# Tree-Structured Indexes

# Introduction

- *As for any index, 3 alternatives for data entries* **k***:
  - Data record with key value **k**
  - <**k**, rid of data record with search key value **k**>
  - <**k**, list of rids of data records with search key **k**>

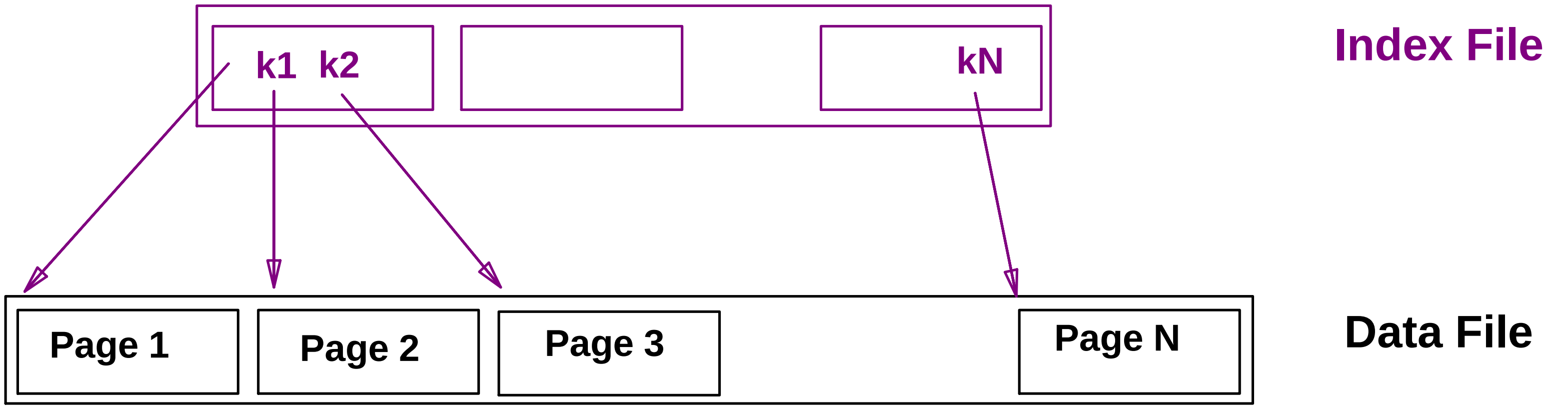
- Choice is orthogonal to the *indexing technique* used to locate data entries **k***.

- Tree-structured indexing techniques support both *range searches* and *equality searches*.

- *ISAM*: static structure; *B+ tree*: dynamic, adjusts gracefully under inserts and deletes.

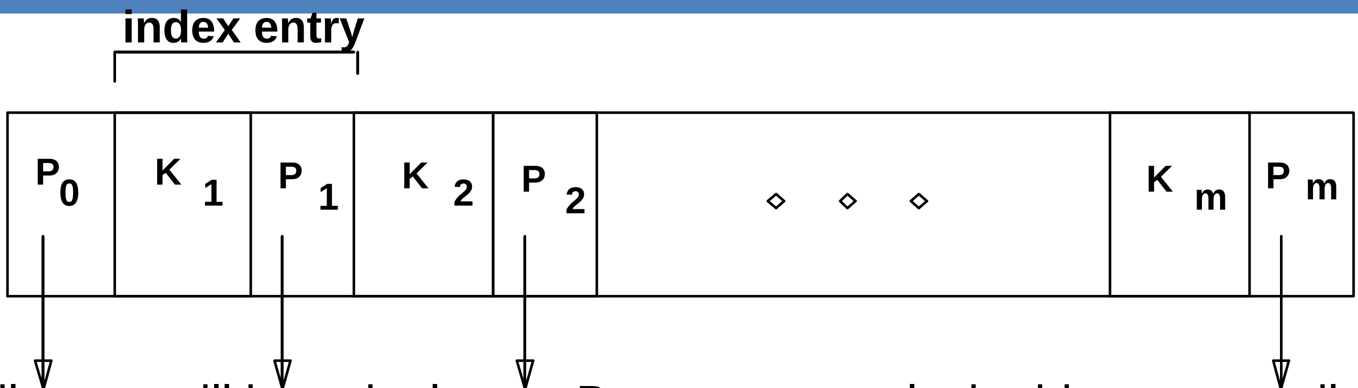
# Range Searches

- ``*Find all students with gpa > 3.0*''
  - If data is in sorted file, do binary search to find first such student, then scan to find others.
  - Cost of binary search can be quite high.
- Simple idea: Create an `index' file.

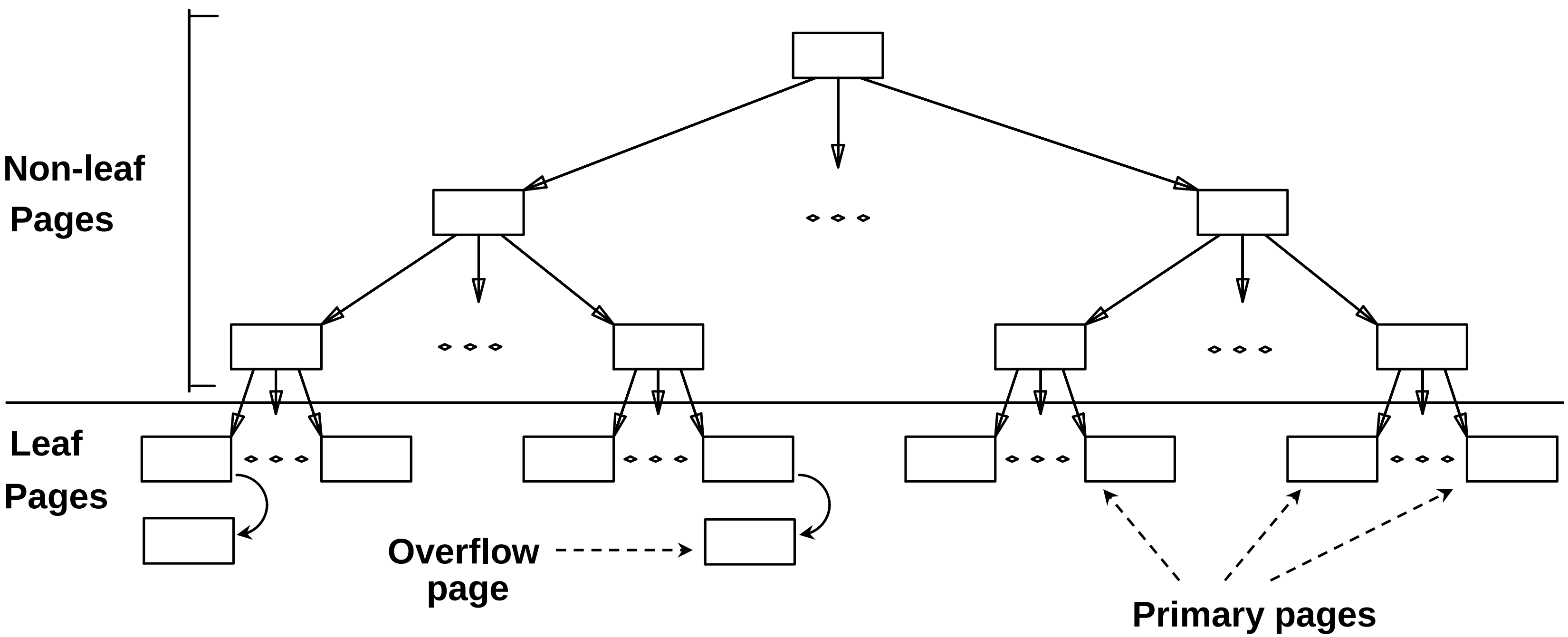



* *Can do binary search on (smaller) index file!*

# ISAM




- Index file may still be quite large.  But we can apply the idea repeatedly!




* *Leaf pages contain data entries.*

# Comments on ISAM

| |
|---|
| **Data Pages** |
| **Index Pages** |
| **Overflow pages** |

- *File creation*: Leaf (data) pages allocated sequentially, sorted by search key; then index pages allocated, then space for overflow pages.

- *Index entries*: <search key value, page id>; they `direct' search for *data entries*, which are in leaf pages.

- ***Search***: Start at root; use key comparisons to go to leaf. Cost $\log_F N$ ; F = # entries/index pg, N = # leaf pgs

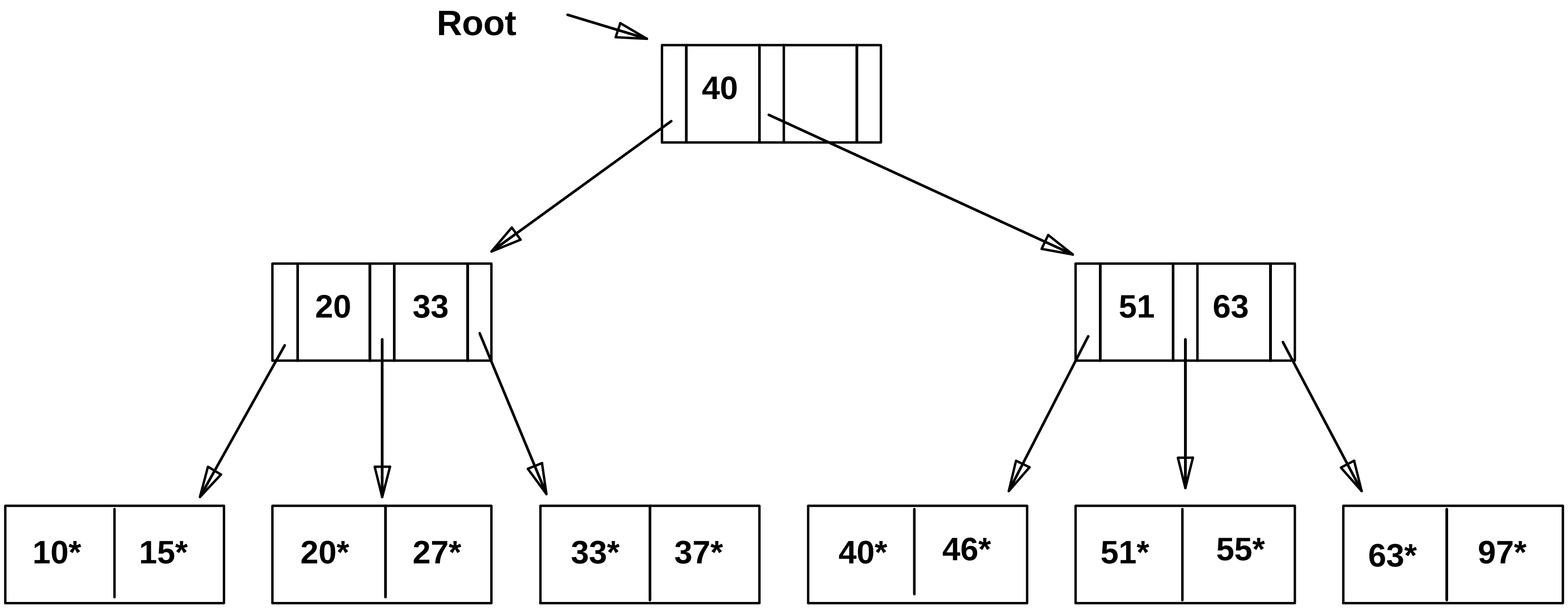
- ***Insert***: Find leaf data entry belongs to, and put it there.

- ***Delete***: Find and remove from leaf; if empty overflow page, de-allocate.

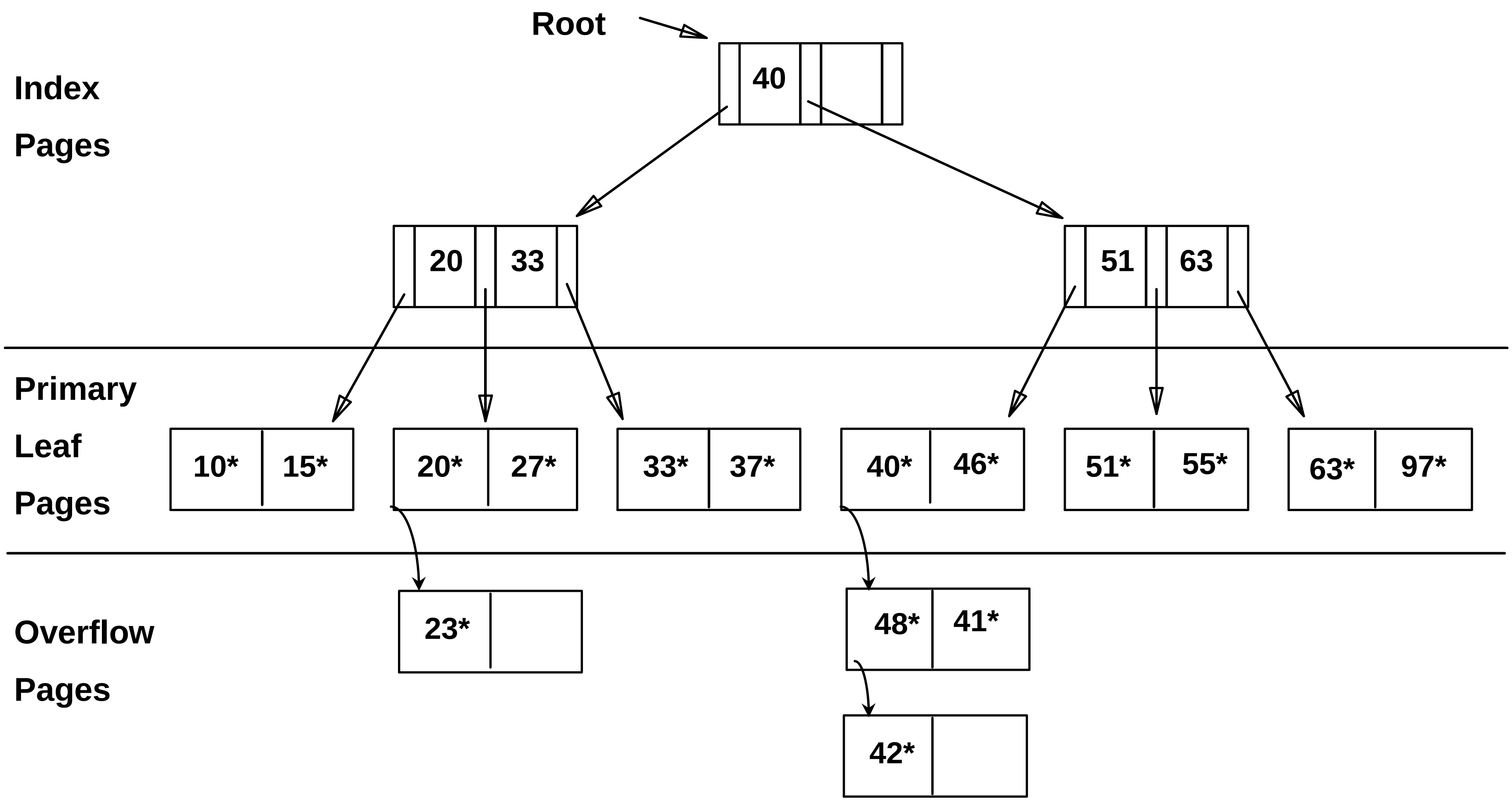
* **Static tree structure**: *inserts/deletes affect only leaf pages.*

# Example ISAM Tree

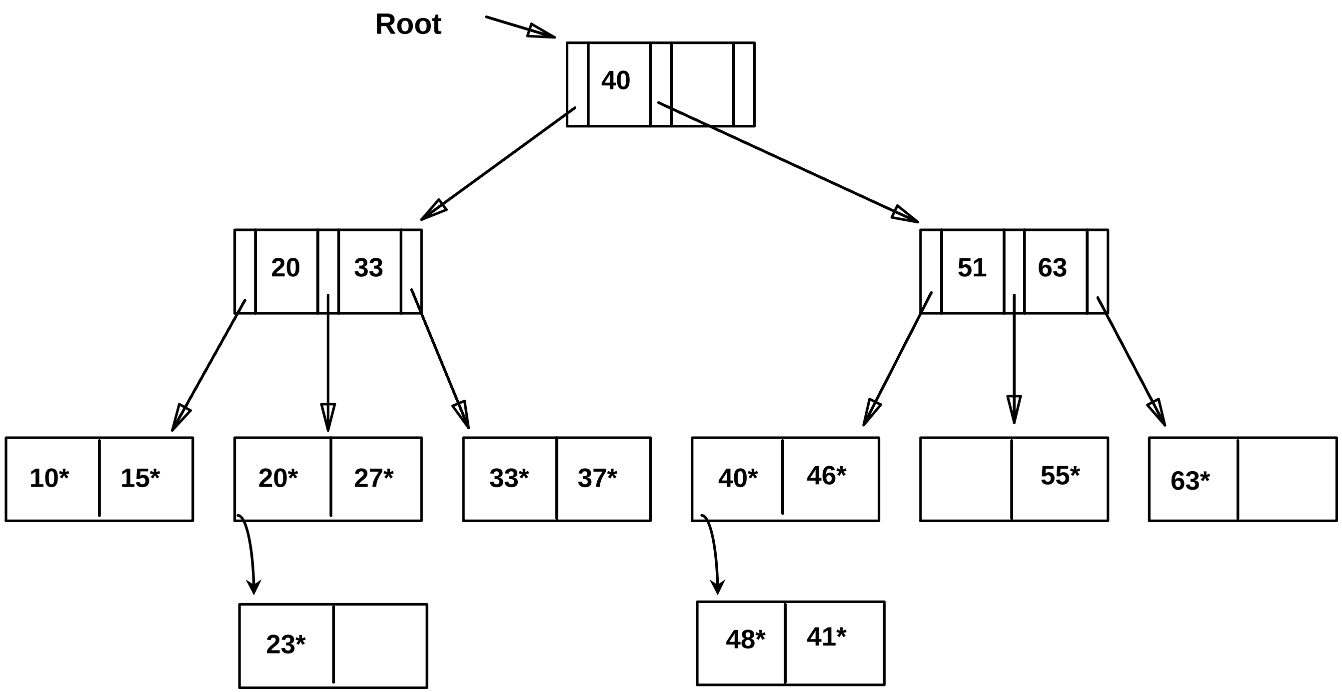
- Each node can hold 2 entries; no need for `next-leaf-page' pointers.  (Why?)



# After Inserting 23*, 48*, 41*, 42* ...



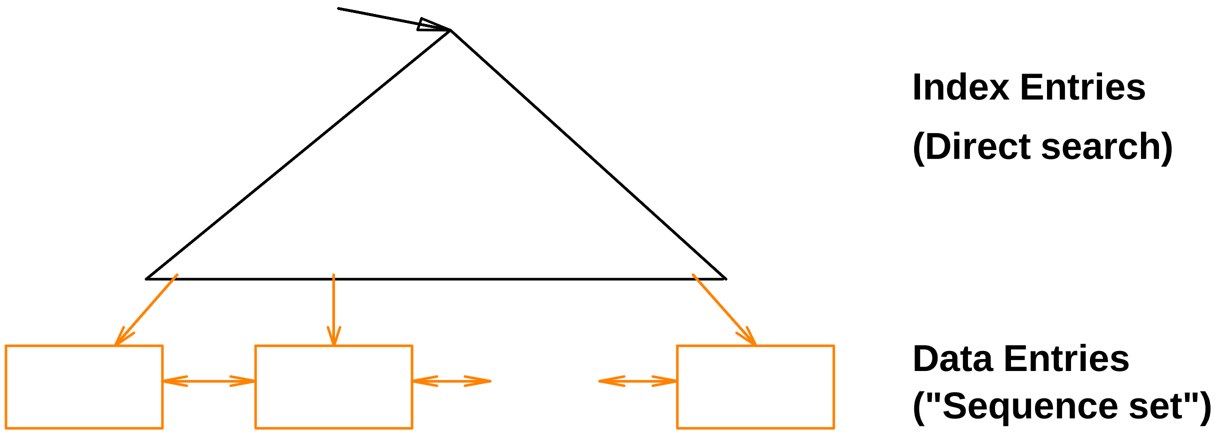
# ... Then Deleting 42*, 51*, 97*




** Note that 51* appears in index levels, but  not in leaf!*

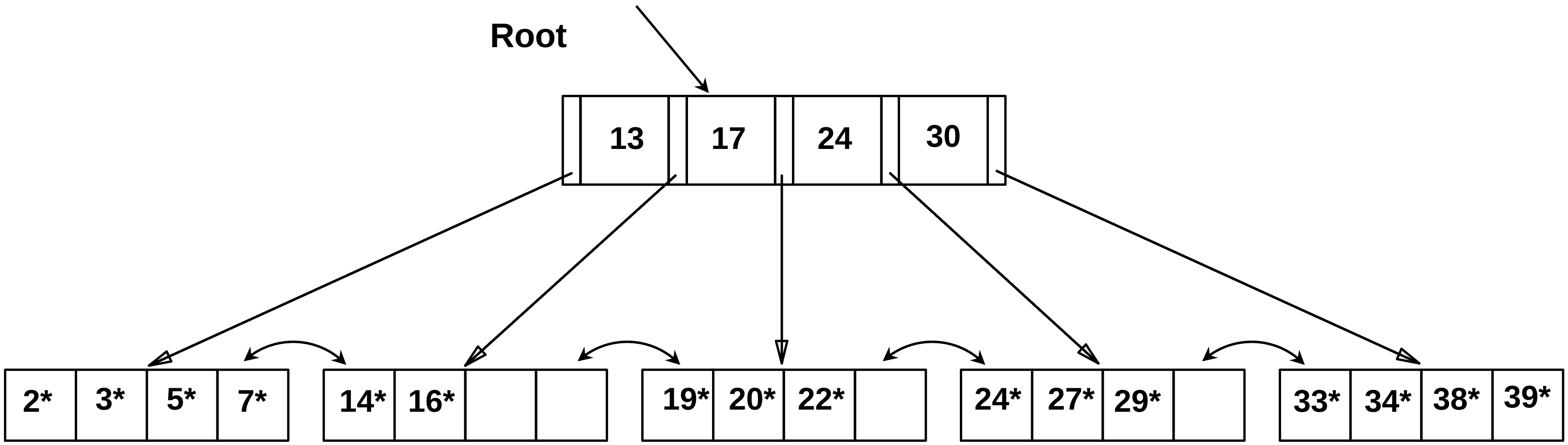
# B+ Tree: Most Widely Used Index

- Insert/delete at $\log_F N$ cost; keep tree *height-balanced*.   (F = fanout, N = # leaf pages)
- Minimum 50% occupancy (except for root).  Each node contains **d** <=  *m*  <= 2**d** entries.  The parameter **d** is called the *order* of the tree.
- Supports equality and range-searches efficiently.



# Example B+ Tree

- Search begins at root, and key comparisons direct it to a leaf (as in ISAM).
- Search for 5*, 15*, all data entries >= 24* ...




* *Based on the search for 15*, we <u>know</u> it is not in the tree!*

# B+ Trees in Practice

- Typical order: 100. Typical fill-factor: 67%.
  - average fanout = 133

- Typical capacities:
  - Height 4: $133^4$ = 312,900,700 records
  - Height 3: $133^3$ = 2,352,637 records

- Can often hold top levels in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes

# Inserting a Data Entry into a B+ Tree

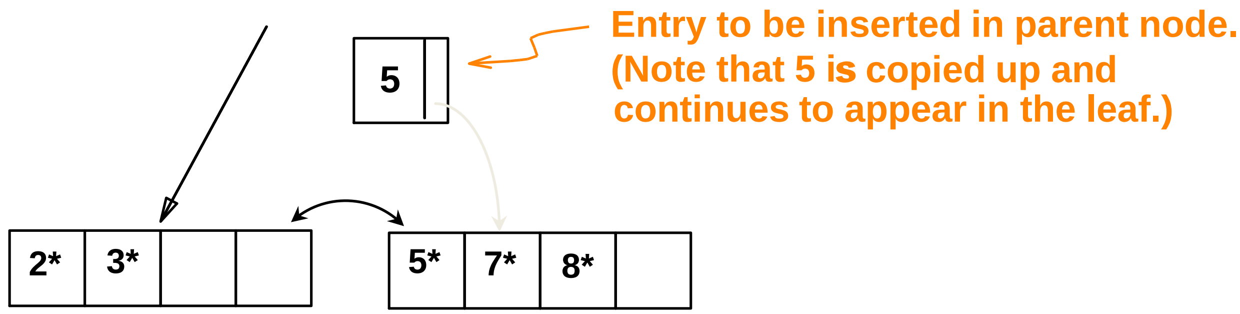
- Find correct leaf *L.*
- Put data entry onto *L*.
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.

- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)

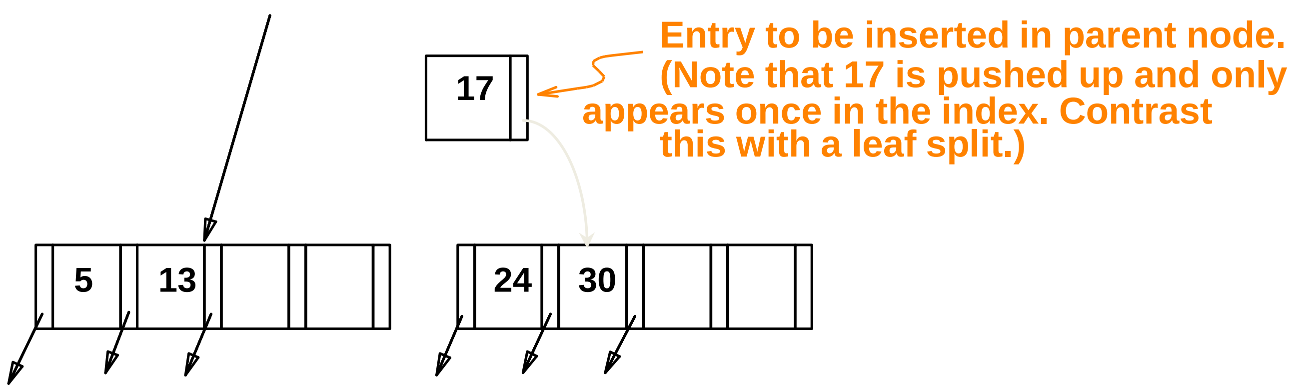
- Splits "grow" tree; root split increases height.
  - Tree growth: gets *wider* or *one level taller at top.*

# Inserting 8* into Example B+ Tree

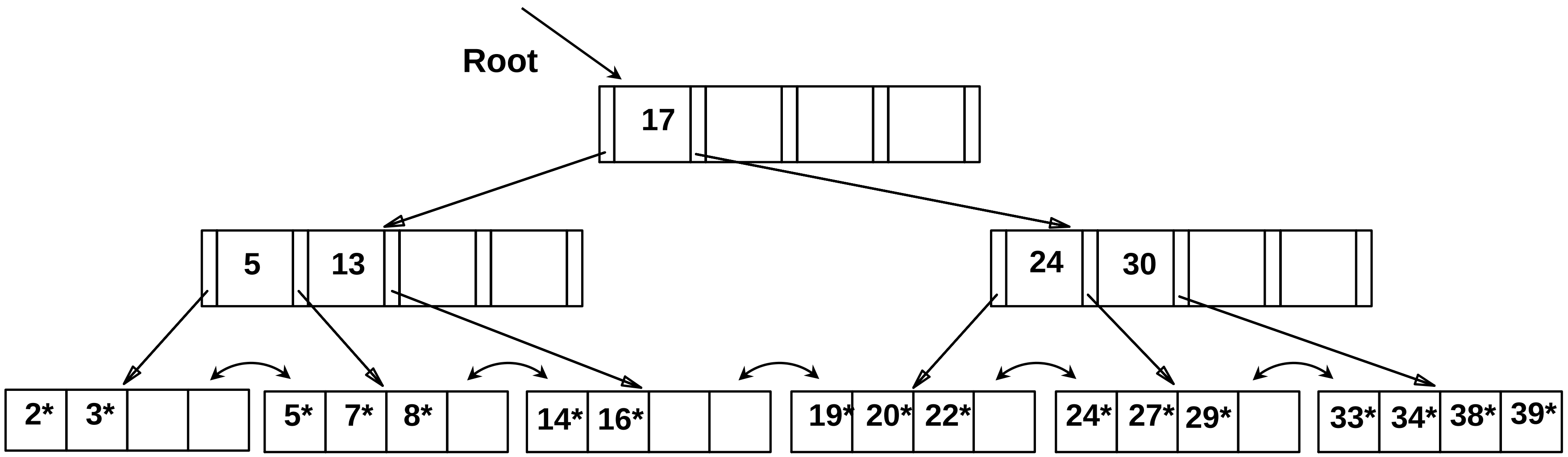
- Observe how minimum occupancy is guaranteed in both leaf and index pg splits.




- Note difference between *copy-up* and *push-up*; be sure you understand the reasons for this.



# Example B+ Tree After Inserting 8*




- Notice that root was split, leading to increase in height.

- In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

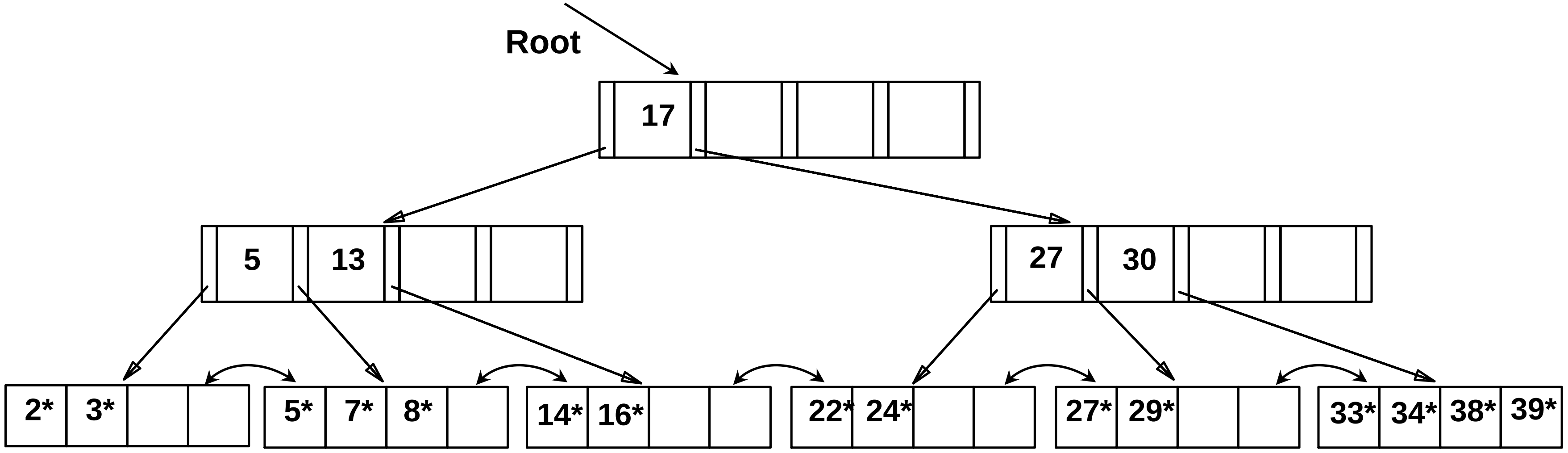
# Deleting a Data Entry from a B+ Tree

- Start at root, find leaf *L* where entry belongs.

- Remove the entry.
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from *sibling* *(adjacent node with same parent as L)*.
    - If re-distribution fails, *merge* *L* and sibling.

- If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.
- Merge could propagate to root, decreasing height.

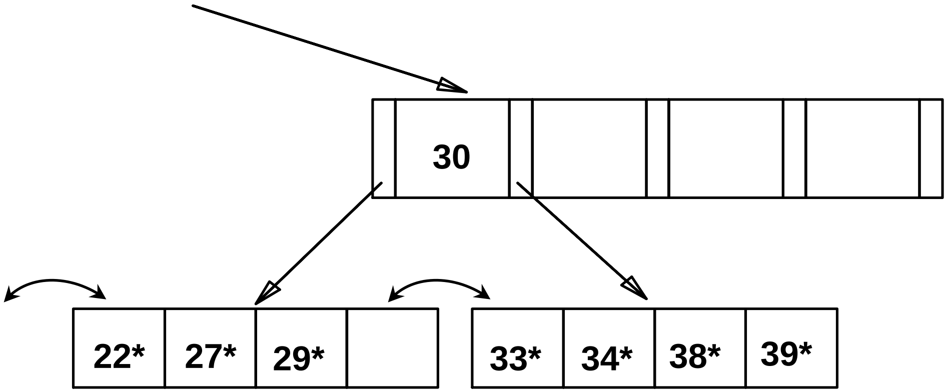
# Example Tree After (Inserting 8*, Then) Deleting 19* and 20* ...

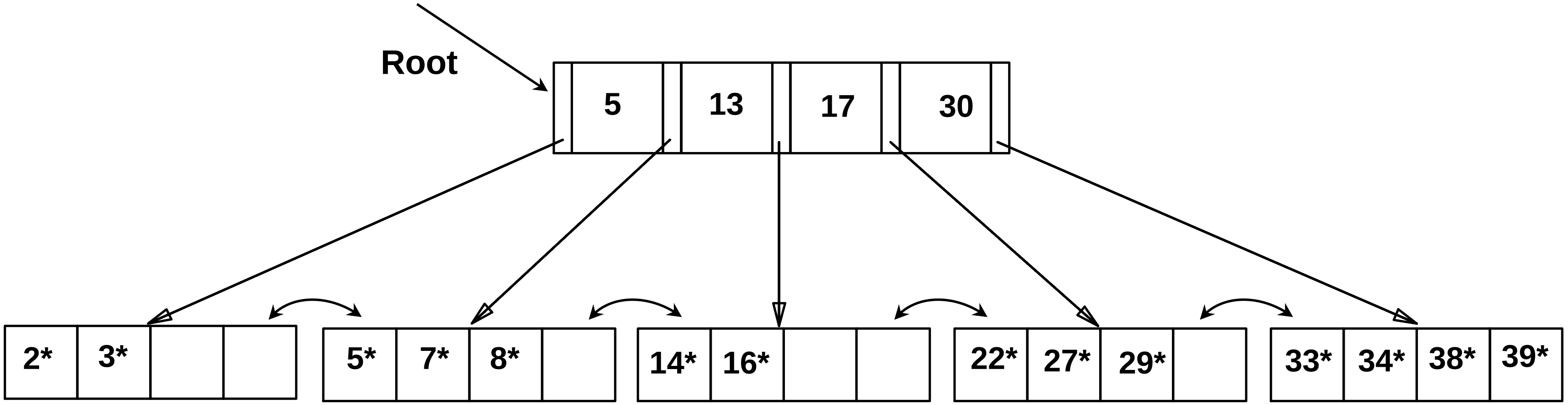



- Deleting 19* is easy.
- Deleting 20* is done with re-distribution. Notice how middle key is *copied up*.

# ... And Then Deleting 24*

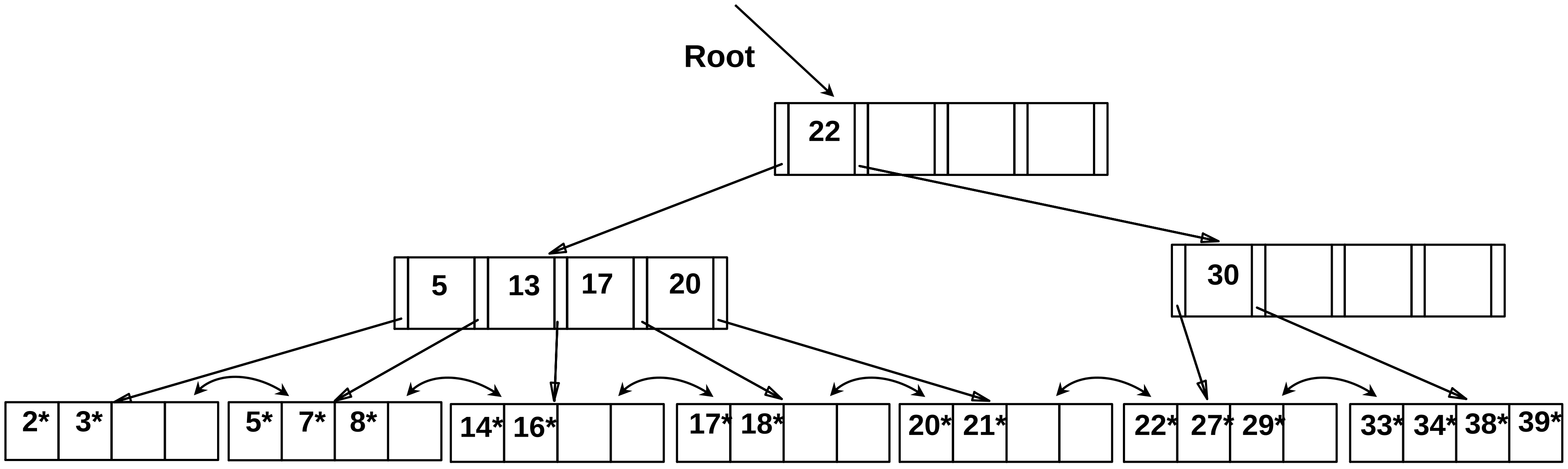
- Must merge.
- Observe *`toss'* of index entry (on right), and *`pull down'* of index entry (below).



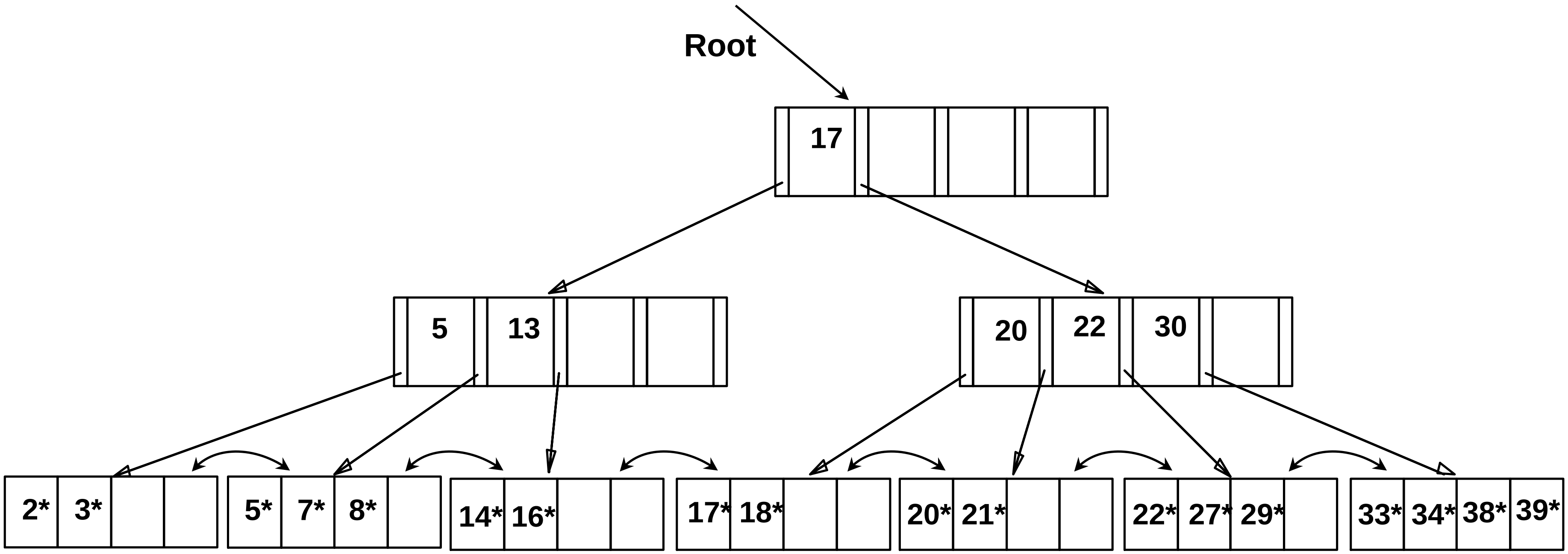
# Example of Non-leaf Re-distribution

- Tree is shown below *during deletion* of 24*. (What could be a possible initial tree?)
- In contrast to previous example, can re-distribute entry from left child of root to right child.



# After Re-distribution

- Intuitively, entries are re-distributed by *`pushing through'* the splitting entry in the parent node.
- It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well for illustration.



# Prefix Key Compression

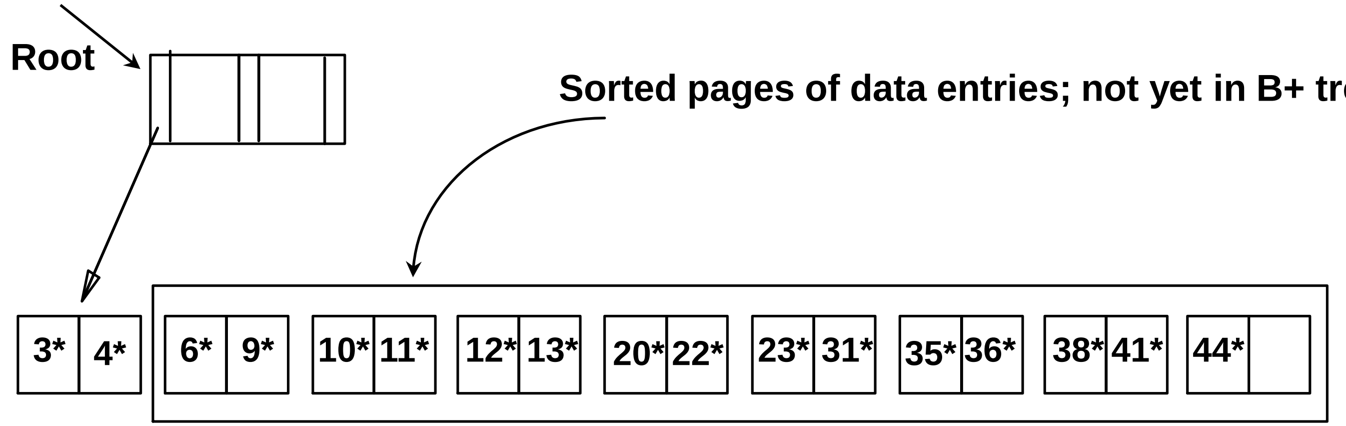
- Important to increase fan-out. (Why?)
- Key values in index entries only `direct traffic'; can often compress them.
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt*, *David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- Insert/delete must be suitably modified.

# Bulk Loading of a B+ Tree

- If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.

- *Bulk Loading* can be done much more efficiently.

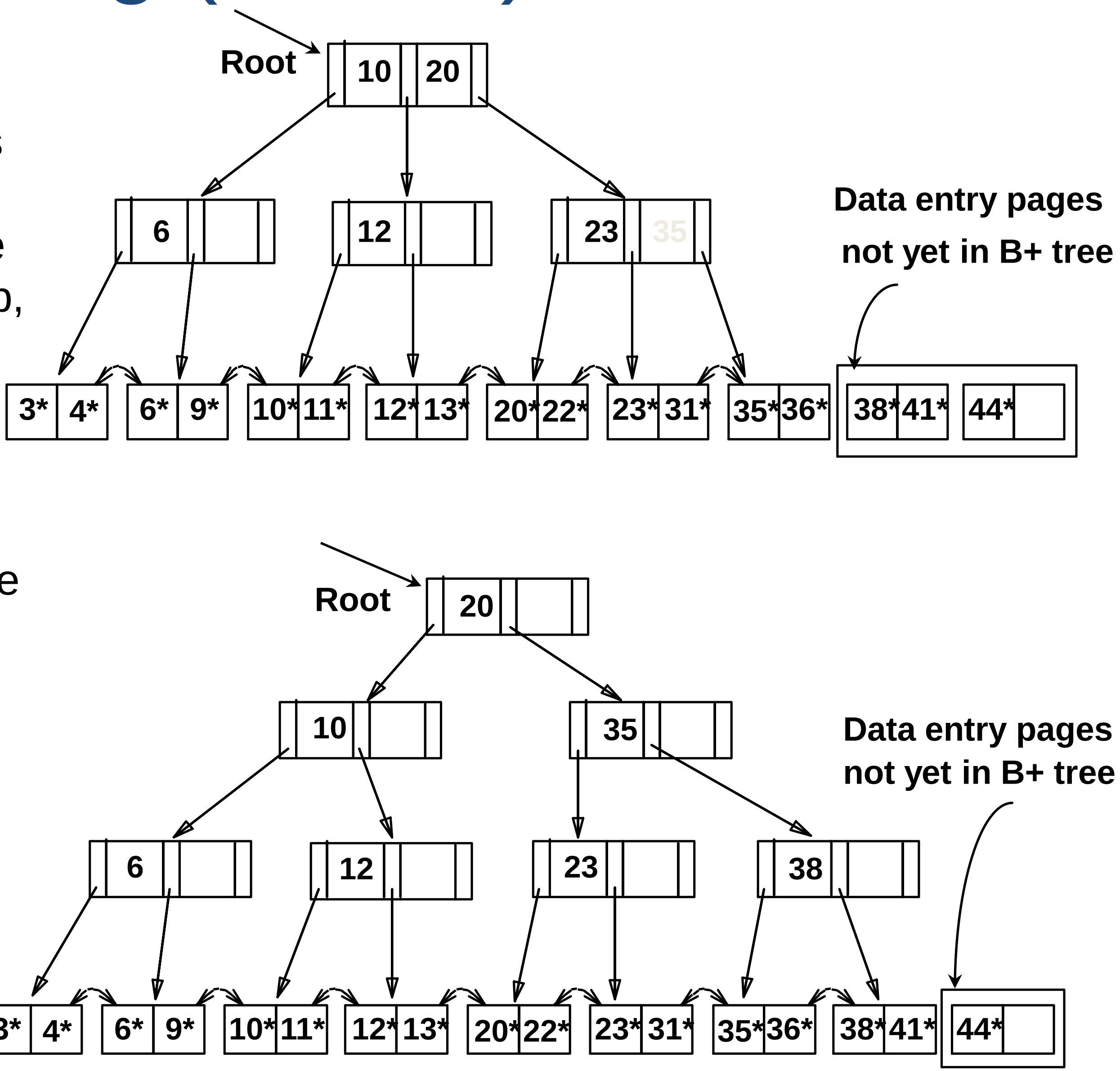
- *Initialization*: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.



# Bulk Loading (Contd.)

- Index entries for leaf pages always entered into right-most index page just above leaf level.  When this fills up, it splits.  (Split may go up right-most path to the root.)

- Much faster than repeated inserts, especially when one considers locking!



# Summary of Bulk Loading

- Option 1: multiple inserts.
  - Slow.
  - Does not give sequential storage of leaves.

- Option 2: *Bulk Loading*
  - Has advantages for concurrency control.
  - Fewer I/Os during build.
  - Leaves will be stored sequentially (and linked, of course).
  - Can control “fill factor” on pages.

# A Note on `Order'

- *Order* (**d**) concept replaced by physical space criterion in practice (`*at least half-full*').
  - Index pages can typically hold many more entries than leaf pages.
  - Variable sized records and search keys mean different nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).

# Summary

- Tree-structured indexes are ideal for range-searches, also good for equality searches.

- ISAM is a static structure.
  - Only leaf pages modified; overflow pages needed.
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant.

- B+ tree is a dynamic structure.
  - Inserts/deletes leave tree height-balanced; $\log_F N$ cost.
  - High fanout (**F**) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.

# Summary (Contd.)

- Typically, 67% occupancy on average.
- Usually preferable to ISAM, modulo *locking* considerations; adjusts to growth gracefully.
- If data entries are data records, splits can change rids!

- Key compression increases fanout, reduces height.

- Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.

- Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.