Optimisation Modern Power Systems 2

Authors

Carlos Canoyra Calero - s250421
Pulin Dhar - s190978

Task	**Carlos**	**Pulin**
Draft 1	50%	50%
Draft 2	50%	50%
Draft 3	50%	50%

Table 1: Contribution of each member to project tasks. Repository: GitHub link

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Contents

1 Problem setup

The client aims to decide weekly fuel purchases for thermal plants amid price fluctuations, uncertain demand and limited storage. Currently, purchasing is reactive with limited foresight on cost–risk trade-offs. Ambiguities remain around forecast accuracy, contract flexibility, and intertemporal effects. We therefore reframe the task as: "Design a decision support framework that minimizes expected procurement cost while maintaining supply reliability under uncertainty.". To solve this problem our consulting team will create 3 different drafts to approach this complex optimization problem, this way the client will be able to see the evolving project and give feedback on the way we should proceed. In addition, this procedure will facilitate the tractability of the project. More information about the client brief and the consulting proposal can be found in the appendix A.5 (Client brief), A.6(Consulting proposal).

2 Data Construction

The data used in this project combines real-world market characteristics with controlled synthetic components, in order to build a realistic yet tractable environment for optimization. Fuel price structures for Oil, Gas, and Coal were inspired by publicly available energy-market sources such as the IEA fuel price statistics [1] and the EEX forward curves [2], together with spot-market behaviour observed in the Nordic power system [3]. For each technology, supplier bids were constructed following a merit-order logic: three suppliers per fuel category, offering ascending price tiers that reflect common procurement practices in thermal generation portfolios. Daily price volatility was encoded through three stylized regimes *Base*, *Spike*, and *Severe* mimicking the short-term fluctuations caused by weather events, geopolitical shocks, or supply bottlenecks, consistent with empirical observations of European commodity markets [4].

Electricity demand was derived from hourly load profiles representative of Denmark (DK1/DK2), scaled to match an imposed weekly energy requirement. The diurnal shape was constructed using normalized 24-hour demand factors based on historical hourly generation shares of fossil technologies (Oil, Gas, Coal) in the Nord Pool dispatch system [5]. Daily demand was then perturbed using a controlled downward volatility factor to emulate seasonal or operational trends.

Electricity prices were generated in a structurally similar manner: hourly prices were created by applying the normalized load multipliers to synthetic daily average prices calibrated around typical ranges observed in the Danish bidding zones. These values reflect both structural price formation principles and the typical volatility range of the Nordic market [6]. Storage capacities, generation limits, and rental costs were informed by engineering data from representative CHP facilities and consistent with the parametric ranges reported in the thermal dispatch literature [7].

Finally, the scenario-generation framework used in Drafts 2 and 3 draws from classical stochastic programming techniques [8]. Each weekly scenario consists of an ordered sequence of daily market regime labels (*Base*, *Spike*, *Severe*) sampled according to calibrated probabilities. This scenario structure captures realistic short-term market uncertainty while remaining computationally tractable for multi-stage optimization and CVaR-based risk management. More information about the procedure can be found in the appendix A.1 (Volatility and Scenario Modeling Across Drafts).

3 Draft 1: Deterministic Cost Minimization Model

The first draft defines a fully deterministic planning model for a thermal power plant operating three technologies (Oil, Gas, Coal). The objective is to decide the optimal daily fuel purchases and generation schedules that minimize the total weekly operating cost while meeting an inelastic weekly electricity demand. No uncertainty is considered at this stage: fuel prices, fuel availability, generation limits, and energy demand are all treated as known and fixed.

Unlike the later drafts, Draft 1 does not include hourly resolution. Electricity demand is aggregated as a *single weekly requirement*, and generation decisions are made at the *daily* level. This results in a simpler yet coherent deterministic fuel-planning model that forms the foundation for the subsequent stochastic and risk-aware extensions.

3.1 Sets and Indices

$$\begin{aligned} T &= \{\text{Oil, Gas, Coal}\} && \text{Technologies} \\ S_t &= \{\text{Suppliers for technology } t\} && \text{Supplier sets} \\ D &= \{1, \ldots, 7\} && \text{Days in the planning horizon} \end{aligned}$$

3.2 Parameters

$$\begin{aligned} \text{DEM}: &\quad \text{Total weekly electricity demand (MWh)} \\ p_{t,s,d}: &\quad \text{Fuel price for technology } t \text{ from supplier } s \text{ on day } d \text{ (EUR/MWh)} \\ \overline{Q}_{t,s,d}: &\quad \text{Maximum daily purchasable fuel from supplier } s \text{ (MWh)} \\ \overline{G}_t: &\quad \text{Maximum daily electricity generation for technology } t \text{ (MWh/day)} \\ \overline{I}_t: &\quad \text{Storage capacity for a technology } t \text{ (MWh)} \\ r_t: &\quad \text{Storage rental cost for a technology } t \text{ (EUR/MWh/day)} \\ I_{t,0}: &\quad \text{Initial inventory for technology } t \text{ (MWh)} \end{aligned}$$

3.3 Decision Variables

$$\begin{aligned} Q_{t,s,d} \geq 0: &\quad \text{Fuel purchased for tech } t \text{ from supplier } s \text{ on day } d \text{ (MWh)} \\ g_{t,d} \geq 0: &\quad \text{Electricity generated by technology } t \text{ on day } d \text{ (MWh)} \\ I_{t,d} \geq 0: &\quad \text{Fuel inventory of tech } t \text{ at the end of day } d \text{ (MWh)} \end{aligned}$$

3.4 Objective Function

$$\min \underbrace{\sum_{t\in T}\sum_{s\in S_t}\sum_{d\in D} p_{t,s,d}\, Q_{t,s,d}}_{\text{fuel purchase cost}} + \underbrace{\sum_{t\in T}\sum_{d\in D} r_t\, I_{t,d}}_{\text{storage cost}}. \tag{1}$$

3.5 Model Constraints

The deterministic model is defined by the following constraints, for all technologies $t \in T$, days $d \in D$, and suppliers $s \in S_t$:

$$\text{(Inventory balance)} \quad I_{t,d} = I_{t,d-1} + \sum_{s\in S_t} Q_{t,s,d} - g_{t,d}, \tag{2}$$

$$\text{(Generation capacity)} \quad g_{t,d} \le \overline{G}_t, \tag{3}$$

$$\text{(Storage capacity)} \quad I_{t,d} \le \overline{I}_t, \tag{4}$$

$$\text{(Supplier limits)} \quad Q_{t,s,d} \le \overline{Q}_{t,s,d}. \tag{5}$$

The total weekly electricity requirement must be met:

$$\sum_{t\in T}\sum_{d\in D} g_{t,d} = \text{DEM}. \tag{6}$$

Finally, the inventory at the end of the week is forced to zero:

$$I_{t,7} = 0 \qquad \forall t \in T. \tag{7}$$

3.6 Discussion

Draft 1 captures the essential physics of the system in its simplest form. Fuel flows through a linear sequence of operations (purchase, storage, and generation) and the plant must meet a fixed, inelastic weekly electricity demand at minimum total cost. Given that no electricity revenue, uncertainty, or emission considerations are included at this stage, the model naturally tends to prioritize Coal and sometimes Gas over Oil as Coal has the lowest fuel price across suppliers followed by Gas. This outcome is consistent with the purely cost-minimizing structure of the formulation. However, this behavior also highlights a key modeling limitation: in realistic power markets, high-emission technologies such as Coal often face lower net revenues once environmental penalties, market pricing, or regulatory constraints are considered. Based on this expectation, we anticipate that including electricity prices, demand variability, and emission-adjusted revenues will significantly alter the dispatch hierarchy. These insights motivate the transition to Draft 2, where electricity revenues and daily demand profiles are introduced, enabling a more realistic economic evaluation of each technology. The numerical results of Draft 1 can be seen in appendix A.2 (Draft 1 (Plots, tables and results)). It is worth to mention that based on the volatility of the suppliers price there are some critical days where Gas is cheaper than coal; due to that, a great use of Gas can be seen.

4 Draft 2: Profit Maximization with Daily and Hourly Demand, Electricity Prices, and Load Deviation Penalties

Draft 2 extends the deterministic framework of Draft 1 by introducing an economically meaningful *profit maximization* objective with full hourly resolution. Electricity demand and electricity prices now vary by hour, allowing the model to capture intra-day operational and market dynamics. Furthermore, the model incorporates explicit *load deviation variables*, which permit the system to over- or under-generate relative to the required hourly demand, at the cost of financial penalties.

4.1 Sets and Indices

Draft 2 keeps the sets from Draft 1 and uses hourly resolution:

$$\begin{aligned}
T &= \{\text{Oil, Gas, Coal}\} && \text{Technologies,}\\
S_t &= \{\text{Suppliers for technology } t\} && \text{Supplier sets,}\\
D &= \{1,\dots,7\} && \text{Days in the horizon,}\\
H &= \{1,\dots,24\} && \text{Hours of each day.}
\end{aligned}$$

4.2 Parameters

In addition to the parameters inherited from Draft 1 (fuel prices, daily supplier limits, storage costs, etc.), Draft 2 introduces the following parameters, all aligned with the optimization model:

$\text{dem}_{d,h}$	:	Hourly demand (MWh)	$\overline{G}_t$	:	Generation capacity (MW)
$\pi_{d,h}$	:	Hourly electricity price (EUR/MWh)	$\overline{I}_t$	:	Storage capacity (MWh)
α_t	:	Revenue factor	r_t	:	Storage rent (EUR/MWh)
$p_{t,s,d}$	:	Fuel price	λ^{under}	:	Under-gen penalty (EUR/MWh)
$\overline{Q}_{t,s,d}$	:	Supplier limit	λ^{over}	:	Over-gen penalty (EUR/MWh)

4.3 Decision Variables

Draft 2 uses the same operational variables as Draft 1, with additional load deviation variables:

$$\begin{aligned}
Q_{t,s,d} \geq 0 &: && \text{Fuel purchased,}\\
g_{t,d,h} \geq 0 &: && \text{Electricity generated,}\\
I_{t,d} \geq 0 &: && \text{End-of-day inventory,}\\
u_{d,h} \geq 0 &: && \text{Under-generation (shortfall from demand),}\\
o_{d,h} \geq 0 &: && \text{Over-generation (excess above demand).}
\end{aligned}$$

4.4 Objective Function

The goal of Draft 2 is to maximize profit, computed as:

$$\begin{aligned} \max \quad & \underbrace{\sum_{d\in D}\sum_{h\in H}\sum_{t\in T}\alpha_t\,\pi_{d,h}\,g_{t,d,h}}_{\text{electricity revenue}} - \underbrace{\sum_{t\in T}\sum_{s\in S_t}\sum_{d\in D}p_{t,s,d}\,Q_{t,s,d}}_{\text{fuel purchase cost}} - \underbrace{\sum_{t\in T}\sum_{d\in D}r_t\,I_{t,d}}_{\text{storage cost}} \\ & - \underbrace{\sum_{d\in D}\sum_{h\in H}\lambda^{\text{under}}\,u_{d,h}}_{\text{under-generation penalty}} - \underbrace{\sum_{d\in D}\sum_{h\in H}\lambda^{\text{over}}\,o_{d,h}}_{\text{over-generation penalty}} . \end{aligned} \tag{8}$$

4.5 Model Constraints

The operational constraints from Draft 1 are preserved and expressed with hourly resolution:

$$I_{t,d} = I_{t,d-1} + \sum_{s\in S_t} Q_{t,s,d} - \sum_{h\in H} g_{t,d,h}, \qquad \text{(inventory balance)} \tag{9}$$

$$g_{t,d,h} \le \overline{G}_t, \qquad \text{(generation capacity)} \tag{10}$$

$$I_{t,d} \le \overline{I}_t, \qquad \text{(storage capacity)} \tag{11}$$

$$Q_{t,s,d} \le \overline{Q}_{t,s,d}, \qquad \text{(supplier limits)} \tag{12}$$

$$\sum_{t\in T} g_{t,d,h} + u_{d,h} - o_{d,h} = \text{dem}_{d,h}, \qquad \text{(load balance with deviations)} \tag{13}$$

with the terminal condition:

$$I_{t,7} = 0, \qquad \forall t \in T. \tag{14}$$

4.6 Discussion

The introduction of load deviation variables provides operational flexibility, allowing the system to deviate from hourly demand at a financial penalty. This prevents infeasibility in cases where fuel availability or capacity constraints restrict generation. Hourly prices and hourly demand create a more realistic operational environment, while the revenue factor α_t captures technology-specific effective pricing (e.g., emission-adjusted revenue). The resulting formulation remains deterministic, but captures detailed operational trade-offs, setting the stage for Draft 3 where full uncertainty and risk measures will be introduced. Based on the results of Draft 2 A.3 (Draft 2 (Plots, tables and results)), we can tell that the optimization model works as expected. Gas has a predominant usage than Coal and Oil due to the good trade-off between "cheap" prices and a good revenue factor based on the emissions. Nevertheless, the great volatility of the prices has not yet been taken into account, so Draft 3 will dive into it to bring a more insightful solution.

5 Draft 3: Scenario-Based Profit Maximization with Fuel Price Volatility and CVaR Risk Aversion

Draft 3 extends the deterministic profit-maximizing framework of Draft 2 to a *multi-scenario* setting with explicit fuel price volatility and risk aversion. Fuel prices now depend on scenario-specific "day-type" patterns (e.g. Base, Spike, Severe), which differ by technology and day. Operational decisions (generation, storage, and load deviations) are allowed to depend on the realized scenario, while fuel purchase decisions remain here-and-now (non-anticipative).

To control exposure to adverse price and demand outcomes, Draft 3 introduces a *Conditional Value-at-Risk (CVaR)* term in the objective. The model maximizes expected profit penalized by the CVaR of downside losses relative to a profit target, with a tunable risk aversion parameter.

5.1 Sets and Indices

Draft 3 keeps the temporal structure from Draft 2 and adds a finite set of scenarios:

$$\begin{aligned}
T &= \{\text{Oil, Gas, Coal}\} && \text{Technologies,}\\
S_t &= \{\text{Suppliers for technology } t\} && \text{Supplier sets,}\\
D &= \{1,\ldots,7\} && \text{Days in the horizon,}\\
H &= \{1,\ldots,24\} && \text{Hours of each day,}\\
W &= \{\omega_1,\ldots,\omega_{|W|}\} && \text{Scenarios (fuel-price paths).}
\end{aligned}$$

5.2 Parameters

The following parameters describe demand, market conditions, technology limits, and risk:

$\text{dem}_{d,h}$	:	Hourly electricity demand (MWh)
$\pi_{d,h}$	:	Hourly electricity market price (EUR/MWh)
α_t	:	Technology-specific revenue factor
$p^{\omega}_{t,s,d}$	:	Scenario fuel price (EUR/MWh)
$\overline{Q}_{t,s,d}$	:	Daily supplier fuel limit (MWh)
$\overline{G}_t$	:	Max hourly generation (MWh)
$\bar{I}_t$	:	Max storage capacity (MWh)
r_t	:	Storage rent (EUR/MWh)
I^0_t	:	Initial inventory (MWh)
λ^{under}	:	Under-generation penalty
λ^{over}	:	Over-generation penalty
P_ω	:	Scenario probability
α	:	CVaR confidence level
λ_{risk}	:	Risk aversion weight
Π^{tar}	:	Target profit (EUR)

5.3 Decision Variables

Fuel purchase decisions are made before the scenario is known, while operational and risk variables are scenario-dependent:

$Q_{t,s,d} \geq 0$: Fuel purchased (MWh)
$g^{\omega}_{t,d,h} \geq 0$: Generation in scenario ω (MWh)
$I^{\omega}_{t,d} \geq 0$: Inventory in scenario ω (MWh)
$f_{t,d} \geq 0$: Fuel available (MWh)
$u^{\omega}_{d,h} \geq 0$: Under-generation (MWh)

$o^{\omega}_{d,h} \geq 0$: Over-generation (MWh)
$L^{\omega} \geq 0$: Scenario loss (EUR)
$s^{\omega} \geq 0$: CVaR tail variable (EUR)
$\eta \in R$: CVaR threshold (EUR)

5.4 Objective Function

Let Π^{ω} denote the profit in scenario $\omega \in W$. For each scenario, profit is defined as revenue minus fuel, storage, and load-deviation costs:

$$\begin{aligned}\Pi^{\omega} = & \underbrace{\sum_{d\in D}\sum_{h\in H}\sum_{t\in T} \alpha_t\, \pi_{d,h}\, g^{\omega}_{t,d,h}}_{\text{electricity revenue in scenario } \omega} - \underbrace{\sum_{t\in T}\sum_{s\in S_t}\sum_{d\in D} p^{\omega}_{t,s,d}\, Q_{t,s,d}}_{\text{fuel purchase cost in scenario } \omega} - \underbrace{\sum_{t\in T}\sum_{d\in D} r_t\, I^{\omega}_{t,d}}_{\text{storage cost in scenario } \omega} \\ & - \underbrace{\sum_{d\in D}\sum_{h\in H} \lambda^{\text{under}}\, u^{\omega}_{d,h}}_{\text{under-generation penalty in scenario } \omega} - \underbrace{\sum_{d\in D}\sum_{h\in H} \lambda^{\text{over}}\, o^{\omega}_{d,h}}_{\text{over-generation penalty in scenario } \omega}\end{aligned} \tag{15}$$

Expected profit is

$$E[\Pi] = \sum_{\omega\in W} P_{\omega}\, \Pi^{\omega}.$$

To model CVaR of the losses relative to a target profit Π^{tar}, we define the loss in scenario ω as

$$L^{\omega} \approx \max\{\Pi^{\text{tar}} - \Pi^{\omega}, 0\}, \quad s^{\omega} \approx \max\{L^{\omega} - \eta, 0\}$$

and CVaR at level α as

$$\text{CVaR}_{\alpha}(L) = \eta + \frac{1}{1-\alpha} \sum_{\omega\in W} P_{\omega}\, s^{\omega}.$$

The objective of Draft 3 is therefore

$$\max \quad \underbrace{\sum_{\omega\in W} P_{\omega}\, \Pi^{\omega}}_{\text{expected profit}} - \lambda_{\text{risk}} \underbrace{\left(\eta + \frac{1}{1-\alpha}\sum_{\omega\in W} P_{\omega}\, s^{\omega}\right)}_{\text{CVaR penalty term}}. \tag{16}$$

5.5 Model Constraints

The operational constraints from Draft 2 are extended to the multi-scenario setting of Draft 3:

$$I_{t,1}^{\omega} = I_t^0 + f_{t,1} - \sum_{h \in H} g_{t,1,h}^{\omega}, \qquad \text{(initial inventory balance)} \tag{17}$$

$$I_{t,d}^{\omega} = I_{t,d-1}^{\omega} + f_{t,d} - \sum_{h \in H} g_{t,d,h}^{\omega}, \qquad \text{(inventory balance for } d > 1) \tag{18}$$

$$f_{t,d} = \sum_{s \in S_t} Q_{t,s,d}, \qquad \text{(fuel availability from purchases)} \tag{19}$$

$$g_{t,d,h}^{\omega} \le \overline{G}_t, \qquad \text{(generation capacity)} \tag{20}$$

$$I_{t,d}^{\omega} \le \overline{I}_t, \qquad \text{(storage capacity)} \tag{21}$$

$$Q_{t,s,d} \le \overline{Q}_{t,s,d}, \qquad \text{(supplier limits)} \tag{22}$$

$$\sum_{t \in T} g_{t,d,h}^{\omega} + u_{d,h}^{\omega} - o_{d,h}^{\omega} = \mathrm{dem}_{d,h}, \qquad \text{(load balance with deviations)} \tag{23}$$

$$I_{t,7}^{\omega} = 0, \qquad \text{(terminal inventory condition)} \tag{24}$$

$$L^{\omega} \ge \Pi^{\text{tar}} - \Pi^{\omega}, \qquad \text{(scenario loss definition)} \tag{25}$$

$$s^{\omega} \ge L^{\omega} - \eta, \qquad \text{(CVaR tail condition)} \tag{26}$$

5.6 Discussion

Compared to Draft 2, Draft 3 introduces three major methodological enhancements that substantially increase the economic realism of the fuel purchasing and operational planning problem. First, fuel price *volatility* is explicitly represented through multiple day-type patterns (e.g. Base, Spike, Severe) which vary across technologies and days. These patterns generate a scenario tree that captures the correlated and time-dependent fluctuations observed in real commodity markets. Second, operational decisions such as generation, storage usage, and load deviations become *scenario-dependent*. This allows the system to adapt its real-time behavior to the realized market conditions while maintaining non-anticipativity for strategic fuel purchasing decisions. Third, an explicit *risk measure* based on Conditional Value-at-Risk (CVaR) is added to the objective. CVaR penalizes poor-performing scenarios, thereby shifting the decision maker toward more robust strategies when fuel price spikes or severe operating conditions occur.

One of the central conceptual features of Draft 3 is the *non-anticipativity* of fuel purchases. While operational variables adjust to the scenario, the purchase quantities $Q_{t,s,d}$ must be identical across all scenarios since they correspond to commitments made before uncertainty unfolds. This creates a strategic trade-off: buying large volumes early reduces exposure to price spikes but increases the chance of over-purchasing in favorable price scenarios; conversely, waiting reduces inventory costs but increases vulnerability to high-price shocks or insufficient supply. The risk aversion parameter λ_{risk} effectively governs this balance. A low value favors high expected profit and more aggressive exploitation of cheap scenarios, while a high value places more weight on avoiding worst-case losses. The CVaR term ensures that excessively risky procurement strategies are penalized even when they yield high expected profit.

Beyond risk-adjusted profit optimization, Draft 3 yields a powerful economic insight through the analysis of *shadow prices* (dual variables) associated with fuel, generation, and inventory constraints. These shadow prices represent the marginal value of an additional unit of resource: one extra MWh of fuel, one extra unit of generation capacity, or one extra MWh of storage space. From the company's perspective, the shadow price of the fuel availability constraint is interpretable as the *willingness-to-pay (WTP)* for an additional unit of fuel at a given day. In other words, it reveals the maximum price at which purchasing an extra MWh remains economically desirable, given the current risk preferences and system conditions. This connection is crucial: while the optimization model determines the optimal purchase quantities, the *shadow prices tell us why*. They provide a transparent, quantitative explanation of the procurement strategy, allowing us to translate optimization outputs into market-facing purchasing guidelines.

In the context of this assignment, the list of WTP prices obtained from the dual variables is arguably the most actionable outcome. Since each WTP value incorporates operational constraints, expected market revenues, storage limitations, and risk preferences, it summarizes a complex stochastic optimization problem into an interpretable economic signal. This enables decision makers to understand how risk aversion affects procurement behavior: higher λ_{risk} leads to higher WTP values on early days (reflecting a desire to insure against price spikes), while more risk-neutral decisions produce more volatile and opportunistic purchasing patterns.

Overall, Draft 3 not only optimizes procurement and operational behavior under uncertainty, but also provides an economically interpretable structure that links optimization results to market decisions. The integration of CVaR risk aversion, scenario-dependent operations, and dual-price interpretation lays the foundation for even more advanced models incorporating demand response, emission constraints, multi-stage procurement, and stochastic expansions of the underlying scenario tree. This framework enables the company to transition from deterministic planning to a fully risk-aware procurement strategy grounded in rigorous quantitative analysis.

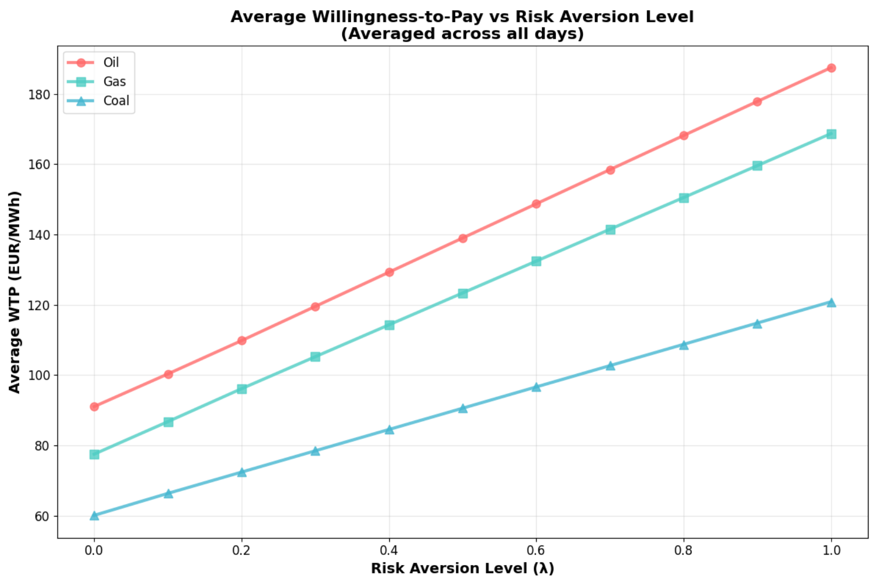

Figure 1: WTP (Average) (All data in: A.4)

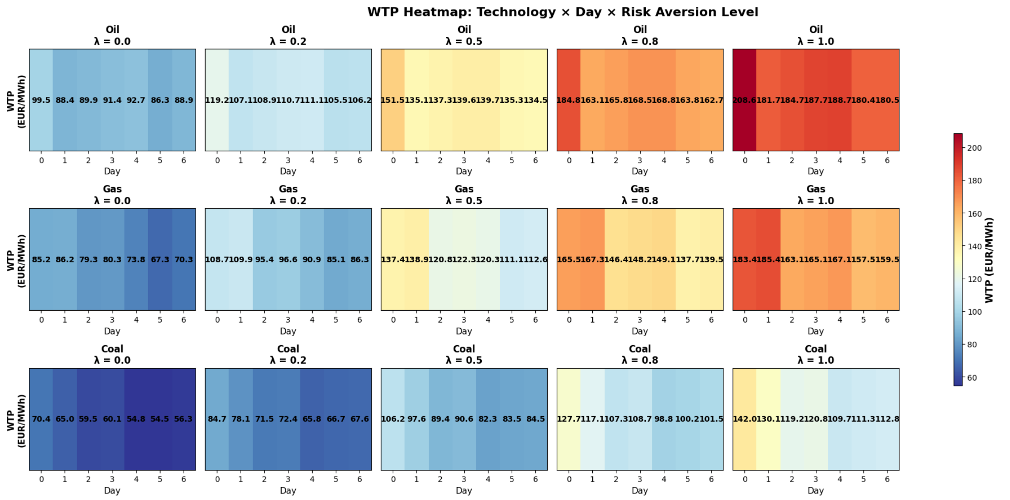

Figure 2: WTP Heatmap (All data in: A.4)

6 Conclusion

This project developed a progressively richer decision-support framework for weekly fuel procurement in thermal power systems, moving from a deterministic formulation to a fully stochastic, risk-aware optimization model. Draft 1 established a baseline model built around perfect information, capturing core system physics such as fuel flows, inventory dynamics, and generation limits. While this simplified model produced the cost-minimizing schedule under ideal conditions, it also revealed unrealistic outcomes particularly the dominance of low-cost fuels such as coal highlighting the need for richer economic signals and uncertainty modelling.

Draft 2 advanced the modelling realism by incorporating hourly demand and electricity price variability, reflecting the operational and market dynamics observed in modern power systems. The addition of load deviation penalties and technology-specific revenue factors enabled a profit-maximization perspective that better aligns with real-world dispatch behavior. This intermediate model demonstrated how time resolution and variable prices influence procurement strategy, and it provided a bridge between deterministic planning and uncertainty-aware decision-making.

Draft 3 introduced scenario-based fuel price volatility and Conditional Value-at-Risk (CVaR), a well-established measure in stochastic optimization. By enforcing non-anticipativity on fuel purchases while allowing scenario-dependent operations, the model effectively separated long-term procurement commitments from real-time operational flexibility. The CVaR term enabled explicit control over downside risk, allowing decision makers to shift smoothly from profit-seeking to robustness-focused strategies depending on their risk aversion. Additionally, analysis of shadow prices yielded economically interpretable willingness-to-pay (WTP) signals, translating complex optimization outputs into actionable purchasing guidance rooted in market fundamentals.

Overall, the modelling progression demonstrates how increasing levels of realism through volatility, time granularity, and risk representation fundamentally reshape optimal procurement decisions. The final framework equips the company with a transparent, quantitative, and risk-aware tool for navigating fuel price uncertainty, improving operational reliability, and supporting financially resilient long-term procurement strategies in a volatile energy market.

A Appendix

A.1 Volatility and Scenario Modeling Across Drafts

A common theme across the three drafts is the progressive use of volatility to move from a purely deterministic setting toward a realistic, risk-aware stochastic model. The same basic idea (a multiplicative perturbation around a baseline value) is first used informally to enrich the data in Draft 1, then extended to demand and electricity prices in Draft 2, and finally embedded in a full scenario structure in Draft 3.

Draft 1: Weekly fuel price variation along the week In Draft 1, the volatility concept is first used at the data level to generate more realistic daily fuel prices for each technology supplier pair along the week. Instead of assuming a single constant price, each daily price is obtained as a perturbation of a baseline value:

$$p^{\mathrm{det}}_{t,s,d} = p^{\mathrm{base}}_{t,s} \left(1 + \varepsilon_{t,s,d}\right),$$

where $p^{\mathrm{base}}_{t,s}$ is a reference fuel price and $\varepsilon_{t,s,d}$ is a small bounded deviation (e.g. sampled from a uniform interval). This produces a vector of prices $\{p^{\mathrm{det}}_{t,s,1}, \dots, p^{\mathrm{det}}_{t,s,7}\}$ for each supplier, capturing the fact that fuel prices fluctuate day-to-day, even though the optimization model in Draft 1 itself remains fully deterministic and single-scenario.

Draft 2: Volatility in demand and electricity prices In Draft 2, the same volatility idea is applied more systematically to both *demand* and *electricity prices*, while the optimization problem is still solved as a single deterministic instance. First, a weekly demand is disaggregated into daily values and then into hourly values using a normalized hourly load profile $\{\phi_h\}_{h\in H}$, with $\sum_{h\in H} \phi_h = 1$. To introduce realistic day-to-day variability and to ensure that demand does not exceed the installed capacity, a *downward-only* multiplicative shock is applied at the daily level:

$$\mathrm{DEM}^{\mathrm{eff}}_d = \mathrm{DEM}^{\mathrm{base}}_d \left(1 - \zeta_d\right), \qquad \zeta_d \sim \mathcal{U}(0, \delta_{\mathrm{dem}}),$$

and hourly demand is then constructed as

$$\mathrm{dem}_{d,h} = \mathrm{DEM}^{\mathrm{eff}}_d \cdot \phi_h.$$

The restriction $\zeta_d \geq 0$ implies that demand can only move *downwards* from the baseline. This is both realistic (days with lower-than-expected demand are common) and necessary to avoid generating hours where $\mathrm{dem}_{d,h}$ would exceed the total available generation capacity, which would make the optimization problem infeasible.

Similarly, daily electricity prices are perturbed around a baseline:

$$\pi^{\mathrm{eff}}_d = \pi^{\mathrm{base}}_d \left(1 + \eta_d\right), \qquad \eta_d \sim \mathcal{U}(-\delta_\pi, \delta_\pi),$$

so that the revenue term in the objective function uses a realistic, mildly volatile price path along the week. At this stage, all these perturbed values are treated as given deterministic inputs: Draft 2 is still a single-scenario model, but with data that mimics market volatility.

Draft 3: Discrete regimes, weekly scenarios, and stochastic optimization Draft 3 takes the final step and embeds volatility into a fully stochastic, multi-scenario framework. Instead of a single perturbed price path, the model considers a finite set of weekly scenarios W, each representing a different combination of "market regimes" over the days of the week. For each technology t and supplier s, the data specifies three regime-dependent price levels per day:

$$p_{t,s,d}^{\text{Base}}, \quad p_{t,s,d}^{\text{Spike}}, \quad p_{t,s,d}^{\text{Severe}},$$

corresponding to normal, moderately stressed, and highly stressed market conditions.

For each scenario $\omega \in W$, and each technology t, a regime label

$$R_{t,d}^{(\omega)} \in \{\text{Base}, \text{Spike}, \text{Severe}\}$$

is assigned for every day d of the week. These regime sequences are constructed so that weeks dominated by Base days are more probable than weeks with many Severe days, reflecting the idea that strongly stressed markets are rarer. Each scenario ω is then given a probability $P(\omega)$, and the fuel price used by the optimizer in scenario ω is

$$p_{t,s,d}^{(\omega)} = p_{t,s,d}^{R_{t,d}^{(\omega)}}.$$

In the current implementation, the hourly demand and electricity price profiles constructed in Draft 2 are kept the same across all scenarios in Draft 3, so that uncertainty enters primarily through the fuel side. This allows us to isolate the impact of fuel-price regime risk on procurement and storage decisions. For each scenario ω, the model computes a scenario-specific profit $\Pi^{(\omega)}$ based on these prices, subject to the same physical constraints on generation, storage, and supplier limits.

Finally, the set of scenario profits is used to build a risk-aware objective based on Conditional Value-at-Risk (CVaR). Defining the loss in scenario ω as $L^{(\omega)} = \max\{0, \Pi^\star - \Pi^{(\omega)}\}$ with respect to a target profit $\Pi^\star$, the CVaR at confidence level α is approximated by introducing an auxiliary variable η (VaR threshold) and slack variables u_ω for each scenario, and enforcing

$$s_\omega \geq L^{(\omega)} - \eta, \qquad s_\omega \geq 0.$$

The CVaR is then

$$\text{CVaR}_\alpha = \eta + \frac{1}{1-\alpha} \sum_{\omega \in W} P(\omega)\, s_\omega,$$

and the Draft 3 objective takes the form

$$\max \; E[\Pi^{(\omega)}] \; - \; \lambda\, \text{CVaR}_\alpha,$$

where $\lambda \geq 0$ controls the degree of risk aversion. In this way, the discrete Base/Spike/Severe regime structure, combined with weekly scenario probabilities and the CVaR term, transforms the deterministic volatility ideas of Drafts 1 and 2 into a fully fledged stochastic, risk-aware optimization model in Draft 3.

A.2 Draft 1 (Plots, tables and results)

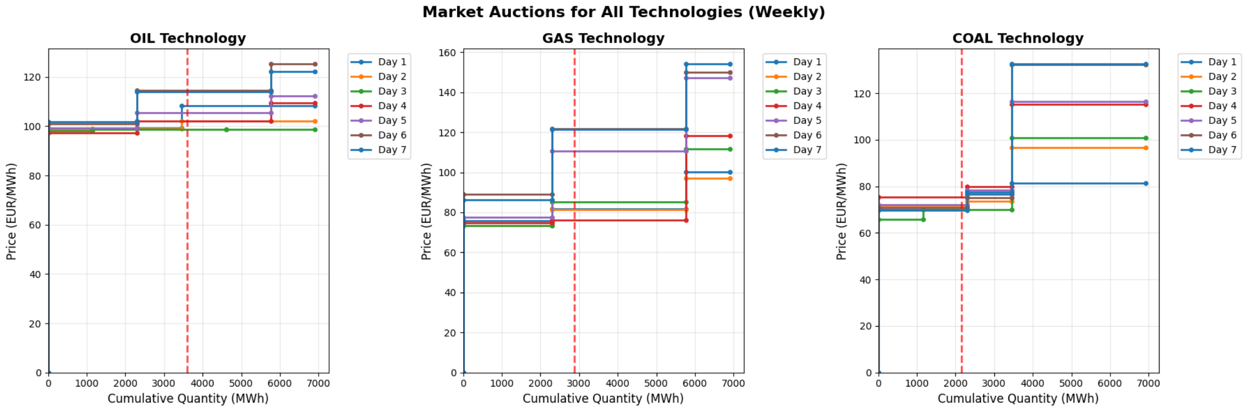

Figure 3: Market auctions

Technology	Max Gen (MWh/day)	# Suppliers	Price Range (EUR/MWh)	Avg. Price	Total Capacity (MWh/day)
Oil	1200.0	3	97.40 – 125.20	105.10	6912.0
Gas	960.0	3	73.20 – 154.00	100.34	6912.0
Coal	720.0	3	65.70 – 132.50	85.82	6912.0

Table 2: Market Auction Summary for Oil, Gas, and Coal Technologies.

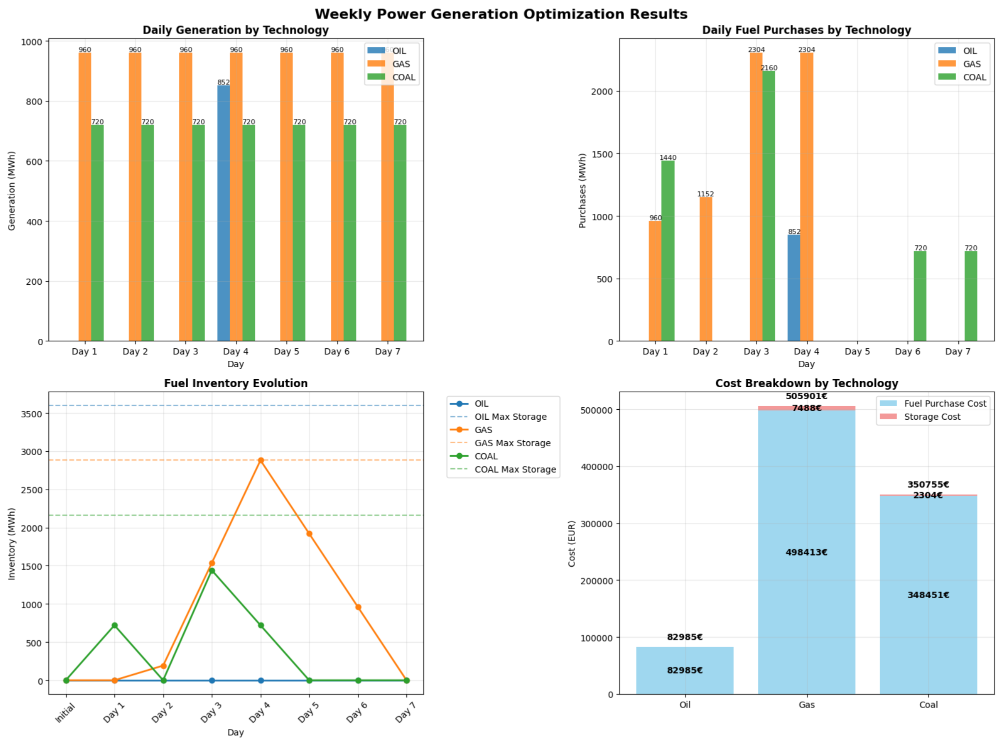

Figure 4: Draft 1 results

Metric	Value
Total Weekly Generation (MWh)	12612.00
Total Weekly Demand (MWh)	12612.00
Total Weekly Purchases (MWh)	12612.00
Total Cost (EUR)	939640.80
Fuel Purchase Cost (EUR)	929848.80 (99.0%)
Storage Cost (EUR)	9792.00 (1.0%)

Table 3: Overall Optimization Results for the Week

Technology	Generation (MWh)	Share of Total (%)	Capacity Utilization (%)
OIL	852.00	6.8%	10.1%
GAS	6720.00	53.3%	100.0%
COAL	5040.00	40.0%	100.0%

Table 4: Generation Breakdown by Technology

Technology	Max Storage (MWh)	Max Usage (% of capacity)	Average Storage (MWh)
OIL	0.00	0.0%	0.00
GAS	2880.00	100.0%	1069.71
COAL	1440.00	66.7%	411.43

Table 5: Storage Utilization by Technology

A.3 Draft 2 (Plots, tables and results)

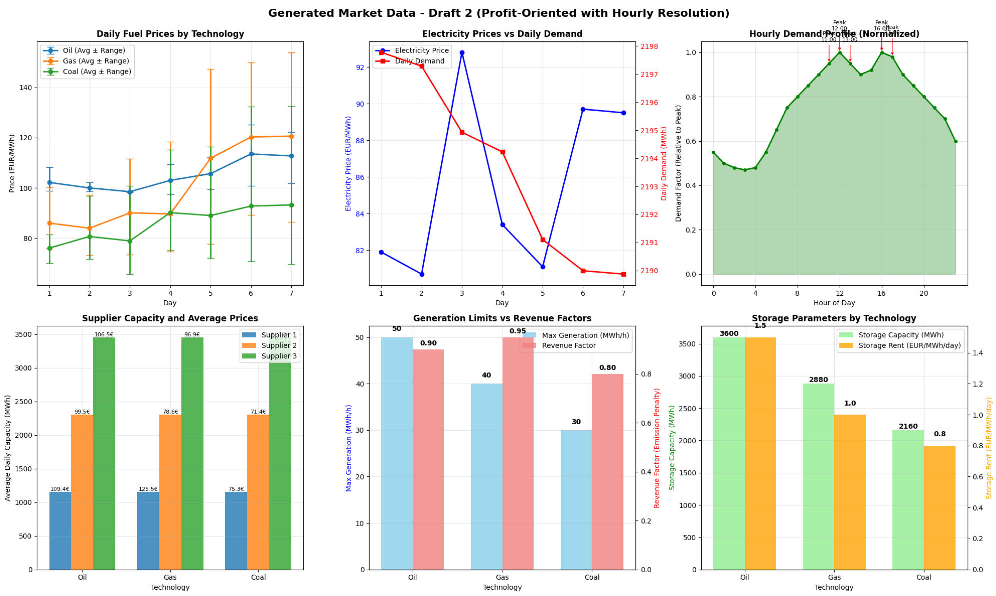

Figure 5: Draft 2 Data visualization (1)

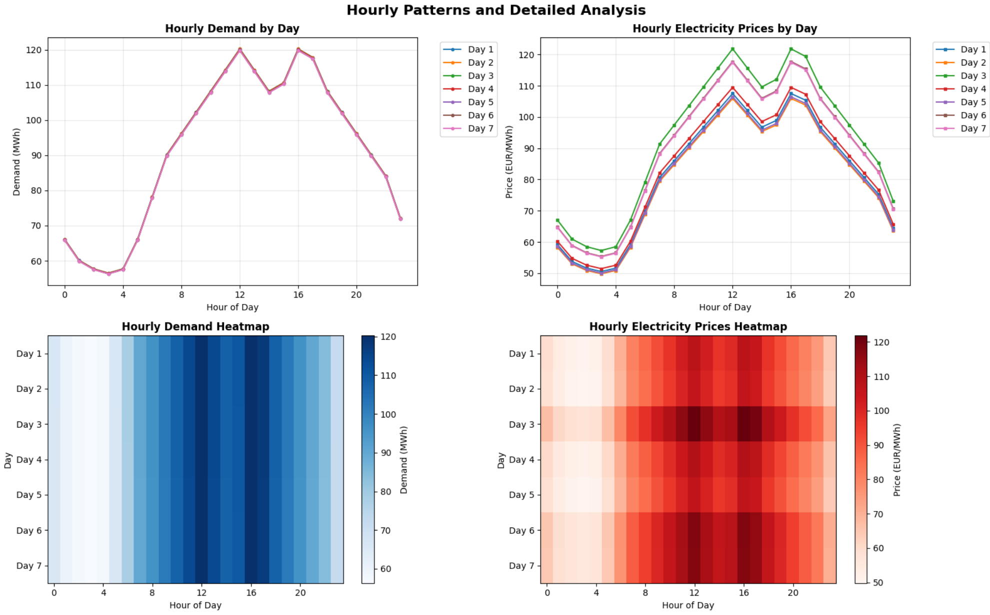

Figure 6: Draft 2 Data visualization (2)

Metric	Value
Weekly target demand (MWh)	15355.2
Daily demand range (MWh)	2189.9 – 2197.8
Hourly factor range	0.470 – 1.000

Table 6: Demand Structure

Metric	Value
Base price (EUR/MWh)	85.0
Daily price range (EUR/MWh)	80.7 – 92.8
Hourly price range (EUR/MWh)	49.8 – 121.8

Table 7: Electricity Price Characteristics

Tech.	Max Gen (MWh/h)	Storage Cap (MWh)	Storage Rent (EUR/MWh/day)
OIL	50	3600	1.5
GAS	40	2880	1.0
COAL	30	2160	0.8

Tech.	Revenue Factor	Avg Fuel Price (EUR/MWh)	Daily Supplier Cap (MWh)
OIL	0.90	105.1	6912.0
GAS	0.95	100.3	6912.0
COAL	0.80	85.8	6912.0

Table 8: Technology Overview for Draft 2

Characteristic	Description
Suppliers per technology	3
Price volatility	Fuel prices include spikes and daily variations.
Demand pattern	Daily variations with an hourly load shape.
Emission penalties	Reflected in revenue factors (Coal < Oil < Gas).

Table 9: Market Characteristics for Draft 2

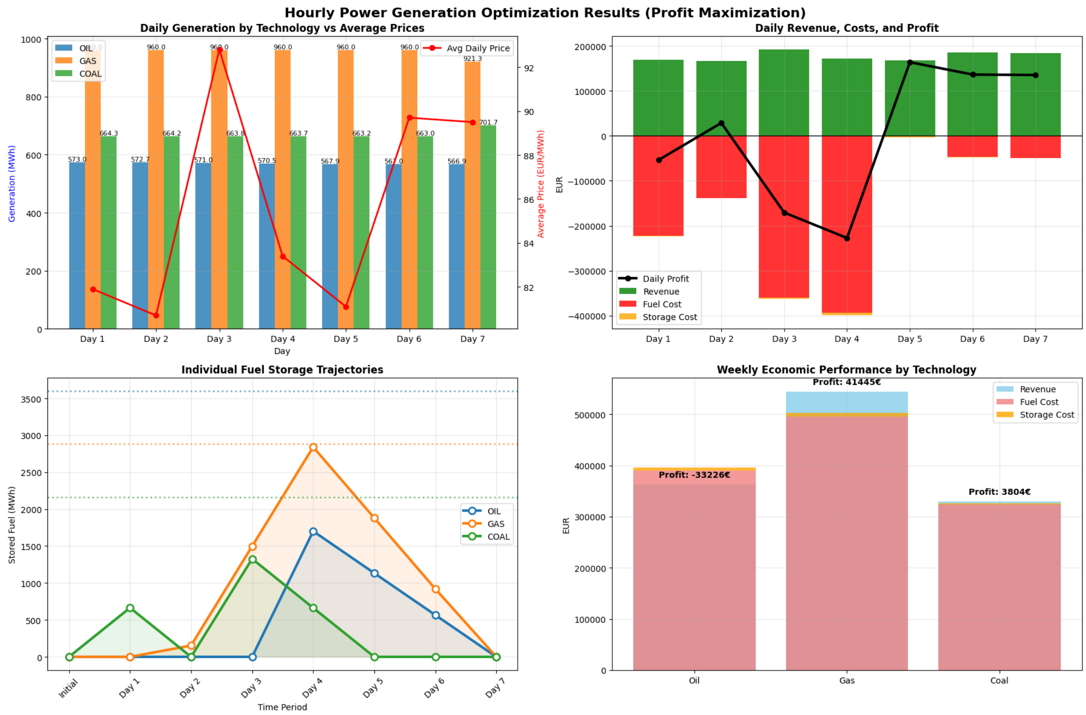

Figure 7: Draft 2 Results visualization (1)

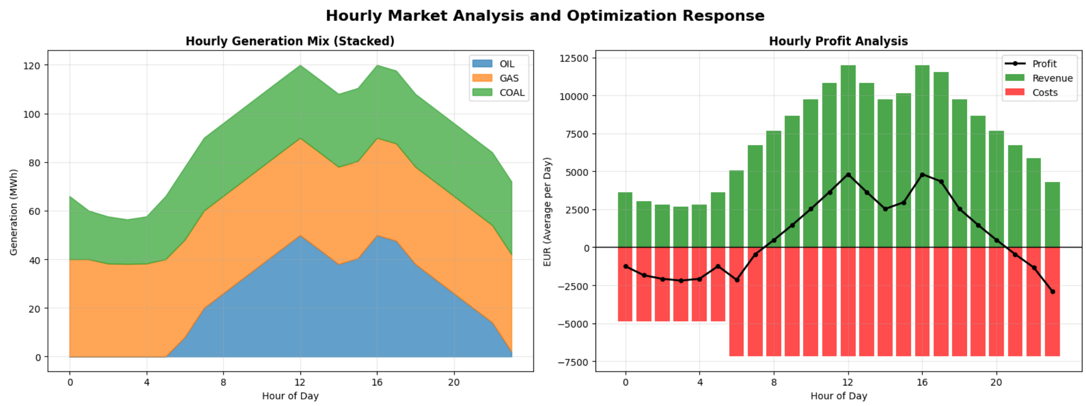

Figure 8: Draft 2 Results visualization (2)

Metric	Value
Total Revenue (EUR)	1,236,163.25
Total Fuel Cost (EUR)	1,209,617.84
Total Storage Cost (EUR)	14,521.84
Total Profit (EUR)	12,023.58
Profit Margin (%)	1.0%

Table 10: Weekly Performance Summary

Technology	Generation (MWh)	Capacity Util. (%)	Revenue (EUR)	Profit (EUR)
OIL	3988.92	47.5%	362,460.07	-33,225.66
GAS	6681.31	99.4%	544,320.58	41,445.45
COAL	4683.89	92.9%	329,382.61	3,803.79

Technology	Profit Margin (%)
OIL	-9.2%
GAS	7.6%
COAL	1.2%

Table 11: Technology-Level Performance Summary

Insight	Value
Peak price hour	12:00 (112.37 EUR/MWh)
Off-peak price hour	03:00 (52.81 EUR/MWh)
Maximum generation hour	12:00 (119.92 MWh)
Price volatility (std dev)	20.23 EUR/MWh

Table 12: Hourly Optimization Insights

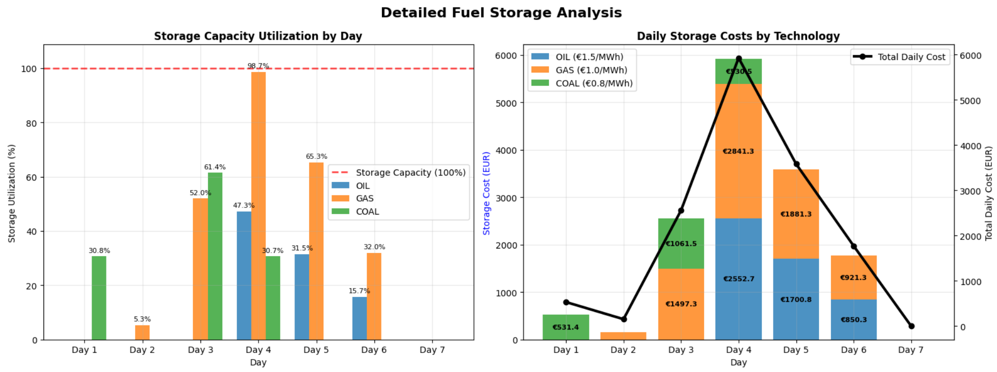

Figure 9: Draft 2 Results visualization (3)

Technology	Capacity (MWh)	Rent (€/MWh/day)	Avg Used (MWh)
OIL	3600	1.5	486.1 (13.5%)
GAS	2880	1.0	1042.1 (36.2%)
COAL	2160	0.8	379.2 (17.6%)

Technology	Max Used (MWh)	Min Used (MWh)	Total Cost (EUR)
OIL	1701.8 (47.3%)	0.0	5103.86
GAS	2841.3 (98.7%)	0.0	7294.54
COAL	1326.9 (61.4%)	0.0	2123.43

Table 13: Fuel Storage Capacity and Utilization by Technology

A.4 Draft 3 (Plots, tables and results)

WTP Analysis Summary for Oil, Gas, and Coal Technologies

Technology	Lambda	Min WTP	Max WTP	Avg WTP	Std WTP
Oil					
Oil	0.0	86.35	99.48	91.04	3.94
Oil	0.1	96.05	109.32	100.32	4.12
Oil	0.2	105.46	119.16	109.80	4.31
Oil	0.3	115.70	129.31	119.54	4.37
Oil	0.4	125.11	140.40	129.27	4.91
Oil	0.5	134.51	151.49	139.01	5.46
Oil	0.6	143.92	162.58	148.74	6.01
Oil	0.7	153.33	173.67	158.48	6.56
Oil	0.8	162.69	184.76	168.20	7.14
Oil	0.9	171.81	196.30	177.89	7.93
Oil	1.0	180.37	208.61	187.49	9.16
Gas					
Gas	0.0	67.31	86.20	77.50	6.72
Gas	0.1	76.28	98.45	86.74	8.27
Gas	0.2	85.08	109.86	96.12	9.20
Gas	0.3	93.58	119.72	105.22	9.60
Gas	0.4	102.35	129.31	114.29	9.86
Gas	0.5	111.13	138.89	123.36	10.14
Gas	0.6	119.90	148.48	132.43	10.42
Gas	0.7	128.68	158.07	141.51	10.71
Gas	0.8	137.71	167.32	150.53	10.81
Gas	0.9	147.33	176.36	159.61	10.64
Gas	1.0	157.54	185.39	168.74	10.35
Coal					
Coal	0.0	54.49	70.39	60.07	5.41
Coal	0.1	60.26	77.56	66.32	5.82
Coal	0.2	65.76	84.72	72.38	6.37
Coal	0.3	71.26	91.88	78.45	6.93
Coal	0.4	76.76	99.05	84.51	7.48
Coal	0.5	82.26	106.21	90.57	8.03
Coal	0.6	87.75	113.37	96.63	8.58
Coal	0.7	93.25	120.54	102.69	9.14
Coal	0.8	98.75	127.70	108.75	9.69
Coal	0.9	104.25	134.86	114.81	10.24
Coal	1.0	109.75	142.02	120.87	10.79

A.5 Phase 1 – Client Brief: Fuel Purchasing Under Price Uncertainty

Group 32 – 46750 Optimization in Modern Power Systems, Assignment 2, Phase 1

Context and Decision Maker

This brief focuses on Stage 2 of *Power Grid*, translated into a real-world setting. We consider a power company responsible for purchasing fuel for its thermal power plants on a weekly basis. The company must make fuel purchasing decisions in an environment with changing fuel prices, uncertain electricity demand, and limited storage capacity. Unlike the board game, real decisions involve forecasts, financial risk, and continuous operations rather than simple turn-based choices.

Decision Problem

The company must decide how much fuel to buy and from which sources. Options include spot market purchases, short-term supply contracts, and drawing from stored fuel. The key challenge is securing enough fuel to operate reliably while managing cost and risk. Buying fuel early can protect against price increases but ties up capital and storage; waiting may save money if prices fall but increases the chance of shortages or high future prices.

Objectives and Trade-Offs

The primary objective is to minimize fuel costs while maintaining reliable supply. Secondary goals include reducing exposure to price volatility, diversifying supply sources, and keeping flexibility for plant operation. There are clear trade-offs: minimizing cost in the short term can increase risk, while overly conservative purchasing can lead to excess inventory and higher overall expenses.

Constraints and Data Needs

Key constraints include weekly budgets, storage capacity, fuel delivery schedules, supplier contract limits, and each plant's fuel requirements. Important data include fuel price forecasts, demand estimates, plant efficiency and fuel use data, storage costs, and supplier reliability. Compared to the game, real settings involve imperfect information, physical limits, and time-coupled decisions.

Desired Optimization Support

We are seeking an optimization model to compare cost and risk across different purchasing and storage strategies. The approach begins with a simple multi-week cost-minimization model with inventory tracking, then expands to include uncertainty in prices and demand. The model should support planning decisions and present results in a clear and interpretable way for company decision makers.

Expected Value

This analysis will help determine when early fuel purchases are beneficial, how much storage is worthwhile, and how various sourcing strategies affect cost and risk. Incorporating uncertainty will support more reliable and cost-effective purchasing decisions and reduce exposure to sudden price increases or supply shortages.

A.6 Phase 2 & 3 – Consulting Proposal: Fuel Procurement Optimization Roadmap

Power Company Client – Stage 2: Power Grid Real-World Translation

1. Problem Reframing and Key Challenges The client must determine weekly fuel purchases for thermal plants amid fluctuating prices, uncertain demand, and limited storage. Current purchasing practices are reactive and offer limited insight into cost–risk trade-offs, with further uncertainties around forecast accuracy, contract flexibility, and intertemporal effects. We therefore reframe the task as: *"Design a decision-support framework that minimizes expected procurement cost while maintaining supply reliability under uncertainty."* The main challenges include uncertainty in prices and demand, intertemporal links between storage and purchasing decisions, the need to balance low-cost strategies with risk exposure, and constraints such as budgets, storage limits, and supplier contracts.

2. Proposed Modeling Roadmap

Model 1 – Deterministic Baseline (Single Week)
Goal: benchmark minimum cost under perfect information.

This model assumes fuel prices and electricity demand are known in advance. It identifies the most cost-effective purchasing plan while respecting supplier and storage limits, providing a reference for ideal, risk-free conditions.

Model 2 – Intertemporal Extension (Multi-Week)
Goal: capture foresight and storage dynamics over time.

This version links decisions across weeks, accounting for how buying or storing fuel now affects future options. It helps assess strategies such as buying early when prices are favorable or using storage to smooth fluctuations, emphasizing coordinated, forward-looking planning.

Model 3 – Uncertainty-Aware Optimization
Goal: evaluate cost–risk trade-offs under uncertain prices and demand.

This model introduces uncertainty via multiple future scenarios and identifies procurement strategies that perform well on average while remaining resilient under unfavorable outcomes. It quantifies flexibility benefits and guides decisions on acceptable risk levels.

3. Realism–Complexity Trade-offs As the models evolve, they capture more realistic operational features but require additional data, computation, and interpretation. Model 1 offers transparency; Model 2 introduces foresight; Model 3 supports robust, risk-informed decision-making.

4. Expected Insights and Client Value Model 1 provides an efficiency benchmark; Model 2 shows how coordinated multi-week planning reduces costs; Model 3 assesses robustness under uncertainty, guiding adaptive, cost-stable procurement strategies. Together, these models form a scalable roadmap that strengthens reliability, reduces volatility exposure, and enhances financial predictability.

A.7 Usage of AI in the assignment

Artificial intelligence was integrated into this project primarily to enhance two areas: refining the written documentation and improving the technical codebase. Its application was strictly limited to improving the report's style, grammar, and synthesis for clarity, and to suggesting improvements on existing code segments to ensure greater structural consistency and best practices. Ultimately, the AI served as a valuable tool to complement and strengthen the analyses and code sections that were actively being developed, rather than generating original content.

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