Midterm Exam

Intertemporal Choice

Fall, 2022

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I: Short Questions

1. Explain *and critique* the argument that the introduction of a Social Security system (a mandatory PAYG pension scheme) reduces the personal saving rate. (By "critique" I mean that you should give at least one well-thought-out *and well-explained* reason why the standard argument might be wrong.)

   Answer:

   The analysis for this question is in `SocSecAndKAccum`. Scores for the critique will be judgmentally assigned, but may include any of the following points:

   - The proposition that 'the introduction of Social Security reduces saving' is inaccurate because before the Social Security system existed there was effectively a private system that worked the same way: Elderly parents lived with their kids, which meant that there were intergenerational transfers (and expectations of such transfers) that had the same saving incentive effects that Social Security did. Thus the 'introduction' of Social Security may not have changed saving because it was just formalizing the existing arrangement
   - Maybe the main reason Social Security was introduced was that there was a lot of evidence that most people were *failing* to save for retirement even though there was NOT a Social Security system. If the previous argument (about an implicit SS system already existing) is ignored, this suggests that people are not forward looking in their saving behavior (they are 'myopic'), and the model assumes that they are perfectly foresighted.
   - In many countries, the financial system at the time the Social Security system is introduced is not sophsticated enough (or trustworthy enough) for the population to be willing to trust it to safely allow them to accumulate assets amounting to several times worth their income and then gradually draw down those assets in old age. (Think Argentina, or Greece, or many countries where the government has effectively confiscated financial assets from households via inflation, financial repression, or other means). For people in those countries, it might be foolish to save for your own retirement because the likelihood is that the government will get the money anyway.

   This does not exhaust the list of potential good answers. These are only some examples.

2. Consider a Diamond OLG model in which the economy has reached the steady state equilibrium with a constant level of population. Suppose that suddenly in period t there is a permanent increase in the level of population (perhaps as a result of a one-time inflow of immigrants). Show the dynamics of k in the model,

and explain whether the surprise increase in the population of workers is good or bad for the old people in period t in this economy, and why.

Answer:

Similar experiments are examined in `OLGModel`. The permanent increase in the level of the population corresponds to one-time negative shock to the K/L ratio; adjustment then works its way back toward the same equilibrium k as before.

The experiment is good for old people because they own the capital and if there are more workers to compete for the use of each unit of capital, then capital is a scarce commodity and will command a high price (thus the old people will earn higher capital income).

Precautionary Saving and Convex Marginal Utility.

Consider a consumer whose last period of life is T and who is trying to decide how much to save in period $T-1$. Suppose the interest factor and the time preference factor are $\mathsf{R} = \beta = 1$ and so consumer's dynamic budget constraint is

$$c_T = a_{T-1} + y_T. \tag{1}$$

and define an end-of-period value function as

$$\mathfrak{v}'_{T-1}(a_{T-1}) = \mathbb{E}_{T-1}[\mathrm{u}'(c_T)] \tag{2}$$

Assuming CRRA utility in periods T and $T-1$, draw a diagram that shows:

3. a) Marginal end-of-period value as a function of a_{T-1} if income is perfectly certain

   b) Marginal end-of-period value as a function of a_{T-1} if income is

$$y_T = \begin{cases} \epsilon & \text{with probability } 0.5 \\ -\epsilon & \text{with probability } 0.5 \end{cases} \tag{3}$$

   c) Draw $u'(m_{T-1} - a_{T-1})$ and explain why a_{T-1} increases as a result of either an increase in risk aversion or an increase in the size of uncertainty ϵ.

Answer:

This question essentially repeats the analysis presented in Carroll and Kimball (2007) discussed in class.

Part II: Medium Question

1. **Consumption with CARA Utility and Labor Income Risk (Caballero (1990)).**

   Consider an individual who lives for two periods, who is born with initial wealth m_1 and will receive a normally distributed uncertain income in the second period of life, $y_2 \sim \mathcal{N}(\bar{y}, \sigma_y^2)$. The individual maximizes:

   $$\mathrm{u}(c_1) + \mathbb{E}[\mathrm{u}(c_2)]$$

   subject to the Intertemporal Budget Constraint

   $$m_2 = m_1 - c_1 + y_2.$$

   Suppose that the consumer's utility function is of the Constant Absolute Risk Aversion form:

   $$\mathrm{u}(c) = -(1/\alpha)e^{-\alpha c}. \tag{4}$$

   a) Show that this utility function exhibits constant absolute risk aversion with risk aversion parameter α (absolute risk aversion is defined as $-\mathrm{u}''(c)/\mathrm{u}'(c)$).

      Answer:

      $$\begin{aligned} \mathrm{u}(c) &= -(1/\alpha)e^{-\alpha c} \\ \mathrm{u}'(c) &= e^{-\alpha c} \\ \mathrm{u}''(c) &= -\alpha e^{-\alpha c} \\ -\mathrm{u}''(c)/\mathrm{u}'(c) &= -\left[-\alpha e^{-\alpha c}/e^{-\alpha c}\right] \\ &= \alpha \end{aligned}$$

   b) Derive the first order condition linking first and second period consumption, and use it to derive an analytical expression for c_1.

      Hint: you will need to use Math Fact `ELogNorm`:

      > If from the viewpoint of period t the stochastic variable $\mathbf{R}_{t+1}$ is lognormally distributed with mean $\mathbf{r}$ and variance $\sigma_{\mathbf{r}}^2$, $\mathbf{r}_{t+1} \sim \mathcal{N}(\mathbf{r}, \sigma_{\mathbf{r}}^2)$, then
      >
      > $$\mathbb{E}_t[e^{\mathbf{r}_{t+1}}] = e^{\mathbf{r}+\sigma_{\mathbf{r}}^2/2} \tag{5}$$

      Answer:

      $$\begin{aligned} \mathrm{u}'(c_1) &= \mathbb{E}_1[\mathrm{u}'(c_2)] \\ e^{-\alpha c_1} &= \mathbb{E}_1[e^{-\alpha(m_1 - c_1 + y_2)}] \end{aligned}$$

$$e^{-2\alpha c_1} = e^{-\alpha m_1}\,\mathbb{E}_1[e^{-\alpha y_2}]$$

But ([ELogNorm]) implies that $\mathbb{E}_1[e^{-\alpha y_2}] = e^{-\alpha\,\mathbb{E}_1[y_2]+\alpha^2\sigma_y^2/2}$, so

$$\begin{aligned} e^{-\alpha 2c_1} &= e^{-\alpha m_1} e^{-\alpha\,\mathbb{E}_1[y_2]+\alpha^2\sigma_y^2/2} && (6)\\ -\alpha 2c_1 &= -\alpha m_1 - \alpha\bar{y} + \alpha^2\sigma_y^2/2 && (7)\\ c_1 &= \frac{1}{2}\left[m_1 + \bar{y} - \alpha\sigma^2/2\right] && (8) \end{aligned}$$

c) Now consider two consumers with different initial wealth but the same second period income and the same risk aversion parameter $\alpha = 2$. Homer's $m_1 = 20,000$ and Mr. Burns's $m_1 = 20,000,000$. Second period income (call it 'Social Security') is normally distributed with mean $\bar{y}_2 = 20,000$ and variance $\sigma_y^2 = 10,000$ for both. Calculate the levels of first-period consumption for Homer and Mr. Burns, and then calculate the effect on consumption for Homer and for Mr. Burns if the uncertainty associated with Social Security income increases to $\sigma_y^2 = 15,000$. Does this result seem plausible to you?

Answer:

Initial consumption plans are:

$$\begin{aligned} c_1^H &= \frac{1}{2}[20,000 + 20,000 - 10,000]\\ &= 10,000 + \frac{1}{2}(20,000 - 10,000)\\ &= 15,000\\ c_1^B &= \frac{1}{2}[20,000,000 + 20,000 - 10,000]\\ &= 10,000,000 + \frac{1}{2}(20,000 - 10,000)\\ &= 10,005,000 \end{aligned}$$

When uncertainty increases, the new consumption plans are:

$$\begin{aligned} c_1^H &= \frac{1}{2}[20,000 + 20,000 - 15,000]\\ &= 10,000 + 2,500\\ c_1^B &= \frac{1}{2}[20,000,000 + 20,000 - 15,000]\\ &= 10,000,000 + 2,500\\ &= 10,002,500 \end{aligned}$$

Thus, the dollar amount by which Homer and Mr. Burns change their consumption in response to the increased Social Security risk is exactly the same.

It seems more plausible that Homer would react more to the risk than Mr. Burns would, because the Social Security risk is a much larger proportion of Homer's lifetime resources.

References

Caballero, Ricardo J. (1990): "Consumption Puzzles and Precautionary Savings," *Journal of Monetary Economics*, 25, 113–136, http://ideas.repec.org/p/clu/wpaper/1988_05.html.

Carroll, Christopher D., and Miles S. Kimball (2007): "Precautionary Saving and Precautionary Wealth," *Palgrave Dictionary of Economics and Finance, 2nd Ed.*, https://www.econ2.jhu.edu/people/ccarroll/papers/PalgravePrecautionary.pdf.