$\boxed{\text{D.6}}$

See video & code

$\boxed{\text{D.4}}$ (A)

Known

$$\dot{x} = 0 = f(x_e u_e) \quad \text{where} \quad x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} \quad u = [F]$$

given

$$f(x,u) = \dot{x} \rightarrow \begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \dot{z} \\ (F - b\dot{z} - kz)/m \end{bmatrix}$$

@ equilibrium

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{z} \\ (F - b\dot{z} - kz)/m \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ F \end{bmatrix} = \begin{bmatrix} \dot{z} \\ b\dot{z} + kz \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ F \end{bmatrix} = \begin{bmatrix} \dot{z} \\ kz \end{bmatrix}$$

$$\begin{bmatrix} z \\ \dot{z} \\ F \end{bmatrix} = \begin{bmatrix} z \\ 0 \\ kz \end{bmatrix}$$

(B) Taylor Series

$$f(x,u) = \cancel{f(x_e u_e)} + \frac{\partial f}{\partial x}\bigg|_{x_e u_e} (x - x_e) + \frac{\partial f}{\partial u}\bigg|_{x_e u_e} (u - u_e)$$

$$f(x_e, u_e) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\frac{\partial f}{\partial x}\bigg|_{x_e u_e} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \qquad \frac{\partial f}{\partial u}\bigg|_{x_e u_e} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$\dot{x} \approx \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \tilde{u}$$

$$\dot{z} \approx \dot{\tilde{z}}$$

$$\ddot{z} \approx -\frac{k}{m}\tilde{z} - \frac{b}{m}\dot{\tilde{z}} + \frac{1}{m}\tilde{F}$$

(c) Let $F = \tilde{F}$

$$\dot{z} = \dot{z}$$

$$\ddot{z} = -\frac{k}{m} z - \frac{b}{m}\dot{z} + \frac{1}{m}\tilde{F}$$

Not that this system is alread linearized
Thus the above is trivial.

$\boxed{D.5}$

(A)
$$s^2 Z(s) = -\frac{k}{m} Z(s) - \frac{b}{m} s Z(s) + \frac{1}{m} F(s)$$

$$Z(s)\left(s^2 + \frac{b}{m}s + \frac{k}{m}\right) = \frac{1}{m} F(s)$$

$$Z(s) = \left(ms^2 + bs + k\right)^{-1} F(s)$$

$$F(s) \longrightarrow \boxed{\frac{1}{ms^2+bs+k}} \longrightarrow Z(s)$$