Technical University of Denmark

Stiffened Plates and Sandwich Constructions
41517

Report A

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March 27, 2023

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Introduction

The aim of this report is to analyse a steel-made stiffened plate panel, which is part of the car deck of the RoRo car ferry Prins Richard, sailing the stretches of sea surrounding Denmark.

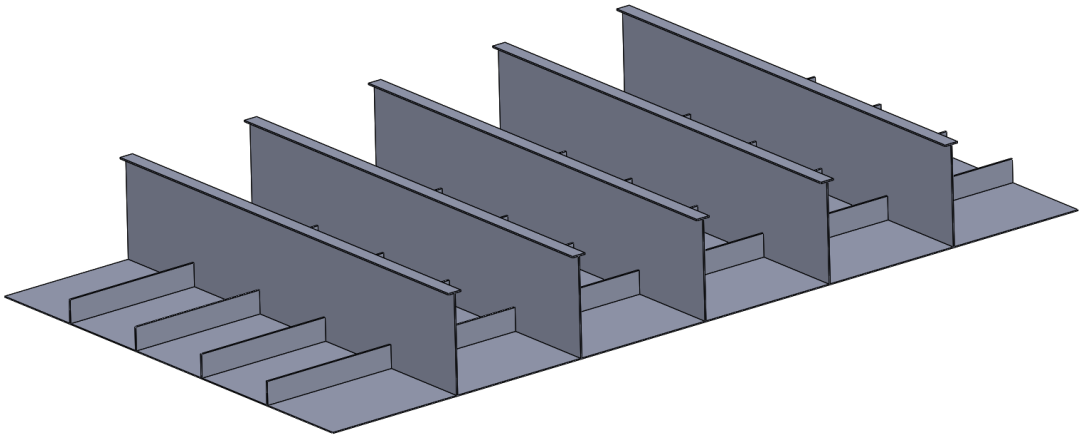

Figure 1: Portion of the car deck panel, proportions are respected

The considered car deck panel, schematically represented is figure 1, is 55.8m long and 9.95m wide, with thickness of the upper plate of 6.5mm. The plate is stiffened with longitudinal flat bars (160x12mm) spaced 663.3mm and transverse beams (T-profiles: web 690x10mm, flange: 100x10mm) spaced 2790mm, as represented in figure 2 (not in scale). In total 14 longitudinals and 19 girders are present in the deck.

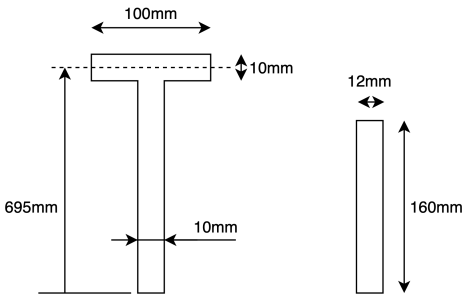

Figure 2: Transversal and longitudinal stiffeners with dimensions

The steel characteristics are reported in table 1. The deck is not supported by columns inside its extension, but only supported along its boundaries by columns and bulkheads.

The report is divided into three main parts, each corresponding to a project phase. In the first phase a separate sub-panel of the car deck plate panel will be analyzed. In the second and third part the analysis is extended to the entire car deck, first analytically and

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Table 1: Steel characteristics

Parameter (symbol)	Value
Young's module (E)	$2.1 \cdot 10^{11} N/m^2$
Poisson ratio (ν)	0.3
Density (ρ)	7850 kg/m^3
Yield Stress (σ_y)	250 MPa

at last numerically using FEM.
As a general introduction to plate structures it is interesting to analyze two commonly used objects, namely an apartment door and a cardboard box, as they present a good example of such structures:

Figure 3: Cardboard box

Figure 4: Simple wooden table

The first object is a cardboard box like the one in figure 3. Assuming the cardboard has a sandwich structure with two layers of flat paper and one corrugated in between, a side panel of the box can be considered simply supported along all four edges, allowing rotation but not translation. The panel is orthotropic due to the varying properties of its layers depending on the direction of loading, and is subjected to both static loading and buckling. Static loading is caused by the pressure of neighboring boxes during transport, requiring the panel to avoid large deflections to protect the box's contents. Buckling occurs when other objects are stacked on the side panel, resulting in in-plane loading. The corrugated panel is strategically placed to increase the stiffness of the vertical side, likely to prevent buckling, as can be observed in the box.

A table such as the one represented in figure 4 is made of four legs and a rectangular steel structure, on top of which a wooden panel is fixed. The rectangular structure acts as a support for the panel, which can be considered as simply supported along all four edges. One could argue that as the panel is not glued to the steel structure, the sides could be seen as free edges, with six fixation points corresponding to the screws, where both rotations and translations are blocked. Considering it as simply supported the analysis is very easy, as the panel only needs to avoid large deflections, due to (probably) static out of plane loads.

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1 Part A: Single Plate Element

1.1 A.3: Analysis methods

In the analysis of plate structures, the most commonly used methods are Navier's and Rayleigh-Ritz method, often referred to as analytical and energy method. The first involves the derivation of a general governing equation for the considered component, while the second method is based on the equilibrium between internal strain energy of the material and energy of external loads. In both methods it is necessary to guess the deflection shape of the component. This is done by choosing a deflection function $w(x_1, x_2)$ such as the one in equations 1.1 and 1.2, respectively for simply supported and fully clamped boundary conditions. There W_{mn} is an unknown, a and b are length and width of the plate and m and n represent the number of half-wavelengths of the deflection pattern in the x_1 and x_2 directions respectively.

$$w\left(x_1, x_2\right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \sin \frac{m\pi x_1}{a} \sin \frac{n\pi x_2}{b} \tag{1.1}$$

$$w\left(x_1, x_2\right) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \left(1 - \cos\left(\frac{2n\pi x_1}{a}\right)\right) \left(1 - \cos\left(\frac{2m\pi x_2}{b}\right)\right) \tag{1.2}$$

These methods are especially useful for simple problems, where it is easy to guess deflection and boundary conditions of the plate. For more complex problems a FEM analysis is often needed, with the disadvantage or requiring more computational power and simulation cost.

1.1.1 Navier's method

Navier's method can only be applied for simply supported boundary condition. It is based on the direct solution of the governing equation for plates, namely equation 1.4.
D_1, D_2 and D_{12} are the stiffness of the plate in the different directions. Here a distinction is made between isotropic and orthotropic plates, for isotropic plates the following applies:

$$D_1 = D_2 = D_{12} = D = \frac{Eh^3}{12\left(1 - v^2\right)} \tag{1.3}$$

$$D_1 \frac{\partial^4 w}{\partial x_1^4} + 2D_{12} \frac{\partial^4 w}{\partial x_1^2 \partial x_2^2} + D_2 \frac{\partial^4 w}{\partial x_2^4} = p \tag{1.4}$$

While for orthotropic plates the rigidities can be calculated with the following equations:

$$D_1 = D\left[1 + b_1 + \frac{12\left(1 - \nu^2\right) a_1 \left(1 + a_2\right) c_1^2}{\left(1 + a_1\right)\left(1 + a_2\right) - \nu^2 a_1 a_2}\right] \tag{1.5}$$

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$$D_2 = D\left[1 + b_2 + \frac{12\left(1-\nu^2\right)a_2\left(1+a_1\right)c_2^2}{\left(1+a_1\right)\left(1+a_2\right)-\nu^2 a_1 a_2}\right] \tag{1.6}$$

$$D_{12} = D\left[1 + \frac{12\left(1-\nu^2\right)\nu a_1 a_2 c_1 c_2}{\left(1+a_1\right)\left(1+a_2\right)-\nu^2 a_1 a_2}\right] + \frac{G}{2}\left(\frac{K_1}{d_1} + \frac{K_2}{d_2}\right) \tag{1.7}$$

All the terms in the three equations above are explained in appendix 4. In the governing equation (1.4) the deflection w is taken from equation 1.1 (simply supported). As w is described by a double sin series summation, a convergence study should be made to find the ideal number of iterations. An example of this study is given in part 1.2 of this document. The deflection will be computed for a specific point, of coordinates x_1 and x_2.

The lateral load p is computed with a similar Fourier transformation, double sine series (equation 1.8). The coefficient P_{mn} is defined in 1.9.

$$p\left(x_1, x_2\right) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} P_{mn} \sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b} \tag{1.8}$$

$$P_{mn} = \frac{4}{ab}\int_0^a\int_0^b p\left(x_1, x_2\right)\sin\frac{m\pi x_1}{a}\sin\frac{n\pi x_2}{b}dx_2 dx_1 \tag{1.9}$$

Substituting 1.1 and 1.9 into the governing equation 1.4 brings to the following formulation for the deflection as a function of external loading:

$$w(x_1, x_2) = \frac{P_{mn}}{\pi^4(D_1(m/a)^4 + 2D_{12}(mn/(ab))^2 + D_2(n/b)^4)}\sin\left(\frac{m\pi x_1}{a}\right)\sin\left(\frac{n\pi x_2}{b}\right) \tag{1.10}$$

In the computation of the P_{mn} coefficient (1.9) if the load is constant the integral reduces to zero when m or n are even. This is very useful in cases as the one analyzed in 1.2, to drastically reduce the computation time.
Once obtained the deflection w of the plate, one can proceed finding the different types of stresses as a function of w with the following equations:

$$\sigma_1 = -\frac{Ex_3}{\left(1-\nu^2\right)}\left(\frac{\partial^2 w}{\partial x_1^2} + \nu\frac{\partial^2 w}{\partial x_2^2}\right) \tag{1.11}$$

$$\sigma_2 = -\frac{Ex_3}{\left(1-\nu^2\right)}\left(\frac{\partial^2 w}{\partial x_2^2} + \nu\frac{\partial^2 w}{\partial x_1^2}\right) \tag{1.12}$$

$$\tau_{12} = -2Gx_3\frac{\partial^2 w}{\partial x_1 \partial x_2} \tag{1.13}$$

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1.1.2 Rayleigh-Ritz's method

In the energy method the deflection is extracted from the principle of minimum potential energy:

$$\partial\Pi = \partial V + \partial U = 0$$

Where V is the potential energy of the external forces acting and U is the strain energy stored in the material. Here only the general equations for U and V are presented in equations 1.14 and 1.15, as later in the document more specific cases are discussed in detail.

$$U_b = \frac{1}{2}\iint_A \left(M_{11}\kappa_1 + M_{22}\kappa_2 + 2M_{12}\chi_{12}\right) dx_1 dx_2 \tag{1.14}$$

$$V = -\left[\iint_A \left(p_x u + p_y v + p_z w\right) dxdy + \sum P\Delta + \sum M\theta\right] \tag{1.15}$$

Where:

- p_x, p_y , p_z Distributed surface forces (pressures)
- P, M Concentrated force and moment
- u, v, w, Δ, Θ Displacement components
- k_1 and k_2 curvature along the two axis, χ_{12} torsional curvature

$$M_1 = D_1\left(\frac{\partial^2 w}{\partial x_1^2} + \nu\frac{\partial^2 w}{\partial x_2^2}\right) \qquad M_2 = D_2\left(\frac{\partial^2 w}{\partial x_2^2} + \nu\frac{\partial^2 w}{\partial x_1^2}\right) \qquad M_{12} = 2D_{12}\frac{\partial^2 w}{\partial x_1 \partial x_2} \tag{1.16}$$

The equations for the moments are in 1.16 where it is evident the dependency with the rigidities in each direction. w is once again approximated by a function, describing the boundary condition for the plate.
One of the greatest advantages of the energy method is that it can consider different boundary conditions such as fully clamped (equation 1.2) or a combination of clamped and simply supported, where different functions are used in each axis. It is interesting to note that using the simply supported boundary condition the energy method reduces to Navier's method. Inserting the formula for w in the energies U and V and deriving the equilibrium equation with respect to W its possible to find the value of the unknown parameter W, obtaining therefore an equation for the deflection w.
Rayleigh-Ritz's method is a more flexible approach than Navier's method and can be applied to more complex geometries and loading conditions, but it may not provide the same level of accuracy as Navier's method. The deflection is still described by a double sin series, but it usually requires a higher amount of iterations to reach a good approximation of the result. So if the boundary condition is simply supported it is usually preferred to use Navier's.

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1.2 A.4: Deflection and stresses due to uniform pressure

The case study for part A of the project is an isotropic plate like the one represented in figure 5, with the following dimensions: length 2800mm, width 670mm and thickness 6.5mm. The material characteristics can be found in table 1.

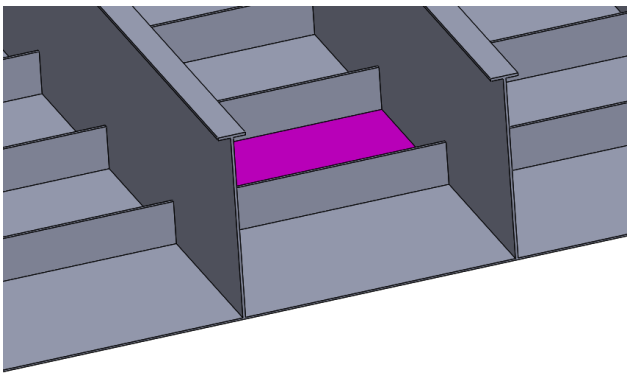

Figure 5: Isotropic plate between stiffeners

The aim of the section is to calculate the deflection and the maximum stresses on the plate due to a uniform static pressure load of $10kN/m^2$. First the simply supported case is implemented with both methods, and later a more in detail analysis using the energy method is given for fully clamped and mixed boundary conditions. The results from the three configurations are anticipated in table 2.

	w [mm]	σ_1 [MPa]	σ_2 [MPa]	τ_{12} [MPa]	σ_{VM} [MPa]
S.S.	4.91	28.41	78.89	29.96	69.98
F.C.	2.73	172.01	243.61	30.84	216.53
Mixed	0.99	15.47	51.57	9.64	45.84

Table 2: Deflection and maximum stresses for uniform p and different B.C.

1.2.1 Simply supported case, uniform pressure

Here the deflection in the center of the plate and the maximum stresses are computed using both methods. The results are collected in table 3 at the end of the sub-chapter.
Navier method is applied as described in section 1.1.1, where the equation for deflection becomes the 1.10. As the load is uniform when computing P_{mn} the integral must be in the whole area of the panel. In this first study it is important to find the required number of iterations, as these values will be used in the rest of the project. It is evident from the study in figure 6 that the deflection converges faster than the stress. In the graph the iterations were 900, but deflection converges at roughly 400 iterations, meaning 20 combinations of m and n, while the stress converges with good approximation at around 650 iterations, meaning 25 combinations. As illustrated one can save computational power by using only the combination of odd m and n for constant p, as anticipated earlier (orange line), this

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results in using only 25% of the iterations. Here and in all this document the stresses are computed using equations 1.11 to 1.13 after finding the deflection w.

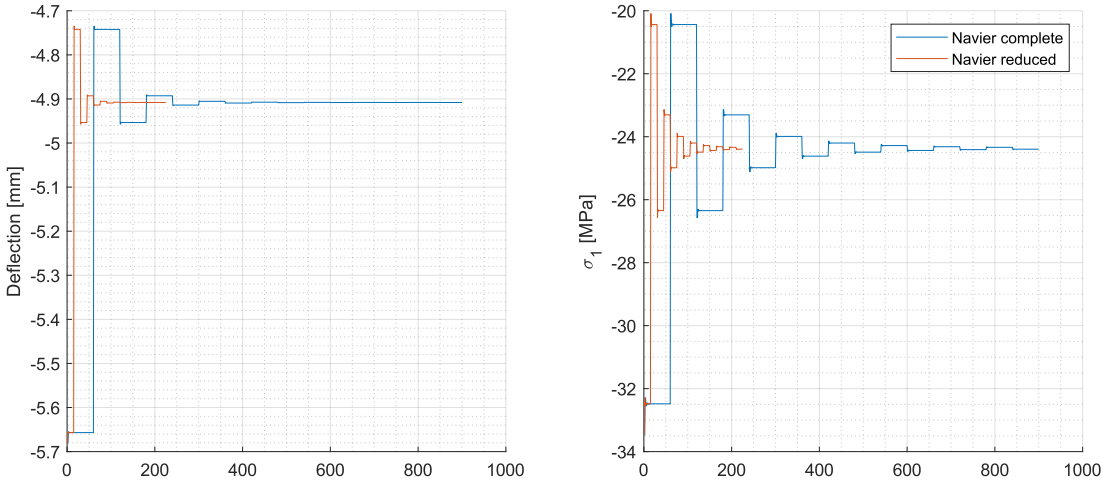

Figure 6: Convergence study for Navier's method

The same process is repeated with the energy method, following the procedure described in section 1.1.2, and obtaining similar results. Once found the need number of iterations it is possible to compute deflection and stresses in all the plate.
To find the maximum stresses on the plate a set of 3D plots was realized. Here only the plots of the deflection and Von Mises stress are included. All the other plots can be found in the appendix 4. Figures 7 and 8 are given by Navier method, but identical results are obtained with the energy method.

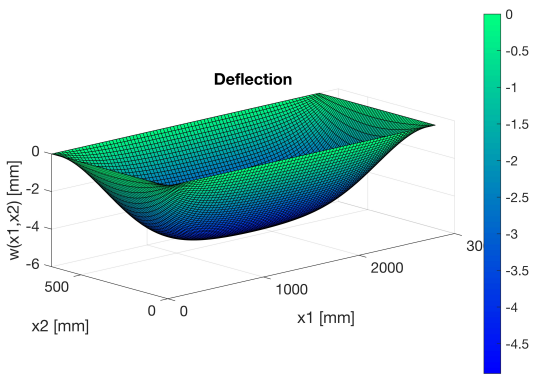

Figure 7: Deflection - S.S. boundaries

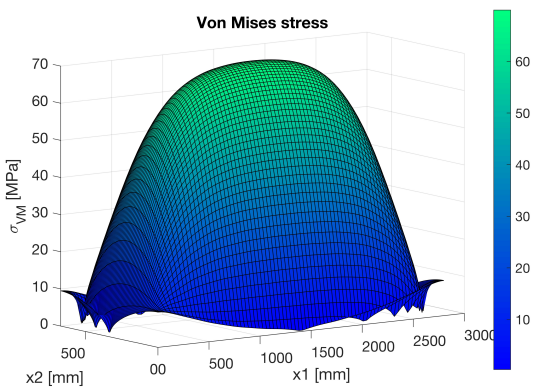

Figure 8: Von Mises stress - S.S. boundaries

The maximum deflection and Von Mises stress appear in the middle of the plate, but this cannot be taken as a general rule. The values of maximum deflection and stress for

the plate are reported in table 3. As illustrated identical results are obtained from the two methods. The results are satisfying both for deflection and Von Mises stress, which is far from yielding of the material. The same values are found applying the energy method.

	w [mm]	σ_1 [MPa]	σ_2 [MPa]	τ_{12} [MPa]	σ_{VM} [MPa]
Navier	4.91	28.41	78.89	29.96	69.98
Energy	4.91	28.41	78.89	29.96	69.98

Table 3: Deflection and maximum stresses for S.S. boundaries, constant p

1.2.2 Clamped case, uniform pressure

As the considered plate is in between stiffeners, it can be treated as fully clamped if those offer stable support. The procedure is the same as described above, but the shape function for w is taken as 1.2. The plots for the deflection and σ_{VM} are in figure 9 and 10 where fewer points were used. Maximum stresses and deflections are in table 4.

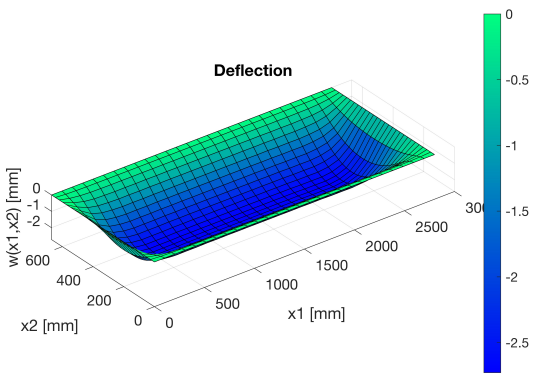

Figure 9: Deflection - F.C. boundaries

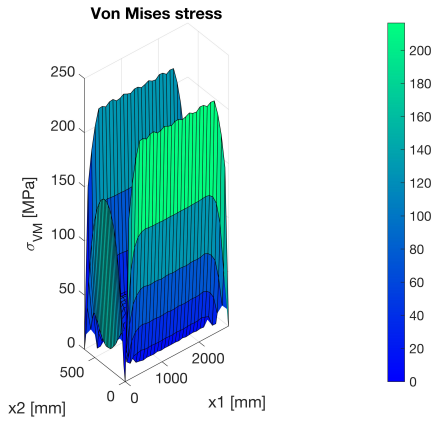

Figure 10: Von Mises - F.C. boundaries

As expected for a clamped boundary condition the maximum Von Mises stresses are along the edges due to the constrain and in the middle due to the plate deflection. The deflection shape is similar to the one for simply supported, but less critical as the plate cannot rotate on the edges. In general this configuration gives worryingly higher stresses compared to the S.S. case, with σ_{VM} over 200MPa, suggesting that the plate thickness might not be sufficient.

	w [mm]	σ_1 [MPa]	σ_2 [MPa]	τ_{12} [MPa]	σ_{VM} [MPa]
Energy	2.73	172.01	243.61	30.84	216.53

Table 4: Deflection and maximum stresses for F.C. boundaries, constant p

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1.2.3 Mixed case, uniform pressure

As the plate fits in between large girders and small longitudinals, a better approximation is to consider it as clamped along the short side (T-stiffeners) and simply supported on the long side (flat bats). This means changing once again the w function, picking a combination of S.S. along x_1 and clamped along x_2. The process is the same, generating the shapes in figure 11 and 12 and the maximum stresses and deflections in table 5.

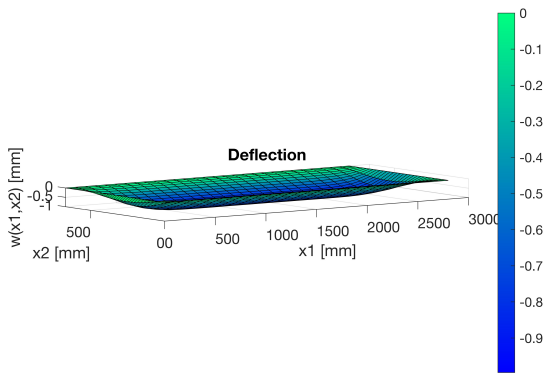

Figure 11: Deflection - Mixed boundaries

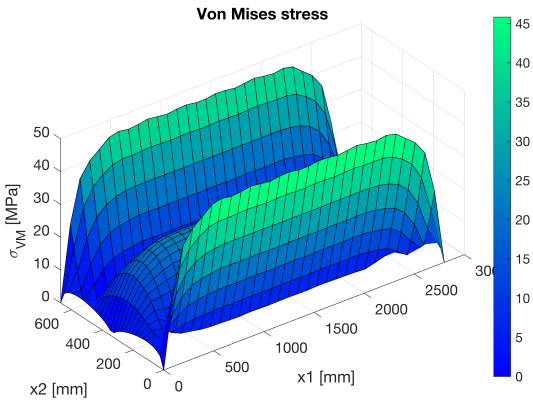

Figure 12: Von Mises - Mixed boundaries

The obtained results are odd. The maximum deflection should lay in between the one for S.S and F.C. to have physical sense. Comparing the Von Mises plot with the previous ones it is also not clear how the maximum stress can be on the free edges, instead of the clamped ones.

	w [mm]	σ_1 [MPa]	σ_2 [MPa]	τ_{12} [MPa]	σ_{VM} [MPa]
Energy	0.99	15.47	51.57	9.64	45.84

Table 5: Deflection and maximum stresses for mixed boundaries, constant p

1.3 A.5: Deflection and stresses due to concentrated pressure

In this new study the load is assumed of 20kN and concentrated in an area of $200x250$mm in the center of the plate. This load is similar to the weight applied by a truck wheel, placed in the middle of the plate in between stiffeners. The procedure of section 1.2 is then repeated following exactly the same steps, with the exception of the boundaries of integration. For Navier's method when calculating P_{mn} with equation 1.10 the boundary conditions need to be changed, reducing the distributed load to the needed $200x250$mm square. For the energy method this is only done for the energy of external forces, V, in equation 1.15, as the energy of the plate U remains the same.

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1.3.1 Simply supported case, concentrated pressure

With the new set-up the process is repeated, obtaining the results in table 6 for simply supported boundary conditions. The new shapes for σ_{VM} and w are illustrated in figures 13 and 14, using 20 combinations of m and n and 30x30 points in the graph.

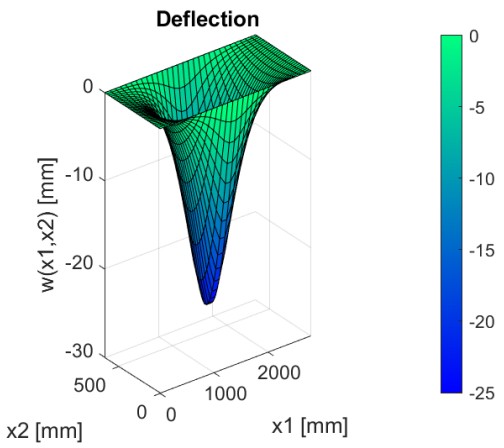

Figure 13: Deflection - S.S. boundaries

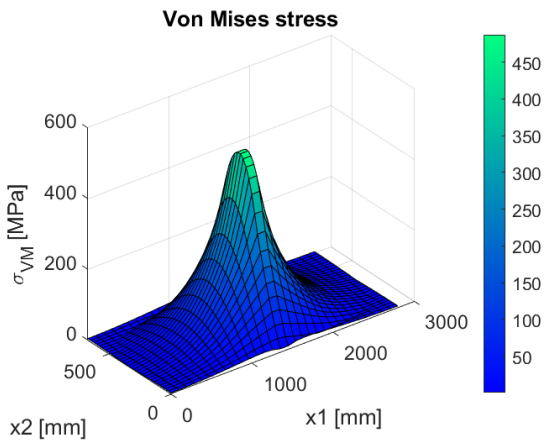

Figure 14: Von Mises - S.S. boundaries

It is evident how both deflections and Von Mises stress are definitely too large, and they would bring the plate to collapse. This suggests that the plate thickness should be increased to a value of at least twice the current $t = 6.5$mm.
Note that the value of $_{VM}$ in the table is for a better convergence of the stress than the one presented in the graph.

	w [mm]	σ_1 [MPa]	σ_2 [MPa]	τ_{12} [MPa]	σ_{VM} [MPa]
Navier	25.57	411.45	553.07	98.72	513.02

Table 6: Deflection and maximum stresses for S.S. boundaries, concentrated p

1.3.2 Clamped case, concentrated pressure

The process is repeated once again using clamped boundary conditions. The obtained results are in table 15 and figures 16 and 7.

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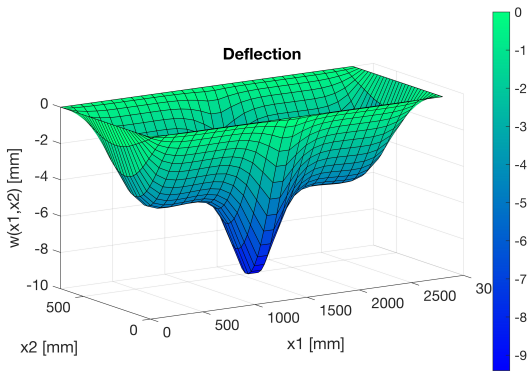

Figure 15: Deflection - F.C. boundaries

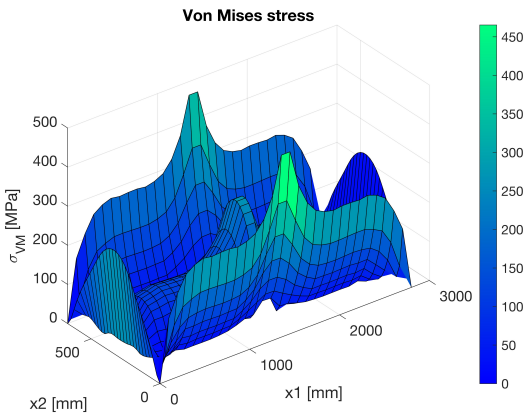

Figure 16: Von Mises - F.C. boundaries

The shape of the deflection is quite odd and suggests that the code probably needed more iterations. The shape of the Von Mises stress corresponds to the B.C., with higher stresses along the edges, where the plate is blocked. The magnitude of deflection is acceptable, while the magnitude of the Von Mises would definitely bring to collapse and suggests redesign is needed.

	w [mm]	σ_1 [MPa]	σ_2 [MPa]	τ_{12} [MPa]	σ_{VM} [MPa]
Energy	9.40	291.34	530.76	46.32	471.75

Table 7: Deflection and maximum stresses for F.C. boundaries, concentrated p

1.3.3 Mixed case, concentrated pressure

The process is repeated once again, with results in table 8 and figure 17 and 18. Similarly to the mixed B.C. case with distributed load the deflection magnitude does not seem logical, as too close to the value for the clamped B.C. For the Von Mises plot the same considerations made in 4 apply.

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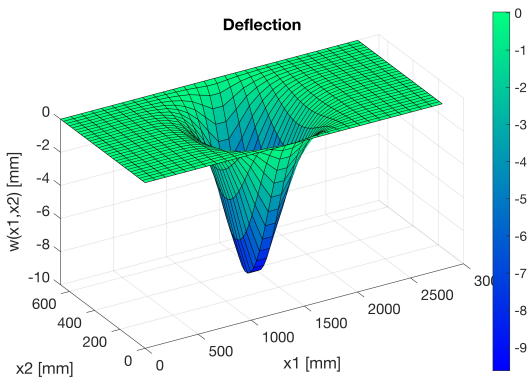

Figure 17: Deflection - Mix boundaries

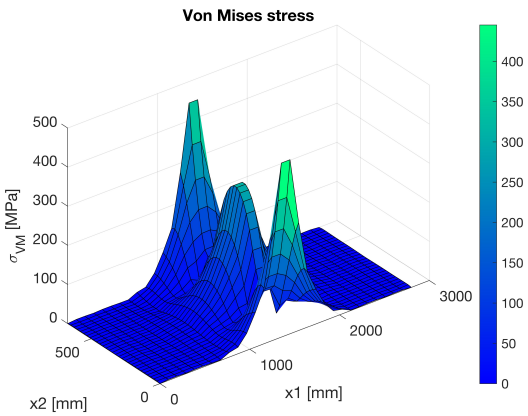

Figure 18: Von Mises - Mix boundaries

	w [mm]	σ_1 [MPa]	σ_2 [MPa]	τ_{12} [MPa]	σ_{VM} [MPa]
Energy	9.62	150.27	500.91	63.46	445.22

Table 8: Deflection and maximum stresses for Mix boundaries, concentrated p

1.4 A.6: Buckling load for plate between stiffeners

When analyzing large structures, like the car deck of a ferry, it is important to consider not only the simple mechanical resistance of the material, but also possible instability phenomenons like buckling.
With buckling one refers to the the sudden deformation of a component due to in plane compression, which brings the structure to instability inducing large deformations, possibly causing the collapse of the structure. This phenomena is generally critical for beams, as those are usually employed to form the skeleton structures, but it is also relevant in plate components.
The approach to buckling analysis consists in evaluating a minimum critical stress, referred to as σ_c, which might cause buckling in the structure. Similarly to what was done in the Navier and Rayleigh-Ritz method a shape function for the deflection is decided depending on the boundary conditions, the critical stress σ_c is then found through a minimisation process of the function.
In this first buckling analysis of the car deck only the plate panel in between the stiffeners is analyzed, so the considered plate is the same $2800x670x6.5$mm plate considered in sections 1.1 and 1.2. The load is assumed as only uniform in plane compression along the x_1 axis, the longest side of the panel, as this in the most critical load for buckling of the plate. All other loads are neglected, and the plate is considered as simply supported, to enhance the conservative approach.

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1.4.1 Analytical solution

The analytical solution consists in the direct derivation of the governing equation for plates, accounting for in plane compression.

$$\frac{\partial^2 M_{11}}{\partial {x_1}^2} + 2\frac{\partial^2 M_{12}}{\partial x_1 \partial x_2} + \frac{\partial^2 M_{22}}{\partial x_2^2} = p + N_{11}\frac{\partial^2 w}{\partial {x_1}^2} + 2N_{12}\frac{\partial^2 w}{\partial x_1 \partial x_2} + N_{22}\frac{\partial^2 w}{\partial {x_2}^2} \tag{1.17}$$

In the case of an isotropic plate in between stiffeners, with only one direction of uniform compressive load this equation becomes:

$$\frac{\partial^4 w}{\partial x_1^4} + 2\frac{\partial^4 w}{\partial x_1^2 \partial x_2^2} + \frac{\partial^4 w}{\partial x_2^4} + \frac{\sigma t}{D}\frac{\partial^2 w}{\partial x_1^2} = 0 \tag{1.18}$$

Where the load along x_1 axis is $N_{11} = -\sigma t$. The isotropic stiffness D is the same as in the previous chapters. The boundary condition is the one of a simply supported plate, equation 1.1. In buckling problems W_{mn} represents the magnitude of the buckling deformation. To simplify the calculation this is not considered in the document, where for buckling only the critical stress is found. Substituting the equation for the deflection into the general equation gives:

$$W_{mn}\left[\left(\frac{m\pi}{a}\right)^4 + 2\left(\frac{m\pi}{a}\right)^2\left(\frac{n\pi}{b}\right)^2 + \left(\frac{n\pi}{b}\right)^4 - \frac{\sigma h}{D}\left(\frac{m\pi}{a}\right)^2\right] = 0 \tag{1.19}$$

Here one is not interested in the solution $W_{mn} = 0$ as it is a trivial solution. The minimum critical tension is found setting the therm in between brackets to zero. This corresponds to having σ_c as a function of m and n. The minimum value will certainly be for $n = 1$.
The term in between brackets is usually referred to as bucking coefficient k, equation 1.21, and it is in this case a function of m.

$$\sigma_c = \frac{D\pi^2}{b^2 t}\left(\frac{mb}{a} + n^2\frac{a}{mb}\right)^2 \qquad \sigma_c = \frac{kD\pi^2}{b^2 t} \tag{1.20}$$

$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2 \tag{1.21}$$

The minimum σ_c is found deriving k. This gives $m = a/b = 4.18$, which is the aspect ratio. This is a very important parameter, which corresponds to the number of half waves that will appear in the plate due to buckling. As the number of half waves needs to be an integer the closest approximation is taken as $m = 4$. With this value of m the buckling

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coefficient is found as $k = 4.008$ and the critical stress is found as $\sigma_c = 71.59$ MPa.
The value of k is a typical value for buckling coefficient of simply supported isotropic plates.

1.4.2 Energy method

For buckling problems the energy method gives a good approximation of the critical stress, because it is relatively easier to guess a deformation shape compared to the simple deflection of a plate described in sections 1.2 and 1.3. In the energy method for buckling, like in the Rayleigh-Ritz method, the equilibrium between strain energy stored in the material U and potential of the external forces V needs to be found. These are described in general therms (but without considering the case of free edges) as:

$$U_{plate} = \frac{1}{2}\int_0^b \int_0^a \left(D_1 \left(\frac{\partial^2 w}{\partial x_1^2}\right)^2 + D_2 \left(\frac{\partial^2 w}{\partial x_2^2}\right)^2 + 2D_{12}\frac{\partial^2 w}{\partial x_1 \partial x_2} \right) dx_1 dx_2 \tag{1.22}$$

$$V_{plate} = -\frac{\sigma_r}{2}\int_0^b \int_0^a \left(f_{11} \left(\frac{\partial w}{\partial x_1}\right)^2 + 2f_{12}\frac{\partial w}{\partial x_1}\frac{\partial w}{\partial x_2} + f_{22} \left(\frac{\partial w}{\partial x_2}\right)^2 \right) dx_1 dx_2 \tag{1.23}$$

In the case analyzed in this chapter $N_{11} = -\sigma_c f_{11} = -\sigma_c t$ and all the other forces are zero. Moreover the plate is isotropic, therefore $D = D_1 = D_2 = D_{12}$. The function describing the deflection is once again 1.1, for simply supported conditions.
From the Matlab code the following values are obtained for U_{plate} and V_{plate}.

$$V_{plate} = -\frac{2W_{mn}^2 \pi^2 m^2 \sigma_c}{10.28} \quad U_{plate} = -\frac{845 W_{mn}^2 \pi^3 (\sin(2\pi m) - 2\pi m)(4489m^2 + 78400)^2}{1690201440256m} \tag{1.24}$$

The critical stress is found from the equilibrium $\Pi = V + U = 0$, inverting the function and obtaining once again σ_c as a function of m (n is again taken a 1).
Minimizing the function for σ_c yields to find $m = 4$ once again and $\sigma_c = 71.59$ MPa.

1.4.3 Energy method - different boundary conditions

So far the plate in between the stiffeners has been considered as simply supported. This is the most conservative approach, as if the edges of the plate are free to rotate buckling will occur at a lower σ_c.
As the plate is in between large T-stiffeners along the short side (x_2) and lighter flat bars on the long side (x_2), a more appropriate boundary condition is to consider it as clamped along x_2 and simply supported along x_1, supposing that the T-stiffeners offer a rigid boundary. The deflection function then becomes:

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$$w\left(x_1, x_2\right) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin\left(\frac{\pi m x_1}{a}\right)\left(1 - \cos\left(\frac{2m\pi x_2}{b}\right)\right) \tag{1.25}$$

Which yields to, from the Matlab code:

$$V_{plate} = \frac{2613 W_{mn}^2 m \sigma_c \pi}{4480}\left(\sin\left(4\pi m\right) - 4\pi m\right) \quad U_{plate} = f(m, W_{mn}^2) \tag{1.26}$$

U_{plate} is intentionally not reported, as the expression is too cumbersome. Also the case of fully clamped is calculated, with deflection function as is equation 1.2.

1.4.4 Comparison and discussion

All the results from the different boundary conditions and methods are reported in table 9, where SS-A is the simply supported with analytical method, SS-E is the same but for energy method, SS+C is simply supported along x_1 and clamped along x_2, and FC is fully clamped. The last two are of course calculated with energy method.

In buckling problems if the critical stress is greater than 50% of the yielding stress of the material, σ_c needs to be corrected to account for plasticity. This is done because if the material operates close to its plastic regime, or over it, buckling will occur for a much lower stress than the one given by the seen methods. This is done with the Johnson - Ostenfeld parabola:

$$\sigma_{c,corrected} = \sigma_y(1 - \frac{\sigma_y}{4\sigma_c}) \tag{1.27}$$

	SS	SS - E	SS + C	FC
σ_c[MPa]	71.592	71.592	167.999	191.096
$\sigma_{c,corrected}$[MPa]			156.993	168.235

Table 9: Critical stress for isotropic plate in different b.c.

As expected SS and SS-E give the same result, while FC is probably a too bold assumption. For the reasons explained above SS+C constitutes the most likely scenario, with a corrected σ_c of 157 MPa. It is also evident how SS is probably a too conservative approach, as it gives an halved σ_c compared to SS+C.

2 Part B: Car Deck Panel

In this second part of the project the full car deck panel is considered, with dimensions $9.95x55.8$ meters, therefore including both the longitudinals and transversal stiffeners. As a

conservative approach the full panel is considered simply supported along all edges. In table 10 all the critical buckling stresses for each component are collected synthetically.

	Full plate	Between stiffeners	Between girders	Tripping
σ_c [MPa]	71.59	417.1	822.9	132.8
$\sigma_{corrected}$ [MPa]		212.5	231	

Table 10: Buckling stresses of the different components

2.1 B.1: Pressure resulting in yielding of deck plate

In this section the load analysis of the car deck structure is carried out. The aim is to find the critical lateral pressure that would cause yielding of the outermost part of the stiffener, for both longitudinal and transversal stiffeners.
The procedure consists in finding the point of maximum deflection of the full panel, to then compute the stress present on the stiffeners in that location and find the pressure p that would cause yielding.

2.1.1 Finding maximum deflection and stress

As the car deck is an orthotropic plate, the first step is to decide how to deal with the stiffeners. The longitudinals are moderate in size, so including them in the plate is a good approximation. The transversal stiffeners are very large, so it is better to consider them as beams, without including them in the plate.
With the aim to compare the results in both configurations (considering the girders as part of the plate or considering them separately) both configurations are implemented, using Navier and energy method respectively. The plate characteristics for the Navier implementation and the Energy method are in equations 2.28 and 2.29 respectively:

$$D = 5.28e04 \qquad D_1 = 4.22e09 \qquad D_2 = 9.30e10 \qquad D_{12} = 1.27e09 \tag{2.28}$$

$$D = 5.28e04 \qquad D_1 = 4.22e09 \qquad D_2 = D \qquad D_{12} = D \tag{2.29}$$

To account for the presence of stiffeners in the energy method, U_{stiff} is calculated by summing up the contribution of all stiffeners with the following equation:

$$U_{stiff} = \frac{1}{2}\int_0^b \left(EI_{x_2}(\frac{\partial^2 w}{\partial x_2^2})^2 + GK_2(\frac{\partial w}{\partial x_2}\frac{\partial w}{\partial x_1})^2 + EI_{f2}(\frac{\partial^2 (d_2 \frac{\partial w}{\partial x_1})}{\partial x_2^2})^2 \right) dx_2 \tag{2.30}$$

This is computed for each stiffener, so substituting x_1 as the position of the j stiffener along the x_1 axis, and summing up all the contribution with a loop.
Once obtained the deflection w, from the equilibrium in the energy method and from the governing equation in Navier, it was possible to find the moment in each point. As the considered plate is now very large, it is difficult to get a good resolution of the deflection points. Instead of doing a full 3D plot of the plate a set of lines were considered along the

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axis: beginning with the x_2 axis the moment was calculated along four lines, corresponding to $x_2 = a/8$, $x_2 = a/4$, $x_2 = a3/8$, $x_2 = a/2$. Note that only half of the plate is considered, as the other half is symmetric. The moment in each point shape is plotted in figure 19, where it is evident how the moment varies along the axis, but always keeping a bell shape. This bell shape in x_2 suggested to compute only one line along x_1, following the peaks of the bells in the middle of the plate. The result of this second computation is in figure 20, where the maximum point was found around $x_2 = 2.5$m. Note that the graph has a scale of $1/100$ in the x axis.

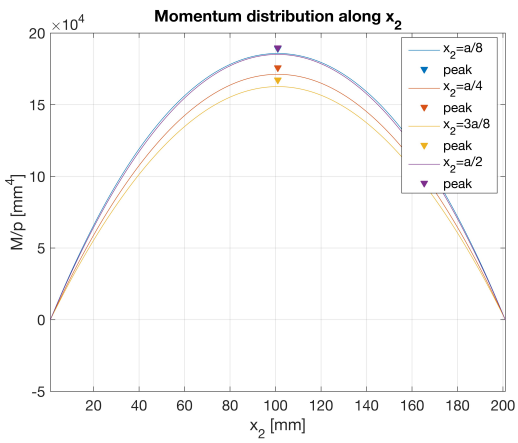

Figure 19: Moment along x_2

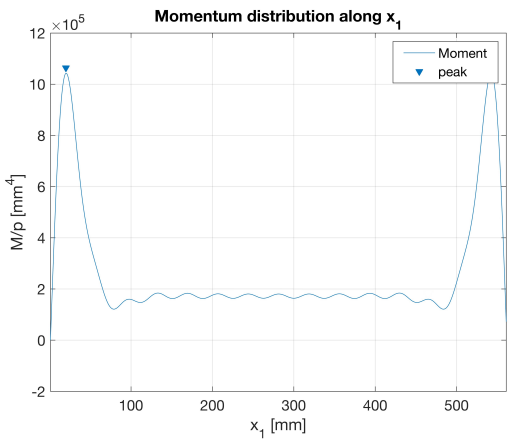

Figure 20: Moment along x_1

With this moment it was possible to compute the critical stress using the effective breath method, considering the effective portion of plate attached to the stiffener. In first part of equation 2.31 the effective breadth b_e is calculated using CL as distance between girders for the longitudinals, and plate width b for the girders. The critical stress is in function of the pressure p, i.e. pressure that would cause yielding.

$$b_e = d \min\left(\frac{1.1}{1 + 2\left(\frac{d}{CL}\right)^2}, 1\right) \qquad A = A_{stiff} + b_e t \qquad e_e = \frac{(-b_e t(t/2) + A_{stiff} e)}{A_x} \tag{2.31}$$

$$\bar{I}x = \frac{fhf^3}{12} + h_f f(h - e_e)^2 + \frac{h_w h_b^3}{12} h_w h_b \left(\frac{h_b}{2} - e_e\right)^2 + \frac{b_e t^3}{12} + b_e t \left(\frac{t}{2} + e_e\right)^2 \tag{2.32}$$

$$W_e = \frac{\bar{I}x}{(h + \frac{hf}{2} - e_e) b_e} \qquad \sigma_{max} = \frac{M_{max}}{W_e} \tag{2.33}$$

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2.1.2 Compare results for Navier and Energy method

The whole process was repeated with Navier method, considering instead of $U = U_{plate} + U_{stiff}$ only U_{plate}, where the plate is an equivalent orthotropic plate accounting for both the stiffeners. The stress distribution graphs were very similar, and the obtained pressures p are reported in table 11.

	Navier method	Energy method
Longitudinals	0.0374MPa	0.0370 MPa
Transversal	0.201MPa	0.1983 MPa

Table 11: Critical pressure p for yielding of stiffeners

Looking at the output of the calculation it seems like approximating the stiffened plate to an orthotropic plate gives a quite good approximation in this case. The most critical components are the longitudinal stiffeners, to evaluate the magnitude of the load that causes yielding a simple consideration is given as follows. A heavy truck weights around 15000kg, which distributed on an area of roughly $2.8x11$ meters results in a distributed pressure of 0.0047MPa, more than five times less the critical one. This is of course for a distributed load, a concentrated load was described in part 1.3 of the document.

2.2 B.2: Buckling stress for plate panel and individual elements

In the following sub-chapter a buckling analysis of the full car deck and its components is carried out, always considering in-plane loading along the longitudinals and uniform over the panel width. Buckling of the plate in between stiffeners is not included here, as it was analyzed in section 1.4.

2.2.1 Buckling of full car deck panel

Once again when considering the full plate smearing out the girders is probably not a good approximation, therefore the implemented procedure accounts for the presence of the large T-stiffeners while considering the flat bars into an equivalent orthotropic plate.
The procedure is similar to the one described in 1.4.2 for the orthotropic plate. The deflection function $w(x_1, x_2)$ is described in equation 1.1, simply supported case.
This time the strain energy of the material is $U = U_{plate} + U_{stiff}$, where U_{plate} is the same as is equation 1.22. Smearing out the longitudinals into the plate gives an orthotropic plate with the same characteristics as in equation 2.28. The contribution of the transversal stiffeners is once again computed with equation 2.30.
As the load is only applied to the longitudinals, along the x_1 axis, $V = V_{plate} + V_{stiff}$ reduces to $V = V_{plate}$. V_{plate} is computed using equation 1.23, as done in section 1.4 for the isotropic plate. The load function is the same, for uniform loads $N_{11} = -\sigma_c f_{11} = -\sigma_c t$. The intermediate results are:

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$$V_{plate} = -\frac{W_{mn}^2 bm\sigma_c \bar{t}\pi \left(\sin\left(2\pi m\right) + 2\pi m\right)}{16a} \tag{2.34}$$

$$U_{plate} = -\frac{W_{mn}^2 \pi^3 \left(\sin(2\pi m) - 2\pi m\right)\left(Da^4 + 2Da^2b^2m^2 + D_1b^4m^4\right)}{16a^3b^3m} \tag{2.35}$$

U_{stiff} is too cumbersome to be included. From these expressions, using the equilibrium, σ_c is found as a function of m. The minimisation of the function yields $m = 12$ and $\sigma_c =$ 417.1MPa. After correction with the Johnson - Ostenfeld parabola the result is $\sigma_{c,corrected} =$ 212.5MPa. Considering also the conservative boundary conditions, this result is satisfying and suggests no redesign is needed. Results are also in table 10

2.2.2 Buckling of panel in between girders

In this section the critical stress σ_c for the panel in between the girders is calculated. The process is the same as for the full car deck panel, modifying the plate dimensions a and b in the code, and removing the transversal stiffeners. U_{plate} is then going to be the one of an orthotropic plate accounting for the longitudinals, and $U_{stiff} = 0$ as the longitudinals are smeared out in the plate. V_{plate} follows the same procedure described above for the orthotropic plate and $V_{stiff} = 0$ despite the load is applied in the x_1 direction, because the longitudinals are included in the plate. The obtained result for this configuration, for $m = 1$, is $\sigma_c =$ 822.9MPa and $\sigma_{c,corrected} =$ 231MPa which is much higher than the one obtained for the full plate. This result is congruent, as the plate in between girders will be less free to deflect than the full plate, therefore resisting more to buckling.

2.2.3 Tripping of longitudinals

Tripping is considered a failure mechanism for plates. It is a phenomena where the stiffener suddenly bends to the side, with large deformation, causing a drastic loss in rigidity of the plate. As in this phenomena the plate deforms together with the beam, considering only the stiffener itself is a too conservative approach.

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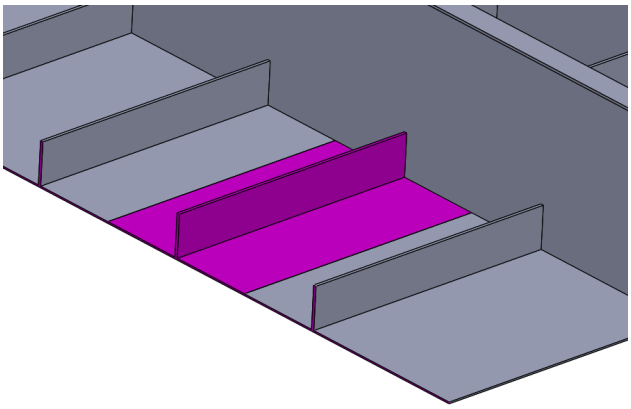

Figure 21: Considered geometry for tripping of the longitudinals

In what follows a portion of $\pm b/2$ of the plate is considered together with the longitudinal in position $x_2 = 0$, as represented in figure 21. Of course only the portion of stiffener in between the girders is considered, so its dimensions are roughly $2800x670$mm. Using the energy method the aim is at the equilibrium $\Pi = U + V$, from which the critical stress is extracted. For this study $U = U_{plate} + U_{stiff}$ and also $V = V_{plate} + V_{stiff}$, as the force is applied on the longitudinal stiffener V_{stiff} has to be considered. The formulation for σ_c becomes:

$$\sigma_{r,c} = \frac{U_{\text{plate}} + U_{\text{stiff}}\big|_{x_2=0}}{V^0_{plate} + V^0_{stiff}\big|_{x_2=0}} \tag{2.36}$$

Where U_{plate} follows equation 1.22 for the isotropic case, as it is the plate in between stiffeners. U_{stiff} is given by equation 2.30 selecting the parameters of the longitudinals and V_{plate} by equation 1.23 where in the load formula the considered thickness is the simple t, for isotropic plate. In the tripping analysis it is important to consider the torsional rigidities as well in U_{stiff}, given by the terms G and K. These rigidities were neglected trough all the other analysis in this document, as open profiles have low torsional rigidity. V_{stiff} is given by equation 2.37 below, where the integration extremes are along x_1 (longitudinals) and in all the equation $x_2 = 0$, as that is the considered stiffener. The deflection function $w(x_1, x_2)$ is taken as always for simply supported.

$$V_{stiff_{equ}} = -\frac{\sigma_c}{2}\int_0^a A_{stiff_1}\left(\frac{\partial w}{\partial x_1}\right)^2 dx_1 - \frac{\sigma_c}{2}\int_0^a I_{p_1}\left(\frac{\partial^2 w}{\partial x_2 \partial x_1}\right)^2 dx_1 \tag{2.37}$$

Here A_{stiff_1} is the area of the stiffener, I_{f1} is the moment of inertia with respect to its vertical axis and I_{p_1} is the polar moment of inertia. Combing the energy components as in equation 2.36, gives a function of m. The minimization process yields $m = 3$, which substituted in the equation gives a minimum buckling stress of $\sigma_c = 132.8$MPa, roughly 50% of the yielding stress.

Compared to the other analysis, tripping of the longitudinals turned out to be a critical point for the car deck. Connecting to the considerations made in section 1.3, the structure

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would probably benefit from a thicker plate, as 6.5mm is quite thin for load carrying. Considering a plate thickness of $t = 10$mm for instance yields to a critical stress of $\sigma_{c,corrected} =$ 170.7MPa with $m = 4$ for tripping of the longitudinals.

3 Part C: FEM

3.1 C.1: FEM analysis of entire car deck panel

The third part of the project includes the analysis of the previous elements by the use of Finite Element Methods. The software selected to execute the method is Solid Works. The FEA simulations in this program are usually simpler than in other similar commercial software, but as it is not a computationally demanding element, Solid Works is capable enough to carry it out.

3.1.1 FEA for exercise A4

First of all, the exercise selected to reply in FEA to compare is A.4 question (distributed load). The material properties have been defined as in the table 1, and the plate dimensions have been replied as in part 1.2 of this report. The component has been modelled as a shell-type element to reduce the computational expensiveness.

Following with the boundary conditions applied, two cases have been computed (simply supported and completely clamped) to knowledge the maximum values obtained, and compare them with the analytical methods. The one for the simply supported model, the edges on the perimeter of the plate have been restricted for translation in the three-axis but, the rotation is still free. The load applied on the plate is a uniformly distributed load of 10000 N/m2.

Moving on with the discretization, the elements used are shell quadratic elements with a size of 62.35 mm, which leads to a 2285 number of nodes and 1086 number of elements.

The results shown below have been chosen to be displayed for the plate: displacement and Von Mises stress.

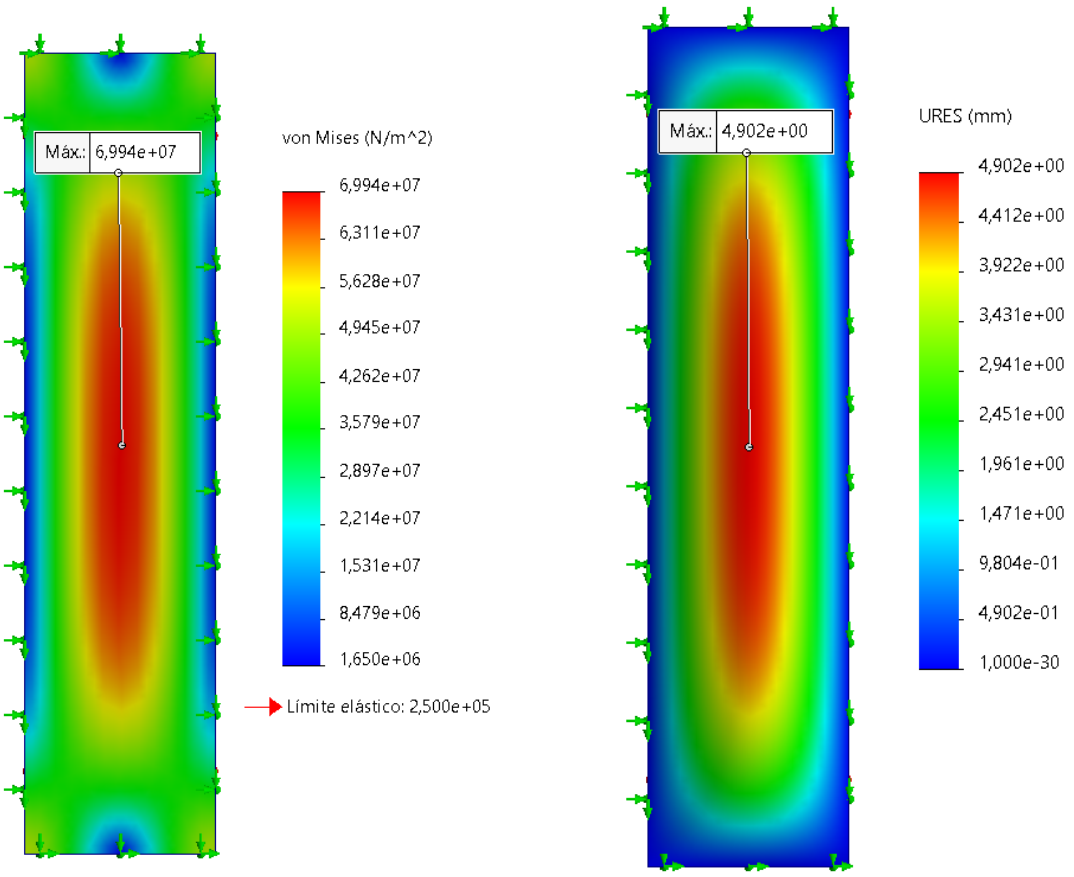

Figure 22: Simply supported plate with distributed load, left Von Mises stress and right, deflection

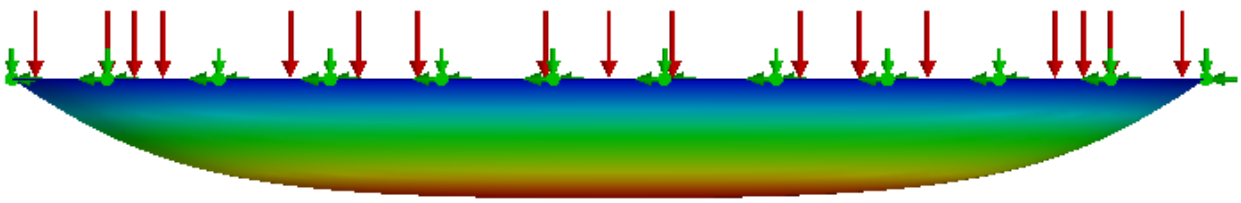

Figure 23: Shape of the defection of the simply supported load

It can be seen in this simulation of figure 23 that the maximun displacement occurs on the middle point of the plate, with a value of 4.9 mm. The maximum Von Mises stress can be found at the same point, with a value of 70 MPa. The shape of the stress increases at the middle of the 4 edges, increases for the points near the edges and goes up again for the centre of the plate. In the figure 23 it is clear that the boundary condition is correctly applied as a simply supported plate because there is no inflexion point along the plate.

Next, the same plate with different boundary conditions has been simulated: clamped edges.

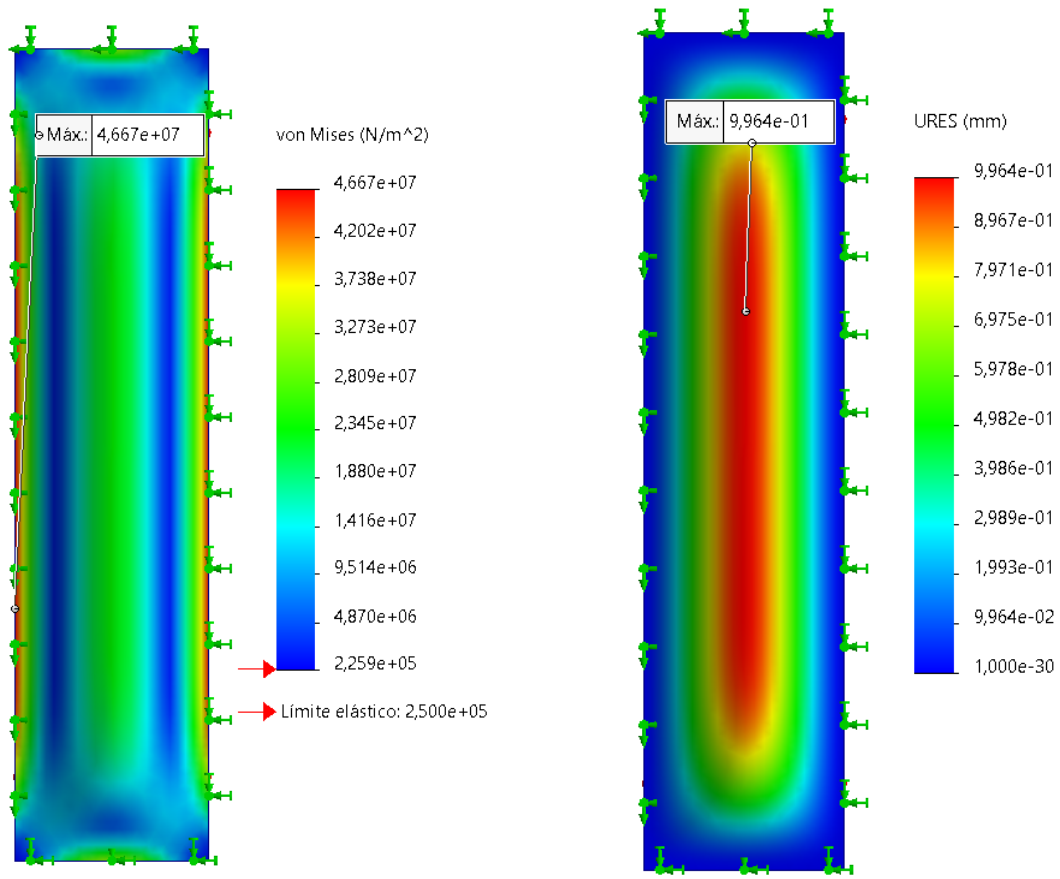

Figure 24: Clamped plate with distributed load, left Von Mises stress and right, deflection

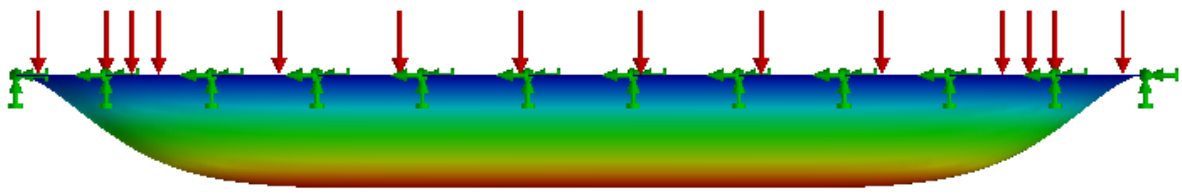

Figure 25: Shape of the defection of the clamped model

In this different case, where the plate is completely clamped in the edges, the maximum Von Mises stress decreases considerably and changes the location, being now in the clamped edge along x_1. The maximum deflection is decreased until less than 1 mm in the same node. The shape that the plate adopts regarding the clamped boundary condition has now inflexion points (figure 25) due to the clamped edge, which does not allow rotation.

Finally, boundary conditions have been mixed.

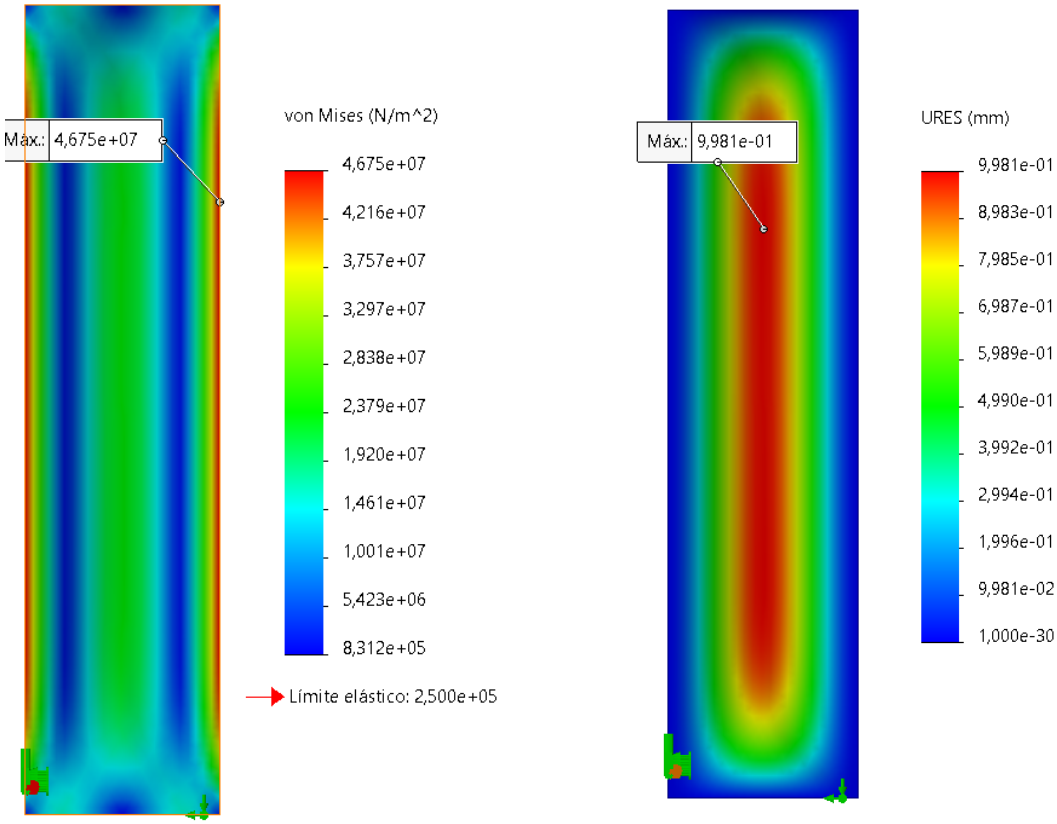

Figure 26: Mixed boundary conditions plate with distributed load, left Von Mises stress and right, deflection

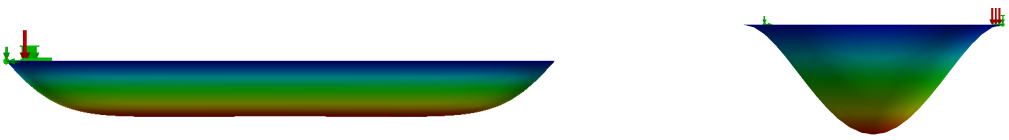

Figure 27: Mixed boundary conditions plate with distributed load, left Von Mises stress and right, deflection

In this specific case, both boundary conditions have been applied to the edges. Clamped along x_1 direction (the long edge) and simply supported along x_2 direction (short part). The results can be found in the table **??**, as well as the similarities between this final simulation and the one with the clamped edges. In this case, it can be seen that one of the sides does have an inflexion point and the other does not, which confirms the applied boundary conditions.

Table 12: Results for simulations for A4

	σ_{VM} [MPa]	w [mm]	σ_1 [MPa]	σ_2 [MPa]	τ_{12} [MPa]
Simply supported	69.94	4.90	28.38	78.88	30.05
Clamped	46.67	0.99	35.44	52.88	6.65
Mixed BC	46.75	0.99	17.51	52.98	9.42

Table 13: Results for simulations for A4

3.1.2 FEA for exercise B1

In this part of the report, the element from exercise B1 has been modelled on SolidWorks to simulate the uniform lateral pressure resulting in yielding. To carry out the FEA, a shell-type model has been implemented with all the stiffeners and the plate. The number of elements used is 7064 and the number of nodes 18433. The edges along all the plate have been defined as simply supported, and the interaction between elements is none penetrating. 37,5 Pa have been applied to the structure with the aim of comparing it with the analytical results. There is a chance that the analytical results are not correct.

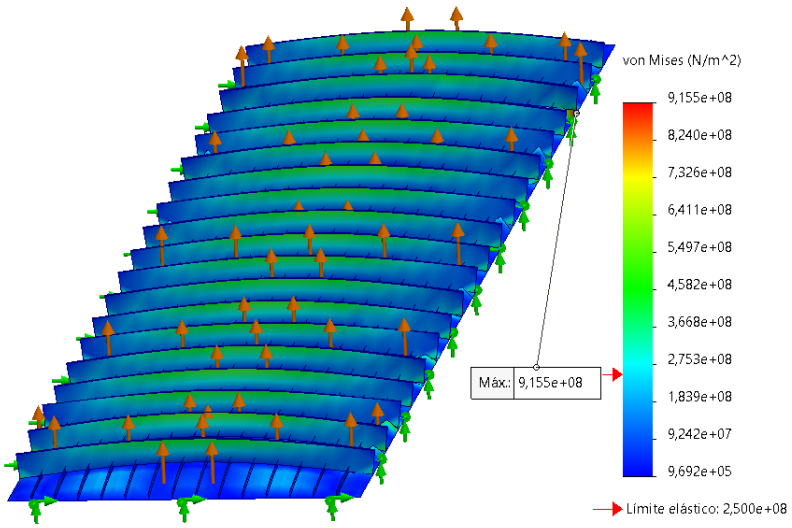

Figure 28: Model for a stiffened plate

The previous figure shows some issues. First of all, the discretization of the mesh is not accurate enough, because the hypothesis made is that the maximum stress will happen around the middle of the plate, in the tip of a stiffener. Due to computational limitations, it could not be simulated with a higher number of elements or nodes.

3.2 C.2: Discussion of the results.

Discussion on the differences between orthotropic plate theory and realistic representation of all stiffeners in a plate panel analysis, eg. using FEM

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4 Appendix

Stiffener equations components

- d_1 = distance between stiffeners along x_1
- c_1 = eccentricity ratio
- b_1 = rigidity ratio
- a_1 = are ratio (stiffener/plate)
- K_1 = torsional rigidity
- G = shear modulus
- ν = Poisson ratio

Plots of plate in between stiffners

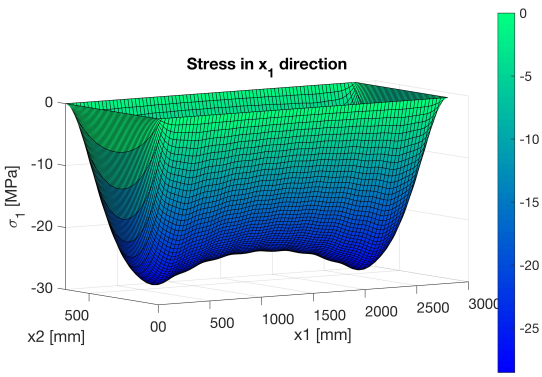

Figure 29: σ_1 - S.S. boundaries

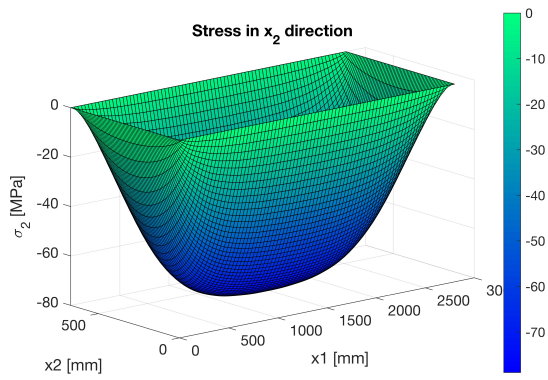

Figure 30: σ_2 - S.S. boundaries

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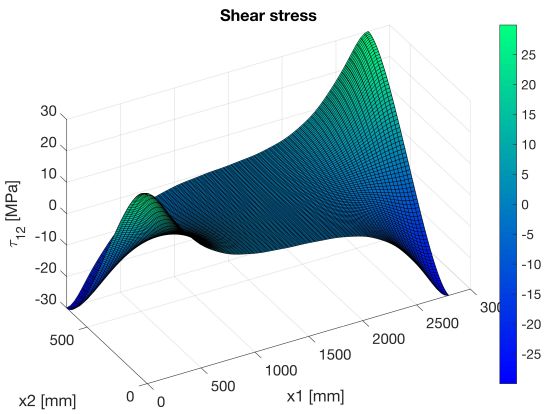

Figure 31: τ_{12} - S.S. boundaries