Dynamic Inefficiency and the Capital Share Coefficient in an OLG Model.
Consider a Diamond (1965) OLG economy like the one in the handout `OLGModel` and the notebook `DiamondOLG`, assuming logarithmic utility and a Cobb-Douglas aggregate production function,

$$\boldsymbol{Y} = \mathrm{F}(\boldsymbol{K}, \boldsymbol{P}\mathrm{N}) \tag{1}$$

where $\boldsymbol{P}$ is a measure of labor productivity that grows according to

$$\boldsymbol{P}_{\tau+1} \quad = \quad \mathrm{G}\boldsymbol{P}_{\tau}. \tag{2}$$

Population growth is zero ($\Xi = 1$; for convenience normalize the population at $\mathbf{N}_{\tau} = 1 \; \forall \; \tau$), and until date t productivity growth has occurred at the rate $\mathrm{g} > 0$ (equivalently, $1 + \mathrm{g} = \mathrm{G} \geq 1$) forever. Under these assumptions, it can be shown that the dynamic process for aggregate $k \equiv \boldsymbol{K}/\boldsymbol{P}\mathbf{N}$ is

$$k_{\tau+1} \quad = \quad \left(\frac{(1-\alpha)\beta}{\mathrm{G}_{\tau+1}(1+\beta)}\right) k_{\tau}^{\alpha} \tag{3}$$

1. Derive the steady-state level of k_{τ} that the economy will have achieved by date t if the rate of productivity growth has always been $\mathrm{G}_{\tau} = \mathrm{G} \; \forall \; \tau$.

   Now suppose that, after an eternity of remaining in the steady state, all of a sudden at the beginning of period t everybody learns that henceforth and forever more, the exponent on capital in the production function will change to $\hat{\alpha} > \alpha$.

2. Define the new steady-state as $\bar{\hat{k}}$. Will this be larger or smaller than the original steady state $\bar{k}$? *Explain your answer.*

3. Next, use a diagram to show how the $k_{\tau+1}(k_{\tau})$ curve changes when the new α takes effect, and show the dynamic adjustment process for the capital stock toward its new steady-state, assuming that the economy was at its original steady state leading up to period t.

4. Define an index of aggregate consumption per efficiency unit of labor in period τ as $\chi_{\tau} = c_{1,\tau} + c_{2,\tau}/\mathsf{G}$, and derive a formula for the sustainable level of χ associated with a given level of k.

5. Derive the conditions under which a marginal increase in α will result in an increase in the steady-state level of χ, and explain in words why this result holds.

References

Diamond, Peter A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126–1150, http://www.jstor.org/stable/1809231.