

Final Exam
Intertemporal Choice
Fall, 2021

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

1. **A Small Open Economy ϕ Model.** The handout **EntrepreneurPF** combines results from **PerfForesightCRRA** and **qModel** to produce a model of the behavior of an entrepreneur who owns a business whose investments incur costs of adjustment. You should be able to skim the handout to get the gist of how the model works (since it is basically a mashup of two frameworks that you are already familiar with). Use the model in the handout to answer the following questions, under the following assumptions:

- We are now reinterpreting the model to describe a small open economy run by a benevolent social planner, rather than a firm run by an entrepreneur
 - Leading up to period t , the economy was in equilibrium with a stable and positive amount of monetary resources \bar{m} and a constant (equilibrium) capital stock \bar{k} .
 - For all three of the experiments below, show the dynamics of consumption, investment, national net worth, and the aggregate capital stock following the shock at period t , and explain why the results look the way they do.
- a) Suppose the country experiences a war that suddenly destroys 10 percent of its physical capital stock, so that $k_t = 0.9\bar{k}$.
- b) Suppose that year t was 2008 and the country had accumulated a substantial amount of monetary assets so that $a_{t-1} \gg 0$, but those assets happened to be deposited in Lehmann Brothers right at the time it collapsed. The value of those assets therefore falls to zero.
- c) Suppose now that what happens at date t is that another country starts producing the same goods that this country has produced before, and the consequence is that this country's production function for the firm suddenly, permanently, and unexpectedly drops; specifically, leading up to period t the firm was in steady state, but in periods $t + 1$ and beyond the production function will be $f_{\geq}(k) = \Psi f_{<}(k)$ for some $\Psi < 1$ where $f_{<}$ and f_{\geq} indicate the production functions before and after the increase in productivity.

2. **Open Source Software and Knowledge Capital.** “Open Source” software is created largely by computer programmers who are not directly paid for their contributions. Some of the most central components of the infrastructure of the internet are open-source (for example, the Apache web server software that powers perhaps half of the world’s webservers; the FireFox browser; the LaTeX typographical system; and much more).

Explaining the production of valuable services by volunteer labor is a challenge for neoclassical economics (to say the least!). This question asks you to make a first stab at the job, using the Lucas (1988) growth model with an aggregate production function

$$Y = AK^\alpha(\ell hL)^{1-\alpha} \quad (1)$$

Recall that in that model, “human capital” h accumulates according to

$$\dot{h}/h = \phi(1 - \ell) \quad (2)$$

where ℓ is the proportion of time that people spend working.

- a) Write a paragraph or two about reasons for and against a reinterpretation of Lucas’s model in which what he calls “human capital” is thought of as “knowledge capital” produced by people in their free-time contributions to open source knowledge projects. Comment, in particular, about what you need to assume about the parameter that captures the degree of externalities in aggregate h in order for this interpretation to have any force.
- b) Suppose a government concludes that the value of open source software in boosting aggregate productivity is very large. Under the appropriate assumptions (that is, the assumptions that make the Lucas model sketched above appropriate for addressing the question), suppose the government wants to evaluate a subsidy that will increase the leisure time of computer programmers (in the belief that in their leisure time they will create more open-source software).
 - Assume that only computer programmers contribute to knowledge capital in the relevant sense (everybody else just contributes raw labor). Also, programmers are “born that way” (they can’t help becoming programmers, and nobody else can become one) so that their proportion in the population remains constant at Π . Show that under these assumptions the effective aggregate production function becomes

$$y = \hat{A}k^\alpha(\ell h)^{1-\alpha} \quad (3)$$

where h is the knowledge capital per capita of the computer programmers and ℓ is the proportion of their hours they spend working.

- Explain why the optimal policy for the government is likely to involve subsidizing the programmers *not* to work. Discuss how would the ap-

appropriate subsidy depends (qualitatively) on the different parameters in the model (including the Cobb-Douglas parameter for capital's share in output).

- Now suppose that labor market imperfections mean that the company that a programmer works for is able to temporarily capture a substantial portion of the “knowledge” benefits of the programmer’s “free time” programming activities, but the amount that the company captures depends on the number of hours that the programmer spends working at the company (the ℓ). Will the company agree with the government about the optimal size of the subsidy? Explain why or why not, and discuss what determines whether the company is likely to benefit (on net) from the externalities mentioned above.
- Assuming that parameter values are such that the subsidy to leisure of programmers has a positive net value, would the model suggest that such a subsidy would have a positive effect on the level of output, on the growth rate of output, or both?
- How might such a subsidy compare, in its effects, to a more traditional policy like a permanent investment tax credit?
- Explain why the government’s optimization problem becomes much harder if some people can pretend to be computer programmers even though their only real talent is in English Literature and they hate programming and in their free time they read Shakespeare instead of coding the latest update to Firefox.

3. Beliefs, Preferences, and Choices.

(The very last question will be given the greatest weight in grading this section; you do not need to have successfully answered the earlier parts to provide a good answer to the last question)

Using data on the stock market investment choices of people in different generations, [Malmendier and Nagel \(2011\)](#) show that “Depression Babies” (DB’s, henceforth), interpreted (say) to include anyone between the ages of 5 and 25 during the Great Depression, tended to be less willing to allocate their savings to “risky” assets than people born at other times (non-DB’s, henceforth).

This question asks you to think about the implications of their research for the consequences of the Great Recession.

As a preliminary, recall that **CRRA-RateRisk** derived a formula according to which the proportion of total net worth a person should invest in risky assets is given by:

$$\varsigma = \left(\frac{\phi}{\rho\sigma_r^2} \right) \quad (4)$$

where ς is the proportion of the portfolio invested in the risky asset, ρ is the coefficient of relative risk aversion, ϕ is the magnitude of the risky return premium, and σ_r^2 is the expected variance of risky return.

- a) Explain the intuition for the role of each term of this equation.
- b) Household surveys have long asked respondents their expectations about the rate of the return to investments in the stock market (interpreted here as “the risky asset”). Suppose that those surveys did not show any difference between DB’s and non-DB’s in ϕ . But such surveys have historically not tried to directly measure either an individual’s risk aversion or perception of the market’s riskiness. Does the theory suggest that there is any way to use portfolio choices to distinguish between the following two ideas: (1) DB’s avoid risky assets because they have high risk aversion; (2) DB’s avoid risky assets because they perceive the stock market to be “riskier” than other (non-DB) people perceive it to be.
- c) Now suppose that individuals’ relative risk aversion coefficients could be estimated in some other way. For example, suppose research finds that non-smokers have risk aversion coefficients higher by about 2 than nonsmokers, holding all other observable characteristics constant (including age). If this fact were assumed to be true about DB’s as well, and the surveys of their portfolio choices also asked whether they were smokers, describe whether and how this new finding might be useful in distinguishing the two hypotheses above.
- d) In recent years (long after most of the DB’s have died), surveys have been developed in which respondents are asked separately about their expectations

of the mean return of stock market investments ϕ and the degree of uncertainty (which can be used to derive an estimate of each person's estimated σ_r^2 at each surveyed moment). Suppose that the data from these surveys show that people's perceptions of stock market uncertainty can be well captured by a regression equation like

$$\sigma_{i,t}^2 = \bar{\sigma}_t^2 + \epsilon_i \quad (5)$$

or in words, everybody has their own particular view σ_i^2 about the variance of the stock market, but those views tend to move up and down in synchrony with the changes in the views of other people. Furthermore, suppose this research shows that there is no systematic difference in the σ_i 's across people in different groups (millennials, Gen-X, Baby Boomers, etc).

That is, if we use $\mathbf{1}$ as an indicator variable for whether a person was a DB or not, ρ_{DB} for the risk aversion of DB number i is:

$$\rho_i = \rho + \mathbf{1}[\Delta\rho_{\text{DB}}] \quad (6)$$

How could you combine these points to go back and do new research using [Malmendier and Nagel \(2011\)](#)'s data to reach a conclusion about whether the DB's avoided stock market investments because they were more risk averse than other people (had a higher ρ) or because they had a higher average assessment of σ^2 ?

- e) Suppose that future research proves that:
- i. The effect of events like the Great Depression or Great Recession is to shift upward people's risk aversion
 - ii. Antidepressant drugs can return people's risk aversion to normal
- Then discuss whether the Great Recession is likely to have a "scarring effect" on the investment choices of people who lived through it.
- f) Suppose that research of the kind described above establishes compellingly that the differences between DB and non-DB generations are attributable to DB's having much higher beliefs about the riskiness of stock market investments. Discuss what implications this could have for
- The correct explanation for the Equity Premium Puzzle
 - Appropriate public policies

4. **Growth Empirics for Buffer Stock Economies.** Answer the following questions using a tractable buffer stock saving model like the one discussed in **TractableBufferStock**.

- a) Consider two economies A and B populated by buffer stock consumers. The economies and parameters are identical in every respect except that the growth rates of wages are respectively G^A and G^B , where $G^A < G^B$. Each economy is in its steady state. Suppose an economist studying the two economies cannot directly observe the growth rate parameter (though the consumers living in the economies can); but the economist can observe the average growth rate outcome for each economy, $\Delta \log Y_i$, and suppose that the reason $G^B > G^A$ is that economy B has a higher level of education. The economist therefore performs a regression of the form:

$$\Delta \log Y_i = \alpha_0 + \alpha_1 E_i \quad (7)$$

where $i \in A, B$ across the two economies and therefore can translate the measurable difference in educational attainment E into its implication for economic growth by constructing $\mathbb{E}[\Delta \log Y_i] = \hat{\alpha}_0 + \hat{\alpha}_1 E_i$ for each country. Suppose the economist then performs a Campbell-Mankiw type regression:

$$\Delta \log C_i = \mu_0 + \mu_1 \mathbb{E}[\Delta \log Y_i] \quad (8)$$

on the data from the two economies. What coefficient estimate μ_1 would the economist find? Why? Would the coefficient μ_1 reflect what Campbell and Mankiw interpreted it as reflecting? Why or why not? Relate your comments to the log-linearized Euler equation.

- b) Now suppose the economist has data on only one country, but that data covers a long span of time. In particular, suppose that the country went through a period of fast growth for many years during the first half of the data sample, and then a period of slow growth for the second half of the data sample (as happened in the post-World-War-2 history of most of Western Europe: a period of rapid growth from 1947-1974, followed by much slower growth thereafter). Assume that during the period of rapid income growth, everyone expected that rapid income growth to continue indefinitely; and when the economy shifts down to slower income growth, everyone perceives the slowdown immediately and adjusts their spending accordingly. Answer the following questions about this economy:
- i. Show what happens to the consumption growth diagram depicting the relation between m_t and expected consumption growth when the growth rate abruptly changes (e.g., when the economy permanently and unexpectedly shifts from rapid growth to slower growth). (Show both any curve shifts, and, using arrows, show the path of consumption growth as it evolves toward its new steady state).
 - ii. Draw diagrams showing the path of the level of c and of C in this economy following the change in growth rates. (Recall that c is the ratio of

consumption to permanent wage income, while C is the absolute level of per-capita consumption. For simplicity, you may wish to assume that the initial growth rate is zero, and the new growth rate is negative.).

- iii. Suppose, again, that the econometrician cannot directly observe the economy's regime change, but suppose there is a survey of consumers that shows households' expectations of income growth. Assume that the data begin in period $-T$ while the regime change happens at date 0; beginning in date $-T$ the economy was at its target ratio, and by n periods after the change the economy has mostly settled down to its new target ratio (the sample ends at period $+T$). Draw a diagram showing the path over time of consumers' expectations of income growth and the path of actual (not expected) consumption growth. Now suppose the econometrician does a regression of the form

$$\Delta \log C_{t+1} = \mu_0 + \mu_1 \mathbb{E}_t[\Delta \log Y_{t+1}]. \quad (9)$$

Discuss how the econometrician's results will depend on the sample period used for estimating this equation. In particular, consider all these possibilities:

- A. A sample that includes only the early and late parts of the history (say, dates $-T$ to $-T/2$ and $T/2$ to T) where $T/2 \gg n$
- B. A sample that includes only the middle parts of the history (say, dates $-n/2$ to $+n/2$), but excludes the date 0 when the regime change occurred.
- C. A sample that includes the entire history (except period 0) when that history is long $T \gg n$
- D. A sample that includes the entire history (except period 0) when that history is short $T < n$

References

- LUCAS, ROBERT E. (1988): “On the Mechanics of Economic Development,” *Journal of Monetary Economics*, 22, 3–42.
- MALMENDIER, ULRIKE, AND STEFAN NAGEL (2011): “Depression Babies: Do Macroeconomic Experiences Affect Risk Taking?,” *Quarterly Journal of Economics*, 126(373–416).