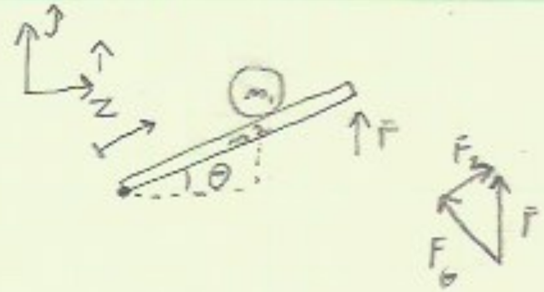
$\boxed{E.2}$



$KE = \sum KE$

Rotational

Beam

$$K = \frac{1}{2} w^T J w$$

$$J = \int_{p \in body} \left((p^T p) I_3 - p p^T \right) dm$$

$$J = \int_0^l \int_0^0 \int_0^0 \left(\left([x \ y \ z] \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} x \\ y \\ z \end{bmatrix} [x y z] \right) \right) dx \, dy \, dz \frac{m}{l}$$

$$J = \frac{m}{l} \int_0^l \int_0^0 \int_0^0 \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -yx & x^2+z^2 & -yz \\ -zx & -zy & x^2+y^2 \end{bmatrix} dx \, dy \, dz$$

$$J = \int_0^l \begin{bmatrix} z^2 & 0 & 0 \\ 0 & z^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} dz$$

$$J = \begin{bmatrix} \frac{1}{3} l^3 & 0 & 0 \\ 0 & \frac{1}{3} l^3 & 0 \\ 0 & 0 & 0 \end{bmatrix} \frac{m}{l} \rightarrow J = \frac{1}{3} l^2 m$$

$$K = \frac{1}{2} \dot{\theta} \frac{1}{3} l^3 \dot{\theta}$$

$$K = \frac{1}{6} l^2 \dot{\theta}^2 m$$

Ball

$$p = \begin{bmatrix} z\cos(\theta) \\ z\sin(\theta) \\ 0 \end{bmatrix} \qquad \dot{p} = \begin{bmatrix} \dot{z}\cos(\theta) - z\dot{\theta}\sin(\theta) \\ \dot{z}\sin(\theta) + z\dot{\theta}\cos(\theta) \\ 0 \end{bmatrix}$$

Know

$$KE = \frac{1}{2} \dot{p}^T m_1 \dot{p}$$

$$KE = \frac{1}{2} m_1 \left((\dot{z}\cos(\theta) - z\dot{\theta}\sin(\theta))^2 + (\dot{z}\sin(\theta) + z\dot{\theta}\cos(\theta))^2 \right)$$

$$KE = \frac{1}{2} m_1 \Big(\dot{z}^2 \cos(\theta)^2 \cancel{- z\dot{z}\dot{\theta}\cos(\theta)\sin(\theta)2} + z^2\dot{\theta}^2 \sin(\theta)^2$$
$$+ (\dot{z}^2 \sin^2(\theta) \cancel{+ z\dot{z}\dot{\theta}\cos(\theta)\sin(\theta)2} + z^2\dot{\theta}^2\cos(\theta)^2 \Big)$$

$$KE = \frac{1}{2} m_1 (\dot{z}^2 + z^2 \dot{\theta}^2)$$

$$K_o = \frac{1}{2}\dot{\theta} m (z^2 + r^2)\dot{\theta}$$

$$K_o = \frac{1}{2} m \dot{\theta} (z^2 + r^2)\dot{\theta}$$

Assume $r \approx 0$ & Translational rotation is 0

$$K_o = \frac{1}{2} m \dot{\theta}^2 Z^2$$

Linear

Beam

$$K = 0$$

Ball

$$K_o = \frac{1}{2} V^T m V$$

$$K_o = \frac{1}{2} \dot{z}\, m\, \dot{z}$$

$$K_o = \frac{1}{2} \dot{Z}^2 m$$

combining

$$KE = \frac{1}{6} \ell^2 \dot{\theta}^2 m_2 + \frac{1}{2} m_1 \dot{\theta}^2 Z^2 + \frac{1}{2} m_1 \dot{Z}^2$$

E.3

(a) $PE = \sin(\theta) \frac{\ell}{2} m_2 + \sin(\theta)(Z) m_1 g$

(b) $q = \begin{bmatrix} z \\ \theta \end{bmatrix}$

(c) $\tau = \begin{bmatrix} 0 \\ F_\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \ell F \cos\theta \end{bmatrix}$ $\qquad b = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

(d) $L = \frac{1}{6} \ell^2 \dot{\theta}^2 m_2 + \frac{1}{2} m_1 \dot{\theta}^2 Z^2 + \frac{1}{2} m_1 \dot{Z}^2 - \sin\theta \frac{\ell}{2} m_2 g - \sin\theta Z m_1 g$

Known

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{q}} L\right) - \frac{\partial}{\partial q} L = \tau - B\dot{q}$$

For $q_1 = z$

$$\frac{d}{dt}(m_1 \dot{z}) - m_1 z \dot{\theta}^2 + \sin\theta \, m_1 g = 0$$

$$m_1 \ddot{z} - m_1 \dot{\theta}^2 z + \sin\theta \, m_1 g = 0$$

For $q_2 = \theta$

$$\frac{d}{dt}\left(\frac{1}{3} l^2 m_2 \dot{\theta} + m_1 z^2 \dot{\theta}\right) + g\cos\theta \frac{l}{2} m_2 + \cos\theta \, z m_1 g$$

$$= l F \cos\theta$$

$$m_1 \dot{z} 2 z \dot{\theta} + \frac{1}{3} l^2 m_2 \ddot{\theta} + m_1 z^2 \ddot{\theta} + \cos\theta \frac{l}{2} m_2 g + \cos\theta \, z m_1 g$$

$$= l F \cos\theta$$

$$2 m_1 \dot{z} z \dot{\theta} + \left(\frac{1}{3} l^2 m_2 + m_1 z^2\right)\ddot{\theta} + \cos\theta\left(\frac{l}{2} m_2 g + g z m_1 - l F\right) = 0$$

<u>check</u>

$\vec{p}_b = z \hat{n}$

$\vec{v}_b = \dot{z}\hat{n} + z \frac{d\hat{n}}{dt}$

$= \dot{z}\hat{n} + z\dot{\theta}\hat{T}$

$\vec{a}_b = \ddot{z}\hat{n} + \dot{z}\frac{d\hat{n}}{dt}$

$+ \dot{z}\dot{\theta}\hat{T} + z\ddot{\theta}\hat{T} + z\dot{\theta}\frac{d\hat{T}}{dt}$

$\vec{a}_b = \ddot{z}\hat{n} + \dot{z}\dot{\theta}\hat{T}$

$+ \dot{z}\dot{\theta}\hat{T} + z\ddot{\theta}\hat{T} - z\dot{\theta}^2\hat{n}$

$$\vec{a} = \begin{bmatrix} \ddot{z} - z\dot{\theta}^2 \\ 2\dot{z}\dot{\theta} + z\ddot{\theta} \end{bmatrix}$$

$\hat{n} = \cos(\theta)\hat{i} + \sin(\theta)\hat{j}$

$\hat{T} = -\sin(\theta)\hat{i} + \cos(\theta)\hat{j}$

$\frac{d\hat{n}}{dt} = -\dot{\theta}\sin(\theta)\hat{i} + \dot{\theta}\cos(\theta)\hat{j}$

$= \dot{\theta}(\cos(\theta)\hat{j} - \sin(\theta)\hat{i})$

$= \dot{\theta}\hat{T}$

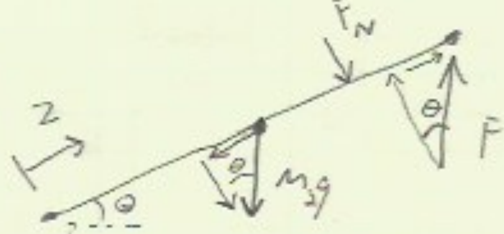
$\frac{d\hat{T}}{dt} = -\dot{\theta}\cos(\theta)\hat{i} - \dot{\theta}\sin(\theta)\hat{j}$

$= -\dot{\theta}(\cos(\theta)\hat{i} + \sin(\theta)\hat{j})$

$= -\dot{\theta}\hat{n}$

$\phi = 90 - \theta$

$\sum F = ma$



$$J\ddot{\theta} = F\ell\cos(\theta) - F_N Z - m_2 g\frac{\ell}{2}\cos(\theta)$$

$$\frac{1}{3}\ell^2 m_2\ddot{\theta} = F\ell\cos(\theta) - F_N Z - m_2 g\frac{\ell}{2}\cos\theta$$

$$m_1(\ddot{Z} - Z\dot{\theta}^2) = -m_1 g\sin(\theta)$$

$$m_1\ddot{Z} = m_1 Z\dot{\theta}^2 - m_1 g\sin(\theta) \checkmark$$

$$m_1(2\dot{Z}\dot{\theta} + Z\ddot{\theta}) = -m_1 g\cos(\theta) + F_N$$

$$F_N = 2m_1\dot{Z}\dot{\theta} + m_1 Z\ddot{\theta} + m_1 g\cos(\theta)$$

$$\frac{1}{3}\ell^2 m_2\ddot{\theta} = F\ell\cos(\theta) - 2m_1 Z\dot{Z}\dot{\theta} - m_1 Z^2\ddot{\theta} - m_1 Z g\cos(\theta) - m_2 g\frac{\ell}{2}\cos(\theta)$$

$$\left(\frac{1}{3}\ell^2 m_2 + m_1 Z^2\right)\ddot{\theta} + 2m_1 Z\dot{Z}\dot{\theta} + \cos(\theta)\left(m_1 Z g + m_2 g\frac{\ell}{2} - F\ell\right) = 0 \checkmark$$