Midterm Exam

Intertemporal Choice

Fall, 2023

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up.* The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I: Short Questions

1. Consider a `TractableBufferStock` model, except that consumers now discount the future for two reasons: They have the usual time discount factor $\beta < 1$, but they also face some probability of dying D between periods (and we will call the probability of not dying $\cancel{\mathsf{D}} = 1 - \mathsf{D}$). (Utility when dead is zero, so expected utility from a future period is the probability of being alive times the discounted utility obtained conditional on being alive.)

   a) Write the employed consumer's optimization problem in the form of a Bellman equation, including the Dynamic Budget Constraint.

   Answer:

$$
\begin{aligned}
\mathrm{v}(m_t) &= \max_{c_t^e} \ \mathrm{u}(c_t^e) + \beta \cancel{\mathsf{D}} \mathrm{v}(m_{t+1}) && (1) \\
& \text{s.t.} \\
m_{t+1} &= (m_t - c_t^e)\mathsf{R} + \ell_{t+1}\mathsf{W}_{t+1}\xi_{t+1} && (2)
\end{aligned}
$$

   or alternatively (and more compactly),

$$
\mathrm{v}(m_t) = \max_{c_t^e} \ \mathrm{u}(c_t^e) + \beta \cancel{\mathsf{D}} \mathrm{v}((m_t - c_t^e)\mathsf{R} + \ell_{t+1}\mathsf{W}_{t+1}\xi_{t+1})
$$

   b) Explain why a 'growth impatience condition' will be required to guarantee that the consumer has a target value for $\check{m}^e$. *After* you explain it in words, produce the formula for the growth impatience condition. (You will not get full credit just for reproducing the formula without an explanation).

   Answer:

   This is just a standard tractable buffer-stock model of consumption, with the sole modification that the effective time discount factor is now $\cancel{\mathsf{D}}\beta$ rather than the usual β.

   In the model in which (aside from mortality risk) the consumer has perfect foresight the 'growth impatience condition' is that

$$
(\cancel{\mathsf{D}}\beta\mathsf{R})^{1/\rho} < \Gamma \qquad (3)
$$

   where Γ is the growth rate of income for the employed consumer conditional on remaining employed. If we redefine $\hat{\beta} = \cancel{\mathsf{D}}\beta$ and $\hat{\text{Þ}}_{\Gamma} = (\hat{\beta}\mathsf{R})^{1/\rho}/\Gamma$ then

$$
\hat{\text{Þ}}_{\Gamma} < 1. \qquad (4)
$$

   But the answer above is for a model without unemployment risk. `TractableBufferStock` showed that the presence of unemployment risk modifies the PF-GIC. The answer above deserves a substantial amount of credit, but the completely correct answer would be

$$\left(\frac{1}{(1-\mho)^{1/\rho}}\right) \;>\; \left(\frac{(\mathsf{R}\hat{\beta})^{1/\rho}}{\Gamma}\right) \tag{5}$$

(this is the condition that is both necessary and sufficient).

The foregoing model captures the behavior of a single individual person. We now want to consider an economy populated by many such consumers. In order to get there, we make two assumptions:

- When a consumer gets hit by the 'unemployment' (or retirement) shock, that consumer leaves the economy and moves somewhere else (probably some place cheaper, as retiring British people move to Spain). The consequence of this assumption is that the wealth remaining in this economy will be the wealth of employed consumers only
- A consumer who dies is replaced by a new 'heir' who is born in the same period and receives the dying consumer's wealth as a bequest. The heir's wage rate will be the same as the parent's wage rate would have been if the parent had not died. Note, however, that the dying person does not care about the utility of the heir; the only reason the bequest occurs is that the dying person's date of death was uncertain until it happened

c) Explain why, even if this economy were originally composed of consumers with different levels of wealth, it would tend toward a steady state in which each 'dynasty' is identical, and describe what the growth rate of aggregate income, wealth, and consumption will be in steady-state in this economy. Discuss how these steady-states can be reconciled with the Euler equation for individual consumption growth.

Answer:

Aggregate labor income, consumption, and wealth will all grow at rate Γ in steady-state. We know this because the growth impatience condition prevents the m_t^e ratio from rising too high, and precautionary saving motives prevent the ratio from falling too low. If m_t^e can't rise or fall forever, then in the long run m_t^e must grow at the same rate as W_t. If m_t and W_t are growing at the same rate, the aggregate budget constraint says that C_t must also grow at that rate.

A few people were tripped up by the fact that some consumers in this economy die, and thought that this might mean that income or wealth or consumption did not grow at rate Γ. But when they 'die', they are replaced by an identical clone who has exactly the same preferences, inherits the same wealth, and has the same permanent income and income uncertainty - in other words, the new 'heir' is identical in every respect to the agent who died, and so from the standpoint of the model the 'death' has no effect on the behavior of the dynasty.

d) Suppose that the probability of death suddenly falls because a new vaccine is invented. Using diagrams like those presented in the tractable buffer stock handout, show and explain the effects on consumption growth, market resources, and income growth in this economy.

Answer:

A decline in D is mathematically identical to a decline in impatience ϑ (an increase in β). The short run effect of a decline in impatience is to cut the level of aggregate consumption (an increase in the aggregate saving rate). As wealth grows from the higher saving rate, the variance term in the consumption Euler equation declines, until the new equilibrium is achieved in which the higher average level of wealth holdings across consumers implies a lower average value of the variance term. The standard diagrams that we used in class to examine the effects of a decline in the time preference rate can be used without modification here to show the effects.

2. **Secular Stagnation.** In 2013, Larry Summers delivered a widely discussed lecture at which he warned that the U.S. and European economies may face a risk that the next decade will be a period of "secular stagnation" like the one that has gripped Japan over the past 25 years. He proposed that a substantial increase in government spending could be an effective way to respond to this risk.

a) If the extra spending advocated by Summers were directly in the form of transfers to households with a high marginal propensity to consume, can the argument described above be completely understood as an assertion that the U.S. is now in a state of "dynamic inefficiency?" (Hint: Mention Ricardian Equivalence in your answer).

Answer:

The economy is dynamically inefficient if the rate of return is less than the "natural" growth rate of the economy (usually, productivity growth plus population growth). If the rise in government spending resulted in a larger amount of government debt, which crowded out private capital (that is, Ricardian Equivalence fails – the transfers are spent by the high MPC consumers, rather than being saved), then the aggregate capital stock would lower and aggregate interest rates would be higher, moving the economy away from the dynamically inefficient state. Note that this is not just a static operation. Due to the presence of debt and GE effects this has intertemporal consequences; it is not just a shuffling around of resources *within* a period. Therefore it can address *dynamic* inefficiency.

b) In fact, Summers did not advocate that the extra spending should all be directed to households who would spend it. He has advocated substantial increases in government investment in infrastructure, education, and other "investment" goods. Explain under what conditions this advice is what would be called for in a dynamically inefficient economy. Explain why this would or would not be an appropriate response.

Answer:

In an OLG model, an economy that is dynamically inefficient already has *too much* capital. If government investment in capital substitutes for private investment in capital (for example, if private firms would build toll roads if the government did not build free roads), the increase in capital from government investments might be counter-productive and drive the interest rate down further. If, however, government investment has the effect of boosting the productivity of private investment (for example, if investment in research spurs technological innovation), then more government investment might help even a neoclassical economy move away from the dynamically inefficient region by boosting the interest rate.

c) Elsewhere, DeLong and Summers (2012) have argued that when the economy is operating far below full employment, an increase in "aggregate demand" may be able to call forth a substantial increase in aggregate supply. This is essentially a Keynesian story. If DeLong and Summers (2012) are right, how would that relate to your answer to question (b) about dynamic inefficiency?

Answer:

If increased aggregate demand calls forth increased aggregate supply, the extra government spending could "pay for itself," at least in part, even if the government capital substitutes (in part or in full) for private capital. The extra output would be a "bonus" that would further reinforce the case for increased government debt.

Part II: Medium Questions

1. **Habit Formation, Sticky Expectations, and Measurement Error.**

Although the baseline `RandomWalk` model of consumption implies that consumption growth is unforecastable, the models in the handouts `Habits` and `StickyExpectations` are both consistent with serial correlation in 'true' consumption growth that takes forms like:

$$\Delta C_{t+1} = \alpha_0 + \alpha_1 \Delta C_t + \epsilon_{t+1} \tag{6}$$

a) Explain what the coefficient α_1 is interpreted as measuring in each of the two theories, and give some intuition for why the coefficient in this regression can be interpreted as measuring that object.

Answer:

In `StickyExpectations` with an interest rate $R = 1$, α_1 is interpreted as the proportion of the population which does not update their expectations in period $t + 1$. In `Habits`, α_1 is the penalty (due to habit formation) that agents incur in period $t + 1$ for their consumption in the previous period.

In practice, a difficulty of estimating either of these models is that actual reported consumption data from government statistical agencies contains measurement error. Suppose that we have data on measures of the true beliefs, collected at date t, about consumption growth from these sources:

- $\mathbb{B}_t^{\text{hhs}}[\Delta C_t]$: Answers from a survey of households who are asked directly about what they believe their consumption growth was
- $\mathbb{B}_t^{\text{fed}}[\Delta C_t]$: Estimates of produced by the Federal Reserve based on aggregate statistics like surveys of retailers

Consider performing regressions of the form

$$\Delta C_{t+1} = \alpha_0 + \alpha_1^{\bullet} \mathbb{B}_t^{\bullet}[\Delta C_t] + \zeta_{t+1} \tag{7}$$

where the $\bullet$ is a stand-in for the different methods of measuring beliefs. This generates a potentially different estimate of α_1 for each of the different measures of consumption beliefs.

a) Suppose first that the estimates of α_1 are similar for all measures of consumption growth beliefs, say $\{\alpha_1^{\text{hhs}}, \alpha_1^{\text{fed}}\} = 0.75$.

i. Under these conditions and using only these data, the two theories are obviously basically indistinguishable. Can you think of any other kinds of data which might be more useful in distinguishing these theories from each other? How would you go about using those data to distinguish the theories?

Answer:

Questions 3 and 4 provide an example. Answers were judged on a case-by-case basis to determine whether the suggested data and methodology could be used to reject one of the theories.

Suppose now that if we somehow had access to data on 'true' consumption data, we would be able to show that households have perfect contemporaneous knowledge of their own consumption, $\mathbb{B}_t^{\text{hhs}}[\Delta C_t] = \Delta C_t$, while the Fed's contemporaneous beliefs are equal to the truth plus some mean-zero measurement error, $\mathbb{B}_t^{\text{fed}}[\Delta C_t] = \Delta C_t + \xi_t$ with some variance $\sigma_\xi^2 > 0$

b) Now suppose the estimates using these beliefs produce different results. In particular, suppose the coefficient estimates fit the pattern $\{\alpha_1^{\text{hhs}} > \alpha_1^{\text{fed}}\}$. Can you reach any new conclusions about the validity of the two theories?

Answer:

Both theories would imply this ranking of results, because mean-zero measurement error biases the estimate of α_1 toward zero regardless of the model which produces the observed consumption patterns. Therefore, they still can't be distinguished from each other.

Suppose that, although Fed's direct measure of consumption expenditures is imperfect, its supervision of the banking sector allows it to measure past income Y_{t-n}, bank balances B_{t-n}, and saving S_{t-n} perfectly (where saving is income minus consumption: $S_{t-n} = \mathsf{r}B_{t-n} + Y_{t-n} - C_{t-n}$ for $n \geq 0$.

c) How can the Fed use these data to improve its $\mathbb{B}_t^{\text{fed}}[\Delta C_t]$?

Answer:

The Fed can back out households' information about their own consumption using:

$$C_{t-n} \quad = \quad \mathsf{r}B_{t-n} + Y_{t-n} - S_{t-n} \tag{8}$$

and thus obtain households' information about their past C_{t-1}.

Now suppose the Fed, again through its banking regulatory powers, has microeconomic data about individual households indexed by i on the same variables that are measured in the aggregate data, for example $c_{t,i}$ etc.

d) Can the Fed use these data to distinguish the two theories? How?

Answer:

Yes! The habit formation theory says that the same equation will hold at the micro as at the macro level. So the Fed can estimate equation 6, now with individual-level data. The rejection of the hypothesis that α_1 at the aggregate level is equal to that at the individual level will be equivalent to the rejection of the habit formation model.

2. **Consumption Dynamics in a Deaton (1992)-Friedman (1957) Model With Transitory and Permanent Shocks.**

Consider a consumer solving the maximization problem

$$\max_{\{\mathrm{C}\}_t^\infty} \quad \mathbb{E}_t\left[\sum_{s=0}^{\infty}\beta^s \mathrm{u}(C_{t+s})\right]$$

subject to the transition equations

$$\begin{aligned} B_{t+1} &= (M_t - C_t)\mathsf{R} \\ P_{t+1} &= P_t + \Psi_{t+1} \\ Y_{t+1} &= P_{t+1} + \Xi_{t+1} \\ M_{t+1} &= B_{t+1} + Y_{t+1} \end{aligned}$$

where Ξ and Ψ are respectively the unpredictable transitory and permanent shocks to the level of income, satisfying $\mathbb{E}_t[\Psi_{t+n}] = \mathbb{E}_t[\Xi_{t+n}] = 0 \ \forall\ n > 0$.

a) Rewrite the problem in Bellman equation form, and derive the Euler equation for consumption.

   Answer:

   This is in `Envelope`.

   For the remainder of the problem, assume that $\mathsf{R}\beta = 1$; the utility function is quadratic; and the consumer's level of resources is too small for consumption ever to reach the 'bliss point.'

b) Show that under these assumptions, $\mathbb{E}_t[C_{t+n}] = C_t \ \forall\ n > 0$.

   Answer:

   This is done in `RandomWalk`.

c) Use the fact that the intertemporal budget constraint must hold in expectation to prove that optimal consumption in period t is given by

$$C_t = (\mathsf{r}/\mathsf{R})(B_t + H_t + \Xi_t) \tag{9}$$

   where, designating the infinite horizon present discounted value by the operator $\mathbb{P}_t(\bullet)$,

$$\begin{aligned} H_t &= \mathbb{E}_t[\mathbb{P}_t(Y)] &(10) \\ &= (\mathsf{R}/\mathsf{r})P_t &(11) \end{aligned}$$

   Answer:

   This is essentially the same as in `PerfForeCRRAModel` because when the expected shocks are all zero, the expectations operator annihilates the future values of those shocks leading to the same formula for expected future income as the formula for actual future income in the perfect foresight model. Note that (9) reduces to

$$C_t = P_t + (\mathsf{r}/\mathsf{R})(B_t + \Xi_t) \tag{12}$$

d) Show that

$$\mathbb{E}_t[\Delta Y_{t+1}] \;=\; -\Xi_t \tag{13}$$

and explain the intuition for this result.

Answer:

$$\begin{aligned} Y_{t+1} - Y_t &= P_{t+1} + \Xi_{t+1} - (P_t - \Xi_t) \\ \mathbb{E}_t[\Delta Y_{t+1}] &= \mathbb{E}_t[\Psi_{t+1}] + \mathbb{E}_t[\Xi_{t+1}] - \Xi_t \\ &= -\Xi_t \end{aligned}$$

If income is high today due to a transitory income shock, it is expected to fall in the following period, and vice versa. This amounts to transitory shocks tending to revert to their mean.

e) The 'Haig-Simons' definition of saving is the change in resources from one period to the next. Consistent with this, define

$$S_t \;=\; B_{t+1}/\mathsf{R} - B_t$$

and show that

$$\begin{aligned} \mathbb{E}_t[\Delta Y_{t+1}] &= -S_t\mathsf{R} - \mathsf{r}B_t &(14)\\ &\approx -S_t\mathsf{R} &(15) \end{aligned}$$

if $\mathsf{r}B_t$ is small. Explain this result intuitively, and discuss its relationship to Friedman's PIH.

Answer:

$$\begin{aligned} S_t &= (M_t - C_t) - B_t \\ &= (B_t + P_t + \Xi_t - C_t) - B_t \\ &= (1 - (\mathsf{r}/\mathsf{R}))\Xi_t - (\mathsf{r}/\mathsf{R})B_t \\ &= (\Xi_t - \mathsf{r}B_t)/\mathsf{R} \\ \mathsf{R}S_t &= \Xi_t - \mathsf{r}B_t \\ -\mathsf{R}S_t - \mathsf{r}B_t &= -\Xi_t \\ &= \mathbb{E}_t[\Delta Y_{t+1}] \end{aligned}$$

The intuition is that a positive transitory shock to income today lifts the level of income above its 'normal' value. If r is 'small' then the amount of extra spending induced by the transitory shock is small and can be neglected (leading to the approximation). That is, the consumer here is setting consumption approximately equal to permanent income P, and spending little out of their transitory income Ξ_t – which is basically what Friedman proposed in his Permanent

Income Hypothesis. Thus, according both to Friedman and to this model, when you get a transitory shock, you save almost all of it. So, the size of the savings is roughly the amount by which income will change next period as it reverts back to its permanent level.

f) Suppose we now construct a forecast of income growth using (15) and perform a regression

$$\Delta C_{t+1} = \alpha_0 + \alpha_1 \mathbb{E}_t[\Delta Y_{t+1}]$$

What should our empirical estimate of α_1 be? How do you reconcile this with the fact that the level of consumption is positively related to the level of income?

Answer:

Nothing about this problem changes the usual conclusion that consumption follows a random walk. So $\alpha_1 = 0$. The fact that the *level* of consumption is related to the *level* of income does not mean that consumption *changes* are predictable even if income changes are predictable (at least one period in advance).

g) Now suppose that consumption data are measured with iid (=white noise = unpredictable) error,

$$\tilde{C}_{t+1} = C_{t+1} + \chi_{t+1}$$

where $\mathbb{E}_t[\chi_{t+n}] = 0 \ \forall \ n > 0$ and the variance of χ is $\mathbb{E}_t[\chi^2_{t+1}] = \sigma^2_\chi$.

Suppose that we wish to use $\Delta\tilde{C}_t$ to forecast $\Delta\tilde{C}_{t+1}$ from an equation of the form

$$\mathbb{E}_t[\Delta\tilde{C}_{t+1}] = \gamma\Delta\tilde{C}_t.$$

Define the expected squared errors from the forecasting equation as

$$SSE = \mathbb{E}_t[(\Delta\tilde{C}_{t+1} - \mathbb{E}_t[\Delta\tilde{C}_{t+1}])^2]$$

Defining the variance of 'true' changes in consumption as $\mathbb{E}_t[(\Delta C_{t+1})^2] = \sigma^2_{\Delta C}$, show that the choice of γ which minimizes the sum of squared errors from such a forecast is

$$\gamma = -\left(\frac{\sigma^2_\chi}{\sigma^2_\chi + \sigma^2_{\Delta C}}\right)$$

Explain why this makes sense intuitively and discuss how it relates to the previous result about forecasting income growth. Hint: Your problem is to find the value of γ which minimizes the expected sum of squared errors:

$$\min_\gamma SSE \tag{16}$$

and you can use the fact that 'true' consumption growth and measurement error are both iid: $\mathbb{E}_t[\Delta C_{t+1}\Delta C_t] = \mathbb{E}_t[\Delta C_{t+1}\chi_t] = \mathbb{E}_t[\Delta C_{t+1}\chi_{t-1}] = \mathbb{E}_t[\chi_{t+1}\Delta C_t] = \mathbb{E}_t[\chi_{t+1}\chi_{t-1}] = 0$.

Answer:

The FOC yields:

$$\begin{aligned} 0 &= -\gamma 2\,\mathbb{E}_t[(\Delta\tilde{C}_{t+1} - \gamma\Delta\tilde{C}_t)(\Delta C_t + \chi_t - \chi_{t-1})] \\ &= \mathbb{E}_t[(\Delta C_{t+1} + \chi_{t+1} - \chi_t - \gamma\Delta\tilde{C}_t)(\Delta C_t + \chi_t - \chi_{t-1})] \\ &= -\sigma_\chi^2 - \gamma(\sigma_\chi^2 + \sigma_{\Delta C}^2) \\ \gamma &= -\left(\frac{\sigma_\chi^2}{\sigma_\chi^2 + \sigma_{\Delta C}^2}\right) \end{aligned}$$

where we have used the fact that the measurement errors are iid and uncorrelated with true consumption growth, $\mathbb{E}_t[\Delta C_{t+1}\Delta C_t] = \mathbb{E}_t[\Delta C_{t+1}\chi_t] = \mathbb{E}_t[\Delta C_{t+1}\chi_{t-1}] = \mathbb{E}_t[\chi_{t+1}\Delta C_t] = \mathbb{E}_t[\chi_{t+1}\chi_{t-1}] = 0$.

Just as changes in income should be unrelated to changes in consumption, ΔC_{t+1} should be unrelated to ΔC_t when consumption is a random walk. Thus, any non-zero estimate of γ is brought about purely by measurement error. Note that in the equation for γ above, when measurement error is negligible ($\sigma_\chi^2 = 0$), we find the desired result.

References

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Friedman, Milton A. (1957): *A Theory of the Consumption Function.* Princeton University Press.