Aggregate Consumption Dynamics. Consider the consumption Euler equation when there is both income uncertainty and rate-of-return uncertainty:

$$1 = \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}(C_{t+1}/C_t)^{-\rho}] \tag{1}$$

1. Using the facts that $(1+\epsilon)^\gamma \approx 1+\gamma\epsilon$ and that $\eta/K \approx \log[(K+\eta)/K]$ for small ϵ and η/K, show that this equation can be approximated by

$$1 \approx \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}(1-\rho\Delta\log C_{t+1})] \tag{2}$$

   Answer:

$$\begin{aligned}
1 &= \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}(C_{t+1}/C_t)^{-\rho}] && (3)\\
&= \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}(\frac{C_t + C_{t+1} - C_t}{C_t})^{-\rho}] && (4)\\
&= \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}(1+(C_{t+1}-C_t)/C_t)^{-\rho}] && (5)\\
&\approx \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}(1+\log[C_{t+1}/C_t])^{-\rho}] && (6)\\
&\approx \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}(1-\rho\Delta\log C_{t+1})] && (7)
\end{aligned}$$

   where the step from (5) to (6) holds by the second fact above assuming that $\eta = C_{t+1} - C_t$ is 'small' relative to C_t.

2. Now define $\mathbb{E}_t[\mathsf{r}_{t+1}] \equiv \log \mathbb{E}_t[\mathsf{R}_{t+1}]$ and show that equation (2) implies

$$\Delta \log C_{t+1} = \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta) + \epsilon_{t+1} \tag{8}$$

   if the covariance between R_{t+1} and C_{t+1} is approximately zero. (Hint: you will need to use the formula $\mathbb{E}[xy] = \mathbb{E}[x]\,\mathbb{E}[y] + \mathrm{cov}(x,y)$.) Explain the reason why the assumption of rational expectations implies that no variable known at time t can be correlated with ϵ_{t+1}.

   Answer:

$$\begin{aligned}
1 &\approx \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}(1-\rho\Delta\log C_{t+1})] && (9)\\
&= \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}]\, \mathbb{E}_t[(1-\rho\Delta\log C_{t+1})] && (10)\\
\frac{1}{\beta\, \mathbb{E}_t[\mathsf{R}_{t+1}]} &\approx 1 - \rho\, \mathbb{E}_t[\Delta \log C_{t+1})] && (11)\\
-\log \beta\, \mathbb{E}_t[\mathsf{R}_{t+1}] &\approx -\rho\, \mathbb{E}_t[\Delta \log C_{t+1})] && (12)\\
\mathbb{E}_t[\Delta \log C_{t+1}] &\approx \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta) && (13)\\
\Delta \log C_{t+1} &\approx \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta) + \epsilon_{t+1} && (14)
\end{aligned}$$

3. Using aggregate data for the U.S., Campbell and Mankiw (1989) find a set of lagged variables that are useful for predicting interest rates and income growth, and then use these variables to construct empirical measures of $\mathbb{E}_t[\mathsf{r}_{t+1}]$ and $\mathbb{E}_t[\Delta \log Y_{t+1}]$. They then estimate an equation of the following form:

$$\Delta \log C_{t+1} = \alpha_0 + \alpha_1\, \mathbb{E}_t[\mathsf{r}_{t+1}] + \alpha_2\, \mathbb{E}_t[\Delta \log Y_{t+1}] + \epsilon_{t+1}$$

4. If the approximations just derived are good and aggregate consumption is determined by a representative agent maximizing utility from aggregate consumption, what values should Campbell and Mankiw have estimated for α_0, α_1, and α_2? (Express your results in terms of the following preference parameters: ϑ is the time preference rate ($\beta = \frac{1}{1+\vartheta}$), and ρ is the coefficient of relative risk aversion). What values did Campbell and Mankiw actually find for α_1 and α_2? What was their explanation for these results?

   Answer:

   The Euler equation for consumption growth in a world without uncertainty is approximated by

   $$\Delta \log C_{t+1} \quad \approx \quad \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta). \tag{15}$$

   Thus, Campbell and Mankiw should have estimated $\alpha_0 = -\vartheta\rho^{-1}$, $\alpha_1 = \rho^{-1}, \alpha_2 = 0$.

   In fact, their estimates were $\alpha_1 \approx 0$ and $\alpha_2 \approx .5$. They argue that the finding that $\alpha_2 = 0.5$ can be explained by assuming that half of consumption is done by Hall-style permanent income consumers and half by "Keynesian" or 'rule-of-thumb' consumers who set $C = Y$ in every period. They basically ignore the finding that $\alpha_1 \approx 0$. The literature since their paper has also largely ignored the fact that it is difficult to find any robust relationship between consumption growth and interest rates.

5. Carroll, Fuhrer, and Wilcox (1994) (CFW) estimate an equation of the form

   $$\Delta \log C_{t+1} \quad = \quad \alpha_0 + \alpha_1 \mathbb{E}_t[\mathsf{r}_{t+1}] + \alpha_2 \mathbb{E}_t[\Delta \log Y_{t+1}] + \alpha_3 S_t + \epsilon_{t+1}$$

   where S_t is 'consumer sentiment' as measured by a monthly survey that asks a random sample of households about their assessment of the current and future state of the economy. S_t tends to be high when the economy is performing well and to be low when the economy is doing badly. CFW use consumer sentiment and other variables to construct expectations for interest rates and income growth, and find a positive and statistically significant coefficient α_3. Are these results consistent with the Campbell-Mankiw model? Explain.

   Answer:

   Consider the two classes of consumers in the Campbell-Mankiw model. The shock to consumption for the PIH consumers must, by the assumption of rational expectations for these consumers, be uncorrelated with any variable known at time t. Since consumers certainly knew their own sentiments at time t, S_t must be uncorrelated with ϵ_t.

   The 'rule-of-thumb' consumers are supposed to set consumption equal to income in each period. But sentiment is being used to help forecast income growth, so any correlation between S_t and consumption growth that happens because S_t can help forecast $\Delta \log Y_{t+1}$ will be captured through the coefficient on $\Delta \log Y_{t+1}$. Thus, consumption growth of the 'rule-of-thumb' consumers should not respond directly to S_t either.

Therefore, neither class of consumers in the economy is supposed to respond to S_t, and so the Campbell-Mankiw model implies that α_3 should be zero.

6. We learned in class that the log-linear approximation to the Euler equation derived in parts (a) and (b) misses the effect of precautionary saving. When a precautionary saving motive is included the approximate Euler equation becomes:

$$\Delta \log C_{t+1} \approx \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta) + (\rho/2)\mathrm{var}_t(\Delta \log C_{t+1}) \tag{16}$$

Discuss whether the CFW empirical results might be caused by precautionary saving effects.

Answer:

The second order approximation to the Euler equation (16) shows that there is a term missing from the approximation used by Campbell and Mankiw, namely the variance component. In order to explain the CFW results, it would have to be the case that when S_t is high, consumers believe that there is a lot of uncertainty about the future, and therefore they have cut back on their level of consumption (but expect faster consumption growth).

The problem is that it seems much more plausible to think that uncertainty and precautionary saving are *low* when consumer sentiment S_t is high, as now. This would imply a negative, not a positive coefficient on S_t. Thus the traditional model of precautionary saving *cannot* explain the positive coefficient on S_t.

7. The model above was solved under the assumption of time-separable utility. When the model is solved under the assumption that there is 'habit formation' in utility (so that consumers 'get used to' a given level of consumption), it turns out that the Euler equation for consumption growth can be approximated by

$$\Delta \log C_{t+1} = \alpha_0 + \alpha_1 \Delta \log C_t + \epsilon_{t+1}$$

Might the CFW results be explained in a model in which habit formation in consumption is important? (I am looking for an informal discussion, not an algebraic derivation).

Answer:

In a world with both habit formation and uncertainty it might be possible to explain the CFW results as follows. Imagine that in period t consumer sentiment deteriorates sharply. In a world without habit formation, current period consumption should deteriorate simultaneously and by the full amount justified by the decline in sentiment. But in a world with habit formation, it is not optimal to do the whole downward adjustment in consumption instantly; instead, households may gradually reduce consumption over time until it eventually reaches its new lower equilibrium value. In that case one might find the result that consumption growth is high for several periods following an increase in sentiment, or vice versa.

References

Campbell, John Y., and N. Gregory Mankiw (1989): "Consumption, Income, and Interest Rates: Reinterpreting the Time-Series Evidence," in *NBER Macroeconomics Annual, 1989*, ed. by Olivier J. Blanchard, and Stanley Fischer, pp. 185–216. MIT Press, Cambridge, MA, http://www.nber.org/papers/w2924.pdf.

Carroll, Christopher D., Jeffrey C. Fuhrer, and David W. Wilcox (1994): "Does Consumer Sentiment Forecast Household Spending? If So, Why?," *American Economic Review*, 84(5), 1397–1408.