

Name \_\_\_\_\_

Final Exam  
Intertemporal Choice

Fall, 2020  
Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

## Short Questions

1. **Capital Market Imperfections and the Fed.** Over the period 2007-2008, the Federal Reserve took several unusual actions in response to developments in the capital markets, including orchestrating the takeover of Bear Sterns by JP Morgan, pledging to be a lender of last resort to investment banks, and joining with the Treasury in a plan for a government takeover of Fannie Mae and Freddie Mac if they should fail.

In the model presented in class on capital market imperfections, the following condition was presented:

$$\gamma > 1 + r + A(c, r, W, \gamma) \quad (1)$$

- a) Explain this condition, and use that model to provide a variety of interpretations of either the reasons for the Fed's intervention or the reasons its actions might be expected to improve the functioning of capital markets.

*Answer:*

Subject to interpretation.

- b) Suppose the “right” diagnosis of the credit market disruptions is that it has been discovered that the cost of verification of financial contracts is higher than had been anticipated. Discuss what this model would predict about the consequences of such an increase in verification costs.

*Answer:*

Discussed in handout.

- c) Give an intuitive explanation for why a decrease in interest rates might not be an effective response to financial market problems caused by financial market imperfections.

*Answer:*

Effects of a reduction in  $r$  are discussed in handout. If the financial disruptions are caused by an increase in  $c$ , an increase in  $r$  is an imperfect fix.

## Long Question.

1. **Predicted Effects of Second Pandemic Stimulus.** On December 20, 2020 a [Washington Post article](#) described the final version of a second pandemic stimulus bill:

- “The legislation includes stimulus checks for millions of Americans of up to \$600 per person. The size of that benefit would be reduced for people who earned more than \$75,000 in 2019 and disappear altogether for those who earned more than \$99,000. The stimulus checks would provide \$600 per adult and child, meaning a family of four would receive \$2,400 up to a certain income.”
- “Congress would also extend federal unemployment benefits of up to \$300 per week, which could start as early as Dec. 27. The deal to extend federal jobless benefits for millions of unemployed Americans at a level of \$300 per week would cover up to 11 weeks of unemployment through March 14.”

This question asks you to discuss predictions of models you learned in class about the effects of the bill.

- a) The first item above indicates that the stimulus checks are strongly “progressive.” That is, for higher income households the amount of the stimulus checks is diminished, until at a household income of \$99,000 or more the stimulus will have been completely phased out.

Suppose somehow the tax authorities had a perfect measure of households’ “permanent income.” Our first questions require you to think about what a buffer stock model like the ones presented in class would say about how the results would differ if the government were able to base stimulus payments on “permanent income” rather than measured income.

- i. **Homotheticity.** Suppose first that the amount of the stimulus was much smaller, say \$1 instead of \$600. Suppose our measure of “effectiveness” of a dollar of stimulus spending is the total amount of extra aggregate spending induced by \$1 of stimulus money spent. What does the baseline version of the model from class say about whether this “progressive” policy would be more effective, less effective, or equally effective compared to an alternative policy that spent the same amount of federal money by distributing the money evenly across households. (Since all households would get a check now, the amount per person would have to be smaller, maybe \$0.50).

*Answer:*

The baseline version of the model is homothetic: A person with permanent income twice as large will have a target buffer stock twice as large and thus on average will be expected to have the

*same* MPC as their comparator. If such a model were literally correct, it would make no difference how the stimulus was allocated across people at different levels of permanent income: The MPC at each level of permanent income would be the same. So the “progressivity” of the stimulus payments would make no difference to their effectiveness.

- ii. **Concavity.** Continue to think about the baseline version of the model, but discuss the consequences of the fact that the size of the stimulus payments is *not* trivial. A family of four with a permanent income of \$12,000 would receive \$2400, but so would a family of four with a permanent income of \$1,200,000. What does the *model* say about which of these should have a larger MPC out of their stimulus check? Why?

*Answer:*

Suppose every household has a ‘target’ buffer stock of 2 months’ worth of income. So, the permanently poor family’s target is \$2400, while the permanently rich family’s target is \$240,000. The stimulus check would be a massive increase in the cash-on-hand for the poor family, but a tiny increase for the rich one. The concavity of the consumption function says that the MPC of the permanently poor family will therefore be less than the MPC for the permanently rich family. So, according to the model, making the tax “progressive” actually makes it less effective - when the measure of income is permanent income.

- iii. **Insurance.** Now suppose we abandon the assumption that the government can measure permanent income; it can only measure actual income received, so that is what it uses in making the calculations of who gets how much stimulus. Would that likely make the stimulus more effective or less effective? Would it improve or worsen the distributional equity (‘fairness’) of the stimulus spending?

*Answer:*

The difference between permanent and actual income is of course transitory shocks. Using measured income would mean giving more stimulus, on average, to households who last year experienced negative transitory shocks to their incomes. Those are people who the model says should have run down their buffer stocks and will be at a point where the marginal propensity to consume is higher. So giving more money to such people will improve the effectiveness of the stimulus. Similarly, it would improve the distributional equity: It would be giving money to people who at the moment need it more (have higher marginal utility of cash).

b) The most important difference between the ‘Tractable’ version of the buffer stock model studied earlier in class and the ‘Full’ version presented late in class is in the assumptions about the unemployment process.

- i. Explain the difference between the assumptions about the nature of unemployment in the two models

*Answer:*

- ii. Consider employed consumers in each of the two models at the same level of market resources  $m_t$ , with consumption functions  $c^{e,T}$  and  $c^{e,F}$  in the ‘Tractable’ and ‘Full’ models respectively, but assume there are no unemployment benefits. For a consumer at any given level of market resources  $m_t$ , explain why the drop in spending upon becoming unemployed would be expected to be much larger for the consumer in the ‘tractable’ model than in the ‘full’ model. That is, even if they had the same market resources in period  $t$ ,  $m_t^T = m_t^F = m_t$  and in period  $t + 1$ ,  $m_{t+1} = (m_t - c_t)R$  the decline in consumption  $c^{e,T}(m_t) - c^{u,T}(m_{t+1}) > c^{e,F}(m_t) - c^{u,F}(m_{t+1})$ .

*Answer:*

- iii. What would a more plausible description of the unemployment process look like? (Hint: In each model, what is the probability that an unemployed person will be employed again in the next period? Is that plausible in either case? Describe briefly how an alternative might work in which the probability of an unemployed person getting a job was, say, 25 percent per quarter in an economic expansion, and 15 percent per quarter in a recession. (What other assumptions would you need to make?)

*Answer:*

- c) The world we live in is like the one described in the “more plausible” model sketched in the last question. The remaining questions ask you to think through what such a model would say about the effects of the unemployment benefit extension described in point 2 above. An earlier unemployment benefit extension, passed originally in early April, provided benefits of up to \$600/wk, but expired in July. Suppose that the typical pre-pandemic earnings of someone who became unemployed during the pandemic were \$300/wk, and suppose that they expect that, whenever they do manage to find a new job, it will again pay \$300/wk. Finally, suppose that, during the spring and summer, everyone had a high degree of uncertainty about how long the pandemic-related job losses would remain, but now that vaccines are actually being distributed, everyone expects the crisis to be mostly over by the time the new extension of UI benefits expires in March.
- i. Would you have expected the spending of the initial UI recipients to fall dramatically immediately after the extended UI benefits came to an end in early July? Why or why not?

*Answer:*

The question posited that (1) the benefits they were receiving were actually twice their normal income; and (2) they were highly uncertain when they would get a job again. Under these circumstances, you would expect unemployed persons with even a modest amount of foresight to save a large proportion of their UI checks, so that they would have some funds to rely upon after the generous period of UI benefits came to an end.

- ii. What would you expect the spending path to have looked like for persons who remained unemployed throughout the period from July to December, but become eligible for UI benefits again in December? Explain any difference in the “MPC” out of UI benefits that an empirical economist might calculate for the two episodes.

*Answer:*

Imagine that such a person (1) became unemployed in April; (2) received enhanced UI benefits through July of \$2400/mo;

(3) spent \$1200/mo (equal to their pre-pandemic “permanent income”). Then they would have saved \$4800. If they had continued spending at a rate of \$1200/mo they would have spent their savings by Nov, and would have nothing left by the time the Dec benefits started up. So, you would expect them to have a very high MPC out of the Dec benefits, but quite a low MPC out of the April benefits.

- iii. What does this example say about the role of “structural modeling” in microeconomic modeling?

*Answer:*

The “MPC out of UI benefits” is not a constant of nature, like the speed of light. It ought to depend on the circumstances under which those benefits are received; the health (or otherwise) of the finances of the recipients; their expectations about their future prospects; and other complex aspects of the problem.

## Medium-Length Questions

1. **qModel and Monopoly.** Steve Ballmer, the Chief Executive Officer of Microsoft, was once asked by the Washington Post whether he believed that Microsoft shareholders might benefit from splitting Microsoft up into multiple “Baby Bill” companies each of which would own a copy of the operating system and Microsoft Office programs. Ballmer replied:

“To split up the company and allow two or more competitors to go head-to-head selling similar products would drive the price of the software so low that neither company could make a profit,” Steve Ballmer said.

“I was an economics major,” Ballmer said. “I learned enough of economics to know that If you have two guys selling the same thing and the marginal cost is zero, the price point on that is a well-known economic fact. And yet you have people say that it will increase shareholder value. I would vehemently dispute any notion there would be enhanced shareholder value on breakup.”

- a) Using  $q$  theory, discuss whether this argument is consistent with Microsoft’s claim in court that it is not a monopoly.

*Answer:*

Abel (1981) and Hayashi (1981) showed that marginal  $q$  and average  $q$  are the same only if there is perfect competition at the level of individual firms. Since Ballmer admits that the marginal cost is zero and the marginal price of Windows is positive, he is implicitly admitting that the market’s high value for Microsoft’s stock (its high average  $q$ ) reflects monopoly power. His argument that there is only one equilibrium price (zero) when there is competition is a further admission that Microsoft has a lot of monopoly power.

- b) Assuming that Ballmer is right about the consequences of a splitup, and that the Hayashi/Abel neoclassical  $q$  model is also right, does Microsoft’s current extremely high value for Tobin’s  $q$  imply that the company should be engaging in large amounts of investment to expand its current lines of business? (Hint: Most sales of Windows are to computer manufacturers who install the program on computers that they sell).

*Answer:*

As we discussed, one circumstance in which marginal  $q$  will be very different from average  $q$  is when the firm has monopoly power. In this case the firm’s ongoing operations may be highly profitable (due to its monopoly), but it may have no ability to expand those profits by increasing production, because marginal revenue will be less than marginal cost.



Think of it this way: If Microsoft decided to double the number of copies of Windows that it wanted to sell, how much would it have to cut the price? Since most copies of Windows are sold on new computers, Microsoft would need to cut the price of Windows enough to double sales of new computers. It seems very likely that Microsoft would have to create a huge subsidy to computer manufacturers to make the price of computers fall far enough to double sales. This amounts to charging a negative price for Windows (say, -\$300). Clearly charging a negative price for its main product would not increase Microsoft's profits!

Thus, if both Ballmer and Hayashi/Abel are right, the high value of  $\varphi$  reflects purely monopoly profits, and *marginal*  $\varphi$  is probably quite low, and so the theory does *not* imply that Microsoft should be doing a lot of investment spending *to expand its current lines of business*, i.e. to sell more copies of Windows or Office.

- c) In fact, Microsoft does engage in very large amounts of “investment” spending, in the form of research and development of new programs (and buying existing companies, sometimes companies whose products compete with Microsoft's and sometimes companies who offer products that Microsoft does not offer). Comment about what this high level of investment spending suggests about Microsoft's views about its ability to obtain monopolies in the future.

*Answer:*

High levels of investment today must be justified by high expected revenues in the future. If Microsoft is engaged in developing new software which in the future will have a zero marginal cost but high marginal price, it must by definition be making those investments in the hopes of obtaining more monopoly power in the future (perhaps in new markets) than it has now, or perhaps in hopes of maintaining its current monopoly power.

- d) Discuss what the  $\varphi$  model would suggest about why it might be a rational decision for Microsoft to give away Internet Explorer for free, given that Microsoft has spent hundreds of millions of dollars in developing the program.

*Answer:*

Once again, this is a rational decision if by giving away IE for free, Microsoft either preserves existing monopoly power or creates new monopoly power in the future. Giving away IE for free can be considered as an ‘investment’ which purchases future monopoly power.

To be fair to Microsoft, the software industry is probably a particularly poor place to try to apply  $\varphi$  theory, because since the

marginal cost of production is close to zero, the whole industry is about monopolistic competition, which  $\phi$  theory doesn't handle very well.

## 2. Government and Growth.

Standard growth models ignore the role of government in determining a country's level of income per capita. Yet looking across countries, it seems clear that countries with honest, efficient, rational governments are more prosperous than countries with corrupt, inefficient, and irrational governments.

Suppose we can capture the effect of government efficiency with a term  $e$  in the per-capita production function:

$$f(k, e) = k^\alpha e^\eta \quad (2)$$

where a country with a more efficient government has a higher value of  $e$ . (Assume the population and the level of productivity are normalized at 1, and  $\eta < 1$ ).

Suppose government expenditures translate one-for-one into productive efficiency  $e$ , and assume that the government must satisfy a balanced budget criterion by the use of lump-sum taxes of amount  $\tau$ :

$$e = \tau. \quad (3)$$

For simplicity, suppose that the capital stock is exogenously fixed at  $k = \bar{k}$  and does not depreciate but cannot be augmented by extra saving (there is an endowment of capital).

- a) Calculate the level of taxes that maximizes per-capita after-tax income  $f(k, e) - \tau$  and explain intuitively the reasons for the effects that the parameters have on the optimal choice of government expenditures.

*Answer:*

$$\max_e \bar{k}^\alpha e^\eta - e \quad (4)$$

has FOC with respect to  $e$  of

$$\bar{k}^\alpha \eta e^{\eta-1} = 1 \quad (5)$$

$$e = (\bar{k}^\alpha \eta)^{1/(1-\eta)} \quad (6)$$

which says that expenditures/taxes will be higher when 1) the capital stock is higher (because there is more productivity to “enhance” by government expenditures; 2) when the coefficient on capital is higher ( $\alpha$  is larger), for the same reasons; 3)  $\eta$  is larger, because the larger is  $\eta$  the smaller is the rate at which government efficiency improvements have diminishing marginal productivity effects.

- b) Now suppose this economy suffers from corruption. Specifically, some of the tax revenues that are raised do not get spent on efficient government

expenditures but instead are wasted. Again using  $e$  for the amount of efficient expenditures, and again imposing the balanced budget constraint, the new level of after-tax income is

$$f(\bar{k}, e) - \underbrace{\tau}_{=e\chi} \quad (7)$$

where  $\chi > 1$  measures the degree of corruption. Thus, taxes paid  $\tau$  exceed expenditures  $e$  (the extra taxes represent waste and corruption). Now calculate the level of  $e$  that maximizes after-tax per capita output. Is it higher or lower than in the honest economy (where  $\chi = 1$ )? Why? Is there a cost to the economy beyond the fact that the tax burden is higher by amount  $\chi$ ? Why?

*Answer:*

The FOC are

$$\bar{k}^\alpha \eta e^{\eta-1} = \chi \quad (8)$$

$$e = \left( \frac{\bar{k}^\alpha \eta}{\chi} \right)^{1/(1-\eta)} \quad (9)$$

and since  $\chi > 1$  this is clearly a smaller number than the  $e$  that was optimal for the honest economy. Notice that after-tax income is lower for *two* reasons: 1) with a lower  $e$  the economy produces less output; 2) with a higher  $\chi$  the effective tax rate is higher. So pretax income is less while taxes are higher.

- c) [Hall and Jones \(1999\)](#) find that, looking across countries in the world, only a very small proportion of the differences in output per capita are explained by differences in capital, natural resources, or other observable factors of production. Discuss how this finding might be related to the modeling choice above to assume a fixed level of capital  $\bar{k}$ . Speculate on whether permitting capital accumulation would be likely to reinforce or to undermine the results from the baseline model.

*Answer:*

The [Hall and Jones \(1999\)](#) finding suggests that capital accumulation is not one of the main influences that make some countries rich and others poor, so an intensive and complex study of optimal intertemporal allocation decisions may not yield much insight about the process of economic growth. While it is not clear from the Hall and Jones finding whether differences in government efficiency *are* important, a growing literature does suggest that differences in the honesty and efficiency of government across countries are quite important.

Permitting capital accumulation would likely reinforce but not change the logic outlined above; in the more efficient economy, the

incentives to save (returns on capital) would be higher, and therefore it is likely that there would be more saving. This effect could act as a “multiplier” on the importance of government efficiency.

3. **Lucas Asset Pricing with CRRA Utility.** Consider a [Lucas \(1978\)](#) model in which, instead of having logarithmic utility, consumers have CRRA utility:  $u(c) = (1 - \rho)^{-1} c^{1-\rho}$ .

- a) Explain the set of assumptions that [Lucas \(1978\)](#) makes that allow him to use the partial equilibrium consumption Euler equation

$$u'(c_t^i) = \beta \mathbb{E}_t^i [u'(c_{t+1}^i) \mathbf{R}_{t+1}] \quad (10)$$

to obtain a general equilibrium asset-pricing equation of the form:

$$P_t = \beta \mathbb{E}_t \left[ \left( \frac{u'(d_{t+1})}{u'(d_t)} \right) (P_{t+1} + d_{t+1}) \right]. \quad (11)$$

*Answer:*

For all answers, see [LucasAssetPrice](#)

- b) Define an object

$$M_{t,t+n} = \beta^n \left( \frac{u'(d_{t+n})}{u'(d_t)} \right) \quad (12)$$

called the ‘stochastic discount factor.’ Analogously, define the cumulative SDF between  $t$  and  $t + 2$  as  $M_{t,t+2} \equiv M_{t,t+1} M_{t+1,t+2}$ , and so on. Using these definitions, explain how to use this object to obtain an equation relating the current asset price to expected future prices via

$$P_t = \mathbb{E}_t [M_{t,t+1} d_{t+1} + M_{t,t+2} d_{t+2} + M_{t,t+3} d_{t+3} + \dots]. \quad (13)$$

*Answer:*

- c) Use (11) to show that if the utility function is of the CRRA form and then the pricing equation reduces to

$$P_t = \beta d_t^\rho \mathbb{E}_t [d_{t+1}^{-\rho} (P_{t+1} + d_{t+1})] \quad (14)$$

*Answer:*

- d) Using (14), show that in the logarithmic utility case where  $\rho = 1$ , the price

eventually reduces to

$$\begin{aligned}
 P_t &= d_t \left( \frac{\beta}{1 - \beta} \right) \\
 &= d_t \left( \frac{1}{1/\beta - 1} \right) \\
 &= d_t \left( \frac{1}{1 + \vartheta - 1} \right) \\
 &= \frac{d_t}{\vartheta}
 \end{aligned}$$

*Answer:*

- e) Now assume that under CRRA utility with  $\rho > 1$ ,  $P_t/d_t^\rho$  is a constant, and assume that the dividend process is white noise (that is,  $\mathbb{E}_t[d_{t+1}] = \mathbb{E}_t[d_{t+2}] = \mathbb{E}_{t+1}[d_{t+2}] \equiv \bar{d} \dots$ ). Under the assumption that the white noise process for dividends is  $d_{t+n} \sim \mathcal{N}(0, \sigma^2) \forall n > 0$ , derive the equilibrium value for  $P/d^\rho$  and discuss the time series relationships between  $P$  and  $d$  and explain how they depend on the value of  $\rho$ .

*Answer:*

See the appendix to [LucasAssetPrice](#).

## References

- HALL, ROBERT E., AND CHARLES I. JONES (1999): “Why Do Some Countries Produce So Much More Output per Worker than Others?,” *Quarterly Journal of Economics*, CXIV, 83–116.
- LUCAS, ROBERT E. (1978): “Asset Prices in an Exchange Economy,” *Econometrica*, 46, 1429–1445, Available at <http://www.jstor.org/stable/1913837>.