Friedman (1957)'s Permanent Income Hypothesis. Answer the following questions under the assumption that Friedman's Permanent Income Hypothesis $\boldsymbol{c}_i = \boldsymbol{p}_i$ is true. Assume that income in each period for each household is $\boldsymbol{y}_{t,i} = \boldsymbol{p}_i + \xi_{t,i}$ where $\xi_{t,i}$ is a mean-0 random ('white noise') transitory shock to income.

1. Suppose that you have consumption and income data for a cross-section of households in a particular year. Suppose further that you know that the variance of transitory income is higher for farmers than it is for members of other occupational groups. Now imagine estimating a Keynesian consumption function $\boldsymbol{c}_i = \alpha_0 + \alpha_1 \boldsymbol{y}_i + \epsilon_i$. How would you expect the estimated coefficients α_1 to differ between the farmers in your sample and the rest of the population? Do you need to know any other information to answer the question definitively?

   Make a copy of the "KeynesFriedmanModigliani" Jupyter notebook, and rename your copy to "FriedmanPIH-[Your-Last-Name]." Using tools in that notebook simulate 50 observations of $\boldsymbol{y}_i$ for each group (farmers and non-farmers), with:

   a) $\sigma^2_{\psi,\text{farmers}} = \sigma^2_{\psi,\text{workers}}$, $\sigma^2_{\xi,\text{farmers}} = 3\sigma^2_{\xi,\text{workers}}$
   b) $\sigma^2_{\psi,\text{farmers}} = 10\sigma^2_{\psi,\text{workers}}$, $\sigma^2_{\xi,\text{farmers}} = 3\sigma^2_{\xi,\text{workers}}$
   c) $\sigma^2_{\psi,\text{farmers}} = 0.1\sigma^2_{\psi,\text{workers}}$, $\sigma^2_{\xi,\text{farmers}} = 3\sigma^2_{\xi,\text{workers}}$

   For purposes of these simulations, assume that $\{\sigma^2_{\psi,\text{workers}}, \sigma^2_{\xi,\text{workers}}\} = 0.01, 0.02$. For each scenario, estimate coefficient α_1 of the Keynesian consumption function, draw the observations and estimated consumption function on a graph with $\boldsymbol{y}$ on the horizontal axis and $\boldsymbol{p}$ on the vertical axis (use different colors for different groups). Make sure you include a 45-degree line (corresponding to $\boldsymbol{c} = \boldsymbol{y}$) on each graph. Relate your results to the first part of this question, and briefly discuss what these graphs tell you. Do the graphs support your answers?

   Answer:

   In class a formula was given relating α_1 to the variance of transitory and permanent shocks:

   $$\alpha_1 = \sigma^2_\psi / (\sigma^2_\psi + \sigma^2_\xi)$$

   where σ^2_ψ is the variance of permanent income and σ^2_ξ is the variance of transitory shocks to income. If σ^2_ψ is the same for farmers and non-farmers, then because σ^2_ξ is greater for farmers $\alpha_{1,\text{farmers}}$ should be less than $\alpha_{1,\text{workers}}$. However, the question provided no information about whether $\sigma^2_{\psi,\text{farmers}}$ is larger or smaller than $\sigma^2_{\psi,\text{workers}}$, so purely on the basis of the information provided in the question it is not possible to know whether $\alpha_{1,\text{farmers}}$ will be greater than or less than (or equal to) $\alpha_{1,\text{workers}}$.

   This point is vividly reflected in Figure 1, where the blue line represents the estimated consumption function for workers and red represents estimated consumption function for farmers. The σ^2_ψ is assumed the same for farmers and non-farmers, but the variance of transitory shock differs:

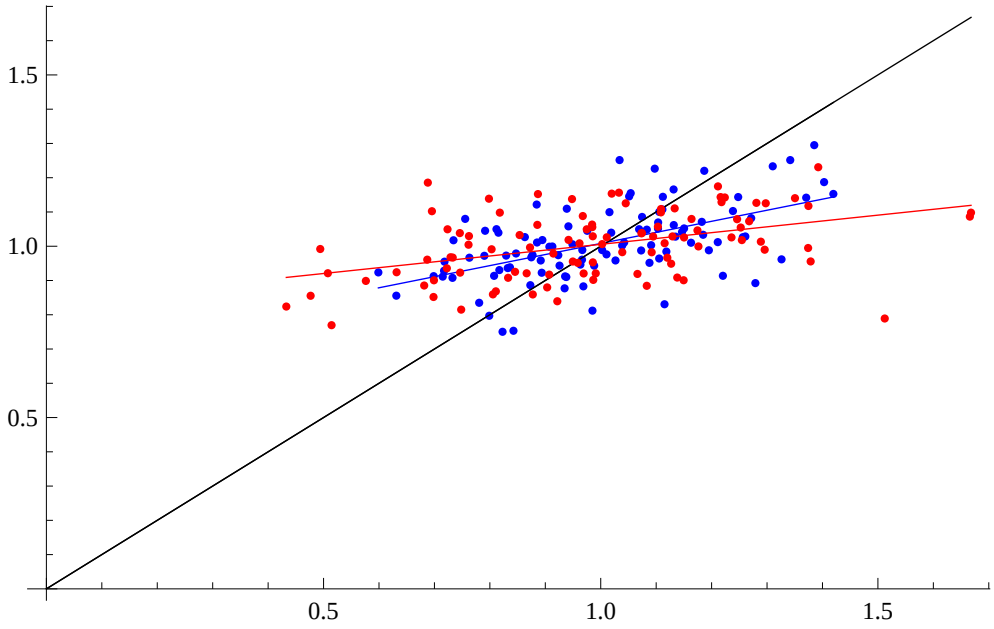

Figure 1 Experiment 1

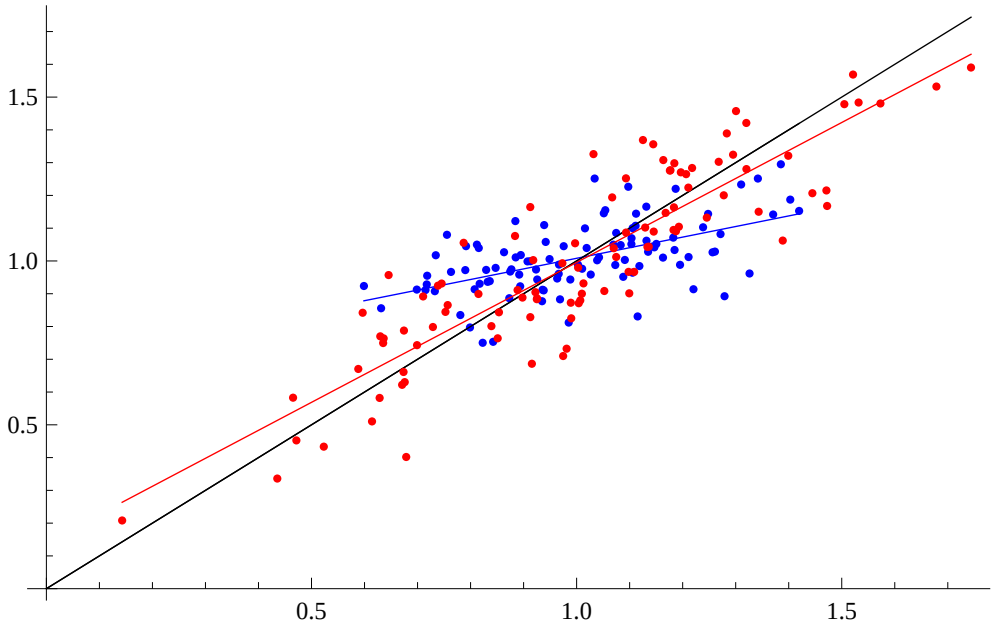

Figure 2 Experiment 2

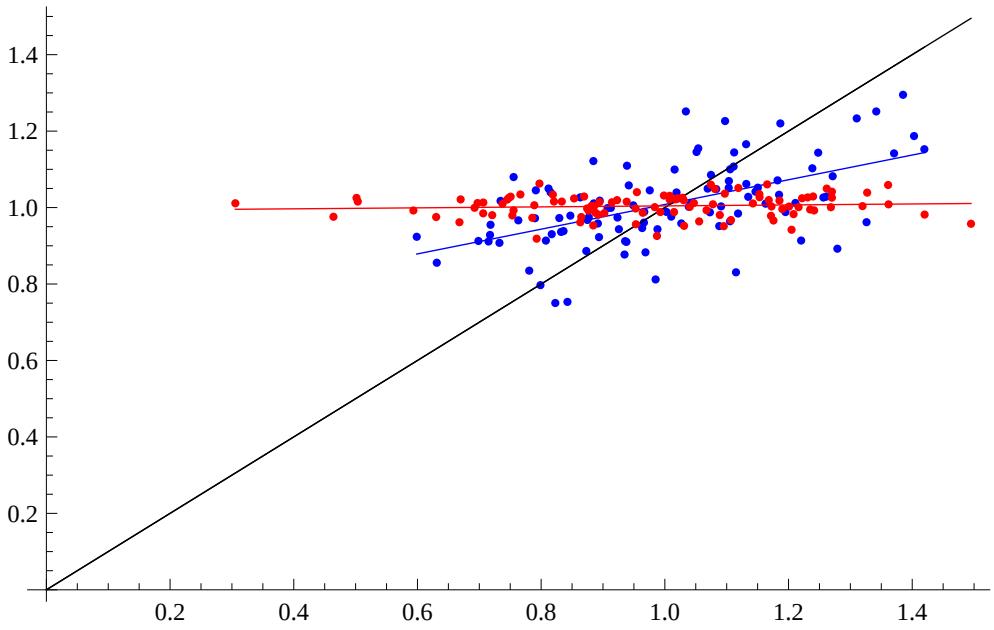

Figure 3 Experiment 3

σ_ξ^2 for farmers is higher than the σ_ξ^2 for workers. The simulations confirm that the MPC for workers is larger than the MPC for farmers (i.e. the blueline is steeper).

The other figures similarly support the intuition behind Friedman's result.

2. Imagine that you observe a set of households for two consecutive periods, t and $t+1$. What relationship should you find between the saving rate at time t and income growth between t and $t+1$?

   Using your Jupyter notebook again, simulate 50 observations of $\boldsymbol{y}_i$ for each of two consecutive periods t and $t+1$. Assume that any individual household's permanent income does not change from period t to $t+1$.

   For each household, calculate the simulated saving rate in period t, the growth rate of income between t and $t+1$, and draw them on a graph (Hint: Put the saving rate in period t on the x-axis, and income growth between t and $t+1$ on the y-axis. Your graph should consist of 50 points). Relate your results to the first part of this question, and briefly discuss what these graphs tell you. Do the graphs support your answers?

   Answer:

   Think of a household which experienced a negative transitory shock in period t of amount $\xi_{t,i}$, i.e. $\boldsymbol{y}_{t,i} = \boldsymbol{p}_i - \xi_{t,i}$. That household would have a negative saving rate in period t, according to Friedman's PIH. Furthermore, in period $t+1$ that household's income is expected to be

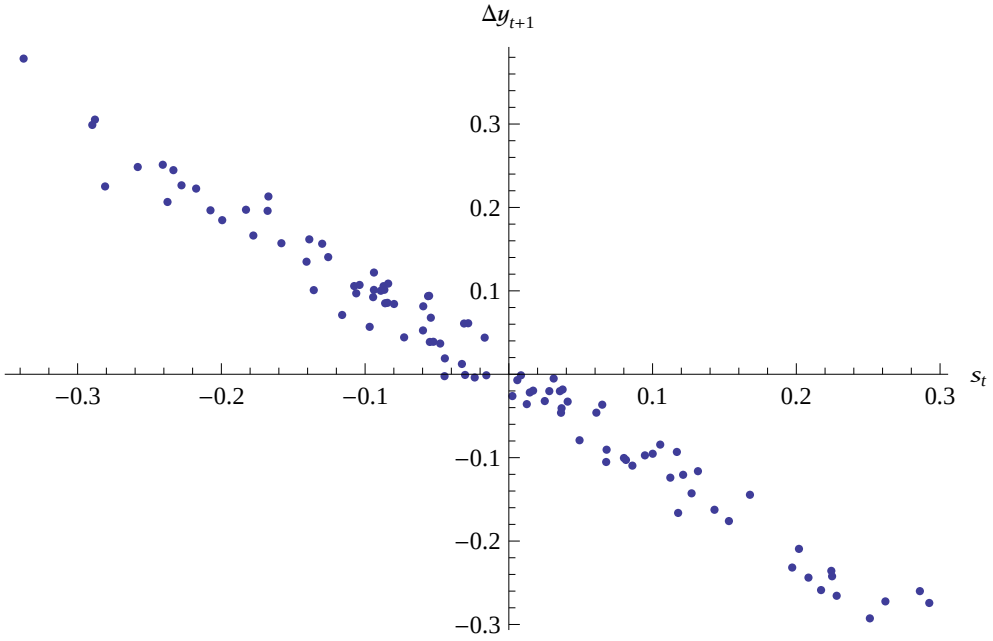

Figure 4 Simulated saving and income growth rate

$\mathbb{E}_t[\boldsymbol{y}_{t+1,i}] = \boldsymbol{p}_i > \boldsymbol{y}_{t,i} = \boldsymbol{p}_i - \xi_{t,i}$, so expected income growth is positive. A similar argument demonstrates that households with positive saving rates in period t should expect negative income growth between t and $t+1$. The conclusion is that there should be a negative correlation between the saving rate in period t and the growth rate of income between t and $t+1$.

The next figure plots the simulated saving rate and income growth rate in next period. Overall, we observe a negative correlation between the saving rate in period t and the growth rate of income between t and $t+1$. A closer look at the figure reveals that this is not *always* the case, however. For instance, if the household after a negative transitory shock suffers from another more severe negative transitory shock, then the *realized* income growth may be negative, although the *expected* income growth is positive.

3. One of the facts that emerged from early household budget studies was that, at any given level of income, Black households had higher saving rates than white households. This seemed puzzling, because other surveys found that Black households had *lower* levels of wealth; and that most households (white or Black) begin life with little wealth.

   Suppose that

   a) White households all have the same permanent income as each other, and all Black households also have the same permanent incomes

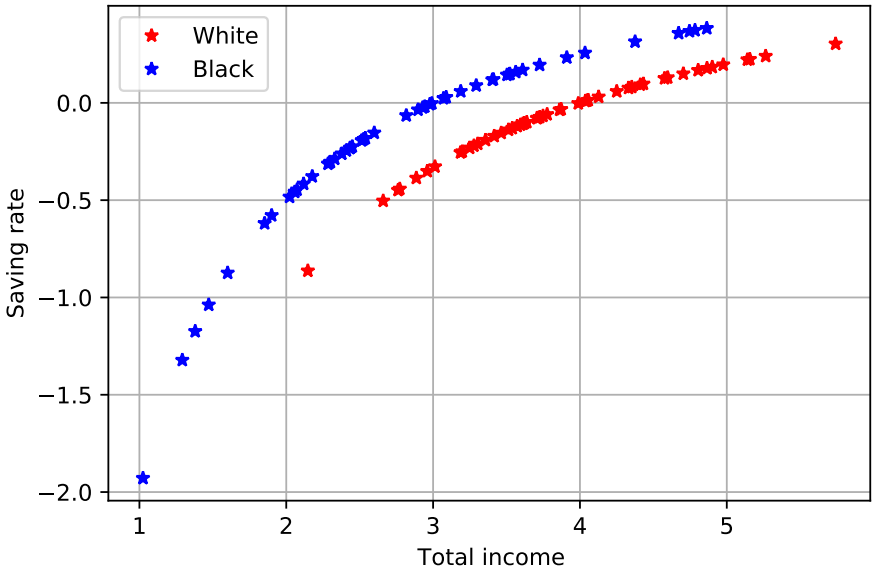

Figure 5 Saving rate at different income levels for different groups.

b) The permanent income of Black households is considerably lower than that of whites.

c) Both groups of households are subject to similarly sized (and large) transitory shocks to income

Do some simulations of a model like this and plot the saving rates by income of the two groups. After pondering the results, explain how the Friedman PIH can explain the two apparently contradictory facts:

a) At any given income level Black households save more

b) Black households end up with lower levels of wealth

Answer:

Figure 5 presents the saving rates of Black and White households at different levels of total income $\boldsymbol{y}$ in a setting that satisfies assumptions a) b) and c).

The figure confirms empirical observation a): at any level of total income, the saving rate of Black households is higher than that of White households. The model's explanation of this fact is that if a Black and a White household have the same *total* income $\boldsymbol{y}$, it must be the case that the Black household is experiencing a higher transitory income shock than its white counterpart (because the Black household's permanent income is lower). Since, in the model, households' flow of savings are exactly

their transitory income shocks, it follows that the Black household will have a higher saving rate.

The model does not have a sensible representation of *wealth* under its usual meaning of accumulated assets: households do not accumulate assets on average since $\mathbb{E}[\boldsymbol{y}_i] = \mathbb{E}[\boldsymbol{c}_i]$. One could think of an interpretation under which White households are *wealthier* in the sense that they have a higher permanent income and will therefore afford a higher level of consumption throughout their lives. This could be a way of interpreting fact b) through the model.

4. In his 1992 State of the Union address, President George H.W. Bush said that, in order to spur consumer spending, he was instructing the IRS to reduce the rate at which income taxes were withheld from taxpayers' wages. Income tax rates were not changed, only the timing of when consumers would pay those taxes. What effect might you expect this change in withholding to have had on consumption?

   Answer:

   According to the PIH, changing withholding rates should have no effect on consumption if the total tax burden is unchanged. Any sensible definition of "transitory" versus "permanent" effects on income would have to classify a change in withholding schedules as having a "transitory" effect on income, since whatever the change in take-home pay is today, come April 15th that change will be exactly offset. One minor caveat: it is true that consumers now got to earn interest on the money that would have been in the government's hands if not for the change in the withholding rate, so perhaps a very small increase in consumption would be justifiable.

References

FRIEDMAN, MILTON A. (1957): *A Theory of the Consumption Function*. Princeton University Press.