D.6

$\dot{x} = Ax + Bu$ where $x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}$ $u = [F]$ $y = z$

$y = Cx + Du$

$\begin{bmatrix} \dot{z} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -K/m & -b/m \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} [F]$

$\dot{z} = \dot{z}$

$\ddot{z} = -\frac{K}{m} z - \frac{b}{m} \dot{z} + \frac{1}{m} F$

$[z] = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \end{bmatrix} + [0][F]$

$\dot{x} = \begin{bmatrix} 0 & 1 \\ -K/m & -b/m \end{bmatrix} x + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u$

$y = [1 \quad 0] x + [0] u$

D.7

given

$$\frac{1}{ms^2 + bs + K} = H(s) = \frac{1}{5s^2 + 0.5s + 3}$$

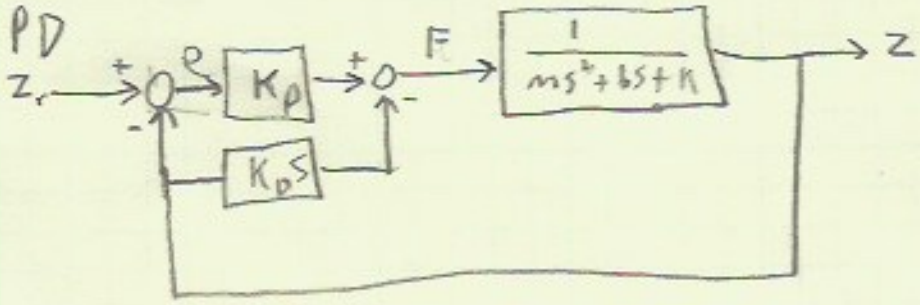
$\Delta(s) = 5s^2 + 0.5s + 3$

$p_{1,2} = \frac{-0.5 \pm \sqrt{0.5^2 - 4(5)3}}{2(5)} = -\frac{1}{20} \pm \frac{1}{10}\sqrt{0.25 - 60} =$

$p_1 = -0.05 + 0.772981j$

$p_2 = -0.05 - 0.772981j$

PD



$e = z_r - z$

$F = eK_p - K_D s z$

$z = F \frac{1}{ms^2 + bs + K}$

$((z_r - z)K_p - K_D s z) \frac{1}{ms^2 + bs + K} = z$

$$K_p Z_r - K_p Z - K_D s Z = Z(ms^2 + bs + K)$$

$$K_p Z_r = Z(ms^2 + (b + K_D)s + (K + K_p))$$

$$Z = \frac{K_p}{ms^2 + (b+K_D)s + (K+K_p)} Z_r$$

$$\Delta(s) = ms^2 + (b+K_D)s + (K+K_p)$$

$$p_{1,2} = \frac{1}{2m}\left(-(b+K_D) \pm \sqrt{(b+K_D)^2 - 4m(K+K_p)}\right)$$

$$p_{1,2} = \frac{1}{10}\left(-(0.5+K_D) \pm \sqrt{(0.5+K_D)^2 - 4(5)(3+K_p)}\right)$$

$$p_{1,2} = \frac{1}{10}\left(-0.5 - K_D \pm \sqrt{0.25 + K_D + K_D^2 - 60 - 20K_p}\right)$$

$$p_{1,2} = \frac{1}{10}\left(-0.5 - K_D \pm \sqrt{K_D^2 - 20K_p + K_D - 60}\right)$$