# Further improvements to Partial Effects Plots

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# Introduction

This notebook reprises selected analyses using GAMs, and then develops nicer partial effects plots than the mgcv defaults. In particular, I'm interested in generating plots of marginal means over data scatter plots. I focus only on graphics that include two related plots of marginal means.

Rachel Lasley-Rasher requested that I assemble plots that lack repeated Y axes. It would probably be possible to construct them directly, assembling a graphic out of two GROBs containing graphs and a third containing the axis title and axis labels, but that would not be easy.

# The Approach

One alternative is to use ggplot2's faceting capabilities. That is the approach I take here. The core idea is that you can assemble a couple of synthetic data frames (one for marginal means, one for raw data) and then produce faceted graphics.

# A couple of Challenges

A few subtleties to make it all work:

- 1. The labels for the faceting factors most match in the two data frames and
- 2. Those labels may have to include expressions that ggplot2 can parse to generate Greek letters, super-scripts, etc. in the facet labels
- 3. Positioning of the facet labels is a bit tricky too.

Unlike the "GAM Analysis Partials" notebook, here I don't rely on a function to generate the graphics in a consistent manner, but instead just work directly with ggplot().

# General Instructions to Authors About Graphics

The instructions to authors suggests figure widths should line up with columns, and proposes figure widths should be:

 $39 \text{ mm} \sim 1.54 \text{ inches } 84 \text{ mm} \sim 3.30 \text{ inches } 129 \text{ mm} \sim 5.04 \text{ inches } 174 \text{ mm} \sim 6.85 \text{ inches } 120 \text{ mm} \sim 1.04 \text{ inches } 120 \text{ inches }$ 

With height not to exceed 235 mm (9.25 inches).

RMarkdown / knitr likes figure dimensions in inches. 174 mm is about 6.85 inches

# Load Libraries

```
library(tidyverse)
                                                  ----- tidyverse 2.0.0 --
#> -- Attaching core tidyverse packages -----
#> v dplyr
             1.1.1
                       v readr
                                   2.1.4
#> v forcats 1.0.0
                       v stringr 1.5.0
#> v ggplot2 3.4.1
                       v tibble
                                  3.2.1
#> v lubridate 1.9.2
                                    1.3.0
                       v tidyr
#> v purrr
              1.0.1
#> -- Conflicts ----- tidyverse_conflicts() --
#> x dplyr::filter() masks stats::filter()
                   masks stats::lag()
#> x dplyr::lag()
#> i Use the conflicted package (<a href="http://conflicted.r-lib.org/">http://conflicted.r-lib.org/</a>) to force all conflicts to become error
library(readxl)
library(mgcv)
                  # for GAM models
#> Loading required package: nlme
#> Attaching package: 'nlme'
#>
#> The following object is masked from 'package:dplyr':
#>
#>
      collapse
#> This is mgcv 1.8-42. For overview type 'help("mgcv-package")'.
library(emmeans)
library(lemon)
#>
#> Attaching package: 'lemon'
#> The following object is masked from 'package:purrr':
      %11%
```

# Set Graphics Theme

theme\_classic() sets ggplot()graphics for no background, no grid lines, etc. in a clean format suitable for (some) publications.

The custom theme bundles a number of other aesthetic choices to make graphics more similar to a preferred design generated in some earlier iteration of the graphics.

```
strip.placement = "outside",
strip.text = element_text(size = 10, color = "gray20"),
panel.spacing.x = unit(-15,"pt"))
}
```

# Input Data

### Folder References

```
data_folder <- "Original_Data"
dir.create(file.path(getwd(), 'figures'), showWarnings = FALSE)</pre>
```

### Load Data

```
filename.in <- "penob.station.data EA 3.12.20.xlsx"
file_path <- file.path(data_folder, filename.in)</pre>
station data <- read excel(file path,
                           sheet="Final", col_types = c("skip", "date",
                                               "numeric", "text", "numeric",
                                               "text", "skip", "skip",
                                               "skip",
                                               rep("numeric", 10),
                                               "text",
                                               rep("numeric", 47),
                                               "text",
                                               rep("numeric", 12))) %>%
 rename_with(~ gsub(" ", "_", .x)) %>%
 rename_with(~ gsub("\\.", "_", .x)) %>%
 rename_with(~ gsub("\\?", "", .x)) %>%
 rename_with(~ gsub("%", "pct", .x)) %>%
 rename_with(~ gsub("_Abundance", "", .x)) %>%
  filter(! is.na(date))
#> New names:
#> * `` -> `...61`
```

```
names(station_data)[10:12]
#> [1] "discharge_week_cftpersec" "discharg_day"
#> [3] "discharge_week_max"
names(station_data)[10:12] <- c('disch_wk', 'disch_day', 'disch_max')</pre>
```

Station names are arbitrary, and Erin previously expressed interest in renaming them from Stations 2, 4, 5 and 8 to Stations 1,2,3,and 4.

The factor() function by default sorts levels before assigning numeric codes, so a convenient way to replace the existing station codes with sequential numbers is to create a factor and extract the numeric indicator values with as.numeric().

```
station_data <- station_data %>%
  mutate(station = factor(as.numeric(factor(station))))
head(station_data)
#> # A tibble: 6 x 76
                           year month month_num season riv_km station station_num
#>
   date.
    \langle dttm \rangle
                          \langle dbl \rangle \langle chr \rangle  \langle dbl \rangle \langle chr \rangle  \langle dbl \rangle \langle fct \rangle
#> 1 2013-05-28 00:00:00 2013 May
                                           5 Spring 22.6 1
                                                                                   1
#> 2 2013-05-28 00:00:00 2013 May
                                             5 Spring 13.9 2
                                             5 Spring 8.12 3
#> 3 2013-05-28 00:00:00 2013 May
                                                                                   3
#> 4 2013-05-28 00:00:00 2013 May
                                             5 Spring
                                                         2.78 4
                                                                                   4
#> 5 2013-07-25 00:00:00 2013 July
                                              7 Summer 22.6 1
                                                                                   1
#> 6 2013-07-25 00:00:00 2013 July
                                             7 Summer 13.9 2
                                                                                   2
#> # i 68 more variables: depth <dbl>, disch_wk <dbl>, disch_day <dbl>,
      disch_max <dbl>, tide_height <dbl>, Full_Moon <dbl>, Abs_Moon <dbl>,
#> # Spring_or_Neap <chr>, ave_temp_c <dbl>, ave_sal_psu <dbl>,
\#> \# ave\_turb\_ntu <dbl>, ave\_do\_mgperl <dbl>, ave\_DO\_Saturation <dbl>,
#> # ave_chl_microqperl <dbl>, sur_temp <dbl>, sur_sal <dbl>, sur_turb <dbl>,
\# sur_do <dbl>, <math>sur_chl <dbl>, bot_temp <dbl>, bot_sal <dbl>, bot_turb <dbl>,
\# #> # bot_do <dbl>, bot_chl <dbl>, max_temp <dbl>, max_sal <dbl>, ...
```

### Subsetting to Desired Data Columns

I base selection of predictor variables here on the ones used in the manuscript.

```
base_data <- station_data %>%
  rename(Date = date,
         Station = station,
         Year = year) %>%
  select(-c(month, month_num)) %>%
  mutate(Month = factor(as.numeric(format(Date, format = '%m')),
                                                 levels = 1:12.
                                                 labels = month.abb),
         DOY = as.numeric(format(Date,format = '%j')),
         season = factor(season, levels = c('Spring', 'Summer', 'Fall')),
         is_sp_up = season == 'Spring' & Station == 1,
         Yearf = factor(Year)) %>%
  rename (Season = season,
         Density = combined_density,
         Temp = ave_temp_c,
         Sal = ave_sal_psu,
         Turb = sur_turb,
         AvgTurb = ave_turb_ntu,
         DOsat = ave_DO_Saturation,
         Chl = ave_chl_microgperl,
         Fish = ^{-}_{-}61,
         RH = Herring
         ) %>%
  select(Date, Station, Year, Yearf, Month, Season, is_sp_up, DOY, riv_km,
         disch_wk, disch_day, disch_max,
         Temp, Sal, Turb, AvgTurb, DOsat, Chl,
         Fish, RH,
         Density, H, SEI,
         Acartia, Balanus, Eurytemora, Polychaete, Pseudocal, Temora) %>%
```

```
arrange(Date, Station)
head(base_data)
#> # A tibble: 6 x 29
     Date
                         Station Year Yearf Month Season is_sp_up
                                                                      DOY riv km
#>
     \langle dttm \rangle
                          <fct>
                                  <dbl> <fct> <fct> <fct> <fct> <lql>
                                                                     <dbl>
                                                                            <dbl>
#> 1 2013-05-28 00:00:00 1
                                  2013 2013 May
                                                                            22.6
                                                    Spring TRUE
                                                                      148
#> 2 2013-05-28 00:00:00 2
                                  2013 2013 May
                                                                      148 13.9
                                                    Spring FALSE
#> 3 2013-05-28 00:00:00 3
                                  2013 2013 May
                                                    Spring FALSE
                                                                      148
                                                                             8.12
#> 4 2013-05-28 00:00:00 4
                                                                             2.78
                                  2013 2013
                                              May
                                                    Spring FALSE
                                                                       148
#> 5 2013-07-25 00:00:00 1
                                  2013 2013
                                              Jul
                                                    Summer FALSE
                                                                      206
                                                                            22.6
#> 6 2013-07-25 00:00:00 2
                                  2013 2013 Jul
                                                    Summer FALSE
                                                                      206 13.9
#> # i 20 more variables: disch_wk <dbl>, disch_day <dbl>, disch_max <dbl>,
       Temp <dbl>, Sal <dbl>, Turb <dbl>, AvgTurb <dbl>, DOsat <dbl>, Chl <dbl>,
       Fish <dbl>, RH <dbl>, Density <dbl>, H <dbl>, SEI <dbl>, Acartia <dbl>,
       Balanus <dbl>, Eurytemora <dbl>, Polychaete <dbl>, Pseudocal <dbl>,
#> #
       Temora <dbl>
```

```
rm(station_data)
```

### Complete Cases

This drops only two samples, one for missing Zooplankton data, one for missing fish data. We need this reduced data set to run The step() function. It makes little sense to try stepwise model selection if each time you add or remove a variable, the sample you are studying changes. Since fish is never an important predictor, we will want need to refit models after stepwise elimination to use the most complete possible data set.

```
complete_data <- base_data %>%
    select(Season, Station, Yearf,
        is_sp_up, Temp, Sal, Turb, Chl, Fish, RH,
        Density, H,
        Acartia, Balanus, Eurytemora, Polychaete, Pseudocal, Temora) %>%
    filter(complete.cases(.))
```

## Reduced Data

The low salinity spring samples are doing something rather different, and they complicate model fitting. Models are far better behaved if we exclude a few extreme samples. These are low salinity low zooplankton samples. We have two complementary ways to specify which samples to omit, without just omitting "outliers". The first is to restrict modeling to "marine" samples over a certain salinity range, and the other is to omit spring upstream samples, which include most of the problematic samples. We eventually decided to go with the first.

# **Functions for Data Preparation**

I developed versions of these two functions in the "GAM Analysis Partials.Rmd" notebook. See that notebook and the "Testing indirection.Rmd" notebook for more of the logic involved.

### Find Evenly Spaced Points

This finds evenly spaced points along the range of a specified variable.

```
find_stops <- function(.dat, .predictor, .nstops = 25) {
   .predictor <- ensym(.predictor)
   r <- range(.dat[[.predictor]])
   stops = seq(r[1], r[2], length.out = .nstops)
   return(stops)
}</pre>
```

# Conduct The Analysis

This calculates marginal means along one predictor variable in a model. Much of the code complexity handles special cases where either the x or y variables are transformed, which changes the way parts of the output are named.

```
marginal_analysis <- function(.dat, .predictor, .model,</pre>
                                .nstops = 25, .logx = TRUE, .transy = TRUE) {
  .predictor <- ensym(.predictor)</pre>
  the_name <- as.character(.predictor)</pre>
  the log name <- paste0("log(", the name, ")")
  # The following finds stops linear in the original predictor scale.
  # That is appropriate for the planned graphics, where both axes are
  # untransformed.
  stops <- find stops(.dat, !!.predictor, .nstops)</pre>
  # browser()
  if (.logx) {
    stopslist <- list(log(stops))</pre>
    names(stopslist) <- the_log_name</pre>
    emms <- emmeans(.model, the_log_name,
                     at = stopslist,
                     type = 'response')
    emms <- as_tibble(emms)</pre>
    #browser()
    emms <- emms %>%
      mutate( !!the name := exp(emms[[the log name]]))
  }
  else {
    #browser()
    stopslist <- list(stops)</pre>
    names(stopslist) <- the_name</pre>
    emms <- emmeans(.model, the name,
```

# Total Zooplankton Density

I fit the simplified model without Station. The full model has the same concurvity problems as before, and here the model fails to converge. While I could alter the convergence criteria to search for a solution, we know the model that includes Station will have concurvity problems, so there is little point.

# Reduced Complexity Model

```
density_gam_reduced<- gam(log(Density) ~</pre>
                        \#s(Temp, bs="ts", k = 5) +
                        \#s(Sal, bs="ts", k = 5) +
                        s(log(Turb), bs="ts", k = 5) +
                        s(log(Chl), bs="ts", k = 5) +
                        \#s(log1p(Fish),bs="ts", k = 5) +
                        s(Yearf, bs = 're'),
                      data = drop_low, family = 'gaussian')
summary(density_gam_reduced)
#> Family: gaussian
#> Link function: identity
#>
#> Formula:
\# log(Density) ~ s(log(Turb), bs = "ts", k = 5) + <math>s(log(Chl), bs = "ts",
#>
      k = 5) + s(Yearf, bs = "re")
#>
#> Parametric coefficients:
             Estimate Std. Error t value Pr(>|t|)
#> (Intercept) 8.1283
                         0.2307 35.23 <2e-16 ***
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#> Approximate significance of smooth terms:
                 edf Ref.df F p-value
#> s(log(Chl)) 0.6072
                     4 0.83 0.122462
#> s(Yearf) 3.6720 4 10.52 1.63e-06 ***
```

```
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#>
R-sq.(adj) = 0.561 Deviance explained = 60.8%
#> GCV = 0.26018 Scale est. = 0.22853 n = 55
```

## Final Graphic

### Generate Separate Marginal Means

#### Name Match

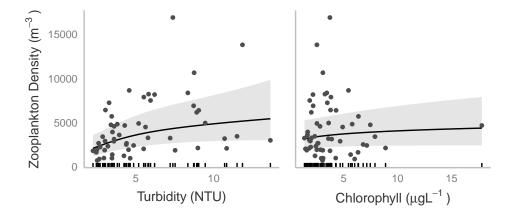
```
names(turb) <- c("log(Pred)", names(turb)[2:6], "Pred")
names(chl) <- c("log(Pred)", names(chl)[2:6], "Pred")</pre>
```

```
emms <- bind_rows(Turbidity = turb, Chlorophyll = chl, .id = "source")</pre>
```

ggplot2 allows you to construct fancier labels by building up something like plotmath expressions. The syntax is a bit obscure, and as far as I can tell, you can only test whether you got it right by plotting the expression.

#### Assemble Data

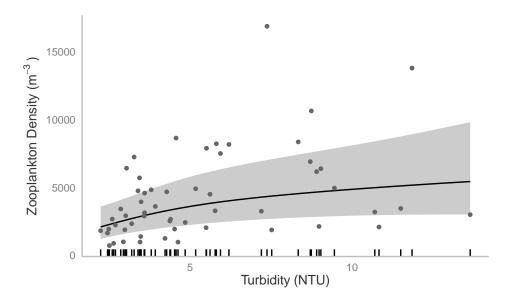
### Draw the Graphic



### Save the Plot

```
ggsave(file='figures/density_4.png',
width = 5.04, height = 2.2)
ggsave('figures/density_4.pdf', device = cairo_pdf,
width = 5.04, height = 2.2)
```

## Supplementary Graphic



### Save Plot

# **Shannon Diversity**

## Model on Reduced Data

```
#> Formula:
\# H \sim s(Temp, bs = "ts", k = 5) + s(Sal, bs = "ts", k = 5) + s(log(Turb), k = 5) + s(l
                      bs = "ts", k = 5) + s(log(Chl), bs = "ts", k = 5) + s(log1p(Fish),
                      bs = "ts", k = 5) + s(Yearf, bs = "re")
#>
#>
#> Parametric coefficients:
#>
                                               Estimate Std. Error t value Pr(>|t|)
#> (Intercept) 1.3310 0.1142 11.66 3.1e-15 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#> Approximate significance of smooth terms:
#>
                                                                              edf Ref.df
                                                                                                                   F p-value
                                                                                              4 4.222 0.002901 **
                                                     1.615e+00
#> s(Temp)
#> s(Sal)
                                                     2.259e-08
                                                                                                       4 0.000 0.257386
                                                                                                      4 0.000 0.608480
#> s(log(Turb)) 1.369e-08
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
\# R-sq.(adj) = 0.417 Deviance explained = 51.1%
\#> GCV = 0.2 Scale est. = 0.1648 n = 55
```

# Final Graphic

### Generate Separate Marginal Means

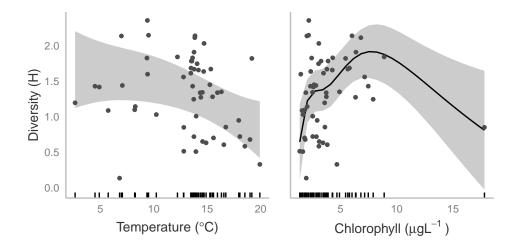
```
names(temp) <- c("Pred", names(temp)[2:6])
names(chl) <- c("log(Pred)", names(chl)[2:6], "Pred")
chl <- chl[,c(7, 2:6)]</pre>
```

#### Name match

### Assemble data

```
fancy_temp <- expression("Temperature (" * degree * "C)")
fancy_chl <- expression("Chlorophyll ("* mu * g * L ^-1 ~")")
dat <- drop_low %>%
  select(Temp, Chl, H) %>%
```

### Generate Plot



###Save the Plot

# Single Species Models

### **Model Choice**

Our model alternatives are similar to the choices we had for the Total Density model. The problem is, we can't use any of the continuous data distributions in GAMS with zero values (at least relying on the canonical link functions) because  $(\log(0) = -\ln f; 1/0 = \ln f, 1/0*0 = \ln f)$ . The easiest solution is to add some finite small quantity to the density data, and predict that. Here we predict  $\log(\text{Density} + 1)$  using Gaussian models.

# **Automating Analysis of Separate Species**

I'm going to automate analysis of all selected species by using a "nested" Tibble. This is a convenient alternative to writing a "for" loop to run multiple identical analyses.

I create a "long" data source, based on the reduced data set that omits low salinity samples.

Next, I create a function to run the analysis. This function takes a data frame or tibble as an argument. The tibble mush have data columns with the correct names.

The initial model fits for some species had a lot of wiggles in them, to an extent that I thought did not make much scientific sense, so I decided to reduce the dimensionality of the GAM smoothers, by adding the parameter k=4. Lowe numbers constrain the GAM to fit smoother lines.

Next, I create the nested tibble, and conduct the analysis on each species....

```
spp_analysis <- spp_data %>%
  group_by(Species) %>%
  nest() %>%
  mutate(gam_mods = map(data, my_gam))
```

And finally, output the model results. I can do that in a "for" loop, but it's Awkward to look through a long list of output, so I step through each species in turn.

### Acartia

```
spp = 'Acartia'
mod <- spp_analysis$gam_mods[spp_analysis$Species == spp][[1]]</pre>
dat <- spp_analysis$data[spp_analysis$Species == spp][[1]]</pre>
summary(mod)
#>
#> Family: qaussian
#> Link function: identity
#>
#> Formula:
\# log1p(Density) ~ s(Temp, bs = "ts", k = 5) + s(Sal, bs = "ts",
   k = 5) + s(log(Turb), bs = "ts", k = 5) + s(log(Chl), bs = "ts",
      k = 5) + s(log1p(Fish), bs = "ts", k = 5) + s(Yearf, bs = "re")
#>
#> Parametric coefficients:
#>
             Estimate Std. Error t value Pr(>|t|)
#> (Intercept) 6.598
                        0.371 17.78 <2e-16 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#> Approximate significance of smooth terms:
#>
                   edf Ref. df F p-value
#> s(Temp)
              3.6631 4 31.950 < 2e-16 ***
                         4 7.570 0.000232 ***
\#>s(Sal)
              3.2713
#> s(Yearf) 3.5153 4 11.237 6.14e-07 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#>
\#> R-sq. (adj) = 0.763 Deviance explained = 81.8%
\#> GCV = 0.93657 Scale est. = 0.70553 n = 55
```

## Final Graphic

```
temp <- marginal_analysis(dat, Temp, mod, .logx = FALSE)
sal <- marginal_analysis(dat, Sal, mod, .logx = FALSE)</pre>
```

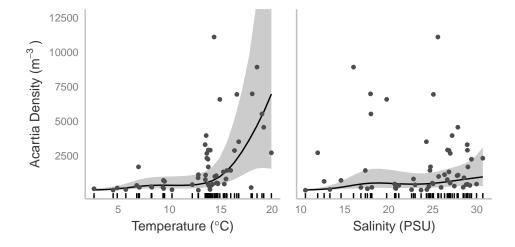
### Generate Separate Marginal Means

```
names(temp) <- c("Pred", names(temp)[2:6])
names(sal) <- c("Pred", names(sal)[2:6])</pre>
```

#### Name match

#### Assemble data

### Generate Plot



#### Save Plot

### Balanus

```
spp = 'Balanus'
mod <- spp_analysis$gam_mods[spp_analysis$Species == spp][[1]]</pre>
dat <- spp_analysis$data[spp_analysis$Species == spp][[1]]</pre>
summary(mod)
#>
#> Family: gaussian
#> Link function: identity
#>
#> Formula:
\# log1p(Density) ~ s(Temp, bs = "ts", k = 5) + s(Sal, bs = "ts",
      k = 5) + s(log(Turb), bs = "ts", k = 5) + s(log(Chl), bs = "ts",
       k = 5) + s(log1p(Fish), bs = "ts", k = 5) + <math>s(Yearf, bs = "re")
#>
#>
#> Parametric coefficients:
             Estimate Std. Error t value Pr(>|t|)
#> (Intercept) 3.6930 0.6478 5.701 8.74e-07 ***
#> Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#> Approximate significance of smooth terms:
                       edf Ref.df F p-value
                9.192e-01 4 2.998 0.00414 **
#> s(Temp)
            1.782e-10
                               4 0.000 0.52552
#> s(Sal)
#> s(log(Turb)) 1.967e+00
#> s(log(Chl)) 1.004e+00
                               4 1.779 0.06016 .
                               4 14.125 2.07e-05 ***
#> s(log1p(Fish)) 1.686e+00
                                4 0.691 0.22444
#> s(Yearf)
               3.568e+00
                                4 7.912 1.75e-05 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#>
\#> R-sq. (adj) = 0.581 Deviance explained = 65.2\%
\#> GCV = 2.7021 Scale est. = 2.2038 n = 55
```

#### Final Graphic

```
temp <- marginal_analysis(dat, Temp, mod, .logx = FALSE)</pre>
```

```
chl <- marginal_analysis(dat, Chl, mod, .logx = TRUE)</pre>
```

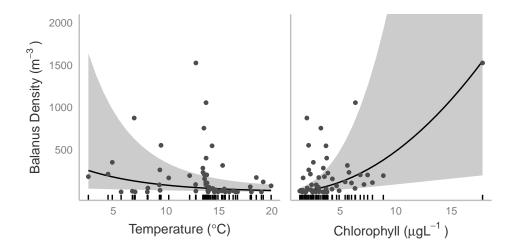
### Generate Separate Marginal Means

```
names(temp) <- c("Pred", names(temp)[2:6])
names(chl) <- c("log(Pred)", names(chl)[2:6], "Pred")
chl <- chl[,c(7, 2:6)]</pre>
```

#### Name match

#### Assemble data

#### Generate Plot



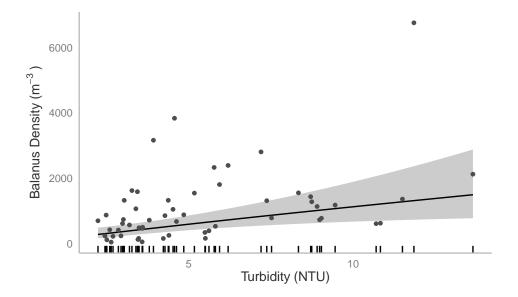
## Save Plot

## Eurytemora

```
spp = "Eurytemora"
mod <- spp_analysis$gam_mods[spp_analysis$Species == spp][[1]]</pre>
dat <- spp_analysis$data[spp_analysis$Species == spp][[1]]</pre>
summary(mod)
#>
#> Family: gaussian
#> Link function: identity
#>
#> Formula:
\#> log1p(Density) \sim s(Temp, bs = "ts", k = 5) + s(Sal, bs = "ts",
      k = 5) + s(log(Turb), bs = "ts", k = 5) + s(log(Chl), bs = "ts",
      k = 5) + s(log1p(Fish), bs = "ts", k = 5) + <math>s(Yearf, bs = "re")
#>
#>
#> Parametric coefficients:
              Estimate Std. Error t value Pr(>|t|)
#> (Intercept) 6.5275 0.1297 50.34 <2e-16 ***
#> ---
#> Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
#> Approximate significance of smooth terms:
                       edf Ref.df F p-value
#> s(Temp)
                 8.514e-10 4 0.000 0.522777
#> s(Sal)
                1.698e+00
                                4 0.439 0.360992
#> s(log(Turb)) 9.561e-01 4 3.326 0.000375 ***
```

### Generate Data

### Generate Graphic



## Save Plot