Name ______________________

Final Exam
180.603
Fall, 2023
Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up.* The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I. Short Questions.

1. **Imperfect Capital Markets and Monetary Policy in 2008.**

a) Write down the condition that determines whether an investment is undertaken in the model of imperfect capital markets discussed in class, and indicate the signs with which each variable affects the wedge between r and the external cost of capital for firms that invest.

Answer:

The condition in the lecture notes is that investment occurs only if:

$$\gamma \quad > \quad 1 + \mathsf{r} + A(c, \mathsf{r}, W, \gamma) \tag{1}$$

where

$$\gamma \quad - \quad \text{expected return factor} \tag{2}$$
$$W \quad - \quad \text{entrepreneur's endowment of wealth} \tag{3}$$
$$c \quad - \quad \text{cost of monitoring} \tag{4}$$

b) Explain how you might interpret the financial crisis that erupted in 2008 as an increase in the cost of monitoring in this model.

Answer:

A good answer would mention that one interpretation of the financial crisis was that it occurred when investors suddenly realized that investments that they had thought they understood (like mortgage-backed securities) were much harder to understand and monitor than they had thought. This can be interpreted as an increase in c. Similar interpretations can be applied to all of the "toxic assets" which were made toxic by the fact that the cost of predicting and understanding their properties was revealed to be much higher than investors had anticipated.

c) If all investment in the economy was determined by a model like this one, explain how a monetary authority like the Fed, which can control r, could offset any effect on investment from the increase in the cost of monitoring.

Answer:

The point here is that, by reducing r enough, the Fed might be able to boost the set of investments that satisfy (1) even with a higher level of c. To the extent that an important part of the crisis reflected a collapse in physical investment, the Fed in principle had a tool to counteract that.

d) Discuss the practical limitations that the Fed might face in pursuing such a response to a financial crisis, if its influence over the economy is restricted to affecting the short-term nominal interest rate. Explain why these limitations might have led the Fed to intervene in markets for other assets (like long-term Treasuries).

Answer:

> Investment spending was not the only component of economic activity that fell dramatically when the crisis came along. Household consumption spending also plummeted. If the decline in consumption was caused by an increase in uncertainty, and if increase in c was large enough, then a decline in short term interest rates to zero might not be enough to restore either investment or consumption. Long-term Treasury securities affect long-term interest rates, which might affect a broader set of investment and consumption decisions than short-term rates, and so could provide more powerful stimulus than would be available from the Fed's usual control only over short-term rates.

e) Suppose the Fed affects interest rates enough to restore investment to its pre-crisis level despite a higher cost of monitoring. What side effects might such an intervention have? Discuss whether, in the context of other developments around 2008-2009, those side effects were likely to be desirable or undesirable.

Answer:

> Possibly the amount of reduction in either short- or both short- and long-term interest rates sufficient to restore investment spending could either overstimulate or understimulate other components of spending (like household consumption). If total spending was overstimulated, the result could be an increase in inflation (arguably desirable, since inflation was well below even the low target rate of 2 percent). If consumption did not respond very much to the low rates, it is possible that low rates might fuel bubbles in markets for other assets (like, say, the stock market). Since prices in many asset markets collapsed in the crisis, restoring those markets might have been desirable.

2. Explain the two mechanisms by which representative agent DSGE models typically achieve fluctuations in consumption expenditures, and discuss the reasons why each of these mechanisms is problematic empirically.

   Answer:

   See `RBC-Prescott`.

3. Consider a tropical economy with two kinds of firms. One firm produces garments in indoor factories, and its production is unaffected by the weather. The other firm produces vegetables and its productivity varies from year to year depending on the intensity of the rainy season. Assuming that investors from outside the country are prohibited from owning shares of the two firms, what would the Lucas asset pricing model imply about which firm's price/dividend ratio will be higher?

Why? Suppose global warming intensifies the size of weather fluctuations from year to year. What would the model imply about the effects of global warming on the firms' relative share prices?

Answer:

> There is no analytical difference between this example and the example given in class comparing firms with differing degrees of correlation in their earnings and the business cycle: The firm whose variability in productivity is greater will have a lower share price. The increase in variability associated with global warming should reduce the share price of the agricultural firm because it increases that firm's riskiness.

Part II. Medium Length Questions.

1. **Government and Growth.**

   Standard growth models ignore the role of government in determining a country's level of income per capita. Yet looking across countries, it seems clear that countries with honest, efficient, rational governments are more prosperous than countries with corrupt, inefficient, and irrational governments.

   Suppose we can capture the effect of government efficiency with a term e in the per-capita production function:

   $$f(k,e) = k^{\alpha}e^{\eta} \tag{5}$$

   where a country with a more efficient government has a higher value of e. (Assume the population and the level of productivity are normalized at 1, and $\eta < 1$).

   Suppose government expenditures translate one-for-one into productive efficiency e, and assume that the government must satisfy a balanced budget criterion by the use of lump-sum taxes of amount τ:

   $$e = \tau. \tag{6}$$

   For simplicity, suppose that the capital stock is exogenously fixed at $k = \bar{k}$ and does not depreciate but cannot be augmented by extra saving (there is an endowment of capital).

   a) Calculate the level of taxes that maximizes per-capita after-tax income $f(k,e) - \tau$ and explain intuitively the reasons for the effects that the parameters have on the optimal choice of government expenditures.

      Answer:

      $$\max_{e} \bar{k}^{\alpha}e^{\eta} - e \tag{7}$$

      has FOC with respect to e of

      $$\bar{k}^{\alpha}\eta e^{\eta-1} = 1 \tag{8}$$

      $$e = \left(\bar{k}^{\alpha}\eta\right)^{1/(1-\eta)} \tag{9}$$

      which says that expenditures/taxes will be higher when 1) the capital stock is higher (because there is more productivity to "enhance" by government expenditures; 2) when the coefficient on capital is higher (α is larger), for the same reasons; 3) η is larger, because the larger is η the smaller is the rate at which government efficiency improvements have diminishing marginal productivity effects.

   b) Now suppose this economy suffers from corruption. Specifically, some of the tax revenues that are raised do not get spent on efficient government expenditures but instead are wasted. Again using e for the amount of efficient expenditures, and again imposing the balanced budget constraint, the new level of after-tax income is

$$f(\bar{k}, e) - \underbrace{\tau}_{=e\chi} \tag{10}$$

where $\chi > 1$ measures the degree of corruption. Thus, taxes paid τ exceed expenditures e (the extra taxes represent waste and corruption). Now calculate the level of e that maximizes after-tax per capita output. Is it higher or lower than in the honest economy (where $\chi = 1$)? Why? Is there a cost to the economy beyond the fact that the tax burden is higher by amount χ? Why?

Answer:

The FOC are

$$\begin{aligned} \bar{k}^\alpha \eta e^{\eta-1} &= \chi & (11)\\ e &= \left(\frac{\bar{k}^\alpha \eta}{\chi}\right)^{1/(1-\eta)} & (12) \end{aligned}$$

and since $\chi > 1$ this is clearly a smaller number than the e that was optimal for the honest economy. Notice that after-tax income is lower for *two* reasons: 1) with a lower e the economy produces less output; 2) with a higher χ the effective tax rate is higher. So pretax income is less while taxes are higher.

c) Hall and Jones (1999) find that, looking across countries in the world, only a very small proportion of the differences in output per capita are explained by differences in capital, natural resources, or other observable factors of production. Discuss how this finding might be related to the modeling choice above to assume a fixed level of capital $\bar{k}$. Speculate on whether permitting capital accumulation would be likely to reinforce or to undermine the results from the baseline model.

Answer:

The Hall and Jones (1999) finding suggests that capital accumulation is not one of the main influences that make some countries rich and others poor, so an intensive and complex study of optimal intertemporal allocation decisions may not yield much insight about the process of economic growth. While it is not clear from the Hall and Jones finding whether differences in government efficiency *are* important, a growing literature does suggest that differences in the honesty and efficiency of government across countries are quite important.

Permitting capital accumulation would likely reinforce but not change the logic outlined above; in the more efficient economy, the incentives to save (returns on capital) would be higher, and therefore it is likely that there would be more saving. This effect could act as a “multiplier” on the importance of government efficiency.

Part III. Long Question.

Brock and Mirman (1972) Multiplied

A large body of recent research has provided support for the old Keynesian idea that, at least under certain conditions,[1] the amount of aggregate output depends partly on the amount of 'aggregate demand.' Under such conditions, any shock that results in a change in a component of 'aggregate demand' will have a 'multiplier' effect (that is, the resulting change in output will be greater than the change in the component of aggregate demand).

Krueger, Mitman, and Perri (2016) propose that the role of a 'consumption multiplier' can be approximately captured by augmentation of the usual productivity term in the production function with a component that is increasing in the level of consumption:

$$\mathrm{F}_t(K_t, \mathrm{L}_t) \;=\; C_t^{\omega} Z_t K_t^{\alpha} \mathrm{L}_t^{1-\alpha} \tag{13}$$

where $0 \leq \omega < 1$ determines the size of the multiplier, and $A_t = C_t^{\omega} Z_t$ would be measured as the level of 'productivity' using conventional approaches. (We might call call Z 'primitive' or 'structural' productivity, as it is what would be manifested if not for the distorting effects of consumption's multiplier).

This question explores the implications of this production function in a growth model in which consumers are not aware of the existence of the multiplier; instead they interpret any changes in output that are attributable to the multiplier as reflecting the usual mysterious shocks to aggregate productivity that drive many other business cycle models. We assume that the 'multiplier effect' takes one time period to manifest itself (most attempts to provide deeper foundations for multipliers would suggest even longer lags).

We examine these ideas in a `BrockMirman` model. Thus, our consumers perceive their problem to be

$$\max \sum_{n=0}^{\infty} \beta^n \log C_{t+n} \tag{14}$$

s.t.

$$Y_t \;=\; A_t K_t^{\alpha} \tag{15}$$

$$K_{t+1} \;=\; Y_t - C_t \tag{16}$$

with Bellman equation

$$\mathrm{V}_t(K_t) = \max_{C_t} \; \log C_t + \beta \mathrm{V}_{t+1}(A_t K_t^{\alpha} - C_t). \tag{17}$$

However, in truth the level of income is given by

$$Y_t \;=\; C_{t-1}^{\omega} Z_t K_t^{\alpha} \tag{18}$$

(questions begin on the next page)

[1]Most notably, when interest rates are stuck at the zero lower bound

1. Show that the consumption function will be

$$C_t = \overbrace{(1-\alpha\beta)}^{\equiv\kappa} Y_t \tag{19}$$

   Answer:

   > Consumers perceive themselves to be solving a perfectly standard BrockMirman model, and so they will behave according to its solution. See the handout for the derivation of the consumption function for the that model.

   For the rest of the question, suppose that consumers believe that the log of productivity $a \equiv \log A$ follows a random walk:

$$a_{t+1} = a_t + \phi_{t+1} \tag{20}$$

   and suppose that consumers observe the current level of output and derive their believed log level of 'productivity' as $a_t = y_t - \alpha k_t = z_t + \omega c_{t-1}$. Finally, assume that z does indeed follow a random walk:

$$z_{t+1} = z_t + \zeta_{t+1}. \tag{21}$$

2. Show that if throughout an indefinitely long prior history the log-level of z has been $z = 0$, there will be 'pseudo-steady-state' values of the logs of the model's main variables, $\{\hat{c}, \hat{y}, \hat{k}, \hat{y}\}$ to which the economy will converge. (Note: Do *not* try actually to do the tedious algebra required to find the solutions; instead, you should give a mathematical argument for why it is likely that such a solution will exist, at least for some parameter values).

   Answer:

   > If we assume that there is a steady state, we have a linear system of four linear equations with four unknowns, which thus can be solved, unless the parameter values are too crazy:

$$\begin{aligned}
\hat{k} &= \log\alpha\beta + \hat{y} \\
\hat{y} &= \hat{a} + \alpha\hat{k} \\
\hat{c} &= \log(1-\alpha\beta) + \hat{y} \\
\hat{a} &= \underbrace{\hat{z}}_{=0 \text{ by assumption}} + \omega\hat{c}
\end{aligned}$$

3. Assume that in period 0, the economy was in the pseudo-steady-state associated with a perpetual past history in which $z_{-n} = 0 \ \forall \ n \geq 0$, $(\{c_0, y_0, k_0, a_0\} = \{\hat{c}, \hat{y}, \hat{k}, \hat{a}\})$. In period 1, a shock of size $\zeta_1 > 0$ occurs (and no other shocks occur thereafter: $\zeta_t = 0 \ \forall \ t \neq 1$). Show that the pattern of income growth is

$$\begin{aligned}
\Delta y_1 &= \zeta_1 \\
\Delta y_2 &= \zeta_1(\alpha+\omega) \\
\Delta y_3 &= \zeta_1(\alpha+\omega)^2
\end{aligned}$$

$$
\begin{aligned}
\vdots &= \vdots \\
\Delta y_n &= \zeta_1(\alpha+\omega)^n
\end{aligned}
$$

Hint: you will want to use these steady-state facts:

$$
\begin{aligned}
\hat{z} &= 0 && (22) \\
\hat{a} &= \hat{z} + \omega\hat{c} = \omega\hat{c} && (23)
\end{aligned}
$$

and the dynamic equations:

$$
\begin{aligned}
z_t &= z_{t-1} + \zeta_t && (24) \\
a_t &= z_t + \omega c_{t-1} && (25) \\
k_t &= \log \alpha\beta + y_{t-1} && (26) \\
y_t &= a_t + \alpha k_t && (27) \\
c_t &= \log(1-\alpha\beta) + y_t. && (28)
\end{aligned}
$$

Further hint: Begin by calculating how the shock ζ_1 changes the period 1 values of a, k, y, and c from their steady state values (for example, $a_1 = \zeta_1 + \omega\hat{c}$), and then use the dynamic equations to move forward.

Answer:

Taking the 'Further hint':

$$
\begin{aligned}
a_1 &= \omega\hat{c} + \zeta_1 && (29) \\
k_1 &= \log \alpha\beta + \hat{y} && (30) \\
y_1 &= \hat{y} + \zeta_1 && (31) \\
c_1 &= \log(1-\alpha\beta) + y_1 = \hat{c} + \zeta_1 && (32)
\end{aligned}
$$

and then using the 'dynamic equations'

$$
\begin{aligned}
z_2 &= z_1 + \overbrace{\zeta_2}^{=0} && (33) \\
a_2 &= z_2 + \omega c_1 && (34) \\
&= \zeta_1 + \omega(\hat{c} + \zeta_1) && (35) \\
&= \zeta_1(1+\omega) + \hat{a} && (36) \\
k_2 &= \log \alpha\beta + y_1 && (37) \\
&= \hat{k} + \zeta_1 && (38) \\
y_2 &= a_2 + \alpha k_2 && (39) \\
&= \zeta_1(1+\omega) + \alpha(\hat{k} + \zeta_1) && (40) \\
&= \zeta_1(1+\omega+\alpha) + \hat{y} && (41) \\
\Delta y_2 &= (\alpha+\omega)\zeta_1 && (42) \\
c_2 &= \log(1-\alpha\beta) + y_2 && (43) \\
&= \hat{c} + \zeta_1(1+\omega+\alpha) && (44)
\end{aligned}
$$

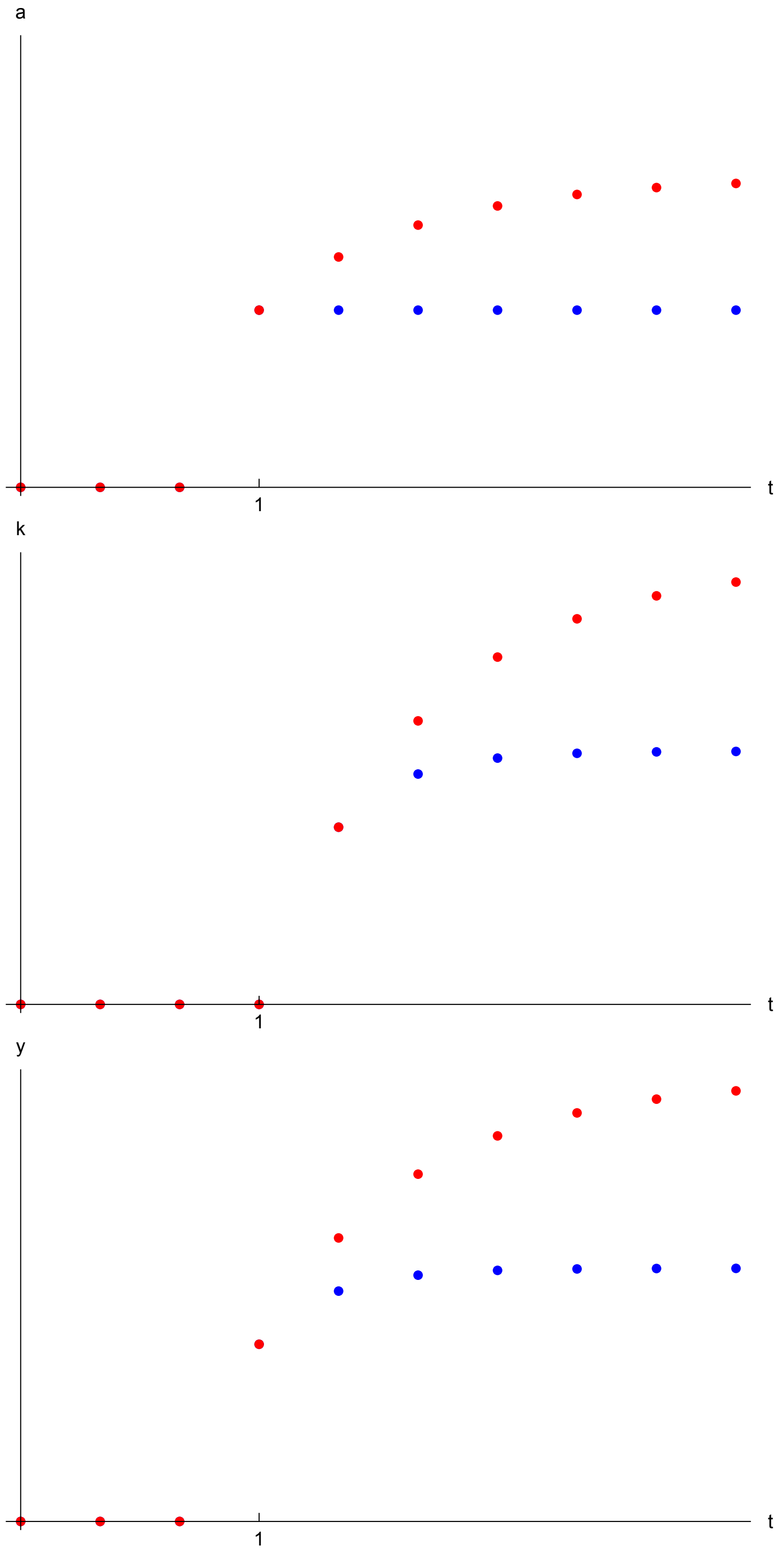
and thereafter generically

$$
\begin{aligned}
a_3 &= z_3 + \omega c_2 && (45)\\
&= \zeta_1 + \omega(\hat{c} + \zeta_1(1 + (\alpha + \omega))) && (46)\\
&= \zeta_1(1 + \omega(1 + (\alpha + \omega))) + \hat{a} && (47)\\
&= a_2 + \zeta_1 \omega(\alpha + \omega) && (48)\\
&= a_1 + \zeta_1 \omega(1 + (\alpha + \omega)) && (49)\\
\Delta a_3 &= \zeta_1 \omega(\alpha + \omega) && (50)\\
k_3 &= \log \alpha\beta + y_2 && (51)\\
&= \log \alpha\beta + \zeta_1(1 + \omega + \alpha) + \hat{y} && (52)\\
&= \hat{k} + \zeta_1(1 + (\omega + \alpha)) && (53)\\
\Delta k_3 &= \zeta_1(\omega + \alpha) && (54)\\
y_3 &= a_3 + \alpha k_3 && (55)\\
&= a_1 + \zeta_1 \left(\omega(1 + (\alpha + \omega)) + \alpha(1 + (\alpha + \omega))\right) && (56)\\
&= a_1 + \zeta_1(\alpha + \omega)(1 + (\alpha + \omega)) && (57)\\
\Delta y_3 &= \Delta a_3 + \alpha \Delta k_3 && (58)\\
&= \zeta_1(\omega(\alpha + \omega)) + \alpha \zeta_1(\alpha + \omega) && (59)\\
&= \zeta_1 \left(\alpha + \omega\right)^2 && (60)
\end{aligned}
$$

and similar analyses show that the pattern continues indefinitely.

4. Draw diagrams showing the dynamics of a, k, and y in the experiment described above. On the same diagrams, show the dynamics that consumers *expect* these variables will follow. Using the comparison of the expected and the actual dynamics, explain the sense in which this model can be said to exhibit a 'consumption multiplier.'

Answer:



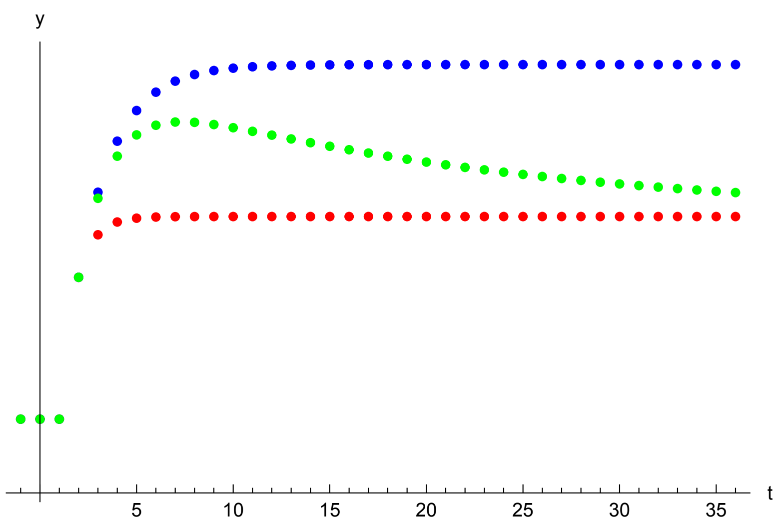
5. Most of the deeper mechanisms by which multipliers are hypothesized to work (e.g., having workers put in extra hours when demand surges, or stickiness of prices) imply that the stimulative effect on productivity eventually dies out. In the framework outlined above, this might be captured by supposing that the multiplier depends not upon the level of C but rather on its ratio to some new 'steady-state' value. That is, the 'productivity' term in the production function is $A_t(C_{t-1}/C^{\text{SS}})^{\omega}$ where C^{SS} catches up slowly but inexorably to actual consumption (for example, if $C_t^{\text{SS}} = \lambda C_{t-1}^{\text{SS}} + (1-\lambda)C_{t-1}$ for some value of λ not much less than 1) so that eventually (in the absence of further shocks) the new 'pseudo-steady-state' would be reached at a point where $C_t/C^{SS} = 1$. Draw the diagrams you would expect this economy to exhibit in response to the same experiment performed earlier.

   Answer:

   If $\lambda = 1$, the model is essentially identical (except for a scale shift) to the one solved above since C^{SS} never moves. If $\lambda = 0$, the 'multiplier' term is $C_{t-1}/C_{t-1} = 1$ so we are back to the standard `BrockMirman` model.

   For a value of λ not much less than one, for the time shortly after the shock the diagrams for this model should look like the ones for the multiplier model; but gradually as C^{SS} catches up to C_{t-1} the levels of all the variables should erode back toward the steady-state values of the basic `BrockMirman` model.

   The figure below shows an approximation of what things might look like for y in this case, with the green dots representing the model in which the multiplier gradually wears off. The figures for k and a will be similar.



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