

### Dynamics of Investment in Response to a Temporary ITC in the $\phi$ Model.

Answer the following questions using an [Abel \(1981\)](#)-[Hayashi \(1982\)](#)  $\phi$  model of investment.

You are expected to answer the questions not just quantitatively (e.g., with figures or numbers) but also conceptually. That is, you must explain, in intuitive terms, *why* the variables do what they do.

1. Leading up to date  $t$ , the economy is in steady state. At date  $t$ , the government unexpectedly introduces a permanent increase in the investment tax credit,  $\zeta \uparrow$ . Show the effects on a phase diagram and show dynamics of investment, capital, share prices, and  $\phi$  following the tax change. Explain why  $\lambda$ , the share price of the firm, *drops* when the ITC is implemented.

*Answer:*

See [qModel](#)-Figure 4 and the ITC discussion therein

2. Leading up to date  $t$ , the economy is in steady state. At date  $t$ , the government unexpectedly introduces a *temporary* increase in the investment tax credit,  $\zeta \uparrow$ . The low ITC will last for two years, and then the ITC will revert back to its normal level. Show the effects on a phase diagram and show dynamics of investment, capital, share prices, and  $\phi$ , and the capital stock under two scenarios: (1) costs of adjustment for the capital stock,  $\omega$ , are high; (2) costs of adjustment are low.

*Answer:*

The way to think about problems like this is, first, to figure out what the long-run phase diagrams will look like, then to figure out what the phase diagram will look like in the short run. A key point to realize is that the  $\Delta k$  phase diagram at any moment of time is a purely mechanical budget equation, so it is usually fairly easy to figure out what happens to the  $\Delta k$  locus. A second point is that the tax terms that enter the equations are always the current value of those taxes; any effects of future taxes have to come through their effects on  $\lambda$  or  $\phi$ .

Upon introduction of the temporary increase in ITC at date  $t$ ,  $\lambda$  jumps down by exactly the amount such that if it evolves according to the new equations of motion it will arrive back at the original saddle path at date  $t + 2$ , and thereafter,  $\lambda$  will move on the original saddle path to the old equilibrium level. The point here is that there can be no anticipated jump in  $\lambda$ . When costs of adjustment for the capital stock are low, the initial jump of  $\lambda$  is big; when costs of adjustment for the capital stock are high, the initial jump of  $\lambda$  is small.

Dynamics of  $\phi$ : At date  $t$ ,  $\phi$  jumps up (note that  $\phi = \lambda/\hat{p}$ , though  $\lambda$  jumps down,  $\hat{p}$  goes down because of the change in ITC). Between date  $t$  and date  $t + 2$ ,  $\phi$  moves in the same direction as  $\lambda$  (northeast). At date  $t + 2$ ,  $\phi$  jumps down to the original saddle path due to the ITC reversal.

Dynamics of  $i$ :  $i$  closely follows the dynamics of  $\varphi$  since  $i = (\iota(\varphi) + \delta)k = (\frac{\varphi-1}{\omega} + \delta)k$ . At date  $t$ ,  $i$  jumps up. Between date  $t$  and date  $t + 2$ ,  $i$  increases gradually. At date  $t + 2$ ,  $i$  jumps down below the initial level and gradually goes back to the initial level.

Dynamics of  $k$ :  $k$  increases between date  $t$  and date  $t + 2$  and then decreases to its original level.

3. Leading up to date  $t$ , the economy is in steady state, and an ITC of 20 percent has existed since the beginning of time. At date  $t$ , the government unexpectedly *announces* that in three years (that is, in year  $t + 5$ ), there will be a *permanent* increase in the investment tax credit,  $\zeta \uparrow$ . Show and explain the effects on a phase diagram and show dynamics of investment, capital, share prices, and  $\varphi$ , and the capital stock under two scenarios: (1) costs of adjustment for the capital stock,  $\omega$ , are high; (2) costs of adjustment are low. EXPLAIN your results

*Answer:*

See **qModel**.

Upon announcement of the permanent increase in ITC in two years at date  $t$ ,  $\lambda$  jumps down and then gravitates to the saddle path dictated by the new ITC. At date  $t + 2$ ,  $\lambda$  will be on the new saddle path when the permanent ITC increase is introduced. Afterwards,  $\lambda$  will move on the new saddle path to the new equilibrium level. The point here is again that there can be no anticipated jump in  $\lambda$ . When costs of adjustment for the capital stock are low, the initial jump of  $\lambda$  is big; when costs of adjustment for the capital stock are high, the initial jump of  $\lambda$  is small.

Dynamics of  $\varphi$ : At date  $t$ ,  $\varphi$  jumps down (note that  $\varphi = \lambda/\hat{p}$ , though  $\lambda$  jumps down, there is no change in  $\hat{p}$ ). Between date  $t$  and date  $t + 2$ ,  $\varphi$  moves in the same direction as  $\lambda$  (southwest). At date  $t + 2$ ,  $\varphi$  jumps up to the new saddle path dictated by the new ITC. Afterwards,  $\varphi$  gravitates to the new equilibrium level.

Dynamics of  $i$ :  $i$  closely follows the dynamics of  $\varphi$  since  $i = (\iota(\varphi) + \delta)k = (\frac{\varphi-1}{\omega} + \delta)k$ . At date  $t$ ,  $i$  jumps down. Between date  $t$  and date  $t + 2$ ,  $i$  decreases gradually. At date  $t + 2$ ,  $i$  jumps up and then gradually decreases to its new equilibrium level.

Dynamics of  $k$ :  $k$  decreases between date  $t$  and date  $t + 2$  and then increases to its new equilibrium level, which is higher than the original equilibrium level.

The point here is that if firms know that the price of capital will be much lower two years in the future, they will have a strong incentive to postpone purchases of capital until that point. However, there is an offsetting effect which is that they know that they will ultimately want to

have a higher capital stock and that means there is an incentive to have a higher capital stock now. Which of these two motives wins out (delay purchases until they are cheap; build capital now so adjustment costs will be lower when you start your future capital-building campaign). For the parameter values chosen for the example in **qModel1**, the former effect is larger than the latter. (My guess is that this is always true, but I'm not completely sure).

- Phase diagrams of  $\lambda_t$  and  $\varphi_t$  are the same as in question a).
- Dynamics if  $\omega$  is high enough so that early investment adjustment dominates:
  - for  $k_t$ , staying at the old steady state before  $t$ ; gradually increases from time  $t$  and asymptotes toward the new higher steady state level.
  - for  $i_t$ , staying at the old steady state level before  $t$ ; jumps up at time  $t$ , evolves to the northeast until time  $t+1$ ; between  $t+1$  and  $t+2$ , an upward jump; and after  $t+2$ , asymptotes downward toward the new steady state level which is higher than the old one.
  - for  $\varphi_t$ , staying at the old steady state level before  $t$ ; jumps up at time  $t$ , evolves to the northeast until time  $t+1$ ; between  $t+1$  and  $t+2$ , an upward jump; and after  $t+2$ , asymptotes downward toward the same equilibrium level one.
  - for  $\lambda_t$ , staying at the old steady state level before  $t$ , jumps up at time  $t$ , evolves to the northeast until time  $t+2$ , and thereafter asymptotes downward toward the new steady state level which is lower than old one.
- Dynamics if  $\omega$  is low enough so that there is no early investment adjustment: all variables stay at the old steady state level until time  $t+2$  when the ITC increase actually takes effect, then
  - $k_t$  gradually increases to higher new steady state level.
  - $i_t$  jumps up at  $t+2$ , then gradually asymptotes toward the new higher steady state level.
  - $\varphi_t$  jumps up at  $t+2$ , then gradually asymptotes toward the original equilibrium level one.
  - $\lambda_t$  jumps downward at  $t+2$ , then gradually decreases toward the lower new steady state level.

4. Explain why the logic of the examples you just went through helps understand why, whenever a member of Congress introduces a bill to increase the investment

tax credit, that bill is always ‘retroactive.’ That is, if the ITC change ever passes, it will apply to investments made during the period between the introduction of the bill and its passage into law.

*Answer:*

Even a member of Congress is smart enough to know that if a cut in the ITC is passed that will not take effect until some future date, firms will delay investment until the tax change comes into effect so they can take advantage of the tax credit. Since Members of Congress do not want to be accused of causing a collapse in corporate investment, they never pass a cut in the ITC that is only effective starting at some future date.

# References

- ABEL, ANDREW B. (1981): “A Dynamic Model of Investment and Capacity Utilization,” *Quarterly Journal of Economics*, 96(3), 379–403.
- HAYASHI, FUMIO (1982): “Tobin’s Marginal Q and Average Q: A Neo-classical Interpretation,” *Econometrica*, 50(1), 213–224, Available at <http://ideas.repec.org/p/nwu/cmsems/457.html>.