

“OpenQuake: Calculate, share, explore”

OpenQuake Engine User Instruction Manual

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PART I

Risk Modeller's Toolkit

Nonlinear Static Method with dispersion information

Nonlinear Static Methods are based on the use of capacity curves resulting from nonlinear static pushover analysis to determine the median seismic intensity values \hat{s}_c corresponding to the attainment of a certain damage state threshold (limit state) and the corresponding dispersion β_{sc} . These parameters are used to represent a fragility curve as the probability of the limit state capacity C being exceeded by the demand D , both expressed in terms of intensity levels (s_c and s respectively), as shown in the following equation:

$$P_{LS}(s) = P(C < D|s) = \Phi\left(\frac{\ln s - \ln \hat{s}_c}{\beta_{sc}}\right) \quad (1.1)$$

The methodology implemented so far in the RMTK allows to consider different shapes of the pushover curve, multilinear and bilinear, record-to-record dispersion, dispersion in the damage state thresholds.

Different input types can be inserted depending on whether the user has already at his disposal an idealised pushover curve or it has to be derived from the raw results of a pushover analysis. Fragility and vulnerability functions can be derived for a single building or for a class of buildings, simply inserting data for many buildings in the inputs.

The intensity measure to be used is S_a and a mapping between any engineering demand parameter (EDP), assumed to describe the damage state thresholds, and the roof displacement should be available from the pushover analysis.

Ruiz-Garcia and Miranda (2007) study on inelastic displacement demand es-

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timation, and Vamvatsikos and Cornell (2006) work on seismic demand estimation with multilinear static pushover curves, have been integrated in two nonlinear static procedures, C_R -based or spo2ida-based, within the same script. In this way the user has the chance to select the procedure with the degree of accuracy consistent with the available input and the type of structural analyses performed.

In section 1.1 the main information necessary to start the analysis are presented. In section 1.2 and section 1.3 the two procedures are explained respectively, from the point of view of the necessary scientific background behind and their step-by-step implementation in the python script.

1.1. How to use the NSP

To start using the nonlinear static procedure with record-to-record variability a command line text editor should be used to enter manually the folder location where the RMTK has been saved. The user should add the path */RMTK/Vulnerability/NSP*, where the nonlinear static method script is located, as shown in the example below:

```
cd path/to/rmtk/folder/RMTK/Vulnerability/NSP
```

From the text editor iPython browser page can be opened with the following command line:

```
ipython-2.7 notebook --pylab=inline
```

Once the iPython page is opened on the browser, the python scripts contained in the NDP directory will be visible. The file *NSM.ipynb* should be selected to start the calculations.

In the initial section of the script "Define Options" the user needs to set the options and to enter the input corresponding to the defined options in the folder *NSP/input*. The main options are the following:

- Type of procedure to perform: either C_R -based or spo2ida-based. The main difference between the two is that C_R -based procedure is applicable to elasto-plastic idealised capacity curve only, while spo2ida-based procedure fits any kind of multilinear curve.

1.1 How to use the NSP

- Type of input: either displacements vs base shear at each time step or idealised pushover curve, as shown in Figures ?? and 1.1.
- Type of output: either fragility curve (probability of exceedance of a set of limit states vs seismic intensity, as shown in Figure XXX) or vulnerability curve (loss ratio vs seismic intensity, as shown in Figure XXY).

These options and others need to be defined in the initial section of the script "Define Options". In section 1.1.1 the alternatives values that the initial variables can assume and their meaning are described in detail, while the parameters to be inserted in the input files are listed below. They are fully described in section 1.2 and section 1.3, within the presentation of each of the two procedures, C_R -based and spo2ida-based.

- Results from a static pushover analysis: either displacements vs base shear at each time step, as shown in Figure ??, or idealised pushover curve, as shown in Figure 1.1.

Figure 1.1 – *Idealised pushover curve.*

- Dynamic parameters of the building: fundamental period of vibration T_1 and corresponding modal participation factor Γ_1 .
- Limit States in terms of roof displacement or inter-storey drift ratio.

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- Consequence function (damage factor for each damage state).

1.1.1. Options

The type of procedure to be performed and the type of inputs at disposal, are set with the variables *an_type* and *in_type* respectively. With the variable *an_type* the user can choose between:

```
an_type = 0 #Cr-based procedure (Ruiz-Garcia and Miranda, 2007)
an_type = 1 #spo2ida-based procedure (Vamvatsikos and Cornell, 2005)
```

With the variable *in_type* the user can choose between:

```
in_type = 0 # idealised pushover curve
in_type = 1 # raw results from a pushover analysis
```

The variable *vulnerability* instead gives the opportunity to decide the type of outputs, whether to stop the process at the derivation of the fragility curves, or to go all the way up to the vulnerability curve definition, applying damage-to-loss functions.

```
vulnerability = 0 # derive fragility curves
vulnerability = 1 # derive vulnerability curve
```

The variable *g* serves the purpose of defining the units that are being used. A floating number must be assigned to the gravity acceleration, compatible with the units used for the period of vibration and for the displacements (if the period is expressed in seconds and displacements are in meters, then $g = 9.81$). The variable *iml* is a numpy array that identifies the intensity measure levels for which loss ratios are computed and provided in the vulnerability curve.

```
g = 9.81
iml = np.linspace(0.1,15,100)
```

The variable *plotflag* allows or inhibits the displaying of plots. It is a python list composed of 4 integers, each one controlling a different plot: idealised pushover curve, 16%-50%-84% ida curves, fragility curves and vulnerability curve respectively. Each integer can take as value either zero or one, whether the corresponding graph has to be displayed or not:

```
plotflag = [1, 1, 1, 1] # plot all the graphs
plotflag = [0, 0, 0, 0] # do not plot any graph
```

The following variables set some of the characteristics of the plots:

- *linew*: integer for defining lines width.
- *fontsize*: fontsize used for labels, graphs etc.
- *units*: list of 3 strings defining displacements, forces and Spectral acceleration units, as ['[kN]', '[m]', '[m/s²']', to be displayed on the axes of the plots.

The last set of variables is needed for spo2ida-based procedure only:

- *pw*: floating number assigning pinching level (from 0 to 1)
- *filletstyle*: integer enabling or not the use of a spline line to link branches of IDA curves

```
filletstyle = 0 # don't use spline curve
filletstyle = 3 # use spline (recommended)
```

- *N*: number of points per segment of IDA curve derived with spo2ida
- *MC*: number of Monte Carlo simulations to account for uncertainty in damage thresholds

1.2. C_R -based procedure

1.2.1. Theoretical background

The aim of this procedure, proposed by Vamvatsikos (2014), is the estimation of the median spectral acceleration value \hat{s}_c , that brings the structure to the attainment of a set of damage states, and the corresponding dispersion beta β_{sc} , the parameters needed for the mathematical representation of fragility in equation 1.1. The aim is achieved making use of the work by Ruiz-Garcia and Miranda (2007), where the inelastic displacement demand is related to the elastic displacement with a simple relationship, and it can thus be easily estimated through a response spectrum analysis and a capacity curve.

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The C_R -based procedure presented herein is applicable to bilinear elasto-plastic capacity curve only, and it is suitable for single building fragility curve estimation, as described in section 1.2.1.1. However the fragility curves derived for single buildings can be combined in a unique fragility curve, which considers the inter-building uncertainty, as described in section 1.2.1.2.

1.2.1.1. Single Building Fragility and Vulnerability function

This procedure provides a simple relationship between median damage state threshold, expressed in terms of top displacement δ_{roof} , at each damage state threshold ds , and the corresponding median elastic Spectral displacement value $\hat{S}_{d,ds}(T_1)$.

$$\hat{\delta}_{roof,ds} = C_R S_{d,ds}(T_1) \Gamma_1 \Phi_1 \quad (1.2)$$

where $\Gamma_1 \Phi_1$ is the first mode participation factor estimated for the first-mode shape normalised by the roof displacement, and C_R is the inelastic displacement ratio (inelastic over elastic spectral displacement), computed by Ruiz-Garcia and Miranda (2007) for nonlinear SDoF systems having hysteretic behaviour representative of the analysed structure, which is a function of the first-mode period of vibration and the relative lateral strength of the system R . Therefore the median Spectral acceleration at the fundamental period of vibration $\hat{S}_{a,ds}(T_1)$ turns out to be expressed as a function of the roof displacement according to the following equation:

$$\hat{S}_{a,ds}(T_1) = \frac{4\pi^2}{\hat{C}_R T^2 \Gamma_1 \Phi_1} \hat{\delta}_{roof,ds} \quad (1.3)$$

Default values of \hat{C}_R parameter estimates are provided by Ruiz-Garcia and Miranda (2007), as result of nonlinear regression analysis of three different measures of central tendency computed from 240 ground motions:

$$\hat{C}_R = 1 + \frac{R - 1}{79.12 T_1^{1.98}} \quad (1.4)$$

and values for R are given as:

$$R_{ds} = \max(0.425(1 - c + \sqrt{c^2 + 2c(2\mu_{ds} - 1) + 1}), 1) \quad (1.5)$$

1.2 C_R -based procedure

where $c = 79.12 T^{1.98}$, and μ_{ds} is the ductility at the damage state threshold of interest.

For what concerns the dispersion of $\hat{S}_{a,ds}$, β_{sc} , the following relationship between S and the median EDP damage threshold $\hat{\theta}$ (Cornell, 2002) is used:

$$\hat{\theta}(s) = aS^b \quad (1.6)$$

so that β_{sc} can be easily derived from the dispersion of θ due to record-to-record variability, $\beta_{\theta d}$, as in the following:

$$\beta_{sc} = \frac{1}{b} \beta_{\theta d} \quad (1.7)$$

The dispersion of θ due to record-to-record variability, $\beta_{\theta d}$ can be easily combined with the dispersion of θ due to uncertainty in the damage state threshold $\beta_{\theta c}$ as shown in the following equation:

$$\beta_{sc} = \frac{1}{b} \sqrt{\beta_{\theta d}^2 + \beta_{\theta c}^2} \quad (1.8)$$

The dispersion of θ can be obtained assuming that d_{roof} and θ are proportional, and they thus share the same dispersion. Moreover the dispersion of d_{roof} is the same as the dispersion of C_R , since they are also proportional. Finally $\beta_{\theta d}$ can be computed with the following equation, which represents Ruiz-Garcia and Miranda's (2007) estimate of C_R dispersion:

$$\sigma_{\ln(C_R)} = \sigma_{\ln(d_{roof})} = \beta_{\theta d} = 1.975 \left[\frac{1}{5.876} + \frac{1}{11.749(T + 0.1)} \right] [1 - \exp(-0.739(R - 1))] \quad (1.9)$$

To derive a discrete vulnerability function at certain intensity measure levels, the input damage-to-loss factors are applied to the probability of occurrence of each damage state, extracted from the probability of exceedance of each damage state described by the fragility function. A value of loss ratio is thus defined for the vector of selected intensity measure levels.

1.2.1.2. Multiple-Building Fragility and Vulnerability function

If multiple buildings have been input to derive fragility curves for a class of buildings all $\mu_{S_a,blg}$ and $\sigma_{S_a,blg}$ are combined in a single lognormal curve. A minimum of 5 buildings should be considered to obtain reliable results for the class. Medians $\hat{S}_{a,ds}$ and total dispersions β_{sc} are converted to logarithmic mean μ_{S_a} and logarithmic standard deviation σ_{S_a} , according to the following equations:

$$\mu_{S_a} = \hat{S}_a e^{\frac{\beta_{sc}^2}{2}} \quad (1.10)$$

$$\sigma_{S_a} = \sqrt[2]{(\beta_{sc}^2 - 1)e^{2 \ln \hat{S}_a + \beta_{sc}^2}} \quad (1.11)$$

A new issue arises when multiple buildings are considered: the S_a at the fundamental period of each building should be converted to a common intensity measure, to be able to combine the different fragility functions. A common intensity measure is selected to be S_a at the period T_{av} , which is a weighted average of the individual buildings fundamental period T_1 . Then each individual fragility needs to be expressed in terms of the common $S_a(T_{av})$, using a spectrum. FEMA P-695 far field set of 22 accelerograms was used to derive a mean uniform hazard spectrum, and the ratio between the S_a at different periods is used to scale the fragility functions. It can be noted that the actual values of the spectrum are not important, but just the spectral shape. Given the linear transformation, the parameters of the scaled fragility curves of each building are simply:

$$\mu_{S_a,blg} = \mu_{S_a,blg} S(T_{av}) / S(T_{1,blg}) \quad (1.12)$$

$$\sigma_{S_a,blg} = \sigma_{S_a,blg} S(T_{av}) / S(T_{1,blg}) \quad (1.13)$$

Finally the parameters of the single lognormal curve for the class of building, logarithmic mean and logarithmic standard deviation, can be computed as the weighted mean of the single means and the SRSS of the inter-building and intra-building standard deviation, the weighted standard deviation of the single means, and the weighted mean of the single standard deviations, respectively, as shown in the following equations:

$$\mu_{S_a,tot} = \sum_{i=0}^{n.blg} w_{blg-i} \mu_{S_a,blg-i} \quad (1.14)$$

$$\sigma_{S_a,tot} = \sqrt{\sum_{i=0}^{n.blg} w_{blg-i} (\mu_{S_a,blg-i} - \mu_{S_a,tot})^2 + \left(\sum_{i=0}^{n.blg} w_{blg-i} \sigma_{S_a,blg-i} \right)^2} \quad (1.15)$$

A single vulnerability function can be also obtained, from the single building vulnerability functions. The input damage-to-loss function is applied to the fragility function derived for each building. For the selected intensity measure levels a value of loss ratio is thus defined for each building. Finally a lognormal distribution of the loss ratios is assumed at each iml and the discrete vulnerability function for the entire class of buildings is represented at each iml by the weighted mean and the weighted standard deviation of all the computed loss ratios.

1.2.2. Inputs

The inputs must be formatted as comma-separated value files (.csv), and saved in the folder *input*, contained in the NSP directory. If any other environment different from Windows is used make sure that the "comma separated values Windows" is selected as saving option when creating the input files.

If multiple buildings want to be analysed to consider the inter-building uncertainty the parameters relative to each building should be added as additional lines in the input tables, as shown in the examples below, otherwise a single line must be input.

If the user has already at disposal an idealised elasto-plastic pushover curve for each building, that is to say that the variable *in_type* has been set to 0, the following data need to be provided in the corresponding csv files:

1. First period of vibration T_1 , corresponding modal participation factor Γ_1 , normalised with respect to the roof displacement, and weight for the combination of different buildings, input in *building_parameters.csv*, as in the example below:

n.building	T_1	Γ_1	weights
1	0.32	1.23	0.2
2	0.40	1.25	0.3
...

2. Roof displacement at each limit state LS and corresponding dispersion $\beta_{\theta c}$

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input in *displacement_profile.csv*, as shown in the example below. If dispersion is unknown, $\beta_{\theta c}$ can be set equal to zero at each LS.

n.building	LS ₁	LS ₂	LS ₃
1	0.066	0.169	0.23
$\beta_{\theta d,1}$	0.1	0.3	0.4
2	0.08	0.172	0.25
$\beta_{\theta d,2}$	0.1	0.3	0.4

- Idealised pushover curve, input in *idealised_curve.csv* as shown below. The only required parameters are the yielding displacement δ_y , the ultimate displacement δ_u and the yielding force F_y .

n.building	d_y	d_u	F_y
1	0.09	0.3	523
2	0.12	0.35	400
...

- Consequence model (loss ratio per each damage state) consistent with the defined set of damage states, input in *consequence.csv*, as in the example below. A single consequence model can be input. This input is needed only if the variable *vulnerability* has been set to 1.

DS ₁	DS ₂	DS ₃
0.2	0.5	1

If the idealised curve are not available, *in_type* = 0 can be selected and the displacements vs base shear at each time step results from a pushover analysis can be input instead. The following data need to be provided in the corresponding csv files:

- T_1 and corresponding Γ_1 , weight for the combination of different buildings, number of storeys and height of each storey, input in *building_parameters.csv*, as in the example below:

n.building	T_1	Γ_1	weights	n.Storey	H ₁	H ₂	...	H _n
1	0.32	1.23	0.2	4	3	3	...	3
2	0.40	1.25	0.3	4	4	2.7	...	2.7
...

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2. Displacements at each storey, at each incremental step of the pushover analysis, input in *displacements_pushover.csv*, as in the example below:

n.building	n.Storey	Step1	Step 2	Storey 3	...	Step n
1	1	0.0001	0.0005	0.001	...	0.01
	2	0.0003	0.0010	0.002	...	0.02
	3	0.0004	0.0016	0.003	...	0.03
	4	0.0006	0.0021	0.004	...	0.04
2	1	0.0001	0.0005	0.001	...	0.01
	2	0.0005	0.0012	0.002	...	0.03

3. Base shear at each incremental step of the pushover analysis input in *reactions_pushover.csv*, as in the example below:

n.building	Step1	Step 2	Storey 3	...	Step n
1	0.35	0.69	1.04	...	29.12
2	0.45	0.78	2.05	...	40.00
...

4. Drift limit state and corresponding dispersion $\beta_{\theta c}$ input in *limits.csv*. If dispersion is unknown, $\beta_{\theta c}$ can be set equal to zero at each limit state.

n.building	LS ₁	LS ₂	LS ₃
1	0.01	0.025	0.0337
$\beta_{\theta d,1}$	0.1	0.2	0.25
2	0.014	0.030	0.0430
$\beta_{\theta d,2}$	0.1	0.2	0.25

5. Consequence model (loss ratio per each damage state) consistent with the defined set of damage states, input in *consequence.csv*, as in the example below. A single consequence model can be input. This input is needed only if the variable *vulnerability* has been set to 1.

DS ₁	DS ₂	DS ₃
0.2	0.5	1

1.2.3. Calculation Steps

The overall workflow of C_R -based procedure is summarised in this section. The option *an_type* must be set equal to 0 and the option *in_type* according to the input at disposal. The corresponding inputs should follow the requirements described in section 1.2.2. At this point the code proceeds with the following steps:

1.
 - a) If *in_type* = 0 roof displacement at limit states and idealised pushover are extracted from *displacement_profile.csv* and *idealised_curve.csv* respectively.
 - b) If *in_type* = 1 results from pushover analysis are extracted from *displacements_pushover.csv* and *reactions_pushover.csv*, and drift limit states from *limits.csv*. The idealised pushover curve is then derived in the *idealisation* function, where the idealisation process is conducted according to FEMA-440. The elastic stiffness is defined as the tangent stiffness passing through the point of the pushover curve where 60% of the maximum base shear is reached, and the perfectly plastic branch is set at an height equal to the maximum base shear. The yielding point is found as the interception between the elastic and the plastic branch.
2. The csv input files are parsed with the function *read_data* according to the defined options. The parameters essential to the analysis are return together with a graphical visualisation of the inputs if the variable *plotflag*[0] is equal to 1.
3. The parameters extracted are used in the *simplified_bilinear* function to derive ductility levels μ_{ds} , median spectral acceleration $\hat{S}_{a,ds}$ and the total dispersion β_{sc} at each limit state through the following steps:
 - The idealised MDoF system is transformed into an equivalent SDoF system, using Γ_1 .
 - Ductility levels μ_{ds} corresponding to each damage threshold, are defined.
 - R and C_R are computed, using eq. 1.5 and 1.4 respectively.
 - $\hat{S}_{a,ds}$ and the corresponding dispersion $\beta_{\theta d}$ are computed using eq. 1.3 and 1.7 respectively.
 - $\beta_{\theta d}$ is combined with dispersion due to uncertainty in the model $\beta_{\theta c}$, if different from zero, to get the total dispersion β_{sc} , using eq. 1.8.

- Medians $\hat{S}_{a,ds}$ and total dispersions β_{sc} are converted to logarithmic mean μ_{S_a} and logarithmic standard deviation σ_{S_a} , according to equations 1.10 and 1.11 respectively.
 - $S_a(T_1)$ is converted to the intensity measure in common with the rest of the buildings, $S_a(T_{av})$, according to eq. 1.13
4. Step 3. is repeated for the number of input buildings.
 5. a) If vulnerability = 0: All $\mu_{S_a,blg}$ and $\sigma_{S_a,blg}$ are combined in a single lognormal curve, whose parameters mean and standard deviation are evaluated according to equations 1.14 and 1.15. Fragility curves for the class of buildings are displayed if the variable $plotflag[2] = 1$, and logarithmic μ and σ are exported in the *outputs* folder.
 - b) If vulnerability =1: Vulnerability curves are derived for each fragility function derived at step 4. For the intensity measure levels defined in the variable *iml* a value of loss ratio is thus defined for each building. They are finally combined in a single mean and its standard deviation, equal to the weighted mean and the weighted standard deviation of the loss ratios, assuming a lognormal distribution, as described in section 1.2.2. Vulnerability curve for the class of buildings is displayed if the variable $plotflag[3] = 1$, and logarithmic μ and σ at each *iml* are exported in the *outputs* folder.

1.3. Spo2ida procedure

1.3.1. Theoretical background

The aim of this procedure is the estimation of the median spectral acceleration value \hat{s}_c , that brings the structure to the attainment of a set of damage states, and the corresponding dispersion beta β_{sc} , the parameters needed for the mathematical representation of fragility in equation 1.1. The aim is achieved making use of the tool *spo2ida* (Vamvatsikos and Cornell, 2006), where static pushover curves are converted into 16%, 50% and 84% IDA curves, using empirical relationships from a large database of incremental dynamic analysis results, as shown in Figure 1.2.

Figure 1.2 – *spo2ida* tool: IDA curves derived from Pushover curve.

The spo2ida-based procedure presented herein is applicable to any kind of multi-linear capacity curve, and it is suitable for single building fragility curve estimation, as described in section ???. However the fragility curves derived for single buildings can be combined in a unique fragility curve, which considers the inter-building uncertainty, as described in section ??.

1.3.1.1. Single-building Fragility and Vulnerability function

Given the idealised capacity curve the spo2ida tool uses an implicit $R - \mu - T$ relation to correlate nonlinear displacement, expressed in terms of ductility μ to the corresponding median capacity in terms of the parameters R . R is the lateral strength ratio, defined as the ratio between the spectral acceleration S_a and the yielding capacity of the system S_{ay} .

Each branch of the capacity curve, hardening, softening and residual plateau, is converted to a corresponding branch of the three ida curves, using the $R - \mu - T$ relation, which is a function of the hardening stiffness, the softening stiffness and the residual force. These parameters are derived from the idealised pushover capacity expressed in μ - R terms, as well as the ductility levels at the onset of each branch. If some of the branches of the pushover curve are missing because of the seismic behaviour of the system, spo2ida can equally work with

bilinear, trilinear and quadrilinear idealisations.

The result of the spo2ida routine is thus a list of ductility levels and corresponding R values at 50%, 16% and 84% percentiles. The distribution of R values at each ductility level, due to the record-to-record variability, is assumed to be lognormal and it can be easily converted to the dispersion of the lognormal distribution with the following equation:

$$\beta_{R(\mu)} = \frac{\ln R(\mu)_{84\%} - \ln R(\mu)_{16\%}}{2} \quad (1.16)$$

Median R and its dispersion at ductility levels corresponding to the damage thresholds can thus be determined, and $\hat{S}_{a,ds}$ can be easily extracted simply multiplying $R_{50\%}(\mu_{ds})$ by the yielding capacity of the system S_{ay} , as shown in the following equation:

$$\hat{S}_{a,ds} = R_{50\%}(\mu_{ds})S_{ay} \quad (1.17)$$

$$S_{ay} = \frac{4\pi^2\delta_{roof,y}}{g\Gamma_1 T_1^1} \quad (1.18)$$

Since \hat{R} and \hat{S}_a are proportional they share the same dispersion.

If dispersion due to uncertainty in the limit state definition β_{θ_c} is different from zero it can not be combined directly with the record-to-record dispersion, but a Monte Carlo sampling of the limit state needs to be performed instead. Different values of ductility limit state are sampled from the lognormal distribution with median the median value of the ductility limit state, and dispersion the input β_{θ_c} . For each of these ductilities the corresponding $R_{16\%}$ - $R_{50\%}$ - $R_{84\%}$ are found and converted into $\hat{S}_{a,ds}$ and β_{θ_d} according to equation 1.17 and 1.16. N random S_a corresponding to the N sampled ductility limit states are computed, and their median and the dispersion are estimated. These parameters constitute the median $\hat{S}_{a,ds}$ and the total dispersion β_{total} for the considered damage state. The procedure is repeated for each damage state.

To derive a discrete vulnerability function at certain intensity measure levels, the input damage-to-loss factors are applied to the probability of occurrence of each damage state, extracted from the probability of exceedance of each damage state described by the set of fragility curves.

If dispersion due to uncertainty in the limit state is different from zero a vulnerability function is derived for the N sets of sampled ductility limit states.

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It results in N loss ratios for each defined intensity measure levels. Finally a lognormal distribution of the loss ratios is assumed at each iml and the vulnerability curve is defined at each iml by the mean and the standard deviation of all the computed loss ratios.

1.3.1.2. Multiple-Building Fragility and Vulnerability function

If multiple buildings have been input to derive a set of fragility curves for a class of buildings all $\mu_{S_a,blg}$ and $\sigma_{S_a,blg}$ are combined in a single lognormal curve for each damage state. A minimum of 5 buildings should be considered to obtain reliable results for the class. The procedure to get $\mu_{S_a,tot}$ and $\sigma_{S_a,tot}$ for the class of building is the same described in section 1.2.1.2, but the $\mu_{S_a,blg}$ and $\sigma_{S_a,blg}$ are those derived from each sampled set of ductility limit state.

A single vulnerability curve can also be obtained, from the single building vulnerability curves. If no dispersion in the limit state is defined, the method is the same described in section 1.2.1.2. Otherwise a vulnerability curve is derived for each building as explained in section 1.3.1.1, considering the sampled set of ductility limit states, that is to say that the mean loss ratio and its standard deviation at each iml, $\mu_{LS,iml,blg}$ and $\sigma_{LS,iml,blg}$ respectively, are found for each building. Finally the mean loss ratio and its standard deviation, $\mu_{LS,iml}$ and $\sigma_{LS,iml}$, are found for the entire class of buildings as the weighted mean of the single $\mu_{LS,iml,blg}$ and the SRSS of the inter-building and intra-building standard deviation, the weighted standard deviation of the single $\mu_{LS,iml,blg}$, and the weighted mean of the single $\sigma_{LS,iml,blg}$.

1.3.2. Inputs

The inputs must be formatted as comma-separated value files (.csv), and saved in the folder *input*, contained in the NSP directory. If any other environment different from Windows is used make sure that the "comma separated values Windows" is selected as saving option when creating the input files.

If multiple buildings want to be analysed to consider the inter-building uncertainty the parameters relative to each building should be added as additional lines in the tables, as shown in the examples below, otherwise a single line must be input.

If the user has already at disposal an idealised multilinear pushover curve for each building, that is to say that the variable *in_type* has been set to 0, the fol-

lowing data need to be provided in the corresponding csv files:

1. First period of vibration T_1 , corresponding modal participation factor Γ_1 , normalised with respect to the roof displacement, and weight for the combination of different buildings, input in *building_parameters.csv*, as in section 1.2.2, input n. 1.
2. Top displacement at each damage state threshold and corresponding dispersion $\beta_{\theta c}$ input in *displacement_profile.csv*, as in section 1.2.2, input n. 2.
3. Idealised pushover curve, input in *idealised_curve.csv* as shown below. The parameters needed to describe the idealised pushover curve are: yielding displacement d_y , displacement at the onset of degradation d_s , displacement at the onset of residual force d_{min} , ultimate displacement d_u , maximum force F_{max} , residual force F_{min} . These parameters are represented in Figure 1.3.

Figure 1.3 – Idealisation of capacity curve using multilinear elasto-plastic form.

n.building	d_y	d_s	d_{min}	d_u	F_{max}	F_{min}
1	0.09	0.3	a	b	523	430
2	0.12	0.35	a	b	400	305

4. Consequence model (loss ratio per each damage state) consistent with the

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defined set of damage states, input in *consequence.csv*, as in section 1.2.2, input n. 5.

If these data are not available, *in_type* = 0 can be selected and the "raw" results from a pushover analysis can be input instead. The same data as in section 1.2.2 for *in_type* = 0 can be input.

1.3.3. Calculations

The overall workflow of spo2ida-based procedure is summarised in this section. The option *an_type* must be set equal to 1 and the option *in_type* according to the input at disposal. The corresponding inputs should follow the requirements described in section 1.3.2. At this point the code proceeds with the following steps:

1. a) If *in_type* = 0, the roof displacement at each limit state and the idealised pushover curve parameters are extracted from *displacement_profile.csv* and *idealised_curve.csv* respectively.
b) If *in_type* = 1 the results from a pushover analysis are extracted from *displacements_pushover.csv* and *reactions_pushover.csv* and drift limit states from *limits.csv*. The idealised pushover curve is then derived in the *idealisation* function, where the idealisation process is conducted according to the Gem Analytical Vulnerability Guidelines.
2. The parameters extracted are used to derive ductility levels μ_{ds} , median spectral acceleration $\hat{S}_{a,ds}$ and the total dispersion β_{sc} at each damage threshold through the following steps:
 - The idealised MDoF system is transformed into an equivalent SDoF system, using Γ_1 , and SDoF capacity curve is expressed in terms of μ -R.
 - The variables for spo2ida tool are extracted from the capacity curve and they are used as input to get the 16%-50%-84% ida curves.
 - The ductility levels μ_{ds} corresponding to each damage threshold are defined, and the corresponding $R_{16\%}$ - $R_{50\%}$ - $R_{84\%}$ are found in ida outputs.
 - $\hat{S}_{a,ds}$ and the corresponding dispersion $\beta_{\theta d}$ are computed using eq. 1.17 and eq. 1.16, respectively.

- Medians $\hat{S}_{a,ds}$ and total dispersions β_{sc} are converted to logarithmic mean μ_{S_a} and logarithmic standard deviation σ_{S_a} , according to equation 1.10 and 1.11, respectively.
 - $S_a(T_1)$ is converted to the intensity measure in common with the rest of the buildings, $S_a(T_{av})$, according to eq. 1.13.
 - If dispersion due to uncertainty in the limit state β_{θ_c} is different from zero different ductility limit states are sampled and corresponding values of $\hat{S}_{a,ds}$ and β_{θ_d} are computed, as described in section 1.3.1.1, but not yet combined together.
3. Step 2. is repeated for the number of input buildings.
4. a) If vulnerability = 0: All $\hat{S}_{a,ds}$ and β_{θ_d} from all the buildings and all the sampled ductility limit states are combined in a single lognormal curve, as described in section 1.3.1.1. Fragility curves for the class of buildings are displayed if the variable $plotflag[2] = 1$, and logarithmic μ and σ are exported in the *outputs* folder.
- b) If vulnerability = 1: For the intensity measure levels defined in the variable *iml* a value of loss ratio is defined for each building and a standard deviation dispersion due to uncertainty in the limit state β_{θ_c} is different from zero. They are finally combined in a single mean and standard deviation, equal to the mean of the loss ratios, and a standard deviation, equal to the SRSS of the inter-building and intra-building standard deviation, as described in section 1.3.1.2. Vulnerability curve for the class of buildings is displayed if the variable $plotflag[3] = 1$, and logarithmic μ and σ at each *iml* are exported in the *outputs* folder.

1.4. R- μ -T procedure

1.4.1. Theoretical background

The aim of this procedure is the estimation of the median spectral acceleration value \hat{s}_c , that brings the structure to the attainment of a set of damage states, and the corresponding dispersion beta β_{sc} , the parameters needed for the mathematical representation of fragility in equation 1.1. The aim is achieved making use of a R- μ -T relationship, between the reduction factor R, the ductility μ and

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period T , which in this case is based on the work of Dolsek and Fajfar (2004). The R - μ - T -based procedure presented herein is applicable to any kind of multi-linear capacity curve, and it is suitable for single building fragility curve estimation, as described in section 1.4.1.1. However the fragility curves derived for single buildings can be combined in a unique fragility curve, which considers the inter-building uncertainty, as described in section ??.

1.4.1.1. Single Building Fragility and Vulnerability function

Nonlinear Dynamic Method

2.1. Nonlinear Dynamic Method for a class of buildings

Nonlinear Dynamic Methods are based on the results of many dynamic analyses, which relate the seismic response of a structure, represented by an Engineering Demand Parameter (EDP), like maximum top displacement, maximum inter-storey drift ratio, maximum top drift etc., to the Intensity Measure Level (IML) of the input accelerograms. Many methods exist in literature to perform a series of dynamic analysis and to post-process the results in order to derive fragility curves. Some of them treat a single building to estimate directly the median seismic intensity value corresponding to the attainment of different damage state threshold (limit state), and the corresponding dispersion (Vamvatsikos and Cornell, 2002, Ellingwood and Kinali, 2009). Others treat a class of buildings, and lead to the evaluation of the probabilities of different damage states for a series of IMLs and thus to the set up of a damage probability matrix (Singhal and Kiremidjian, 1996, Silva et al., 2013).

The last approach have been implemented in the DPM-based procedure, explained in section 2.2, from the point of view of the necessary scientific background behind and their step-by-step implementation in the python script.

2.2. DPM-based procedure

2 Nonlinear Dynamic Method

2.2.1. How to use the NDM

To start using the nonlinear dynamic method a command line text editor should be used to enter manually the folder location where the RMTK has been saved. The user should add the path */RMTK/Vulnerability/NDP*, where the nonlinear dynamic method script is located, as shown in the example below:

```
cd path/to/rmtk/folder/RMTK/Vulnerability/NDP
```

From the text editor iPython browser page can be opened with the following command line:

```
ipython-2.7 notebook --pylab=inline
```

Once the iPython page is opened on the browser, the python scripts contained in the NDP directory will be visible. The file *NDM.ipynb* should be selected to start the calculations.

In the initial section of the script "Define Options" the user needs to set the options and to enter the input corresponding to the defined options in the folder *NDP/input*. In section 2.2.3 the alternatives values that the initial variables can assume and their meaning are described in detail, while the parameters to be inserted in the input files are fully described in section 2.2.4.

2.2.2. Theoretical background

This procedure performs the post-processing part of a set of dynamic analyses to first assemble a damage probability matrix, and then use this data for the derivation of a fragility function. Therefore the results of a set of dynamic analyses previously run have to be input to start the process. A list of intensity measure associated to each accelerogram, and corresponding EDP for each structure of the building class can be input, along with the set of limit states expressed in terms of the same EDP. The EDPs resulting from the dynamic analyses are compared with the limit state displacements and a global damage state is assigned to each structure. Thus, for each record, the number of frames in each damage state can be obtained. The distribution of buildings in each damage state is organised in the damage probability matrix, with a number of rows equal to the number of ground motion records and a number of columns equal to the number of damage states.

The processing of the data continue with the estimation of the cumulative fraction of structures in each damage state, summing the percentages of frames belonging to all the subsequent damage states. A lognormal cumulative distribution function, expressing the probability of exceeding each damage state in a continuous fashion, is then fit to these results, leading to the statistical parameters of the fragility curves. The regression analysis is carried out using the maximum likelihood method.

This function have the advantage of accounting for the record-to-record variability by the use of many ground motion records, and the inter-building variability subjecting to the same set of accelerograms hundreds of structures representing the entire building class.

To derive a discrete vulnerability function at certain intensity measure levels, the input damage-to-loss factors are applied to the probability of occurrence of each damage state, extracted from the probability of exceedance of each damage state described by the fragility function. For the vector of selected intensity measure levels a value of loss ratio is thus defined.

2.2.3. Options

In the Options the user has to define first of all the type of inputs at disposal, setting the variable *in_type*, choosing between entering a damage count matrix, which corresponds to a damage probability matrix, where the probabilities of each damage state are substituted by the count of buildings in that damage state, or IML and corresponding EDPs for each dynamic analysis

```
in_type = 0 # damage count matrix
in_type = 1# IMLs and EDPs
```

The variable *vulnerability* instead gives the opportunity to decide the type of outputs, whether to stop the process at the derivation of the fragility curves, or to go all the way up to the vulnerability curve definition, applying damage-to-loss functions.

```
vulnerability = 0 # derive fragility curves
vulnerability = 1 # derive vulnerability curve
```

The variable *iml* is a numpy array that identifies the intensity measure levels for which loss ratios are computed and provided in the vulnerability curve.

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```
iml = np.linspace(0.1,15,100)
```

The variable *plotflag* allows or inhibits the displaying of plots. It is a python list composed of 2 integers, each one controlling a different plot: fragility and vulnerability function respectively. Each integer can take as value either zero or one, whether the corresponding graph has to be displayed or not:

```
plotflag = [1, 1] # plot all the graphs  
plotflag = [0, 0] # do not plot any graph
```

The following variables set some of the characteristics of the plots:

- IMlabel: list of one strings defining the IM on the x axis as ['Sa(Tel)-m/s²']
- linewidth: integer for defining lines width.
- fontsize : fontsize used for labels, graphs etc.

2.2.4. Inputs

The inputs must be formatted as comma-separated value files (.csv), and saved in the folder *input*, contained in the NDP directory. If any other environment different from Windows is used make sure that the "comma separated values Windows" is selected as saving option when creating the input files.

Two types of input can be entered, whether the results of the set of dynamic analyses performed have already been organised in a damage probability matrix for or not. In the former case the variable *in_type* should be set to 0 and the damage count matrix should be input in the csv file *dcm.csv*. The first two columns refer to the number of record and the corresponding intensity measure level, the following columns report the number of buildings in each damage state, as shown in the following table:

n.records	Intensity Measure Level	DS ₀	DS ₁	DS ₂
1	49.852	30	16	54
2	47.056	54	15	31
3	33.012	59	10	31
4	82.125	24	26	50
...
5	37.499	58	5	37

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In the latter case the variable *in_type* should be set to 1, and the results of the set of dynamic analyses should be entered in the *edp.csv* file in the following fashion: number of record, corresponding IML, and corresponding EDPs for each building subjected to that record.

n.records	IML	$\text{edp}_{blg,1}$	$\text{edp}_{blg,2}$...	$\text{edp}_{blg,n}$
1	69.209	-0.00069	0.00031	...	0.00131
2	75.470	0.00102	0.00202	...	0.00302
3	62.233	-0.00090	0.00010	...	0.00110
4	168.47	-0.00246	-0.00146	...	-0.00046
5	67.612	0.00095	0.00195	...	0.00295
...
n	34.484	0.00036	0.00136	...	0.00236

In this case also the limit states must be input in the *limits.csv* file. Each line of the file corresponds to the limit states of a building, but if all the buildings share the same limits a single line can be input.

n.building	LS ₁	LS ₂	LS ₃	LS ₄
1	0.003	0.010	0.025	0.0337
2	0.004	0.015	0.020	0.035
3	0.002	0.019	0.027	0.032
...
n	0.0024	0.016	0.025	0.03

2.2.5. Calculations

The overall workflow of the DPM-based procedure is summarised in this section. The option *in_type* should be set according to the input at disposal. The corresponding inputs should follow the requirements described in section ???. At this point the code proceeds with the following steps:

1. In the function *read_data* the inputs are read and the damage count matrix is returned.
 - a) If *in_type* = 0, the damage count matrix is extracted directly from the *dcm.csv* file.
 - b) If *in_type* = 1 the IMLs of the records used in the dynamic analyses and the corresponding EDPs are extracted from *edp.csv* and the limit

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states for each building, expressed in terms of the same EDP, are extracted from *limits.csv*. These data are converted into a damage count matrix according to the method described in section 2.2.2.

2. The parameters extracted are used to derive the Probability of Exceedance (PoE) of each limit state for each IML, as described in section 2.2.2.
3. The PoEs are fitted with a lognormal function using the maximum likelihood method. The mean μ_{lnIML} and standard deviation σ_{lnIML} of the corresponding normal distribution for the entire class of buildings are found for each limit state.
4. The μ_{lnIML} and σ_{lnIML} are converted to logarithmic μ_{IML} and σ_{IML} . The fragility curves for the class of buildings are displayed if the variable *plotflag*[0] = 1, and the logarithmic μ_{IML} and σ_{IML} are exported in the *outputs* folder.
5. If vulnerability =1: For the intensity measure levels defined in the variable *iml* a value of loss ratio is defined, according to section 2.2.2. A vulnerability curve for the class of buildings is displayed if the variable *plotflag*[1] = 1, and the loss ratios at each *iml* are exported in the *outputs* folder.