Name ____________________

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Midterm Exam

Intertemporal Choice

Fall, 2021

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I: Short Questions

1. A recent literature stimulated by Bloom (2009) has argued that shocks to aggregate and idiosyncratic uncertainty are a major source of business cycle fluctuations. Explain, using both math and words, why this argument cannot be investigated using a model in which a representative consumer with quadratic utility makes optimal consumption choices.

   Answer:

   > A consumer with quadratic utility who makes optimal choices has no precautionary saving motive. Therefore an increase in aggregate uncertainty would have no effect. Consumers with quadratic utility DO exhibit risk aversion – but aggregate risk is uninsurable (because everybody is subject to it) so in a representative agent model there are no effects that come through the risk aversion channel. (In principle, if this were a heterogeneous agent model with an increase in idiosyncratic risk, there could be economic effects that would come through an increased appetite of consumers for consumption insurance).

2. **CARA Utility and Impatience Over Infinite Horizons**. For a consumer with Constant Absolute Risk Aversion utility,

$$\mathrm{u}(c) = -\alpha^{-1} e^{-\alpha c} \tag{1}$$

calculate the expected change in consumption if $\mathsf{R}\beta$ is not equal to 1. Discuss why the long-run implications of this result are disturbing for consumers for whom $\mathsf{R}\beta < 1$ in an infinite-horizon framework. (Hint: Where will consumption go?)

Answer:

CARA utility is $\mathrm{u}(c) = -\alpha^{-1} e^{-\alpha c}$ so $\mathrm{u}'(c) = e^{-\alpha c}$.

$$\begin{aligned}
\mathrm{u}'(c_t) &= \mathsf{R}\beta \mathrm{u}'(c_{t+1}) \\
e^{-\alpha c_t} &= \mathsf{R}\beta e^{-\alpha c_{t+1}} \\
-\alpha c_t &= (\mathsf{r} - \vartheta) - \alpha c_{t+1} \\
\Delta c_{t+1} &= (\mathsf{r} - \vartheta)/\alpha
\end{aligned}$$

The model says that the absolute change in the level of consumption is the same in every period, and is either a positive or a negative number. If the consumer is return impatient ($\vartheta > \mathsf{r}$) then this implies that from any starting point, the **level** of consumption will asymptote to negative infinity. What could that possibly mean? (It's bad enough just for consumption to become negative. Negative **infinity**? Really?)

3. **Dynamic Inefficiency and Japan's Woes.**

   Define dynamic inefficiency and explain why a plausible way of thinking about Japan's economic problems after 1990 is to argue that Japan's economy has been dynamically inefficient. Explain why the fact that Japan's capital/output ratio is not much higher than the U.S. capital/output ratio presents a challenge to the dynamic inefficiency interpretation of Japan's problems, and discuss how the question of dynamic inefficiency is connected not just to the level of aggregate capital but also to the efficiency with which financial markets allocate capital to productive uses.

   Answer:

   An economy is dynamically inefficient in steady-state if the net rate of return on a marginal unit of capital accumulation (after depreciation) is less than the population growth rate plus the rate of productivity growth. While both population growth and productivity growth have been low in Japan and are projected to remain low, the rate of return may have been even lower or even negative. Thus the theory of dynamic inefficiency would say that the situation of Japanese consumers could be improved if there were less saving.

   In the standard model, the interest rate equals the marginal product of capital, i.e. $\mathsf{r} = \mathrm{f}'(k)$. If we have $k_U = k_J$ (where U and J stand for U.S. and Japan), then the fact that $\mathsf{r}_U >> \mathsf{r}_J$ suggests that the technology for converting marginal capital into marginal output is *different* in the U.S. and in Japan. That is, it is incorrect to assume that $\mathrm{f}_U = \mathrm{f}_J$ because if they have the same $\bar{k} = k_U = k_J$ yet different values of $\mathrm{f}'(k)$, only be captured by assuming $\mathrm{f}'_u \neq \mathrm{f}'_j$ so $\mathrm{f}_u \neq \mathrm{f}_j$.

   Thus, the possible explanation of Japan's problems is that Japan's high saving has been inefficient not because it increased the capital stock to an excessive level, but rather because it was being used inefficiently. If Japanese capital had been used better, perhaps the economy would not have been dynamically inefficient.

   This perspective suggests that if financial market reforms could boost the rate of return to saving in Japan, that could raise the net rate of return and eliminate the dynamic inefficiency.

4. Consider a Diamond OLG model in which the economy has reached the steady state equilibrium with a constant level of population. Suppose that suddenly in period t there is a permanent increase in the level of population (perhaps as a result of a one-time inflow of immigrants). Show the dynamics of k in the model, and explain whether the surprise increase in the population of workers is good or bad for the old people in period t in this economy, and why.

   Answer:

   Similar experiments are examined in `OLGModel`. The permanent increase in the level of the population corresponds to one-time negative shock to the K/L ratio; adjustment then works its way back toward the same equilibrium k as before.

   The experiment is good for old people because they own the capital and if there are more workers to compete for the use of each unit of capital, then capital is a scarce commodity and will command a high price (thus the old people will earn higher capital income).

Part II: Medium Question

Social Security and Dynamic Inefficiency. Consumers born in period t in a small open economy solve the problem

$$\begin{aligned}\max_{\{c_{1,t}\}} \quad & \log c_{1,t} + \beta \log c_{2,t+1} \\ \text{s.t.} \quad & \\ y_{1,t} &= \mathsf{W}_t - z_{1,t} \\ c_{2,t+1} &= \mathsf{R}_{t+1}(y_{1,t} - c_{1,t}) - z_{2,t+1}\end{aligned}$$

where a consumer supplies 1 unit of labor when young earning wage rate W_t and $z_{1,t}$ reflects lump-sum taxes. Interest rates are set by global capital markets exogenously and fixed at $\mathsf{R}_t = \bar{\mathsf{R}} \; \forall \; t$, and wages are constant at $\mathsf{W}_t = \bar{\mathsf{W}}$. A Social Security system is run using a strictly "Pay As You Go" system: Benefits paid to the elderly (generation 2) in any period $t+n$ must be raised by taxation on the young generation in that period (generation 1):

$$Z_{2,t+n} = -Z_{1,t+n} \tag{2}$$

where upper-case variables reflect the total amounts paid and received by all persons of a given generation: $Z_{\bullet,t+n} = z_{\bullet,t+n} L_{t+n}$. That is, each member of the generation that is young at date $t+n$ pays $z_{1,t+n}$ in taxes, and the proceeds are evenly divided among the elderly.

The labor force (the population of young people) grows according to

$$L_{t+1} = \Xi_{t+1} L_t. \tag{3}$$

The PAYG system is not allowed to change the tax rate on the young unless there is a "surprise" that causes the system to become "unbalanced." What this means is that the system tells young people that they should expect a "return" of $z_{2,t+1}/z_{1,t}$ on Social Security "savings" that is based on an assumption that the future rate of population growth will be constant at some value Ξ, and tries to keep that rate of return stable across generations.

If a "surprise" occurs at date t, then it is possible to allow for changes to the current tax rate on the young generation, and to announce a different tax rate for next period's young generation, but no further changes in expected tax rates are allowed beyond that. In other words, $z_{1,t}$ and $z_{1,t+1}$ can be changed, but $z_{1,t+1+n} = z_{1,t+1} \; \forall \; n > 0$.

1. What return should young people in period $t-1$ expect?

   Answer:

   > As discussed in the lecture notes, the only possible return factor that a "Pay As You Go" social security system with constant per-capita taxes can produce is the product of the technological growth and population growth factors. With no productivity growth, the return factor in this economy will be Ξ.

2. Suppose that at date t there is a surprise: The projected rate of growth of the population permanently falls, causing a projected "imbalance" as of the beginning of period t (before $z_{1,t}$ has been set). Explain why this constitutes an "imbalance."

   Answer:

   > The system can only produce a return factor equal to the population growth factor. Therefore, with lower population growth, it cannot continue to pay the old rate of Ξ that it had promised.

3. Define the 'generational account' of the generation that is young at date $t+n$ as $z_{t+n} = z_{1,t+n} + \mathsf{R}^{-1} z_{2,t+n+1}$, and define the 'Social Security return' of that generation as $z_{2,t+n+1}/z_{1,t+n}$. Explain why the 'Social Security return' of the generation born at $t+1$ must decline when the population growth rate declines.

   Answer:

   > Because projected future changes in z_1 beyond period $t+1$ are prohibited, the generation born in $t+1$ will earn a 'Social Security return' equal to the new, lower population growth rate.

4. Using this framework, discuss the government's options for reacting to the population growth shock. Suppose there are three principles of 'fairness' that the government might adopt:

   a) Taxes should be (re)set such that, after the reset, every generation is expected to receive the same SS return as every other generation
   b) Taxes should be (re)set such that the generation young at time t will get the same rate of return it would have been projected to get in a projection made in period $t-1$
      - That is, the generation born at t should get the same return as the one born at $t-1$ even though population growth has changed
   c) Taxes should be (re)set so that the only generation that experiences a rate of return different from the 'sustainable' return expected when they were young is the generation that is young when the surprise occurs.

   Describe the tax policies that will achieve each of these objectives, and explain how they will affect the 'old' (people who were young in period $t-1$), the 'young' (people who are young in period t) and the 'unborn' (people who will be young in periods $t+1$ and thereafter. (Assume that the government is only allowed to break promises about 'Social Security return' when a surprise happens; assume further that the government is prohibited from adjusting Social Security taxes for any reason except to implement one of the rules above).

   Answer:

   > Adopt the following notation:
   >
   > - z_0 denotes the per-capita taxes that were paid by generations $t-1$, $t-2$, $t-3$,...

- z_1 denotes the per-capita taxes that will be paid by the generation born at t.
- z_2 denotes the per-capita taxes that will be paid by the generations born in $t+1$, $t+2$, $t+3$,...

The tax authorities decide on z_1 and z_2 to achieve each of the goals above. Here is how they could do it.

a) Since per-capita taxes will be constant after t, all the future generations will get the same return factor: the new population growth factor Ξ_{t+1}. Therefore, we must only worry about the generation born at t, whose return factor will be $z_2\Xi_{t+1}/z_1$. From this expression, it is clear that setting $z_1 = z_2$ will give them the same return factor as future generations.

   The return factor for the old (born at $t-1$) will be $z_1\Xi_t/z_0$, so they will benefit from the change if $z_1 > z_0$.

b) This principle requires that generation t gets a return factor of Ξ_t. For this to be the case, it must be true that

$$\frac{z_2\Xi_{t+1}}{z_1} = \Xi_t \quad \leftrightarrow \quad \frac{z_2}{z_1} = \frac{\Xi_t}{\Xi_{t+1}} < 1$$

   This tells us that the only way to give generation t the rate of return that they were promised is to allow them to pay lower taxes than future generations will. As before, whether the change will benefit or hurt the old generation depends on how z_1 compares with z_0.

c) The only generations that could possibly get returns different from 'sustainable' levels are those born in periods $t-1$ and t. Thus, this principle's only restriction is that the return for generation $t-1$ must be Ξ_t, which will be true only if $z_1 = z_0$.

   The effect of the surprise on generation t will depend on how z_1 compares to z_2.

5. Discuss the relationship between these choices and considerations of 'dynamic efficiency.' In particular, discuss how the level of world interest rates is related to the question of which responses to the population slowdown might affect the well-being of the different generations differently.

   Answer:

   If the economy is dynamically inefficient, the return that people get from the social security system (Ξ) is higher than the one they would get from their own private savings ($\bar{\mathsf{R}}$).

   People will benefit from the system as long as:

   - The economy is dynamically inefficient so that they get a higher return than they otherwise would.

- The forced savings (through taxes) are not higher than the level of savings they would have privately chosen.

Thus, if the economy is not dynamically inefficient, a lower level of taxes will always be better because it requires people to save less at the lower rate of return that social security gives them.

If the economy is dynamically inefficient, then we would have to compare the tax level with the amount that people would privately save at the social security return rate in order to know whether they are being hurt.

6. Briefly discuss whether these conclusions would change if the economy were a Diamond (1965) closed economy rather than a small open economy, and if so how and why.

   Answer:

   The difference would be that in an open economy private savings would affect the return factor R. Higher social security taxes will crowd out private savings, lower the level of aggregate capital, and increase R. This reduction of the aggregate stock of capital is a way in which a social security system can reduce the degree to which an economy is dynamically inefficient, if it is.

   This channel would have to be taken into account by the tax authority when it configures the social security system. If preferences and technology are such that the economy is dynamically inefficient, a large social security system would improve welfare. However, the new and lower Ξ increases the threshold for capital per-capita at which the economy becomes dynamically inefficient, and this might be an argument for a smaller social security system that allows the economy to operate with a higher level of capital. We would have to know more about the economy (for instance, whether it was dynamically inefficient to begin with) to give a definitive answer.

References

Bloom, Nicholas (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77(3), 623–685.

Diamond, Peter A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126–1150, http://www.jstor.org/stable/1809231.