Lab 1. Introduction to modelling

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Specialization: Automation

Objective

Familiarize yourself with the Simulink software environment and basic methods for modeling linear electrical circuits.

Theoretical information

A mathematical model of a linear electric circuit as a linear stationary system can be represented in the form of a scalar differential equation of the *nth* order (input-output model) or in the form of a system of *n* differential equations of the 1st order (input-state-output model).

The input-output model has the form

$$y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_1\dot{y} + a_0 y = b_m u^{(m)} + b_{m-1}u^{(m-1)} + \ldots + b_1\dot{u} + b_0 u\,, \tag{1}$$

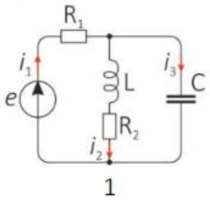
where *y* is the output variable, *u* is the input signal, *n* is the order of the system, *m* is the order of the derivative of the output variable, which explicitly depends on *u* ($m \le n$), a_j, b_j are constant coefficients.

Provided that $m \le n$, the input-state-output model can be represented as

$$\begin{cases} \dot{x}_1 = \alpha_{11}x_1 + \alpha_{12}x_2 + \ldots + \alpha_{1n}x_n + \beta_1 u, \\ \dot{x}_2 = \alpha_{21}x_1 + \alpha_{22}x_2 + \ldots + \alpha_{2n}x_n + \beta_2 u, \\ \ldots \\ \dot{x}_n = \alpha_{n1}x_1 + \alpha_{n2}x_2 + \ldots + \alpha_{nn}x_n + \beta_n u, \\ y = c_1x_1 + c_2x_2 + \ldots + c_nx_n, \end{cases} \tag{2}$$

where x_j are the coordinates of the state vector, α_{ij} and β_j are constant coefficients. System (2) can be represented in a compact vector-matrix form

Schemes



Option	Scheme	Circuit parameters				Source voltage waveform ω=50*2*π
		R_1, [Ohm]	R_2, [Ohm]	L, [mH]	C, [uF]	
6	1	12	15	20	10	$e(t)=24$, $e(t)=E_m \sin(\omega t)$

6	$W_1(s)=\frac{I_2(s)}{E(s)}$, $W_2(s)=\frac{U_C(s)}{E(s)}$	$x=[i_2 \quad U_C]^T$	$x=[1 \quad 1]^T$

1. Build a simulation circuit.

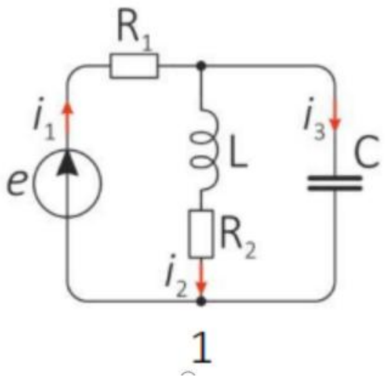


Figure 1. Equivalent circuit.

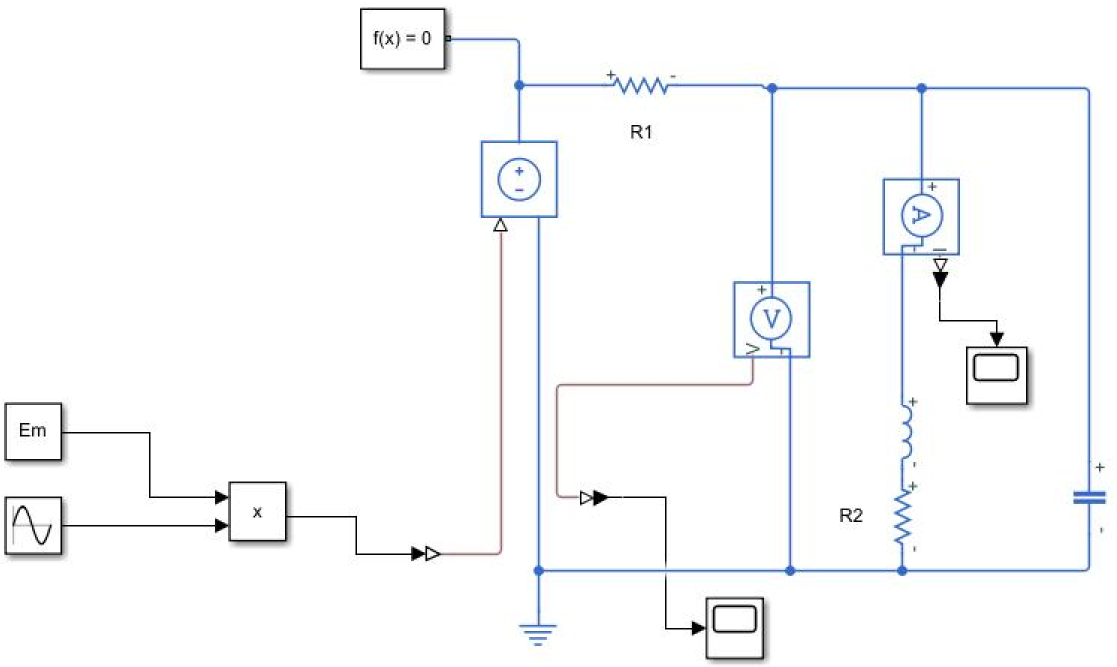


Figure 2. Simulation circuit.

2. Component equations.

$$\frac{dU_C}{dt} = \frac{I_C}{C}$$

$$U_R = R \cdot I_R$$

$$U_L = L\frac{dI_L}{dt}$$

3. Topological equations.

$$\mathrm{e}(t) - u_{R1} - u_L - u_{R2} = 0$$

$$u_L + u_{R2} - u_C = 0$$

$$i_1 = i_2 + i_3$$

4. State-space model.

$$\boldsymbol{\frac{dx_1}{dt} = \frac{di_2}{dt}, \quad \frac{dx_2}{dt} = \frac{du_C}{dt}}$$

$$\boldsymbol{e(t) = u_{R1} + u_C = i_1R_1 + u_C = \left(C\frac{du_C}{dt} + i_2\right)R_1 + u_C}$$

$$\boldsymbol{e(t) = CR_1\frac{du_C}{dt} + i_2R_1 + u_C}$$

$$\boldsymbol{\frac{du_C}{dt} = \frac{e(t) + i_2R_1 - u_C}{CR_1} =- \frac{1}{C}i_2 - \frac{1}{CR_1}u_C + \frac{e(t)}{CR_1}}$$

Solution: $\boldsymbol{a_{12} =- \frac{1}{C}, \quad a_{22} =- \frac{1}{CR_1}, \quad b_{21} = \frac{1}{CR_1}}$

$$\frac{di_2}{dt} = \frac{u_L}{L} = \frac{u_C - i_2R_2}{L} =- \frac{R_2}{L}i_2 + \frac{1}{L}u_C$$

Solution: $a_{11} =- \frac{R_2}{L}, \quad a_{12} = \frac{1}{L}, \quad b_{11} = 0$

5. Simulink simulation of the circuit and the state-space model using the predetermined input and zero initial conditions.

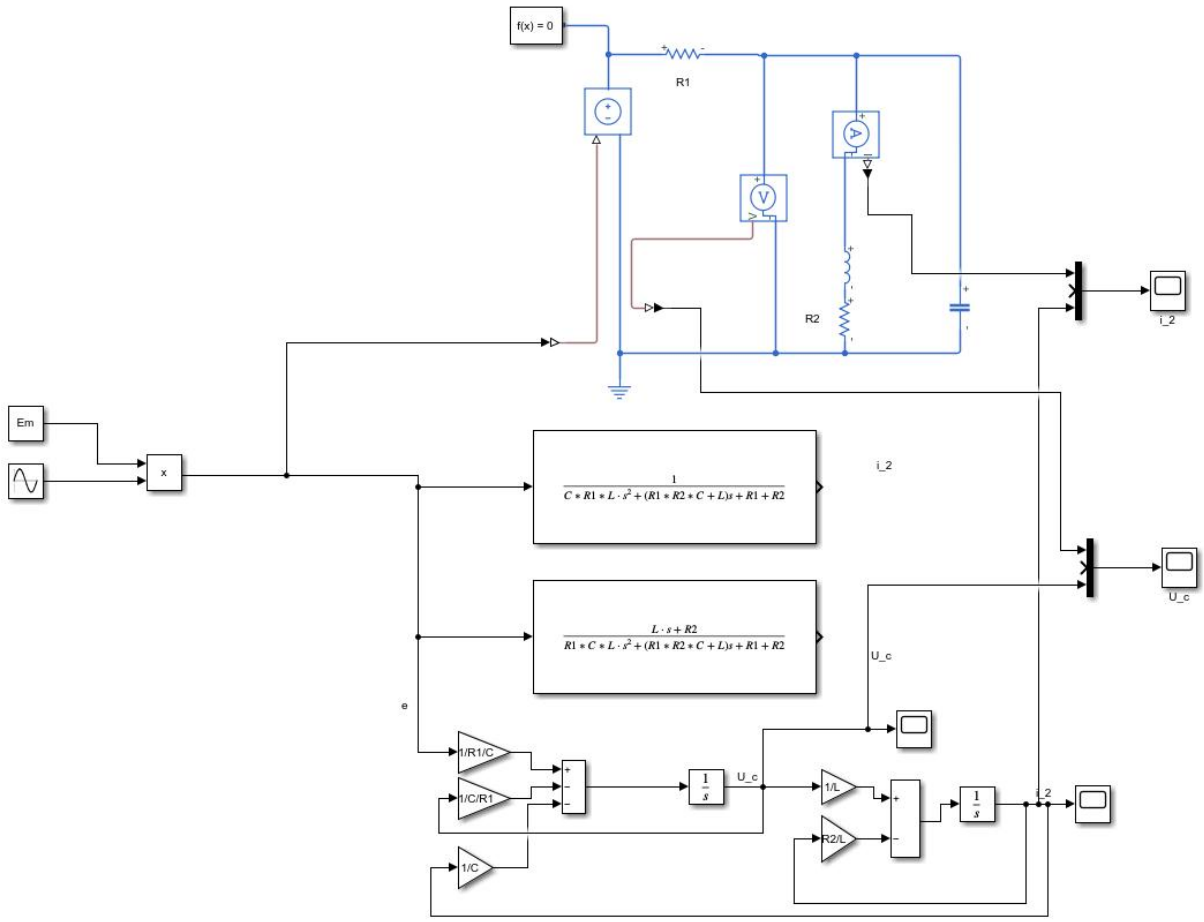


Figure 3 Modelling with sinusoidal input voltage

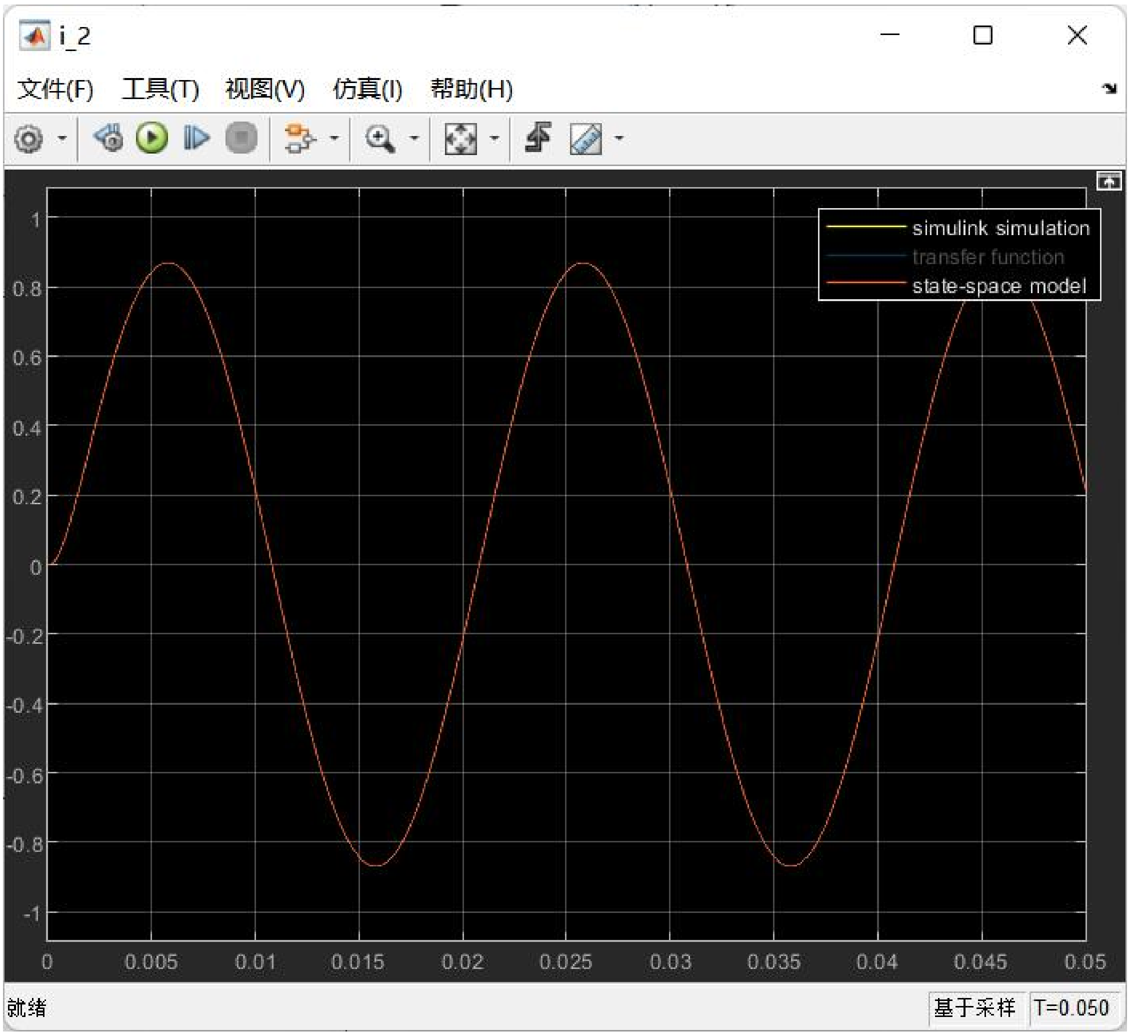


Figure 4 i_2 of Simulink simulation of the circuit and the state-space model

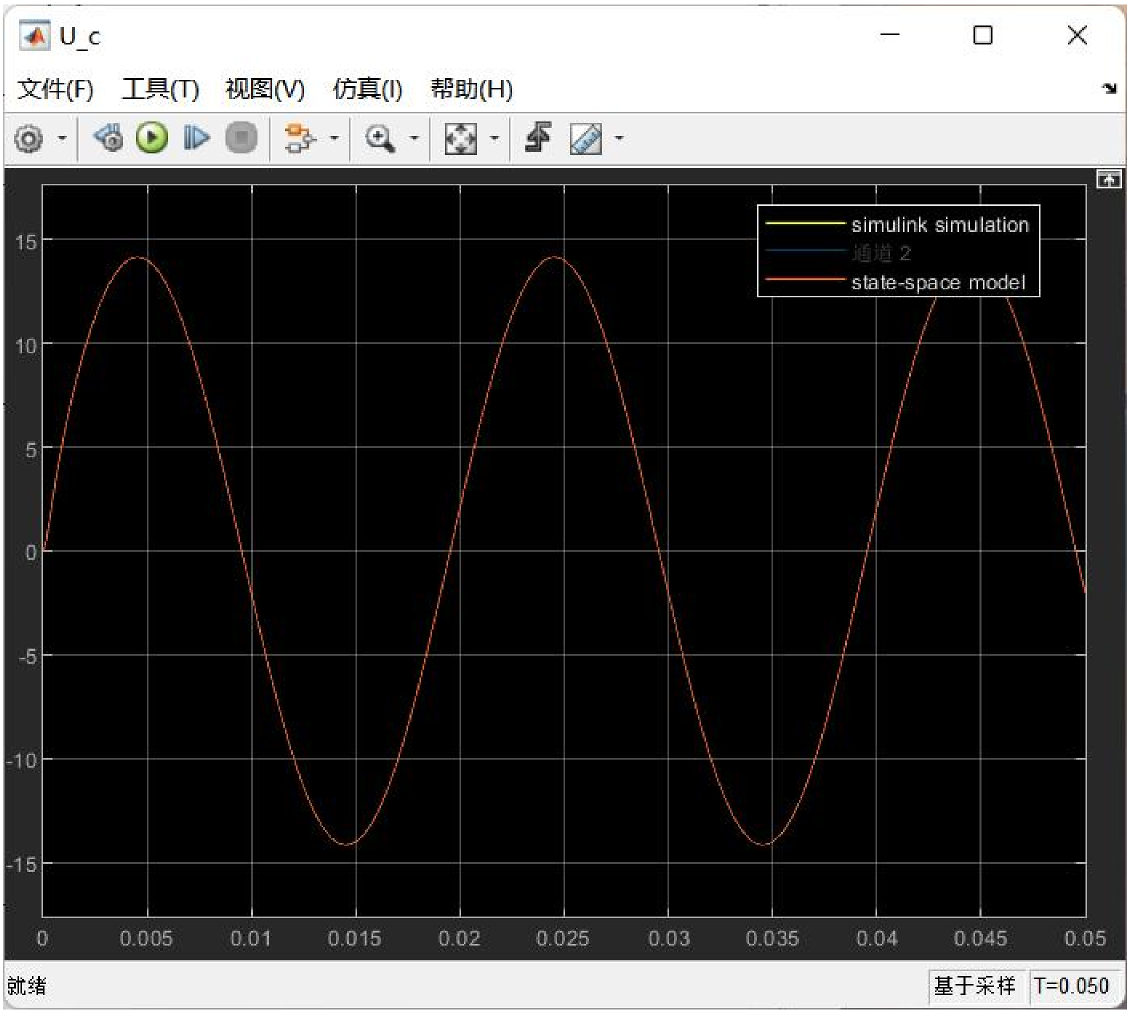


Figure 5 U_c of Simulink simulation of the circuit and the state-space model

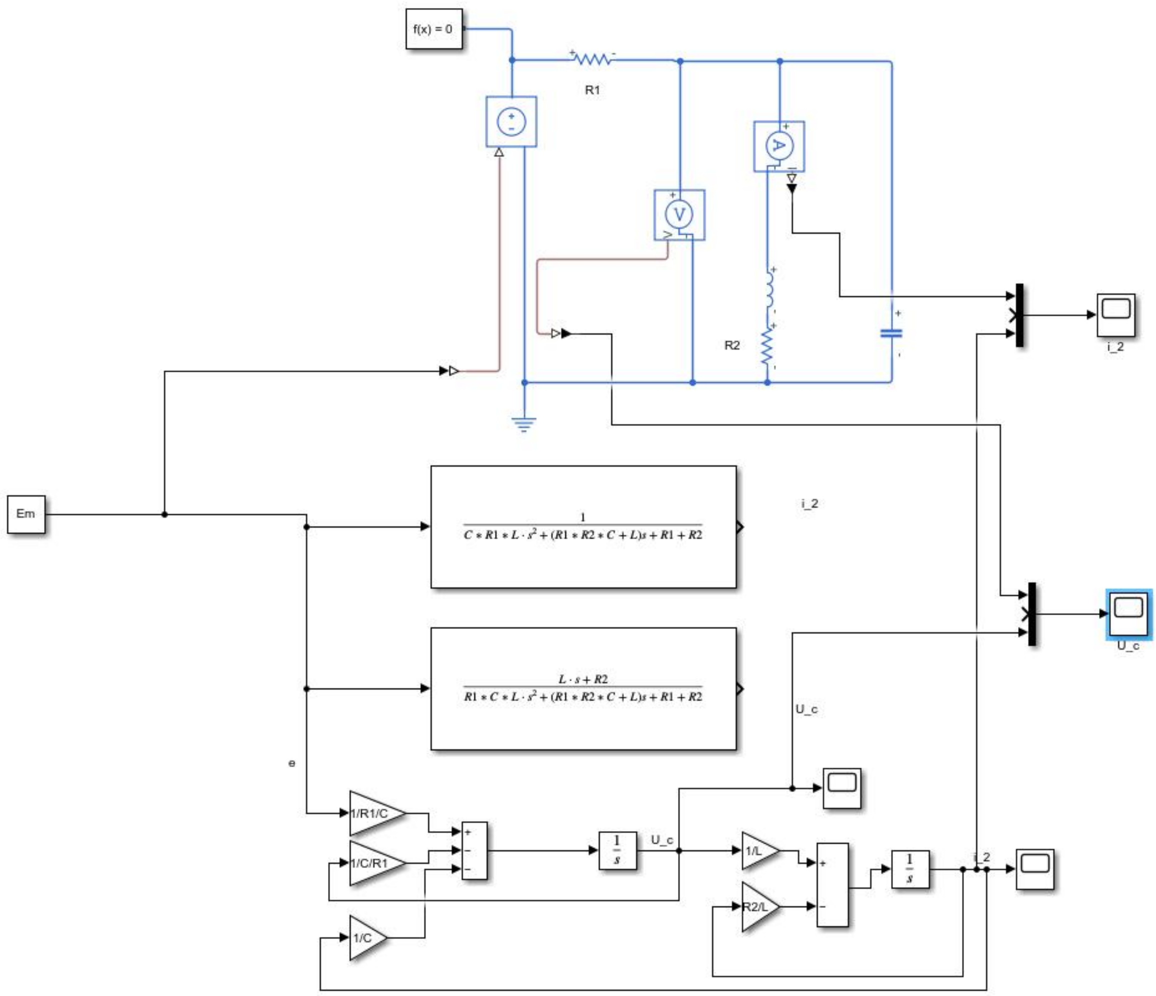


Figure 6 Modelling with constant voltage

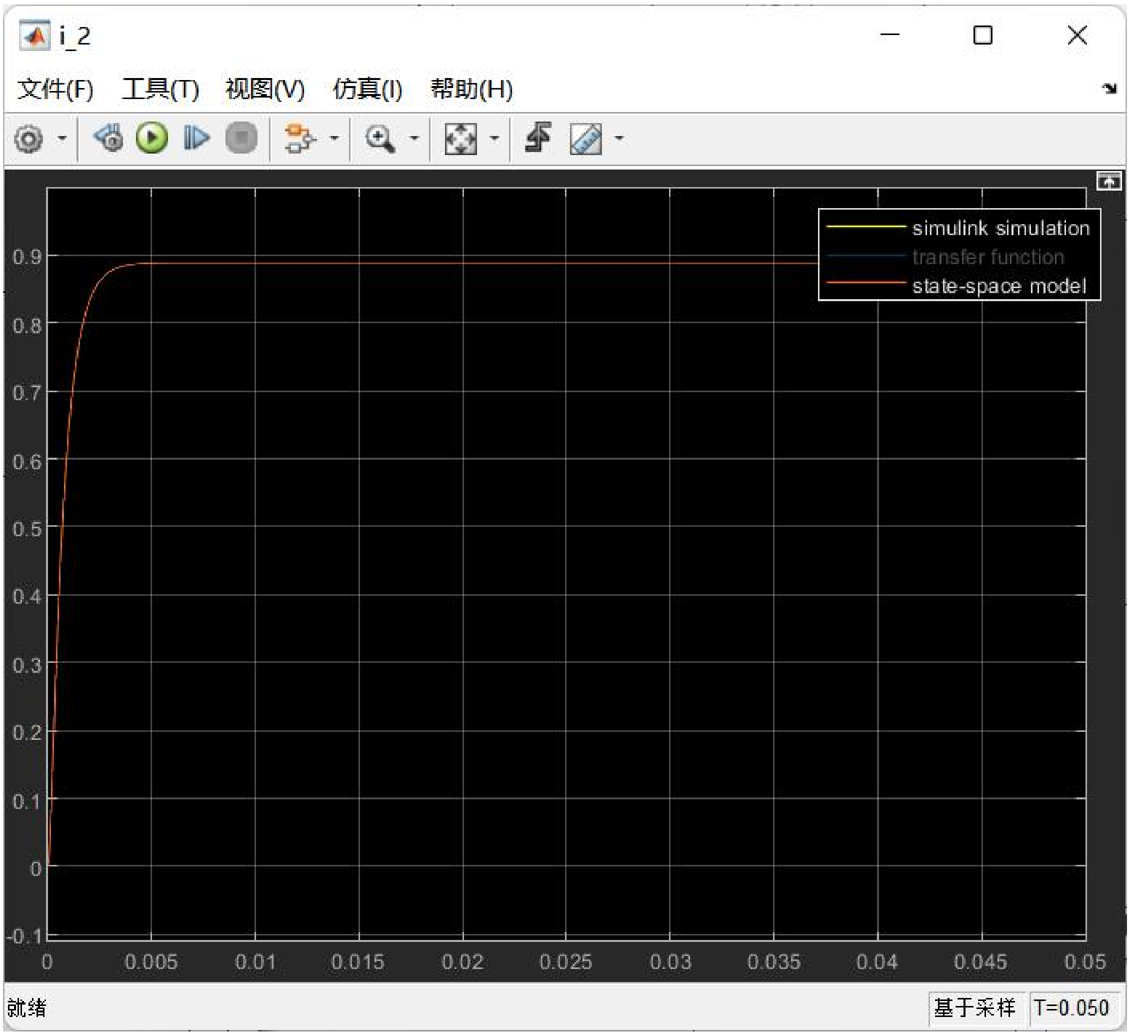


Figure 7 i_2 of Simulink simulation of the circuit and the state-space model

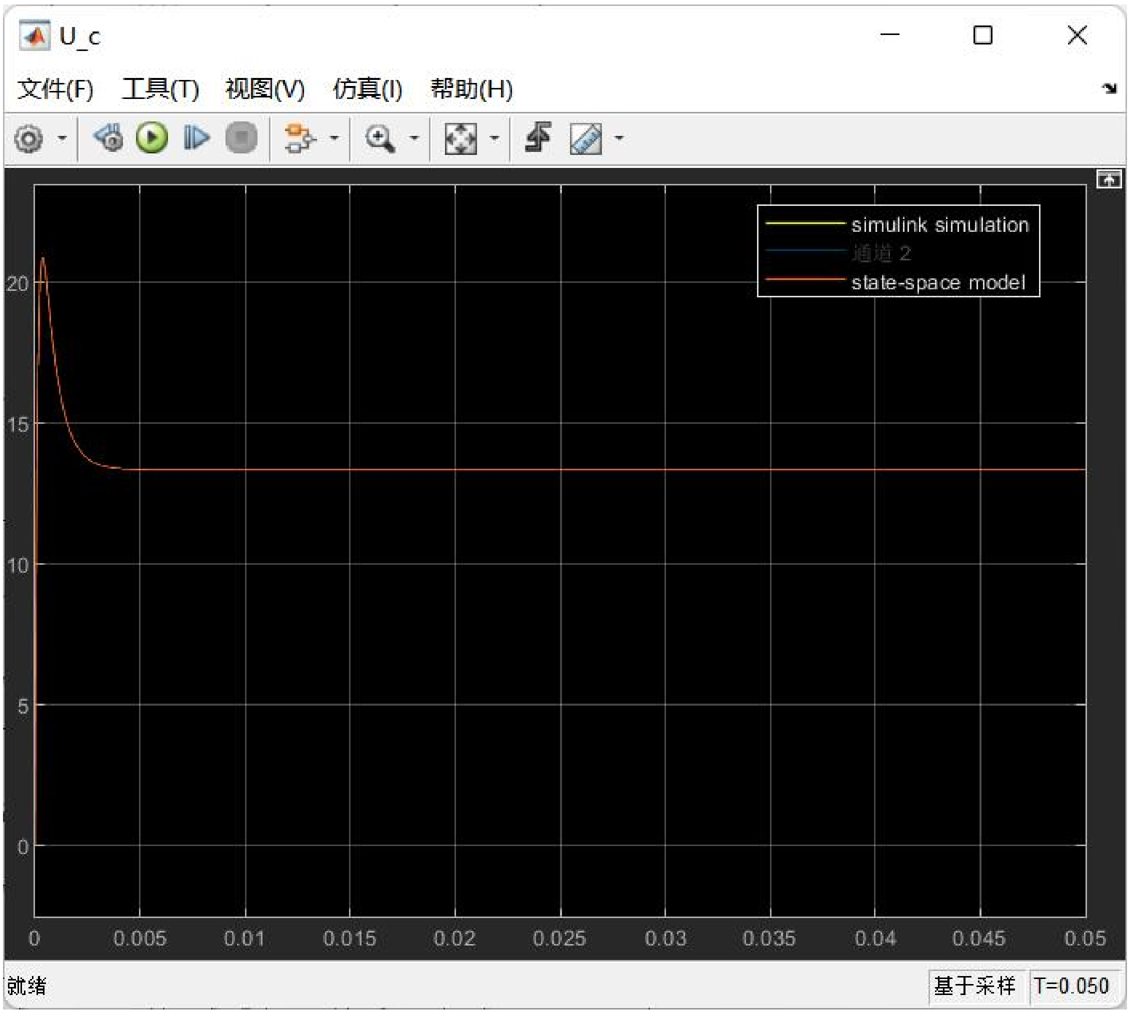


Figure 8 U_c of Simulink simulation of the circuit and the state-space model

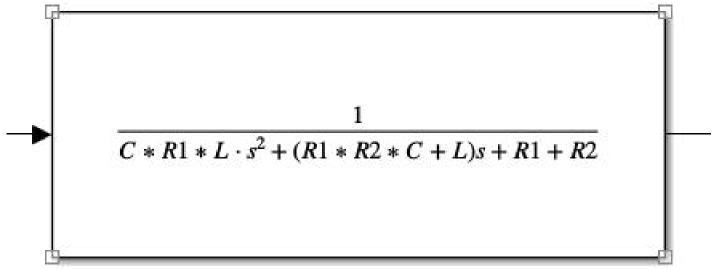
6. “Input-output" model.

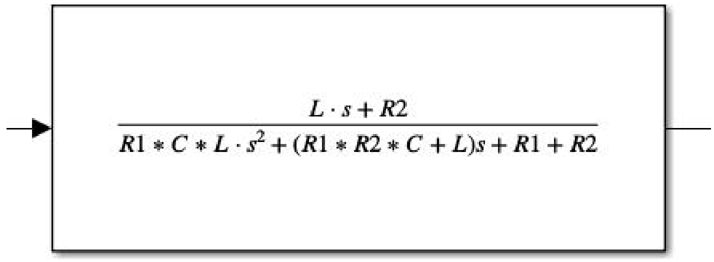
$$si_2 = \frac{U_C - i_2 R_2}{L}$$

$$sU_C = -\frac{i_2}{C} - \frac{U_C}{CR_1} + \frac{1}{CR_1}e$$

$$U_C(s) = \frac{LS + R_2}{CR_1LS^2 + (CR_1R_2 + L)S + R_1 + R_2}e(s)$$

$$i_2(s) = \frac{1}{CR_1LS^2 + (CR_1R_2 + L)S + R_1 + R_2}e(s)$$





7. Simulink simulation of the circuit and the resulting transfer functions using the predetermined input

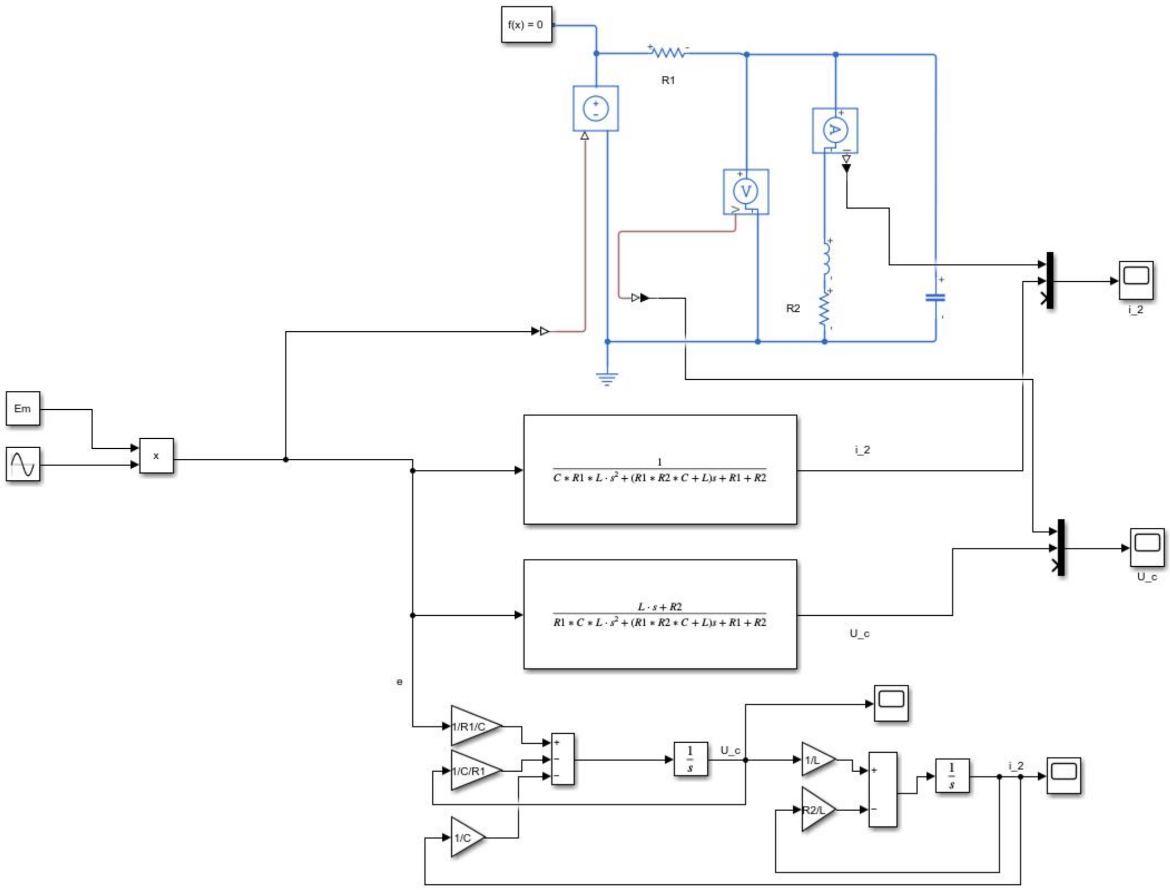


Figure 9 Modelling with sinusoidal input voltage

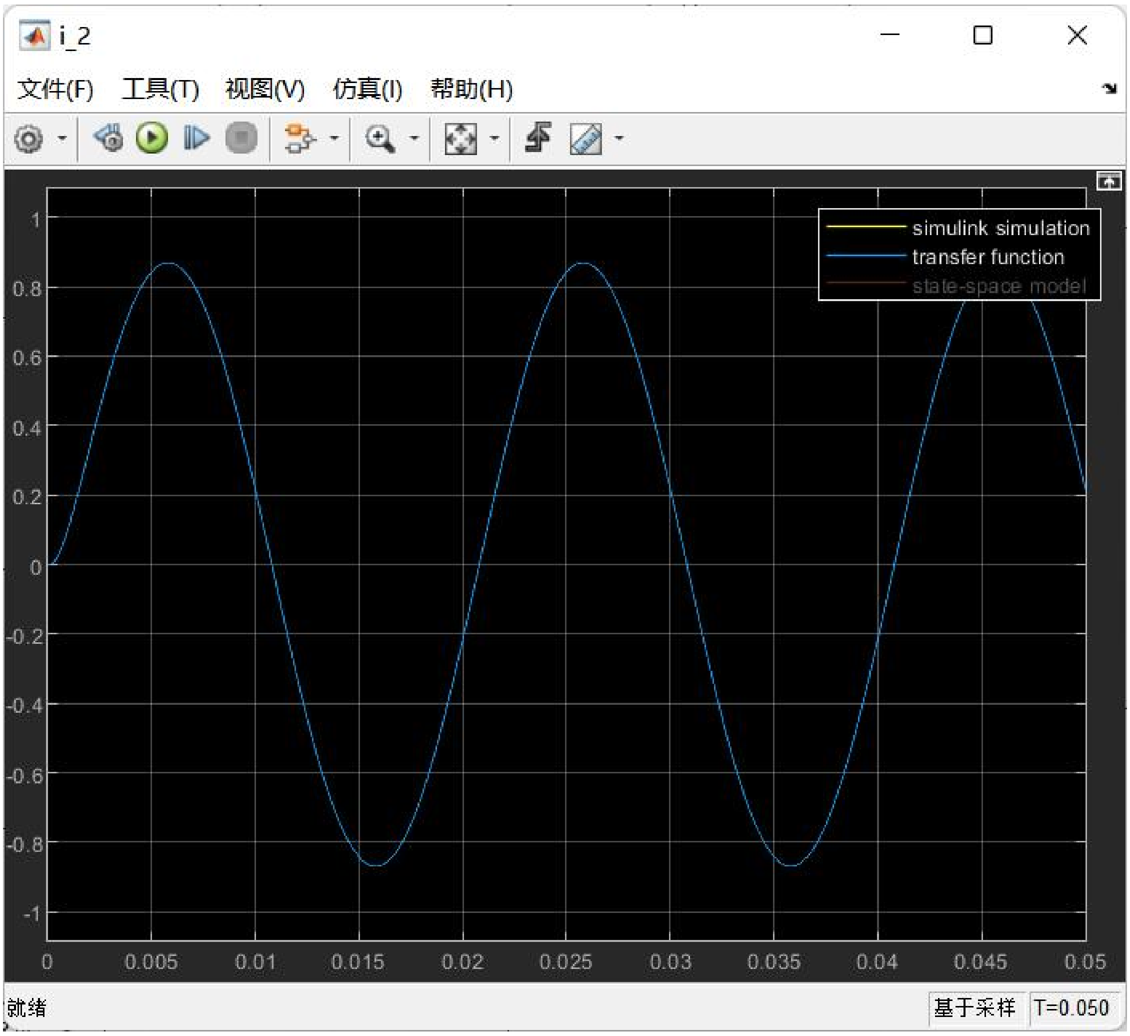


Figure 10 i_2 of Simulink simulation of the circuit and transfer functions

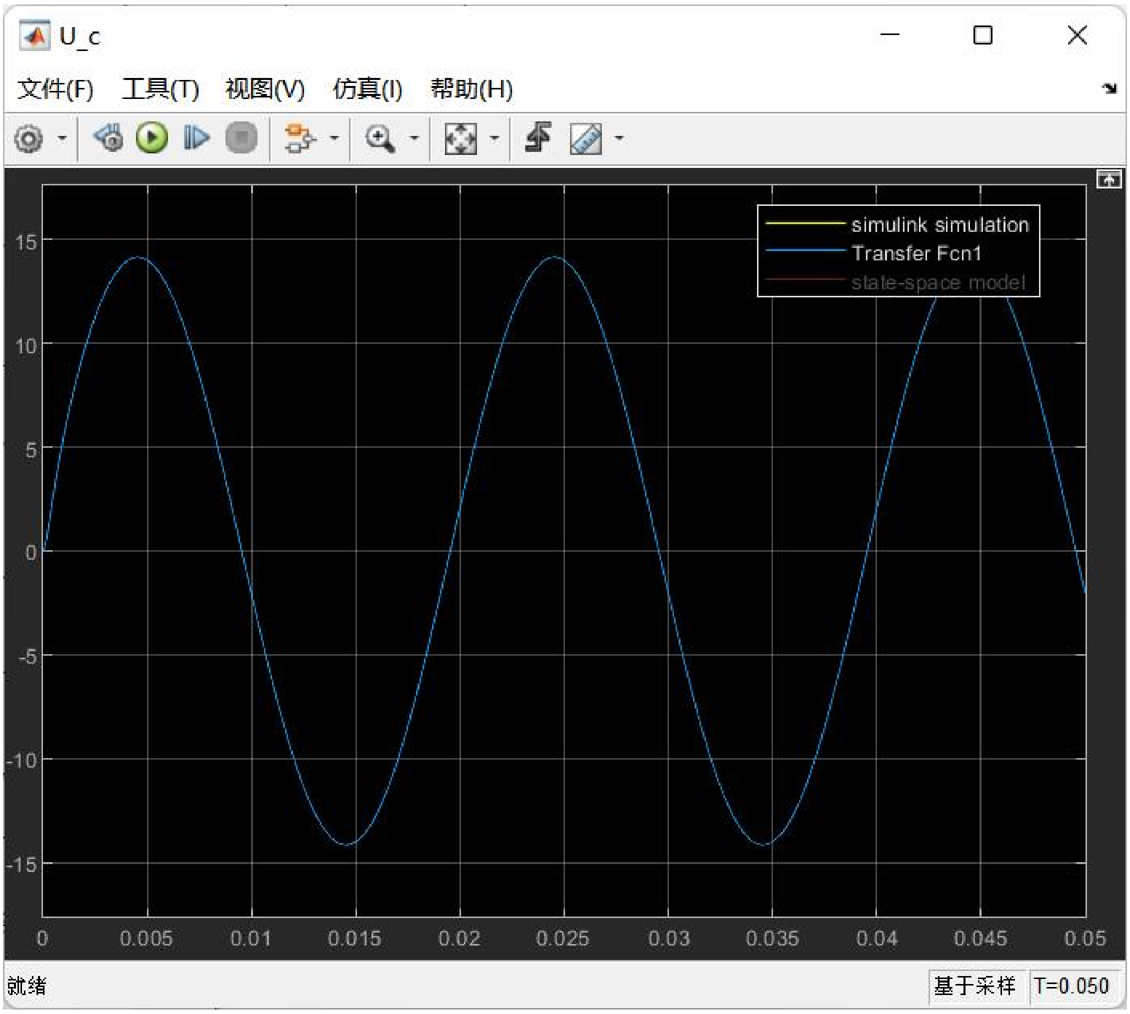


Figure 11 U_c of Simulink simulation of the circuit and transfer functions

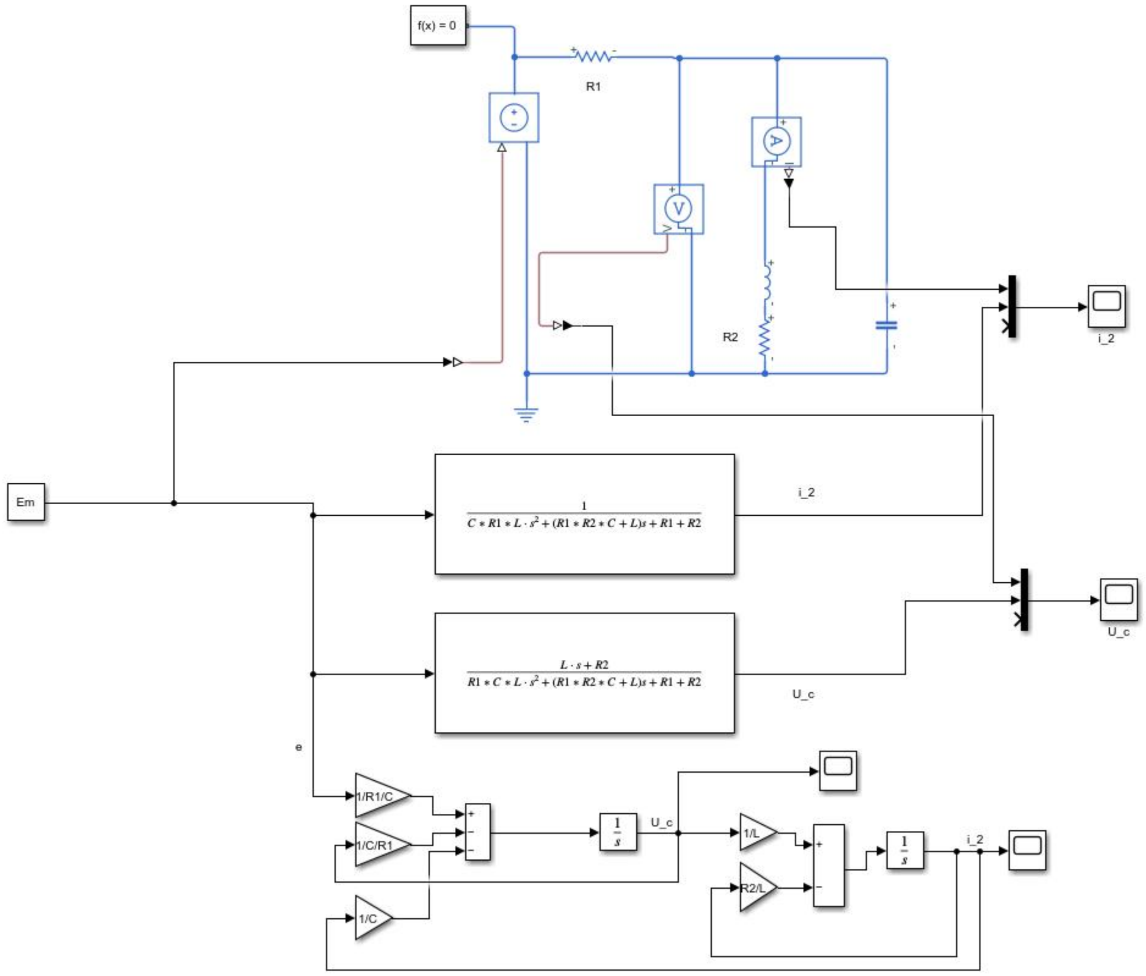


Figure 12 Modelling with constant voltage

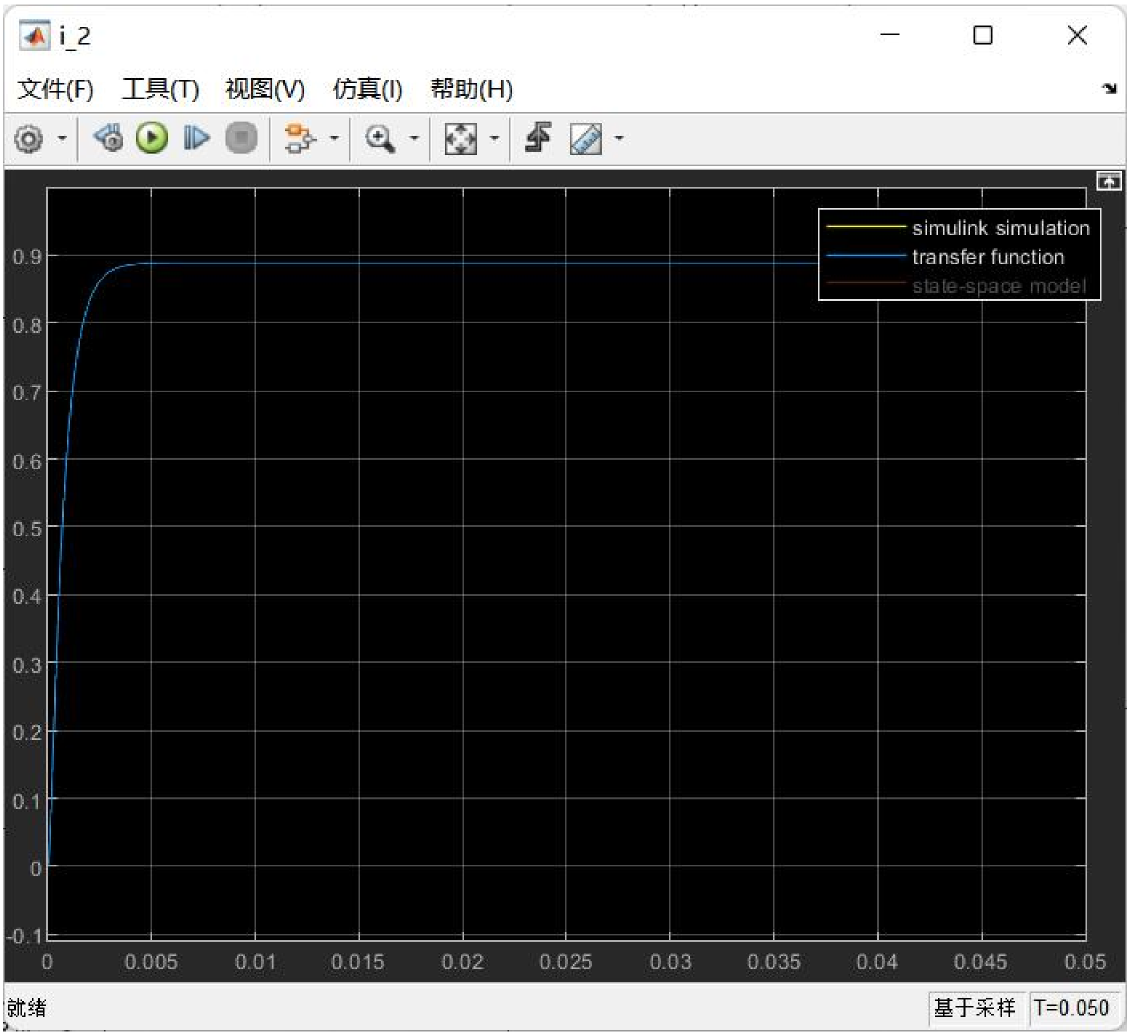


Figure 13 i_2 of Simulink simulation of the circuit and transfer functions

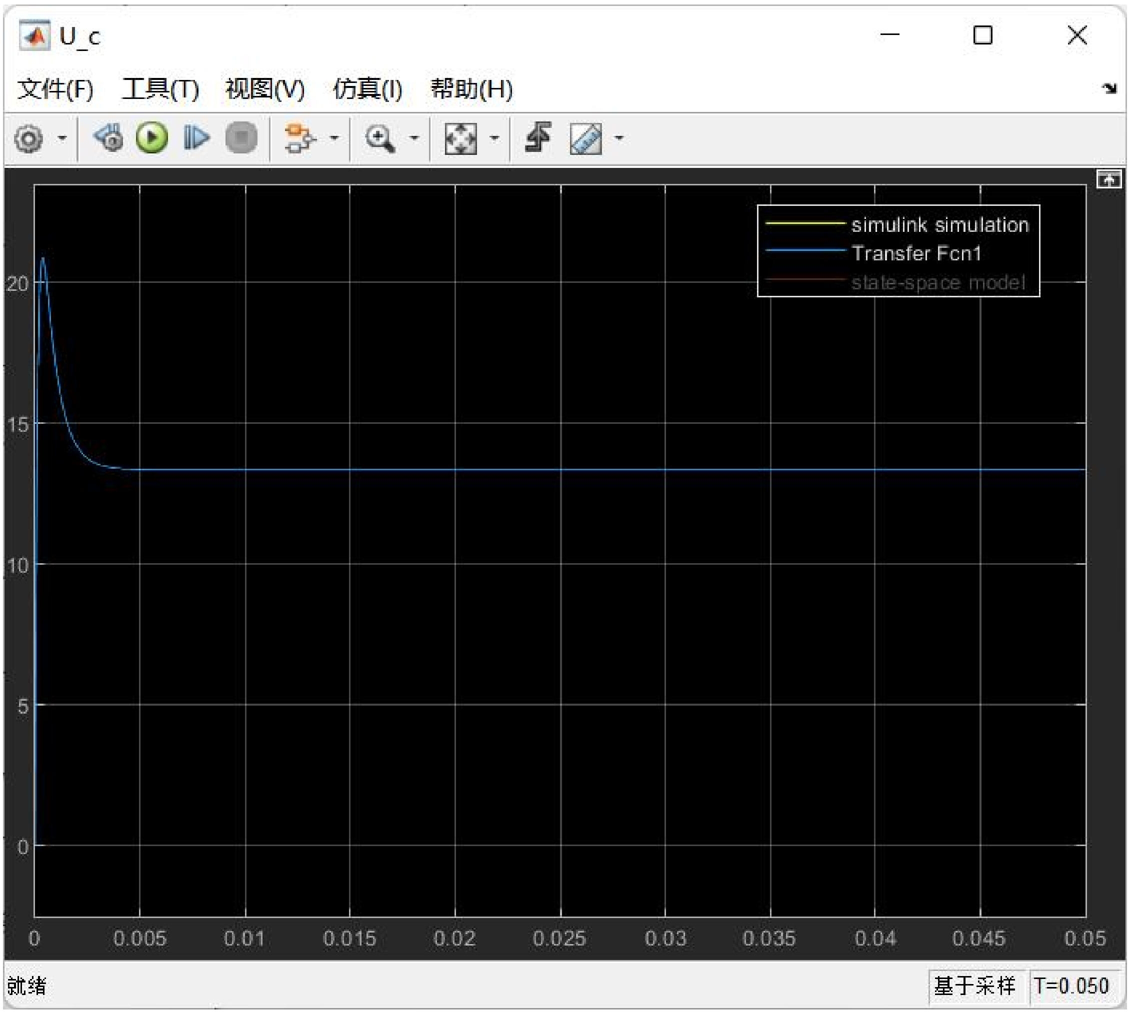


Figure 14 U_c of Simulink simulation of the circuit and transfer functions

8. Simulation of the circuit and the state-space model with zero input and non-zero initial conditions.

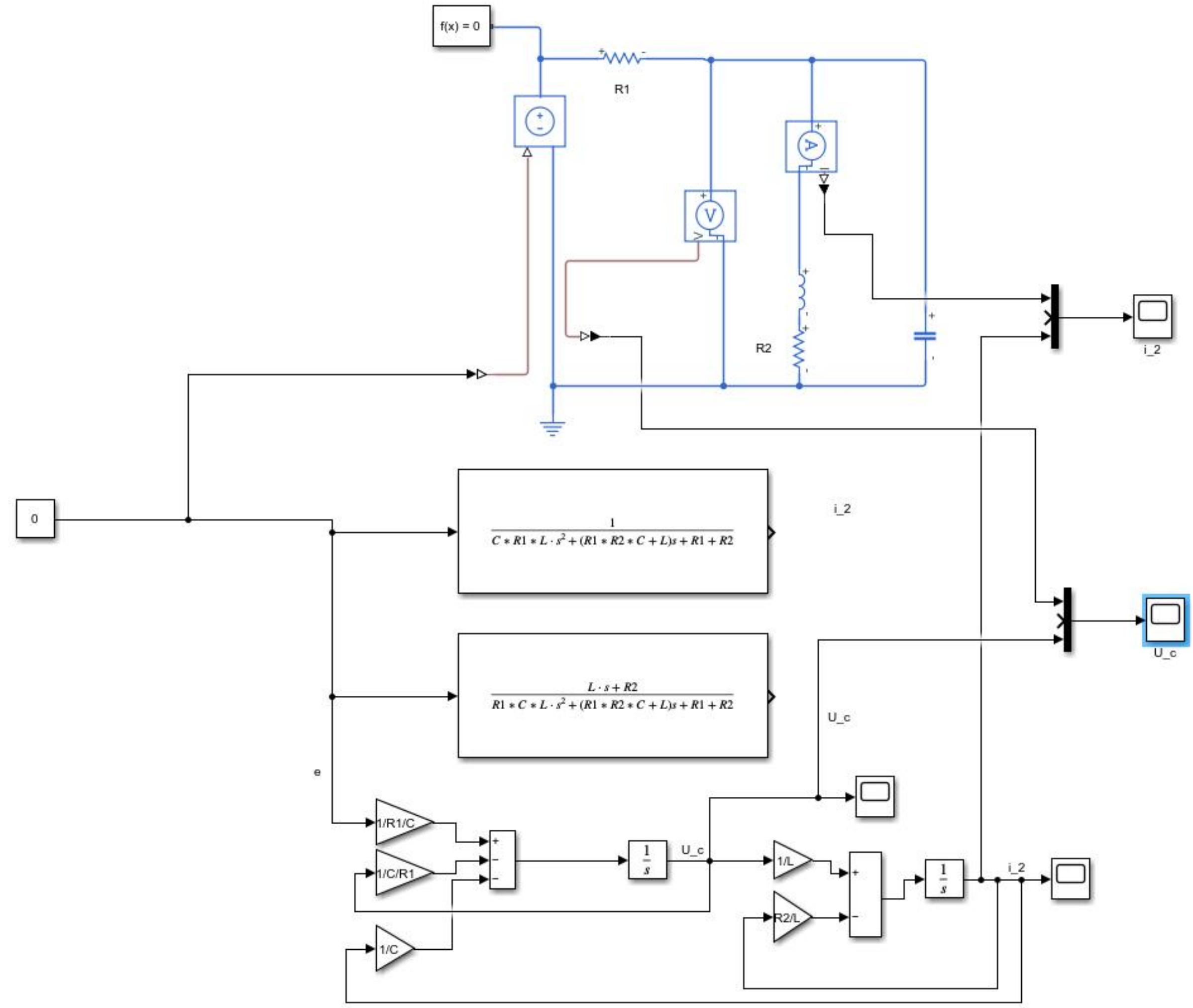


Figure 15 Modelling with zero voltage and nonzero initial conditions

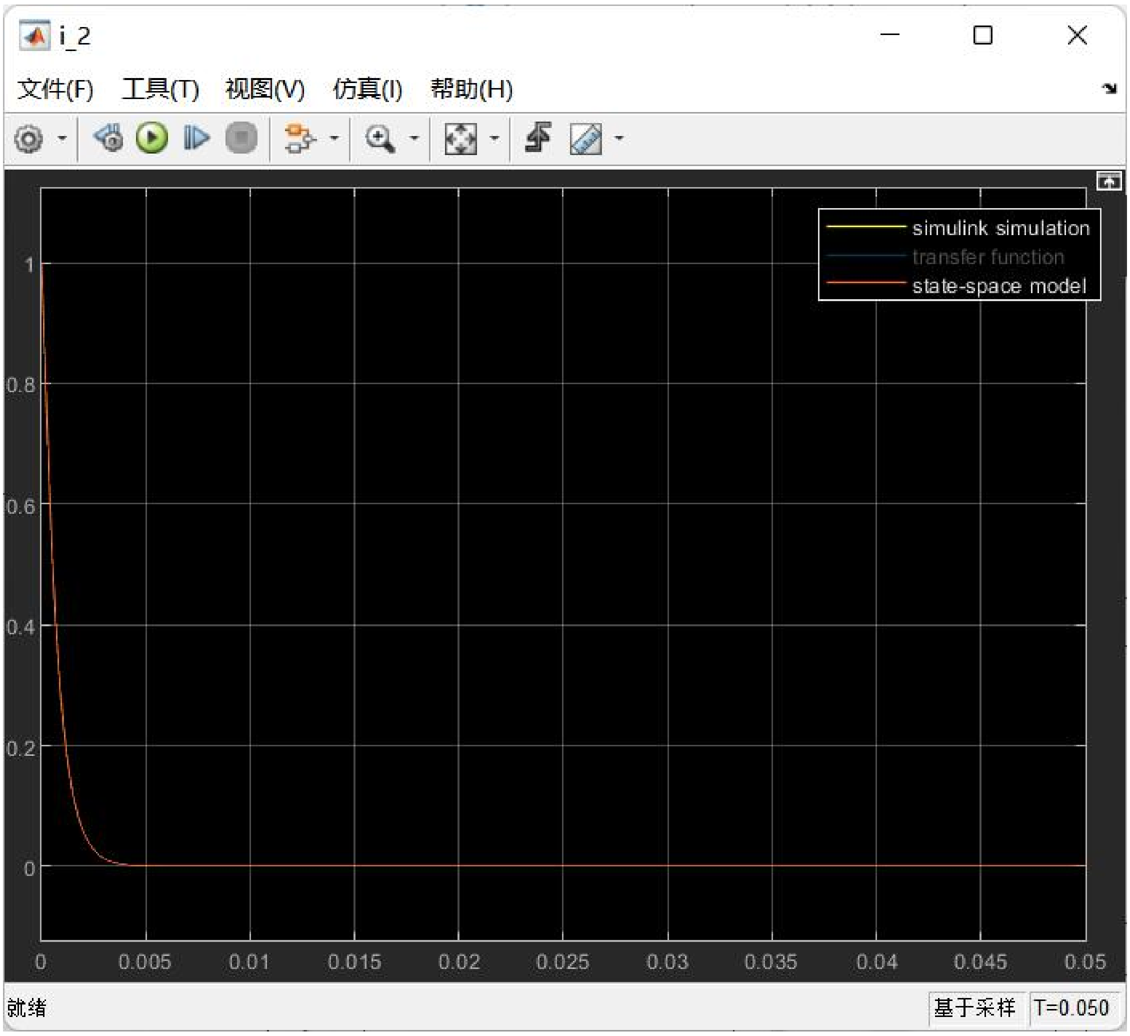


Figure 16 i_2 of Simulink simulation of the circuit and the state-space model

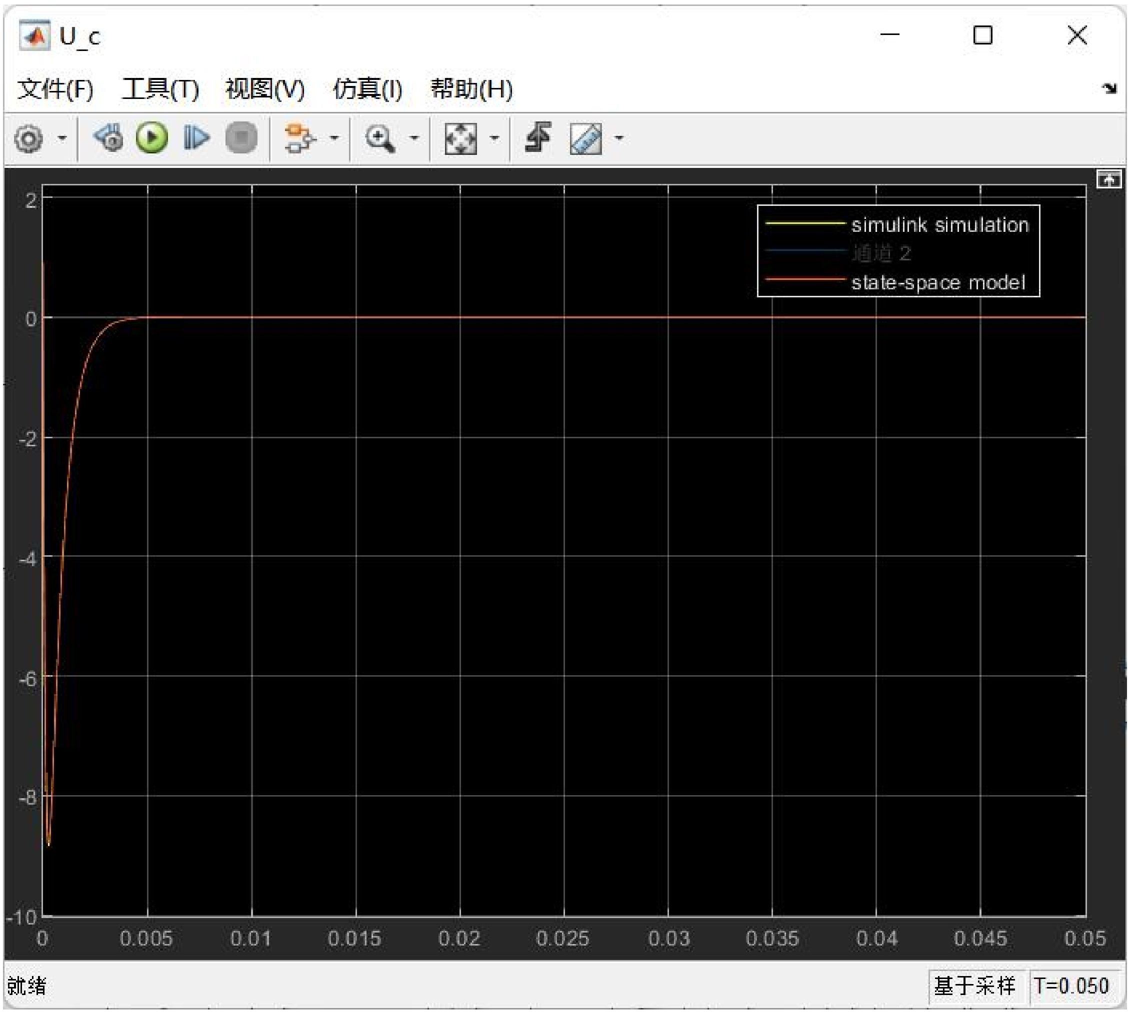


Figure 17 U_c of Simulink simulation of the circuit and the state-space model

Conclusions:

In this lab, I successfully implemented and simulated a linear electrical circuit using three different modeling approaches in Simulink: Simscape circuit, state-space representation, and transfer function.

Through the experiments, I observed that all three models produced similar transient and steady-state responses, confirming their theoretical equivalence in circuit analysis.

And I applied different types of input signals to the circuit, including sinusoidal voltage, constant voltage, and zero input with nonzero initial conditions.For sinusoidal voltage,The circuit responded with a steady-state sinusoidal output, maintaining the same frequency as the input but with an amplitude change and a phase shift. For constant voltage,The circuit exhibited transient behavior before reaching a steady-state condition.For zero input with nonzero initial conditions,I observed the system’s natural response, which depends entirely on the circuit’s initial energy stored in capacitors and inductors.