# LU Factorization of a Penta-diagonal Matrix

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## 1 LU Factorization

$$LU = A = \begin{bmatrix} 2 & 0 & -1 & & & & \\ 0 & 2 & 0 & -1 & & & \\ -1 & 0 & 2 & 0 & -1 & & \\ & -1 & 0 & 2 & 0 & -1 & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & & & & & \\ 0 & 1 & & & & \\ \frac{-1}{2} & 0 & 1 & & & \\ 0 & \frac{-1}{2} & 0 & 1 & & \\ 0 & 0 & \frac{-2}{3} & 0 & 1 & \\ 0 & 0 & 0 & \frac{-2}{3} & 0 & 1 \\ & & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$

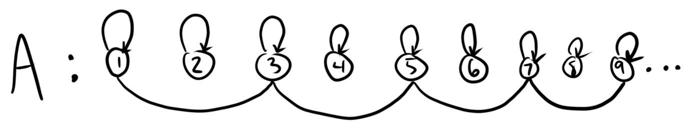
$$U = \begin{bmatrix} 2 & 0 & -1 & & & & \\ & 2 & 0 & -1 & & & \\ & & \frac{3}{2} & 0 & -1 & & \\ & & & \frac{3}{2} & 0 & -1 & \\ & & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$

## 2 Fill-in

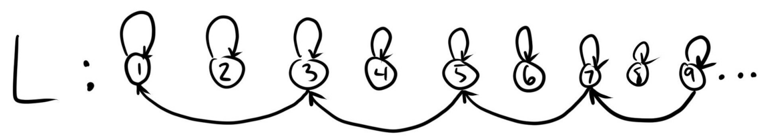
During LU factorization, there is no fill-in. All terms in the penta-diagonal matrix A that were zero to begin with remain zero. The only elements that were altered enough to change the graph were the upper and lower diagonal elements that became zero to form upper and lower triangular matrices.

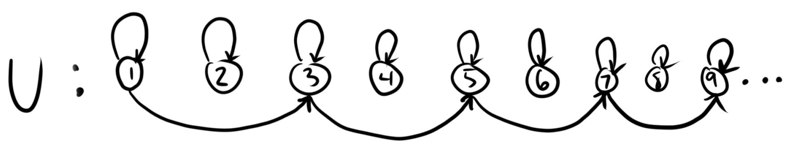
# 3 Graphs

Before LU factorization




After LU Factorization:




The graph of the original matrix, A, is non-directed, meaning that when a line is drawn on graph A between 1 and 3, $\langle 1,3\rangle$ and $\langle 3,1\rangle$ are both non-zero values on the matrix.

L and U are both directed, but in opposite directions. Since L is a lower triangular matrix, a tuple $\langle a,b\rangle$ can only be valid if a $\leq b$

# 4 Relating Fill-In to The Graph

Since there is no fill in, there are no lines added to the graphs of L and U that were not originally on A. The reduction of some non-zero elements to zeroes reduces some lines, making the non-directed graph into a directed one.