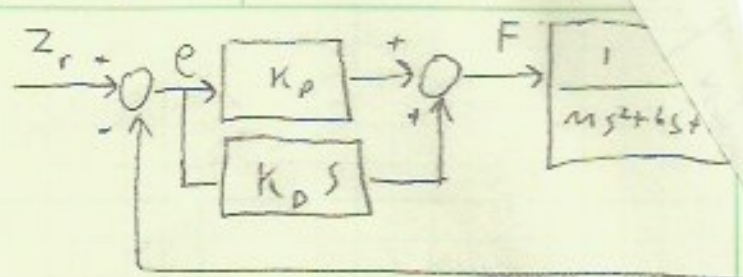


D.9



$$E = Z_r - Z$$

$$F = E(K_p - K_D s)$$

$$Z = F \frac{1}{ms^2 + bs + k}$$

$$Z = E(K_p - K_D s) \frac{1}{ms^2 + bs + k}$$

$$Z = - \frac{K_D s - K_p}{ms^2 + bs + k} E$$

$$Z_r - E = - \frac{K_D s - K_p}{ms^2 + bs + k} E$$

$$Z_r = \left( 1 - \frac{K_D s - K_p}{ms^2 + bs + k} \right) E$$

$$E = \frac{1}{1 - \frac{K_D s - K_p}{ms^2 + bs + k}} Z_r$$

Assume poles in left half plane

Final value theorem

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s)$$

$$= \lim_{s \rightarrow 0} s \frac{1}{1 + \frac{K_D s + K_p}{ms^2 + bs + k}} Z_r$$

Step

$$= \lim_{s \rightarrow 0} s \frac{1}{1 + \frac{K_D s + K_p}{ms^2 + bs + k}} \frac{1}{s} = \frac{1}{1 + \frac{K_p}{k}}$$

Finite error

Ramp

$$= \lim_{s \rightarrow 0} s \frac{1}{1 + \frac{K_D s + K_p}{ms^2 + bs + k}} \frac{1}{s^2} = \infty$$

Infinite error

Parabola

$$= \lim_{s \rightarrow 0} s \frac{1}{1 + \frac{K_D s + K_p}{ms^2 + bs + k}} \frac{1}{s^3} = \infty$$

Infinite error

System type = 0

Known

$$E(s) = \frac{1}{1+P(s)C(s)} Z_r(s) = \frac{1}{1 + \frac{K_0 s^2 + K_1 s + K_i}{s(m s^2 + b s + k)}} Z_r$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s E(s) = \lim_{s \rightarrow 0} s \frac{s}{s + \frac{K_0 s^2 + K_1 s + K_i}{m s^2 + b s + k}} Z_r$$

Step

$$= \lim_{s \rightarrow 0} s \frac{s}{s + \frac{K_0 s^2 + K_1 s + K_i}{m s^2 + b s + k}} \frac{1}{s} = 0$$

Ramp

$$= \lim_{s \rightarrow 0} s \frac{s}{s + \frac{K_0 s^2 + K_1 s + K_i}{m s^2 + b s + k}} \frac{1}{s^2} = \frac{K}{K_i}$$

Parabola

$$= \lim_{s \rightarrow 0} s \frac{s}{s + \frac{K_0 s^2 + K_1 s + K_i}{m s^2 + b s + k}} \frac{1}{s^3} = \infty$$

system type = 1

Known

$$E(s) = \frac{1}{1+P(s)C(s)} R + \frac{1}{1+P(s)C(s)} N + \frac{1}{1+P(s)C(s)} D_{out} + \frac{P(s)}{1+P(s)C(s)} D_{in}$$

$$E_{D_{in}} = \frac{P(s)}{1+P(s)C(s)} D_{in} = \frac{1}{1 + \frac{K_0 s + K_1}{m s^2 + b s + k}} \frac{A}{s}$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \frac{1}{1 + \frac{K_0 s + K_1}{m s^2 + b s + k}} \frac{A}{s} = \frac{A}{1 + K_1/k}$$

$$\lim_{t \rightarrow \infty} e(t) = \lim_{s \rightarrow 0} s \frac{s}{s + \frac{K_0 s^2 + K_1 s + K_i}{m s^2 + b s + k}} \frac{A}{s} = 0$$