# **FAI** Item Selection

```
suppressPackageStartupMessages({
  library("tidyverse")
  library("TAM")
  library("psych")
  library("lordif")
  library("ggridges")
  library("ggpubr")
  library("paletteer")
  library("readxl")
  library("mice")
  library("VIM")
  library("paran")
  library("lavaan")
  library("polycor") # for hetcor()
  library("multilevel")
  library("miceRanger")
  library("papaja")
  theme_set(theme_apa())
  library("outForest")
  library("lavaanExtra")
  # Caricare le librerie necessarie
  library(glmnet) # Per LASSO regression
  library(randomForest) # Per valutare importanza delle variabili
  library(Boruta) # Per selezione robusta delle variabili
  library(missRanger)
  library(caret)
  library(pROC)
  library(lavaan)
  library(semTools)
})
library("here")
```

here() starts at /Users/corrado/\_repositories/meyer\_fai

```
# Increase max print
options(max.print = .Machine$integer.max)
source(here("functions", "fai_funs.R"))
```

# Import data

```
fai_s <- read_xlsx(
  here("data", "raw", "FAI_TOT_2020_corrected.xlsx"), col_names = TRUE
)</pre>
```

Demographic information.

```
demo_info <- recode_demo_info(fai_s)</pre>
```

Categorize disease severity.

```
# Function to categorize disease severity
categorize_severity <- function(disease) {</pre>
  disease <- tolower(disease) # Normalize text (case insensitive)
  if (grepl("allergia|asma lieve|rinite", disease)) {
   return("Lieve")
  } else if (grepl("diabete|colite ulcerosa|artrite|cardiopatia|insufficienza renale|iperten
   return("Moderata")
  } else if (grepl("fibrosi cistica|leucemia|epilessia|distrofia|sindrome nefrosica|osteosar
   return("Grave")
  } else if (grepl("tumore|cancro|metastasi|cuore ipoplasico|malattia metabolica|aciduria|en
   return("Critica")
  } else {
    return(NA) # Unclassified
 }
}
# Apply the function to categorize chronic diseases
demo_info <- demo_info %>%
 mutate(severity_level = factor(
    sapply(chronic_disease, categorize_severity),
```

In the original administration, there is no psychological criterion variable available. The only external variable that could serve as a criterion is death risk, based on the assumption that coping is more challenging for a life-threatening disease compared to a non-life-threatening condition.

```
table(demo_info$death_risk)
```

0 1 264 230

# **Data Wrangling**

Extract item responses (columns 51 to 247).

```
items <- fai_s[, 51:247]
colnames(items) <- paste0("item_", seq_len(ncol(items))) # Rename items</pre>
```

Identify and remove items with excessive missing values.

```
n_nas <- colSums(is.na(items))
bad_items <- names(n_nas[n_nas > 50])  # Adjust threshold as needed
items_filtered <- items %>%
    dplyr::select(-all_of(bad_items))
```

Combine cleaned item data with demographic info.

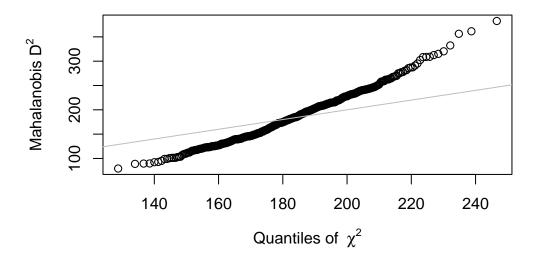
```
mydata <- bind_cols(demo_info, items_filtered) %>%
  mutate(subj_id = as.factor(seq_len(nrow(.))))
```

# **Identify and Remove Problematic Participants**

```
d_num <- mydata %>%
  dplyr::select(starts_with("item_")) %>%
  dplyr::select(where(is.numeric)) # Ensure numeric selection

# Mahalanobis Distance
mahal_out <- careless::mahad(d_num)</pre>
```

# Q-Q plot of Mahalanobis D<sup>2</sup> vs. quantiles of $\chi^2_{nvar}$



```
mahal_cutoff <- boxplot(mahal_out, plot = FALSE)$stats[5]
bad_mahal <- mydata$subj_id[mahal_out > mahal_cutoff]

# Longstring Analysis
longstring_out <- careless::longstring(d_num)
longstring_cutoff <- boxplot(longstring_out, plot = FALSE)$stats[5]
bad_longstring <- mydata$subj_id[longstring_out > longstring_cutoff]

# IRV Analysis
irv_out <- careless::irv(d_num)
irv_cutoff <- boxplot(irv_out, plot = FALSE)$stats[5]
bad_irv <- mydata$subj_id[irv_out > irv_cutoff]

# Person Total Correlation
cm <- colMeans(d_num, na.rm = TRUE)</pre>
```

```
person_tot_cor <- apply(d_num, 1, function(x) cor(x, cm, use = "complete.obs"))
person_tot_cutoff <- boxplot(person_tot_cor, plot = FALSE)$stats[1]
bad_person_tot <- mydata$subj_id[person_tot_cor < person_tot_cutoff]

# Combine all flagged participants
bad_ids <- unique(c(bad_mahal, bad_longstring, bad_irv, bad_person_tot))

# Remove problematic participants before imputation
d_clean <- mydata %>%
    dplyr::filter(!subj_id %in% bad_ids)
```

# Multiple Imputation

```
dim(df)
```

[1] 453 208

#### Item Selection Based on Clinical Criteria

```
# Define selected items based on clinical criteria
selected items <- c(
 "i_86", "i_57", "i_5", "i_85", "i_81",
 "i_105", "i_48", "i_133", "i_129", "i_39", "i_103",
  "i 143", "i 79".
 "i_111", "i_34", "i_119", "i_116", "i_23", "i_45", "i_41",
  "i_186", "i_38", "i_128", "i_7", "i_16", "i_29",
 "i_137", "i_96", "i_194"
# Convert to match column names in `imp`
selected_items_corrected <- paste0("item_", sub("^i_", "", selected_items))</pre>
# Ensure only existing columns are selected
selected_items_corrected <- intersect(selected_items_corrected, colnames(imp))</pre>
# Select the matching columns
imp_selected <- imp %>%
  dplyr::select(any_of(selected_items_corrected))
# Validate the selection
if (length(selected items_corrected) != length(selected_items)) {
 warning("Some selected items were not found in the dataset.")
}
print(names(imp_selected))
 [1] "item 86" "item 57" "item 5" "item 85" "item 81" "item 105"
 [7] "item_48" "item_133" "item_129" "item_39" "item_103" "item_143"
[13] "item_79" "item_111" "item_34" "item_119" "item_116" "item_23"
[19] "item_45" "item_41" "item_186" "item_38" "item_128" "item_7"
[25] "item_16" "item_29" "item_137" "item_96" "item_194"
```

#### **Define Target Matrix for Factor Structure (29x6)**

```
# Initialize a 29x6 matrix filled with zeros
TARGET <- matrix(0, nrow = length(selected_items_corrected), ncol = 6)</pre>
```

```
# Assign factor loadings based on clinical criteria
TARGET[1:5, 1] <- 1  # F1
TARGET[6:11, 2] <- 1  # F2
TARGET[12:13, 3] <- 1  # F3
TARGET[14:20, 4] <- 1  # F4
TARGET[21:26, 5] <- 1  # F5
TARGET[27:29, 6] <- 1  # F6

# Add row names for clarity
rownames(TARGET) <- selected_items_corrected
colnames(TARGET) <- pasteO("F", 1:6)

# Print the target rotation matrix
print(TARGET)</pre>
```

```
F1 F2 F3 F4 F5 F6
item_86
    1 0 0 0 0 0
item_57
     1 0 0 0 0 0
    1 0 0 0 0 0
item 5
item_85
     1 0 0 0 0 0
     1 0 0 0 0 0
item_81
item_105 0 1 0 0 0 0
item_143 0 0 1 0 0 0
item_79 0 0 1 0 0 0
item_111 0 0 0 1 0 0
item_119 0 0 0 1 0 0
item_116 0 0 0 1 0 0
0 0 0 1 0 0
item_45
item_41 0 0 0 1 0 0
item_186 0 0 0 0 1 0
item_128  0  0  0  0  1  0
item_7 0 0 0 0 1 0
     0 0 0 0 1 0
item_16
```

### Define the ESEM Model in Lavaan Syntax

```
model <- '
   efa("efa1")*f1 +
   efa("efa1")*f2 +
   efa("efa1")*f3 +
   efa("efa1")*f4 +
    efa("efa1")*f5 +
    efa("efa1")*f6 =~
       item_86 + item_57 + item_5 + item_85 + item_81 +
       # F2
       item_105 + item_48 + item_133 + item_129 + item_39 + item_103 +
       # F3
      item_{143} + item_{79} +
       # F4
      item_111 + item_34 + item_119 + item_116 + item_23 + item_45 + item_41 +
       # F5
       item_186 + item_38 + item_128 + item_7 + item_16 + item_29 +
       # F6
       item_137 + item_96 + item_194
```

# Fit the ESEM Model with Target Rotation

```
fit1 <- sem(
  model = model,
  data = imp_selected,
  ordered = TRUE,  # Use ordered estimation if Likert-type items
  rotation = "target",
  rotation.args = list(target = TARGET)
)</pre>
```

```
fit_indices <- fitMeasures(fit1, c(
    "chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr",
    "chisq.scaled", "df.scaled", "pvalue.scaled", "cfi.robust", "tli.robust",
    "rmsea.robust", "srmr"
))

# Display selected fit indices (Standard & Robust)
cat("\nModel Fit Indices (Standard & Robust):\n")</pre>
```

## Model Fit Indices (Standard & Robust):

```
fit_indices_df <- data.frame(
   Measure = names(fit_indices),
   Value = round(fit_indices, 3)
)
print(fit_indices_df)</pre>
```

```
Measure
                 Value
1
          chisq 235.901
2
            df 247.000
       pvalue 0.683
3
            cfi 1.000
5
            tli 1.001
6
          rmsea 0.000
7
           srmr 0.037
8
  chisq.scaled 461.729
      df.scaled 247.000
10 pvalue.scaled 0.000
     cfi.robust 0.922
12
     tli.robust 0.872
13 rmsea.robust 0.059
14
           srmr 0.037
```

```
# Extract Standardized Factor Loadings
# Get standardized solution
std_solution <- standardizedSolution(fit1)

# Filter only loadings (Lambda matrix)
std_loadings <- std_solution %>%
filter(op == "=~") %>%
```

```
dplyr::select(lhs, rhs, est.std) %>%
  arrange(lhs, desc(abs(est.std))) # Sorted by factor and magnitude

colnames(std_loadings) <- c("Factor", "Item", "Standardized Loading")

# Display standardized loadings
cat("\nStandardized Factor Loadings:\n")</pre>
```

# Standardized Factor Loadings:

# print(std\_loadings)

|    | Factor | Item                 | Standardized.Loading |
|----|--------|----------------------|----------------------|
| 1  | f1     | item_81              | 0.577                |
| 2  | f1     | item_86              | 0.571                |
| 3  | f1     | item_48              | 0.511                |
| 4  | f1     | $\mathtt{item\_105}$ | 0.508                |
| 5  | f1     | item_57              | 0.461                |
| 6  | f1     | item_38              | 0.358                |
| 7  | f1     | item_39              | 0.341                |
| 8  | f1     | item_5               | 0.340                |
| 9  | f1     | $\mathtt{item\_103}$ | 0.312                |
| 10 | f1     | $item_194$           | -0.264               |
| 11 | f1     | $\mathtt{item\_186}$ | 0.221                |
| 12 | f1     | $\mathtt{item\_137}$ | -0.221               |
| 13 | f1     | item_7               | 0.177                |
| 14 | f1     | item_41              | 0.170                |
| 15 | f1     | item_96              | -0.168               |
| 16 | f1     | item_85              | 0.167                |
| 17 | f1     | item_34              | 0.164                |
| 18 | f1     | ${\tt item\_119}$    | -0.157               |
| 19 | f1     | $item_45$            | 0.134                |
| 20 | f1     | item_29              | 0.123                |
| 21 | f1     | item_23              | -0.116               |
| 22 | f1     | $\mathtt{item\_116}$ | 0.077                |
| 23 | f1     | $\mathtt{item\_129}$ | 0.067                |
| 24 | f1     | item_79              | 0.065                |
| 25 | f1     | $\mathtt{item\_143}$ | 0.065                |
| 26 | f1     | $item_111$           | -0.053               |
| 27 | f1     | $\mathtt{item\_133}$ | 0.031                |
|    |        |                      |                      |

| 28 | f1 | item_128             | -0.022 |
|----|----|----------------------|--------|
| 29 | f1 | item_16              | 0.007  |
| 30 | f2 | $item_129$           | 0.908  |
| 31 | f2 | $item_133$           | 0.836  |
| 32 | f2 | item_81              | 0.722  |
| 33 | f2 | item_105             | 0.594  |
| 34 | f2 | $item_48$            | 0.533  |
| 35 | f2 | $item_103$           | 0.500  |
| 36 | f2 | item_5               | 0.475  |
| 37 | f2 | item_39              | 0.400  |
| 38 | f2 | $item_116$           | 0.399  |
| 39 | f2 | item_29              | 0.344  |
| 40 | f2 | $item_111$           | 0.274  |
| 41 | f2 | item_86              | 0.260  |
| 42 | f2 | item_23              | 0.239  |
| 43 | f2 | $item_137$           | 0.202  |
| 44 | f2 | item_96              | 0.196  |
| 45 | f2 | item_57              | 0.191  |
| 46 | f2 | item_34              | 0.171  |
| 47 | f2 | $item_45$            | 0.144  |
| 48 | f2 | item_85              | 0.120  |
| 49 | f2 | item_79              | 0.101  |
| 50 | f2 | item_16              | -0.093 |
| 51 | f2 | $\mathtt{item\_119}$ | 0.086  |
| 52 | f2 | item_41              | -0.066 |
| 53 | f2 | item_7               | -0.047 |
| 54 | f2 | $\mathtt{item\_143}$ | -0.045 |
| 55 | f2 | $\mathtt{item\_128}$ | -0.037 |
| 56 | f2 | $\mathtt{item\_194}$ | -0.033 |
| 57 | f2 | $\mathtt{item\_186}$ | -0.027 |
| 58 | f2 | item_38              | 0.023  |
| 59 | f3 | $\mathtt{item\_143}$ | 0.928  |
| 60 | f3 | item_79              | 0.852  |
| 61 | f3 | $\mathtt{item\_194}$ | 0.236  |
| 62 | f3 | $\mathtt{item\_186}$ | 0.226  |
| 63 | f3 | item_85              | 0.213  |
| 64 | f3 | $item_45$            | -0.200 |
| 65 | f3 | item_86              | -0.181 |
| 66 | f3 | item_39              | -0.173 |
| 67 | f3 | $\mathtt{item\_116}$ | -0.152 |
| 68 | f3 | item_5               | 0.140  |
| 69 | f3 | item_38              | 0.135  |
| 70 | f3 | $item_111$           | -0.121 |
|    |    |                      |        |

| 71  | f3 | item_105             | 0.119  |
|-----|----|----------------------|--------|
| 72  | f3 | $\mathtt{item\_128}$ | -0.106 |
| 73  | f3 | item_16              | -0.102 |
| 74  | f3 | item_48              | 0.089  |
| 75  | f3 | item_7               | -0.077 |
| 76  | f3 | item_23              | -0.075 |
| 77  | f3 | item_96              | -0.071 |
| 78  | f3 | $\mathtt{item\_103}$ | -0.062 |
| 79  | f3 | $item_41$            | -0.060 |
| 80  | f3 | $\mathtt{item\_133}$ | 0.054  |
| 81  | f3 | $item_34$            | -0.045 |
| 82  | f3 | $item_119$           | -0.029 |
| 83  | f3 | $\mathtt{item\_129}$ | 0.023  |
| 84  | f3 | item_81              | -0.018 |
| 85  | f3 | item_57              | -0.012 |
| 86  | f3 | item_29              | 0.009  |
| 87  | f3 | $item_137$           | 0.003  |
| 88  | f4 | $item_119$           | 0.836  |
| 89  | f4 | $item_111$           | 0.803  |
| 90  | f4 | item_34              | 0.775  |
| 91  | f4 | item_23              | 0.705  |
| 92  | f4 | item_41              | 0.595  |
| 93  | f4 | item_39              | 0.422  |
| 94  | f4 | $item_103$           | 0.378  |
| 95  | f4 | $item_116$           | 0.281  |
| 96  | f4 | $item_194$           | 0.232  |
| 97  | f4 | item_48              | 0.229  |
| 98  | f4 | item_85              | 0.187  |
| 99  | f4 | $item_143$           | -0.182 |
| 100 | f4 | item_38              | -0.170 |
| 101 | f4 | item_137             | 0.155  |
| 102 |    | item_96              | 0.139  |
| 103 | f4 | $item_128$           | 0.129  |
| 104 | f4 | item_86              | 0.105  |
| 105 | f4 | item_29              | -0.082 |
| 106 | f4 | item_57              | -0.082 |
| 107 | f4 | item_105             | 0.070  |
| 108 | f4 | item_7               | -0.060 |
| 109 | f4 | item_133             | 0.060  |
| 110 | f4 | item_45              | -0.060 |
| 111 | f4 | item_79              | -0.055 |
| 112 | f4 | item_129             | -0.054 |
| 113 | f4 | item_16              | 0.033  |
|     |    |                      |        |

| 114 | f4 | item_81              | -0.030 |
|-----|----|----------------------|--------|
| 115 | f4 | item_5               | -0.029 |
| 116 | f4 | item_186             | -0.021 |
| 117 | f5 | item_7               | 1.015  |
| 118 | f5 | item_128             | 0.800  |
| 119 | f5 | item_16              | 0.664  |
| 120 | f5 | item_186             | 0.481  |
| 121 | f5 | item_38              | 0.426  |
| 122 | f5 | $item_45$            | 0.397  |
| 123 | f5 | $item_23$            | -0.318 |
| 124 | f5 | item_86              | 0.307  |
| 125 | f5 | item_85              | 0.238  |
| 126 | f5 | $\mathtt{item\_137}$ | 0.178  |
| 127 | f5 | $\mathtt{item\_103}$ | -0.166 |
| 128 | f5 | item_57              | 0.161  |
| 129 | f5 | $\mathtt{item\_143}$ | 0.161  |
| 130 | f5 | item_79              | 0.160  |
| 131 | f5 | item_39              | 0.157  |
| 132 | f5 | $item_194$           | 0.138  |
| 133 | f5 | item_81              | 0.121  |
| 134 | f5 | $\mathtt{item\_129}$ | 0.114  |
| 135 | f5 | $item_111$           | -0.100 |
| 136 | f5 | item_34              | -0.096 |
| 137 | f5 | ${\tt item\_119}$    | -0.091 |
| 138 | f5 | $\mathtt{item\_105}$ | 0.091  |
| 139 | f5 | item_96              | 0.077  |
| 140 | f5 | item_5               | -0.064 |
| 141 | f5 | $item_41$            | -0.060 |
| 142 | f5 | item_29              | 0.057  |
| 143 | f5 | $\mathtt{item\_133}$ | -0.033 |
| 144 | f5 | $\mathtt{item\_116}$ | -0.022 |
| 145 | f5 | $item_48$            | -0.021 |
| 146 | f6 | $\mathtt{item\_137}$ | 0.842  |
| 147 | f6 | item_96              | 0.758  |
| 148 | f6 | $\mathtt{item\_116}$ | 0.492  |
| 149 | f6 | $\mathtt{item\_194}$ | 0.399  |
| 150 | f6 | item_81              | -0.286 |
| 151 | f6 | $\mathtt{item\_103}$ | 0.256  |
| 152 | f6 | item_23              | 0.242  |
| 153 | f6 | item_39              | 0.223  |
| 154 | f6 | item_38              | 0.223  |
| 155 | f6 | item_57              | -0.221 |
| 156 | f6 | item_86              | -0.216 |
|     |    |                      |        |

```
157
       f6 item_48
                                  -0.199
158
       f6 item_34
                                   0.167
159
       f6 item_128
                                   0.130
160
       f6 item_45
                                  0.130
161
       f6 item 143
                                  0.126
162
       f6 item_41
                                  -0.120
163
       f6 item 186
                                  0.108
164
       f6
            item_7
                                   0.106
165
       f6 item_111
                                  -0.097
166
       f6 item_133
                                  0.082
167
       f6 item_79
                                  0.056
168
       f6 item_16
                                  0.033
169
       f6 item_119
                                  -0.017
170
       f6 item_85
                                  -0.011
171
       f6
            item_5
                                  0.010
172
       f6 item_129
                                  -0.009
173
       f6 item_105
                                  0.005
174
                                   0.000
       f6 item_29
```

```
# Extract Interfactor Correlations
# Extract standardized correlations (Phi matrix)
interfactor_corr <- std_solution %>%
    dplyr::filter(op == "~~" & lhs != rhs) %>% # Only factor correlations
    dplyr::select(lhs, rhs, est.std) %>%
    arrange(desc(abs(est.std))) # Sort by magnitude

colnames(interfactor_corr) <- c("Factor 1", "Factor 2", "Standardized Correlation")
# Display interfactor correlations
cat("\nInterfactor Correlations:\n")</pre>
```

#### Interfactor Correlations:

# print(interfactor\_corr)

```
4
         f3
                   f5
                                          0.517
5
         f5
                   f6
                                         -0.405
6
         f4
                   f6
                                         -0.394
7
         f2
                   f6
                                          0.349
                                         -0.282
8
         f3
                   f6
9
         f1
                   f5
                                         -0.267
10
         f2
                   f4
                                         -0.212
11
         f2
                   f3
                                         -0.203
12
         f1
                   f3
                                         -0.147
13
         f1
                   f4
                                         -0.121
14
         f2
                   f5
                                         -0.043
15
         f1
                   f2
                                          0.010
```

# **Classification Accuracy**

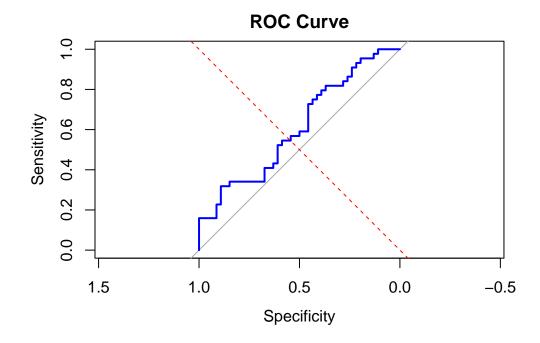
```
# Extract manifest variables (indicators) from the fitted model
indicator_names <- lavNames(fit1, type = "ov")</pre>
# Select corresponding variables from imputed data
XX <- imp_selected |>
  dplyr::select(all_of(indicator_names))
# Convert to matrix for modeling
X <- as.matrix(XX)</pre>
y <- df$death_risk
# Ensure death_risk is a binary factor
df$death_risk <- as.factor(df$death_risk)</pre>
# Merge imp_selected with the outcome variable
df_model <- imp_selected %>%
  mutate(death_risk = df$death_risk)
# Split data into training (80%) and testing (20%) sets
set.seed(123) # For reproducibility
trainIndex <- createDataPartition(df model$death_risk, p = 0.8, list = FALSE)
train_data <- df_model[trainIndex, ]</pre>
test_data <- df_model[-trainIndex, ]</pre>
# Fit logistic regression model
logit_model <- glm(death_risk ~ ., data = train_data, family = binomial)</pre>
```

```
# Predict probabilities on test set
pred_probs <- predict(logit_model, test_data, type = "response")</pre>
# Convert probabilities to class labels (0.5 threshold)
pred_classes <- factor(ifelse(pred_probs > 0.5, 1, 0), levels = levels(test_data$death_risk)
# Compute classification accuracy
accuracy <- mean(pred_classes == test_data$death_risk)</pre>
cat("Classification Accuracy:", round(accuracy, 3), "\n")
Classification Accuracy: 0.556
# Compute AUC (Area Under the Curve)
roc_curve <- roc(test_data$death_risk, pred_probs)</pre>
Setting levels: control = 0, case = 1
Setting direction: controls < cases
auc_value <- auc(roc_curve)</pre>
cat("AUC:", round(auc_value, 3), "\n")
AUC: 0.611
# Permutation Test: Evaluating Accuracy Against Chance Level
set.seed(123)
null_accuracies <- replicate(1000, {</pre>
  shuffled_risk <- sample(train_data$death_risk) # Shuffle class labels</pre>
  # Fit model on shuffled labels
  null_model <- glm(shuffled_risk ~ ., data = train_data, family = binomial)</pre>
  # Predict using null model
  null_preds <- predict(null_model, test_data, type = "response")</pre>
  null_classes <- factor(ifelse(null_preds > 0.5, 1, 0), levels = levels(test_data$death_ris
  mean(null_classes == test_data$death_risk) # Compute accuracy on actual test labels
})
```

```
# Compute p-value: Proportion of null models performing as well as or better than the real m p_value <- mean(null_accuracies >= accuracy) cat("P-value for classification above chance:", round(p_value, 4), "\n")
```

P-value for classification above chance: 0.332

```
# Plot ROC Curve
plot(roc_curve, col = "blue", main = "ROC Curve")
abline(a = 0, b = 1, lty = 2, col = "red") # Reference line for random classification
```



# Remove the worse items

```
selected_items <- c(
   "i_86", "i_57", "i_5", "i_81",
   "i_105", "i_48", "i_133", "i_129", "i_39", "i_103",
   "i_143", "i_79",
   "i_111", "i_34", "i_119", "i_116", "i_23", "i_41",
   "i_186", "i_38", "i_128", "i_7", "i_16",
   "i_137", "i_96", "i_194"</pre>
```

```
)
# Convert to match column names in `imp`
selected_items_corrected <- paste0("item_", sub("^i_", "", selected_items))</pre>
# Ensure only existing columns are selected
existing_items <- intersect(selected_items_corrected, colnames(imp))</pre>
# Issue a warning if any items are missing
missing_items <- setdiff(selected_items_corrected, existing_items)</pre>
if (length(missing_items) > 0) {
warning("The following items were not found in the dataset: ", paste(missing_items, collapse)
}
# Select the matching columns
imp_selected <- imp %>%
  dplyr::select(all_of(existing_items))
# Validate the selection
print(names(imp_selected))
 [1] "item_86" "item_57" "item_5"
                                       "item_81" "item_105" "item_48"
 [7] "item_133" "item_129" "item_39" "item_103" "item_143" "item_79"
[13] "item_111" "item_34" "item_119" "item_116" "item_23" "item_41"
[19] "item_186" "item_38" "item_128" "item_7"
                                                 "item_16" "item_137"
[25] "item_96" "item_194"
# Define Target Matrix for Factor Structure (26x6)
# Initialize a 26x6 matrix filled with zeros
TARGET <- matrix(0, nrow = length(existing_items), ncol = 6)</pre>
# Assign factor loadings dynamically
factor_assignments <- list(</pre>
 F1 = c(1:4),
 F2 = c(5:10),
 F3 = c(11:12),
 F4 = c(13:18),
  F5 = c(19:23),
  F6 = c(24:26)
```

```
# Assign 1s based on factor structure
for (factor in names(factor_assignments)) {
   TARGET[factor_assignments[[factor]], as.numeric(substr(factor, 2, 2))] <- 1
}

# Add row and column names for clarity
rownames(TARGET) <- existing_items
colnames(TARGET) <- pasteO("F", 1:6)

# Print the target rotation matrix
print(TARGET)</pre>
```

```
F1 F2 F3 F4 F5 F6
item_86
    1 0 0 0 0 0
item_57
     1 0 0 0 0 0
item_5 1 0 0 0 0 0
item_81 1 0 0 0 0 0
item_105 0 1 0 0 0 0
item_143 0 0 1 0 0 0
item_111 0 0 0 1 0 0
item_119 0 0 0 1 0 0
item_116 0 0 0 1 0 0
item_23 0 0 0 1 0 0
item_41
    0 0 0 1 0 0
item_186 0 0 0 0 1 0
0 0 0 0 1 0
item_7
    0 0 0 0 1 0
item_16
item_137 0 0 0 0 0 1
    0 0 0 0 0 1
item_96
item_194 0 0 0 0 0 1
```

```
# Define the ESEM Model in Lavaan Syntax
model <- '
    efa("efa1")*f1 +
    efa("efa1")*f2 +
    efa("efa1")*f3 +
    efa("efa1")*f4 +
    efa("efa1")*f5 +
    efa("efa1")*f6 =~
       # F1
       item_86 + item_57 + item_5 + item_81 +
       item_105 + item_48 + item_133 + item_129 + item_39 + item_103 +
       # F3
       item_{143} + item_{79} +
       # F4
       item 111 + item 34 + item 119 + item 116 + item 23 + item 41 +
       item_186 + item_38 + item_128 + item_7 + item_16 +
       # F6
       item_137 + item_96 + item_194
# Fit the ESEM Model with Target Rotation
fit2 <- sem(
  model = model,
  data = imp_selected,
  ordered = TRUE, # Use ordered estimation if Likert-type items
 rotation = "target",
  rotation.args = list(target = TARGET)
)
fit2 indices <- fitMeasures(fit2, c(</pre>
  "chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr",
  "chisq.scaled", "df.scaled", "pvalue.scaled", "cfi.robust", "tli.robust",
  "rmsea.robust", "srmr"
))
# Display selected fit indices (Standard & Robust)
cat("\nModel Fit Indices (Standard & Robust):\n")
```

#### Model Fit Indices (Standard & Robust):

```
fit2_indices_df <- data.frame(
    Measure = names(fit2_indices),
    Value = round(fit2_indices, 3)
)
print(fit2_indices_df)</pre>
```

```
Value
        Measure
          chisq 182.195
1
2
             df 184.000
         pvalue
3
                  0.524
4
                 1.000
            cfi
5
            tli
                  1.000
6
          rmsea 0.000
7
           srmr
                  0.035
8
   chisq.scaled 391.429
9
      df.scaled 184.000
10 pvalue.scaled 0.000
11
     cfi.robust 0.930
12
     tli.robust 0.877
13 rmsea.robust 0.063
14
           srmr 0.035
```

```
# Extract Standardized Factor Loadings
# Get standardized solution
std_solution <- standardizedSolution(fit2)

# Filter only loadings (Lambda matrix)
std_loadings <- std_solution %>%
    filter(op == "=~") %>%
    dplyr::select(lhs, rhs, est.std) %>%
    arrange(lhs, desc(abs(est.std))) # Sorted by factor and magnitude

colnames(std_loadings) <- c("Factor", "Item", "Standardized Loading")

# Display standardized loadings
cat("\nStandardized Factor Loadings:\n")</pre>
```

# Standardized Factor Loadings:

# print(std\_loadings)

|    | Factor | Item                 | Standardized.Loading |
|----|--------|----------------------|----------------------|
| 1  | f1     | item_81              | 0.562                |
| 2  | f1     | item_86              | 0.553                |
| 3  | f1     | $\mathtt{item\_105}$ | 0.485                |
| 4  | f1     | $item_48$            | 0.474                |
| 5  | f1     | item_57              | 0.436                |
| 6  | f1     | item_5               | 0.336                |
| 7  | f1     | item_39              | 0.327                |
| 8  | f1     | item_38              | 0.322                |
| 9  | f1     | $item_103$           | 0.316                |
| 10 | f1     | item_194             | -0.314               |
| 11 | f1     | item_119             | -0.255               |
| 12 | f1     | item_137             | -0.196               |
| 13 | f1     | item_186             | 0.157                |
| 14 | f1     | item_7               | 0.147                |
| 15 | f1     | item_111             | -0.139               |
| 16 | f1     | item_23              | -0.134               |
| 17 | f1     | item_96              | -0.127               |
| 18 | f1     | item_41              | 0.117                |
| 19 | f1     | item_34              | 0.106                |
| 20 | f1     | item_116             | 0.106                |
| 21 | f1     | item_129             | 0.082                |
| 22 | f1     | item_79              | -0.064               |
| 23 | f1     | $item_143$           | -0.059               |
| 24 | f1     | item_128             | -0.050               |
| 25 | f1     | $item_133$           | 0.026                |
| 26 | f1     | item_16              | -0.003               |
| 27 | f2     | $item_129$           | 0.890                |
| 28 | f2     | $item_133$           | 0.848                |
| 29 | f2     | item_81              | 0.738                |
| 30 | f2     | $item_105$           | 0.632                |
| 31 | f2     | item_48              | 0.559                |
| 32 | f2     | $item_103$           | 0.502                |
| 33 | f2     | item_5               | 0.475                |
| 34 | f2     | item_39              | 0.416                |
| 35 | f2     | item_116             | 0.382                |
| 36 | f2     | $item_111$           | 0.298                |
| 37 | f2     | item_86              | 0.283                |
| 38 | f2     | item_57              | 0.213                |
| 39 | f2     | item_23              | 0.195                |

| 40 | f2 | item_137             | 0.192  |
|----|----|----------------------|--------|
| 41 | f2 | item_96              | 0.186  |
| 42 | f2 | $item_34$            | 0.153  |
| 43 | f2 | item_79              | 0.123  |
| 44 | f2 | $item_119$           | 0.111  |
| 45 | f2 | item_16              | -0.107 |
| 46 | f2 | item_38              | 0.079  |
| 47 | f2 | $item_41$            | -0.077 |
| 48 | f2 | $item_7$             | -0.036 |
| 49 | f2 | $item_128$           | -0.032 |
| 50 | f2 | $item_194$           | -0.032 |
| 51 | f2 | $item_186$           | 0.018  |
| 52 | f2 | $\mathtt{item\_143}$ | -0.002 |
| 53 | f3 | $item_143$           | 0.952  |
| 54 | f3 | item_79              | 0.787  |
| 55 | f3 | $item_186$           | 0.227  |
| 56 | f3 | $item_194$           | 0.206  |
| 57 | f3 | item_86              | -0.176 |
| 58 | f3 | $item_116$           | -0.154 |
| 59 | f3 | item_38              | 0.143  |
| 60 | f3 | $\mathtt{item\_105}$ | 0.132  |
| 61 | f3 | $item_128$           | -0.129 |
| 62 | f3 | item_39              | -0.122 |
| 63 | f3 | $item_5$             | 0.122  |
| 64 | f3 | $item_16$            | -0.112 |
| 65 | f3 | $item_111$           | -0.112 |
| 66 | f3 | $item_48$            | 0.111  |
| 67 | f3 | item_23              | -0.109 |
| 68 | f3 | $item_41$            | -0.084 |
| 69 | f3 | $item_7$             | -0.080 |
| 70 | f3 | $item_34$            | -0.067 |
| 71 | f3 | $item_103$           | -0.066 |
| 72 | f3 | $item_133$           | 0.053  |
| 73 | f3 | item_57              | -0.051 |
| 74 | f3 | item_96              | -0.050 |
| 75 | f3 | $item_137$           | 0.018  |
| 76 | f3 | $item_119$           | -0.018 |
| 77 | f3 | item_81              | -0.018 |
| 78 | f3 | item_129             | 0.010  |
| 79 | f4 | item_119             | 0.837  |
| 80 | f4 | item_34              | 0.792  |
| 81 | f4 | $item_111$           | 0.792  |
| 82 | f4 | item_23              | 0.750  |
|    |    |                      |        |

| 83  | f4 | item_41              | 0.614  |
|-----|----|----------------------|--------|
| 84  | f4 | item_39              | 0.376  |
| 85  | f4 | $item_103$           | 0.357  |
| 86  | f4 | item_116             | 0.254  |
| 87  | f4 | item_38              | -0.250 |
| 88  | f4 | item_194             | 0.228  |
| 89  | f4 | $item_48$            | 0.226  |
| 90  | f4 | item_7               | -0.172 |
| 91  | f4 | item_57              | -0.138 |
| 92  | f4 | $\mathtt{item\_143}$ | -0.135 |
| 93  | f4 | $\mathtt{item\_137}$ | 0.116  |
| 94  | f4 | item_96              | 0.103  |
| 95  | f4 | $\mathtt{item\_186}$ | -0.077 |
| 96  | f4 | $\mathtt{item\_129}$ | -0.069 |
| 97  | f4 | item_81              | -0.061 |
| 98  | f4 | $\mathtt{item\_133}$ | 0.039  |
| 99  | f4 | item_86              | 0.033  |
| 100 | f4 | item_16              | -0.029 |
| 101 | f4 | item_5               | -0.023 |
| 102 | f4 | item_79              | 0.022  |
| 103 | f4 | $\mathtt{item\_128}$ | 0.021  |
| 104 | f4 | $\mathtt{item\_105}$ | 0.020  |
| 105 | f5 | item_7               | 1.084  |
| 106 | f5 | $\mathtt{item\_128}$ | 0.906  |
| 107 | f5 | item_16              | 0.723  |
| 108 | f5 | $\mathtt{item\_186}$ | 0.504  |
| 109 | f5 | item_38              | 0.455  |
| 110 | f5 | item_86              | 0.324  |
| 111 | f5 | item_23              | -0.323 |
| 112 | f5 | item_57              | 0.199  |
| 113 | f5 | $item_137$           | 0.198  |
| 114 | f5 | $\mathtt{item\_103}$ | -0.175 |
| 115 | f5 | $item_194$           | 0.173  |
| 116 | f5 | item_79              | 0.123  |
| 117 | f5 | $item_111$           | -0.122 |
| 118 | f5 | item_39              | 0.121  |
| 119 | f5 | $\mathtt{item\_119}$ | -0.106 |
| 120 | f5 | $item_143$           | 0.102  |
| 121 | f5 | item_34              | -0.101 |
| 122 | f5 | item_96              | 0.092  |
| 123 | f5 | item_5               | -0.088 |
| 124 | f5 | item_48              | -0.085 |
| 125 | f5 | item_133             | -0.077 |
|     |    |                      |        |

```
127
        f5 item_81
                                   0.067
128
        f5 item_129
                                   0.059
129
        f5 item_41
                                  -0.049
130
        f5 item 116
                                  -0.028
131
        f6 item_137
                                   0.822
132
        f6 item 96
                                   0.733
133
        f6 item_116
                                   0.488
134
        f6 item_194
                                   0.403
135
        f6 item_81
                                  -0.275
136
        f6 item_23
                                   0.268
137
        f6 item_103
                                   0.251
138
        f6 item_39
                                   0.207
139
        f6 item_86
                                  -0.206
        f6 item_48
140
                                  -0.205
141
        f6 item_38
                                   0.202
142
        f6 item_57
                                  -0.200
143
        f6 item_34
                                   0.193
144
        f6 item_128
                                   0.157
145
        f6
             item 7
                                   0.129
146
        f6 item_143
                                   0.126
147
        f6 item 111
                                  -0.107
148
        f6 item_186
                                   0.101
                                  -0.093
149
        f6 item_41
150
        f6 item_79
                                   0.076
151
        f6 item_133
                                   0.071
152
        f6 item_16
                                   0.060
153
        f6 item_119
                                  -0.028
154
        f6
             item_5
                                   0.019
155
        f6 item_105
                                  -0.008
156
        f6 item_129
                                  -0.005
# Extract Interfactor Correlations
# Extract standardized correlations (Phi matrix)
interfactor_corr <- std_solution %>%
```

0.073

126

 $f5 item_105$ 

colnames(interfactor\_corr) <- c("Factor 1", "Factor 2", "Standardized Correlation")</pre>

dplyr::filter(op == "~~" & lhs != rhs) %>% # Only factor correlations

dplyr::select(lhs, rhs, est.std) %>%

arrange(desc(abs(est.std))) # Sort by magnitude

```
# Display interfactor correlations
cat("\nInterfactor Correlations:\n")
```

#### Interfactor Correlations:

```
print(interfactor_corr)
```

|    | Factor.1 | Factor.2 | Standardized.Correlation |
|----|----------|----------|--------------------------|
| 1  | f4       | f5       | 0.710                    |
| 2  | f3       | f4       | 0.678                    |
| 3  | f3       | f5       | 0.551                    |
| 4  | f1       | f6       | 0.497                    |
| 5  | f5       | f6       | -0.426                   |
| 6  | f4       | f6       | -0.365                   |
| 7  | f2       | f6       | 0.345                    |
| 8  | f3       | f6       | -0.237                   |
| 9  | f2       | f3       | -0.202                   |
| 10 | f1       | f5       | -0.187                   |
| 11 | f2       | f4       | -0.171                   |
| 12 | f1       | f2       | -0.049                   |
| 13 | f2       | f5       | -0.021                   |
| 14 | f1       | f4       | 0.016                    |
| 15 | f1       | f3       | 0.015                    |

#### Combine the first two factors

Also, add additional items to maximize classification accuracy. These are the items that maximally contribute to accurate classification: item\_22 item\_23 item\_26 item\_40 item\_99 item\_121 item\_125 item\_139 item\_157 item\_159 item\_164 item\_187.

```
selected_items <- c(
   "i_86", "i_57", "i_5", "i_81",
   "i_105", "i_48", "i_133", "i_129", "i_39", "i_103",
   "i_143", "i_79",
   "i_111", "i_34", "i_119", "i_23", "i_41", "i_139",
   "i_186", "i_38", "i_128", "i_7", "i_16", "i_125",
   "i_137", "i_96", "i_194", "i_159"
)</pre>
```

```
# Convert to match column names in `imp`
selected_items_corrected <- paste0("item_", sub("^i_", "", selected_items))</pre>
# Ensure only existing columns are selected
existing_items <- intersect(selected_items_corrected, colnames(imp))</pre>
# Issue a warning if any items are missing
missing_items <- setdiff(selected_items_corrected, existing_items)</pre>
if (length(missing_items) > 0) {
 warning("The following items were not found in the dataset: ", paste(missing_items, collap.
}
# Select the matching columns
imp_selected <- imp %>%
  dplyr::select(all_of(existing_items))
# Validate the selection
print(names(imp_selected))
 [1] "item_86" "item_57" "item_5" "item_81" "item_105" "item_48"
 [7] "item_133" "item_129" "item_39" "item_103" "item_143" "item_79"
[13] "item_111" "item_34" "item_119" "item_23" "item_41" "item_139"
[19] "item 186" "item 38" "item 128" "item 7"
                                                 "item 16" "item 125"
[25] "item_137" "item_96" "item_194" "item_159"
# Define Target Matrix for Factor Structure
# Determine the number of rows dynamically
num_items <- length(existing_items)</pre>
TARGET <- matrix(0, nrow = num_items, ncol = 5)</pre>
# Assign factor loadings dynamically
factor_assignments <- list(</pre>
  F1 = 1:10,  # Caratteristiche bambino e richieste caregiving
 F2 = 11:12, # Percezione cura
  F3 = 13:18, # Fattori intrapsichici
 F4 = 19:24, # Coping
  F5 = 25:28 # Iperprotezione
)
# Assign 1s based on factor structure
```

```
for (factor in names(factor_assignments)) {
   TARGET[factor_assignments[[factor]], as.numeric(substr(factor, 2, 2))] <- 1
}

# Add row and column names for clarity
rownames(TARGET) <- existing_items
colnames(TARGET) <- paste0("F", 1:5)

# Print the target rotation matrix
print(TARGET)</pre>
```

```
F1 F2 F3 F4 F5
item_86
     1 0 0 0 0
item_57
    1 0 0 0 0
item_5
     1 0 0 0 0
item_105 1 0 0 0 0
item_133 1 0 0 0 0
item_129 1 0 0 0 0
item_143 0 1 0 0 0
item_111 0 0 1 0 0
item_119 0 0 1 0 0
item_23 0 0 1 0 0
item_139 0 0 1 0 0
item_186  0  0  0  1  0
item_128  0  0  0  1  0
item_7 0 0 0 1 0
item_16
     0 0 0 1 0
item_125 0 0 0 1 0
item_137 0 0 0 0 1
item_96  0  0  0  0  1
item_159 0 0 0 0 1
```

```
# Define the ESEM Model in Lavaan Syntax
model <- '
    efa("efa1")*f1 +
   efa("efa1")*f2 +
    efa("efa1")*f3 +
    efa("efa1")*f4 +
    efa("efa1")*f5 =~
       # F1
       item_86 + item_57 + item_5 + item_81 +
       item_105 + item_48 + item_133 + item_129 + item_39 + item_103 +
       # F2
       item_143 + item_79 +
       # F3
      item_111 + item_34 + item_119 + item_23 + item_41 + item_139 +
       item_186 + item_38 + item_128 + item_7 + item_16 + item_125 +
       # F4
       item_137 + item_96 + item_194 + item_159
# Fit the ESEM Model with Target Rotation
fit3 <- sem(
 model = model,
  data = imp_selected,
 ordered = TRUE, # Use ordered estimation if Likert-type items
 rotation = "target",
  rotation.args = list(target = TARGET)
)
fit3_indices <- fitMeasures(fit3, c(</pre>
  "chisq", "df", "pvalue", "cfi", "tli", "rmsea", "srmr",
  "chisq.scaled", "df.scaled", "pvalue.scaled", "cfi.robust", "tli.robust",
  "rmsea.robust", "srmr"
))
# Display selected fit indices (Standard & Robust)
```

#### Model Fit Indices (Standard & Robust):

cat("\nModel Fit Indices (Standard & Robust):\n")

```
fit3_indices_df <- data.frame(
   Measure = names(fit3_indices),
   Value = round(fit3_indices, 3)
)
print(fit3_indices_df)</pre>
```

```
Measure
                  Value
          chisq 342.662
1
2
             df 248.000
         pvalue
3
                  0.000
4
            cfi
                  0.994
5
            tli
                0.990
6
          rmsea
                  0.029
7
                  0.044
           srmr
8
  chisq.scaled 648.584
9
      df.scaled 248.000
10 pvalue.scaled 0.000
     cfi.robust 0.864
     tli.robust 0.793
12
13 rmsea.robust 0.081
14
           srmr 0.044
```

```
# Extract Standardized Factor Loadings
# Get standardized solution
std_solution <- standardizedSolution(fit3)

# Filter only loadings (Lambda matrix)
std_loadings <- std_solution %>%
    filter(op == "=~") %>%
    dplyr::select(lhs, rhs, est.std) %>%
    arrange(lhs, desc(abs(est.std))) # Sorted by factor and magnitude

colnames(std_loadings) <- c("Factor", "Item", "Standardized Loading")

# Display standardized loadings
cat("\nStandardized Factor Loadings:\n")</pre>
```

Standardized Factor Loadings:

# print(std\_loadings)

|    | Factor | Item                 | Standardized.Loading |
|----|--------|----------------------|----------------------|
| 1  | f1     | item_81              | 0.973                |
| 2  | f1     | $\mathtt{item\_105}$ | 0.830                |
| 3  | f1     | $\mathtt{item\_129}$ | 0.805                |
| 4  | f1     | item_48              | 0.750                |
| 5  | f1     | $\mathtt{item\_133}$ | 0.730                |
| 6  | f1     | item_39              | 0.705                |
| 7  | f1     | $\mathtt{item\_103}$ | 0.693                |
| 8  | f1     | item_86              | 0.640                |
| 9  | f1     | item_5               | 0.577                |
| 10 | f1     | item_57              | 0.433                |
| 11 | f1     | item_34              | 0.349                |
| 12 | f1     | item_38              | 0.314                |
| 13 | f1     | item_7               | 0.304                |
| 14 | f1     | $\mathtt{item\_143}$ | -0.295               |
| 15 | f1     | item_41              | 0.265                |
| 16 | f1     | item_96              | 0.251                |
| 17 | f1     | item_23              | 0.211                |
| 18 | f1     | $\mathtt{item\_194}$ | -0.204               |
| 19 | f1     | ${\tt item\_128}$    | 0.193                |
| 20 | f1     | $\mathtt{item\_111}$ | 0.188                |
| 21 | f1     | $\mathtt{item\_159}$ | 0.184                |
| 22 | f1     | $\mathtt{item}\_137$ | 0.172                |
| 23 | f1     | item_79              | -0.166               |
| 24 | f1     | $item_186$           | 0.160                |
| 25 | f1     | $\mathtt{item\_139}$ | 0.116                |
| 26 | f1     | item_16              | 0.083                |
| 27 | f1     | $\mathtt{item\_125}$ | 0.072                |
| 28 | f1     | $\mathtt{item\_119}$ | -0.034               |
| 29 | f2     | $\mathtt{item\_143}$ | 0.746                |
| 30 | f2     | item_79              | 0.641                |
| 31 | f2     | $\mathtt{item\_125}$ | 0.317                |
| 32 | f2     | item_129             | -0.314               |
| 33 | f2     | $\mathtt{item\_133}$ | -0.279               |
| 34 | f2     | $item_186$           | 0.255                |
| 35 | f2     | $\mathtt{item\_194}$ | 0.244                |
| 36 | f2     | $\mathtt{item\_137}$ | 0.239                |
| 37 | f2     | $\mathtt{item\_139}$ | 0.192                |
| 38 | f2     | item_81              | -0.185               |
| 39 | f2     | item_38              | 0.170                |

| 40 | f2 | item_96   | 0.168  |
|----|----|-----------|--------|
| 41 | f2 | $item_41$ | 0.164  |
| 42 | f2 | item_111  | -0.152 |
| 43 | f2 | $item_34$ | 0.107  |
| 44 | f2 | item_159  | 0.096  |
| 45 | f2 | item_86   | -0.079 |
| 46 | f2 | item_5    | 0.056  |
| 47 | f2 | item_128  | -0.052 |
| 48 | f2 | $item_57$ | -0.050 |
| 49 | f2 | item_105  | 0.049  |
| 50 | f2 | $item_16$ | -0.047 |
| 51 | f2 | $item_23$ | 0.026  |
| 52 | f2 | $item_48$ | 0.016  |
| 53 | f2 | item_119  | -0.015 |
| 54 | f2 | item_39   | 0.013  |
| 55 | f2 | item_103  | 0.011  |
| 56 | f2 | $item_7$  | -0.006 |
| 57 | f3 | $item_41$ | 0.959  |
| 58 | f3 | item_119  | 0.787  |
| 59 | f3 | item_111  | 0.768  |
| 60 | f3 | item_139  | 0.767  |
| 61 | f3 | $item_34$ | 0.657  |
| 62 | f3 | $item_23$ | 0.609  |
| 63 | f3 | $item_48$ | 0.359  |
| 64 | f3 | item_39   | 0.344  |
| 65 | f3 | item_38   | -0.299 |
| 66 | f3 | $item_79$ | 0.227  |
| 67 | f3 | item_103  | 0.214  |
| 68 | f3 | item_159  | -0.184 |
| 69 | f3 | $item_7$  | -0.161 |
| 70 | f3 | item_133  | 0.149  |
| 71 | f3 | item_194  | 0.117  |
| 72 | f3 | item_81   | 0.111  |
| 73 | f3 | item_105  | 0.093  |
| 74 | f3 | item_186  | -0.092 |
| 75 | f3 | item_143  | 0.088  |
| 76 | f3 | $item_57$ | -0.086 |
| 77 | f3 | item_129  | 0.075  |
| 78 | f3 | $item_16$ | -0.063 |
| 79 | f3 | item_137  | -0.054 |
| 80 | f3 | $item_5$  | 0.051  |
| 81 | f3 | item_125  | -0.049 |
| 82 | f3 | item_86   | 0.047  |
|    |    |           |        |

| 83  | f3 | item_128             | -0.019 |
|-----|----|----------------------|--------|
| 84  | f3 | item_96              | -0.011 |
| 85  | f4 | $item_7$             | 1.058  |
| 86  | f4 | ${\tt item\_128}$    | 0.878  |
| 87  | f4 | $\mathtt{item\_125}$ | 0.742  |
| 88  | f4 | $item_16$            | 0.698  |
| 89  | f4 | $\mathtt{item\_186}$ | 0.670  |
| 90  | f4 | item_38              | 0.548  |
| 91  | f4 | item_86              | 0.427  |
| 92  | f4 | item_23              | -0.343 |
| 93  | f4 | $\mathtt{item\_143}$ | 0.317  |
| 94  | f4 | item_79              | 0.305  |
| 95  | f4 | item_57              | 0.291  |
| 96  | f4 | $\mathtt{item\_105}$ | 0.259  |
| 97  | f4 | $item_41$            | -0.256 |
| 98  | f4 | item_81              | 0.254  |
| 99  | f4 | $\mathtt{item\_139}$ | -0.232 |
| 100 | f4 | item_96              | -0.155 |
| 101 | f4 | $\mathtt{item\_129}$ | 0.145  |
| 102 | f4 | $item_194$           | 0.138  |
| 103 | f4 | $item_48$            | 0.114  |
| 104 | f4 | item_39              | 0.106  |
| 105 | f4 | $\mathtt{item\_159}$ | -0.104 |
| 106 | f4 | $item_111$           | -0.080 |
| 107 | f4 | $\mathtt{item\_137}$ | -0.066 |
| 108 | f4 | item_34              | -0.061 |
| 109 | f4 | $\mathtt{item\_119}$ | -0.059 |
| 110 | f4 | $\mathtt{item\_103}$ | -0.032 |
| 111 | f4 | item_5               | 0.017  |
| 112 | f4 | $\mathtt{item\_133}$ | -0.010 |
| 113 |    | $\mathtt{item\_137}$ | 0.665  |
| 114 | f5 | $\mathtt{item\_159}$ | 0.586  |
| 115 | f5 | item_96              | 0.463  |
| 116 | f5 | $\mathtt{item\_194}$ | 0.455  |
| 117 | f5 | $\mathtt{item\_139}$ | -0.430 |
| 118 | f5 | $\mathtt{item\_143}$ | 0.364  |
| 119 | f5 | $item_41$            | -0.364 |
| 120 | f5 | item_79              | 0.337  |
| 121 | f5 | $\mathtt{item\_133}$ | 0.325  |
| 122 | f5 | item_86              | -0.248 |
| 123 | f5 | item_23              | 0.219  |
| 124 | f5 | item_81              | -0.197 |
| 125 | f5 | $\mathtt{item\_129}$ | 0.194  |
|     |    |                      |        |

```
126
       f5 item_119
                                  0.180
127
       f5 item_57
                                 -0.174
128
       f5 item_111
                                  0.140
129
       f5
           item_7
                                 -0.125
130
       f5 item 48
                                 -0.119
131
       f5 item_103
                                  0.107
132
       f5 item_34
                                  0.107
       f5 item_16
133
                                 -0.099
134
       f5 item_39
                                  0.077
135
       f5 item_128
                                 -0.056
136
       f5 item_5
                                  0.049
137
       f5 item_38
                                  0.044
138
       f5 item_186
                                  0.032
139
       f5 item_105
                                  0.030
140
       f5 item_125
                                  0.015
```

```
# Extract Interfactor Correlations
# Extract standardized correlations (Phi matrix)
interfactor_corr <- std_solution %>%
   dplyr::filter(op == "~~" & lhs != rhs) %>% # Only factor correlations
   dplyr::select(lhs, rhs, est.std) %>%
   arrange(desc(abs(est.std))) # Sort by magnitude

colnames(interfactor_corr) <- c("Factor 1", "Factor 2", "Standardized Correlation")
# Display interfactor correlations
cat("\nInterfactor Correlations:\n")</pre>
```

### Interfactor Correlations:

# print(interfactor\_corr)

|   | Factor.1 | Factor.2 | Standardized.Correlation |
|---|----------|----------|--------------------------|
| 1 | f3       | f4       | 0.751                    |
| 2 | f1       | f3       | -0.451                   |
| 3 | f1       | f4       | -0.424                   |
| 4 | f1       | f5       | 0.335                    |
| 5 | f1       | f2       | 0.235                    |
| 6 | f2       | f5       | -0.225                   |

```
7
         f4
                   f5
                                          0.128
         f2
                   f4
                                         -0.071
8
9
         f3
                   f5
                                          0.031
10
         f2
                   f3
                                         -0.010
```

#### **Classification Accuracy**

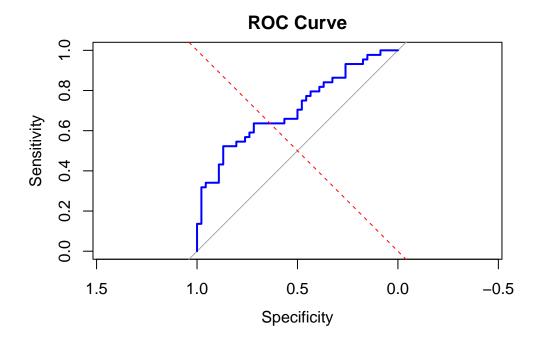
```
# Get observed variable names from the fitted model
indicator_names <- lavNames(fit3, type = "ov")</pre>
# Select corresponding variables from imputed dataset
XX <- imp_selected |>
  dplyr::select(all_of(indicator_names))
# Convert to matrix for modeling
X <- as.matrix(XX)</pre>
y <- df$death_risk # Outcome variable
# Ensure death_risk is a binary factor
df$death_risk <- factor(df$death_risk)</pre>
# Merge imputed data with the criterion variable
df_model <- imp_selected %>%
  mutate(death_risk = df$death_risk)
# Validate the factor levels
if (length(levels(df_model$death_risk)) != 2) {
  stop("Error: The death_risk variable must be binary.")
}
# Train-Test Split
set.seed(123) # For reproducibility
trainIndex <- createDataPartition(df_model$death_risk, p = 0.8, list = FALSE)</pre>
train_data <- df_model[trainIndex, ]</pre>
test_data <- df_model[-trainIndex, ]</pre>
# Fit Logistic Regression Model
logit_model <- glm(death_risk ~ ., data = train_data, family = binomial)</pre>
# Predict probabilities on test set
pred_probs <- predict(logit_model, test_data, type = "response")</pre>
```

```
# Convert probabilities to class labels using dynamic factor levels
threshold <- 0.5
pred_classes <- factor(ifelse(pred_probs > threshold, levels(df$death_risk)[2], levels(df$death_risk)
                        levels = levels(df$death_risk))
# Compute classification accuracy
accuracy <- mean(pred_classes == test_data$death_risk, na.rm = TRUE)
cat("Classification Accuracy:", round(accuracy, 3), "\n")
Classification Accuracy: 0.622
# Compute AUC (Area Under the Curve)
roc_curve <- roc(test_data$death_risk, pred_probs)</pre>
Setting levels: control = 0, case = 1
Setting direction: controls < cases
auc_value <- auc(roc_curve)</pre>
cat("AUC:", round(auc_value, 3), "\n")
AUC: 0.71
# Permutation Test: Evaluating Accuracy Against Chance Level
set.seed(123)
null_accuracies <- replicate(1000, {</pre>
  shuffled_risk <- sample(train_data$death_risk) # Shuffle class labels</pre>
  # Fit model on shuffled labels
  null_model <- glm(shuffled_risk ~ ., data = train_data, family = binomial)</pre>
  # Predict using null model
  null_preds <- predict(null_model, test_data, type = "response")</pre>
  null_classes <- factor(ifelse(null_preds > threshold, levels(df$death_risk)[2], levels(df$
                          levels = levels(df$death_risk))
  mean(null_classes == test_data$death_risk, na.rm = TRUE) # Compute accuracy on actual test
})
```

```
# Compute p-value: Proportion of null models performing as well as or better than the real models p-value <- mean(null_accuracies >= accuracy) cat("P-value for classification above chance:", round(p_value, 4), "\n")
```

P-value for classification above chance: 0.123

```
# Plot ROC Curve
plot(roc_curve, col = "blue", main = "ROC Curve")
abline(a = 0, b = 1, lty = 2, col = "red") # Reference line for random classification
```



### Interpreting the Results

### Classification Accuracy:

Measures the percentage of correct classifications in the test set. If accuracy > 0.70, the model has reasonably good predictive power.

#### AUC (Area Under the Curve):

AUC = 0.5: Model performs at chance level (random guessing). AUC > 0.7: Good discrimination between positive/negative cases. AUC = 1: Excellent classification performance.

# Permutation Test (p-value):

Tests whether the classifier performs significantly better than chance. If p < 0.05: The model significantly outperforms random classification. If p  $\,$  0.5: The model is not better than random guessing.