ITMO

High-Pass Filters

Image Processing

High-Pass Filters

Key Terms

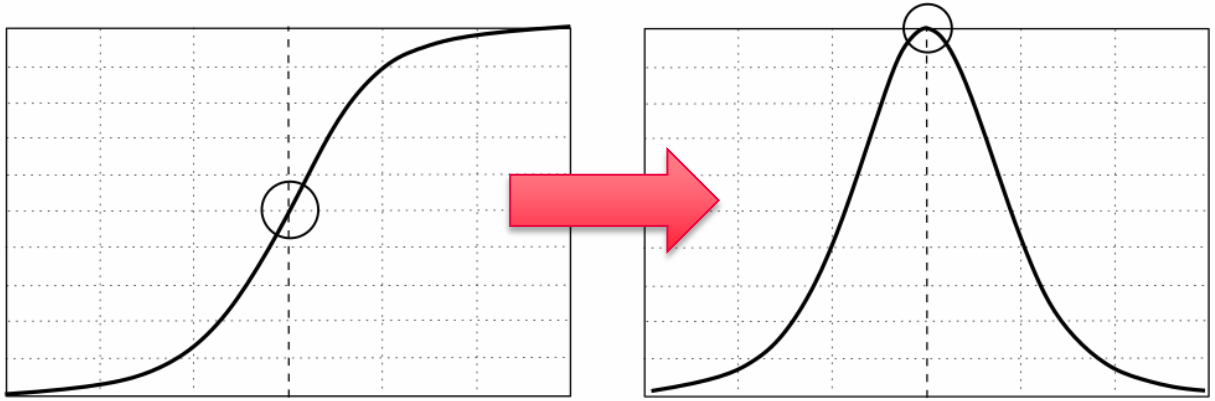
- What is **high-pass** components of the digital image?

High-pass
(edges)

Low-pass
(flat regions)

Edge Detection Idea

- How to detect edges?
- Derivative!



What is a Derivative?

- Mathematical definition:

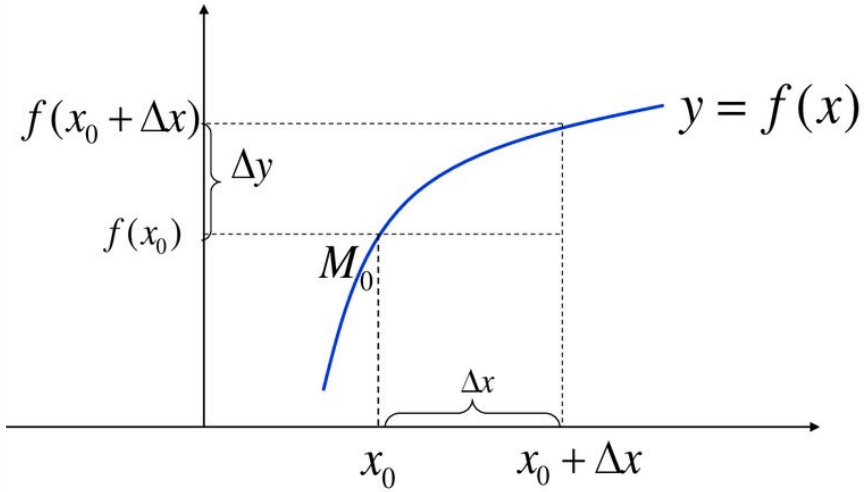
$$f'(x) = \lim_{\Delta x \to 0} \left(\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \right),$$

where $f(x_0)$ – function,

x_0 – argument,

$f(x_0 + \Delta x)$ – function increment,

Δx – argument increment.



How to use it?

- Let represent the image *I* as a continuous function of *intensity (brightness)*.
- Then find derivative of the *I*.
- We know edges! Royal flush!

- Almost…

Double Trouble

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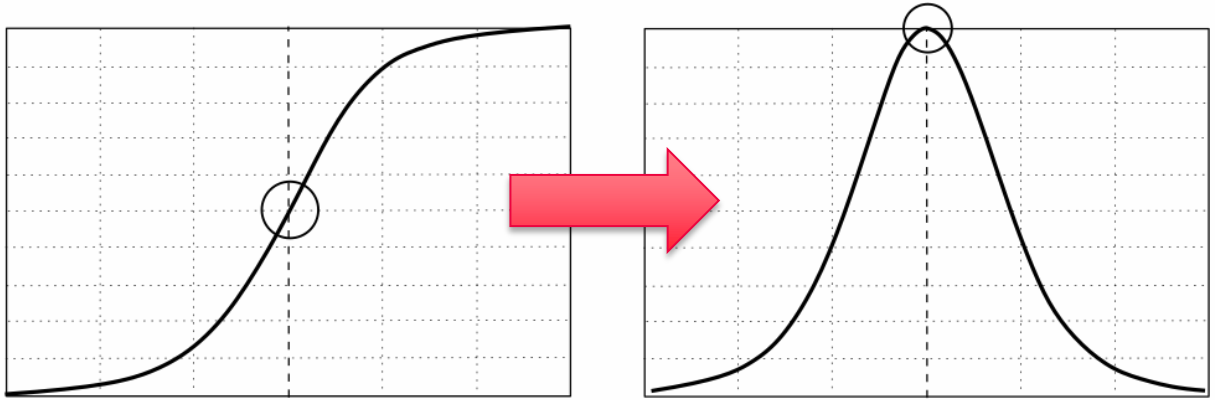
1. Image is discrete, so...
   - we have fixed discrete step!
2. Image has two axes, in which direction...
   - in both!

Discrete Realization

- What is a minimum value of Δx?
  - **1 pixel.**
- How to estimate rate of fastest brightness increasing (decreasing)?
  - **Find gradient** (derivative in each pixel) on Ox (G_x) and Oy (G_y) axes.
- How to find direction of the gradient?
  - $\mathbf{atan}\left(\frac{G_y}{G_x}\right)$
- Full House!

Resume of High-Pass Filters

- Used to highlight intensity differences and create edge filters.
- A sharp change in intensity can be determined by analyzing the first derivative of the intensity function.
- Sum of coefficients in a mask should be equal to zero.
- High-Pass filters can be called as "*differential operators*".



What is a Minimum Size of a Mask?

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2x2

Roberts Operator

- Used 2x2 masks:

$$G_x: \begin{bmatrix} +1 & -1 \\ 0 & 0 \end{bmatrix}, G_y: \begin{bmatrix} +1 & 0 \\ -1 & 0 \end{bmatrix},$$

or

$$G_x: \begin{bmatrix} +1 & 0 \\ 0 & -1 \end{bmatrix}, G_y: \begin{bmatrix} 0 & +1 \\ -1 & 0 \end{bmatrix}.$$

- As a result, obtain gradient estimation in the directions G_x, G_y.
- The gradient modulus (magnitude) of all edge detectors:

$$G = \sqrt{G_x^2 + G_y^2} = |G_x| + |G_y|.$$

- Gradient direction (maximum brightness difference):

$$\operatorname{atan}\left(\frac{G_y}{G_x}\right).$$

Prewitt Operator

- Used 3x3 masks:

$$G_x: \begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix}, G_y: \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{bmatrix}.$$

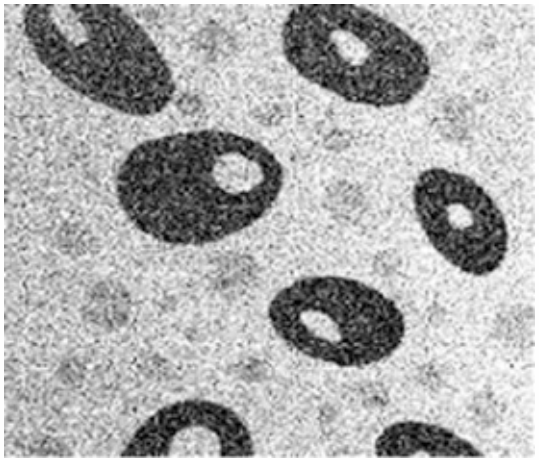
Sobel Operator

- Used 3x3 masks:

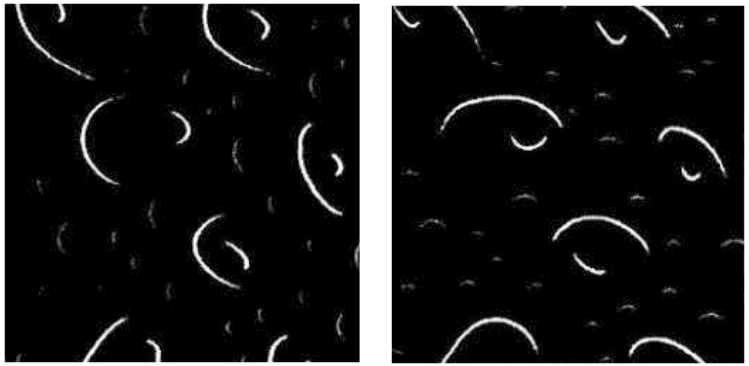
$$G_x: \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}, G_y: \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}.$$

- Used different weights of the mask.

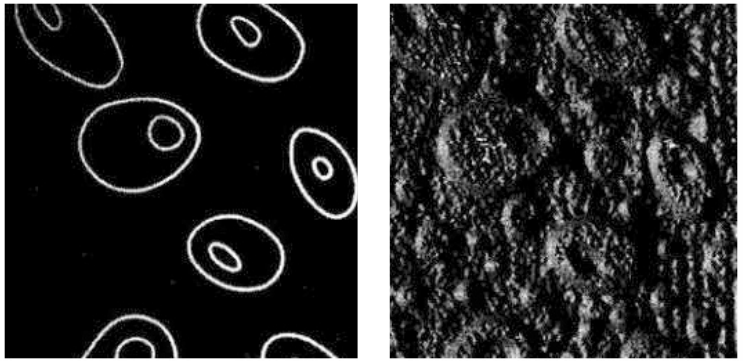
Sobel Operator



Source image



Sobel Operator – convolution on G_x and G_y



Sobel Operator – magnitude and gradient directions

Scharr Operator

- Used 3x3 masks:

$$G_x: \begin{bmatrix} +3 & 0 & -3 \\ +10 & 0 & -10 \\ +3 & 0 & -3 \end{bmatrix}, G_y: \begin{bmatrix} +3 & +10 & +3 \\ 0 & 0 & 0 \\ -3 & -10 & -3 \end{bmatrix}.$$

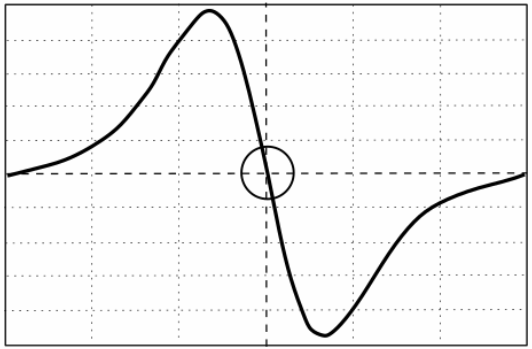
- Used different weights of the mask.

Laplace Operator

- The approximation of the second derivatives along the *Ox* and *Oy* axes is used:

$$L\big(f(x,y)\big) = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$$ – Laplacian of the image $f(x,y)$

- The gradient is calculated independently of the direction, so the edges are more accurately detected.



Laplace Operator

$L\big(f(x,y)\big) = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$ – Laplacian of the image $f(x,y)$

$$L\big(f(x,y)\big) = \big[\big(f(x,y) - f(x-1,y)\big) - \big(f(x+1,y) - f(x,y)\big)\big] +$$
$$+\big[\big(f(x,y) - f(x,y-1)\big) - \big(f(x,y+1) - f(x,y)\big)\big] =$$
$$= -f(x,y-1) - f(x-1,y) - f(x,y+1) - f(x+1,y) + 4f(x,y)$$

Activity Time

What is a Mask?

$L\big(f(x,y)\big) = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$ – Laplacian from the image $f(x,y)$

$$L\big(f(x,y)\big) =$$
$$= -f(x,y-1) - f(x-1,y) - f(x,y+1) - f(x+1,y) + 4f(x,y)$$

$$w(s,t) = ?$$

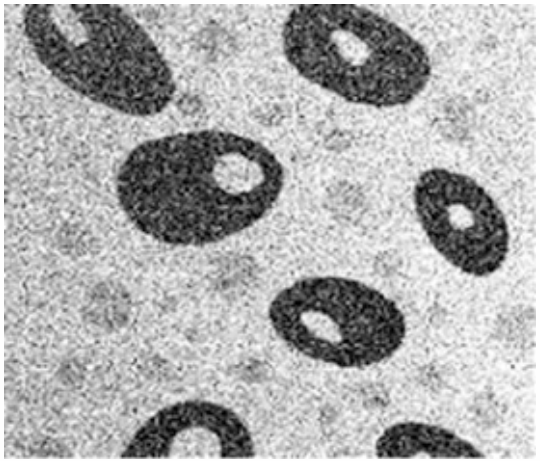
What is a Mask?

$L\big(f(x,y)\big) = \frac{d^2 f}{dx^2} + \frac{d^2 f}{dy^2}$ – Laplacian from the image $f(x,y)$

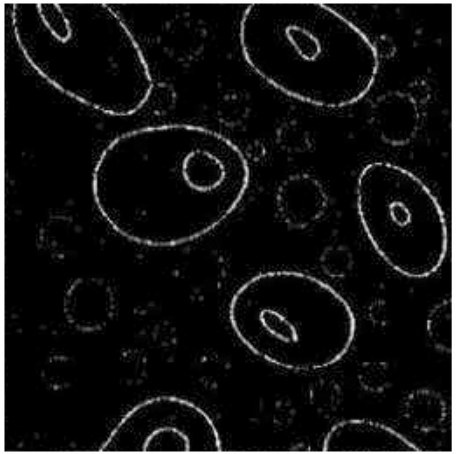
$$L\big(f(x,y)\big) =$$
$$= -f(x,y-1) - f(x-1,y) - f(x,y+1) - f(x+1,y) + 4f(x,y)$$

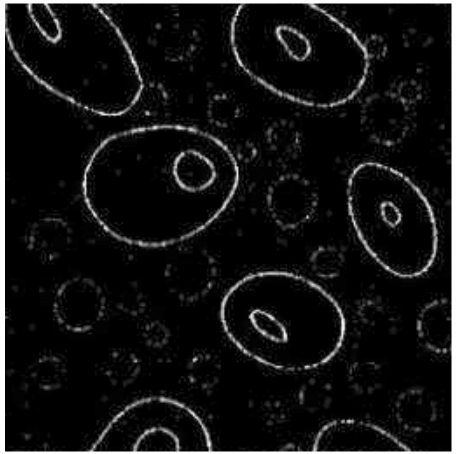
$$w(s,t) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & +4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

Laplace Operator



Source image

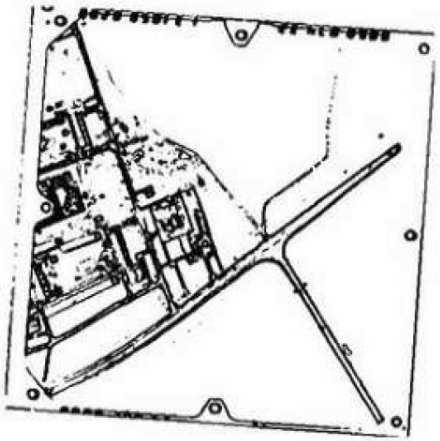




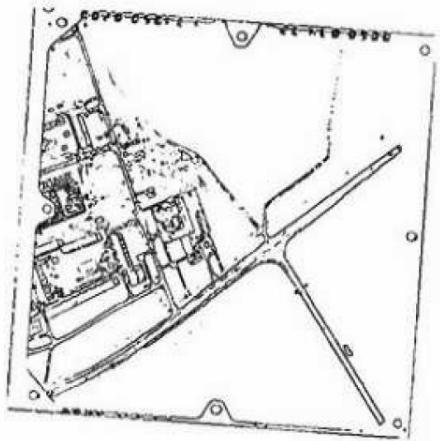
Laplace Operator – masks are 3x3 and 5x5

Thinning Edge

- **Task:** obtaining a contour with a unit width.
- **Thinning requirements:**
  - if the contour is connected, then the result must be connected;
  - the middle line should go through the points with the highest intensity value.

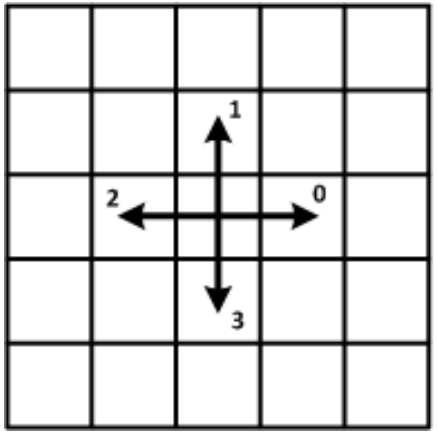


Detected Contour

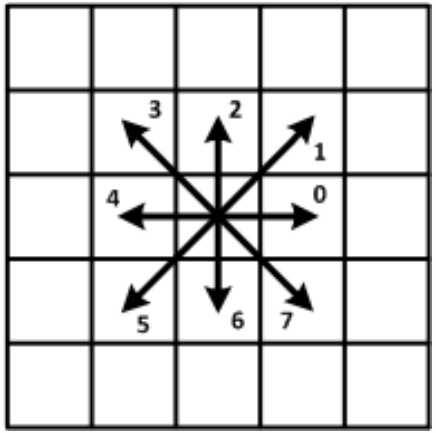


Thinned Contour

Principle Directions



By four connection



By eight connection

Connected Regions

- Connected region – region with the one intensity of pixel.
- How many connected regions on the piece of the image *I*:

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

1. By eight connection;
2. By four connection;

?

Connected Regions

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By eight connection

2 regions

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By four connection

7 regions

Activity Time

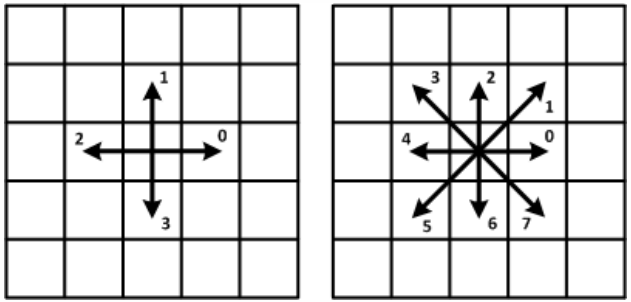
Connected Regions

- How many connected regions on the pieces of the image:

$$I1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}, \qquad I2 = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

1. By eight connection;
2. By four connection;

?



Canny Algorithm

Purpose: obtaining a contour with a unit width.

Algorithm:

1. Image blurring with a Gaussian filter.
2. Calculation of the gradients of all pixels by the Sobel filter:
   - The direction of the gradient is rounded in steps of **45 degrees**.
3. Suppression of non-maximum gradient modulus:
   - A pixel is **edge** if its gradient is larger than its neighbors,
   - otherwise, pixel is **non-maximum**.

Canny Algorithm

4. Performing **Double Threshold Filtering:**
   - if pixel value above the upper threshold T_2, the pixel is **edge**;
   - if pixel value less than the lower threshold T_1, the pixel is **not edge**;
   - if pixel value between thresholds T_1 and T_2, the pixel is **ambiguous**.
5. **Refine edges** by tracing an ambiguity region:
   - If pixel from the region is connected by eight connection with edge, pixel is **edge;**
   - otherwise, pixel is **not edge.**

Canny Algorithm

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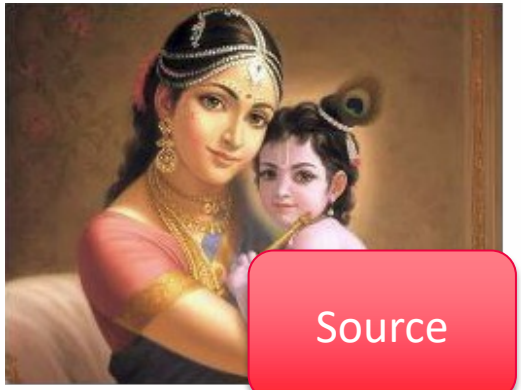
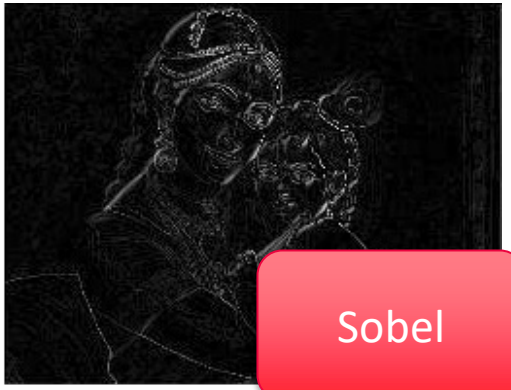


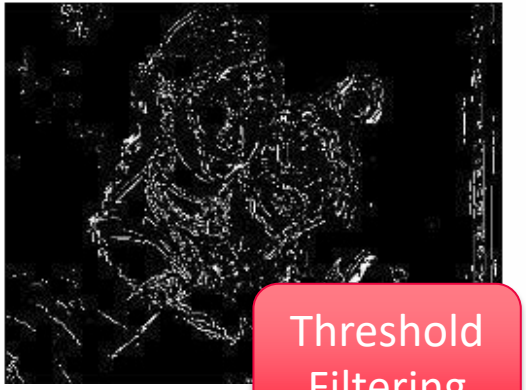
Source

RGB->Gray
Gauss



Sobel

Non-maximums
suppression



Threshold
Filtering

Refine
Edges

THANK YOU FOR YOUR TIME!

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