

**Growth Empirics for Buffer Stock Economies.** Answer the following questions using a tractable buffer stock saving model like the one discussed in **TractableBufferStock**.

1. Consider two economies  $A$  and  $B$  populated by buffer stock consumers. The economies and parameters are identical in every respect except that the growth rates of wages are respectively  $G^A$  and  $G^B$ , where  $G^A < G^B$ . Each economy is in its steady state. Suppose an economist studying the two economies cannot directly observe the growth rate parameter (though the consumers living in the economies can); but the economist can observe the average growth rate outcome for each economy,  $\Delta \log Y_i$ , and suppose that the reason  $G^B > G^A$  is that economy  $B$  has a higher level of education. The economist therefore performs a regression of the form:

$$\Delta \log Y_i = \alpha_0 + \alpha_1 E_i \quad (1)$$

where  $i \in A, B$  across the two economies and therefore can translate the measurable difference in educational attainment  $E$  into its implication for economic growth by constructing  $\mathbb{E}[\Delta \log Y_i] = \hat{\alpha}_0 + \hat{\alpha}_1 E_i$  for each country. Suppose the economist then performs a Campbell-Mankiw type regression:

$$\Delta \log C_i = \mu_0 + \mu_1 \mathbb{E}[\Delta \log Y_i] \quad (2)$$

on the data from the two economies. What coefficient estimate  $\mu_1$  would the economist find? Why? Would the coefficient  $\mu_1$  reflect what Campbell and Mankiw interpreted it as reflecting? Why or why not? Relate your comments to the log-linearized Euler equation.

2. Now suppose the economist has data on only one country, but that data covers a long span of time. In particular, suppose that the country went through a period of fast growth for many years during the first half of the data sample, and then a period of slow growth for the second half of the data sample (as happened in the post-World-War-2 history of most of Western Europe: a period of rapid growth from 1947-1974, followed by much slower growth thereafter). Assume that during the period of rapid income growth, everyone expected that rapid income growth to continue indefinitely; and when the economy shifts down to slower income growth, everyone perceives the slowdown immediately and adjusts their spending accordingly. Answer the following questions about this economy:
  - a) Show what happens to the consumption growth diagram depicting the relation between  $m_t$  and expected consumption growth when the growth rate abruptly changes (e.g., when the economy permanently and unexpectedly shifts from rapid growth to slower growth). (Show both any curve shifts, and, using arrows, show the path of consumption growth as it evolves toward its new steady state).
  - b) Draw diagrams showing the path of the level of  $c$  and of  $C$  in this economy following the change in growth rates. (Recall that  $c$  is the ratio of consump-

tion to permanent wage income, while  $C$  is the absolute level of per-capita consumption. For simplicity, you may wish to assume that the initial growth rate is zero, and the new growth rate is negative.).

- c) Suppose, again, that the econometrician cannot directly observe the economy's regime change, but suppose there is a survey of consumers that shows households' expectations of income growth. Assume that the data begin in period  $-T$  while the regime change happens at date 0; beginning in date  $-T$  the economy was at its target ratio, and by  $n$  periods after the change the economy has mostly settled down to its new target ratio (the sample ends at period  $+T$ ). Draw a diagram showing the path over time of consumers' expectations of income growth and the path of actual (not expected) consumption growth. Now suppose the econometrician does a regression of the form

$$\Delta \log C_{t+1} = \mu_0 + \mu_1 \mathbb{E}_t[\Delta \log Y_{t+1}]. \quad (3)$$

Discuss how the econometrician's results will depend on the sample period used for estimating this equation. In particular, consider all these possibilities:

- i. A sample that includes only the early and late parts of the history (say, dates  $-T$  to  $-T/2$  and  $T/2$  to  $T$ ) where  $T/2 \gg n$
- ii. A sample that includes only the middle parts of the history (say, dates  $-n/2$  to  $+n/2$ ), but excludes the date 0 when the regime change occurred.
- iii. A sample that includes the entire history (except period 0) when that history is long  $T \gg n$
- iv. A sample that includes the entire history (except period 0) when that history is short  $T < n$

## References