Final Exam

Intertemporal Choice

Fall, 2022

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

1. **'Real' and 'Financial' Explanations of Asset Price Movements**

   A large literature at the intersection of finance and macroeconomics aims to explain movements of stock prices. Roughly speaking, much of that literature attempts to identify movements that can be explained by (a) 'real' and observable factors like news about a firm's profits ('cash flow') and dividends, versus (b) other unobservable factors that are often called 'discount rate' shocks. (The literature typically assumes that the 'discount rate' or, synonymously, the 'stochastic discount factor,' is impossible to observe *except* through its consequences for asset prices.)

   a) Interpret the foregoing statement using the Lucas (1978) model. In that framework, what would you identify as 'real' and what would you identify as 'financial' factors?

      Answer:

      `LucasAPM` shows that in the version of Lucas's model with log utility, asset prices are given by:

      $$\mathsf{P}_t = \frac{d_t}{\vartheta} \tag{1}$$

      which provides a very direct interpretation: Movements in prices that are caused by changes in dividends (in the model, the productivity of the trees) are 'real' movements, while those due to unobserved factors must be attributed to changes in ϑ.

   b) Explain why it is problematic, for the status of finance as a science, to attribute all movements in asset prices that are not explainable as reflecting observable 'real' factors to movements in a variable which is in principle unobservable ('the stochastic discount rate').

      Answer:

      The hypothesis that imputed movements of an unobservable variable (the 'stochastic discount factor') 'explain' those movements of an observable variable (asset prices) which cannot be explained by other observable phenomena ('real' measurable variables) begs the question of how the hypothesis can be disproven. If the stochastic discount factor can never be directly observed, and its movements can explain all movements in observable variables that are not otherwise explained, how can the theory be proven to be wrong?

   c) Using the insights from the prior questions, explain why Robert Shiller's work in attempting to find credible ways of surveying market participants to measure their beliefs and preferences and attitudes would, if successful, be useful.

      Answer:

      If beliefs and preferences and attitudes could be directly measured by surveys or other methods, that would either bolster or undermine the

presumption that movements in asset prices always reflect changes in beliefs or preferences or attitudes. Either outcome would be progress relative to the situation described above.

2. **Open Source Software and Knowledge Capital.** "Open Source" software is created in part by computer programmers who are not directly paid for their contributions. Some of the most central components of the infrastructure of the internet are open-source (for example, the Apache web server software that powers perhaps half of the world's webservers; the FireFox browser; the LaTeX typographical system; and much more). Assume (for the purposes of this question) that all open source software is created in this way.

   Explaining the production of valuable services by both paid and by volunteer labor is a challenge for neoclassical economics (to say the least!). This question asks you to make a first stab at the job, using the Lucas (1988) growth model with an aggregate production function

   $$Y = AK^{\alpha}(\ell h L)^{1-\alpha} \tag{2}$$

   Recall that in that model, "human capital" h accumulates according to

   $$\dot{h}/h \;=\; \phi(1-\ell) \tag{3}$$

   where ℓ is the proportion of time that people spend working.

   a) Write a paragraph or two about reasons for and against a reinterpretation of Lucas's model in which what he calls "human capital" is thought of as "knowledge capital" produced by people in their free-time contributions to open source software projects. Comment, in particular, about what you need to assume about the parameter that captures the degree of externalities in aggregate h in order for this interpretation to have any force.

      Answer:

      The essential distinction is between human capital, which is embodied in an individual mind and hence cannot be accumulated forever without bound, and knowledge capital, which goes into things like textbooks and can be accumulated without bound. This carries over into whether we can interpret ℓ as a proportion of the population or a proportion of an individual's time. To the extent that the programmer learns something useful, *or establishes a brand for themselves as being comptetent*, an open source contribution is genunine human capital. To the extent that the contribution is entirely altruistic (some contributions are made anonymously, and in people's spare time), it is best understood as a contribution to "knowledge."

   b) Suppose a government concludes that the value of open source software in boosting aggregate productivity is very large. Under the appropriate assumptions (that is, the assumptions that make the Lucas model sketched above appropriate for addressing the question), suppose the government wants to evaluate a subsidy that will increase the leisure time of computer programmers (in the belief that in their leisure time they will create more open-source software).

i. Assume that only computer programmers contribute to knowledge capital in the relevant sense (everybody else just contributes raw labor). Also, programmers are "born that way" (they can't help becoming programmers, and nobody else can become one) so that their proportion in the population remains constant at Π. Show that under these assumptions the effective aggregate production function becomes

$$y = \overbrace{A(1-\Pi)^{1-\alpha}}^{\hat{A}} k^{\alpha}(\ell h)^{1-\alpha} \tag{4}$$

where Π is the proportion of programmers, h is the knowldege capital per capita of the programmers and ℓ is the proportion of their hours they spend working.

Answer:

First note that if only a fraction $1-\Pi$ of the population are workers then the production function becomes

$$\begin{aligned} Y &= AK^{\alpha}(\ell h\{(1-\Pi)L\})^{1-\alpha} \\ &= AK^{\alpha}(\ell h)^{1-\alpha}(1-\Pi)^{1-\alpha}LL^{-\alpha} \\ y &= Ak^{\alpha}(\ell h)^{1-\alpha}(1-\Pi)^{1-\alpha} \\ &= \hat{A}k^{\alpha}(\ell h)^{1-\alpha} \end{aligned}$$

where y and k are per capita terms, and $\hat{A} = A(1-\Pi)^{1-\alpha}$. (Workers and programmers have exactly the same tastes for work and leisure so ℓ will be the same for both groups).

ii. Explain why the optimal policy for the government is likely to involve subsidizing the programmers *not* to work. Discuss how would the appropriate subsidy depends (qualitatively) on the different parameters in the model (including the Cobb-Douglas parameter for capital's share in output).

Answer:

Ultimately the programmers' contribution to knowledge creation is more important than their raw labor. As discussed further below, more raw labor has a *level* effect, but more time spent in knowledge creating activities has a *growth* effect. To the extent that this growth happens in the future and hence depends on preferences towards the future, the trade-off will depend on β; a higher value will imply a higher subsidy, as the consumers care more about future growth. As capital's share of income declines and capital becomes less important to output, human capital becomes more important, and so as α falls the subsidy should rise. A larger Π will mean that a single dollar of subsidy will go farther, since it increases $\hat{A}$ and hence the marginal product of human capital. Finally, a higher ϕ means that the externalities from

human capital accumulation are larger, and so a higher subsidy is warranted.

iii. Now suppose that labor market imperfections mean that the company that a programmer works for is able to temporarily capture a substantial portion of the "knowledge" benefits of the programmer's "free time" programming activities, but the amount that the company captures depends on the number of hours that the programmer spends working at the company (the ℓ); but continue to assume that everything the programmer does "on the clock" (paid) is proprietary and forever secret. Will the company agree with the government about the optimal size of the subsidy? Explain why or why not, and discuss what determines whether the company is likely to benefit (on net) from the externalities mentioned above.

Answer:

No, the company has two incentives for ℓ to be higher: the value of the raw labor and the greater ability to capture benefits from the programmer's "free time." The social planner only has the former incentive, and since she takes the externality into account she has incentives to reduce ℓ. Hence the optimal subsidy is larger than what the company would like. However, if the externality is large (ϕ is large), then the benefits of greater aggregate knowledge capital will offset the loss of their monopolizing capacity.

iv. Assuming that parameter values are such that the subsidy to leisure of programmers has a positive net social value, would the model suggest that such a subsidy would have a positive effect on the level of output, on the growth rate of output, or both?

Answer:

Both. The subsidy would lower ℓ the moment it went into effect, which would directly lower the level of output; however, since we have assumed that that parameter values are such that the subsidy has a positive net value, the benefits of higher knowledge capital would offset this and output would increase (although this might take time to materialize, since h moves continuously). Since knowledge capital is accumulating at a faster rate after the subsidy, growth is higher.

v. How might such a subsidy compare, in its effects, to a more traditional policy like a permanent investment tax credit? (Assume that firms in the production side of this economy would be the recipients).

Answer:

In a standard growth model, a permanent ITC will increase investment flows and hence increase capital to a new steady state

level. In a model like Romer's with positive externalities from capital investment, there is also a permanent increase in the steady state growth rate. The subsidy here is like the ITC of the Romer model, in the sense that there is a permanent growth effect. However, since this subsidy has direct labor market effects, there is also a level effect that depends on the parameters.

vi. Explain why the government's optimization problem becomes much harder if some people can pretend to be computer programmers even though their only real talent is in English Literature and they hate programming and in their free time they read Shakespeare instead of coding the latest update to Firefox.

Answer:

The government faces the problem that they would be paying Lit people to withdraw their raw labor from the market, reducing output, but not gaining any of the knowledge capital in return. In this case they need to deal additionally with the adverse selection problem, meaning that they need to either design optimal screening or sorting devices, or pay verification costs to companies who monitor people claiming to be programmers. In either case, as in the imperfect capital markets a la Romer, the contract design problem becomes much more difficult.

3. **Beliefs, Preferences, and Choices.**

(The very last question will be given the greatest weight in grading this section; you do not need to have successfully answered the earlier parts to provide a good answer to the last question)

Using data on the stock market investment choices of people in different generations, Malmendier and Nagel (2011) show that "Depression Babies" (DB's, henceforth), interpreted (say) to include anyone between the ages of 5 and 25 during the Great Depression, tended to be less willing to allocate their savings to "risky" assets than people born at other times (non-DB's, henceforth).

This question asks you to think about the implications of their research for the consequences of the Great Recession.

As a preliminary, recall that `CRRA-RateRisk` derived a formula according to which the proportion of total net worth a person should invest in risky assets is given by:

$$\varsigma = \left(\frac{\phi}{\rho\sigma_{\mathbf{r}}^2}\right) \tag{5}$$

where ς is the proportion of the portfolio invested in the risky asset, ρ is the coefficient of relative risk aversion, ϕ is the magnitude of the risky return premium, and $\sigma_{\mathbf{r}}^2$ is the expected variance of risky return.

a) Explain the intuition for the role of each term of this equation.

Answer:

See the CRRA-Portfolio handout.

b) Household surveys have long asked respondents their expectations about the rate of the return to investments in the stock market (interpreted here as "the risky asset"). Suppose that those surveys did not show any difference between DB's and non-DB's in ϕ. But such surveys have historically not tried to directly measure either an individual's risk aversion or perception of the market's riskiness. Does the theory suggest that there is any way to use portfolio choices to distinguish between the following two ideas: (1) DB's avoid risky assets because they have high risk aversion; (2) DB's avoid risky assets because they perceive the stock market to be "riskier" than other (non-DB) people perceive it to be.

Answer:

According to (5), what matters for portfolio allocation is the *product* of relative risk aversion and the variance of returns. Therefore, even given knowledge of ϕ it is impossible to use portfolio allocation choices to distinguish the hypothesis of high risk aversion from the hypothesis of perceptions of high riskiness.

c) Now suppose that individuals' relative risk aversion coefficients could be estimated in some other way. For example, suppose research finds that

non-smokers have risk aversion coefficients higher by about 2 than smokers, holding all other observable characteristics constant (including age). If this fact were assumed to be true about DB's as well, and the surveys of their portfolio choices also asked whether they were smokers, describe whether and how this new finding might be useful in distinguishing the two hypotheses above.

Answer:

A first thing to do is to look at whether DB's were less likely to be "smokers" than non-DB's (controlling for trends in smoking). If so, that would suggest that they were generally more risk averse. If not, not.

Then we can look at the differences in portfolio shares between "smoker" and "non-smoker" DB's, vs "smoker" and "non-smoker" non-DB's. If there is no difference in the effect of risk aversion (as instrumented by smoking) on portfolio share, the implication would be that (if the formula is right) the DB's invested less in the stock market not because they were more risk averse but because they believed that $\sigma^2_{\mathbf{r}}$ was higher.

d) In recent years (long after most of the DB's have died), surveys have been developed in which respondents are asked separately about their expectations of the mean return of stock market investments ϕ and the degree of uncertainty (which can be used to derive an estimate of each person's estimated $\sigma^2_{\mathbf{r}}$ at each surveyed moment). Suppose that the data from these surveys show that people's perceptions of stock market uncertainty can be well captured by a regression equation like

$$\sigma^2_{i,t} = \bar{\sigma}^2_t + \sigma^2_i \tag{6}$$

or in words, everybody has their own particular view σ^2_i about the variance of the stock market (some people are born optimists, and some are born pessimists), but those views move up and down in synchrony with the changes in the views of other people.

On the other hand, if we use $\mathbf{1}$ as an indicator variable for whether a person was a DB or not, ρ_{DB} for the risk aversion of DB number i is:

$$\rho_i = \rho + \mathbf{1}[\Delta\rho_{\text{DB}}] \tag{7}$$

How could you combine these points to go back and do new research using Malmendier and Nagel (2011)'s data to reach a conclusion about whether the DB's avoided stock market investments because they were more risk averse than other people (had a higher ρ) or because they had a higher average assessment of σ^2?

Answer:

The risky share formula for the DB's will be

$$\varsigma = \left(\frac{\phi}{(\rho + \Delta\rho_{\text{DB}})\sigma_t^2}\right) \tag{8}$$

or

$$\log \varsigma = \log \phi - \left(\log \sigma_t^2 + \log(\rho + (\Delta\rho_{DB})\mathbf{1}(\text{DB})\right) \tag{9}$$

which in principle we could estimate from

$$\log \varsigma_{i,t} = \gamma_0 + \gamma_1 \log \phi_{i,t}\gamma_2 \mathbf{1}(\text{DB}) \tag{10}$$

Basically, the point is that if their risk aversion coefficient is higher, not only will the DB's have a lower portfolio share on average, that portfolio share will be more responsive to changes to the perceived degree of riskiness of the stock market. That is, DB's are more likely to "get out" when the market goes through a spell of high volatility.

e) Suppose that future research proves that:

i. The effect of events like the Great Depression or Great Recession is to shift upward people's risk aversion

ii. Antidepressant drugs can return people's risk aversion to normal

Then discuss whether the Great Recession is likely to have a "scarring effect" on the investment choices of people who lived through it.

Answer:

While antidepressant drugs can balance out the spike in risk aversion through the great depression, this drug might not be widely available. Nevertheless, the experience of the great depression impacts arguably everyone. Hence, scarring effects should be visible on the aggregate with a share of households who show no effects.

f) Suppose that research of the kind described above establishes compellingly that the differences between DB and non-DB generations are attributable to DB's having much higher beliefs about the riskiness of stock market investments. Discuss what implications this could have for

- The correct explanation for the Equity Premium Puzzle
- Appropriate public policies

Answer:

- Our standard portfolio choice model says the portfolio share is:

$$_i = \frac{\phi_i}{\rho_i \sigma_i^2} \tag{11}$$

If, as the question supposes, we can measure ϕ_i and σ_i^2, then we can back out ρ_i from

$$\rho_i = \frac{\phi_i}{\rho_{ii}} \tag{12}$$

We could then see how large the differences in risk aversion would need to be in order to explain the differences in portfolio shares. If the differences in required ρ were reasonable, this would lend (some) credence to the theory that differences across people in risk aversion largely explain their portfolio choices (given their measured beliefs).

- Appropriate public policies: Possible answers can explain policies that improve information to households such as public awareness campaigns, financial education in school. Unfortunately, the available evidence finds that such efforts are almost completely ineffective. Somewhat more effective might be to insist that, at least for retirement savings, the decisionmaking should be delegated to someone whose knowledge and beliefs are better calibrated – which is in fact what has (gradually) been happening, for example with the strong move in 401(k) plans to making the default be for employees to automatically be opted into "target date" funds (like, a fund designed to achieve a reasonable risk/reward profile for someone expected to retire in, say, 2055).

4. **Growth Empirics for Buffer Stock Economies.** Answer the following questions using a tractable buffer stock saving model like the one discussed in TractableBufferStock.

   a) Consider two economies A and B populated by buffer stock consumers. The economies and parameters are identical in every respect except that the growth rates of wages are respectively G^A and G^B, where $\mathrm{G}^A < \mathrm{G}^B$. Each economy is in its steady state. Suppose an economist studying the two economies cannot directly observe the growth rate parameter (though the consumers living in the economies can); but the economist can observe the average growth rate outcome for each economy, $\Delta \log Y_i$, and suppose that the reason $\mathrm{G}^B > \mathrm{G}^A$ is that economy B has a higher level of education. The economist therefore performs a regression of the form:

$$\Delta \log Y_i \quad = \quad \alpha_0 + \alpha_1 E_i \tag{13}$$

   where $i \in A, B$ across the two economies and therefore can translate the measurable difference in educational attainment E into its implication for economic growth by constructing $\mathbb{E}[\Delta \log Y_i] = \hat{\alpha}_0 + \hat{\alpha}_1 E_i$ for each country. Suppose the economist then performs a Campbell-Mankiw type regression:

$$\Delta \log C_i \quad = \quad \mu_0 + \mu_1 \, \mathbb{E}[\Delta \log Y_i] \tag{14}$$

   on the data from the two economies. What coefficient estimate μ_1 would the economist find? Why? Would the coefficient μ_1 reflect what Campbell and Mankiw interpreted it as reflecting? Why or why not? Relate your comments to the log-linearized Euler equation.

   Answer:

   The education variable will correctly predict income growth since it is perfectly correlated with income growth across the two countries.

   In a buffer-stock economy, the growth rate of aggregate consumption matches the growth rate of aggregate income, so the coefficients will be $\mu_0 = 0$ and $\mu_1 = 1$.

   But in Campbell and Mankiw's model, the regression coefficient μ_1 was supposed to reflect the proportion of households who set consumption exactly equal to income in each period, and did not engage in intertemporally optimizing behavior. Here, *all* consumers are interetemporally optimizing; none are Keynesian $C = Y$ consumers. So the coefficient does *not* correctly reveal the fraction of households who fail to intertemporally optimize, because that fraction is zero.

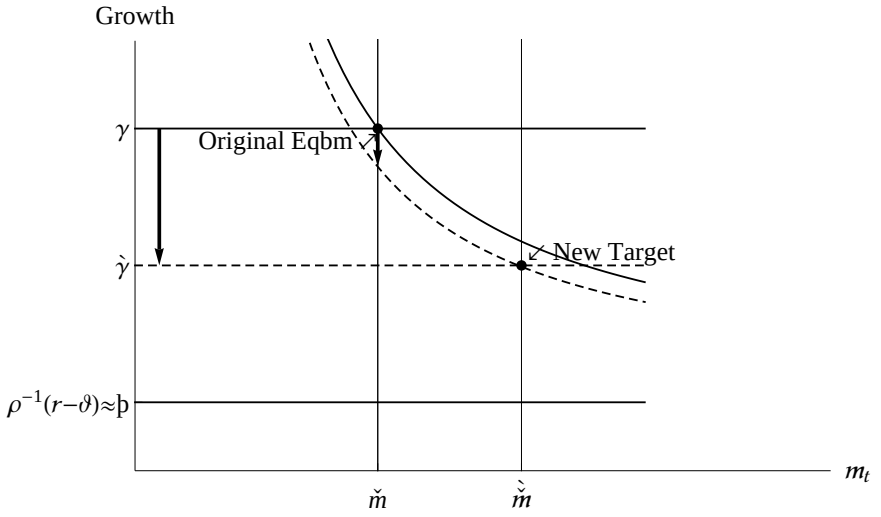
   The log-linearized Euler equation says that $\Delta \log C_{t+1} = \rho^{-1}(\mathsf{r} - \vartheta)$ while here we have $\Delta \log C_{t+1} = \gamma$. Thus, the results violate the log-linearized Euler equation. This can be interpreted as reflecting the *endogeneity* of higher-order terms in the Euler equation.

b) Now suppose the economist has data on only one country, but that data covers a long span of time. In particular, suppose that the country went through a period of fast growth for many years during the first half of the data sample, and then a period of slow growth for the second half of the data sample (as happened in the post-World-War-2 history of most of Western Europe: a period of rapid growth from 1947-1974, followed by much slower growth thereafter). Assume that during the period of rapid income growth, everyone expected that rapid income growth to continue indefinitely; and when the economy shifts down to slower income growth, everyone perceives the slowdown immediately and adjusts their spending accordingly. Answer the following questions about this economy:

i. Show what happens to the consumption growth diagram depicting the relation between m_t and expected consumption growth when the growth rate abruptly changes (e.g., when the economy permanently and unexpectedly shifts from rapid growth to slower growth). (Show both any curve shifts, and, using arrows, show the path of consumption growth as it evolves toward its new steady state).

Answer:

The actual numerical solution is shown in the figure below:



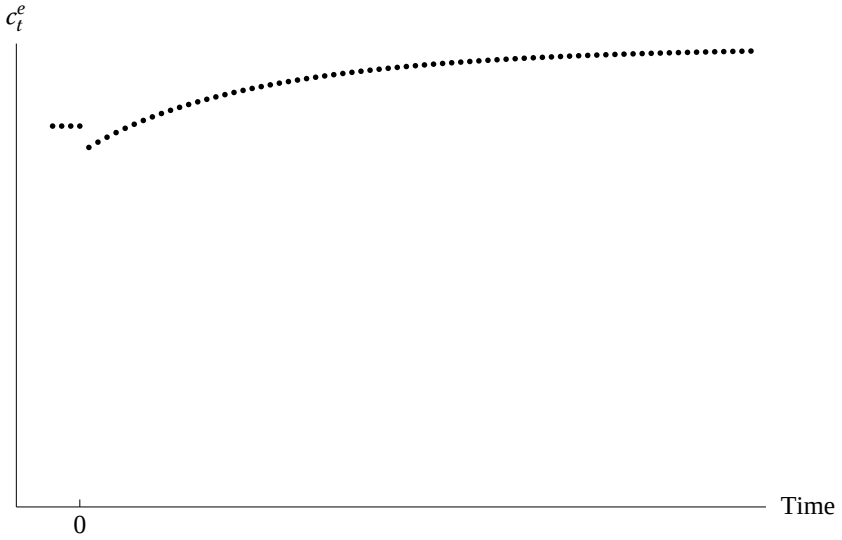
and the major shift is of course in the expected growth rate of permanent income γ. As you can see the expected consumption growth curve shifts down a little bit; but this is a result of subtle higher-order effects that I did not emphasize or explain in class or in the handout, and so if you just assumed that the expected consumption growth curve remained fixed you will get full credit for your answer (so long as, conditional on that assumption, your analysis was correct).

Your analysis should have shown that after the initial decline in consumption growth, arrows should be drawn on the dashed line in the above diagram showing the growth rate evolving down toward its new equilibrium.

ii. Draw diagrams showing the path of the level of c and of C in this economy following the change in growth rates. (Recall that c is the ratio of consumption to permanent wage income, while C is the absolute level of per-capita consumption. For simplicity, you may wish to assume that the initial growth rate is zero, and the new growth rate is negative.).

Answer:

The ratio of consumption to permanent labor income looks like:



While the level looks similar but with the portion after the jump at time zero rotated clockwise downward.

iii. Suppose, again, that the econometrician cannot directly observe the economy's regime change, but suppose there is a survey of consumers that shows households' expectations of income growth. Assume that the data begin in period $-T$ while the regime change happens at date 0; beginning in date $-T$ the economy was at its target ratio, and by n periods after the change the economy has mostly settled down to its new target ratio (the sample ends at period $+T$). Draw a diagram showing the path over time of consumers' expectations of income growth and the path of actual (not expected) consumption growth. Now suppose the econometrician does a regression of the form

$$\Delta \log C_{t+1} \quad = \quad \mu_0 + \mu_1 \, \mathbb{E}_t[\Delta \log Y_{t+1}]. \tag{15}$$

Discuss how the econometrician's results will depend on the sample period used for estimating this equation. In particular, consider all these possibilities:

A. A sample that includes only the early and late parts of the history (say, dates $-T$ to $-T/2$ and $T/2$ to T) where $T/2 >> n$

B. A sample that includes only the middle parts of the history (say, dates $-n/2$ to $+n/2$), but excludes the date 0 when the regime change occurred.

C. A sample that includes the entire history (except period 0) when that history is long $T >> n$

D. A sample that includes the entire history (except period 0) when that history is short $T < n$

Answer:

Expected income growth first stays constant before date 0, drops discretely to a lower level at date 0, and remains at the lower level after that; actual consumption growth first stays at the same level as the expected income growth before date 0, decreases gradually over a period of time starting from date 0, and finally reaches the lower level of income growth rate and remains at that level after that.

A. This sample essentially is like the comparison in the early part of the problem between two economies with different equilibrium growth rates that are both in their steady states: We will get $\mu_0 \approx 0$ and $\mu_1 \approx 1$.

B. During the first part of the sample, we have consumption growth matching income growth except for period 0 in which there is a big drop in consumption which will be a big negative error in the equation. Thereafter, consumption growth will be slightly above income growth as the consumer builds up consumption toward the higher c^e target ratio. So the average consumption growth in the later period is not as much lower as the average income growth is; this means that our coefficient on income growth will be biased toward zero (too small), though it will still have a positive coefficient. Note that if we were to include date 0 in the sample, the results are ambiguous – think of the case where only periods -1, 0, and 1 are included; in that case in period -1 we would have consumption growth matching expected income growth, in period 0 we would have consumption growth hugely below expected income growth (because the regime change is a negative surprise), and in period 1 we would have consumption growth slightly exceeding a much lower level of income growth. It is *possible* that average consumption growth in periods -1 and 0 will be lower than in

period 1, while expected income growth in those periods will have been higher than in period 1, in which case we would actually get a negative coefficient on income growth. (This latter scenario was not asked in the question and is provided only to clarify the logic of the model; you did not need to describe it to get credit for your answer to the question).

C. If the history is long, it will be dominated by the steady-state results and we should expect the coefficients to approach the unbiased case

D. If the history is short, it will be dominated by the transition results and we should expect the coefficients to approach the biased case

References

Lucas, Robert E. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46, 1429–1445, Available at http://www.jstor.org/stable/1913837.

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