E.b

See attached

E.4 (a)

$$\overbrace{\qquad\qquad f(x,u)\qquad\qquad}$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{Z} \\ \ddot{\theta} \\ \ddot{Z} \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \dot{Z} \\ \dfrac{(Fl - m_1 Z g - m_2 g \frac{l}{2})\cos(\theta) - 2 m_1 Z \dot{Z}\dot{\theta})}{\frac{1}{3}l^2 m_2 + m_1 Z^2} \\ Z\dot{\theta}^2 - g\sin(\theta) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ \dot{Z} \\ (Fl - m_1 Z g - m_2 g \frac{l}{2}) \\ -g\sin(\theta) \end{bmatrix} \quad \left[\; l^2 m_2 + m_1 Z^2 \;\right]$$

$\longrightarrow \theta = 0$

$$\begin{bmatrix} \theta \\ Z \\ \dot{\theta} \\ \dot{Z} \\ F \end{bmatrix} = \begin{bmatrix} 0 \\ Z \\ 0 \\ 0 \\ (m_1 Z g + m_2 g \frac{l}{2})/l \end{bmatrix}$$

$\underline{Let}$

$A = (Fl - m_1 Z g - m_2 g \frac{l}{2})$

$B = \frac{1}{3}l^2 m_2 + m_1 Z^2$

(B)

$$\frac{\partial f}{\partial x} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ & & \frac{\partial f}{\partial x_3} & \\ -g\cos(\theta) | \dot{\theta}^2 | 2 Z\dot{\theta} | 0 \end{bmatrix}_e = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{\partial f}{\partial Z}|_e & 0 & 0 \\ -g & 0 & 0 & 0 \end{bmatrix}$$

$$f = \left(\frac{1}{3}\ell^2 m_2 + m_1 z^2\right)^{-1}\left(\left(F\ell - m_1 zg - m_2 g\frac{\ell}{2}\right)\cos(\theta) - 2m_1 z\dot{z}\dot{\theta}\right)$$

$$\frac{\partial f}{\partial \theta} = \frac{\left(m_1 zg + m_2 g\frac{\ell}{2} - F\ell\right)}{\frac{1}{3}\ell^2 m_2 + m_1 z^2}\sin(\theta) \overset{\ell\,\theta_e}{=} 0$$

$$\frac{\partial f}{\partial z} = \frac{-gm}{\left(\frac{1}{3}\ell^2 m_2 + m_1 z^2\right)} \leftarrow \text{from } Ti \rightarrow \partial q$$

$$+ \left(\frac{1}{3}\ell^2 m_2 + m_1 z^2\right)^{-1}\left(-m_1 g\cos(\theta) - 2m_1 \dot{z}\dot{\theta}\right)$$

$$= \frac{-2m_1 z \cdot 2}{\left(\frac{1}{3}\ell^2 m_2 + m_1 z^2\right)^2}\left(\ell(m_1 zg + m_2 z^2)\frac{F\ell - m_1 g\frac{\ell}{2}}{\frac{1}{3}\ell^2 m_2 + m_1 z^2}\right) - \frac{m_1 g}{\frac{1}{3}\ell^2 m_2 + m_1 z^2}$$

$$\frac{\partial f}{\partial \dot{\theta}} = -\left(\frac{1}{3}\ell^2 m_2 + m_1 z^2\right)^{-1} 2m_1 z\dot{z} = 0$$

$$\frac{\partial f}{\partial \dot{z}} = -\left(\frac{1}{3}\ell^2 m_2 + m_1 z^2\right)^{-1} 2m_1 z\dot{\theta} = 0$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} 0 \\ 0 \\ \ell\cos(\theta)/\left(\frac{1}{3}\ell^2 m_2 + m_1 z^2\right) \\ 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{z} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dfrac{\ell}{\frac{1}{3}\ell m_2 + m_1 z^2} \\ 0 \end{bmatrix}$$

$$\ddot{\theta} = \frac{-gm_1 z}{\frac{1}{3}\ell^2 m_2 + m_1 z_e^2}\tilde{z} + \frac{\ell}{\frac{1}{3}\ell^2 m_2 + m_1 z_e^2}\tilde{F} \quad\blacktriangleleft$$

$$\left(\frac{1}{3}\ell^2 m_2 + m_1 z_e^2\right)\ddot{\theta} = \ell\tilde{F} - gm_1\tilde{z}$$

$$\ddot{\tilde{\theta}} = \dot{\tilde{\theta}}$$

$$\dot{\tilde{Z}} = \dot{Z}$$

$$\left(\tfrac{1}{3}\ell^2 m_2 + m_1 Z_c^2\right)\ddot{\tilde{\theta}} = \ell\,\tilde{F} - g m_1 \tilde{Z}$$

$$\ddot{\tilde{Z}} = -g\tilde{\theta}$$

E.5   Ⓐ & Ⓑ

$$\left(\tfrac{1}{3}\ell^2 m_2 + m_1 Z_c^2\right) s^2 \tilde{\theta}(s) = \ell\,\tilde{F}(s) - g m_1 \tilde{Z}(s)$$

$$s^2 \tilde{Z}(s) = -g\tilde{\theta}(s) \;\rightarrow\; \tilde{\theta}(s) = -\frac{s^2}{g}\tilde{Z}(s) \rightarrow \tilde{Z}(s) = -\frac{g}{s^2}\tilde{\theta}(s)$$

$$-\left(\left(\tfrac{1}{3}\ell^2 m_2 + m_1 Z_c^2\right) s^4/g - g m_1\right)\tilde{Z}(s) = \ell\,\tilde{F}(s)$$

$$\tilde{Z}(s) = \frac{\ell}{g m_1 - \frac{1}{g}\left(\tfrac{1}{3}\ell^2 m_2 + m_1 Z_c^2\right) s^4}\,\tilde{F}(s)$$

E.4   Ⓒ   can Not be linearized because
their is no input in the $\ddot{Z}$
equation

E.5   Ⓒ   Tf becomes

$$\tilde{Z}(s) = \frac{\ell g}{\left(\tfrac{1}{3}\ell^2 m_2 + m_1 Z_c^2\right) s^4}\,\tilde{F}(s)$$

Ⓓ

$$\tilde{F}(s) \longrightarrow \boxed{\frac{\ell}{\left(\tfrac{1}{3}\ell^2 m_2 + m_1 Z_c^2\right) s^2}} \xrightarrow{\;\tilde{\theta}(s)\;} \boxed{-\frac{g}{s^2}} \rightarrow \tilde{Z}(s)$$