ITMO UNIVERSITY

Mathematic modelling of electrical systems dynamic

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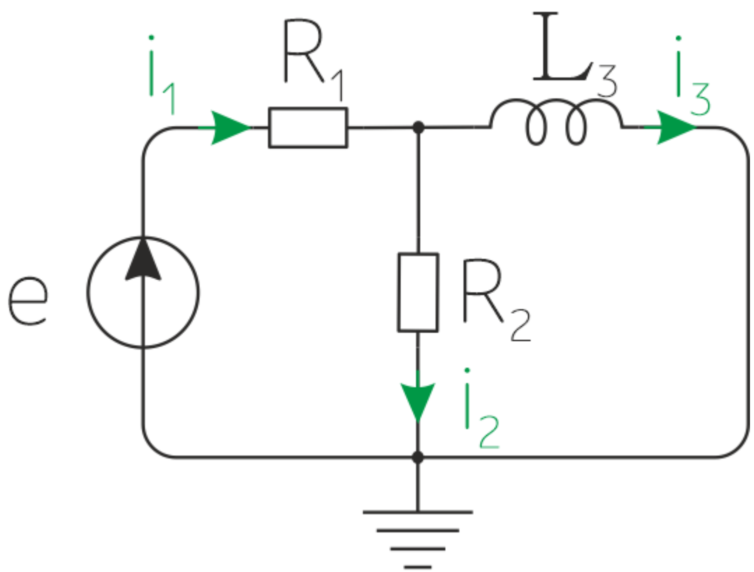
- Mathematic modelling of electrical system with inductance
- Mathematic modelling of electrical system with capacitance
- Mathematic modelling of second order electrical system

Mathematatic modelling of electrical system with inductance

Faculty of Control Systems and Industrial Robotics , ITMO University , Saint Petersburg

Electrical circuits with inductance

Get the state-space model



Electrical circuits with inductance

1. Component equations:

- Dissipative elements.

$${}_1 = {}_1 \cdot {}_1$$

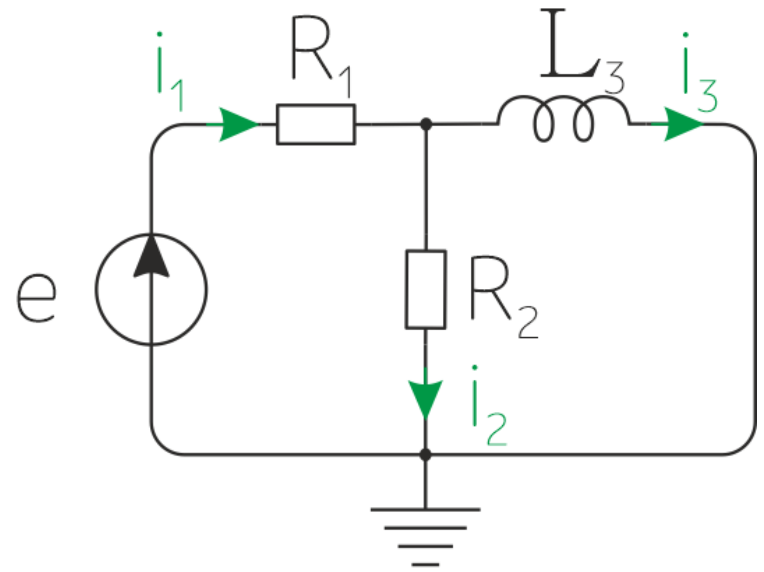
$${}_2 = {}_2 \cdot {}_2$$

- Elastic elements.

$${}_3 = {}_3 \cdot \frac{{}_3}{}$$

- Inertial elements.

$$\frac{}{}$$



Electrical circuits with inductance

2. Topological equations:

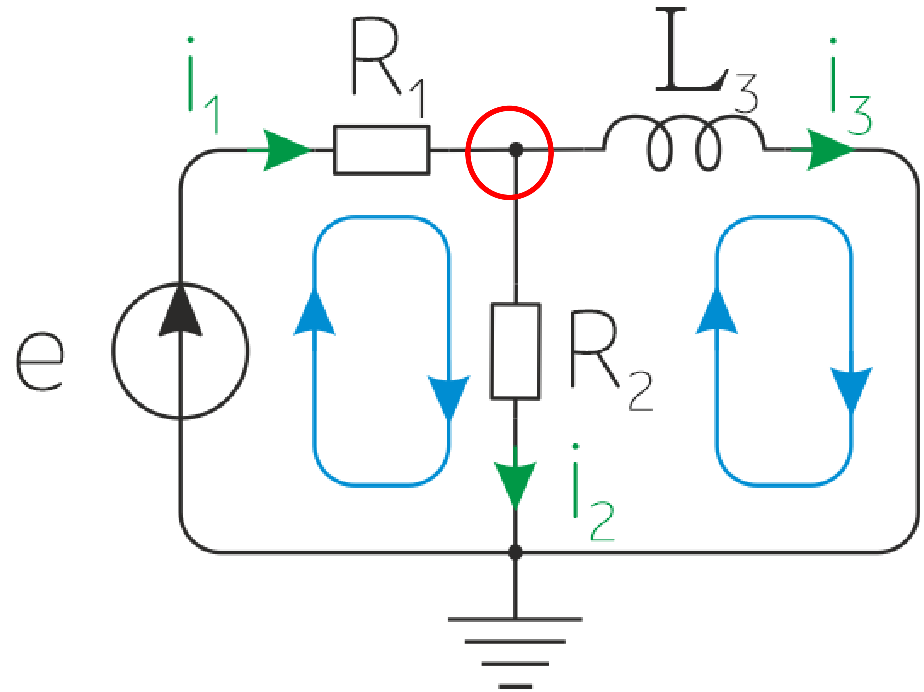
- Equilibrium equations.

$${}_1 + {}_2 =$$

$$- {}_2 + {}_3 =$$

- Condition of continuity of flow type coordinates.

$${}_1 - {}_2 - {}_3 =$$



Electrical circuits with inductance

3. State-space model:

$$\frac{}{} = \quad + $$

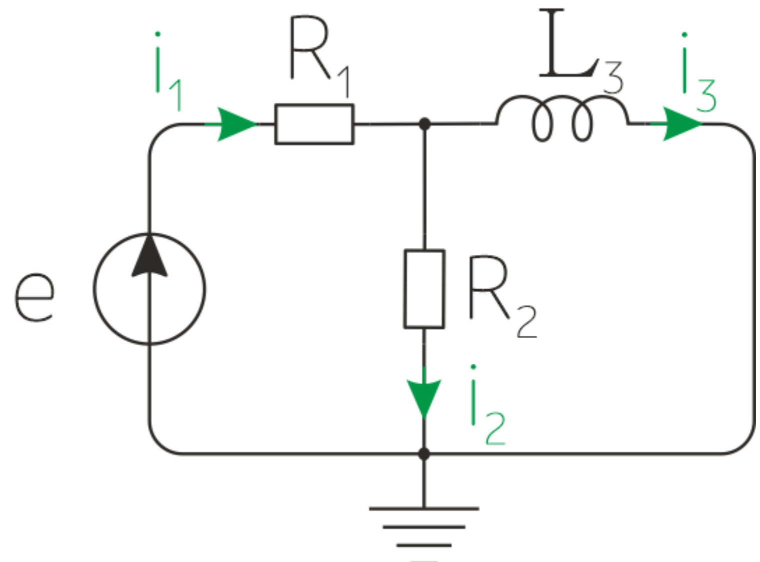
$$= \quad +$$

State vector x:

$$= \quad {}_3$$

Output vector y:

$$= \quad {}_3$$



Electrical circuits with inductance

3.1 Select the required number of equations for the model:

Three component equations.

$_1 = {}_1 \cdot {}_1$

$_2 = {}_2 \cdot {}_2$

$_3 = {}_3 \cdot \frac{{}_3}{}$

Three topological equations.

$_1 + {}_2 =$

$- {}_2 + {}_3 =$

$_1 - {}_2 - {}_3 =$

Six circuit variables.

$_1 \quad {}_2 \quad {}_3$

$_1 \quad {}_2 \quad {}_3$

One state variable.

$_3$

Electrical circuits with inductance

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$- {}_2 + {}_3 =$$

$${}_2 = {}_2 \cdot {}_2$$

$${}_2 = {}_1 - {}_3$$

$${}_1 = \frac{{}_1}{{}_1}$$

$${}_1 = \quad - \quad {}_2$$

$${}_3 = {}_3 \cdot \frac{{}_3}{\quad}$$

Electrical circuits with inductance

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$-\ {}_2 + \ {}_3 =$$

$${}_1 = \frac{{}_1}{{}_2 + \ {}_1} + \frac{{}_2 \ {}_1}{{}_2 + \ {}_1}\ {}_3$$

$${}_1 = \frac{}{{}_2 + \ {}_1} + \frac{{}_2}{{}_2 + \ {}_1}\ {}_3$$

$${}_2 = \frac{}{{}_2 + \ {}_1} - \frac{{}_1}{{}_2 + \ {}_1}\ {}_3$$

$${}_2 = \frac{{}_2}{{}_2 + \ {}_1} - \frac{{}_1 \ {}_2}{{}_2 + \ {}_1}\ {}_3$$

$${}_3 = \ {}_3 \cdot \frac{{}_3}{}$$

Electrical circuits with inductance

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$-\frac{{}_2}{{}_2+{}_1}+\frac{{}_1\ {}_2}{{}_2+{}_1}{}_3+{}_3\cdot\frac{{}_3}{\ }=$$

$$\frac{{}_3}{\ }=-\frac{{}_1\ {}_2}{{}_3\left({}_2+{}_1\right)}{}_3+\frac{{}_2}{{}_3\left({}_2+{}_1\right)}$$

Electrical circuits with inductance

3.3 Express the output vector in terms of the state vector and substitute input vector:

$$\frac{{}_3}{} = -\frac{{}_1\;{}_2}{{}_3\left(\;{}_2 + \;{}_1\right)}\;{}_3 + \frac{{}_2}{{}_3\left(\;{}_2 + \;{}_1\right)}$$

$${}_3 = \;{}_3\frac{{}_3}{} = -\frac{{}_1\;{}_2}{\left(\;{}_2 + \;{}_1\right)}\;{}_3 + \frac{{}_2}{\left(\;{}_2 + \;{}_1\right)}$$

Electrical circuits with inductance

3.4 Write the state space model:

$$\frac{{}_3}{} = -\frac{{}_1\ {}_2}{{}_3\left(\ {}_2 + \ {}_1\right)}\ {}_3 + \frac{{}_2}{{}_3\left(\ {}_2 + \ {}_1\right)}$$

$${}_3 = -\frac{{}_1\ {}_2}{\left(\ {}_2 + \ {}_1\right)}\ {}_3 + \frac{{}_2}{\left(\ {}_2 + \ {}_1\right)}$$

Electrical circuits with inductance

Algorithm for getting the state-space model:

1. Write component equations.
2. Write topological equations.
3. State-space model.

   3.1 Select the required number of equations for the model.

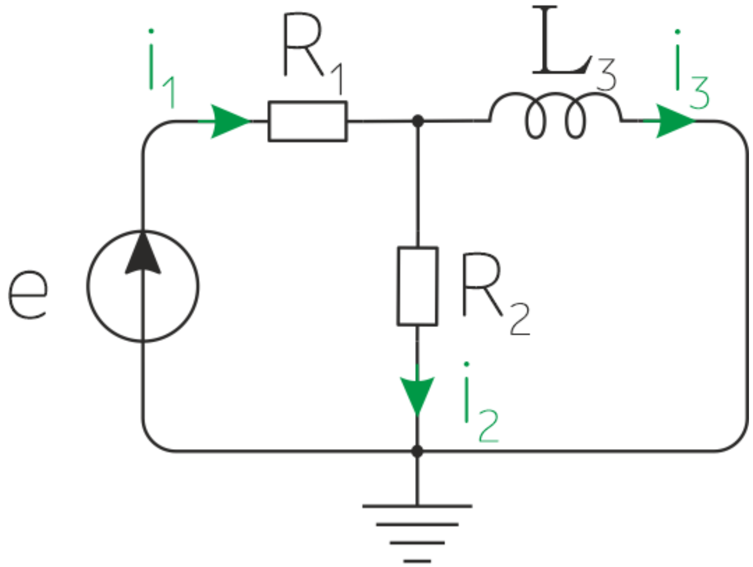
   3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation.

   3.3 Express the output vector in terms of the state vector and substitute input vector.

   3.4 Write the state space model.

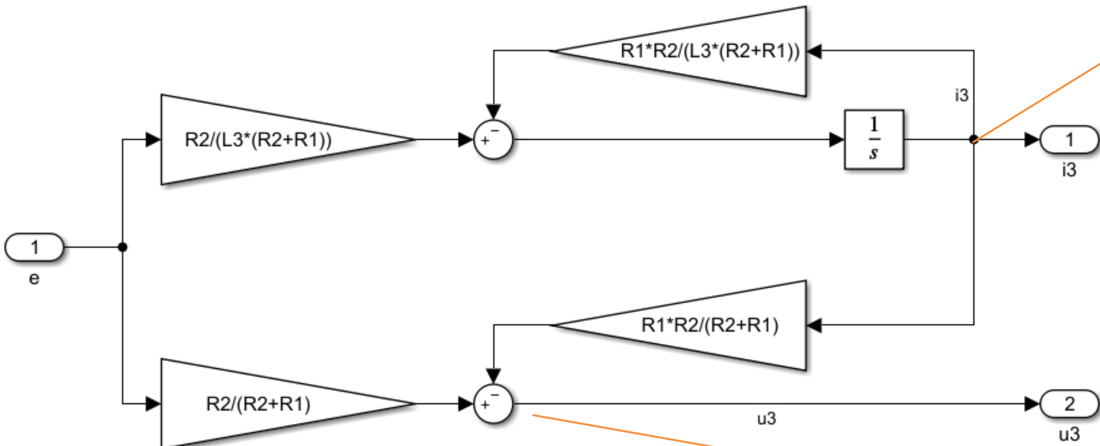
Electrical circuits with inductance

Modelling in Matlab
(zero initial conditions)



Electrical circuits with inductance

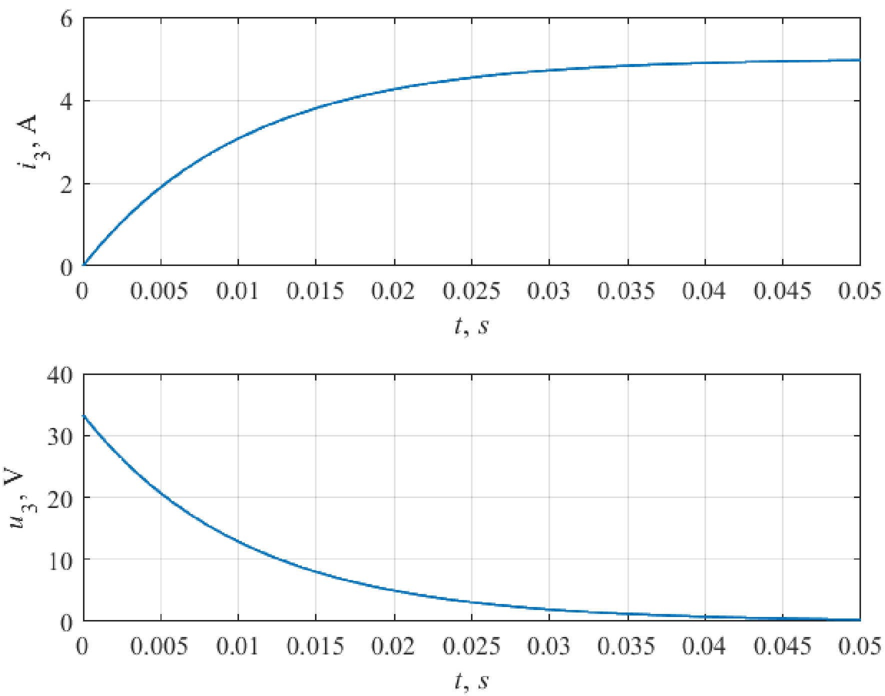
$$\frac{di_3}{dt} = -\frac{R_1 R_2}{L_3 (R_2 + R_1)} i_3 + \frac{R_2}{L_3 (R_2 + R_1)} e$$



$$u_3 = -\frac{R_1 R_2}{(R_2 + R_1)} i_3 + \frac{R_2}{(R_2 + R_1)} e$$

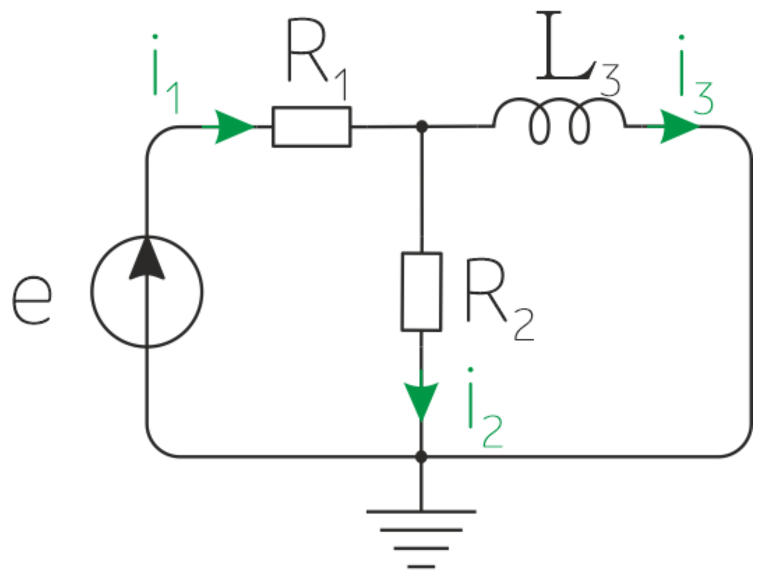
Electrical circuits with inductance

Graphs of the transients in the circuit
(zero initial condition)



Electrical circuits with inductance

Get the transfer function



Electrical circuits with inductance

4. Transfer function:

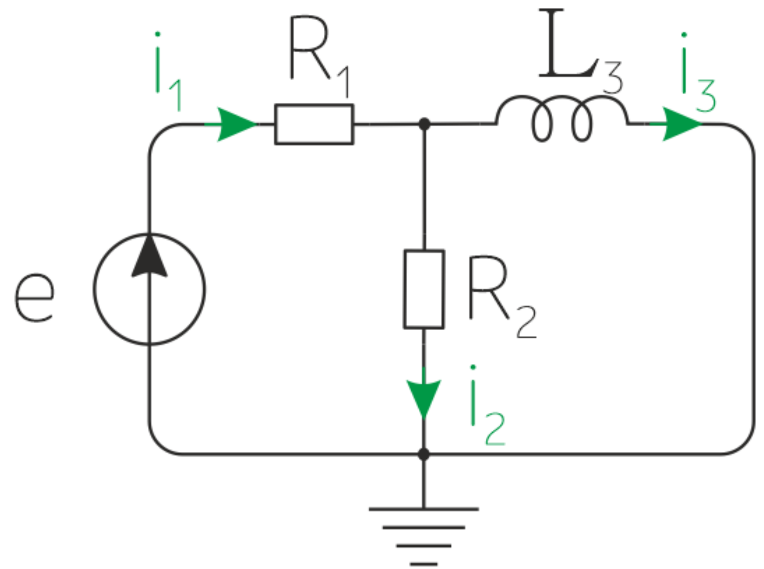
$= \overline{}$

Input:

$=$

Output:

$= \quad _3$



4.1 Perform Laplace transform for the state-space model (zero initial conditions):

$$\frac{{}_{3}}{} = -\frac{{}_{1}\;{}_{2}}{{}_{3}\left({}_{2}+{}_{1}\right)}\,{}_{3} + \frac{{}_{2}}{{}_{3}\left({}_{2}+{}_{1}\right)}$$

$${}_{3} = -\frac{{}_{1}\;{}_{2}}{\left({}_{2}+{}_{1}\right)}\,{}_{3} + \frac{{}_{2}}{\left({}_{2}+{}_{1}\right)}$$

$${}_{3} = -\frac{{}_{1}\;{}_{2}}{{}_{3}\left({}_{2}+{}_{1}\right)}\,{}_{3} + \frac{{}_{2}}{{}_{3}\left({}_{2}+{}_{1}\right)}$$

$${}_{3} = -\frac{{}_{1}\;{}_{2}}{\left({}_{2}+{}_{1}\right)}\,{}_{3} + \frac{{}_{2}}{\left({}_{2}+{}_{1}\right)}$$

4.2 Express the output in terms of input:

$$ {}_{3} = \frac{{}_{2}}{{}_{3}\left({}_{2} + {}_{1} \right) + {}_{1}\ {}_{2}} $$

$$ {}_{3} = \frac{{}_{3}\ {}_{2}}{{}_{3}\left({}_{2} + {}_{1} \right) + {}_{1}\ {}_{2}} $$

4.3 Write the transfer function:

$$= \frac{_3}{\quad} = \frac{_3 \quad _2}{_3\left(\; _2 + \; _1 \right) + \; _1 \; _2} = \frac{\quad}{\quad + \quad}$$

$$= \frac{_3}{_1}$$

$$= \; _3 \frac{\left(\; _2 + \; _1 \right)}{_1 \; _2}$$

Electrical circuits with inductance

Algorithm for getting the transfer function:

4. Transfer function.

   4.1 Perform Laplace transform for the state-space model (zero initial conditions).

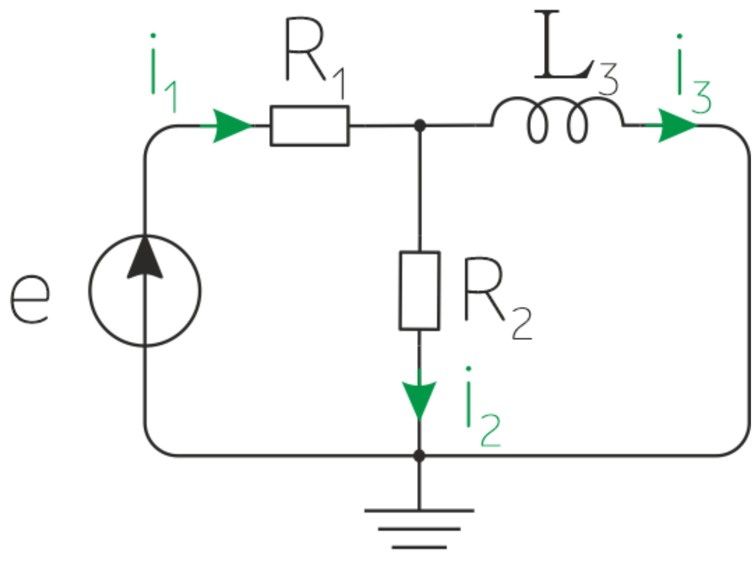
   4.2 Express the output in terms of input.

   4.3 Write the transfer function:

Attendance

Electrical circuits with inductance

Modelling in Matlab
(nonzero initial conditions)



Electrical circuits with inductance

State-space model:

$$\frac{{}_3}{} = -\frac{{}_1\;{}_2}{{}_3\left(\;{}_2 + \;{}_1\right)}\;{}_3 + \frac{{}_2}{{}_3\left(\;{}_2 + \;{}_1\right)}$$

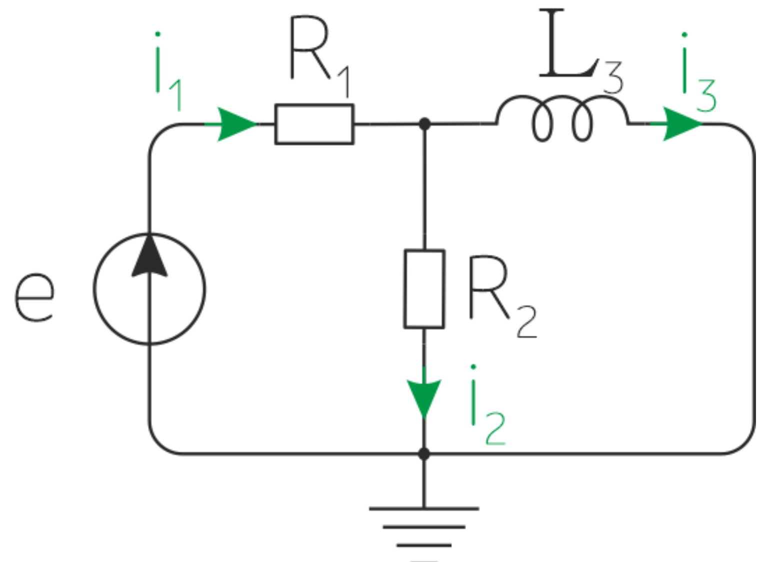
$${}_3 = -\frac{{}_1\;{}_2}{\left(\;{}_2 + \;{}_1\right)}\;{}_3 + \frac{{}_2}{\left(\;{}_2 + \;{}_1\right)}$$

Initial conditions:

$${}_2 =$$

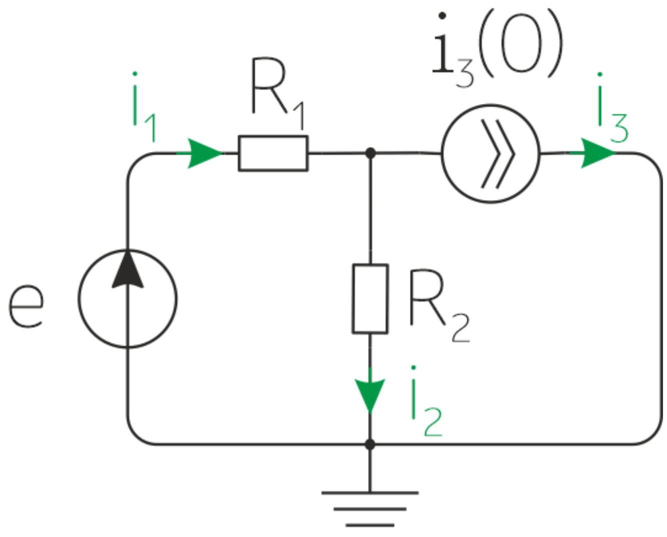
State space vector x:

$$= \quad {}_3$$

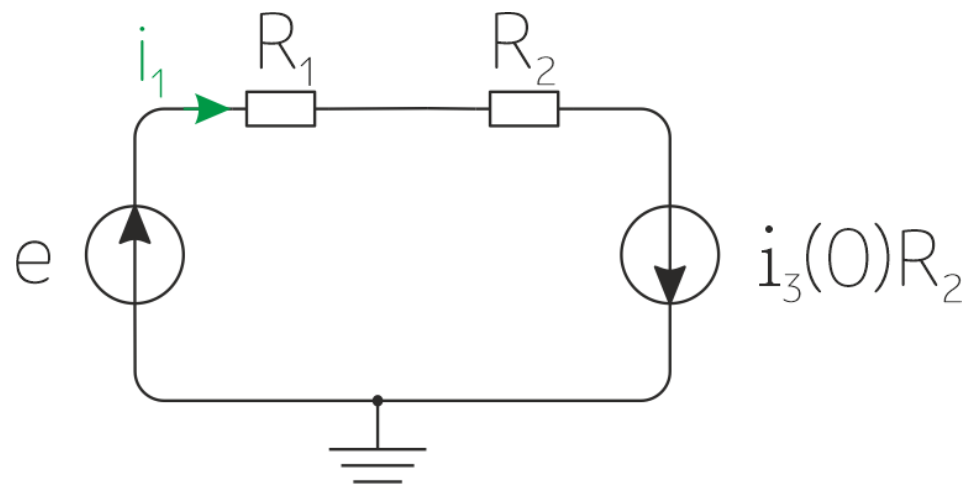


Electrical circuits with inductance

Circuit at time t = 0+:



Equivalent transform of current source:



$$ {}_1 = \frac{+ \, {}_3 \quad \cdot \quad {}_2}{{}_1 + \quad {}_2} $$

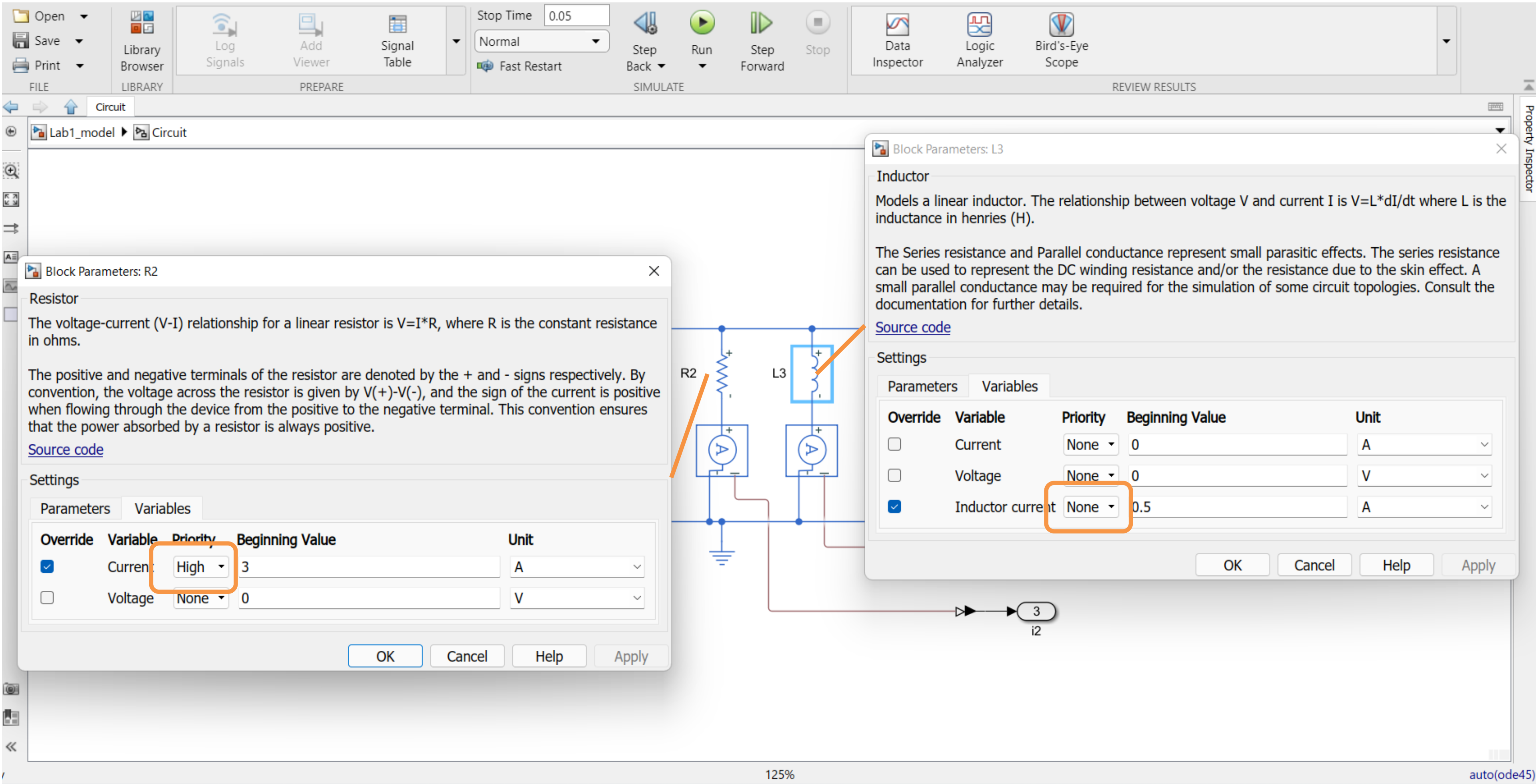
$$ {}_1 = {}_2 \quad + \, {}_3 $$

$$ {}_3 \quad \frac{{}_1}{{}_1 + \quad {}_2} = {}_2 \quad + \quad \frac{}{{}_1 + \quad {}_2} $$

$$ \boxed{{}_3 \quad = \frac{{}_1 + \quad {}_2}{{}_1} \, {}_2 \quad + \quad \frac{}{{}_1}} $$

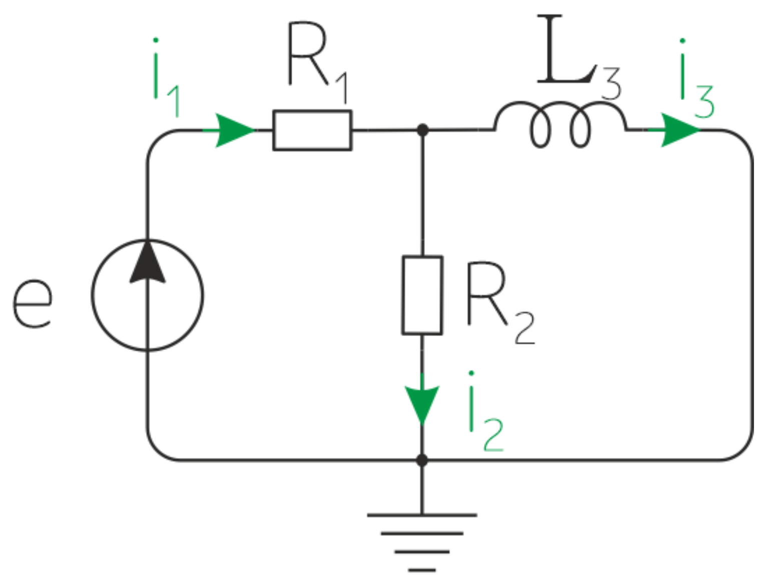
Electrical circuits with inductance

Nonzero initial conditions in Matlab



Electrical circuits with inductance

Parametric identification using transient response



Electrical circuits with inductance

Transfer function:

$$= \frac{{}_{3}}{\quad} = \frac{{}_{i}}{\quad + \quad}$$

$$_{i} = \frac{\quad}{_{1}}$$

$$= {}_{3}\frac{(\ _{2} + \ _{1})}{_{1} \ _{2}}$$

Parameters:

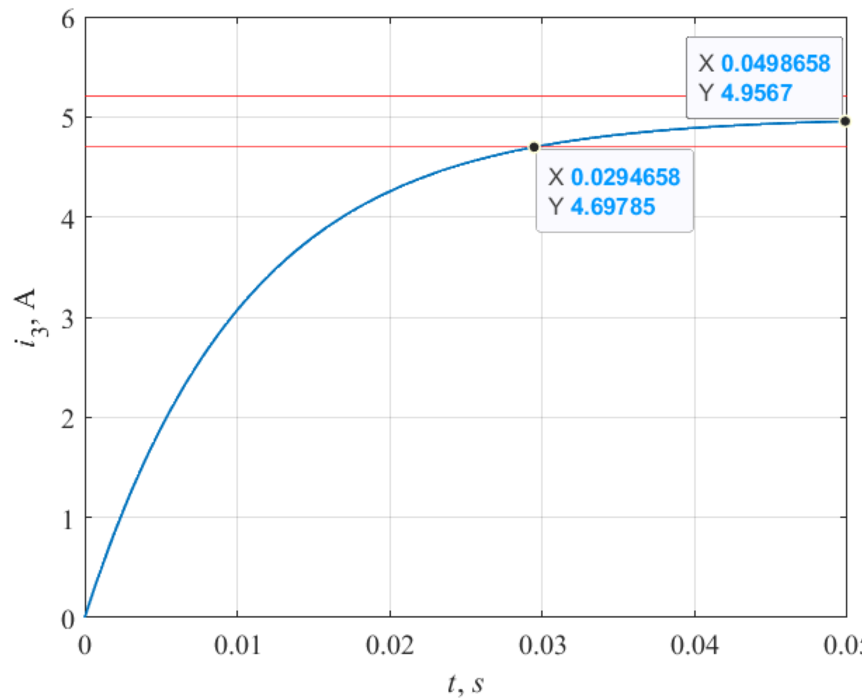
$$_{i} \cdot \quad =$$

$$\cdot \quad =$$

$$\frac{\quad}{_{1}} =$$

$${}_{3}\frac{(\ _{2} + \ _{1})}{_{1} \ _{2}} =$$

Transient response:



Electrical circuits with inductance

Transfer function:

$$= \frac{3}{\quad} = \frac{u}{\quad + \quad}$$

$$_u = \frac{3}{1}$$

$$= {}_3 \frac{(\ {}_2 + \ {}_1)}{{}_1\ {}_2}$$

Parameters:

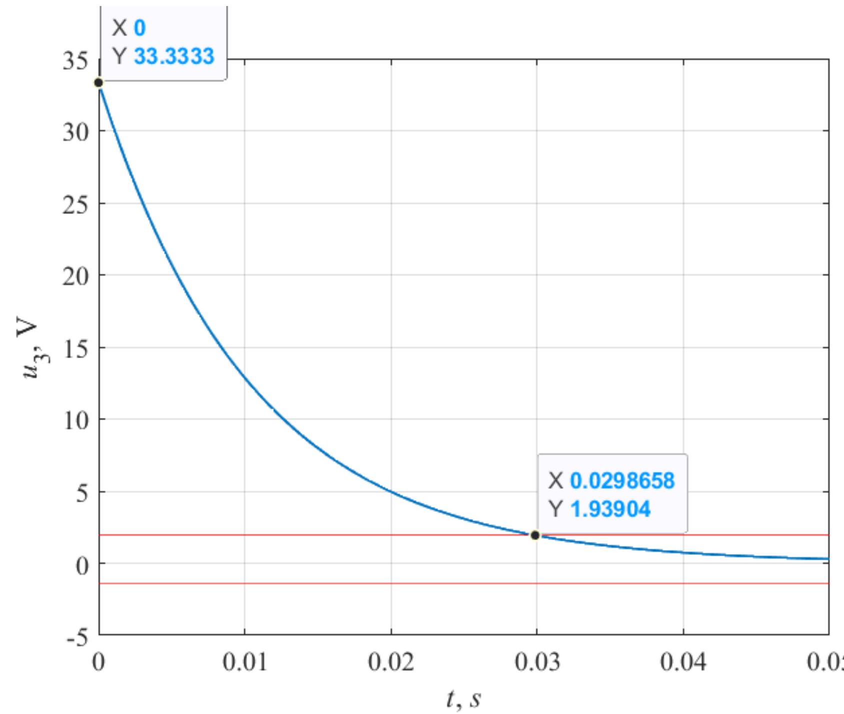
$$\frac{u}{\quad} \cdot \quad =$$

$$\cdot \quad =$$

$$\frac{\cdot \ {}_2}{{}_2 + \ {}_1} =$$

$${}_3 \frac{(\ {}_2 + \ {}_1)}{{}_1\ {}_2} =$$

Transient response:



Electrical circuits with inductance

State-space model:

$$\frac{\ {}_3}{\ } = -\frac{\ {}_1\ {}_2}{\ {}_3(\ {}_2 + \ {}_1)}\ {}_3 + \frac{\ {}_2}{\ {}_3(\ {}_2 + \ {}_1)}$$

$$\ {}_3 = -\frac{\ {}_1\ {}_2}{(\ {}_2 + \ {}_1)}\ {}_3 + \frac{\ {}_2}{(\ {}_2 + \ {}_1)}$$

$$\frac{\ {}_3}{\ } = -\frac{\ }{\ }\ {}_3 + \frac{\ {}_i}{\ }$$

$$\ {}_3 = -\frac{\ {}_u}{\ {}_i}\ {}_3 + \frac{\ {}_u}{\ }$$

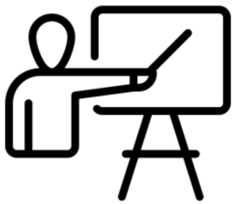
Parameters:

$$\frac{\ }{\ {}_1} =$$

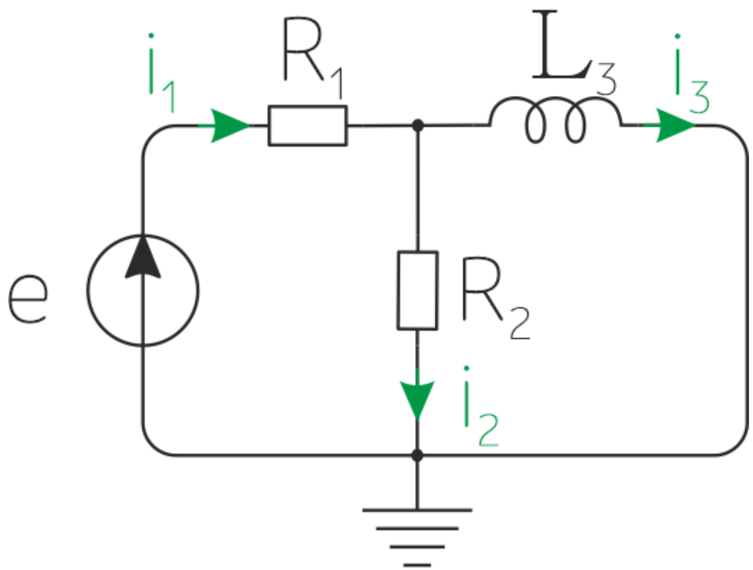
$$\frac{\dot{}\ \ {}_2}{\ {}_2 + \ {}_1} =$$

$$\ {}_3\frac{(\ {}_2 + \ {}_1)}{\ {}_1\ {}_2} =$$

Electrical circuits with inductance



Parametric identification using frequency response



Electrical circuits with inductance

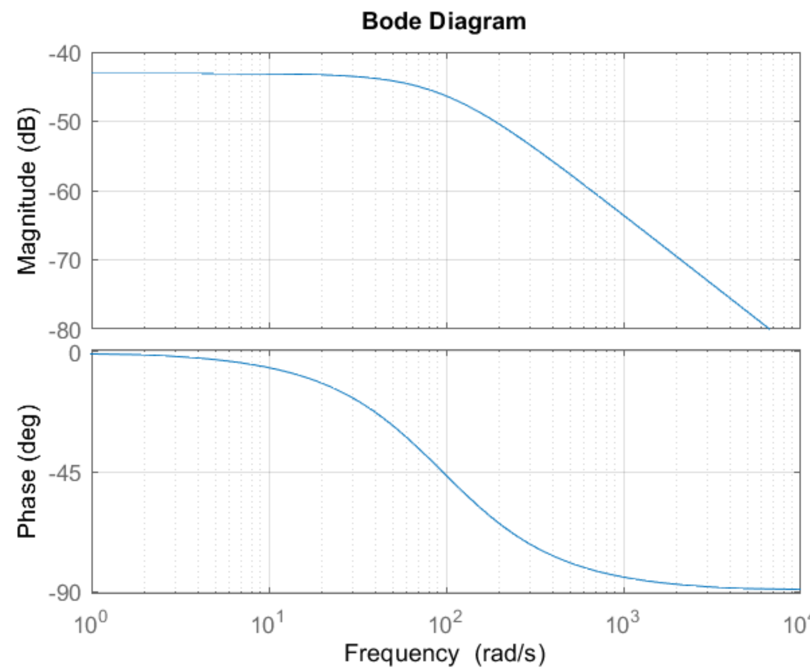
Transfer function:

$$= \frac{{}_3}{\quad} = \frac{{}_i}{\quad + \quad}$$

$${}_i = \frac{\quad}{{}_1}$$

$$= {}_3 \frac{\left({}_2 + {}_1 \right)}{{}_1 \; {}_2}$$

Frequency response:



Electrical circuits with inductance

Parameters:

$\omega_c = \quad /$

$= \frac{}{\omega_c} =$

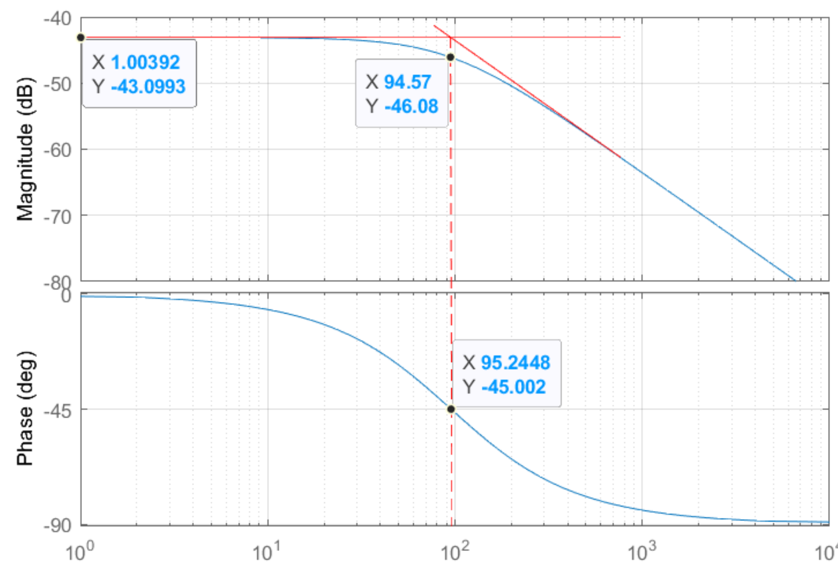
$= -$

$_i = {}^{H}/_{20} =$

$\frac{}{_1} =$

$_3 \frac{(\ _2 + \ _1)}{_1 \ _2} =$

Frequency response:



Electrical circuits with inductance

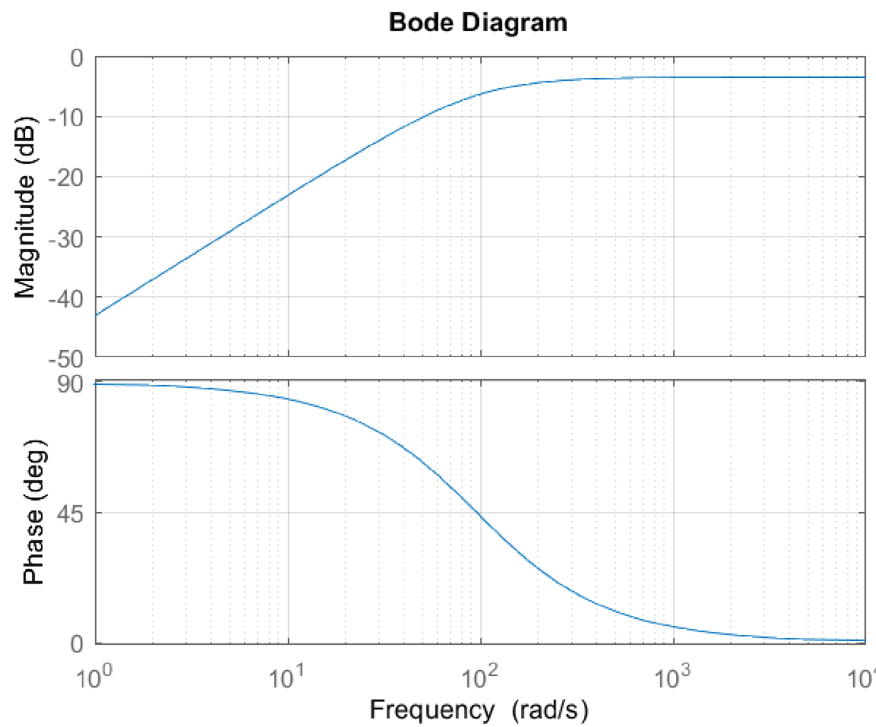
Transfer function:

$$= \frac{{}_3}{\quad} = \frac{{}_u}{\quad + \quad}$$

$${}_u = \frac{{}_3}{{}_1}$$

$$= {}_3 \frac{\left(\ {}_2 + \ {}_1\right)}{{}_1 \ {}_2}$$

Frequency response:



Electrical circuits with inductance

Parameters:

$\omega_c =$ /

$= \frac{}{\omega_c} =$

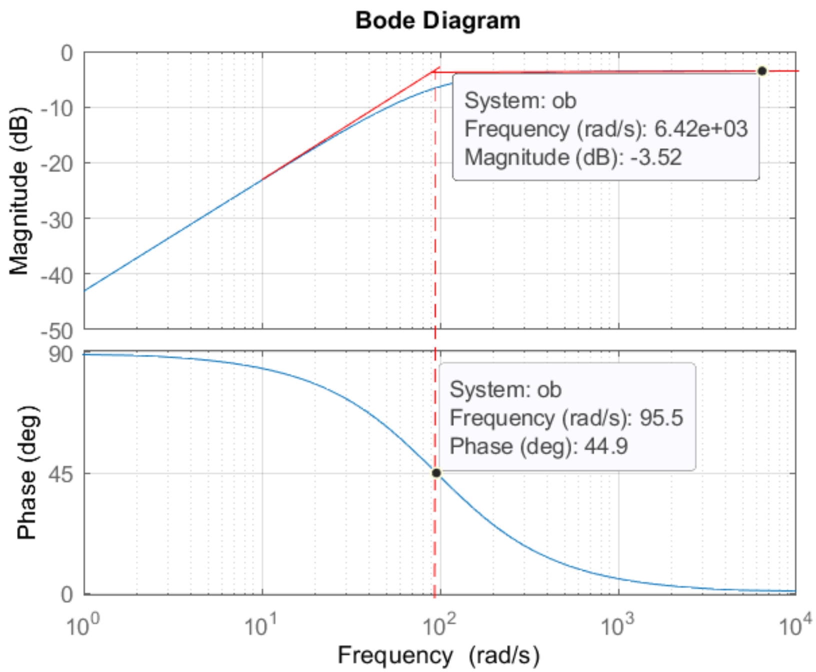
$= -$

$\frac{u}{} = \ ^{H}/_{20} =$

$\frac{2}{2 + 1} =$

$3 \frac{(2 + 1)}{1 \ 2} =$

Frequency response:



Electrical circuits with inductance

State-space model:

$$\frac{{}_3}{} = -\frac{{}_1\ {}_2}{{}_3\left({}_2+{}_1\right)}{}_3+\frac{{}_2}{{}_3\left({}_2+{}_1\right)}$$

$${}_3 = -\frac{{}_1\ {}_2}{\left({}_2+{}_1\right)}{}_3+\frac{{}_2}{\left({}_2+{}_1\right)}$$

$$\frac{{}_3}{} = -\frac{}{}{}_3+\frac{{}_i}{}$$

$${}_3 = -\frac{{}_u}{{}_i}{}_3+\frac{{}_u}{}$$

Parameters:

$$\frac{}{{}_1} =$$

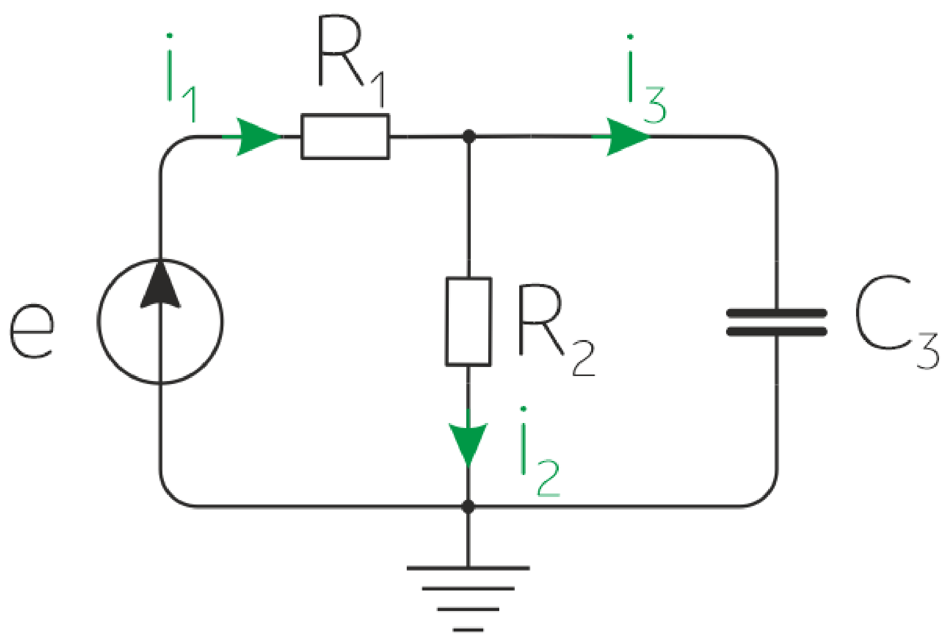
$$\frac{{}_2}{{}_2+{}_1} =$$

$${}_3\frac{\left({}_2+{}_1\right)}{{}_1\ {}_2} =$$

Mathematatic modelling of electrical system with capacitance

Electrical circuits with capacitance

Get the state-space model



Electrical circuits with capacitance

1. Component equations:

- Dissipative elements.

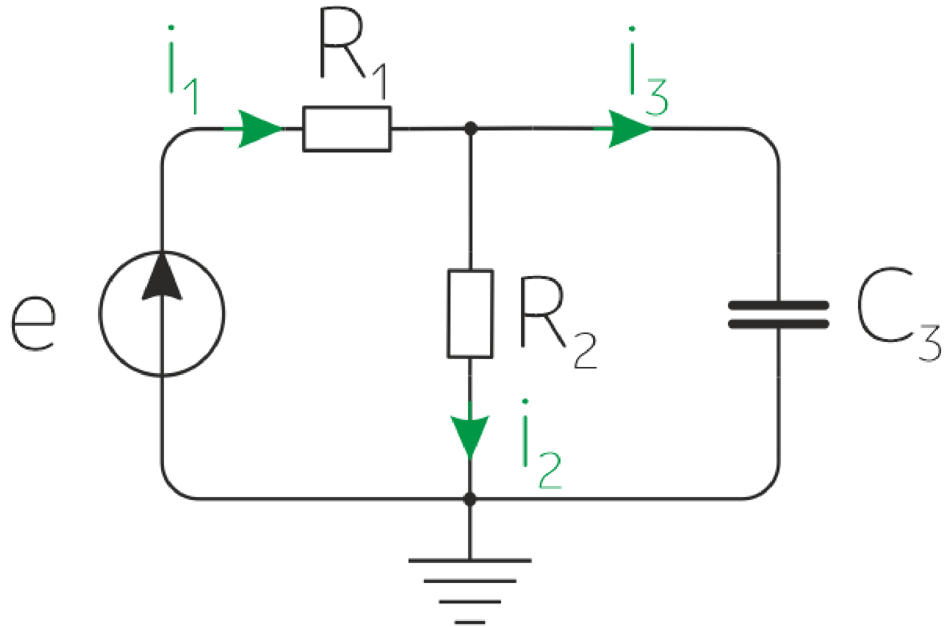
$$_{1} = {}_{1} \cdot {}_{1}$$

$$_{2} = {}_{2} \cdot {}_{2}$$

- Elastic elements.

- Inertial elements.

$$_{3} = {}_{3}^{-1} \int {}_{3}$$



Electrical circuits with capacitance

2. Topological equations:

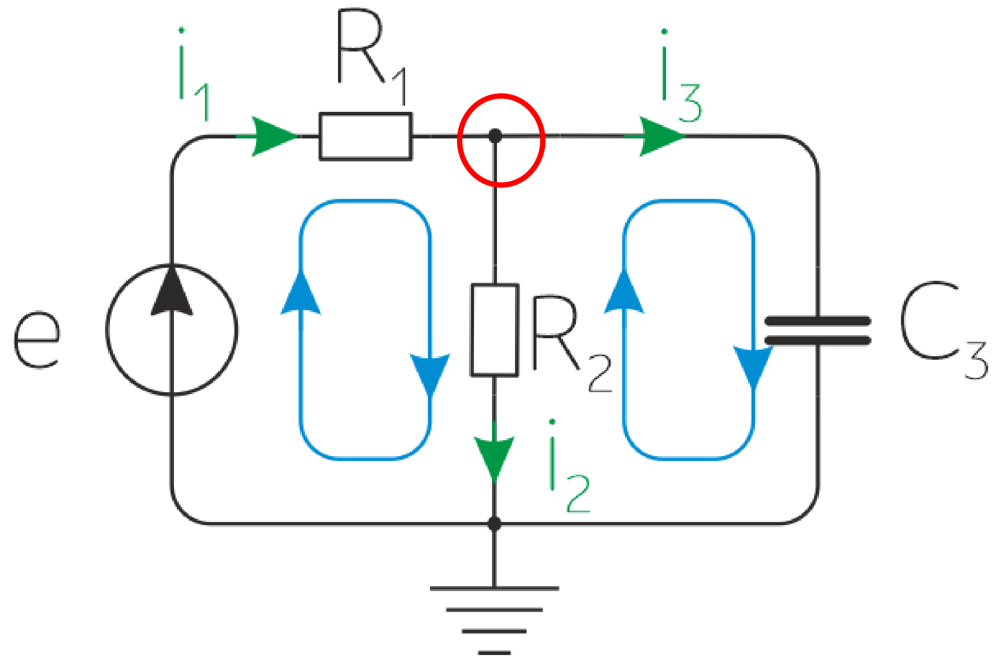
- Equilibrium equations.

$$ {}_{1} + {}_{2} = $$

$$ - {}_{2} + {}_{3} = $$

- Condition of continuity of flow type coordinates.

$$ {}_{1} - {}_{2} - {}_{3} = $$



Electrical circuits with capacitance

3. State-space model:

— = +

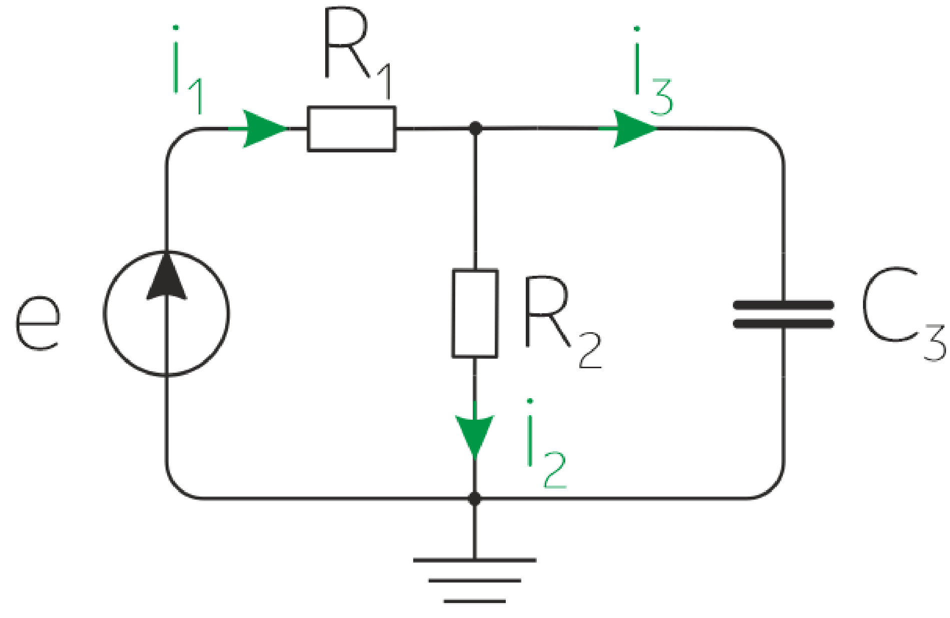
= +

State vector x:

= 3

Output vector y:

= 3



Electrical circuits with capacitance

3.1 Select the required number of equations for the model:

Three component equations.

$${}_{1} = {}_{1} \cdot {}_{1}$$

$${}_{2} = {}_{2} \cdot {}_{2}$$

$${}_{3} = {}_{3}^{-1} \int {}_{3}$$

Three topological equations.

$${}_{1} + {}_{2} =$$

$$- {}_{2} + {}_{3} =$$

$${}_{1} - {}_{2} - {}_{3} =$$

Six circuit variables.

${}_{1}$ ${}_{2}$ ${}_{3}$

${}_{1}$ ${}_{2}$ ${}_{3}$

One state variable.

${}_{3}$

Electrical circuits with capacitance

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$- {}_2 + {}_3 =$$

$${}_2 = {}_2 \cdot {}_2$$

$${}_2 = {}_1 - {}_3$$

$${}_1 = \frac{1}{1}$$

$${}_1 = - {}_2$$

$${}_3 = {}_3^{-1} \int {}_3$$

Electrical circuits with capacitance

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$-\ _2 + \ _3 =$$

$$_2 = \frac{}{_2 + \ _1} - \frac{_1 \ _3}{_2 + \ _1}\frac{_3}{}$$

$$_1 = \frac{}{_2 + \ _1} + \frac{_2 \ _3}{_2 + \ _1}\frac{_3}{}$$

$$_1 = \frac{_1}{_2 + \ _1} + \frac{_1 \ _2 \ _3}{_2 + \ _1}\frac{_3}{}$$

$$\boxed{_2 = \frac{_2}{_2 + \ _1} - \frac{_1 \ _2 \ _3}{_2 + \ _1}\frac{_3}{}}$$

$$_3 = \ _3 \cdot \frac{_3}{}$$

Electrical circuits with capacitance

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$-\frac{{}_{2}}{{}_{2}+{}_{1}}+\frac{{}_{1}\;{}_{2}}{{}_{2}+{}_{1}}\cdot\frac{{}_{3}}{}+{}_{3}=$$

$$\frac{{}_{3}}{}=-\frac{{}_{1}+{}_{2}}{{}_{2}\;{}_{1}\;{}_{3}}\;{}_{3}+\frac{}{{}_{1}\;{}_{3}}$$

Electrical circuits with inductance

3.3 Express the output vector in terms of the state vector and substitute input vector:

$$\frac{{}_3}{\quad} = -\frac{{}_1 + {}_2}{{}_2\ {}_1\ {}_3}\ {}_3 + \frac{\quad}{{}_1\ {}_3}$$

$${}_3 = {}_3\frac{{}_3}{\quad} = -\frac{{}_1 + {}_2}{{}_2\ {}_1}\ {}_3 + \frac{\quad}{{}_1}$$

Electrical circuits with inductance

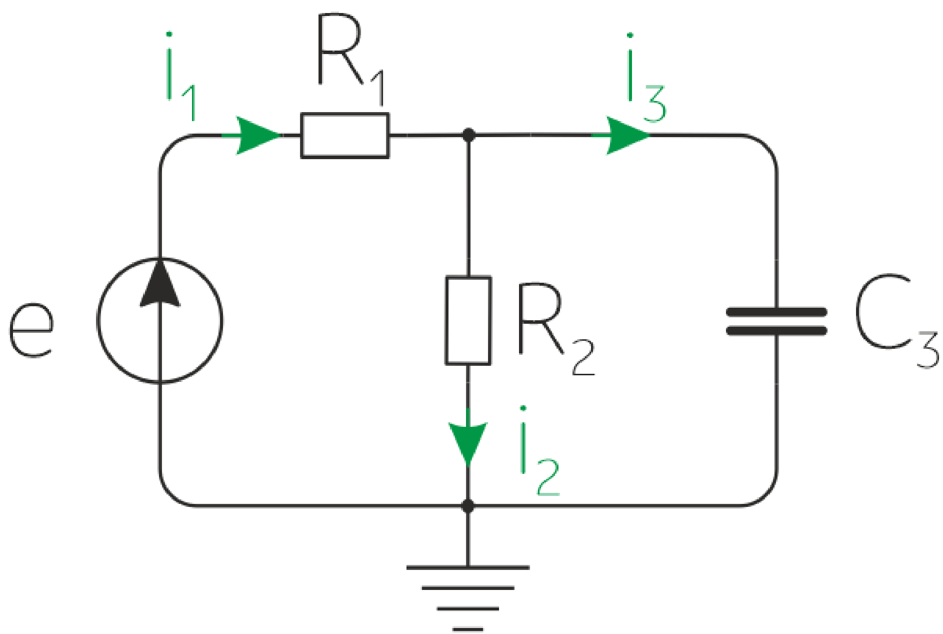
3.4 Write the state space model:

$$\frac{{}_{3}}{} = -\frac{{}_{1} + {}_{2}}{{}_{2}\;{}_{1}\;{}_{3}}\,{}_{3} + \frac{}{{}_{1}\;{}_{3}}$$

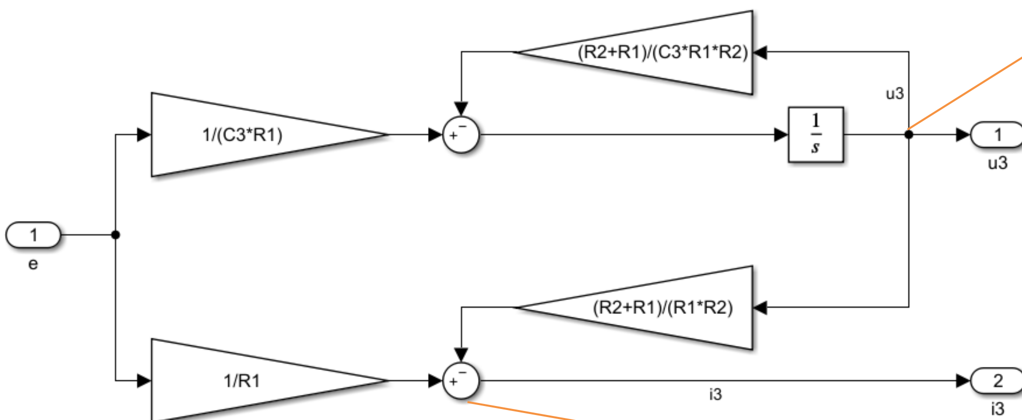
$${}_{3} = -\frac{{}_{1} + {}_{2}}{{}_{2}\;{}_{1}}\,{}_{3} + \frac{}{{}_{1}}$$

Electrical circuits with capacitance

Modelling in Matlab
(zero initial conditions)



Electrical circuits with capacitance

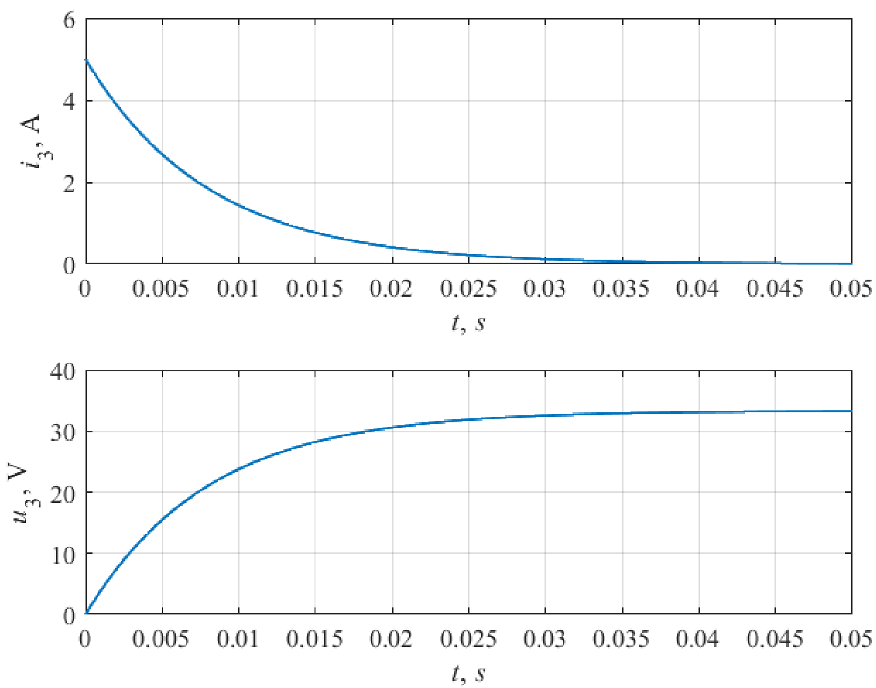


$$\frac{3}{} = -\frac{1 + 2}{2 \quad 1 \quad 3}\ 3 + \frac{}{1 \quad 3}$$

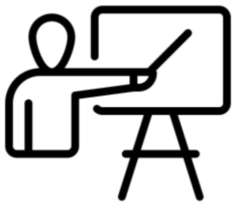
$$3 = -\frac{1 + 2}{2 \quad 1}\ 3 + \frac{}{1}$$

Electrical circuits with capacitance

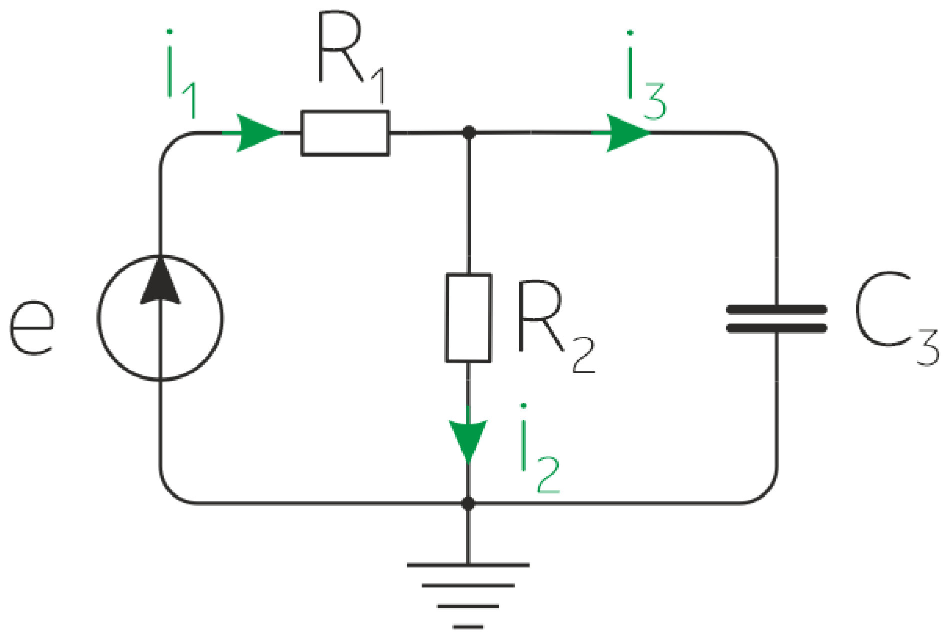
Graphs of the transients in the circuit
(zero initial condition)



Electrical circuits with capacitance



Get the transfer function



Electrical circuits with capacitance

4. Transfer function:

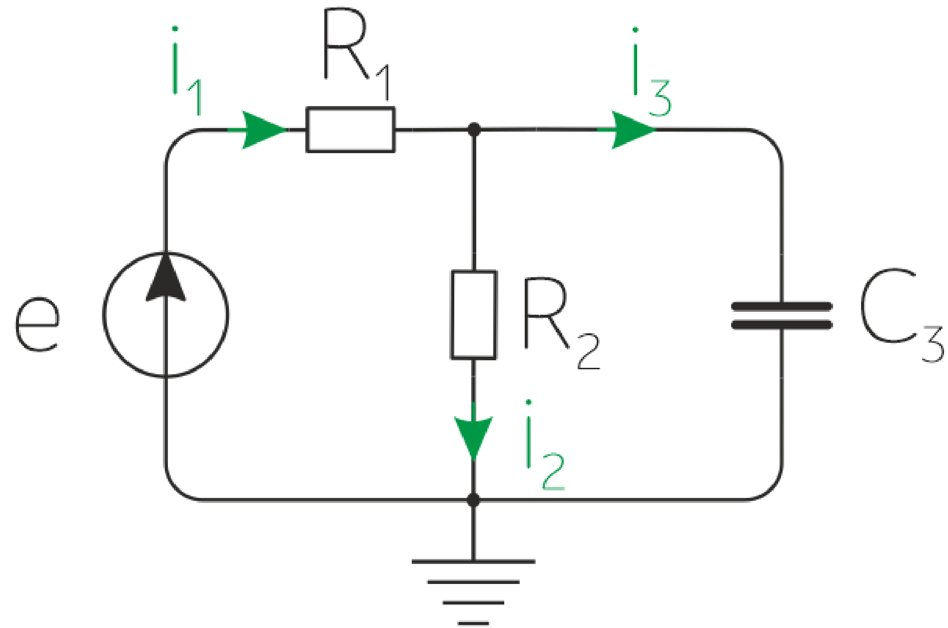
$$= \frac{\quad}{\quad}$$

Input:

$$=$$

Output:

$$= \;_{3}$$



Electrical circuits with capacitance

4.1 Perform Laplace transform for the state-space model (zero initial conditions):

$$\frac{\ _3}{} = -\frac{_1 + \ _2}{_2\ _1\ _3}\ _3 + \frac{}{_1\ _3}$$

$$_3 = -\frac{_1 + \ _2}{_2\ _1}\ _3 + \frac{}{_1}$$

$$_3 \quad = -\frac{_1 + \ _2}{_2\ _1\ _3}\ _3 \quad + \frac{}{_1\ _3}$$

$$_3 \quad = -\frac{_1 + \ _2}{_2\ _1}\ _3 \quad + \frac{}{_1}$$

Electrical circuits with capacitance

4.2 Express the output in terms of input:

$$ {}_{3} = \frac{{}_{2}}{{}_{2}\;{}_{1}\;{}_{3} + \;{}_{1} + \;{}_{2}} $$

$$ {}_{3} = \frac{{}_{3}\;{}_{2}}{{}_{2}\;{}_{1}\;{}_{3} + \;{}_{1} + \;{}_{2}} $$

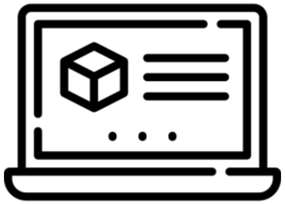
4.3 Write the transfer function:

$$= \frac{{}_3}{\quad} = \frac{{}_3 \quad {}_2}{{}_2 \quad {}_1 \quad {}_3 + \quad {}_1 + \quad {}_2} = \frac{\quad}{+}$$

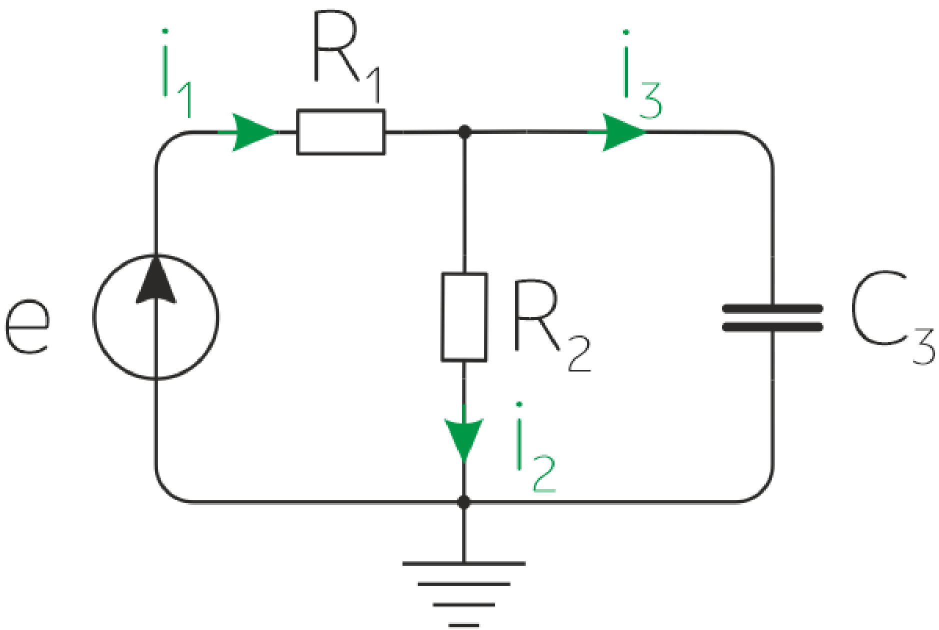
$$= \frac{{}_3 \quad {}_2}{{}_1 + \quad {}_2}$$

$$= \quad {}_3 \frac{{}_1 \quad {}_2}{\left(\quad {}_2 + \quad {}_1 \right)}$$

Electrical circuits with capacitance



Modelling in Matlab
(nonzero initial conditions)



Electrical circuits with capacitance

State-space model:

$$\frac{{}_3}{} = -\frac{{}_1 + {}_2}{{}_2\ {}_1\ {}_3}\,{}_3 + \frac{}{{}_1\ {}_3}$$

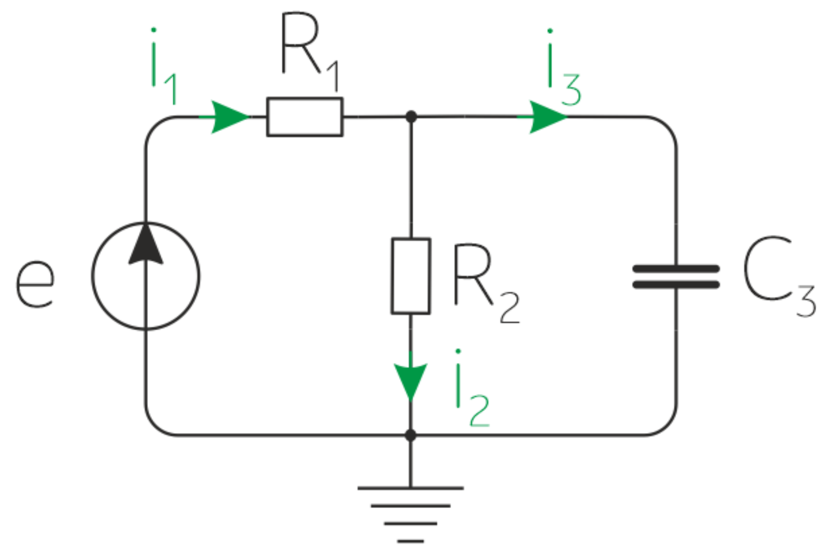
$$ {}_3 = -\frac{{}_1 + {}_2}{{}_2\ {}_1}\,{}_3 + \frac{}{{}_1}$$

Initial conditions:

$${}_1 =$$

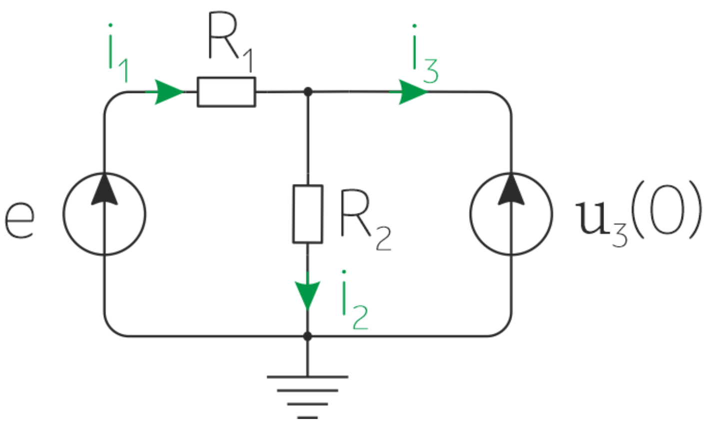
State space vector x:

$$= \quad {}_3$$



Electrical circuits with capacitance

Circuit at time t = 0+:

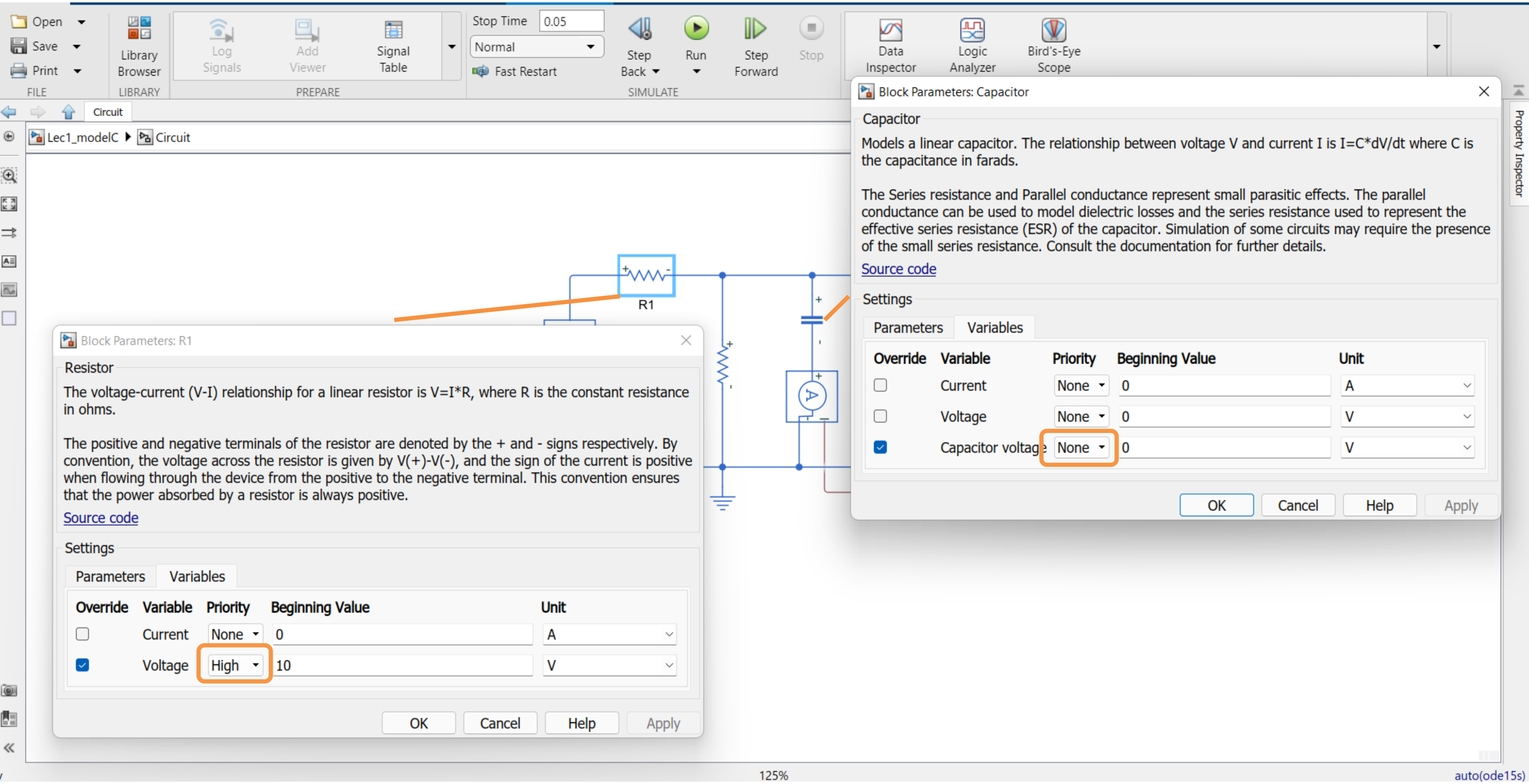


$$_{1} + _{3} =$$

$$\boxed{_{3} = \quad - _{1}}$$

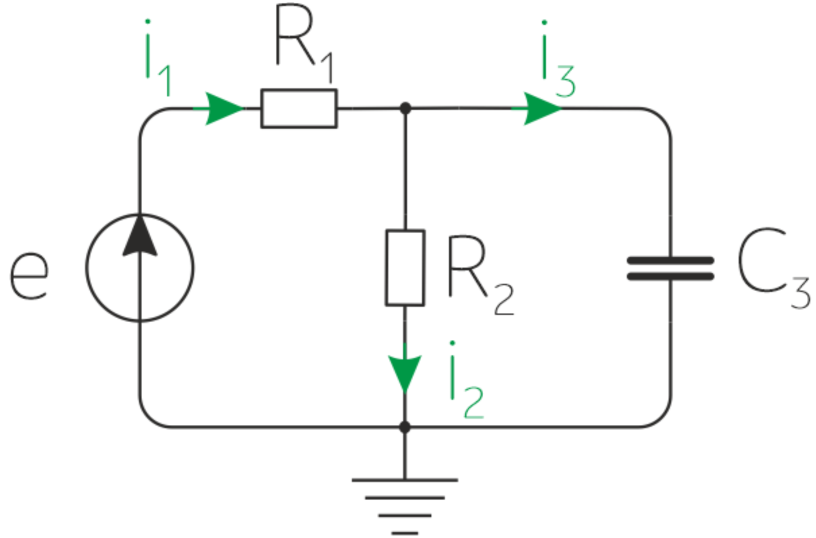
Electrical circuits with capacitance

Nonzero initial conditions in Matlab



Electrical circuits with capacitance

Parametric identification using transient response



Electrical circuits with capacitance

Transfer function:

$$= \frac{{}_{3}}{\quad} = \frac{{}_{u}}{\quad + \quad}$$

$$_{u} = \frac{{}_{2}}{{}_{1} + {}_{2}}$$

$$= \quad {}_{3}\frac{{}_{1}\quad{}_{2}}{\left(\quad{}_{2} + \quad{}_{1}\right)}$$

Parameters:

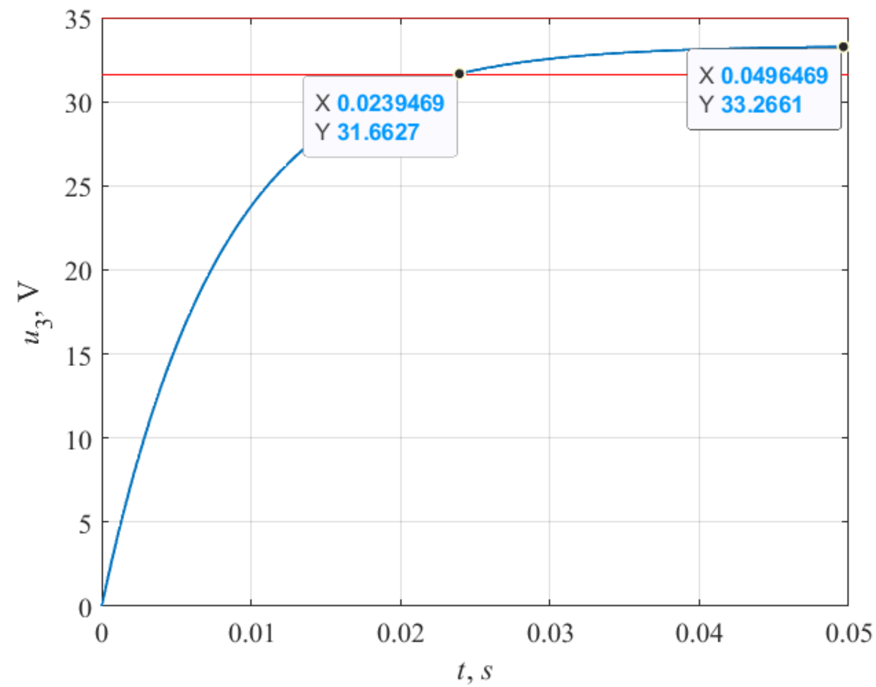
$$_{u} \cdot \quad =$$

$$\cdot \quad =$$

$$\frac{\cdot \quad {}_{1}}{{}_{2} + \quad {}_{1}} =$$

$$_{3}\frac{{}_{1}\quad{}_{2}}{\left(\quad{}_{2} + \quad{}_{1}\right)} =$$

Transient response:



Electrical circuits with capacitance

Transfer function:

$$= \frac{3}{\quad} = \frac{i}{+}$$

$$_i = \frac{3 \quad 2}{1 + \quad 2}$$

$$= \quad 3 \frac{1 \quad 2}{(\quad 2 + \quad 1)}$$

Parameters:

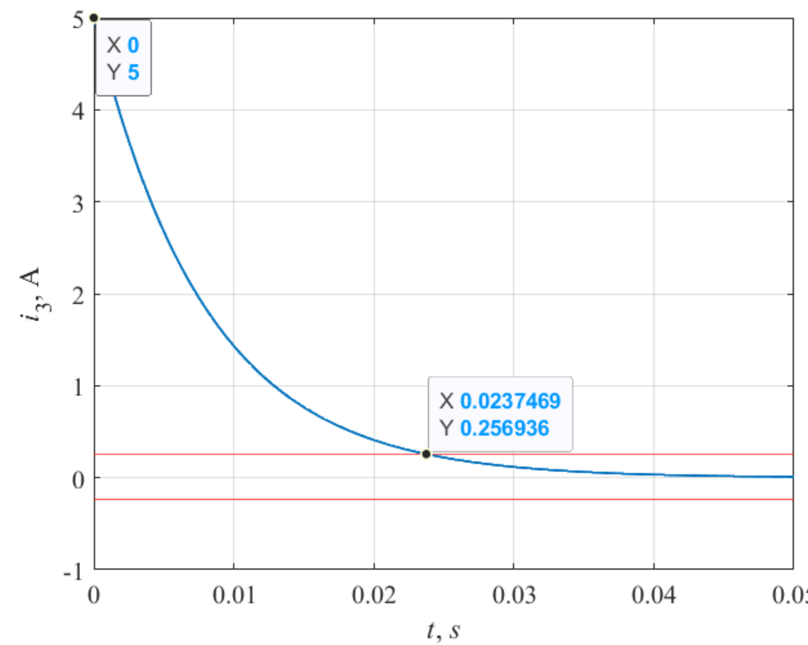
$$\frac{i}{\quad} \cdot \quad =$$

$$\cdot \quad =$$

$$\frac{\quad}{1} =$$

$$3 \frac{1 \quad 2}{(\quad 2 + \quad 1)} =$$

Transient response:



Electrical circuits with capacitance

State-space model:

$$\frac{\quad {}_3}{\qquad} = -\frac{{}_1 + {}_2}{{}_2 \;\; {}_1 \;\; {}_3}\, {}_3 + \frac{\qquad}{{}_1 \;\; {}_3}$$

$${}_3 = -\frac{{}_1 + {}_2}{{}_2 \;\; {}_1}\, {}_3 + \frac{\quad}{{}_1}$$

$$\frac{\quad {}_3}{\qquad} = - \; {}_3 + \frac{u}{\quad}$$

$${}_3 = -\frac{i}{u}\, {}_3 + \frac{u}{1}$$

Parameters:

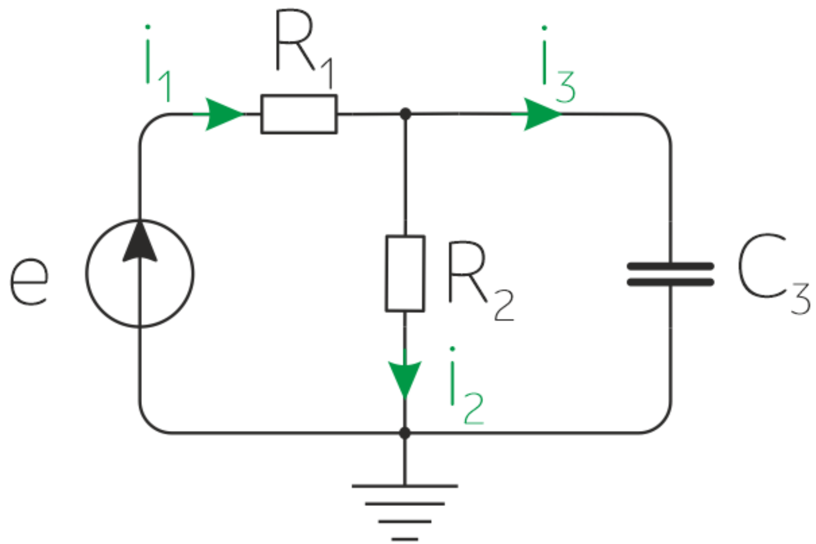
$$\frac{\quad}{1} =$$

$$\frac{\cdot \;\; {}_1}{{}_2 + \;\; {}_1} =$$

$${}_3 \frac{{}_1 \;\; {}_2}{\left({}_2 + \;\; {}_1 \right)} =$$

Electrical circuits with capacitance

Parametric identification using frequency response



Electrical circuits with capacitance

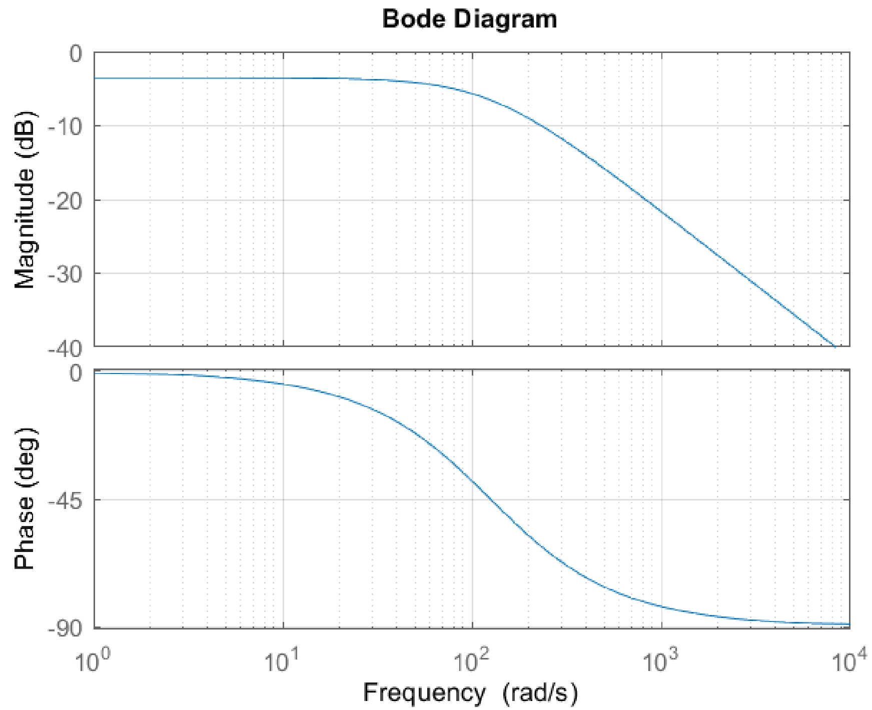
Transfer function:

$$= \frac{_3}{\quad} = \frac{_u}{\quad + \quad}$$

$$_u = \frac{_2}{_1 + \quad _2}$$

$$= \quad _3 \frac{_1 \quad _2}{\left(\quad _2 + \quad _1 \right)}$$

Frequency response:



Electrical circuits with capacitance

Parameters:

$$\omega_c = \quad /$$

$$= \frac{}{\omega_c} =$$

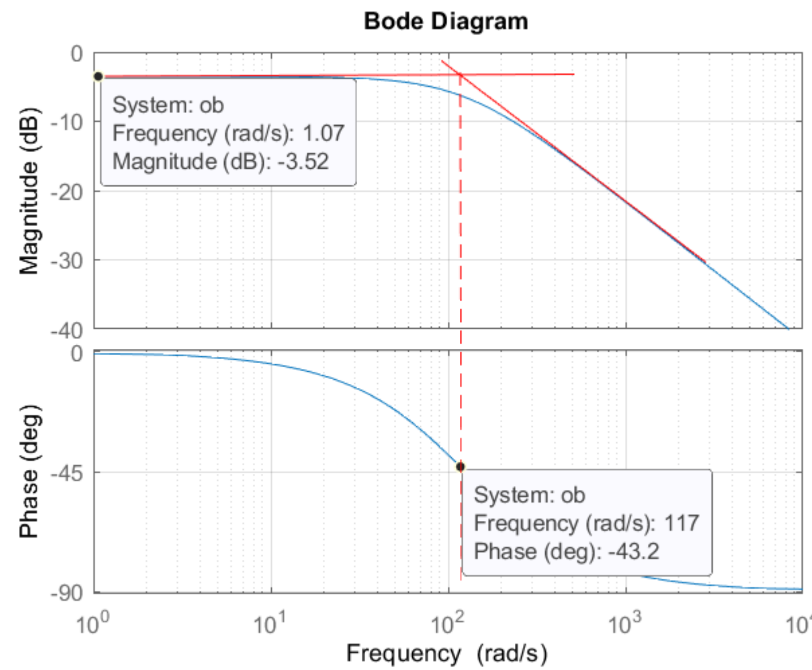
$$= -$$

$$_u = \quad {}^{H}\!/_{20} =$$

$$\frac{1}{_2 + \, _1} =$$

$$_3 \frac{_1 \; _2}{(_2 + \, _1)} =$$

Frequency response:



Electrical circuits with capacitance

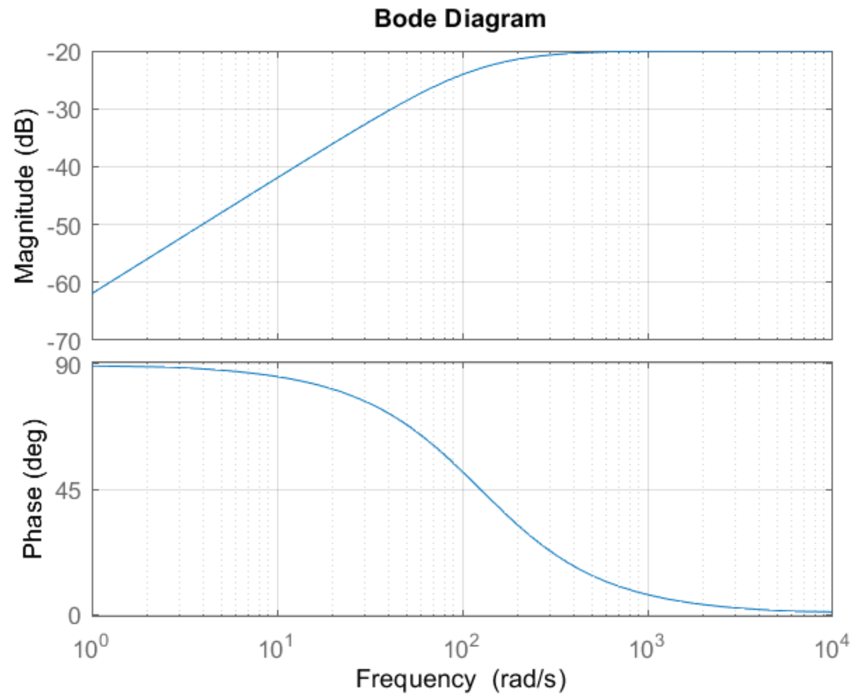
Transfer function:

$$= \frac{{}_3}{\quad} = \frac{{}_i}{\quad + \quad}$$

$$_i = \frac{{}_3 \quad {}_2}{{}_1 + \quad {}_2}$$

$$= \quad {}_3 \frac{{}_1 \quad {}_2}{\left(\quad {}_2 + \quad {}_1 \right)}$$

Frequency response:



Electrical circuits with capacitance

Parameters:

$\omega_c =$ /

$= \frac{\quad}{\omega_c} =$

$= -$

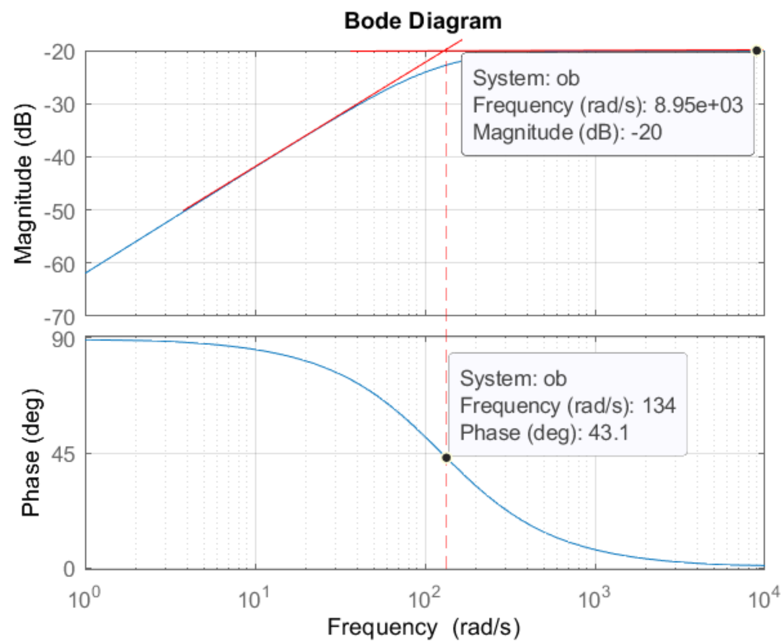
$\frac{\ \ i}{\quad} = \ {}^{H}\!/_{20} =$

$\frac{\quad}{\quad} =$

1

$3\frac{1\quad 2}{(\ _2 + \ _1)} =$

Frequency response:



Electrical circuits with capacitance

State-space model:

$$\frac{{}_3}{\quad} = -\frac{{}_1 + {}_2}{{}_2\ {}_1\ {}_3}\ {}_3 + \frac{\quad}{{}_1\ {}_3}$$

$${}_3 = -\frac{{}_1 + {}_2}{{}_2\ {}_1}\ {}_3 + \frac{\quad}{{}_1}$$

Parameters:

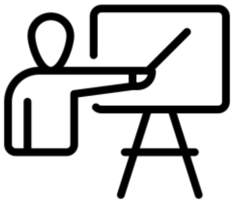
$$\frac{\quad}{{}_1} =$$

$$\frac{{}_1}{{}_2 + {}_1} =$$

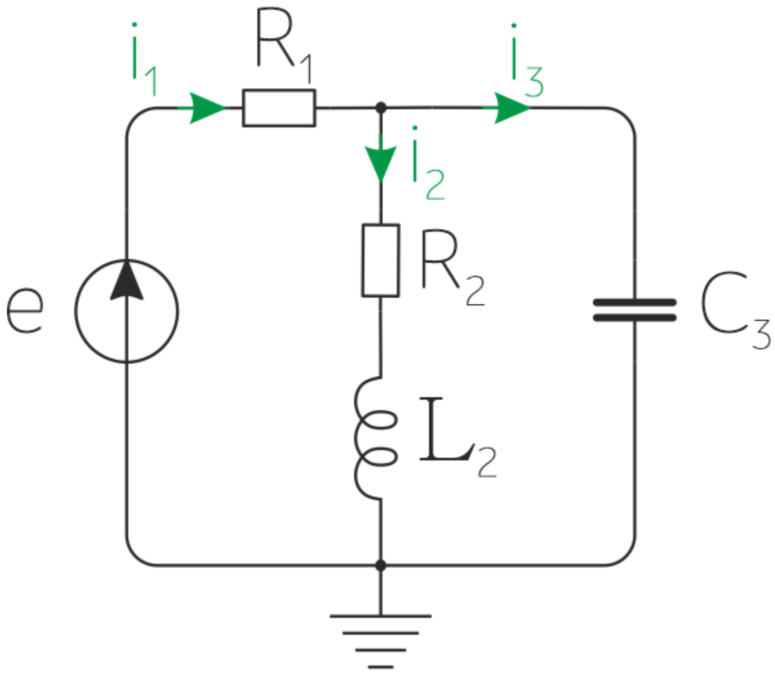
$${}_3\frac{{}_1\ {}_2}{(\ {}_2 + {}_1)} =$$

Mathematic modelling of second order electrical system

Second order electrical circuits



Get the state-space model



Second order electrical circuits

1. Component equations:

- Dissipative elements.

$$_1 = {}_1 \cdot {}_1$$

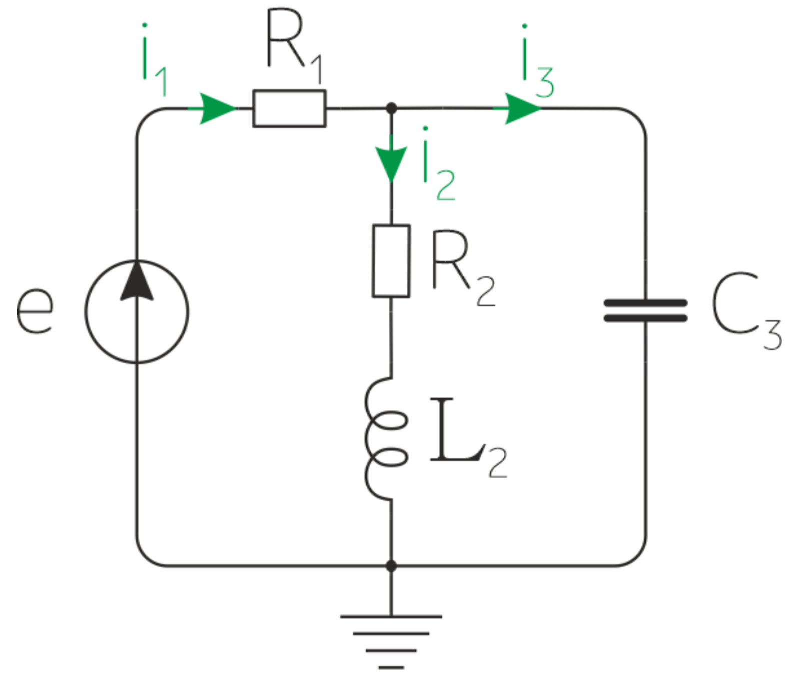
$$_{R2} = {}_2 \cdot {}_2$$

- Elastic elements.

$$_{L2} = {}_2 \cdot \frac{{}_2}{\ }$$

- Inertial elements.

$$_3 = {}_3^{-1} \int {}_3$$



Second order electrical circuits

2. Topological equations:

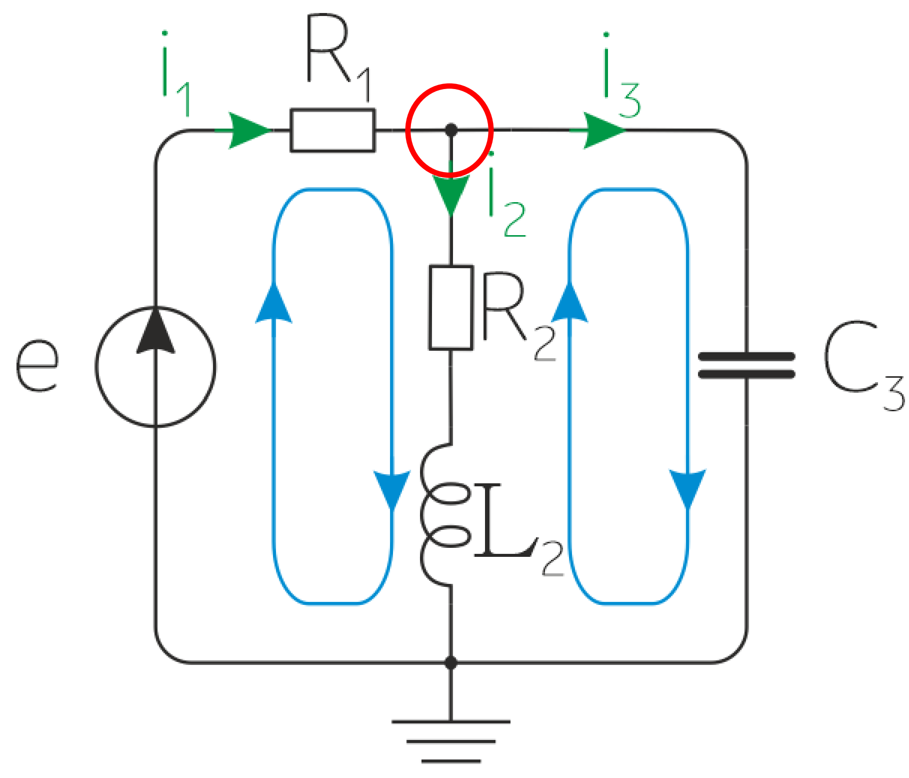
- Equilibrium equations.

$$_1 + \;_{R2} + \;_{L2} =$$

$$- \;_{R2} - \;_{L2} + \;_3 =$$

- Condition of continuity of flow type coordinates.

$$_1 - \;_2 - \;_3 =$$



Second order electrical circuits

3. State-space model:

$$\frac{}{} = \quad +$$

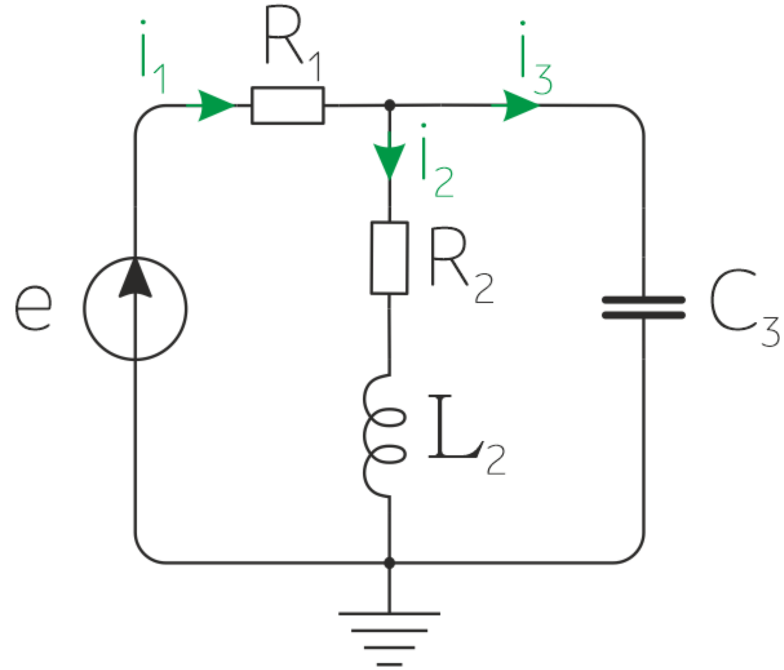
$$= \quad +$$

State vector x:

$$= \begin{bmatrix} {}_{2} \\ {}_{3} \end{bmatrix}$$

Output vector v:

$$= \quad {}_{L2}$$



Second order electrical circuits

3.1 Select the required number of equations for the model:

Four component equations.	Three topological equations.	Six circuit variables.	Two state variables.
${}_1 = {}_1 \cdot {}_1$	${}_1 + {}_{R2} + {}_{L2} =$	${}_1 \quad {}_2 \quad {}_3$	${}_2 \quad {}_3$
${}_{R2} = {}_2 \cdot {}_2$	$- {}_{R2} - {}_{L2} + {}_3 =$	${}_1 \quad {}_2 \quad {}_3$	
${}_{L2} = {}_2 \cdot \frac{{}_2}{}$	${}_1 - {}_2 - {}_3 =$		
${}_3 = {}_3^{-1} \int {}_3$			

Second order electrical circuits

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$- {}_{R2} - {}_{L2} + {}_{3} =$$

$$ {}_{1} - {}_{2} - {}_{3} =$$

$$ {}_{R2} = {}_{2} \cdot {}_{2}$$

$$ {}_{L2} = {}_{2} \cdot \frac{{}_{2}}{}$$

$$ {}_{1} = \frac{1}{{}_{1}}$$

$$ {}_{1} = - {}_{R2} - {}_{L2}$$

$$ {}_{3} = {}_{3}^{-1} \int {}_{3}$$

Second order electrical circuits

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$-\ {}_{R2} -\ {}_{L2} +\ {}_{3} =$$

$$ {}_{1} -\ {}_{2} -\ {}_{3} =$$

$$ {}_{R2} =\ {}_{2}\ {}_{2}$$

$$ {}_{L2} =\ {}_{2} \frac{{}_{2}}{\ }$$

$$ {}_{1} = -\ {}_{2}\ {}_{2} -\ {}_{2} \frac{{}_{2}}{\ } +$$

$$ {}_{1} = - \frac{{}_{2}}{{}_{1}}\ {}_{2} - \frac{{}_{2}}{{}_{1}} \cdot \frac{{}_{2}}{\ } + \frac{\ }{{}_{1}}$$

$$ {}_{3} =\ {}_{3} \cdot \frac{{}_{3}}{\ }$$

3.2 Express the remaining parameters in terms of the state vector parameters and substitute it to the equation:

$$\begin{cases} - {}_{2}{}_{2} - {}_{2}\frac{2}{} + {}_{3} = \\ -\frac{2}{1}{}_{2} - {}_{2} - \frac{2}{1}\cdot\frac{2}{} + \frac{}{1} - {}_{3}\frac{3}{} = \end{cases}$$

$$\begin{cases} \frac{2}{} = -\frac{2}{2}{}_{2} + \frac{}{2}{}_{3} = \\ \frac{3}{} = -\frac{}{3}{}_{2} - \frac{}{1\ \ 3}{}_{3} + \frac{}{1\ \ 3} \end{cases}$$

3.3 Express the output vector in terms of the state vector and substitute input vector:

$$\begin{cases} \dfrac{{}_2}{} = -\dfrac{{}_2}{{}_2}\,{}_2 + \dfrac{}{{}_2}\,{}_3 = \\[2ex] \dfrac{{}_3}{} = -\dfrac{}{{}_3}\,{}_2 - \dfrac{}{{}_1\,{}_3}\,{}_3 + \dfrac{}{{}_1\,{}_3} \end{cases}$$

$${}_{L2} = {}_2\dfrac{{}_2}{} = -{}_2\,{}_2 + {}_3$$

Second order electrical circuits

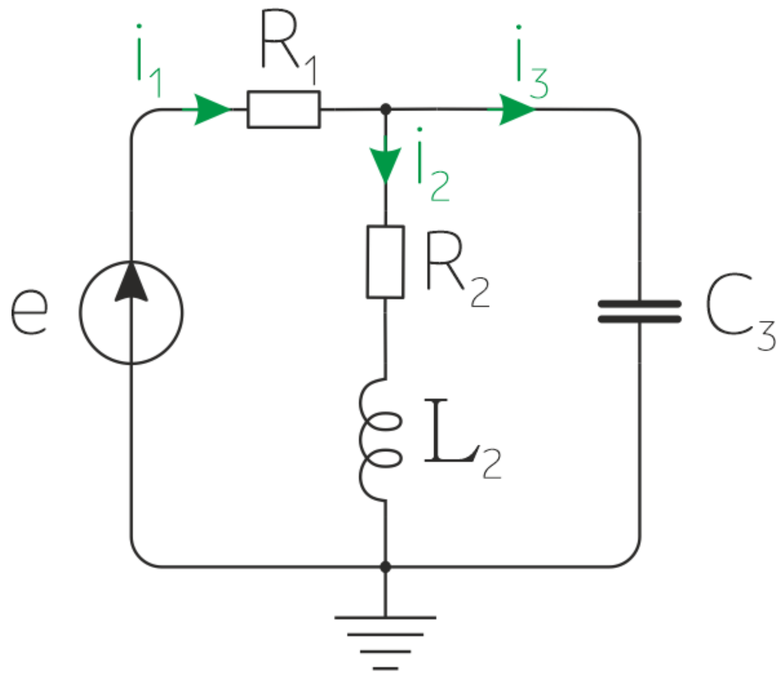
3.4 Write the state space model:

$$\begin{cases} \dfrac{\quad_2}{\quad} = -\dfrac{\quad_2}{\quad_2}\,{}_2 + \dfrac{\quad}{\quad_2}\,{}_3 \\[2ex] \dfrac{\quad_3}{\quad} = -\dfrac{\quad}{\quad_3}\,{}_2 - \dfrac{\quad}{\quad_1\quad_3}\,{}_3 + \dfrac{\quad}{\quad_1\quad_3} \end{cases}$$

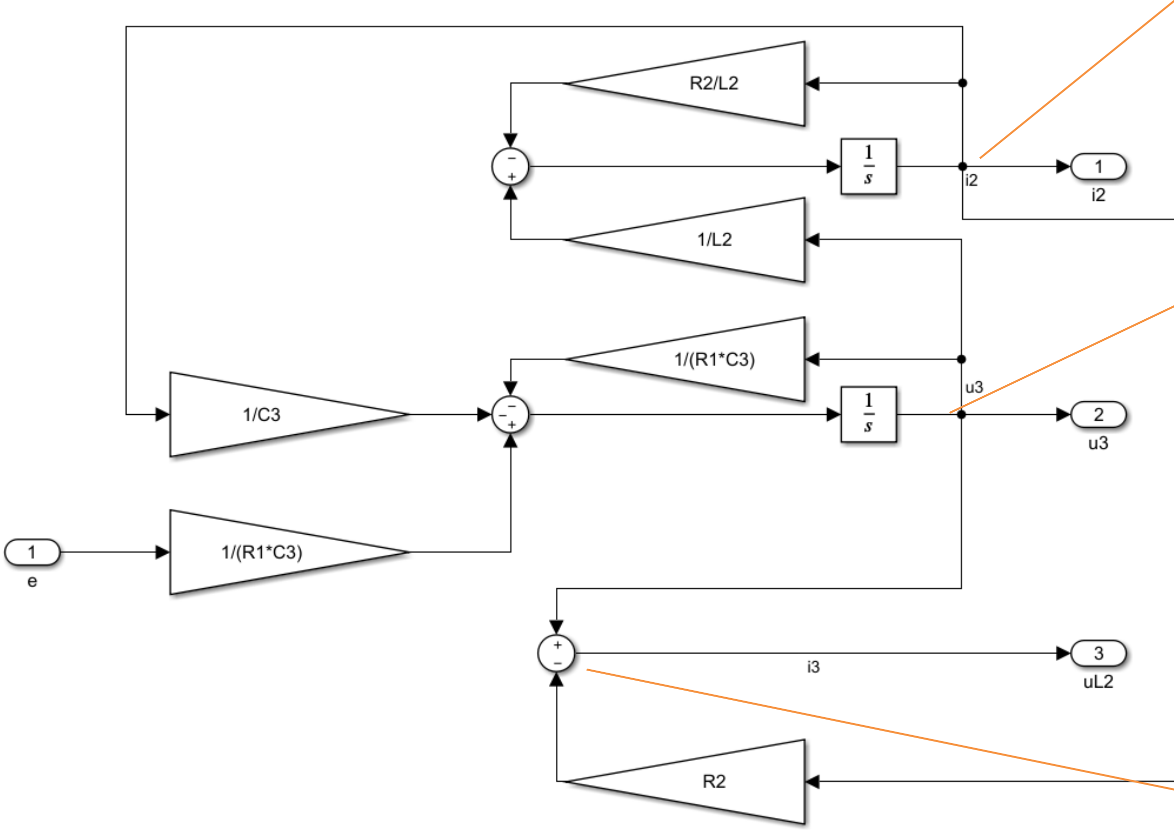
$$_{L2} = -\ {}_2\,{}_2 + \ {}_3$$

Second order electrical circuits

Modelling in Matlab
(zero initial conditions)



Second order electrical circuits



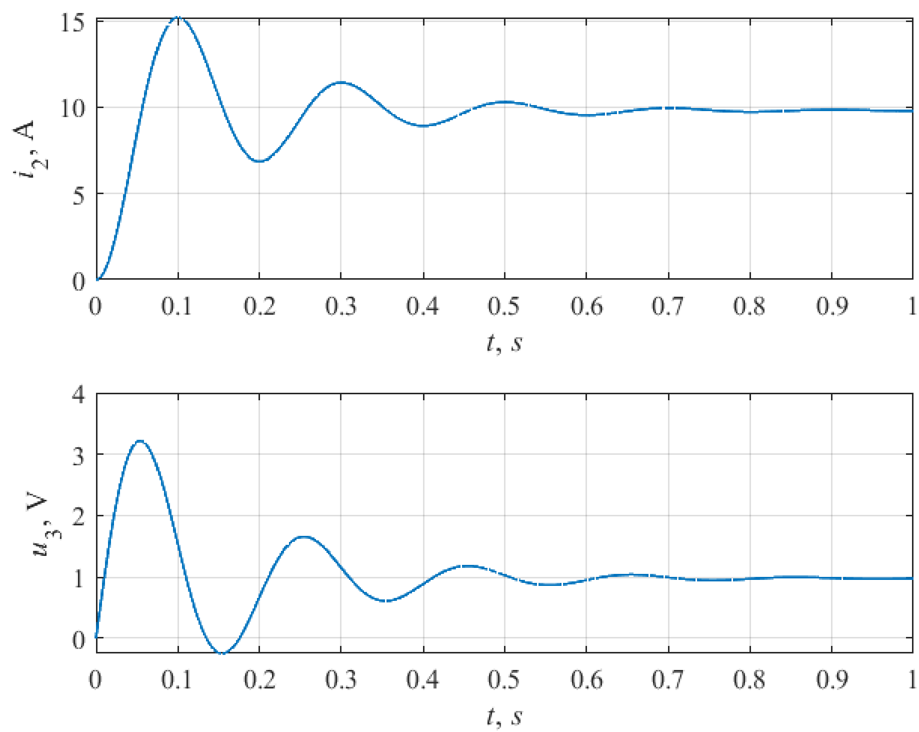
$$\frac{{}_2}{} = -\frac{{}_2}{{}_2}\,{}_2 + \frac{}{{}_2}\,{}_3$$

$$\frac{{}_3}{} = -\frac{}{{}_3}\,{}_2 - \frac{}{{}_1\,{}_3}\,{}_3 + \frac{}{{}_1\,{}_3}$$

$$_{L2} = -\,{}_2\,{}_2 + {}_3$$

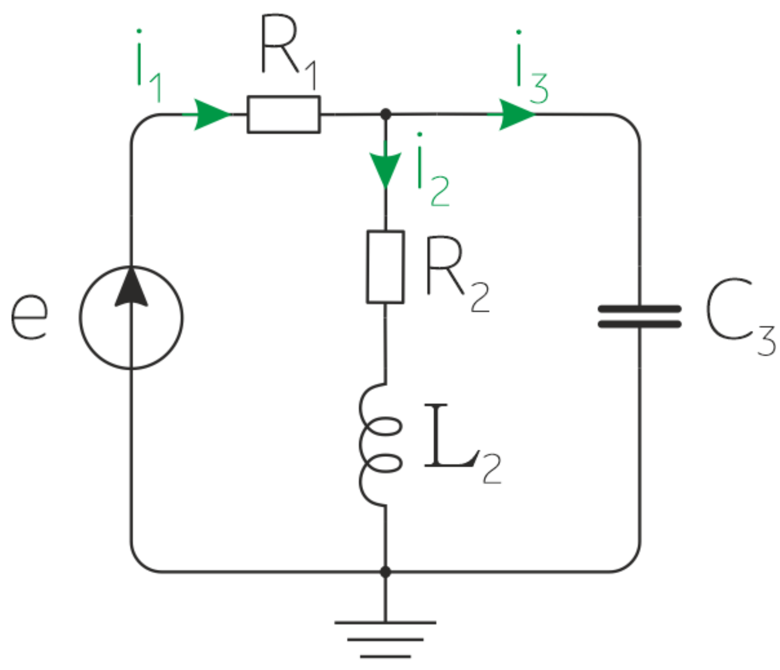
Second order electrical circuits

Graphs of the transients in the circuit (zero initial condition)



Second order electrical circuits

Get the transfer function



Second order electrical circuits

4. Transfer function:

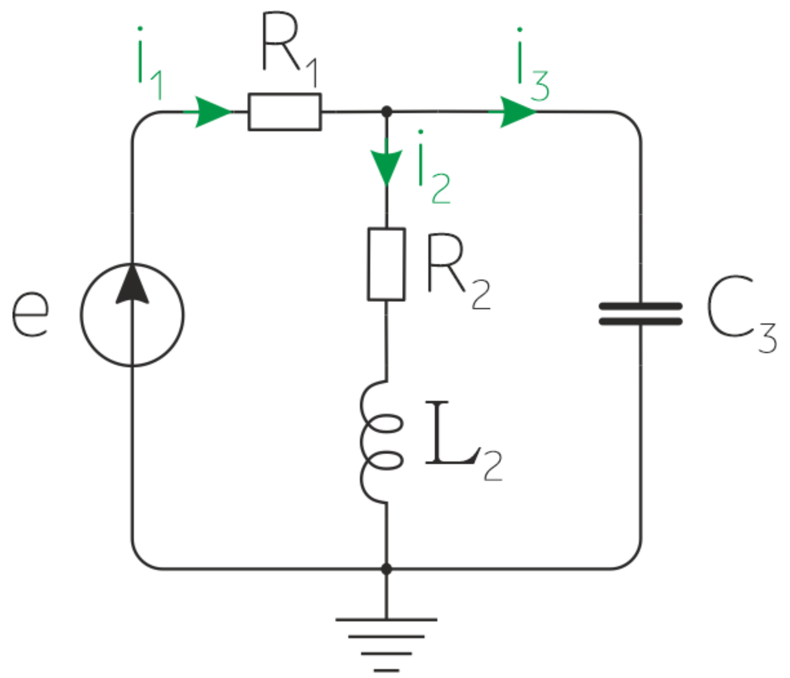
$$= \overline{}$$

Input:

$=$

Output:

$= \quad _{L2}$



4.1 Perform Laplace transform for the state-space model (zero initial conditions):

$$\begin{cases} \dfrac{\ _2}{\ } = -\dfrac{\ _2}{\ _2}\ _2 + \dfrac{\ }{\ _2}\ _3 \\[2ex] \dfrac{\ _3}{\ } = -\dfrac{\ }{\ _3}\ _2 - \dfrac{\ }{\ _1\ _3}\ _3 + \dfrac{\ }{\ _1\ _3} \end{cases}$$

$$_{L2} = -\ _2\ _2 + \ _3$$

$$\Downarrow$$

$$\begin{cases} \ _2 = -\dfrac{\ _2}{\ _2}\ _2 + \dfrac{\ }{\ _2}\ _3 \\[2ex] \ _3 = -\dfrac{\ }{\ _3}\ _2 - \dfrac{\ }{\ _1\ _3}\ _3 + \dfrac{\ }{\ _1\ _3} \end{cases}$$

$$_{L2} = -\ _2\ _2 + \ _3$$

4.2 Express the output in terms of input:

$$_{2} = \frac{}{_{1}\;_{3}\;_{2}\;^{2} + \;_{1}\;_{2}\;_{3} + \;_{2} \;+\; _{1} + \;_{2}}$$

$$_{3} = \frac{_{2} + \;_{2}}{_{1}\;_{3}\;_{2}\;^{2} + \;_{1}\;_{2}\;_{3} + \;_{2} \;+\; _{1} + \;_{2}}$$

$$_{L2} = \frac{^{2}}{_{1}\;_{3}\;_{2}\;^{2} + \;_{1}\;_{2}\;_{3} + \;_{2} \;+\; _{1} + \;_{2}}$$

4.3 Write the transfer function:

$$= \frac{L2}{\quad} = \frac{{}^{2}\big/\left({}_{1} + {}_{2}\right)}{{}_{1}\ {}_{3}\ {}_{2}\big/\left({}_{1} + {}_{2}\right)\ {}^{2} + {}_{1}\ {}_{2}\ {}_{3} + {}_{2}\big/\left({}_{1} + {}_{2}\right) + } = \frac{\quad}{{}^{2}\ {}^{2} + \cdot\ \cdot \xi \cdot + }$$

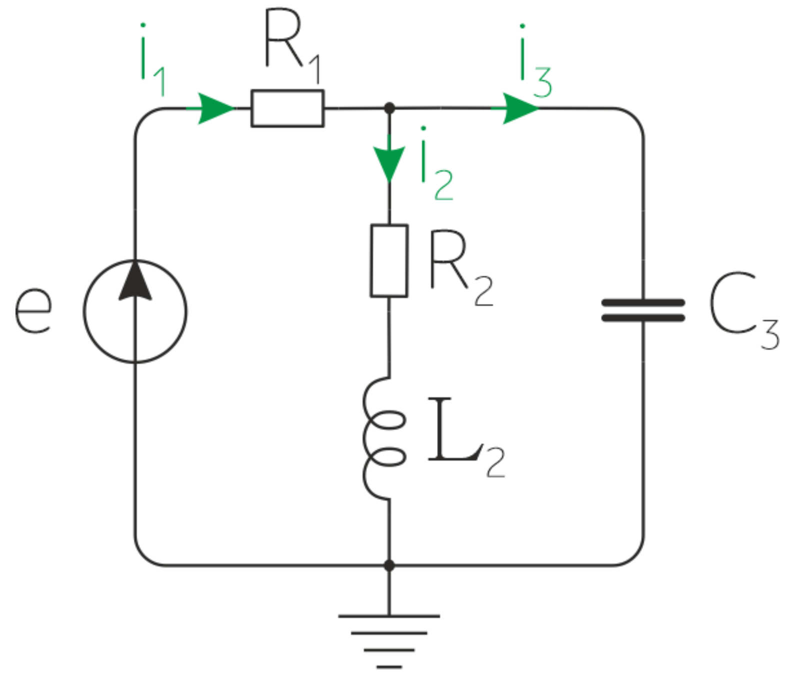
$$= \frac{{}^{2}}{{}_{1} + {}_{2}}$$

$$= \sqrt{\frac{{}_{1}\ {}_{3}\ {}_{2}}{\left({}_{2} + {}_{1}\right)}}$$

$$\xi = \frac{{}_{1}\ {}_{2}\ {}_{3} + {}_{2}}{\cdot\sqrt{{}_{1}\ {}_{3}\ {}_{2}\left({}_{1} + {}_{2}\right)}}$$

Second order electrical circuits

Modelling in Matlab
(nonzero initial conditions)



Second order electrical circuits

State-space model:

$$\begin{cases} \frac{\ _2}{} = -\frac{\ _2}{\ _2}\ _2 + \frac{}{\ _2}\ _3 \\ \frac{\ _3}{} = -\frac{}{\ _3}\ _2 - \frac{}{\ _1\ _3}\ _3 + \frac{}{\ _1\ _3} \end{cases}$$

$$_{L2} = -\ _2\ _2 + \ _3$$

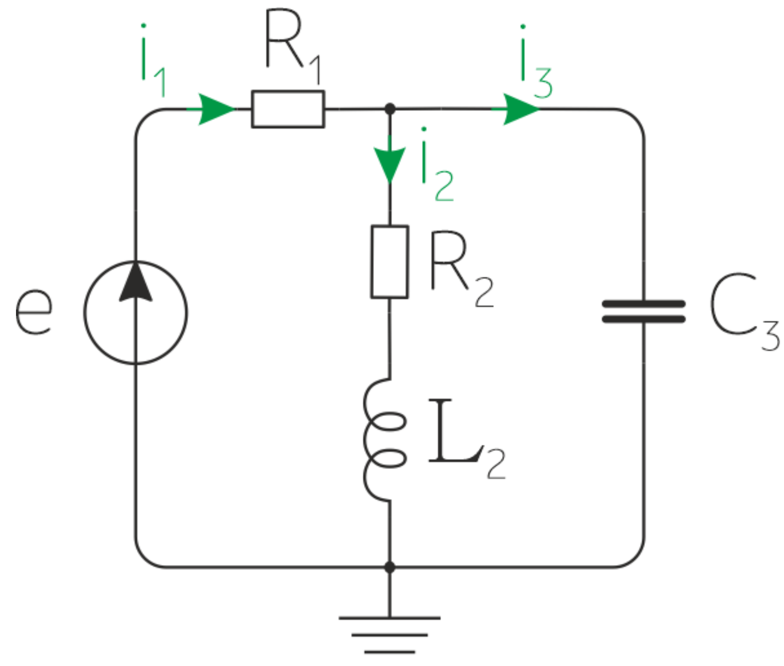
Initial conditions:

$$_1 =$$

$$_3 =$$

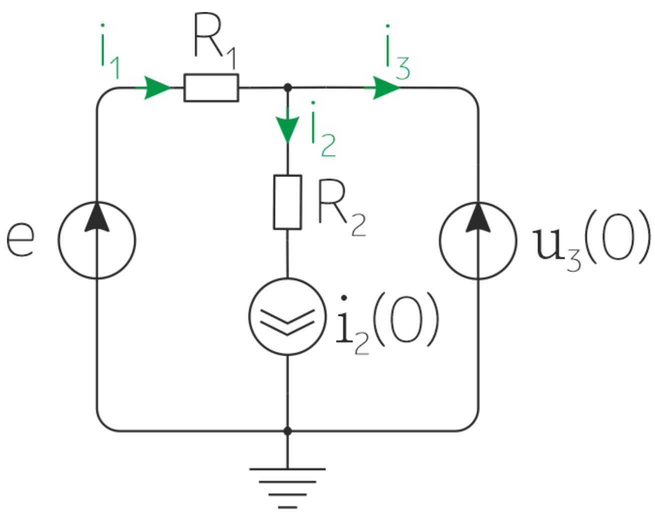
State space vector x:

$$= \begin{bmatrix} _2 \\ _3 \end{bmatrix}$$



Second order electrical circuits

Circuit at time t = 0+:



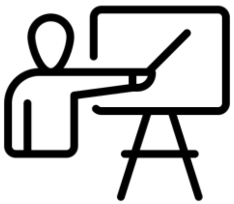
$${}_1 \quad + {}_3 \quad =$$

$${}_1 \quad = {}_2 \quad + {}_3$$

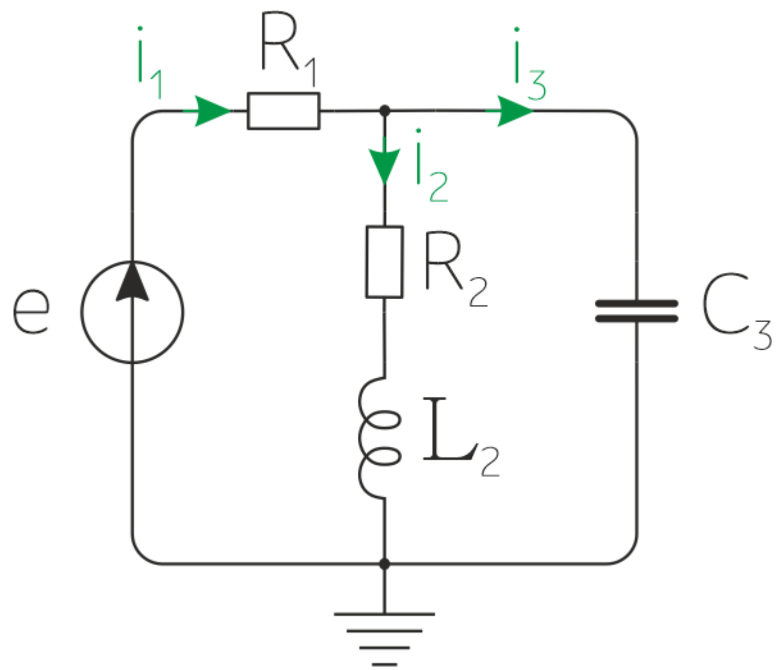
$${}_3 \quad = \quad - {}_1$$

$${}_2 \quad = \frac{{}_1}{{}_1} - {}_3$$

Second order electrical circuits



Parametric identification using transient response



Second order electrical circuits

Transfer function:

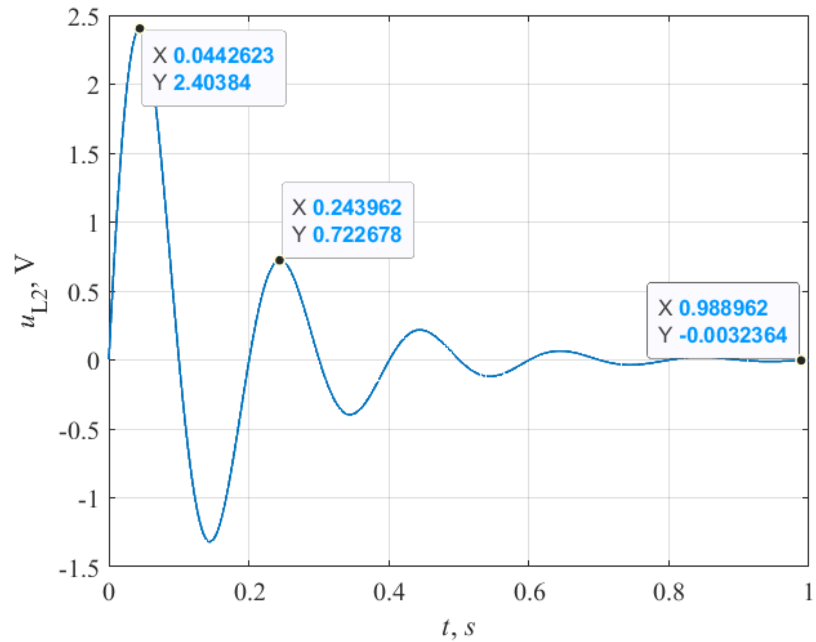
$$= \frac{L2}{\quad} = \frac{uL2}{{}^{2}\ {}^{2} + \ \cdot \ \cdot \xi \cdot \ +}$$

$$_{uL2} = \frac{2}{_{1} + \ _{2}}$$

$$= \sqrt{\frac{1\ \ 3\ \ 2}{\left(\ _{2} + \ _{1}\right)}}$$

$$\xi = \frac{1\ \ 2\ \ 3 + \ \ 2}{\cdot\sqrt{1\ \ 3\ \ 2\left(\ _{1} + \ _{2}\right)}}$$

Transient response:



Parameters:

$$_{r} = \quad - \quad =$$

$$\delta = -\frac{\frac{\quad -}{\quad -}}{r}$$

$$= \frac{\quad}{\sqrt{\delta^{2} + \ _{r}^{-2}}} =$$

$$\xi = -\delta \cdot \quad =$$

Second order electrical circuits

Transfer function:

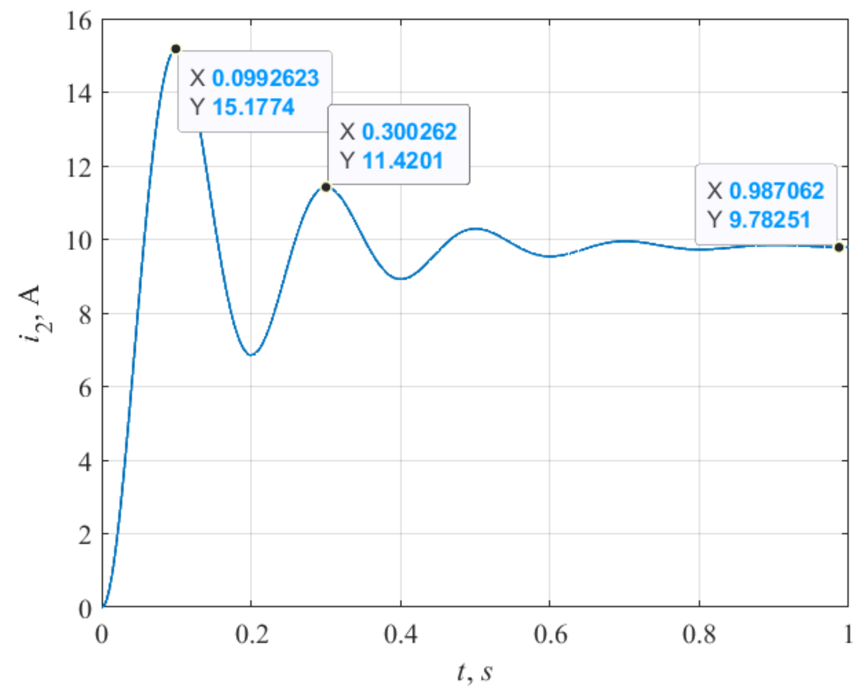
$$=\frac{2}{}=\frac{i2}{2\ \ 2+\ \cdot\ \cdot\xi\cdot\ +}$$

$$_{i2}=\frac{}{_1+\ _2}$$

$$=\sqrt{\frac{1\ \ 3\ \ 2}{(\ _2+\ _1)}}$$

$$\xi=\frac{1\ \ 2\ \ 3+\ \ 2}{\cdot\sqrt{1\ \ 3\ \ 2(\ _1+\ _2)}}$$

Transient response:



Parameters:

$$_r=\ \ -\ \ =$$

$$\delta=-\frac{\frac{-}{-}}{_r}$$

$$_{i2}\cdot\ \ =$$

$$=\frac{}{\sqrt{\delta^2+\ _r^{-2}}}=$$

$$\xi=-\delta\cdot\ \ =$$

$$_{i2}=\frac{}{}$$

Second order electrical circuits

Transfer function:

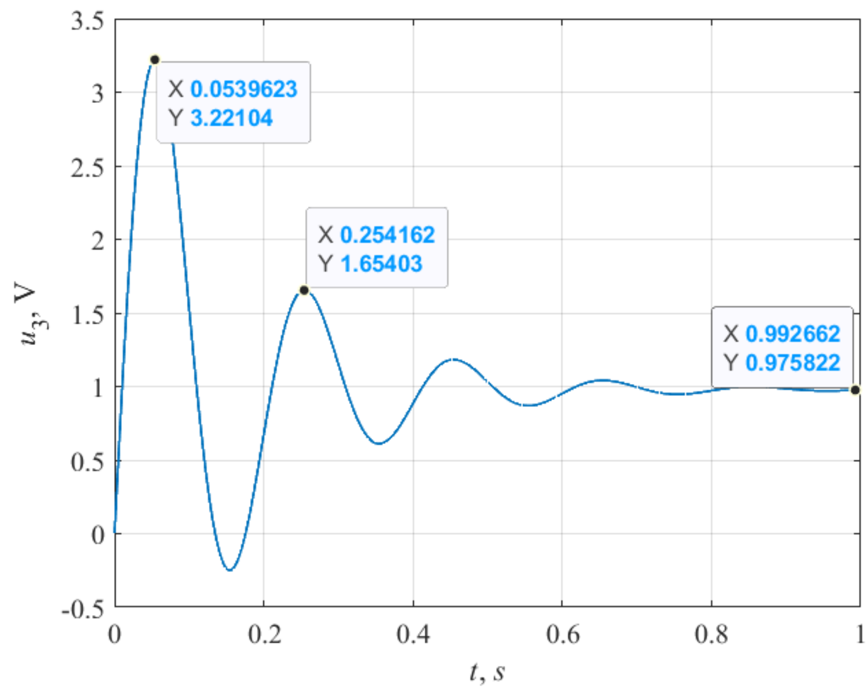
$$= \frac{\ \ _3}{\ } = \frac{_{u3}\left(\ _f\ +\ \right)}{^2\ ^2 + \cdot\ \cdot \xi \cdot\ +}$$

$$_{u3} = \frac{\ _2}{_1 + \ _2}$$

$$= \sqrt{\frac{\ _1\ _3\ _2}{\left(\ _2 + \ _1\right)}} \qquad _f = \frac{\ _2}{\ _2}$$

$$\xi = \frac{\ _1\ _2\ _3 + \ _2}{\cdot \sqrt{\ _1\ _3\ _2\left(\ _1 + \ _2\right)}}$$

Transient response:



Parameters:

$$_r = \quad - \quad =$$

$$\delta = -\frac{\frac{-}{\ }}{\frac{-}{_r}}$$

$$_{u3} \cdot \quad =$$

$$= \frac{\ }{\sqrt{\delta^2 + \ _r^{-2}}} =$$

$$\xi = -\delta \cdot \quad =$$

$$_{u3} = \frac{\ }{\ }$$

Second order electrical circuits

State-space model:

$$\begin{cases} \frac{\ {}_2}{\ } = -\frac{{}_2}{{}_2}\ {}_2 + \frac{\ }{{}_2}\ {}_3 \\ \frac{\ {}_3}{\ } = -\frac{\ }{{}_3}\ {}_2 - \frac{\ }{{}_1\ {}_3}\ {}_3 + \frac{\ }{{}_1\ {}_3} \end{cases}$$

$${}_{L2} = -\ {}_2\ {}_2 + \ {}_3$$

Parameters:

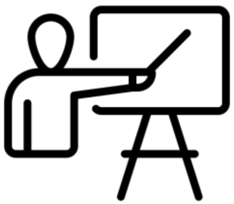
$$\sqrt{\frac{{}_1\ {}_3\ {}_2}{(\ {}_2 + \ {}_1)}} =$$

$$\frac{{}_1\ {}_2\ {}_3 + \ {}_2}{\cdot \sqrt{{}_1\ {}_3\ {}_2(\ {}_1 + \ {}_2)}} =$$

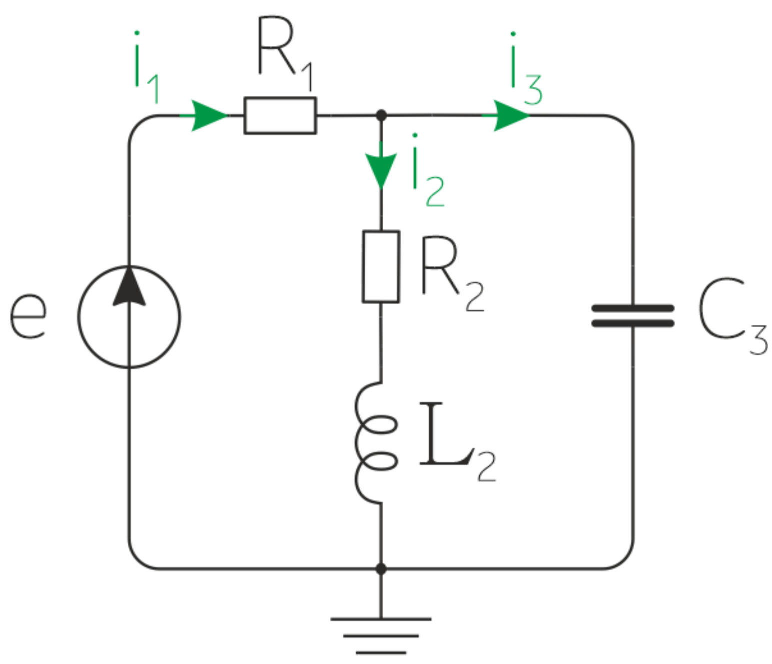
$$\frac{\ }{{}_1 + \ {}_2} = \frac{\ }{\ }$$

$$\frac{{}_2}{{}_1 + \ {}_2} = \frac{\ }{\ }$$

Second order electrical circuits



Parametric identification using frequency response



Second order electrical circuits

Transfer function:

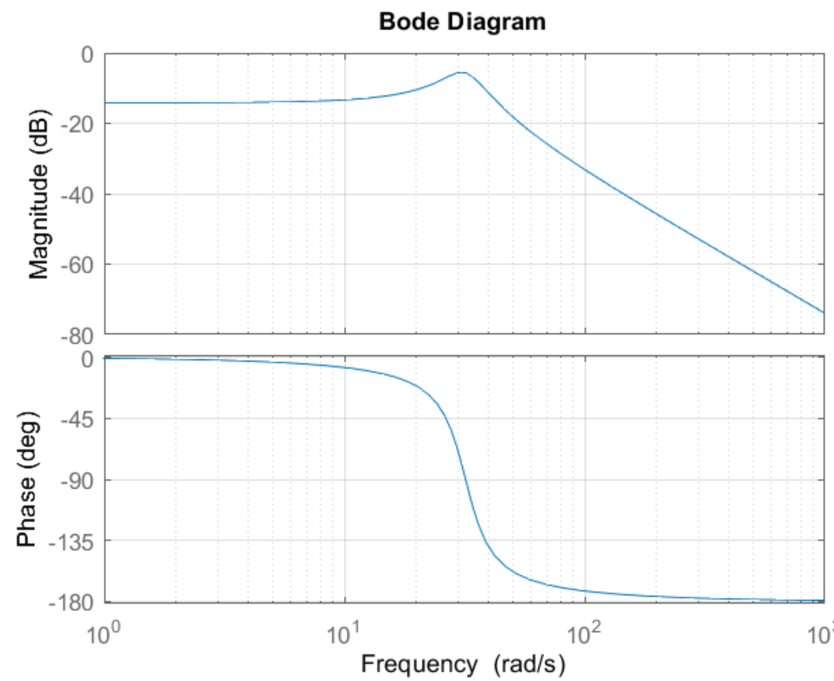
$$= \frac{\quad ^{2}\quad}{} = \frac{\quad _{i2}\quad}{\ ^{2}\ ^{2} + \cdot \quad \cdot \xi \cdot \quad +}$$

$$_{i2} = \frac{}{_{1} + \quad _{2}}$$

$$= \sqrt{\frac{_{1}\ _{3}\ _{2}}{(\ _{2} + \quad _{1})}}$$

$$\xi = \frac{_{1}\ _{2}\ _{3} + \quad _{2}}{\cdot \sqrt{_{1}\ _{3}\ _{2}(\ _{1} + \quad _{2})}}$$

Frequency response:



Second order electrical circuits

Parameters:

$$\omega_c = \quad /$$

$$= \frac{\quad}{\omega_c} =$$

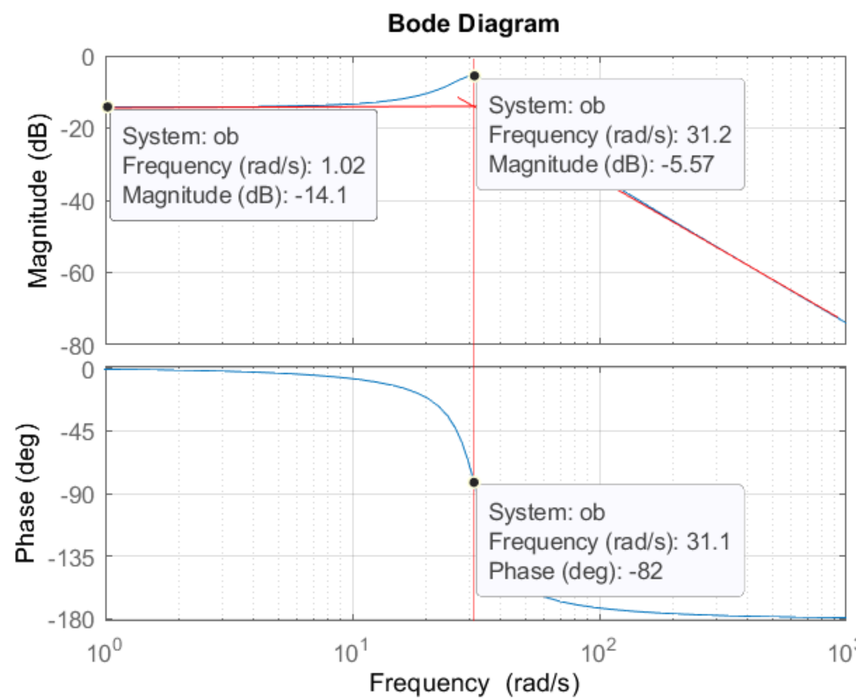
$$_1 = -$$

$$_{i2} = {}^{H_1}\!/_{20} =$$

$$_2 = -$$

$$\frac{_{i2}}{\xi} = {}^{H_2}\!/_{20} =$$

Frequency response:



Second order electrical circuits

Transfer function:

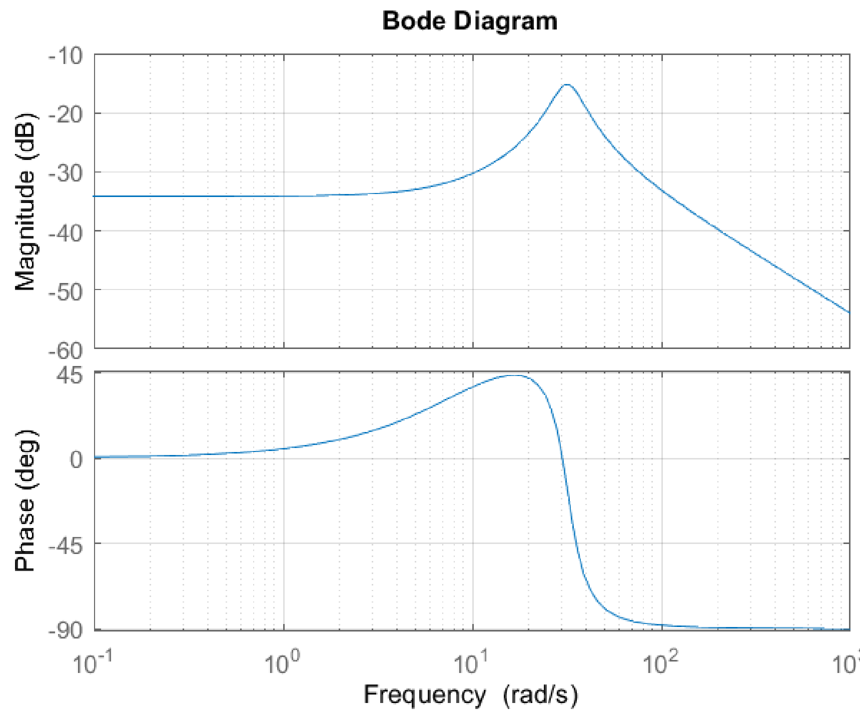
$$= \frac{_3}{\quad} = \frac{_{u3}\left(\;_f\; + \;\right)}{^2\;^2 + \;\cdot\; \cdot\xi\cdot\; +}$$

$$_{u3} = \frac{_2}{_1 + \;_2}$$

$$= \sqrt{\frac{_1\;_3\;_2}{\left(\;_2 + \;_1\right)}} \qquad _f = \frac{_2}{_2}$$

$$\xi = \frac{_1\;_2\;_3 + \;_2}{\cdot\sqrt{\;_1\;_3\;_2\left(\;_1 + \;_2\right)}}$$

Frequency response:



Second order electrical circuits

Parameters:

$$\omega_f = \quad /$$

$$_f = \frac{}{\omega_f} =$$

$$\omega_c = \quad /$$

$$= \frac{}{\omega_c} =$$

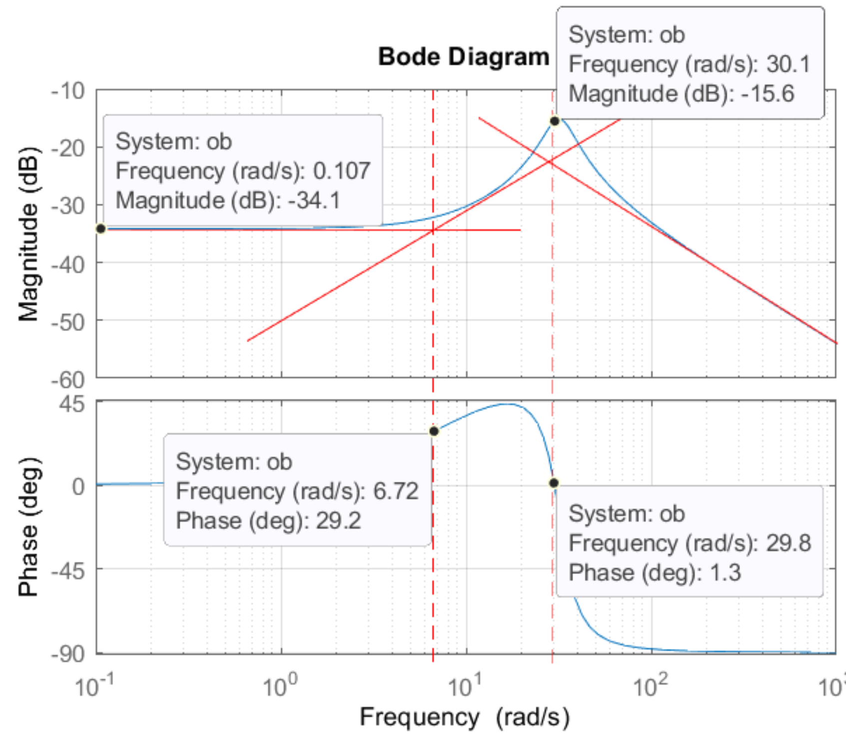
$$_1 = -$$

$$_{u3} = \quad ^{H_1/20} =$$

$$_2 = -$$

$$\frac{_{u3}\sqrt{\left(\frac{_f}{}\right)^2 + }}{\xi} = \quad ^{H_2/20} =$$

Frequency response:



Second order electrical circuits

State-space model:

$$\begin{cases} \dfrac{{}_2}{} = -\dfrac{{}_2}{{}_2}\,{}_2 + \dfrac{}{{}_2}\,{}_3 \\ \dfrac{{}_3}{} = -\dfrac{}{{}_3}\,{}_2 - \dfrac{}{{}_1\;{}_3}\,{}_3 + \dfrac{}{{}_1\;{}_3} \end{cases}$$

$$L2 = -\;{}_2\,{}_2 + \;{}_3$$

Parameters:

$$\sqrt{\dfrac{{}_1\;{}_3\;{}_2}{\left({}_2 + {}_1\right)}} =$$

$$\dfrac{{}_2}{{}_2} = \;f$$

$$\dfrac{{}_1\;{}_2\;{}_3 + {}_2}{\cdot\sqrt{{}_1\;{}_3\;{}_2\left({}_1 + {}_2\right)}} = \xi$$

$$\dfrac{}{{}_1 + {}_2} = \;i2$$

$$\dfrac{{}_2}{{}_1 + {}_2} = \;u3$$