The Income, Substitution, and Human Wealth Effects in a Two Period Lifetime (Fisher (1930)). For a consumer who solves the following maximization problem,

$$\begin{aligned} \mathrm{v}(y_1) &= \max_{c_1} \quad \mathrm{u}(c_1) + \beta \mathrm{u}(c_2) \\ &\text{s.t.} \\ c_2 &= (y_1 - c_1)\mathsf{R} + y_2 \\ \mathrm{u}(c) &= \frac{c^{1-\rho}}{1-\rho}. \end{aligned}$$

1. Solve for consumption and saving in the first period as a function of $\mathsf{R}, y_1, y_2, \rho$, and β.

   Answer:

$$\begin{aligned} c_1 &= (y_1 + y_2/\mathsf{R})/(1 + \mathsf{R}^{-1}(\beta \mathsf{R})^{1/\rho}) \\ s_1 &= y_1 - c_1 = y_1 - [y_1 + y_2/\mathsf{R}]/[1 + \mathsf{R}^{-1}(\beta \mathsf{R})^{1/\rho}] \end{aligned}$$

2. Suppose that all noncapital income is earned in the first period of life, i.e. $y_2 = 0$. How does the sign of the derivative of saving with respect to the interest rate depend on the value of ρ (consider the cases of $\rho \to 0, 0 < \rho < 1, \rho = 1, \rho > 1$, and $\rho \to \infty$)? Give a verbal explanation.

   Answer:

$$\begin{aligned} c_1 &= \frac{y_1 + y_2/\mathsf{R}}{1 + \mathsf{R}^{-1+1/\rho}\beta^{1/\rho}} && (1) \\ &= \frac{y_1}{1 + \mathsf{R}^{-1+1/\rho}\beta^{1/\rho}} && (2) \end{aligned}$$

   Consider the term $\mathsf{R}^{-1+1/\rho}$.

   If $\rho = 1$ this becomes $\mathsf{R}^0 = 1$, and consumption and saving do not depend at all on the value of R. This is the log utility case.

   If $\rho > 1$, the exponent on R is negative so that an increase in R reduces the value of the denominator of equation (2), causing c_1 to rise and s_1 to fall. The opposite occurs if $\rho < 1$.

   The verbal explanation is as follows. If ρ is large, then households are unwilling to substitute future consumption for current consumption. In the limit, as ρ approaches infinity, the consumer will set consumption equal in the two periods. In that case, the only effect the interest rate will have on consumption in either period will be due to the income effect: higher R means that the consumer can save less today in order to achieve the same level of consumption tomorrow, thus boosting consumption in both periods. Thus if ρ is large, the "income effect" will be relatively more powerful than the "substitution effect" and thus an increase in R will boost consumption in the first period.

By contrast, if ρ is small, the consumer will be very willing to substitute consumption in one period for consumption in the other. In the limit as ρ approaches zero, the consumer will tend to concentrate all consumption in whichever period affords maximum (discounted) return. In this case an increase in R will give the consumer a relatively powerful incentive to shift consumption from period 1 to period 2. Thus if ρ is small (less than one), an increase in R should reduce current consumption.

Log utility is the special case where the income and substitution effects are exactly balanced.

3. Now suppose that all noncapital income is earned in the second period of life, i.e. $y_1 = 0, y_2 > 0$. For a given value of ρ, how does the responsiveness of consumption and saving to the interest rate compare to the case when all noncapital income was earned in the first period? What is the name of the additional effect on saving? (Hint: There is a particular value of ρ which makes this question easy.)

   Answer:

   Let's choose $\rho = 1$, log utility, to examine this question. In that case consumption reduces to:

$$
\begin{aligned}
c_1 &= \frac{y_2/\mathsf{R}}{1+\beta} \\
s_1 &= y_1 - c_1 \\
&= -\frac{y_2/\mathsf{R}}{1+\beta}
\end{aligned}
$$

   It is clear from this equation that an increase in R will reduce current consumption and increase saving. This is the "human wealth effect." If much or all of lifetime noncapital income is in the future, current consumption should be a strong negative function of interest rates, because the present discounted value of future noncapital income will be very sensitive to the rate at which future noncapital income is discounted.

4. Now suppose that noncapital income is earned equally in both periods, $y_1 = y_2$. How does the response of consumption and saving to interest rates depend on ρ? Describe and explain the special results obtained as ρ approaches infinity and and as it approaches zero.

   Answer:

$$c_1 = y_1 \frac{1 + 1/\mathsf{R}}{1 + \mathsf{R}^{-1+1/\rho}\beta^{1/\rho}} \tag{3}$$

As ρ approaches zero, the substitution effect again becomes very powerful and, as in the case where all noncapital income is concentrated in the first period of life, the allocation of consumption between periods of life becomes very sensitive to the interest rate.

As ρ approaches infinity, however, $(\mathsf{R}\beta)^{1/\rho}$ approaches one and the consumption expression approaches

$$\begin{aligned} c_1 &= y_1 \frac{1 + 1/\mathsf{R}}{1 + 1/\mathsf{R}} \\ \Rightarrow c_1 &= y_1 \end{aligned}$$

Thus as ρ gets large, in this special case consumption becomes perfectly insensitive to the interest rate. This happens because if $y_1 = y_2$, the human wealth effect and the income effect are of exactly equal and opposite sizes, and cancel each other out.

References

FISHER, IRVING (1930): *The Theory of Interest*. MacMillan, New York.