Corey Caskey | CS 4646 – ML for Trading | Martingale HW – 8/30/18

1. In Experiment 1, estimate the probability of winning $80 within 1000 sequential bets. Explain your reasoning.

   The probability of winning $80 in 1000 sequential bets is approximately 1.0. Based on Figure 1, each of the 10 simulations of 1000 bets converged to $80 in under 200 bets, which would presume a guaranteed winning strategy. However, this is just a small sample of the total number of possible win/loss combinations that could occur with the Monte Carlo approach. While unlikely that a player would lose a large number of sequential bets such that they never recovered enough to reach $80, the possibility exists and thus forces our probability to be slightly lower than 1.0.

2. In Experiment 1, what is the expected value of our winnings after 1000 sequential bets? Explain your reasoning.

   The expected value after 1000 sequential bets would be $80. Based on Figure 1 and the explanation from question 1, as well as after running 1000 simulations and reaching $80 every time, we conclude that the probability of achieving $80 is infinitesimally close to 1.0. In this infinite case of earnings between $-\infty$ and 80, we have the following formula: $E[\text{Earnings over 1000 bets}] = \sum_{i=-\infty}^{80} i \cdot p_i$. Again, since $80 dominates the probability, the equation would become $E[\text{Earnings over 1000 bets}] = 80 \cdot 1.0$. Furthermore, this is the value Figure 2 converges to over the course of the bets.

3. In Experiment 1, does the standard deviation reach a maximum value then converge or stabilize as the number of sequential bets increases? Explain why it does (or does not).

   Over the course of 1000 bets, the standard deviation appears to move alongside the slope of the mean graph in Figure 2. While there are several spikes indicating short periods of massive bets due to repeated losses, they are surrounded by smaller peaks that generally stabilize as more bets are placed. This follows intuition that, initially, if you start out with $0 in earnings, any losses will immediately send you into the negatives, whereas later on when your average winnings are more positive, any stint of repeated losses may keep you in positive earnings longer and, thus, prevent the larger variation in winnings until it stabilizes when the threshold is reached.

4. In Experiment 2, estimate the probability of winning \$80 within 1000 sequential bets. Explain your reasoning.

   The probability of winning \$80 within 1000 sequential bets using the more realistic simulation is approximately 0.633, which was obtained after running 1000 simulations. From Figure 4 and Figure 5, this probability seems to make sense. Over the course of 1000 bets, on average, we are not losing too much money; ergo, a portion of the episodes must achieve positive earnings and potentially even the \$80 threshold. Furthermore, Figure 5 shows that the median earnings converge to \$80, which means that *at least* half of the simulations resulted in the \$80 winnings.

5. In Experiment 2, what is the expected value of our winnings after 1000 sequential bets? Explain your reasoning.

   The expected value after 1000 sequential bets would be about $-\$43$. After running a simulation of 1000 sequential bets, it was found that the end result was either \$80 or $-\$256$, meaning that any other potential earnings at 1000 bets falling between those two thresholds occur with an infinitesimally small probability. Within this simulation, it was found that \$80 had a probability of 0.633 and $-\$256$ had a probability of 0.367, which leads the expected value to be $E[\text{Earnings after 1000 bets}] = (0.633 \cdot 80) + (0.367 \cdot -256) = -43.312$. Again, this makes sense because Figure 4 also converges to this value over time.

6. In Experiment 2, does the standard deviation reach a maximum value then converge or stabilize as the number of sequential bets increases? Explain why it does (or does not).

   It does appear that the standard deviation stabilizes over the course of the 1000 sequential bets and ultimately converges to a value. There are no massive fluctuations, or even small ones, like Experiment 1 had. This makes sense because instead of reaching one threshold (\$80), Experiment 2 is testing for two of them: the \$80 winning threshold and the $-\$256$ losing threshold. Ergo, any massive losses are ultimately stabilized by hitting that "empty wallet" baseline and any continuous winning is stabilized by hitting the maximum earnings baseline.

7. Graphs

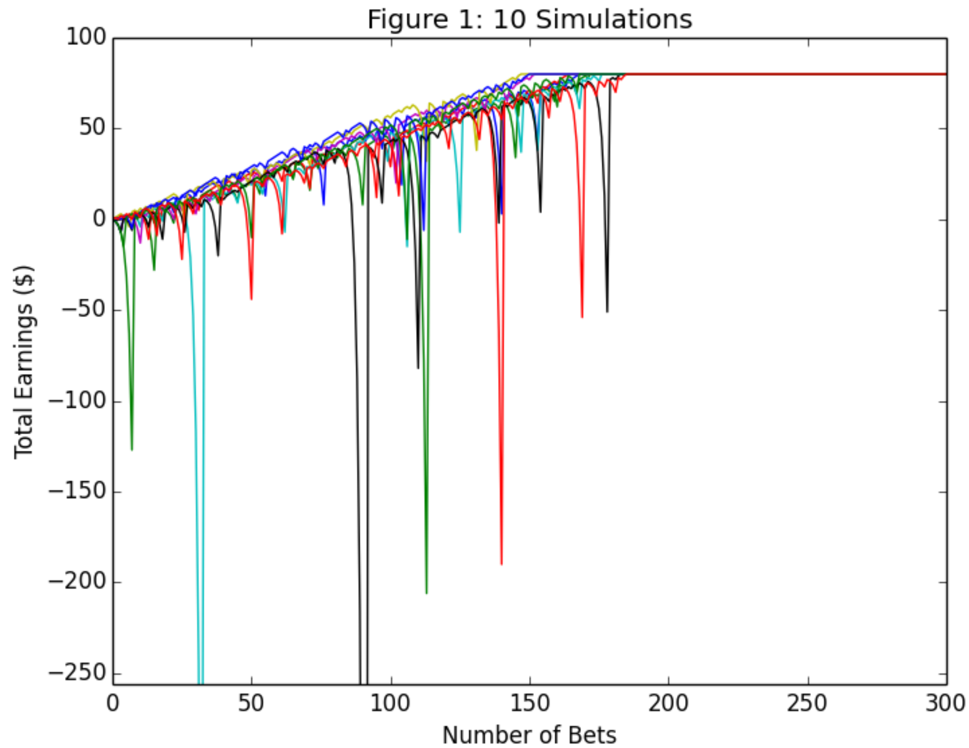

Figure 1: 10 Simulations
Total Earnings ($)
Number of Bets
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300

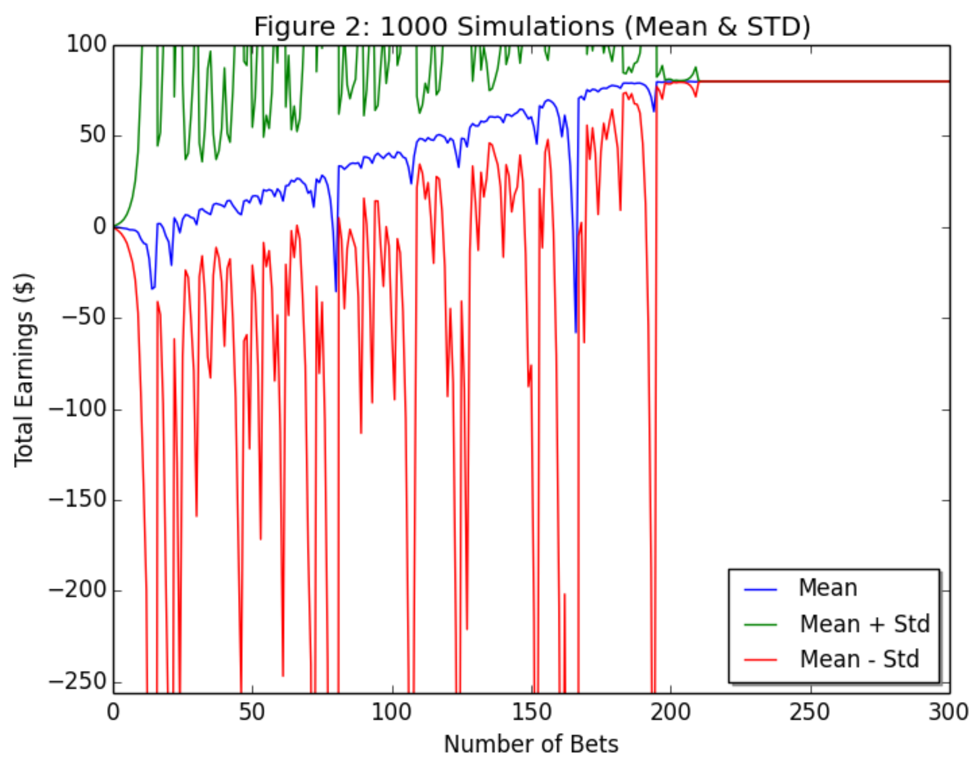

Figure 2: 1000 Simulations (Mean & STD)
Total Earnings ($)
Number of Bets
Mean
Mean + Std
Mean - Std
100
50
0
−50
−100
−150
−200
−250
0
50
100
150
200
250
300

Figure 3: 1000 Simulations (Median & Std)

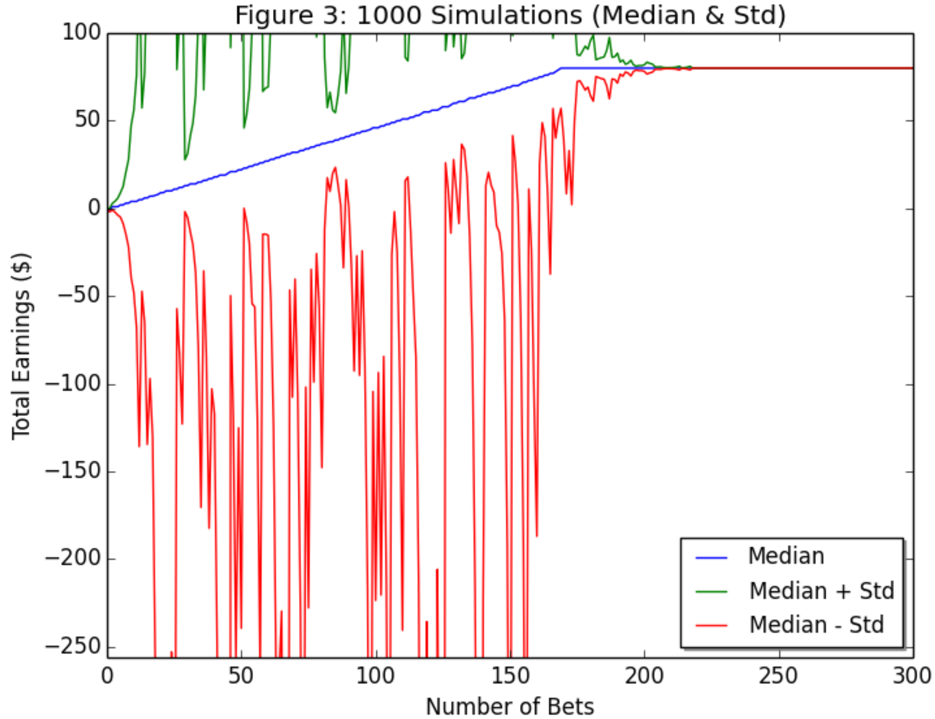

Figure 4: 1000 Realistic Simulations (Mean & STD)

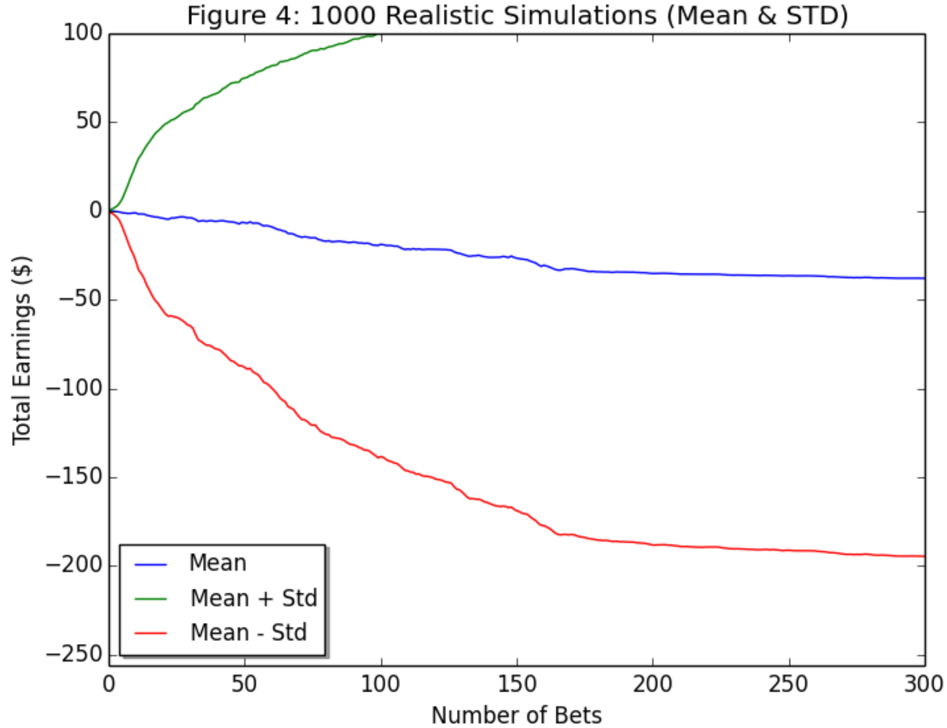

Figure 5: 1000 Realistic Simulations (Median & Std)