Lab 2.
Simulation components of dynamic systems

1. My name and HDU ID

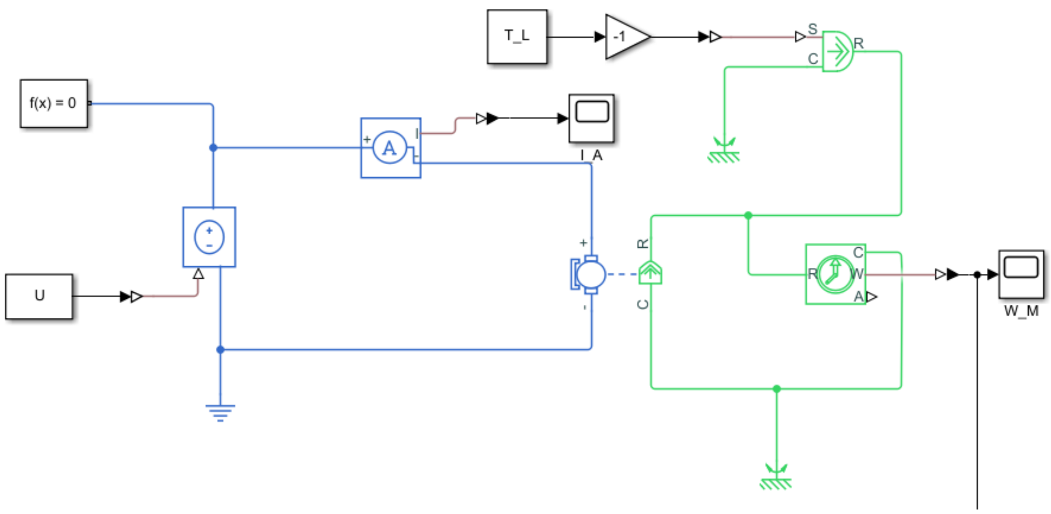
Name:Li Xin

HDU ID:22320404

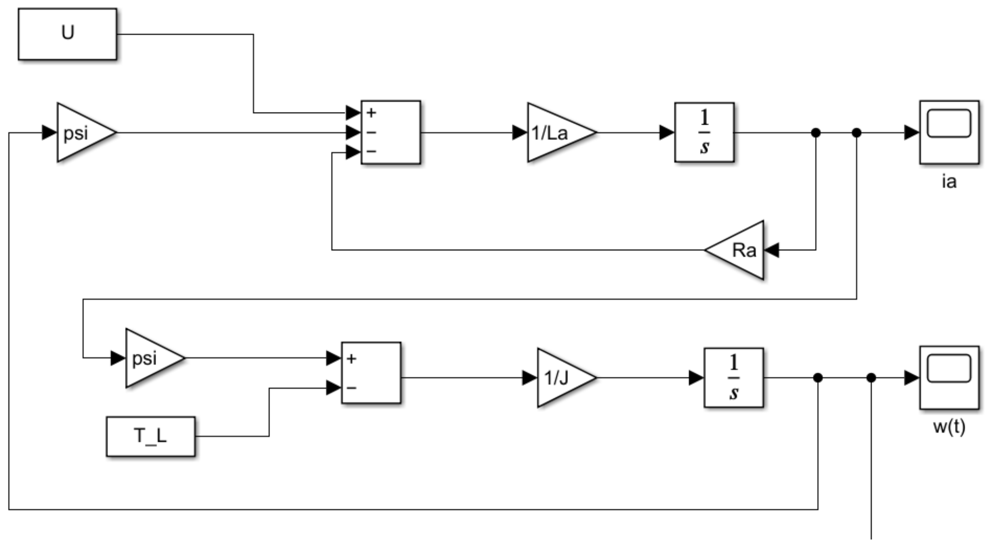
2. My variant and initial data

No. In ITMO	Surname, First name in Russian	No. In HDU	Surname, First name in English	Surname, First name in Chinese	Gender	Variant	U	psi	R	L	J1	J2	T_load
375334	Ли Синь	22320404	LI XIN	李馨	Female	6	12	0.28648	0.35	0.00035	0.00035	0.00234	0.68755

3. Simscape model of DC-motor



4. Block diagram model of DC-motor



5. Transfer functions of DC-motor

$$L_a s I_a(s) = U(s) - R_a I_a(s) - \Psi w(s)$$

$$J s w(s) = \Psi I_a(s) - T_L(s)$$

$$I_a(s) = \frac{U(s) - \Psi w(s)}{L_a s + R_a}$$

$$J s w(s) = \Psi \frac{U(s) - \Psi w(s)}{L_a s + R_a} - T_L(s)$$

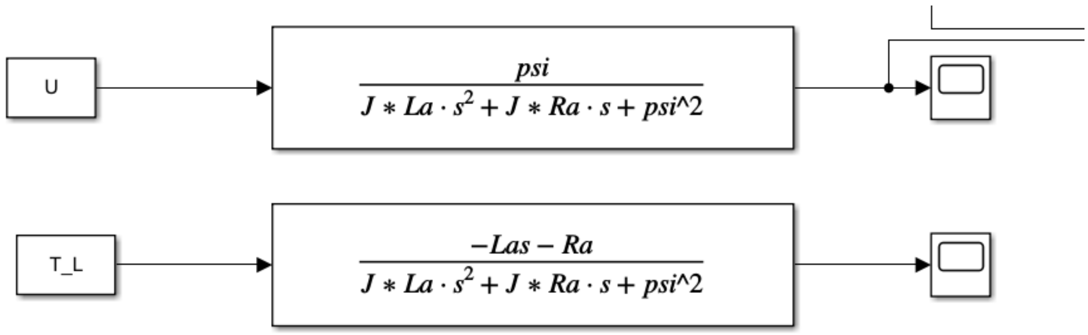
$$\text{Let } T_L = 0$$

$$W_1(s) = \frac{w(s)}{U(s)} = \frac{\Psi}{J L_a s^2 + J R_a s + \Psi^2}$$

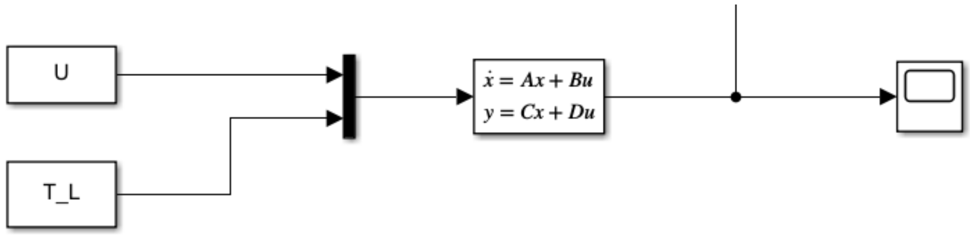
$$\text{Let } U(s) = 0$$

$$\Omega\,(s) = \frac{-(L_a s + R_a)}{Js(L_a s + R_a) + \Psi^2} T_L(s)$$

$$W_2(s) = \frac{-(L_a s + R_a)}{Js(L_a s + R_a) + \Psi^2}$$



6. State space model of DC-motor



$$L_a \frac{di_a(t)}{dt} = U - R_a i_a(t) - \Psi\,\omega(t)$$

$$J \frac{d\omega(t)}{dt} = \Psi\, i_a(t) - T_L$$

$$x_1 = i_a, \quad x_2 = \omega$$

$$\dot{x_1} = \frac{di_a}{dt}, \quad \dot{x_2} = \frac{d\omega}{dt}$$

$$\dot{x_1} = \frac{1}{L_a}\left(U - R_a x_1 - \Psi\, x_2\right)$$

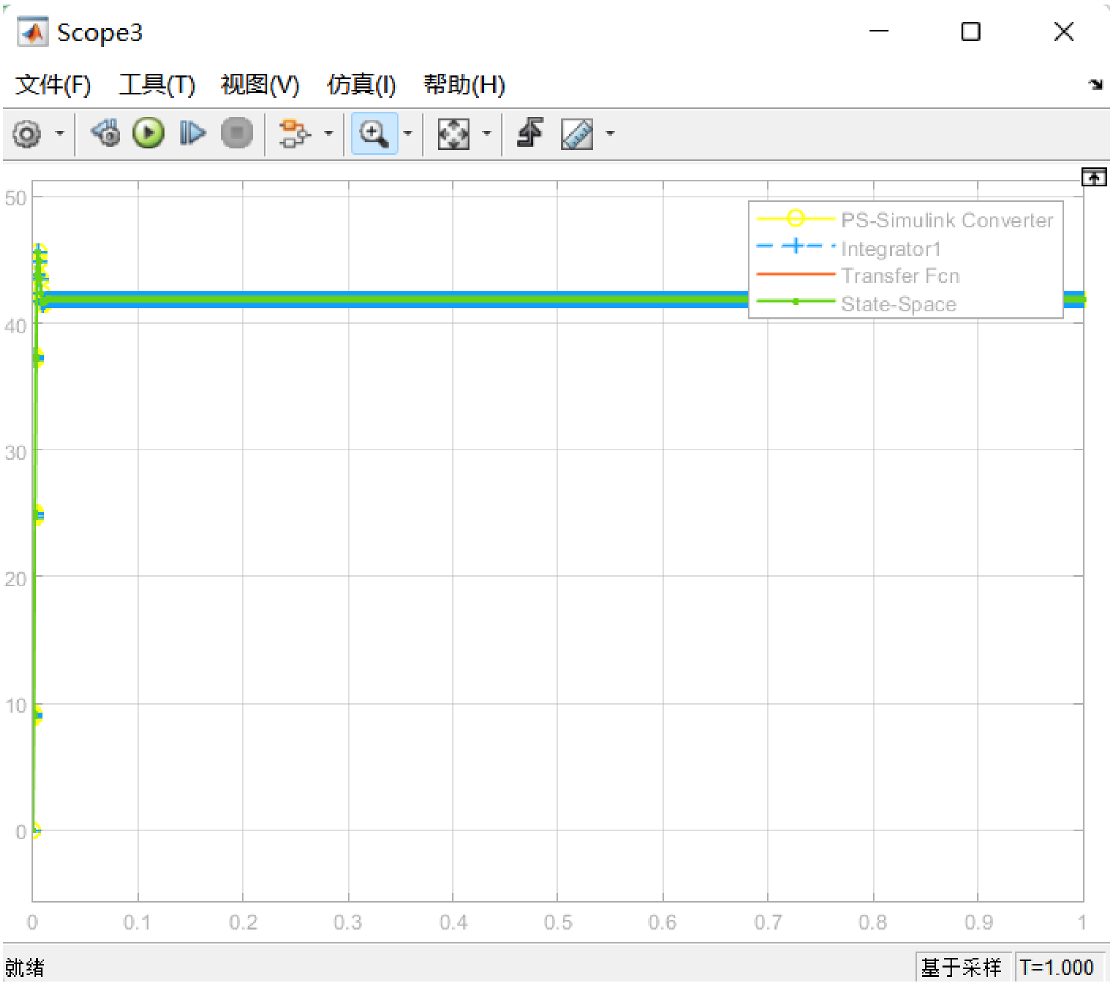
$$\dot{x_2} = \frac{1}{J}\left(\Psi\, x_1 - T_L\right)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{\Psi}{L_a} \\ \frac{\Psi}{J} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} U \\ T_L \end{bmatrix}$$

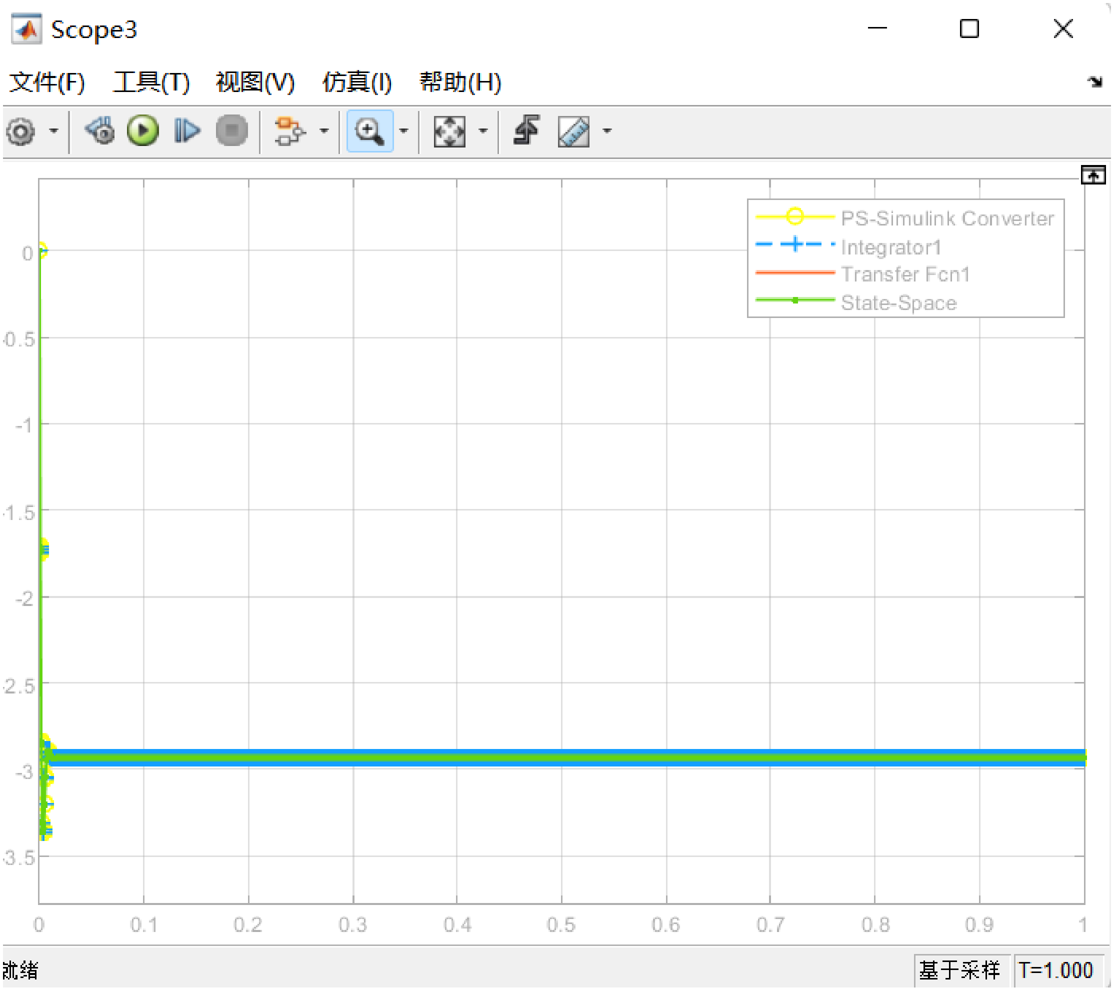
$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \end{bmatrix} \begin{bmatrix} U \\ T_L \end{bmatrix}$$

$$A = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{\Psi}{L_a} \\ \frac{\Psi}{J} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

7. Simulation results for 2 cases



With rated voltage and zero load torque



With zero voltage and rated load torque

8. Calculation of transient response function based on transfer function of DC-motor for two values of inertia

$$JL_a s^2 + JR_a s + \Psi^2 = 0$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\omega_n^2 = \frac{\Psi^2}{JL_a}$$

$$2\zeta\omega_n = \frac{JR_a}{JL_a} = \frac{R_a}{L_a}$$

$$\omega_n = \sqrt{\frac{\Psi^2}{JL_a}}$$

$$\zeta = \frac{R_a}{2}\sqrt{\frac{J}{\Psi^2 L_a}}$$

$$\zeta 1 = 0.6109 < 1$$

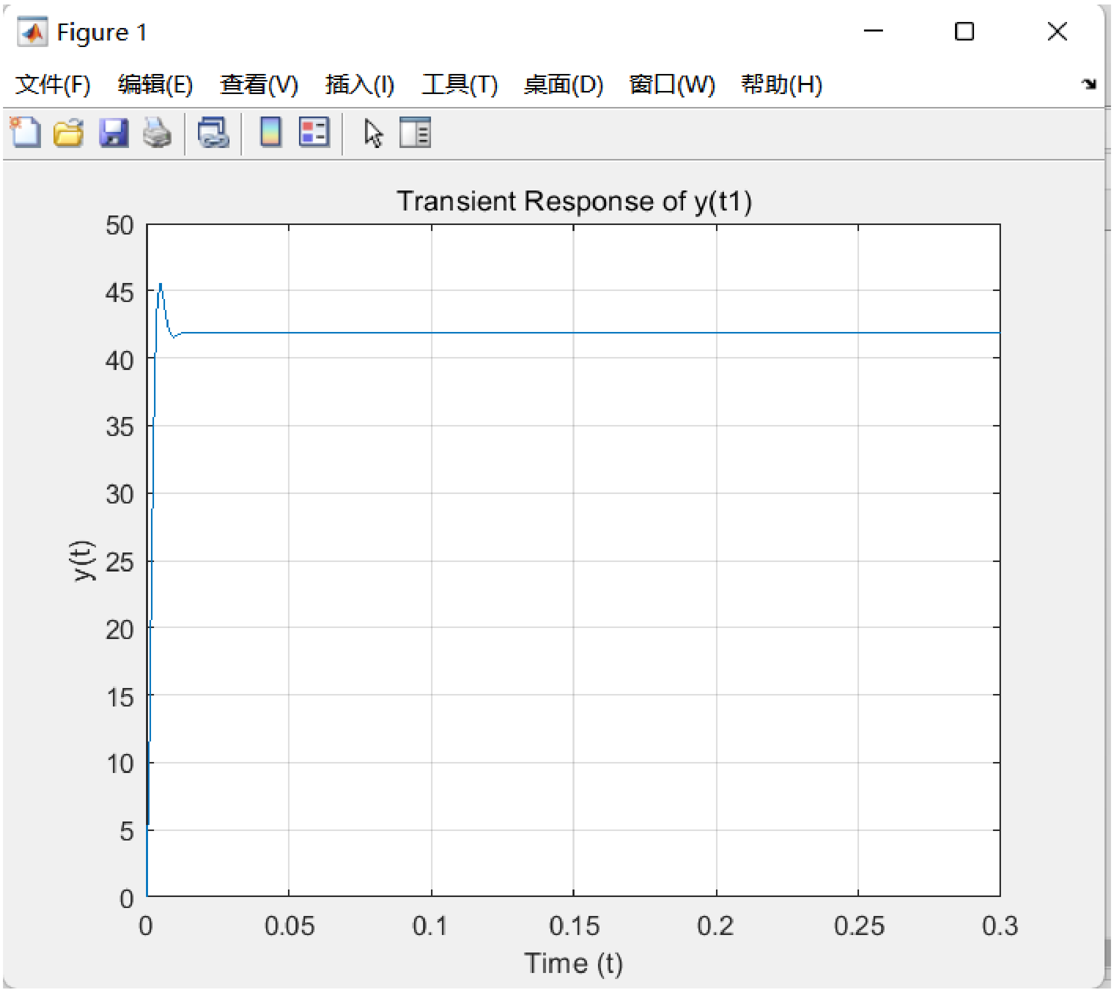
$$\zeta 2 = 1.5795 > 1$$

So the transient response function ω(t) for U(t) = U_rated , J=J1 is underdamped.

After inverse Laplace transform,

$$\omega(t) = \frac{U_{rated}}{\varphi}\left(1 - e^{-\zeta\omega_n t}\left(\cos(\omega_d t) + \frac{\zeta}{\sqrt{1-\zeta}}\sin(\omega_d t)\right)\right)$$

9. Graphs of transient responses



With J=J1

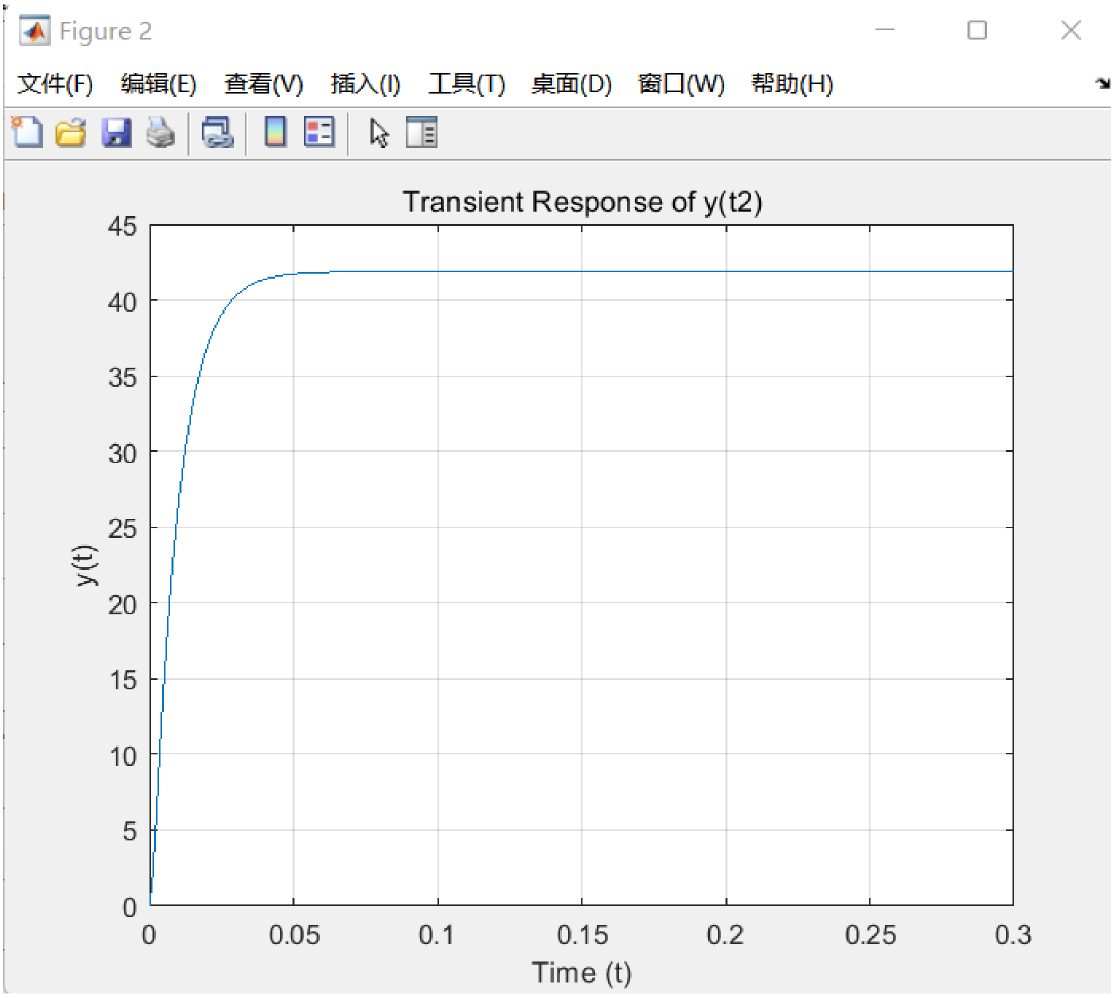


Figure 2
文件(F) 编辑(E) 查看(V) 插入(I) 工具(T) 桌面(D) 窗口(W) 帮助(H)
Transient Response of y(t2)
y(t)
Time (t)
0
5
10
15
20
25
30
35
40
45
0
0.05
0.1
0.15
0.2
0.25
0.3

With J=J2

10. Values of rise time, maximum overshoot and settling time

For J1

Rise Time (10% to 90%): 0.003 s

Max Overshoot: 8.85%

Settling Time (5% tolerance): 0.008 s

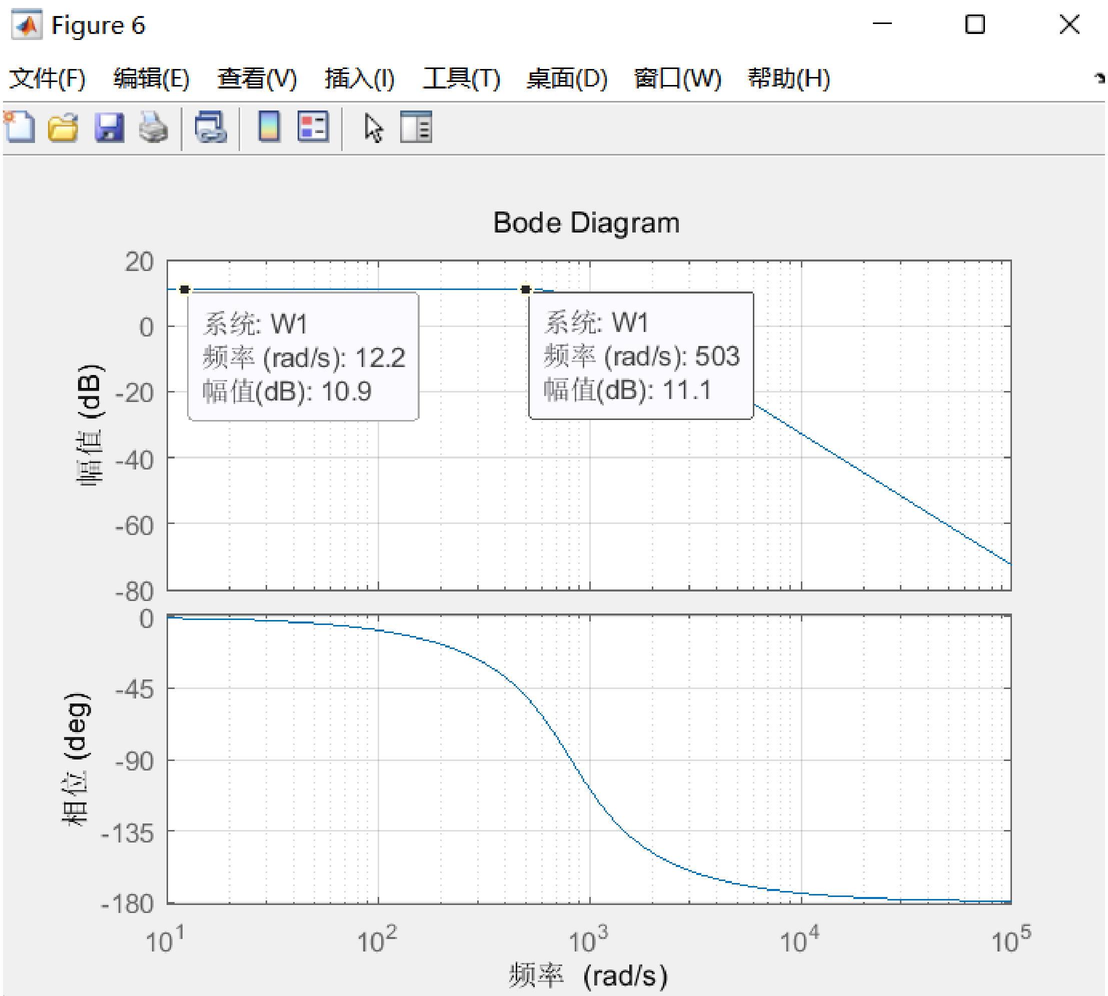
For J2:

Rise Time (10% to 90%): 0.0197 s

Max Overshoot: 0.00%

Settling Time (5% tolerance): 0.0358 s

11. Bode plot of underdamped model of DC-motor



12.Values of the static gain and damped natural frequency calculated from Bode plots

Static Gain K =$\frac{10.9}{20} = 10^{0.545}$=3.50

Damped Natural Frequency wd = 503 rad/s

13.conclusion

Model Validation

The derived transfer function and state-space model of the DC motor align with the physical principles of electromechanical systems. The transfer function and state-space matrices were successfully validated through Simulink simulations, confirming their accuracy in describing the motor's dynamics under both no-load and rated load conditions.

Impact of Inertia (*J*) on Transient Response

For $J=J1$: The system exhibited **underdamped behavior** ($\zeta 1=0.61$), resulting in a fast rise time (0.003s) but with an overshoot of 8.85%.

For $J=J2$: The system became **overdamped** ($\zeta 2=1.58$), eliminating overshoot at the cost of slower response (rise time 0.0197s).

Frequency Domain Analysis

The Bode plot of the underdamped model ($J=J1$) revealed a **static gain K=3.50** and **damped natural frequency ωd=503rad/s**. These values align with theoretical predictions, confirming the system's bandwidth and resonance characteristics.