Name ____________________

Midterm Exam

Intertemporal Choice

Fall, 2022

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up.* The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I: Short Questions

1. Explain *and critique* the argument that the introduction of a Social Security system (a mandatory PAYG pension scheme) reduces the personal saving rate. (By "critique" I mean that you should give at least one well-thought-out *and well-explained* reason why the standard argument might be wrong.)

2. Consider a Diamond OLG model in which the economy has reached the steady state equilibrium with a constant level of population. Suppose that suddenly in period t there is a permanent increase in the level of population (perhaps as a result of a one-time inflow of immigrants). Show the dynamics of k in the model, and explain whether the surprise increase in the population of workers is good or bad for the old people in period t in this economy, and why.

3. **Precautionary Saving and Convex Marginal Utility.**

   Consider a consumer whose last period of life is T and who is trying to decide how much to save in period $T-1$. Suppose the interest factor and the time preference factor are $\mathsf{R} = \beta = 1$ and so consumer's dynamic budget constraint is

$$c_T = a_{T-1} + y_T. \tag{1}$$

   and define an end-of-period value function as

$$\mathfrak{v}'_{T-1}(a_{T-1}) = \mathbb{E}_{T-1}[\mathrm{u}'(c_T)] \tag{2}$$

   Assuming CRRA utility in periods T and $T-1$, draw a diagram that shows:

   a) Marginal end-of-period value as a function of a_{T-1} if income is perfectly certain

   b) Marginal end-of-period value as a function of a_{T-1} if income is

$$y_T = \begin{cases} \epsilon & \text{with probability } 0.5 \\ -\epsilon & \text{with probability } 0.5 \end{cases} \tag{3}$$

   c) Draw $u'(m_{T-1} - a_{T-1})$ and explain why a_{T-1} increases as a result of either an increase in risk aversion or an increase in the size of uncertainty ϵ.

   (The next page has been left blank for your figure and discussion.)

Part II: Medium Question

1. **Consumption with CARA Utility and Labor Income Risk (Caballero (1990)).**

   Consider an individual who lives for two periods, who is born with initial wealth m_1 and will receive a normally distributed uncertain income in the second period of life, $y_2 \sim \mathcal{N}(\bar{y}, \sigma_y^2)$. The individual maximizes:

$$\mathrm{u}(c_1) + \mathbb{E}[\mathrm{u}(c_2)]$$

   subject to the Intertemporal Budget Constraint

$$m_2 = m_1 - c_1 + y_2.$$

   Suppose that the consumer's utility function is of the Constant Absolute Risk Aversion form:

$$\mathrm{u}(c) = -(1/\alpha)e^{-\alpha c}. \tag{4}$$

   a) Show that this utility function exhibits constant absolute risk aversion with risk aversion parameter α (absolute risk aversion is defined as $-\mathrm{u}''(c)/\mathrm{u}'(c)$).

   b) Derive the first order condition linking first and second period consumption, and use it to derive an analytical expression for c_1.

      Hint: you will need to use Math Fact `ELogNorm`:

      If from the viewpoint of period t the stochastic variable $\mathbf{R}_{t+1}$ is lognormally distributed with mean $\mathbf{r}$ and variance $\sigma_{\mathbf{r}}^2$, $\mathbf{r}_{t+1} \sim \mathcal{N}(\mathbf{r}, \sigma_{\mathbf{r}}^2)$, then

$$\mathbb{E}_t[e^{\mathbf{r}_{t+1}}] = e^{\mathbf{r}+\sigma_{\mathbf{r}}^2/2} \tag{5}$$

   c) Now consider two consumers with different initial wealth but the same second period income and the same risk aversion parameter $\alpha = 2$. Homer's $m_1 = 20,000$ and Mr. Burns's $m_1 = 20,000,000$. Second period income (call it 'Social Security') is normally distributed with mean $\bar{y}_2 = 20,000$ and variance $\sigma_y^2 = 10,000$ for both. Calculate the levels of first-period consumption for Homer and Mr. Burns, and then calculate the effect on consumption for Homer and for Mr. Burns if the uncertainty associated with Social Security income increases to $\sigma_y^2 = 15,000$. Does this result seem plausible to you?

References

CABALLERO, RICARDO J. (1990): "Consumption Puzzles and Precautionary Savings," *Journal of Monetary Economics*, 25, 113–136, http://ideas.repec.org/p/clu/wpaper/1988_05.html.