Name ____________________

Final Exam
Intertemporal Choice
Fall, 2024

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up.* The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Short Questions.

1. **Labor Supply and Consumption.** In a Real Business Cycle model where labor supply and consumption c are both chosen freely subject to a budget constraint, define z as the proportion of time spent in leisure activities (that is, not working) and W, r, and ϑ as the wage, interest, and time preference rates.

   a) Assuming that the 'hat' operator is equivalent to a difference in logs (e.g., $\hat{\bullet}_{t+1} \equiv \log \bullet_{t+1} - \log \bullet_t$), explain the intuition behind the result that

   $$\hat{z}_{t+1} \approx -\hat{\mathsf{W}}_{t+1} + (\mathsf{r}_{t+1} - \vartheta) \tag{1}$$

   when preferences satisfy the "balanced growth" condition in a perfect foresight model and utility is logarithmic.

   Answer:

   See `RBC-Prescott`. The model suggests z fluctuates for two reasons: The first reason is a response of labor supply to temporary deviations of wages from their permanent level (the Cobb-Douglas assumption means the response of leisure to permanent changes in W is zero, but that assumption does not rule out temporary increases in labor supply to take advantage of temporary high wages). The second reason is intertemporal trade off between consumption of different periods. The idea is that in a period with high interest rates you will want to reduce both your c and your z in order to earn more cash which can be invested to take advantage of the high interest rate (and vice versa).

   b) Discuss the relationship between this result and empirical evidence under two hypotheses about the reasons for employment fluctuations over the business cycle:

      i. They are driven mostly by temporary fluctuations in wages

      ii. They are driven mostly by optimizing responses to interest rate shocks

   Answer:

      i. If this were true, we would see a large elasticity of labor supply with respect to wages; but microeconomic evidence has estimated that elasticity to be small;

      ii. If this is true, there should at the same time be a strong consumption response. Business cycle fluctuations should show that consumption is high when work hours are low and vice versa, which is the opposite of the true pattern

   c) Use the logic of the first order condition to discuss whether you would expect the addition of uncertainty about future consumption to change the implications of the frictionless model about the relationship between movements in consumption and equilibrium labor supply.

Answer:

The two first order conditions are:

$$\frac{\mathrm{u}'(z_t)}{\mathrm{u}'(c_t)} = \mathsf{W}_t \tag{2}$$

$$\mathrm{u}'(c_t) = \beta\,\mathbb{E}_t[\mathbf{R}_{t+1}\mathrm{u}'(c_{t+1})] \tag{3}$$

Introducing uncertainty about future consumption does not change any part of either equation other than adding an expectation sign to the second. You might imagine two effects of uncertainty: A direct (precautionary) effect, and an effect that comes through any consequence uncertainty might have for interest rates. The direct effect implies that, for precautionary reasons, consumption would fall and leisure would fall – which makes sense for an individual, whose fears might make them both cut back on their spending and want to work extra hours, but which does not make sense in the aggregate, because periods of falling consumption tend to be periods of falling employment (and therefore rising leisure). The effects through interest rates are the same as in the model without uncertainty.

2. **Capital Market Imperfections and the Fed.** Over the period 2007-2008, the Federal Reserve took several unusual actions in response to developments in the capital markets, including orchestrating the takeover of Bear Sterns by JP Morgan, pledging to be a lender of last resort to investment banks, and joining with the Treasury in a plan for a government takeover of Fannie Mae and Freddie Mac if they should fail.

   In the model presented in class on capital market imperfections, the following condition was presented:

$$\gamma > 1 + \mathsf{r} + A(c, \mathsf{r}, W, \gamma) \tag{4}$$

   a) Explain this condition, and use that model to provide a variety of interpretations of either the reasons for the Fed's intervention or the reasons its actions might be expected to improve the functioning of capital markets.

      Answer:

      Subject to interpretation.

   b) Suppose the "right" diagnosis of the credit market disruptions is that it has been discovered that the cost of verification of financial contracts is higher than had been anticipated. Discuss what this model would predict about the consequences of such an increase in verification costs.

      Answer:

      Discussed in handout.

   c) Give an intuitive explanation for why a decrease in interest rates might not be an effective response to financial market problems caused by financial market imperfections.

Answer:

Effects of a reduction in r are discussed in handout. If the financial disruptions are caused by an increase in c, an increase in r is an imperfect fix.

3. **Lucas Random Walk**. In the Lucas (1978) asset pricing model (APM), does aggregate consumption follow a random walk? Explain why or why not. Explain the relationship of your answer to the consumption Euler equation.

   Answer:

   In the Lucas APM, the dynamics of aggregate consumption are required to match the dynamics of aggregate income. This means that if aggregate income is predictable, aggregate consumption is equally predictable. This is consistent with the consumption Euler equation because the predictable movements in consumption correspond exactly to observable movements in interest rates.

   So, consumption does not follow a random walk in the LAPM model.

   Mathematically, it remains true that

   $$\mathrm{u}'(C_t) \quad = \quad \beta\,\mathbb{E}_t[\mathbf{R}_{t+1}\mathrm{u}'(C_{t+1})] \tag{5}$$

   by the usual derivations. But the whole point of the LAPM is that it provides a way of *calculating* the $\mathbf{R}_{t+1}$ which will make current consumption balance with current income. The only value of $\mathbf{R}_{t+1}$ that is consistent with the model is the one that induces the representative consumer to want to consume an amount that exactly matches current income.

   To take a concrete example, suppose the weather in the coconut economy is bad this year, but is expected to be normal next year. In this case, the interest rate will be very high, because everybody wants to consume today and the only thing that can induce them not to do so is a high interest rate.

4. **A Small Open Economy With Costs of Adjustment.** Consider a perfect foresight Ramsey/Cass-Koopmans model with no technological progress and a constant population $L_t = 1 \; \forall \; t$. The social planner has a CRRA felicity function $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$ and maximizes geometrically discounted intertemporally separable utility. The economy is open to world capital markets, so that the social planner can borrow and lend risklessly according to the global interest factor R. Domestic labor can be combined with capital to produce output according to

$$\begin{aligned} \mathrm{F}(K, L) &= K^{\alpha}(AL)^{1-\alpha} &(6) \\ \mathrm{f}(k) &= k^{\alpha} &(7) \end{aligned}$$

and domestic labor and capital markets are perfectly competitive where z is the level of productivity per unit of labor.

However, capital is subject to costs of adjustment of the usual q-model kind (convex in $(i - \delta k)/k$).

a) Construct a phase diagram for the "corporate" sector of this economy showing the relationship between q and k. (Note that since there are no taxes, q and the marginal value of shares λ are the same). Are there any substantive differences between the q diagram for this small open economy and the q diagram for a firm that we studied in class (aside from our setting of taxes to zero here)?

Answer:

This is identical to the q diagram for the standard q model.

b) Suppose that leading up to period t the country is in steady state, but at date t there is a sudden brief unanticipated war in which 10 percent of the capital stock is destroyed. Show the dynamics of k, c, and i leading up to and subsequent to date t.

Answer:

This is identical to the experiment of the destruction of 10 percent of the firm's capital stock in the entrepreneur's handout. See that handout for the answer.

c) Suppose that leading up to period t the country is in steady state, but at date t an international criminal gang (somehow) steals 10 percent of the shares of the firms who own the aggregate capital stock (that is, the value of the theft is the same as the value of the destruction of capital in the previous question). Show the dynamics of k, c, and i leading up to and subsequent to this theft. Explain the resons for any similarities and differences in the results of this question and the previous one.

Answer:

There is no effect whatsoever on the country's capital stock or investment; those are determined solely by the profit-maximizing behavior of the firms, and the capital stock was already at the profit-maximizing level. The theft of financial assets means that the dividends that would previously have been paid to domestic owners will instead be paid to the international criminal gang (because they own the shares). So, the residents are poorer by the amount of the lost dividends, but the assumption of efficient capital markets means that there are no effects on the capital stock or investment.

Medium-Length Questions

1. **Growth In An Economy With Savers and Spenders (Mankiw (2000)).** Consider an economy with two kinds of consumers: "Keynesian" consumers set their c equal to their current labor income and own no capital; they constitute proportion $\wp$ of the population, while "Ramsey" consumers solve a traditional perfect foresight dynamic optimizing problem to determine their c and constitute the remaining $\not\wp = 1 - \wp$ of the population L. There is no population growth or productivity growth, and the aggregate production function is Cobb-Douglas in labor and capital, $K^{\alpha}L^{1-\alpha}$; both capital and labor markets are perfect implying that interest rates and wages are given by

$$\begin{aligned} \mathsf{r} &= \alpha K^{\alpha-1}L^{1-\alpha} \\ &= \alpha k^{\alpha-1} \\ &= \mathrm{f}'(k) \\ \mathsf{w} &= (1-\alpha)K^{\alpha}L^{-\alpha} \\ &= (1-\alpha)k^{\alpha} \end{aligned}$$

where as usual we define capital per capita as $k = K/\mathrm{L} = (K_K + K_R)/(\mathrm{L}_K + \mathrm{L}_R) = K_R/((1+\wp-\wp)\mathrm{L}) = K_R/\mathrm{L}$, where, e.g., $K_K = 0$ because since individual Keynesian consumers are assumed to own zero capital, the aggregate capital owned by Keynesian consumers must also be zero. Assume CRRA utility $u(C) = C^{1-\rho}/(1-\rho)$ for all consumers.

Thus the continuous-time maximization problem for the Ramsey consumers is

$$\max \quad \int_t^{\infty} \mathrm{u}(C_R)e^{-\vartheta t}dt \tag{8}$$

$$\dot{K}_R = (\mathrm{f}'(k) - \delta)K_R + \mathsf{w}\mathrm{L}_R - C_R \tag{9}$$

where δ is the depreciation rate and ϑ is the time preference rate.

a) Show that the steady-state capital stock in this model is the same as in an economy entirely populated by 'Ramsey' consumers. Discuss the plausibility of this result if the proportion of the population that is Ramsey is very small.

Answer:

Many students proceed by trying to rework the intertemporal budget constraint, as follows. Since all capital is owned by the Ramsey consumers, we can substitute $K = K_R$ in the dynamic budget constraint; dividing by L, it becomes

$$\begin{aligned} \dot{K}_R/\mathrm{L} &= (\mathrm{f}'(k) - \delta)k + \mathsf{w}(1-\wp) - (C_R/\mathrm{L}_R)(\mathrm{L}_R/\mathrm{L}) \qquad (10) \\ (\dot{K}_R/\mathrm{L}_R)(\mathrm{L}_R/\mathrm{L}) &= (\mathrm{f}'(k) - \delta)k + \mathsf{w}(1-\wp) - c_R(1-\wp) \\ (1-\wp)\dot{k}_R &= (\mathrm{f}'(k) - \delta)k + (\mathsf{w} - c_R)(1-\wp) \end{aligned}$$

This is all true, but not relevant: The equilibrium outcome will be determined by the point at which $\dot{c}_R/c_R = 0$, just as it is in the regular Ramsey model.

The first order condition from the optimization problem yields a standard c Euler equation:

$$\frac{\dot{c_R}}{c_R} = \rho^{-1}(\mathrm{f}'(k) - \delta - \vartheta) \tag{11}$$

and the solution to this equation will clearly come at the point where $\mathrm{f}'(k) = \delta + \vartheta$. Since this equation does not depend on the proportion of the population that is Ramsey, the steady-state capital stock is the same regardless of what proportion of the population consists of Ramsey consumers.

A famous paper by Uzawa (1968) first made the point that if an economy contains multiple infinite-horizon consumers with different time preference rates, then in steady-state the entire capital stock will belong to the most-patient agent. The result for this problem can be interepreted as an example of an Uzawa economy in which the Keynesian consumers are simply interpreted as more impatient than the Ramsey consumers.

Of course, this result leans very heavily on the assumption that the Ramsey consumers are infinitely lived. If there is only one Ramsey consumer and everyone else is Keynesian, it might take millions of years for the capital stock to reach the Ramsey predicted level.

I can't resist quoting an alternative answer provided on one of the exams: "If the Ramsey population is very small, then they would receive large profits from their capital. This formulation is implausible as we all know that if there were a small number of owners of capital exploiting the working class, eventually there would be a revolution and everyone would share the capital and be very happy. The capitalists should really try stock options to prevent such needless bloodshed." I gave the author full credit, on the grounds that he is merely repeating the point I made about the "rule of law."

Now assume that the population is divided half-and-half between Ramsey and Keynesian consumers: $\mathrm{L}_R = \mathrm{L}_K = (1/2)$.

b) Calculate three measures of inequality for this economy: the fraction of total labor income going to the 'poor' (the Keynesian consumers), their fraction of aggregate wealth, and their fraction of aggregate income. (Assume that capital's share in GDP is $\alpha = 1/3$.) How does the ranking of these three measures of inequality compare with the ranking in real economies?

Answer:

Since labor is supplied inelastically by both poor and rich, they earn exactly the same labor income so the ratio $\mathsf{w}\mathrm{L}_K/(\mathsf{w}\mathrm{L}_R + \mathsf{w}\mathrm{L}_K) = 1/2$.

In steady-state, the Ramsey consumers own the entire capital stock and the Keynesian consumers own no capital, so the ratio of wealth of the poor to aggregate wealth is zero.

The total income of the Keynesian consumers is their labor income. The total income of the Ramsey consumers is their labor income plus the entire amount of aggregate capital income.

$$
\begin{aligned}
\frac{y_K}{y_K + y_R} &= \frac{\mathsf{w}\mathrm{L}_K}{\mathsf{w}(\mathrm{L}_K + \mathrm{L}_R) + \alpha k^\alpha} && (12)\\
&= \frac{(1-\alpha)k^\alpha \mathrm{L}_K}{(1-\alpha)k^\alpha(\mathrm{L}_K + \mathrm{L}_R) + \alpha k^\alpha} && (13)\\
&= \frac{(1-\alpha)\mathrm{L}_K}{(1-\alpha)(\mathrm{L}_K + \mathrm{L}_R) + \alpha} && (14)\\
&= \frac{(1-\alpha)/2}{(1-\alpha) + \alpha} && (15)\\
&= \frac{(1-\alpha)}{2} && (16)\\
&= \frac{2/3}{2} && (17)\\
&= (1/3) && (18)
\end{aligned}
$$

Thus, the greatest inequality is in wealth, followed by total income, with the least inequality in labor income. This is the same ranking as prevails in real economies.

c) Discuss whether a model like this is consistent with the evidence that aggregate c growth exhibits excess sensitivity to lagged information.

Answer:

This is just a version of the Campbell-Mankiw model, which was originally proposed precisely in order to explain excess sensitivity. Thus the model is certainly consistent with the evidence on excess sensitivity.

d) Consider an economy like this one which has reached the steady-state level of the capital stock. Suppose that the preferences of the Keynesian consumers are identical to the preferences of the Ramsey consumers and the only reason the Keynesian consumers have always set $c_K = y_K$ is because they did not have access to financial markets so that they were unable to save. Discuss what would happen in this economy if the Keynseian consumers were suddenly granted access to financial markets. Discuss in particular the dynamics of inequality.

Answer:

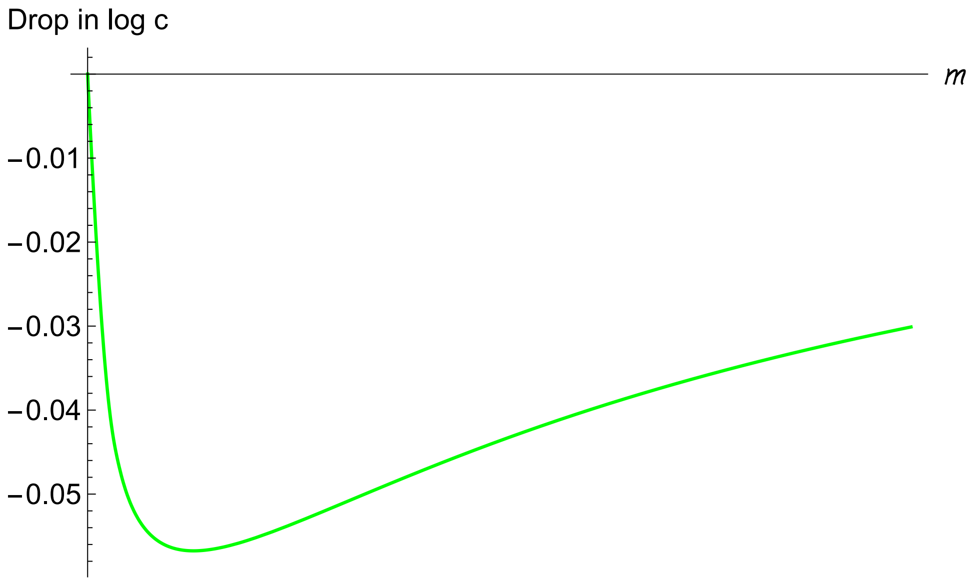
Rather than blindly setting $c_K = y_K$, the Keynesian consumers now will obey the optimality conditions for c,

$$\frac{\dot{c}_K}{c_K} = \rho^{-1}(\mathrm{f}'(k) - \delta - \vartheta) \tag{19}$$

which is the same as the optimality condition for the Ramsey consumers. But we assumed that the economy had reached the steady-state capital stock, so that $\mathrm{f}'(k) = \delta + \vartheta$. Thus the Keynesian consumers will have optimal c growth of zero, and so they will *choose* to continue setting c equal to income even though they are now unconstrained! Thus, the inequality that came about during the period when the Keynesian consumers were constrained will be perpetuated forever after, because steady-state interest rates must be at exactly the right level to make everybody want to spend exactly their income.

2. **Whose Consumption Falls Most When Uncertainty Increases?**

   In the model of TractableBufferStock, when there is an increase in the degree of uncertainty $\mho$, every employed consumer's consumption will fall. But the same increase in $\mho$ will affect people at different m's differently. The figure below shows the immediate change in log c that results from an unexpected doubling of the $\mho$ parameter from its baseline.



   Characterizing the behavior of 'the poor' as being the limiting value as $m \downarrow 0$, 'the rich' as the limit as $m \uparrow \infty$, and 'the middle' as the people at points substantially away from either limit (for example, at the target level of wealth), answer the following.

   a) To what limit does the drop in consumption go as $m \uparrow \infty$? Why?

      Answer:

      > The limiting behavior of the rich is the solution to the perfect foresight problem, both before and after the increase in $\mho$. So in the limit, the effect of increasing labor income uncertainty for 'the rich' is zero.

   b) Explain why the drop in c gets very small as $m \downarrow 0$. Hint: an appendix to TractableBufferStock shows that

$$\lim_{m \downarrow 0} \kappa^e = \frac{\mho\varphi}{1+\mho\varphi} \tag{20}$$

      for some positive constant φ likely to be not too far from 1.

      Answer:

      > The limiting behavior of the poor is close to the spend-everything 45 degree line both before and after the increase in $\mho$ so at any given

level of m the value of c is just a bit less than m. It drops a small amount in response to the increase in uncertainty, but not very much.

c) Explain intuitively (in words) why consumption of people in 'the middle' is the place to expect the largest precautionary effect.

Answer:

The limiting behavior of the poor is close to the spend-everything 45 degree line both before and after the increase in $\mho$ so at any given level of m the value of c is just a bit less than m. It drops a small amount in response to the increase in uncertainty, but not very much.

The 'middle' people whose buffer stock is at its target, are holding their buffer stock only for precautionary reasons. (They satisfy the growth impatience condition so in the absence of the precautionary motive their m would asymptote to zero).

d) In light of these results, explain the circumstances in which a Campbell-Mankiw model would or would not provide an adequate approximation to the behavior of the economy over the business cycle, if the true model is the tractable buffer stock model.

Answer:

The Campbell-Mankiw model will certainly not be adequate if one of the things that happens over the business cycle is systematic movements in uncertainty, since that model cannot capture any effect of uncertainty. The only circumstances in which the model would be completely correct would be if the entire population were divided into 'the poor' with almost exactly zero wealth, and 'the rich' with so much wealth that any precautionary effects might be negligible.

References

Lucas, Robert E. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429–1445, Available at http://www.jstor.org/stable/1913837.

Mankiw, N. Gregory (2000): "The Savers-Spenders Theory of Fiscal Policy," in *American Economic Review Papers and Proceedings*.

Uzawa, Hirofumi (1968): "Time Preference, the Consumption Function, and Optimum Asset Holdings," in *Value, Capital, and Growth: Papers in Honor of Sir John Hicks*, ed. by J. N. Wolfe, pp. 485–504. Edinborough University Press, Chicago.