Name _____

Final Exam

Intertemporal Choice

Fall, 2024

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Short Questions.

1. **Labor Supply and Consumption.** In a Real Business Cycle model where labor supply and consumption c are both chosen freely subject to a budget constraint, define z as the proportion of time spent in leisure activities (that is, not working) and W, r, and ϑ as the wage, interest, and time preference rates.

   a) Assuming that the 'hat' operator is equivalent to a difference in logs (e.g., $\hat{\bullet}_{t+1} \equiv \log \bullet_{t+1} - \log \bullet_t$), explain the intuition behind the result that

   $$\hat{z}_{t+1} \approx -\hat{\mathsf{W}}_{t+1} + (\mathsf{r}_{t+1} - \vartheta) \tag{1}$$

   when preferences satisfy the "balanced growth" condition in a perfect foresight model and utility is logarithmic.

   b) Discuss the relationship between this result and empirical evidence under two hypotheses about the reasons for employment fluctuations over the business cycle:

      i. They are driven mostly by temporary fluctuations in wages

      ii. They are driven mostly by optimizing responses to interest rate shocks

c) Use the logic of the first order condition to discuss whether you would expect the addition of uncertainty about future consumption to change the implications of the frictionless model about the relationship between movements in consumption and equilibrium labor supply.

Capital Market Imperfections and the Fed. Over the period 2007-2008, the Federal Reserve took several unusual actions in response to developments in the capital markets, including orchestrating the takeover of Bear Sterns by JP Morgan, pledging to be a lender of last resort to investment banks, and joining with the Treasury in a plan for a government takeover of Fannie Mae and Freddie Mac if they should fail.

In the model presented in class on capital market imperfections, the following condition was presented:

$$\gamma > 1 + \mathsf{r} + A(c, \mathsf{r}, W, \gamma) \tag{2}$$

2. a) Explain this condition, and use that model to provide a variety of interpretations of either the reasons for the Fed's intervention or the reasons its actions might be expected to improve the functioning of capital markets.

b) Suppose the "right" diagnosis of the credit market disruptions is that it has been discovered that the cost of verification of financial contracts is higher than had been anticipated. Discuss what this model would predict about the consequences of such an increase in verification costs.

c) Give an intuitive explanation for why a decrease in interest rates might not be an effective response to financial market problems caused by financial market imperfections.

3. **Lucas Random Walk**. In the Lucas (1978) asset pricing model (APM), does aggregate consumption follow a random walk? Explain why or why not. Explain the relationship of your answer to the consumption Euler equation.

4. **A Small Open Economy With Costs of Adjustment.** Consider a perfect foresight Ramsey/Cass-Koopmans model with no technological progress and a constant population $L_t = 1 \ \forall \ t$. The social planner has a CRRA felicity function $\mathrm{u}(\bullet) = \bullet^{1-\rho}/(1-\rho)$ and maximizes geometrically discounted intertemporally separable utility. The economy is open to world capital markets, so that the social planner can borrow and lend risklessly according to the global interest factor R. Domestic labor can be combined with capital to produce output according to

$$\mathrm{F}(K, L) = K^{\alpha}(AL)^{1-\alpha} \tag{3}$$

$$\mathrm{f}(k) = k^{\alpha} \tag{4}$$

and domestic labor and capital markets are perfectly competitive where z is the level of productivity per unit of labor.

However, capital is subject to costs of adjustment of the usual q-model kind (convex in $(i - \delta k)/k$).

a) Construct a phase diagram for the "corporate" sector of this economy showing the relationship between q and k. (Note that since there are no taxes, q and the marginal value of shares λ are the same). Are there any substantive differences between the q diagram for this small open economy and the q diagram for a firm that we studied in class (aside from our setting of taxes to zero here)?

b) Suppose that leading up to period t the country is in steady state, but at date t there is a sudden brief unanticipated war in which 10 percent of the capital stock is destroyed. Show the dynamics of k, c, and i leading up to and subsequent to date t.

c) Suppose that leading up to period t the country is in steady state, but at date t an international criminal gang (somehow) steals 10 percent of the shares of the firms who own the aggregate capital stock (that is, the value of the theft is the same as the value of the destruction of capital in the previous question). Show the dynamics of k, c, and i leading up to and subsequent to this theft. Explain the resons for any similarities and differences in the results of this question and the previous one.

Medium-Length Questions

1. **Growth In An Economy With Savers and Spenders (Mankiw (2000)).** Consider an economy with two kinds of consumers: "Keynesian" consumers set their c equal to their current labor income and own no capital; they constitute proportion $\wp$ of the population, while "Ramsey" consumers solve a traditional perfect foresight dynamic optimizing problem to determine their c and constitute the remaining $\not\wp = 1 - \wp$ of the population L. There is no population growth or productivity growth, and the aggregate production function is Cobb-Douglas in labor and capital, $K^\alpha L^{1-\alpha}$; both capital and labor markets are perfect implying that interest rates and wages are given by

$$\begin{aligned} \mathsf{r} &= \alpha K^{\alpha-1} L^{1-\alpha} \\ &= \alpha k^{\alpha-1} \\ &= \mathrm{f}'(k) \\ \mathsf{w} &= (1-\alpha) K^\alpha L^{-\alpha} \\ &= (1-\alpha) k^\alpha \end{aligned}$$

where as usual we define capital per capita as $k = K/\mathrm{L} = (K_K + K_R)/(\mathrm{L}_K + \mathrm{L}_R) = K_R/((1+\wp-\wp)\mathrm{L}) = K_R/\mathrm{L}$, where, e.g., $K_K = 0$ because since individual Keynesian consumers are assumed to own zero capital, the aggregate capital owned by Keynesian consumers must also be zero. Assume CRRA utility $u(C) = C^{1-\rho}/(1-\rho)$ for all consumers.

Thus the continuous-time maximization problem for the Ramsey consumers is

$$\max \qquad \int_t^\infty \mathrm{u}(C_R) e^{-\vartheta t} dt \tag{5}$$

$$\dot{K}_R = (\mathrm{f}'(k) - \delta) K_R + \mathsf{w}\mathrm{L}_R - C_R \tag{6}$$

where δ is the depreciation rate and ϑ is the time preference rate.

a) Show that the steady-state capital stock in this model is the same as in an economy entirely populated by 'Ramsey' consumers. Discuss the plausibility of this result if the proportion of the population that is Ramsey is very small.

(space to continue your answer)

Now assume that the population is divided half-and-half between Ramsey and Keynesian consumers: $L_R = L_K = (1/2)$.

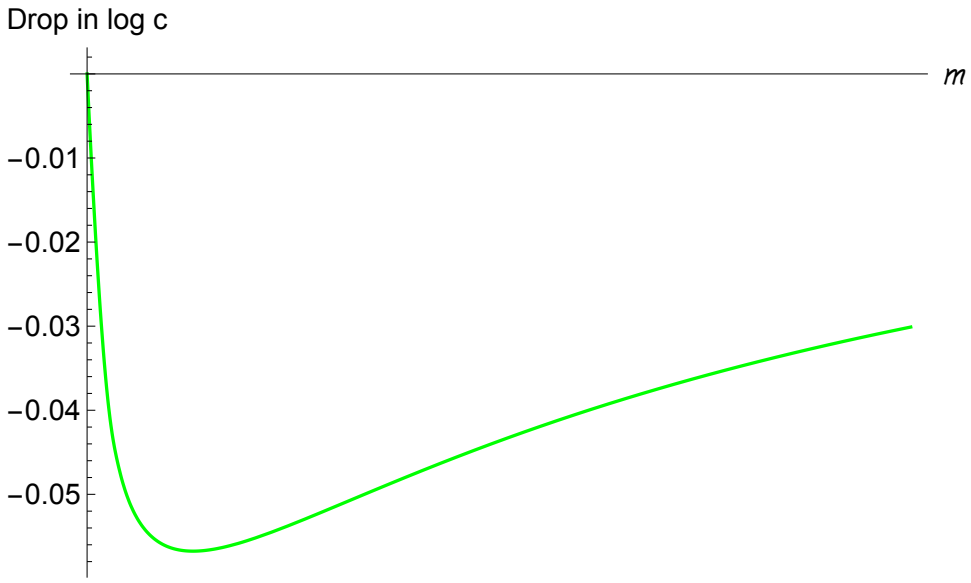
b) Calculate three measures of inequality for this economy: the fraction of total labor income going to the 'poor' (the Keynesian consumers), their fraction of aggregate wealth, and their fraction of aggregate income. (Assume that capital's share in GDP is $\alpha = 1/3$.) How does the ranking of these three measures of inequality compare with the ranking in real economies?

c) Discuss whether a model like this is consistent with the evidence that aggregate c growth exhibits excess sensitivity to lagged information.

d) Consider an economy like this one which has reached the steady-state level of the capital stock. Suppose that the preferences of the Keynesian consumers are identical to the preferences of the Ramsey consumers and the only reason the Keynesian consumers have always set $c_K = y_K$ is because they did not have access to financial markets so that they were unable to save. Discuss what would happen in this economy if the Keynseian consumers were suddenly granted access to financial markets. Discuss in particular the dynamics of inequality.

2. **Whose Consumption Falls Most When Uncertainty Increases?**

In the model of `TractableBufferStock`, when there is an increase in the degree of uncertainty $\mho$, every employed consumer's consumption will fall. But the same increase in $\mho$ will affect people at different m's differently. The figure below shows the immediate change in log c that results from an unexpected doubling of the $\mho$ parameter from its baseline.



Characterizing the behavior of 'the poor' as being the limiting value as $m \downarrow 0$, 'the rich' as the limit as $m \uparrow \infty$, and 'the middle' as the people at points substantially away from either limit (for example, at the target level of wealth), answer the following.

a) To what limit does the drop in consumption go as $m \uparrow \infty$? Why?

b) Explain why the drop in c gets very small as $m \downarrow 0$. Hint: an appendix to `TractableBufferStock` shows that

$$\lim_{m \downarrow 0} \kappa^{e} \quad = \quad \frac{\mho \varphi}{1 + \mho \varphi} \tag{7}$$

for some positive constant φ likely to be not too far from 1.

c) Explain intuitively (in words) why consumption of people in 'the middle' is the place to expect the largest precautionary effect.

d) In light of these results, explain the circumstances in which a Campbell-Mankiw model would or would not provide an adequate approximation to the behavior of the economy over the business cycle, if the true model is the tractable buffer stock model.

References

Robert E. Lucas. Asset prices in an exchange economy. *Econometrica*, 46:1429–1445, December 1978. URL `http://www.jstor.org/stable/1913837`. Available at `http://www.jstor.org/stable/1913837`.

N. Gregory Mankiw. The savers-spenders theory of fiscal policy. In *American Economic Review Papers and Proceedings*, 2000.