

Income Growth Over the Lifetime Versus Between Generations (Modigliani (1986), Carroll and Summers (1991)). This question concerns the effects on aggregate saving of income growth over the lifetime versus income growth between generations. Consider an overlapping generations economy in which each individual lives for two periods. Population is constant, so that the population growth factor between generations is $\Xi = 1$; normalize population itself to $\mathbf{N} = 1$ per generation. The individuals' noncapital incomes in each period are exogenous. The first period noncapital income of an individual born at time t is $y_{t,1}$, and the second-period noncapital income of the same individual is $y_{t+1,2} = Xy_{t,1}$ where X can be greater than or less than one. The consumer solves the optimization problem:

$$\max \quad \log(c_{t,1}) + \beta \log(c_{t+1,2}) \quad (1)$$

$$\text{s.t.} \quad (2)$$

$$c_{t+1,2} = (y_{t,1} - c_{t,1})R + y_{t+1,2}. \quad (3)$$

Finally, between generations the first period noncapital incomes grow by a factor $G = (1 + g)$ so that:

$$y_{t+1,1} = Gy_{t,1}$$

For the purposes of this question, consider this to be an open economy so that the aggregate interest rate R and noncapital incomes y are fixed (that is, don't try to derive their values from an aggregate production function).

1. How does an increase in the growth rate of noncapital income over the lifetime, X , affect the saving of young households? Explain.

Answer:

$$c_{t,1} = \frac{y_{t,1} + y_{t+1,2}/R}{1 + \beta} \quad (4)$$

$$= y_{t,1} \left[\frac{1 + X/R}{1 + \beta} \right] \quad (5)$$

$$s_{t,1} = y_{t,1} - c_{t,1} \quad (6)$$

$$= y_{t,1} \left[1 - \frac{1 + X/R}{1 + \beta} \right] \quad (7)$$

$$= y_{t,1} \left[\frac{\beta - X/R}{1 + \beta} \right] \quad (8)$$

So an increase in X reduces the saving of young households. Higher X reduces the need for saving to provide for consumption in the second period of life, because with higher X you will be richer in the second period of life anyway.

2. Calculate the level of aggregate saving $S_t = K_{t+1} - K_t$ as a function of $y_{t,1}$, and then calculate the aggregate saving rate out of noncapital income $\sigma_t = S_t/Y_t =$

$S_t/(y_{t,1} + y_{t,2})$. How is the aggregate saving rate related to the growth rate of income between generations, G ? How and why does the answer depend on the relationship between β and X/R ?

Answer:

The aggregate capital stock in period $t + 1$ is given by the saving of the young in period t :

$$K_{t+1} = Ls_{t,1} = s_{t,1} \quad (9)$$

Aggregate saving is given by the change in the aggregate capital stock:

$$S_t = K_{t+1} - K_t \quad (10)$$

Using the expression for $s_{t,1}$ derived above, this is:

$$S_t = [y_{t,1} - y_{t,1-1}] \left(\frac{\beta - X/R}{1 + \beta} \right) \quad (11)$$

If $y_{t,1} = Gy_{t,1-1}$ this is:

$$S_t = (G - 1)y_{t,1-1} \left[\frac{\beta - X/R}{1 + \beta} \right] \quad (12)$$

Aggregate income is equal to the sum of the income of the young and the income of the old,

$$Y_t = y_{t,1} + y_{t,2} \quad (13)$$

$$= Gy_{t,1-1} + Xy_{t,1-1} \quad (14)$$

$$= y_{t,1-1}[G + X] \quad (15)$$

Thus the aggregate saving rate σ_t is

$$\sigma_t = \left(\frac{G - 1}{G + X} \right) \left[\frac{\beta - X/R}{1 + \beta} \right] \quad (16)$$

How does the saving rate vary with G ?

$$\frac{d\sigma_t}{dG} = \frac{(G + X) - (G - 1)}{(G + X)^2} \left[\frac{\beta - X/R}{1 + \beta} \right] \quad (17)$$

$$= \frac{1 + X}{(G + X)^2} \left[\frac{\beta - X/R}{1 + \beta} \right] \quad (18)$$

The sign of this relationship depends on whether $\beta > X/R$ or, equivalently, whether $\beta R > X$. Recall that βR is the growth rate of consumption over the lifetime and X is the growth rate of income. If the growth rate of consumption is greater than the growth rate of income, this means that the level of consumption in the first period of life must be below the level of income - that is, young people must be savers. The parameter G determines how much richer the young are than the old. If the young are savers, then making them richer than the old increases

the aggregate saving rate. If the young are dissavers, then making them richer than the old reduces the aggregate saving rate.

3. Thus far in the problem, we have assumed that X is independent of G ; that is, we have assumed that the rate at which income grows during your lifetime is unrelated to the rate at which each generation's income exceeds the income of the previous generation. Now assume that $X = \gamma G$ for some constant γ . Also assume (for simplicity) that $\beta = 1/R$ (qualitative results are the same when $\beta \neq 1/R$, but the analysis is messier). Now when does an increase in G increase or reduce the aggregate saving rate?

Answer:

Since $X = \gamma G$ and $\beta = 1/R$, we have $X/R = \gamma G\beta$, which can be substituted into (16):

$$\sigma_t = \left(\frac{G-1}{G+\gamma G} \right) \left[\frac{\beta - \gamma G\beta}{1+\beta} \right] \quad (19)$$

$$= \left(\frac{G-1}{G} \right) (1 - \gamma G) \left[\frac{\beta}{(1+\gamma)(1+\beta)} \right] \quad (20)$$

$$= \left(\frac{G-1+\gamma G - \gamma G^2}{G} \right) \left[\frac{\beta}{(1+\gamma)(1+\beta)} \right] \quad (21)$$

$$= (1 - 1/G + \gamma - \gamma G) \left[\frac{\beta}{(1+\gamma)(1+\beta)} \right] \quad (22)$$

So the derivative of the saving rate with respect to G is

$$\frac{d\sigma_t}{dG} = ((1/G^2) - \gamma) \left[\frac{\beta}{(1+\gamma)(1+\beta)} \right] \quad (23)$$

which will be positive only if $1/G^2 > \gamma$.

4. Empirical evidence shows that the ratio of the income of the old (people aged 55-85) to income of the young (people aged 25-55) is about 0.7 in both the US and in Japan. From the late 1940s to the late 1980s, Japan's economic growth rate was about 4 percent per year in per-capita terms. Over the same period income growth in the US was about 1 percent per capita. Japan's aggregate saving rate was also much higher than the US saving rate during this period. Discuss whether the overlapping generations model can explain Japan's high saving rate as being the result of its rapid growth rate (continue to assume $\beta = 1/R$). (Hint: start by figuring out the OLG model's implications for the ratio of the income of the old to income of the young.)

Answer:

(The figures below show some evidence supporting the view that the ratio of consumption of the old to that of the young is not very different across countries.)

$$y_{t,2} = Xy_{t,1-1} \quad (24)$$

$$= Xy_{t,1}/G \quad (25)$$

$$\frac{y_{t,2}}{y_{t,1}} = X/G \quad (26)$$

The fact that the income of the old is about 0.7 times the income of the young implies that $X = 0.7G$ in both countries. The US per capita growth rate of 1 percent per year implies an increase in per-capita income between generations of $G = 1.01^{30} \approx 1.35$. The Japanese per capita growth rate translates to $G = 1.04^{30} \approx 3.24$.

$$\sigma_{US} = (1 - 1/1.35 + 0.7 - 0.7 * 1.35) \left[\frac{\beta}{(1.7)(1 + \beta)} \right] \quad (27)$$

$$= 0.014 \left[\frac{\beta}{(1.7)(1 + \beta)} \right] \quad (28)$$

$$\sigma_{Japan} = (1 - 1/3.24 + 0.7 - 0.7 * 3.24) \left[\frac{\beta}{(1.7)(1 + \beta)} \right] \quad (29)$$

$$= -0.87 \left[\frac{\beta}{(1.7)(1 + \beta)} \right] \quad (30)$$

Thus the Japanese saving rate should actually be strongly negative, and certainly much lower than the US saving rate.¹ You can also see that this must be true because from equation (23) if $\gamma = 0.7$ the derivative of the saving rate with respect to G is negative for any G such that

$$1/G^2 < 0.7 \quad (31)$$

$$1/0.7 < G^2 \quad (32)$$

$$1.19 < G. \quad (33)$$

Since we calculated $G = 1.34$ for the US and $G = 3.24$ for Japan, as we increase G from the US value to the Japanese value the saving rate must be steadily falling.

The reason the model implies a negative saving rate for Japan is simple: the young Japanese consumers know that their incomes will be much higher when they are old than when they are young, so they borrow large amounts while young and repay those loans when old.

Thus, the OLG model is not a good candidate for explaining why Japan's saving rate has been so high during its period of rapid growth.

¹You may wonder how the model can imply a negative steady-state saving rate forever. The reason this does not violate the economy's Intertemporal Budget Constraint is that the economy is growing so fast that it can always repay its borrowing in period t with some of its much greater period $t + 1$ income.

Figure 4a

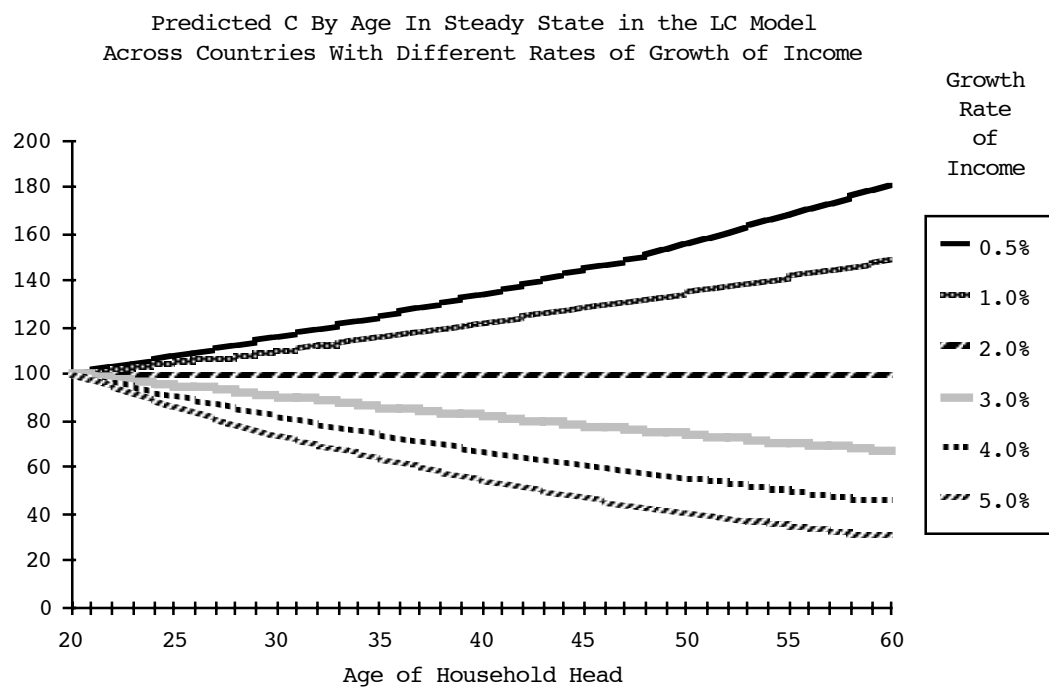
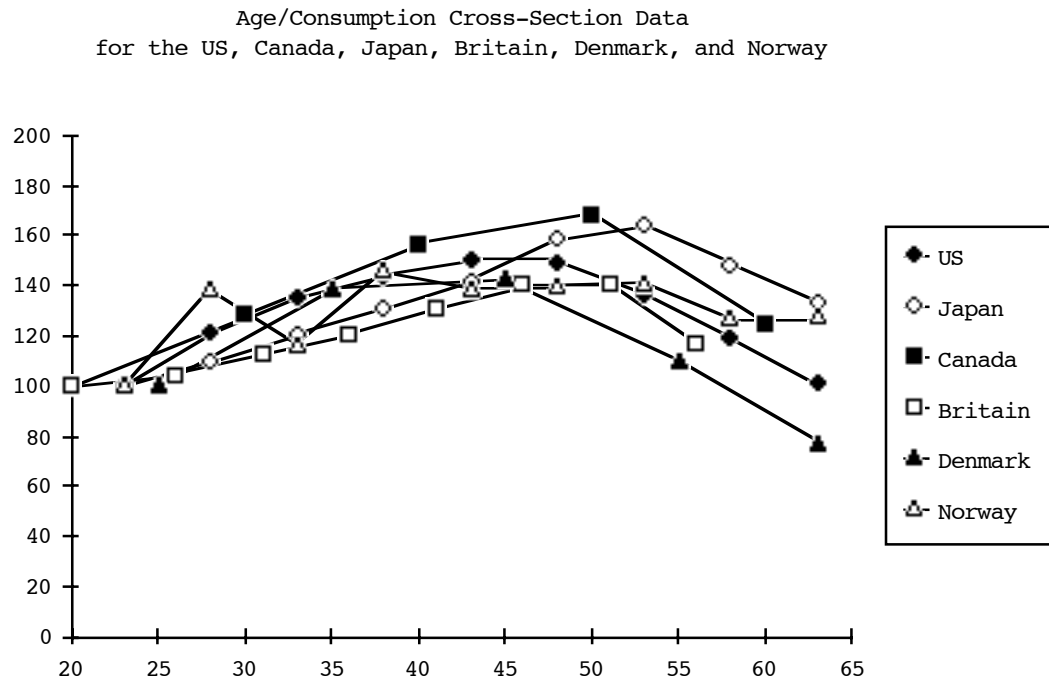


Figure 4b



References

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