"OpenQuake: Calculate, share, explore"

OpenQuake Engine User Instruction Manual

Contents

PART I

Risk Modeller's Toolkit

CHAPTER 1

Nonlinear Static Method with dispersion information

Nonlinear Static Methods are based on the use of capacity curves resulting from nonlinear static pushover analysis to determine the median seismic intensity values $\hat{s}_c$ corresponding to the attainment of a certain damage state threshold (limit state) and the corresponding dispersion β_{sc}. These parameters are used to represent a fragility curve as the probability of the limit state capacity C being exceeded by the demand D, both expressed in terms of intensity levels (s_c and s respectively), as shown in the following equation:

$$P_{LS}(s) = P(C < D|s) = \Phi(\frac{lns - ln\hat{s}_c}{\beta_{sc}}) \tag{1.1}$$

The methodology implemented so far in the RMTK allows to consider different shapes of the pushover curve, multilinear and bilinear, record-to-record dispersion, dispersion in the damage state thresholds.

Different input types can be inserted depending on whether the user has already at his disposal an idealised pushover curve or it has to be derived from the raw results of a pushover analysis. Fragility and vulnerability functions can be derived for a single building or for a class of buildings, simply inserting data for many buildings in the inputs.

The intensity measure to be used is S_a and a mapping between any engineering demand parameter (EDP), assumed to describe the damage state thresholds, and the roof displacement should be available from the pushover analysis.

Ruiz-Garcia and Miranda (2007) study on inelastic displacement demand estimation, Vamvatsikos and Cornell (2006) and Dolsek and Fajfar (2004) work on seismic demand estimation with multilinear static pushover curves, have been integrated in three nonlinear static procedures, C_R-based, spo2ida-based and R-mu-T-based, within the same script. In this way the user has the chance to select the procedure with the degree of accuracy consistent with the available input and the type of structural analyses performed.

In section **??** the main information necessary to start the analysis are presented. In sections **??**, **??** and **??** the three procedures are explained respectively, from the point of view of the necessary scientific background behind and their step-by-step implementation in the python script.

1.1. How to use the NSP

To start using the nonlinear static procedure with record-to-record variability a command line text editor should be used to enter manually the folder location where the RMTK has been saved. The user should add the path */RMTK/Vulnerability/NSP*, where the nonlinear static method script is located, as shown in the example below:

```
cd path/to/rmtk/folder/RMTK
```

From the text editor iPython browser page can be opened with the following command line:

```
ipython-2.7 notebook --pylab=inline
```

Once the iPython page is opened on the browser, the python scripts contained in the RMTK directory will be visible. The file *NSM_dispersion.ipynb* should be selected to start the calculations.

In the initial section of the script "Define Options" the user needs to set the options and to enter the input corresponding to the defined options in the folder *NSP/input*. The main options are the following:

- Type of procedure to perform: either C$_R$-based, spo2ida-based or R-mu-T-based. The main difference between the three is that C$_R$-based procedure is applicable to elasto-plastic idealised capacity curve only, while spo2ida-based and R-mu-T-based procedure fit any kind of multilinear curve. Among the two procedures for multilinear capacity curves the main difference lies in the spo2ida-based ability to compute more accurately the record-to-record dispersion.

- Type of input: either displacements vs base shear at each time step or idealised pushover curve, as shown in Figures **??** and **??**.

- Type of output: either fragility curve (probability of exceedance of a set of limit states vs seismic intensity, as shown in Figure XXX) or vulnerability curve (loss ratio vs seismic intensity, as shown in Figure XXY).

These options and others need to be defined in the initial section of the script "Define Options". In section **??** the alternatives values that the initial variables can assume and their meaning are described in detail, while the parameters to be inserted in the input files are listed below. They are fully described in section **??** section **??** and section **??**, within the presentation of each of the three procedures.

- Results from a static pushover analysis: either displacements vs base shear at each time step, as shown in Figure **??**, or idealised pushover curve, as shown in Figure **??**.

- Dynamic parameters of the building: fundamental period of vibration T_1 and corresponding modal participation factor Γ_1.
- Limit States in terms of roof displacement or inter-storey drift ratio.
- Consequence function (damage factor for each damage state).

1.1.1. Options

The type of procedure to be performed and the type of inputs at disposal, are set with the variables *an_type* and *in_type* respectively. With the variable *an_type* the user can choose between:

```
an_type = 0 #Cr-based procedure (Ruiz-Garcia and Miranda, 2007)
an_type = 1 #spo2ida-based procedure (Vamvatsikos and Cornell, 2006)
an_type = 2 #R-$mu$-T-based procedure (Dolsek and Fajfar, 2004)
```

With the variable *in_type* the user can choose between:

```
in_type = 0 # idealised pushover curve
in_type = 1 # raw results from a pushover analysis
```

The variable *vulnerability* instead gives the opportunity to decide the type of outputs, whether to stop the process at the derivation of the fragility curves, or to go all the way up to the vulnerability curve definition, applying damage-to-loss functions.

```
vuln = 0 # derive fragility curves
vuln = 1 # derive vulnerability curve
```

The variable *g* serves the purpose of defining the units that are being used. A floating number must be assigned to the gravity acceleration, compatible with the units used for the period of vibration and for the displacements (if the period is expressed in seconds and displacements are in meters, then $g = 9.81$). The variable *iml* is a numpy array that identifies the intensity measure levels for which loss ratios are computed and provided in the vulnerability curve.

```
g = 9.81
iml = np.linspace(0.1,15,100)
```

The variable *plotflag* allows or inhibits the displaying of plots. It is a python list composed of 4 integers, each one controlling a different plot: idealised pushover curve, 16%-50%-84% ida curves, fragility curves and vulnerability curve respectively. Each integer can take as value either zero or one, whether the corresponding graph has to be displayed or not:

```
plotflag = [1, 1, 1, 1] # plot all the graphs
plotflag = [0, 0, 0, 0] # do not plot any graph
```

The following variables set some of the characteristics of the plots:

- *linew*: integer for defining lines width.
- *fontsize*: fontsize used for labels, graphs etc.
- *units*: list of 3 strings defining displacements, forces and Spectral acceleration units, as ['[kN]', '[m]', '[m/s^2]'], to be displayed on the axes of the plots.

The following variables are needed for spo2ida-based procedure only:

- *N*: number of points per segment of IDA curve derived with spo2ida
- *MC*: number of Monte Carlo simulations to account for uncertainty in damage thresholds

The last set of variables is needed for R-μ-T-based procedure only:

- *Tc*: constant accel-constant velocity corner period of a Newmark-Hall type spectrum. Default value is 0.5. *Td*: constant velocity-constant displacement corner period of a Newmark-Hall type spectrum. Default value is 1.8.

1.2. C$_R$-based procedure

1.2.1. Theoretical background

The aim of this procedure, proposed by Vamvatsikos (2014), is the estimation of the median spectral acceleration value $\hat{S}_{a,ds}$, that brings the structure to the attainment of a set of damage states ds, and the corresponding dispersion beta β_{S_a}, the parameters needed for the mathematical representation of fragility in equation **??**. The aim is achieved making use of the work by Ruiz-Garcia and Miranda (2007), where the inelastic displacement demand is related to the elastic displacement with a simple relationship, and it can thus be easily estimated through a response spectrum analysis and a capacity curve.

The C$_R$-based procedure presented herein is applicable to bilinear elasto-plastic capacity curve only, and it is suitable for single building fragility curve estimation, as described in section **??**. However the fragility curves derived for single buildings can be combined in a unique fragility curve, which considers the inter-building uncertainty, as described in section **??**.

1.2.1.1. Single Building Fragility and Vulnerability function

This procedure provides a simple relationship between median damage state threshold, expressed in terms of top displacement δ_{roof}, at each damage state threshold ds, and the corresponding median elastic Spectral displacement value $\hat{S}_{d,ds}(T_1)$.

$$\hat{\delta}_{roof,ds} = C_R S_{d,ds}(T_1)\Gamma_1\Phi_1 \tag{1.2}$$

where $\Gamma_1\Phi_1$ is the first mode participation factor estimated for the first-mode shape normalised by the roof displacement, and C$_R$ is the inelastic displacement ratio (inelastic over elastic spectral displacement), computed by Ruiz-Garcia and Miranda (2007) for nonlinear SDoF systems having hysteretic behaviour representative of the analysed structure, which is a function of the first-mode period of vibration and the relative lateral strength of the system R. Therefore the median Spectral acceleration at the fundamental period of vibration $\hat{S}_{a,ds}(T_1)$ turns out to be expressed as a function of the roof displacement according to the following equation:

$$\hat{S}_{a,ds}(T_1) = \frac{4\pi^2}{\hat{C}_R T^2 \Gamma_1 \Phi_1} \hat{\delta}_{roof,ds} \tag{1.3}$$

Default values of $\hat{C}_R$ parameter estimates are provided by Ruiz-Garcia and Miranda (2007), as result of nonlinear regression analysis of three different measures of central tendency computed from 240 ground motions:

$$\hat{C}_R = 1 + \frac{R-1}{79.12 T_1^{1.98}} \tag{1.4}$$

and values for R are given as:

$$R_{ds} = max(0.425(1 - c + \sqrt{c^2 + 2c(2\mu_{ds} - 1) + 1}), 1) \tag{1.5}$$

where c = 79.12 $T^{1.98}$, and μ_{ds} is the ductility at the damage state threshold of interest.

For what concerns the dispersion of $\hat{S}_{a,ds}$, β_{S_a}, the following relationship between S and the median EDP damage threshold $\hat{\theta}$ (Cornell, 2002) is used:

$$\hat{\theta}(S_a) = a S_a^b \tag{1.6}$$

so that β_{S_a} can be easily derived from the dispersion of θ due to record-to-record variability, $\beta_{\theta d}$, as in the following:

$$\beta_{S_a} = \frac{1}{b} \beta_{\theta d} \tag{1.7}$$

The dispersion of θ due to record-to-record variability, $\beta_{\theta d}$ can be easily combined with the dispersion of θ due to uncertainty in the damage state threshold $\beta_{\theta c}$ as shown in the following equation:

$$\beta_{S_a} = \frac{1}{b} \sqrt{\beta_{\theta d}^2 + \beta_{\theta c}^2} \tag{1.8}$$

The dispersion of θ can be obtained assuming that d_{roof} and θ are proportional, and they thus share the same dispersion. Moreover the dispersion of d_{roof} is the same as the dispersion of C_R, since they are also proportional. Finally $\beta_{\theta d}$ can be computed with the following equation, which represents Ruiz-Garcia and Miranda's (2007) estimate of C_R dispersion:

$$\sigma_{\ln(C_R)} = \sigma_{\ln(d_{roof})} = \beta_{\theta d} = 1.975[\frac{1}{5.876} + \frac{1}{11.749(T+0.1)}][1-\exp(-0.739(R-1))] \tag{1.9}$$

To derive a discrete vulnerability function at certain intensity measure levels, the input damage-to-loss factors are applied to the probability of occurance of each damage state, extracted from the probability of exceedance of each damage state described by the fragility function. A value of loss ratio is thus defined for the vector of selected intensity measure levels.

1.2.1.2. Multiple-Building Fragility and Vulnerability function

If multiple buildings have been input to derive fragility function for a class of buildings all $\hat{S}_{a,blg}$ and $\beta_{S_a,blg}$ are combined in a single lognormal curve. A minimum of 5 buildings should be considered to obtain reliable results for the class. A new issue arises when multiple buildings are considered: the S_a at the fundamental period of each building should be converted to a common intensity measure, to be able to combine the different fragility functions. A common intensity measure is selected to be S_a at the period T_{av}, which is a weighted average of the individual buildings fundamental period T_1. Then each individual fragility needs to be expressed in terms of the common $S_a(T_{av})$, using a spectrum. FEMA P-695 far field set of 22 accelerograms was used to derive a mean uniform hazard spectrum, and the ratio between the S_a at different periods is used to scale the fragility functions. It can be noted that the actual values of the spectrum are not important, but just the spectral shape. Median $\hat{S}_a$ is converted to the mean $\mu_{ln(S_a)}$ of the corresponding normal distribution ($\mu_{ln(S_a)} = ln(\hat{S}_a)$) and, simply scaled to the common intensity measure as follows:

$$\mu_{ln(S_a),blg} = \mu_{ln(S_a),blg} S(T_{av})/S(T_{1,blg}) \tag{1.10}$$

$$\beta_{S_a,blg} = \beta_{S_a,blg} S(T_{av})/S(T_{1,blg}) \tag{1.11}$$

Finally the parameters of the single lognormal curve for the class of buildings, mean and dispersion, can be computed as the weighted mean of the single means and the weighted SRSS of the inter-building and intra-building standard deviation, the standard deviation of the single means and the single dispersions respectively, as shown in the following equations:

$$\mu_{ln(S_a),tot} = \sum_{i=0}^{n.blg} w_{blg-i} \mu_{ln(S_a),blg-i} \tag{1.12}$$

$$\beta_{S_a,tot} = \sqrt{\sum_{i=0}^{n.blg} w_{blg-i}((\mu_{ln(S_a),blg-i} - \mu_{ln(S_a),tot})^2 + \beta^2_{S_a,blg-i})} \tag{1.13}$$

The mean $\mu_{ln(S_a)}$ and total dispersion β_{S_a} of the fragility function of the class are converted to logarithmic mean μ_{S_a} and logarithmic covariance cov_{S_a} (standard deviation σ_{S_a} over μ_{S_a}), according to the following equations:

$$\hat{S}_a = e^{\mu_{ln(S_a)}} \tag{1.14}$$

$$\mu_{S_a} = \hat{S}_a e^{\frac{\beta^2_{S_a}}{2}} \tag{1.15}$$

$$\sigma_{S_a} = \sqrt[2]{(\beta^2_{S_a} - 1)e^{2\ln\hat{S}_a + \beta^2_{S_a}}} \tag{1.16}$$

$$cov_{S_a} = \frac{\sigma_{S_a}}{\mu_{S_a}} \tag{1.17}$$

A single vulnerability function can be also obtained, from the single building vulnerability functions. The input damage-to-loss function is applied to the fragility function derived for each building. For the selected intensity measure levels a value of loss ratio LR_{blg} is thus defined for each building. A discrete vulnerability function for the entire class of buildings is represented at each iml by a mean LR, $\mu_{LR,tot}$, equal tot the weighted LR_{blg}, and a standard deviation, $\sigma_{LR,tot}$, equal to the weighted standard deviation of all the computed LR_{blg}. The $\sigma_{LR,tot}$ of the fragility function of the class is converted to covariance cov_{LR} (standard deviation $\sigma_{LR,tot}$ over $\mu_{LR,tot}$).

1.2.2. Inputs

The inputs must be formatted as comma-separated value files (.csv), and saved in the folder *input*, contained in the NSP directory. If any other environment different from Windows is used make sure that the "comma separated values Windows" is selected as saving option when creating the input files.

If multiple buildings want to be analysed to consider the inter-building uncertainty the parameters relative to each building should be added as additional lines in the input tables, as shown in the examples below, otherwise a single line must be input.

If the user has already at disposal an idealised elasto-plastic pushover curve for each building, that is to say that the variable *in_type* has been set to 0, the following data need to be provided in the corresponding csv files:

1. First period of vibration T_1, corresponding modal participation factor Γ_1, normalised with respect to the roof displacement, and weight for the combination of different buildings, input in *building_parameters.csv*, as in the example below:

2. Roof displacement at each limit state LS and corresponding dispersion $\beta_{\theta c}$ input in *displacement_profile.csv*, as shown in the example below. If dispersion is unknown, $\beta_{\theta c}$ can be set equal to zero at each LS.

3. Idealised pushover curve, input in *idealised_curve.csv* as shown below. The only required parameters are the yielding displacement δ_y, the ultimate displacement δ_u and the yielding force F_y.

4. Consequence model (loss ratio per each damage state) consistent with the defined set of damage states, input in *consequence.csv*, as in the example below. A single consequence model can be input. This input is needed only if the variable *vulnerability* has been set to 1.

If the idealised curve are not available, *in_type* $= 0$ can be selected and the displacements vs base shear at each time step results from a pushover analysis can be input instead. The following data need to be provided in the corresponding csv files:

1. T_1 and corresponding Γ_1, weight for the combination of different buildings, number of storeys and height of each storey, input in *building_parameters.csv*, as in the example below:

2. Displacements at each storey, at each incremental step of the pushover analysis, input in *displacements_pushover.csv*, as in the example below:

3. Base shear at each incremental step of the pushover analysis input in *reactions_pushover.csv*, as in the example below:

4. Drift limit state and corresponding dispersion $\beta_{\theta c}$ input in *limits.csv*. If dispersion is unknown, $\beta_{\theta c}$ can be set equal to zero at each limit state.

5. Consequence model (loss ratio per each damage state) consistent with the defined set of damage states, input in *consequence.csv*, as in the example below. A single consequence model can be input. This input is needed only if the variable *vulnerability* has been set to 1.

1.2.3. Calculation Steps

The overall workflow of C$_R$-based procedure is summarised in this section. The option *an_type* must be set equal to 0 and the option *in_type* according to the input at disposal. The corresponding inputs should follow the requirements described in section **??**. At this point the code proceeds with the following steps:

1. a) If *in_type* $= 0$ roof displacement at limit states and idealised pushover are extracted from *displacement_profile.csv* and *idealised_curve.csv* respectively.

   b) If *in_type* $= 1$ results from pushover analysis are extracted from *displacements_pushover.csv* and *reactions_pushover.csv*, and drift limit states from *limits.csv*. The idealised pushover curve is then derived in the *idealisation* function, where the idealisation process is conducted according to FEMA-440. The elastic stiffness is defined as the tangent stiffness passing through the point of the pushover curve where 60% of the maximum base shear is reached, and the perfectly plastic branch is set at an height equal to the maximum base shear. The yielding point is found as the interception between the elastic and the plastic branch.

2. The csv input files are parsed with the function *read_data* according to the defined options. The parameters essential to the analysis are return together with a graphical visualisation of the inputs if the variable *plotflag*[0] is equal to 1.

3. The parameters extracted are used in the *simplified_bilinear* function, within the *fragility_process* function, to derive ductility levels μ_{ds}, median spectral acceleration $\hat{S}_{a,ds}$ and the total dispersion β_{S_a} at each limit state through the following steps:
   - The idealised MDoF system is transformed into an equivalent SDoF system, using Γ_1.
   - Ductility levels μ_{ds} corresponding to each damage threshold, are defined.
   - R and C$_R$ are computed, using eq. **??** and **??** respectively.
   - $\hat{S}_{a,ds}$ and the corresponding dispersion $\beta_{\theta d}$ are computed using eq. **??** and **??** respectively.
   - $\beta_{\theta d}$ is combined with dispersion due to uncertainty in the model $\beta_{\theta c}$, if different from zero, to get the total dispersion β_{S_a}, using eq. **??**.
   - $\hat{S}_a(T_1)$ is converted to mean $\mu_{ln(S_a)}(T_1)$ and then to the intensity measure in common with the rest of the buildings, $\mu_{ln(S_a(T_{av}))}$, according to eq. **??**

4. Step 3. is repeated for the number of input buildings.

5. a) If vulnerability = 0: All $\mu_{ln(S_a),blg}$ and $\beta_{S_a,blg}$ are combined in a single lognormal curve, whose parameters are evaluated according to equations **??** and **??**. Mean $\mu_{ln(S_a)}$ and total dispersion β_{S_a} are then converted to logarithmic mean μ_{S_a} and logarithmic covariance cov_{S_a}, according to equations **??** and **??** respectively. Fragility curves for the class of buildings are displayed if the variable *plotflag*[2] = 1, and logarithmic μ_{S_a} and cov_{S_a} are exported in the *outputs* folder.

   b) If vulnerability =1: Vulnerability curves are derived for each fragility function derived at step 4. For the intensity measure levels defined in the variable *iml* a value of loss ratio is thus defined for each building. They are finally combined in a single mean and its standard deviation, equal to the weighted mean and the weighted standard deviation of the loss ratios, as described in section **??**. Vulnerability curve for the class of buildings is displayed if the variable *plotflag*[3] = 1. The $\sigma_{LR,tot}$ of the fragility function of the class is converted to covariance cov_{LR} and μ_{LR} and cov_{LR} at each iml are exported in the *outputs* folder.

1.3. Spo2ida-based procedure

1.3.1. Theoretical background

The aim of this procedure is the estimation of the median spectral acceleration value $\hat{S}_{a,ds}$, that brings the structure to the attainment of a set of damage states ds, and the corresponding dispersion beta β_{S_a}, the parameters needed for the mathematical representation of fragility in equation **??**. The aim is achieved making use of the tool spo2ida (Vamvatsikos and Cornell, 2006), where static pushover curves are converted into 16%, 50% and 84% IDA curves, using empirical relationships from a large database of incremental dynamic analysis results, as shown in Figure **??**.

The spo2ida-based procedure presented herein is applicable to any kind of multilinear capacity curve, and it is suitable for single building fragility curve estimation, as described in section **??**. However the fragility curves derived for single buildings can be combined in a unique fragility curve, which considers the inter-building uncertainty, as described in section **??**.

1.3.1.1. Single-building Fragility and Vulnerability function

Given the idealised capacity curve the spo2ida tool uses an implicit R-μ-T relation to correlate nonlinear displacement, expressed in terms of ductility μ to the corresponding medians capacity in terms of the parameters R. R is the lateral strength ratio, defined as the ratio between the spectral spectral acceleration S_a and the yielding capacity of the system S_{ay}.

Each branch of the capacity curve, hardening, softening and residual plateau, is converted to a corresponding branch of the three ida curves, using the R-μ-T relation, which is a function of the hardening stiffness, the softening stiffness and the residual force. These parameters are derived from the idealised pushover capacity expressed in μ-R terms, as well as the ductility levels at the onset of each branch. If some of the branches of the pushover curve are missing because of the seismic behaviour of the system, spo2ida can equally work with bilinear, trilinear and quadrilinear idealisations.

The result of the spo2ida routine is thus a list of ductility levels and corresponding R values at 50%, 16% and 84% percentiles. The distribution of R values at each ductility level, due to the record-to-record variability, is assumed to be lognormal and it can be easily converted to the dispersion of the lognormal distribution with the following equation:

$$\beta_{R(\mu)} = \frac{\ln R(\mu)_{84\%} - \ln R(\mu)_{16\%}}{2} \tag{1.18}$$

$\beta_{R(\mu)}$ represents also the record-to-record variability of S_a at different ductility levels $\beta_{S_a,d}$. Median R and its dispersion at ductility levels corresponding to the damage thresholds can thus be determined, and $\hat{S}_{a,ds}$ can be easily extracted simply multiplying $R_{50\%}(\mu_{ds})$ by the yielding capacity of the system S_{ay}, as shown in the following equation:

$$\hat{S}_{a,ds} = R_{50\%}(\mu_{ds}) S_{ay} \tag{1.19}$$

$$S_{ay} = \frac{4\pi^2 \delta_{roof,y}}{g \Gamma_1 T_1^1} \tag{1.20}$$

Since $\hat{R}$ and $\hat{S}_a$ are proportional they share the same dispersion.

If dispersion due to uncertainty in the limit state definition $\beta_{\theta c}$ is different from zero it can not be combined directly with the record-to-record dispersion, but a Monte Carlo sampling of the limit state needs to be performed instead. Different values of ductility limit state are sampled from the lognormal distribution with median the median value of the ductility limit state, and dispersion the input $\beta_{\theta c}$. For each of these ductilities the corresponding $R_{16\%}$-$R_{50\%}$-$R_{84\%}$ are found and converted into $\hat{S}_{a,ds}$ and $\beta_{\theta d}$ according to equation **??** and **??**. N random S_a corresponding to the N sampled ductility limit states are computed, and their median and the dispersion are estimated. These parameters constitute the median $\hat{S}_{a,ds}$ and the total dispersion β_{S_a} for the considered damage state. The procedure is repeated for each damage state.

To derive a discrete vulnerability function at certain intensity measure levels, the input damage-to-loss factors are applied to the probability of occurance of each damage state, extracted from the probability of exceedance of each damage state described by the set of fragility curves.

If dispersion due to uncertainty in the limit state is different from zero a vulnerability function is derived for the N sets of sampled ductility limit states. It results in N loss ratios for each defined intensity measure levels. Finally a lognormal distribution of the loss ratios is assumed at each iml and the vulnerability curve is defined at each iml by the mean and the standard deviation of all the computed loss ratios.

1.3.1.2. Multiple-Building Fragility and Vulnerability function

If multiple buildings have been input to derive a set of fragility curves for a class of buildings all $\hat{S}_{a,blg}$ and $\beta_{S_a,blg}$ are combined in a single lognormal curve for each damage state. A minimum of 5 buildings should be considered to obtain reliable results for the class. The procedure to get $\mu_{S_a,tot}$ and $cov_{S_a,tot}$ for the class of building is the same described in section **??**, but the $\hat{S}_{a,blg}$ and $\beta_{S_a,blg}$ are those derived from each sampled set of ductility limit state.

A single vulnerability curve can also be obtained, from the single building vulnerability curves. If no dispersion in the limit state is defined, the method is the same described in section **??**. Otherwise a vulnerability curve is derived for each building as explained in section **??**, considering the sampled set of ductility limit states, that is to say that the mean loss ratio and its standard deviation at each iml, $\mu_{LR,iml,blg}$ and $\sigma_{LR,iml,blg}$ respectively, are found for each building. Finally the mean loss ratio and its standard deviation, $\mu_{LR,iml}$ and $\sigma_{LR,iml}$, are found for the entire class of buildings as the weighted mean of the single $\mu_{LR,iml,blg}$ and the weighted SRSS of the inter-building and intra-building standard deviation, the standard deviation of the single means $\mu_{LR,iml,blg}$ and the single dispersions $\sigma_{LR,iml,blg}$ respectively, as described in eq **??**, substituting loss ratio to spectral acceleration.

1.3.2. Inputs

The inputs must be formatted as comma-separated value files (.csv), and saved in the folder *input*, contained in the NSP directory. If any other environment different from Windows is used make sure that the "comma separated values Windows" is selected as saving option when creating the input files.

If multiple buildings want to be analysed to consider the inter-building uncertainty the parameters relative to each building should be added as additional lines in the tables, as shown in the examples below, otherwise a single line must be input.

If the user has already at disposal an idealised multilinear pushover curve for each building, that is to say that the variable *in_type* has been set to 0, the following data need to be provided in the corresponding csv files:

1. First period of vibration T_1, corresponding modal participation factor Γ_1, normalised with respect to the roof displacement, and weight for the combination of different buildings, input in *building_parameters.csv*, as in section **??**, input n. 1.

2. Top displacement at each damage state threshold and corresponding dispersion $\beta_{\theta c}$ input in *displacement_profile.csv*, as in section **??**, input n. 2.

3. Idealised pushover curve, input in *idealised_curve.csv* as shown below. The parameters needed to describe the idealised pushover curve are: yielding displacement d_y, displacement at the onset of degradation d_s, displacement at the onset of residual force d_{min}, ultimate displacement d_u, maximum force F_{max}, residual force F_{min}. These parameters are represented in Figure **??**.

4. Consequence model (loss ratio per each damage state) consistent with the defined set of damage states, input in *consequence.csv*, as in section **??**, input n. 5.

If these data are not available, *in_type* $= 0$ can be selected and the "raw" results from a pushover analysis can be input instead. The same data as in section **??** for *in_type* $= 0$ can be input.

1.3.3. Calculation Steps

The overall workflow of spo2ida-based procedure is summarised in this section. The option *an_type* must be set equal to 1 and the option *in_type* according to the input at disposal. The corresponding inputs should follow the requirements described in section **??**. At this point the code proceeds with the following steps:

1. a) If *in_type* $= 0$, the roof displacement at each limit state and the idealised pushover curve parameters are extracted from *displacement_profile.csv* and *idealised_curve.csv* respectively.

   b) If *in_type* $= 1$ the results from a pushover analysis are extracted from *displacements_pushover.csv* and *reactions_pushover.csv* and drift limit states from limits.csv. The idealised pushover curve is then derived in the *idealisation* function, where the idealisation process is conducted according to the Gem Analytical Vulnerability Guidelines.

2. The parameters extracted are used to derive ductility levels μ_{ds}, median spectral acceleration $\hat{S}_{a,ds}$ and the total dispersion β_{S_a} at each damage threshold through the following steps:
   - The idealised MDoF system is transformed into an equivalent SDoF system, using Γ_1, and SDoF capacity curve is expressed in terms of μ-R.
   - The variables for spo2ida tool are extracted from the capacity curve and they are used as input to get the 16%-50%-84% ida curves.
   - The ductility levels μ_{ds} corresponding to each damage threshold are defined, and the corresponding $R_{16\%}$-$R_{50\%}$-$R_{84\%}$ are found in ida outputs.
   - $\hat{S}_{a,ds}$ and the corresponding dispersion $\beta_{S_{a,d}}$ are computed using eq. **??** and eq. **??**, respectively.
   - If dispersion due to uncertainty in the limit state $\beta_{\theta c}$ is different from zero different ductility limit states are sampled for each median ductility level μ_{ds} and corresponding values of $\hat{S}_{a,ds}$ and $\beta_{S_{a,d}}$ are computed, as described in section **??**, but not yet combined together.
3. All $\hat{S}_{a,ds}(T_1)$ are converted to mean $\mu_{ln(S_{a,ds})}(T_1)$ and then to the intensity measure in common with the rest of the buildings, $\mu_{ln(S_{a,ds}(T_{av}))}$, according to eq. **??**.
4. Step 2. and 3. are repeated for the number of input buildings.
5. a) If vulnerability $= 0$: All $\hat{S}_{a,ds}$ and $\beta_{S_{a,d}}$ from all the buildings and all the sampled ductility limit states are combined in a single lognormal curve, as described in section **??**. Mean $\mu_{ln(S_a)}$ and total dispersion β_{S_a} are then converted to logarithmic mean μ_{S_a} and logarithmic covariance cov_{S_a}, according to equations **??** and **??** respectively. Fragility curves for the class of buildings are displayed if the variable *plotflag*[2] $= 1$, and logarithmic μ_{S_a} and cov_{S_a} are exported in the *outputs* folder.

   b) If vulnerability $=1$: For the intensity measure levels defined in the variable *iml* a value of loss ratio $\mu_{LR,iml,blg}$ is defined for each building and a standard deviation $\sigma_{LR,iml,blg}$, if dispersion due to uncertainty in the limit state $\beta_{\theta c}$ is different from zero. They are finally combined in a single mean and standard deviationas described in section **??**. Vulnerability curve for the class of buildings is displayed if the variable *plotflag*[3] $= 1$, and μ_{LR} and cov_{LR} at each iml are exported in the *outputs* folder.

1.4. R-μ-T procedure

1.4.1. Theoretical background

The aim of this procedure is the estimation of the median spectral acceleration value $\hat{s}_c$, that brings the structure to the attainment of a set of damage states, and the corresponding dispersion beta β_{sc}, the parameters needed for the mathematical representation of fragility in equation **??**. The aim is achieved making use of a R-μ-T relationship, between the reduction factor R, the ductility μ and period T, which is based on the work of Dolsek and Fajfar (2004). The R-μ-T-based procedure presented herein is applicable to any kind of multi-linear capacity curve, and it is suitable for single building fragility curve estimation, as described in section **??**. However the fragility curves derived for single buildings can be combined in a unique fragility curve, which considers the inter-building uncertainty, as described in section **??**.

1.4.1.1. Single Building Fragility and Vulnerability function

The spectral value at each damage state threshold ds $\hat{S}_{a,ds}$ is found from the top displacement representing that ds attainment $\hat{\delta}_{roof,ds}$, as explained in C_R-based procedure and reported the following equation:

$$\hat{S}_{a,ds}(T_1) = \frac{4\pi^2}{\hat{C}_R T^2 \Gamma_1 \Phi_1} \hat{\delta}_{roof,ds} \tag{1.21}$$

The value of C_R, the ratio between the inelastic and the elastic spectral displacement, is found from equation **??**.

$$\hat{C}_R = \frac{\mu_{LS}}{R_{LS}} \tag{1.22}$$

where μ_{ds} and R_{ds} are the ductility level and the reduction factor at each ds attainment. According to the results of an extensive parametric study using three different sets of recorded and semi-artificial ground motions Dolsek and Fajfar (2004) related the ductility demand, μ , and reduction factor, R , through the following formula:

$$\mu = \frac{1}{c}(R - R_0) + \mu_0 \tag{1.23}$$

In the proposed model, μ is linearly dependent on R within two reduction factor intervals. The parameter c defines the slope of the Rμ relation, and depends on the initial period of the system T, the ratio r_u, the reduction factor R and the corner periods T_c and T_d. T_c and T_d are the corner periods between the constant acceleration and constant velocity part of the idealized elastic spectrum, and between the constant velocity and constant displacement part of the idealized elastic spectrum respectively.