DISTRIBUTED SIMULATION AND DISTRIBUTED INFERENCE

Algorithms, Tradeoffs, and a Conjecture

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Joint work with **Jayadev Acharya** (Cornell University) and **Himanshu Tyagi** (IISc Bangalore)

A STORY

Boaty McBoatface is starting its first mission today!
It's going to Antarctica to study global warming, not to play.

The world's oceans are changing, you see.
It's freezing down there, but not as cold as it used to be.

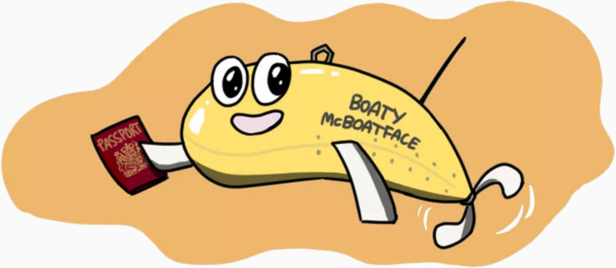

Illustration ©Dami Lee

Boaty's findings will be sent to scientists with care,
By way of a radio link, but with a certain flair.

Illustration ©Dami Lee

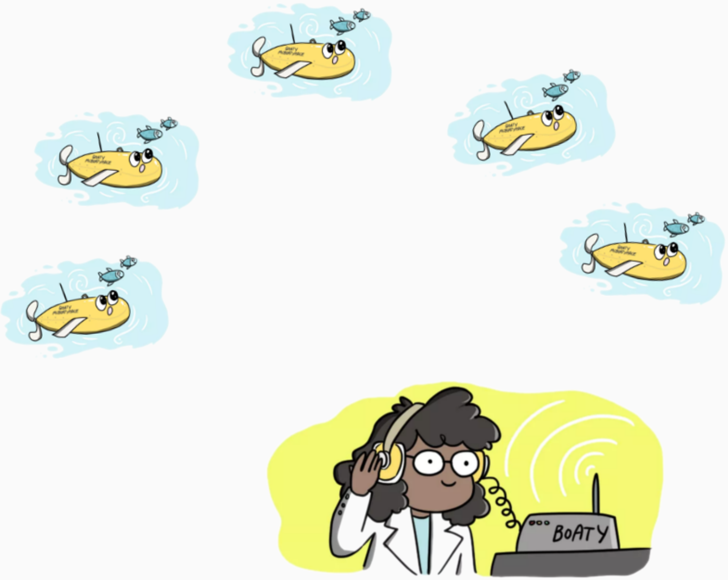

Illustration ©Dami Lee

McBoatfaces are expensive

What is the most ship-efficient protocol to reliably test whether the distribution of temperatures matches the one on record?

DISTRIBUTED INFERENCE

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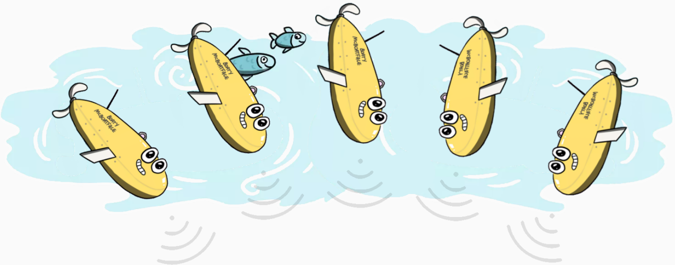
Question

As a function of k, ℓ, and all relevant parameters of $\mathcal{P}$, how many players n are required?

THE SETTING, CONT'D



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- Different flavors: public-coin, pairwise-coin, private-coin

“SIMULATE-AND-INFER”

ONE APPROACH TO SOLVE IT ALL

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Begging the question

Can the referee simulate independent samples from p using the messages from the players?

NO APPROACH TO SOLVE IT ALL?

Theorem

For every $k \geq 1$ and $\ell < \log k$, there exists no SMP with ℓ bits of communication per player for distributed simulation over $[k]$ with any finite number of players. (Even allowing public-coin and interactive protocols.)

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Proof.

By contradiction, [...] pigeonhole principle [...]. $\square$

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Theorem

For every $k, \ell \geq 1$, there exists a private-coin protocol with ℓ bits of communication per player for distributed simulation over $[k]$, with expected number of players $O(k/2^\ell \vee 1)$. Moreover, this is optimal even allowing public-coin and interactive protocols.

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Proof.

Case $\ell = 1$. Player $2i - 1$ and $2i$ both send 1 if their sample "hits" i; the referee outputs i if (i) player $2i - 1$ is the only odd player sending 1, and player $2i$ sends 0.

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(and some complications to bound this away from 1). □

ONE APPROACH TO SOLVE IT ALL!

Corollary (Informal)

For any inference task $\mathcal{P}$ over k-ary distributions with sample complexity s in the non-distributed model, there is a private-coin protocol for $\mathcal{P}$, with ℓ bits of communication per player, and $n = O(s \cdot k/2^{\ell})$ players.

Illustration ©Dami Lee

ONE APPROACH TO SOLVE IT ALL!

Corollary (Learning in Total Variation)

For every $k, \ell \leq \log_2 k$, there is a private-coin protocol for learning k-ary distributions with ℓ bits per player, and $n = O(\frac{k^2}{2^\ell \varepsilon^2})$ players. (And this is optimal, even for public-coin and interactive protocols.)

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Corollary (Testing Uniformity)

For every $k, \ell \leq \log_2 k$, there is a private-coin protocol for testing uniformity over $[k]$ with ℓ bits per player, and $n = O(\frac{k^{3/2}}{2^\ell \varepsilon^2})$ players.

ONE APPROACH TO REALLY, REALLY SOLVE IT ALL?

Natural Question

Is this “simulate-and-infer” approach optimal?

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Conjecture (The Flying Pony Question)

Does the simulate-and-infer scheme that simulates independent samples *compressed to the size** of the problem using private-coin protocols, and sends them to the referee who then infers from them, always require the lowest number of players?

NO FLYING PONY

The answer is no:

Theorem

There exists an inference task $\mathcal{P}$ over k-ary distributions with $2^{\mathsf{size}(\mathcal{P})} \cdot \mathsf{samplecomplexity}(\mathcal{P}) = \Omega(k^{3/2})$, yet for which there is a 1-bit private-coin protocol with $n = O(k)$ players.

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Proof.

Promise problem: p is either uniform, or uniform on an arbitrary subset of $k/2$ elements.

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PUBLIC-COIN UNIFORMITY TESTING

Must decide:

$$p = u_k \text{ (uniform)}$$

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(and be correct on any p with probability at least 2/3)

Fundamental property of distributions, building block for testing many others. [BKR04, Gol16, CDGR17]

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- general “simulate-and-infer” scheme gives private-coin protocol with $n = O(k^{3/2}/\varepsilon^2)$ players (optimal?)
- what if we allow public coins?

Theorem (Upper Bound)

For every $k, \ell \leq \log_2 k$, there is a public-coin protocol for testing uniformity over $[k]$ with ℓ bits per player, and $n = O\left(\frac{k}{2^{\ell/2}\varepsilon^2}\right)$ players.

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Theorem (Lower Bound)

This is optimal.

Theorem (Warm Up)

For every k, there is a public-coin protocol for testing uniformity over $[k]$ with $\ell = 1$ bit per player, and $n = O\left(\frac{k}{\varepsilon^3} \log \frac{1}{\varepsilon}\right)$ players.

Theorem (Warm Up)

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Proof.

Starting point: if p is ε-far from uniform, by definition,

$$\mathbb{E}_{x\sim u}[|p(x) - 1/k|] > \varepsilon/k\,.$$

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Same starting point. Now, by a better averaging argument (Levin's work investment strategy), there exists $1 \leq j \leq L := \log_2(1/\varepsilon)$ s.t.

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and therefore [...] (also, don't pay for the union bound!) $\square$

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Starting point: for a set $S \subseteq [k]$ of $2^\ell - 1$ elements with $p(S) \simeq 2^\ell/k$, testing uniformity of the conditional distribution p_S would cost

$$(k/2^\ell) \cdot \sqrt{2^\ell}/\varepsilon^2 = k/(2^{\ell/2}\varepsilon^2)$$

samples, by rejection sampling.

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samples, by rejection sampling. Now, if p is ε-far from uniform then, on a u.a.r. set $S \subseteq [k]$ of $2^\ell - 1$ elements, p_S is ε-far from uniform on expectation. Then, same ideas as before: Levin's strategy+careful allocation of the failure probabilities. □

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By Le Cam's two-point method, consider a distribution over "hard instances":

$$\forall 1 \leq i \leq k/2, \qquad p(2i-1), p(2i) = \left(\frac{1 \pm \varepsilon}{k}, \frac{1 \mp \varepsilon}{k}\right)$$

uniformly and independently at random. (Paninski's construction [Pan08]).

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CONCLUSION

- General framework for distributed inference problems over discrete distributions, in the communication-starved regime

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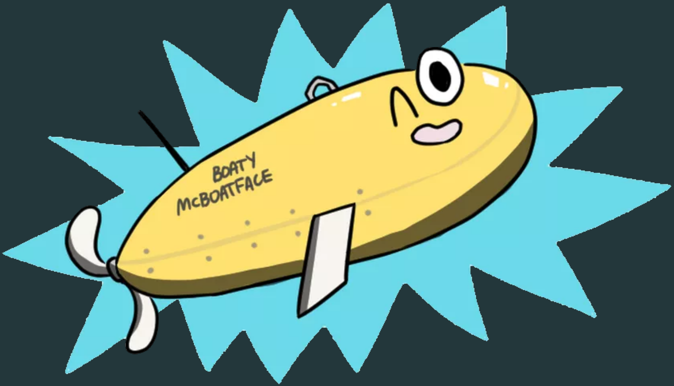
- Tight bounds for distributed simulation (and distributed learning [DGL+17, HMÖW18, HÖW18])
- First work on distributed testing
- Optimal protocols for public-coin uniformity testing
- Many questions and directions to explore

Thank you



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