



# ITMO UNIVERSITY

Mathematic modelling of electrical systems  
dynamic

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Aleksandr Mamatov

# Content

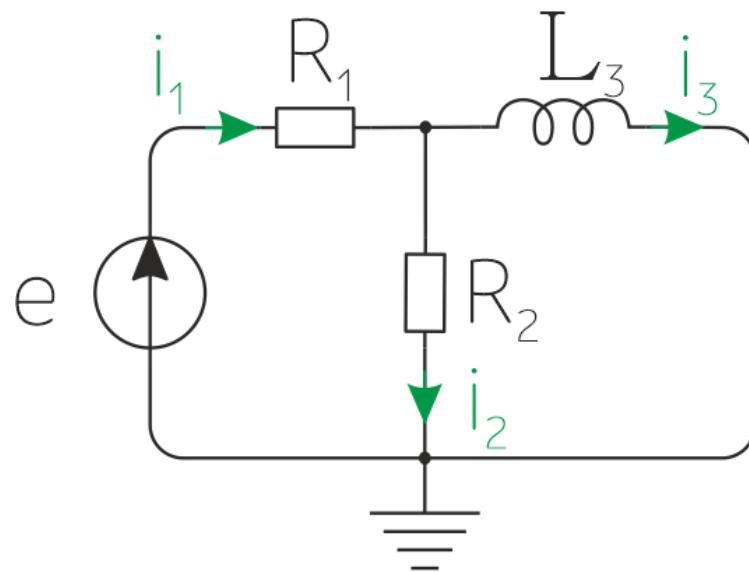
- Mathematic modelling of electrical svstem with inductance
- Mathematic modelling of electrical svstem with capacitance
- Mathematic modelling of second order electrical svstem

# Mathematic modelling of electrical system with inductance

# Electrical circuits with inductance



Get the state-space model



# Electrical circuits with inductance

1. Component equations:

- Dissipative elements.

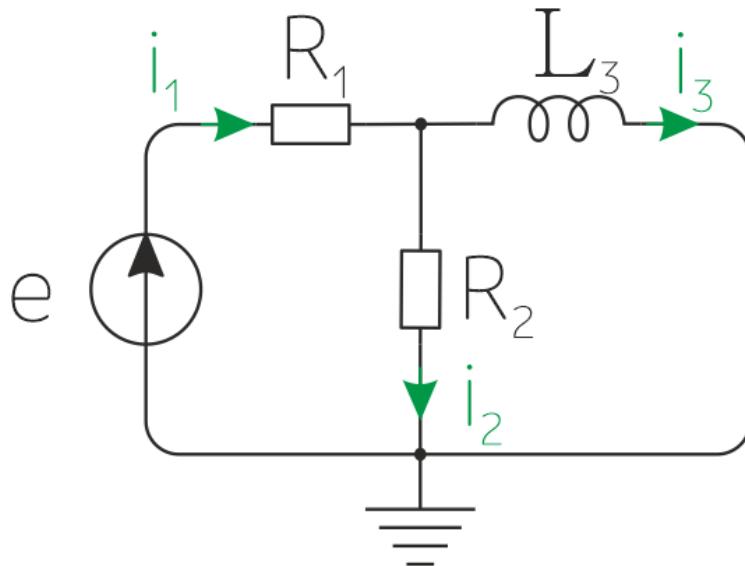
$$i_1 = R_1 \cdot v_1$$

$$i_2 = R_2 \cdot v_2$$

- Elastic elements.

$$v_3 = L_3 \cdot \frac{di_3}{dt}$$

- Inertial elements.



# Electrical circuits with inductance

2. Topological equations:

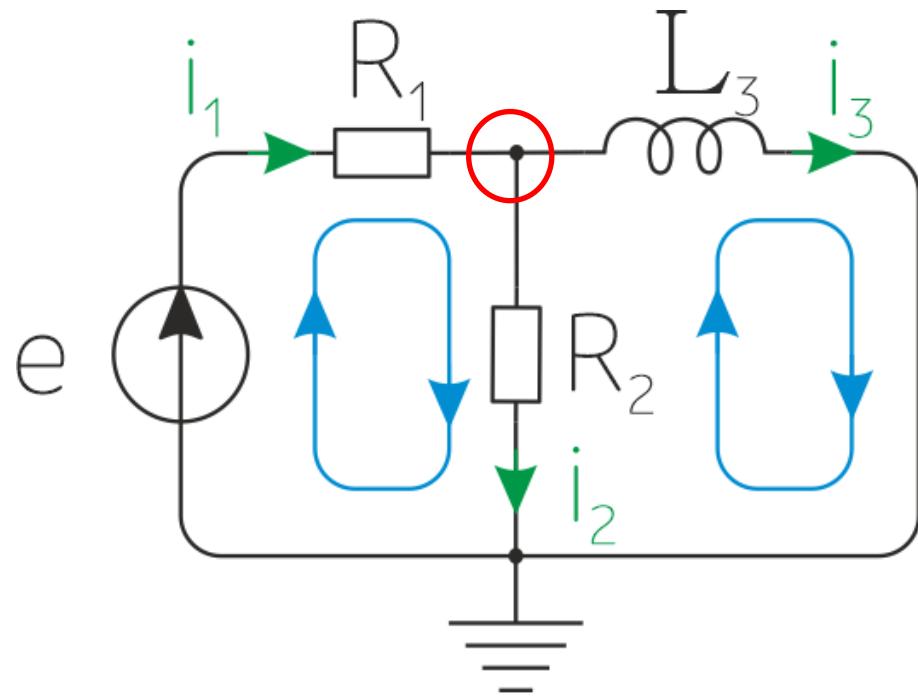
- Equilibrium equations.

$$i_1 + i_2 =$$

$$-i_2 + i_3 =$$

- Condition of continuity of flow type coordinates.

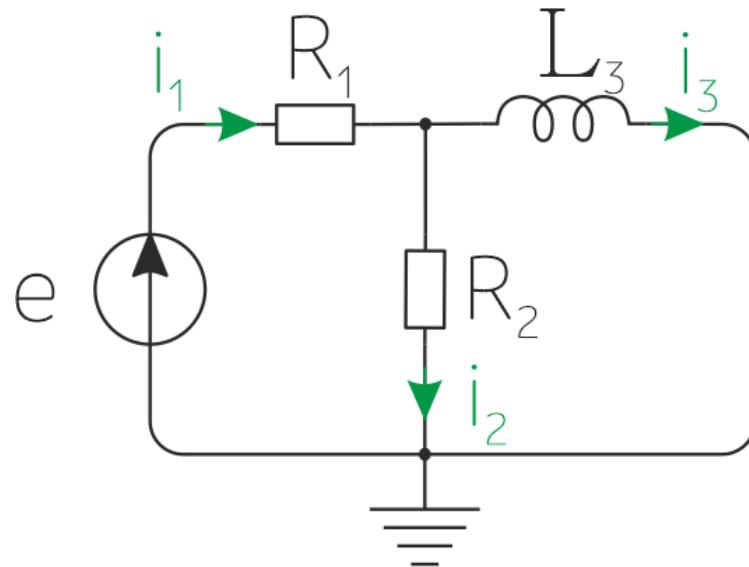
$$i_1 - i_2 - i_3 =$$



# Electrical circuits with inductance

3. State-space model:

$$\begin{array}{c} \text{---} = + \\ = + \end{array}$$



State vector  $x$ :

$$= \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

Output vector  $v$ :

$$= \begin{pmatrix} e \\ i_3 \end{pmatrix}$$

# Electrical circuits with inductance

3.1 Select the required number of equations for the model:

Three component  
equations.

Three topological  
equations.

Six circuit  
variables.

One state  
variable.

$$1 = 1 \cdot 1$$

$$1 + 2 =$$

1 2 3

3

$$2 = 2 \cdot 2$$

$$- 2 + 3 =$$

1 2 3

$$3 = 3 \cdot -\frac{3}{3}$$

$$1 - 2 - 3 =$$

# Electrical circuits with inductance

3.2 Express the remaining parameters in terms of the state vector  
parameters and substitute it to the equation:

$$z = z \cdot z$$

$$z = z - z$$

$$-z + z =$$

$$z = \frac{1}{1}$$

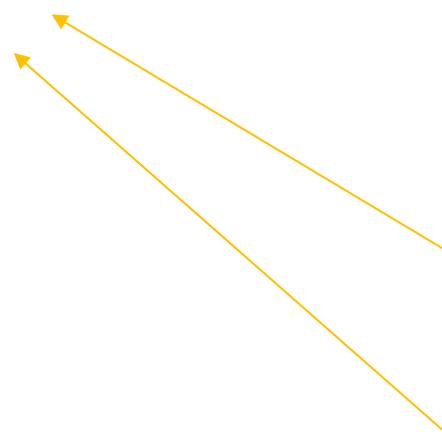
$$z = -z$$

$$z = z \cdot \frac{z}{z}$$

# Electrical circuits with inductance

3.2 Express the remaining parameters in terms of the state vector  
parameters and substitute it to the equation:

$$- \dot{x}_2 + \dot{x}_3 =$$



$$x_1 = \frac{1}{x_2 + x_1} + \frac{x_2}{x_2 + x_1} x_3$$

$$x_1 = \frac{1}{x_2 + x_1} + \frac{x_2}{x_2 + x_1} x_3$$

$$x_2 = \frac{1}{x_2 + x_1} - \frac{1}{x_2 + x_1} x_3$$

$$x_2 = \frac{1}{x_2 + x_1} - \frac{1}{x_2 + x_1} x_3$$

$$x_3 = x_3 \cdot \frac{1}{x_2 + x_1}$$

# Electrical circuits with inductance

3.2 Express the remaining parameters in terms of the state vector  
parameters and substitute it to the equation:

$$-\frac{z_2}{z_2 + z_1} + \frac{1}{z_2 + z_1} z_3 + z_3 \cdot \frac{z_3}{z_3} =$$

$$\frac{z_3}{z_3} = -\frac{1}{z_3(z_2 + z_1)} z_3 + \frac{2}{z_3(z_2 + z_1)}$$

# Electrical circuits with inductance

3.3 Express the output vector in terms of the state vector and substitute input vector:

$$\underline{\dot{x}}_3 = -\frac{1 \ 2}{3(-2 + 1)} \underline{x}_3 + \frac{2}{3(-2 + 1)}$$

$$x_3 = x_3 - \frac{3}{3(-2 + 1)} \underline{x}_3 + \frac{2}{(-2 + 1)}$$

# Electrical circuits with inductance

3.4 Write the state space model:

$$\dot{x}_3 = -\frac{1 \ 2}{3(2 + 1)} x_3 + \frac{2}{3(2 + 1)}$$

$$x_3 = -\frac{1 \ 2}{(2 + 1)} x_3 + \frac{2}{(2 + 1)}$$

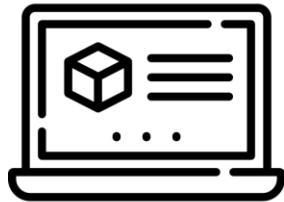
# Electrical circuits with inductance



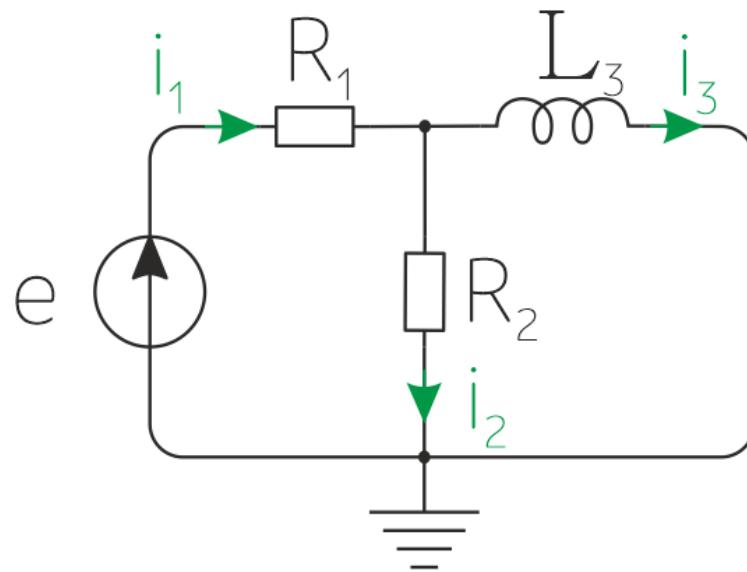
Algorithm for getting the state-space model:

1. Write component equations.
2. Write topological equations.
3. State-space model.
  - 3.1 Select the required number of equations for the model.
  - 3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation.
  - 3.3 Express the output vector in terms of the state vector and substitute input vector.
  - 3.4 Write the state space model.

# Electrical circuits with inductance

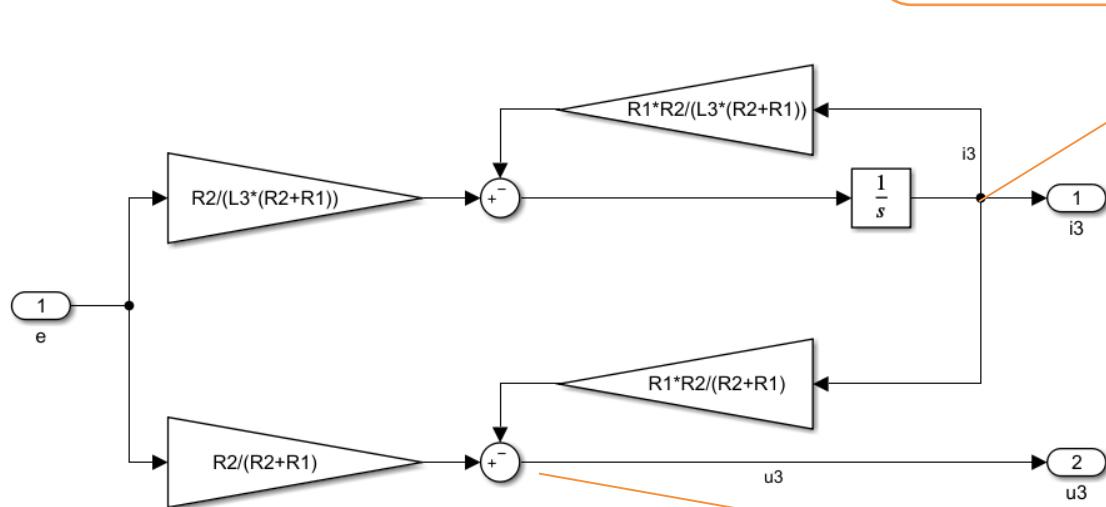


Modelling in Matlab  
(zero initial conditions)



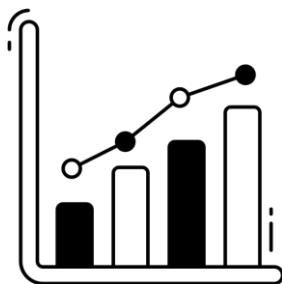
# Electrical circuits with inductance

$$\underline{3} = -\frac{1}{3} \left( \frac{2}{2+1} \right) \underline{3} + \frac{2}{3} \left( \frac{2}{2+1} \right)$$

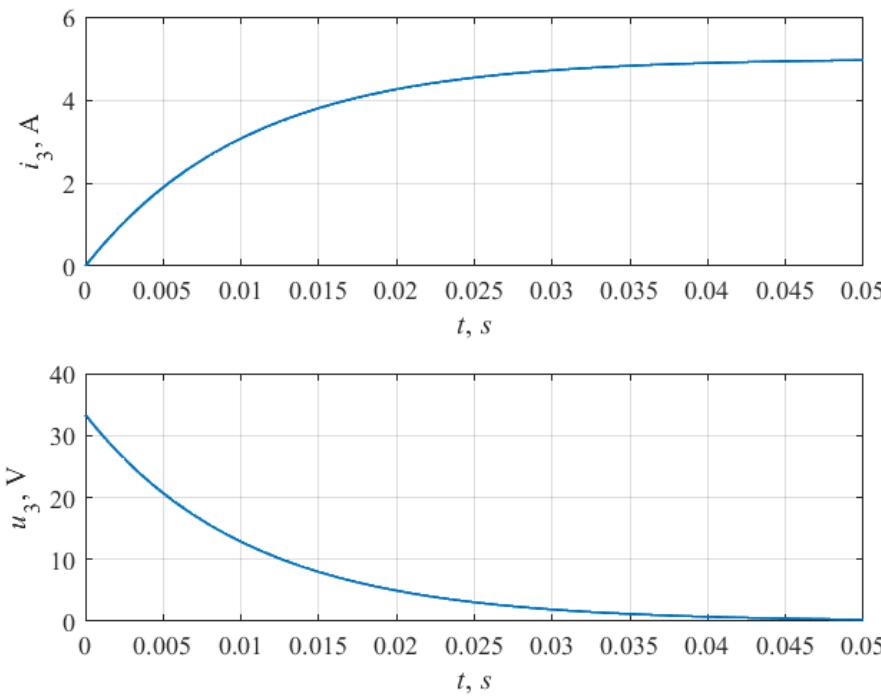


$$\underline{3} = -\frac{1}{\left( \frac{2}{2+1} \right)} \underline{3} + \frac{2}{\left( \frac{2}{2+1} \right)}$$

# Electrical circuits with inductance



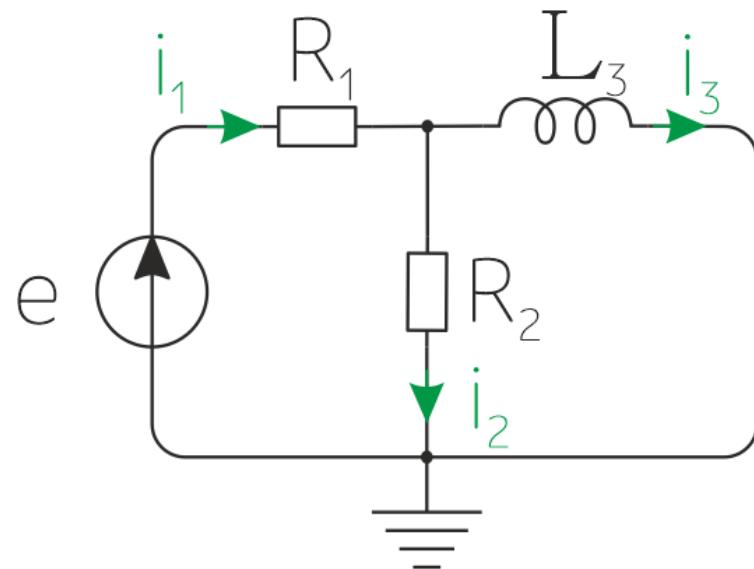
Graphs of the transients in the circuit  
(zero initial condition)



# Electrical circuits with inductance



Get the transfer function



# Electrical circuits with inductance

4. Transfer function:

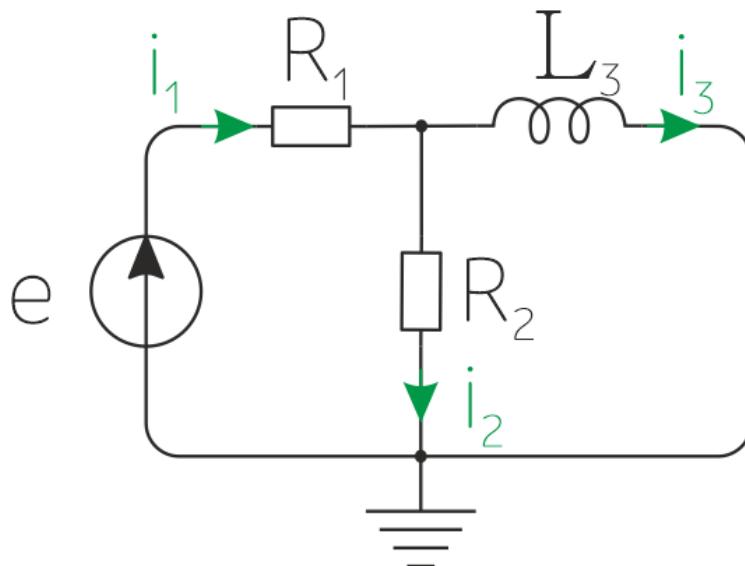
= \_\_\_\_\_

Input:

=

Output:

=  $i_3$



# Electrical circuits with inductance

4.1 Perform Laplace transform for the state-space model (zero initial conditions):

$$\underline{\dot{x}_3} = -\frac{1 \ 2}{3(-2 + 1)} \underline{x_3} + \frac{2}{3(-2 + 1)}$$

$$\underline{\dot{x}_3} = -\frac{1 \ 2}{(-2 + 1)} \underline{x_3} + \frac{2}{(-2 + 1)}$$



$$\underline{\dot{x}_3} = -\frac{1 \ 2}{3(-2 + 1)} \underline{x_3} + \frac{2}{3(-2 + 1)}$$

$$\underline{\dot{x}_3} = -\frac{1 \ 2}{(-2 + 1)} \underline{x_3} + \frac{2}{(-2 + 1)}$$

# Electrical circuits with inductance

4.2 Express the output in terms of input:

$$z_3 = \frac{z_2}{z_3(z_2 + z_1) + z_1 z_2}$$

$$z_3 = \frac{z_3 z_2}{z_3(z_2 + z_1) + z_1 z_2}$$

# Electrical circuits with inductance

4.3 Write the transfer function:

$$\begin{aligned} &= \frac{3}{s + \frac{1}{3}} = \frac{3}{s + \frac{3}{2} + \frac{2}{1}} = \frac{3}{s + \frac{1}{2} + \frac{2}{1}} \\ &= \frac{3}{s + \frac{1}{\frac{1}{2} + \frac{2}{1}}} \end{aligned}$$

# Electrical circuits with inductance



Algorithm for getting the transfer function:

4. Transfer function.

4.1 Perform Laplace transform for the state-space model  
(zero initial conditions).

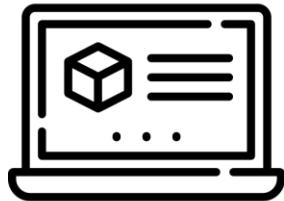
4.2 Express the output in terms of input.

4.3 Write the transfer function:

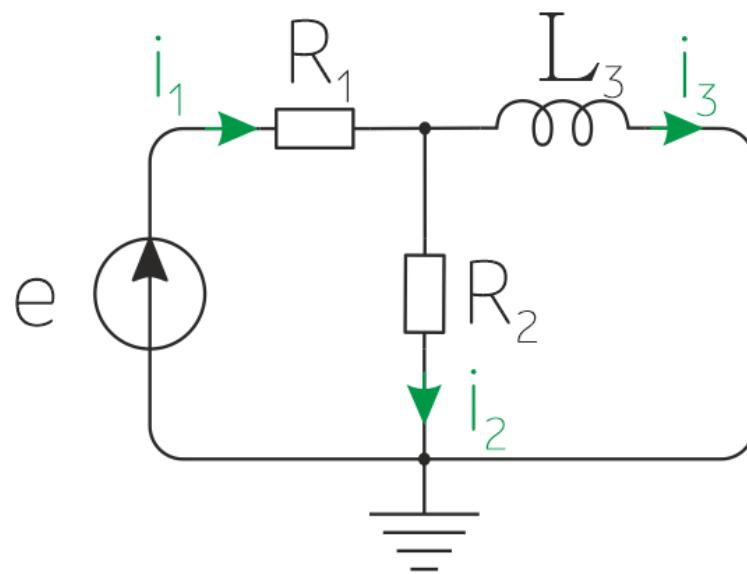
# Attendance



# Electrical circuits with inductance



Modelling in Matlab  
(nonzero initial conditions)



# Electrical circuits with inductance

State-space model:

$$\dot{x}_3 = -\frac{1 \ 2}{3(2 + 1)} x_3 + \frac{2}{3(2 + 1)}$$

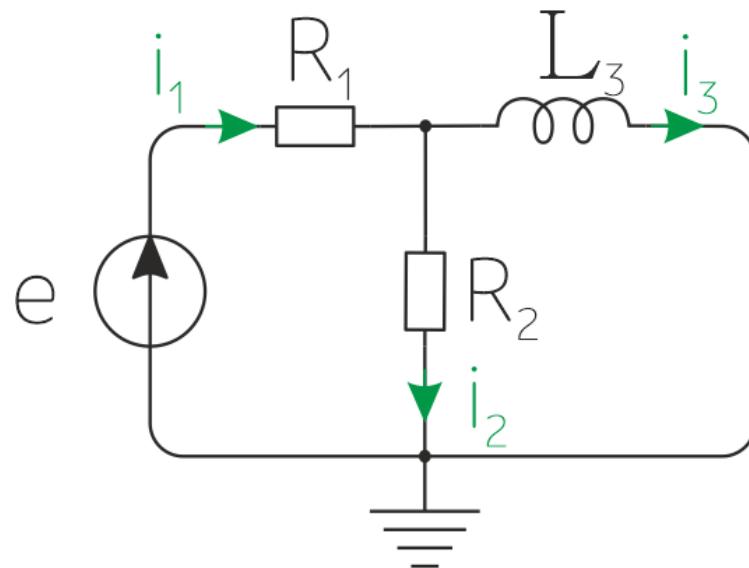
$$x_3 = -\frac{1 \ 2}{(2 + 1)} x_3 + \frac{2}{(2 + 1)}$$

Initial conditions:

$$x_2 =$$

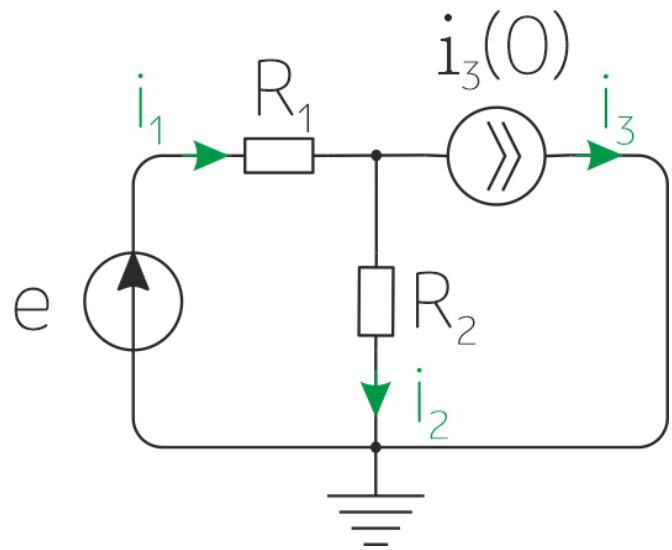
State space vector x:

$$= x_3$$

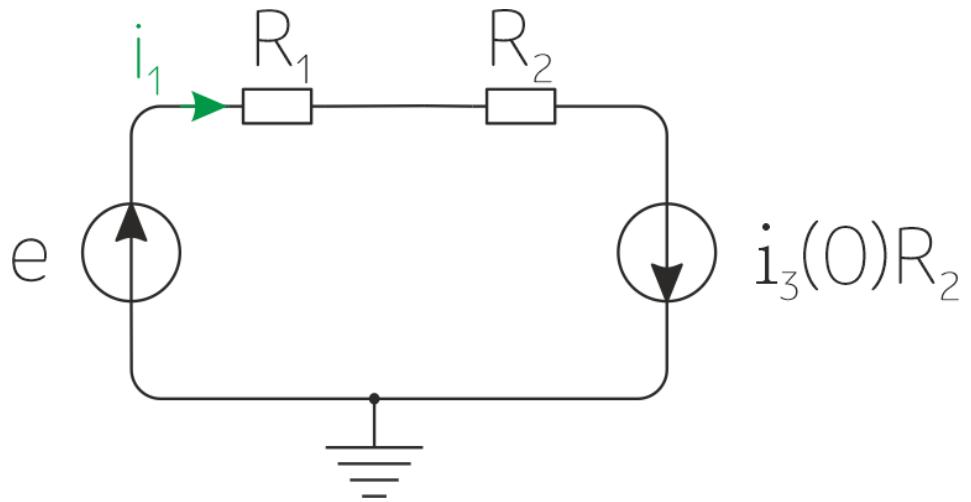


# Electrical circuits with inductance

Circuit at time  $t = 0+$ :



Equivalent transform  
of current source:



$$i_1 = \frac{e}{R_1 + R_2}$$
$$i_1 = i_2 + i_3$$



$$i_3 = \frac{1}{R_1 + R_2} = i_2 + \frac{1}{R_1 + R_2}$$
$$i_3 = -\frac{1}{R_1 + R_2} i_2 + \frac{1}{R_1 + R_2}$$

# Electrical circuits with inductance

## Nonzero initial conditions in Matlab

The screenshot shows the MATLAB/Simulink environment with a circuit model open. The circuit consists of a resistor R2 (labeled i2) in series with an inductor L3. A voltage source is connected across the inductor. The 'Block Parameters: R2' dialog is open, showing settings for 'Current' (Priority: High, Value: 3) and 'Voltage' (Priority: None, Value: 0). The 'Block Parameters: L3' dialog is also open, specifically the 'Settings' tab, which includes a 'Source code' section and a table for overriding parameters. The 'Inductor current' row in the table has 'Override' checked, 'Variable' set to 'Inductor current', 'Priority' set to 'None', 'Beginning Value' set to '0.5', and 'Unit' set to 'A'. An orange box highlights the 'Beginning Value' field for the inductor current.

Block Parameters: R2

Resistor

The voltage-current (V-I) relationship for a linear resistor is  $V=I \cdot R$ , where  $R$  is the constant resistance in ohms.

The positive and negative terminals of the resistor are denoted by the + and - signs respectively. By convention, the voltage across the resistor is given by  $V(+)-V(-)$ , and the sign of the current is positive when flowing through the device from the positive to the negative terminal. This convention ensures that the power absorbed by a resistor is always positive.

[Source code](#)

Settings

Parameters Variables

Override	Variable	Priority	Beginning Value	Unit
<input checked="" type="checkbox"/>	Current	High	3	A
<input type="checkbox"/>	Voltage	None	0	V

OK Cancel Help Apply

Block Parameters: L3

Inductor

Models a linear inductor. The relationship between voltage  $V$  and current  $I$  is  $V=L \cdot dI/dt$  where  $L$  is the inductance in henries (H).

The Series resistance and Parallel conductance represent small parasitic effects. The series resistance can be used to represent the DC winding resistance and/or the resistance due to the skin effect. A small parallel conductance may be required for the simulation of some circuit topologies. Consult the documentation for further details.

Source code

Settings

Parameters Variables

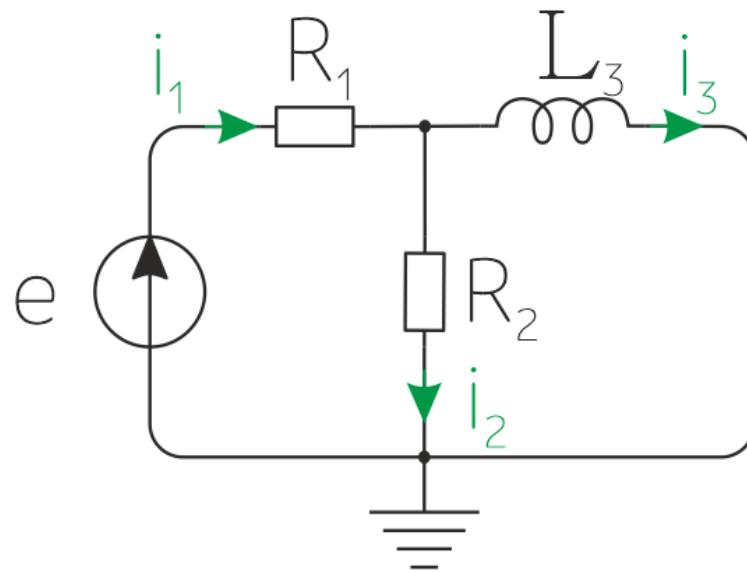
Override	Variable	Priority	Beginning Value	Unit
<input type="checkbox"/>	Current	None	0	A
<input type="checkbox"/>	Voltage	None	0	V
<input checked="" type="checkbox"/>	Inductor current	None	0.5	A

OK Cancel Help Apply

# Electrical circuits with inductance



Parametric identification using  
transient response



# Electrical circuits with inductance

Transfer function:

$$= \frac{z}{+} = \frac{i}{+}$$

$$i = \frac{1}{1} = \frac{(z_2 + z_1)}{z_3 (1 - z_1 z_2)}$$

Parameters:

$$i \cdot =$$

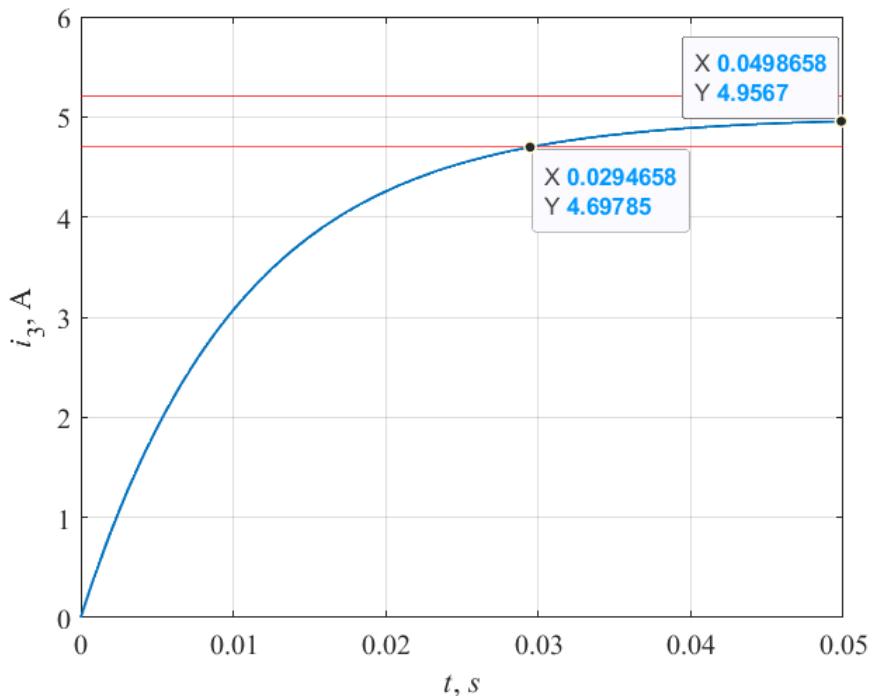
$$\cdot =$$

$$=$$

$$1$$

$$\frac{(z_2 + z_1)}{z_3 (1 - z_1 z_2)} =$$

Transient response:



# Electrical circuits with inductance

Transfer function:

$$= \frac{3}{+} = \frac{u}{+}$$

$$u = \frac{3}{1} \\ = \frac{3}{3} \left( \frac{2 + 1}{1 + 2} \right)$$

Parameters:

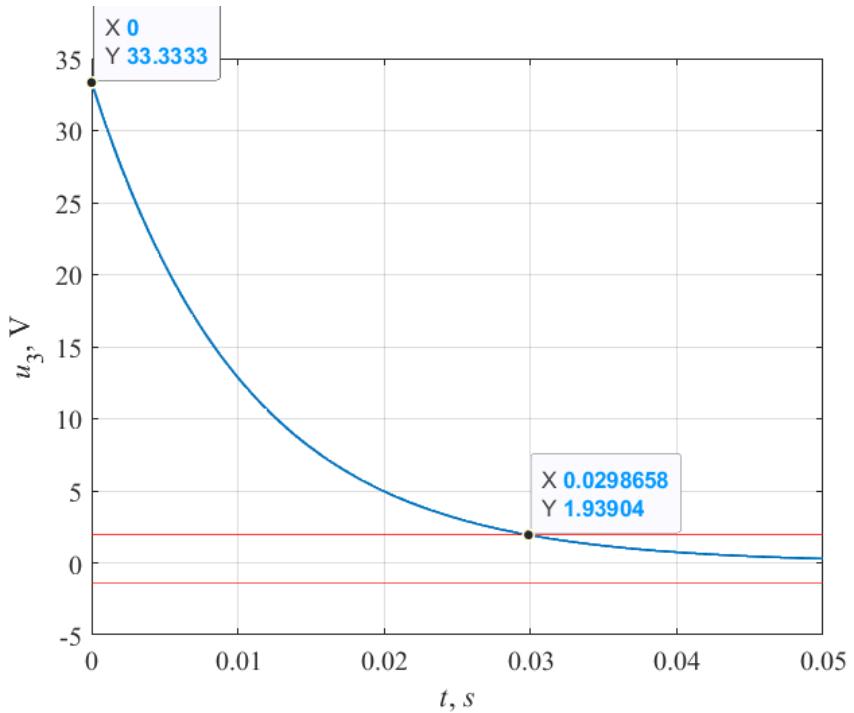
$$\frac{u}{+} =$$

$$\cdot =$$

$$\frac{\cdot}{2 + 1} =$$

$$= \frac{\left( \frac{2 + 1}{1 + 2} \right)}{3}$$

Transient response:



# Electrical circuits with inductance

State-space model:

$$\dot{x}_3 = -\frac{1 \ 2}{3(2 + 1)} x_3 + \frac{2}{3(2 + 1)}$$

$$x_3 = -\frac{1 \ 2}{(2 + 1)} x_3 + \frac{2}{(2 + 1)}$$

Parameters:

$$=$$

$$1$$

$$\frac{\cdot}{2 + 1} =$$

$$3 \frac{(2 + 1)}{1 \ 2} =$$

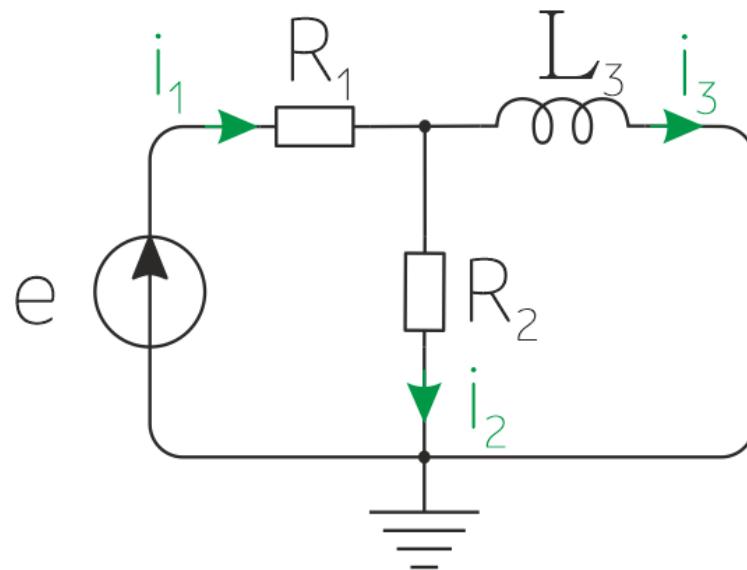
$$\dot{x}_3 = -x_3 + \frac{i}{u}$$

$$x_3 = -\frac{u}{i} x_3 + \frac{u}{i}$$

# Electrical circuits with inductance



Parametric identification using  
frequency response



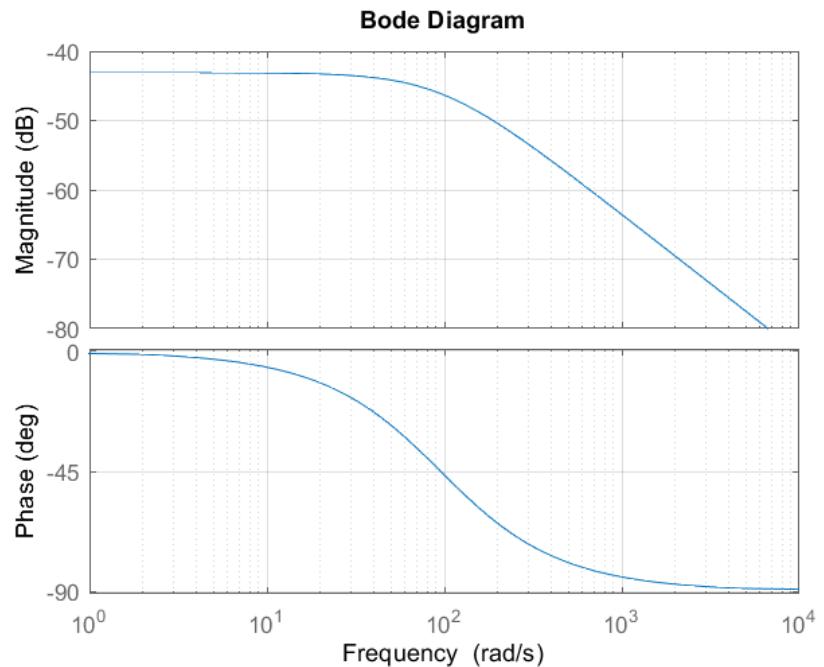
# Electrical circuits with inductance

Transfer function:

$$= \frac{3}{+} = \frac{i}{+}$$

$$i = \frac{1}{1} \\ = \frac{(2 + 1)}{3(1 + 2)}$$

Freauency response:



# Electrical circuits with inductance

Parameters:

$$\omega_c =$$



$$= \text{---} =$$

$$\omega_c$$

$$= \text{---}$$

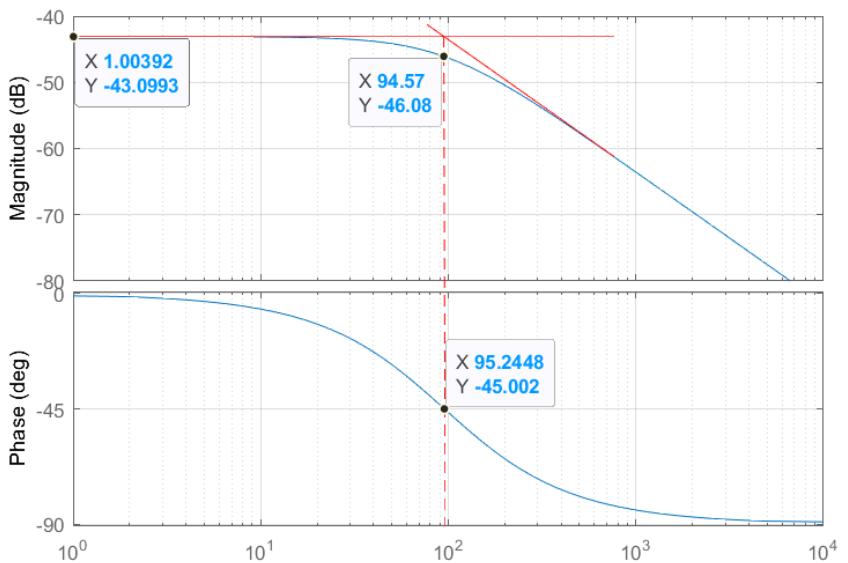
$$i = \frac{H/20}{\omega_c} =$$

$$\text{---} =$$

$$1$$

$$3 \frac{\left( \text{---}_2 + \text{---}_1 \right)}{\text{---}_1 \text{---}_2} =$$

Freauency response:



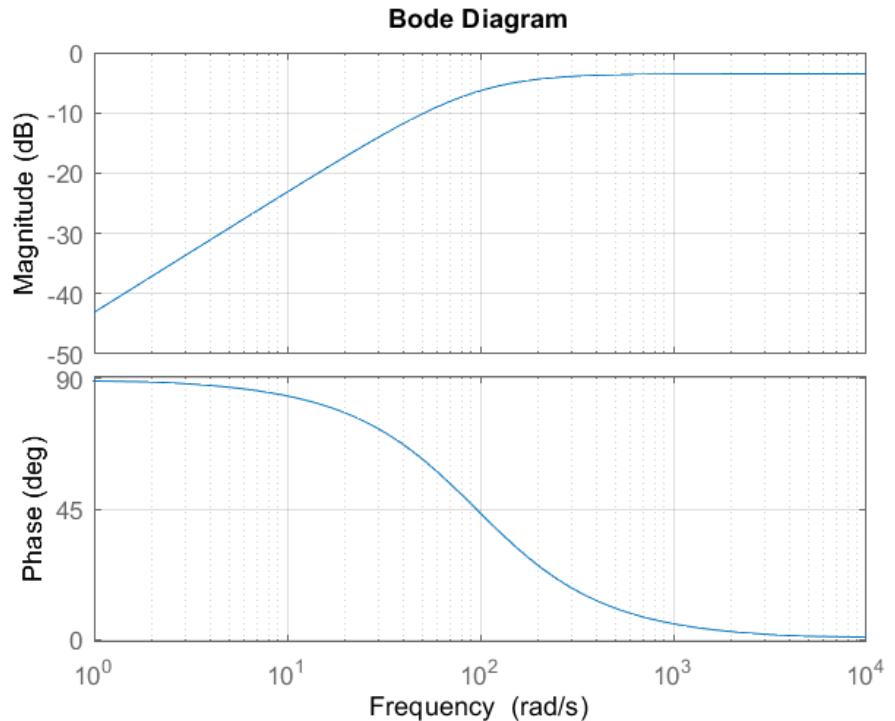
# Electrical circuits with inductance

Transfer function:

$$= \frac{3}{+} = \frac{u}{}$$

$$u = \frac{3}{1} \\ = \frac{3}{3} \left( \frac{2 + 1}{1 2} \right)$$

Frequency response:



# Electrical circuits with inductance

Parameters:

$$\omega_c = \frac{1}{L}$$

$$= \frac{1}{C} = \omega_c$$

$$= -\frac{H}{20} =$$

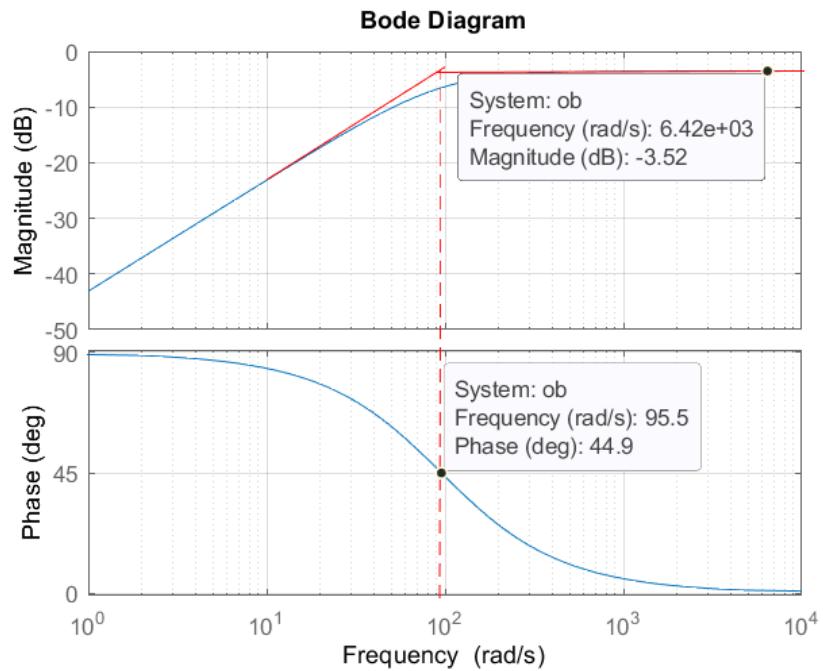
$$\frac{2}{s^2 + s} =$$

$$s^2 + s$$

$$3 \frac{(s^2 + s)}{s^2 + 2s} =$$

$$s^2 + 2s$$

Frequency response:



# Electrical circuits with inductance

State-space model:

$$\underline{z}_3 = -\frac{1 \ 2}{3(2 + 1)} \underline{z}_3 + \frac{2}{3(2 + 1)}$$

$$\underline{z}_3 = -\frac{1 \ 2}{(2 + 1)} \underline{z}_3 + \frac{2}{(2 + 1)}$$

$$\underline{z}_3 = -\underline{z}_3 + \frac{i}{u}$$

$$\underline{z}_3 = -\frac{u}{i} \underline{z}_3 + \frac{u}{i}$$

Parameters:

$$- =$$

$$1$$

$$\frac{2}{2 + 1} =$$

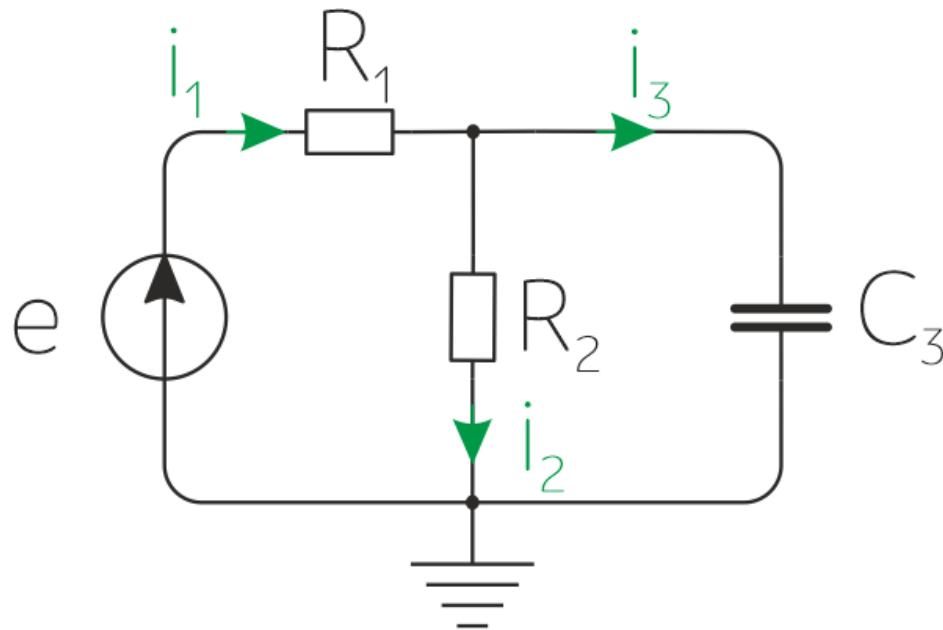
$$\frac{3(2 + 1)}{1 \ 2} =$$

# Mathematic modelling of electrical system with capacitance

# Electrical circuits with capacitance



Get the state-space model



# Electrical circuits with capacitance

1. Component equations:

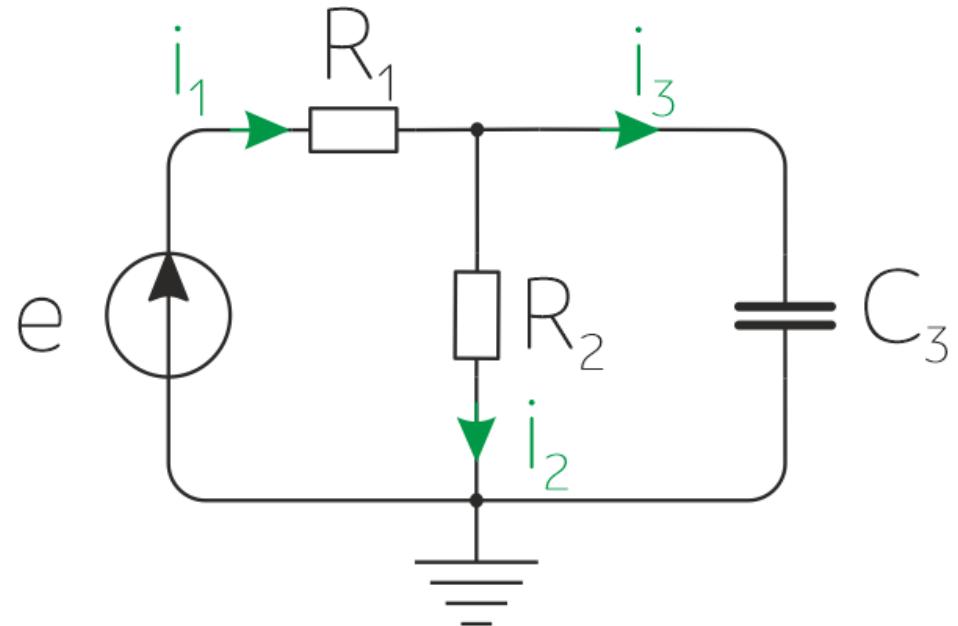
- Dissipative elements.

$$i_1 = e_1 \cdot R_1$$

$$e_2 = i_2 \cdot R_2$$

- Elastic elements.
- 

$$i_3 = e_3^{-1} \int i_3$$



- Inertial elements.

# Electrical circuits with capacitance

2. Topological equations:

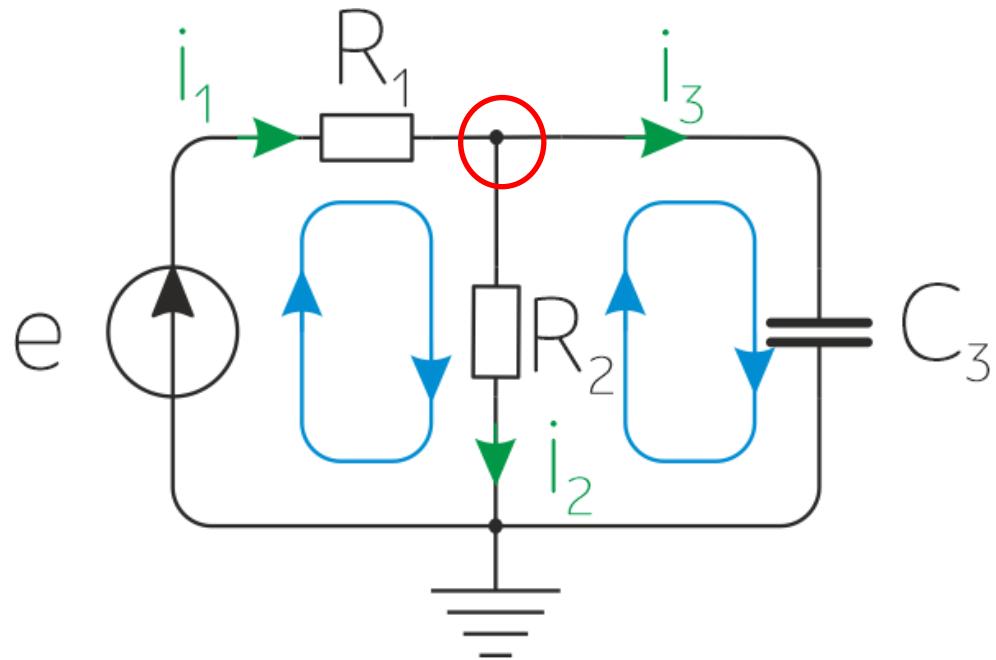
- Equilibrium equations.

$$i_1 + i_2 =$$

$$-i_2 + i_3 =$$

- Condition of continuity of flow type coordinates.

$$i_1 - i_2 - i_3 =$$



# Electrical circuits with capacitance

3. State-space model:

$$= +$$

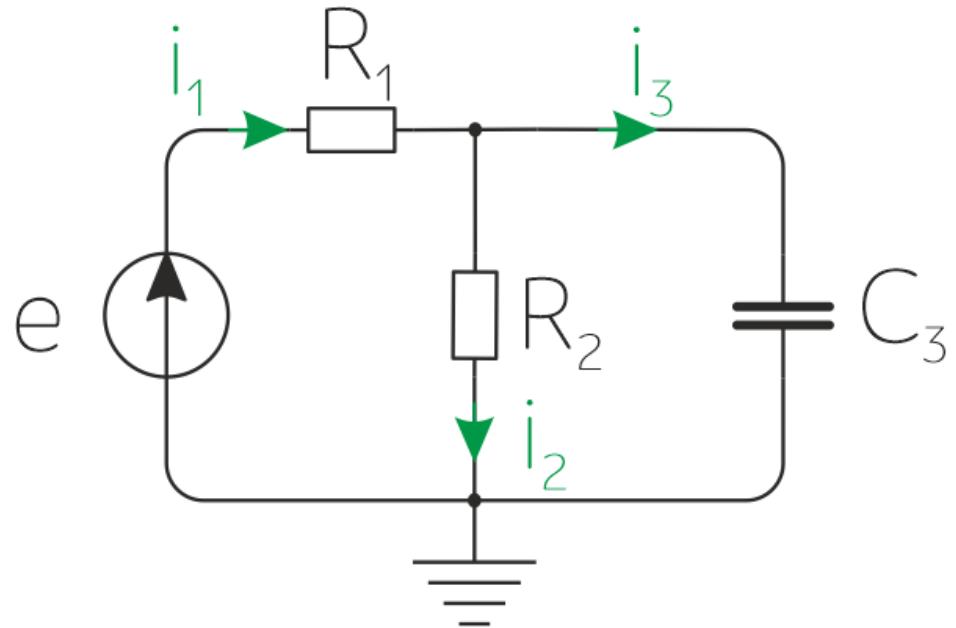
$$= +$$

State vector  $x$ :

$$= \begin{pmatrix} i_1 \\ i_2 \end{pmatrix}$$

Output vector  $v$ :

$$= \begin{pmatrix} e \\ v_3 \end{pmatrix}$$



# Electrical circuits with capacitance

3.1 Select the required number of equations for the model:

Three component  
equations.

Three topological  
equations.

Six circuit  
variables.

One state  
variable.

$$1 = 1 \cdot 1$$

$$1 + 2 =$$

1 2 3

3

$$2 = 2 \cdot 2$$

$$- 2 + 3 =$$

1 2 3

$$3 = 3^{-1} \int 3$$

$$1 - 2 - 3 =$$

# Electrical circuits with capacitance

3.2 Express the remaining parameters in terms of the state vector  
parameters and substitute it to the equation:

$$z = z \cdot z$$

$$z = z - z$$

$$-z + z =$$

$$z = \frac{1}{1}$$

$$z = -z$$

$$z = z^{-1} \int z$$

# Electrical circuits with capacitance

3.2 Express the remaining parameters in terms of the state vector  
parameters and substitute it to the equation:

$$- \underline{z}_2 + \underline{z}_3 =$$

$$\underline{z}_2 = \frac{}{\underline{z}_2 + \underline{z}_1} - \frac{\underline{z}_1}{\underline{z}_2 + \underline{z}_1} \underline{z}_3$$

$$\underline{z}_1 = \frac{}{\underline{z}_2 + \underline{z}_1} + \frac{\underline{z}_2}{\underline{z}_2 + \underline{z}_1} \underline{z}_3$$

$$\underline{z}_1 = \frac{1}{\underline{z}_2 + \underline{z}_1} + \frac{1 \ 2 \ 3}{\underline{z}_2 + \underline{z}_1} \underline{z}_3$$

$$\underline{z}_2 = \frac{2}{\underline{z}_2 + \underline{z}_1} - \frac{1 \ 2 \ 3}{\underline{z}_2 + \underline{z}_1} \underline{z}_3$$

$$\underline{z}_3 = \underline{z}_3 \cdot \underline{z}_3$$

# Electrical circuits with capacitance

3.2 Express the remaining parameters in terms of the state vector  
parameters and substitute it to the equation:

$$-\frac{2}{2+1} + \frac{1}{2+1} \cdot \underline{\underline{3}} + \underline{\underline{3}} =$$

$$\underline{\underline{3}} = -\frac{1}{2+1} \underline{\underline{2}} \underline{\underline{3}} + \underline{\underline{1}} \underline{\underline{3}}$$

# Electrical circuits with inductance

3.3 Express the output vector in terms of the state vector and substitute input vector:

$$\underline{\underline{3}} = -\frac{\begin{matrix} + \\ 1 & 2 \end{matrix}}{\begin{matrix} 2 & 1 & 3 \end{matrix}} \underline{\underline{3}} + \underline{\underline{}}$$

$$\underline{\underline{3}} = \underline{\underline{3}} - \frac{\begin{matrix} + \\ 1 & 2 \end{matrix}}{\begin{matrix} 2 & 1 \end{matrix}} \underline{\underline{3}} + \underline{\underline{1}}$$

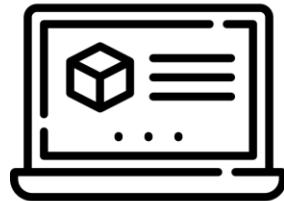
# Electrical circuits with inductance

3.4 Write the state space model:

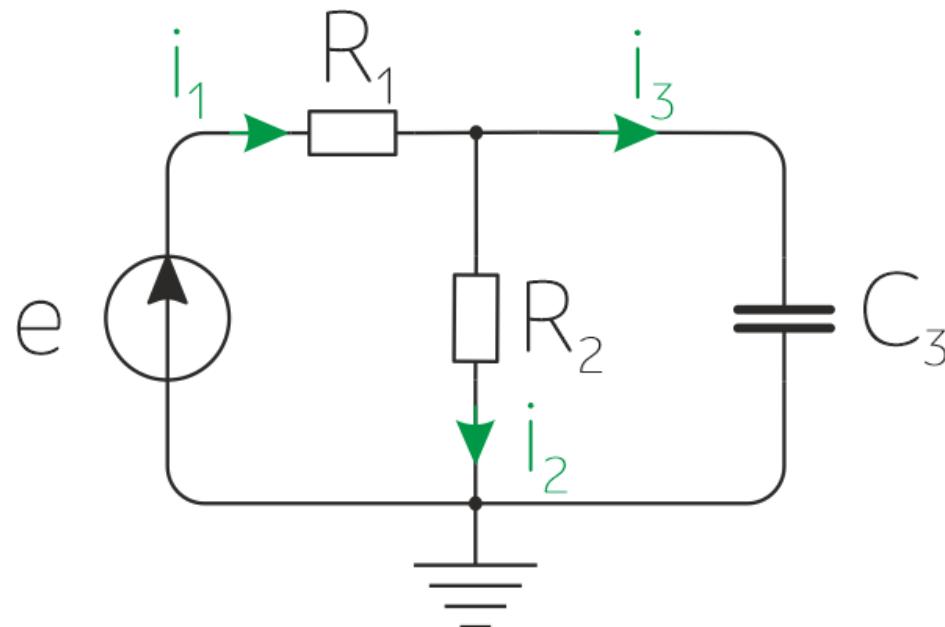
$$\underline{z_3} = -\frac{1+2}{2 \ 1 \ 3} z_3 + \underline{\quad}$$

$$z_3 = -\frac{1+2}{2 \ 1} z_3 + \underline{\quad}$$

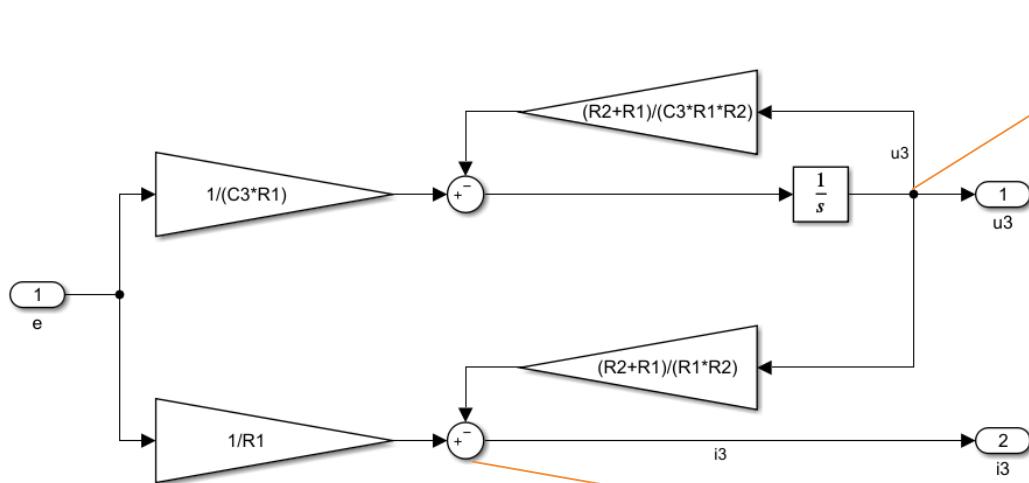
# Electrical circuits with capacitance



Modelling in Matlab  
(zero initial conditions)



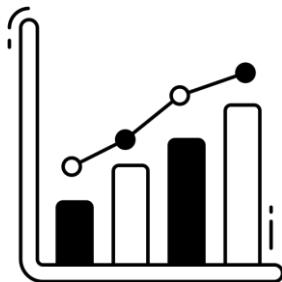
# Electrical circuits with capacitance



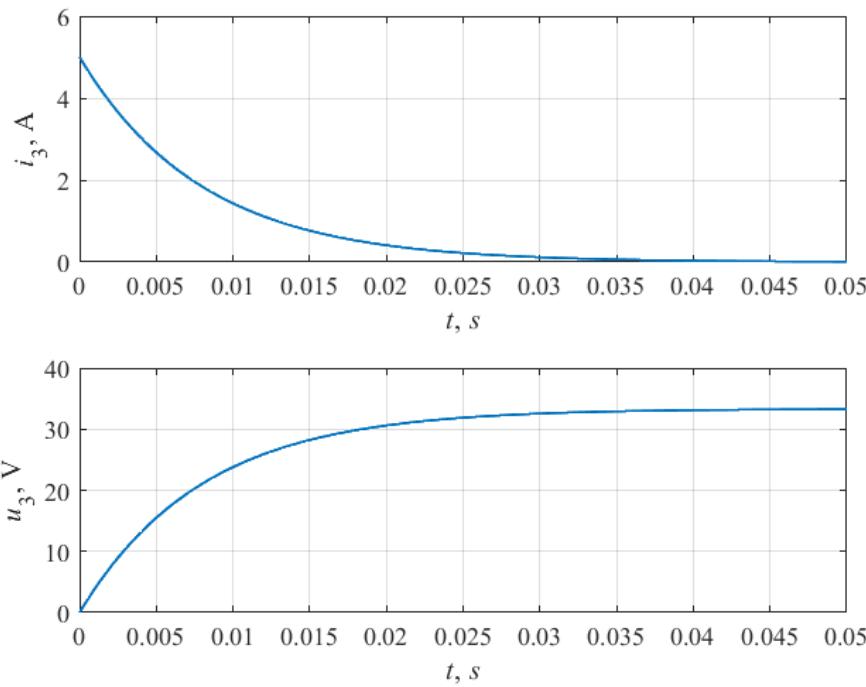
$$3 = -\frac{1 + 2}{2 \ 1 \ 3} 3 + \frac{1}{1 \ 3}$$

$$3 = -\frac{1 + 2}{2 \ 1} 3 + \frac{1}{1}$$

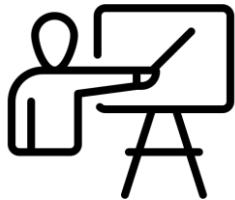
# Electrical circuits with capacitance



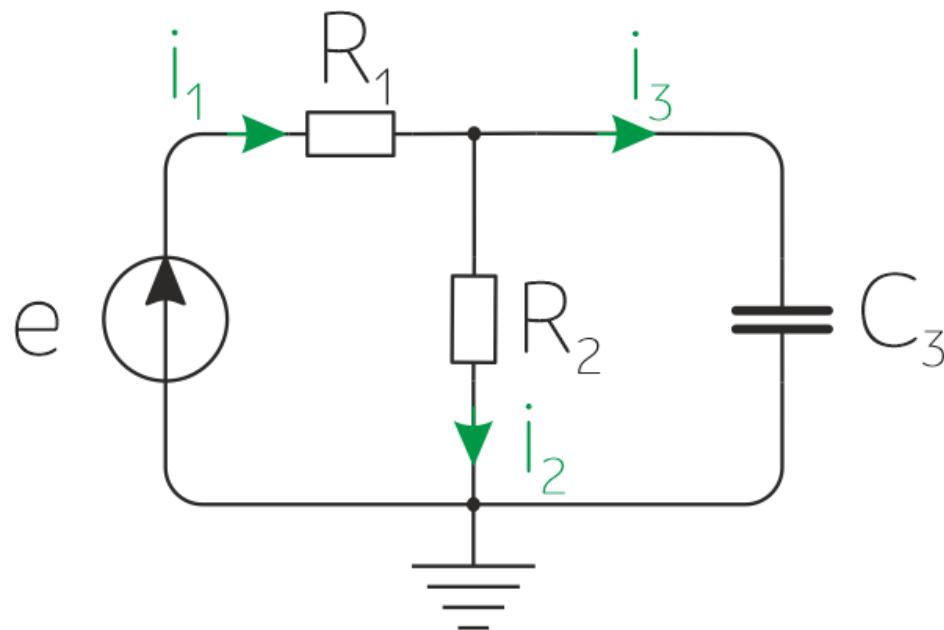
Graphs of the transients in the circuit  
(zero initial condition)



# Electrical circuits with capacitance



Get the transfer function



# Electrical circuits with capacitance

4. Transfer function:

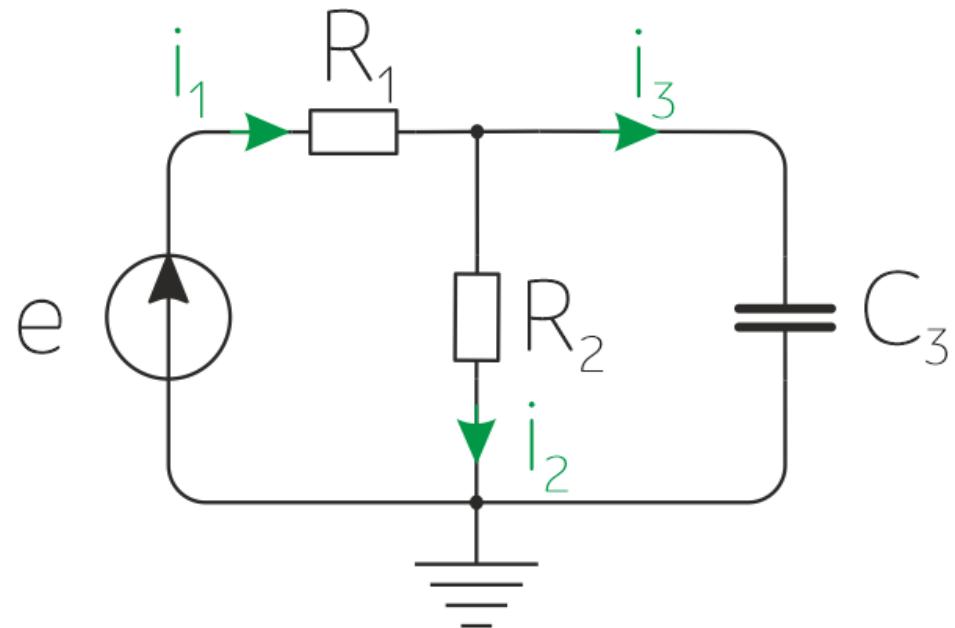
$$= \underline{\hspace{1cm}}$$

Input:

$$=$$

Output:

$$= \underline{\hspace{1cm}}_3$$



# Electrical circuits with capacitance

4.1 Perform Laplace transform for the state-space model (zero initial conditions):

$$\frac{3}{z} = -\frac{1 + 2}{2 \ 1 \ 3} z + \frac{1}{1 \ 3}$$

$$\frac{3}{z} = -\frac{1 + 2}{2 \ 1} z + \frac{1}{1}$$



$$\frac{3}{z} = -\frac{1 + 2}{2 \ 1 \ 3} z + \frac{1}{1 \ 3}$$

$$\frac{3}{z} = -\frac{1 + 2}{2 \ 1} z + \frac{1}{1}$$

# Electrical circuits with capacitance

4.2 Express the output in terms of input:

$$3 = \frac{2}{2 \ 1 \ 3 + 1 + 2}$$

$$3 = \frac{3 \ 2}{2 \ 1 \ 3 + 1 + 2}$$

# Electrical circuits with capacitance

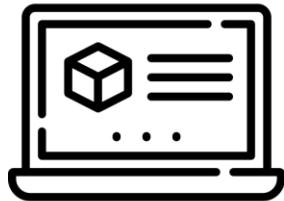
4.3 Write the transfer function:

$$= \frac{3}{2 \ 1 \ 3} = \frac{3 \ 2}{+ \ 1 + \ 2} = \frac{}{+}$$

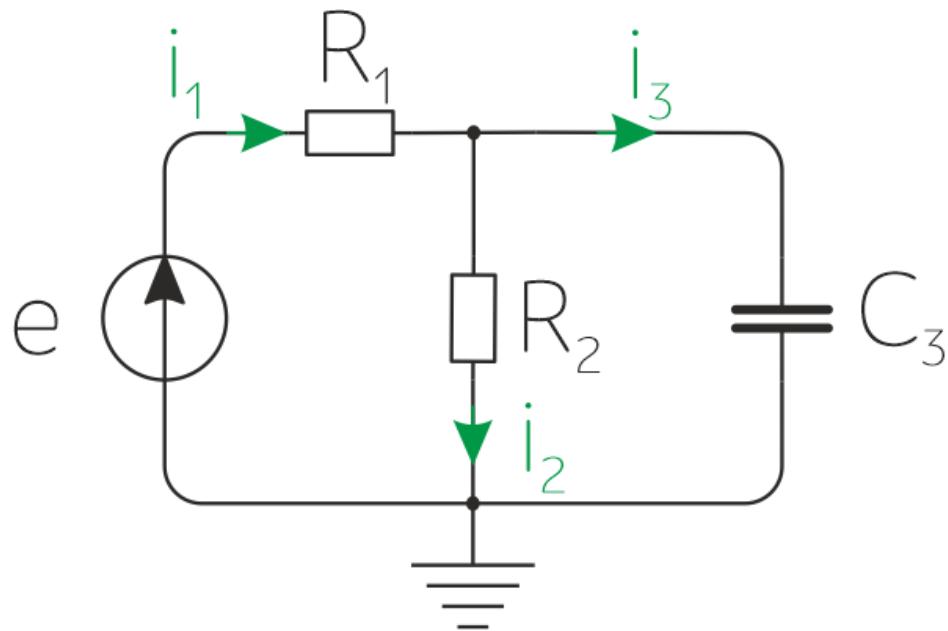
$$= \frac{3 \ 2}{1 + 2}$$

$$= \frac{1 \ 2}{3 \left( \frac{1}{2} + \frac{1}{1} \right)}$$

# Electrical circuits with capacitance



Modelling in Matlab  
(nonzero initial conditions)

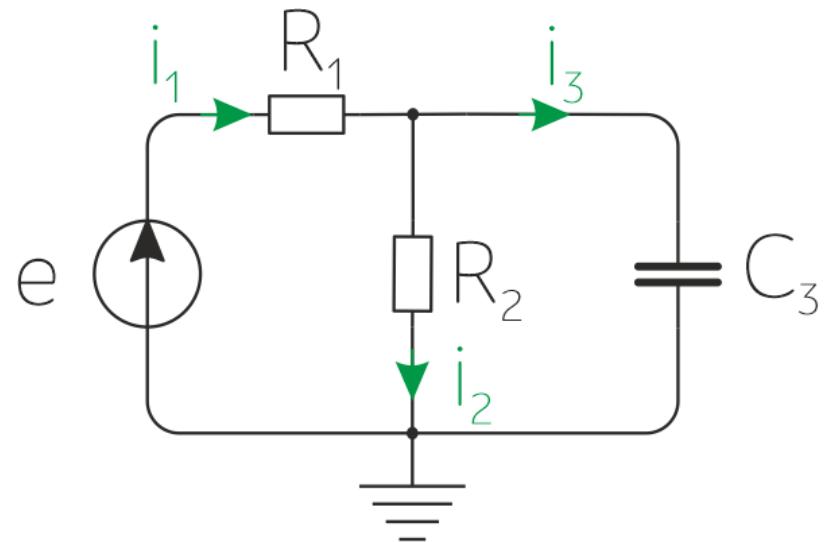


# Electrical circuits with capacitance

State-space model:

$$\dot{x}_3 = -\frac{1+2}{2 \ 1 \ 3} x_3 + \dots$$

$$x_3 = -\frac{1+2}{2 \ 1} x_3 + \dots$$



Initial conditions:

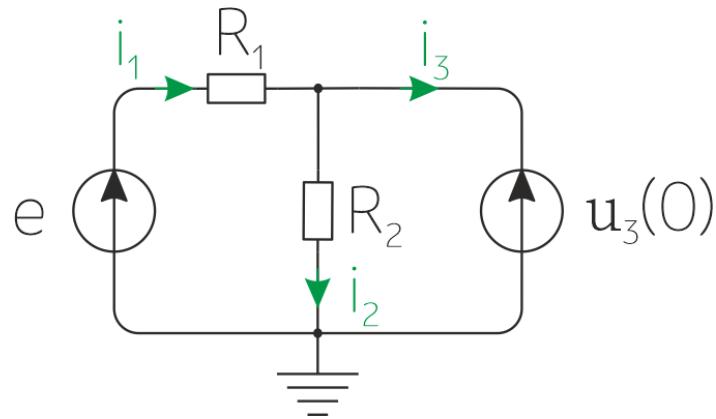
$$x_1 =$$

State space vector x:

$$x = \begin{pmatrix} x_1 \\ x_3 \end{pmatrix}$$

# Electrical circuits with capacitance

Circuit at time  $t = 0+$ :



$$i_1 + i_3 =$$

$$i_3 = -i_1$$

# Electrical circuits with capacitance

## Nonzero initial conditions in Matlab

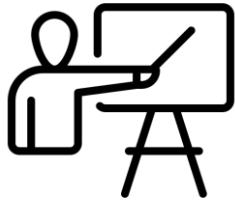
The screenshot shows the MATLAB/Simulink environment with a circuit diagram open. The circuit consists of a resistor R1 (labeled 'R1') in series with a capacitor (indicated by a circle with a plus sign). The circuit is connected to a voltage source (indicated by a circle with a minus sign) and ground.

Two parameter dialogs are displayed:

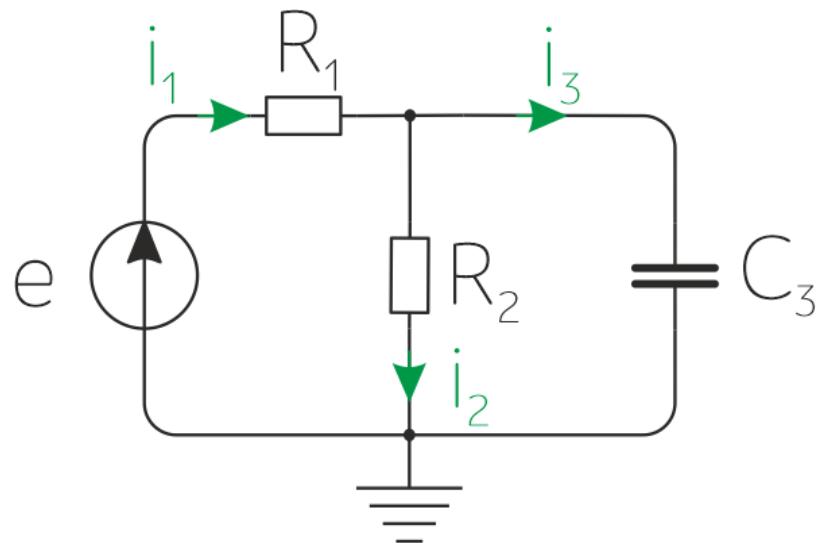
- Block Parameters: Resistor (R1)**: Shows settings for 'Current' (Priority: None, Beginning Value: 0, Unit: A) and 'Voltage' (Priority: High, Beginning Value: 10, Unit: V). The 'Voltage' setting is highlighted with a red box.
- Block Parameters: Capacitor**: Shows the description of a capacitor and its parameters. The 'Capacitor voltage' setting is also highlighted with a red box.

The 'Capacitor voltage' setting in both dialogs is set to 'None' with a value of 0, which corresponds to the 'High' priority setting of 10 in the Resistor dialog.

# Electrical circuits with capacitance



Parametric identification using  
transient response



# Electrical circuits with capacitance

Transfer function:

$$= \frac{3}{+} = \frac{u}{+}$$

$$u = \frac{2}{1 + 2}$$

$$= \frac{1}{3} \frac{2}{(2 + 1)}$$

Parameters:

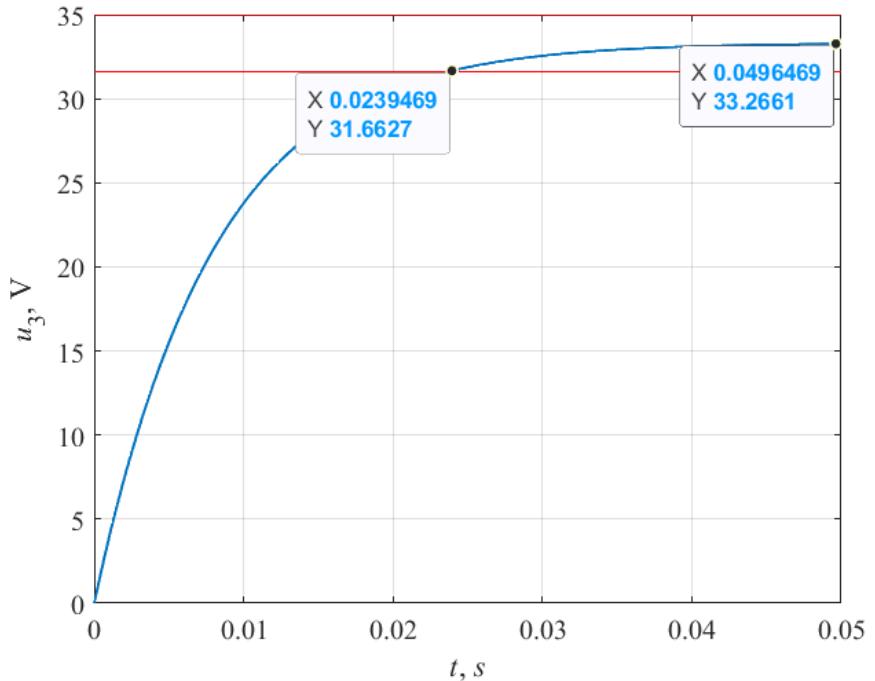
$$u \cdot =$$

$$\cdot =$$

$$\frac{\cdot 1}{2 + 1} =$$

$$\frac{1}{3} \frac{2}{(2 + 1)} =$$

Transient response:



# Electrical circuits with capacitance

Transfer function:

$$= \frac{3}{+} = \frac{i}{+}$$

$$i = \frac{3 \cdot 2}{1 + 2}$$

$$= \frac{1 \cdot 2}{(2 + 1)}$$

Parameters:

$$\frac{i}{\cdot} =$$

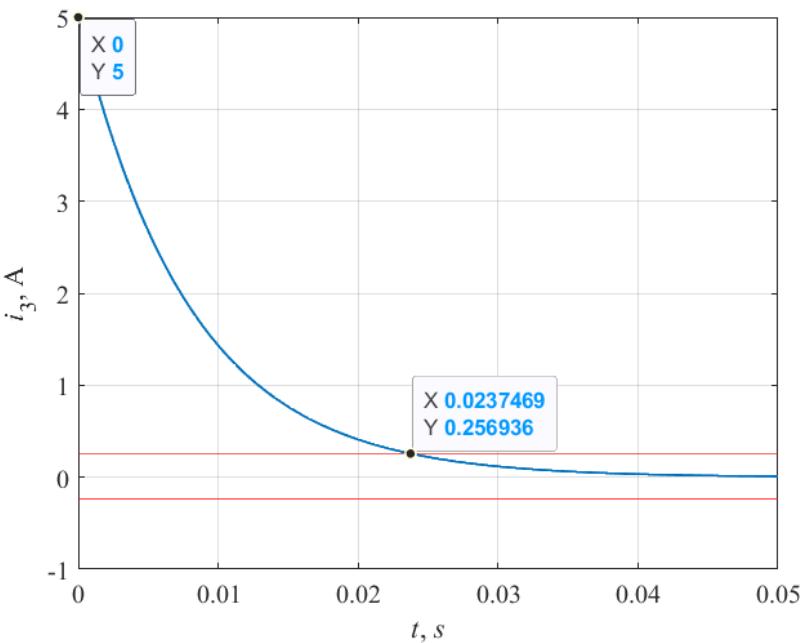
$$\cdot =$$

$$=$$

$$1$$

$$\frac{1 \cdot 2}{(2 + 1)} =$$

Transient response:



# Electrical circuits with capacitance

State-space model:

$$\underline{\underline{z}}_3 = -\frac{1+2}{2 \ 1 \ 3} \underline{\underline{z}}_3 + \underline{\underline{u}}$$

$$\underline{\underline{z}}_3 = -\frac{1+2}{2 \ 1} \underline{\underline{z}}_3 + \underline{\underline{u}}$$

$$\underline{\underline{z}}_3 = -\underline{\underline{z}}_3 + \underline{\underline{u}}$$

$$\underline{\underline{z}}_3 = -\frac{i}{u} \underline{\underline{z}}_3 + \frac{u}{1}$$

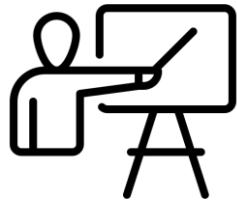
Parameters:

$$\underline{\underline{z}} =$$
  
1

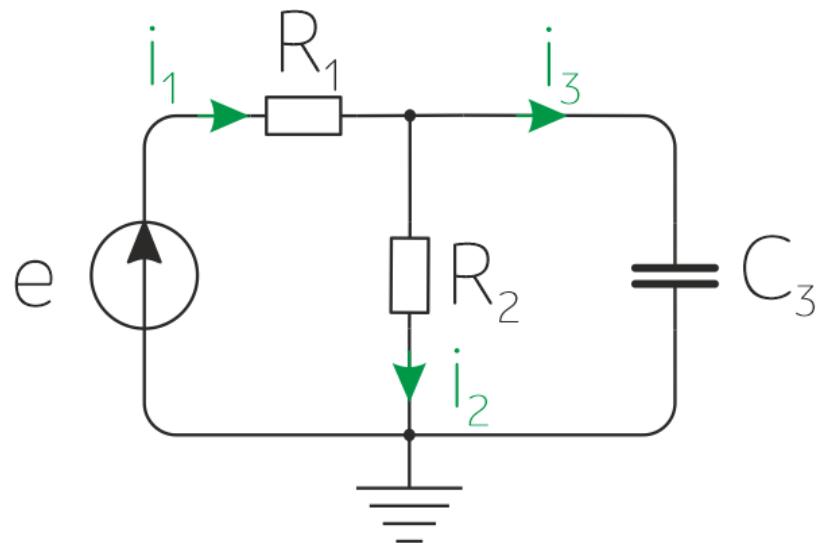
$$\frac{\cdot}{2} \frac{1}{+} \underline{\underline{z}}_1 =$$

$$\underline{\underline{z}}_3 \frac{1 \ 2}{(2 + 1)} =$$

# Electrical circuits with capacitance



Parametric identification using  
frequency response



# Electrical circuits with capacitance

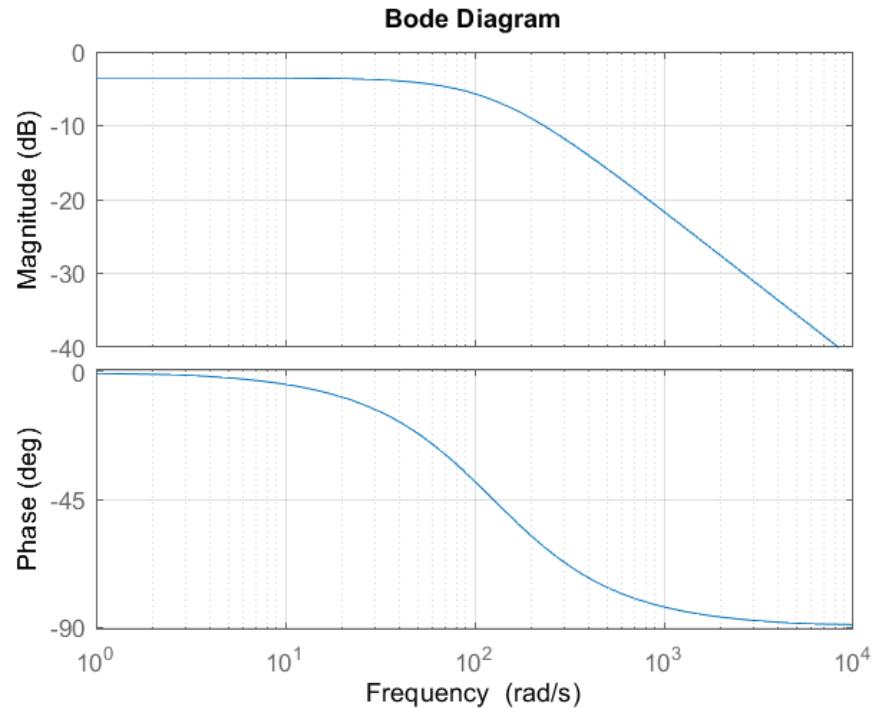
Transfer function:

$$= \frac{3}{+} = \frac{u}{+}$$

$$u = \frac{2}{1 + 2}$$

$$= \frac{3}{\left( \frac{1}{2} + \frac{2}{1} \right)}$$

Frequency response:



# Electrical circuits with capacitance

Parameters:

$$\omega_c = \frac{1}{RC}$$

$$= \frac{1}{C} = \omega_c$$

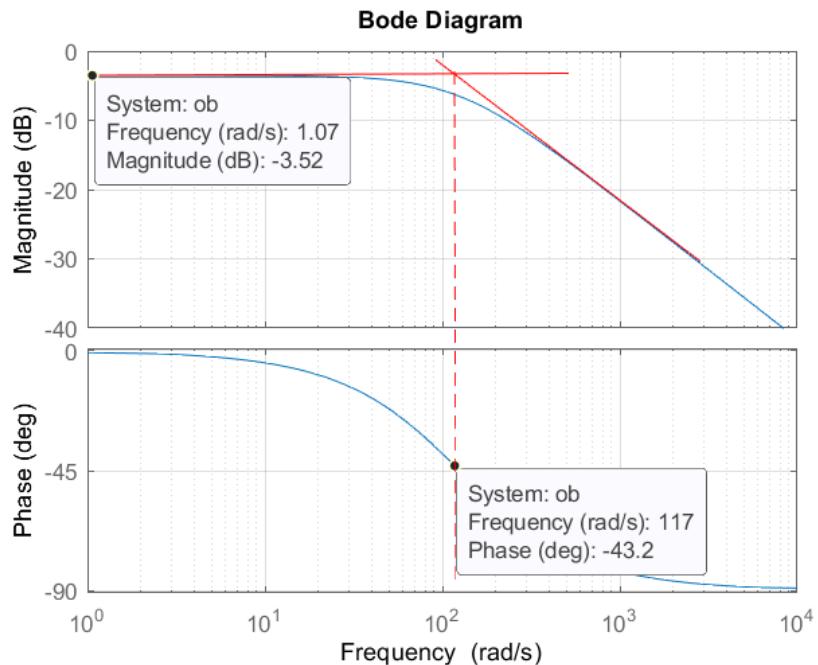
= -

$$u = \frac{H}{20} =$$

$$\frac{1}{2 + 1} =$$

$$^3 \frac{1 \ 2}{(2 + 1)} =$$

Frequency response:



# Electrical circuits with capacitance

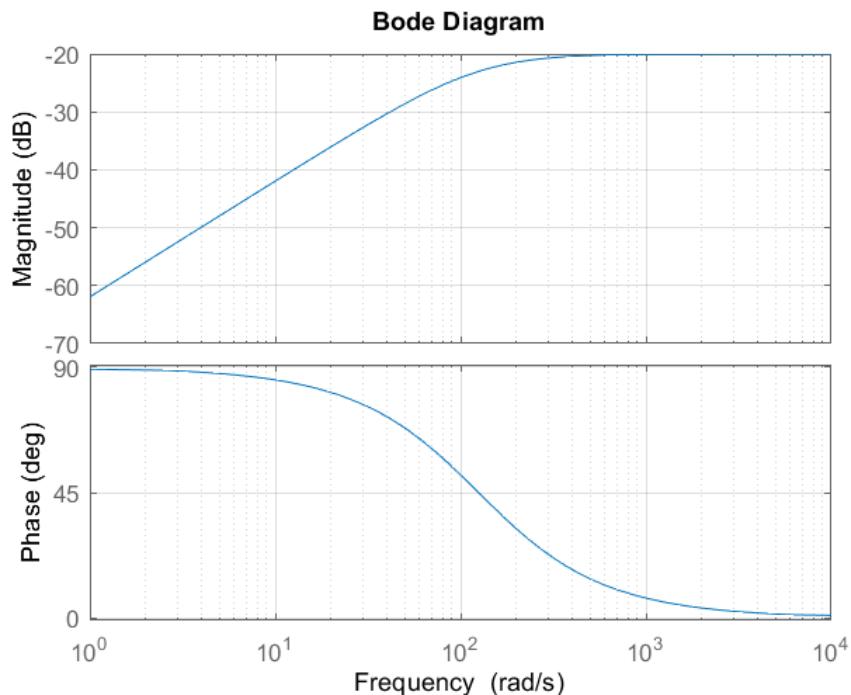
Transfer function:

$$= \frac{3}{+} = \frac{i}{+}$$

$$i = \frac{3 \cdot 2}{1 + 2}$$

$$= \frac{3}{\left( \frac{1}{2} + \frac{1}{1} \right)}$$

Frequency response:



# Electrical circuits with capacitance

Parameters:

$$\omega_c = \frac{1}{RC}$$

$$= \frac{1}{C} = \omega_c$$

= -

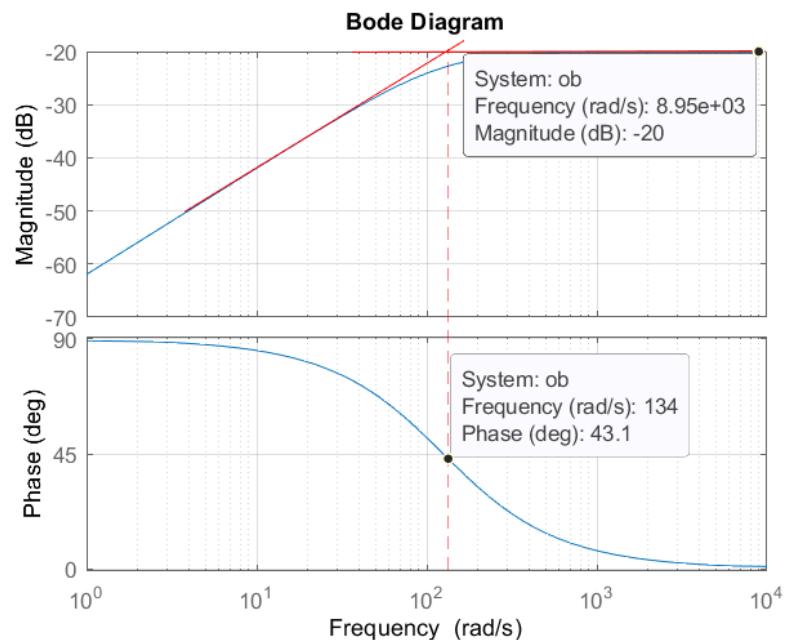
$$-i = \frac{H}{20} =$$

$$=$$

1

$$^3 \left( \frac{1}{C_2} + \frac{1}{C_1} \right) =$$

Frequency response:



# Electrical circuits with capacitance

State-space model:

$$\dot{x}_3 = -\frac{1}{2}x_1 + \frac{2}{3}x_3 + u$$

$$x_3 = -\frac{1}{2}x_1 + \frac{2}{3}x_3 + u$$

Parameters:

$$u =$$

$$1$$

$$\frac{1}{2}x_1 + 1 =$$

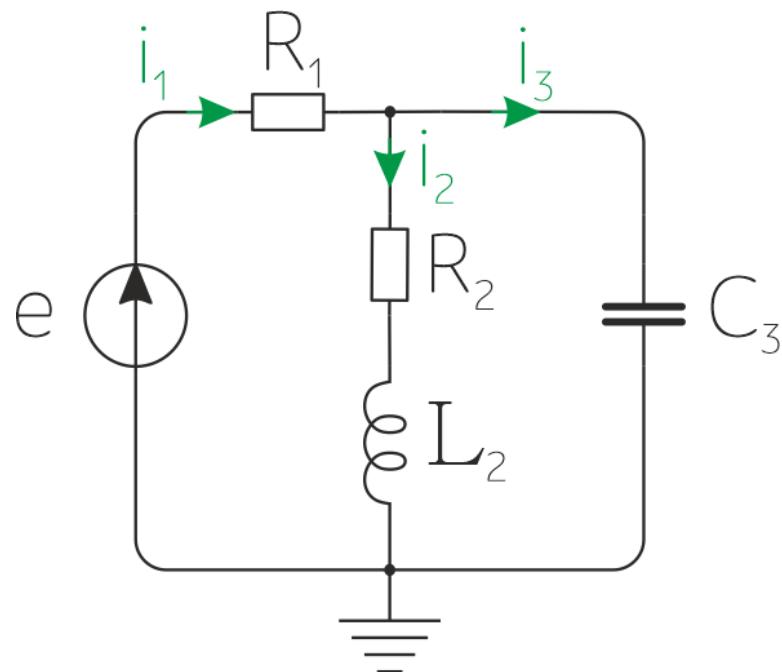
$$3 \left( \frac{1}{2}x_1 + 1 \right) =$$

# Mathematic modelling of second order electrical system

# Second order electrical circuits



Get the state-space model



# Second order electrical circuits

1. Component equations:

- Dissipative elements.

$$i_1 = R_1 \cdot v_1$$

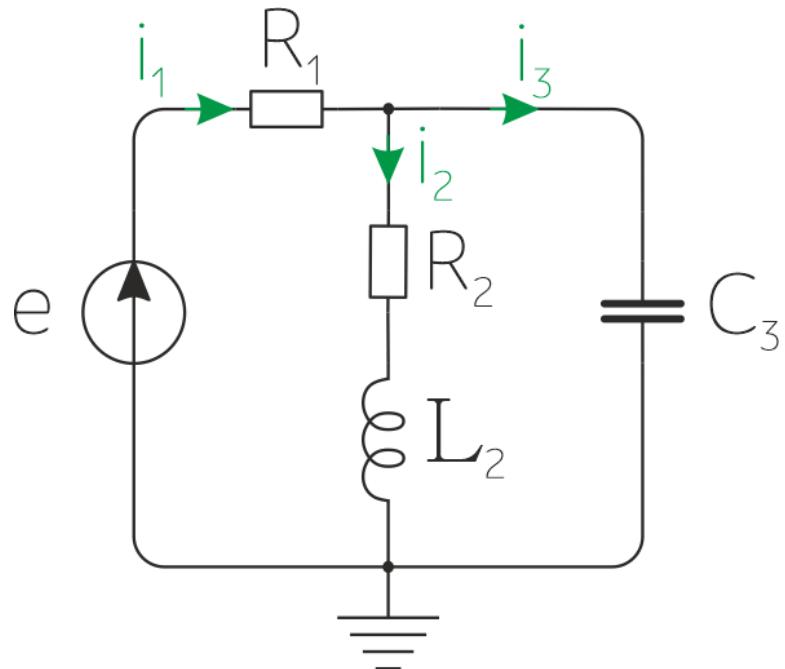
$$R_2 = v_2 \cdot i_2$$

- Elastic elements.

$$L_2 = v_2 \cdot \frac{i_2^2}{2}$$

- Inertial elements.

$$m_3 = m_3^{-1} \int v_3$$



# Second order electrical circuits

2. Topological equations:

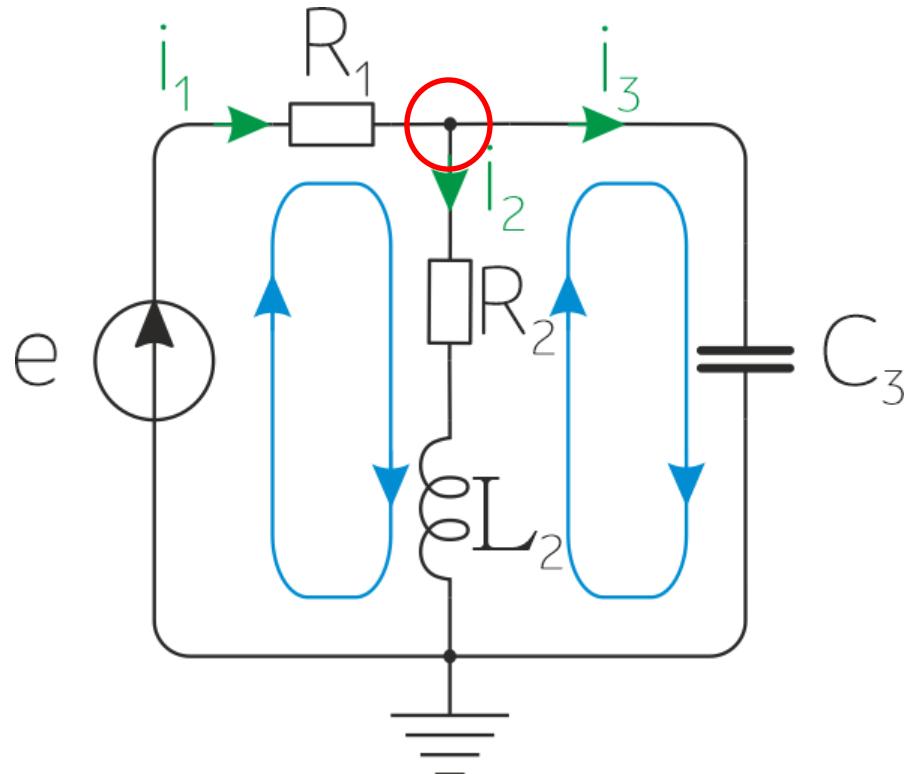
- Equilibrium equations.

$$i_1 + R_2 + L_2 =$$

$$-R_2 - L_2 + i_3 =$$

- Condition of continuity of flow type coordinates.

$$i_1 - i_2 - i_3 =$$



# Second order electrical circuits

3. State-space model:

$$= +$$

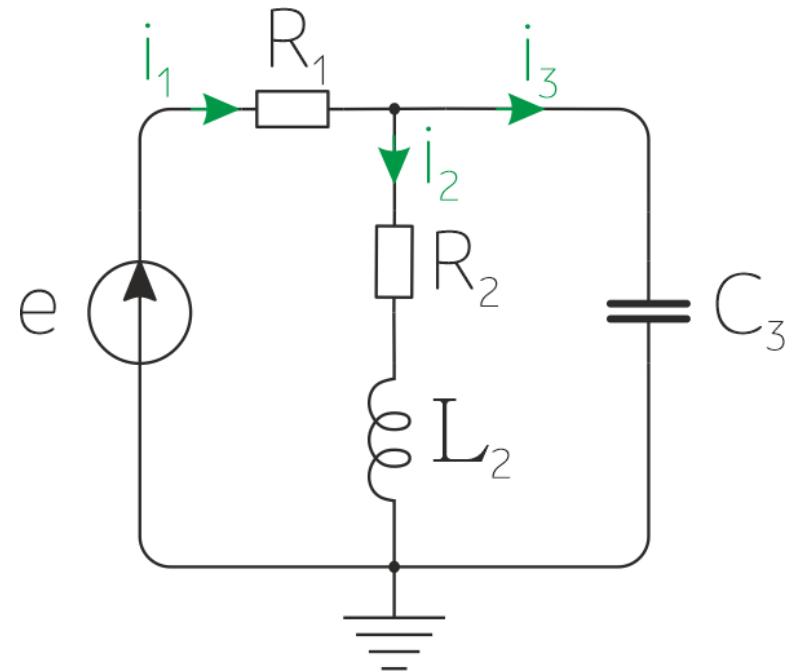
$$= +$$

State vector  $x$ :

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Output vector  $v$ :

$$= L_2$$



# Second order electrical circuits

3.1 Select the required number of equations for the model:

Four component equations.

Three topological equations.

Six circuit variables.

Two state variables.

$$1 = 1 \cdot 1$$

$$R2 = 2 \cdot 2$$

$$L2 = 2 \cdot \underline{2}$$

$$3 = 3^{-1} \int 3$$

$$1 + R2 + L2 =$$

$$- R2 - L2 + 3 =$$

$$1 - 2 - 3 =$$

1 2 3

2 3

1 2 3

## Second order electrical circuits

3.2 Express the remaining parameters in terms of the state vector  
parameters and substitute it to the equation:

$$\begin{aligned} R_2 &= \underline{\underline{z}}_2 \cdot \underline{\underline{z}}_2 \\ L_2 &= \underline{\underline{z}}_2 \cdot \underline{\underline{z}}_2^2 \\ -R_2 - L_2 + z_3 &= \\ z_1 - z_2 - z_3 &= \frac{1}{\underline{\underline{z}}_1} \\ z_1 &= -R_2 - L_2 \\ z_3 &= z_3^{-1} \int z_3 \end{aligned}$$

# Second order electrical circuits

3.2 Express the remaining parameters in terms of the state vector  
parameters and substitute it to the equation:

$$\begin{aligned} & R_2 = 2 \cdot 2 \\ & L_2 = 2 \cdot 2 \\ -R_2 - L_2 + 3 &= \\ 1 - 2 - 3 &= \\ & 1 = -2 \cdot 2 - 2 \cdot 2 + \\ & 1 = -\frac{2}{1} \cdot 2 - \frac{2}{1} \cdot \frac{2}{1} + \frac{2}{1} \\ & 3 = 3 \cdot \frac{3}{3} \end{aligned}$$

# Second order electrical circuits

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$\left\{ \begin{array}{l} -\frac{2}{2} - \frac{2}{2} + \frac{2}{3} = \\ -\frac{2}{1} - \frac{2}{2} - \frac{2}{1} \cdot \frac{2}{1} + \frac{2}{1} - \frac{3}{3} = \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{2}{2} = -\frac{2}{2} - \frac{2}{2} + \frac{2}{3} = \\ \frac{3}{3} = -\frac{2}{3} - \frac{2}{1} \cdot \frac{2}{3} + \frac{2}{1} - \frac{3}{3} = \end{array} \right.$$

# Second order electrical circuits

3.3 Express the output vector in terms of the state vector and substitute input vector:

$$\begin{cases} \frac{d^2}{dt^2}x_2 = -\frac{2}{2}x_2 + \frac{1}{2}x_3 = \\ \frac{d^3}{dt^3}x_3 = -\frac{1}{3}x_2 - \frac{1}{1}x_3 + \frac{1}{3} \end{cases}$$

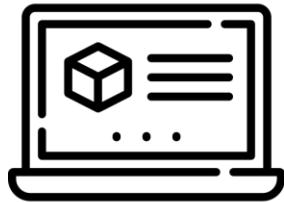
$$L_2 = \frac{d^2}{dt^2}x_2 = -\frac{2}{2}x_2 + \frac{1}{3}x_3$$

# Second order electrical circuits

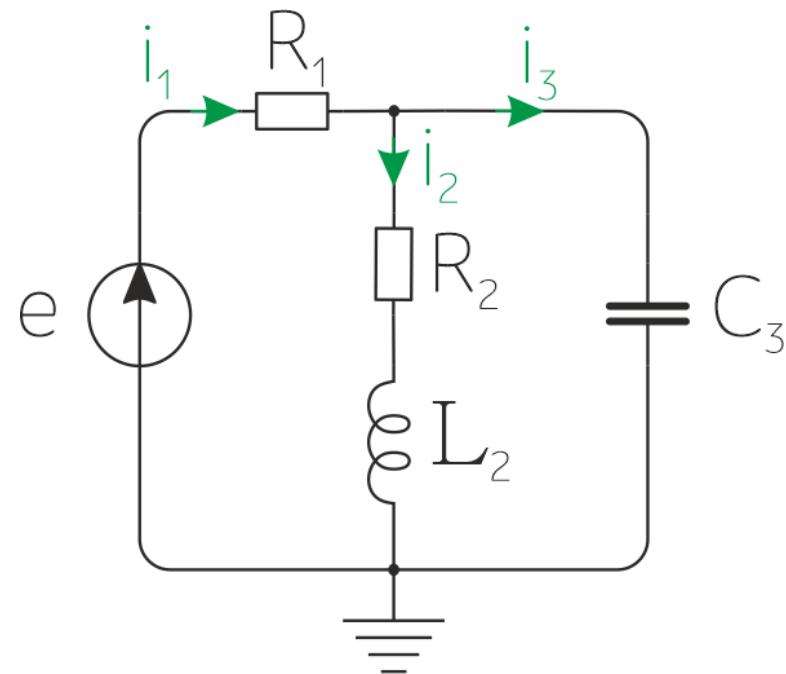
3.4 Write the state space model:

$$\left\{ \begin{array}{l} \frac{2}{s^2} = -\frac{2}{2} s + \frac{1}{2} \\ \frac{3}{s^3} = -\frac{1}{3} s - \frac{1}{1} s + \frac{1}{3} \\ L_2 = -\frac{1}{2} s + \frac{1}{3} \end{array} \right.$$

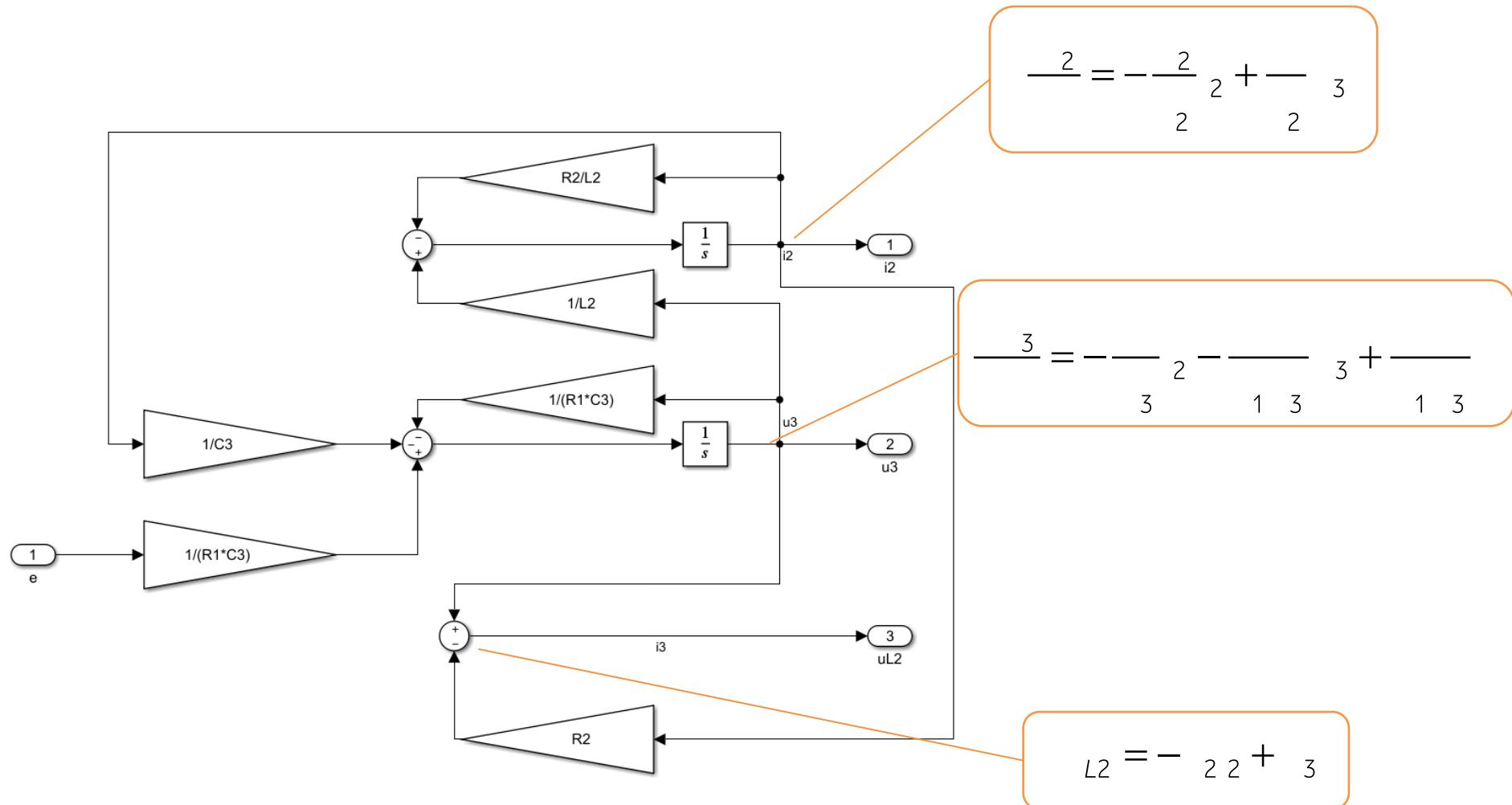
# Second order electrical circuits



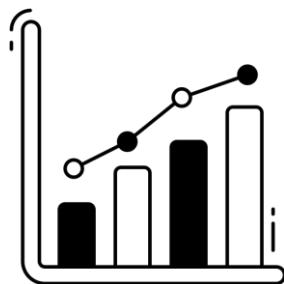
Modelling in Matlab  
(zero initial conditions)



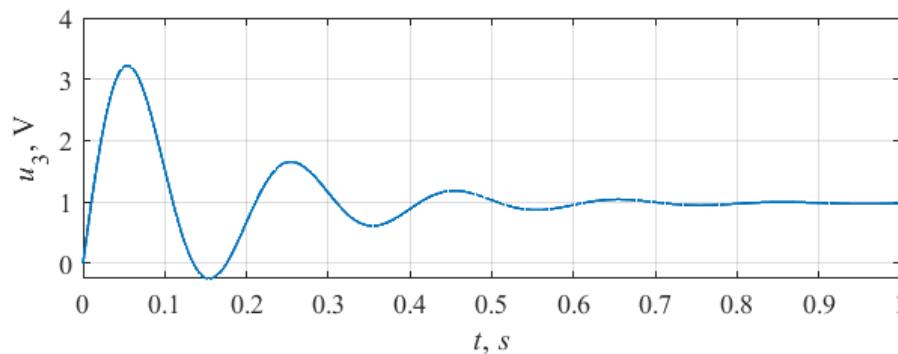
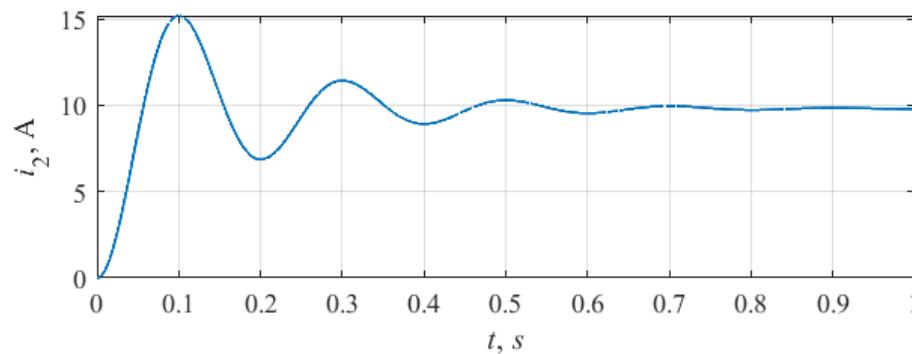
# Second order electrical circuits



# Second order electrical circuits



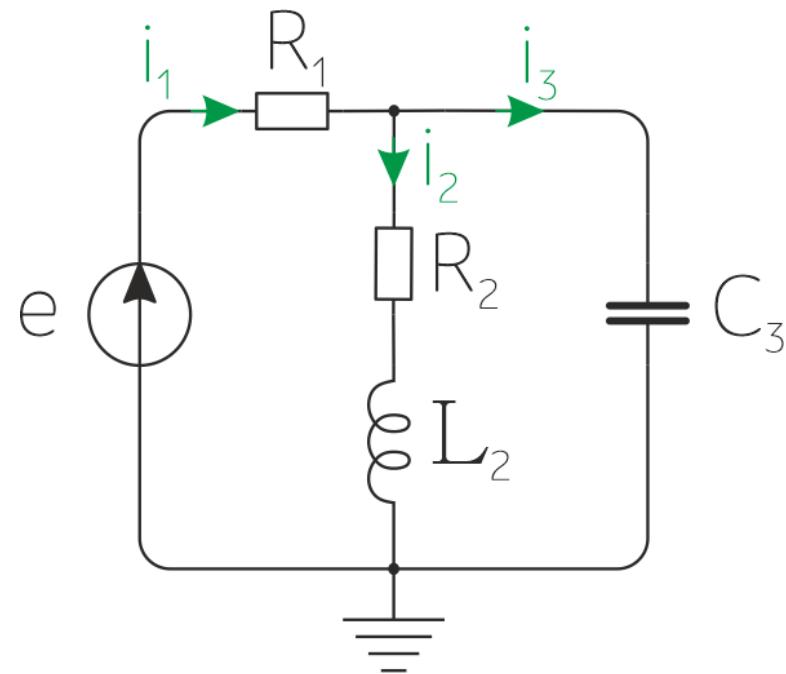
Graphs of the transients in the circuit  
(zero initial condition)



# Second order electrical circuits



Get the transfer function



# Second order electrical circuits

4. Transfer function:

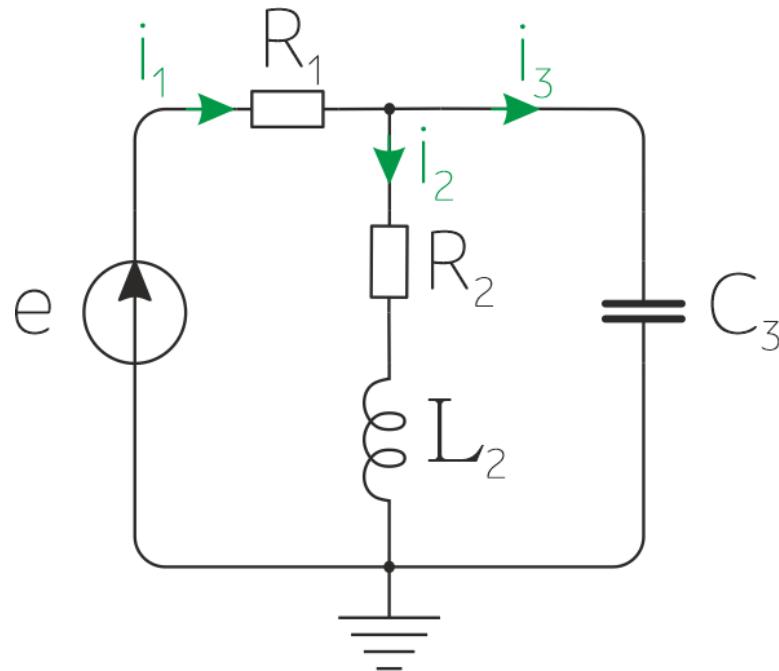
= —

Input:

=

Output:

=  $L_2$



# Second order electrical circuits

4.1 Perform Laplace transform for the state-space model (zero initial conditions):

$$\left\{ \begin{array}{l} \frac{2}{2} = -\frac{2}{2} s + \frac{1}{2} \\ \frac{3}{3} = -\frac{1}{3} s - \frac{1}{1} s + \frac{1}{3} \\ L_2 = -\frac{2}{2} s + \frac{1}{3} \end{array} \right.$$



$$\left\{ \begin{array}{l} 2 = -\frac{2}{2} s + \frac{1}{2} \\ 3 = -\frac{1}{3} s - \frac{1}{1} s + \frac{1}{3} \\ L_2 = -\frac{2}{2} s + \frac{1}{3} \end{array} \right.$$

# Second order electrical circuits

4.2 Express the output in terms of input:

$$2 = \frac{2}{1 \ 3 \ 2 \ 2 + 1 \ 2 \ 3 + 2 + 1 + 2}$$

$$3 = \frac{2 + 2}{1 \ 3 \ 2 \ 2 + 1 \ 2 \ 3 + 2 + 1 + 2}$$

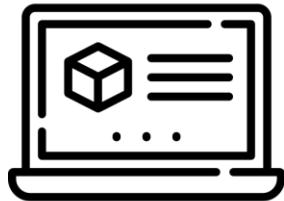
$$L2 = \frac{2}{1 \ 3 \ 2 \ 2 + 1 \ 2 \ 3 + 2 + 1 + 2}$$

## Second order electrical circuits

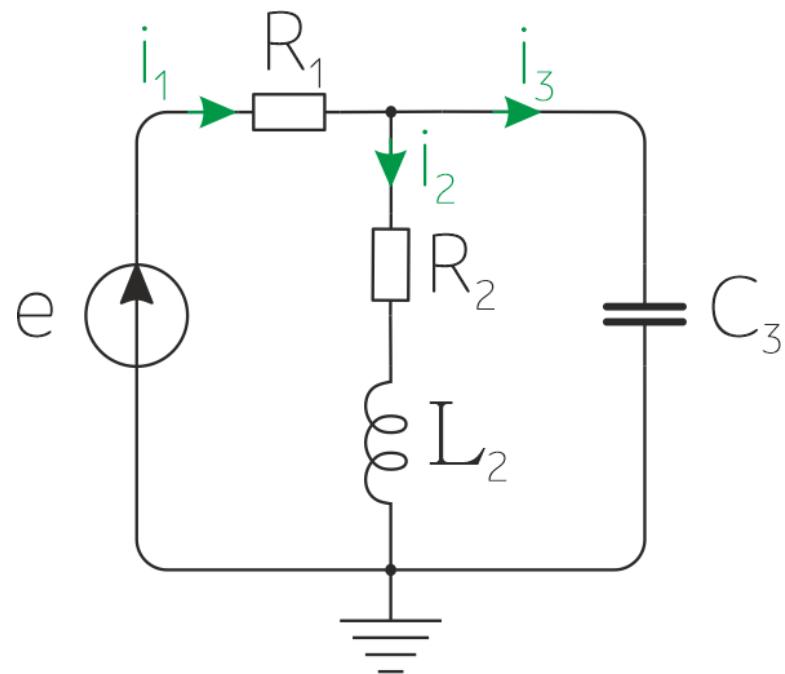
4.3 Write the transfer function:

$$\begin{aligned} &= \frac{L_2}{1 \ 3 \ 2} = \frac{\cancel{2} \left( \cancel{1} + \cancel{2} \right)}{\cancel{1} \ \cancel{3} \ \cancel{2} \left( \cancel{1} + \cancel{2} \right)^2 + \cancel{1} \ \cancel{2} \ \cancel{3} + \cancel{2} \left( \cancel{1} + \cancel{2} \right) +} = \frac{\cancel{2}^2 + \cdot \cdot \xi \cdot +}{\cancel{1}^2 + \cdot \cdot} \\ &= \frac{2}{1 + 2} \\ &= \sqrt{\frac{1 \ 3 \ 2}{2 + 1}} \\ \xi &= \frac{1 \ 2 \ 3 + 2}{\cdot \sqrt{1 \ 3 \ 2 \left( 1 + 2 \right)}} \end{aligned}$$

# Second order electrical circuits



Modelling in Matlab  
(nonzero initial conditions)



# Second order electrical circuits

State-space model:

$$\begin{cases} \frac{d^2}{dt^2} = -\frac{2}{2} i_2 + \frac{1}{2} i_3 \\ \frac{d^3}{dt^3} = -\frac{1}{3} i_2 - \frac{1}{1} i_3 + \frac{1}{3} i_3 \\ L_2 = -\frac{1}{2} i_2 + \frac{1}{3} i_3 \end{cases}$$

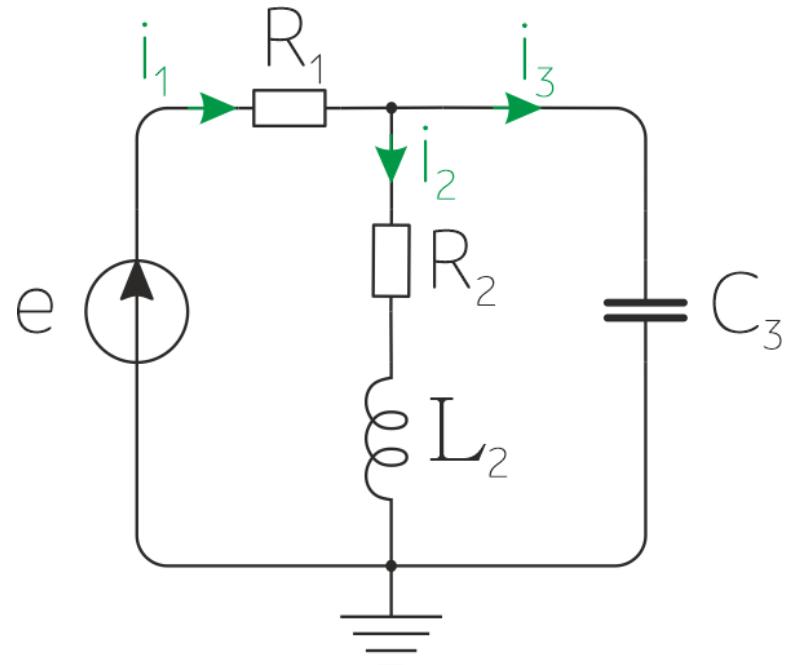
Initial conditions:

$$i_1 =$$

$$i_3 =$$

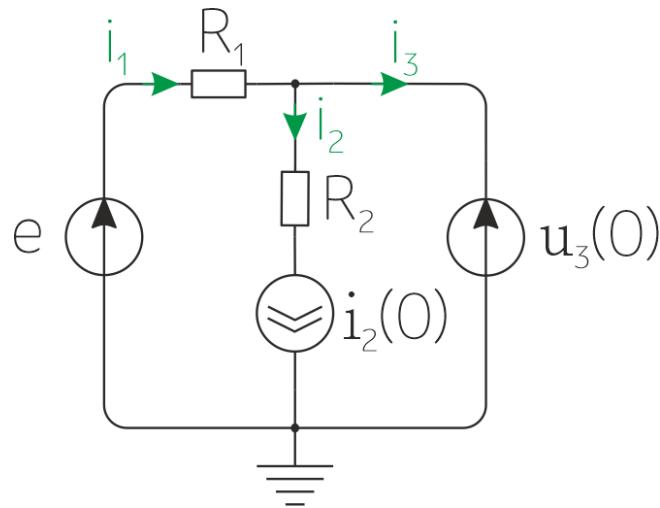
State space vector x:

$$= \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$



# Second order electrical circuits

Circuit at time  $t = 0+$ :



$$i_1 + i_3 =$$
  
$$i_1 = i_2 + i_3$$

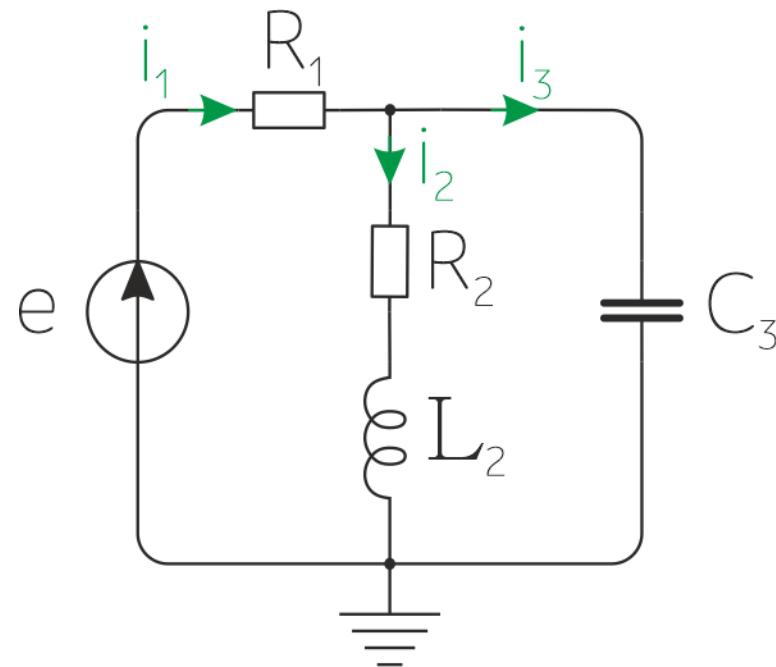


$$i_3 = -i_1$$
$$i_2 = \frac{1}{1} - i_3$$

# Second order electrical circuits



Parametric identification using  
transient response



# Second order electrical circuits

Transfer function:

$$= \frac{L_2}{r^2 + \cdot \cdot \cdot \xi \cdot \cdot \cdot} = \frac{uL_2}{r^2 + \cdot \cdot \cdot \xi \cdot \cdot \cdot}$$

$$uL_2 = \frac{2}{r_1 + r_2}$$

$$= \sqrt{\left( \frac{1}{r_2} + \frac{2}{r_1} \right)}$$

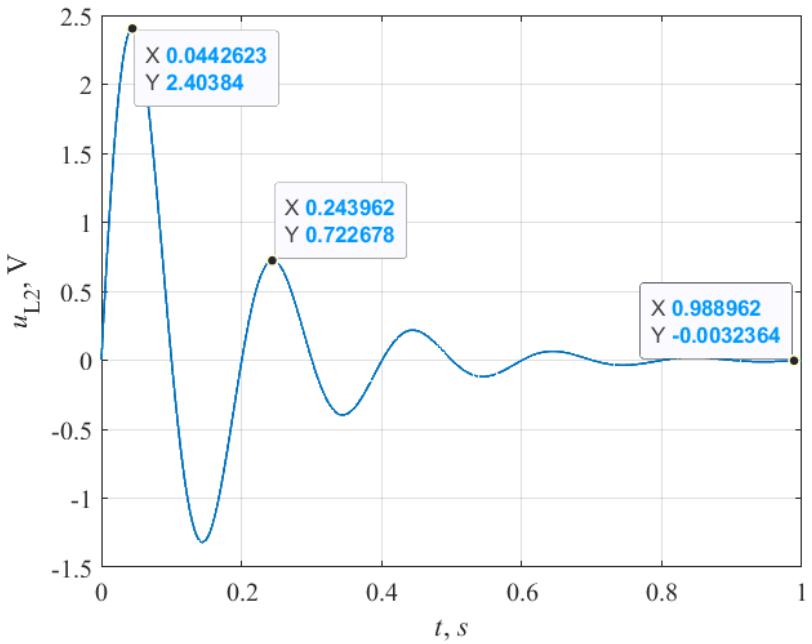
$$\xi = \frac{1}{\sqrt{\frac{1}{r_1} + \frac{2}{r_2}}}$$

Parameters:

$$r = \frac{-}{-} =$$

$$\delta = -\frac{-}{r} =$$

Transient response:



$$= \frac{-}{\sqrt{\delta^2 + \frac{-2}{r}}} =$$

$$\xi = -\delta \cdot =$$

# Second order electrical circuits

Transfer function:

$$= \frac{2}{s^2 + \cdot \cdot \cdot \xi \cdot \cdot \cdot} = \frac{i_2}{s^2 + \cdot \cdot \cdot \xi \cdot \cdot \cdot}$$

$$i_2 = \frac{2}{s^2 + \cdot \cdot \cdot} = \sqrt{\frac{1 \cdot 3 \cdot 2}{(s^2 + \cdot \cdot \cdot)}}$$

$$\xi = \frac{1 \cdot 2 \cdot 3 + 2}{\cdot \sqrt{1 \cdot 3 \cdot 2(s^2 + \cdot \cdot \cdot)}} =$$

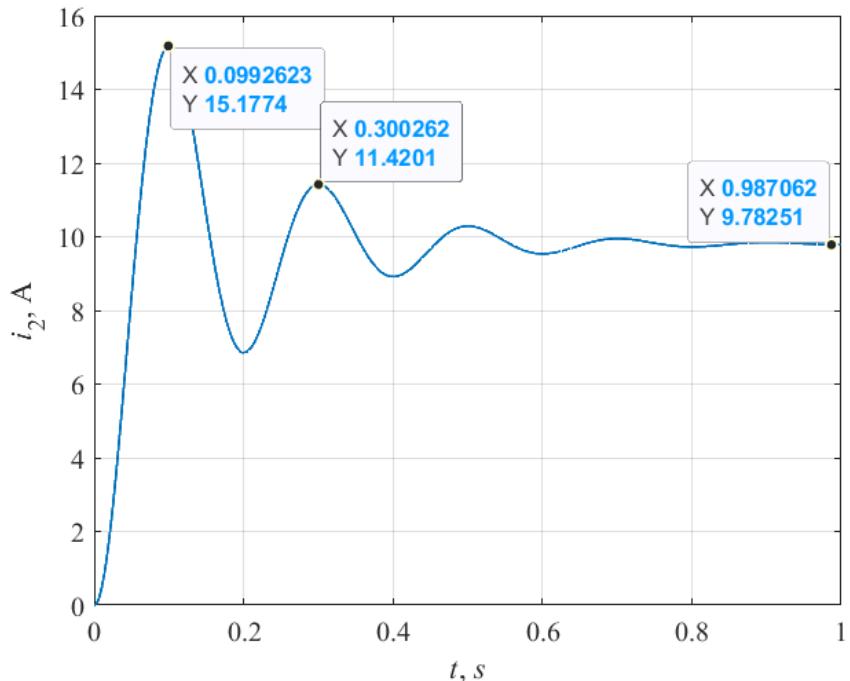
Parameters:

$$r = \frac{-}{-} =$$

$$\delta = \frac{-}{-} = \frac{r}{r}$$

$$i_2 \cdot \cdot \cdot =$$

Transient response:



$$= \frac{-}{\sqrt{\delta^2 + \frac{r^2}{r}}} =$$

$$\xi = -\delta \cdot \cdot \cdot =$$

$$i_2 \cdot \cdot \cdot =$$

# Second order electrical circuits

Transfer function:

$$= \frac{3}{s^2 + 2s + 2} = \frac{u_3(f + )}{s^2 + 2s + 2}$$

$$u_3 = \frac{2}{1 + 2} = \sqrt{\frac{1 \ 3 \ 2}{(2 + 1)}} \quad f = \frac{2}{2}$$

$$\xi = \frac{1 \ 2 \ 3 + 2}{\cdot \sqrt{1 \ 3 \ 2 (1 + 2)}}$$

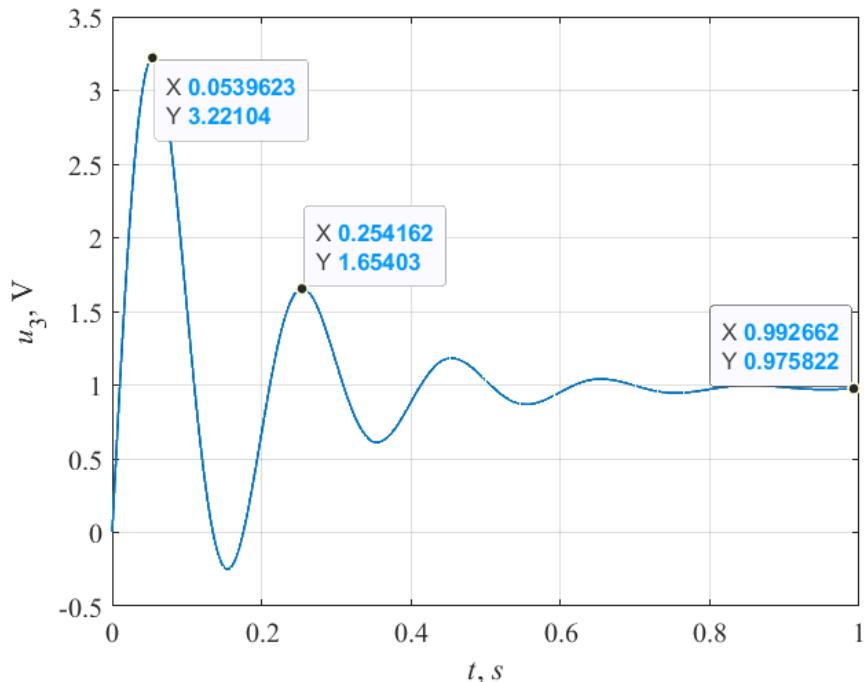
Parameters:

$$r = \underline{\underline{-}} =$$

$$\delta = \underline{\underline{-}} = \frac{r}{r}$$

$$u_3 \cdot =$$

Transient response:



$$= \frac{1}{\sqrt{\delta^2 + \frac{r^2}{4}}} =$$

$$\xi = -\delta \cdot =$$

$$u_3 = \underline{\underline{-}}$$

# Second order electrical circuits

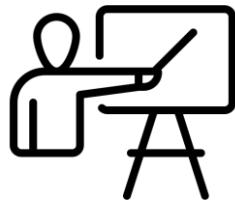
State-space model:

$$\begin{cases} \dot{x}_1 = -\frac{2}{2}x_1 + \frac{2}{2}x_2 \\ \dot{x}_2 = -\frac{3}{3}x_1 - \frac{1}{1}x_2 + \frac{3}{3}x_3 \\ L_2 = -\frac{2}{2}x_2 + \frac{3}{3}x_3 \end{cases}$$

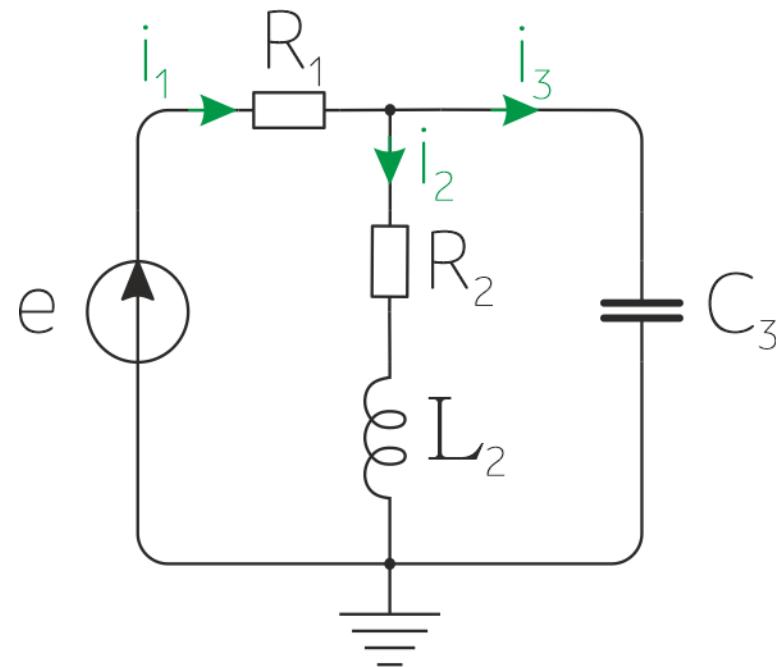
Parameters:

$$\begin{aligned} \sqrt{\frac{1 & 3 & 2}{2 + 1}} &= \\ \frac{1 & 2 & 3 + 2}{\cdot \sqrt{1 & 3 & 2 (1 + 2)}} &= \\ \frac{2}{1 + 2} &= \\ \frac{2}{1 + 2} &= \end{aligned}$$

# Second order electrical circuits



Parametric identification using  
frequency response



# Second order electrical circuits

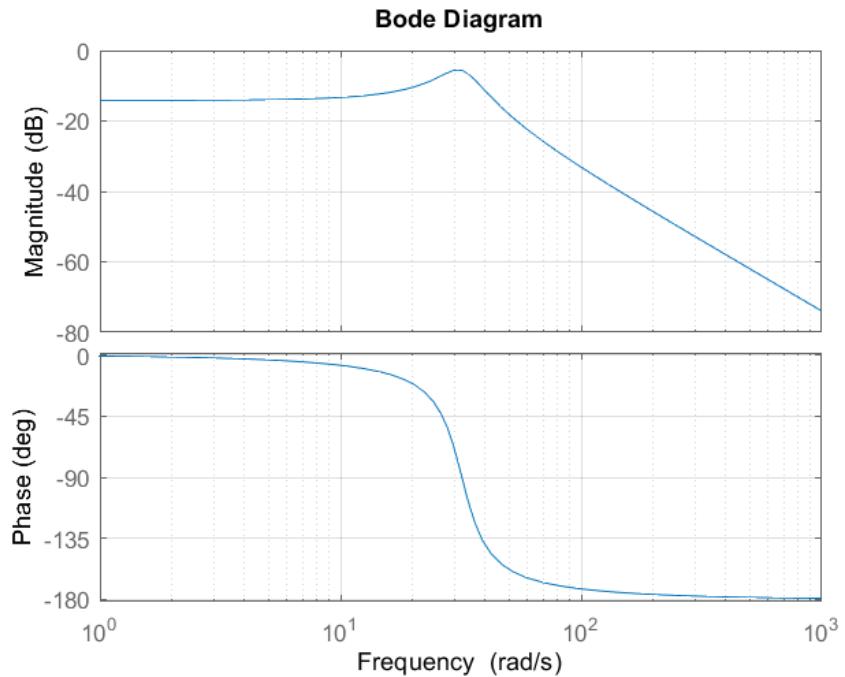
Transfer function:

$$= \frac{2}{s^2 + \cdot \cdot \cdot \xi \cdot \cdot \cdot} = \frac{i2}{s^2 + \cdot \cdot \cdot \cdot \cdot \xi \cdot \cdot \cdot}$$

$$i2 = \frac{1}{s_1 + s_2}$$
$$= \sqrt{\frac{1 \ 3 \ 2}{(s_2 + s_1)}}$$

$$\xi = \frac{1 \ 2 \ 3 + s_2}{\cdot \sqrt{1 \ 3 \ 2(s_1 + s_2)}}$$

Frequency response:



# Second order electrical circuits

Parameters:

$$\omega_c = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{RC} = \omega_c$$

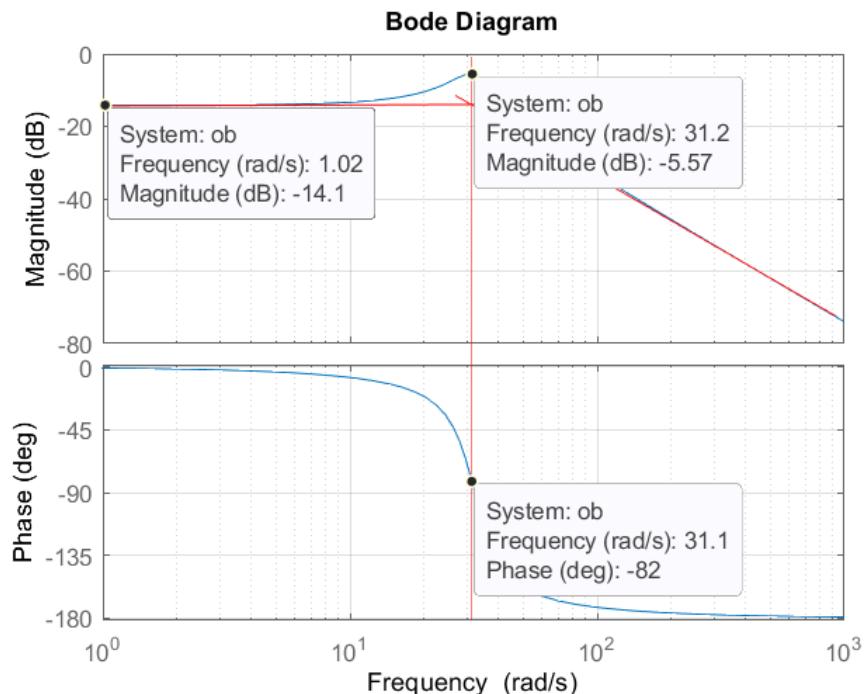
$$i_1 = -$$

$$i_2 = \frac{H_1/20}{R} =$$

$$i_2 = -$$

$$\frac{i_2}{\xi} = \frac{H_2/20}{R} =$$

Frequency response:



# Second order electrical circuits

Transfer function:

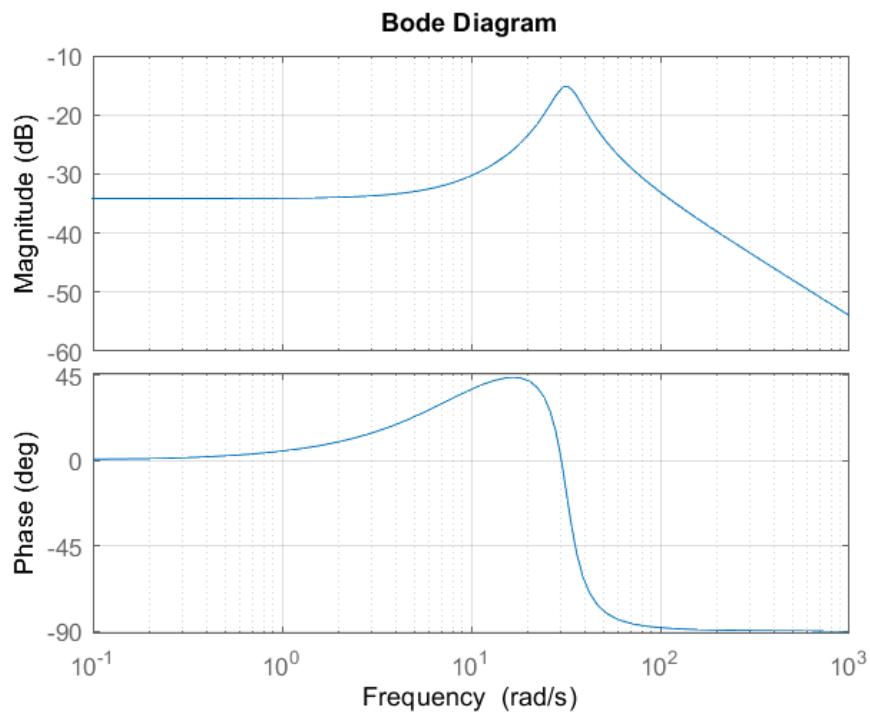
$$= \frac{u_3}{s^2 + \cdot \cdot \cdot \xi \cdot +} = \frac{u_3 ( f + )}{s^2 + 2 \cdot 2}$$

$$u_3 = \frac{2}{1 + 2}$$

$$= \sqrt{\frac{1 \ 3 \ 2}{(2 + 1)}} \quad f = \frac{2}{2}$$

$$\xi = \frac{1 \ 2 \ 3 + 2}{\cdot \sqrt{1 \ 3 \ 2 (1 + 2)}}$$

Frequency response:



# Second order electrical circuits

Parameters:

$$\omega_f =$$



$$f = \frac{1}{\omega_f} =$$

$$\omega_c =$$



$$= \frac{1}{\omega_c} =$$

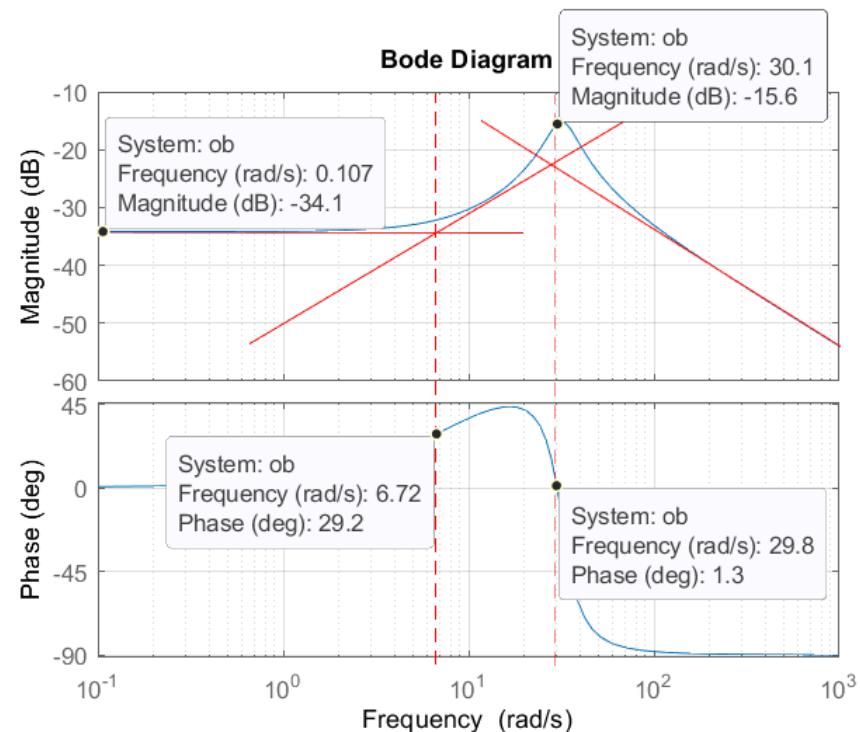
$$_1 = -$$

$$u_3 = \frac{H_1}{20} =$$

$$_2 = -$$

$$u_3 \sqrt{\left(\frac{-f}{\xi}\right)^2 + 1} = \frac{H_2}{20} =$$

Frequency response:



# Second order electrical circuits

State-space model:

$$\begin{cases} \dot{x}_1 = -\frac{2}{2}x_1 + \frac{2}{2}x_2 \\ \dot{x}_2 = -\frac{3}{3}x_2 - \frac{1}{1}x_3 + \frac{1}{3}x_3 \\ L_2 = -\frac{2}{2}x_2 + \frac{1}{3}x_3 \end{cases}$$

Parameters:

$$\sqrt{\frac{1}{2} + \frac{3}{1}} =$$

$$\frac{2}{2} = f$$

$$\frac{1}{2} + \frac{3}{1} = \xi$$

$$\frac{1}{1} + \frac{2}{2} = i_2$$

$$\frac{2}{1} + \frac{2}{2} = u_3$$