Name _____

Midterm Exam

Intertemporal Choice

Fall, 2021

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I: Short Questions

1. A recent literature stimulated by Bloom (2009) has argued that shocks to uncertainty are a major source of business cycle fluctuations. Explain, using both math and words, why this argument cannot be investigated using a model in which a representative consumer with quadratic utility makes optimal consumption choices.

   Answer:

2. **CARA Utility and Impatience Over Infinite Horizons**. For a consumer with Constant Absolute Risk Aversion utility,

$$\mathrm{u}(c) = -\alpha^{-1}e^{-\alpha c} \tag{1}$$

calculate the expected change in consumption if $\mathsf{R}\beta$ is not equal to 1. Discuss why the long-run implications of this result are disturbing for consumers for whom $\beta < \mathsf{R}$ in an infinite-horizon framework. (Hint: Where will consumption go?)

Answer:

CARA utility is $\mathrm{u}(c) = -\alpha^{-1}e^{-\alpha c}$ so $\mathrm{u}'(c) = e^{-\alpha c}$.

$$\begin{aligned}
\mathrm{u}'(c_t) &= \mathsf{R}\beta\mathrm{u}'(c_{t+1}) \\
e^{-\alpha c_t} &= \mathsf{R}\beta e^{-\alpha c_{t+1}} \\
-\alpha c_t &= (\mathsf{r} - \vartheta) - \alpha c_{t+1} \\
\Delta c_{t+1} &= (\mathsf{r} - \vartheta)/\alpha
\end{aligned}$$

The model says that the absolute change in the level of consumption is the same in every period, and is either a positive or a negative number. If the consumer is return impatient ($\vartheta > \mathsf{r}$) then this implies that from any starting point, the **level** of consumption will asymptote to negative infinity. What could that possibly mean? (It's bad enough just for consumption to become negative. Negative **infinity**? Really?)

3. **Dynamic Inefficiency and Japan's Woes.**

Define dynamic inefficiency and explain why a plausible way of thinking about Japan's economic problems after 1990 is to argue that Japan's economy has been dynamically inefficient. Explain why the fact that Japan's capital/output ratio is not much higher than the U.S. capital/output ratio presents a challenge to the dynamic inefficiency interpretation of Japan's problems, and discuss how the question of dynamic inefficiency is connected not just to the level of aggregate capital but also to the efficiency with which financial markets allocate capital to productive uses.

Answer:

An economy is dynamically inefficient in steady-state if the net rate of return on a marginal unit of capital accumulation (after depreciation) is less than the population growth rate plus the rate of productivity growth. While both population growth and productivity growth have been low in Japan and are projected to remain low, the rate of return may have been even lower or even negative. Thus the theory of dynamic inefficiency would say that the situation of Japanese consumers could be improved if there were less saving.

In the standard model, the interest rate equals the marginal product of capital, i.e. $\mathsf{r} = \mathrm{f}'(k)$. If we have $k_U = k_J$ (where U and J stand for U.S. and Japan), then the fact that $\mathsf{r}_U >> \mathsf{r}_J$ suggests that the technology for converting marginal capital into marginal output is *different* in the U.S. and in Japan. That is, it is incorrect to assume that $\mathrm{f}_U = \mathrm{f}_J$ because if they have the same $\bar{k} = k_U = k_J$ yet different values of $\mathrm{f}'(k)$, only be captured by assuming $\mathrm{f}'_u \neq \mathrm{f}'_j$ so $\mathrm{f}_u \neq \mathrm{f}_j$.

Thus, the possible explanation of Japan's problems is that Japan's high saving has been inefficient not because it increased the capital stock to an excessive level, but rather because it was being used inefficiently. If Japanese capital had been used better, perhaps the economy would not have been dynamically inefficient.

This perspective suggests that if financial market reforms could boost the rate of return to saving in Japan, that could raise the net rate of return and eliminate the dynamic inefficiency.

4. Consider a Diamond OLG model in which the economy has reached the steady state equilibrium with a constant level of population. Suppose that suddenly in period t there is a permanent increase in the level of population (perhaps as a result of a one-time inflow of immigrants). Show the dynamics of k in the model, and explain whether the surprise increase in the population of workers is good or bad for the old people in period t in this economy, and why.

   Answer:

   Similar experiments are examined in `OLGModel`. The permanent increase in the level of the population corresponds to one-time negative shock to the K/L ratio; adjustment then works its way back toward the same equilibrium k as before.

   The experiment is good for old people because they own the capital and if there are more workers to compete for the use of each unit of capital, then capital is a scarce commodity and will command a high price (thus the old people will earn higher capital income).

Part II: Medium Question

Social Security and Dynamic Inefficiency. Consumers born in period t in a small open economy solve the problem

$$\begin{aligned} \max_{\{c_{1,t}\}} \quad & \log c_{1,t} + \beta \log c_{2,t+1} \\ \text{s.t.} \quad & \\ y_{1,t} &= \mathsf{W}_t - z_{1,t} \\ c_{2,t+1} &= \mathsf{R}_{t+1}(y_{1,t} - c_{1,t}) - z_{2,t+1} \end{aligned}$$

where a consumer supplies 1 unit of labor when young earning wage rate W_t and $z_{1,t}$ reflects lump-sum taxes. Interest rates are set by global capital markets exogenously and fixed at $\mathsf{R}_t = \bar{\mathsf{R}} \; \forall \; t$, and wages are constant at $\mathsf{W}_t = \bar{\mathsf{W}}$. A Social Security system is run using a strictly "Pay As You Go" system: Benefits paid to the elderly (generation 2) in any period $t+n$ must be raised by taxation on the young generation in that period (generation 1):

$$Z_{2,t+n} = -Z_{1,t+n} \tag{2}$$

where upper-case variables reflect the total amounts paid and received by all persons of a given generation: $Z_{\bullet,t+n} = z_{\bullet,t+n} L_{t+n}$. That is, each member of the generation that is young at date $t+n$ pays $z_{1,t+n}$ in taxes, and the proceeds are evenly divided among the elderly.

The labor force (the population of young people) grows according to

$$L_{t+1} = \Xi_{t+1} L_t. \tag{3}$$

The PAYG system is not allowed to change the tax rate on the young unless there is a "surprise" that causes the system to become "unbalanced." What this means is that the system tells young people that they should expect a "return" of $z_{2,t+1}/z_{1,t}$ on Social Security "savings" that is based on an assumption that the future rate of population growth will be constant at some value Ξ, and tries to keep that rate of return stable across generations.

If a "surprise" occurs at date t, then it is possible to allow for changes to the current tax rate on the young generation, and to announce a different tax rate for next period's young generation, but no further changes in expected tax rates are allowed beyond that. In other words, $z_{1,t}$ and $z_{1,t+1}$ can be changed, but $z_{1,t+1+n} = z_{1,t+1} \; \forall \; n > 0$.

1. What return should young people in period $t-1$ expect?

   Answer:

   Ξ

2. Suppose that at date t there is a surprise: The projected rate of growth of the population permanently falls, causing a projected "imbalance" as of the beginning of period t (before $z_{1,t}$ has been set). Explain why this constitutes an "imbalance."

   Answer:

The system cannot continue to pay the old rate of Ξ when the population is now growing at a new rate.

3. Define the 'generational account' of the generation that is young at date $t+n$ as $z_{t+n} = z_{1,t+n} + \mathsf{R}^{-1} z_{2,t+n+1}$, and define the 'Social Security return' of that generation as $z_{2,t+n+1}/z_{1,t+n}$. Explain why the 'Social Security return' of the generation born at $t+1$ must decline when the population growth rate declines.

   Answer:

   Because projected future changes in z_1 beyond period $t+1$ are prohibited, the generation born in $t+1$ will earn a 'Social Security return' equal to the new, lower population growth rate.

4. Using this framework, discuss the government's options for reacting to the population growth shock. Suppose there are three principles of 'fairness' that the government might adopt:

   a) Taxes should be (re)set such that, after the reset, every generation is expected to receive the same SS return as every other generation

   b) Taxes should be (re)set such that no generation should get a lower return than it would have been projected to get in the prior period (that is, before the surprise)

      - That is, the generation born at t should get the same return as the one born at $t-1$ even though population growth has changed

   c) Taxes should be (re)set so that the only generation that experiences a rate of return different from the 'sustainable' return is the generation that is young when the surprise occurs

   Describe the tax policies that will achieve each of these objectives, and explain how they will affect the 'old' (people who were young in period $t-1$), the 'young' (people who are young in period t) and the 'unborn' (people who will be young in periods $t+1$ and thereafter. (Assume that the government is only allowed to break promises about 'Social Security return' when a surprise happens; assume further that the government is prohibited from

   Answer:

5. Discuss the relationship between these choices and considerations of 'dynamic efficiency.' In particular, discuss how the level of world interest rates is related to the question of which responses to the population slowdown might affect the well-being of the different generations differently.

   Answer:

   Assuming the economy is dynamically efficient even under the faster population growth rate, a slowdown in population growth will require the generational account of the unborn generations to worsen.

a) Screw the old (reduce taxes on the young today)

b) Screw the young (keep taxes on the young the same today, but promise them a lower rate of return)

c) Screw the unborn (keep taxes on the young the same today, and promise them the same rate of return, financed by a higher tax rate on unborn generations)

6. Briefly discuss whether these conclusions would change if the economy were a closed economy rather than a small open economy, and if so how and why.

   Answer:

References

Bloom, Nicholas (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77(3), 623–685.