

Music and Probability

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Abstract

The idea is that we can create new music using Markov chains. In each case, each experiment is performed many times in the same way, and where the outcome of one trial depends only on the previous trial*.

Markov chains also known as Markov processes can be used as mathematical model of a wide variety of situations in biology, business, chemistry, engineering and elsewhere.

Introduction

- Likelihood of typing "c" is $\frac{1}{27}$.
- Likelihood of typing "ca" is $\frac{1}{27} * \frac{1}{27}$.
- For example take the following sentence:
"Applications to Markov chains"
The likelihood to create the entire phrase is $(\frac{1}{27})^{29}$
or 1 in 3.2325791×10^{41}

Markov Chains in music

In the End-Linkin Park.

Note-sequence

Original: D# A# A# F# F F F F F# D#
Randomized 1 : D# D# A# F# F F F F F F
Randomized 2: A# A# A# F# D# A# A# A# A# F#

Example 1

In the End-Linkin Park probability matrix.

Probability matrix

$$M = \begin{matrix} & \begin{matrix} D\sharp & A\sharp & F\sharp & F \end{matrix} \\ \begin{matrix} D\sharp \\ A\sharp \\ F\sharp \\ F \end{matrix} & \begin{pmatrix} 0.50 & 0.50 & 0 & 0 \\ 0 & 0.50 & 0.50 & 0 \\ 0.50 & 0 & 0 & 0.50 \\ 0 & 0 & 0.25 & 0.75 \end{pmatrix} \end{matrix}$$

What is probability matrix? It is a compilation of non-negative vectors whose entries add up to 1.

A stochastic matrix is a square matrix whose rows are probability vectors.

Example 2

Play trial 1 and 2, attempt to guess the original song, then play the original.

Note-sequence

Original: G A B E D — B A G G
Trial 1: G G A B E — D B A G
Trial 2: E D B E D — B A G G

Pattern outcome

- Using a probability matrix, one can determine the probability of coming up with a specific set of states. In this case, a specific string of notes.
- This approach starts opening areas of study such as plagiarism and legal issues.
- Examples:
 - Led Zeppling v. Spirit
 - "Blurred Lines" v. Marvin Gaye
 - "Bring It On Home," by Led Zeppelin (1969) vs. "Bring It On Home," by Sonny Boy Williamson (written by Willie Dixon) (1966)
 - "Whole Lotta Love," by Led Zeppelin (1969) vs. "You Need Love," by Muddy Waters (written by Willie Dixon) (1962)

Plagiarism

With credit given to Steven J. Leon and his book "Linear Algebra with Applications"

Matrix norms have been established to be an adequate way to measure distance between two given matrices A and B . The objective is to find the product $(B - A)^T(B - A)$, then find the trace of the resulting matrix, and then take the square root which will result in the matrix norm, or the "distance" from A to B .

References

- Leon, Steven J. "7.4 Matrix Norms and Condition Numbers." Linear Algebra with Applications. Eighth ed. Upper Saddle River, NJ: Prentice Hall, 2010. 403-09. Print.
- Lay, David C. Linear Algebra and its applications. Fourth ed. University of Maryland: Addison-Wesley, 2012. Print.

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