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A Greedy Heuristic for Maximizing the Lifetime of Wireless Sensor Networks based on Disjoint Weighted Dominating Sets

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Abstract: Dominating sets are among the most well-studied concepts in graph theory, with many real-world applications especially in the area of wireless sensor networks. One way to increase network lifetime in wireless sensor networks consists in assigning sensors to disjoint dominating node sets, which are then sequentially used by a sleep-wake cycling mechanism. This paper presents a greedy heuristic for solving a weighted version of the maximum disjoint dominating sets problem for energy conservation purposes in wireless sensor networks. Moreover, an integer linear programming model is presented. Experimental results based on a large set of 640 problem instances show, first, that the integer linear programming model is only useful for small problem instances. Moreover, they show that our algorithm outperforms recent local search algorithms from the literature with respect to both solution quality and computation time.

Keywords: greedy algorithm; disjoint dominating set; set problem; lifetime maximization; wireless sensors networks.

Citation: Balbal, S.; Bouamama, S.; Blum, C. A greedy heuristic for maximizing the lifetime of WSNs based on disjoint DSs. *Algorithms* **2021**, *1*, 0. https://doi.org/

Received:
Accepted:
Published:

1. Introduction

Wireless sensor networks (WSNs) have received an increasing attention in the last decade due to their potential applications in various fields such as environmental monitoring, medical and health applications, security surveillance and emergency operations [1]. They are generally composed of a rather large number of small devices, called sensors, with a limited low-power supply that depletes rather quickly. One of the most challenging issues in WSNs is extending the lifetime of the network while providing sufficient sensing coverage and communication reliability. The lifetime of a network is defined as the time (duration) in which the network is fully functional with respect to the tasks that have to be fulfilled, that is, the time duration in which the overall sensing coverage is maintained. Therefore, WSN lifetime significantly depends on the energy consumption of sensors in the context of limited energy availability.

According to [2], power saving strategies can usually be classified under one of the following mechanisms:

- Sleep-wake cycling (also called duty-culing): sensors switch between active and sleep mode.
- Power control by adjusting the transmission range of wireless nodes.
- Energy efficient routing, respectively data gathering.
- Reducing the amount of transmitted data and avoiding useless activity.

This study focuses on the first possibility, in the following way. The problem of extending (prolonging) WSN lifetime is addressed by organizing the sensors into a number of disjoint

subsets that are activated successively. We require that each of these subsets is a dominating set of the communication graph defined by a sensor network. Note that, given the locations of the sensors, the communication graph is obtained by joining two sensors with an edge in case they can communicate with each other via their wireless antennas. The requirement of a subset of sensors being a dominating set is introduced such that each subset of nodes is able to cover the whole coverage area.

The maximization of network lifetime has been studied in the literature from different perspectives. Most of them addressed the problem either via the set K-cover problem (also known as the target coverage problem) or as the domatic partition problem (also known as the maximum disjoint dominating sets problem). Slijepcevic and Potkonjak [3] were the first ones to model the problem as a set K-cover problem and proved its NP-hardness using a polynomial time transformation from the minimum cover problem. The set K-cover problem is defined over a bipartite graph of two node sets (sensors and targets) with the aim to partition the sensors into a maximum number of disjoint sets (covers) over all targets. These sensor covers are then activated one after the other to effectively extend the WSN lifetime. The sensors of an active cover are able to monitor the entire set of targets. Due to the hardness of the problem, a variety of approximate algorithms have been devised, also for different variants of the problem. Examples include a greedy heuristic [3], memetic algorithms [4–6], cuckoo search [7] and a genetic algorithm [8].

As mentioned before, the maximization of the lifetime of WSNs is often approached through the domatic partition problem in which the goal is to find a partition of the nodes of a given network—represented as a simple, undirected graph—into a maximum number of disjoint dominating sets. Throughout this paper, we refer to the domatic partition problem as maximum disjoint dominating sets (MDDS) problem. The MDDS problem, which belongs to the large and important familiy of dominating set problems [9–11], is known to be NP-hard for general graphs [12]. Cardei et *al.* [13] proved the NP-completeness of one of its variants known as the 3-disjoint dominating sets problem that asks for deciding whether or not a given graph contains three disjoint dominating sets. The same authors showed that, unless P = NP, the MDDS problem has no polynomial-time approximation with a performance guarantee of less than 1.5. They also introduced a heuristic approach based on graph coloring. In the same year, Feige et *al.* [14] proved that every graph with maximum degree Δ and minimum degree δ includes a domatic partition (a collection of disjoint dominating sets) of size $(1-o(1))(\delta+1)/\ln\Delta$. Here, the term $o(1)$ tends to zero as n increases. Moreover, they showed that there is no approximation algorithm for the MDDS problem with an approximation ratio of $(1+o(1))\ln n$ unless $NP \subseteq DTIME(n^{O(\log\log n)})$. The same authors also provided a centralized algorithm to find a domatic partition of size $W(\delta/\ln\Delta)$. Based on this later work, Moscibroda and Wattenhöfer [15] defined the MDDS problem as the maximum cluster-lifetime problem and introduced a randomized, distributed algorithm that has—with high probability—a performance ratio of $O(\log(n))$. Nguyen and Huynh [16] demonstrated that the 3-disjoint dominating sets problem remains NP-complete even for planar unit disk graphs and studied the performance comparison of four greedy heuristics for the MDDS problem. In [17], a greedy heuristic of time complexity $O(n^3)$ is described. More recently, Pino et *al.* [18] focused on a weighted variant of the MDDS problem which is henceforth labelled the maximum weighted disjoint Dominating Sets (MWDDS) problem. They devised three local search algorithms. Each local search algorithm is seeded with an initial feasible solution generated by a modified greedy heuristic introduced in [17] and iteratively swaps nodes between different dominating sets in the solution, in the hope to increase the total lifetime of the network. These local search approaches differ in the way they realize the swaps.

The main contributions of this paper can be summarized as follows:

- We introduce an integer linear programming (ILP) model for the MWDDS problem.
- We propose an efficient greedy heuristic for the MWDDS problem.

- We conduct comprehensive experiments on a set of 640 problem instances in order to investigate the performance and the advantages of our greedy heuristic in comparison to the application of CPLEX and to recent state-of-the-art methods.

The rest of this paper is organized as follows. In Section 2 we provide a technical description of the MWDDS problem and recall some required notations and basic definitions. In Section 4, we introduce a new greedy algorithm. Next, an ILP model for the MWDDS problem is presented in Section 3. Finally, in Section 5 we present and discuss the experimental results, while Section 6 summarizes the work and offers directions for future work.

2. The Maximum Weighted Disjoint Dominating Sets Problem

Let $G = (V, E, lifetime)$ be a simple, undirected, node-weighted graph, where V (with $|V| = n$) is the set of nodes, $E \subseteq V \times V$ (with $|E| = m$) is the set of edges, and $lifetime : V \to \mathbb{R}^+$ is a weight function that assigns a positive weight value $lifetime(v) > 0$ to each node $v \in V$. Note that such a graph may represent the communication graph of a WSN, and the node weight may represent the lifetime of the node. However, before technically introducing the MWDDS problem, let us briefly recall some basic definitions and notations used throughout this paper. For more details on graph theoretic aspects, the readers may refer to [19]. We say that two nodes $v \neq u \in V$ are neighbors (adjacent to each other) if and only if there exists an edge between them, that is, $(v, u) \in E$. Given a node $v \in V$, $N(v) := \{u \in V \mid (v, u) \in E\}$ denotes the set of neighbors of v, known as the *open neighborhood* of v in G. Furthermore, the *closed neighborhood* of a node $v \in V$, denoted by $N[v]$, contains the nodes adjacent to v plus node v itself, that is, $N[v] := N(v) \cup \{v\}$. The degree $\deg(v)$ of v is the number of neighbors of v, that is, $\deg(v) := |N(v)|$. More generally, given a subset of nodes $D \subseteq V$, the open neighborhood of D denoted by $N(D)$ is defined as $\bigcup_{v \in D} N(v)$ and its closed neighborhood, denoted by $N[D]$, is defined as $N[D] := N(D) \cup D$.

Definition 1 (Dominating set)**.** *A subset $D \subseteq V$ of the nodes of an undirected graph $G = (V, E)$ is called a* dominating set *of G if and only if each node $v \in V \setminus D$ has at least one neighbor in D. Each node in a dominating set D is called a* dominator*, otherwise it is called a* dominatee*. A dominator dominates (covers) itself and all its neighbors.*

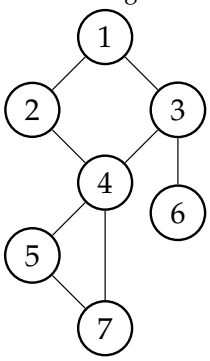
Definition 2 (Domatic partition)**.** *A set $\mathcal{D} = \{D_1, D_2, \cdots, D_k\}$ of subsets $D_i \subseteq V$ is called a domatic partition of $G = (V, E)$ if each D_i ($i = 1, \ldots, k$) is a dominating set of G and for all $1 \leq i < j \leq k, D_i \cap D_j = \emptyset$, that is, if all sets in $\mathcal{D}$ are pairwise disjoint.*

Definition 3 (Domatic number)**.** *The domatic number of a graph $G = (V, E)$ is determined as $|\mathcal{D}^*|$ where $\mathcal{D}^* := argmax\{|\mathcal{D}| \mid \mathcal{D}$ is a domatic partition of $G\}$. In other words, the domatic number of G is the size of the largest domatic partition of G. It is known that the domatic number of a graph G is limited from above by $\delta + 1$, where δ is the minimum degree of the nodes in V.*

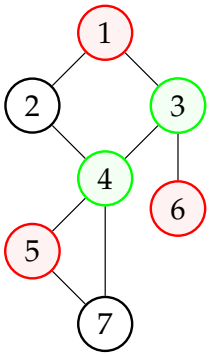
Given an undirected graph $G = (V, E)$, the maximum disjoint dominating set (MDDS) problem requires to find a domatic partition of G of maximal size. Furthermore, given a graph $G = (V, E, lifetime)$ as introduced above, the maximum weighted disjoint dominating sets (MWDDS) problem is defined as follows. Any domatic partition $\mathcal{D}$ of G is a valid solution. The objective function value of a valid solution $\mathcal{D} = \{D_1, \ldots, D_{|\mathcal{D}|}\}$ is defined as $f(\mathcal{D}) := \sum_{i=1}^{|\mathcal{D}|} \min\{lifetime(v) \mid v \in D_i\}$. In other words, the quality of a subset D_i is determined as the minimum lifetime of all its nodes. The objective is to find a valid solution $\mathcal{D}^*$ that maximizes $f()$. More formally, the problem can be expressed as follows:

node label	1	2	3	4	5	6	7
node weight	0.804	0.175	0.775	0.424	0.782	0.021	0.257

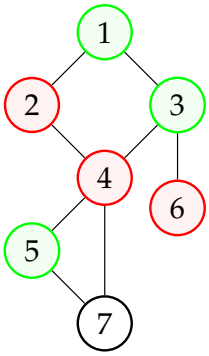
(a) Table containing the node weights, that is, the lifetimes of the nodes.



(b) A problem instance.



(c) A feasible solution $\mathcal{D} = \{\{3,4\},\{1,5,6\}\}$ with $f(\mathcal{D})$= 0.445.



(d) The optimal solution $\mathcal{D}^* = \{\{1,3,5\},\{2,4,6\}\}$ with $f(\mathcal{D}^{opt}) = 0.796$.

Figure 1. An illustrative example of the MWDDS problem.

$$\textbf{max} \quad f(\mathcal{D}) = \sum_{i=1}^{|\mathcal{D}|} \min\{lifetime(v) \mid v \in D_i\} \tag{1}$$

$$\textbf{s.t.} \quad D_i \subseteq V, \quad i = 1, \cdots, k \tag{2}$$

$$N[D_i] = V, \quad i = 1, \cdots, k \tag{3}$$

$$D_i \cap D_j = \emptyset, \quad 1 \leq i < j \leq k \tag{4}$$

Equation (3) ensures that each set D_i is a dominating set and equation (4) asks for all subsets of $\mathcal{D}$ to be pairwise disjoint. Figure 1 provides an illustrative example of the MWDDS problem. Figure 1a and Figure 1b provide a problem instance and the node weights, respectively. Note that the node weights (lifetime values) are normalized between 0 and 1. While Figure 1c shows a feasible solution $\mathcal{D} := \{D_1 = \{3,4\}, D_2 = \{1,5,6\}\}$ with objective function value $f(\mathcal{D}) = 0.445$, Figure 1d shows the optimal solution for this problem instance, that is, $\mathcal{D}^* := \{D_1 = \{1,3,5\}, D_2 = \{2,4,6\}\}$ with objective function value $f(\mathcal{D}^{opt}) = 0.796$. Note that, since this graph contains a node of degree 1, any valid solution contains at most two disjoint dominating sets.

3. An ILP Model for the MWDDS Problem

Our ILP model for the MWDDS problem requires the following sets of variables. First, we introduce a binary variable x_{ij} for each node v_i ($i = 1, \ldots, n$) and each possible disjoint set D_j ($j = 1, \ldots, \delta(G) + 1$). Hereby, $\delta(G) := \min\{deg(v) \mid v \in V\}$.[1] When $x_{ij} = 1$, node v_i is assigned to the $j-$th dominating set. Second, a binary variable y_j ($j = 1, \ldots, \delta(G) + 1$) indicates whether or not the $j-$th set is opened. Finally, a real-valued variable $z_j \in [0, M]$ ($M \in \mathbb{R}^+$) stores the weight of the $j-$th dominating set. Hereby, $M := \max\{lifetime(v) \mid v \in V\}$. Based on these variables, the MWDDS can be stated as an ILP in the following way.

[1] Remember that the maximum number of disjoint dominating sets in a graph is $\delta(G) + 1$.

$$\textbf{max} \quad \sum_{j=1}^{\delta(G)+1} z_j \tag{5}$$

$$\textbf{s. t.} \quad \sum_{j=1}^{\delta(G)+1} x_{ij} \leq 1 \qquad i = 1, \ldots, n \tag{6}$$

$$\sum_{v_k \in N(v_i)} x_{kj} \geq y_j - x_{ij} \qquad j = 1, \ldots, \delta(G)+1 \tag{7}$$

$$y_j \geq x_{ij} \qquad i = 1, \ldots, n \text{ and } j = 1, \ldots, \delta(G)+1 \tag{8}$$

$$x_{ij} \cdot lifetime(v_i) + (1 - x_{ij}) \cdot M \geq z_j \qquad i = 1, \ldots, n \text{ and } j = 1, \ldots, \delta(G)+1 \tag{9}$$

$$y_j \cdot M \geq z_j \qquad j = 1, \ldots, \delta(G)+1 \tag{10}$$

$$y_j \geq y_{j+1} \qquad j = 1, \ldots, \delta(G) \tag{11}$$

$$z_j \geq z_{j+1} \qquad j = 1, \ldots, \delta(G) \tag{12}$$

The objective function sums over the weight values of all dominating sets. Note that, for this objective function to be correct, the weight value of those dominating sets that are not opened must be set to zero by respective constraints. Constraints (6) require that each node of the graph is assigned to at most one dominating set. This assures the disjointness of the dominating sets. Next, constraints (7) are the so-called dominating set constraints, that is, they ensure that, if the $j-$th set is opened, then the set of nodes assigned to the j-th set form a dominating set of G. Further, constraints (8) make sure that nodes can only be assigned to opened dominating sets. Constraints (9) are responsible for correctly determining the weights of the openend dominating sets. In other words, the value of variable z_j of the $j-$th dominating set (if open) is set to the minimum of the lifetime-values of all nodes assigned to this set. Next, constraints (10) ensure that the weight values of dominating sets that are not opened are set to zero. Finally, the last two sets of constraints—that is, constraints (11) and (12)—are not necessary for the correctness of the ILP. They are tie-breaking constraints that ensure that (1) if k dominating sets are openend, they are assigned to sets $1, \ldots, k$ and (2) the openend dominating sets are ordered according to a non-increasing weight value.

4. Proposed Greedy heuristic

Greedy algorithms are constructive heuristics for building feasible solutions to combinatorial optimization problems. They start from an empty solution and iteratively add the most favorable component from a set of feasible extensions of the current partial solution until a complete solution is reached. That is, the component to be added at each construction step is chosen deterministically according to some greedy function, which is a measure for the goodness of the solution components. In general, greedy heuristics do not require overly large computation times. The main drawback of a greedy heuristic is that the single constructed solution is most likely not optimal.

The pseudo-code of our greedy algorithm, henceforth labelled GH-MWDDS, is presented in Algorithm 1. At the start of the algorithm, the following sets are initialized. Solution $\mathcal{D}$ is initialized to the empty set. Furthermore, set V_{rem}, which contains all nodes that may still be added to a dominating set, is initialized to V. Finally, set V_{done}, which collects all nodes that already form part of a dominating set in $\mathcal{D}$, is initialized to the empty set. Counter k counts the number of dominating sets that are being constructed. It is initialized to zero. At each main iteration $k \geq 1$, the algorithm generates a dominating set D_k consisting of nodes from set V_{rem} which, as mentioned above, contains all nodes of G that do not form part of any of the already generated dominating sets D_i, $i = 1, \ldots, k-1$; see lines 5 and 21. The algorithm stops once no further dominating set can be generated from the nodes in V_{rem}. This is the case if at least one node $v \in V$ and all its neighbors form

Algorithm 1 GH-MWDDS: A Greedy heuristic for the MWDDS problem

Input: a simple, node-weighted, undirected graph $G = (V, E, lifetime)$
Output: a family of disjoint dominating sets $\mathcal{D} = \{D_1, D_2, \cdots, D_k\}$

1: $\mathcal{D} \leftarrow \emptyset$
2: $k \leftarrow 0$
3: $V_{\text{rem}} \leftarrow V$
4: $V_{\text{done}} \leftarrow \emptyset$
5: **while** not exists $v \in V$ s.t. $N[v]$ is a subset of V_{done} **do**
6: $k \leftarrow k + 1$
7: **for** each node $v \in V$ **do**
8: $color(v) \leftarrow$ WHITE
9: **end for**
10: $D_k \leftarrow \emptyset$
11: **while** D_k is not a dominating set of G (that is, $N[D_k] \neq V$) **do**
12: $v^* \leftarrow \text{argmax}\{score(v) \mid v \in \{\mathcal{G}(D_k) \cup \mathcal{W}(D_k)\} \cap V_{\text{rem}}\}$
13: $D_k \leftarrow D_k \cup \{v^*\}$
14: $V_{\text{rem}} \leftarrow V_{\text{rem}} \setminus \{v^*\}$
15: **for** each node $u \in N(v^*)$ **do**
16: **if** ($color(u) =$ WHITE) **then**
17: $color(v) \leftarrow$ GRAY
18: **end if**
19: **end for**
20: $color(v^*) \leftarrow$ BLACK
21: **end while**
22: Reduce(D_k, V_{rem}, V_{done}) {optional}
23: $\mathcal{D} \leftarrow \mathcal{D} \cup \{D_k\}$
24: $V_{\text{done}} \leftarrow V_{\text{done}} \cup D_k$
25: **end while**
26: **return** $\mathcal{D} = \{D_1, D_2, \cdots, D_k\}$.

part of V_{done}. More formally, the algorithm stops if there is at least one $v \in V$ such that $N[v]$ is a subset of V_{done} (see line 5).

The construction of a dominating set D_k at round k of the algorithm is done as follows. First, D_k is initialized to the empty set. At each moment, each node of the input graph G belongs to one of the following sets:

- *Black nodes*: nodes that are part of D_k.
- *Gray nodes*: nodes in $\mathcal{G}(D_k) := N(D_k) \setminus D_k$, that is, nodes that are not contained in D_k but that are adjacent to at least one black node.
- *White nodes*: nodes in $\mathcal{W}(D_k) := V \setminus N[D_k]$, that is, nodes that are neither gray nor black.

Remember, in this context, that $N(D_k)$ and $N[D_k]$ denote the open neighborhood and the closed neighborhood of D_k, respectively. Given these definitions, D_k is iteratively generated by adding at each construction step exactly one node from $\{\mathcal{G}(D_k) \cup \mathcal{W}(D_k)\} \cap V_{\text{rem}}$, that is, a nodes that is (1) either gray or white, and (2) that is not already added to one of the dominating sets from previous rounds. In order to make this choice, all nodes from $\{\mathcal{G}(D_k) \cup \mathcal{W}(D_k)\} \cap V_{\text{rem}}$ are evaluated by means of greedy function $score()$, which is defined as follows:

$$score(v) := lifetime(v) * white\_degree(v) \qquad \forall v \in \{\mathcal{G}(D_k) \cup \mathcal{W}(D_k)\} \cap V_{\text{rem}} \tag{13}$$

$$white\_degree(v) := |N[v] \cap \mathcal{W}(D_k)| \tag{14}$$

In other words, the greedy function of a node v is obtained by multiplying the lifetime of the node with the number of white nodes from the closed neighborhood of v.

To further improve the quality of solutions, the dominating set D_k might be reduced by removing redudant nodes before adding D_k to solution $\mathcal{D}$. This is done in the optional function Reduce$(D_k, V_{\text{rem}}, V_{\text{done}})$; see line 22 of the pseudo-code. Formally, a node $v \in D_k$ is redundant if each node from its closed neighborhood $N[v]$ is dominated by at least two nodes from D_k, that is, $N[v] \subset \bigcup_{u \in D_k \setminus \{v\}} N[u]$ [20]. Redundant nodes are identified and removed in an iterative way. Moreover, sets V_{rem} and V_{done} are updated accordingly.

4.1. Time Complexity of GH-MWDDS

In this section, we analyze the complexity of our greedy algorithm, without the optional reduction mechanism from line 22. The main while loop (line 5) of the algorithm is iterated k times (rounds), where k refers to the number of disjoint dominating sets returned by the algorithm. Since the maximum number of dominating sets in a domatic partition of a graph $G = (V, E)$ is less or equal to $\delta + 1$ [14], where δ is the minimum degree of any node of G, it holds that $k \leq \delta + 1$. As a result, checking the condition in line 5 and executing lines 7-9 is both done in $O(\delta \cdot n)$ time. Further, n is clearly an upper bound for the number of times that the inner while loop (lines 11-21) is iterated. To determine the time complexity of line 12, we proceed as follows. Remember that for any node v the calculation of its $white\_degree(v)$ value requires $deg(v)$ operations. Moreover, for choosing the first node v_1^* of a dominating set, line 12 requires a time of $deg(v_1) + deg(v_2) + \cdots + deg(v_n)$. The well-known Handshaking Lemma states that $\sum_{v \in V} deg(v) = 2m$, where $m = |E|$. For choosing the second node v_2^*, line 12 requires $m - deg(v_1^*)$ time, etc. Therefore, the total time complexity of line 12 is $O(n\ m)$. In summary, we can conclude that our greedy algorithm has a worst-case time complexity of $O(\delta\ n) + O(n\ m) = O(max(\delta\ n, n\ m)) = O(n\ m)$.

5. Experimental Evaluation

The proposed greedy algorithm (GH-MWDDS) was implemented in ANSI C++ using GCC 7.4.0 for compiling the software. Its performance was compared with three recent local search approaches available from the related literature [18]: Local Search (LS), Fixed Depth (FD), and Variable Depth (VD). In order to conduct a fair comparison to the three local search algorithms (LS, FD and VD) we used the original source code provided by the authors of [18]. GH-MWDDS and the three local search algorithms were experimentally evaluated on a laptop equipped with a 64-bit 2.5-GHz Intel® Core™ i5-7200U processor and 8 GB of RAM. We applied GH-MWDDS to a set of 640 problem instances that are described below. Moreover, in order to test the usefulness of our ILP model, we applied the ILP solver ILOG CPLEX 20.01 in single-threaded mode to all problem instances. The time limit for each application of CPLEX was set to 2 CPU hours. Moreover, the experiments with CPLEX were performed on a cluster of machines with two Intel® Xeon® Silver 4210 CPUs with 10 cores of 2.20GHz and 92 Gbytes of RAM.

5.1. Problem Instances

We used the source code provided in [18] for generating 640 diverse problem instances. Each problem instance was generated according to two control parameters: the size of the network n (number of nodes) and its average degree d, where $d = 2m/n$ and m is the number of edges. More specifically, we considered five different network sizes: 50, 100, 150, 200 and 250 nodes. For each network size, we considered between 5 to 8 different average degrees in a way such that networks over a range from sparse to dense networks are generated. In addition, each node (sensor) of the network was given a random real value between 0 and 1 as node weight (lifetime). Then, for each combination of n and d, 20 different networks were randomly generated resulting in a total of 640 problem instances for the experimental evaluation.

5.2. Results and Discussion

Table 1 reports on the performance comparison of GH-MWDDS (without the optional reduction procedure) and LS, FD, VD, and CPLEX. The first two columns indicate the

problem instance type. More specifically, the number of nodes (n) is provided in the first column, whereas the average degree (d) is shown in the second column. Obviously, the higher d, the more dense is the network. Remember that for each combination of n and d—that is, for each table row—there are 20 randomly generated problem instances. The values shown in the tables represent the average results obtained for these 20 instances per table row. Table columns 3 and 4 contain the results of CPLEX. While the first one of these columns indicates the average solution quality obtained, the second column provides the average gap (in percent) between the found results and the upper bounds. The results of the other algorithms are provided in two columns for each algorithm. These columns report on the average solution quality (*Value*) and the average computation time (*Time(s)*) in seconds. Moreover, the 11-th column of the table indicates the average size of solutions ($|DP|$) in terms of the average number of disjoint dominating sets. Since the three algorithms LS, FD and VD start with the same greedy solution as initial solution, and the number of disjoint dominating sets does not change during local search, the column with heading $|DP|$ holds for all three algorithms. Finally, in the last column of the table, the average size of the solutions of GH-MWDDS is given.

In addition to evaluating GH-MWDDS, we also tested the following two additional versions. Henceforth we refer to the version of GH-MWDDS that removes redundant nodes in line 22 as GH-MWDDS$^+$. In addition, the algorithm can be easily adapted in order to solve the maximum disjoint dominating sets (MDDS) problem, instead of the MWDDS. The resulting greedy algorithm, which is called GH-MDDS, is obtained by replacing the $score(\cdot)$ function in line 12 of Algorithm 1 by the $white\_degree(\cdot)$ function of Equation 14. The experiential results obtained with GH-MDDS and GH-MWDDS$^+$ are provided in Table 2, whose structure is very similar to the one of Table 1.

From the results displayed in Tables 1 and 2, the following observations can me made:

- First of all, the ILP model (solved by CPLEX) is only useful in the context of the smallest problem instances. In fact, CPLEX obtains the best results for all networks with 50 nodes. As an additional information we can say that CPLEX was able to solve 8 out of 20 problem instances with 50 nodes and an average degree of 15 to optimality. In contrast, CPLEX already starts to fail for the instances with 100 nodes, for which the results are already inferior to those of GH-MWDDS. In fact, for most of the problem instances (starting from 150 nodes and an average degree of 60) CPLEX is unable to find anything but the trivial solution that contains no dominating set.
- Concerning GH-MWDDS, we can observe that it dominates the three competitors from the literature for all problem instances in terms of solution quality as well as computational time. It is significantly better than the best of the other three approaches, while requiring a computation time that is approximately five orders of magnitude smaller than the time needed by the other methods. The improvement ratio of our algorithm with respect to VD (the best approach from the literature) is about 5.37.
- As expected, most solutions suffer from the existence of redundant nodes. As a result, GH-MWDDS$^+$ is able to produce solutions whose size (in terms of the number of disjoint dominating sets) is increased with respect to the solutions produced by GH-MWDDS. This is achieved without compromising the quality of the generated solutions. On the contrary, the quality of the obtained solutions is slighly improved, at the cost of a negligible amount of computation time.
- In addition, from Table 2 we can observe that GH-MDDS and GH-MWDDS$^+$ achieve an overall average solution quality (see that last row of Table 2) of 3.667 and 9.515, respectively, and an average solution size (in terms of the number of disjoint dominating sets) of 24.089 and 21.541, respectively. It is worthwhile to note that GH-MDDS always produces solutions with the maximum possible size (equal to $\delta(G) + 1$). However, the quality of these solutions (as measured by the objective function of the MWDDS problem) is, of course, lower than the quality of the solutions produced by GH-MWDDS. This is because GH-MDDS is primarily focused on finding solutions of large size,

Table 1. Numerical results of comparison between LS, FD, VD and GH-MWDDS

n	d	CPLEX		LS		FD		VD		|DP|	GH-MWDDS		
		Value	Gap(%)	Value	Time(s)	Value	Time(s)	Value	Time(s)		Value	Time(s)	|DP|
50	15	**2.779**	32.839	0.395	0.003	0.477	0.019	0.555	1.984	2.55	2.155	0.000	5.75
	20	**3.922**	121.088	0.649	0.004	0.825	0.055	1.014	7.651	3.55	3.353	0.001	8.05
	25	**5.098**	158.038	1.011	0.006	1.245	0.055	1.490	4.885	4.05	4.595	0.001	10.05
	30	**6.432**	200.567	1.664	0.007	2.460	0.124	2.780	13.117	6.30	5.926	0.001	12.90
	35	**7.929**	197.434	2.200	0.012	2.914	0.204	3.166	25.048	6.85	7.376	0.000	15.30
100	20	2.467	405.669	0.283	0.023	0.333	0.188	0.423	30.458	2.45	**2.784**	0.001	7.00
	30	3.202	>1000.0	0.431	0.017	0.535	0.165	0.576	31.796	2.95	**4.501**	0.003	10.90
	40	3.338	>1000.0	0.859	0.021	1.259	0.394	1.341	183.921	4.75	**6.679**	0.002	14.80
	50	3.857	>1000.0	1.361	0.038	2.018	0.796	2.407	225.704	6.00	**8.500**	0.001	18.65
	60	5.804	823.022	2.019	0.027	2.884	0.830	3.226	362.775	7.15	**11.131**	0.001	23.40
150	30	0.055	>1000.0	0.287	0.053	0.326	0.316	0.326	106.602	2.85	**4.098**	0.002	10.65
	40	0.028	>1000.0	0.557	0.050	0.670	0.595	0.745	156.994	3.80	**5.816**	0.003	14.05
	50	0.011	>1000.0	0.615	0.077	0.927	1.139	1.009	182.630	3.95	**7.479**	0.003	17.50
	60	0	>1000.0	0.848	0.046	1.506	2.576	1.521	646.268	4.90	**9.281**	0.002	21.00
	70	0	>1000.0	1.373	0.066	2.293	4.219	2.464	1028.650	6.65	**11.388**	0.004	24.45
	80	0	>1000.0	1.689	0.065	2.729	2.599	3.036	549.900	7.20	**13.508**	0.005	28.90
	90	0	>1000.0	2.209	0.055	3.817	3.793	4.193	906.030	9.05	**15.485**	0.006	32.95
200	40	0	>1000.0	0.256	0.049	0.317	0.648	0.370	81.750	2.65	**5.424**	0.004	13.65
	50	0	>1000.0	0.374	0.099	0.475	1.252	0.483	186.900	3.45	**6.784**	0.005	16.60
	60	0	>1000.0	0.678	0.112	0.897	2.083	0.917	313.850	3.75	**8.656**	0.005	20.35
	70	0	>1000.0	1.004	0.170	1.652	6.463	1.680	3112.950	5.95	**10.351**	0.006	23.65
	80	0	>1000.0	0.840	0.076	1.624	5.874	1.795	1629.400	5.35	**12.447**	0.008	27.15
	90	0	>1000.0	1.153	0.109	1.924	5.260	2.045	1364.400	5.65	**13.978**	0.012	30.55
	100	0	>1000.0	1.503	0.091	2.836	9.396	3.098	3024.830	7.65	**15.868**	0.009	34.30
250	50	0	>1000.0	0.266	0.136	0.314	0.867	0.329	557.676	2.95	**6.681**	0.016	16.85
	60	0	>1000.0	0.526	0.221	0.892	7.638	0.945	1400.788	4.55	**8.244**	0.012	19.70
	70	0	>1000.0	0.689	0.264	1.050	6.906	1.326	2380.366	5.10	**9.636**	0.013	22.95
	80	0	>1000.0	0.652	0.241	1.163	9.801	1.445	647.763	5.40	**11.520**	0.013	26.15
	90	0	>1000.0	0.841	0.324	1.448	18.394	1.591	1242.663	5.80	**12.938**	0.013	29.55
	100	0	>1000.0	1.117	0.228	1.981	20.789	2.443	2210.880	6.45	**14.733**	0.016	32.70
	120	0	>1000.0	1.259	0.146	2.577	10.442	2.781	2249.350	7.20	**18.348**	0.016	39.70
	140	0	>1000.0	2.135	0.121	4.275	16.885	4.713	6624.596	10.50	**22.478**	0.020	47.25
Avg		1.404		0.992	0.092	1.583	4.399	1.757	984.143	5.231	**9.442**	**0.006**	**21.169**

Table 2. Numerical results of comparison between GH-MDDS, GH-MWDDS and GH-MWDDS^{+}

n	d	GH-MDDS			GH-MWDDS			GH-MWDDS^{+}		
		Value	Time(s)	|DP|	Value	Time(s)	|DP|	Value	Time(s)	|DP|
50	15	1.111	0.001	**6.75**	2.155	0.000	5.75	**2.191**	0.002	6.150
	20	1.548	0.001	**9.05**	3.353	0.001	8.05	**3.450**	0.000	8.350
	25	2.630	0.001	**11.85**	4.595	0.001	10.05	**4.672**	0.000	10.550
	30	3.510	0.001	**14.60**	5.926	0.001	12.90	**6.042**	0.000	13.350
	35	4.772	0.001	**17.25**	7.376	0.000	15.30	**7.546**	0.000	15.900
100	20	0.841	0.001	**8.50**	2.784	0.001	7.00	**2.842**	0.001	7.400
	30	1.714	0.004	**12.70**	4.501	0.003	10.90	**4.580**	0.001	11.150
	40	2.900	0.002	**17.15**	6.679	0.002	14.80	**6.794**	0.002	15.200
	50	3.908	0.002	**21.40**	8.500	0.001	18.65	**8.525**	0.002	19.100
	60	6.111	0.004	**26.30**	11.131	0.001	23.40	**11.174**	0.002	23.600
150	30	1.036	0.003	**12.00**	4.098	0.002	10.65	**4.141**	0.002	10.900
	40	1.769	0.002	**16.20**	5.816	0.003	14.05	**5.872**	0.002	14.350
	50	2.553	0.003	**20.00**	7.479	0.003	17.50	**7.570**	0.002	17.850
	60	3.647	0.004	**24.10**	9.281	0.002	21.00	**9.371**	0.005	21.450
	70	4.789	0.002	**28.10**	11.388	0.004	24.45	**11.446**	0.005	25.050
	80	6.468	0.005	**33.00**	13.508	0.005	28.90	**13.611**	0.003	29.150
	90	7.143	0.006	**36.55**	15.485	0.006	32.95	**15.589**	0.005	33.550
200	40	1.350	0.006	**15.50**	5.424	0.004	13.65	**5.486**	0.005	13.950
	50	1.719	0.006	**18.95**	6.784	0.005	16.60	**6.848**	0.006	17.050
	60	2.551	0.007	**23.35**	8.656	0.005	20.35	**8.710**	0.008	20.550
	70	3.566	0.008	**26.80**	10.351	0.006	23.65	**10.395**	0.008	24.100
	80	4.466	0.015	**30.70**	12.447	0.008	27.15	**12.529**	0.003	27.400
	90	5.368	0.009	**34.80**	13.978	0.012	30.55	**14.046**	0.009	31.000
	100	6.425	0.011	**38.50**	15.868	0.009	34.30	**15.993**	0.009	34.650
250	50	1.643	0.013	**19.05**	6.681	0.016	16.85	**6.783**	0.008	16.950
	60	2.062	0.010	**22.70**	8.244	0.012	19.70	**8.285**	0.010	19.950
	70	2.613	0.012	**26.15**	9.636	0.013	22.95	**9.699**	0.013	23.150
	80	3.348	0.013	**29.55**	11.520	0.013	26.15	**11.571**	0.013	26.500
	90	4.030	0.016	**33.50**	12.938	0.013	29.55	**12.978**	0.016	29.900
	100	5.024	0.016	**37.15**	14.733	0.016	32.70	**14.800**	0.016	33.150
	120	7.134	0.017	**45.05**	18.348	0.016	39.70	**18.418**	0.018	40.250
	140	9.584	0.020	**53.60**	22.478	0.020	47.25	**22.514**	0.020	47.700
Avg		3.667	0.007	**24.089**	9.442	0.006	21.169	**9.515**	0.006	21.541

without taking into consideration the problem-specific knowledge of the lifetime values of the nodes. Therefore, a solution to the MWDDS problem with the maximum size does not necessarily correspond to the best MWDDS-solution. For example, in the case where $n = 250$ and $d = 140$, for which GA-MDDS finds a solutions with an average value 9.584 and an average size 53.6, GH-MWDDS$^+$ finds solutions with an average value 22.514 and average size 47.7.

- From above observations we can say that the poor performance of the three local search algorithms proposed by Pino et *al.* [18] is mainly due to greedy heuristic used for producing the initial solutions for their local search approaches. Since this greedy heuristic was developed for the MDDS problem, and the local search approaches try to improve these solutions by making swaps among nodes of different disjoint dominating sets, they are limited to the size of the solutions found by the initial greedy heuristic, which cannot be changed later. As can be seen by our results, this way of proceeding limits too much the possibility of the local search algorithms to find improvements.

In summary we can say that our GH-MWDDS approach is clearly a new state-of-the-art method for the MWDDS problem.

6. Conclusions

This paper deals with the network lifetime maximization problem using the approach of computing disjoint dominating sets in a given network in which sensors are characterized by different lifetime values. In other words, we considered a version of the weighted disjoint dominating sets problem. In this context, we have proposed a greedy heuristic that benefits from the use of problem-specific knowledge. To assess the performance of our algorithm, 640 problem instances of different sizes (both sparse and dense graphs) were evaluated and the obtained results were compared with those of three recently published local search algorithms. Computational results show the superiority of our greedy algorithm over recent approaches available from the literature. In addition we stated the considered problem as an integer linear programming model. However, the results of applying CPLEX to solve the model have shown that this approach is only useful for the smallest ones of the considered networks.

In the future we plan to develop well-working metaheuristics on the basis of the developed greedy heuristic in order to further improve the obtained results. Options include ant colony optimization or iterated greedy algorithms, which are both based on the step-by-step construction of solutions. Another line of research will focus on hybrid techniques such as construct, merge, solve & adapt (CMSA) [21], which make it possible to take profit from ILP solvers such as CPLEX even in the context of large problem instances.

Author Contributions: The authors contributed equally in all aspects of this research. All authors have read and agreed to the published version of the manuscript

Funding: This work was supported by project CI-SUSTAIN funded by the Spanish Ministry of Science and Innovation (PID2019-104156GB-I00).

Acknowledgments: The authors would like to thank Tayler Pino for providing the code of the three proposed local search algorithms; S. Bouamama is grateful to DGRSDT-MESRS (Algeria), for financial support.

Conflicts of Interest: The authors declare no conflict of interest.

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