MAE: 259B: Mechanics of Slender Structures and Soft Robots Homework 2

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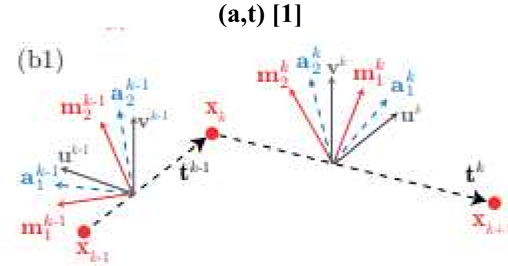
Abstract— **Homework 2 solutions: This assignment simulated the discrete twisting of rods. It began by modeling simple director and reference frame transformations then computing the reference twist. The process was then built up to simulate a curved rod twisting under the influence of gravity with respect to time. All solutions were simulated in Matlab and the code is stored at: https://github.com/ccampbell007/MAE259B-Homework2**

I. Introduction and Approach

The approach to this project was to initially solve the Chapter 6 problems in order to validate the code with the given answers in the textbook before implementing it into a larger solution for Chapter 7 [1]. It was difficult to troubleshoot bugs with no known values so it was critical to have confidence in the equations used. The code was written, debugged and troubleshooted without success. In particular, the simulation was unstable and would experience diverging error terms. Each diverging term would be tracked to help identify bugs, but eventually it became unstable after many timesteps within the elastic energy functions and it was difficult to troubleshoot further. The code was gradually updated to more closely resemble the code demonstrated in class in order to reduce uncertainty, but the error remained [2]. However, it was now possible to troubleshoot the code and it was determined that too large of a timestep was causing the error. Once this was corrected, the code behaved normally. Much of the original code is left commented to show the process developed.

The twist of a rod is computed by first defining a reference frame [1, 3]. This frame is rotated spatially across each node, but does not change with the rod's twist with respect to time. An additional parameter, the material reference frame, does vary with twist over time as shown in Figure 1 [1]. The reference frames and ultimately twist of a rod were calculated in Chapter 6 using nodal locations, reference directors, material directors and angle of twist. These techniques were then built upon in Chapter 7 [1]. The frames were continuously computed using Netwon's method to simulate the actual nodal locations and deformation vs time [1].

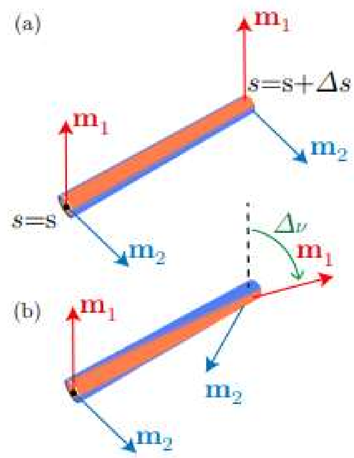
Figure 1: Space-parallel reference frame (u,v), material reference frame (m) and time parallel reference frame (a,t) [1]



II. Problems

A. Chapter 6: Problem 1

Figure 2: Problem One System Model [1]



Problem 1 calculated the integrated twist of a rod given the node locations and material reference frame as shown in Figure 2. The edges were calculated followed by the tangent vectors. Next the parallel transport and signed angle functions were used to calculate the reference twist.

B. Chapter 6: Problem 2

Problem 2 involved calculating the reference twist by calculating the angle of twist at each node. The angle at each node was calculated by first finding the tangent vectors. The reference directors were translated across the rod using parallel transport then the singed angle was computed to find the angle of twist [3]. Finally, the integrated twist was found by taking the difference in angles of twist [1].

C. Chapter 6: Problem 3

Problem 3 gave the degree of freedom vector coordinates at various timesteps [1]. The simulation proceeded by calculating the (x,y,z,theta) coordinates, the edges, tangents, reference frame directors via space parallel transport, and material reference frames using the cross product. From there, the material reference frame and angle of twist were calculated using parallel transport and by taking the signed angle between the reference frames and tangents for each node [1].

D. Chapter 7

The Chapter 7 simulation used Newton's method to simulate a curved rod that experienced gravity force in the negative z direction [1]. The simulation began by defining the initial node locations, reference frame directors and material directors. Next the material reference frames tangents and reference twist for each node were calculated for each timestep using Newton's method. The stretching, bending and torsion elastic energies were included in this calculation [1, 3].

The deformation of the rod at t = 0 and t=5 are shown in Figure 3 and

Figure 4 respectively. The z coordinate position vs time of the last node is shown in Figure 5.

Figure 3: Rod Deformation at Time = 0

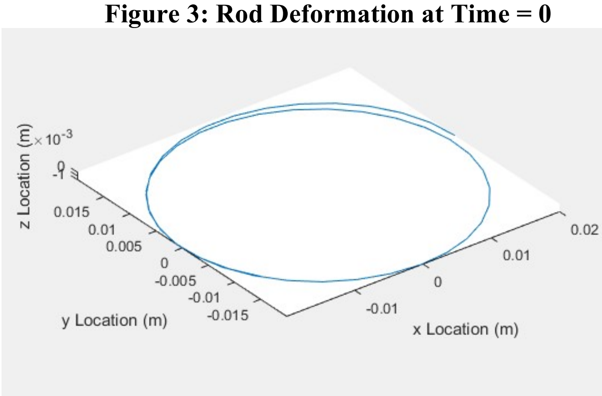


Figure 4: Rod Deformation at T = 5s

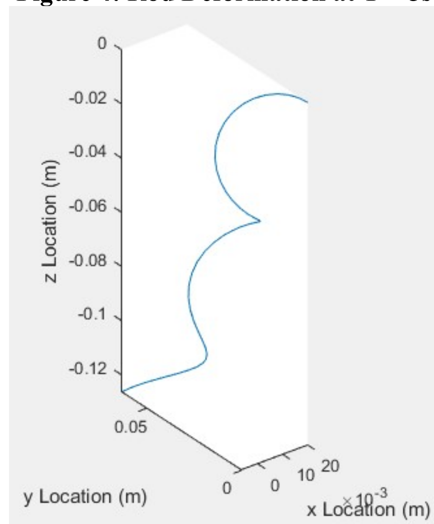
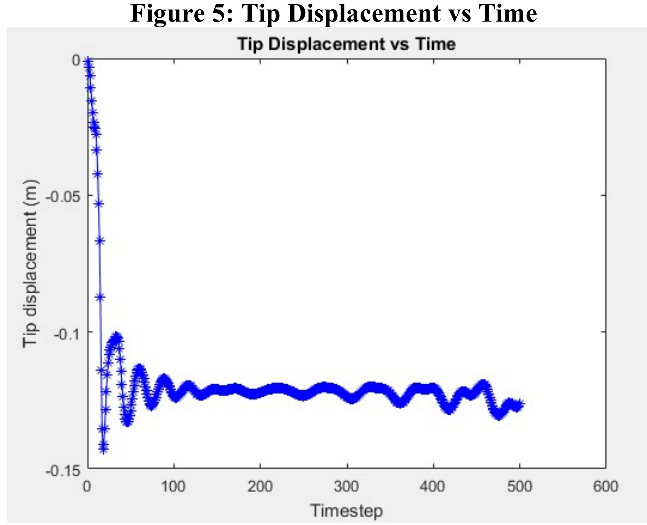


Figure 5: Tip Displacement vs Time



III. CONCLUSION

The discrete twist of a rod can be simulated using Newton's Method similar to the method previously used to model beam bending. The twisting computations are more challenging to implement, but once working, can be combined with the bending simulations to model a variety of complex situations.

IV. REFERENCES

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