

The Income, Substitution, and Human Wealth Effects in a Two Period Lifetime (Fisher (1930)). For a consumer who solves the following maximization problem,

$$\begin{aligned} v(y_1) &= \max_{c_1} u(c_1) + \beta u(c_2) \\ \text{s.t.} \\ c_2 &= (y_1 - c_1)R + y_2 \\ u(c) &= \frac{c^{1-\rho}}{1-\rho}. \end{aligned}$$

1. Solve for consumption and saving in the first period as a function of R, y_1, y_2, ρ , and β .
2. Suppose that all noncapital income is earned in the first period of life, i.e. $y_2 = 0$. How does the sign of the derivative of saving with respect to the interest rate depend on the value of ρ (consider the cases of $\rho \rightarrow 0, 0 < \rho < 1, \rho = 1, \rho > 1$, and $\rho \rightarrow \infty$)? Give a verbal explanation.
3. Now suppose that all noncapital income is earned in the second period of life, i.e. $y_1 = 0, y_2 > 0$. For a given value of ρ , how does the responsiveness of consumption and saving to the interest rate compare to the case when all noncapital income was earned in the first period? What is the name of the additional effect on saving? (Hint: There is a particular value of ρ which makes this question easy.)
4. Now suppose that noncapital income is earned equally in both periods, $y_1 = y_2$. How does the response of consumption and saving to interest rates depend on ρ ? Describe and explain the special results obtained as ρ approaches infinity and as it approaches zero.

References

FISHER, IRVING (1930): *The Theory of Interest*. MacMillan, New York.