Name ____________________

Midterm Exam

Intertemporal Choice

Fall, 2020

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

In questions that ask about response of the economy to a shock, a perfect answer requires you to depict:

1. The initial state of the economy/variables of interest before the shock.

2. The steady state at which the variables of interest will settle many periods after the shock.

3. The transitional behavior that the variables of interest follow from the moment the shock happens until the new steady state is reached. A particularly important aspect of these dynamics is a clear depiction of moments at which variables "jump" from one level to another abruptly as opposed to moments at which there are small and progressive adjustments to their levels.

These items can be captured by clearly labeled plots with the variable of interest on the y axis and time on the x axis. An alternative is, for instance, a phase diagram in which the trajectory of the variables in the x and y axes is simultaneously represented through an (x,y) line with the flow of time represented using arrows or annotations.

Part I: Short Questions

Explain why the Mankiw (1982) model of durable goods expounded in Durables implies that expenditures on durable goods will be more volatile than spending on nondurable goods. In particular, how and why is the degree of volatility related to the degree of durability?

Answer:

This is answered in Durables. You were not required to reproduce the derivations in order to obtain full credit; however, you did need to understand that the conclusion comes about because the ratio of the *stock* of durable goods d to the *flow* of nondurables c is constant, so that when there is a change in c it is necessary to change the stock of d. The handout illustrates why changing a stock requires a relatively large amount of expenditure. It is likely that in order to obtain full credit you will have needed to have exhibited an equation like

$$x_t/x_{t-1} = \frac{\epsilon_t + \delta}{\delta} \tag{1}$$

although it is possible to obtain full credit by a particularly clear textual explanation of the implications of this equation without actually reproducing the equation itself.

Fisherian Separation (Fisher (1930)). Explain what is meant by the "Fisherian separation" proposition, then explain how relaxing each of the following assumptions in the Fisher model might undermine the proposition:

1. Liquidity constraints
2. Uncertainty
3. Failure to optimize intertemporally
4. Time inconsistent preferences

Answer:

Fisherian separation holds when the profile of consumption is not dependent on the profile of income. In other words, the timing of consumption is independent of the timing of income (up to PDV constraints).

1. If there are liquidity constraints, then a dollar today and a dollar tomorrow are not equivalent. I could use the dollar today directly; but because of liquidity constraints I may be unable to borrow money based on future income and so my consumption is smaller. Thus not only the PDV of income matters for consumption, the timing does as well.

2. Because consumers are risk averse, uncertain income in the future is not equivalent to income today for planning purposes. I will tend to save more and consume less when my income is in the risky future, and consume more and save less when my income is in the present, so the timing of income is important.
3. If consumers fail to optimize intertemporally and are simple Keynesians, spending nine tenths of current income, then the timing of income will influence the timing of consumption directly.
4. With time inconsistent preferences, if the consumer receives more income today she may not be able to help herself and consume more of it than she would have had she received the income later in life; thus not only the PDV of income matters.

Part II

Buffer Stock Responses To Shocks.

In August 2007 global financial markets experienced turmoil triggered by problems in the U.S. subprime mortgage market. Several theories were advanced for the events in financial markets, and some of those theories had important implications for other aspects of macroeconomic behavior, including consumption dynamics. This question asks you to use the TractableBufferStock model to analyze the consequences for consumption of various theories of asset market movements. Specifically, for each theory, you should draw diagrams showing the level of consumption and the growth rate of consumption you would expect to be implied by the theory for continuing-employed consumers if the shock hits in period t and employed consumers were at their target level of wealth before that date.

1. The first theory is that there was an increase in the degree of uncertainty. (While the connection of financial uncertainty to the kind of uncertainty considered in the buffer stock model $\mho$ is tenuous, the right way to think about $\mho$ is as a proxy for all kinds of uncertainty, including financial uncertainty).

   Answer:

   > An increase in $\mho$ increases the target level of assets; to achieve this higher target, it is necessary for consumers to cut consumption for a while so that they can build up their assets. Thus, the short-run consequence is an immediate drop in the level of consumption. In the medium run, consumption growth will be faster, and in the long run consumption will asymptote into a higher level reflecting the larger equilibrium income on the larger stock of assets.

2. The next theory is that there was a sudden, worldwide increase in the coefficient of relative risk aversion ρ. In showing the effects of this, you may assume that the interest rate is equal to the time preference rate, if you find that such an assumption helps clarify your thinking.

Answer:

Surprisingly few people got this right, because most seem to have memorized the formula $\Delta c^e_{t+1} \approx \rho^{-1}(\mathsf{r} - \vartheta) + \mho \nabla c^e_{t+1}$, which is merely a first order approximation (or reflects the case where ρ is fixed at $\rho = 1$), and therefore misses the second-order term which captures the effect of prudence. (Hence the hint 'if you find ...' – the lesson is to NEVER ignore the hint; one this kind of question, you should use your mind, not just your brain.)

If $\mathsf{r} = \vartheta$ and $\rho \geq 1$ then an increase in ρ unambiguously shifts the $\mathbb{E}_t[C_{t+1}/C_t]$ curve upward, and shifts the consumption function downward for any initial value of m. Therefore the consequence is a drop in the *level* of consumption as the more-prudent households strive to build up their savings toward a new higher target buffer stock.

3. The next theory is that the losses in the subprime market are like a transitory negative shock to wealth.
Answer:

The consumption function does not change in response to a transitory negative shock to m; after the shock, m will evolve back toward its steady state. Consumption will drop upon impact, then asymptote back toward the same level that it had before.

4. The final hypothesis is that the era of the 'global savings glut' is coming to an end as the baby boom generation approaches retirement. This corresponds to an increase in the rate of time preference ϑ.
   Answer:

   The reaction to a change in ϑ is given in `TractableBufferStock`.

Part III

Productivity Growth and Dynamic Inefficiency in the OLG Model.
Consider a Diamond (1965) OLG economy like the one in the handout `OLGModel`, assuming logarithmic utility and a Cobb-Douglas aggregate production function,

$$Y = \mathrm{F}(K, PL) \tag{2}$$

where P_t is a measure of labor productivity that grows by

$$P_{t+1} = \mathrm{G}P_t \tag{3}$$

from period to period. Assume that population growth is zero ($\Xi = 1$; for convenience normalize the population at $L_\tau = 1 \;\forall\; \tau$), and assume that productivity growth has occurred at the rate $\mathsf{g} = \mathrm{G} - 1$ forever.

One unit of the quantity PL is called an 'efficiency unit' of labor: It reflects a unit of labor input to the production process.

1. Assume that $\mathrm{F}(K, PL)$ is a Constant Returns to Scale function, and show how to rewrite the capital accumulation equation

$$K_{t+1} = A_{1,t} \tag{4}$$

in per-efficiency-unit terms as

$$k_{t+1} = a_{1,t}/\mathrm{G} \tag{5}$$

Answer:

In an OLG economy, aggregate capital in period $t+1$ is equal to the savings from period t:

$$\begin{aligned}
K_{t+1} &= A_{1,t} & (6)\\
K_{t+1} &= a_{1,t}P_t & (7)\\
\left(\frac{K_{t+1}}{P_t}\right) &= a_{1,t} & (8)\\
\left(\frac{K_{t+1}}{P_{t+1}}\right)\left(\frac{P_{t+1}}{P_t}\right) &= a_{1,t} & (9)\\
k_{t+1} &= a_{1,t}/\mathrm{G}_{t+1} & (10)
\end{aligned}$$

2. Show that under these assumptions, the process for aggregate k dynamics is

$$k_{t+1} = \left(\frac{(1-\alpha)\beta}{\mathrm{G}_{t+1}(1+\beta)}\right)k_t^\alpha \tag{11}$$

Answer:

$a_{1,t}$ can be found from:

$$C_{1,t} = \frac{W_{1,t} + \overbrace{W_{2,t+1}}^{=0}/R_{t+1}}{1+\beta} \tag{12}$$

$$c_{1,t} = \frac{w_t}{1+\beta} \tag{13}$$

$$\begin{aligned} a_{1,t} &= w_{1,t} - c_{1,t} & (14)\\ &= w_{1,t}(1 - 1/(1+\beta)) & (15)\\ &= w_{1,t}(\beta/(1+\beta)) & (16)\\ &= (1-\alpha)k_t^{\alpha}\left(\frac{\beta}{1+\beta}\right) & (17) \end{aligned}$$

Thus (10) and (17) can be combined to yield:

$$\overbrace{k_{t+1}}^{=a_{1,t}/\mathrm{G}_{t+1}} = k_t^{\alpha}\left[\frac{(1-\alpha)\beta}{\mathrm{G}_{t+1}(1+\beta)}\right] \tag{18}$$

as required.

3. Derive the steady-state level of k_t that the economy achieves if the rate of productivity growth is constant at $\mathrm{G}_t = \mathrm{G}\ \forall\ t$.

   Answer:

   The steady-state will be the place where $k_{t+1} = k_t = \bar{k}$. Substituting into equation (11):

$$\bar{k} = \bar{k}^{\alpha}\left[\frac{(1-\alpha)\beta}{\mathrm{G}(1+\beta)}\right] \tag{19}$$

$$\bar{k}^{1-\alpha} = \left[\frac{(1-\alpha)\beta}{\mathrm{G}(1+\beta)}\right] \tag{20}$$

$$\bar{k} = \left[\frac{(1-\alpha)\beta}{\mathrm{G}(1+\beta)}\right]^{1/(1-\alpha)} \tag{21}$$

Now suppose that the economy had been growing at this constant rate G since the beginning of time, but all of a sudden at the beginning of period t everybody learns that henceforth and forever more, productivity will grow at a faster rate than before, $\hat{\mathrm{G}} > \mathrm{G}$.

4. Define the new steady-state as $\bar{\hat{k}}$. Will this be larger or smaller than the original steady state $\bar{k}$? *Explain your answer.*

   Answer:

   Since $(1-\alpha) > 0$, equation (21) implies that a larger value of G implies a lower steady-state capital stock per efficiency unit. The reason is that

Figure 1 Convergence of OLG Economy After Increase in G

with faster productivity growth, the efficiency units of labor provided by the young generation are larger relative to the size of the capital stock saved by the previous generation, so the *ratio* of capital to efficiency units of labor is smaller.

5. Next, use a diagram to show how the $k_{t+1}(k_t)$ curve changes when the new growth rate takes effect, and show the dynamic adjustment process for the capital stock toward its new steady-state, assuming that the economy was at its original steady state leading up to period t.

   Answer:

   Defining the original steady-state capital stock as $\bar{k}$ and the new steady-state capital stock as $\bar{\bar{k}}$, the convergence process looks as indicated in figure 1.

6. Define an index of aggregate consumption per efficiency unit of labor in period t as $\chi_t = c_{1,t} + c_{2,t}/\mathrm{G}$, and derive a formula for the sustainable level of χ associated with a given level of k.

   Answer:

$$\begin{aligned}
K_{t+1} &= K_t + K_t^{\alpha} P_t^{1-\alpha} - C_{1,t} - C_{2,t} && (22)\\
\left(\frac{K_{t+1}}{P_t}\right) &= k_t + k_t^{\alpha} - c_{1,t} - \frac{c_{2,t}P_{t-1}}{P_t} && (23)\\
\left(\frac{K_{t+1}}{P_{t+1}}\frac{P_{t+1}}{P_t}\right) &= k_t + k_t^{\alpha} - c_{1,t} - c_{2,t}/\mathrm{G} && (24)\\
k_{t+1}\mathrm{G} &= k_t + k_t^{\alpha} + \chi_t && (25)
\end{aligned}$$

   The sustainable level of χ is the level $\bar{\chi}$ such that $k_{t+1} = k_t = k$:

$$\begin{aligned}
(1+\mathsf{g})k &= k + k^{\alpha} - \bar{\chi} && (26)\\
\bar{\chi} &= k^{\alpha} - \mathsf{g}k. && (27)
\end{aligned}$$

7. Derive the conditions under which a marginal increase in the productivity growth rate g will result in an increase in the steady-state level of χ, and explain in words why this result holds. (You can leave the term $\partial\bar{k}/\partial\mathsf{g}$ unevaluated in your answer, using only what we know about this term from above).

   Answer:

   There are two effects of an increase in g. First, for a given k, the sustainable amount of $\chi(k)$ declines, because the faster productivity growth means that to keep capital per efficiency unit constant the economy must save more (each efficiency unit of labor must be supplied with its own capital; faster growth of efficiency units therefore requires faster growth of capital). Second, with a faster g the endogenous saving rate and steady-state capital-per-capita $\bar{k}$ will change. Whether steady-state consumption per capita rises or falls depends on the balance between these two things.

Steady-state χ can be written as $\bar{\chi}(\bar{k})$. We are interested in

$$\begin{aligned} \left(\frac{d\bar{\chi}}{d\mathsf{g}}\right) &= \left(\frac{\partial\bar{\chi}}{\partial\mathsf{g}}\right) + \left(\frac{\partial\bar{\chi}}{\partial\bar{k}}\frac{\partial\bar{k}}{\partial\mathsf{g}}\right) && (28) \\ &= -\bar{k} + \left(\frac{\partial\bar{\chi}}{\partial\bar{k}}\frac{\partial\bar{k}}{\partial\mathsf{g}}\right) && (29) \end{aligned}$$

But we know (from above) that $\partial\bar{k}/\partial\mathsf{g}$ is negative; since $-\bar{k} < 0$, (29) can *possibly* be positive only if

$$\begin{aligned} \left(\frac{\partial\bar{\chi}}{\partial\bar{k}}\right) &< 0 && (30) \\ \alpha\bar{k}^{\alpha-1} - \mathsf{g} &< 0 && (31) \\ \bar{r} &< \mathsf{g}, && (32) \end{aligned}$$

where $\bar{r} = f'(\bar{k}) = \alpha\bar{k}^{\alpha-1}$. This is just the dynamic efficiency condition.

In words: We know from above that a higher value of g will decrease the steady-state capital stock per efficiency unit. We know from our analysis in class that the only circumstance in which a *decrease* in capital per efficiency unit will directly result in an *increase* in consumption per efficiency unit is if the dynamic efficiency condition fails to hold. So in order for there to be any hope of an increase in g increasing $\bar{\chi}$, the economy must start out as being dynamically inefficient.

However, dynamic inefficiency is not enough - the second term in (29) must be larger than $\bar{k}$ in order to offset the negative effect of faster g on $\bar{\chi}$. The economy must be *sufficiently* dynamically inefficient that the increase in the raw marginal product of capital that comes from lower $\bar{k}$ *more than offsets* the capital-dilution effect from the requirement to equip the new efficiency units of labor with capital. In math, faster growth increases consumption per efficiency unit when

$$\begin{aligned} \left(\frac{d\bar{\chi}}{d\mathsf{g}}\right) &> 0 && (33) \\ -\bar{k} + (\bar{r} - \mathsf{g})\left(\frac{\partial\bar{k}}{\partial\mathsf{g}}\right) &> 0 && (34) \\ (\bar{r} - \mathsf{g})\left(\frac{\partial\bar{k}}{\partial\mathsf{g}}\right) &> \bar{k}. && (35) \end{aligned}$$

References

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MANKIW, N. GREGORY (1982): "Hall's Consumption Hypothesis and Durable Goods," *Journal of Monetary Economics*, 10(3), 417–425.