$\boxed{E.6}$

See attached

$\boxed{E.4}$ (a)

$$\begin{bmatrix}\dot\theta\\ \dot z\\ \ddot\theta\\ \ddot z\end{bmatrix} = \overbrace{\begin{bmatrix}\dot\theta\\ \dot z\\ \dfrac{(F\ell - m_1 z g - m_2 g \frac{\ell}{2})\cos(\theta) - 2 m_1 z \dot z \dot\theta)}{\frac{1}{3}\ell^2 m_2 + m_1 z^2}\\ z\dot\theta^2 - g\sin(\theta)\end{bmatrix}}^{f(x,u)}$$

$$\begin{bmatrix}0\\0\\0\\0\end{bmatrix} = \begin{bmatrix}\dot\theta\\ \dot z\\ (F\ell - m_1 z g - m_2 g\frac{\ell}{2})\\ -g\sin(\theta)\end{bmatrix}$$

$\rightarrow \theta = 0$

$$\begin{bmatrix}\theta\\ z\\ \dot\theta\\ \dot z\\ F\end{bmatrix} = \begin{bmatrix}0\\ z\\ 0\\ 0\\ (m_1 z g + m_2 g\frac{\ell}{2})/\ell\end{bmatrix}$$

Let

$A = (F\ell - m_1 z g - m_2 g\frac{\ell}{2})$

$B = \frac{1}{3}\ell^2 m_2 + m_1 z^2$

(B)

$$\frac{\partial f}{\partial x} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ & \frac{\partial f}{\partial x_3} & & \\ -g\cos(\theta) & \dot\theta^2 & 2z\dot\theta & 0\end{bmatrix}_e = \begin{bmatrix}0&0&1&0\\0&0&0&1\\0&\left.\frac{\partial f}{\partial z}\right|_e&0&0\\-g&0&0&0\end{bmatrix}$$

$$f = \left(\tfrac{1}{3} l^2 m_2 + m_1 Z^2\right)^{-1}\left(\left(Fl - m_1 Z g - m_2 g \tfrac{l}{2}\right)\cos(\theta) - 2 m_1 Z \dot{Z} \dot{\theta}\right)$$

$$\frac{\partial f}{\partial \theta} = \frac{\left(m_1 Z g + m_2 g \frac{l}{2} - Fl\right)}{\frac{1}{3} l^2 m_2 + m_1 Z^2} \sin(\theta) \overset{@\,\theta_e}{=} 0$$

$$\frac{\partial f}{\partial Z} = \frac{-g m}{\left(\frac{1}{3} l^2 m_2 + m_1 Z^2\right)} \leftarrow \text{from TI-89}$$

$$\frac{\partial f}{\partial \dot{\theta}} = -\left(\tfrac{1}{3} l^2 m_2 + m_1 Z^2\right)^{-1} 2 m_1 Z \dot{Z} = 0$$

$$\frac{\partial f}{\partial \dot{Z}} = -\left(\tfrac{1}{3} l^2 m_2 + m_1 Z^2\right)^{-1} 2 m_1 Z \dot{\theta} = 0$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ 0 \\ l\cos(\theta)/\left(\frac{1}{3} l^2 m_2 + m_1 Z^2\right) \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{l}{\frac{1}{3} l m_2 + m_1 Z^2} \\ 0 \end{bmatrix}$$

$$\ddot{\theta} = \frac{-g m_1}{\frac{1}{3} l^2 m_2 + m_1 Z_e^2} \tilde{Z} + \frac{l}{\frac{1}{3} l^2 m_2 + m_1 Z_e^2} \tilde{F} \longleftarrow$$

$$\left(\tfrac{1}{3} l^2 m_2 + m_1 Z_e^2\right)\ddot{\theta} = l \tilde{F} - g m_1 \tilde{Z}$$

$$\dot{\tilde{\theta}} = \dot{\theta}$$
$$\dot{\tilde{z}} = \dot{z}$$
$$\left(\tfrac{1}{3} l^2 m_2 + m_1 z_c^2\right) \ddot{\tilde{\theta}} = l \tilde{F} - g m_1 \tilde{z}$$
$$\ddot{\tilde{z}} = -g \tilde{\theta}$$

$\boxed{E.5}$ A & B

$$\left(\tfrac{1}{3} l^2 m_2 + m_1 z_c^2\right) s^2 \tilde{\theta}(s) = l \tilde{F}(s) - g m_1 \tilde{z}(s)$$

$$s^2 \tilde{z}(s) = -g \tilde{\theta}(s) \rightarrow \tilde{\theta}(s) = -\frac{s^2}{g} \tilde{z}(s) \rightarrow \tilde{z}(s) = -\frac{g}{s^2} \tilde{\theta}(s)$$

$$-\left(\left(\tfrac{1}{3} l^2 m_2 + m_1 z_c^2\right) s^4/g - g m_1\right) \tilde{z}(s) = l \tilde{F}(s)$$

$$\tilde{z}(s) = \frac{l}{g m_1 - \frac{1}{g}\left(\frac{1}{3} l^2 m_2 + m_1 z_c^2\right) s^4} \tilde{F}(s)$$

$\boxed{E.4}$ C can Not be linearized because their is no input in the $\ddot{z}$ equation

$\boxed{E.5}$ C Tf becomes

$$\tilde{z}(s) = \frac{l g}{\left(\frac{1}{3} l^2 m_2 + m_1 z_c^2\right) s^4} \tilde{F}(s)$$

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$$\tilde{F}(s) \longrightarrow \boxed{\frac{l g}{\left(\frac{1}{3} l^2 m_2 + m_1 z_c^2\right) s^4}} \xrightarrow{\tilde{z}(s)} \boxed{-\frac{g}{s^2}} \rightarrow \tilde{\theta}(s)$$