F.6 See Attached

$M = 2m_r + m_c$

$J = (2m_r d^2 + J_c)$

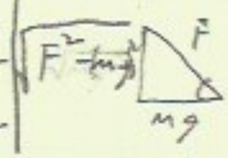
F.4 (A)

$$\begin{bmatrix} \ddot{z} \\ \ddot{h} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -(\sin(\theta)F + \mu\dot{z})/M \\ (\cos(\theta)F - Mg)/M \\ d\tau/J \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{z} \\ \dot{h} \\ \dot{\theta} \\ -\sin(\theta)F/m \\ (\cos(\theta)F - mg)/M \\ \tau \end{bmatrix} = \begin{bmatrix} \\ \end{bmatrix}$$

$0 = \sin(\theta)F$

$F\cos(\theta) = mg \rightarrow \cos(\theta) = \frac{mg}{F}$



$0 = \sqrt{F^2 - (mg)^2}/F \; F \rightarrow F = mg$

$1 = \cos(\theta) \rightarrow \theta = 0$

$$\boxed{\theta = 0,\ z = z_c,\ h = h_c,\ \dot{\theta} = 0,\ \dot{z} = 0,\ \dot{h} = 0,\ F = mg,\ \tau = 0}$$

(B)

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \dot{z} \\ \dot{h} \\ \ddot{\theta} \\ \ddot{z} \\ \ddot{h} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \dot{z} \\ \dot{h} \\ d\tau/J \\ -(\sin(\theta)F + \mu\dot{z})/M \\ (\cos(\theta)F - mg)/m \end{bmatrix} \qquad x = \begin{bmatrix} \theta \\ z \\ h \\ \dot{\theta} \\ \dot{z} \\ \dot{h} \end{bmatrix}$$

7/8

$$\frac{\partial \dot{x}}{\partial x} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\cos\theta \frac{F}{m} & 0 & 0 & 0 & -\mu/m & 0 \\ -\sin\theta \frac{F}{m} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_e \qquad \frac{\partial \dot{x}}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & d/J \\ \frac{-\sin(\theta)}{m} & 0 \\ \frac{\cos(\theta)}{m} & 0 \end{bmatrix}_e$$

at equilibrium

$$\left.\frac{\partial \dot{x}}{\partial x}\right|_e = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -g & 0 & 0 & 0 & -\mu/m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \left.\frac{\partial \dot{x}}{\partial u}\right|_e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & d/J \\ -0 & 0 \\ m^{-1} & 0 \end{bmatrix}$$

$$\dot{\tilde{\underline{x}}} = \dot{\underline{x}}(\underline{x}_e) + \left.\frac{\partial \dot{\underline{x}}}{\partial \underline{x}}\right|_e \tilde{\underline{x}} + \left.\frac{\partial \dot{x}}{\partial u}\right|_e \tilde{\underline{\tau}} \qquad \tilde{\underline{x}} = \begin{bmatrix} \tilde{\theta} \\ \tilde{z} \\ \tilde{h} \\ \dot{\tilde{\theta}} \\ \dot{\tilde{z}} \\ \dot{\tilde{h}} \end{bmatrix} \quad \tilde{\underline{\tau}} = \begin{bmatrix} \tilde{F} \\ \tilde{\tau} \end{bmatrix}$$

$$\dot{\tilde{\Theta}} = \dot{\tilde{\Theta}}$$

$$\dot{\tilde{z}} = \dot{\tilde{z}}$$

$$\dot{\tilde{h}} = \dot{\tilde{h}}$$

$$\ddot{\tilde{\Theta}} = \frac{d}{J}\tilde{\tau}$$

$$\ddot{\tilde{z}} = -g\tilde{\theta} - \frac{\mu}{m}\dot{\tilde{z}}$$

$$\ddot{\tilde{h}} = \frac{1}{m}\tilde{F}$$

(c) Can Not be Linearized by feedback Because of coupling. i.e. Linearizing $\ddot{z}$ would force $\ddot{h}$ to be unLinear and Viceversa

F.5 (A)

$$s^2 \tilde{\Theta}(s) = \frac{d}{J} \Upsilon(s)$$

$$s^2 \tilde{Z}(s) = -g \tilde{\Theta}(s) - \frac{\mu}{M} s \tilde{Z}(s)$$

$$s^2 \tilde{h}(s) = \frac{1}{M} \tilde{F}(s)$$

(B) $$\tilde{h}(s) = \frac{1}{s^2 M} \tilde{F}(2)$$

(C) $$\tilde{\Theta}(s) = \frac{d}{J s^2} \tilde{\Upsilon}(s)$$

$$\tilde{Z}(s) = \frac{-g}{s^2 + \frac{\mu}{M} s} \tilde{\Theta}(s)$$

$$\tilde{Z}(s) = \frac{-g d}{J s^4 + J \frac{\mu}{M} s^3} \tilde{\Upsilon}(s)$$