

A segmented and observable Yield Curve

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Dynamic term structure models

Term Structure of Interest rates Intro

- ▶ Central banks.
 - ▶ Expectations of market participants on changes in interest rates.
 - ▶ General shape of the curve.
- ▶ Academic finance, practitioners.
 - ▶ Term and risk premiums (non-observable).
 - ▶ Reference curves for pricing (tracking and curve fitting).

Approaches

- ▶ Statistical (parametric and spline-based).
 - ▶ Arbitrage restrictions are not enforced.
 - ▶ Good statistical representation of the data (forecasting performance).
- ▶ Structural, Affine term structure models.
 - ▶ Impose no-arbitrage restrictions.
 - ▶ Poor forecasting performance.

Statistical Dynamic term structure models

- ▶ Three factor exponential model, Nelson and Siegel (1987)
- ▶ Four factor extension, Sveenson (1994)
- ▶ Dynamic extension, Diebold and Li (2006)

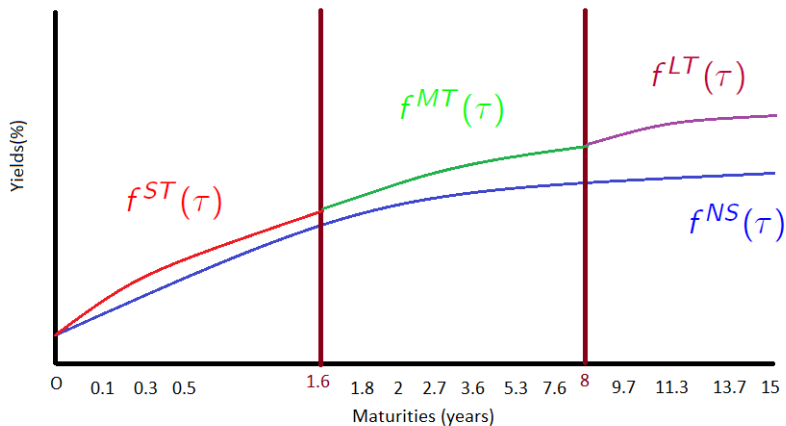
Factor loading's are exponential functions with 3 to 4 time varying factors estimated using the complete set of existing maturities.

Segmented term structure models, Almeida et al. 2018

- ▶ Preferred habitat theory, local shocks influence interest rates for each maturities.
- ▶ Equilibrium yields determined by supply and demand of different types of investors along the curve.
- ▶ At the same time investors act as arbitrators guaranteeing a smooth curve

A class of models that separate the yield curve into segments, which their own local shocks, but which are simultaneously connected.

Segmented term structure models



Estimation

The yield curve is estimated over N maturities, τ_i $i \in [1, N]$.

- ▶ knots, $\phi = \{\tau_1, \tau_{1.6}, \tau_8, \tau_N\}$
- ▶ observed yields, $\tilde{\tau} = \{\tau_2, \dots, \tau_{N-1}\}$

The term structure is represented as a factor model,

$$y_t(\tilde{\tau}) = W(\tilde{\tau})B_t + \varepsilon_t(\tilde{\tau})$$

where $W(\tilde{\tau})$ time invariant factor loadings and B_t are the time varying factors. In the Nelson and Siegel model there are 3 factors.

Estimation

In the segmented model

- ▶ For each segment there are B_t time varying factors, K number of factors and S the number of segments, then $K \times S$ total number of factors to estimate.
- ▶ At the knots there is no estimation error, $y_t(\phi) = W(\phi)B_t$

Estimate with constraints that impose smoothness across the segments

$$y_t(\tau) = W(\tau)B_t, \text{ s.t. } R(\phi)B_t = 0$$

- ▶ $f^{ST}(\tau_{1.6}) = f^{MT}(\tau_{1.6}); f^{MT}(\tau_8) = f^{LT}(\tau_8)$
- ▶ $f'^{ST}(\tau_{1.6}) = f'^{MT}(\tau_{1.6}); f'^{MT}(\tau_8) = f'^{LT}(\tau_8)$
- ▶ $f''^{ST}(\tau_{1.6}) = f''^{MT}(\tau_{1.6}), f''^{MT}(\tau_8) = f''^{LT}(\tau_8)$
- ▶ $f_t^i(\tau_i) = y_t(\tau_i)$

Estimation

In the segmented model

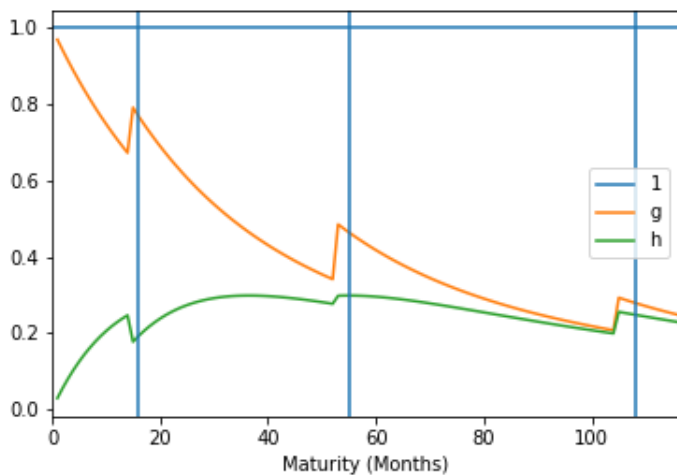
- ▶ Since restrictions are linear and $R(\phi)$ has no specific ordering, re-write the model and partition the parameter space,
 $R_1(\tilde{\phi})B_{1,t} + R_2(\tilde{\phi})B_{2,t} = 0$
- ▶ At estimation reduce the parameter space from $K \times S$ to $K + 1$
- ▶ Recover latent yields (inner knots), $y_t(\tau_{1.6}), y_t(\tau_8)$

Strongly segmented model (factor loading's also change across the segments)

- ▶ slope $\frac{1 - \exp^{-\lambda \Lambda_i(\tau)}}{\lambda \Lambda_i(\tau)}$
- ▶ curvature $\frac{1 - \exp^{-\lambda \Lambda_i(\tau)}}{\lambda \Lambda_i(\tau)} - \exp^{-\lambda \Lambda_i(\tau)}$

where $\Lambda_i(\tau) = \tau - \tau_{i-1}(1 - p)$, τ_{i-1} lower knot and $p \in [0, 1]$, controls the degree of loading segmentation.

Strongly segmented loadings, $p = 0.5$



Yields and Bond prices in the Literature

- ▶ Most papers on term structure modeling use synthetic smoothed interpolated yields, not directly observable data. See Gurkaynak et al. (2007), US Treasury yield curve since 1961.
- ▶ U.S yield curve: T-bills, Treasury notes and bonds.
- ▶ More recently there is an interest in working directly on large panels of bond prices: a one step approach (non-linear state space model), Andreasen et al. (2019).
- ▶ The practice in the Colombian bond market is to chose a reference bond for a particular maturity ("referencia especifica").
- ▶ Arbitrary matching across time for representative bonds.
- ▶ Number of available bonds for < 1 year is very unstable over time.

Observable yields

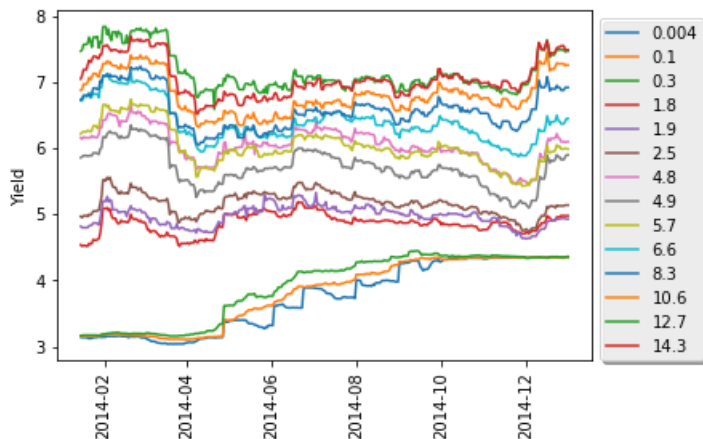
Daily bond price data from January 2013 to September 2018, from Precia S.A. Sovereign bonds in local currency that are trading in the secondary market.

1. Use a simple estimate of the approximate yield.

$$y(\tau) = \frac{\frac{\text{Coupon/Interest Payment}(\text{Par Value} - \text{Bond Price})}{\text{Years to maturity}}}{\frac{(\text{Par Value} + \text{Bond Price})}{2}}$$

2. Estimate model for each year with available maturities (no arbitrary historical matching). On average 14 observable yields.
3. For the short part of the curve, reference short rate in the money market (IBR), overnight, one month, three months and six months.

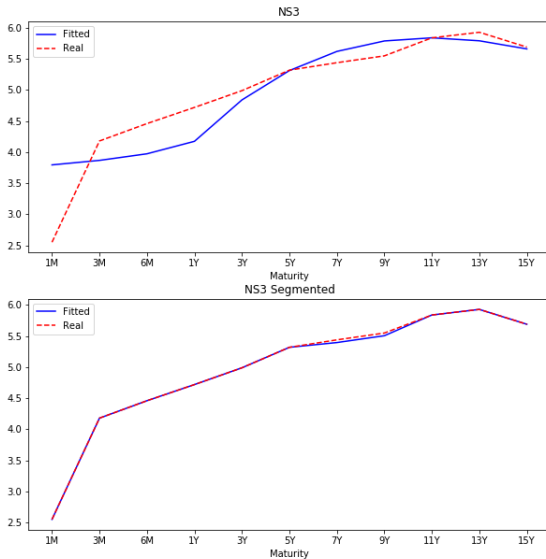
Observed approximate yields, 2014



Segmented and observable yield curve

- ▶ Nelson and Siegel three factor model
- ▶ Dynamic part each factor follows an independent AR(1) process. Is this enough (Hamilton and Wu, 2014)? AutoArima
- ▶ Three segments: short, medium and long.
- ▶ 9 factors: 3 factors from NS and 3 segments.
- ▶ On average 14 observable yields and two interior knots (unobserved yields) at 1.6 and 8 years.
- ▶ Classical NS, Segmented NS, Weakly Segmented NS with smoothness constraints, Strongly Segmented NS with smoothness constraints.
- ▶ Out-of-sample rolling window exercise; horizon: day, week, month.
- ▶ Out-of-sample forecasting benchmark: Random walk.
- ▶ Forecast each year but link the forecast (across years) using the estimated factors (dimension reduction). Matching

Model fit, comparison



In-sample, RMSE in bp, 2014

years	NS3	NS3S	NS3WS	NS3SS
0.004	12.8	78.8	6.3	5.7
0.1	9.6	75.1	3.5	2.3
0.3	10.5	70.6	8.4	6.6
1.8	14.6	59.1	8.4	8.3
1.9	19.9	58.7	12.7	3.6
2.5	14.7	50.0	5.2	3.6
4.8	17.3	39.1	13.6	21.8
4.9	20.1	29.5	23.6	15.4
5.7	10.7	25.9	10.3	5.5
6.6	6.9	23.9	10.9	9.9
8.3	18.1	2.3	6.4	17.3
10.6	17.6	8.5	21.0	8.7
12.7	12.4	11.0	7.5	17.2
14.3	8.7	4.8	8.2	2.7
	13.9	38.4	10.4	9.2

In-sample, RMSE in bp, 2017

years	NS3	NS3S	NS3WS	NS3SS
0.004	31.9	170.7	24.7	11.2
0.1	20.3	174.2	14.3	5.1
0.3	5.6	176.0	5.0	6.2
0.5	15.2	179.2	16.7	13.5
1.8	21.5	150.0	14.0	13.9
1.9	34.8	152.7	26.3	9.7
2.7	16.2	121.1	6.2	5.0
3.6	7.4	84.5	15.7	9.9
5.3	6.8	53.5	6.8	5.1
7.6	15.4	18.6	11.7	6.1
9.7	14.0	0.7	9.0	12.3
11.3	6.7	1.7	13.9	9.7
13.7	5.5	1.7	4.3	2.0
15.5	9.5	0.7	13.0	14.7
	15.1	91.8	13	8.9

Out-of-Sample RMSE in bp, 2014

years	day			week			month		
	NS	SM	RW	NS	SM	RW	NS	SM	RW
0.004	15.7	7.5	15.5	15.6	8.8	14.6	14.9	12.8	12.7
0.1	11.4	3.7	11.5	10.7	6.0	10.5	12.6	14.6	10.0
0.3	14.3	8.5	14.5	13.7	10.1	14.1	17.1	17.5	15.6
1.8	19.5	9.5	19.3	22.3	12.1	21.1	31.1	18.4	26.6
1.9	24.8	5.4	24.6	26.4	8.6	25.8	33.5	15.3	29.7
2.5	18.9	5.7	18.8	21.4	9.6	20.4	30.4	18.4	26.0
4.8	18.5	22.8	18.7	19.5	23.7	20.2	30.3	31.6	29.3
4.9	21.6	17.1	21.3	24.8	20.1	23.2	38.3	34.0	34.2
5.7	12.3	7.4	11.9	16.1	11.5	14.2	31.1	28.0	27.0
6.6	9.2	12.3	9.2	12.9	15.7	12.8	27.8	29.5	26.4
8.3	21.1	19.5	21.2	24.7	23.6	24.8	38.0	37.0	37.7
10.6	21.0	11.4	20.9	24.3	16.4	24.1	37.7	30.9	36.9
12.7	15.7	20.0	15.8	17.8	21.7	18.2	28.3	28.9	27.7
14.3	10.7	6.0	10.6	14.3	12.6	14.1	27.9	26.8	25.7
	16.8	11.2	16.7	18.9	14.3	18.4	28.5	24.6	26.1

Diebold and Mariano p-values, 2014

years	day		week		month	
	NS	RW	NS	RW	NS	RW
0.004	0.00	0.00	0.00	0.00	0.65	0.90
0.1	0.00	0.00	0.00	0.00	0.00	0.00
0.3	0.00	0.00	0.00	0.00	0.00	0.01
1.8	0.00	0.00	0.00	0.00	0.00	0.00
1.9	0.00	0.00	0.00	0.00	0.00	0.00
2.5	0.00	0.00	0.00	0.00	0.00	0.00
4.8	0.00	0.00	0.00	0.00	0.00	0.00
4.9	0.00	0.00	0.00	0.00	0.00	0.61
5.7	0.00	0.00	0.00	0.00	0.01	0.07
6.6	0.00	0.00	0.00	0.00	0.00	0.00
8.3	0.00	0.00	0.05	0.06	0.49	0.45
10.6	0.00	0.00	0.00	0.00	0.00	0.00
12.7	0.00	0.00	0.00	0.00	0.02	0.08
14.3	0.00	0.00	0.01	0.01	0.74	0.08

Ho: forecast method 1 = forecast method 2

Out-of-Sample RMSE in bp, 2017

years	day			week			month		
	NS	SM	RW	NS	SM	RW	NS	SM	RW
0.004	31.9	13.1	33.5	25.5	11.6	30.8	22.8	26.5	21.0
0.1	19.6	6.1	21.1	15.3	10.1	18.9	27.6	33.0	13.7
0.3	7.7	9.6	7.1	15.9	17.2	9.2	40.8	41.9	20.7
0.5	20.0	18.2	18.3	29.3	25.3	21.7	53.7	48.9	35.1
1.8	27.8	15.5	26.8	34.0	16.0	28.7	52.8	31.4	37.8
1.9	39.3	12.0	37.9	45.9	17.2	39.8	65.0	37.5	48.7
2.7	18.9	7.6	17.8	25.4	13.5	20.5	41.8	30.5	29.8
3.6	9.4	11.9	9.5	13.5	14.4	12.0	26.3	26.1	18.9
5.3	9.0	8.2	9.5	12.9	14.5	13.2	23.2	28.8	21.4
7.6	16.6	8.5	17.1	17.6	12.4	18.8	22.7	23.5	23.7
9.7	15.1	14.0	15.6	16.7	18.5	18.0	22.7	30.3	25.0
11.3	8.6	11.2	9.2	11.0	15.1	12.2	18.2	26.1	20.3
13.7	7.9	4.6	7.3	13.0	11.1	11.4	22.4	23.1	20.1
15.5	13.3	15.7	12.3	17.4	18.2	14.7	26.3	25.5	21.0
	17.5	11.2	17.3	20.9	15.4	19.3	33.3	31.0	25.5

Diebold and Mariano p-values, 2017

years	day		week		month	
	NS	RW	NS	RW	NS	RW
0.004	0.00	0.00	0.00	0.00	0.37	0.00
0.1	0.00	0.00	0.00	0.00	0.01	0.00
0.3	0.00	0.00	0.04	0.00	0.86	0.00
0.5	0.00	0.30	0.00	0.00	0.00	0.00
1.8	0.00	0.00	0.00	0.00	0.00	0.00
1.9	0.00	0.00	0.00	0.00	0.00	0.00
2.7	0.00	0.00	0.00	0.00	0.00	0.56
3.6	0.00	0.00	0.16	0.00	0.27	0.00
5.3	0.05	0.00	0.01	0.04	0.00	0.00
7.6	0.00	0.00	0.00	0.00	0.72	0.57
9.7	0.08	0.01	0.11	0.72	0.00	0.00
11.3	0.00	0.00	0.00	0.00	0.00	0.00
13.7	0.00	0.00	0.00	0.33	0.94	0.00
15.5	0.00	0.00	0.53	0.00	0.36	0.00

Ho: forecast method 1 = forecast method 2

Results and comparison, US (synthetic) and Canadian data

- ▶ Dufee (2002), $\text{RMSE} \in (28, 52)$ bp, monthly data forecast horizon 3 months for maturities up to 10 years, with affine term structure models.
- ▶ Ang and Piazzesi (2003), $\text{RMSE} \in (18, 30)$ bp, monthly data forecast horizon 1 month for maturities up to 3 years, with affine term structure model with macro variables.
- ▶ Andreasen et al. (2019), $\text{RMSE} \in (4, 10)$ bp (in-sample), $\text{RMSE} \in (40, 55)$ bp (out-of-sample), monthly data forecast horizon 3 months for maturities up to 30 years, with affine Nelson and Siegel model.

Results and comparison, Colombian (synthetic) data

- ▶ Maldonado et al. (2014), $\text{RMSE} \in (21, 57)$ bp, daily data forecast horizon 1 day for maturities up to 13 years, with dynamic Nelson-Siegel model.
- ▶ Velasquez and Restrepo (2016), $\text{RMSE} \in (0, 18)$ bp (in-sample), $\text{RMSE} \in (6, 17.5)$ bp (out-of-sample), daily data forecast horizon 1 day for maturities up to 10 years, with affine term structure model.

Results, Colombian (bond prices) data

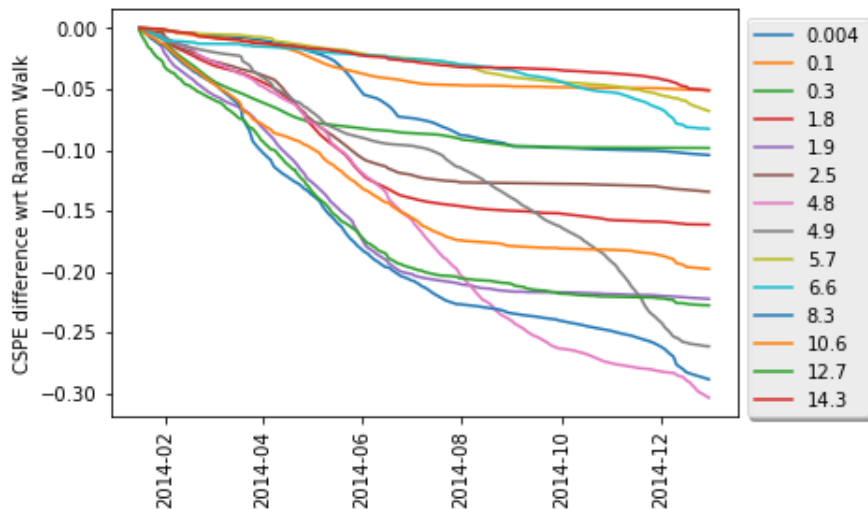
Segmented dynamic Nelson-Siegel model.

- ▶ In-sample, RMSE $\in (2.3, 18)$ bp, daily data for maturities up to 15 years.
- ▶ Out-of-sample, RMSE $\in (2.1, 19)$ bp daily data forecast horizon 1 day for maturities up to 15 years

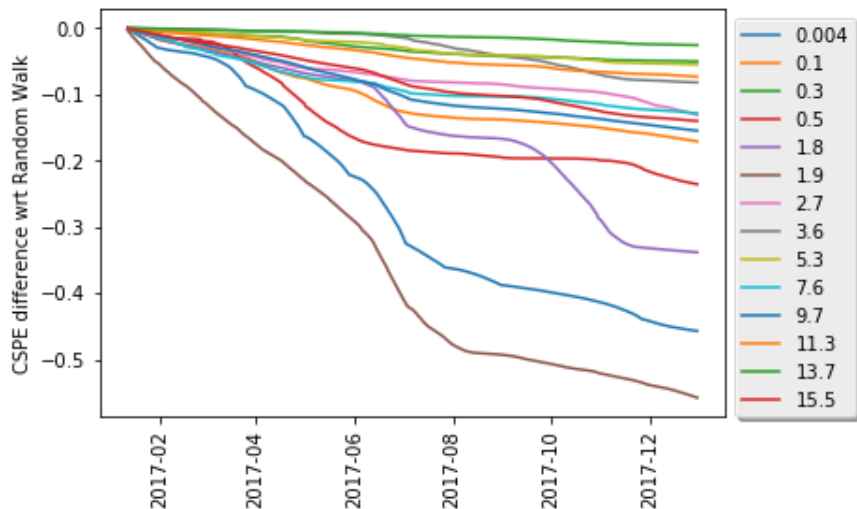
Outperforms dynamic Nelson and Siegel model both in and out-of-sample but does not systematically beat the random walk model, according to cumulative square prediction error.

$$CSPE_t = \sum_{i=1}^t ((\hat{y}_i^{RW}(\tau) - y_i(\tau))^2 - (\hat{y}_i^{NS3SS}(\tau) - y_i(\tau))^2)$$

Cumulative square error prediction model, 1 day 2014



Cumulative square error prediction model, 1 day 2017



Conclusions and research

- ▶ The segmented term structure model provides a better fit for the yield curve than the standard Nelson and Siegel model, because it is able to capture the local dynamics of each segment.
- ▶ The out-of-sample results show that in some cases the segmented model outperforms the random walk approach (statistically significant the difference), but unfortunately this result is not systematic.
- ▶ We provide a first approach that avoids using synthetic estimates of the yields based on a particular model. Our estimates based on each year of available bond does not preclude the construction of historical reference yield since this is accomplished with the lower dimensional factors.

Conclusions and research

- ▶ Estimating more factors must come at a costs of uncertainty around these estimates (at each segment we have few observations). We do not measure the trade-off.
- ▶ A different risk function for performance (a proper trade-off) between a segmented and a non-segmented model should be constructed on the basis of different criteria (for example robustness of the curve with respect to small changes in bond prices at different maturities), instead of the typical bias and variance trade-off. This has largely been omitted in the literature.
- ▶ Equal predictive accuracy test for nested models, Clark and West (2007).

In-sample, RMSE in bp, 2015

years	NS3	NS3S	NS3WS	NS3SS
0.004	10.2	288.3	8.2	6.6
0.1	4.9	280.9	3.6	2.5
0.3	6.8	267.3	6.9	7.1
1.5	9.0	213.7	9.2	7.9
3.8	14.3	109.0	17.6	9.2
3.9	19.5	139.0	16.0	24.3
4.7	9.9	103.3	8.0	9.9
5.6	5.6	68.6	5.5	7.2
7.3	11.7	30.8	3.3	17.8
9.6	10.2	1.2	7.3	8.0
11.7	7.1	4.5	10.3	5.6
13.3	9.6	4.4	6.6	5.9
15.7	9.1	1.2	11.2	12.5
	15.1	91.8	13	8.9

In-sample, RMSE in bp, 2016

years	NS3	NS3S	NS3WS	NS3SS
0.004	9.7	97.2	8.6	12.8
0.1	7.6	94.9	4.1	3.9
0.3	12.1	90.4	9.6	9.1
0.5	12.6	82.8	12.4	14.4
2.8	11.4	42.7	6.8	9.5
2.9	37.6	60.3	26.9	21.5
3.7	14.4	42.7	5.6	6.6
4.6	7.9	26.7	11.5	9.1
6.3	5.7	22.1	6.4	5.2
8.6	21.4	0.9	3.2	4.4
10.7	15.5	3.2	18.5	3.8
12.3	4.4	3.2	11.2	4.6
14.7	16.8	0.8	16.1	5.6
	15.1	91.8	13	8.9

In-sample, RMSE in bp, 2018

years	NS3	NS3S	NS3WS	NS3SS
0.004	14.4	167.4	14.8	10.9
0.1	6.5	171.4	6.8	4.9
0.3	8.4	178.4	8.3	6.5
0.5	20.0	182.2	20.0	14.4
1.7	7.4	126.7	7.3	8.9
2.6	13.7	79.4	12.9	13.6
4.3	5.3	47.2	4.5	7.2
6.6	4.3	11.7	4.3	5.0
7.9	3.7	5.4	1.7	3.5
8.7	2.1	0.7	1.9	13.1
10.3	8.9	1.7	9.2	10.7
12.7	2.3	1.7	2.6	3.4
14.5	6.0	0.7	6.0	12.1
	15.1	91.8	13	8.9

Out-of-Sample RMSE in bp, 2015

years	day			week			month		
	NS	SM	RW	NS	SM	RW	NS	SM	RW
0.004	12.4	9.2	12.4	11.7	8.9	11.8	12.1	13.0	11.2
0.1	6.0	3.4	6.1	7.4	7.2	7.0	14.1	16.7	13.1
0.3	11.6	12.9	11.4	15.4	17.6	14.3	25.4	28.3	24.0
1.5	11.9	9.7	11.9	14.1	14.4	13.9	20.7	26.7	19.7
3.8	18.4	13.5	18.0	23.8	20.2	22.5	37.5	33.8	34.8
3.9	20.9	25.1	21.2	21.6	25.7	22.2	22.3	25.0	23.2
4.7	12.4	12.8	12.4	16.2	17.3	15.8	24.9	25.4	23.1
5.6	9.0	10.6	8.6	16.2	18.1	15.0	30.2	32.0	27.2
7.3	14.8	21.4	14.7	18.8	24.6	18.1	30.9	36.4	28.2
9.6	13.3	9.6	13.3	18.3	13.0	17.7	31.1	22.1	28.3
11.7	10.4	8.1	10.3	15.3	12.9	14.8	27.7	25.2	24.9
13.3	11.2	8.2	11.4	13.6	12.8	14.1	21.9	24.8	21.2
15.7	11.9	14.5	11.9	15.5	20.5	15.1	27.3	36.6	25.9
	12.6	12.2	12.6	16.0	16.4	15.6	25.1	26.6	23.4

Out-of-Sample RMSE in bp, 2016

years	day			week			month		
	NS	SM	RW	NS	SM	RW	NS	SM	RW
0.004	13.6	16.0	11.2	16.4	26.6	10.2	28.8	63.9	8.0
0.1	13.4	11.7	9.8	17.8	25.4	9.6	31.8	62.2	11.2
0.3	16.2	14.5	12.8	20.9	25.7	13.9	34.1	60.5	17.6
0.5	17.8	19.0	14.8	23.2	28.4	17.3	36.3	60.5	23.0
2.8	15.4	11.1	14.5	20.9	17.2	18.0	34.3	30.3	26.0
2.9	40.1	22.7	39.7	44.5	28.5	42.4	56.3	40.8	49.8
3.7	16.8	9.6	16.3	24.3	18.6	21.3	41.0	32.7	32.5
4.6	9.8	11.2	9.9	17.2	17.7	15.7	34.8	30.1	26.6
6.3	9.4	8.6	9.3	18.6	16.5	16.9	36.6	29.1	27.5
8.6	22.6	8.3	22.9	25.0	15.7	25.4	34.5	27.2	29.5
10.7	17.3	8.0	17.2	21.6	15.9	21.3	33.3	27.3	26.6
12.3	9.8	9.4	7.9	19.2	17.1	15.5	36.7	31.1	24.8
14.7	20.0	8.9	19.6	25.5	15.8	23.6	40.4	29.7	29.3
	17.1	12.2	15.8	22.7	20.7	19.3	36.8	40.4	25.6

Out-of-Sample RMSE in bp, 2018

years	day			week			month		
	NS	SM	RW	NS	SM	RW	NS	SM	RW
0.004	16.4	11.6	15.6	17.7	10.9	14.7	12.0	9.3	8.7
0.1	7.5	5.0	6.7	9.9	5.1	6.0	8.7	8.8	5.3
0.3	8.6	7.3	8.9	10.9	9.2	10.4	15.3	15.6	16.5
0.5	20.0	15.1	20.6	20.0	16.0	21.5	24.0	20.4	26.3
1.7	8.5	9.8	8.6	11.3	11.2	10.3	17.7	18.0	16.4
2.6	15.4	14.6	14.6	18.5	16.1	15.0	24.2	22.0	16.4
4.3	7.3	8.5	6.4	11.8	10.8	8.6	21.2	18.6	15.2
6.6	6.2	6.0	5.6	10.2	8.8	8.2	18.3	15.6	14.0
7.9	4.3	4.6	4.7	7.8	7.9	7.7	16.8	16.1	15.3
8.7	4.0	13.8	3.7	9.2	15.9	8.0	18.4	22.4	16.5
10.3	9.8	11.8	10.0	12.3	14.4	12.5	20.5	22.1	20.7
12.7	4.0	4.6	4.0	8.4	7.9	7.9	17.0	14.6	16.3
14.5	7.2	12.6	7.2	10.5	13.9	10.6	15.9	16.0	15.9
	9.2	9.6	9	12.2	11.4	10.9	17.7	16.9	15.7

Out-of-Sample RMSE in bp, 2014 AutoArima

years	day ARMA(1,1)			day AUTO ARIMA		
	NS	SM	RW	NS	SM	RW
0.004	15.7	7.5	15.5	17.6	10.0	15.5
0.1	11.4	3.7	11.5	14.2	6.4	11.5
0.3	14.3	8.5	14.5	16.8	9.2	14.5
1.8	19.5	9.5	19.3	21.0	10.0	19.3
1.9	24.8	5.4	24.6	26.2	7.4	24.6
2.5	18.9	5.7	18.8	20.5	6.9	18.8
4.8	18.5	22.8	18.7	21.0	23.6	18.7
4.9	21.6	17.1	21.3	22.4	17.2	21.3
5.7	12.3	7.4	11.9	13.7	8.0	11.9
6.6	9.2	12.3	9.2	12.6	13.5	9.2
8.3	21.1	19.5	21.2	22.6	19.3	21.2
10.6	21.0	11.4	20.9	22.3	11.4	20.9
12.7	15.7	20.0	15.8	18.7	21.0	15.8
14.3	10.7	6.0	10.6	13.8	6.6	10.6
	16.8	11.2	16.7	18.8	12.2	16.7

Motivacion

El mercado de bonos proporciona el principal mecanismo de financiamientos de las empresas y el sector publico a nivel mundial. El mercado de bonos soberanos de corto y largo plazo a su vez representa una importante porcion dentro del mercado de renta fija, especialmente en los mercados menos desarrollados.

- ▶ T-bills \leq 1 año (US)
- ▶ $2 < \text{T-notes} \leq 10$ años (US)
- ▶ Treasury bonds > 10 años (US)
- ▶ TES: **1,2,4,5,6,8,10,15** años

Tasa de rendimiento a su madurez (Yield to maturity): tasa que hace que el valor presente de los pagos del bono sea igual al precio.

El mercado nos genera unos precios para estos bonos en cada momento del tiempo. Estrategia comprar y mantener hasta el periodo de maduración (buy and hold).

Los precios de los bonos contienen informacion

Estructura a plazos de la tasa de interes

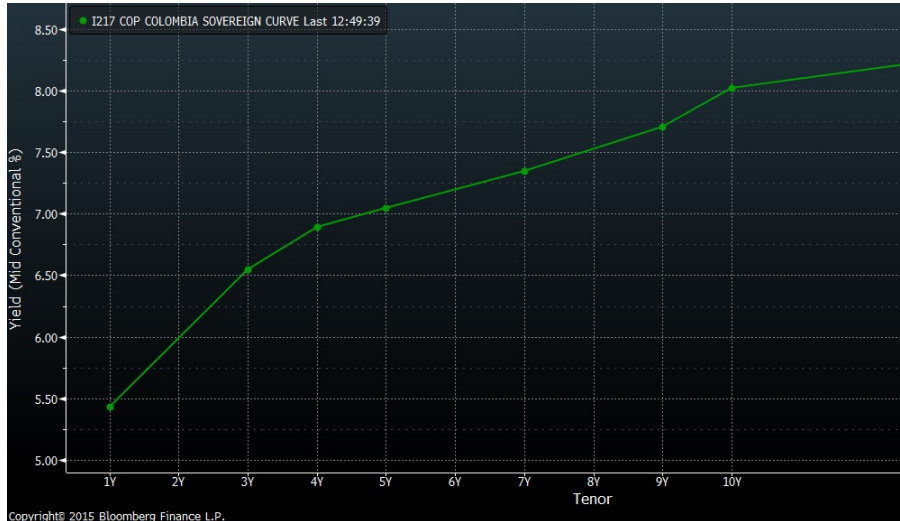
La estructura a plazos de la tasa de interes es la relacion entre los rendimientos al vencimiento de bonos distintos (de la misma calidad crediticia con diferente maduracion) y su vencimiento. Tiene una representacion grafica (curva)

- ▶ **curva spot**
- ▶ curva de descuento (efecto cupon)
- ▶ **curva forward**

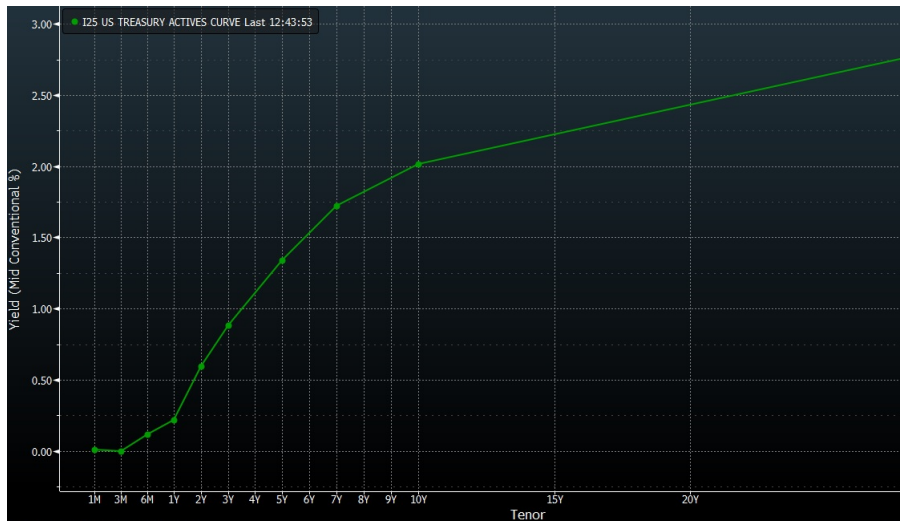
Tasa forward: tasa que se infiere a partir de igualar el retorno de una inversion a dos años vs. adquirir un bono a un año y realizar un roll-over por el otro año. Por lo tanto hay una relacion entre la tasa forward y la spot.

En esta presentacion nos enfocamos en la curva spot.

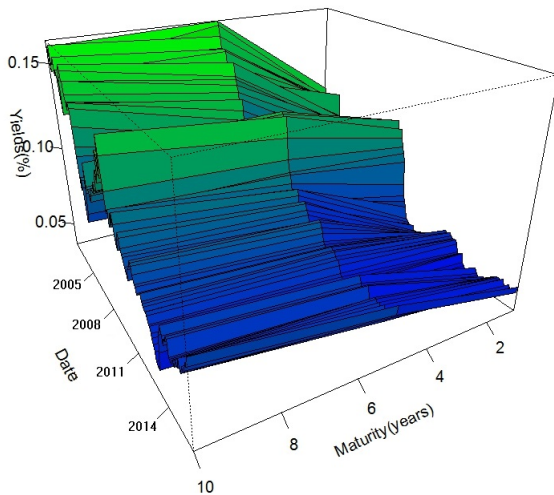
Representacion grafica de la curva-Bloomberg, Colombia



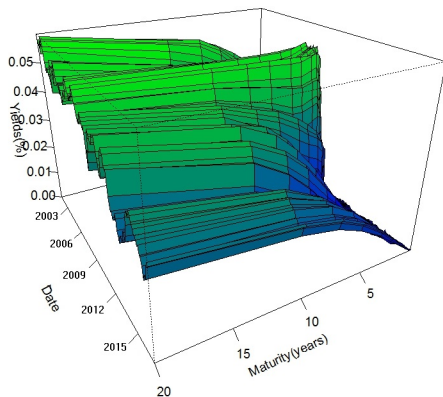
Representacion grafica de la curva-Bloomberg, US



Representacion grafica de la curva a traves del tiempo, Colombia



Representacion grafica de la curva a traves del tiempo, US



Historical Yields: matching bonds or factors

matching bonds

bond 1Y

bond 2Y

:

bond 5Y

bond A 3Y

bond B 3Y

bond 3.5Y

bond 4Y

bond 1Y

bond 2Y

bond 4Y

2014

2015

2016

historical series 1Y, 2Y, 5Y

matching factors

bond 1Y

bond 2.5Y

bond 3Y, 7Y....

3 factors_t

bond 1.6Y

bond 2.4Y

bond 2.9Y,....

3 factors_t

2014

2015

2016

rolling window forecast with respect to the factors to forecast across years.

Notice that since it is rolling at some point I stop using year_{t-1}'s factors