Final Exam
Intertemporal Choice

Fall, 2021
Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

1. **A Small Open Economy ϱ Model.** The handout `EntrepreneurPF` combines results from `PerfForesightCRRA` and `qModel` to produce a model of the behavior of an entrepreneur who owns a business whose investments incur costs of adjustment. You should be able to skim the handout to get the gist of how the model works (since it is basically a mashup of two frameworks that you are already familiar with). Use the model in the handout to answer the following questions, under the following assumptions:

   - We are now reinterpreting the model to describe a small open economy run by a benevolent social planner, rather than a firm run by an entrepreneur

   - Leading up to period t, the economy was in equilibrium with a stable and positive amount of monetary resources $\bar{m}$ and a constant (equilibrium) capital stock $\bar{k}$.

   - For all three of the experiments below, show the dynamics of consumption, investment, national net worth, and the aggregate capital stock following the shock at period t, and explain why the results look the way they do.

   a) Suppose the country experiences a war that suddenly destroys 10 percent of its physical physical capital stock, so that $k_t = 0.9\bar{k}$.

   Answer:

   > The key insight about the perfect foresight entrepreneur's model is that Fisherian separation holds. So, any question that is about investment can be answered identically to the answer in a ϱ model, and any question about consumption reflects the answer that would apply in `PerfForesightCRRA`.

   > So, for investment and capital, the answer is identical to the experiment in which 10 percent of the entrepreneur's capital stock is destroyed in the handout.

   > For consumption the answer is that there is an instantaneous jump in c to the level consistent with the new PDV of profits, and consumption is projected never to change thereafter.

b) Suppose that year t was 2008 and the country had accumulated a substantial amount of monetary assets so that $a_{t-1} >> 0$, but those assets happened to be deposited in Lehmann Brothers right at the time it collapsed. The value of those assets therefore falls to zero.

Answer:

In this case, there is no impairment in the productivity of the physical assets, so this is equivalent to a wealth shock in `PerfForesightCRRA`: An immediate and permanent change in c.

c) Suppose now that what happens at date t is that another country starts producing the same goods that this country has produced before, and the consequence is that this country's production function for the firm suddenly, permanently, and unexpectedly drops; specifically, leading up to period t the firm was in steady state, but in periods $t + 1$ and beyond the production function will be $f_{\geq}(k) = \Psi f_{<}(k)$ for some $\Psi < 1$ where $f_{<}$ and $f_{\geq}$ indicate the production functions before and after the increase in productivity.

Answer:

In this case, the decline in productivity has effects on investment in the capital stock identical to those that transpire in `qModel` in response to a productivity shock.

This event reduces the PDV of profits from the entrepreneurial venture. The consequence is identical to the consequence of any decline in net worth (the sum of human and nonhuman wealth) in `PerfForesightCRRA`: An immediate downward jump in consumption followed by no further changes.

2. **Open Source Software and Knowledge Capital.** "Open Source" software is created largely by computer programmers who are not directly paid for their contributions. Some of the most central components of the infrastructure of the internet are open-source (for example, the Apache web server software that powers perhaps half of the world's webservers; the FireFox browser; the LaTeX typographical system; and much more).

Explaining the production of valuable services by volunteer labor is a challenge for neoclassical economics (to say the least!). This question asks you to make a first stab at the job, using the Lucas (1988) growth model with an aggregate production function

$$Y = AK^\alpha(\ell h L)^{1-\alpha} \tag{1}$$

Recall that in that model, "human capital" h accumulates according to

$$\dot{h}/h \quad = \quad \phi(1-\ell) \tag{2}$$

where ℓ is the proportion of time that people spend working.

a) Write a paragraph or two about reasons for and against a reinterpretation of Lucas's model in which what he calls "human capital" is thought of as "knowledge capital" produced by people in their free-time contributions to open source knowledge projects. Comment, in particular, about what you need to assume about the parameter that captures the degree of externalities in aggregate h in order for this interpretation to have any force.

Answer:

This is covered in the handout. The essential distinction is between human capital, which is embodied in an individual mind and hence cannot be accumulated forever without bound, and knowledge capital, which goes into things like textbooks and can be accumulated without bound. This carries over into whether we can interpret ℓ as a proportion of the population or a proportion of an individual's time.

b) Suppose a government concludes that the value of open source software in boosting aggregate productivity is very large. Under the appropriate assumptions (that is, the assumptions that make the Lucas model sketched above appropriate for addressing the question), suppose the government wants to evaluate a subsidy that will increase the leisure time of computer programmers (in the belief that in their leisure time they will create more open-source software).

- Assume that only computer programmers contribute to knowledge capital in the relevant sense (everybody else just contributes raw labor). Also,

programmers are "born that way" (they can't help becoming program-
mers, and nobody else can become one) so that their proportion in the
population remains constant at Π. Show that under these assumptions
the effective aggregate production function becomes

$$y = \hat{A}k^\alpha(\ell h)^{1-\alpha} \tag{3}$$

where h is the knowldege capital per capita of the computer programmers
and ℓ is the proportion of their hours they spend working.

Answer:

First note that if only a fraction $1 - \Pi$ of the population are
workers then the production function becomes

$$
\begin{aligned}
Y &= AK^\alpha(\ell h \{(1 - \Pi)L\})^{1-\alpha} \tag{4}\\
Y &= AK^\alpha(\ell h)^{1-\alpha}(1 - \Pi)^{1-\alpha}LL^{-\alpha} \tag{5}\\
y &= Ak^\alpha(\ell h)^{1-\alpha}(1 - \Pi)^{1-\alpha} \tag{6}\\
y &= \hat{A}k^\alpha(\ell h)^{1-\alpha} \tag{7}
\end{aligned}
$$

where y and k are per capita terms, and $\hat{A} = A(1 - \Pi)^{1-\alpha}$

- Explain why the optimal policy for the government is likely to involve
  subsidizing the programmers *not* to work. Discuss how would the ap-
  propriate subsidy depends (qualitatively) on the different parameters in
  the model (including the Cobb-Douglas parameter for capital's share in
  output).

  Answer:

  Ultimately the programmers' contribution to knowledge creation
  is more important than their raw labor. As discussed further
  below, more raw labor has a level effect, but more time spent in
  knowledge creating activities has a growth effect. To the extent
  that this growth happens in the future and hence depends on
  preferences towards the future, the trade-off will depend on β; a
  higher value will imply a higher subsidy, as the consumers care
  more about future growth. As capital's share of income declines
  and capital becomes less important to output, human capital
  becomes more important, and so as α falls the subsidy should rise.
  A larger Π will mean that a single dollar of subsidy will go farther,
  since it increases $\hat{A}$ and hence the marginal product of human
  capital. Finally, a higher ϕ means that the externalities from
  human capital accumulation are larger, and so a higher subsidy
  is warranted.

- Now suppose that labor market imperfections mean that the company
  that a programmer works for is able to temporarily capture a substantial
  portion of the "knowledge" benefits of the programmer's "free time" pro-
  gramming activities, but the amount that the company captures depends

on the number of hours that the programmer spends working at the company (the ℓ). Will the company agree with the government about the optimal size of the subsidy? Explain why or why not, and discuss what determines whether the company is likely to benefit (on net) from the externalities mentioned above.

Answer:

No, the company has two incentives for ℓ to be higher: the value of the raw labor and the greater ability to capture benefits from the programmer's "free time." The social planner only has the former incentive, and since she takes the externality into account she has incentives to reduce ℓ. Hence the optimal subsidy is likely larger than what the company would like. However, if the externality is large (ϕ is large), then the benefits of greater aggregate knowledge capital will offset the loss of their monopolizing capacity.

- Assuming that parameter values are such that the subsidy to leisure of programmers has a positive net value, would the model suggest that such a subsidy would have a positive effect on the level of output, on the growth rate of output, or both?

Answer:

Both. The subsidy would lower ℓ the moment it went into effect, which would directly lower the level of output; however, since we have assumed that that parameter values are such that the subsidy has a positive net value, the benefits of higher knowledge capital would offset this and output would increase (although this might take time to materialize, since h moves continuously). Since knowledge capital is accumulating at a faster rate after the subsidy, growth is higher.

- How might such a subsidy compare, in its effects, to a more traditional policy like a permanent investment tax credit?

Answer:

In a standard growth model, a permanent ITC will increase investment flows and hence increase capital to a new steady state level. In a model like Romer's with positive externalities from capital investment, there is also a permanent increase in the steady state growth rate. The subsidy here is like the ITC of the Romer model, in the sense that there is a permanent growth effect. However, since this subsidy has direct labor market effects, there is also a level effect that depends on the parameters.

- Explain why the government's optimization problem becomes much harder if some people can pretend to be computer programmers even though their only real talent is in English Literature and they hate

programming and in their free time they read Shakespeare instead of coding the latest update to Firefox.

Answer:

> The government faces the problem that they would be paying Lit people to withdraw their raw labor from the market, reducing output, but not gaining any of the knowledge capital in return. In this case they need to deal additionally with the adverse selection problem, meaning that they need to either design optimal screening or sorting devices, or pay verification costs to companies who monitor people claiming to be programmers. In either case, as in the imperfect capital markets a la Romer, the contract design problem becomes much more difficult.

3. **Beliefs, Preferences, and Choices.**

(The very last question will be given the greatest weight in grading this section; you do not need to have successfully answered the earlier parts to provide a good answer to the last question)

Using data on the stock market investment choices of people in different generations, Malmendier and Nagel (2011) show that "Depression Babies" (DB's, henceforth), interpreted (say) to include anyone between the ages of 5 and 25 during the Great Depression, tended to be less willing to allocate their savings to "risky" assets than people born at other times (non-DB's, henceforth).

This question asks you to think about the implications of their research for the consequences of the Great Recession.

As a preliminary, recall that `CRRA-RateRisk` derived a formula according to which the proportion of total net worth a person should invest in risky assets is given by:

$$\varsigma = \left(\frac{\phi}{\rho \sigma_{\mathbf{r}}^2} \right) \tag{8}$$

where ς is the proportion of the portfolio invested in the risky asset, ρ is the coefficient of relative risk aversion, ϕ is the magnitude of the risky return premium, and $\sigma_{\mathbf{r}}^2$ is the expected variance of risky return.

a) Explain the intuition for the role of each term of this equation.

b) Household surveys have long asked respondents their expectations about the rate of the return to investments in the stock market (interpreted here as "the risky asset"). Suppose that those surveys did not show any difference between DB's and non-DB's in ϕ. But such surveys have historically not tried to directly measure either an individual's risk aversion or perception of the market's riskiness. Does the theory suggest that there is any way to use portfolio choices to distinguish between the following two ideas: (1) DB's avoid risky assets because they have high risk aversion; (2) DB's avoid risky assets because they perceive the stock market to be "riskier" than other (non-DB) people perceive it to be.

Answer:

According to (8), what matters for portfolio allocation is the *product* of relative risk aversion and the variance of returns. Therefore, even given knowledge of ϕ it is impossible to use portfolio allocation choices to distinguish the hypothesis of high risk aversion from the hypothesis of perceptions of high riskiness.

c) Now suppose that individuals' relative risk aversion coefficients could be estimated in some other way. For example, suppose research finds that non-smokers have risk aversion coefficients higher by about 2 than nonsmokers, holding all other observable characteristics constant (including age). If this fact were assumed to be true about DB's as well, and the surveys of their

portfolio choices also asked whether they were smokers, describe whether and how this new finding might be useful in distinguishing the two hypotheses above.

Answer:

A first thing to do is to look at whether DB's were less likely to be "smokers" than non-DB's (controlling for trends in smoking). If so, that would suggest that they were generally more risk averse. If not, not.

Then we can look at the differences in portfolio shares between "smoker" and "non-smoker" DB's, vs "smoker" and "non-smoker" non-DB's. If there is no difference in the effect of risk aversion (as instrumented by smoking) on portfolio share, the implication would be that (if the formula is right) the DB's invested less in the stock market not because they were more risk averse but because they believed that $\sigma_{\mathbf{r}}^2$ was higher.

d) In recent years (long after most of the DB's have died), surveys have been developed in which respondents are asked separately about their expectations of the mean return of stock market investments ϕ and the degree of uncertainty (which can be used to derive an estimate of each person's estimated $\sigma_{\mathbf{r}}^2$ at each surveyed moment). Suppose that the data from these surveys show that people's perceptions of stock market uncertainty can be well captured by a regression equation like

$$\sigma_{i,t}^2 = \bar{\sigma}_t^2 + \epsilon_i \tag{9}$$

or in words, everybody has their own particular view σ_i^2 about the variance of the stock market, but those views tend to move up and down in synchrony with the changes in the views of other people. Furthermore, suppose this research shows that there is no systematic difference in the σ_i's across people in different groups (millenials, Gen-X, Baby Boomers, etc).

That is, if we use **1** as an indicator variable for whether a person was a DB or not, ρ_{DB} for the risk aversion of DB number i is:

$$\rho_i = \rho + \mathbf{1}[\Delta\rho_{\mathrm{DB}}] \tag{10}$$

How could you combine these points to go back and do new research using Malmendier and Nagel (2011)'s data to reach a conclusion about whether the DB's avoided stock market investments because they were more risk averse than other people (had a higher ρ) or because they had a higher average assessment of σ^2?

Answer:

The risky share formula for the DB's will be

$$\varsigma = \left(\frac{\phi}{(\rho + \Delta\rho_{\mathrm{DB}})\sigma_t^2}\right) \tag{11}$$

or

$$\log\varsigma = \log\phi - \left(\log\sigma_t^2 + \log(\rho + (\Delta\rho_{DB})\mathbf{1}(\mathrm{DB})\right) \tag{12}$$

which in principle we could estimate from

$$\log\varsigma_{i,t} = \gamma_0 + \gamma_1\log\phi_{i,t}\gamma_2\mathbf{1}(\mathrm{DB}) \tag{13}$$

Basically, the point is that if their risk aversion coefficient is higher, not only will the DB's have a lower portfolio share on average, that portfolio share will be more responsive to changes to the perceived degree of riskiness of the stock market. That is, DB's are more likely to "get out" when the market goes through a spell of high volatility.

e) Suppose that future research proves that:

   i. The effect of events like the Great Depression or Great Recession is to shift upward people's risk aversion

   ii. Antidepressant drugs can return people's risk aversion to normal

   Then discuss whether the Great Recession is likely to have a "scarring effect" on the investment choices of people who lived through it.

f) Suppose that research of the kind described above establishes compellingly that the differences between DB and non-DB generations are attributable to DB's having much higher beliefs about the riskiness of stock market investments. Discuss what implications this could have for

   • The correct explanation for the Equity Premium Puzzle

   • Appropriate public policies

4. **Growth Empirics for Buffer Stock Economies.** Answer the following questions using a tractable buffer stock saving model like the one discussed in `TractableBufferStock`.

a) Consider two economies A and B populated by buffer stock consumers. The economies and parameters are identical in every respect except that the growth rates of wages are respectively G^A and G^B, where $G^A < G^B$. Each economy is in its steady state. Suppose an economist studying the two economies cannot directly observe the growth rate parameter (though the consumers living in the economies can); but the economist can observe the average growth rate outcome for each economy, $\Delta \log Y_i$, and suppose that the reason $G^B > G^A$ is that economy B has a higher level of education. The economist therefore performs a regression of the form:

$$\Delta \log Y_i = \alpha_0 + \alpha_1 E_i \tag{14}$$

where $i \in A, B$ across the two economies and therefore can translate the measurable difference in educational attainment E into its implication for economic growth by constructing $\mathbb{E}[\Delta \log Y_i] = \hat{\alpha}_0 + \hat{\alpha}_1 E_i$ for each country. Suppose the economist then performs a Campbell-Mankiw type regression:

$$\Delta \log C_i = \mu_0 + \mu_1 \mathbb{E}[\Delta \log Y_i] \tag{15}$$

on the data from the two economies. What coefficient estimate μ_1 would the economist find? Why? Would the coefficient μ_1 reflect what Campbell and Mankiw interpreted it as reflecting? Why or why not? Relate your comments to the log-linearized Euler equation.

Answer:

The education variable will correctly predict income growth since it is perfectly correlated with income growth across the two countries.

In a buffer-stock economy, the growth rate of aggregate consumption matches the growth rate of aggregate income, so the coefficients will be $\mu_0 = 0$ and $\mu_1 = 1$.

But in Campbell and Mankiw's model, the regression coefficient μ_1 was supposed to reflect the proportion of households who set consumption exactly equal to income in each period, and did not engage in intertemporally optimizing behavior. Here, *all* consumers are interetemporally optimizing; none are Keynesian $C = Y$ consumers. So the coefficient does *not* correctly reveal the fraction of households who fail to intertemporally optimize, because that fraction is zero.

The log-linearized Euler equation says that $\Delta \log C_{t+1} = \rho^{-1}(r - \vartheta)$ while here we have $\Delta \log C_{t+1} = \gamma$. Thus, the results violate the log-linearized Euler equation. This can be interpreted as reflecting the *endogeneity* of higher-order terms in the Euler equation.

b) Now suppose the economist has data on only one country, but that data covers a long span of time. In particular, suppose that the country went through a period of fast growth for many years during the first half of the data sample, and then a period of slow growth for the second half of the data sample (as happened in the post-World-War-2 history of most of Western Europe: a period of rapid growth from 1947-1974, followed by much slower growth thereafter). Assume that during the period of rapid income growth, everyone expected that rapid income growth to continue indefinitely; and when the economy shifts down to slower income growth, everyone perceives the slowdown immediately and adjusts their spending accordingly. Answer the following questions about this economy:

   i. Show what happens to the consumption growth diagram depicting the relation between m_t and expected consumption growth when the growth rate abruptly changes (e.g., when the economy permanently and unexpectedly shifts from rapid growth to slower growth). (Show both any curve shifts, and, using arrows, show the path of consumption growth as it evolves toward its new steady state).

Answer:

The actual numerical solution is shown in the figure below:
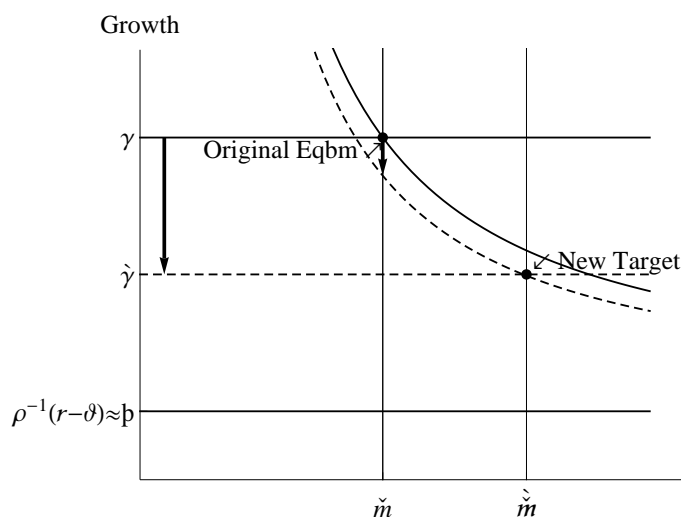
and the major shift is of course in the expected growth rate of permanent income γ. As you can see the expected consumption growth curve shifts down a little bit; but this is a result of subtle higher-order effects that I did not emphasize or explain in class or in the handout, and so if you just assumed that the expected consumption growth curve remained fixed you will get full credit for your answer (so long as, conditional on that assumption, your analysis was correct).

Your analysis should have shown that after the initial decline in consumption growth, arrows should be drawn on the dashed line in the above diagram showing the growth rate evolving down toward its new equilibrium.

ii. Draw diagrams showing the path of the level of c and of C in this economy following the change in growth rates. (Recall that c is the ratio of consumption to permanent wage income, while C is the absolute level of per-capita consumption. For simplicity, you may wish to assume that the initial growth rate is zero, and the new growth rate is negative.).

Answer:

The ratio of consumption to permanent labor income looks like:
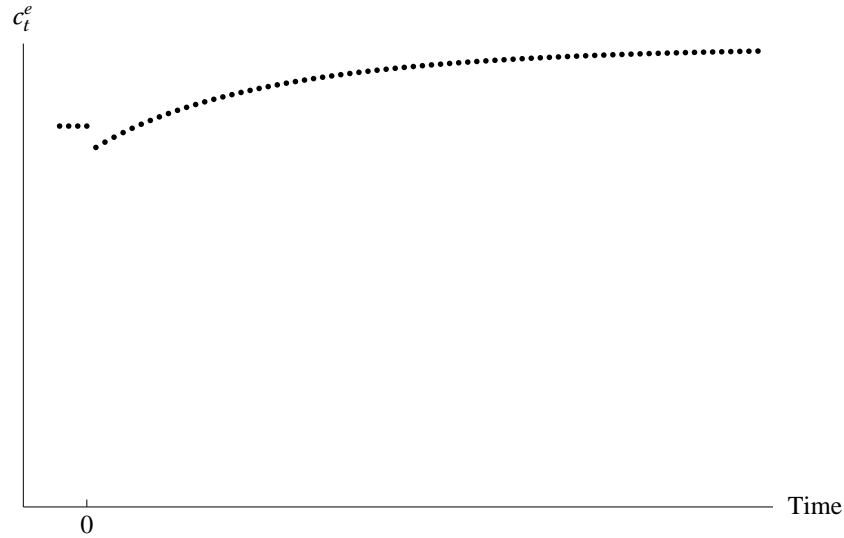
While the level looks similar but with the portion after the jump at time zero rotated clockwise downward.

iii. Suppose, again, that the econometrician cannot directly observe the economy's regime change, but suppose there is a survey of consumers that shows households' expectations of income growth. Assume that the data begin in period $-T$ while the regime change happens at date 0; beginning in date $-T$ the economy was at its target ratio, and by n periods after the change the economy has mostly settled down to its new target ratio (the sample ends at period $+T$). Draw a diagram showing the path over time of consumers' expectations of income growth and the path of actual (not expected) consumption growth. Now suppose the econometrician does a regression of the form

$$\Delta \log C_{t+1} = \mu_0 + \mu_1 \, \mathbb{E}_t[\Delta \log Y_{t+1}]. \qquad (16)$$

Discuss how the econometrician's results will depend on the sample period used for estimating this equation. In particular, consider all these possibilities:

A. A sample that includes only the early and late parts of the history (say, dates $-T$ to $-T/2$ and $T/2$ to T) where $T/2 >> n$

B. A sample that includes only the middle parts of the history (say, dates $-n/2$ to $+n/2$), but excludes the date 0 when the regime change occurred.

C. A sample that includes the entire history (except period 0) when that history is long $T >> n$

D. A sample that includes the entire history (except period 0) when that history is short $T < n$

Answer:

Expected income growth first stays constant before date 0, drops discretely to a lower level at date 0, and remains at the lower level after that; actual consumption growth first stays at the same level as the expected income growth before date 0, decreases gradually over a period of time starting from date 0, and finally reaches the lower level of income growth rate and remains at that level after that.

A. This sample essentially is like the comparison in the early part of the problem between two economies with different equilibrium growth rates that are both in their steady states: We will get $\mu_0 \approx 0$ and $\mu_1 \approx 1$.

B. During the first part of the sample, we have consumption growth matching income growth except for period 0 in which there is a big drop in consumption which will be a big negative error in the equation. Thereafter, consumption growth will be slightly above income growth as the consumer builds up consumption toward the higher c^e target ratio. So the average consumption growth in the later period is not as much lower as the average income growth is; this means that our coefficient on income growth will be biased toward zero (too small), though it will still have a positive coefficient. Note that if we were to include date 0 in the sample, the results are ambiguous – think of the case where only periods -1, 0, and 1 are included; in that case in period -1 we would have consumption growth matching expected income growth, in period 0 we would have consumption growth hugely below expected income growth (because the regime change is a negative surprise), and in period 1 we would have consumption growth slightly exceeding a much lower level of income growth. It is *possible* that average consumption growth in periods -1 and 0 will be lower than in

period 1, while expected income growth in those periods will have been higher than in period 1, in which case we would actually get a negative coefficient on income growth. (This latter scenario was not asked in the question and is provided only to clarify the logic of the model; you did not need to describe it to get credit for your answer to the question).

C. If the history is long, it will be dominated by the steady-state results and we should expect the coefficients to approach the unbiased case

D. If the history is short, it will be dominated by the transition results and we should expect the coefficients to approach the biased case

References

LUCAS, ROBERT E. (1988): "On the Mechanics of Economic Development," *Journal of Monetary Economics*, 22, 3–42.

MALMENDIER, ULRIKE, AND STEFAN NAGEL (2011): "Depression Babies: Do Macroeconomic Experiences Affect Risk Taking?," *Quarterly Journal of Economics*, 126(373–416).