Growth Under Total Utility Maximization. In the continuous-time Ramsey (1928)/Cass-Koopmans model presented in class, the social planner's objective was to maximize

$$\int_0^\infty \mathrm{u}(c_t)e^{-\nu t}dt \tag{1}$$

where $c_t = C_t/\mathrm{L}_t$ is per-capita consumption. Thus, the social planner is assumed to care about per-capita consumption. (For short, refer to this as the "per-capita" model.) An alternative would be to assume that the social planner's utility is given by total utility of all households in the population. In that case, the maximand would be

$$\int_0^\infty \mathrm{L}_t\mathrm{u}(c_t)e^{-\nu t}dt. \tag{2}$$

Answer the following questions about the optimum if this equation characterizes the social planner's maximand. Assume that the aggregate budget constraint is $Y_t = \mathrm{F}(K_t, \mathrm{L}_t) = \dot{K}_t + C_t + \delta K_t$, population growth is constant at $\dot{\mathrm{L}}_t/\mathrm{L}_t = \xi$, the CRS production function is $\mathrm{F}(K, \mathrm{L}) = K^\alpha \mathrm{L}^{1-\alpha}$, and there is no productivity growth.

1. Rewrite the aggregate budget constraint in per-capita terms, and then write down the maximization problem and the first order conditions for the optimum.
2. Assume CRRA utility $\mathrm{u}(c) = c^{1-\rho}/(1-\rho)$. Derive the optimal growth rate of per-capita consumption, and explain the intuition for why it's different from the per-capita model.
3. Solve for the golden rule and modified golden rule levels of capital and consumption. How do they compare to the per-capita model?

References

RAMSEY, FRANK (1928): "A Mathematical Theory of Saving," *Economic Journal*, 38(152), 543–559.