Saving, Interest Rates, and Growth. PerfForesightCRRA shows that in a perfect foresight infinite horizon CRRA-utility consumption model, the formula for consumption

$$c_t = (h_t \boldsymbol{p}_t + b_t)\kappa \tag{1}$$

implies that

$$\mathbf{c}_t \approx \overbrace{\mathbf{p}_t \left(1 - \text{þ}_\gamma/(\mathsf{r}-\gamma)\right)}^{\text{from human wealth}} + \overbrace{(\mathsf{r} - \rho^{-1}(\mathsf{r}-\vartheta))\mathbf{b}_t}^{\text{from nonhuman wealth}} \tag{2}$$

where

$$\begin{aligned} \gamma &= \text{Growth rate of permanent noncapital income} \\ \text{þ}_\gamma &= \rho^{-1}(\mathsf{r}-\vartheta) - \gamma \\ &\equiv \text{'growth impatience rate'} \end{aligned} \tag{3}$$

ExplainRIC

1. Explain, in words, why the 'return impatience condition' $\text{þ}_\mathsf{r} < 0$ needs to be imposed in order for the model to have a sensible solution. Explain, in words, why the 'finite human wealth' condition, $\gamma < \mathsf{r}$ needs to be imposed. Explain, in words, what imposing the 'growth impatience condition' $\text{þ}_\gamma < 0$ accomplishes and why it might be desirable to impose that condition in an infinite-horizon model. (Reminder: $\text{þ}_\mathsf{r} \equiv \rho^{-1}(\mathsf{r}-\vartheta) - \mathsf{r}$).

DeriveSavRate

2. Use (2) and the fact that the level of saving $\mathbf{s}$ can be defined as total income minus total consumption:

$$\mathbf{s}_t = \mathsf{r}a_{t-1} + \mathbf{p}_t - \mathbf{c}_t \tag{4}$$

to show that the ratio of saving to permanent labor income $\boldsymbol{p}_t$ is approximately

$$s_t \approx \text{þ}_\gamma/(\mathsf{r}-\gamma) + \rho^{-1}(\mathsf{r}-\vartheta)a_{t-1} \tag{5}$$

and that the ratio of saving to total income (the 'saving rate') is

$$\varsigma_t = \left(\frac{\text{þ}_\gamma/(\mathsf{r}-\gamma) + \rho^{-1}(\mathsf{r}-\vartheta)a_{t-1}}{1 + \mathsf{r}a_{t-1}}\right). \tag{6}$$

SavSens

3. Use the tools available at Econ-ARK/HARK to plot the relationship between the saving rate and assets under various combinations of parameter values, with the objective of exploring the sensitivity of saving to the model's calibration. (You probably will find it useful to start with the notebook PerfForesightCRRA-SavingRate).

4. Opinions about the long-term growth rate of income, γ, are widely divergent today. Some scholars (e.g., Robert Gordon (2012)) believe that productivity growth in the U.S. is likely to be slow, perhaps 1 percent, over the next 50 years, while others (e.g., Erik Brynjolfsson and McAfee (2014)) are considerably more optimistic, projecting little if any slowdown from the growth rate of 2.5 percent a year that has characterized the postwar period. Furthermore, beliefs about long-run growth have become considerably more pessimistic in the period since the Great Recession.

GrowthCastsDoubt

a) Explain why these facts cast doubt upon this model as a useful or reliable guide to understanding actual saving choices. Relate this point to the argument of Summers (1981) about the magnitude of the human wealth effect in perfect foresight models.

WhyHigher

b) Explain why the saving rate is higher in a `TractableBufferStock` model than in the perfect foresight model

PFvsBS

c) Using the same Jupyter notebook as before, compare Compare the degree of sensitivity of the saving rate to the growth rate of income in this model compared to the perfect foresight model. Give the intuition for any differences.

References

Brynjolfsson, Erik, and Andrew McAfee (2014): *The second machine age: work, progress, and prosperity in a time of brilliant technologies*. WW Norton & Company.

Gordon, Robert J. (2012): "Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds," Working Paper 18315, National Bureau of Economic Research.

Summers, Lawrence H. (1981): "Capital Taxation and Accumulation in a Life Cycle Growth Model," *American Economic Review*, 71(4), 533–544, http://www.jstor.org/stable/1806179.