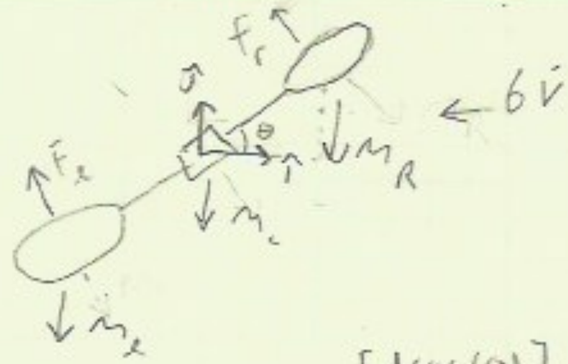
F.2



Rotational

$$KE = \frac{1}{2}\omega^T J \omega$$

$$J = \sum J$$

given J_c

$$P_R = \begin{bmatrix} d\cos(\theta) \\ d\sin(\theta) \\ 0 \end{bmatrix} \quad P_L = -P_R$$

$$J = m\left((P^T P) I_3 - P P^T\right)$$

$$J_R = m_r\left(\left([d\cos\theta, d\sin\theta, 0]\begin{bmatrix} d\cos\theta \\ d\sin\theta \\ 0 \end{bmatrix}\right)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} d\cos\theta \\ d\sin\theta \\ 0 \end{bmatrix}[d\cos\theta, d\sin\theta, 0]\right)$$

$$J_R = m_r\left(\begin{bmatrix} d^2 & 0 & 0 \\ 0 & d^2 & 0 \\ 0 & 0 & d^2 \end{bmatrix} - \begin{bmatrix} d^2\cos^2\theta & d^2\sin\theta\cos\theta & 0 \\ d^2\cos\theta\sin\theta & d^2\sin^2\theta & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$$

Because Rotation is in the $\hat{K}$ direction

$$J_R = J_L = m_r d^2 \quad \& \quad J_c = J_c$$

$$KE = \frac{1}{2}\dot{\theta}^2(m_r d^2 2 + J_c)$$

Translational

$$P = \begin{bmatrix} z \\ h \\ 0 \end{bmatrix} \qquad V = \begin{bmatrix} \dot{z} \\ \dot{h} \\ 0 \end{bmatrix}$$

$$KE = \frac{1}{2}V^T m V$$

$$KE = \frac{1}{2}[\dot{z}\ \dot{h}\ 0](m_R + m_L + m_c)\begin{bmatrix} \dot{z} \\ \dot{h} \\ 0 \end{bmatrix}$$

$$KE = \frac{1}{2}(m_R + m_L + m_c)(\dot{z}^2 + \dot{h}^2)$$

Combine

$$KE = \dot{\theta}^2 m_r d^2 + \frac{1}{2}\dot{\theta}^2 J_c + \frac{1}{2}(2m_r + m_c)\dot{z}^2 + \frac{1}{2}(2m_r + m_c)\dot{h}^2$$

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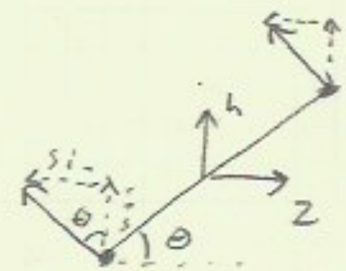
F.3

(a) $PE = (2m_r + m_c) g h$

$$L = \dot{\theta}^2 m_r d^2 + \frac{1}{2}\dot{\theta}^2 J_c + \frac{1}{2}(2m_r + m_c)\dot{z}^2 + \frac{1}{2}(2m_r + m_c)\dot{h}^2 - (2m_r + m_c) g h$$

(b) $q = \begin{bmatrix} z \\ h \\ \theta \end{bmatrix} \quad b = \begin{bmatrix} \mu \\ 0 \\ 0 \end{bmatrix}$

(c) $\tau = \begin{bmatrix} -\sin(\theta)(f_l + f_r) \\ \cos(\theta)(f_l + f_r) \\ (f_r - f_l) d \end{bmatrix}$



(d) $\frac{d}{dt}\left(\frac{\partial}{\partial q} L\right) - \frac{\partial}{\partial q} L = \tau - b\dot{q}$

for z

$$\frac{d}{dt}\left((2m_r + m_c)\dot{z}\right) - 0 = -\sin(\theta)(f_l + f_r) - \mu\dot{z}$$

$$(2m_r + m_c)\ddot{z} + \mu\dot{z} = -\sin(\theta)(f_l + f_r)$$

for h

$$\frac{d}{dt}\left((2m_r + m_c)\dot{h}\right) + (2m_r + m_c) g = \cos(\theta)(f_l + f_r) - 0$$

$$(2m_r + m_c)\ddot{h} + (2m_r + m_c) g = \cos(\theta)(f_l + f_r)$$

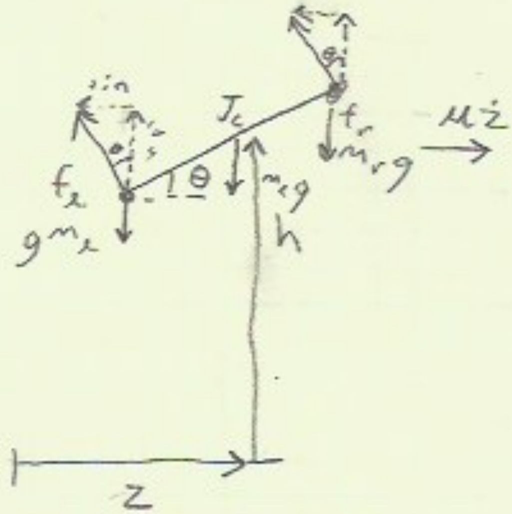
for θ

$$\frac{d}{dt}\left(2m_r d^2 \dot{\theta} + J_c \dot{\theta}\right) - 0 = (f_r - f_l) d - 0$$

$$2m_r d^2 \ddot{\theta} + J_c \ddot{\theta} = (f_r - f_l) d$$

Check

$ma = \sum F$



$$-\sin(\theta)(f_l + f_r) - \mu \dot{z} = (2m_r + m_c)\ddot{z}$$

$$(2m_r + m_c)\ddot{z} + \mu \dot{z} = -\sin(\theta)(f_l + f_r)$$ ✓

$$(2m_r + m_c)\ddot{h} = \cos(\theta)(f_l + f_r) - (2m_r + m_c)g$$

$$(2m_r + m_c)\ddot{h} + (2m_r + m_c)g = \cos(\theta)(f_l + f_r)$$ ✓

$$(2m_r d^2 + J_c)\ddot{\theta} = d(f_r - f_l)$$ ✓