Carly
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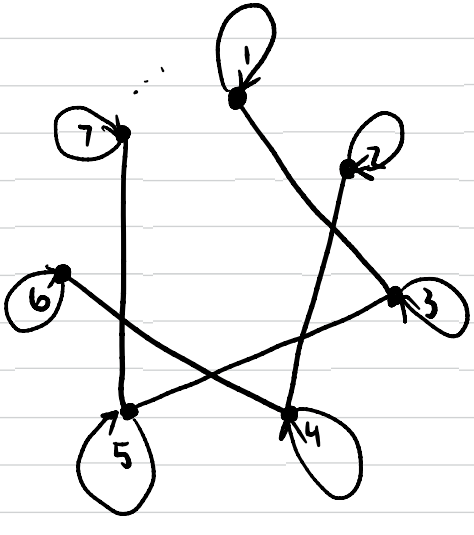
LU factorization of a penta-diagonal matrix:

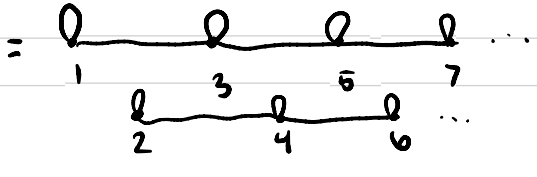
$$\begin{bmatrix} 2 & 0 & -1 & & \\ 0 & 2 & 0 & -1 & \\ -1 & 0 & 2 & 0 & -1 \\ & -1 & 0 & \ddots & \ddots \\ & & & \ddots & \end{bmatrix} = L \cdot U$$



since the matrix is symmetrical across the diagonal, the graph is undirected

$\deg(1, 2, n, n-1) = 3$

$\deg(\text{else}) = 4$



$$A: \begin{bmatrix} 2 & 0 & -1 & 0 & & \\ 0 & 2 & 0 & -1 & 0 & \\ -1 & 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 2 & 0 & -1 \\ & & & & & \ddots \end{bmatrix} \qquad L_1 = \begin{bmatrix} 1 & 0 & & & & \\ 0 & 1 & & & 0 & \\ 1/2 & 0 & 1 & & & \\ 0 & & & 1 & & \\ 0 & & & & 1 & \\ & & & & & \ddots \end{bmatrix}$$

$$L_2 = \begin{bmatrix} 1 & & & & \\ 0 & 1 & & 0 & \\ 0 & 0 & 1 & & \\ 0 & 1/2 & 0 & \ddots & \\ \vdots & \vdots & 0 & & \end{bmatrix} \qquad L_3 = \begin{bmatrix} 1 & & & & \\ 0 & 1 & & 0 & \\ 0 & 0 & 1 & & \\ 0 & 0 & 0 & 1 & \\ 0 & \vdots & 1/2 & 0 & \ddots \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & & & \\ 0 & 1 & & 0 \\ -1/2 & 0 & 1 & \\ 0 & -1/2 & -1/2 & \ddots \end{bmatrix}$$

$$U = \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ & 2 & 0 & -1 & 0 \\ & & 3/2 & 0 & -1 \\ & & & 3/2 & 0 \\ 0 & & & & 3/2 & \ddots \end{bmatrix}$$

Since there is always an oblique grid of zeroes, they never get filled in. L and U graphs only differ from A in that they are directed, due to the fact that they are Lower & Upper diagonal matrices