Predictable and Unpredictable Changes in Consumption in a Perfect Foresight Model.

In representative agent (RA) macroeconomic models, a key channel by which the central bank's control of interest rates affects aggregate demand is through its effect on consumption. For a small open economy, consumption behavior of the RA can be well approximated by the solution for a perfect foresight consumer, which PerfForesightCRRA[1] shows can be written approximately as:

$$c_t \approx \boldsymbol{p}_t \left(1 - \text{þ}_\gamma/(\mathsf{r} - \gamma)\right) - \text{þ}_\mathsf{r} b_t \tag{1}$$

where

$$\text{þ}_\gamma \equiv \rho^{-1}(\mathsf{r} - \vartheta) - \gamma$$

is the 'growth patience' rate while the 'return patience' rate is

$$\text{þ}_\mathsf{r} \equiv \rho^{-1}(\mathsf{r} - \vartheta) - \mathsf{r}.$$

1. a) Briefly:
      i. Explain why we must assume that $\text{þ}_\mathsf{r} < 0$ and $\text{þ}_\gamma < 0$
      ii. Use these formulae to describe the three channels by which a change in r should affect the level of c

   b) Discuss why, for quantitatively plausible calibrations in which r and ϑ are roughly the same size, this model implies that the effects of interest rates on consumption should be very large.

   Consider now an economy in which consumers always behave as though they *believe* that interest rates will remain constant at the current level. But, every now and then a completely unexpected shock occurs that changes the interest rate; subsequently, consumers believe interest rates will remain forever at their new level. (A shock of this kind is sometimes called an 'MIT shock'). Mathematically,

$$\mathbb{E}_t[\mathsf{r}_{t+n}] = \mathsf{r}_t \ \forall \ n > 0$$

2. Assume that leading up to time t, interest rates had been constant at some level $\underline{\mathsf{r}}$. At date t there is an MIT shock that causes interest rates to go up to $\bar{\mathsf{r}} > \underline{\mathsf{r}}$. Explain why this model implies the equation that Hall (1988) estimated:

$$\Delta \log c_{t+1} \approx \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \nu) + \epsilon_{t+1} \tag{2}$$

   for $\mathbb{E}_t[\epsilon_{t+1}] = 0$ if our expectations of interest rates come from surveys of consumers. (Hint: Consider separately the 'normal' periods in which there is no shock to interest rates, and the 'rare' periods in which there are such shocks).

[1] Notation: Bank balances are b, noncapital income is $\boldsymbol{p}_t$, relative risk aversion is ρ, the permanent rate of income growth is γ, etc.

3. Given your analysis above, explain what forces will be the chief contributors to the magnitude of the ϵ shocks?

    a) Answer this question analytically first, under the convenient assumption that the time preference rate is $\nu = \underline{\mathsf{r}}$ and that leading up to t bank balances were constant at $b = 1$ and permanent income was $\boldsymbol{p}_t = 1$.

    b) Now answer the question quantitatively, under the convenient assumption that $\underline{\mathsf{r}} = 0.02$ and $\bar{\mathsf{r}} = 0.04$ and $\rho = 2$ and $\gamma = 0$.

    c) After you have done the algebraic and quantitative analysis, draw a diagram showing the level of consumption from the period leading up to t through an interval after t. On the diagram, label changes that correspond to the combined income, substitution, and human wealth effects in period $t+1$, and to the intertemporal substitution channel of predictable consumption growth in periods before t and after $t+1$.

References

Hall, Robert E. (1988): "Intertemporal Substitution in Consumption," *Journal of Political Economy*, XCVI, 339–357, Available at .