STEC Project

Cristian Cardona

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Abstract

While musical preferences are quite varied, music possesses many properties that we can quantify. As such, mathematicians look for mathematical tools to describe the artistic creativity and talent in songs we enjoy. In this project, Markov chains are used to compare and contrast songs from different artists. Given a set of states (in this music project, states will represent notes or chords), a Markov chain represents the probability of going from one state to the next. Given a Markov chain, we can then produce a random song.

Introduction

The project will address three research questions:

- Does an artist have a similar Markov chain across all of his/her songs?
- What are the differences and similarities of the Markov chain between two different artists?
- Can we distinguish a random song produced by the Markov chain from the actual song?

Markov Chains

A *Markov Chain* extends the idea of a single probabilistic experiment on the outcome space Ω to a sequence of experiments on Ω, one for every $t =$ 0, 1, . . . Letting X_t denote the tth iteration. Future states are determined by the present state only, this being determined by the *memoryless* property of the Markov processes.

Music and probabilites

Randomness in music can occur in several ways. Various composers have used chance events such as dice rolls to help produce music that is performed by humans on standard instruments.

- The music game.
- David Cope.
- Emmy Howell.
- Experiments in Musical Intelligence.

Markov Chains in music

In the End-Linkin Park.

Note-sequence

Original:	D♯ A♯ A♯ F♯ F F F F F♯ D♯
Randomized 1 :	D♯ D♯ A♯ F♯ F F F F F F
Randomized 2:	A♯ A♯ A♯ F♯ D♯ A♯ A♯ A♯ A♯ F♯

Example 1

In the End-Linkin Park probability matrix.

Probability matrix

$$M = \begin{array}{c} \\ D\sharp \\ A\sharp \\ F\sharp \\ F \end{array} \begin{array}{c} \begin{array}{cccc} D\sharp & A\sharp & F\sharp & F \end{array} \\ \begin{pmatrix} 0.50 & 0.50 & 0 & 0 \\ 0 & 0.50 & 0.50 & 0 \\ 0.50 & 0 & 0 & 0.50 \\ 0 & 0 & 0.25 & 0.75 \end{pmatrix} \end{array}$$

What is probability matrix? It is a compilation of non-negative vectors whose entries add up to 1.
A stochastic matrix is a square matrix whose rows are probability vectors.

Methods Used

Some of the methods used to identify the differences between the songs were:

- Euclidean
- Absolute Value Sum
- Minkowski
- Euclidean
- Manhattan / City Block Distance
- Maximum
- Canberra

The Euclidean distance between two points p and q is the length of the line segment connecting them. In cartesian coordinates, if $p = (p1, p2, ..., pn)$ and $q = (q1, q2, ..., qn)$ are two points in Euclidean n-space, then the distance (d) from p to q, or from q to p is given by the Pythagorean formula:

$$d(p,q) = d(q,p) = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + \cdots + (q_n - p_n)^2}$$

$$= \sqrt{\sum_{i=1}^{n} (q_n - p_n)^2}$$

The Manhattan or city-block space, there exist many equal-length paths between two points in city-block space.

Euclidean distance and city-block distance are special cases (different values of k) fo the Minkowski metric. Note: $k = 1$ gives city-block distance. $k = 2$ gives Euclidean distance. As k increases, increasing emphasis is given to large differences in individual dimensions.

Plagiarism

With credit given to Steven J. Leon and his book "Linear Algebra with Applications"
Matrix norms have been established to be an adequate way to measure distance between two given matrices A and B. The objective is to find the product $(B-A)^T(B-A)$, then find the trace of the resulting matrix, and then take the square root which will result in the matrix norm, or the "distance" from A to B.

Refences

Leon, Steven J. "7.4 Matrix Norms and Condition Numbers."
Linear Algebra with Applications.
Eighth ed. Upper Saddle River, NJ: Prentice Hall, 2010. 403-09.
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Special thanks to the Department of Mathematics of Georgia Gwinnett College
1000 University Center Ln, Lawrenceville, GA 30043

Cristian Cardona, Student.