ITMO

Morphological Analysis

Image Processing

Morphological Analysis

Morphology

- ***Morphology is the science of form***.
- The word morphology (in the field of image processing) denotes mathematical methods of image analysis based on meaningful brightness-geometric models.
- The most significant achievements in the creation of a "general theory of form":
  - **J. Serra "mathematical morphology";**
  - **Yu.P. Pyt'ev "morphological analysis of images".**
- For a long time, these two morphologies were considered fundamentally different.

Morphology

- **What is a form?**
  - An informal definition (according to Yu.P. Pyt'ev) is what is present in all images of a given scene or object, regardless of the conditions of their registration.

Morphology

- **What is a form?**
  - The images show the same scene, but the brightness is different at each point of the field of view. The form is unchanged, defined as an invariant of image brightness transformations, simulating the change in the registration conditions.

Morphology

- **What is a form?**
  - The scenes are distinguished by the presence of an object:
    - on the right image there is no bead in the lower left quarte.

Morphology

- **Methods of morphological analysis** are methods for solving problems of recognition, classifying objects, evaluating objects parameters, highlighting differences in scenes from their images (signals). Based on the concept of a waveform.
- **Mathematical morphology** is based on set theory.

Basic Concepts of Set Theory

- $X = \{x\}, Y = \{y\}$ – sets;
- $Z = \{z: z \in X\ are\ z \in Y\} = X \cup Y$ – union of sets (elements of z belong to X or Y);
- $Z \subset X$ the set Z includes X;
- $Z = \{z: z \in X\ и\ z \in Y\} = X \cap Y$ – intersection of sets (elements of z belong to X and Y);
- $Z = X^C = \{z: z \notin X\}$ – complement of the set X;
- $Z = \{z: z \in X, z \notin Y\} = X \backslash \mathrm{Y}$ – difference of sets;
- $Z = \emptyset$ – empty set (contains no elements).

- **The ratios are valid:**
  - $(X \cup Y)^C = X^C \cap Y^C$;
  - $(X \cap Y)^C = X^C \cup Y^C$;
  - $X \backslash \mathrm{Y} = X \cap Y^C$.

Mathematical Morphology
(J. Serra)

- Let a Euclidean space E^N be given.
    - Consider some transformation $\Psi: E^N \to E^N$.
- An operator Ψ is called increasing if:
    - $X \subset Y \Longrightarrow \Psi(X) \subset \Psi(Y), X, Y \subset E^N$.
- The operator Ψ is called dilation if:
    - $\Psi(\cup X_i) = \cup \Psi(X_i), \forall X_i \subset E^N$.
- The operator Ψ is called erosion if:
    - $\Psi(\cap X_i) = \cap \Psi(X_i), \forall X_i \subset E^N$.

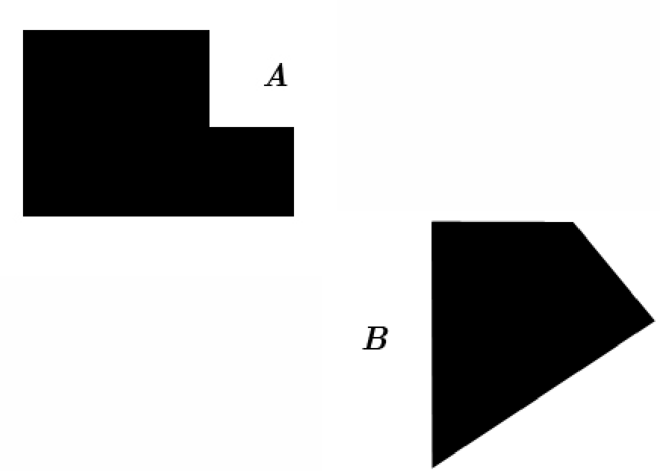
Mathematical Morphology

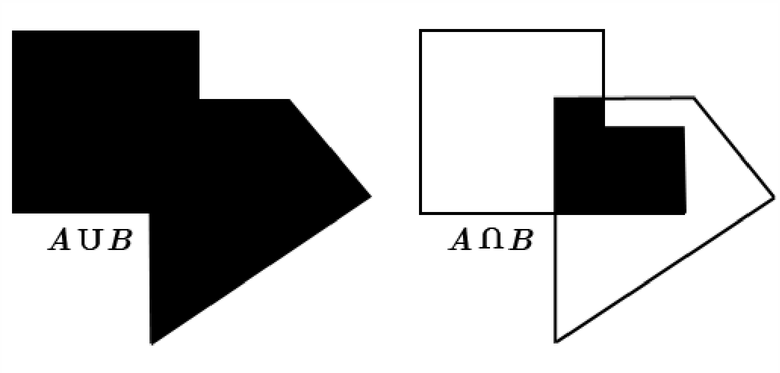
- If $\Psi(X) \supseteq X$ – operator Ψ is called extensive.
- If $\Psi(X) \subseteq X$ – operator Ψ is called anti-extensive.
- If $\Psi(\Psi(X)) \supseteq \Psi(X)$ – operator Ψ is increasing.
- If $\Psi(\Psi(X)) \subseteq \Psi(X)$ – operator Ψ is decreasing.
- If $\Psi\big(\Psi(X)\big) = \Psi(X)$ – operator Ψ is equivalent.

- ***Morphological filters*** are a set of operators that are both equivalent and increasing.

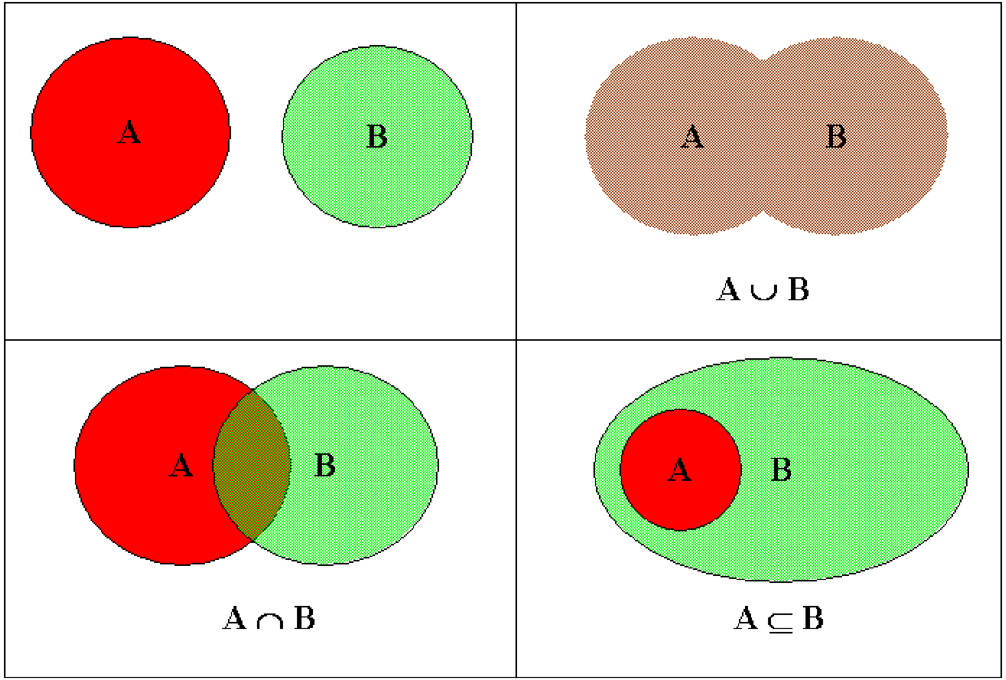
Morphological Operations
on Binary Images

- For binary images, logical operations are in one-to-one correspondence with operations on sets:
  - The *intersection* operation is logical *multiplication*;
  - The *union* operation is logical *addition*.





In the "style" of Euler's circles



Morphological Operations

on Binary Images

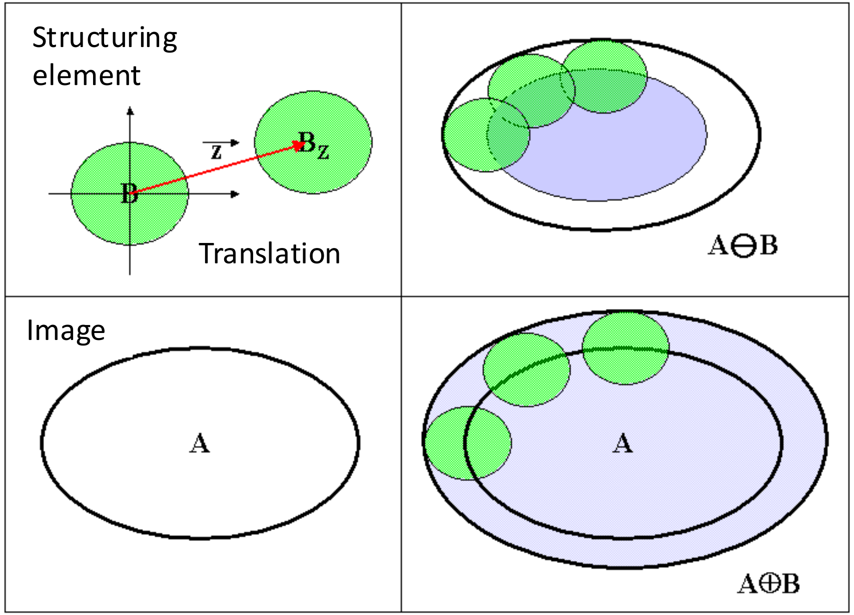
- We define the translation of a set *(two-dimensional binary image)* $A \subset E$ along some displacement vector $z \in E$ as a transformation:
$$A_z = \{q | a \in A, q = a + z\}.$$
- The addition of two-dimensional points (pixels) in this case is understood as the addition of their Cartesian coordinates.
- Let there be given two binary images $A, B \subset E$. Operation:
$$A \oplus B = \{a + b | a \in A, b \in B\} = \cup B_a = \cup A_b$$

is called the *Minkowski addition*. Operation:
$$A \ominus B = \{z | B_z \subseteq A\} = \cap A_z$$

is called the *Minkowski subtraction*.

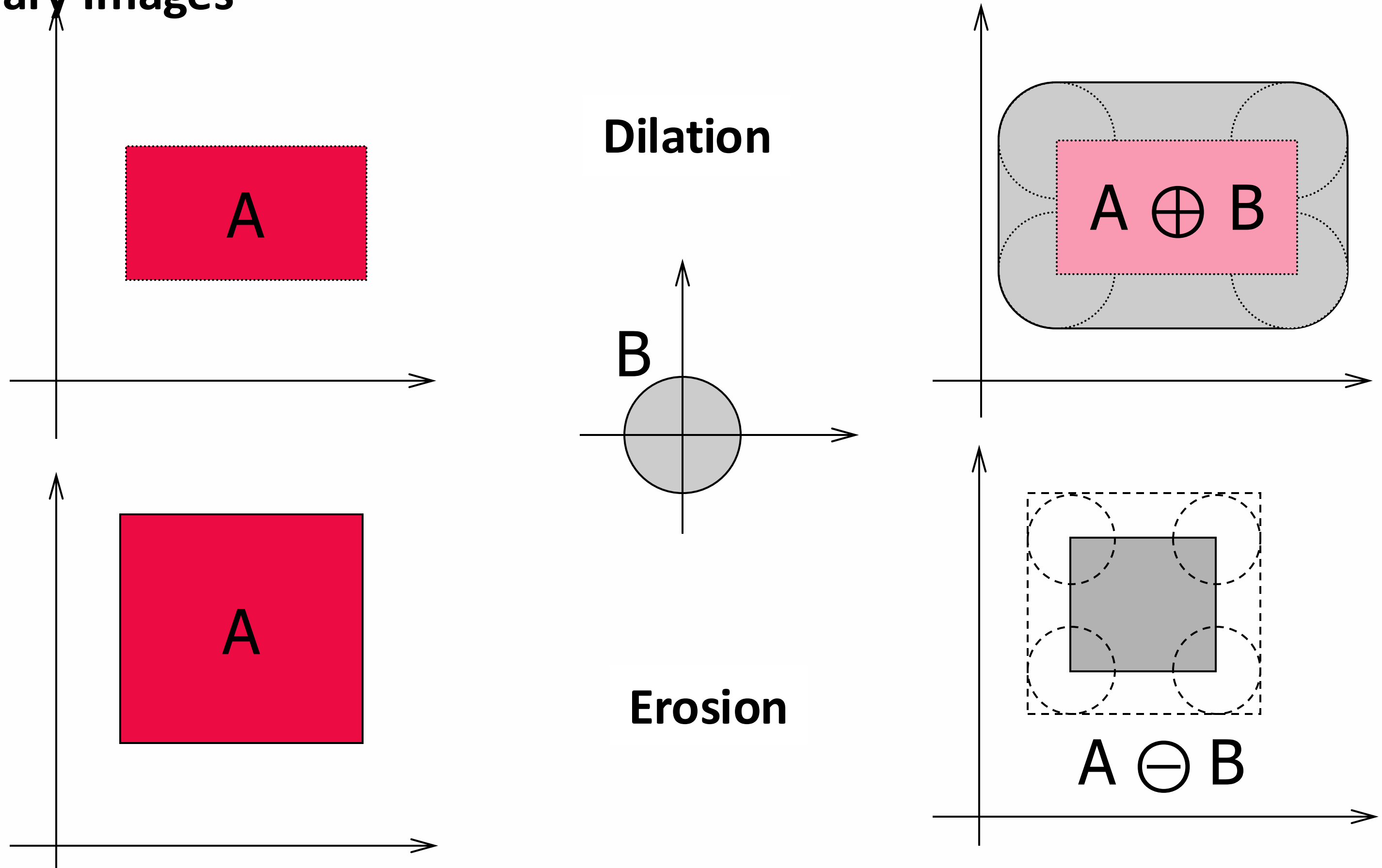
Morphological Operations
on Binary Images

- The set B is called a *structuring element*.
- Minkowski addition is called **dilation**, and subtraction is called **erosion** A by structuring element B.



Morphological Operations

on Binary Images



Morphological Operations
on Binary Images

- The basic operations of mathematical morphology are dilation and erosion by double relatives in relation to each other:

$$A \ominus B = (A^C \oplus B^V)^C,$$

where A^C is the complement to A, and $B^V = \{-b|b \in B\}$.

- Thus, all theorems proved for one operation can be represented in the double form of another operation.

Morphological Operations

Properties

- Commutability property:
  - $A \oplus B = B \oplus A$;
  - $A \ominus B \neq B \ominus A$.
- Associativity property:
  - $A \oplus (B \oplus C) = (A \oplus B) \oplus C$;
  - $A \ominus (B \oplus C) = (A \ominus B) \ominus C$.
- Distribution property:
  - $(\cup A_i) \oplus B = \cup (A_i \oplus B), (\cup A_i) \ominus B = \cup (A_i \ominus B)$;
  - $(\cap A_i) \oplus B = \cap (A_i \oplus B), (\cap A_i) \ominus B = \cap (A_i \oplus B)$.
- Scale invariance property:
  - $\lambda A \oplus \lambda B = \lambda(A \oplus B)$;
  - $\lambda A \ominus \lambda B = \lambda(A \ominus B)$.

Morphological Operations
on Binary Images

- ***Materon's theorem****:* any translation-invariant increasing operator Ψ, can be represented as a union of erosion:

$$\Psi(X) = \cup\, X \ominus B, B \in k(\Psi),$$

where $k(\Psi)$ – is the kernel of $\Psi(X)$, i.e. a set of structuring elements B such that $\Psi(B)$ contains the origin, X – an image with a structuring element B.

- The dual form of Materon's theorem:

$$\Psi(X) = \cap\, X \oplus B, B \in k(\Psi^*),$$

where $\Psi^*(X) = \Psi(X^C)^C$.

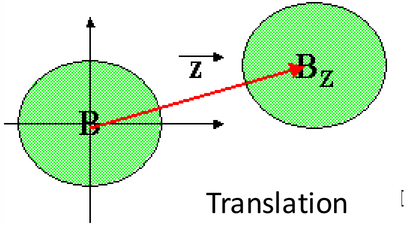
- Any morphological filter can be represented as a union of erosions or intersection of dilations.

Basic Morphological
Operations

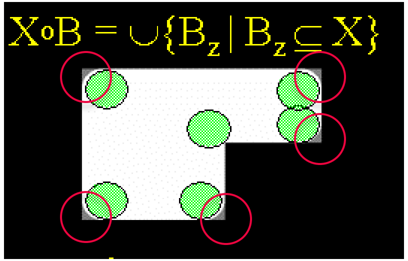
- The operation of ***open*** X to B is a sequential application of erosion and dilation:
$$X \circledcirc B = (X \ominus B) \oplus B.$$
- The *open* operator is anti-extensive and increasing.
- The operation of ***close*** X to B is a sequential application of dilation and erosion:
$$X \odot B = (X \oplus B) \ominus B.$$
- The *close* operator is extensive and increasing.
- *Both* operators are equivalent and, by definition, are morphological filters.

Basic Morphological
Operations

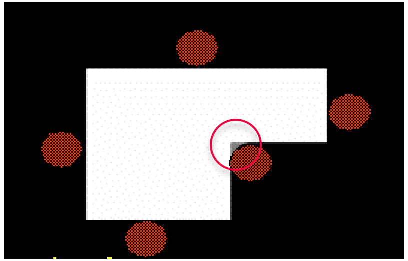
Structuring element



Source Image



Open



Close

Filtering Example

- An example of filtering: the image shows a rectangular object with "shape defects" such as internal "holes" and external "protrusions".

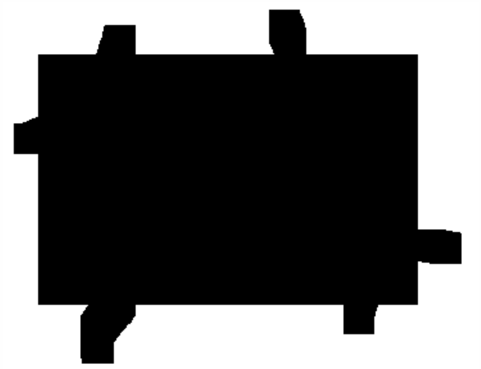
- We will use a rectangular structuring element, since the object is rectangular.

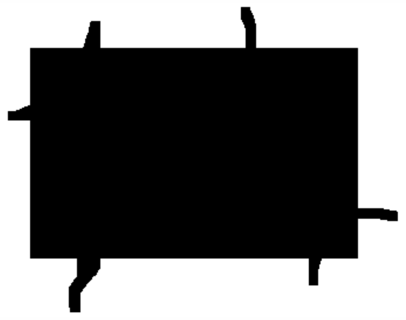
Filtering Example

- Consider removing external shape defects, using open:
  - At the first stage, erosion is performed, which removes ("eats") the external "protrusions". In this case, the external size of the object decreases, and internal defects increase.
  - After applying the dilation, the object's shape is restored.
  - As a result of the open operation as a whole, the external dimensions and shape of the object are restored, but the internal shape defects remain.

Filtering Example

- Consider removing internal shape defects, using close:
  - At the first stage, the dilation is performed, which removes (or "fills") the internal "holes" and "channels". In this case, the external size of the object increases along with the protrusions.
  - After applying erosion, the object's shape is restored.
  - As a result of the close operation, the external dimensions and shape of the object are restored, but the external shape defects remain.



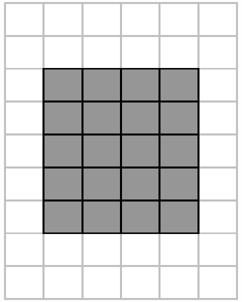


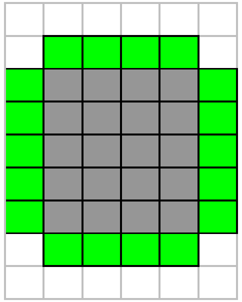
Filtering Example

- Elimination of both "holes" and "protrusions":
  - Sequential application to the original image of the open, and then to the result – the close operations with the same rectangular structuring element:

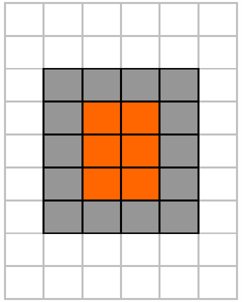
Basic Operations

A



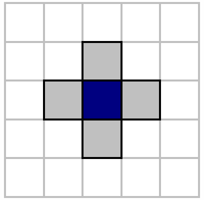


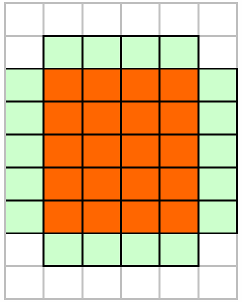
Dilation



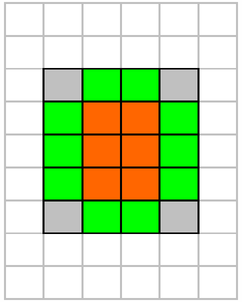
Erosion

B





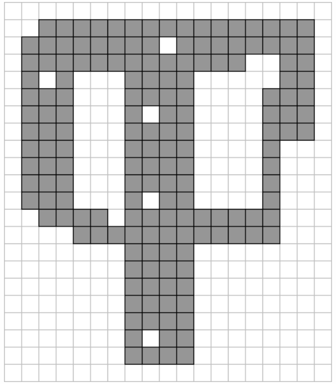
Close



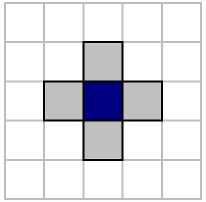
Open

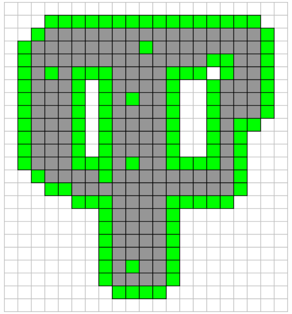
Basic Operations

A

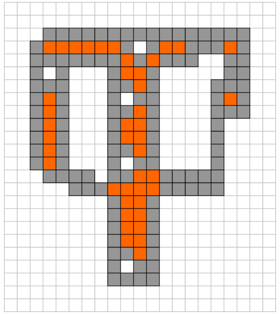


B

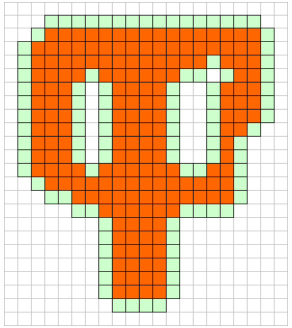




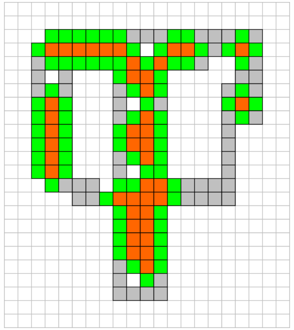
Dilation



Erosion



Close



Open

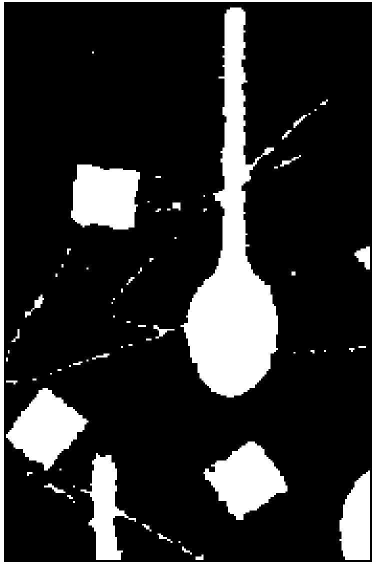
Filtering Example

- Let an object with internal defects be given:

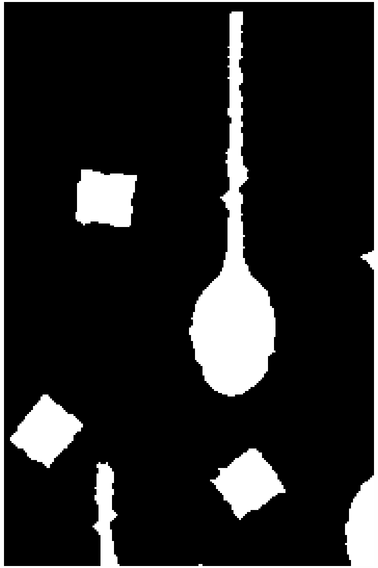
Erosion Example

- Result with different structuring element:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & [1] & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



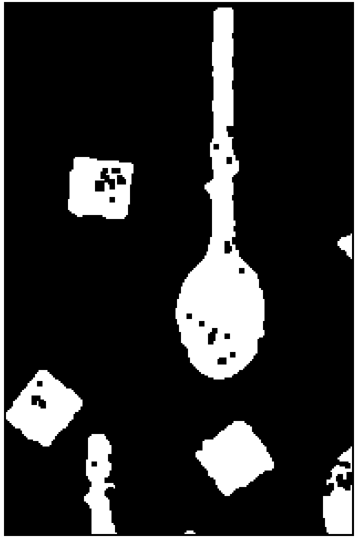
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & [1] & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



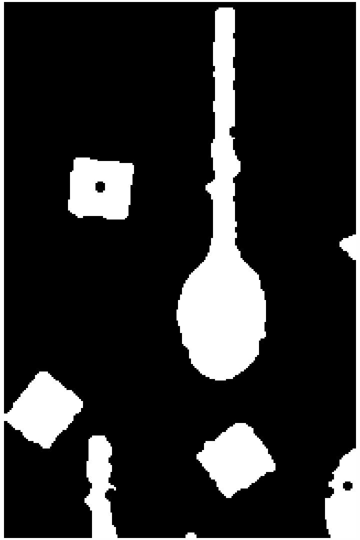
$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & [1] & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Close Example

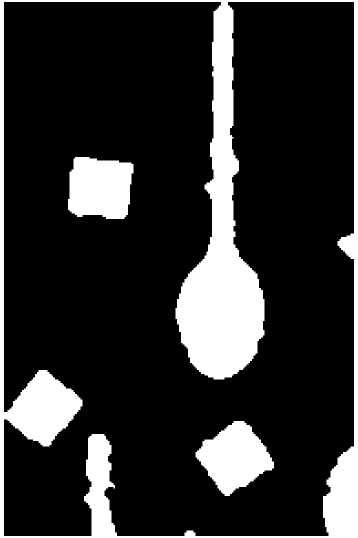
- Result with different structuring element:



$$\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$



$$\begin{bmatrix}0&1&1&1&0\\1&1&1&1&1\\1&1&1&1&1\\1&1&1&1&1\\0&1&1&1&0\end{bmatrix}$$

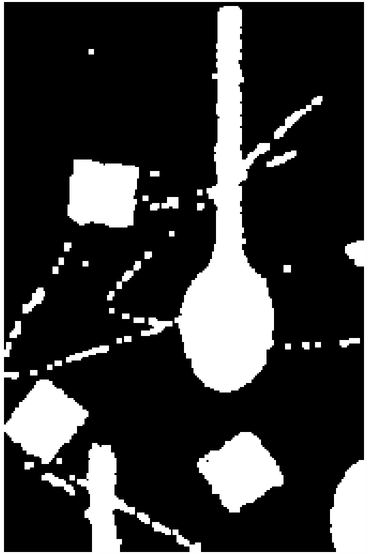


$$\begin{bmatrix}0&0&1&1&1&0&0\\0&1&1&1&1&1&0\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\1&1&1&1&1&1&1\\0&1&1&1&1&1&0\\0&0&1&1&1&0&0\end{bmatrix}$$

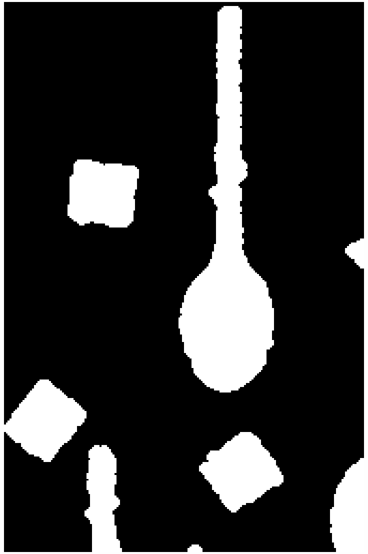
Open Example

- Result with different structuring element:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



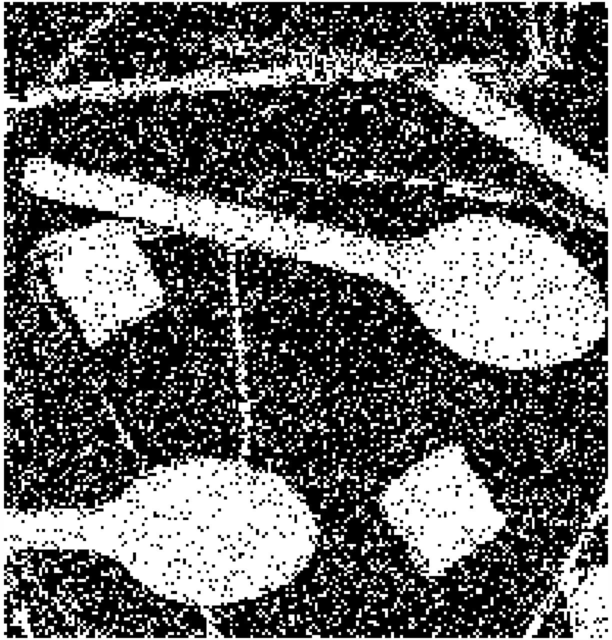
$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

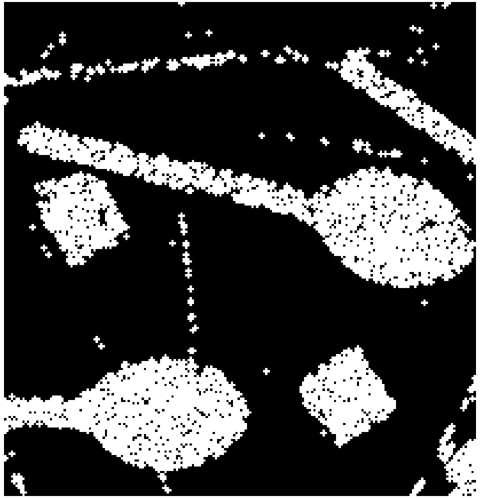
Open Example

- Let a very noisy object be given:

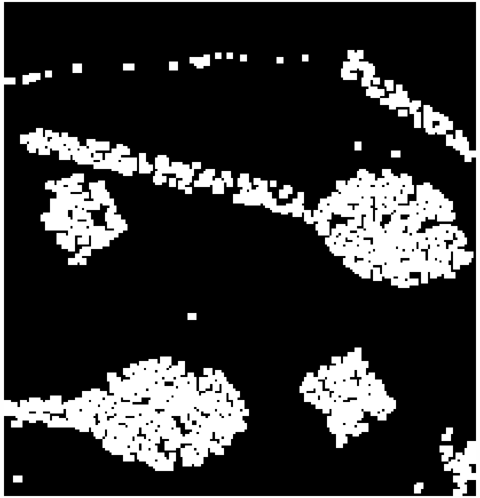


Open Example

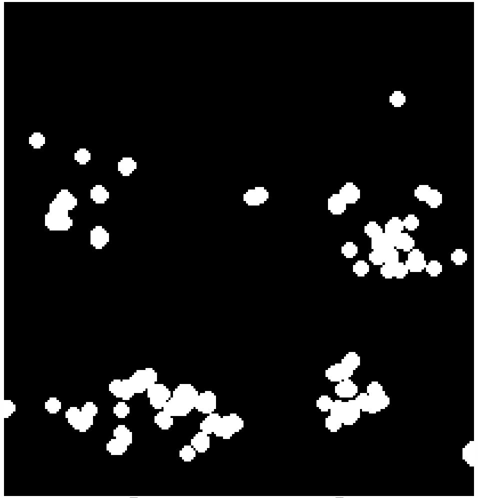
- Result with different structuring element:



$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Thinning Lines

A

B $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Removing background noise (erosion)

Removing dark spots on the print (open)

Thinning Lines

A

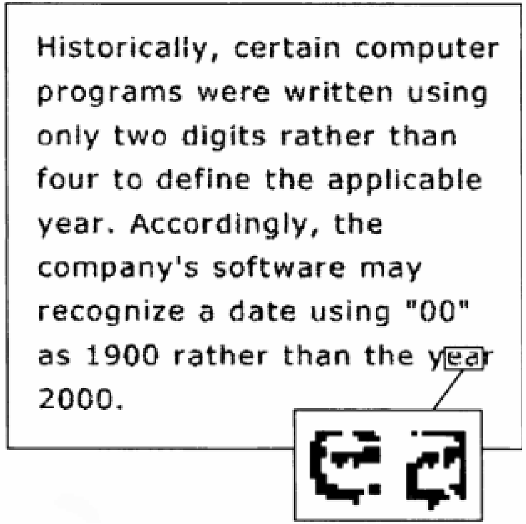
B $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

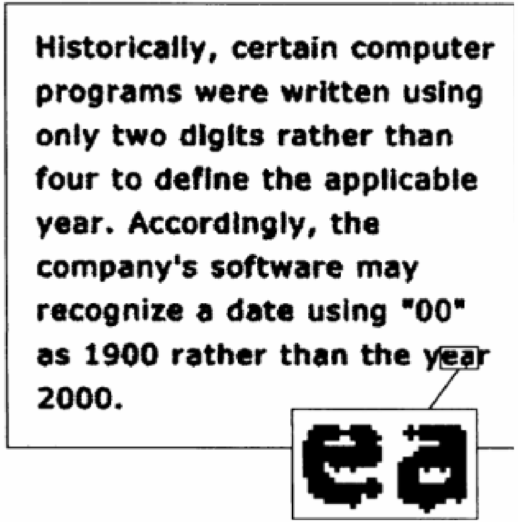
Removing gaps in prints (open then dilation)

Restoring the width of the print stripes (erosion)

Eliminating Gaps

- Let it be known that the maximum gap length is two pixels.
- Let's select a structuring element: $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, and apply dilation.


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

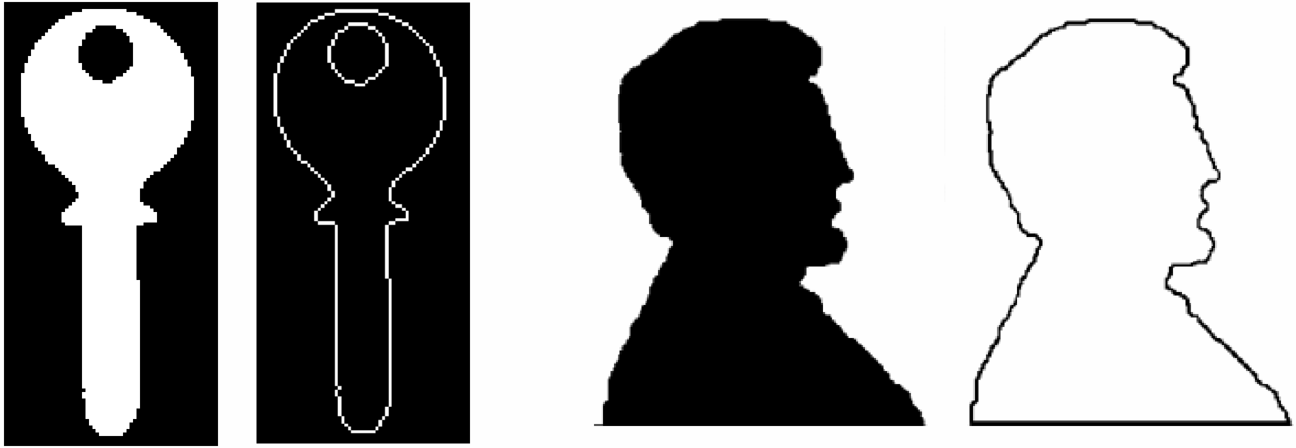
Contour Formation

- Example: morphological selection of the binary image border.
- Formation of the inner contour:

$$C = A - (A \ominus B).$$

- Formation of the outer contour:

$$C = (A \oplus B) - A.$$

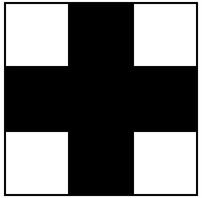


Morphological Skeletonization

- $skel(A)$ – the erosion of the image A by a structuring element *B:*

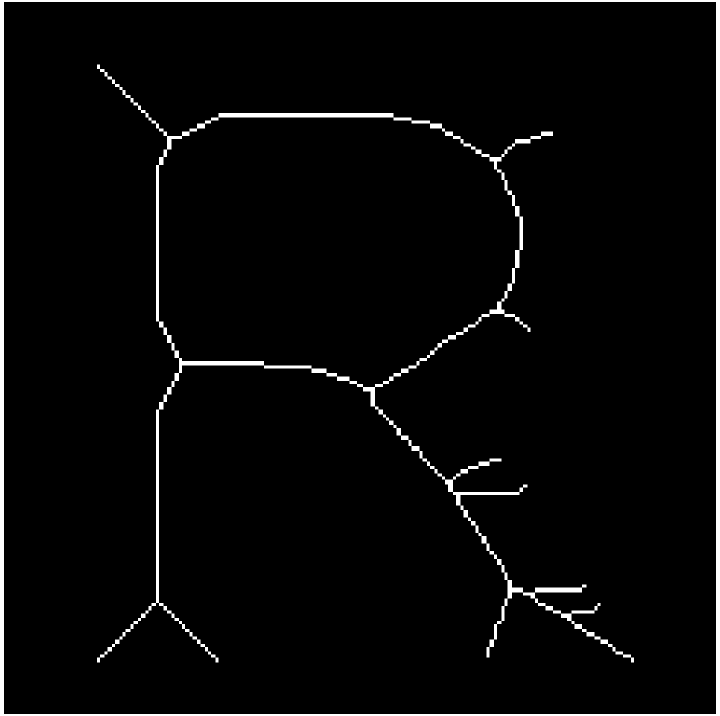
$$E_n(A) = A \ominus nB,$$
$$K_n(A) = E_n(A) - (E_n(A) \ominus B_1) \oplus B_1,$$

where B_1 has the form:

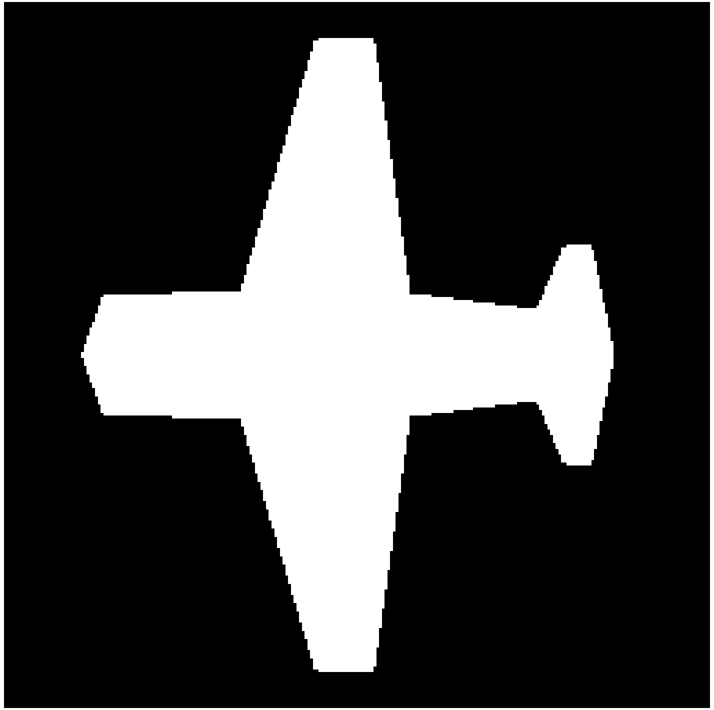


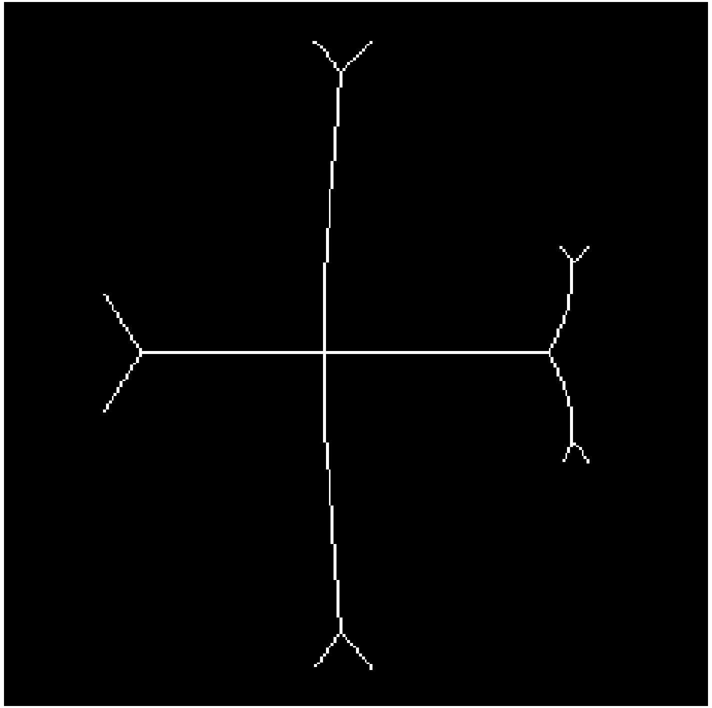
$skel(A) = \cup\, K_n(A), n = 0 \ldots N-1$, where N is such that $E_{N-1} \neq \emptyset$, and $E_N = \emptyset$

Morphological Skeletonization

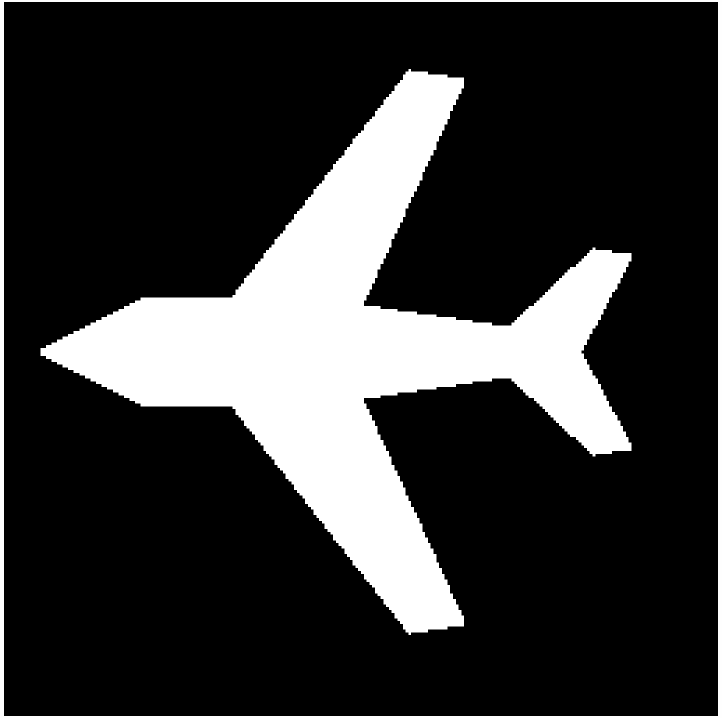


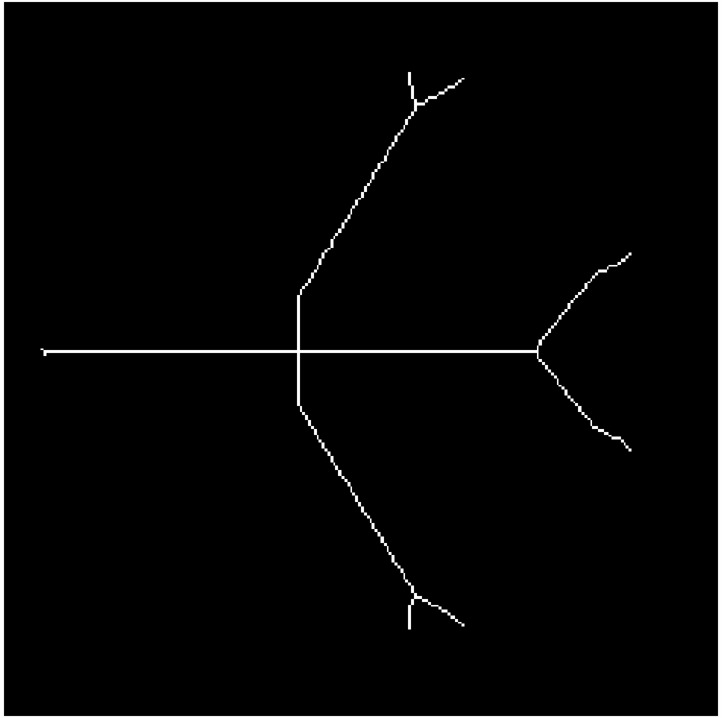
Morphological Skeletonization





Morphological Skeletonization



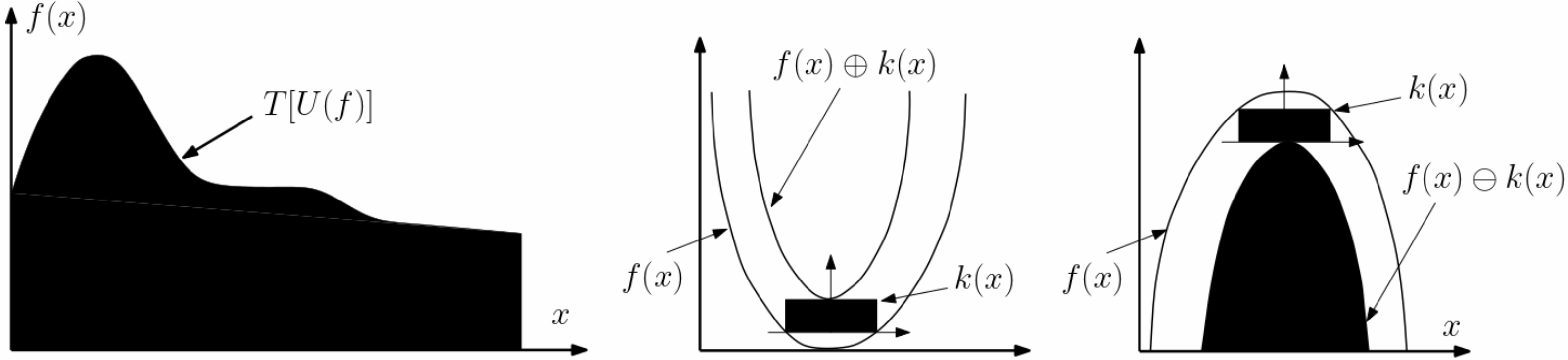


Morphology of
Grayscale Images

- Image as a function $f: F \to E, F \subset E^{N-1}$,
  - where N – is the dimension of space (in the case of two-dimensional images, $F \subset E^2$), sets the intensity of the image on F.
- *The shadow* f is the set $U(f) \subset F \times E$, defined as:
$$U = \{(x, y) \in F \times E | y \leq f(x)\}.$$
- *The surface of the set* $A \subseteq F \times E$ is the set $T[A]: F \to E$, defined at each point as:
$$T[A](x) = \max y\,, (x, y) \in A.$$
- The relationship between shadow and surface:
$$T[U(f)] = f.$$

Morphology of
Grayscale Images

- Geometric representation of shadow and surface:



- Dilation f by k is called:

$$f \oplus k = T[U(f) \oplus U(k)] = \max\{f(x-z) + k(z)\}.$$

- Erosion f by k is called:

$$f \ominus k = T[U(f) \ominus U(k)] = \min\{f(x+z) - k(z)\}.$$

Morphology of
Grayscale Images

- Erosion expands many of the dark pixels in a grayscale image, while dilation expands many of the bright pixels.

Original image

Erosion

Dilation

Morphology of

Grayscale Images

- The expressions for the open and close operations are completely equivalent to the case of binary images.
- Open operation f by k:

$$f \circledcirc k = (f \ominus k) \oplus k = \max(\min(f)) .$$

  - For removing small light details.
- Close operation f by k:

$$f \odot k = (f \oplus k) \ominus k = \min(\max(f)) .$$

  - For removing small dark details.

Morphology of
Grayscale Images


Original image


Dilation


Erosion


Open


Close

Morphology of
Grayscale Images

- **Image smoothing** is the sequential application of opening and closing.

$$\min(\max(\max(\min f)))$$

Original image

Smoothed image

Morphology of
Grayscale Images

- Morphological gradient

$$\frac{1}{2}(\max(f) - \min(f))$$

Original image

Processed image

Morphology of
Grayscale Images

- Laplace morphological filter

$$\frac{1}{2}(\max(f) - 2f + \min(f))$$

Original image

Processed image

THANK YOU
FOR YOUR TIME!

ITsMOre than a UNIVERSITY

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