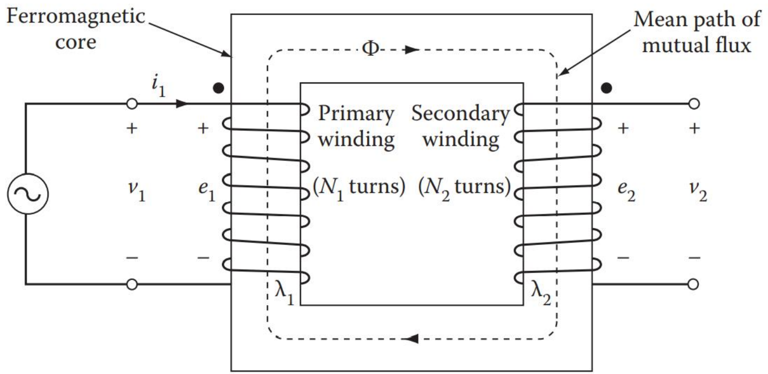
Ideal transformer

$$v_1 = e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi}{dt} \qquad (3)$$

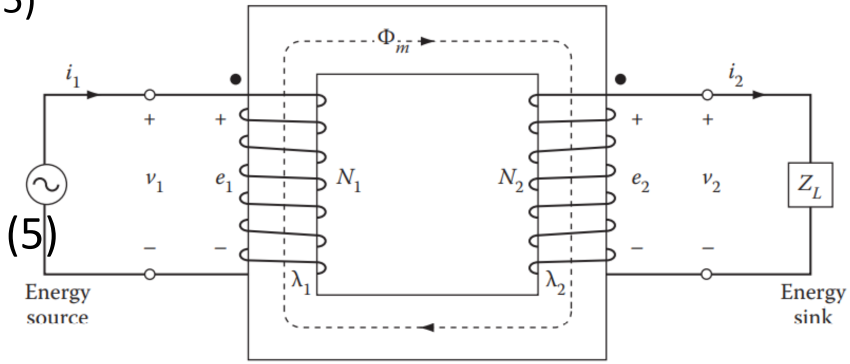
$$v_2 = e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi}{dt} \qquad (4)$$

$$\boxed{\frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{N_1}{N_2} = a} \text{ or in rms form } \boxed{\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a} \qquad (5)$$

where **a** is known as the **«turns ratio»** or the **«ratio of transformation»**



«Ideal» transformer with no load



(5)

«Ideal» transformer with load

Ideal transformer

Since the core of an ideal transformer is infinitely permeable, the net mmf will always be zero:

$$F_{net} = N_1 i_1 - N_2 i_2 = \Phi \cdot R = 0$$

where R is the reluctance of the magnetic core.

Since the reluctance of a magnetic core of a well-designed modern transformer is very small (almost zero) before the core is saturated:

$$N_1 i_1 - N_2 i_2 = 0 \Rightarrow N_1 i_1 = N_2 i_2 \Rightarrow \boxed{\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}}$$

or in rms form

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a}} \qquad (6)$$

Ideal transformer

From equations (5) and (6) it can be expressed:

$$V_1 = aV_2 \qquad I_1 = \frac{I_2}{a} \quad \Rightarrow \quad V_1 I_1 = aV_2 \cdot \frac{I_2}{a} = V_2 I_2$$

In an ideal transformer, the input power (VA) is equal to the output power (VA).

This is the power invariance principle which means that the volt-amperes are conserved.

If **a = 1**, the transformer is known as the **isolating transformer**.

If **a < 1**, the transformer is known as a **step-up transformer**.

If **a > 1**, the transformer is known as a **step-down transformer**.

Ideal transformer

Impedance Transfer through a Transformer

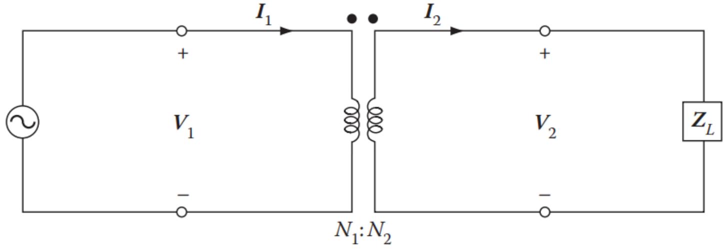
A load impedance: $\mathbf{Z}_L = \dfrac{\mathbf{V}_2}{\mathbf{I}_2}$

Notification:

$\mathbf{Z}$ - vector (complex) value

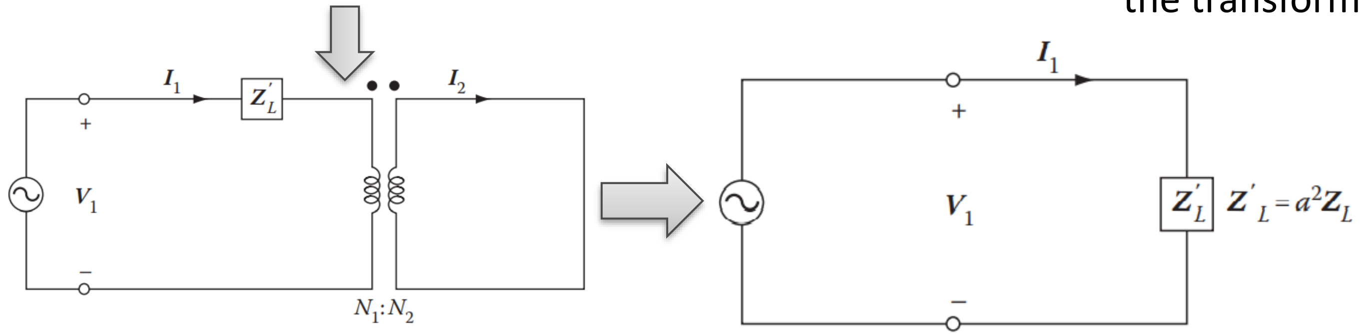
Z - scalar value

Impedance transfer to primary winding:



$$\mathbf{Z}'_L = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \frac{a\mathbf{V}_2}{\mathbf{I}_2 / a} = a^2 \frac{\mathbf{V}_2}{\mathbf{I}_2} = a^2 \mathbf{Z}_L$$

a² is known as the impedance ratio of the transformer



Ideal transformer

Impedance Transfer through a Transformer

Transferring an impedance from one side of the transformer to the other is known as **referring the impedance to the other side**. Thus, Z'_L is known as the load impedance referred to the primary side.

An impedance located at the primary side of a transformer can also be **referred to the secondary side** as:

$$\mathbf{Z}'_1 = \frac{\mathbf{Z}_1}{a^2}$$

It can be used in **impedance matching** to determine the maximum power transfer from a source with an internal impedance Z_S to a load impedance Z_L.

It is necessary to select the turns ratio so that:

$$\mathbf{Z}'_L = \left(\frac{N_1}{N_2}\right)^2 \mathbf{Z}_L = a^2\mathbf{Z}_L = \mathbf{Z}_S$$

Ideal transformer

Input and Output Powers of an Ideal Transformer

The **input power:**

$$P_{in} = V_1 I_1 \cos\theta_1$$

The **output power**:

$$P_{out} = V_2 I_2 \cos\theta_2$$

In an **ideal transformer**:

$$\theta_1 = \theta_2 = \theta$$

θ_1 is the angle between the primary voltage and the primary current.

θ_2 is the angle between the secondary voltage and the secondary current.

The same **power factor** is seen by both the primary and secondary windings:

$$P_{out} = V_2 I_2 \cos\theta = \frac{V_1}{a} \cdot a I_1 \cdot \cos\theta = V_1 I_1 \cos\theta = P_{in}$$

In an ideal transformer, the output power is equal to its input power, because an ideal transformer has no internal power losses.

The same argument can be extended to reactive and apparent powers:

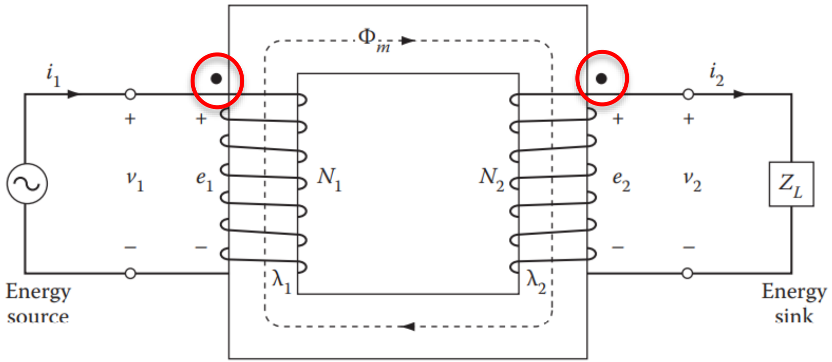
$$Q_{out} = V_2 I_2 \sin\theta = V_1 I_1 \sin\theta = Q_{in}$$

$$S_{out} = V_2 I_2 = V_1 I_1 = S_{in}$$

Dot Convention in Transformers

The dots near the upper end of each winding are known as the **polarity marks**.

The dot convention implies that (1) *currents entering at the dotted terminals will result in mmfs that will produce fluxes in the same direction*, and (2) *voltages from the dotted to undotted terminals have the same sign.*



«Ideal» transformer with load

Since the current i_1 flows into the dotted end of the primary winding and the current i_2 flows out of the dotted end of the secondary winding, the mmfs will be subtracted from each other.

Thus, it can be said that the transformer has a **subtractive polarity**. Here, current i2 is flowing in the direction of the induced current, according to Lenz's law.

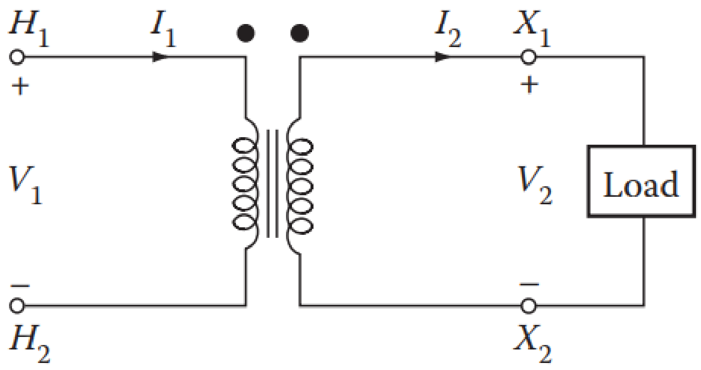
Dot Convention in Transformers

The terminals on the high-voltage side are labeled H_1 and H_2, while those on the low-voltage side are identified as X_1 and X_2.

The terminal with subscript 1 in this convention (known as the standard method of marking transformer terminals) is equivalent to the dotted terminal in the dot-polarity notation.

In a transformer where H_1 and X_1 terminals are adjacent, as shown in Figure, the transformer is said to have **subtractive polarity**.

If terminals H_1 and X_1 are diagonally opposite, the transformer is said to have **additive polarity**.



Polarity markings of a single-phase two-winding transformer

Dot Convention in Transformers

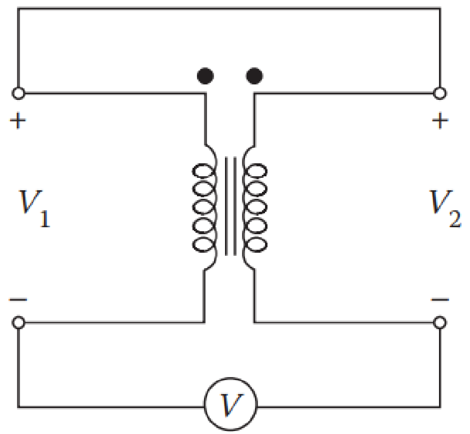
Transformer polarities can be found by performing a simple test:

Two adjacent terminals of high- and low-voltage windings are connected together, and a small voltage is applied to the high-voltage winding, as shown in Figure.

Then the voltage between the high- and low-voltage winding terminals that are not connected together are measured.

The polarity is **subtractive** if the voltage **V** reading **is less** than the voltage $\mathbf{V_1}$ which is applied to the high voltage winding.

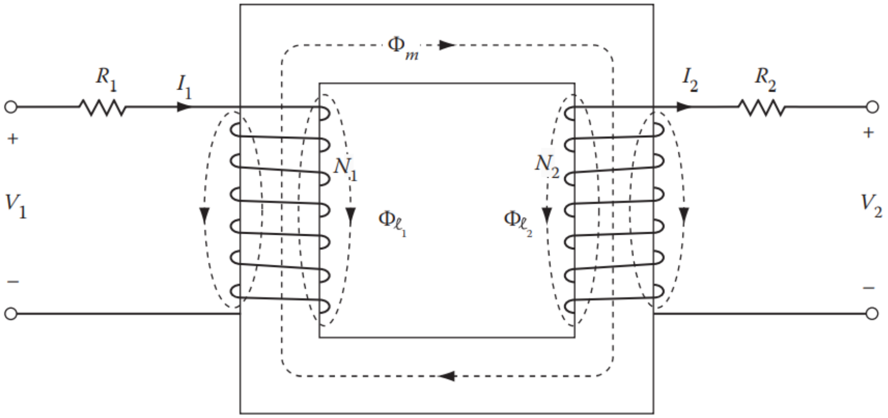
The polarity is **additive** if the voltage **V** reading **is greater** than the applied voltage $\mathbf{V_1}$



Polarity test

Real transformer

- the primary and secondary winding resistances R_1 and R_2 are not negligible;
- the leakage fluxes Φ_{l1} and Φ_{l2} exist;
- the core losses are not negligible;
- the permeability of the core material is not infinite and therefore a considerable mmf is required to establish mutual flux Φ_m in the core;
- the core material saturates.

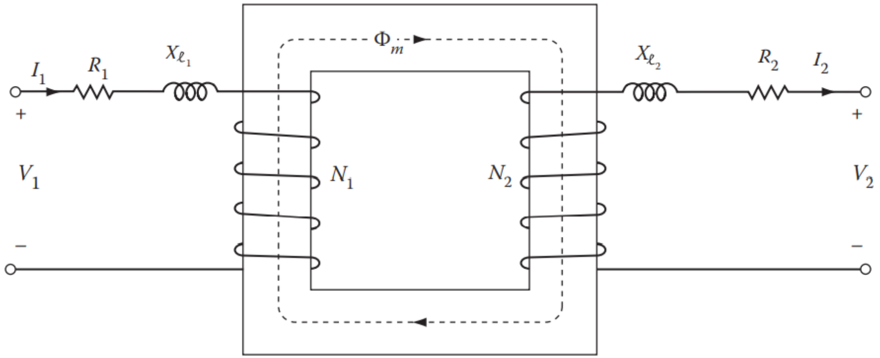


Real transformer

X_{l1} and X_{l2} are leakage inductances which correspond to leakage fluxes Φ_{l1} and Φ_{l2} :

$$X_{\ell 1} = \omega L_{\ell 1} = \omega N_1^2 P_{\ell 1} = \omega N_1 \frac{\Phi_{\ell 1}}{I_1}$$

$$X_{\ell 2} = \omega L_{\ell 2} = \omega N_2^2 P_{\ell 2} = \omega N_2 \frac{\Phi_{\ell 2}}{I_2}$$

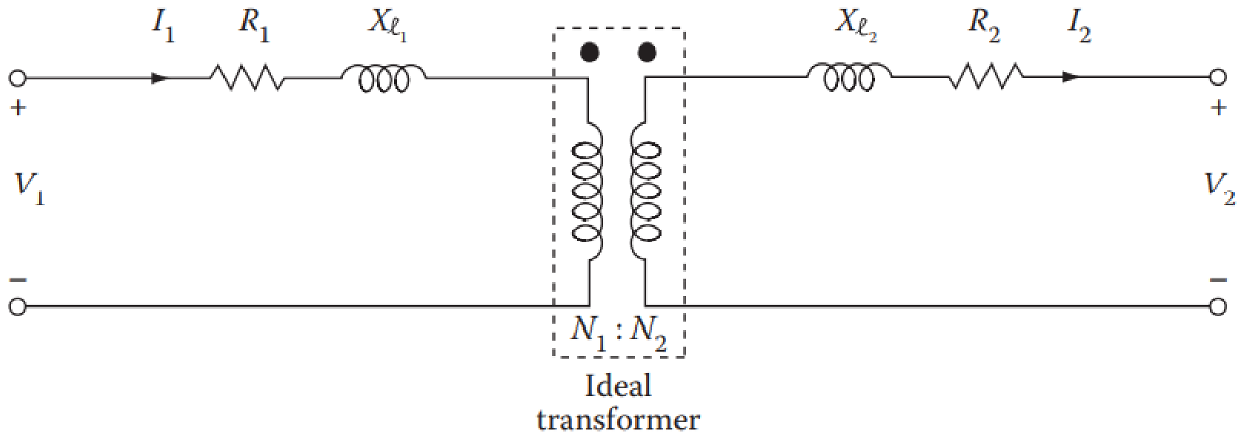


P_{l1} is the permeance of the leakage flux path of the primary winding

P_{l2} is the permeance of the leakage flux path of the secondary winding

Real transformer

In such a representation, the transformer windings are tightly coupled by a mutual flux, and represented as shown in Figure



Real transformer

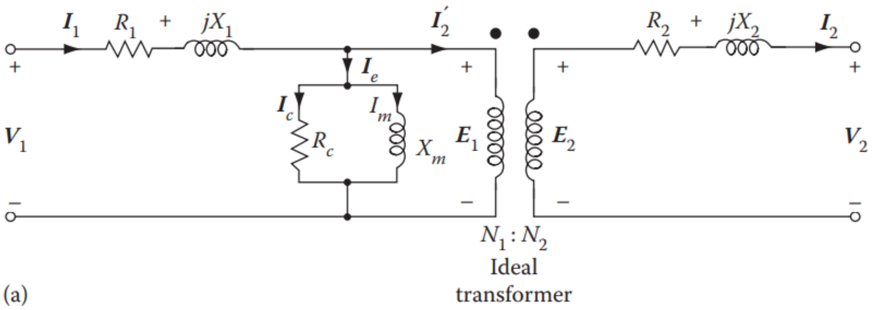
Transformer-equivalent circuits

I'_2 is the load current referred to primary and can be expressed as:

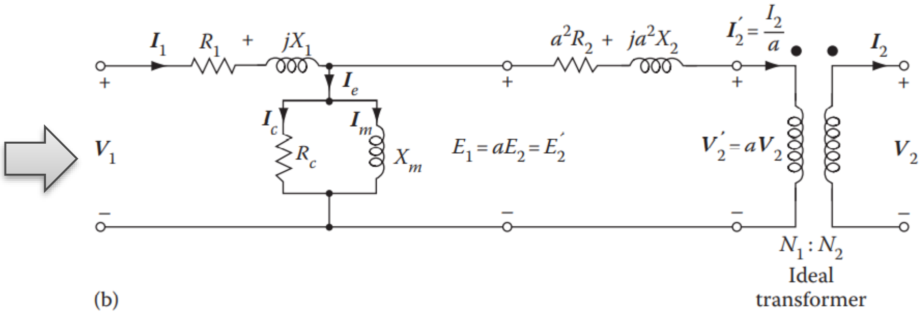
$$\mathbf{I}'_2 = \frac{N_2}{N_1}\mathbf{I}_2 = \frac{\mathbf{I}_2}{a}$$

Therefore, the primary current can be expressed in terms of phasor summation as:

$$\mathbf{I}_1 = \mathbf{I}'_2 + \mathbf{I}_e$$



Equivalent circuit of real transformer



Equivalent circuit of real transformer referred to primary

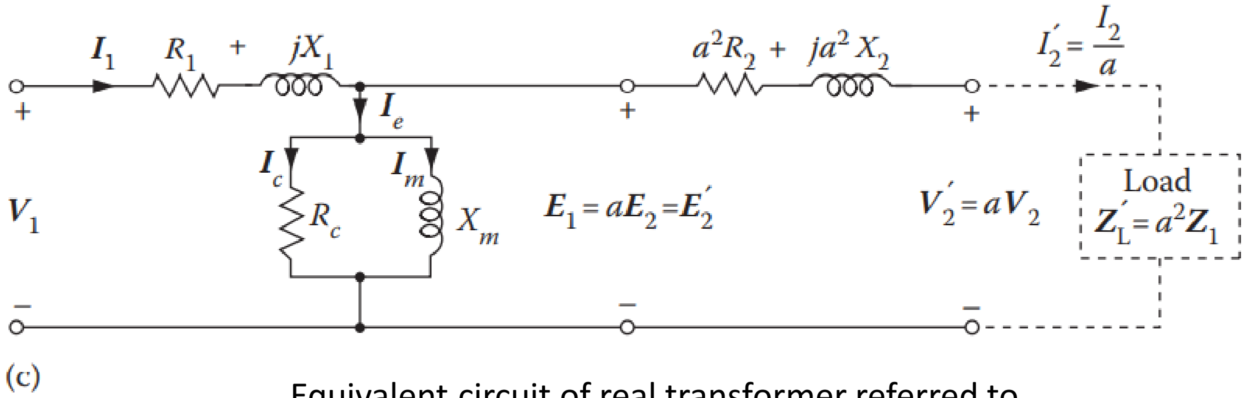
Real transformer

Transformer-equivalent circuits

***I**$_e$* is the excitation current (i.e., the additional primary current) needed to develop the resultant mutual flux. ***I**$_e$* is non-sinusoidal and can be expressed as:

$$\mathbf{I}_e = \mathbf{I}_m + \mathbf{I}_c$$

where ***I**$_c$* is the core-loss component of the excitation current supplying the hysteresis and eddy-current losses in the core, ***I**$_m$* is the magnetizing component of the excitation current needed to magnetize the core.



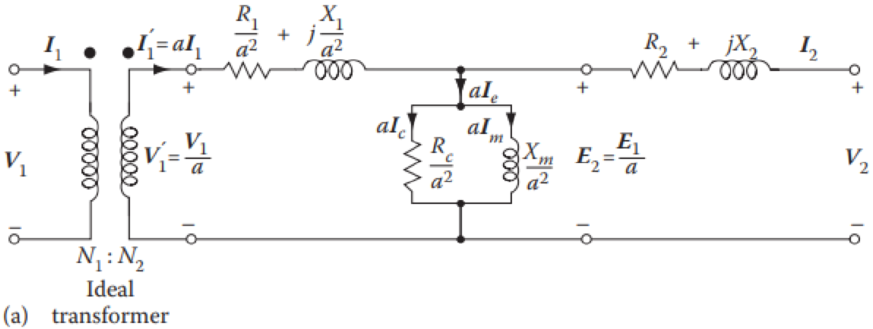
Equivalent circuit of real transformer referred to primary without ideal transformer

Real transformer

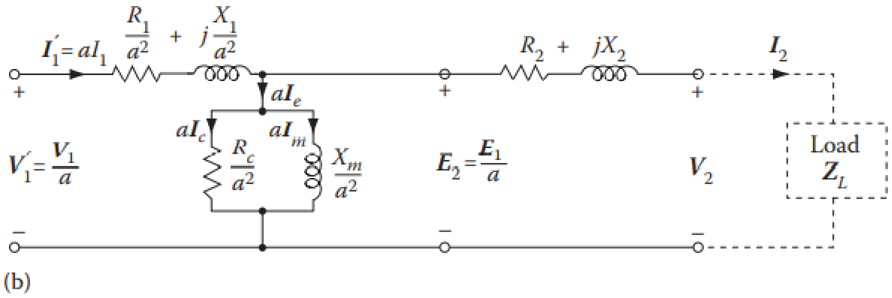
Transformer-equivalent circuits

$\boldsymbol{I_c}$ is in phase with the counter-emf $\boldsymbol{E_1}$ and $\boldsymbol{I_m}$ lags E_1 by 90°.

Therefore, the core-loss component and the magnetizing component are modeled by a resistance R_c and an inductance X_m, respectively, that are connected across the primary voltage source.



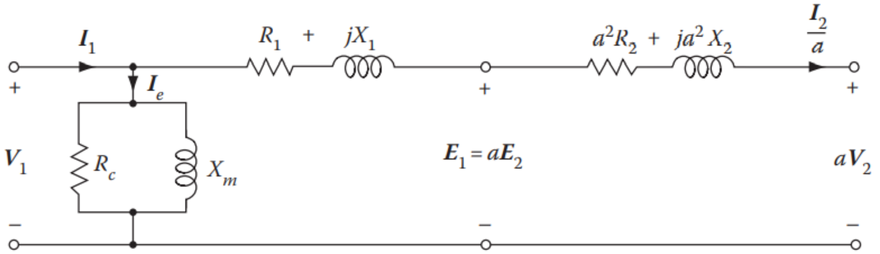
Equivalent circuit of real transformer referred to secondary



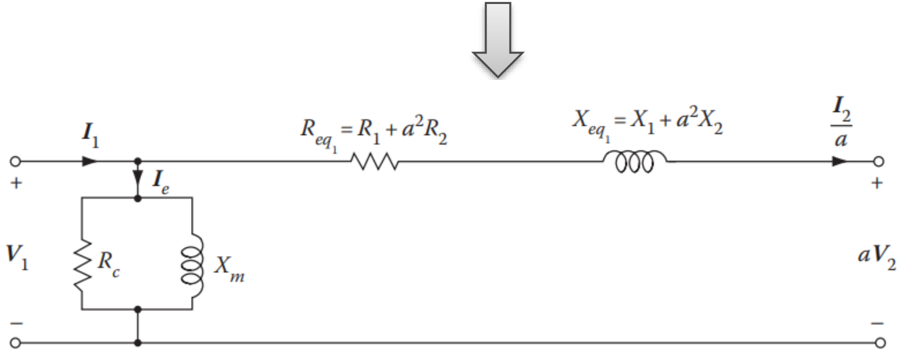
Equivalent circuit of real transformer referred to secondary without ideal transformer

Real transformer

Approximation of equivalent circuit of a real transformer



Approximate equivalent circuit referred to the primary



Approximate equivalent circuit referred to the primary: collecting R's and X's together

Real transformer

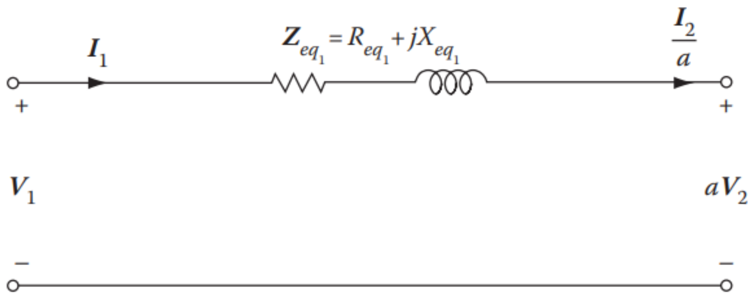
Approximation of equivalent circuit of a real transformer

As shown in Figure, if the equivalent impedance is **referred to the primary**:

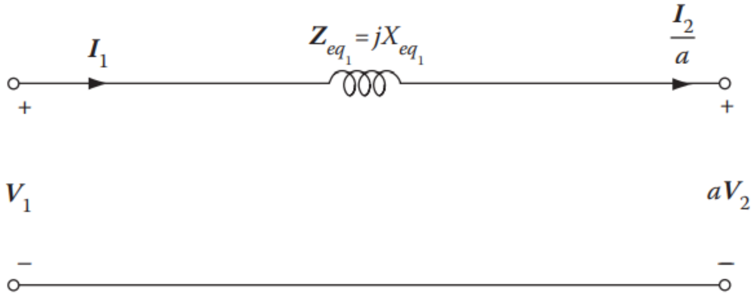
$$\mathbf{Z}_{eq1} = (R_1 + a^2 R_2) + j(X_1 + a^2 X_2)$$

$$\mathbf{Z}_{eq1} = R_{eq1} + jX_{eq1}$$

The R_{eq1} and X_{eq1} are the equivalent resistance and reactance referred to the primary



(c)



(d)

Real transformer

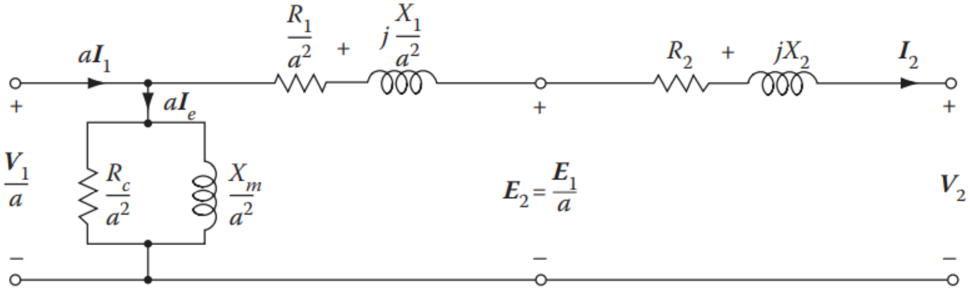
Approximation of equivalent circuit of a real transformer

As shown in Figure, if the equivalent impedance is **referred to the secondary**

$$\mathbf{Z}_{eq2} = \left(\frac{R_1}{a^2} + R_2\right) + j\left(\frac{X_1}{a^2} + X_2\right)$$

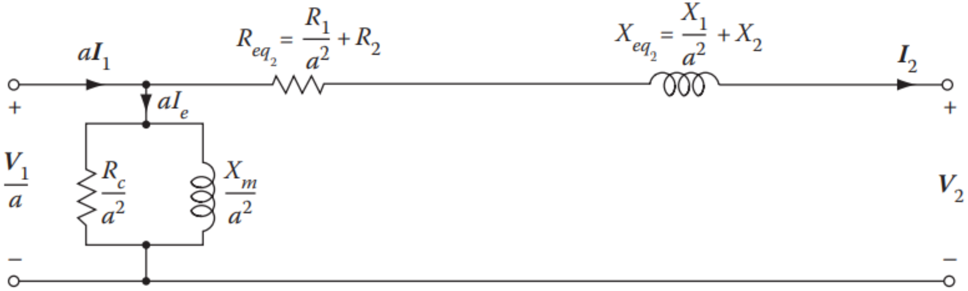
$$\mathbf{Z}_{eq2} = R_{eq2} + jX_{eq2}$$

The terms R_{eq2} and X_{eq2} represent the equivalent resistance and reactance values referred to the secondary, respectively.



(a)

Approximate equivalent circuit referred to the secondary



(b)

Approximate equivalent circuit referred to the secondary: collecting R's and X's together

Real transformer

Approximation of equivalent circuit of a real transformer

It is interesting to notice that

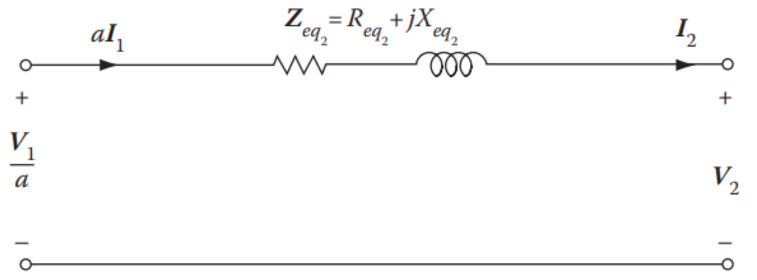
$$\frac{\mathbf{Z}_{eq1}}{\mathbf{Z}_{eq2}} = \frac{R_{eq1}}{R_{eq2}} = \frac{X_{eq1}}{X_{eq2}} = a^2$$

In power transformers, the equivalent resistance R_{eq} is small in comparison to the equivalent reactance X_{eq}.

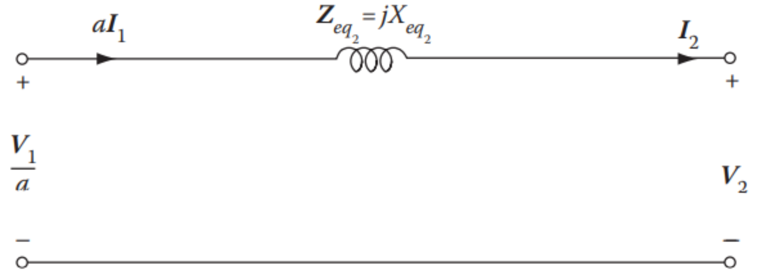
Therefore, the transformer can only be represented by its equivalent reactance X_{eq}:

$$\mathbf{Z}_{eq1} = jX_{eq1}$$

$$\mathbf{Z}_{eq2} = jX_{eq2}$$



(c)



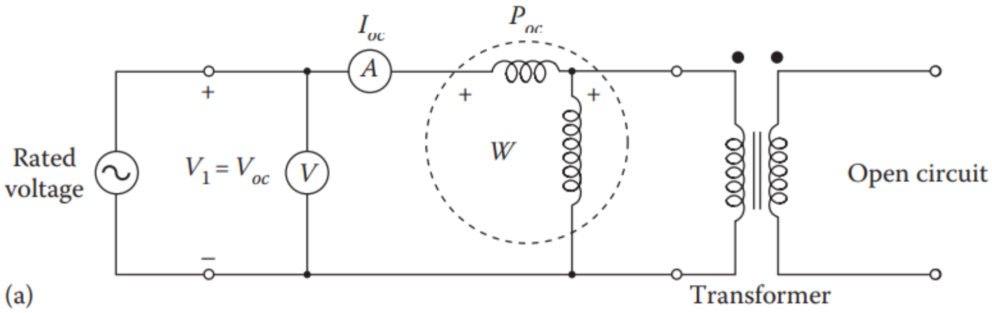
(d)

Real transformer

Determination of equivalent circuit parameters

Open-circuit test

The purpose of the open-circuit test is to determine the excitation admittance of the transformer equivalent circuit, the no-load loss, the no-load excitation current, and the no-load power factor.



Wiring diagram for the open-circuit test

Such an open-circuit test is performed by applying ***rated voltage*** to one of the windings, with the other winding (or windings) open-circuited.

The input ***power, current***, and ***voltage*** are measured, as shown in Figure.

Real transformer

Determination of equivalent circuit parameters

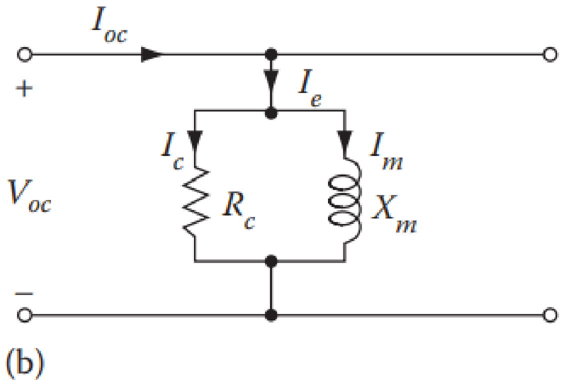
Open-circuit test

This results in an approximate equivalent circuit, as shown in Figure. Also ignored is the (primary) power loss due to the excitation current. Therefore, the excitation admittance can be expressed as:

$$\mathbf{Y_e} = \mathbf{Y_{oc}} = \frac{\mathbf{I_{oc}}}{\mathbf{U_{oc}}} \angle -\theta_{oc} \quad (7)$$

Here θ_{oc} is the angle of the admittance found from the open-circuit power factor PF_{oc} as

$$PF_{oc} = \cos\theta_{oc} = \frac{P_{oc}}{U_{oc} I_{oc}} \Rightarrow \theta_{oc} = \arccos(PF_{oc}) = \arccos\left(\frac{P_{oc}}{U_{oc} I_{oc}}\right)$$



Equivalent circuit of open-circuit test

Real transformer

Determination of equivalent circuit parameters

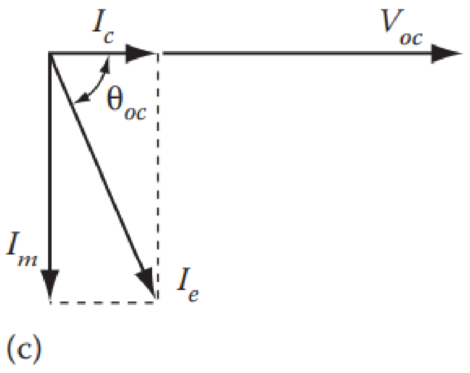
Open-circuit test

For a given transformer, the PF_{oc} is always lagging. For this reason, there is a negative sign in front of θ_{oc} in Equation (7).

If the excitation admittance is expressed in rectangular coordinates:

$$\mathbf{Y}_e = \mathbf{Y}_{oc} = G_c - jB_m = \frac{1}{R_c} - j\frac{1}{X_m}$$

$$R_c = \frac{1}{G_c} \qquad X_m = \frac{1}{B_m}$$



Phasor diagram of open-circuit test

Alternatively, the core-loss conductance and the susceptance can be found, respectively, from:

$$G_c \approx G_{oc} = \frac{P_{oc}}{V_{oc}^2} \qquad B_m \approx B_{oc} = \sqrt{Y_{oc}^2 - G_{oc}^2}$$

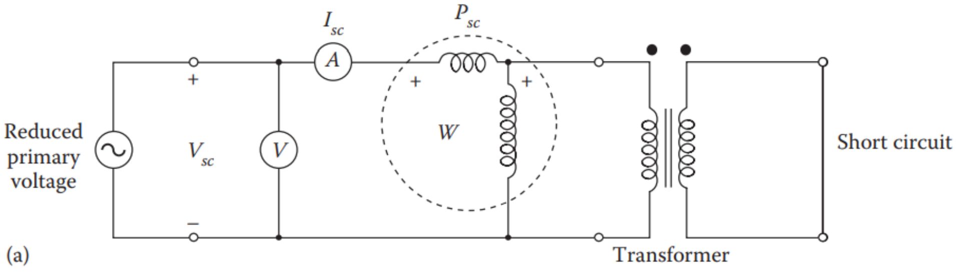
Real transformer

Determination of equivalent circuit parameters

Short-circuit test

The purpose of the short-circuit test is to determine the equivalent resistance and reactance of the transformer under rated conditions.

This test is performed by short-circuiting one winding (usually the low-voltage winding) and applying a reduced voltage to the other winding, as shown in Figure.



Wiring diagram for the short-circuit test

The reduced input voltage is adjusted until the ***current*** in the shorted winding is equal to its ***rated value***. The input voltage, current, and power are measured as before. The applied voltage V_{sc} is only a small percentage of the rated voltage and is sufficient to circulate rated current in the windings of the transformer. Usually, this voltage is about 2%–12% of the rated voltage

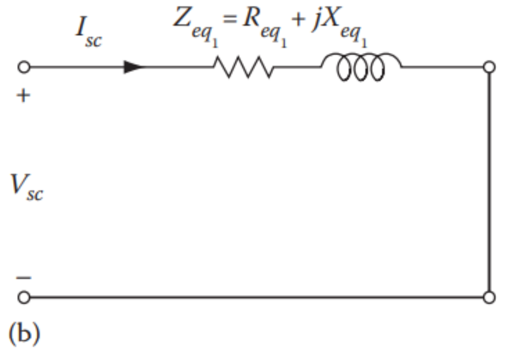
Real transformer

Determination of equivalent circuit parameters

Short-circuit test

The shunt branch representing excitation admittance does not appear in this equivalent circuit. The series impedance Z_{sc} can be found from:

$$\mathbf{Z}_{sc} = \mathbf{Z}_{eq1} = \frac{\mathbf{V}_{sc}}{\mathbf{I}_{sc}} = \frac{V_{sc}\angle 0^\circ}{I_{sc}\angle -\theta_{sc}} \qquad (8)$$



Equivalent circuit of short-circuit test

The short-circuit power factor is lagging and determined from:

$$PF_{sc} = \cos\theta_{sc} = \frac{P_{sc}}{U_{sc}I_{sc}} \Rightarrow \theta_{sc} = \arccos(PF_{sc}) = \arccos\left(\frac{P_{sc}}{U_{sc}I_{sc}}\right)$$

For a given transformer, the PF_{sc} is always lagging. For this reason, there is a negative sign in front of θ_{sc} in Equation (8).

Equation (8) can be expressed as: $$\mathbf{Z}_{eq1} = \mathbf{Z}_{sc} = \frac{\mathbf{U}_{sc}}{\mathbf{I}_{sc}}\angle\theta_{sc} = R_{eq1} + jX_{eq1} \Rightarrow \begin{aligned} R_{eq1} &= R_1 + a^2 R_2 \\ X_{eq1} &= X_1 + a^2 X_2 \end{aligned}$$

Real transformer

Determination of equivalent circuit parameters

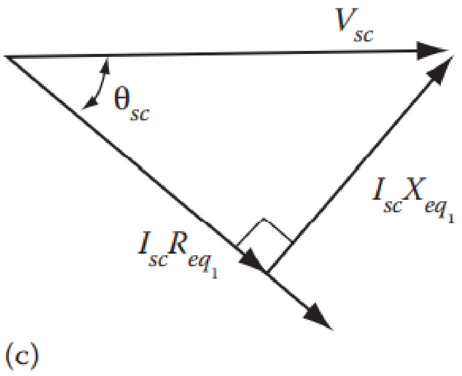
Short-circuit test

Alternatively, the equivalent circuit resistance and reactance can be found from:

$$R_{eq1} \approx R_{sc} = \frac{P_{sc}}{I_{sc}^2}$$

$$Z_{eq1} = \frac{U_{sc}}{I_{sc}}$$

$$X_{eq1} \approx X_{sc} = \sqrt{Z_{eq1}^2 - R_{eq1}^2}$$



Phasor diagram of short-circuit test

Figure shows the phasor diagram under short-circuit conditions. In a well-designed transformer, when all impedances are referred to the same side (in this case, to the primary side):

$$R_1 = a^2 R_2 = R'_2 \approx \frac{R_{eq1}}{2} \quad X_1 = a^2 X_2 = X'_2 \approx \frac{X_{eq1}}{2}$$

Real transformer

Performance characteristics of a transformer

The main use of the equivalent circuit of a given transformer is to determine its performance characteristics, which are basically its voltage regulation and its efficiency.

Voltage regulation of a transformer

The voltage regulation of a transformer is the change in the magnitude of secondary terminal voltage from no load to full load when the primary voltage is constant. It is usually expressed as a percentage of the full-load value as:

$$\%V_{REG} = \frac{V_{2(no\ load)} - V_{2(load)}}{V_{2(load)}} \times 100$$

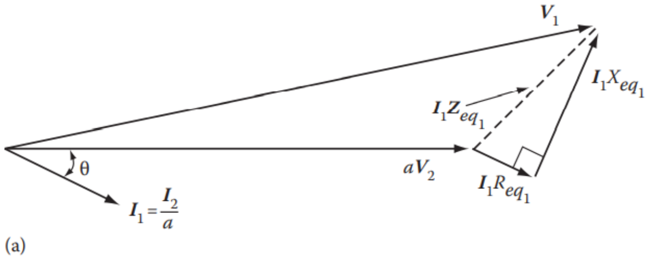
$$V_{2(no\ load)} = \frac{V_1}{a} \quad \Rightarrow \quad \%V_{REG} = \frac{\frac{V_1}{a} - V_{2(load)}}{V_{2(load)}} \times 100$$

Real transformer

Voltage regulation of a transformer

The voltage regulation is affected by the magnitude and power factor of the load as well as by the internal impedance (i.e., the leakage impedance) of the transformer.

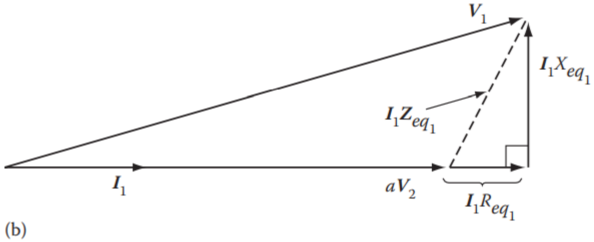
Phasor diagrams referred to primary side



Lagging power factor

$$\%V_{REG} > 0$$

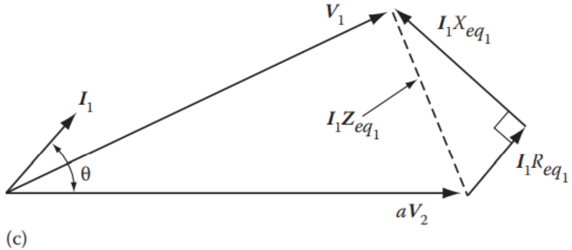
RL-load



Unity power factor

$$\%V_{REG} > 0$$

R-load



Leading power factor

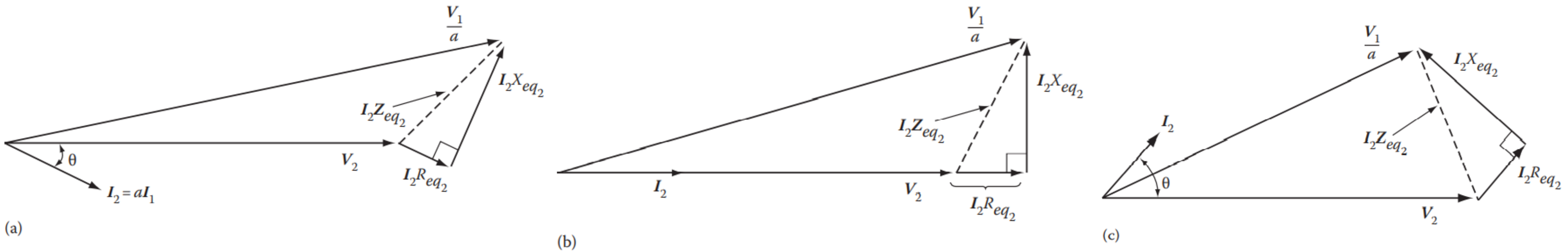
$$\%V_{REG} < 0$$

RC-load

Real transformer

Voltage regulation of a transformer

Phasor diagrams referred to secondary side



Lagging power factor	Unity power factor	Leading power factor
$\%V_{REG} > 0$	$\%V_{REG} > 0$	$\%V_{REG} < 0$
RL-load	R-load	RC-load

Primary voltage

$$\mathbf{V}_1 = a\mathbf{V}_2 + \mathbf{I}_1\mathbf{Z}_{eq1} = a\mathbf{V}_2 + \mathbf{I}_1 R_{eq1} + j\mathbf{I}_1 X_{eq1}$$

referred to primary side

$$\mathbf{V}_1 / a = \mathbf{V}_2 + \mathbf{I}_2\mathbf{Z}_{eq2} = \mathbf{V}_2 + \mathbf{I}_2 R_{eq2} + j\mathbf{I}_2 X_{eq2}$$

referred to secondary side

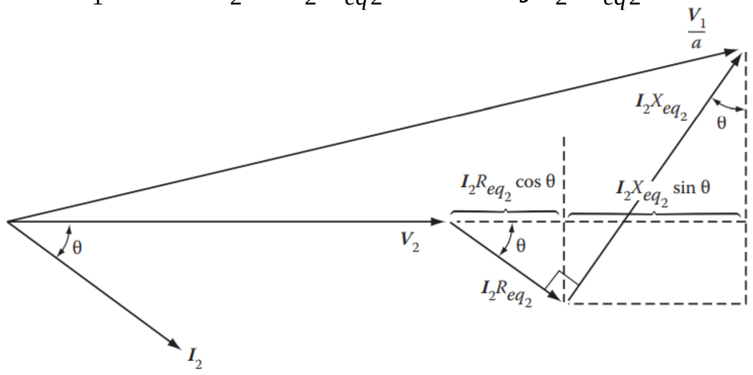
Real transformer

Voltage regulation of a transformer

It is possible to use an approximate value for the primary voltage by taking into account only the horizontal components in the phasor diagram, as shown in Figure.

Therefore, when all quantities are referred to the secondary side of the transformer, the approximate value of the primary voltage is:

$$\mathbf{V}_1 / a = \mathbf{V}_2 + \mathbf{I}_2 R_{eq2} \cos\theta + j\mathbf{I}_2 X_{eq2} \sin\theta$$



Phasor diagram showing the derivation of the approximate equation for V_1/a

Real transformer

Voltage regulation of a transformer

The voltage regulation also can be found from short-circuit and open-circuit tests data:

for RL-load

$$V_{2(no\ load)} - V_{2(load)} = \Delta V_2 = \beta \frac{V_{sc}}{a} (\cos\theta_{sc} \cos\theta + \sin\theta_{sc} \sin\theta)$$

for RC-load

$$\Delta V_2 = \beta \frac{V_{sc}}{a} (\cos\theta_{sc} \cos\theta - \sin\theta_{sc} \sin\theta)$$

for R-load

$$\Delta V_2 = \beta \frac{V_{sc}}{a} (\cos\theta_{sc})$$

$$\beta = \frac{I_2}{I_{2N}} \approx \frac{I_1}{I_{1N}}$$

Transformers used in power system applications are usually designed with taps on one winding in order to change its turns ratio over a small range.

Real transformer

Transformer Efficiency

The efficiency of any equipment can be defined as the ratio of output power to input power. Therefore, the efficiency (η) is:

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{loss}} = \frac{P_{in} - P_{loss}}{P_{in}} = 1 - \frac{P_{loss}}{P_{in}}$$

The power losses in a given transformer are the core losses, which can be considered constant for a given voltage and frequency, and the copper losses, caused by the resistance of the windings.

The core losses are the sum of ***hysteresis*** and ***eddy-current*** losses. Therefore, the input power can be expressed as:

$$P_{in} = P_{out} + P_{cu} + P_{core}$$

Real transformer

Transformer Efficiency

Output power can be expressed as:

$$P_{out} = V_2 I_2 \cos\theta = S_{out} \cos\theta$$

Here, the cosθ is the load power factor.

Therefore, the percent efficiency of the transformer is:

$$\eta = \frac{V_2 I_2 \cos\theta}{V_2 I_2 \cos\theta + P_{cu} + P_{core}}$$

the copper losses

$$P_{cu} = R_1 I_1^2 + R_2 I_2^2 = I_1^2 R_{eq1} = I_2^2 R_{eq2} \approx P_{sc}$$

the core losses

$$P_{core} \approx P_{oc}$$

Real transformer

Transformer Efficiency

The current, voltage, and equivalent circuit parameters must be referred to the same side of the transformer. The ***maximum efficiency*** is achieved when the core loss is equal to the copper loss, that is:

$$P_{cu} = P_{core}$$

In general, the efficiency of transformers at a rated load is very high and increases with their ratings. For example, transformers as small as 1 kVA may have an efficiency of 90%. Power transformer efficiencies vary from 95% to 99%. In a well-designed transformer, both core losses and copper losses are extremely small, so that efficiency is very high. For example, efficiency for very large transformers is about 99%.

Real transformer

Transformer nameplate rating

Among the information provided by the nameplate of a transformer are its apparent power (in terms of the kVA rating or the MVA rating), voltage ratings, and impedance.

For example, a typical transformer may have 25 kVA, 2400/120V. Here, the voltage ratings point out that the transformer has two windings: one rated for 2400V and the other for 120V.

Also, the given 25 kVA rating indicates that each winding is designed to carry 25 kVA. Thus, the current rating for the high-voltage winding is 25,000VA/2,400V =10.42 A, but for the low-voltage winding is 25,000VA/120V=208.33A.

The kVA rating always refers to the output kVA measured at the secondary (load) terminals. The input kVA will be slightly more due to the losses involved.

Thank you!

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