Simple digital circuits design
Practice

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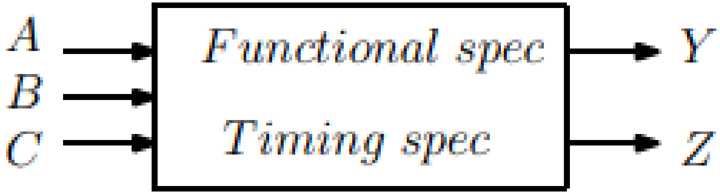
Introduction

In digital electronics, a *circuit* is a network that processes discrete valued variables.



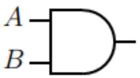
- A, B, C are inputs;
- Y, Z are outputs;
- a *functional specification* describing the relationship between inputs and outputs;
- a *timing specification* describing the delay between inputs changing and outputs responding.

Digital circuits are classified as *combinational* or *sequential:*

- a combinational circuit's outputs depend only on the current values of the inputs;
- a sequential circuit's outputs depend on both current and previous values of the inputs (sequential circuits have memory).

Introduction

Combinational (logic) blocks

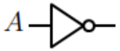


$Y = A \cdot B$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$Y = A + B$

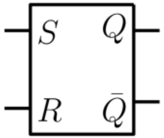
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1



$Y = \bar{A}$

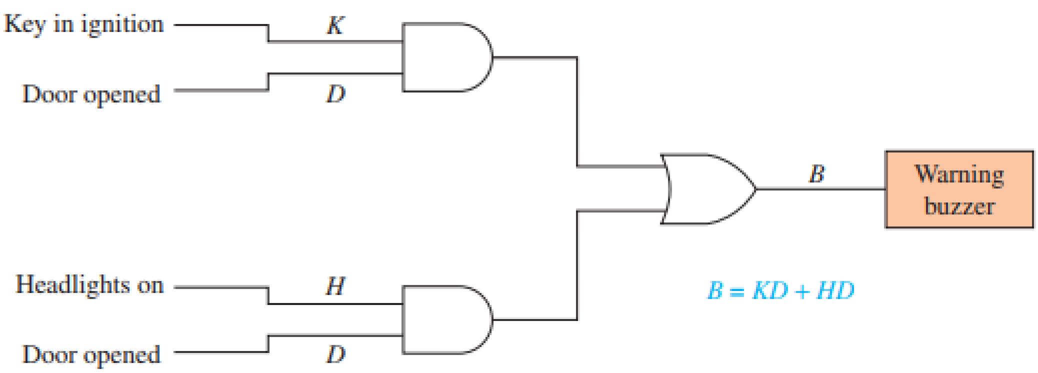
A	Y
0	0
1	1

Sequential blocks

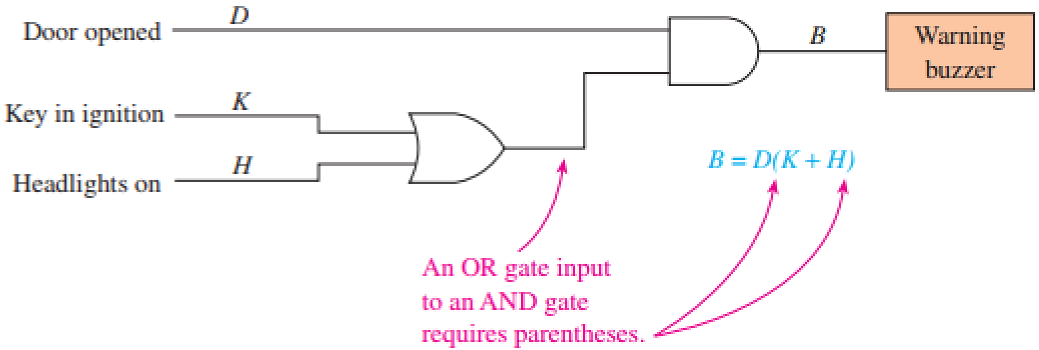


Input		*Output*	
In_1	In_2	Q_{n+1}	$\bar{Q}_{n+1}$
0	0	Q_n	$\bar{Q}_n$
0	1	0	1
1	0	1	0
1	1	Not allowed	

Combinational logic employs the use of two or more of the basic logic gates to form a more useful, complex function.

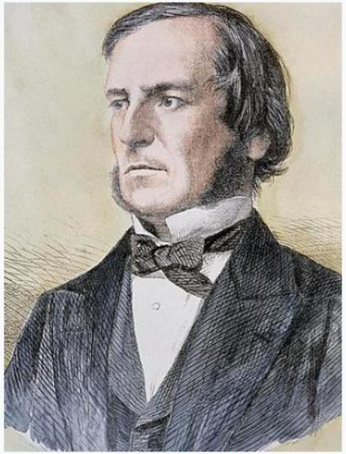


Boolean algebra



Boolean algebra is a mathematical system with logic notation used to describe different interconnections of digital circuits.

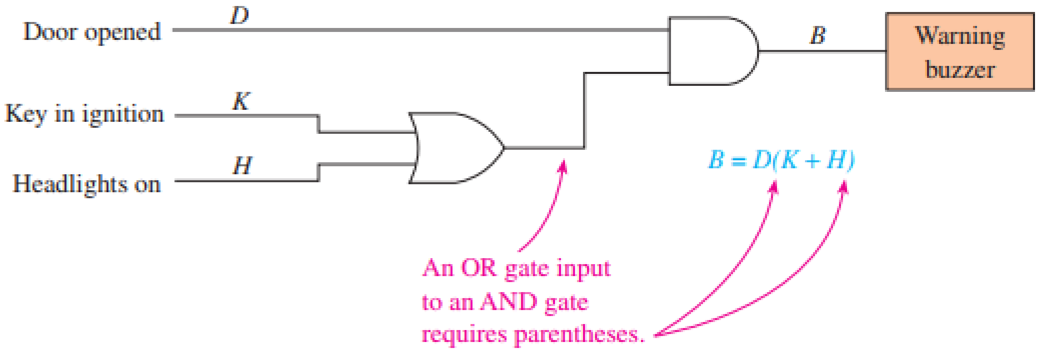
George Boole



Boole, c. 1860

Born	2 November 1815 Lincoln, Lincolnshire, England
Died	8 December 1864 (aged 49) Ballintemple, Cork, Ireland
Education	Bainbridge's Commercial Academy[1]
Spouse	Mary Everest Boole
Era	19th-century philosophy
Region	Western philosophy
School	British algebraic logic[2]

Boolean algebra



Boolean algebra is a mathematical system with logic notation used to describe different interconnections of digital circuits.

Basic Boolean Identities - basic Identities in Boolean algebra

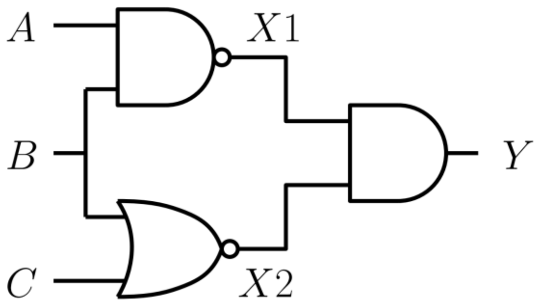
№	Identity
1	X+0=X
2	X+1=1
3	X+X=X
4	$X+\overline{X}=1$
5	X·0=0
6	X·1=X
7	X·X=X
8	$X\cdot\overline{X}=0$
9	$\overline{\overline{X}} = X$

Basic Boolean Identities

№	Identity	Comments
1	X+Y=Y+X	Commutative
2	X·Y=Y·X	Commutative
3	X+(Y+Z)=(X+Y)+Z	Associative
4	X·(Y·Z)=(X·Y)·Z	Associative
5	X·(Y+Z)=X·Y+X·Z	Distributive
6	X+Y·Z=(X+Y)·(X+Z)	Distributive
7	X+X·Y=X	Absorption
8	X·(X+Y)=X	Absorption
9	$X \cdot Y+\bar{X} \cdot Z+Y \cdot Z=X \cdot Y+\bar{X} \cdot Z$	Consensus
10	$\overline{X+Y+Z}=\bar{X}\cdot\bar{Y}\cdot\bar{Z}$	DeMorgan
	$\overline{X\cdot Y\cdot Z}=\bar{X}+\bar{Y}+\bar{Z}$	DeMorgan

Example

Derive the Boolean function for the combinational network



$Y = ???$

Example

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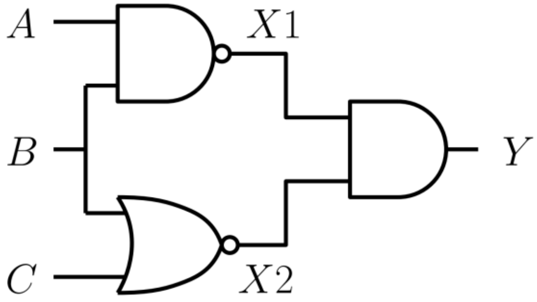

$$Y = X1 \cdot X2$$

Example

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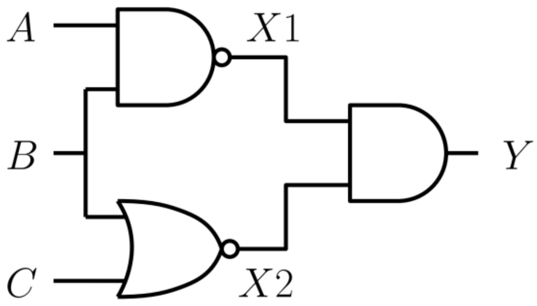

$$Y = X1 \cdot X2$$

$$X1 = \overline{A \cdot B}$$

$$X2 = \overline{B + C}$$

Example

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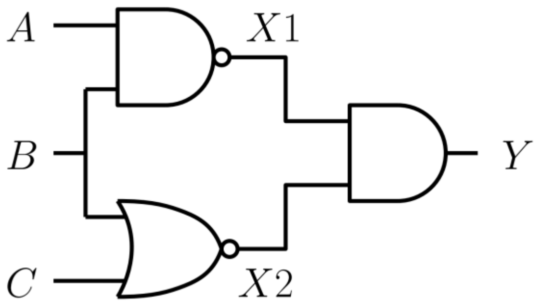

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$$Y = \overline{AB} \cdot \overline{B + C}$$

ITMO

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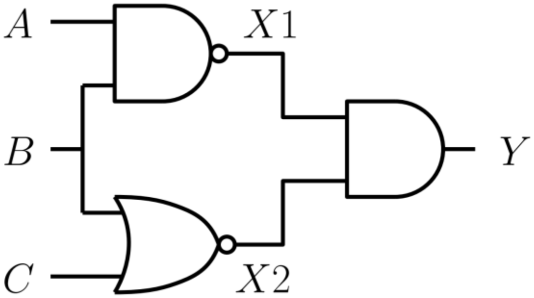

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Is it the final result?

Example

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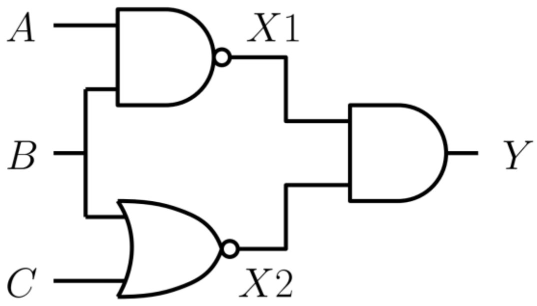

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Is it the final result?

NO, we can make some transformations…

Example

Augustus De Morgan

Born	27 June 1806 Madurai, Carnatic, Madras Presidency, (present-day India)
Died	18 March 1871 (aged 64) London, England
Nationality	British
Alma mater	Trinity College, Cambridge
Known for	De Morgan's laws De Morgan algebra De Morgan hierarchy Relation algebra Universal algebra
	Scientific career
Fields	Mathematician and logician
Institutions	University College London University College School

We are going to apply De Morgan’s theorem.

In the form of an equation, **De Morgan’s theorem** is stated as follows:

$$\overline{A \cdot B} = \overline{A} + \overline{B}$$

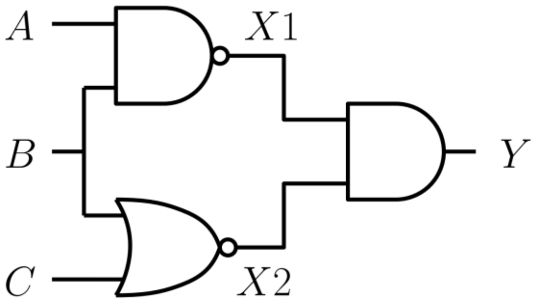
$$\overline{A + B} = \overline{A} \cdot \overline{B}$$

We would like to have the final equation in a form called the **sum-of-products (SOP) form**.

Also we can obtain the final equation in a form called the **Product-of-sums (POS) form.**

We will find more simple equation for our case in the SOP form, this form is very useful form for building truth tables and Karnaugh maps

Example

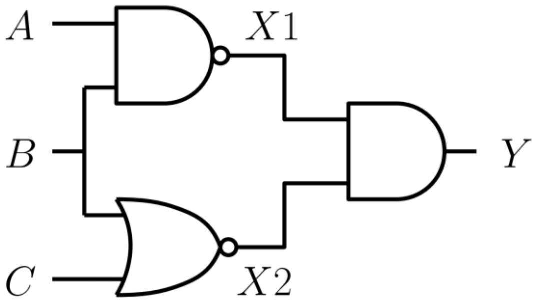


$$Y = \overline{AB} \cdot \overline{B + C}$$

First step – We apply De Morgan's theorem

$$Y = \overline{AB} \cdot \overline{B + C} = (\bar{A} + \bar{B}) \cdot (\bar{B} \cdot \bar{C})$$

Example



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First step – We apply De Morgan's theorem

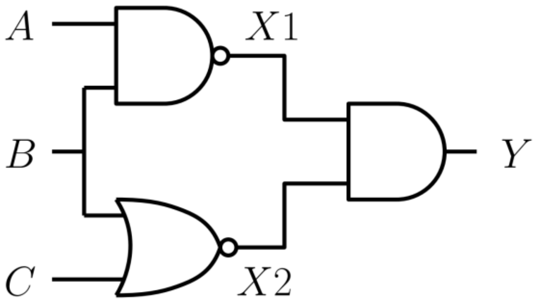
$$Y = \overline{AB} \cdot \overline{B + C} = (\bar{A} + \bar{B}) \cdot (\bar{B} \cdot \bar{C})$$

Second step – We use Boolean algebra rules

1) X·(Y+Z)=X·Y+X·Z

$$Y = \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{B} \cdot \bar{B} \cdot \bar{C}$$

Example



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$$Y = \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{B} \cdot \bar{B} \cdot \bar{C}$$

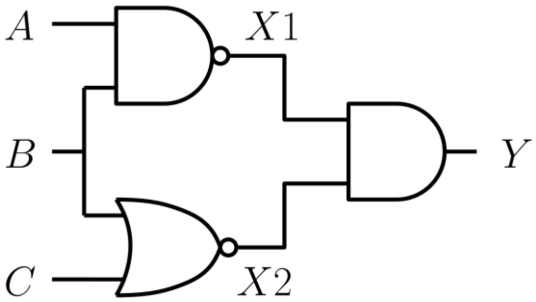
2) X·X=X

$$Y = \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{B} \cdot \bar{C}$$

$$Y = \bar{B} \cdot \bar{C} \cdot (\bar{A} + 1)$$

7	X·X=X
5	X·(Y+Z)=X·Y+X·Z

Example



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$$Y = \bar{A} \cdot \bar{B} \cdot \bar{C} + \bar{B} \cdot \bar{C}$$

$$Y = \bar{B} \cdot \bar{C} + (\bar{A} + 1)$$

3) X+1=1

$$Y = \bar{B} \cdot \bar{C}$$

This is the final result!!!

Example

True table for example

A	B	C	$Y = \overline{AB} \cdot \overline{B + C}$	$Y = \bar{B} \cdot \bar{C}$
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		

Karnaugh mapping

Maurice Karnaugh	
Born	October 4, 1924 New York City, U.S.
Died	November 8, 2022 (aged 98) The Bronx, New York, U.S.
Nationality	American
Known for	Karnaugh map
Spouse	Linn Blank (m. 1970)

Boolean algebra and De Morgan's theorem, let us to minimize Boolean functions, but we need have a lot of practice for make more simple solution.

Karnaugh mapping is a *systematic approach*, which will always produce the simplest configuration possible for the logic circuit.

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Karnaugh, Maurice (November 1953). "The Map Method for Synthesis of Combinational Logic Circuits". Transactions of the American Institute of Electrical Engineers, Part I: Communication and Electronics. 72 (5): 593–599. doi:10.1109/TCE.1953.6371932

Karnaugh mapping

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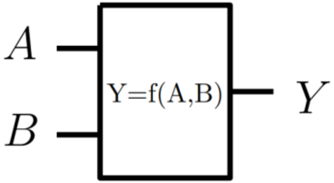
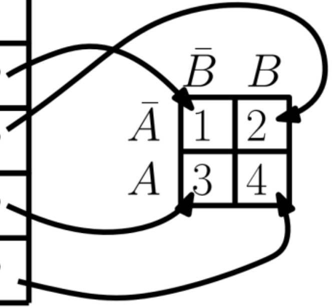
That is the Karnaugh map and how can we use it???

Karnaugh mapping

Karnaugh map for two input parameters.

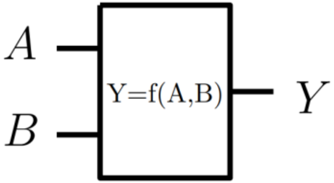


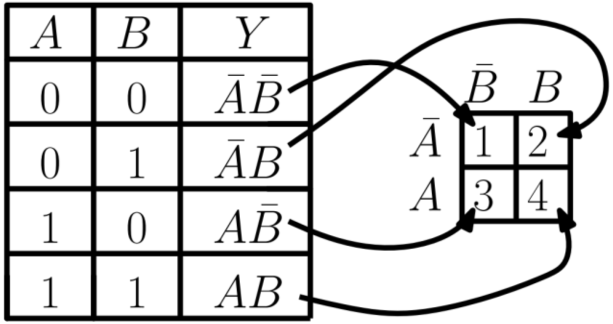
A	B	Y
0	0	$\bar{A}\bar{B}$
0	1	$\bar{A}B$
1	0	$A\bar{B}$
1	1	AB



Karnaugh mapping

Karnaugh map for two input parameters.

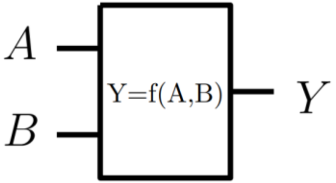


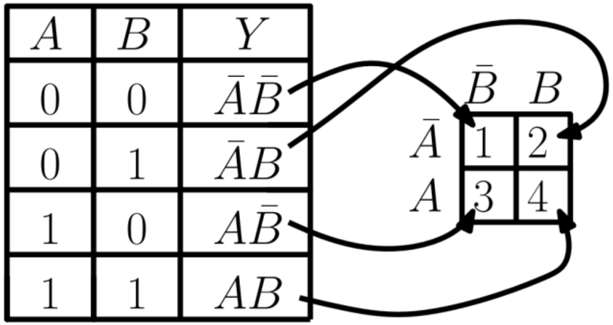


And how can we use it???

Karnaugh mapping

Karnaugh map for two input parameters.

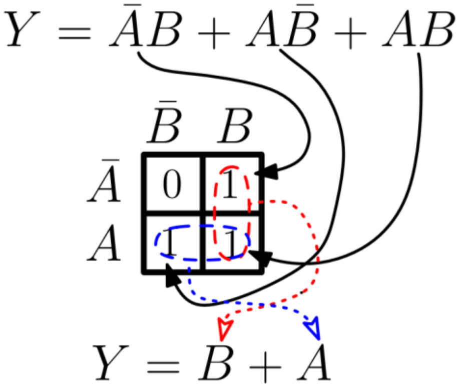




And how can we use it???

Simple example – consider Boolean equation

$Y = \bar{A}B + A\bar{B} + AB$



Karnaugh mapping

Consider the Karnaugh map for three input parameters.

$$Y = A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$$

	$\bar{C}$	C
$\bar{A}\bar{B}$	1	1
$\bar{A}B$		
AB	1	
$A\bar{B}$	1	

$$Y = A\bar{C} + \bar{A}\bar{B}$$

Consider the Karnaugh map for four input parameters.

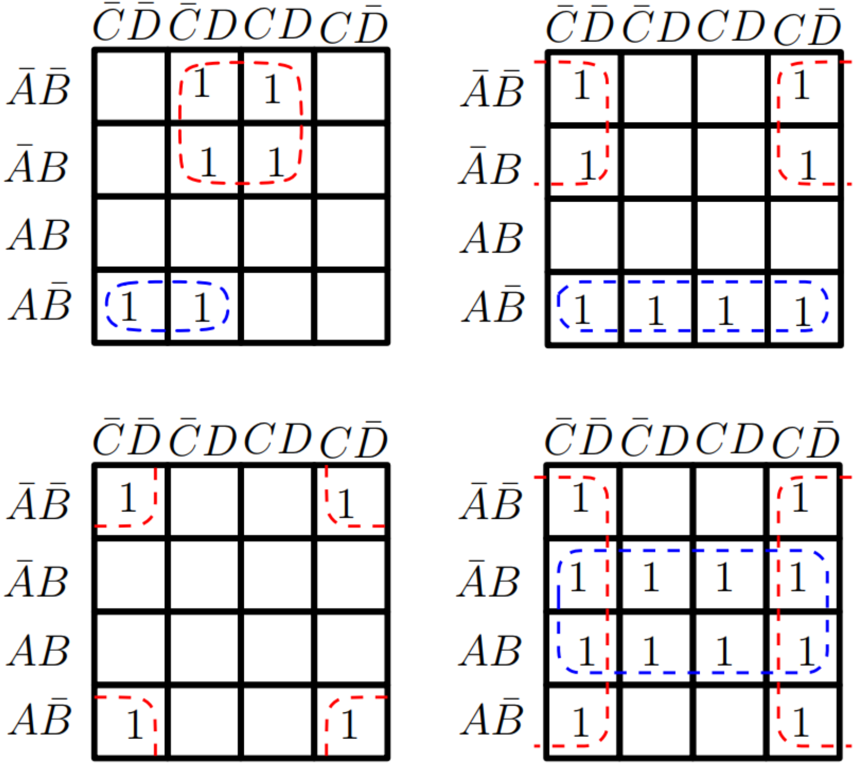
$$Y = A\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD + \bar{A}BCD + A\bar{B}\bar{C}D$$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$		1	1	
$\bar{A}B$		1	1	
AB				
$A\bar{B}$	1	1		

$$Y = A\bar{B}\bar{C} + \bar{A}D$$

Karnaugh mapping

Which looping technics can we use in the Karnaugh map?



$$Y = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

	$\bar{C}$	C
$\bar{A}\bar{B}$		
$\bar{A}B$	1	1
AB	1	1
$A\bar{B}$	1	1

$$Y = A + B$$

Example

$$Y = \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC$$

A	B	C	Y

$$Y = A + B$$

A	B	Y

References

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Thank you for your attention!