# Many Eggs, More Baskets: New Insights from New Models

## Thesis Proposal

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Columbia University – 2016

# Introduction

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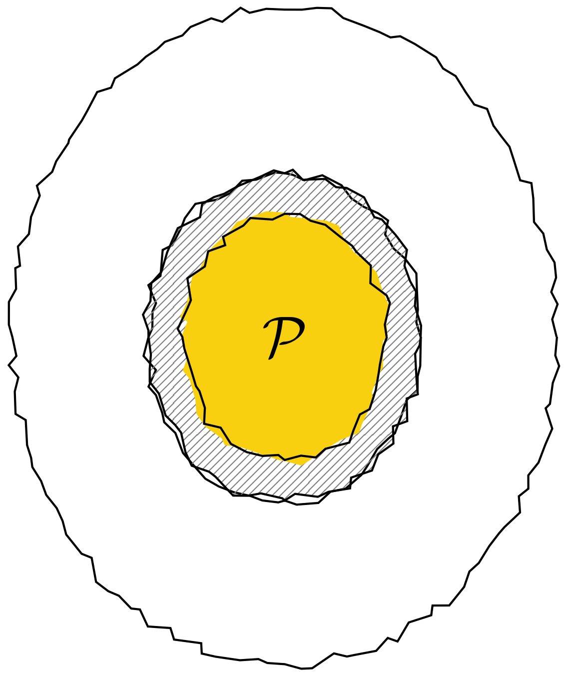
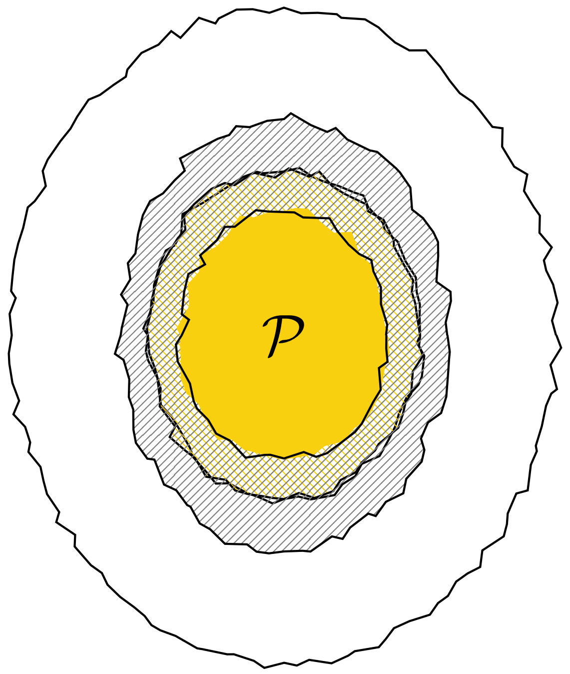
**Property testing:** what can we say about an object while barely looking at it?



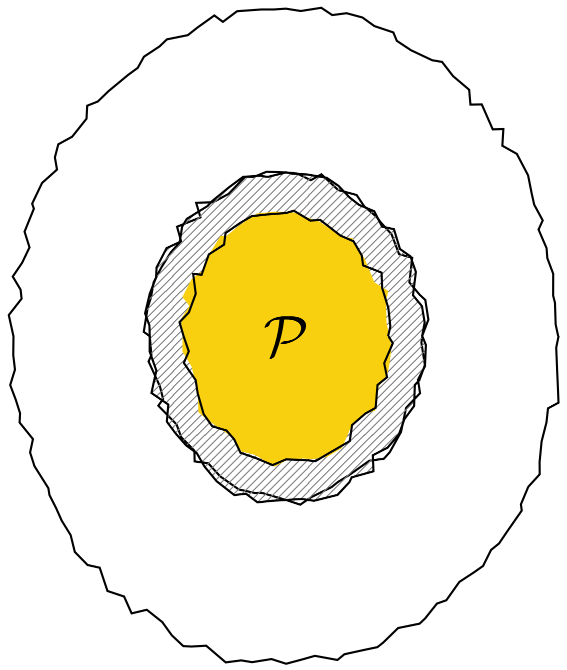

"Is it in the yolk?"

**Tolerant testing:** robust version of property testing




**(Typically harder.)**

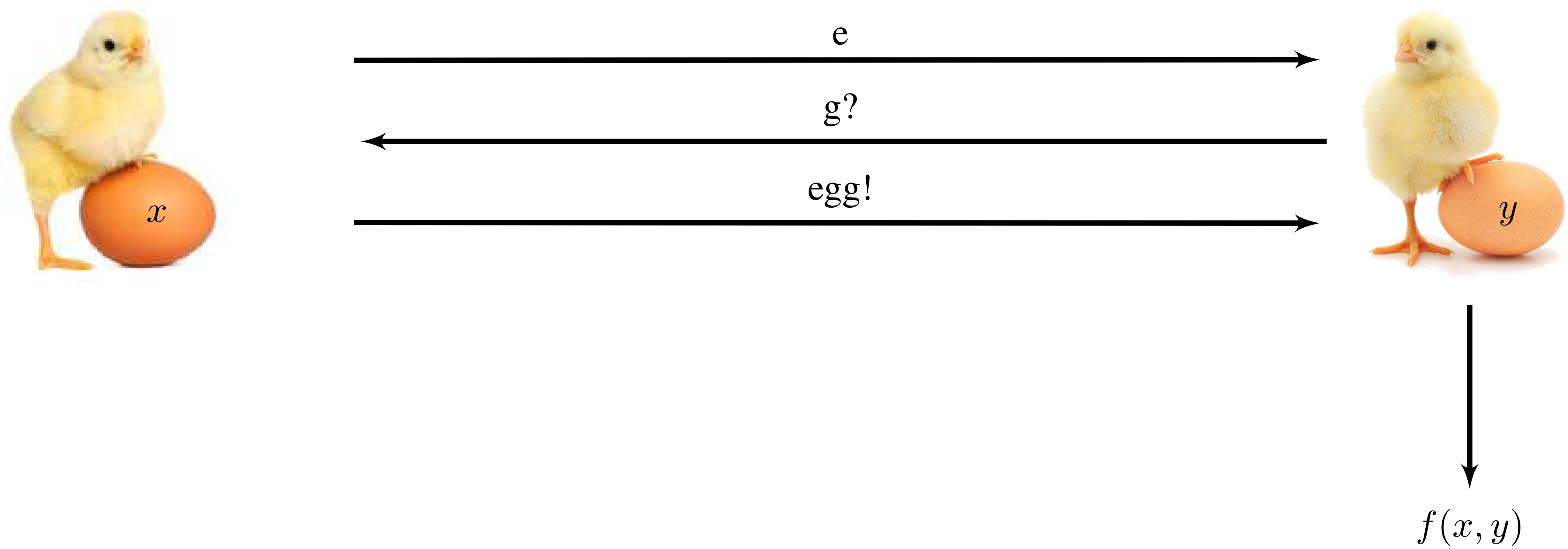
**Distribution testing:** property testing for probability distributions




**Different metric, objects, and type of access.**

# Introduction

**Communication complexity:**



# Outline of the talk

# In and Beyond Distribution Testing

# The standard setting

$\Delta(\Omega)$: all distributions over (finite) domain $\Omega$ of size $n$. **Property:** subset $\mathcal{P} \subseteq \Delta(\Omega)$. **Tester:** randomized algorithm (knows $n$, $\mathcal{P}$).

Given independent samples from a distribution $D \in \Delta(\Omega)$, and parameter $\varepsilon \in (0,1)$, output accept or reject:

- If $D \in \mathcal{P}$, accept with probability at least 2/3; *(in the yolk)*
- If $\ell_1(D, \mathcal{P}) > \varepsilon$, reject with probability at least 2/3; *(definitely white)*
- otherwise, whatever (make an omelet).

**Goal:** take $o(n)$ samples, ideally $O_\varepsilon(1)$.
(time efficiency is secondary, yet not frowned upon.)

[BFF+01, BKR04, BFR+10, GGR98]

# The challenges



**Unified frameworks:** how to get past the *ad hoc*, property-specific results (upper and lower bounds) to get *generic* approaches?

**Strong lower bounds:** how to get around the hardness results in the standard sampling model (e.g. [VV10a])?

**Strong assumptions:** how to get rid of (some) of the assumptions – can we deal with *limited independence*?

# Several chickens with one stone

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Many individual results on specific properties:

- Uniformity
- Identity
- Equivalence
- Independence
- Monotonicity
- Poisson Binomial Distributions
- and more…

…but almost none on general frameworks.

# Several chickens with one stone

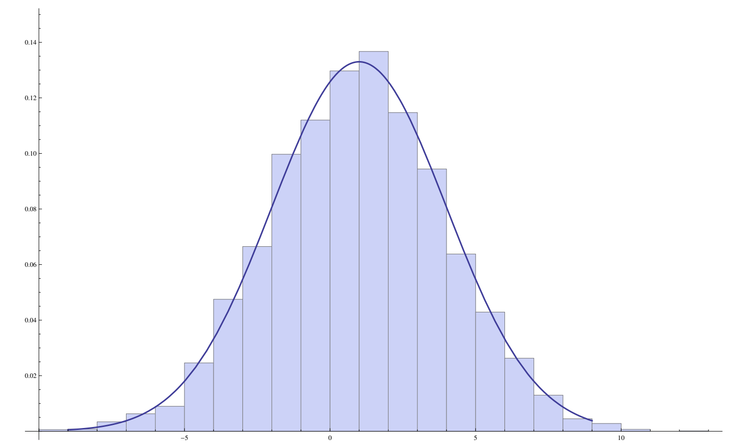
How to get past the *ad hoc*, property-specific results (upper and lower bounds) to get *generic* approaches?

**1. Abstract structural properties of the properties.**

# Several chickens with one stone

A unified approach to things:

- define a structural criterion (parameterized by some quantity $L$) of classes of distributions



- obtain a single testing algorithm $\mathcal{T}$ that takes $L$ as input

$$D \rightsquigarrow \mathcal{T}(L) \rightsquigarrow \text{accept/reject}$$

- Prove existential result for your favorite class $\mathcal{C}$:

$$\mathcal{C} \rightsquigarrow L(\mathcal{C}, \varepsilon)$$

- Use $\mathcal{T}$ to test $\mathcal{C}$

$$D \rightsquigarrow \mathcal{T}(L(\mathcal{C}, \varepsilon)) \rightsquigarrow \text{accept/reject}$$

# Several chickens with another stone

How to get past the *ad hoc*, property-specific results (upper and lower bounds) to get *generic* approaches?

**2. Do the (supposedly) impossible.**

# Several chickens with another stone

Other generic frameworks:

- Upper bounds by learning-and-testing [ADK15, Can16]

$$\chi^2 \leq \varepsilon^2 \text{ vs. } \mathrm{d}_{\mathrm{TV}} > \varepsilon$$

- Upper bounds *via* $\ell_2$ testing and randomized mapping [DK16]

$$D \in \Delta([n]) \rightsquigarrow F(D) \in \Delta([N]) \rightsquigarrow \ell_2\text{-testing}$$

- Lower bounds *via* blackbox reductions [CDGR15]

$$\mathcal{C}^{\text{Hard}} \subseteq \mathcal{C} \rightsquigarrow \text{testing}(\mathcal{C}^{\text{Hard}}) \preceq \text{testing}(\mathcal{C})$$

- Lower bounds *via* information theory [DK16]

# Several chickens with a big rock

How to get past the *ad hoc*, property-specific results (upper and lower bounds) to get *generic* approaches?

**3. Ask Alice and Bob.**

# Communication Compleggsity

# Several chickens with a big rock

Approach *à la* [BBM11]: reduction from communication complexity:

- select the right communication setting:
$$A \to B, \quad A \leftrightarrow B, \quad A \to R \leftarrow B \dots$$
- choose a hard enough communication problem:
$$\textsc{Disjointness}, \textsc{Gap-Hamming}, \textsc{Equality}^*, \text{something new} \dots$$
- create distance from the CC inputs:
$$(a,b) \in \mathcal{Y} \rightsquigarrow D_{a,b} \in \mathcal{P} \qquad (a,b) \in \mathcal{N} \rightsquigarrow \ell_1(D_{a,b}, \mathcal{P}) > \varepsilon$$
- simulate access from the CC inputs
$$A \to B \rightsquigarrow s \sim D_{a,b}$$

# Strengthening the oracle

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How to get around the hardness results in the sampling model?

**Question the model.**

# Strengthening the oracle

Changing the model of access to $D$:

- with evaluation queries to the pmf: [RS09] ("property-testing"-style)

$$x \in \Omega \rightsquigarrow D(x)$$

- with sampling and evaluation queries to the pmf: [BDKR05, GMV06, CR14]

$$? \rightsquigarrow x \sim D \quad \text{and} \quad x \in \Omega \rightsquigarrow D(x)$$

- with sampling and evaluation queries to the cdf: [BKR04, CR14, Can15]

$$? \rightsquigarrow j \sim D \quad \text{and} \quad j \in [n] \rightsquigarrow \sum_{i=1}^{j} D(i)$$

- with conditional sampling: [CFGM13, CRS15, ADK15, Can15]

$$S \subseteq \Omega \rightsquigarrow x \sim D_S$$

# Results: the Sunny Side (Up)

**Informally:** across the models and flavors, exponential sample complexity improvements – sometimes even from $n^{\Omega(1)}$ to constant. Some hardness remains, still – and most importantly, *all rules of thumbs are down.*

**Conditional sampling:** identity and closeness testing are no longer related ($O_\varepsilon(1)$ vs. $(\log\log n)^{\Omega(1)}$). Tolerant uniformity testing and entropy estimation are, similarly, worlds apart.

**Testing with queries:** Testing uniformity, identity and closeness becomes easy: the challenge now seems to lie in tolerant testing, or in testing against classes.

**Challenges:** Understanding how these new models relate, and develop generic tools to analyze them.

# Weakening the assumptions

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How to get rid of (some) of the assumptions – can we deal with limited independence?

**Semi-adversarial setting, capturing real-life situations: memory pages, hard drive, clustered data…**

# Weakening the assumptions

Work in the external memory model of [AIOR09]:

- multiset $S \subseteq [n]$ of $m$ datapoints, clustered (arbitrarily) in blocks of size $B$

$$|S_1| = |S_2| = \cdots = |S_{m/B}| = B$$

- random access to the blocks, reading a full block has unit cost

$$i \rightsquigarrow S_i$$

- want to test properties of the dataset: of $D \in \Delta([n])$ induced by $S$

$$D(i) = \frac{1}{m} \sum_{j=1}^{m} \mathbf{1}_{\{s_j = i\}}$$

- take advantage of this egg in our beer: optimal in $m, B, \varepsilon, n$?

# Other and Future work

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Other “neglected” or novel settings: what fails to be addressed or captured - and ought to be?

**Imperfect communication, Sampling correction, Robust function testing**

# Proposed Timeline

| Timeline | Work | Progress |
| --- | --- | --- |
| Sep. 2012–May 2016 | Unified testing, Conditional sampling (COND), Extended access model, Sampling correctors, Communication with Imperfect randomness (ISR) | completed |
| Spring 2016 | Submit full version of ISR to IEEE-IT | in review |
| Spring 2016 | Submit full version of (second) COND to ToC | in review |
| Spring 2016 | Submit full version of Sampling Correctors to SICOMP | in progress |
| Jan. 2016–July 2016 | Limited Independence | in progress |
| Oct. 2015–Aug. 2016 | Lower bounds via CC | in progress |
| Dec. 2015–Oct. 2016 | Tolerant Junta testing | in progress |
| Fall 2016 | Followup results on conditional sampling? | |
| Dec. 2016–March 2017 | Thesis writing | |
| Spring 2017 | **Thesis defense** | |
| *Spring 2017–Forever* | *Stay here?* | *(wishful)* |

# The ~~End~~

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