F.6 F.4 See Attached

$M = 2m_r + m_c$

$J = (2m_r d^2 + J_c)$

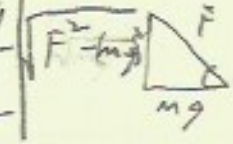
④

$$\begin{bmatrix} \ddot{z} \\ \ddot{h} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -(\sin(\theta) F + M \dot{z}) / M \\ (\cos(\theta) F - Mg)/M \\ d\Upsilon / J \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{z} \\ \dot{h} \\ \dot{\theta} \\ -\sin(\theta) F/m \\ (\cos(\theta) F - mg)/M \\ \Upsilon \end{bmatrix} = \begin{bmatrix} \end{bmatrix}$$

$0 = \sin(\theta) F$

$F\cos(\theta) = mg \rightarrow \cos(\theta) = \frac{mg}{F}$



$0 = \sqrt{F^2 - (mg)^2}/F \; F \rightarrow F = mg$

$1 = \cos(\theta) \rightarrow \theta = 0$

$\theta = 0, \; z = z_c, \; h = h_c, \; \dot{\theta} = 0, \; \dot{z} = 0, \; \dot{h} = 0, \; F = mg, \; \Upsilon = 0$

⑧

$$\dot{x} = \begin{bmatrix} \dot{\theta} \\ \dot{z} \\ \dot{h} \\ \ddot{\theta} \\ \ddot{z} \\ \ddot{h} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \dot{z} \\ \dot{h} \\ d\Upsilon/J \\ -(\sin(\theta) F + M\dot{z})/M \\ (\cos(\theta) F - mg)/m \end{bmatrix} \qquad x = \begin{bmatrix} \theta \\ z \\ h \\ \dot{\theta} \\ \dot{z} \\ \dot{h} \end{bmatrix}$$

$$\frac{\partial \dot{x}}{\partial x} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -\cos\theta \frac{F}{m} & 0 & 0 & 0 & M/M & 0 \\ -\sin\theta \frac{F}{M} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}_e \quad \frac{\partial \dot{x}}{\partial u} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & d/J \\ \frac{-\sin(\theta)}{m} & 0 \\ \frac{\cos(\theta)}{m} & 0 \end{bmatrix}_e$$

at equilibrium

$$\frac{\partial \dot{x}}{\partial x}\Big|_e = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -g & 0 & 0 & 0 & M/M & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \frac{\partial \dot{x}}{\partial u}\Big|_e = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & d/J \\ -0 & 0 \\ M^{-1} & 0 \end{bmatrix}$$

$$\tilde{\dot{x}} = \dot{x}(x_e) + \frac{\partial \dot{x}}{\partial x}\Big|_e \tilde{x} + \frac{\partial \dot{x}}{\partial u}\Big|_e \tilde{u} \qquad \tilde{x} = \begin{bmatrix} \tilde{\theta} \\ \tilde{z} \\ \tilde{h} \\ \dot{\tilde{\theta}} \\ \dot{\tilde{z}} \\ \dot{\tilde{h}} \end{bmatrix} \quad \tilde{u} = \begin{bmatrix} \tilde{T} \\ \tilde{F} \end{bmatrix}$$

$$\dot{\tilde{\theta}} = \dot{\tilde{\theta}}$$
$$\dot{\tilde{z}} = \dot{\tilde{z}}$$
$$\dot{\tilde{h}} = \dot{\tilde{h}}$$
$$\ddot{\tilde{\theta}} = \frac{d}{J}\tilde{T}$$
$$\ddot{\tilde{z}} = -g\tilde{\theta} + \frac{M}{m}\tilde{z}$$
$$\ddot{\tilde{h}} = \frac{1}{M}\tilde{F}$$

(c) Can Not be linearized by feedback Because of coupling. i.e. Linearizing $\ddot{z}$ would force $\ddot{h}$ to be unlinear and vicaversa

F.5 (A)

$$s^2\tilde{\theta}(s) = \frac{d}{J}\tilde{\tau}(s)$$

$$s^2\tilde{z}(s) = -g\tilde{\theta}(s) + \frac{u}{M}s\tilde{z}(s)$$

$$s^2\tilde{h}(s) = \frac{1}{M}\tilde{F}(s)$$

(B) $$\tilde{h}(s) = \frac{1}{s^2 M}\tilde{F}(s)$$

(C) $$\tilde{\theta}(s) = \frac{d}{Js^2}\tilde{\tau}(s)$$

$$\tilde{z}(s) = \frac{-g}{s^2 - \frac{u}{M}s}\tilde{\theta}(s)$$

$$\tilde{z}(s) = \frac{-gd}{Js^4 - J\frac{u}{M}s^3}\tilde{\tau}(s)$$