Growth Empirics for Buffer Stock Economies. Answer the following questions using a tractable buffer stock saving model like the one discussed in TractableBufferStock.

1. Consider two economies A and B populated by buffer stock consumers. The economies and parameters are identical in every respect except that the growth of wages are respectively G^A and G^B, where $\mathrm{G}^A < \mathrm{G}^B$. Each economy is in its steady state. Suppose an economist studying the two economies cannot directly observe the growth factor parameter (though the consumers living in the economies can); but the economist can observe the average growth rate outcome for each economy, $\Delta \log Y_i$, and suppose that the reason $\mathrm{G}^B > \mathrm{G}^A$ is that economy B has a higher level of education. The economist therefore performs a regression of the form:

$$\Delta \log Y_i \quad = \quad \alpha_0 + \alpha_1 E_i \tag{1}$$

where $i \in A, B$ across the two economies and therefore can translate the measurable difference in educational attainment E into its implication for economic growth by constructing $\mathbb{E}[\Delta \log Y_i] = \hat{\alpha}_0 + \hat{\alpha}_1 E_i$ for each country. Suppose the economist then performs a Campbell-Mankiw type regression:

$$\Delta \log C_i \quad = \quad \mu_0 + \mu_1 \, \mathbb{E}[\Delta \log Y_i] \tag{2}$$

on the data from the two economies. What coefficient estimate μ_1 would the economist find? Why? Would the coefficient μ_1 reflect what Campbell and Mankiw interpreted it as reflecting? Why or why not? Relate your comments to the log-linearized Euler equation.

Answer:

The education variable will correctly predict income growth since it is perfectly correlated with income growth across the two countries.

In a buffer-stock economy, the growth rate of aggregate consumption matches the growth rate of aggregate income, so the coefficients will be $\mu_0 = 0$ and $\mu_1 = 1$.

But in Campbell and Mankiw's model, the regression coefficient μ_1 was supposed to reflect the proportion of households who set consumption exactly equal to income in each period, and did not engage in intertemporally optimizing behavior. Here, *all* consumers are interetemporally optimizing; none are Keynesian $C = Y$ consumers. So the coefficient does *not* correctly reveal the fraction of households who fail to intertemporally optimize, because that fraction is zero.

The log-linearized Euler equation says that $\Delta \log C_{t+1} = \rho^{-1}(\mathsf{r} - \vartheta)$ while here we have $\Delta \log C_{t+1} = \gamma$. Thus, the results violate the log-linearized Euler equation. This can be interpreted as reflecting the *endogeneity* of higher-order terms in the Euler equation.

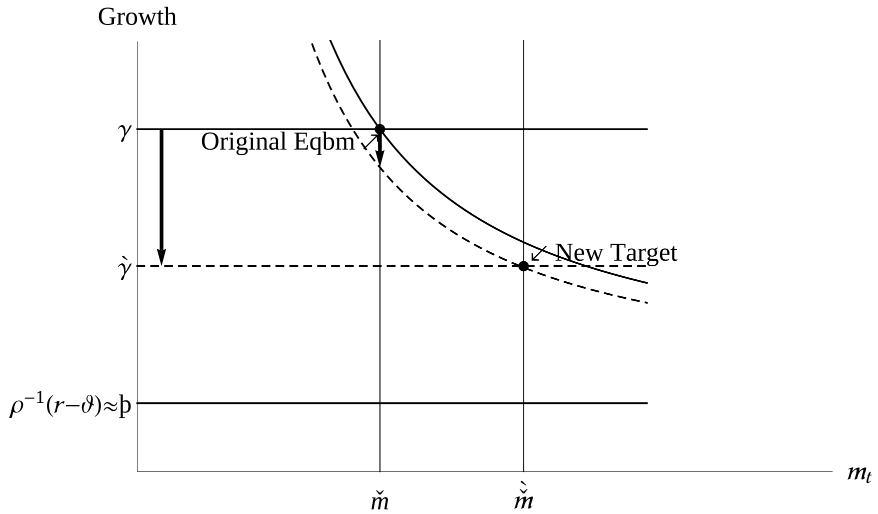
2. Now suppose the economist has data on only one country, but that data covers a long span of time. In particular, suppose that the country went through a period

of fast growth for many years during the first half of the data sample, and then a period of slow growth for the second half of the data sample (as happened in the post-World-War-2 history of most of Western Europe: a period of rapid growth from 1947-1974, followed by much slower growth thereafter). Assume that during the period of rapid income growth, everyone expected that rapid income growth to continue indefinitely; and when the economy shifts down to slower income growth, everyone perceives the slowdown immediately and adjusts their spending accordingly. Answer the following questions about this economy:

a) Show what happens to the consumption growth diagram depicting the relation between m_t and expected consumption growth when the growth rate abruptly changes (e.g., when the economy permanently and unexpectedly shifts from rapid growth to slower growth). (Show both any curve shifts, and, using arrows, show the path of consumption growth as it evolves toward its new steady state).

Answer:

The actual numerical solution is shown in the figure below:



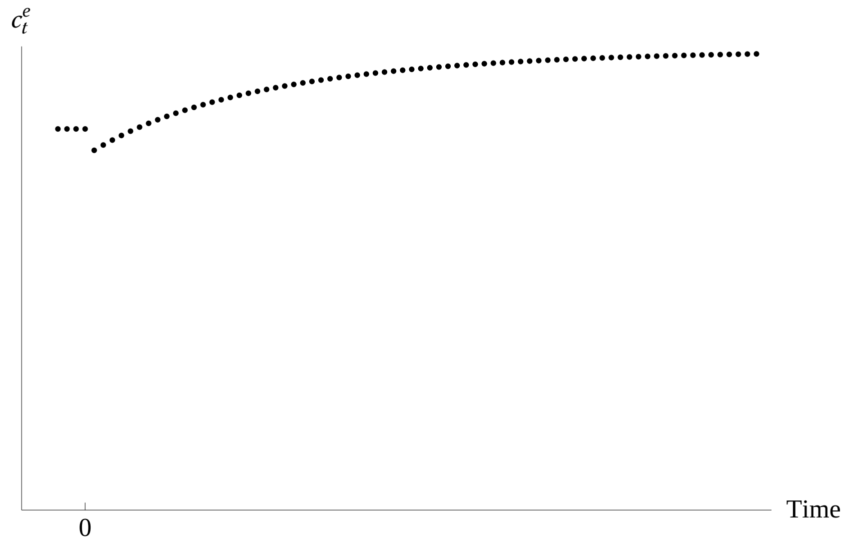
and the major shift is of course in the expected growth rate of permanent income γ. As you can see the expected consumption growth curve shifts down a little bit; but this is a result of subtle higher-order effects that I did not emphasize or explain in class or in the handout, and so if you just assumed that the expected consumption growth curve remained fixed you will get full credit for your answer (so long as, conditional on that assumption, your analysis was correct).

Your analysis should have shown that after the initial decline in consumption growth, arrows should be drawn on the dashed line in the above diagram showing the growth rate evolving down toward its new equilibrium.

b) Draw diagrams showing the path of the level of c and of C in this economy following the change in growth . (Recall that c is the ratio of consumption to permanent wage income, while C is the absolute level of per-capita consumption. For simplicity, you may wish to assume that the initial growth rate is zero, and the new growth rate is negative.).

Answer:

The ratio of consumption to permanent labor income looks like:



While the level looks similar but with the portion after the jump at time zero rotated clockwise downward.

c) Suppose, again, that the econometrician cannot directly observe the economy's regime change, but suppose there is a survey of consumers that shows households' expectations of income growth. Assume that the data begin in period $-T$ while the regime change happens at date 0; beginning in date $-T$ the economy was at its target ratio, and by n periods after the change the economy has mostly settled down to its new target ratio (the sample ends at period $+T$). Draw a diagram showing the path over time of consumers' expectations of income growth and the path of actual (not expected) consumption growth. Now suppose the econometrician does a regression of the form

$$\Delta \log C_{t+1} \quad = \quad \mu_0 + \mu_1 \, \mathbb{E}_t[\Delta \log Y_{t+1}]. \tag{3}$$

Discuss how the econometrician's results will depend on the sample period used for estimating this equation. In particular, consider all these possibilities:

i. A sample that includes only the early and late parts of the history (say, dates $-T$ to $-T/2$ and $T/2$ to T) where $T/2 >> n$

ii. A sample that includes only the middle parts of the history (say, dates $-n/2$ to $+n/2$), but excludes the date 0 when the regime change occurred.

iii. A sample that includes the entire history (except period 0) when that history is long $T >> n$

iv. A sample that includes the entire history (except period 0) when that history is short $T < n$

Answer:

Expected income growth first stays constant before date 0, drops discretely to a lower level at date 0, and remains at the lower level after that; actual consumption growth first stays at the same level as the expected income growth before date 0, decreases gradually over a period of time starting from date 0, and finally reaches the lower level of income growth rate and remains at that level after that.

i. This sample essentially is like the comparison in the early part of the problem between two economies with different equilibrium growth that are both in their steady states: We will get $\mu_0 \approx 0$ and $\mu_1 \approx 1$.

ii. During the first part of the sample, we have consumption growth matching income growth except for period 0 in which there is a big drop in consumption which will be a big negative error in the equation. Thereafter, consumption growth will be slightly above income growth as the consumer builds up consumption toward the higher c^e target ratio. So the average consumption growth in the later period is not as much lower as the average income growth is; this means that our coefficient on income growth will be biased toward zero (too small), though it will still have a positive coefficient. Note that if we were to include date 0 in the sample, the results are ambiguous – think of the case where only periods -1, 0, and 1 are included; in that case in period -1 we would have consumption growth matching expected income growth, in period 0 we would have consumption growth hugely below expected income growth (because the regime change is a negative surprise), and in period 1 we would have consumption growth slightly exceeding a much lower level of income growth. It is *possible* that average consumption growth in periods -1 and 0 will be lower than in period 1, while expected income growth in those periods will have been higher than in period 1, in which case we would actually get a negative coefficient on income growth. (This latter scenario was not asked in the question and is provided only to clarify the logic of the model; you did not need to describe it to get credit for your answer to the question).

iii. If the history is long, it will be dominated by the steady-state results and we should expect the coefficients to approach the unbiased case

iv. If the history is short, it will be dominated by the transition results and we should expect the coefficients to approach the biased case