Aggregate Consumption Dynamics. Consider the consumption Euler equation when there is both income uncertainty and rate-of-return uncertainty:

$$1 \quad = \quad \beta \, \mathbb{E}_t[\mathsf{R}_{t+1}(C_{t+1}/C_t)^{-\rho}] \tag{1}$$

1. Using the facts that $(1+\epsilon)^\gamma \approx 1+\gamma\epsilon$ and that $\eta/K \approx \log[(K+\eta)/K]$ for small ϵ and η/K, show that this equation can be approximated by

$$1 \quad \approx \quad \beta \, \mathbb{E}_t[\mathsf{R}_{t+1}(1-\rho\Delta \log C_{t+1})] \tag{2}$$

2. Now define $\mathbb{E}_t[\mathsf{r}_{t+1}] \equiv \log \mathbb{E}_t[\mathsf{R}_{t+1}]$ and show that equation (2) implies

$$\Delta \log C_{t+1} \quad = \quad \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta) + \epsilon_{t+1} \tag{3}$$

   if the covariance between R_{t+1} and C_{t+1} is approximately zero. (Hint: you will need to use the formula $\mathbb{E}[xy] = \mathbb{E}[x]\,\mathbb{E}[y] + \text{cov}(x,y)$.) Explain the reason why the assumption of rational expectations implies that no variable known at time t can be correlated with ϵ_{t+1}.

3. Using aggregate data for the U.S., Campbell and Mankiw (1989) find a set of lagged variables that are useful for predicting interest rates and income growth, and then use these variables to construct empirical measures of $\mathbb{E}_t[\mathsf{r}_{t+1}]$ and $\mathbb{E}_t[\Delta \log Y_{t+1}]$. They then estimate an equation of the following form:

$$\Delta \log C_{t+1} \quad = \quad \alpha_0 + \alpha_1 \, \mathbb{E}_t[\mathsf{r}_{t+1}] + \alpha_2 \, \mathbb{E}_t[\Delta \log Y_{t+1}] + \epsilon_{t+1}$$

4. If the approximations just derived are good and aggregate consumption is determined by a representative agent maximizing utility from aggregate consumption, what values should Campbell and Mankiw have estimated for α_0, α_1, and α_2? (Express your results in terms of the following preference parameters: ϑ is the time preference rate ($\beta = \frac{1}{1+\vartheta}$), and ρ is the coefficient of relative risk aversion). What values did Campbell and Mankiw actually find for α_1 and α_2? What was their explanation for these results?

5. Carroll, Fuhrer, and Wilcox (1994) (CFW) estimate an equation of the form

$$\Delta \log C_{t+1} \quad = \quad \alpha_0 + \alpha_1 \, \mathbb{E}_t[\mathsf{r}_{t+1}] + \alpha_2 \, \mathbb{E}_t[\Delta \log Y_{t+1}] + \alpha_3 S_t + \epsilon_{t+1}$$

   where S_t is 'consumer sentiment' as measured by a monthly survey that asks a random sample of households about their assessment of the current and future state of the economy. S_t tends to be high when the economy is performing well and to be low when the economy is doing badly. CFW use consumer sentiment and other variables to construct expectations for interest rates and income growth, and find a positive and statistically significant coefficient α_3. Are these results consistent with the Campbell-Mankiw model? Explain.

6. We learned in class that the log-linear approximation to the Euler equation derived in parts (a) and (b) misses the effect of precautionary saving. When a precautionary saving motive is included the approximate Euler equation becomes:

$$\Delta \log C_{t+1} \approx \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \vartheta) + (\rho/2)\mathrm{var}_t(\Delta \log C_{t+1}) \quad (4)$$

Discuss whether the CFW empirical results might be caused by precautionary saving effects.

7. The model above was solved under the assumption of time-separable utility. When the model is solved under the assumption that there is 'habit formation' in utility (so that consumers 'get used to' a given level of consumption), it turns out that the Euler equation for consumption growth can be approximated by

$$\Delta \log C_{t+1} = \alpha_0 + \alpha_1 \Delta \log C_t + \epsilon_{t+1}$$

Might the CFW results be explained in a model in which habit formation in consumption is important? (I am looking for an informal discussion, not an algebraic derivation).

References

Campbell, John Y., and N. Gregory Mankiw (1989): "Consumption, Income, and Interest Rates: Reinterpreting the Time-Series Evidence," in *NBER Macroeconomics Annual, 1989*, ed. by Olivier J. Blanchard, and Stanley Fischer, pp. 185–216. MIT Press, Cambridge, MA, http://www.nber.org/papers/w2924.pdf.

Carroll, Christopher D., Jeffrey C. Fuhrer, and David W. Wilcox (1994): "Does Consumer Sentiment Forecast Household Spending? If So, Why?," *American Economic Review*, 84(5), 1397–1408.