Color Selection
Color Temperature

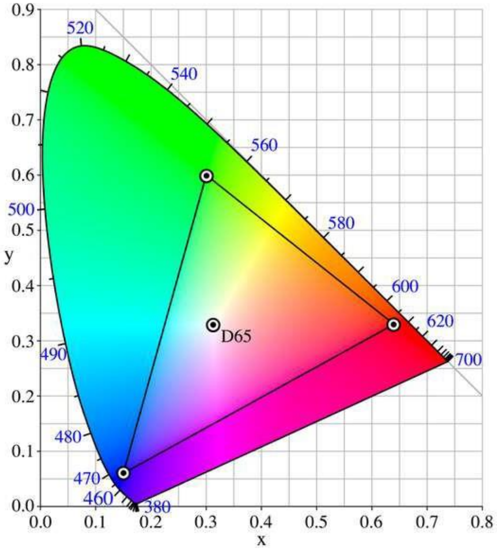
Image Processing

Outline

- White point
- Color temperature
- Chromatic adaptation
- Creation of spectrum by color

Specifying RGB color space

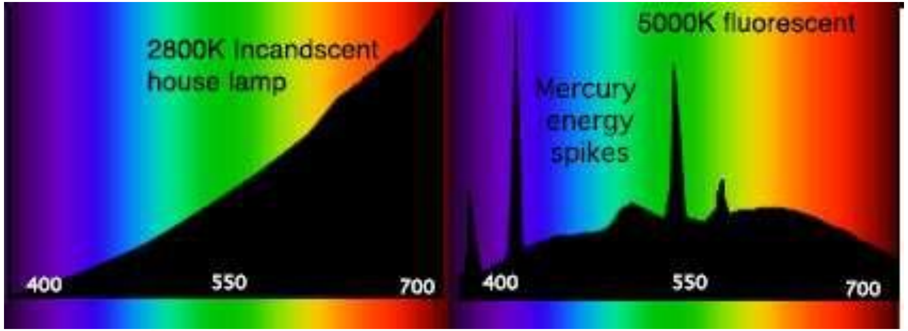
- Three base colors define an additive color space
- For a complete specification, usually define
  - *xy* coordiates for r,g,b base colors
  - **white point (relative luminance)**



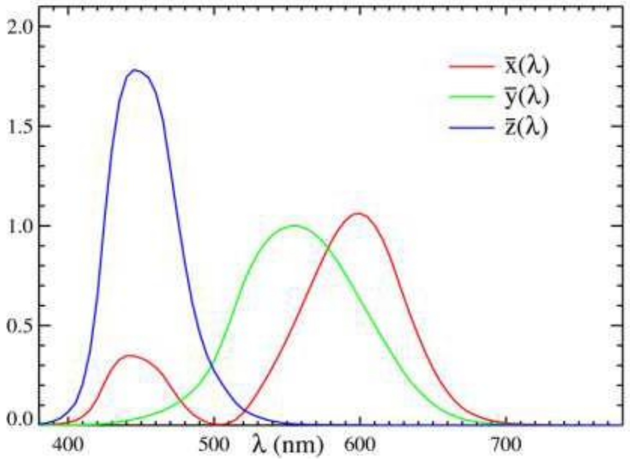
sRGB color space
(base colors and **white point**)

The light source and a white point

- **The illuminant** is defined by a spectral distribution
- For each illuminant, you can find a **white point** - this is the illuminant color in the CIE xyY color space
- We need to calculate the xy chromatic coordinates from the illuminant spectrum



How to calculate the xy chromatic coordinates from spectrum?



$$X = \int_{\lambda=380}^{780} C(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = \int_{\lambda=380}^{780} C(\lambda)\bar{y}(\lambda)d\lambda$$

$$Z = \int_{\lambda=380}^{780} C(\lambda)\bar{z}(\lambda)d\lambda$$

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

Ideal black body

- The ideal black body is a body that absorbs all radiation incident on it
- So, this body does not reflect or transmit anything
- However, it emits different radiation spectra depending on its temperature
- You can calculate the white point for an ideal black body and express it in terms of its temperature in Kelvin
- Thermal radiation of an object (Sun, lamp, etc.) can be expressed in terms of the temperature of a black body

Planck's law

- *u(ν,T)* – radiance density distribution in the *[ν; ν+dν]* range
- *u*(*λ*,*T*) – radiance density distribution in the *[λ; λ +dλ]* range

$$u(\nu,T)\,d\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_BT}-1}d\nu$$

$$u(\lambda,T)\,d\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda k_BT}-1}d\lambda$$

  - h – Planck's constant
  - c – speed of light
  - k_B – Boltzmann constant
  - ν – radiation frequency
  - λ – radiation wavelength ($\lambda = c/\nu$)
  - T – temperature in Kelvin

Wien's displacement law

$$\lambda_{max} = \frac{0.0028999}{T}$$

$$h = 6.62607015 \cdot 10^{-34}\,J/Hz$$

$$k_B = 1.380649 \cdot 10^{-23}\,J/K$$

$$c = 299792458\,m/s$$

Planck's law

We can calculate black body spectrum and convert it to color

$$X = \int_{\lambda=380}^{780} C(\lambda)\bar{x}(\lambda)d\lambda$$

$$Y = \int_{\lambda=380}^{780} C(\lambda)\bar{y}(\lambda)d\lambda$$

$$Z = \int_{\lambda=380}^{780} C(\lambda)\bar{z}(\lambda)d\lambda$$

$$x = \frac{X}{X+Y+Z}$$

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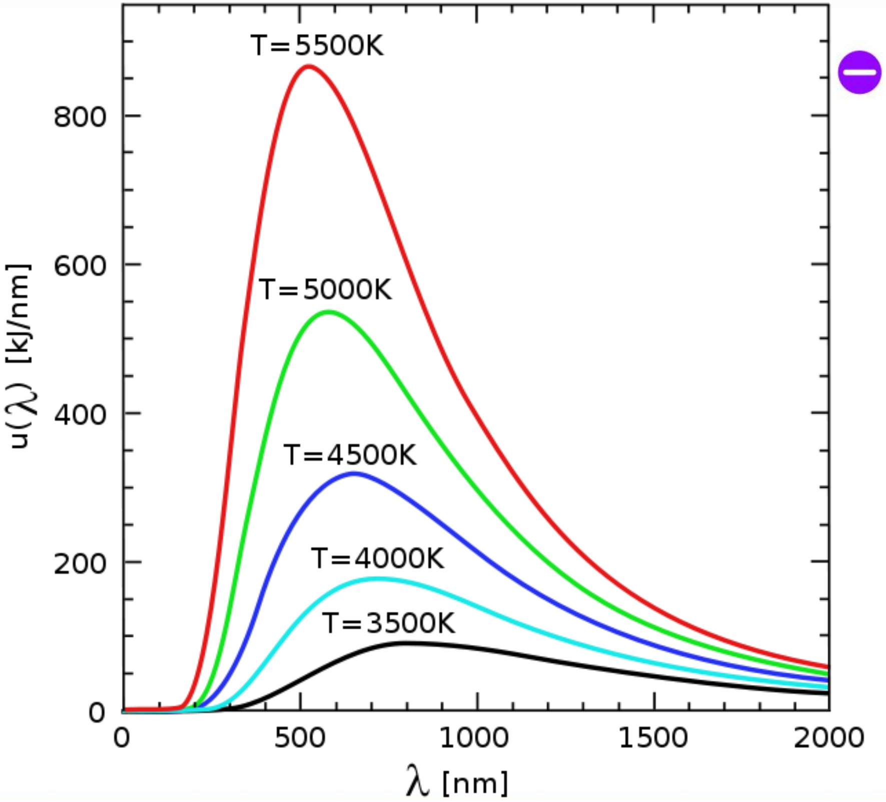
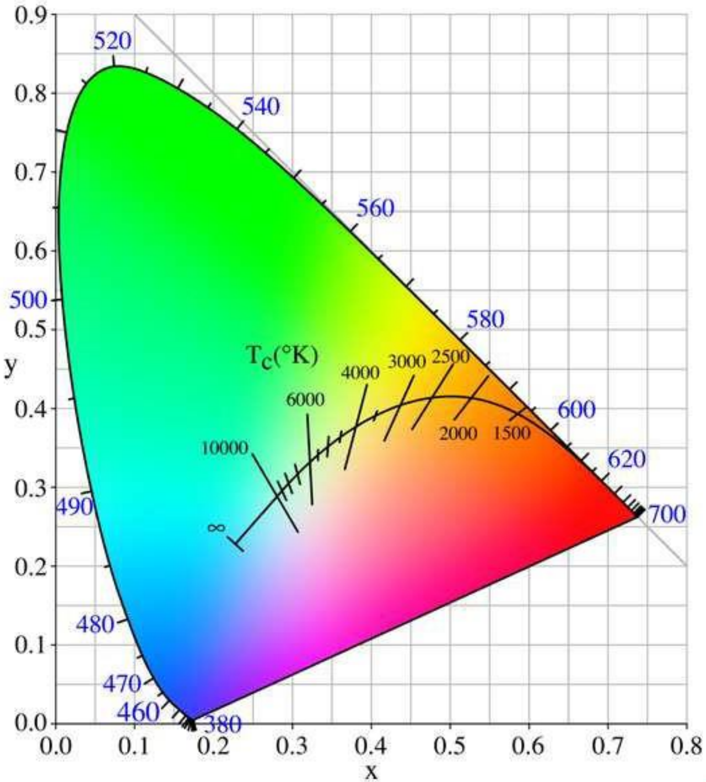


Color temperature: examples

- 1600 K: sunrise and sunset
- 1800 K: candle
- 2800 K: incandescent lightbulb
- 3200 K: studio lamps
- 5200 K: midday sunlight
- 5500 K: averaged daylight
- 6000 K: cloudy sky
- 20000 K: bright clear blue sky
- 28000 - 30000 K: lightning



Correlated color temperature

- In the real world, only incandescent bulbs have properties which are close to an ideal black body
- Therefore, it is necessary to expand the color temperature definition system to include illuminants (light sources) which are close enough to an ideal black body in terms of their color

- The correlated color temperature for a given light source is the color temperature of an ideal black body that is the closest in terms of color to a given source (at the same luminance level and observation conditions)

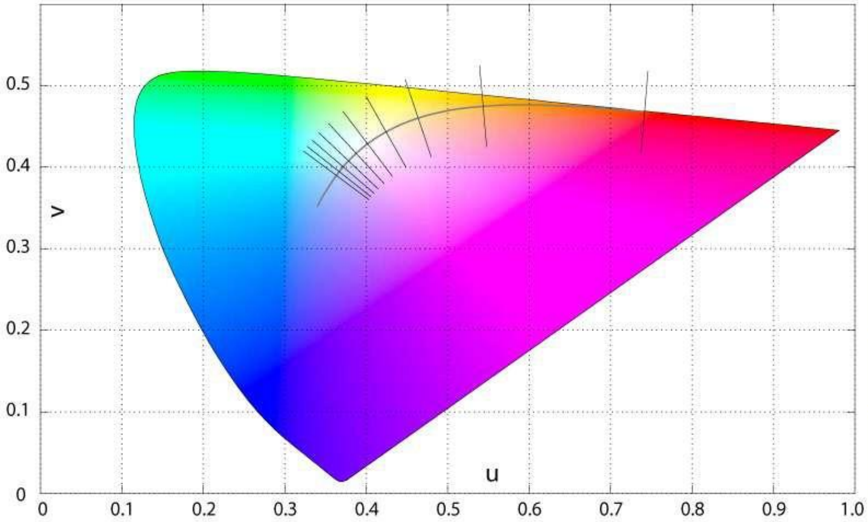
Uniform Color Space

- For a given illuminant, you need to find the closest point on the Planck curve
- For proximity in distance to match perceived proximity, it is necessary to transform the color to a different color space

CIE 1960 UCS color space

$$u = \frac{5.5932x + 1.9116y}{12y - 1.882x + 2.9088}$$

$$v = \frac{7.8972y}{12y - 1.882x + 2.9088}$$



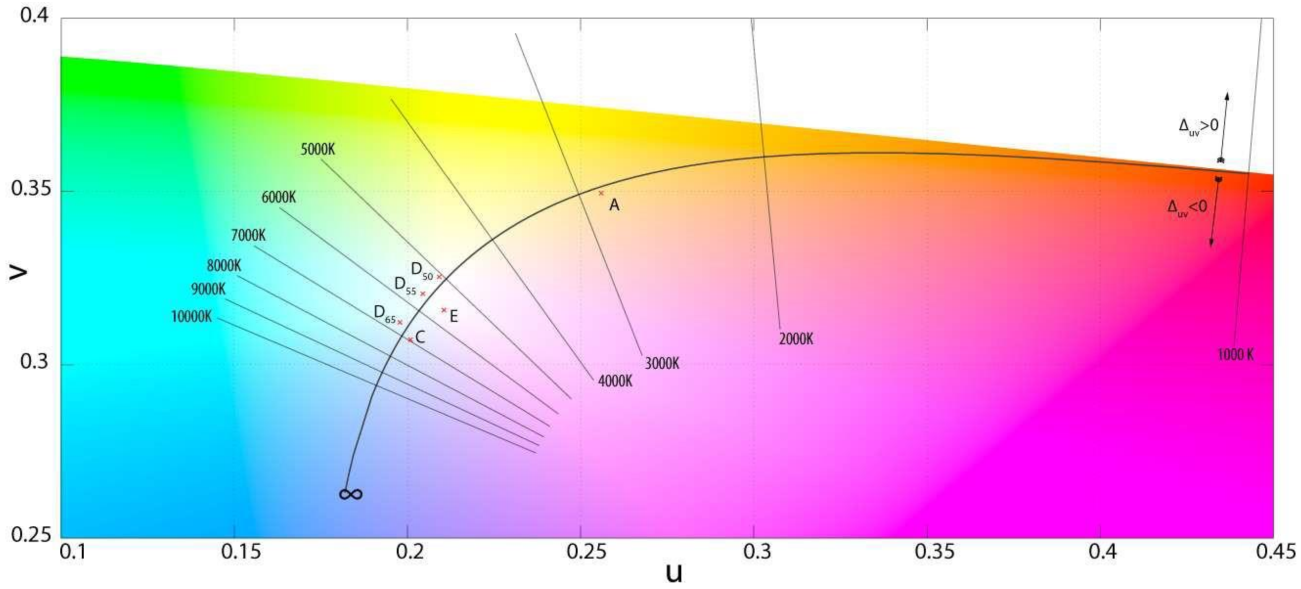
In this space, the distance from the illuminant to the Planck curve is calculated along the perpendicular to the Planck curve

CIE 1960 UCS color space is not used for anything else

CIE 1960 UCS color space

Any source can be described by its **CCT** + **ΔUV**

This makes sense only for light sources that are close to the Planck curve

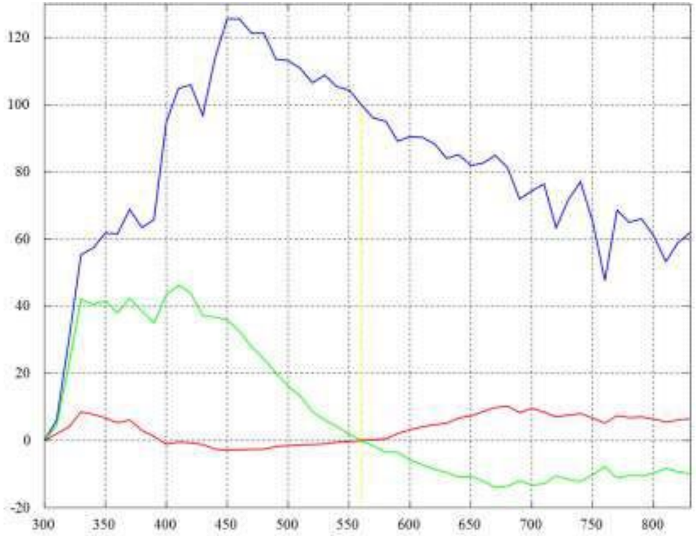


White points: some standard CIE illuminants

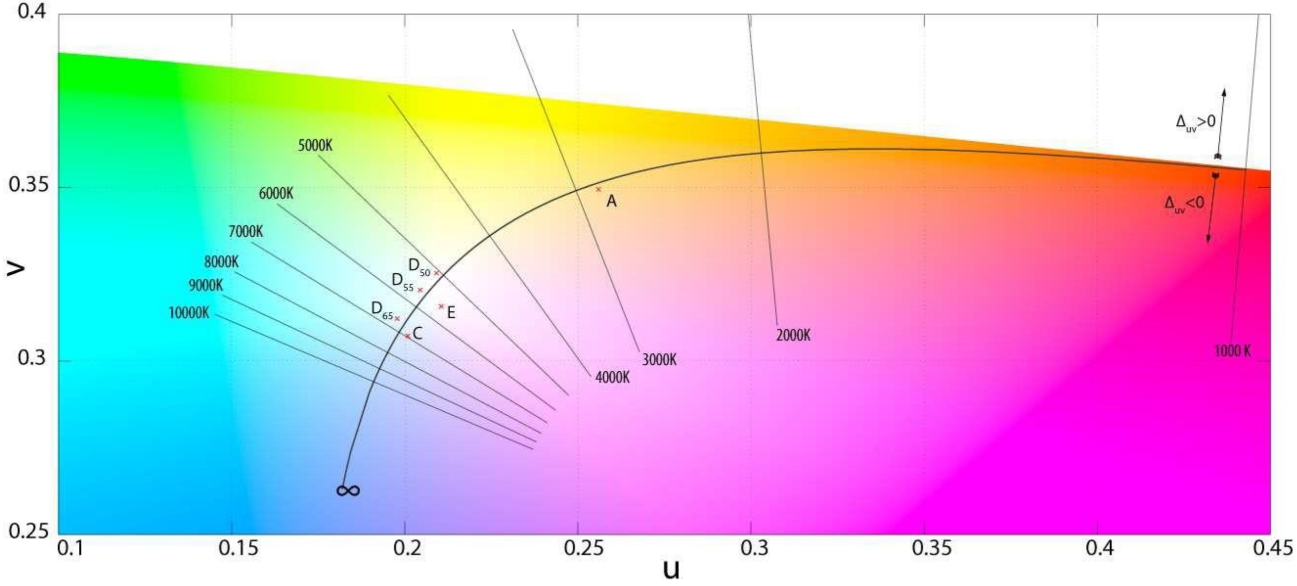
Name	CIE 1931 xy		CCT	Notes
	x	y		
E	1/3	1/3	5400	Point of equal energy
D55	0.33242	0.34743	5500	
D65	0.31271	0.32902	6500	TV, sRGB
D75	0.29902	0.31485	7500	
A	0.44757	0.40745	2856	Incandescent lightbulb

Series D standard illuminant

- The result of analyzing a large number of daylight spectra
- Based on the PCA (Principal Component Analysis) analysis of the results, it was revealed that it is possible to construct various daylight spectra based on varying three parameters
- S0 (blue) – middle range spectrum
- S1 (green) – yellow-blue spectrum (sun, clouds)
- S2 (red) – purple-green spectrum (water vapours)

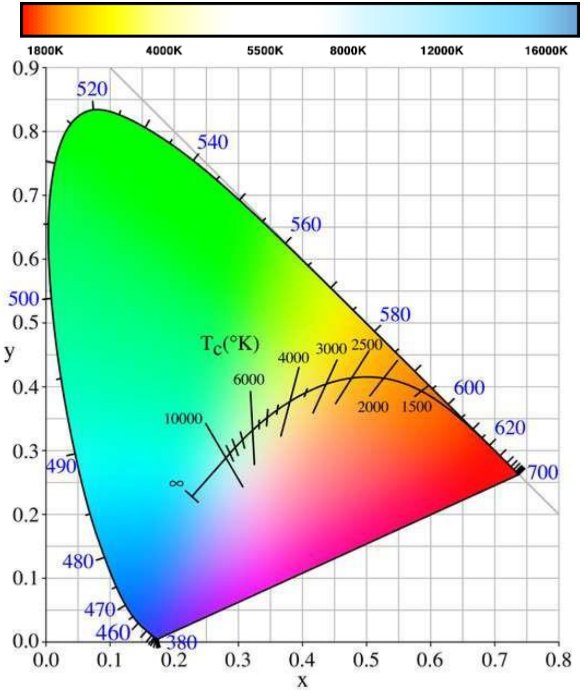


Standard light sources on CIE 1960 UCS



Color temperature

- Color temperature is a characteristic of visible light
- Color temperature is acquired by comparison of color with the color of the heated black body (black body radiator)
- Since most of the light sources are based on the radiation of a heated body, it is convenient to describe them using color temperature
  - Can be matched with a real light



Specifying RGB color space

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- For a complete specification, usually define
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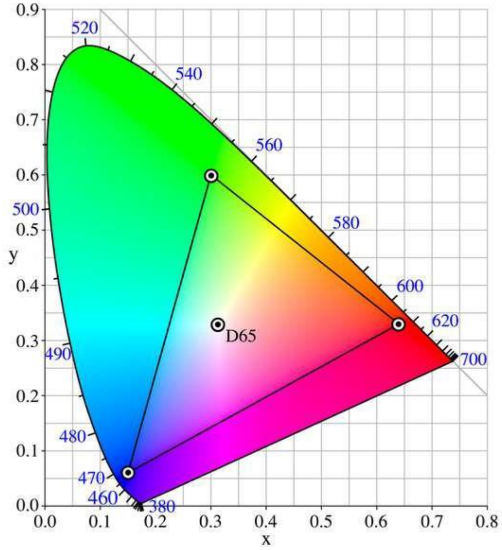


sRGB color space
(base colors and **white point**)

Specifying RGB color space: white point

- White point is the color that is considered as white under the given conditions
- For a monitor, it is the color emitted by color light sources at maximum luminance (1,1,1)
  - Sets the relative phosphors luminances
- Typically, the monitor white point is selected as the white point of one of the standard CIE sources (often it is D65)
- There are standard white points
  - CIE common white points

The transition between color spaces defined by the white point

Input:

1. *x,y* chromatic coordinates of three base colors
2. *x,y* chromatic coordinates of the white point

(x_r, y_r)

(x_g, y_g)

(x_b, y_b)

(x_w, y_w)

Solution:

1. Search for the transformation matrix for transition RGB->XYZ
2. Calculate a conversion to RGB for the white point, because we know that in RGB it will be equal to (1,1,1)

The transition between color spaces defined by the white point

1. Search for the transformation matrix M

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M \begin{pmatrix} r \\ g \\ b \end{pmatrix}$$

$$M = \begin{pmatrix} X_r & X_g & X_b \\ Y_r & Y_g & Y_b \\ Z_r & Z_g & Z_b \end{pmatrix}$$

The transition between color spaces defined by the white point

Compute the CIE XYZ coordinates based on the CIE xyY coordinates

$$X_i = \frac{x_i Y_i}{y_i}$$

$$Y_i = Y_i$$

$$Z_i = \frac{(1 - x_i - y_i) Y_i}{y_i} = \frac{z_i Y_i}{y_i}$$

Don’t be confused: CIE XYZ and CIE xyY are different color spaces!

The transition between color spaces defined by the white point

2. Calculate a conversion to RGB for the white point

$$\begin{pmatrix} \frac{x_w}{y_w} \\ 1 \\ \frac{z_w}{y_w} \end{pmatrix} = \begin{pmatrix} \frac{x_r Y_r}{y_r} & \frac{x_g Y_g}{y_g} & \frac{x_b Y_b}{y_b} \\ Y_r & Y_g & Y_b \\ \frac{z_r Y_r}{y_r} & \frac{z_g Y_g}{y_g} & \frac{z_b Y_b}{y_b} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{x_w}{y_w} \\ 1 \\ \frac{z_w}{y_w} \end{pmatrix} = \begin{pmatrix} \frac{x_r}{y_r} & \frac{x_g}{y_g} & \frac{x_b}{y_b} \\ 1 & 1 & 1 \\ \frac{z_r}{y_r} & \frac{z_g}{y_g} & \frac{z_b}{y_b} \end{pmatrix} \begin{pmatrix} Y_r & 0 & 0 \\ 0 & Y_g & 0 \\ 0 & 0 & Y_b \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{x_w}{y_w} \\ 1 \\ \frac{z_w}{y_w} \end{pmatrix} = \begin{pmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_r \end{pmatrix} \begin{pmatrix} Y_r/y_r & 0 & 0 \\ 0 & Y_g/y_g & 0 \\ 0 & 0 & Y_b/y_b \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

The transition between color spaces defined by the white point

$$\begin{pmatrix} \frac{x_w}{y_w} \\ 1 \\ \frac{z_w}{y_w} \end{pmatrix} = \begin{pmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_r \end{pmatrix} \begin{pmatrix} Y_r/y_r & 0 & 0 \\ 0 & Y_g/y_g & 0 \\ 0 & 0 & Y_b/y_b \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$F = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} W = \begin{pmatrix} \frac{x_w}{y_w} \\ 1 \\ \frac{z_w}{y_w} \end{pmatrix} K = \begin{pmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_r \end{pmatrix} G = \begin{pmatrix} Y_r/y_r & 0 & 0 \\ 0 & Y_g/y_g & 0 \\ 0 & 0 & Y_b/y_b \end{pmatrix},$$

$$\begin{aligned} W &= KGF \Rightarrow GF = K^{-1}W, V = GF \Rightarrow V = K^{-1}W \\ M &= KG \end{aligned}$$

The transition between color spaces defined by the white point

The resulting algorithm for calculating the transformation matrix

1. Fill **W** and **K** matrices with input data
2. Calculate **V=K^{-1}W**
3. Calculate the **G** matrix from vector **V**

   As **V=GF**, where **F** is a unit vector and **G** is a diagonal matrix
4. **M=KG**.

$$F = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$W = \begin{pmatrix} \frac{x_w}{y_w} \\ 1 \\ \frac{z_w}{y_w} \end{pmatrix}$$

$$K = \begin{pmatrix} x_r & x_g & x_b \\ y_r & y_g & y_b \\ z_r & z_g & z_r \end{pmatrix}$$

$$G = \begin{pmatrix} Y_r/y_r & 0 & 0 \\ 0 & Y_g/y_g & 0 \\ 0 & 0 & Y_b/y_b \end{pmatrix}$$

Chromatic adaptation

- Human color perception allows adapting to a wide range of the light source colors (chromatic adaptation)
- If you smoothly change the color of the light source, the human will perceive the color of the surface as the same (color constancy)
- At the same time color measurements will show different spectra and different CIE XYZ values

Adaptation problems are visualization problems

- When observing some real scene, the observer can be considered adapted to the lighting of that scene
- When observing the image of this scene on the computer screen, the observer is adapted to the illumination conditions of the room and the parameters of the monitor (white point)
- As results, the adaptation is different and the images look different for an observer

Chromatic adaptation correction

So, we must correct it. We know:

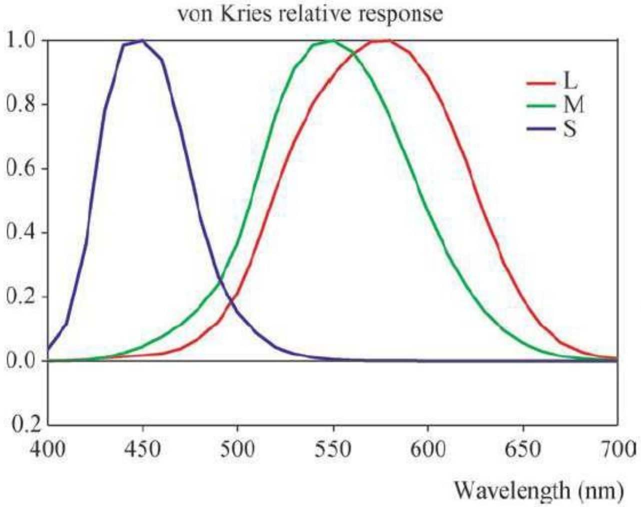
- Image white point
- Observation conditions white point (the one the observer is adopted right now)

- It is necessary to create a transformation that transforms the color of each pixel in the image to the desired adaptation

Von Kries coefficient law

- In 1902, Von Kries supposed that chromatic adaptation occurs independently on three types of cones (L, M, S)
  - Based on the Helmholtz's trichromatic theory
- Therefore, the chromatic adaptation can be modeled by converting XYZ to LMS (cone response domain) and individually scaling the components

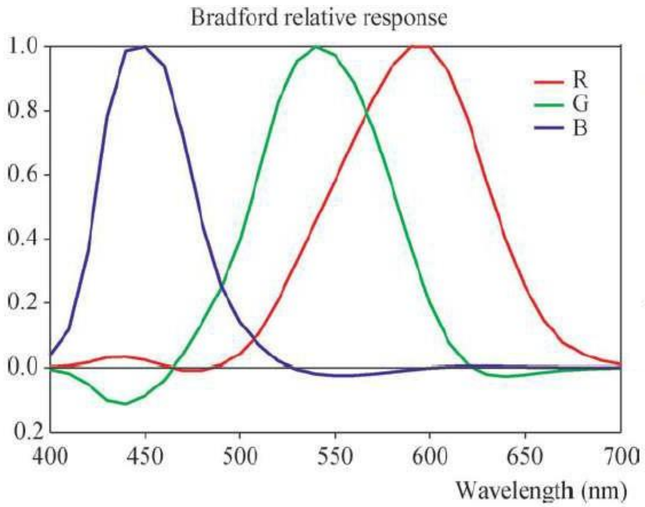
Von Kries transformation matrix



$$M_{\text{vonKries}} = \begin{bmatrix} 0.3897 & 0.6890 & -0.0787 \\ -0.2298 & 1.1834 & 0.0464 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

$$M_{\text{vonKries}}^{-1} = \begin{bmatrix} 1.9102 & -1.1121 & 0.2019 \\ 0.3710 & 0.6291 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 \end{bmatrix}$$

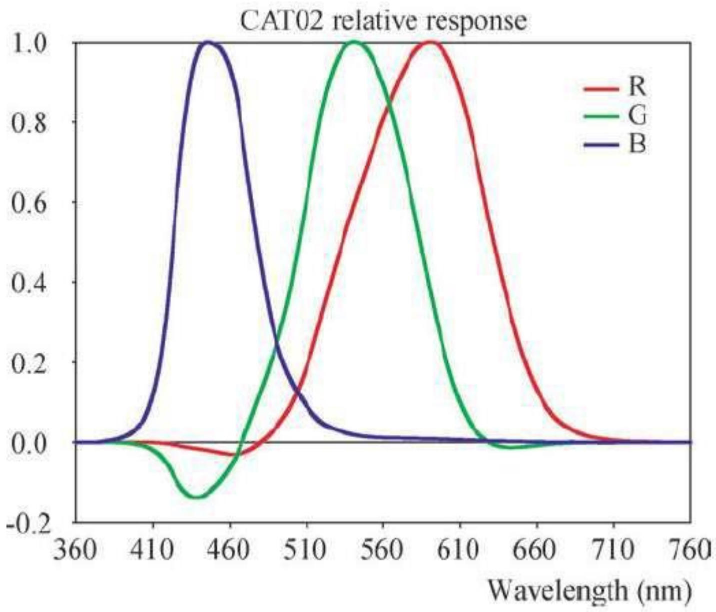
Bradford transformation matrix



$$M_{\text{Bradford}} = \begin{bmatrix} 0.8951 & 0.2664 & -0.1614 \\ -0.7502 & 1.7135 & 0.0367 \\ 0.0389 & -0.0685 & 1.0296 \end{bmatrix}$$

$$M_{\text{Bradford}}^{-1} = \begin{bmatrix} 0.9870 & -0.1471 & 0.1600 \\ 0.4323 & 0.5184 & 0.0493 \\ -0.0085 & 0.0400 & 0.9685 \end{bmatrix}$$

CAT02 transformation matrix



$$M_{CAT02} = \begin{bmatrix} 0.7328 & 0.4296 & -0.1624 \\ -0.7036 & 1.6975 & 0.0061 \\ 0.0030 & 0.0136 & 0.9834 \end{bmatrix}$$

$$M_{CAT02}^{-1} = \begin{bmatrix} 1.0961 & -0.2789 & 0.1827 \\ 0.4544 & 0.4735 & 0.0721 \\ -0.0096 & -0.0057 & 1.0153 \end{bmatrix}$$

Chromatic transformation matrix calculation

1. Calculate the LMS coordinates for white points
2. Calculate the scale factors for each of L, M, S coordinate axes
3. For each pixel in XYZ color space
    1. Transform it to LMS
    2. Scale it
    3. Transform it back to XYZ

$$\begin{bmatrix} \rho_S \\ \gamma_S \\ \beta_S \end{bmatrix} = M_{cat} \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix}$$

$$\begin{bmatrix} \rho_D \\ \gamma_D \\ \beta_D \end{bmatrix} = M_{cat} \begin{bmatrix} X_D \\ Y_D \\ Z_D \end{bmatrix},$$

Chromatic transformation matrix calculation

- You can calculate a single transformation matrix in matrix form:

$$M = M_{\text{cat}}^{-1} \begin{bmatrix} \rho_{\text{D}}/\rho_{\text{S}} & 0 & 0 \\ 0 & \gamma_{\text{D}}/\gamma_{\text{S}} & 0 \\ 0 & 0 & \beta_{\text{D}}/\beta_{\text{S}} \end{bmatrix} M_{\text{cat}}$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Chromatic adaptation: Examples

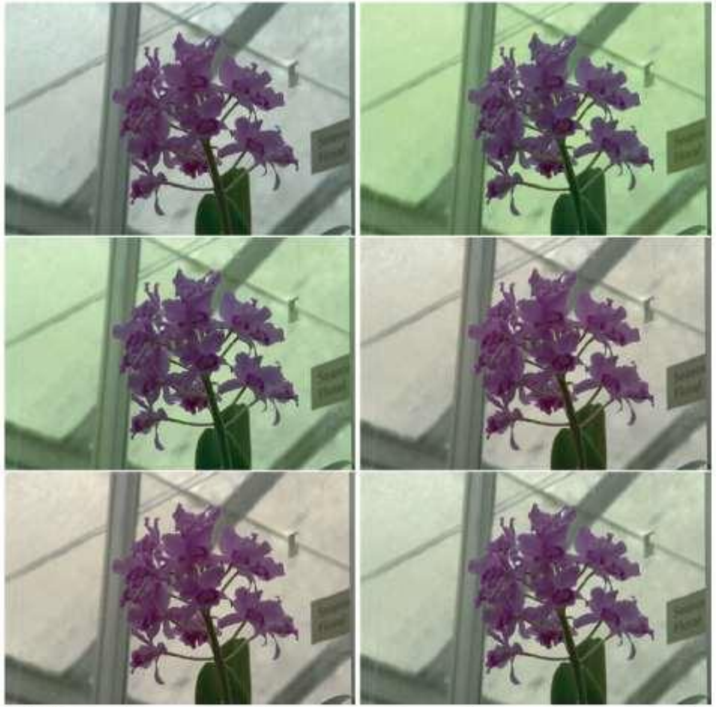


FIGURE 2.15 CAT02 chromatic adaptation. In reading order: original image, followed by five images chromatically adapted from D_{65} to incandescent, tungsten, D_{50}, E, and F2.

Chromatic adaptation: different matrices

Calculating spectrum from color

- Often don’t know the spectrum of a light source
- The problem arises of obtaining the spectrum of the light source by its color
- Obviously, this can be solved in many ways (metametrism)

Calculating spectrum from color: the task

- **On input** we have a triplet R=(r,g,b) for a specified color space
- **On output** we should have a S(λ) spectrum so that an artificial light souce with this spectrum should give the R color in a specified color space

Calculating spectrum from color: the general solution

1. Convert the color to a CIE 1931 XYZ color space:
   $X = (x, y, z)$. $X = MR$.
   – Next, will select some spectrum to match this color
2. Choose three basic spectra $F_1(\lambda)$, $F_2(\lambda)$, $F_3(\lambda)$
   – The resulting spectrum will be obtained by their combination
3. Calculate spectrum weight coefficients

Calculating spectrum from color: step 1

Convert from RGB to XYZ

– Apply the transformation matrix for one of standard RGB color spaces
– Or apply the transformation matrix for the currently used monitor phosphors

– $X = (x, y, z)$. $X = MR$.

Calculating spectrum from color: step 2

Choose three basic spectra F1(λ), F2(λ), F3(λ)

- Select three linearly independent functions
- For example, use delta functions for three wavelengths:

  $\delta_1(\lambda), \delta_2(\lambda), \delta_3(\lambda)$

  Then the resulting spectrum would look like:

  $S(\lambda) = a_1 * \delta_1(\lambda) + a_2 * \delta_2(\lambda) + a_3 * \delta_3(\lambda)$

- The goal is to find a_1, a_2, a_3 coefficients so that the S(λ) spectrum would correspond to (*x, y, z*) color

Calculating spectrum from color: step 3

Calculate α_1, α_2, α_3:

$$X = \int x(\lambda)S(\lambda)d\lambda =$$
$$= x(\lambda_1)a_1 + x(\lambda_2)a_2 + x(\lambda_3)a_3$$

$$Y = \int y(\lambda)S(\lambda)d\lambda =$$
$$= y(\lambda_1)a_1 + y(\lambda_2)a_2 + y(\lambda_3)a_3$$

$$Z = \int z(\lambda)S(\lambda)d\lambda =$$
$$= z(\lambda_1)a_1 + z(\lambda_2)a_2 + z(\lambda_3)a_3$$

Calculating spectrum from color: step 3

Calculate α_1, α_2, α_3:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \bar{x}(\lambda_1) & \bar{x}(\lambda_2) & \bar{x}(\lambda_3) \\ \bar{y}(\lambda_1) & \bar{y}(\lambda_2) & \bar{y}(\lambda_3) \\ \bar{z}(\lambda_1) & \bar{z}(\lambda_3) & \bar{z}(\lambda_3) \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

C matrix

- $A=C^{-1}X$
- $X = MR$.
- => **$A=C^{-1}MR$**

Calculating spectrum from color: conclusion

Method features and problems:

- The proposed solution based on a spectrum with delta functions can cause a problem with visualization, since of possible spectrum segmentation
- Other continuous functions can be taken as basic ones
- **Can get a negative spectrum values!**

Additional reading

- Judd, Deane B., et al. 1964. Spectral distribution of typical daylight as a function of correlated color temperature
- Glassner, A. 1989. How to derive a spectrum from an RGB triplet. Computer Graphics and Applications

THANK YOU FOR YOUR TIME!

ITsMOre than a UNIVERSITY

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