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# **MARKSCHEME**

**November 2010**

**MATHEMATICAL STUDIES**

**Standard Level**

**Paper 2**

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**Paper 2 Markscheme  
Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

**1 Abbreviations**

The markscheme may make use of the following abbreviations:

**M** Marks awarded for **Method**

**A** Marks awarded for an **Answer** or for **Accuracy**

**G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, irrespective of working shown.

**R** Marks awarded for clear **Reasoning**

**AG** **Answer Given** in the question and consequently, marks not awarded.

**ft** Marks that can be awarded as **follow through** from previous results in the question

In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**. Marks to be awarded for unsupported answers are designated **G** in the mark scheme as such answers will usually arise from working performed on a graphic display calculator.

**2 Method of Marking**

- (a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the document ‘Guidance for e-marking Mathematical Studies SL 2010’.
- (b) Marks must be noted on candidates’ scripts as in the markscheme and show the breakdown of individual marks using the **abbreviations** (**M1**), (**A2**) *etc*;
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.
- (e) If correct working results in a correct answer but then further working is developed, full marks are **not** always awarded. In most such cases it will be a single final answer mark that is lost. Full marks **can** be awarded if the candidate demonstrates clear understanding of the task and the result. If in doubt, consult your team leader.
- (f) Candidate drawn graphs will have a single (**A1**) available for scales and labels combined. This can be awarded if all these are present and correct, even if no graph is drawn, however, the mark should not be awarded if the scale shown is inappropriate to, or inadequate for, the required missing graph. In papers which have two candidate drawn graphs, consistent errors in showing labels or scales can follow through on the second graph, though not if the error is complete omission of these features.

**Please note:** Assignment of marks to the answers in all the following examples is for demonstration purposes only. Marks for actual examination questions will not necessarily follow the same pattern.

Question: Using Pythagoras to find a side of a triangle:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ <b>(M1)(A1)</b> (3.61 3s.f.)  <b>OR</b> Answer only <b>(G2)</b>	Case (i) $\sqrt{13}$ or 3.61 or both  Case (ii) $\sqrt{9+4} = \sqrt{13}$ = 6.50	<b>(G2)</b>  <b>(M1)</b> <b>(A0)</b>

Question: Calculate the gradient of the line passing through the points (5,3) and (0,9).

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5} = -\frac{6}{5}$ <b>(M1)(A1)</b>  <b>OR</b> Answer only <b>(G1)</b>	(i) $-6/5$  (ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ Gradient is $-6/5$ $y = -6x/5 + 9$  (iii) $\frac{9-3}{0-5} = -\frac{6}{5}$ $y = -6x/5 + 9$	<b>(G1)</b>  <b>(M1)</b> <b>(A1)</b> (There is clear understanding of the gradient.)  <b>(M1)</b> <b>(A0)</b> (There is confusion about what is required.)

Question: sine rule used to find angle  $A$ , with angle  $B$  and side  $b$  known but side  $a$  is first calculated using Pythagoras in an adjoining triangle.

Markscheme	Candidate's Script	Marking
$a = \sqrt{25 + 36} = \sqrt{61}$ (M1)(A1)	Case (i) $a = \sqrt{61}$	(G2)
OR answer only (G2)	$A = 55.9^\circ$	(A2)
$\frac{\sin(A)}{\sqrt{61}} = \frac{\sin(32)}{5}$ (M1)(A1)	Case (ii) $A = 55.9^\circ$ (with no mention of $a$ )	(A2)
$A = 55.9^\circ$ (A1)		
OR answer only (A2)		

### 3 Follow-through (ft) Marks

Errors made at any step of a solution can affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with '**(ft)**' appended to the eligible mark(s).

- If an answer resulting from follow through is extremely unrealistic (e.g. negative distances or wrong by a large order of magnitude) then the final  $A$  mark should not be awarded. If in doubt, contact your team leader.
- If a question is transformed by an error into a **different, much simpler question** then follow through might not apply or might be reduced. In this situation consult your team leader and record the decision on the candidate's script.
- To award follow through marks for a question part, **there must be working present for that part** and not just an answer based on the follow through. An isolated follow through answer, with no working, must be regarded as incorrect and receives no marks **even if it seems approximately correct**.
- Inadvertent use of radians will be penalised the first time it occurs. Subsequent use, even in later questions will normally be allowed follow through marks unless the answer is unrealistic. Cases of this kind will be addressed on an individual basis.

Question: An investment problem with two different rates of interest and a total amount of \$600 split across the rates in consecutive periods:

Markscheme	Candidate's Script	Marking
(a) \$ 600 × 1.02 = \$ 612 <b>OR</b> answer only	Case (i) (a) Final amount after 1 <sup>st</sup> period = \$ 600 × 1.02 = \$ 602	<b>(M1)</b> <b>(A0)</b>
(b) \$(\frac{612}{2} \times 1.02) + (\frac{612}{2} \times 1.04)\$ = \$ 630.36 <b>OR</b> answer only <i>Note: The (M1) is for splitting the value from (a) and forming a sum of products.</i>	(b) Amount after 2 <sup>nd</sup> period = 301 × 1.02 + 301 × 1.04 = \$ 620.06  but note Case (ii) an <b>(M0)</b> almost always prohibits the <b>associated (ft)</b> so	<b>(M1)</b> <b>(A1)(ft)</b>
Here the <b>(ft)</b> indicates a possible follow through from part (a).	(a) \$ 600 × 1.02 = \$ 602  (b) \$ 602 × 1.04 = \$ 626.08  Case (iii) (a) \$ 600 × 1.02 = \$ 602  (b) No working. 620.06 given as answer.  Case (iv) (a) \$ 612  (b) \$ 630.36	<b>(M1)(A0)</b>  <b>(M0)(A0)(ft)</b>  <b>(M1)(A0)</b>  <b>(G0)(ft)</b>  <b>(G2)</b>  <b>(G1)</b>

Question: Using trigonometry to calculate angles and sides of triangles.

Markscheme	Candidate's Script	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> (M1)(A1)</p> <p><math>A = 22.0^\circ</math> (A1)</p> <p>OR answer only (A2)</p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math></p> <p><math>A = 41.8^\circ</math></p>	<p>(M1) (A0) (use of sine rule but with wrong values)</p> <p>(A0)</p> <p>(Note: the 2<sup>nd</sup> (A1) here was not marked (ft) and cannot be awarded, because there was an earlier error in the same question part.)</p>
<p>(b) <math>x = 7 \tan A</math> (M1)</p> <p><math>= 2.83</math> (A1)(ft)</p> <p>OR answer 2.83 only (G1)</p>	<p>(b) case (i) <math>x = 7 \tan A</math></p> <p><math>= 6.26</math></p> <p>but</p> <p>case (ii) 6.26</p>	<p>(M1) (A1)(ft)</p> <p>(G0)</p>

#### 4 Using the Markscheme

This markscheme presents a particular way in which each question might be worked and how it should be marked.

- (a) As **A** marks are normally **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark, unless otherwise instructed in the markscheme. (See the finance example above).

Similarly **(A1)(R0)** cannot be awarded for an answer which is accidentally correct for the wrong reasons given.

**Example:** Question: (a)  $\chi^2$  calculated followed by (b) degrees of freedom found and (c) and (d) comparison to critical value. (Dependence of **A** and **R** marks.)

Markscheme	Candidate's Script	Marking
(a) $\chi_{calc}^2 = 3.92$ (A1)	Case (i)	
(b) $n = 4$ (A1)	(a) $\chi_{calc}^2 = 3.92$	(A1)
(c) $\chi_{crit}^2 = 9.488$ (A1)(ft)	(b) $n = 4$	(A1)
(d) Do not reject null hypothesis (A1)(ft) because $\chi_{calc}^2 < \chi_{crit}^2$ (R1)(ft)	(c) Don't know?	(A0)
	(d) Do not reject null hypothesis (A0)(ft) because $\chi_{calc}^2 > 0$ (R0)(ft)	(A0)(ft)
		((A0) was awarded here because the reason is wrong.)



Case (ii)	
(a) $\chi_{calc}^2 = 3.92$	(A1)
(b) $n = 4$	(A1)
(c) $\chi_{crit}^2 = 4.488$	(A0)
(d) Do not reject null hypothesis because $\chi_{calc}^2 < \chi_{crit}^2$	(A1)(ft) (R1)(ft)
Case (iii)	
(a) $\chi_{calc}^2 = 3.92$	(A1)
(b) $n = 1$	(A0)
(c) $\chi_{crit}^2 = 3.841$	(A1)(ft)
(d) Reject null hypothesis because $\chi_{calc}^2 > \chi_{crit}^2$	(A1)(ft) (R1)(ft)

- (b) **Alternative methods** have not always been included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method in a manner that is consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by ‘**OR**’ etc. This includes alternatives obtained with a graphic display calculator. In such cases, alternative **G** mark assignments for answer only will not be repeated if this is redundant.

**Example:** Question to find the coordinates of a vertex of a given quadratic.

Working	Marks
$f(x) = 2x^2 + 7x - 3$  $x = -\frac{b}{2a} = -\frac{7}{4}$ <b>(M1)</b> for use of $-b/2a$ , <b>(A1)</b> for correct answer $f(-7/4) = -\frac{146}{16} = -\frac{73}{8}$  <b>(M1)</b> for using $f(-7/4)$ , <b>(A1)</b> for answer.  Coordinates are $(-7/4, -73/8)$  <b>OR</b>  $(-7/4, -73/8)$ (with no working at all)	   <b>(M1)(A1) or (G2)</b>  <b>(M1)(A1)(ft)</b>  <b>or (G1)</b>  <b>(A1)(ft)</b>   <b>OR</b>  <b>(G2)(G1)</b>

<p><b>OR</b></p> <p><math>f'(x) = 4x + 7, \quad 4x + 7 = 0</math></p> <p>so <math>x = -7/4</math></p> <p><i>(M1) for attempting to take a derivative and setting it to 0</i></p> <p><i>(A1) for answer</i></p> <p><math>f(-7/4) = -\frac{146}{16} = -\frac{73}{8}</math></p> <p><i>(M1) for using <math>f(-7/4)</math>, (A1) for answer.</i></p> <p>Coordinates are <math>(-7/4, -73/8)</math></p>	<p><b>OR</b></p> <p><i>(M1)</i></p> <p><i>(A1)</i></p> <p><i>(M1)(A1)(ft)</i></p> <p><i>(A1)(ft)</i></p>
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- (c) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .  
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.
- (d) As this is an international examination, all valid **alternative forms of notation** should be accepted.  
Some examples of these are:  
Decimal points: 1.7; 1'7; 1 · 7 ; 1,7.  
Different descriptions of an interval:  $3 < x < 5$ ; (3, 5); ] 3, 5 [.  
Different forms of notation for set properties (e.g. complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A$   
Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ .  
 $p \Rightarrow q$ ;  $p \rightarrow q$ ;  $q \Leftarrow p$ .
- (e) Discretionary **(d)** marks: There will be rare occasions where the markscheme does not cover the work seen. In such cases, **(d)** should be used to indicate where an examiner has used discretion. It must be accompanied by a brief note to explain the decision made.

## 5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers must be given exactly or correct to 3 significant figures.

A penalty known as an **ACCURACY PENALTY (AP)** is applied if an answer is either

- (i) rounded incorrectly to 3 significant figures or
- (ii) rounded correctly or incorrectly to some other level of accuracy.

This penalty is applied to the **final answer** of a question part only. It applies **also** when an exact answer is incorrectly rounded.

**THE ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER!** Subsequent accuracy errors can be **ignored** and full marks awarded if all else is correct. Please see section G in the guidance document which clearly explains, with the use of screenshots how this works in scoris.

An accuracy penalty must be recorded in proximity to the incorrect answer as **(AI)(AP)**. This is different to what we have done previously awarding A0AP. This mark is not deducted in the item box but from the final whole paper total automatically in scoris.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. This is **NOT** an accuracy penalty. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. If the rounding is incorrect, an accuracy penalty should be applied as detailed above. Exact answers such as  $\frac{1}{4}$  can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of  $\pi$  and answers taking the form of square roots of integers (**even if exact squares**) or any rational power of an integer (*e.g.*  $\sqrt{13}$ ,  $2^{\frac{2}{3}}$ ,  $\sqrt[4]{5}$ ,  $\sqrt{9}$ ) may be accepted as exact answers. All other powers (*e.g.* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

Answers **with no supporting working** which are written correct to more than 3 significant figures should be marked according to the scheme for correct answers with no working, but with an **(AP)** then applied. When this happens, **(A2)** or **(G2)** can be split if necessary (*e.g.* **(A1)(A1)(AP)** or **(G1)(G1)(AP)**). If there is no working shown, and answers are given to the correct two significant figures, apply the **(AP)**. However, do not accept answers to one significant figure without working.

An accuracy penalty should not be applied to an answer that is already incorrect for some other reason.

### Special cases

An answer taken directly from the IB chi-squared statistical table can be given and used to the same level of accuracy as appears in the table (3 decimal places) or correct to 3 significant figures.

For judging equivalence between 3 significant figures and use of minutes and seconds for angles, guidelines have been issued to paper setters. This problem will be dealt with on an individual basis as the need arises.

**Examples:** The Pythagoras example used before:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ (M1)(A1) (3.61 3s.f.) <b>OR</b> answer only (G2)	(i) 4 (ii) 3.60555 or 3.6 (iii) $\sqrt{9+4} = \sqrt{13}$ = 3.6 (iv) $\sqrt{9+4} = \sqrt{13}$ = 3.60555 (v) $\sqrt{9+4} = \sqrt{13} = 3.60$ (vi) $\sqrt{9+4} = \sqrt{14} = 3.74$	(G0) (G1)(G1)(AP) (M1) (A1)(AP) (M1) (A1)(AP) (M1)(A1)(AP) (M1)(A0)

If the accuracy is specified in the question e.g. give your answer correct to 4 decimal places, then there would be one extra mark available as follows:

Markscheme	Candidates' Scripts	Marking
$\sqrt{9+4} = \sqrt{13}$ (M1)(A1) <b>OR</b> answer only (G2) (Note: requires more than 4 d.p.) = 3.6056 (4 d.p.) (A1)(ft) <b>OR</b> answer only (G2) <b>OR</b> answer 3.606 or 3.61 only (G1)	(i) 3.605551 = 3.6056 (4 d.p.) (ii) $\sqrt{9+4} = \sqrt{13}$ = 3.606 (iii) 3.60555 (iv) 3.6056 (v) $\sqrt{9+4} = \sqrt{14}$ = 3.7417 (vi) $\sqrt{9-4} = \sqrt{5}$ = 2.2361 (vii) 3.606	(G2)(A1) (M1)(A1) (A0) (G2)(A0) (G2) (M1)(A0) (A1)(ft) (M0)(A0) (A1)(ft) (Note: this is a special case, where the initial (M0) does not determine the final (A0) because the correction to 4dp is an entirely new task.) (G1)

### Premature Rounding

Accuracy errors in a final answer, which result from premature rounding earlier in the same question part, should not receive an accuracy penalty. There are two situations. If there is a mark available for a prematurely rounded answer and the rounding occurs at this stage, then the inappropriate rounding should be penalised with **(A0)** but the answer can then be allowed to follow through to the end of the question. If the first stage of the answer is correct but rounded further on, then it should be penalised at an appropriate place close to where it is rounded. Some discretion should be used to deny a **(ft)** mark if the rounding is very bad and the answer far from its required value.

**Example:** Question: sine rule used to find angle  $A$ , with angle  $B$  and side  $b$  known but side  $a$  is first calculated using Pythagoras in an adjoining triangle.

Markscheme	Candidate's Script	Marking
$a = \sqrt{25 + 36} = \sqrt{61}$ <b>(M1)(A1)</b> <b>OR</b> answer only <b>(G2)</b> $\frac{\sin(A)}{\sqrt{61}} = \frac{\sin(32)}{5}$ <b>(M1)(A1)(ft)</b> $A = 55.9^\circ$ <b>(A1)(ft)</b> <b>OR</b> answer only <b>(G2)</b>	(i) $a = \sqrt{25 + 36} = \sqrt{61}$ $= 7.8$ $\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = 55.8^\circ$ (ii) $a = \sqrt{25 + 36} = \sqrt{61}$ $\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = 55.8^\circ$ (iii) $a = \sqrt{25 + 36} = \sqrt{61}$ $\frac{\sin(A)}{7.8} = \frac{\sin(32)}{5}$ $A = \sin^{-1}(0.83) = 56.1^\circ$ (iv) $a = \sqrt{25 + 36} = \sqrt{61} = 8$ $\frac{\sin(A)}{8} = \frac{\sin(32)}{5}$	<b>(M1)</b> <b>(A0)</b>  <b>(M1)(A1)(ft)</b>  <b>(A1)(ft)</b>  <b>(M1)(A1)</b>  <b>(M1)(A0)</b>  <b>(A1)(ft)</b>  <b>(M1)(A1)</b>  <b>(M1)(A0)</b>  <b>(A0)</b>  <b>(M1)(A0)</b>  <b>(M1)(A1)(ft)</b>

	$A = 58.0^\circ$  (v) $a = 7.8$  $A = 55.8^\circ$	<b>(A0)(ft)</b> <i>(The rounding is severe and the answer quite far from correct).</i> <b>(G0)</b>  <b>(G0)(ft)</b> <i>(there is no working to justify the follow through.)</i>
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## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places, but could differ in rare instances depending on the currency in question.

A penalty known as a FINANCIAL ACCURACY PENALTY **(FP)** is applied if an answer does not adhere to the specification in the question. This penalty is applied to the final answer of a question part only. Please see section G in the guidance document which clearly explains, with the use of screenshots how this works in scoris.

**THE FINANCIAL ACCURACY PENALTY IS APPLIED AT MOST ONCE PER PAPER!**  
Subsequent financial accuracy errors can be ignored and full marks awarded, if all else is correct.

A financial accuracy penalty must be recorded in proximity to the incorrect answer as **(A1)(FP)**. This is different to what we have done previously awarding **(A0)(FP)**. This mark is not deducted in the item box but from the final whole paper total automatically by scoris.

The financial accuracy penalty is imposed only for rounding to the wrong level of accuracy and NOT for incorrect rounding to the required number of places. The latter would incur a normal accuracy penalty **(AP)**.

**No single answer can receive two penalties. If both types of error are present then (FP) takes priority.**

Please see the examples below.

NOTE: The financial accuracy penalty will be flagged in the markscheme at the start of each answer where it could apply, with the words “Financial accuracy penalty **(FP)** applies in part (a)...”. If this instruction is not present, then do not apply the penalty. An **(FP)** will also be present in the left hand column next to where it applies.

**Example:** A financial question demands accuracy correct to 2dp.  
Prior to rounding the answer is \$231.6189

Markscheme	Candidate's Script	Marking
Financial accuracy penalty <b>(FP)</b> applies in this question		
\$231.62 <b>(AI)</b>	\$231.62 or 231.62	<b>(AI)</b> (No unit penalty (see section 7 below) for missing \$ symbol.)
	231.6 or 232	<b>(AI)(FP)</b> (Correct rounding process but incorrect level.)
	231.61	<b>(AI)(AP)</b> (Incorrect rounding process to correct level.)
	231	<b>(AI)(FP)</b> (Both types of error occurred but <b>(FP)</b> takes priority.)
	232.00	<b>(AI)(AP)</b> (It's not clear whether nearest dollar or 2dp was really intended but we interpret as 2dp rounded incorrectly.)

## 7 Units in answers

A penalty known as a UNIT PENALTY (**UP**) is applied if an answer does not include the correct units. This applies both to missing units and to incorrect units. This penalty is applied to the final answer of a question part only.

**THE UNIT PENALTY IS APPLIED AT MOST ONCE PER PAPER!** Subsequent unit errors can be ignored and full marks awarded if all else is correct. Please see section G in the guidance document which clearly explains, with the use of screenshots how this works in scoris.

**THE UNIT PENALTY IS APPLIED AT MOST ONCE PER PAPER!** Subsequent unit errors can be ignored and full marks awarded if all else is correct.

A unit penalty must be recorded in proximity to the incorrect answer as **(A1)(UP)**. This is different to what we have done previously awarding **(A0)(UP)**. This mark is not deducted in the item box but from the final whole paper total automatically in scoris.

NOTE: The unit penalty will be flagged in the markscheme at the start of each answer where it could apply, with the words “Unit penalty (**UP**) applies in parts (a)...”. If this instruction is not present, then do not apply the penalty. A (**UP**) will also be present in the left hand column next to where it applies.

NOTE: In this context, symbols for currency such as \$ or GBP etc are not considered units. Candidates are encouraged to include them but should not be penalised if they are missing.

Missing degree symbols and percentage symbols are also not eligible for a unit penalty.

**No single answer can receive two penalties. If an answer is rounded incorrectly and also has wrong or missing units, apply the accuracy penalty (AP) only. If the (AP) has already been used, such an answer is eligible for the unit penalty.**

**Example:** A question has answer to part (i) of 66.2 cm. The answer before rounding is 66.213 cm. Part (ii) involves dividing by 60 with units of  $\text{cm s}^{-1}$ . Assume that the (**UP**) has not been used previously.

Markscheme	Candidate's Script	Marking
<i>Unit penalty (<b>UP</b>) applies in part (i) and (ii).</i>		
(i) 66.2 cm <b>(A1)</b>	(i) 66.2cm	<b>(A1)</b>
(ii) 1.10 $\text{cm s}^{-1}$ <b>(A1)</b>	(ii) 1.10 $\text{cm s}^{-1}$	<b>(A1)</b>
	(i) 66.2	<b>(A1)(UP)</b>
	(ii) 1.10	<b>(A1)</b>
	(i) 66.2 cm	<b>(A1)</b>
	(ii) 1.10	<b>(A1)(UP)</b>
	(i) 66	<b>(A1)(AP)</b> if ( <b>AP</b> ) not used previously but <b>(A1)(UP)</b> otherwise.
	(ii) 1.1	<b>(A1)(UP)</b> if ( <b>AP</b> )



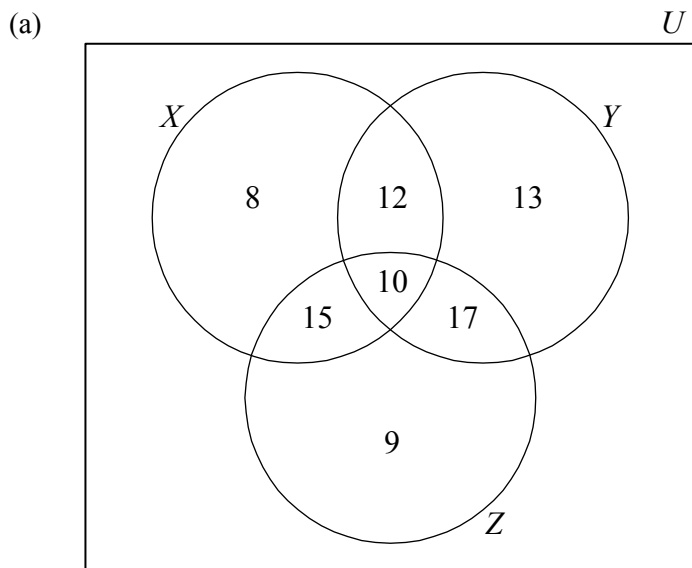
		used in part (i) but <b>(AI)(ft)</b> for correct follow through to exact answer if <b>(UP)</b> used in part (i).
(i)	66	<b>(AI)(AP)</b> if <b>(AP)</b> not used previously but <b>(AI)(UP)</b> otherwise.
(ii)	$1.1 \text{ cms}^{-1}$	<b>(AI)(ft)</b>

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment ‘I used my GDC’ cannot receive a method mark.

## QUESTION 1

### Part A



(A1) for rectangle and three intersecting circles

(A1) for 10, (A1) for 8, 13 and 9, (A1) for 12, 15 and 17

(A4) [4 marks]

(b)  $100 - (9 + 12 + 13 + 15 + 10 + 17 + 8) = 16$

(M1)(A1)(ft)(G2) [2 marks]

**Note:** Follow through from their diagram.

(c)  $\frac{51}{100} (0.51)$   
 $= 51\%$

(A1)(ft)

(A1)(ft)(G2) [2 marks]

**Note:** Follow through from their diagram.

- (d) **Note:** The following statements are correct. Please note that the connectives are important. It is not the same (had cereal) and (not bread) and (had cereal) or (not bread). The parentheses are not needed but are there to facilitate the understanding of the propositions.

(had cereal) and (did not have bread)

(had cereal only) or (had cereal and fruit only)

(had either cereal or (fruit and cereal)) and (did not have bread)

(A1)(A1) [2 marks]

**Notes:** If the statements are correct but the connectives are wrong then award at most (A1)(A0).

For the statement (had only cereal) and (cereal and fruit) award (A1)(A0).

For the statement had cereal and fruit award (A0)(A0).

continued...

Question 1 continued

- (e)  $\frac{54}{100}$  (0.54, 54 %) (A1)(ft)(A1)(ft)(G2) [2 marks]

**Note:** Award (A1)(ft) for numerator, follow through from their diagram, (A1)(ft) for denominator. Follow through from total or denominator used in part (c).

- (f)  $\frac{10}{100} \times \frac{9}{99} = \frac{1}{110}$  (0.00909, 0.909 %) (A1)(ft)(M1)(A1)(ft)(G2) [3 marks]

**Notes:** Award (A1)(ft) for their correct fractions, (M1) for multiplying two fractions, (A1)(ft) for their correct answer.  
Answer 0.009 with no working receives no marks.  
Follow through from denominator in parts (c) and (e) and from their diagram.

Part B

- (a)  $H_0$ : The (average) number of meals per day a student has and gender are independent (A1) [1 mark]

**Note:** For “independent” accept “not associated” but do not accept “not related” or “not correlated”.

- (b) 2 (A1) [1 mark]

- (c) 5.99 (accept 5.991) (A1)(ft) [1 mark]

**Note:** Follow through from their part (b).

- (d)  $\frac{28 \times 45}{100} = 12.6 = 13$  or  $\frac{28}{100} \times \frac{25}{100} \times 100 = 12.6 = 13$  (M1)(A1)(AG) [2 marks]

**Notes:** Award (M1) for correct formula and (A1) for correct substitution.  
Unrounded answer must be seen for the (A1) to be awarded.

- (e) 0.0321 (G2) [2 marks]

**Note:** For 0.032 award (G1)(G1)(AP).  
For 0.03 with no working award (G0).

- (f)  $0.0321 < 5.99$  or  $0.984 > 0.05$  (R1)  
accept  $H_0$  (A1)(ft) [2 marks]

**Note:** If reason is incorrect both marks are lost, do not award (R0)(A1).

Total [24 marks]

**QUESTION 2**

(a)  $\frac{9-1}{0-(-4)}$   
 $=2$

**(M1)**

**(A1)(G2) [2 marks]**

**Notes:** Award **(M1)** for correct substitution into the gradient formula.

(b)  $-6$

**(A1) [1 mark]**

**Note:** Accept  $(0, -6)$ .

(c)  $y = -\frac{1}{2}x - 1$  (or equivalent)

**(A1)(ft)(A1)**

**Notes:** Award **(A1)(ft)** for gradient, **(A1)** for correct y-intercept.  
 Follow through from their gradient in (a).

$x + 2y + 2 = 0$

**(A1)(ft)**

**Notes:** Award **(A1)(ft)** from their gradient and their y-intercept.  
 Accept any multiple of this equation with integer coefficients.

**OR**

$y - 1 = -\frac{1}{2}(x + 4)$  (or equivalent)

**(A1)(ft)(A1)**

**Note:** Award **(A1)(ft)** for gradient, **(A1)** for any point on the line correctly substituted in equation.

$x + 2y + 2 = 0$

**(A1)(ft) [3 marks]**

**Notes:** Award **(A1)(ft)** from their equation.  
 Accept any multiple of this equation with integer coefficients.

(d)  $D(2, -2)$  **or**  $x = 2, y = -2$

**(A1) [1 mark]**

**Note:** Award **(A0)** if brackets not present.

(e)  $R(6, 6)$  **or**  $x = 6, y = 6$

**(A1)(A1) [2 marks]**

**Note:** Award at most **(A0)(A1)(ft)** if brackets not present and absence of brackets has not already been penalised in part (d).

*continued...*

*Question 2 continued*

(f) (i)  $DR = \sqrt{8^2 + 4^2}$  **(M1)**

$DR = \sqrt{80}$  (8.94) **(A1)(ft)(G2)**

**Note:** Award **(M1)** for correct substitution into the distance formula. Follow through from their D and R.

(ii)  $\text{Area} = \frac{\sqrt{80} \times \sqrt{45}}{2}$  **(M1)**

$= 30$  (30.0) **(A1)(ft)(G2)**     **[4 marks]**

**Note:** Award **(M1)** for correct substitution in the area of triangle formula. Follow through from their answer to part (f) (i).

**Total [13 marks]**

**QUESTION 3** Unit penalty (UP) applies in parts (a), (c) and (e).

(a)  $AB^2 = 10^2 + 8^2 - 2 \times 10 \times 8 \times \cos 150^\circ$

(M1)(A1)

(UP)  $AB = 17.4 \text{ km}$

(A1)(G2) [3 marks]

**Note:** Award (M1) for substitution into correct formula, (A1) for correct substitution, (A1) for correct answer.

(b)  $\frac{8}{\sin \hat{CAB}} = \frac{17.4}{\sin 150^\circ}$

(M1)(A1)

$\hat{CAB} = 13.3^\circ$

(A1)(ft)(G2)

**Notes:** Award (M1) for substitution into correct formula, (A1) for correct substitution, (A1) for correct answer. Follow through from their answer to part (a).

(UP) (c)  $AD = 8.70 \text{ km}$  (8.7 km)

(A1)(ft) [1 mark]

**Note:** Follow through from their answer to part (a).

(d)  $DT = \tan(13.29\dots^\circ) \times 8.697\dots = 2.0550\dots$

(M1)(A1)

$= 2.06$

(AG)

[2 marks]

**Notes:** Award (M1) for correct substitution in the correct formula, award (A1) for the unrounded answer seen. If 2.06 not seen award at most (M1)(AO).

continued...

(UP) (e)  $\sqrt{8.70^2 + 2.06^2} + 8.70 + 2.06$

(AI)(MI)

= 19.7 km

(AI)(ft)(G2) [3 marks]

**Note:** Award (AI) for AT, (MI) for adding the three sides of the triangle ADT, (AI)(ft) for answer. Follow through from their answer to part (c).

(f)  $\frac{19.7}{70} \times 60 + 10$

(MI)(MI)

= 26.9

(AI)(ft)

**Note:** Award (MI) for time on road in minutes, (MI) for adding 10, (AI)(ft) for unrounded answer. Follow through from their answer to (e).

= 27 (nearest minute)

(AI)(ft)(G3) [4 marks]

**Note:** Award (AI)(ft) for their unrounded answer given to the nearest minute.

**Total [16 marks]**

# QUESTION 4

**Part A** *Financial penalty (FP) applies in parts (b) and (d).  
Accuracy penalty applies in part (e) if answer not given correct  
to 2 decimal places*

(a)  $4000 \times 0.97 = 3880.00$  (3880) (M1)(A1)(G2)

**Note:** Award (M1) for multiplication of correct numbers.

**OR**

$3\% \text{ of } 4000 = 120$

(A1)

$4000 - 120 = 3880.00$  (3880)

(A1)(G2)

[2 marks]

(FP) (b)  $3880 \times 0.3071 = 1191.55$

(M1)(A1)(ft)(G2)

[2 marks]

**Note:** Award (M1) for multiplication of correct numbers. Follow through from their answer to part (a).

(c)  $\frac{400}{0.3125}$

(M1)

$= 1280.00$  (1280)

(A1)(G2)

[2 marks]

**Note:** Award (M1) for division of correct numbers.

(FP) (d) 63.20

(A1)(ft)

[1 mark]

**Note:** Follow through (their (c) –1216.80).

(e)  $t = \frac{63.20 \times 100}{1280}$

(M1)

$t = 4.94$

(A1)(ft)(G2)

[2 marks]

**Note:** Follow through from their answers to parts (c) and (d).

*continued...*



Question 4 continued

**Part B** *Financial penalty (FP) applies in part (b).  
Accuracy penalty applies in part (c) if answer not given correct to  
2 decimal places.*

$$(a) \quad A = 1000 \left( 1 + \frac{3.5}{2 \times 100} \right)^6 = 1109.7023... \quad (M1)(A1)(A1)$$

$$= 1109.70 \quad (AG)$$

**Notes:** Award **(M1)** for substitution into correct formula, **(A1)** for correct substitution, **(A1)** for unrounded answer. If 1109.70 not seen award at most **(M1)(A1)(A0)**.

**OR**

$$I = 1000 \left( 1 + \frac{3.5}{2 \times 100} \right)^6 - 1000 = 109.7023 \quad (M1)(A1)$$

$$A = 1109.7023... \quad (A1)$$

$$= 1109.70 \quad (AG) \quad [3 \text{ marks}]$$

**Note:** Award **(M1)** for substitution into correct formula, **(A1)** for correct substitution, **(A1)** for unrounded answer.

**(FP)** (b) 109.70 (A1) [1 mark]

**Note:** No follow through here.

$$(c) \quad \frac{1000 \times 3 \times r}{100} = 109.70 \quad (A1)(M1)$$

$$r = 3.66 \quad (A1)(ft)(G2) \quad [3 \text{ marks}]$$

**Notes:** Award **(A1)** for 109.70 seen and used, **(M1)** for correct substitutions in simple interest formula, **(A1)** for answer.  
Follow through from their answer to (b).  
If 1109.70 or other is used as interest award at most **(A0)(M1)(A0)**.

**Total [16 marks]**

**QUESTION 5**

(a) 30 (A1) [1 mark]

(b)  $f'(x) = 3x^2 - 6x - 24$  (A1)(A1)(A1) [3 marks]

**Note:** Award (A1) for each term. Award at most (A1)(A1) if extra terms present.

(c)  $f'(1) = -27$  (M1)(A1)(ft)(G2) [2 marks]

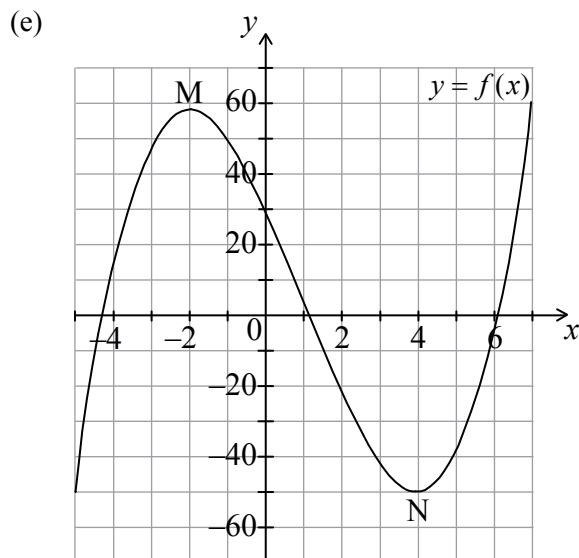
**Note:** Award (M1) for substituting  $x = 1$  into their derivative.

(d) (i)  $f'(x) = 0$   
 $3x^2 - 6x - 24 = 0$   
 $x = 4; x = -2$  (M1)  
(A1)(ft)(A1)(ft)

**Notes:** Award (M1) for either  $f'(x) = 0$  or  $3x^2 - 6x - 24 = 0$  seen.  
Follow through from their derivative.  
Do not award the two answer marks if derivative not used.

(ii) M(-2, 58) accept  $x = -2, y = 58$  (A1)(ft)  
N(4, -50) accept  $x = 4, y = -50$  (A1)(ft) [5 marks]

**Note:** Follow through from their answer to part (d) (i).



(A1) for window  
(A1) for a smooth curve with the correct shape  
(A1) for axes intercepts in approximately the correct positions  
(A1) for M and N marked on diagram and in approximately correct position

(A4) [4 marks]

**Note:** If window is not indicated award at most (A0)(A1)(A0)(A1)(ft).

continued...

*Question 5 continued*

- (f) (i)  $3x^2 - 6x - 24 = 21$  (M1)  
 $3x^2 - 6x - 45 = 0$  (M1)  
 $x = 5; x = -3$  (A1)(ft)(A1)(ft)(G3)

**Note:** Follow through from their derivative.

**OR**

- Award (A1) for  $L_1$  drawn tangent to the graph of  $f$  on their (A1)(ft)  
 sketch in approximately the correct position ( $x = -3$ ),  
 (A1) for a second tangent parallel to their  $L_1$ , (A1)(ft)  
 (A1) for  $x = -3$ , (A1) for  $x = 5$ . (A1)(A1)

**Note:** If only  $x = -3$  is shown without working award (G2).  
 If both answers are shown irrespective of working  
 award (G3).

- (ii)  $f(5) = -40$  (M1)(A1)(ft)(G2) [6 marks]

**Notes:** Award (M1) for attempting to find the image of their  $x = 5$ .  
 Award (A1) only for  $(5, -40)$ .  
 Follow through from their  $x$ -coordinate of B **only if it has  
 been clearly identified** in (f) (i).

**Total [21 marks]**



# **MARKSCHEME**

**November 2011**

**MATHEMATICAL STUDIES**

**Standard Level**

**Paper 2**

Deadline for all marking to be completed is 06/12/11  
**Ignore deadline on IBIS**

*This markscheme is **confidential** and for the exclusive use of examiners in this examination session.*

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**Paper 2 Markscheme**  
**Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

**1 Abbreviations**

**M** Marks awarded for **Method**

**A** Marks awarded for an **Answer** or for **Accuracy**

**R** Marks awarded for clear **Reasoning**

**G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, irrespective of working shown.

**AG** **Answer Given** in the question and consequently, marks not awarded.

**ft** Marks that can be awarded as **follow through** from previous results in the question.

**2 Method of Marking**

- (a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the **A0** annotation, otherwise full annotations must be shown.
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.
- (e) If correct working results in a correct answer but then further working is developed, full marks may **not** always be awarded. Full marks **will** be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

**Example:** Calculate the gradient of the line passing through the points (5, 3) and (0, 9) .

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5} \quad (M1)$ <p><i>Award (M1) for correct substitution in gradient formula</i></p> $= -\frac{6}{5} \quad (A1)$	<p>(i) <math>\frac{9-3}{0-5} = -\frac{6}{5}</math></p> <p>Gradient is <math>= -\frac{6}{5}</math></p> <p style="text-align: right;"><i>(There is clear understanding of the gradient.)</i></p> <p><math>y = -\frac{6}{5}x + 9</math></p> <p>(ii) <math>\frac{9-3}{0-5} = -\frac{6}{5}</math></p> <p><math>y = -\frac{6}{5}x + 9</math></p> <p style="text-align: right;"><i>(There is confusion about what is required.)</i></p>	<p><b>(M1)</b></p> <p><b>(A1)</b></p> <p><b>(M1)</b></p> <p><b>(A0)</b></p>

### 3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Mark schemes will indicate where it is appropriate to apply follow through in a question with ‘(ft)’.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (*e.g.* negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate’s use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

Markscheme	Candidates’ Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> (M1)(A1)</p> <p>Award (M1) for substitution in sine rule formula, (A1) for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) (A1)(G2)</p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math></p> <p><math>A = 41.8^\circ</math></p> <p>(Note: the 2<sup>nd</sup> (A1) here was not marked (ft) and cannot be awarded because there was an earlier error in the <b>same</b> question part.)</p>	<p>(M1)(A0)</p> <p>(use of sine rule but with wrong values)</p> <p>(A0)</p>
<p>(b) <math>x = 7 \tan (22.0243\dots^\circ)</math> (M1)</p> <p><math>= 2.83</math> (2.83163...) (A1)(ft)</p>	<p>(b) case (i) <math>x = 7 \tan 41.8^\circ</math></p> <p><math>= 6.26</math></p> <p><b>but</b> case (ii) <math>6.26</math></p>	<p>(M1)</p> <p>(A1)(ft)</p> <p>(G0)</p> <p>since no working shown</p>

#### 4 Using the Markscheme

- (a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.  
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.
- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

**Example:** Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b></p> <p><i>Award (M1) for substitution in sine rule formula, (A1) for correct substitutions.</i></p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(i) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math></p> <p><math>A = 22.0^\circ</math></p> <p>(ii) <math>A = 22.0^\circ</math></p> <p><b>Note:</b> <b>G</b> marks are used only if no working has been shown and the answer is correct.</p>	<p><b>(M1)(A1)</b></p> <p><b>(A1)</b></p> <p><b>(G2)</b></p>

- (d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.  
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' etc.
- (e) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .  
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.  
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:  
the 3 significant figure answer worked through from full calculator display;  
the exact value (for example  $\sqrt{3}$  if applicable);  
the full calculator display in the form 2.83163... as in the example above.  
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.



- (f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7;  $1 \cdot 7$  ; 1,7 .

Different descriptions of an interval:  $3 < x < 5$ ; (3, 5); ] 3, 5 [ .

Different forms of notation for set properties (*e.g.* complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A; U \setminus A$ .

Different forms of logic notation:  $\neg p$  ;  $p'$  ;  $\tilde{p}$  ;  $\bar{p}$  ;  $\sim p$ .  
 $p \Rightarrow q$  ;  $p \rightarrow q$  ;  $q \Leftarrow p$  .

- (g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through scoris to the team leader.

As from Nov 11 there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5 Accuracy of Answers

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award **(AI)** and ignore subsequent rounding.

**Note:** The unrounded answer may appear in either the working box or on the final answer line.

2. If the candidate's unrounded answer is **not** seen then award **(AI)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarised in the table below and illustrated in the examples following.

	If candidates final answer is given...				
	Exact or correct to 3 or more sf	<b>Incorrect to 3sf</b>	Correct to 2sf <sup>3</sup>	Incorrect to 2sf	Correct or incorrect to 1sf
Unrounded answer seen <sup>1</sup>	Award the final <b>(AI)</b> irrespective of correct or incorrect rounding				
Unrounded answer not seen <sup>2</sup>	<b>(AI)</b>	<b>(A0)</b>	<b>(AI)</b>	<b>(A0)</b>	<b>(A0)</b>
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen <sup>3</sup>			

**Examples:**

<b>Markscheme</b>	<b>Candidates' Scripts</b>	<b>Marking</b>
9.43 (9.43398...) (AI)	(i) 9.43398... is seen followed by 9; 9.4; 9.43; 9.434 etc (correctly rounded)	(AI)
	(ii) 9.43398... is seen followed by 9.433; 9.44 etc (incorrectly rounded)	(AI)
	(iii) 9.4	(AI)
	(iv) 9	(A0) (correct to 1sf)
	(v) 9.3	(A0) (incorrectly rounded to 2sf)
	(vi) 9.44	(A0) (incorrectly rounded to 3sf)

<b>Markscheme</b>	<b>Candidates' Scripts</b>	<b>Marking</b>
7.44 (7.43798...) (AI)	(i) 7.43798... is seen in the working box followed by 7; 7.4; 7.44; 7.438 etc (correctly rounded)	(AI)
	(ii) 7.43798... is seen followed by 7.437; 7.43 etc (incorrectly rounded)	(AI)
	(iii) 7.4	(AI)
	(iv) 7	(A0) (correct to 1sf)
	(v) 7.5	(A0) (incorrectly rounded to 2sf)
	(vi) 7.43	(A0) (incorrectly rounded to 3sf)

**Example:** ABC is a right angled triangle with angle  $ABC = 90^\circ$ ,  $AC = 32$  cm and  $AB = 30$  cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b>  <i>Award (M1) for correct substitution in Pythagorus' formula</i></p> <p><math>= 11.1 (\sqrt{124}, 11.1355...)</math> (cm) <b>(A1)</b></p>	<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b></p> <p>11 (cm) <b>(A1)</b>  <i>(2 sf answer only seen, but correct)</i></p>	
<p>(b) <math>\text{Area} = \frac{1}{2} \times 30 \times 11.1355...</math> <b>(M1)</b>  <i>Award (M1) for correct substitution in area of triangle formula</i></p> <p><math>= 167(167.032...)</math> (cm<sup>2</sup>) <b>(A1)(ft)</b></p>	<p>(b) case (i) <math>\text{Area} = \frac{1}{2} \times 30 \times 11</math> <b>(M1)</b>  <i>(working shown)</i></p> <p><math>= 165</math> (cm<sup>2</sup>) <b>(A1)(ft)</b></p> <p>case (ii) <math>= 165</math> (cm<sup>2</sup>) <b>(M0)(A0)(ft)</b>  <i>(No working shown, the answer 11 is treated as a ft, so no marks awarded here)</i></p>	

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**. Exact answers such as  $\frac{1}{4}$  can be written as decimals to less than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (e.g.  $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ .) may be accepted as exact answers. All other powers (e.g. of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:** A financial question demands accuracy correct to 2dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) <b>(AI)</b>	(i) 231.6	<b>(A0)</b>
	(ii) 232	<b>(A0)</b> <i>(Correct rounding to incorrect level)</i>
	(iii) 231.61	<b>(A0)</b>
	(iv) 232.00	<b>(A0)</b> <i>(Parts (iii) and (iv) are both incorrect rounding to correct level)</i>

## 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded **(AI)** under the accuracy rules given in Section 5.

**Example:**

Markscheme	Candidates' Scripts	Marking
(a) 37000 m <sup>2</sup> <b>(AI)</b>	(a) 36000 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect answer so units not considered)</i>
(b) 3200 m <sup>3</sup> <b>(AI)</b>	(b) 3200 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect units)</i>

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

**QUESTION 1** *Units not required in this question*

(a) 65 (A1) [1 mark]

(b) (i) 54 (km h<sup>-1</sup>) (G2)

**Note:** If the answer to part (b)(i) is consistent with the answer to part (a) then award (G2)(ft) even if no working seen.

(ii) 19.2 (19.2093...) (G1) [3 marks]

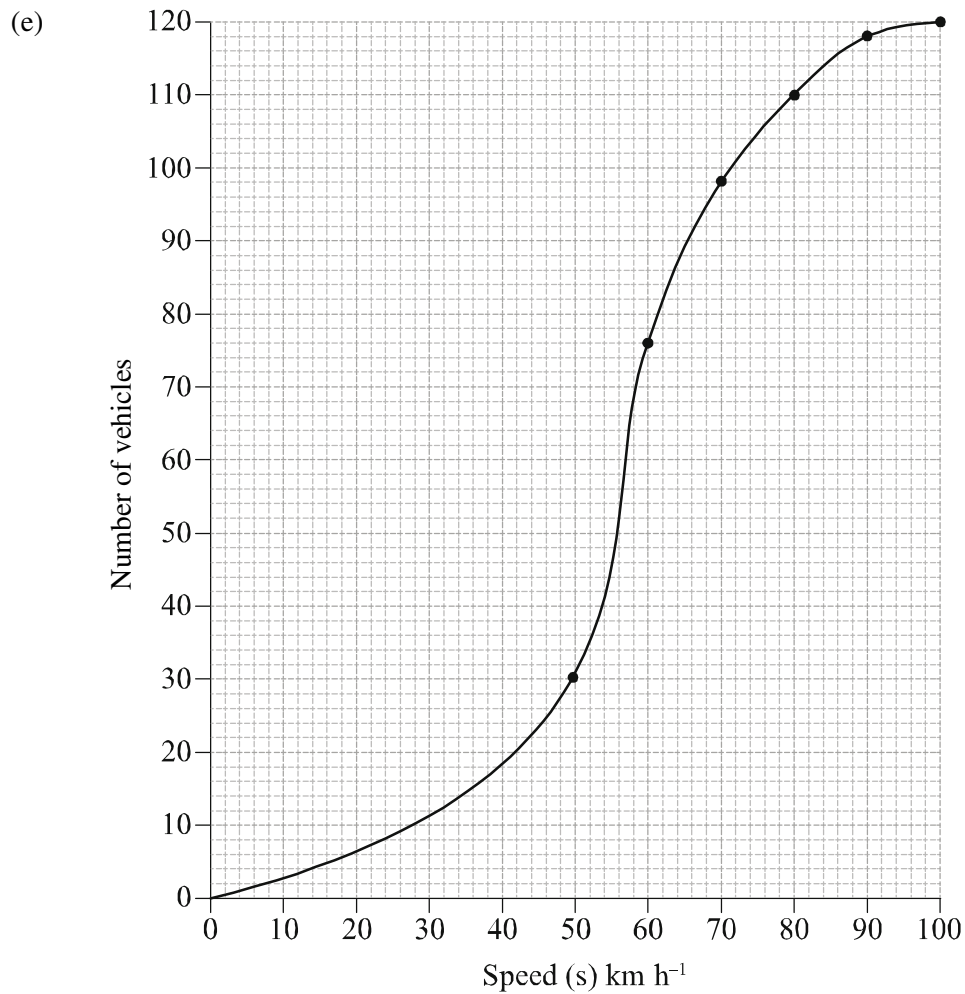
**Note:** Accept 19, do not accept 20.

(c) 76 (A1) [1 mark]

(d)  $a = 76$ ,  $b = 98$  (A1)(ft)

**Note:** Follow through from their answer to part (c) for  $a$  and  $b = \text{their } a + 22$ .

$c = 118$  (A1) [2 marks]



(A1)(A1)(ft)(A1)(ft)(A1) [4 marks]

continued...

*Question 1 continued*

**Notes:** Award **(AI)** for axes labelled and correct scales. If the axes are reversed do not award this mark but follow through.  
 Award **(A2)(ft)** for **their** 6 points correct, **(AI)(ft)** for at least 3 of these points correct.  
 Award **(AI)** for smooth curve drawn through all points **including** (0, 0). If either the  $x$  or the  $y$  axis has a break in it to zero, do not award this final mark.

(f) (i) 57 (km h<sup>-1</sup>) (±2) **(MI)(AI)(ft)(G2)**

**Note:** Award **(MI)** for clear indication of median on their graph  
 Follow through from their graph.  
 If their answer is consistent with their incorrect graph but there is no working present on graph then no marks are awarded.

(ii) 90 vehicles (±2) **(MI)(AI)(ft)(G2)** [4 marks]

**Note:** Award **(MI)** for clear indication of method on their graph.  
 Follow through from their graph.  
 If their answer is consistent with their incorrect graph but there is no working present on graph then no marks are awarded.

(g) 50 + 19.2 = 69.2 **(AI)(ft)**

24 (±2) drivers will be fined **(MI)(AI)(ft)(G2)** [3 marks]

**Notes:** Follow through from their graph and from their part (b)(ii).  
 Award **(MI)** for indication of method on their graph.  
 If their answer is consistent with their incorrect graph but there is no working present on graph then no marks are awarded.

**Total [18 marks]**

**QUESTION 2**

(a) (i)  $\frac{100}{400} \left( \frac{1}{4}, 0.25, 25 \% \right)$  (AI)

(ii)  $\frac{90}{400} \left( \frac{9}{40}, 0.225, 22.5 \% \right)$  (AI)

(iii)  $\frac{20}{400} \left( \frac{1}{20}, 0.05, 5 \% \right)$  (AI)(AI)

**Note:** Award (AI) for numerator, (AI) for denominator.

(iv)  $\frac{120}{400} \left( \frac{3}{10}, 0.3, 30 \% \right)$  (AI)(AI)

**Note:** Award (AI) for numerator, (AI) for denominator.

(v)  $\frac{30}{110} \left( \frac{3}{11}, 0.273, 27.3 \% \right) (0.272727\dots)$  (AI)(AI) [8 marks]

**Note:** Award (AI) for numerator, (AI) for denominator. Accept 0.27, do not accept 0.272, do not accept 0.3.

(b)  $\frac{1}{20} \neq \frac{1}{4} \times \frac{9}{40}$  (RI)(ft)

**Note:** The fractions must be used as part of the reason.  
Follow through from (a)(i), (a)(ii) and (a)(iii).

Pam is not correct (AI)(ft) [2 marks]

**Notes:** Do not award (R0)(AI).  
Accept the events are not independent (dependent).

*continued...*



Question 2 continued

- (c) (i) The mathematics course and language of examination are independent. (AI)

**Notes:** Accept “There is no association between Mathematics course and language”.  
Do not accept “not related”, “not correlated”, “not influenced”.

- (ii)  $\frac{110}{400} \times \frac{150}{400} \times 400 \left( = \frac{110 \times 150}{400} \right)$  (M1)  
 $= 41.25$  (AI)  
 $= 41.3$  (AG) [3 marks]

**Note:** 41.25 and 41.3 must be seen to award final (AI).

- (d) (i) 7.67 (7.67003...) (G2)

**Note:** Accept 7.7, do not accept 8 or 7.6.  
Award (GI) if formula with all nine terms seen but their answer is not one of those above.

- (ii) 4 (GI)  
 (iii) 9.488 (AI)(ft) [4 marks]

**Notes:** Accept 9.49 or 9.5, do not accept 9.4 or 9.  
Follow through from their degrees of freedom.

- (e)  $7.67 < 9.488$  (RI)

**OR**

- $p = 0.104\dots$ ,  $p > 0.05$  (RI)  
 Accept (Do not reject)  $H_0$  (Pam’s belief is correct) (AI)(ft) [2 marks]

**Notes:** Follow through from part (d). Do not award (R0)(AI).

**Total [19 marks]**

**QUESTION 3**

$$(a) \quad \cos ADB = \frac{12^2 + 20^2 - 28^2}{2(12)(20)} \quad (M1)(A1)$$

**Notes:** Award **(M1)** for substituted cosine rule formula, **(A1)** for correct substitutions.

$$\angle ADB = 120^\circ \quad (A1)(G2) \quad [3 \text{ marks}]$$

$$(b) \quad \text{Area} = \frac{(12)(20)\sin 120^\circ}{2} \quad (M1)(A1)(ft)$$

**Notes:** Award **(M1)** for substituted area formula, **(A1)(ft)** for their correct substitutions.

$$= 104 \text{ cm}^2 \quad (103.923 \dots \text{cm}^2) \quad (A1)(ft)(G2) \quad [3 \text{ marks}]$$

**Note:** The final answer is  $104 \text{ cm}^2$ , **the units are required.**  
Accept  $100 \text{ cm}^2$ .

$$(c) \quad \frac{\sin BCD}{12} = \frac{\sin 60^\circ}{13} \quad (A1)(ft)(M1)(A1)$$

**Note:** Award **(A1)(ft)** for their 60 seen, **(M1)** for substituted sine rule formula, **(A1)** for correct substitutions.

$$BCD = 53.1^\circ \quad (53.0736 \dots) \quad (A1)(G3) \quad [4 \text{ marks}]$$

**Note:** Accept 53, do not accept 50 or 53.0.

*continued...*

Question 3 continued

(d) Using triangle ABC

$$\frac{\sin BAC}{13} = \frac{\sin 53.1^\circ}{28}$$

(M1)(A1)(ft)

**OR**

Using triangle ABD

$$\frac{\sin BAD}{12} = \frac{\sin 120^\circ}{28}$$

(M1)(A1)(ft)

**Note:** Award (M1) for substituted sine rule formula (one of the above), (A1)(ft) for their correct substitutions. Follow through from (a) or (c) as appropriate.

$$BAC = BAD = 21.8^\circ \quad (21.7867\dots)$$

(A1)(ft)(G2)

**Notes:** Accept 22, do not accept 20 or 21.7.  
Accept equivalent methods, for example cosine rule.

$180^\circ - (53.1^\circ + 21.8^\circ) \neq 90^\circ$ , hence triangle ABC is not right angled

(R1)(AG)

**OR**

$$\frac{CD}{\sin 66.9^\circ} = \frac{13}{\sin 60^\circ}$$

(M1)(A1)(ft)

**Note:** Award (M1) for substituted sine rule formula, (A1)(ft) for their correct substitutions. Follow through from (a) and (c).

$$CD = 13.8 \quad (13.8075\dots)$$

(A1)(ft)

$13^3 + 28^2 \neq 33.8^2$ , hence triangle ABC is not right angled.

(R1)(ft)(AG)

[4 marks]

**Note:** The complete statement is required for the final (R1) to be awarded.

**Total [14 marks]**

**QUESTION 4**

- (a) (i)  $x = 0$  (AI)(AI)

**Note:** Award (AI) for  $x =$  a constant, (AI) for the constant in their equation being 0.

- (ii)  $-1.58$  ( $-1.58454\dots$ ) (G1)

**Note:** Accept  $-1.6$ , do not accept  $-2$  or  $-1.59$ .

- (iii)  $(2.06, 4.49)$  ( $2.06020\dots, 4.49253\dots$ ) (G1)(G1) [5 marks]

**Note:** Award at most (G1)(G0) if brackets not used.  
Award (G0)(G1)(ft) if coordinates are reversed.

**Note:** Accept  $x = 2.06$ ,  $y = 4.49$ .

**Note:** Accept 2.1, do not accept 2.0 or 2. Accept 4.5, do not accept 5 or 4.50.

- (b)  $f'(x) = 2x - 2 - \frac{9}{x^2}$  (AI)(AI)(AI)(AI) [4 marks]

**Notes:** Award (AI) for  $2x$ , (AI) for  $-2$ , (AI) for  $-9$ , (AI) for  $x^{-2}$ .  
Award a maximum of (AI)(AI)(AI)(A0) if there are extra terms present.

- (c)  $f'(x) = \frac{x^2(2x-2)}{x^2} - \frac{9}{x^2}$  (M1)

**Note:** Award (M1) for taking the correct common denominator.

$$= \frac{(2x^3 - 2x^2)}{x^2} - \frac{9}{x^2} \quad (M1)$$

**Note:** Award (M1) for multiplying brackets or equivalent.

$$= \frac{2x^3 - 2x^2 - 9}{x^2} \quad (AG) \quad [2 \text{ marks}]$$

**Note:** The final (M1) is not awarded if the given answer is not seen.

- (d)  $f'(1) = \frac{2(1)^3 - 2(1) - 9}{(1)^2}$  (M1)  
 $= -9$  (AI)(G2) [2 marks]

**Note:** Award (M1) for substitution into **given** (or their correct)  $f'(x)$ .  
There is no follow through for use of their incorrect derivative.

continued...

Question 4 continued

(e)  $\frac{1}{9}$  (AI)(ft) [1 mark]

**Note:** Follow through from part (d).

(f)  $y - 8 = \frac{1}{9}(x - 1)$  (M1)(M1)

**Notes:** Award (M1) for substitution of their gradient from (e), (M1) for substitution of given point. Accept all forms of straight line.

$y = \frac{1}{9}x + \frac{71}{9}$  ( $y = 0.111111...x + 7.88888...$ ) (AI)(ft)(G3) [3 marks]

**Note:** Award the final (AI)(ft) for a correctly rearranged formula of their straight line in (f).  
Accept 0.11x, do not accept 0.1x.  
Accept 7.9, do not accept 7.88. do not accept 7.8.

(g)  $-2.50, 3.61$  ( $-2.49545..., 3.60656...$ ) (AI)(ft)(AI)(ft) [2 marks]

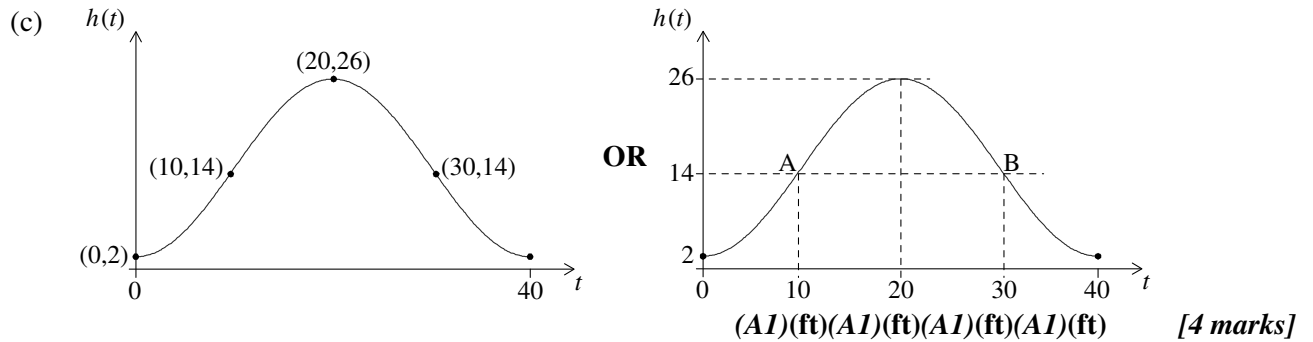
**Notes:** Follow through from their line  $L$  from part (f) even if no working shown.  
Award at most (A0)(AI) (ft) if their correct coordinate pairs given.

**Note:** Accept  $-2.5$ , do not accept  $-2.49$ . Accept  $3.6$ , do not accept  $3.60$ .

**Total [19 marks]**

**QUESTION 5** *Units not required in this question*

- (a) (i) 14 m (AI)
- (ii) 26 m (AI) [2 marks]
- (b) A:10, B:30 (AI)(AI) [2 marks]



**Note:** Award (AI)(ft) for coordinates of each point clearly indicated either by scale or by coordinate pairs. Points need not be labelled A and B in the second diagram.  
Award a maximum of (AI)(A0)(AI)(ft)(AI)(ft) if coordinates are reversed. Do not penalise reversed coordinates if this has already been penalised in Q4(a)(iii).

- (d) (i)  $a = -12$  (A2)(ft)

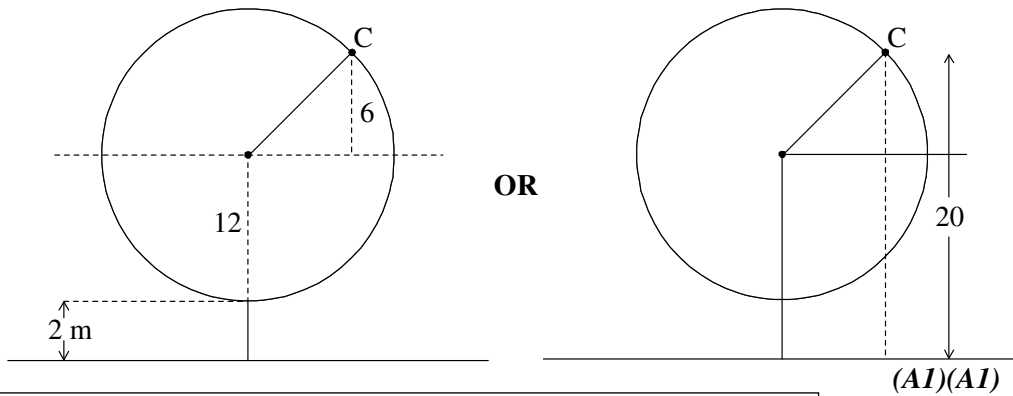
**Note:** Follow through from (a) even if no working seen.

- (ii)  $b = \frac{360}{40}$  (M1)  
 $= 9$  (AI)(G2)
- (iii)  $c = 14$  (AI) [5 marks]

*continued...*

Question 5 continued

(e) (i)



**Note:** Award **(AI)** for C in correct quadrant, **(AI)** for clear indication of 20 m above ground.

(ii)  $90 + \sin^{-1}\left(\frac{6}{12}\right)$  **(M1)(M1)**

**Note:** Award **(M1)** for adding 90, **(M1)** for use of trigonometric ratio with appropriate values.

$= 120^\circ$  **(AI)(G2)**

**OR**

$20 = -12 \cos(9T) + 14$  **(M1)(M1)**

**Note:** Award **(M1)** for substituting values, **(M1)** for equating to 20.

$9T = 120^\circ$  **(AI)(ft)(G2)**

**Note:** Follow through from their  $a$ ,  $b$  and  $c$  in part (d). The final answer must be an obtuse angle.  $9T$  does not have to be seen.

(iii)  $\frac{120}{360} \times 40$  **(M1)**

$= 13\frac{1}{3}$  seconds (13.3333...) **(AI)(ft)(G2)**

**OR**

$9T = 120^\circ$  **(M1)**

$T = 13\frac{1}{3}$  **(AI)(ft)(G2)** [7 marks]

**Notes:** Follow through from their answer to part (e)(ii).  
The final answer must be consistent with their diagram.  
Accept 13.

**Total [20 marks]**



# **MARKSCHEME**

**November 2012**

**MATHEMATICAL STUDIES**

**Standard Level**

**Paper 2**



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**Paper 2 Markscheme**  
**Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

**1 Abbreviations**

**M** Marks awarded for **Method**

**A** Marks awarded for an **Answer** or for **Accuracy**

**R** Marks awarded for clear **Reasoning**

**G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

**AG** **Answer Given** in the question and consequently, marks not awarded.

**ft** Marks that can be awarded as **follow through** from previous results in the question.

**2 Method of Marking**

- (a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the **A0** annotation, otherwise full annotations must be shown.
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.
- (e) If correct working results in a correct answer but then further working is developed, full marks may **not** always be awarded. Full marks **will** be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

**Example:** Calculate the gradient of the line passing through the points (5, 3) and (0, 9) .

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5} \quad (M1)$ <p>Award (M1) for correct substitution in gradient formula</p> $= -\frac{6}{5} \quad (A1)$	<p>(i) <math>\frac{9-3}{0-5} = -\frac{6}{5}</math></p> <p>Gradient is <math>= -\frac{6}{5}</math></p> <p>(There is clear understanding of the gradient.)</p> <p><math>y = -\frac{6}{5}x + 9</math></p> <p>(ii) <math>\frac{9-3}{0-5} = -\frac{6}{5}</math></p> <p><math>y = -\frac{6}{5}x + 9</math></p> <p>(There is confusion about what is required.)</p>	<p>(M1)</p> <p>(A1)</p> <p>(M1)</p> <p>(A0)</p>

### 3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **‘(ft)’**.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (for example, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate’s use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies**.
- (f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

Markscheme	Candidates’ Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> (MI)(AI)</p> <p>Award (MI) for substitution in sine rule formula, (AI) for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) (AI)(G2)</p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math></p> <p><math>A = 41.8^\circ</math></p> <p>(Note: the 2<sup>nd</sup> (AI) here was not marked (ft) and cannot be awarded because there was an earlier error in the <b>same</b> question part.)</p>	<p>(MI)(A0)</p> <p>(use of sine rule but with wrong values)</p> <p>(A0)</p>
<p>(b) <math>x = 7 \tan (22.0243\dots)</math> (MI)</p> <p><math>= 2.83</math> (2.83163...) (AI)(ft)</p>	<p>(b) case (i) <math>x = 7 \tan 41.8^\circ</math></p> <p><math>= 6.26</math></p> <p><b>but</b> case (ii) <math>6.26</math></p>	<p>(MI)</p> <p>(AI)(ft)</p> <p>(G0)</p> <p>since no working shown</p>

#### 4 Using the Markscheme

- (a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.  
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.
- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

**Example:** Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b> Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(i) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <math>A = 22.0^\circ</math></p> <p>(ii) <math>A = 22.0^\circ</math> <b>Note: G marks are used only if no working has been shown and the answer is correct.</b></p>	<p><b>(M1)(A1)</b>  <b>(A1)</b>  <b>(G2)</b></p>

- (d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.  
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' etc.
- (e) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .  
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.  
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:  
the 3 significant figure answer worked through from full calculator display;  
the exact value (for example  $\sqrt{3}$  if applicable);  
the full calculator display in the form 2.83163... as in the example above.  
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

- (f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Different descriptions of an interval:  $3 < x < 5$ ;  $(3, 5)$ ;  $] 3, 5 [$  .

Different forms of notation for set properties (for example, complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A; U \setminus A$ .

Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ .

$$p \Rightarrow q; p \rightarrow q; q \Leftarrow p .$$

- (g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through scoris to the team leader.

As from Nov 11 there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5 Accuracy of Answers

**Incorrect accuracy should be penalized once only in each question according to the rules below.**

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award **(AI)** and ignore subsequent rounding.
2. If the candidate's unrounded answer is **not** seen then award **(AI)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.  
**Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.
3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given ...				
	Exact or correct to 3 or more sf	<b>Incorrect to 3 sf</b>	Correct to 2 sf <sup>3</sup>	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen <sup>1</sup>	Award the final <b>(AI)</b> irrespective of correct or incorrect rounding				
Unrounded answer not seen <sup>2</sup>	<b>(AI)</b>	<b>(A0)</b>	<b>(AI)</b>	<b>(A0)</b>	<b>(A0)</b>
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen <sup>3</sup>			

**Examples:**

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398...) ( <i>AI</i> )	(i) 9.43398... is seen followed by 9; 9.4; 9.43; 9.434 <i>etc.</i> (correctly rounded)	( <i>AI</i> )
	(ii) 9.43398... is seen followed by 9.43; 9.44 <i>etc.</i> (incorrectly rounded)	( <i>AI</i> )
	(iii) 9.4	( <i>AI</i> )
	(iv) 9	( <i>A0</i> ) (correct to 1 sf)
	(v) 9.3	( <i>A0</i> ) (incorrectly rounded to 2 sf)
	(vi) 9.44	( <i>A0</i> ) (incorrectly rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798...) ( <i>AI</i> )	(i) 7.43798... is seen followed by 7; 7.4; 7.44; 7.438 <i>etc.</i> (correctly rounded)	( <i>AI</i> )
	(ii) 7.43798... is seen followed by 7.437; 7.43 <i>etc.</i> (incorrectly rounded)	( <i>AI</i> )
	(iii) 7.4	( <i>AI</i> )
	(iv) 7	( <i>A0</i> ) (correct to 1 sf)
	(v) 7.5	( <i>A0</i> ) (incorrectly rounded to 2 sf)
	(vi) 7.43	( <i>A0</i> ) (incorrectly rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b>  Award <b>(M1)</b> for correct substitution in Pythagoras' formula</p> <p><math>= 11.1 \left( \sqrt{124}, 11.1355... \right) (\text{cm})</math> <b>(A1)</b></p>	<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b></p> <p>11 (cm) <b>(A1)</b>  <i>(2 sf answer only seen, but correct)</i></p>	
<p>(b) <math>\text{Area} = \frac{1}{2} \times 30 \times 11.1355...</math> <b>(M1)</b>  Award <b>(M1)</b> for correct substitution in area of triangle formula</p> <p><math>= 167(167.032...) (\text{cm}^2)</math> <b>(A1)(ft)</b></p>	<p>(b) case (i) <math>\text{Area} = \frac{1}{2} \times 30 \times 11</math> <b>(M1)</b>  <i>(working shown)</i></p> <p><math>= 165 (\text{cm}^2)</math> <b>(A1)(ft)</b></p> <p>case (ii) <math>= 165 (\text{cm}^2)</math> <b>(M0)(A0)(ft)</b>  <i>(No working shown, the answer 11 is treated as a ft, so no marks awarded here)</i></p>	

For example, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	(G2)
7.68 (7.67543...) (A2)	(b) 7.67	(G1)
	(c) 7.6	(G1)
	(d) 8	(G0)
	(e) 7	(G0)
	(e) 7.66	(G0)



Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ (A2) $(y = 0.887686...x + 13.4895...)$ If an answer is not in the form of an equation award at most (A1)(A0).	(a) $y = 0.89x + 13$  (b) $y = 0.88x + 13$  (c) $y = 0.88x + 14$  (d) (i) $y = 0.9x + 13$  (ii) $y = 0.8x + 13$  (e) $0.88x + 13$	(G2) <i>(both accepted)</i>  (G1) <i>(one rounding error)</i>  (G1) <i>(rounding error repeated)</i>  (G1) <i>(1 sf not accepted)</i>  (G0) <i>(one rounding error and not an equation)</i>

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ (A1)(A1) $(2.06020..., 4.49253...)$	(a) $(2.1, 4.5)$  (b) $(2.0, 4.4)$  (c) $(2.06, 4.4)$  (d) $(2, 4.4)$	(A1)(A1) <i>(both accepted)</i>  (A1) <i>(same rounding error twice)</i>  (A1) <i>(one rounding error)</i>  (A0) <i>(1sf not accepted, one rounding error)</i>

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**.

Exact answers such as  $\frac{1}{4}$  can be written as decimals to fewer than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (for example,  $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ ,) may be accepted as exact answers. All other powers (for example, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (M0).

## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:** A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) <b>(A1)</b>	(i) 231.6	<b>(A0)</b>
	(ii) 232	<b>(A0)</b> <i>(Correct rounding to incorrect level)</i>
	(iii) 231.61	<b>(A0)</b>
	(iv) 232.00	<b>(A0)</b> <i>(Parts (iii) and (iv) are both incorrect rounding to correct level)</i>

## 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

**Example:**

Markscheme	Candidates' Scripts	Marking
(a) 37000 m <sup>2</sup> <b>(A1)</b>	(a) 36000 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect answer so units not considered)</i>
(b) 3200 m <sup>3</sup> <b>(A1)</b>	(b) 3200 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect units)</i>

**If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.**

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

# QUESTION 1

(a) (i)  $\frac{71+79+\dots}{12}$  (M1)

72.4  $\left(72.4166\dots, \frac{869}{12}\right)$  (A1)(G2)

**Note:** Award (M1) for correct substitution into the mean formula.

(ii) 4.77 (4.76896...) (G1)

(iii)  $72.4 + 4.77 = 77.17$  (M1)

**Note:** Award (M1) for adding their mean to their standard deviation.

Two golfers (A1)(ft)(G2) [5 marks]

**Note:** Follow through from their answers to parts (i) and (ii).

(b) 0.990 (0.99014...) (G2) [2 marks]

(c)  $y = 1.01x + 0.816$  ( $y = 1.01404\dots x + 0.81618\dots$ ) (G1)(G1)

**Notes:** Award (G1) for  $1.01x$  and (G1) for  $0.816$ . If the answer is not an equation award a maximum of (G1)(G0).

**OR**

$y - 74.25 = 1.01(x - 72.4)$  ( $y - 74.25 = 1.01404\dots(x - 72.4166\dots)$ ) (A1)(A1) [2 marks]

**Notes:** Award (A1) for  $1.01$  correctly substituted in the equation, and (A1)(ft) for correct substitution of  $(72.4, 74.25)$  in the equation.  
Follow through from their part (a)(i).  
If the final answer is not an equation award a maximum of (A1)(A0).

(d)  $y = 1.01404\dots \times 70 + 0.81618\dots$  (M1)

**Note:** Award (M1) for substitution of 70 into their regression line equation from part (c).

$y = 72$  (71.7989...) (A1)(ft)(G2) [2 marks]

**Note:** Follow through from their part (c).

continued...

*Question 1 continued*

- (e) No, equation cannot be (reliably) used as 89 is outside the data range. (AI)(RI)

**OR**

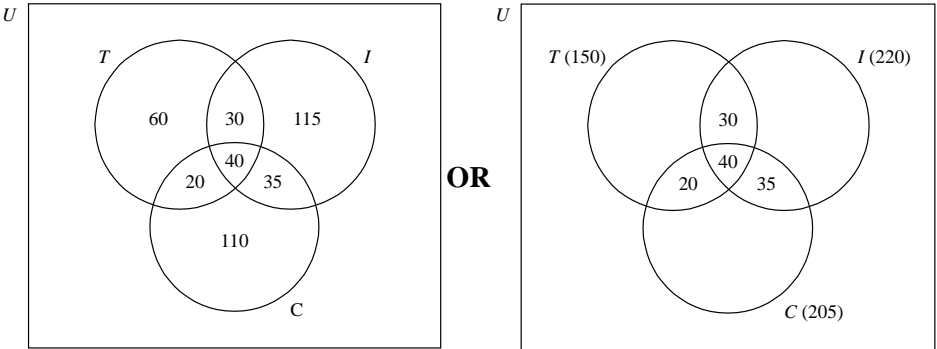
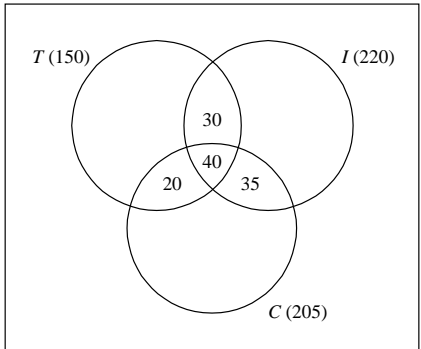
Yes, but the result is not valid/not reliable as 89 is outside the data range/as we extrapolate

(AI)(RI) [2 marks]

<b>Note:</b> Do not award (AI)(R0).
-------------------------------------

**Total [13 marks]**

## QUESTION 2

(a)  **OR** 

(AI)(AI)(AI)(AI) [4 marks]

**Notes:** Award (AI) for labelled sets  $T$ ,  $C$ , and  $I$  included inside an enclosed universal set. (Label  $U$  is not essential.)  
 Award (AI) for central entry 40.  
 (AI) for 20, 30 and 35 in the other intersecting regions.  
 (AI) for 60, 110 and 115 or  $T(150)$ ,  $C(205)$ ,  $I(220)$ .

*In parts (b), (c) and (d) follow through from their diagram.*

- (b) (i) 110 (AI)(ft)  
 (ii) 35 (AI)(ft) [2 marks]  
 (c) 60 (AI)(ft) [1 mark]  
 (d)  $450 - (60 + 20 + 40 + 30 + 115 + 35 + 110)$  (MI)

**Note:** Award (MI) for subtracting all their values from 450.

- $= 40$  (AI)(ft)(G2) [2 marks]  
 (e) (i)  $\frac{230}{450} \times \frac{229}{449}$  (AI)(MI)

**Note:** Award (AI) for correct fractions, (MI) for multiplying their fractions.

$$\frac{52670}{202050} \left( \frac{5267}{20205}, 0.261, 26.1\% \right) (0.26067\dots) \quad (AI)(G2)$$

**Note:** Follow through from their Venn diagram in part (a).

*continued...*

Question 2 continued

$$(ii) \quad \frac{220}{450} \times \frac{230}{449} + \frac{230}{450} \times \frac{220}{449} \quad (AI)(AI)$$

**Note:** Award **(AI)** for addition of their products, **(AI)** for two correct products.

**OR**

$$\frac{230}{450} \times \frac{220}{449} \times 2 \quad (AI)(AI)$$

**Notes:** Award **(AI)** for their product of two fractions multiplied by 2,  
**(AI)** for correct product of two fractions multiplied by 2.  
 Award **(A0)(A0)** if correct product is seen not multiplied by 2.

$$\frac{2024}{4041} \quad (0.501, 50.1\%) \quad (0.50086...) \quad (AI)(G2) \quad [6 \text{ marks}]$$

**Note:** Follow through from their Venn diagram in part (a) and/or their 230 used in part (e)(i).

**Note:** For consistent use of replacement in parts (i) and (ii) award at most **(A0)(MI)(A0)** in part (i) and **(AI)(ft)(AI)(AI)(ft)** in part (ii).

$$(f) \quad (i) \quad x + 9y = 13050 \quad (AI)$$

$$(ii) \quad \begin{array}{l} x = 900 \\ y = 1350 \end{array} \quad \begin{array}{l} (AI)(ft) \\ (AI)(ft) \end{array} \quad [3 \text{ marks}]$$

**Notes:** Follow through from their equation in (f)(i). Do not award **(AI)(ft)** if answer is negative.  
 Award **(MI)(A0)** for an attempt at solving simultaneous equations algebraically but incorrect answer obtained.

continued...

*Question 2 continued*

(g)  $49500 = 900 + 1350n$  (AI)(ft)

**Notes:** Award (AI)(ft) for setting up correct equation. Follow through from candidate's part (f).

$n = 36$  (AI)(ft)

The total number of months is 37. (AI)(ft)(G2)

**Note:** Award (GI) for 36 seen as final answer with no working. The value of  $n$  must be a positive integer for the last two (AI)(ft) to be awarded.

**OR**

$49500 = 900 + 1350(n - 1)$  (A2)(ft)

**Notes:** Award (A2)(ft) for setting up correct equation. Follow through from candidate's part (f).

$n = 37$  (AI)(ft)(G2) [3 marks]

**Note:** The value of  $n$  must be a positive integer for the last (AI)(ft) to be awarded.

**Total [21 marks]**

**QUESTION 3** *Units are required in part (c) only.*

(a)  $BC^2 = 5^2 + 7^2 - 2(5)(7)\cos 112^\circ$  (MI)(AI)

**Note:** Award (MI) for substitution in cosine formula, (AI) for correct substitutions.

$BC = 10.0 \text{ (m)} \text{ (10.0111...)} \quad (AI)(G2) \quad [3 \text{ marks}]$

**Note:** If radians are used, award at most (MI)(AI)(A0).

(b)  $\frac{\sin 40^\circ}{10.0111...} = \frac{\sin \hat{DCB}}{6}$  (MI)(AI)(ft)

**Notes:** Award (MI) for substitution in sine formula, (AI)(ft) for their correct substitutions. Follow through from their part (a).

$\hat{DCB} = 22.7^\circ \text{ (22.6589...)} \quad (AI)(ft)$

**Notes:** Award (A2) for  $22.7^\circ$  seen without working. Use of radians results in unrealistic answer. Award a maximum of (MI)(AI)(ft)(A0)(ft). Follow through from their part (a).

$\hat{DBC} = 117^\circ \text{ (117.341...)} \quad (AI)(ft)(G3)$

**Notes:** Do not penalize if use of radians was already penalized in part (a). Follow through from their answer to part (a).

**OR**

From use of cosine formula  
 $DC = 13.8 \text{ (m)} \text{ (13.8346...)} \quad (AI)(ft)$

**Note:** Follow through from their answer to part (a).

$\frac{\sin \alpha}{13.8346...} = \frac{\sin 40^\circ}{10.0111...} \quad (MI)$

**Note:** Award (MI) for correct substitution in the correct sine formula.

$\alpha = 62.7^\circ \text{ (62.6589...)} \quad (AI)(ft)$

**Note:** Accept  $62.5^\circ$  from use of 3sf.

$\hat{DBC} = 117 \text{ (117.341...)} \quad (AI)(ft) \quad [4 \text{ marks}]$

**Note:** Follow through from their part (a). Use of radians results in unrealistic answer, award a maximum of (AI)(MI)(A0)(A0).

*continued...*



Question 3 continued

(c)  $\text{Area ABDC} = \frac{1}{2}(5)(7)\sin 112^\circ + \frac{1}{2}(6)(10.0111\dots)\sin 117.341\dots^\circ$  (M1)(A1)(ft)(M1)

**Note:** Award (M1) for substitution in both **triangle** area formulae, (A1)(ft) for their correct substitutions, (M1) for seen or implied addition of their two **triangle** areas. Follow through from their answer to part (a) and (b).

$= 42.9 \text{ m}^2$  (42.9039...) (A1)(ft)(G3) [4 marks]

**Notes:** Answer is  $42.9 \text{ m}^2$  *i.e.* the units are required for the final (A1)(ft) to be awarded. Accept  $43.0 \text{ m}^2$  from using 3sf answers to parts (a) and (b). Do not penalize if use of radians was previously penalized.

(d)  $42.9039\dots \times 0.5$  (M1)(M1)

**Note:** Award (M1) for 0.5 seen (or equivalent), (M1) for multiplication of their answer in part (c) with their value for depth.

$= 21.5 \text{ (m}^3\text{)}$  (21.4519...) (A1)(ft)(G3) [3 marks]

**Note:** Follow through from their part (c) **only if working is seen**. Do not penalize if use of radians was previously penalized. Award at most (A0)(M1)(A0)(ft) for multiplying by 50.

(e) (i)  $\pi(0.15)^2(0.4)$  (M1)(A1)

**OR**

$\pi \times 15^2 \times 40$  (28274.3...) (M1)(A1)

**Notes:** Award (M1) for substitution in the correct volume formula. (A1) for correct substitutions.

$= 0.0283 \text{ (m}^3\text{)}$  (0.0282743...,  $0.09\pi$ )

(ii)  $\frac{21.4519\dots}{0.0282743\dots}$  (M1)

**Note:** Award (M1) for correct division of their volumes.

$= 759$  (A1)(ft)(G2) [5 marks]

**Notes:** Follow through from their parts (d) and (e)(i). Accept 760 from use of 3sf answers.  
Answer must be a positive integer for the final (A1)(ft) mark to be awarded.

**Total [19 marks]**

#### QUESTION 4

- (a) (i)  $\frac{220}{500} \left( \frac{11}{25}, 0.44, 44 \% \right)$  (AI)(GI)
- (ii)  $\frac{180}{500} \left( \frac{9}{25}, 0.36, 36 \% \right)$  (AI)(GI)
- (iii)  $\frac{40}{500} \left( \frac{2}{25}, 0.08, 8 \% \right)$  (AI)(AI)(G2)
- (iv)  $\frac{55}{280} \left( \frac{11}{56}, 0.196, 19.6 \% \right)$  (AI)(AI)(G2) [6 marks]

**Note:** Award (AI) for numerator, (AI) for denominator. Award (A0)(A0) if answers are given as incorrect reduced fractions without working.

- (b) “The size of the television screen is independent of gender.” (AI) [1 mark]

**Note:** Accept “not associated”, do not accept “not correlated”.

- (c)  $\frac{180}{500} \times \frac{220}{500} \times 500$  OR  $\frac{180 \times 220}{500}$  (MI)
- = 79.2 (AI)
- = 79 (AG) [2 marks]

**Note:** Both the unrounded and the given answer must be seen for the final (AI) to be awarded.

- (d) 3 (AI) [1 mark]
- (e)  $\chi^2_{calc} = 104$  (103.957...) (G2) [2 marks]

**Note:** Award (MI) if an attempt at using the formula is seen but incorrect answer obtained.

- (f) 11.345 (AI)(ft) [1 mark]

**Notes:** Follow through from their degrees of freedom.

- (g)  $\chi^2_{calc} > \chi^2_{crit}$  OR  $p < 0.01$  (RI)
- Do not accept  $H_0$ . (AI)(ft) [2 marks]

**Note:** Do not award (R0)(AI)(ft). Follow through from their parts (d), (e) and (f).

**Total [15 marks]**

### QUESTION 5

- (a)  $x = 0$  (AI)(AI) [2 marks]

**Notes:** Award (AI) for  $x = \text{constant}$ , (AI) for 0. Award (A0)(A0) if answer is not an equation.

- (b)  $b - \frac{2}{x^3}$  (AI)(AI)(AI) [3 marks]

**Note:** Award (AI) for  $b$ , (AI) for  $-2$ , (AI) for  $\frac{1}{x^3}$  (or  $x^{-3}$ ).  
Award at most (AI)(AI)(A0) if extra terms seen.

- (c)  $3 = b - \frac{2}{(1)^3}$  (M1)(M1)

**Note:** Award (M1) for substituting 1 into their gradient function, (M1) for equating their gradient function to 3.

$b = 5$  (AG) [2 marks]

**Note:** Award at most (M1)(A0) if final line is not seen or  $b$  does not equal 5.

- (d)  $g(1) = 3$  or  $(1, 3)$  (seen or implied from the line below) (AI)

$3 = 3 \times 1 + c$  (M1)

**Note:** Award (M1) for correct substitution of their point  $(1, 3)$  and gradient 3 into equation  $y = mx + c$ . Follow through from their point of tangency.

$y = 3x$  (AI)(ft)(G2)

**OR**

$y - 3 = 3(x - 1)$  (M1)(AI)(ft)(G2) [3 marks]

**Note:** Award (M1) for substitution of gradient 3 and their point  $(1, 3)$  into  $y - y_1 = m(x - x_1)$ , (AI)(ft) for correct substitutions. Follow through from their point of tangency. Award at most (AI)(M1)(A0)(ft) if further incorrect working seen.

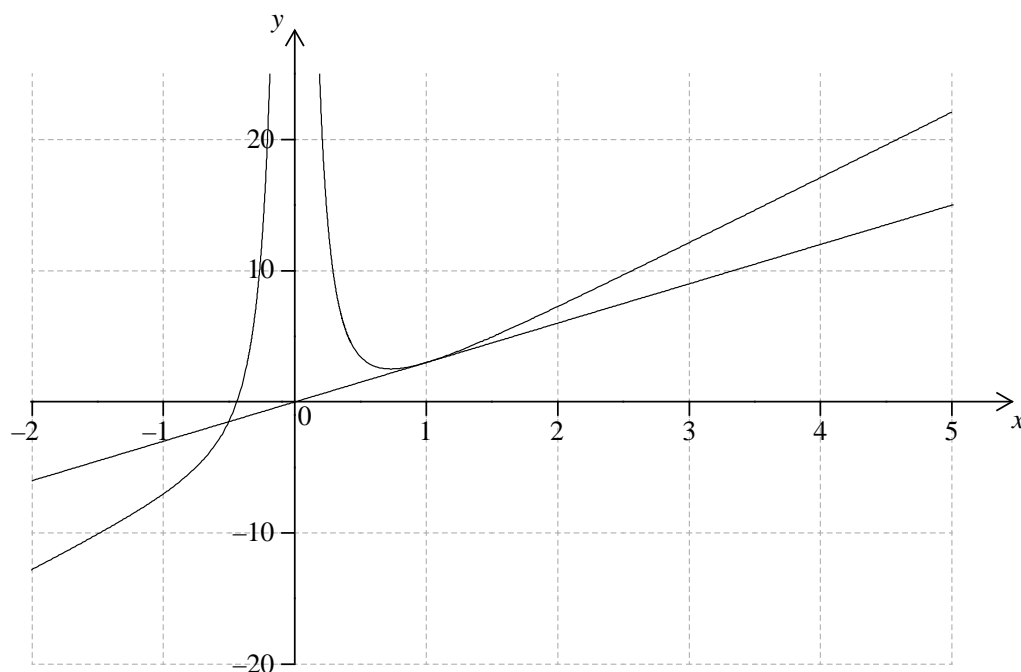
- (e)  $(-0.439, 0)$   $((-0.438785\dots, 0))$  (G1)(G1) [2 marks]

**Notes:** If no parentheses award at most (G1)(G0).  
Accept  $x = -0.439$ ,  $y = 0$ .

continued...

Question 5 continued

(f) (i)



Award **(AI)** for labels and some indication of scale in the stated window.

Award **(AI)** for correct general shape (curve must be smooth and must not cross the y-axis)

Award **(AI)(ft)** for x-intercept consistent with their part (e).

Award **(AI)** for local minimum in the first quadrant. **(AI)(AI)(AI)(ft)(AI)**

(ii) Tangent to curve drawn at approximately  $x = 1$

**(AI)(AI)** [6 marks]

**Note:** Award **(AI)** for a line tangent to curve approximately at  $x = 1$ .

Must be a straight line for the mark to be awarded.

Award **(AI)(ft)** for line passing through the origin. Follow through from their answer to part (d).

(g)  $(0.737, 2.53)$   $((0.736806\dots, 2.52604\dots))$

**(GI)(GI)** [2 marks]

**Notes:** Do not penalize for lack of parentheses if already penalized in (e).

Accept  $x = 0.737$ ,  $y = 2.53$ .

(h)  $0.737 < x < 5$  **OR**  $(0.737; 5)$

**(AI)(AI)(ft)** [2 marks]

**Notes:** Award **(AI)** for correct strict or weak inequalities with  $x$  seen if the interval is given as inequalities, **(AI)(ft)** for 0.737 and 5 or their value from part (g).

**Total [22 marks]**



# **MARKSCHEME**

**November 2013**

**MATHEMATICAL STUDIES**

**Standard Level**

**Paper 2**

*This markscheme is **confidential** and for the exclusive use of examiners in this examination session.*

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**Paper 2 Markscheme**  
**Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

**1 Abbreviations**

**M** Marks awarded for **Method**

**A** Marks awarded for an **Answer** or for **Accuracy**

**R** Marks awarded for clear **Reasoning**

**G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

**AG** **Answer Given** in the question and consequently, marks not awarded.

**ft** Marks that can be awarded as **follow through** from previous results in the question.

**2 Method of Marking**

- (a) All marking must be done in scoris using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the **A0** annotation, otherwise full annotations must be shown.
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.
- (e) If correct working results in a correct answer but then further working is developed, full marks may **not** always be awarded. Full marks **will** be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

**Example:** Calculate the gradient of the line passing through the points (5, 3) and (0, 9) .

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5} \quad (M1)$ <p>Award (M1) for correct substitution in gradient formula</p> $= -\frac{6}{5} \quad (A1)$	<p>(i) <math>\frac{9-3}{0-5} = -\frac{6}{5}</math></p> <p>Gradient is <math>= -\frac{6}{5}</math></p> <p>(There is clear understanding of the gradient.)</p> $y = -\frac{6}{5}x + 9$ <p>(ii) <math>\frac{9-3}{0-5} = -\frac{6}{5}</math></p> $y = -\frac{6}{5}x + 9$ <p>(There is confusion about what is required.)</p>	<p>(M1)</p> <p>(A1)</p> <p>(M1)</p> <p>(A0)</p>

### 3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Mark schemes will indicate where it is appropriate to apply follow through in a question with **‘(ft)’**.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (*e.g.* negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate’s use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies.**
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

Markscheme	Candidates’ Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> (M1)(A1)</p> <p>Award (M1) for substitution in sine rule formula, (A1) for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) (A1)(G2)</p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math></p> <p><math>A = 41.8^\circ</math></p> <p>(Note: the 2<sup>nd</sup> (A1) here was not marked (ft) and cannot be awarded because there was an earlier error in the <b>same</b> question part.)</p>	<p>(M1)(A0)</p> <p>(use of sine rule but with wrong values)</p> <p>(A0)</p>
<p>(b) <math>x = 7 \tan (22.0243\dots)</math> (M1)</p> <p><math>= 2.83</math> (2.83163...) (A1)(ft)</p>	<p>(b) case (i) <math>x = 7 \tan 41.8^\circ</math></p> <p><math>= 6.26</math></p> <p><b>but</b> case (ii) <math>6.26</math></p>	<p>(M1)</p> <p>(A1)(ft)</p> <p>(G0)</p> <p>since no working shown</p>



#### 4 Using the Markscheme

- (a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.  
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.
- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

**Example:** Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b> Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(i) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <math>A = 22.0^\circ</math></p> <p>(ii) <math>A = 22.0^\circ</math> <b>Note:</b> <b>G</b> marks are used only if no working has been shown and the answer is correct.</p>	<p><b>(M1)(A1)</b></p> <p><b>(A1)</b></p> <p><b>(G2)</b></p>

- (d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.  
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' etc.
- (e) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .  
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.  
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:  
the 3 significant figure answer worked through from full calculator display;  
the exact value (for example  $\sqrt{3}$  if applicable);  
the full calculator display in the form 2.83163... as in the example above.  
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

- (f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Different descriptions of an interval:  $3 < x < 5$ ;  $(3, 5)$ ;  $] 3, 5 [$  .

Different forms of notation for set properties (*e.g.* complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A; U \setminus A$ .

Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ .

$p \Rightarrow q$ ;  $p \rightarrow q$ ;  $q \Leftarrow p$  .

- (g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through scoris to the team leader.

There will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5 Accuracy of Answers

**Incorrect accuracy should be penalized once only in each question according to the rules below.**

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the candidate's unrounded answer is seen and would round to the required 3 sf answer, then award **(AI)** and ignore subsequent rounding.
  2. If the candidate's unrounded answer is **not** seen then award **(AI)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.
- Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.
3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given ...				
	Exact or correct to 3 or more sf	<b>Incorrect to 3 sf</b>	Correct to 2 sf <sup>3</sup>	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen <sup>1</sup>	Award the final <b>(AI)</b> irrespective of correct or incorrect rounding				
Unrounded answer not seen <sup>2</sup>	<b>(AI)</b>	<b>(A0)</b>	<b>(AI)</b>	<b>(A0)</b>	<b>(A0)</b>
Treatment of subsequent parts	As per MS	Treat as follow through, only if working is seen <sup>3</sup>			

**Examples:**

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398...) ( <i>AI</i> )	(i) 9.43398... is seen followed by 9; 9.4; 9.43; 9.434 <i>etc.</i> (correctly rounded)	( <i>AI</i> )
	(ii) 9.43398... is seen followed by 9.433; 9.44 <i>etc.</i> (incorrectly rounded)	( <i>AI</i> )
	(iii) 9.4	( <i>AI</i> )
	(iv) 9	( <i>A0</i> ) (correct to 1 sf)
	(v) 9.3	( <i>A0</i> ) (incorrectly rounded to 2 sf)
	(vi) 9.44	( <i>A0</i> ) (incorrectly rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798...) ( <i>AI</i> )	(i) 7.43798... is seen followed by 7; 7.4; 7.44; 7.438 <i>etc.</i> (correctly rounded)	( <i>AI</i> )
	(ii) 7.43798... is seen followed by 7.437; 7.43 <i>etc.</i> (incorrectly rounded)	( <i>AI</i> )
	(iii) 7.4	( <i>AI</i> )
	(iv) 7	( <i>A0</i> ) (correct to 1 sf)
	(v) 7.5	( <i>A0</i> ) (incorrectly rounded to 2 sf)
	(vi) 7.43	( <i>A0</i> ) (incorrectly rounded to 3 sf)

**Example:** ABC is a right angled triangle with angle  $ABC = 90^\circ$ ,  $AC = 32$  cm and  $AB = 30$  cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b>  Award <b>(M1)</b> for correct substitution in Pythagoras' formula</p> <p><math>= 11.1 \left( \sqrt{124}, 11.1355... \right) \text{ (cm)} \text{ (A1)}</math></p> <p>(b) <math>\text{Area} = \frac{1}{2} \times 30 \times 11.1355...</math> <b>(M1)</b>  Award <b>(M1)</b> for correct substitution in area of triangle formula</p> <p><math>= 167(167.032...) \text{ (cm}^2\text{)} \text{ (A1)(ft)}</math></p>	<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b>    11 (cm) <b>(A1)</b>  (2 sf answer only seen, but correct)</p> <p>(b) case (i) <math>\text{Area} = \frac{1}{2} \times 30 \times 11</math> <b>(M1)</b>      = 165 (cm<sup>2</sup>) <b>(A1)(ft)</b></p> <p>case (ii) <math>= 165 \text{ (cm}^2\text{)}</math> <b>(M0)(A0)(ft)</b>  (No working shown, the answer 11 is treated as a ft, so no marks awarded here)</p>	

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

e.g. Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	<b>(G2)</b>
7.68 (7.67543...) <b>(A2)</b>	(b) 7.67	<b>(G1)</b>
	(c) 7.6	<b>(G1)</b>
	(d) 8	<b>(G0)</b>
	(e) 7	<b>(G0)</b>
	(e) 7.66	<b>(G0)</b>

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ (A2) $(y = 0.887686...x + 13.4895...)$ If an answer is not in the form of an equation award at most (A1)(A0).	(a) $y = 0.89x + 13$  (b) $y = 0.88x + 13$  (c) $y = 0.88x + 14$  (d) (i) $y = 0.9x + 13$  (ii) $y = 0.8x + 13$  (e) $0.88x + 13$	(G2) <i>(both accepted)</i>  (G1) <i>(one rounding error)</i>  (G1) <i>(rounding error repeated)</i>  (G1) <i>(1 sf not accepted)</i>  (G0) <i>(one rounding error and not an equation)</i>

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ (A1)(A1) $(2.06020..., 4.49253...)$	(a) $(2.1, 4.5)$  (b) $(2.0, 4.4)$  (c) $(2.06, 4.4)$  (d) $(2, 4.4)$	(A1)(A1) <i>(both accepted)</i>  (A1) <i>(same rounding error twice)</i>  (A1) <i>(one rounding error)</i>  (A0) <i>(1sf not accepted, one rounding error)</i>

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**.

Exact answers such as  $\frac{1}{4}$  can be written as decimals to fewer than three significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (*e.g.*  $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ ,) may be accepted as exact answers. All other powers (*e.g.* of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a (ft) mark regardless of an immediately preceding (M0).

## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:** A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) <b>(A1)</b>	(i) 231.6	<b>(A0)</b>
	(ii) 232	<b>(A0)</b> (Correct rounding to incorrect level)
	(iii) 231.61	<b>(A0)</b>
	(iv) 232.00	<b>(A0)</b> (Parts (iii) and (iv) are both incorrect rounding to correct level)

## 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

**Example:**

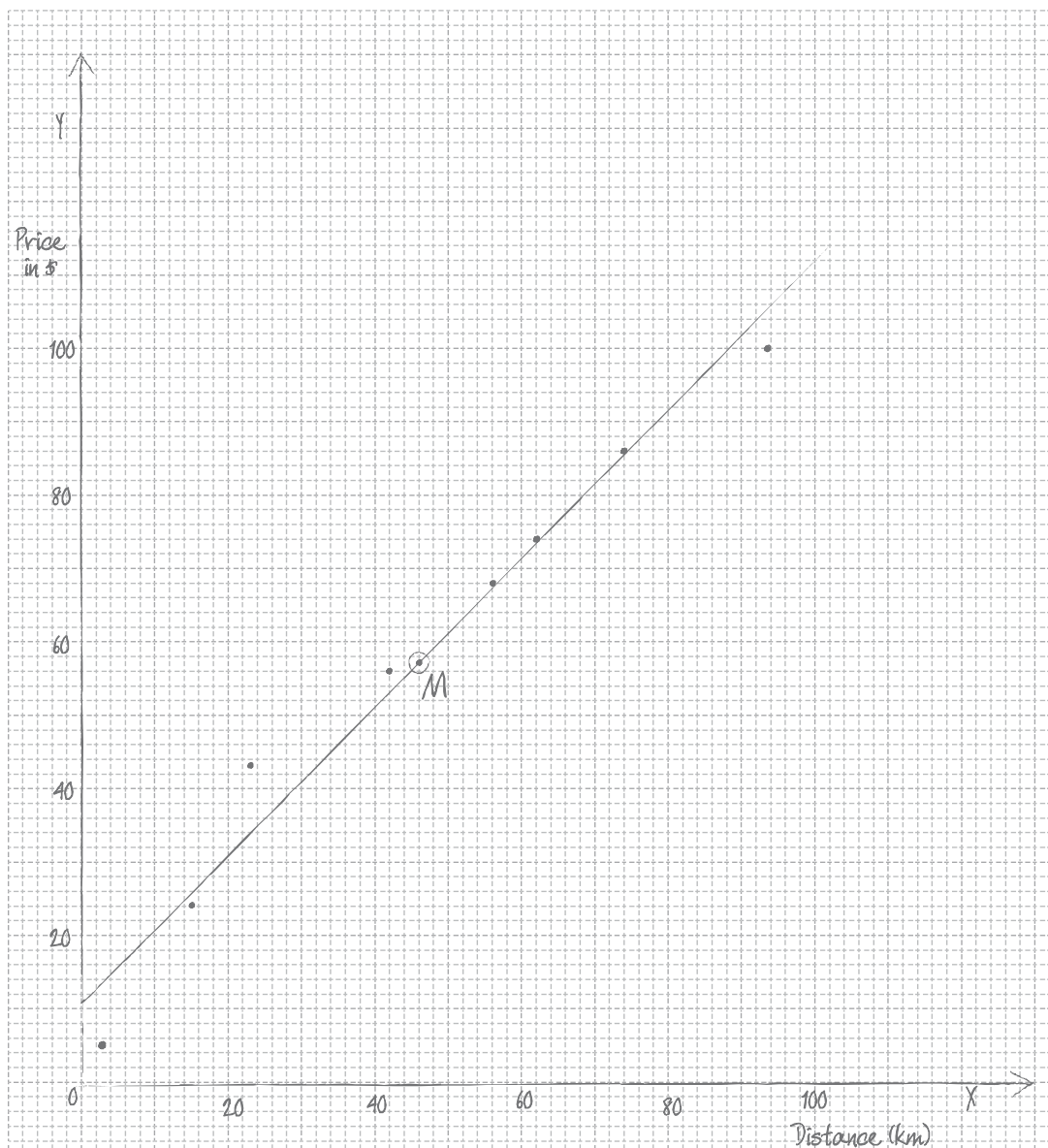
Markscheme	Candidates' Scripts	Marking
(a) 37000 m <sup>2</sup> <b>(A1)</b>	(a) 36000 m <sup>2</sup>	<b>(A0)</b> (Incorrect answer so units not considered)
(b) 3200 m <sup>3</sup> <b>(A1)</b>	(b) 3200 m <sup>2</sup>	<b>(A0)</b> (Incorrect units)

**If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.**

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a)



(A4) [4 marks]

**Notes:** Award (A1) for correct scale and labels (accept  $x$  and  $y$ ).

Award (A3) for 7 or 8 points plotted correctly.

Award (A2) for 5 or 6 points plotted correctly.

Award (A1) for 3 or 4 points plotted correctly.

Award at most (A1)(A2) if points are joined up.

If axes are reversed, award at most (A0)(A3).

If graph paper is not used, award at most (A1)(A0).

(b) (i)  $(\bar{x} =) 46$

(G1)

(ii)  $(\bar{y} =) 57$

(G1) [2 marks]

(c) M(46, 57) plotted and labelled on the scatter diagram

(A1)(ft) [1 mark]

**Notes:** Follow through from their part (b).  
Accept  $(\bar{x}, \bar{y})$  as the label.

continued...



*Question 1 continued*

(d) (i) 0.986 (0.986322...) (G1)

(ii)  $y = 1.01x + 10.3$  ( $y = 1.01431...x + 10.3412...$ ) (G1)(G1)

**Notes:** Award (G1) for  $1.01x$ , (G1) for  $10.3$ .  
Award (G1)(G0) if not written in the form of an equation.

**OR**

$(y - 57) = 1.01(x - 46)$  ( $y - 57 = 1.01431...(x - 46)$ ) (G1)(G1)(ft) [3 marks]

**Note:** Award (G1) for  $1.01$ , (G1) for their  $57$  and  $46$ .

(e) straight line drawn on the scatter diagram (A1)(ft)(A1)(ft) [2 marks]

**Notes:** The line must be straight for either of the two marks to be awarded.

Award (A1)(ft) passing through their M plotted in (c).  
Award (A1)(ft) for correct y-intercept (between 9 and 12).

Follow through from their y-intercept found in part (d).  
If part (d) is used, award (A1)(ft) for their intercept ( $\pm 1$ ).

(f)  $y = 1.01431... \times 76 + 10.3412...$  (M1)

**Note:** Award (M1) for substitution of  $76$  into their regression line.

$= 87.4295...$  (A1)(ft)

**Note:** Follow through from part (d). If 3 sf values are used the value is  $87.06$ .

\$87 (A1)(ft)(G2) [3 marks]

**Notes:** The final (A1) is awarded for their answer given correct to the nearest dollar.  
Method, followed by the answer of  $87$  earns (M1)(G2). It is not necessary to see the interim step.  
Where the candidate uses their graph instead of the equation, and arrives at an answer other than  $87$ , award, at most, (G1)(ft).  
If the candidate uses their graph and arrives at the required answer of  $87$ , award (G2)(ft).

*continued...*

*Question 1 continued*

- (g) 76 is within the range of distances given in the data **OR** the correlation coefficient is close to 1.

**(R1)** [1 mark]

**Notes:** Award **(R1)** if **either** condition is given.  
 Sufficient to indicate that 76 is ‘within the data range’ and the correlation is ‘strong’.  
 Allow  $r^2$  close to 1.  
 Do **not** accept “within the range of prices”.

- (h) Percentage error =  $\frac{87-80}{80} \times 100$

**(M1)**

**Note:** Award **(M1)** for correct substitution into formula.

8.75%

**(A1)(ft)(G2)** [2 marks]

**Notes:** Follow through from their answer to part (f).  
 Accept either the rounded or unrounded answer to part (f).  
 If no integer value seen in part (f), follow through from their unrounded answer to part (f).  
 Answer must be positive.

**Total: [18 marks]**

2. *Units are required in part (b).*

(a)  $AC^2 = 30^2 + 24^2 - 2 \times 30 \times 24 \times \cos 35^\circ$  (M1)(A1)

**Note:** Award (M1) for substituted cosine rule formula, (A1) for correct substitutions.

$AC = 17.2 \text{ cm} \quad (17.2168\dots)$  (A1)(G2) [3 marks]

**Notes:** Use of radians gives 52.7002... Award (M1)(A1)(A0).  
No marks awarded in this part of the question where candidates assume that angle  $ACB = 90^\circ$ .

(b) Area of triangle  $ABC = \frac{1}{2} \times 24 \times 30 \times \sin 35^\circ$  (M1)(A1)

**Notes:** Award (M1) for substitution into area formula, (A1) for correct substitutions.  
**Special Case:** Where a candidate has assumed that angle  $ACB = 90^\circ$  in part (a), award (M1)(A1) for a correct alternative substituted formula for the area of the triangle  $\left( ie \frac{1}{2} \times \text{base} \times \text{height} \right)$ .

$= 206 \text{ cm}^2 \quad (206.487\dots \text{cm}^2)$  (A1)(G2) [3 marks]

**Notes:** Use of radians gives negative answer,  $-154.145\dots$  Award (M1)(A1)(A0).  
**Special Case:** Award (A1)(ft) where the candidate has arrived at an area which is correct to the standard rounding rules from their lengths (units required).

(c)  $206.487\dots \times 25 \times 2600$  (M1)

**Note:** Award (M1) for multiplication of their answer to part (b) by 25 and 2600.

13 421 688.61 (A1)

**Note:** Accept unrounded answer of 13 390 000 for use of 206.

13 400 000 (AG) [2 marks]

**Note:** The final (A1) cannot be awarded unless both the unrounded and rounded answers are seen.

*continued...*

*Question 2 continued*

(d)  $1.34 \times 10^7$

(A2) [2 marks]

**Notes:** Award (A2) for the correct answer.  
Award (A1)(A0) for 1.34 and an incorrect index value.  
Award (A0)(A0) for any other combination (including answers such as  $13.4 \times 10^6$ ).

(e)  $2 \times 206.487 \dots + 24 \times 25 + 30 \times 25 + 17.2168 \dots \times 25$

(M1)(M1)

**Note:** Award (M1) for multiplication of their answer to part (b) by 2 for area of two triangular ends, (M1) for three correct rectangle areas using 24, 30 and their 17.2.

2193.26...

(A1)

**Note:** Accept 2192 for use of 3 sf answers.

2190

(AG) [3 marks]

**Note:** The final (A1) cannot be awarded unless both the unrounded and rounded answers are seen.

(f)  $\frac{2190 \times 2600}{22 \times 10000}$

(M1)(M1)

**Notes:** Award (M1) for multiplication by 2600 and division by 22, (M1) for division by 10000.  
The use of 22 may be implied *ie* division by 2200 would be acceptable.

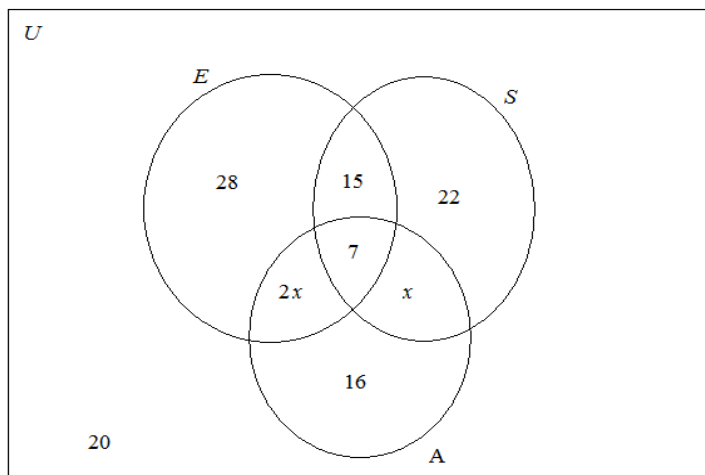
25.9 litres (25.8818...)

(A1)(G2) [3 marks]

**Note:** Accept 26.

**Total: [16 marks]**

3. (a)



(A1)(A1)(A1)(A1)(A1) [5 marks]

**Notes:** Award (A1) for rectangle and three labelled intersecting circles.  
Award (A1) for 7 in correct place.  
Award (A1) for 28, 22 and 16 in the correct places.  
Award (A1) for 15,  $x$  and  $2x$  in the correct places.  
Award (A1) for 20 in the correct place.  
Accept 4 and 8 instead of  $x$  and  $2x$ .  
Do not penalize if  $U$  is omitted from the diagram.

(b)  $3x = 120 - (20 + 28 + 15 + 22 + 7 + 16)$  (M1)

**Note:** Award (M1) for setting up a correct equation involving  $x$ , the 120 and values from their diagram.

$x = 4$  (A1)(ft)(G2) [2 marks]

**Note:** Follow through from part (a). For the follow through to be awarded  $x$  must be a positive integer.

(c) (Women who had visited) Europe **or** South America and (but had) **not** (visited) Asia (A1)(A1) [2 marks]

**Notes:** Award (A1) for “(visited) Europe **or** South America” (or both).  
Award (A1) for “and (but) had **not** visited Asia”.  
 $E$ (urope) union  $S$ (outh America) intersected with not  $A$ (sia) earns no marks, (A0).

(d) 20 (A1) [1 mark]

**Note:** Award (A0) for the embedded answer of  $n(20)$ .

(e)  $\frac{58}{120} \left( \frac{29}{60}, 0.483, 48.3\% \right) (0.48333...)$  (A1)(ft)(A1)(G2) [2 marks]

**Note:** Award (A1)(ft) for numerator, follow through from their value of  $x$ , or their diagram, (A1) for denominator.

continued...

*Question 3 continued*

(f)  $\frac{15}{35} \left( \frac{3}{7}, 0.429, 42.9\% \right) (0.428571\dots)$  (AI)(ft)(AI)(ft)(G2) [2 marks]

**Note:** Award (AI)(ft) for numerator, (AI)(ft) for denominator, follow through from their value of  $x$  or their diagram.

(g)  $\frac{48}{120} \times \frac{47}{119}$  (AI)(ft)(MI)

**Notes:** Award (AI)(ft) for two correct fractions, follow through from their denominator in part (e), follow through the numerator from their answer to part (b) or from their diagram, (MI) for multiplication of their two fractions.

$= \frac{2256}{14280} \left( \frac{94}{595}, 0.158, 15.8\% \right) (0.157983\dots)$  (AI)(ft)(G2) [3 marks]

**Notes:** Award (AI)(MI)(AI) for correct fractions, correctly multiplied together with an answer of 0.16.

Award (A0)(MI)(A0) for  $\frac{48}{120} \times \frac{48}{120} = 0.16$ .

Award (GI) for an answer of 0.16 with no working seen.

**Total: [17 marks]**

4. (a)  $\frac{3}{4}(-2)^4 - (-2)^3 - 9(-2)^2 + 20$  (M1)

**Note:** Award (M1) for substituting  $x = -2$  in the function.

$= 4$  (A1)(G2) [2 marks]

**Note:** If the coordinates  $(-2, 4)$  are given as the answer award, at most, (M1)(A0). If no working shown award (G1). If  $x = -2$ ,  $y = 4$  seen then award full marks.

(b)  $3x^3 - 3x^2 - 18x$  (A1)(A1)(A1) [3 marks]

**Note:** Award (A1) for each correct term, award at most (A1)(A1)(A0) if extra terms seen.

(c)  $f'(3) = 3 \times (3)^3 - 3 \times (3)^2 - 18 \times 3$  (M1)

**Note:** Award (M1) for substitution in their  $f'(x)$  of  $x = 3$ .

$= 0$  (A1)

**OR**

$3x^3 - 3x^2 - 18x = 0$  (M1)

**Note:** Award (M1) for equating their  $f'(x)$  to zero.

$x = 3$  (A1)

$f'(x_1) = 3 \times (x_1)^3 - 3 \times (x_1)^2 - 18 \times x_1 < 0$  where  $0 < x_1 < 3$  (M1)

**Note:** Award (M1) for substituting a value of  $x_1$  in the range  $0 < x_1 < 3$  into their  $f'$  and showing it is negative (decreasing).

$f'(x_2) = 3 \times (x_2)^3 - 3 \times (x_2)^2 - 18 \times x_2 > 0$  where  $x_2 > 3$  (M1)

**Note:** Award (M1) for substituting a value of  $x_2$  in the range  $x_2 > 3$  into their  $f'$  and showing it is positive (increasing).

continued...

*Question 4 continued*

**OR**

*With or without a sketch:*

Showing  $f'(x_1) > f'(3)$  where  $x_1 < 3$  and  $x_1$  is close to 3. **(M1)**

Showing  $f'(x_2) > f'(3)$  where  $x_2 > 3$  and  $x_2$  is close to 3. **(M1)**

**Note:** If a sketch of  $f(x)$  is drawn **in this part of the question and**  $x = 3$  is identified as a stationary point on the curve, then  
 (i) award, at most, **(M1)(A1)(M1)(M0)** if the stationary point has been found;  
 (ii) award, at most, **(M0)(A0)(M1)(M0)** if the stationary point has not been previously found.

Since the gradients go from negative (decreasing) through zero to positive (increasing) it is a local minimum **(R1)(AG)**

**Note:** Only award **(R1)** if the first two marks have been awarded ie  $f'(3)$  has been shown to be equal to 0.

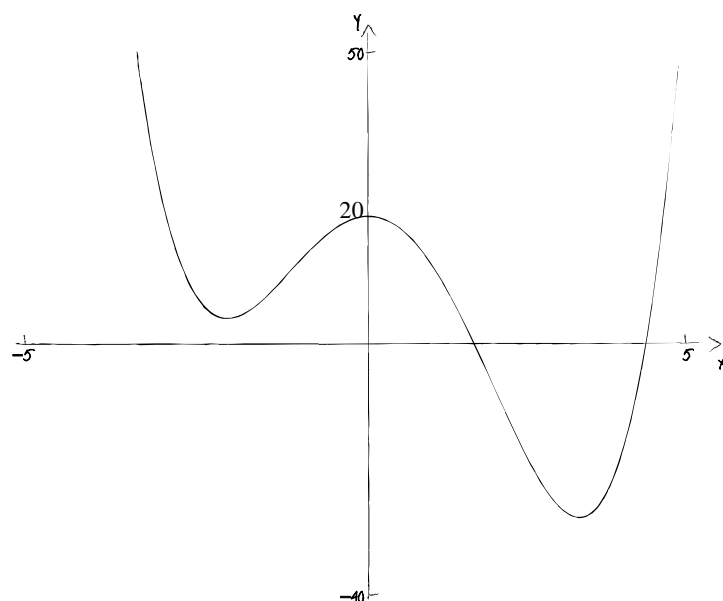
**[5 marks]**

*continued...*



Question 4 continued

(d)



(A1)(A1)(A1)(A1)

**Notes:** Award (A1) for labelled axes and indication of scale on both axes.  
Award (A1) for smooth curve with correct shape.  
Award (A1) for local minima in 2<sup>nd</sup> and 4<sup>th</sup> quadrants.  
Award (A1) for y intercept (0, 20) seen and labelled. Accept 20 on y-axis.  
Do **not** award the third (A1) mark if there is a turning point on the x-axis.  
If the derivative function is sketched then award, at most, (A1)(A0)(A0)(A0).  
For a smooth curve (with correct shape) there should be **ONE** continuous thin line, no part of which is straight and no (one to many) mappings of x.

[4 marks]

(e) (0, 20)

(G1)(G1)

**Note:** If parentheses are omitted award (G0)(G1).

OR

$$x = 0, y = 20$$

(G1)(G1) [2 marks]

**Note:** If the derivative function is sketched in part (d), award (G1)(ft)(G1)(ft) for (-1.12, 12.2).

(f)  $f'(2) = 3(2)^3 - 3(2)^2 - 18(2)$

(M1)

**Notes:** Award (M1) for substituting  $x = 2$  into their  $f'(x)$ .

$$= -24$$

(A1)(ft)(G2) [2 marks]

continued...

Question 4 continued

(g) (i) Gradient of perpendicular  $= \frac{1}{24}$  (0.0417, 0.041666...) (AI)(ft)(GI)

**Note:** Follow through from part (f).

(ii)  $y + 12 = \frac{1}{24}(x - 2)$  (MI)(MI)

**Note:** Award (MI) for correct substitution of (2, –12), (MI) for correct substitution of their perpendicular gradient into equation of line.

OR

$$-12 = \frac{1}{24} \times 2 + d \quad (MI)$$

$$d = -\frac{145}{12}$$

$$y = \frac{1}{24}x - \frac{145}{12} \quad (MI)$$

**Note:** Award (MI) for correct substitution of (2, –12) and gradient into equation of a straight line, (MI) for correct substitution of the perpendicular gradient and correct substitution of  $d$  into equation of line.

$b = -24, c = -290$  (AI)(ft)(AI)(ft)(G3) [5 marks]

**Note:** Follow through from parts (f) and g(i).  
To award (ft) marks,  $b$  and  $c$  must be integers.  
Where candidate has used 0.042 from g(i), award (AI)(ft) for –288.

**Total: [23 marks]**

5. *The first answer not given to two decimal places is not awarded the final (A1). Incorrect rounding is not penalized thereafter.*

(a)  $37\,500 \times 0.7234$  (M1)  
 $= 27\,127.50$  (A1)(G2) [2 marks]

(b)  $6947.50$  (A1)(ft)(G1) [1 mark]

**Note:** Follow through from part (a) irrespective of whether working is seen.

(c)  $\frac{6947.50 \times 4.5 \times 4}{100} + 6947.50$  (M1)(M1)

**Note:** Award (M1) for their correctly substituted simple interest formula, (M1) for addition of their part (b).

$= 8198.05$  (A1)(ft)(G2) [3 marks]

**Note:** Follow through from part (b).

(d)  $27\,127.50 \times 0.91$  (A1)(M1)

**Note:** Award (A1) for 0.91 seen or equivalent, (M1) for their 27 127.50 multiplied by 0.91

**OR**

$27\,127.50 - 0.09 \times 27\,127.50$  (A1)(M1)

**Note:** Award (A1) for  $0.09 \times 27\,127.50$  seen, and (M1) for  $27\,127.50 - 0.09 \times 27\,127.50$ .

$= 24\,686.03$  (A1)(ft)(G2) [3 marks]

**Note:** Follow through from part (a).

*continued...*

Question 5 continued

(e)  $27127.50 \times \left(1 - \frac{9}{100}\right)^4$  (MI)(AI)(ft)

**Notes:** Award (MI) for substituted compound interest formula, (AI)(ft) for correct substitution.  
Follow through from part (a).

**OR**

$27127.50 \times (0.91)^4$  (MI)(AI)(ft)

**Notes:** Award (MI) for substituted geometric sequence formula, (AI)(ft) for correct substitution.  
Follow through from part (a).

**OR** (lists (i))

24686.03, 22464.28..., 20442.50..., 18602.67... (MI)(AI)(ft)

**Notes:** Award (MI) for at least the 2<sup>nd</sup> term correct (calculated from their (a)  $\times 0.91$ ). Award (AI)(ft) for four correct terms (rounded or unrounded).  
Follow through from part (a).  
Accept list containing the last three terms only (24686.03 may be implied).

**OR** (lists(ii))

$27127.50 - (2441.47... + 2221.74... + 2021.79... + 1839.82...)$  (MI)(AI)(ft)

**Notes:** Award (MI) for subtraction of four terms from 27127.50.  
Award (AI) for four correct terms (rounded or unrounded).  
Follow through from part (a).

$= 18602.67$   
 $= 18600$

(AI)  
(AG) [3 marks]

**Note:** The final (AI) is not awarded unless both the unrounded and rounded answers are seen.

continued...

Question 5 continued

$$(f) \quad \frac{18600 + 8198.05}{0.8694} - 30500 \quad (M1)(M1)(M1)$$

**Note:** Award **(M1)** for their answer to part (c) added to 18 600, **(M1)** for  $\frac{18600 + (\text{any value})}{0.8694}$ , **(M1)** for the difference between  $\frac{18600 + (\text{any value})}{0.8694}$  and 30 500.

**OR**

$$\frac{18600 + 8198.05 - 30500 \times 0.8694}{0.8694} \quad (M1)(M1)(M1)$$

**Note:** Award **(M1)** for their answer to part (c) added to 18 600, **(M1)** for difference between (their answer to part (c) added to 18 600) and  $(30500 \times 0.8694)$ , **(M1)** for dividing the resultant value by 0.8694.  
If the value for the exchange rate used is 0.7234, then award, at most, **(M1)(M0)(M1)**.

$$= 323.61 \quad (A1)(ft)(G3) \quad [4 \text{ marks}]$$

**Note:** Follow through from their part (c).  
Award **(A1)(ft)** for final answer provided it is positive, and dependant on all three method marks.

**Total: [16 marks]**



# **MARKSCHEME**

**November 2014**

**MATHEMATICAL STUDIES**

**Standard Level**

**Paper 2**

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**Paper 2 Markscheme**  
**Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

**1 Abbreviations**

**M** Marks awarded for **Method**

**A** Marks awarded for an **Answer** or for **Accuracy**

**R** Marks awarded for clear **Reasoning**

**G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

**AG** **Answer Given** in the question and consequently, marks not awarded.

**ft** Marks that can be awarded as **follow through** from previous results in the question.

**2 Method of Marking**

- (a) All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- (b) If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the **A0** annotation, otherwise full annotations must be shown.
- (c) Working crossed out by the candidate should not be awarded any marks.
- (d) Where candidates have written two solutions to a question, only the first solution should be marked.
- (e) If correct working results in a correct answer but then further working is developed, full marks may **not** always be awarded. Full marks **will** be awarded if the candidate shows correct working leading to the correct answer. See also section 4(c).

**Example:** Calculate the gradient of the line passing through the points (5, 3) and (0, 9) .

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5}$ ( <b>M1</b> ) Award ( <b>M1</b> ) for correct substitution in gradient formula  $= -\frac{6}{5}$ ( <b>A1</b> )	(i) $\frac{9-3}{0-5} = -\frac{6}{5}$ Gradient is $= -\frac{6}{5}$ (There is clear understanding of the gradient.) $y = -\frac{6}{5}x + 9$	( <b>M1</b> )  ( <b>A1</b> )
	(ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ $y = -\frac{6}{5}x + 9$ (There is confusion about what is required.)	( <b>M1</b> )  ( <b>A0</b> )



### 3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Mark schemes will indicate where it is appropriate to apply follow through in a question with **‘(ft)’**.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (*eg*, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate’s use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies**.
- (f) Inadvertent use of radians will be penalised the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

Markscheme	Candidates’ Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> (MI)(AI)</p> <p>Award (MI) for substitution in sine rule formula, (AI) for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) (AI)(G2)</p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math></p> <p><math>A = 41.8^\circ</math></p> <p>(Note: the 2<sup>nd</sup> (AI) here was not marked (ft) and cannot be awarded because there was an earlier error in the <b>same</b> question part.)</p>	<p>(MI)(A0)</p> <p>(use of sine rule but with wrong values)</p> <p>(A0)</p>
<p>(b) <math>x = 7 \tan (22.0243\dots)</math> (MI)</p> <p><math>= 2.83</math> (2.83163...) (AI)(ft)</p>	<p>(b) case (i) <math>x = 7 \tan 41.8^\circ</math></p> <p><math>= 6.26</math></p> <p><b>but</b> case (ii) <math>6.26</math></p>	<p>(MI)</p> <p>(AI)(ft)</p> <p>(G0)</p> <p>since no working shown</p>

#### 4 Using the Markscheme

- (a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.  
The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.
- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

**Example:** Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b></p> <p><i>Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</i></p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(i) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math></p> <p><math>A = 22.0^\circ</math></p> <p>(ii) <math>A = 22.0^\circ</math> <b>(G2)</b></p> <p><i><b>Note:</b> <b>G</b> marks are used only if no working has been shown and the answer is correct.</i></p>	<p><b>(M1)(A1)</b></p> <p><b>(A1)</b></p>

- (d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.  
Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' etc.
- (e) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .  
On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.  
Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:  
the 3 significant figure answer worked through from full calculator display;  
the exact value  $\left( \text{for example } \frac{2}{3} \text{ if applicable} \right)$ ;  
the full calculator display in the form 2.83163... as in the example above.  
Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

- (f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval:  $3 < x < 5$ ;  $(3, 5)$ ;  $] 3, 5 [$  .

Different forms of notation for set properties (eg, complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A; U \setminus A$ .

Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ .

$p \Rightarrow q$ ;  $p \rightarrow q$ ;  $q \Leftarrow p$  .

Significance level may be written as  $\alpha$  .

- (g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5 Accuracy of Answers

**Incorrect accuracy should be penalized once only in each question according to the rules below.**

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award **(AI)** and ignore subsequent rounding.
  2. If the candidate's unrounded answer is **not** seen then award **(AI)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.
- Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.
3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples for

	If candidates final answer is given ...					
	Exact or to 4 or more sf (and would <b>round to the correct 3 sf</b> )	<b>Correct to 3 sf</b>	<b>Incorrect to 3 sf</b>	Correct to 2 sf <sup>3</sup>	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen <sup>1</sup>	Award the final <b>(AI)</b> irrespective of correct or incorrect rounding					
Unrounded answer not seen <sup>2</sup>	<b>(AI)</b>	<b>(AI)</b>	<b>(A0)</b>	<b>(AI)</b>	<b>(A0)</b>	<b>(A0)</b>
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen. <sup>3</sup>			

**Examples:**

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398...) ( <i>AI</i> )	(i) 9.43398... is seen followed by 9; 9.4; 9.43; 9.434 <i>etc.</i> (correctly rounded)	( <i>AI</i> )
	(ii) 9.43398... is seen followed by 9.433; 9.44 <i>etc.</i> (incorrectly rounded)	( <i>AI</i> )
	(iii) 9.4	( <i>AI</i> )
	(iv) 9	( <i>A0</i> ) (correct to 1 sf)
	(v) 9.3	( <i>A0</i> ) (incorrectly rounded to 2 sf)
	(vi) 9.44	( <i>A0</i> ) (incorrectly rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798...) ( <i>AI</i> )	(i) 7.43798... is seen followed by 7; 7.4; 7.44; 7.438 <i>etc.</i> (correctly rounded)	( <i>AI</i> )
	(ii) 7.43798... is seen followed by 7.437; 7.43 <i>etc.</i> (incorrectly rounded)	( <i>AI</i> )
	(iii) 7.4	( <i>AI</i> )
	(iv) 7	( <i>A0</i> ) (correct to 1 sf)
	(v) 7.5	( <i>A0</i> ) (incorrectly rounded to 2 sf)
	(vi) 7.43	( <i>A0</i> ) (incorrectly rounded to 3 sf)

**Example:** ABC is a right angled triangle with angle  $ABC = 90^\circ$ ,  $AC = 32$  cm and  $AB = 30$  cm . Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b>  <i>Award (M1) for correct substitution in Pythagoras' formula</i></p> <p><math>= 11.1 \left( \sqrt{124}, 11.1355... \right) (\text{cm})</math> <b>(A1)</b></p>	<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b></p> <p>11 (cm) <b>(A1)</b>  <i>(2 sf answer only seen, but correct)</i></p>	
<p>(b) <math>\text{Area} = \frac{1}{2} \times 30 \times 11.1355...</math> <b>(M1)</b>  <i>Award (M1) for correct substitution in area of triangle formula</i></p> <p><math>= 167(167.032...) (\text{cm}^2)</math> <b>(A1)(ft)</b></p>	<p>(b) case (i) <math>\text{Area} = \frac{1}{2} \times 30 \times 11</math> <b>(M1)</b>  <i>(working shown)</i></p> <p><math>= 165 (\text{cm}^2)</math> <b>(A1)(ft)</b></p> <p>case (ii) <math>= 165 (\text{cm}^2)</math> <b>(M0)(A0)(ft)</b>  <i>(No working shown, the answer 11 is treated as a ft, so no marks awarded here)</i></p>	

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	(G2)
7.68 (7.67543...) (A2)	(b) 7.67	(G1)
	(c) 7.6	(G1)
	(d) 8	(G0)
	(e) 7	(G0)
	(e) 7.66	(G0)

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ (A2) $(y = 0.887686...x + 13.4895...)$ If an answer is not in the form of an equation award at most (A1)(A0).	(a) $y = 0.89x + 13$  (b) $y = 0.88x + 13$  (c) $y = 0.88x + 14$  (d) (i) $y = 0.9x + 13$  (ii) $y = 0.8x + 13$  (e) $0.88x + 13$ (G0)	(G2) <i>(both accepted)</i>  (G1) <i>(one rounding error)</i>  (G1) <i>(rounding error repeated)</i>  (G1) <i>(1 sf not accepted)</i>  <i>(one rounding error and not an equation)</i>

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ (A1)(A1) $(2.06020..., 4.49253...)$	(a) $(2.1, 4.5)$  (b) $(2.0, 4.4)$  (c) $(2.06, 4.4)$  (d) $(2, 4.4)$	(A1)(A1) <i>(both accepted)</i>  (A1) <i>(same rounding error twice)</i>  (A1) <i>(one rounding error)</i>  (A0) <i>(1sf not accepted, one rounding error)</i>

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**.

Exact answers such as  $\frac{1}{4}$  can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (eg,  $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ , ) may be accepted as exact answers. All other powers (eg, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:** A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) <b>(AI)</b>	(i) 231.6	<b>(A0)</b>
	(ii) 232	<b>(A0)</b> <i>(Correct rounding to incorrect level)</i>
	(iii) 231.61	<b>(A0)</b>
	(iv) 232.00	<b>(A0)</b> <i>(Parts (iii) and (iv) are both incorrect rounding to correct level)</i>

## 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded **(AI)** under the accuracy rules given in Section 5.

**Example:**

Markscheme	Candidates' Scripts	Marking
(a) 37000 m <sup>2</sup> <b>(AI)</b>	(a) 36000 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect answer so units not considered)</i>
(b) 3200 m <sup>3</sup> <b>(AI)</b>	(b) 3200 m <sup>2</sup>	<b>(A0)</b> <i>(Incorrect units)</i>

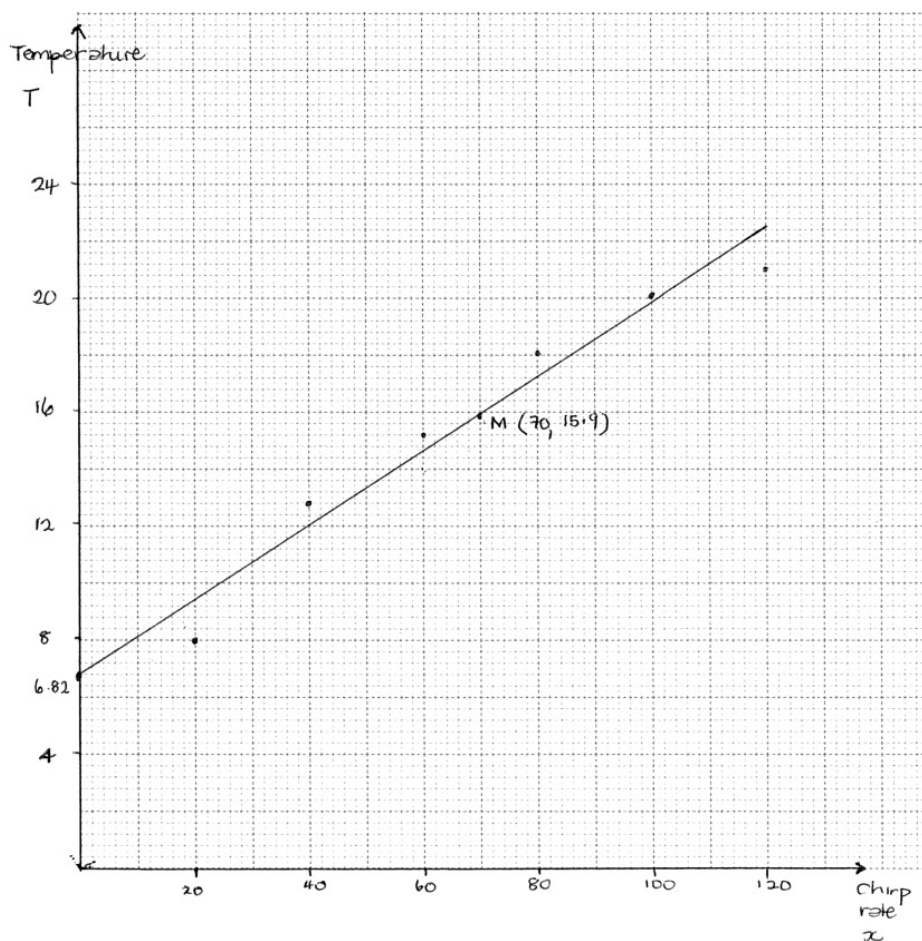
**If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.**

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.



1. (a)



(A4) [4 marks]

**Notes:** Award (A1) for correct scales and labels.  
Award (A3) for all six points correctly plotted,  
(A2) for four or five points correctly plotted,  
(A1) for two or three points correctly plotted.  
Award at most (A0)(A3) if axes reversed.  
Accept tolerance  $\pm 0.2$  for  $T$ -axis

(b) 0.977 (0.977324...)

(G2) [2 marks]

**Notes:** Award (G1) for 0.97.

(c) (Very) strong positive correlation

(A1)(ft)(A1)(ft) [2 marks]

**Notes:** Award (A1) for (very) strong, (A1) for positive.  
Follow through from part (b).

continued ...

*Question 1 continued*

- (d)  $T = 0.129x + 6.82$  (G2) [2 marks]

**Notes:** Award (G1) for  $0.129x$ , (G1) for  $+6.82$ .  
Award a maximum of (G0)(G1) if the answer is not an equation.

- (e)  $0.129 \times 70 + 6.82$  (M1)

**Note:** Award (M1) for substitution of 70 into their equation of regression line.

**OR**

$$\frac{8 + 12.8 + \dots + 21.1}{6}$$

$$= 15.9 \text{ (15.85)}$$

(M1)

(A1)(ft)(G2) [2 marks]

**Note:** Follow through from part (d) without working.

- (f) regression line through (70, 15.9) (A1)(ft)

**Note:** Accept  $15.9 \pm 0.2$ .  
Follow through from part (e).

with  $T$ -intercept, 6.82

(A1)(ft) [2 marks]

**Note:** Follow through from part (d). Accept  $6.82 \pm 0.2$ .  
In case the regression line is not straight (ruler not used), award (A0)(A1)(ft) if line passes through both their (70, 15.9) and (0, 6.82), otherwise award (A0)(A0).  
Do not penalize if line does not intersect the  $T$ -axis.

- (g)  $T = 0.45z + 10$  (A1) [1 mark]

*continued ...*

*Question 1 continued*

(h) (i)  $0.45(20) + 10$  (M1)

**Note:** Award (M1) for correct substitution of 20 into their formula from part (g).

$= 19 \text{ (}^{\circ}\text{C)}$  (A1)(ft)(G2)

**Note:** Follow through from part (g).

(ii)  $18.2 \text{ (}^{\circ}\text{C)}$  (A1)

(iii)  $\left| \frac{19 - 18.2}{18.2} \right| \times 100\%$  (M1)(A1)(ft)

**Note:** Award (M1) for substitution in the percentage error formula, (A1) for correct substitution.

$4.40\% \text{ (4.39560...)}$  (A1)(ft)(G2) [6 marks]

**Notes:** Follow through from parts (h)(i) and (h)(ii).

**Total: [21 marks]**

2. (a) (i)  $\frac{1}{3}$  (0.333333..., 33.3333...%) (AI)
- (ii)  $\frac{1}{2}$  (0.5, 50%) (AI)
- (iii)  $\frac{1}{4}$  (0.25, 25%) (AI) [3 marks]

- (b) (i)  $\frac{1}{3} \times \frac{1}{4}$  (M1)
- $= \frac{1}{12}$  (0.0833333..., 8.33333...%) (AI)(G2)
- (ii)  $\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{4}$  (AI)(ft)(M1)

**Note:** Award (AI)(ft) for their three correct products seen, (M1) for addition of their products.

$$= \frac{1}{3} \text{ (0.333333..., 33.3333...%)} \quad (AI)(ft)(G2)$$

**Note:** Follow through from their parts (a)(i) and (a)(iii).

(iii)  $1 - \frac{1}{12} - \frac{1}{3}$  (M1)

**Note:** Follow through from parts (b)(i) and (b)(ii).

**OR**

$$\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{4} \quad (M1)$$

**Note:** Follow through from parts (a)(i) and (a)(ii).

$$= \frac{7}{12} \text{ (0.583333..., 58.3333...%)} \quad (AI)(ft)(G2) \quad [7 \text{ marks}]$$

*continued ...*

*Question 2 continued*

- (c) Sonya is not correct.

**(A1)(ft)**

The probability that Mike escapes is  $\frac{7}{12}$ , which is

greater than  $\frac{5}{12}$  (or greater than  $\frac{1}{2}$ ).

**(R1)(ft)**

**[2 marks]**

**Notes:** Do not award **(A1)(R0)**.

Follow through from their answers to part (b).

- (d)  $\frac{\frac{1}{3}}{\frac{7}{12}}$

**(A1)(A1)(ft)**

**Note:** Award **(A1)** for correct numerator, **(A1)** for correct denominator.

$$= \frac{4}{7} \left( \frac{12}{21}, 0.571428..., 57.1428...\% \right)$$

**(A1)(ft)(G2)**

**[3 marks]**

**Note:** Follow through from their answer to part (b)(iii).

**Total: [15 marks]**

3. (a)  $\frac{\pi l^2}{2} = 39.27$  (M1)(A1)

**Note:** Award (M1) for equating the formula for area of a semicircle to 39.27, award (A1) for correct substitution of  $l$  into the formula for area of a semicircle.

$l = 5$  (m) (AG) [2 marks]

(b) (i)  $5 \times \pi$  (M1)  
 $= 15.7$  (15.7079... ,  $5\pi$ ) (m) (A1)(G2)

(ii)  $2\pi r = 15.7079...$  OR  $5\pi r = 39.27$  (M1)  
 $(r =) 2.5$  (m) (A1)(ft)(G2)

**Note:** Follow through from part (b)(i).

(iii)  $(h^2 =) 5^2 - 2.5^2$  (M1)

**Notes:** Award (M1) for correct substitution into Pythagoras' theorem.  
 Follow through from part (b)(ii).

$(h =) 4.33$  (4.33012...) (m) (A1)(ft)(G2) [6 marks]

(c)  $9.33 - 2 \times r$  (A1) [1 mark]

(d)  $V = \frac{\pi r^2}{3} \times (9.33 - 2r)$  (M1)

**Note:** Award (M1) for correct substitution in the volume formula.

$V = 3.11\pi r^2 - \frac{2}{3}\pi r^3$  (AG) [1 mark]

(e)  $6.22\pi r - 2\pi r^2$  (A1)(A1) [2 marks]

**Notes:** Award (A1) for  $6.22\pi r$ , (A1) for  $-2\pi r^2$ .  
 If extra terms present, award at most (A1)(A0).

continued ...

*Question 3 continued*

(f) (i)  $6.22\pi r - 2\pi r^2 = 0$  (M1)

**Note:** Award (M1) for setting their derivative from part (e) to 0.

$r = 3.11 \text{ (m)}$  (A1)(ft)(G2)

**Notes:** Award (A1) for identifying 3.11 as the answer.  
Follow through from their answer to part (e).

(ii)  $\frac{1}{3}\pi(3.11)^3$  OR  $3.11\pi(3.11)^2 - \frac{2}{3}\pi(3.11)^3$  (M1)

**Note:** Award (M1) for correct substitution into the correct volume formula

$31.5 \text{ (m}^3\text{)} \text{ (31.4999...)}$  (A1)(ft)(G2) [4 marks]

**Note:** Follow through from their answer to part (f)(i).

**Total: [16 marks]**

4. (a) (i)  $\cos \hat{A}CB = \frac{10^2 + 12^2 - 15^2}{2 \times 10 \times 12}$  (M1)(A1)

**Note:** Award (M1) for substituted cosine rule, (A1) for correct substitution.

$\hat{A}CB = 85.5^\circ$  (85.4593...) (A1)(G2)

(ii)  $\hat{D}CE = \hat{A}CB$  and  $\hat{A}CB = 85.5^\circ$  (85.4593...) (A1)

**OR**

$\hat{B}CE = 180^\circ - 85.5^\circ = 94.5^\circ$  and  $\hat{D}CE = 180^\circ - 94.5^\circ = 85.5^\circ$  (A1)

**Notes:** Both reasons must be seen for the (A1) to be awarded.

$\hat{D}CE = 85.5^\circ$  (AG) [4 marks]

(b) (i)  $\hat{D}EC = \frac{180^\circ - 85.5^\circ}{3}$  (M1)

$\hat{D}EC = 31.5^\circ$  (A1)(G2)

(ii)  $\frac{\sin(31.5^\circ)}{9} = \frac{\sin(85.5^\circ)}{DE}$  (M1)(A1)(ft)

**Note:** Award (M1) for substituted sine rule, (A1) for correct substitution.

$DE = 17.2$  (km) (17.1718...) (A1)(ft)(G2) [5 marks]

*continued ...*



*Question 4 continued*

(c)  $0.5 \times 17.1718... \times 9 \times \sin(63^\circ)$  (A1)(ft)(M1)(A1)(ft)

**Note:** Award (A1)(ft) for 63 seen, (M1) for substituted triangle area formula, (A1)(ft) for  $0.5 \times 17.1718... \times 9 \times \sin(\text{their angle CDE})$ .

**OR**

(triangle height =)  $9 \times \sin(63^\circ)$  (A1)(ft)(A1)(ft)  
 $0.5 \times 17.1718... \times 9 \times \sin(\text{their angle CDE})$  (M1)

**Note:** Award (A1)(ft) for 63 seen, (A1)(ft) for correct triangle height with their angle CDE, (M1) for  $0.5 \times 17.1718... \times 9 \times \sin(\text{their angle CDE})$ .

$= 68.9 \text{ km}^2$  (68.8509...) (A1)(ft)(G3) [4 marks]

**Notes:** Units are required for the last (A1)(ft) mark to be awarded.  
 Follow through from parts (b)(i) and (b)(ii).  
 Follow through from their angle CDE **within this part of the question.**

**Total: [13 marks]**

5. (a) 6 (m) (A1)(G1) [1 mark]

(b) (i) 8 (A1)(ft)

(ii) 10 (A1)(ft)(G2) [2 marks]

**Note:** Follow through from part (a).

(c) 2 (m) (A1)(ft) [1 mark]

**Note:** Follow through from parts (a) and (b).

(d) (i)  $2 \times 24 = 6 + 2(n - 1)$  **OR**  $24 = 3 + (n - 1)$  (M1)

**Note:** Award (M1) for correct substitution in arithmetic sequence formula.

$n = 22$  (A1)(ft)(G1)

**Note:** Follow through from parts (a) and (c).

(ii)  $\frac{(6 + 48)}{2} \times 22$  (M1)(A1)(ft)

**Note:** Award (M1) for substitution in arithmetic series formula, (A1)(ft) for correct substitution.

$= 594$  (A1)(ft)(G2) [5 marks]

**Note:** Follow through from parts (a) and (d)(i).

*continued ...*

*Question 5 continued*

$$(e) \quad \frac{[2 \times 6 + 2(n-1)] \times n}{2} = 940 \quad (M1)(A1)(ft)$$

**Notes:** Award *(M1)* for substitution in arithmetic series formula, *(A1)* for their correct substituted formula equated to 940. Follow through from parts (a) and (c).

$$n^2 + 5n - 940 = 0$$

$$n = 28.2611\dots$$

$$n = 28$$

*(A1)(ft)(G2) [3 marks]*

$$(f) \quad \frac{[2 \times 6 + 2(28-1)] \times 28}{2} \quad (M1)$$

**Notes:** Award *(M1)* for substituting their 28 into the arithmetic series formula.

$$= 16 \text{ (m)}$$

*(A1)(ft)(G2) [2 marks]*

**Total: [14 marks]**

6. (a)  $(0, 4)$  (A1) [1 mark]

**Notes:** Accept  $x = 0, y = 4$ .

- (b) (i)  $(a, 4)$  (A1)(ft)

**Notes:** Follow through from part (a).

- (ii)  $\frac{4}{a}$  (A1)(ft) [2 marks]

**Note:** Follow through from part (b)(i).

*continued ...*

Question 6 continued

(c) (i)  $-\frac{a}{4}$  (AI)(ft)

**Note:** Follow through from part (b)(ii).

(ii)  $y = -\frac{a}{4}x + c$  (MI)

**Note:** Award (MI) for substitution of their gradient from part (c)(i) in the equation.

$$4 = -\frac{a}{4} \times a + c$$

$$c = \frac{1}{4} \times a^2 + 4$$

$$y = -\frac{a}{4}x + \frac{1}{4}a^2 + 4$$
 (AI)

**OR**

$$y - 4 = -\frac{a}{4}(x - a)$$
 (MI)

**Note:** Award (MI) for substitution of their gradient from part (c)(i) in the equation.

$$y = -\frac{ax}{4} + \frac{a^2}{4} + 4$$
 (AI)

$$4y = -ax + a^2 + 16$$

$$4y + ax - a^2 - 16 = 0$$
 (AG) [3 marks]

**Note:** Both the simplified and the not simplified equations must be seen for the final (AI) to be awarded.

continued ...

*Question 6 continued*

(d) (i)  $2a$  (AI)

(ii)  $\frac{4x}{2} = 3 \times 2a$  (MI)

**Note:** Award (MI) for correct equation.

$x = 3a$  (AI)(ft)

**Note:** Follow through from part (d)(i).

**OR**

$0 - 4 = -\frac{a}{4}(x - a)$  (MI)

**Note:** Award (MI) for correct substitution of their gradient and the coordinates of their point into the equation of a line.

$\frac{16}{a} = x - a$   
 $x = a + \frac{16}{a}$  (AI)(ft)

**Note:** Follow through from parts (b)(i) and (c)(i).

**OR**

$4 \times 0 + ax - a^2 - 16 = 0$  (MI)

**Note:** Award (MI) for correct substitution of the coordinates of A(x, 0) into the equation of line AB.

$ax - a^2 - 16 = 0$   
 $x = a + \frac{16}{a}$  OR  $x = \frac{(a^2 + 16)}{a}$  (AI)(GI) [3 marks]

*continued ...*

*Question 6 continued*

(e)  $4(0) + a(3a) - a^2 - 16 = 0$  **(M1)**

**Note:** Award **(M1)** for correct substitution of their  $3a$  from part (d)(ii) into the equation of line AB.

**OR**

$$\frac{1}{2} \left( a + \frac{16}{a} \right) \times 4 = 3 \left( \frac{4a}{2} \right) \quad \text{**(M1)**}$$

**Note:** Award **(M1)** for area of triangle AOB (with their substituted  $a + \frac{16}{a}$  and 4) equated to three times their area of triangle AOB.

$a = 2.83 \left( 2.82842..., 2\sqrt{2}, \sqrt{8} \right)$  **(A1)(ft)(G1)** *[2 marks]*

**Note:** Follow through from parts (d)(i) and (d)(ii).

**Total: [11 marks]**

# **Markscheme**

**November 2015**

**Mathematical Studies**

**Standard level**

**Paper 2**



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**Paper 2 Markscheme**  
**Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

**1 Abbreviations**

**M** Marks awarded for **Method**

**A** Marks awarded for an **Answer** or for **Accuracy**

**R** Marks awarded for clear **Reasoning**

**G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

**AG** **Answer Given** in the question and consequently, marks not awarded.

**ft** Marks that can be awarded as **follow through** from previous results in the question.

**2 Method of Marking**

- All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the **A0** annotation, otherwise full annotations must be shown.
- Working crossed out by the candidate should not be awarded any marks.
- Where candidates have written two solutions to a question, only the first solution should be marked.
- If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

**Example:**

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	Award the final <b>(A1)</b> (ignore the further working)
2.	$(x-6)(x+1)$	$x=6$ and $-1$	Do <b>not</b> award the final <b>(A1)</b>

**Example:** Calculate the gradient of the line passing through the points (5, 3) and (0, 9).

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5}$ <b>(M1)</b> Award <b>(M1)</b> for correct substitution in gradient formula $= -\frac{6}{5}$ <b>(A1)</b>	(i) $\frac{9-3}{0-5} = -\frac{6}{5}$ <b>(M1)</b> Gradient is $= -\frac{6}{5}$ <b>(A1)</b> (There is clear understanding of the gradient.) $y = -\frac{6}{5}x + 9$	
	(ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ <b>(M1)</b> $y = -\frac{6}{5}x + 9$ <b>(A0)</b> (There is confusion about what is required.)	

### 3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Mark schemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (eg, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies**.
- (f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b>  Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math></p> <p><math>A = 41.8^\circ</math></p> <p><b>(Note: the 2<sup>nd</sup> (A1) here was not marked (ft) and cannot be awarded because there was an earlier error in the same question part.)</b></p>	<p><b>(M1)(A0)</b>  <i>(use of sine rule but with wrong values)</i></p> <p><b>(A0)</b></p>
<p>(b) <math>x = 7 \tan (22.0243\dots)</math> <b>(M1)</b>  <math>= 2.83</math> (2.83163...) <b>(A1)(ft)</b></p>	<p>(b) case (i) <math>x = 7 \tan 41.8^\circ</math>  <math>= 6.26</math></p> <p><b>but</b> case (ii) <math>6.26</math></p>	<p><b>(M1)</b>  <b>(A1)(ft)</b>  <b>(G0)</b>  <i>since no working shown</i></p>

#### 4 Using the Markscheme

- (a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.

The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

**Example:** Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b></p> <p>Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(i) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math></p> <p><math>A = 22.0^\circ</math></p> <p>(ii) <math>A = 22.0^\circ</math></p> <p><b>Note: G marks are used only if no working has been shown and the answer is correct.</b></p>	<p><b>(M1)(A1)</b></p> <p><b>(A1)</b></p> <p><b>(G2)</b></p>

- (d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' etc.

- (e) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value  $\left( \text{for example } \frac{2}{3} \text{ if applicable} \right)$ ;

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

- (f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval:  $3 < x < 5$ ;  $(3, 5)$ ;  $] 3, 5 [$  .

Different forms of notation for set properties (eg, complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A; U \setminus A$ .

Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ .  
 $p \Rightarrow q$ ;  $p \rightarrow q$ ;  $q \Leftarrow p$  .

Significance level may be written as  $\alpha$  .

- (g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5 Accuracy of Answers

**Incorrect accuracy should be penalized once only in each question according to the rules below.**

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award **(A1)** and ignore subsequent rounding.
2. If the candidate's unrounded answer is **not** seen then award **(A1)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.

**Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given ...					
	Exact or to 4 or more sf (and would <b>round to the correct 3 sf</b> )	<b>Correct to 3 sf</b>	<b>Incorrect to 3 sf</b>	Correct to 2 sf <sup>3</sup>	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen <sup>1</sup>	Award the final <b>(A1)</b> irrespective of correct or incorrect rounding					
Unrounded answer not seen <sup>2</sup>	<b>(A1)</b>	<b>(A1)</b>	<b>(A0)</b>	<b>(A1)</b>	<b>(A0)</b>	<b>(A0)</b>
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen. <sup>3</sup>			

**Examples:**

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398...) <b>(A1)</b>	(i) 9.43398... is seen followed by 9; 9.4; 9.43; 9.434 <i>etc.</i> (correctly rounded)	<b>(A1)</b>
	(ii) 9.43398... is seen followed by 9.433; 9.44 <i>etc.</i> (incorrectly rounded)	<b>(A1)</b>
	(iii) 9.4	<b>(A1)</b>
	(iv) 9 <i>sf)</i>	<b>(A0)</b> <i>(correct to 1</i>
	(v) 9.3	<b>(A0)</b> <i>(incorrectly rounded to 2 sf)</i>
	(vi) 9.44	<b>(A0)</b> <i>(incorrectly rounded to 3 sf)</i>

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798...) <b>(A1)</b>	(i) 7.43798... is seen followed by 7; 7.4; 7.44; 7.438 <i>etc.</i> (correctly rounded)	<b>(A1)</b>
	(ii) 7.43798... is seen followed by 7.437; 7.43 <i>etc.</i> (incorrectly rounded)	<b>(A1)</b>
	(iii) 7.4	<b>(A1)</b>
	(iv) 7	<b>(A0)</b> <i>(correct to 1 sf)</i>
	(v) 7.5	<b>(A0)</b> <i>(incorrectly rounded to 2 sf)</i>
	(vi) 7.43	<b>(A0)</b> <i>(incorrectly rounded to 3 sf)</i>

**Example:** ABC is a right angled triangle with angle  $ABC = 90^\circ$ ,  $AC = 32$  cm and  $AB = 30$  cm. Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b> Award <b>(M1)</b> for correct substitution in Pythagoras' formula</p> <p><math>= 11.1 (\sqrt{124}, 11.1355...)</math> (cm) <b>(A1)</b></p> <p>(b) <math>\text{Area} = \frac{1}{2} \times 30 \times 11.1355...</math> <b>(M1)</b> Award <b>(M1)</b> for correct substitution in area of triangle formula</p> <p><math>= 167 (167.032...) (\text{cm}^2)</math> <b>(A1)(ft)</b></p>	<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b>  11 (cm) <b>(A1)</b> (2 sf answer only seen, but correct)</p> <p>(b) case (i) <math>\text{Area} = \frac{1}{2} \times 30 \times 11</math> <b>(M1)</b> (working shown)  <math>= 165 (\text{cm}^2)</math> <b>(A1)(ft)</b></p> <p>case (ii) <math>= 165 (\text{cm}^2)</math> <b>(M0)(A0)(ft)</b> (No working shown, the answer 11 is treated as a ft, so no marks awarded here)</p>	

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	<b>(G2)</b>
7.68 (7.67543...) <b>(A2)</b>	(b) 7.67	<b>(G1)</b>
	(c) 7.6	<b>(G1)</b>
	(d) 8	<b>(G0)</b>
	(e) 7	<b>(G0)</b>
	(e) 7.66	<b>(G0)</b>



Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ <b>(A2)</b> $(y = 0.887686...x + 13.4895...)$ If an answer is not in the form of an equation award at most <b>(A1)(A0)</b> .	(a) $y = 0.89x + 13$	<b>(G2)</b> (both accepted)
	(b) $y = 0.88x + 13$	<b>(G1)</b> (one rounding error)
	(c) $y = 0.88x + 14$	<b>(G1)</b> (rounding error repeated)
	(d) (i) $y = 0.9x + 13$	<b>(G1)</b> (1 sf not accepted)
	(ii) $y = 0.8x + 13$	
	(e) $0.88x + 13$	<b>(G0)</b> (one rounding error and not an equation)

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ <b>(A1)(A1)</b> $(2.06020..., 4.49253...)$	(a) $(2.1, 4.5)$	<b>(A1)(A1)</b> (both accepted)
	(b) $(2.0, 4.4)$	<b>(A1)</b> (same rounding error twice)
	(c) $(2.06, 4.4)$	<b>(A1)</b> (one rounding error)
	(d) $(2, 4.4)$	<b>(A0)</b> (1sf not accepted, one rounding error)

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**.

Exact answers such as  $\frac{1}{4}$  can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (eg,  $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ , ) may be accepted as exact answers. All other powers (eg, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In **all** such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.

## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:** A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) <b>(A1)</b>	(i) 231.6	<b>(A0)</b>
	(ii) 232	<b>(A0)</b> (Correct rounding to incorrect level)
	(iii) 231.61	<b>(A0)</b>
	(iv) 232.00	<b>(A0)</b> (Parts (iii) and (iv) are both incorrect rounding to correct level)

## 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

**Example:**

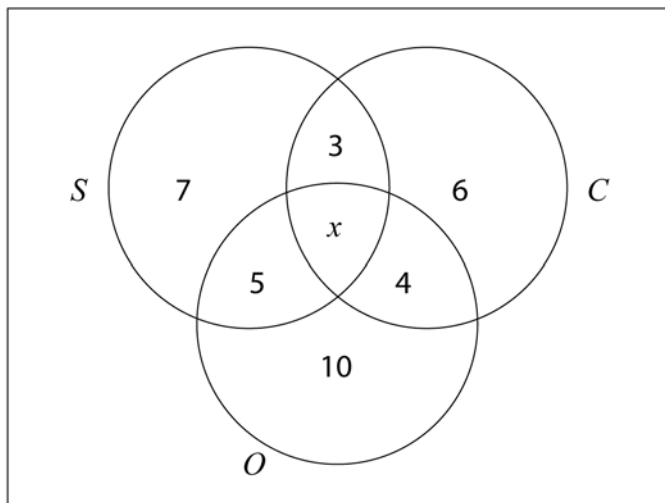
Markscheme	Candidates' Scripts	Marking
(a) 37000 m <sup>2</sup> <b>(A1)</b>	(a) 36000 m <sup>2</sup>	<b>(A0)</b> (Incorrect answer so units not considered)
(b) 3200 m <sup>3</sup> <b>(A1)</b>	(b) 3200 m <sup>2</sup>	<b>(A0)</b> (Incorrect units)

**If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.**

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a) U



(A1)(A1)(A1)(A1)

**Notes:** Award (A1) for 3 labelled intersecting circles and rectangle, and (A1) for  $x$ , (A1) for 3, 4, 5, (A1) for 6, 7, 10 (all in the correct place). Award (A0) for the appearance of 2 rather than  $x$  on the diagram.

[4 marks]

(b)  $50 - (7 + 3 + 6 + 4 + 5 + 10 + 13)$  OR  $50 - 48$

(M1)

**Note:** Award (M1) for setting up a correct expression involving 50 and values from **their** diagram.

$(x =) 2$

(A1)(ft)(G2)

**Note:** Follow through from their Venn diagram in part (a).

[2 marks]

(c) (i)  $\frac{17}{50}$  (0.34, 34%)

(A1)(ft)(A1)(G2)

**Note:** Award (A1)(ft) for correct numerator, (A1) for 50. 17 on its own earns (A0). Follow through from parts (a) and (b).

(ii)  $\frac{21}{50}$  (0.42, 42%)

(A1)(ft)(A1)(G2)

**Note:** Award (A1)(ft) for correct numerator, (A1) for 50. 21 on its own earns (A0). Follow through from parts (a) and (b).

continued...

Question 1 continued

(iii) they are not independent

(A1)(ft)

$$\frac{17}{50} \times \frac{21}{50} \neq \frac{7}{50}$$

(R1)

**Note:** Do not accept  $0.1428 = 0.14$  through premature rounding.  
Do not award **(A1)(R0)**.  
Follow through from parts (c)(i), (c)(ii) and part (b).

[6 marks]

(d)  $\frac{5}{15} \left( \frac{1}{3}, 0.333, 33.3\% \right)$

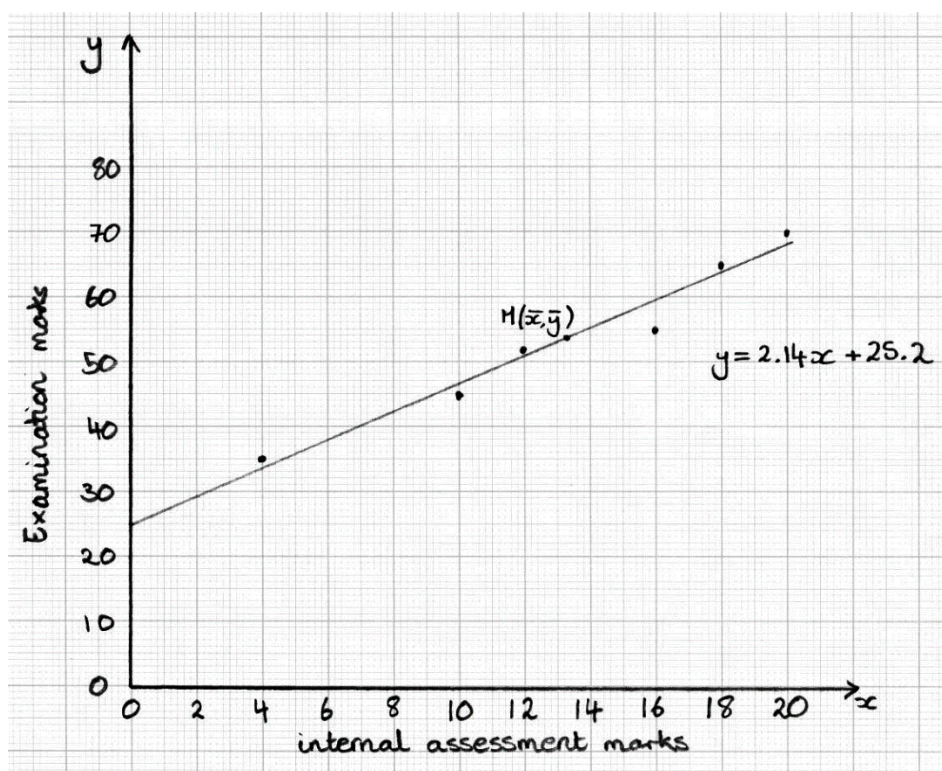
(A1)(ft)(A1)(ft)(G2)

**Note:** Award **(A1)(ft)** for numerator, **(A1)(ft)** for denominator. Follow through from parts (a) and (b).

[2 marks]

**Total [14 marks]**

2. (a)



(A3)

**Notes:** Award (A1) for correct labels and scales.

Accept  $x$  and  $y$  labelled on the axes as sufficient for labelling.

Award (A2) for all 6 points correct, (A1) for 4 or 5 points correct, otherwise (A0).

Award, at most, (A0)(A2) if axes are the wrong way round.

Award, at most, (A0)(A2) if the points are joined by lines.

If graph paper is not used, award at most (A1)(A0).

[3 marks]

(b) (i) 0.981 (0.981193...)

(G2)

(ii) (very) strong  
positive

(A1)(ft)  
(A1)(ft)

**Note:** Follow through from part (b)(i).

[4 marks]

(c) (i) 13.3 (13.3333...)

(A1)

(ii) 53.7 (53.6666...)

(A1)

[2 marks]

continued...

Question 2 continued

- (d) point M labelled, in correct position (A1)(A1)(ft)

**Note:** Award (A1)(ft) for point plotted in correct position, (A1) for point labelled M or  $(\bar{x}, \bar{y})$  or (13.3, 53.7). Follow through from part (c).  
 Allow tolerance of  $\pm 0.3$  on both values for  $(\bar{x}, \bar{y})$  for (A1)(ft).  
 An incorrectly plotted point (outside of tolerance) earns (A0)(A0).  
 If graph paper not used, award at most (A1)(A0).

[2 marks]

- (e)  $y = 2.14x + 25.2$  ( $y = 2.13846\dots x + 25.1538\dots$ ) (A1)(A1)(G2)

**Note:** Award (A1) for “2.14x” and (A1) for “25.2”.  
 Award at most (A1)(A0) if not written as an equation.

[2 marks]

- (f)  $y = 2.13846\dots \times (8) + 25.1538\dots$  (M1)  
 $y = 42.3$  (42.2615...) (A1)(ft)(G2)

**Notes:** Accept either 42 or 43. (as marks are discrete values).  
 Follow through from part (e). Do not award (M1) for finding an estimate by using the graph.  
 If the candidate has an incorrect equation in part (e) and there is no attempt to substitute 8 into this equation then (M0)(A0).

[2 marks]

- (g) straight line goes through point (their  $\bar{x}$ , their  $\bar{y}$ ) (A1)(ft)  
 y-intercept of the straight line is (their) 25.2 (A1)(ft)

**Notes:** Accept  $25.2 \pm 1.5$  for the y-intercept.  
 Follow through from parts (d) and (e).  
 The line does not need to be labelled.  
 The line does not need to intercept the y-axis. However, the extension of their line should intersect the axis at the correct point.  
 If graph paper is not used, award at most (A1)(A0).

[2 marks]

- (h) not reliable (A1)  
 30 marks is outside the given range (R1)

OR

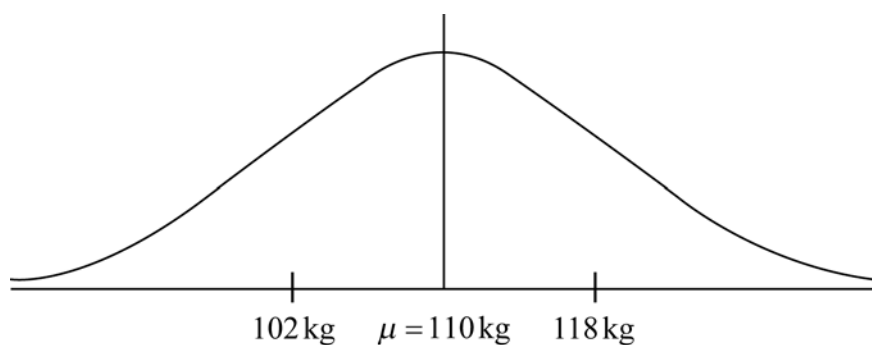
- maximum mark of the internal assessment is 20 in the sample (R1)

**Note:** Do not award (A1)(R0).

[2 marks]

Total [19 marks]

3. (a)



(A2)

**Note:** Award (A1) for normal curve with mean of 110 (kg) indicated and (A1) for labelling 118 (kg) (one standard deviation above the mean) and 102 (kg) (one standard deviation below the mean).

[2 marks]

(b) (i) 2

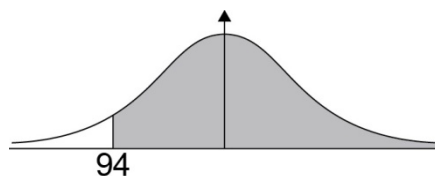
(A1)

**Note:** Do not accept –2.

(ii) 0.977 (0.977249..., 97.7249...%)

(G2)

**Notes:** Do not accept 0.975.  
Award (G2) for 0.98 unless this comes from 0.975.  
If (G0), award (M1) for  $P(\text{weight} > 94)$ .  
If (G0), award (M1) for correct region indicated on a labelled diagram.

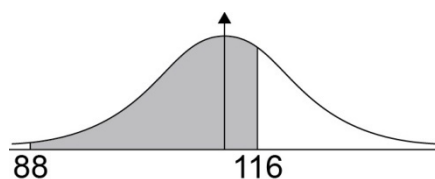


[3 marks]

(c) (i) 0.770 (0.770392..., 77.0392...%)

(G2)

**Notes:** If (G0), award (M1) for  $P(88 < \text{weight} < 116)$ .  
If (G0), award (M1) for correct region indicated on a labelled diagram.



continued...

Question 3 continued

(ii)  $0.770392... \times 160$

(M1)

**Note:** Award (M1) for multiplying their probability from part (c)(i) by 160.

$= 123$  (123.262...)

(A1)(ft)(G2)

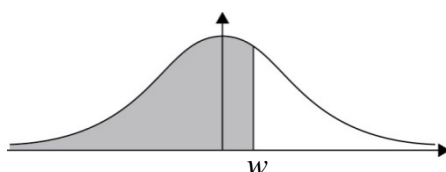
**Notes:** Accept either 123 or 124 (whole number of sheep).  
Accept 123.2 as the unrounded answer if 0.770 is used.  
Follow through from part (c)(i).

[4 marks]

(d)  $w = 115$  (115.395...)

(G2)

**Notes:** If (G0), award (M1) for  $P(\text{weight} < w) = 0.75$ .  
If (G0), award (M1) for a vertical line drawn to the right of the mean with the area to the left of this line shaded. The  $w$  may not be seen.



[2 marks]

(e)  $a = 106$  (105.804...)

(G1)

$b = 114$  (114.195...)

(G1)

**Note:** If their answers are not identified by the letters  $a$  and  $b$  and the order is incorrect (114 followed by 106) then award, at most, (G0)(G1).

[2 marks]

Total [13 marks]



4. Units are required in part (b) for full marks to be awarded.

$$(a) \quad \cos \hat{BAC} = \frac{140^2 + 120^2 - 110^2}{2(120)(140)} \quad (M1)(A1)$$

**Notes:** Award **(M1)** for substitution into the cosine rule, **(A1)** for correct substitution.

$$\hat{BAC} = 49.3^\circ \text{ (49.3236...)} \quad (A1)(G2) \quad [3 \text{ marks}]$$

$$(b) \quad \frac{1}{2} \times 140 \times 120 \times \sin(\text{their } \hat{BAC}) \quad (M1)(A1)(ft)$$

**Note:** Award **(M1)** for substitution into the area formula, **(A1)(ft)** for correct substitution.

$$= 6370 \text{ (6370.58...)} \text{ m}^2 \quad (A1)(ft)(G2)$$

**Note:** The answer is  $6370 \text{ m}^2$ , the units are **required**. Follow through from part (a). Award **(G1)** for 6370 given without units and no working.

[3 marks]

$$(c) \quad \begin{aligned} BDC &= 180 - (80 + 40) \\ &= 60^\circ \end{aligned} \quad \begin{array}{l} (A1) \\ (AG) \end{array}$$

[1 mark]

$$(d) \quad \frac{DC}{\sin 80^\circ} = \frac{110}{\sin 60^\circ} \quad (M1)(A1)$$

**Note:** Award **(M1)** for substitution into the sine rule, **(A1)** for correct substitution.

$$DC = 125 \text{ (m) (125.087...)} \quad (A1)(G2) \quad [3 \text{ marks}]$$

continued...

Question 4 continued

(e)  $\frac{DB}{\sin 40^\circ} = \frac{110}{\sin(60^\circ)}$  (M1)(A1)

**Note:** Award (M1) for substitution into the sine rule, (A1) for correct substitution.  
Accept, as equivalent, the sine rule using  $40^\circ$ ,  $80^\circ$  and  $DC = 125$ ,  
awarding (M1)(A1)(ft). Follow through from part (d).

$DB = 81.6449\dots$  (A1)(G2)

**OR**

$DB^2 = (\text{their } DC)^2 + 110^2 - 2(110)(\text{their } DC)\cos 40^\circ$  (M1)(A1)(ft)

**Note:** Award (M1) for substitution into the cosine rule, (A1)(ft) for correct substitution.

$DB = 81.6449\dots$  (A1)(ft)(G2)

**Either**  $DB + AB = 221.644\dots$  (m) **or**  $DC + CA = 245.087\dots$  (m) (A1)(ft)

**Both**  $DB + AB = 221.644\dots$  (m) **and**  $DC + CA = 245.087\dots$  (m) (A1)(ft)

**Notes:** Follow through from part (d).

so the route that passes through B is shorter (AG)

**Note:** The final (A1)(ft) should only be awarded if the journey lengths are consistent with the demand of the question **and** the conclusion is stated.

[5 marks]

**Total [15 marks]**

5. (a) increasing (A1)  
gradient (of tangent line is) positive (R1)

**Note:** Do not award (A1)(R0).  
Accept the word 'derivative' for gradient.

[2 marks]

- (b) 150 (A1)

[1 mark]

- (c)  $3nx^2 - 60x - 240$  (A1)(A1)(A1)

**Note:** Award (A1) for each correct term. Award up to (A1)(A1)(A0) if additional terms are seen.

[3 marks]

- (d) (4, -650) (A1)

**Note:** Award (A0) if parentheses are missing. Accept  $x = 4$ ,  $y = -650$ .

[1 mark]

- (e)  $0 = 3n(4)^2 - 60(4) - 240$  (M1)

**Note:** Award (M1) for the correct substitution of  $x = 4$  or  $x = -2$  into their derivative and equating to zero.

$$0 = 48n - 240 - 240 \quad (A1)(ft)$$

**Note:** Accept equivalent simplifications. This (A1)(ft) should only be awarded if the final answer of  $n = 10$  is seen following correct simplification. Follow through from part (c).

OR

$$-650 = n(4)^3 - 30(4)^2 - 240(4) + 150 \quad (M1)$$

**Notes:** Award the (M1) for correct substitution of their (4, -650) into the cubic. Accept other correct coordinates e.g. (-5, -650), (-1, 350) or (5, -550) for the method mark.

$$-650 = 64n - 480 - 960 + 150 \quad (A1)(ft)$$

**Note:** Accept equivalent simplifications. This (A1)(ft) should only be awarded if the final answer of  $n = 10$  is seen following correct simplification. Follow through from part (d).

$$n = 10 \quad (AG)$$

**Note:** Substituting  $n = 10$  at the outset invalidates the method, so (M0)(A0).

[2 marks]

continued...

Question 5 continued

(f) (i)  $30(-1)^2 - 60(-1) - 240$  (M1)

**Note:** Award (M1) for correct substitution into their derivative.

$= -150$  (A1)(ft)(G2)

**Note:** Follow through from part (c).

(ii)  $y - 350 = -150(x + 1)$  OR  $350 = -150 \times (-1) + c$  (M1)

**Note:** Answer must be given as an equation. Award (M1) for substituting their gradient from part (f)(i) and the point  $(-1, 350)$ .

$150x + y - 200 = 0$  (A1)(ft)(G2)

**Note:** Accept  $-150x - y + 200 = 0$ .  
Do not accept  $y = -150x + 200$ .  
Follow through from their equation  $y - 350 = -150(x + 1)$   
OR  $y = -150x + 200$ , correctly rearranged.

(iii)  $\frac{1}{150}$  (A1)(ft)

**Note:** Award (A1)(ft) for the negative reciprocal of their answer to part (f)(i).

[5 marks]

(g) 6.39 (6.38898...) (G1)

**Note:** Award (G0) if answer is given as coordinates (unless  $x$  is clearly identified).

[1 mark]

(h)  $-3.97$  ( $-3.97340\dots$ ) and  $-0.809$  ( $-0.808518\dots$ ) (G1)(ft)(G1)(ft)

**Note:** If  $x = 7.78$  ( $7.78192\dots$ ) also seen award at most (G1)(ft)(G0).  
Award, at most, (G0)(G1)(ft) if answers given as coordinates, and this was not penalized in part (g). Follow through from part (b).

[2 marks]

Total [17 marks]

6. (a) volume =  $\pi 4^2 \times 15$  (M1)

**Note:** Award (M1) for correct substitution into volume of a cylinder formula.

$$= 754 \text{ (cm}^3\text{)} \quad ((753.982\dots, 240\pi)) \quad (\text{A1})(\text{G2})$$

**Note:** If  $r = 8 \text{ cm}$  is used, then award (M0)(A0) for this part.

[2 marks]

- (b)  $85 = \frac{1}{3} \pi (4)^2 \times h$  (M1)

**Notes:** Award (M1) for correct substitution into volume of a cone formula (85 must be seen).  
If  $r = 8 \text{ cm}$  was penalized in part (a), accept  $r = 8 \text{ cm}$  for method here.

$$(h =) 5.07 \text{ (cm)} (5.07306\dots) \quad (\text{A1})(\text{ft})(\text{G2})$$

**Notes:** Follow through only for  $r = 8 \text{ cm}$ . Answer is then  $(h =) 1.26826\dots$

[2 marks]

- (c) (radius of the hemisphere =) 4.52693... (A1)(ft)

**Notes:** Award (A1)(ft) for  $24.6 - 5.07306\dots - 15$ . If  $r = 8 \text{ cm}$  used, this value is 8.33173... Follow through from part (b).

$$(V =) \frac{1}{2} \left( \frac{4}{3} \pi (4.52693\dots)^3 \right) \quad (\text{A1})(\text{ft})(\text{M1})$$

**Notes:** Award (A1)(ft) for the correct substitution of their radius of the hemisphere into volume of sphere formula.

Award (M1) for multiplying volume of sphere formula by  $\frac{1}{2}$ .

$$= 194 \text{ (cm}^3\text{)} (194.299\dots) \quad (\text{A1})(\text{ft})(\text{G3})$$

**Notes:** Accept volume = 195 (cm<sup>3</sup>) from the use of  $r = 4.53$ .  
If  $r = 8 \text{ cm}$  is used, the answer is 1211.33... (cm<sup>3</sup>).  
Follow through from their radius of the hemisphere unless radius = 4 used.  
If radius of hemisphere = 4 cm used, award at most (A0)(A1)(ft)(M1)(A0).

[4 marks]

continued...

Question 6 continued

(d)  $(l = ) \sqrt{5.07306...^2 + 4^2}$  (M1)

**Note:** Award **(M1)** for the correct substitution of their part (b) into Pythagoras' theorem.  
If  $r = 8$  cm has been penalized previously, accept  $r = 8$  cm for method here.

curved surface area  $= \pi(4)\sqrt{5.07306...^2 + 4^2}$  (M1)

**Note:** Award **(M1)** for correct substitution of their calculated  $l$  into the curved surface area formula. Candidates may have calculated  $l$ , which equals 6.46033...  
Do not accept  $\pi \times 4 \times 5.07306...$  for method.

$= 81.1830...$  (A1)(ft)

**Notes:** If  $r = 8$  cm is used, the answer is 203.572...  
Follow through from part (b).

$= 81 \text{ (cm}^2\text{)}$  (A1)(ft)(G3)

**Notes:** The final **(A1)(ft)** is awarded for correctly rounding their 81.1830...

[4 marks]

Total [12 marks]

# **Markscheme**

**November 2016**

**Mathematical Studies**

**Standard level**

**Paper 2**

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**Paper 2 Markscheme**  
**Instructions to Examiners**

**Notes:** If in doubt about these instructions or any other marking issues, contact your team leader for clarification.

**1 Abbreviations**

**M** Marks awarded for **Method**

**A** Marks awarded for an **Answer** or for **Accuracy**

**R** Marks awarded for clear **Reasoning**

**G** Marks awarded for correct solutions obtained from a **Graphic Display Calculator**, when no working shown.

**AG** **Answer Given** in the question and consequently, marks not awarded.

**ft** Marks that can be awarded as **follow through** from previous results in the question.

**2 Method of Marking**

- All marking must be done in RM Assessor using the mathematical studies annotations and in accordance with the current document for guidance in e-marking Mathematical Studies SL. It is essential that you read this document before you start marking.
- If a question part is completely correct use the number tick annotations to award full marks. If a part is completely wrong use the **A0** annotation, otherwise full annotations must be shown.
- Working crossed out by the candidate should not be awarded any marks.
- Where candidates have written two solutions to a question, only the first solution should be marked.
- If correct working results in a correct answer but then further working is developed, indicating a lack of mathematical understanding full marks should **not** be awarded. In most such cases it will be a single final answer mark that is lost. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal.

**Example:**

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	Award the final <b>(A1)</b> (ignore the further working)
2.	$(x-6)(x+1)$	$x=6$ and $-1$	Do <b>not</b> award the final <b>(A1)</b>

**Example:** Calculate the gradient of the line passing through the points (5, 3) and (0, 9) .

Markscheme	Candidates' Scripts	Marking
$\frac{9-3}{0-5}$ <b>(M1)</b> Award <b>(M1)</b> for correct substitution in gradient formula $= -\frac{6}{5}$ <b>(A1)</b>	(i) $\frac{9-3}{0-5} = -\frac{6}{5}$ <b>(M1)</b> Gradient is $= -\frac{6}{5}$ <b>(A1)</b> (There is clear understanding of the gradient.) $y = -\frac{6}{5}x + 9$	
	(ii) $\frac{9-3}{0-5} = -\frac{6}{5}$ <b>(M1)</b> $y = -\frac{6}{5}x + 9$ <b>(A0)</b> (There is confusion about what is required.)	

### 3 Follow-through (ft) Marks

Errors made at any step of a solution affect all working that follows. To limit the severity of the penalty, **follow through (ft)** marks can be awarded. Markschemes will indicate where it is appropriate to apply follow through in a question with **'(ft)'**.

- (a) Follow through applies only from one part of a question to a subsequent part of the question. Follow through does not apply within the same part.
- (b) If an answer resulting from follow through is extremely unrealistic (eg, negative distances or incorrect by large order of magnitude) then the final **A** mark should not be awarded.
- (c) If a question is transformed by an error into a **different, much simpler question** then follow through may not apply.
- (d) To award follow through marks for a question part, **there must be working present for that part**. An isolated follow through answer, without working is regarded as incorrect and receives no marks **even if it is approximately correct**.
- (e) The exception to the above would be in a question which is testing the candidate's use of the GDC, where working will not be expected. **The markscheme will clearly indicate where this applies**.
- (f) Inadvertent use of radians will be penalized the first time it occurs. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for the use of radians.

**Example:** Finding angles and lengths using trigonometry

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b>  Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(a) <math>\frac{\sin A}{4} = \frac{\sin 30}{3}</math></p> <p><math>A = 41.8^\circ</math></p> <p><b>(Note: the 2<sup>nd</sup> (A1) here was not marked (ft) and cannot be awarded because there was an earlier error in the <b>same</b> question part.)</b></p>	<p><b>(M1)(A0)</b>  (use of sine rule but with wrong values)</p> <p><b>(A0)</b></p>
<p>(b) <math>x = 7 \tan (22.0243\dots^\circ)</math> <b>(M1)</b>  <math>= 2.83</math> (2.83163...) <b>(A1)(ft)</b></p>	<p>(b) case (i) <math>x = 7 \tan 41.8^\circ</math>  <math>= 6.26</math></p> <p><b>but</b> case (ii) <math>6.26</math></p>	<p><b>(M1)</b>  <b>(A1)(ft)</b>  <b>(G0)</b>  since no working shown</p>

#### 4 Using the Markscheme

- (a) **A** marks are **dependent** on the preceding **M** mark being awarded, it is **not** possible to award **(M0)(A1)**. Once an **(M0)** has been awarded, all subsequent **A** marks are lost in that part of the question, even if calculations are performed correctly, until the next **M** mark.

The only exception to this will be for an answer where the accuracy is specified in the question – see section 5.

- (b) **A** marks are **dependent** on the **R** mark being awarded, it is **not** possible to award **(A1)(R0)**. Hence the **(A1)** cannot be awarded for an answer which is correct when no reason or the wrong reason is given.
- (c) In paper 2 candidates are expected to demonstrate their ability to communicate mathematics using appropriate working. Answers which are correct but not supported by adequate working will **not always receive full marks**, these unsupported answers are designated **G** in the mark scheme as an alternative to the full marks. Example **(M1)(A1)(A1)(G2)**.

**Example:** Using trigonometry to calculate an angle in a triangle.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math> <b>(M1)(A1)</b></p> <p>Award <b>(M1)</b> for substitution in sine rule formula, <b>(A1)</b> for correct substitutions.</p> <p><math>A = 22.0^\circ</math> (22.0243...) <b>(A1)(G2)</b></p>	<p>(i) <math>\frac{\sin A}{3} = \frac{\sin 30}{4}</math></p> <p><math>A = 22.0^\circ</math></p> <p>(ii) <math>A = 22.0^\circ</math></p> <p><b>Note: G marks are used only if no working has been shown and the answer is correct.</b></p>	<p><b>(M1)(A1)</b></p> <p><b>(A1)</b></p> <p><b>(G2)</b></p>

- (d) **Alternative methods** may not always be included. Thus, if an answer is wrong then the working must be carefully analysed in order that marks are awarded for a different method consistent with the markscheme.

Where alternative methods for complete questions are included in the markscheme, they are indicated by '**OR**' etc.

- (e) Unless the question specifies otherwise, accept **equivalent forms**. For example:  $\frac{\sin \theta}{\cos \theta}$  for  $\tan \theta$ .

On the markscheme, these equivalent numerical or algebraic forms will sometimes be written in brackets after the required answer.

Where numerical answers are required as the final answer to a part of a question in the markscheme, the scheme will show, in order:

the 3 significant figure answer worked through from full calculator display;

the exact value  $\left( \text{for example } \frac{2}{3} \text{ if applicable} \right)$ ;

the full calculator display in the form 2.83163... as in the example above.

Where answers are given to 3 significant figures and are then used in subsequent parts of the question leading to a **different** 3 significant figure answer, these solutions will also be given.

- (f) As this is an international examination, all valid **alternative forms of notation** should be accepted. Some examples of these are:

Decimal points: 1.7; 1'7; 1·7; 1,7 .

Decimal numbers less than 1 may be written with or without a leading zero: 0.49 or .49 .

Different descriptions of an interval:  $3 < x < 5$ ; (3, 5); ] 3, 5 [ .

Different forms of notation for set properties (eg, complement):  $A'$ ;  $\bar{A}$ ;  $A^c$ ;  $U - A$ ;  $(A; U \setminus A$ .

Different forms of logic notation:  $\neg p$ ;  $p'$ ;  $\tilde{p}$ ;  $\bar{p}$ ;  $\sim p$ .

$p \Rightarrow q$ ;  $p \rightarrow q$ ;  $q \Leftarrow p$  .

Significance level may be written as  $\alpha$  .

- (g) Discretionary marks: There will be very rare occasions where the markscheme does not cover the work seen. In such cases the annotation DM should be used to indicate where an examiner has used discretion. Discretion should be used sparingly and if there is doubt and exception should be raised through RM Assessor to the team leader.

As with previous sessions there will be no whole paper penalty marks for accuracy AP, financial accuracy FP and units UP. Instead these skills will be assessed in particular questions and the marks applied according to the rules given in sections 5, 6 and 7 below.

## 5 Accuracy of Answers

**Incorrect accuracy should be penalized once only in each question according to the rules below.**

Unless otherwise stated in the question, all numerical answers should be given exactly or correct to 3 significant figures.

1. If the **candidate's answer** is seen to 4 sf or greater **and** would round to the required 3 sf answer, then award **(A1)** and ignore subsequent rounding.
2. If the candidate's unrounded answer is **not** seen then award **(A1)** if the answer given is **correctly** rounded to 2 or more significant figures, otherwise **(A0)**.

**Note:** If the candidate's unrounded answer is **not** seen and the answer is given correct to 1 sf (correct or not), the answer will be considered wrong and will not count as incorrect accuracy. If this answer is used in subsequent parts, then working must be shown for further marks to be awarded.

3. If a correct 2 sf answer is used in subsequent parts, then working **must** be shown for further marks to be awarded. (This treatment is the same as for following through from an incorrect answer.)

These 3 points (see numbers in superscript) have been summarized in the table below and illustrated in the examples following.

	If candidates final answer is given ...					
	Exact or to 4 or more sf (and would <b>round to</b> <b>the correct 3 sf</b> )	<b>Correct</b> <b>to 3 sf</b>	<b>Incorrect</b> <b>to 3 sf</b>	Correct to 2 sf <sup>3</sup>	Incorrect to 2 sf	Correct or incorrect to 1 sf
Unrounded answer seen <sup>1</sup>	Award the final <b>(A1)</b> irrespective of correct or incorrect rounding					
Unrounded answer not seen <sup>2</sup>	<b>(A1)</b>	<b>(A1)</b>	<b>(A0)</b>	<b>(A1)</b>	<b>(A0)</b>	<b>(A0)</b>
Treatment of subsequent parts	As per MS		Treat as follow through, only if working is seen. <sup>3</sup>			

**Examples:**

Markscheme	Candidates' Scripts	Marking
9.43 (9.43398...) <b>(A1)</b>	(i) 9.43398... is seen followed by 9; 9.4; 9.43; 9.434 etc. (correctly rounded)	<b>(A1)</b>
	(ii) 9.43398... is seen followed by 9.433; 9.44 etc. (incorrectly rounded)	<b>(A1)</b>
	(iii) 9.4	<b>(A1)</b>
	(iv) 9 sf)	<b>(A0)</b> (correct to 1 sf)
	(v) 9.3	<b>(A0)</b> (incorrectly rounded to 2 sf)
	(vi) 9.44	<b>(A0)</b> (incorrectly rounded to 3 sf)

Markscheme	Candidates' Scripts	Marking
7.44 (7.43798...) <b>(A1)</b>	(i) 7.43798... is seen followed by 7; 7.4; 7.44; 7.438 etc. (correctly rounded)	<b>(A1)</b>
	(ii) 7.43798... is seen followed by 7.437; 7.43 etc. (incorrectly rounded)	<b>(A1)</b>
	(iii) 7.4	<b>(A1)</b>
	(iv) 7	<b>(A0)</b> (correct to 1 sf)
	(v) 7.5	<b>(A0)</b> (incorrectly rounded to 2 sf)
	(vi) 7.43	<b>(A0)</b> (incorrectly rounded to 3 sf)

**Example:** ABC is a right angled triangle with angle  $ABC = 90^\circ$ ,  $AC = 32$  cm and  $AB = 30$  cm . Find (a) the length of BC, (b) The area of triangle ABC.

Markscheme	Candidates' Scripts	Marking
<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b> Award <b>(M1)</b> for correct substitution in Pythagoras' formula</p> <p><math>= 11.1 (\sqrt{124}, 11.1355...)</math> (cm) <b>(A1)</b></p> <p>(b) <math>\text{Area} = \frac{1}{2} \times 30 \times 11.1355...</math> <b>(M1)</b> Award <b>(M1)</b> for correct substitution in area of triangle formula</p> <p><math>= 167 (167.032...) (\text{cm}^2)</math> <b>(A1)(ft)</b></p>	<p>(a) <math>BC = \sqrt{32^2 - 30^2}</math> <b>(M1)</b>  11 (cm) <b>(A1)</b> (2 sf answer only seen, but correct)</p> <p>(b) case (i) <math>\text{Area} = \frac{1}{2} \times 30 \times 11</math> <b>(M1)</b> (working shown)  <math>= 165 (\text{cm}^2)</math> <b>(A1)(ft)</b></p> <p>case (ii) <math>= 165 (\text{cm}^2)</math> <b>(M0)(A0)(ft)</b> (No working shown, the answer 11 is treated as a ft, so no marks awarded here)</p>	

Certain answers obtained from the GDC are worth 2 marks and working will not be seen. In these cases only one mark should be lost for accuracy.

eg, Chi-squared, correlation coefficient, mean

Markscheme	Candidates' Scripts	Marking
Chi-squared	(a) 7.7	<b>(G2)</b>
7.68 (7.67543...) <b>(A2)</b>	(b) 7.67	<b>(G1)</b>
	(c) 7.6	<b>(G1)</b>
	(d) 8	<b>(G0)</b>
	(e) 7	<b>(G0)</b>
	(e) 7.66	<b>(G0)</b>

Regression line

Markscheme	Candidates' Scripts	Marking
$y = 0.888x + 13.5$ <b>(A2)</b> $(y = 0.887686...x + 13.4895...)$  If an answer is not in the form of an equation award at most <b>(A1)(A0)</b> .	(a) $y = 0.89x + 13$  (b) $y = 0.88x + 13$  (c) $y = 0.88x + 14$  (d) (i) $y = 0.9x + 13$  (ii) $y = 0.8x + 13$  (e) $0.88x + 13$	<b>(G2)</b> <i>(both accepted)</i>  <b>(G1)</b> <i>(one rounding error)</i>  <b>(G1)</b> <i>(rounding error repeated)</i>  <b>(G1)</b> <i>(1 sf not accepted)</i>  <b>(G0)</b> <i>(one rounding error and not an equation)</i>

Maximum/minimum/points of intersection

Markscheme	Candidates' Scripts	Marking
$(2.06, 4.49)$ <b>(A1)(A1)</b> $(2.06020..., 4.49253...)$	(a) $(2.1, 4.5)$  (b) $(2.0, 4.4)$  (c) $(2.06, 4.4)$  (d) $(2, 4.4)$	<b>(A1)(A1)</b> <i>(both accepted)</i>  <b>(A1)</b> <i>(same rounding error twice)</i>  <b>(A1)</b> <i>(one rounding error)</i>  <b>(A0)</b> <i>(1sf not accepted, one rounding error)</i>

Rounding of an exact answer to 3 significant figures **should be accepted if performed correctly**.

Exact answers such as  $\frac{1}{4}$  can be written as decimals to fewer than 3 significant figures if the result is still exact. Reduction of a fraction to its lowest terms is **not** essential, however where an answer simplifies to an integer this is expected. Fractions that include a decimal in the numerator and/or the denominator are acceptable for showing correct substitution, but not as a final answer.

Ratios of  $\pi$  and answers taking the form of square roots of integers or any rational power of an integer (eg,  $\sqrt{13}, 2^{\frac{2}{3}}, \sqrt[4]{5}$ , ) may be accepted as exact answers. All other powers (eg, of non-integers) and values of transcendental functions such as sine and cosine must be evaluated.

**If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.** In all such cases the final mark is not awarded if the rounding does not follow the instructions given in the question. A mark for specified accuracy can be regarded as a **(ft)** mark regardless of an immediately preceding **(M0)**.



## 6 Level of accuracy in finance questions

The accuracy level required for answers will be specified in all questions involving money. This will usually be either whole units or two decimal places. The first answer not given to the specified level of accuracy will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one mark per paper can be lost for incorrect accuracy in a financial question.

**Example:** A financial question demands accuracy correct to 2 dp.

Markscheme	Candidates' Scripts	Marking
\$231.62 (231.6189) <b>(A1)</b>	(i) 231.6	<b>(A0)</b>
	(ii) 232	<b>(A0)</b> (Correct rounding to incorrect level)
	(iii) 231.61	<b>(A0)</b>
	(iv) 232.00	<b>(A0)</b> (Parts (iii) and (iv) are both incorrect rounding to correct level)

## 7 Units in answers

There will be specific questions for which the units are required and this will be indicated clearly in the markscheme. The first correct answer with no units or incorrect units will not be awarded the final **A** mark. The markscheme will give clear instructions to ensure that only one or two mark per paper can be lost for lack of units or incorrect units.

The units are considered only when the numerical answer is awarded **(A1)** under the accuracy rules given in Section 5.

**Example:**

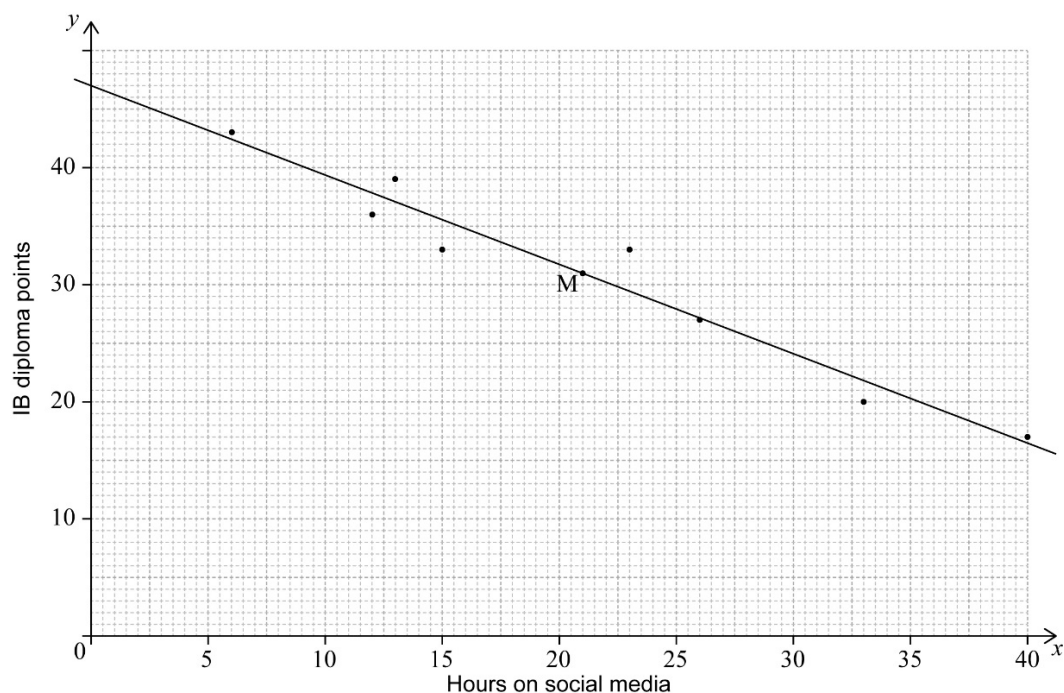
Markscheme	Candidates' Scripts	Marking
(a) 37000 m <sup>2</sup> <b>(A1)</b>	(a) 36000 m <sup>2</sup>	<b>(A0)</b> (Incorrect answer so units not considered)
(b) 3200 m <sup>3</sup> <b>(A1)</b>	(b) 3200 m <sup>2</sup>	<b>(A0)</b> (Incorrect units)

**If no method is shown and the answer is correct but with incorrect or missing units award G marks with a one mark penalty.**

## 8 Graphic Display Calculators

Candidates will often be obtaining solutions directly from their calculators. They must use mathematical notation, not calculator notation. No method marks can be awarded for incorrect answers supported only by calculator notation. The comment 'I used my GDC' cannot receive a method mark.

1. (a)



(A4)

**Notes:** Award (A1) for correct scale and labelled axes.

Award (A3) for 7 or 8 points correctly plotted,

(A2) for 5 or 6 points correctly plotted,

(A1) for 3 or 4 points correctly plotted.

Award at most (A0)(A3) if axes reversed.

Accept  $x$  and  $y$  sufficient for labelling.

If graph paper is not used, award (A0).

If an inconsistent scale is used, award (A0). Candidates' points should be read from this scale **where possible** and awarded accordingly.

A scale which is too small to be meaningful (ie mm instead of cm) earns (A0) for plotted points.

[4 marks]

(b) (i)  $\bar{x} = 21$

(A1)

(ii)  $\bar{y} = 31$

(A1)

[2 marks]

(c)  $(\bar{x}, \bar{y})$  correctly plotted on graph  
this point labelled M

(A1)(ft)

(A1)

**Note:** Follow through from parts (b)(i) and (b)(ii).  
Only accept M for labelling.

[2 marks]

continued...

Question 1 continued

(d)  $-0.973$  ( $-0.973388\dots$ ) (G2)

**Note:** Award (G1) for 0.973, without minus sign.

[2 marks]

(e)  $y = -0.761x + 47.0$  ( $y = -0.760638\dots x + 46.9734\dots$ ) (A1)(A1)(G2)

**Notes:** Award (A1) for  $-0.761x$  and (A1) +47.0. Award a maximum of (A1)(A0) if answer is not an equation.

[2 marks]

(f) line on graph (A1)(ft)(A1)(ft)

**Notes:** Award (A1)(ft) for **straight line** that passes through their M, (A1)(ft) for line (extrapolated if necessary) that passes through (0, 47.0). If M is not plotted or labelled, follow through from part (e).

[2 marks]

(g)  $y = -\frac{2}{3}(34) + \frac{125}{3}$  (M1)

**Note:** Award (M1) for correct substitution.

19 (points) (A1)(G2)

[2 marks]

(h) extrapolation (R1)

**OR**

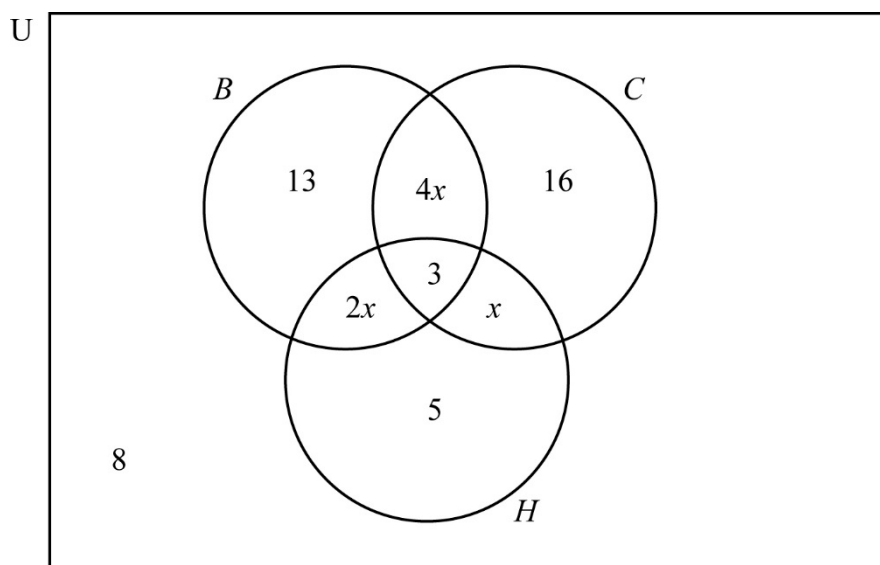
34 hours is outside the given range of data (R1)

[1 mark]

**Note:** Do not accept 'outlier'.

**Total [17 marks]**

2. (a)



(A5)

**Notes:** Award (A1) for rectangle and three labelled intersecting circles (U need not be seen),

(A1) for 3 in the correct region,

(A1) for 8 in the correct region,

(A1) for 5, 13 and 16 in the correct regions,

(A1) for  $x$ ,  $2x$  and  $4x$  in the correct regions.

[5 marks]

(b)  $8 + 13 + 16 + 3 + 5 + x + 2x + 4x = 66$

(M1)

**Note:** Award (M1) for **either** a completely correct equation **or** adding all the terms from **their** diagram in part (a) and equating to 66.

Award (M0)(A0) if their equation has no  $x$ .

$$7x = 66 - 45 \quad \text{OR} \quad 7x + 45 = 66$$

(A1)

**Note:** Award (A1) for adding their like terms correctly, **but only** when the solution to their equation is equal to 3 and is consistent with their original equation.

$$x = 3$$

(AG)

**Note:** The conclusion  $x = 3$  must be seen for the (A1) to be awarded.

[2 marks]

(c) 15

(A1)(ft)

**Note:** Follow through from part (a). The answer must be an integer.

[1 mark]

continued...

Question 2 continued

(d) (i)  $\frac{42}{66} \left( \frac{7}{11}, 0.636, 63.6\% \right)$

(A1)(ft)(A1)(G2)

**Note:** Award **(A1)(ft)** for numerator, **(A1)** for denominator.  
Follow through from their Venn diagram.

(ii)  $\frac{3}{9} \left( \frac{1}{3}, 0.333, 33.3\% \right)$

(A1)(A1)(ft)(G2)

**Note:** Award **(A1)** for numerator, **(A1)(ft)** for denominator.  
Follow through from their Venn diagram.

**[4 marks]**

**Total [12 marks]**

3. (a)  $2 \times 4 - 1 - 7 = 0$  (or equivalent) (R1)

**Note:** For (R1) accept substitution of  $x = 1$  or  $y = 4$  into the equation followed by a confirmation that  $y = 4$  or  $x = 1$ .

(since the point satisfies the equation of the line,) A lies on  $L_1$  (A1)

**Note:** Do not award (A1)(R0).

[2 marks]

- (b)  $\frac{1+5}{2}$  OR  $\frac{4+12}{2}$  seen (M1)

**Note:** Award (M1) for at least one correct substitution into the midpoint formula.

(3, 8) (A1)(G2)

**Notes:** Accept  $x = 3$ ,  $y = 8$ .

Award (M1)(A0) for  $\left(\frac{1+5}{2}, \frac{4+12}{2}\right)$ .

Award (G1) for each correct coordinate seen without working.

[2 marks]

- (c)  $\sqrt{(5-1)^2 + (12-4)^2}$  (M1)

**Note:** Award (M1) for a correct substitution into distance between two points formula.

$= 8.94 \left(4\sqrt{5}, \sqrt{80}, 8.94427\dots\right)$  (A1)(G2)

[2 marks]

- (d) gradient of AC =  $\frac{12-4}{5-1}$  (M1)

**Note:** Award (M1) for correct substitution into gradient formula.

$= 2$  (A1)

**Note:** Award (M1)(A1) for gradient of AC = 2 with or without working

gradient of the normal =  $-\frac{1}{2}$  (M1)

**Note:** Award (M1) for the negative reciprocal of their gradient of AC.

continued...

Question 3 continued

$$y - 8 = -\frac{1}{2}(x - 3) \quad \text{OR} \quad 8 = -\frac{1}{2}(3) + c \quad (\text{M1})$$

**Note:** Award **(M1)** for substitution of their point and gradient into straight line formula.

This **(M1)** can **only** be awarded where  $-\frac{1}{2}$  (gradient) is correctly determined as the gradient of the normal to AC.

$$2y - 16 = -(x - 3) \quad \text{OR} \quad -2y + 16 = x - 3 \quad \text{OR} \quad 2y = -x + 19 \quad (\text{A1})$$

**Note:** Award **(A1)** for correctly removing fractions, **but only** if their equation is equivalent to the given equation.

$$2y + x - 19 = 0 \quad (\text{AG})$$

**Note:** The conclusion  $2y + x - 19 = 0$  must be seen for the **(A1)** to be awarded. Where the candidate has **shown** the gradient of the normal to  $AC = -0.5$ , award **(M1)** for  $2(8) + 3 - 19 = 0$  and **(A1)** for (therefore)  $2y + x - 19 = 0$ . Simply substituting (3, 8) into the equation of  $L_2$  with no other prior working, earns no marks.

[5 marks]

(e) (6, 6.5) (A1)(A1)(G2)

**Note:** Award **(A1)** for 6, **(A1)** for 6.5. Award a maximum of **(A1)(A0)** if answers are not given as a coordinate pair. Accept  $x = 6$ ,  $y = 6.5$ . Award **(M1)(A0)** for an attempt to solve the two simultaneous equations  $2y - x - 7 = 0$  and  $2y + x - 19 = 0$  algebraically, leading to at least one incorrect or missing coordinate.

[2 marks]

(f) 3.3541 (A1)

**Note:** Answer must be to 5 significant figures.

[1 mark]

continued...

Question 3 continued

$$(g) \quad 2 \times \frac{1}{2} \times \sqrt{80} \times \frac{\sqrt{45}}{2} \quad (M1)(M1)$$

**Notes:** Award **(M1)** for correct substitution into area of triangle formula.  
 If their triangle is a quarter of the rhombus then award **(M1)** for multiplying their triangle by 4.  
 If their triangle is a half of the rhombus then award **(M1)** for multiplying their triangle by 2.

OR

$$\frac{1}{2} \times \sqrt{80} \times \sqrt{45} \quad (M1)(M1)$$

**Notes:** Award **(M1)** for doubling MD to get the diagonal BD, **(M1)** for correct substitution into the area of a rhombus formula.  
 Award **(M1)(M1)** for  $\sqrt{80} \times$  their (f).

$$= 30 \quad (A1)(ft)(G3)$$

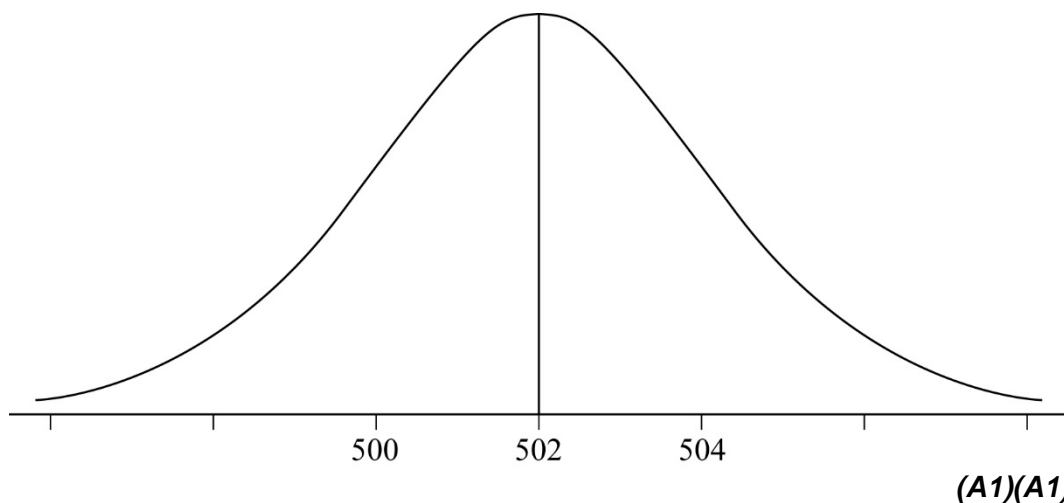
**Notes:** Follow through from parts (c) and (f).  
 $8.94 \times 3.3541 = 29.9856...$

[3 marks]

Total [17 marks]



4. (a)



**Notes:** Award **(A1)** for bell shape with mean of 502.  
Award **(A1)** for an indication of standard deviation eg 500 and 504.

[2 marks]

(b) (i) 0.921 (0.920968..., 92.0968...%) (G2)

**Note:** Award **(M1)** for a diagram showing the correct shaded region.

(ii)  $1500 \times 2 \times 0.920968...$  (M1)

= (\$) 2760 (2762.90...) (A1)(ft)(G2)

**Note:** Follow through from their answer to part (b)(i).

[4 marks]

(c)  $1500 \times 0.16 \times 0.079031...$  (M1)

**Notes:** Award **(A1)** for  $1500 \times 0.16 \times$  their  $(1 - 0.920968...)$ .

OR

$(1500 - 1381.45) \times 0.16$  (M1)

**Notes:** Award **(M1)** for  $(1500 - \text{their } 1381.45) \times 0.16$ .

= (\$) 19.0 (18.9676...) (A1)(ft)(G2)

[2 marks]

(d) 347 (grams) (346.614...) (G3)

**Notes:** Award **(G2)** for an answer that rounds to 346.  
Award **(G1)** for 353.385... seen without working (for finding the top 3 %).

[3 marks]

Total [11 marks]

5. (a)  $(BD =) \sqrt{95^2 + 40^2}$  (M1)

**Note:** Award (M1) for correct substitution into Pythagoras' theorem.

$= 103 \text{ (m)} \quad (103.077..., 25\sqrt{17})$  (A1)(G2)

[2 marks]

(b)  $\cos \hat{B}AD = \frac{105^2 + 70^2 - (103.077...)^2}{2 \times 105 \times 70}$  (M1)(A1)(ft)

**Note:** Award (M1) for substitution into cosine rule, (A1)(ft) for their correct substitutions. Follow through from part (a).

$(\hat{B}AD) = 68.9^\circ \quad (68.8663...)$  (A1)(ft)(G2)

**Note:** If their 103 used, the answer is 68.7995...

[3 marks]

(c)  $(\text{Area of } ABD =) \frac{1}{2} \times 105 \times 70 \times \sin(68.8663...)$  (M1)(A1)(ft)

**Notes:** Award (M1) for substitution into the trig form of the area of a triangle formula.  
Award (A1)(ft) for their correct substitutions.  
Follow through from part (b).  
If  $68.8^\circ$  is used the area =  $3426.28... \text{ m}^2$ .

$= 3430 \text{ m}^2 \quad (3427.82...)$  (A1)(ft)(G2)

[3 marks]

(d)  $\text{area of } ABCD = \frac{1}{2} \times 40 \times 95 + 3427.82...$  (M1)

**Note:** Award (M1) for correctly substituted area of triangle formula added to their answer to part (c)

$= 5330 \text{ m}^2 \quad (5327.83...)$  (A1)(ft)(G2)

[2 marks]

continued...

Question 5 continued

$$(e) \quad \frac{1}{2} \times 105 \times AP \times \sin(68.8663\dots) = 0.5 \times 5327.82\dots \quad (M1)(M1)$$

**Notes:** Award **(M1)** for the correct substitution into triangle formula.  
Award **(M1)** for equating their triangle area to half their part (d).

$$(AP =) 54.4 \text{ (m)} \quad (54.4000\dots) \quad (A1)(ft)(G2)$$

**Notes:** Follow through from parts (b) and (d).

**[3 marks]**

$$(f) \quad BP^2 = 105^2 + (54.4000\dots)^2 - 2 \times 105 \times (54.4000\dots) \times \cos(68.8663\dots) \quad (M1)(A1)(ft)$$

**Notes:** Award **(M1)** for substituted cosine rule formula.

Award **(A1)(ft)** for their correct substitutions. Accept the exact fraction  $\frac{53}{147}$

in place of  $\cos(68.8663\dots)$ .

Follow through from parts (b) and (e).

$$(BP =) 99.3 \text{ (m)} \quad (99.3252\dots) \quad (A1)(ft)(G2)$$

**Notes:** If 54.4 and  $\cos(68.9)$  are used the answer is 99.3567...

**[3 marks]**

**Total [16 marks]**

6. (a)  $(A =) \pi r^2 + 2\pi rh$  (A1)(A1)

**Note:** Award (A1) for either  $\pi r^2$  OR  $2\pi rh$  seen. Award (A1) for two correct terms added together.

[2 marks]

- (b) 500000 (A1)

**Notes:** Units **not** required.

[1 mark]

- (c)  $500000 = \pi r^2 h$  (A1)(ft)

**Notes:** Award (A1)(ft) for equating  $\pi r^2 h$  to their part (b).  
Do not accept  $V = \pi r^2 h$  unless  $V$  is explicitly defined as their part (b).

[1 mark]

- (d)  $A = \pi r^2 + 2\pi r \left( \frac{500000}{\pi r^2} \right)$  (A1)(ft)(M1)

**Note:** Award (A1)(ft) for their  $\frac{500000}{\pi r^2}$  seen.

Award (M1) for correctly substituting **only**  $\frac{500000}{\pi r^2}$  into a **correct** part (a).

Award (A1)(ft)(M1) for rearranging part (c) to  $\pi rh = \frac{500000}{r}$  and substituting for  $\pi rh$  in expression for  $A$ .

$$A = \pi r^2 + \frac{1000000}{r} \quad (\text{AG})$$

**Notes:** The conclusion,  $A = \pi r^2 + \frac{1000000}{r}$ , must be consistent with their working seen for the (A1) to be awarded.  
Accept  $10^6$  as equivalent to 1000000.

[2 marks]

- (e)  $2\pi r - \frac{1000000}{r^2}$  (A1)(A1)(A1)

**Note:** Award (A1) for  $2\pi r$ , (A1) for  $\frac{1}{r^2}$  or  $r^{-2}$ , (A1) for  $-1000000$ .

[3 marks]

continued...

Question 6 continued

$$(f) \quad 2\pi r - \frac{1000000}{r^2} = 0 \quad (M1)$$

**Note:** Award **(M1)** for equating their part (e) to zero.

$$r^3 = \frac{1000000}{2\pi} \quad \text{OR} \quad r = \sqrt[3]{\frac{1000000}{2\pi}} \quad (M1)$$

**Note:** Award **(M1)** for isolating  $r$ .

**OR**

sketch of derivative function (M1)  
with its zero indicated (M1)

$$(r =) 54.2 \text{ (cm)} \quad (54.1926...) \quad (A1)(ft)(G2)$$

[3 marks]

$$(g) \quad \pi(54.1926...) ^2 + \frac{1000000}{(54.1926...)} \quad (M1)$$

**Note:** Award **(M1)** for correct substitution of their part (f) into the given equation.

$$= 27700 \text{ (cm}^2\text{)} \quad (27679.0...) \quad (A1)(ft)(G2)$$

[2 marks]

$$(h) \quad \frac{27679.0...}{2000} \quad (M1)$$

**Note:** Award **(M1)** for dividing their part (g) by 2000.

$$= 13.8395... \quad (A1)(ft)$$

**Notes** Follow through from part (g).

$$14 \text{ (cans)} \quad (A1)(ft)(G3)$$

**Notes:** Final **(A1)** awarded for rounding up their 13.8395... to the next integer.

[3 marks]

**Total [17 marks]**