Midterm Exam
Intertemporal Choice
Fall, 2025

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I: Long Question

1. Consider a consumer with quadratic utility $u(c) = -(1/2)(c - \wp)^2$ for whom the interest and discount factors satisfy $R\beta = 1$. The consumer wishes to maximize the discounted sum of intertemporally separable utility from consumption

$$\sum_{s=t}^{\infty} \mathbb{E}_t \left[\beta^{s-t} u(c_s) \right] \tag{1}$$

subject to a budget constraint

$$m_{t+1} = (m_t - c_t)R + y_{t+1}$$

where m is market resources, y is labor income, and $R = 1 + r$ is the interest factor.

   a) Write the problem in Bellman equation form, and show how to use the Envelope condition and the first order condition to obtain the Euler equation for consumption.

b) Show that the Euler equation $u'(c_t) = R\beta\mathbb{E}_t[u'(c_{t+1})]$ implies that the level of consumption follows a random walk,

$$\Delta c_{t+1} = \epsilon_{t+1}$$

where $\mathbb{E}_t[\epsilon_{t+1}] = 0$.

Now assume that income is subject to transitory and permanent shocks:

$$y_{t+1} = p_{t+1} + \theta_{t+1}$$
$$p_{t+1} = p_t + \psi_{t+1}$$

where the shocks are mutually and serially uncorrelated ($\mathbb{E}_t[\psi_{t+n}] = \mathbb{E}_t[\theta_{t+n}] = \mathbb{E}_t[\psi_{t+m}\theta_{t+n}] = 0 \ \forall \ m, n > 0$).

c) Defining end-of-period human wealth as the PDV of expected future income,

$$\hbar_t = R^{-1}\mathbb{E}_t\left[\sum_{s=t+1}^{\infty} R^{t+1-s}y_s\right]$$

show that

$$\hbar_t = p_t/r$$

Define 'balances' b as the amount of resources the consumer has in hand before receiving income in the current period:

$$b_{t+1} \quad = \quad (b_t + y_t - c_t)\mathsf{R}$$

and define total wealth o as the sum of asset balances b, current income y, and human wealth $\mathfrak{h}$,

$$o_{t+1} \quad = \quad b_{t+1} + y_{t+1} + \mathfrak{h}_{t+1},$$

d) Use the intertemporal budget constraint to show that

$$c_t \quad = \quad p_t + (b_t + \theta_t)(\mathsf{r}/\mathsf{R}) \tag{2}$$

Explain why this equation implies a marginal propensity to consume out of shocks to permanent income ψ_t of 1, and out of transitory income θ of (r/R).

Part II: Short Question

1. **Small Open Economy Models of A Shock to Capital.** In the spring of 2011, Japan was hit by a terrible earthquake, tsunami, and resulting nuclear accident that destroyed a substantial amount of capital. This question asks what effect different models might lead you to expect for consumption spending in response to this disaster. Assume that before the shock, Japan was at its steady-state level of the capital/output ratio, and that the shock is perceived as a one-time destruction of a fixed amount of aggregate capital. Describe the dynamics of consumption per capita under the following models:

    a) A Diamond 2-Period OLG model

b) A Hall Random Walk model

c) A model with habit formation in aggregate consumption

d) A model where the reason aggregate consumption is normally sluggish is 'sticky information' (hint: you do not need to assume that information is *always* equally sticky in every period)

Part III

Predictable and Unpredictable Changes in Consumption in a Perfect Foresight Model.

   In representative agent (RA) macroeconomic models, a key channel by which the central bank's control of interest rates affects aggregate demand is through its effect on consumption. For a small open economy, consumption behavior of the RA can be well approximated by the solution for a perfect foresight consumer, which `PerfForesightCRRA`[1] shows can be written approximately as:

$$\boldsymbol{c}_t \;\approx\; \boldsymbol{p}_t \left(1 - \text{þ}_\gamma/(\text{r} - \gamma)\right) - \text{þ}_\text{r}\boldsymbol{b}_t \tag{3}$$

where

$$\text{þ}_\gamma \;\equiv\; \rho^{-1}(\text{r} - \vartheta) - \gamma$$

is the 'growth patience' rate while the 'return patience' rate is

$$\text{þ}_\text{r} \;\equiv\; \rho^{-1}(\text{r} - \vartheta) - \text{r}.$$

1. a) Briefly:

   i. Explain why we must assume that $\text{þ}_\text{r} < 0$ and $\text{þ}_\gamma < 0$

   ii. Use these formulae to describe the three channels by which a change in r should affect the level of $\boldsymbol{c}$

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[1]Notation: Bank balances are $\boldsymbol{b}$, noncapital income is $\boldsymbol{p}_t$, relative risk aversion is ρ, the permanent rate of income growth is γ, etc.

b) Discuss why, for quantitatively plausible calibrations in which r and ϑ are roughly the same size, this model implies that the effects of interest rates on consumption should be very large.

Consider now an economy in which consumers always behave as though they *believe* that interest rates will remain constant at the current level. But, every now and then a completely unexpected shock occurs that changes the interest rate; subsequently, consumers believe interest rates will remain forever at their new level. (An n 'MIT shock'). Mathematically,

$$\mathbb{E}_t[\mathsf{r}_{t+n}] \;\; = \;\; \mathsf{r}_t \; \forall \, n > 0$$

2. Assume that leading up to time t, interest rates had been constant at some level $\underline{\mathsf{r}}$. At date t there is an MIT shock that causes interest rates to go up permanently to $\bar{\mathsf{r}} > \underline{\mathsf{r}}$. Explain why this model implies the equation that Hall (1988) estimated:

$$\Delta \log c_{t+1} \;\; \approx \;\; \rho^{-1}(\mathbb{E}_t[\mathsf{r}_{t+1}] - \nu) + \epsilon_{t+1} \tag{4}$$

for $\mathbb{E}_t[\epsilon_{t+1}] = 0$ if our expectations of interest rates come from surveys of consumers. (Hint: Consider separately the 'normal' periods in which there is no shock to interest rates, and the 'rare' periods in which there are such shocks).

3. Given your analysis above, explain what forces will be the chief contributors to the magnitude of the ϵ shocks?

   a) Answer this question analytically first, under the convenient assumption that the time preference rate is $\nu = \underline{\mathsf{r}}$ and that leading up to t bank balances were constant at $b = 1$ and permanent income was $\boldsymbol{p}_t = 1$.

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b) Now answer the question quantitatively, under the convenient assumption that $\underline{r} = 0.02$ and $\bar{r} = 0.04$ and $\rho = 2$ and $\gamma = 0$.

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c) After you have done the algebraic and quantitative analysis, draw a diagram showing the level of consumption from the period leading up to t through an interval after t. On the diagram, label changes that correspond to the combined income, substitution, and human wealth effects in period $t+1$, and to the intertemporal substitution channel of predictable consumption growth in periods before t and after $t+1$.

References

HALL, ROBERT E. (1988): "Intertemporal Substitution in Consumption," *Journal of Political Economy*, XCVI, 339–357, Available at http://www.stanford.edu/~rehall/Intertemporal-JPE-April-1988.pdf.