

Quick Sort

Sorting Algorithm

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Introduction

Why would we ever want to sort anything?

Brief Description of QuickSort

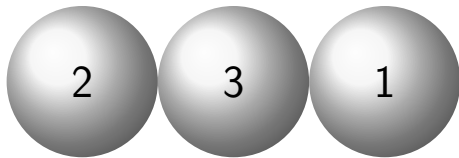
- ▶ QuickSort is a recursive algorithm based on the “divide and conquer” technique. It works by placing the “middle” element in the proper place, i.e., in the “middle”, and then doing the same thing for the left and right halves. This “middle” element is known as the **PIVOT**.
- ▶ Think of sorting a stack of books by number of pages. Pick a book at random (the pivot, or “middle”), and place all the books with fewer pages on the left, and all the books with more pages on the right. Repeat for the left and right sides.
- ▶ Question: How would you do this for a deck of cards?

Description of QuickSort

1. Call the QuickSort() procedure, passing a list as the only parameter.
2. Start with the list parameter. Pick one element of the list to be the pivot. (First, last, middle, random — it doesn't matter.)
3. Iterate through the list placing all elements less than the pivot on the left and all elements greater than the pivot on the right. (Equal elements can go either on the left, right, or — in my demonstration — in the middle.)
4. Continue the same process for the left and right sides. Call the same procedure passing the left or right side as the parameter. This is the recursive step.
5. Stop the recursion when a list has zero or one elements and return the list. This stops the recursion.

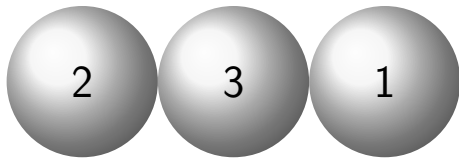
Quick Sort Examples

Example 1

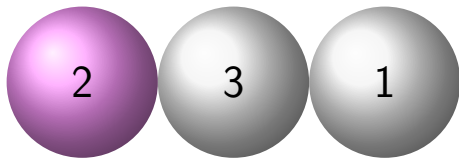


Choose a pivot, use first element

Example 1

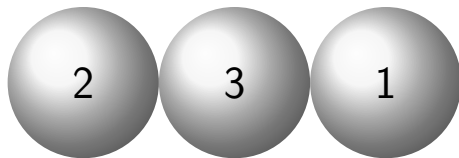


Choose a pivot, use first element

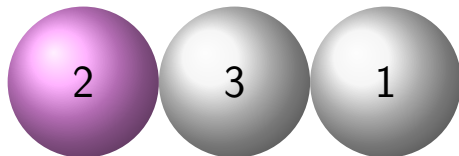


Sort right and left lists

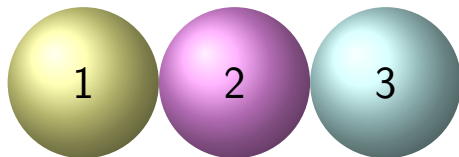
Example 1



Choose a pivot, use first element

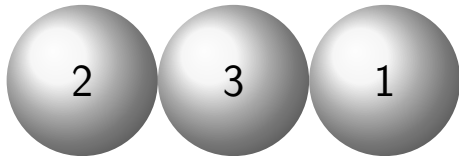


Sort right and left lists



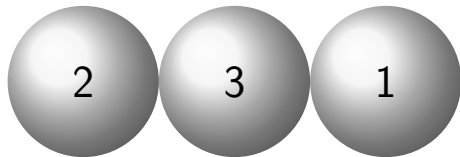
R & L stopped, List is sorted

Example 2

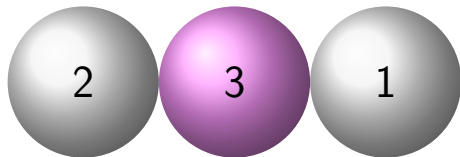


Choose a pivot, use middle

Example 2

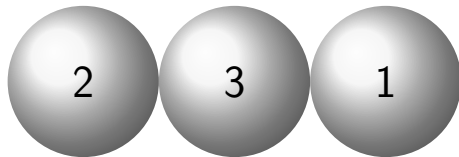


Choose a pivot, use middle

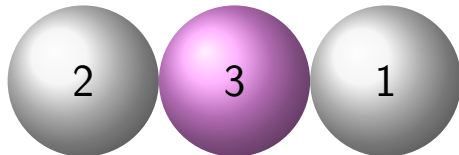


Sort right and left

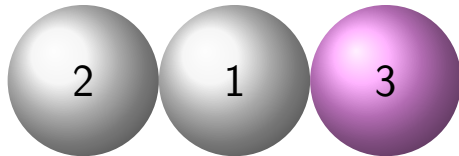
Example 2



Choose a pivot, use middle

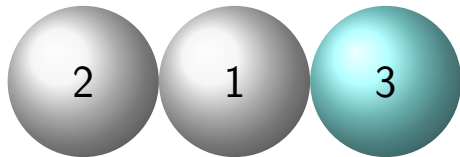


Sort right and left



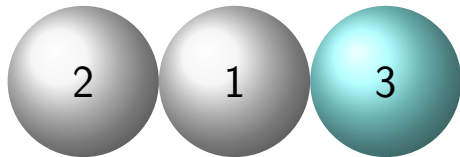
Sort left list

Example 2, continued

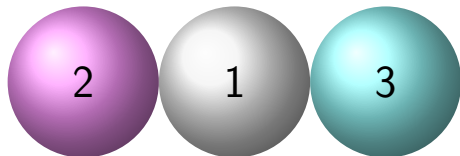


Choose a pivot, use middle

Example 2, continued

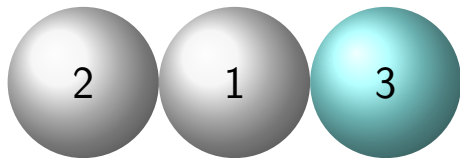


Choose a pivot, use middle

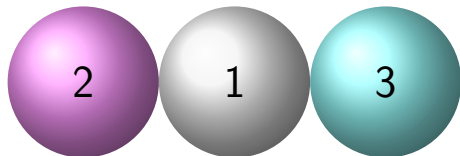


Sort right and left

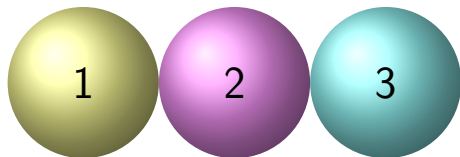
Example 2, continued



Choose a pivot, use middle

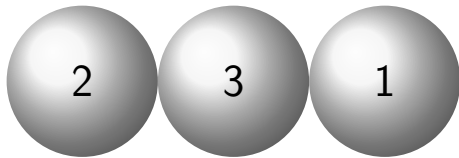


Sort right and left



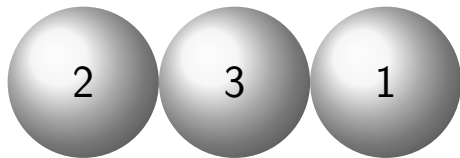
List is sorted

Example 3

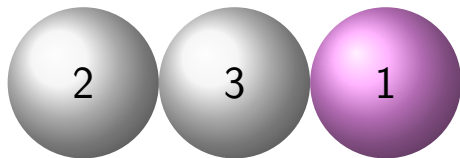


Choose a pivot, use last

Example 3

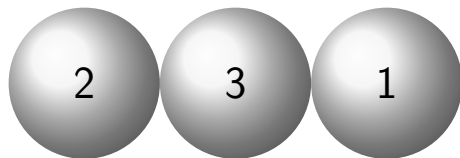


Choose a pivot, use last

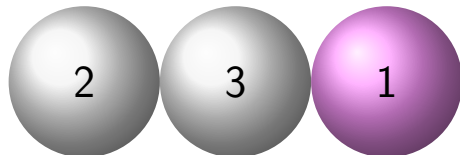


Sort right and left

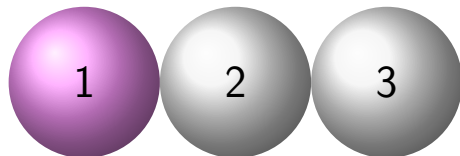
Example 3



Choose a pivot, use last



Sort right and left



Sort not done, $R > 1$

Example 3, continued



Choose a pivot, use last

Example 3, continued



Choose a pivot, use last



Sorted, $R == 1$, $L == 0$

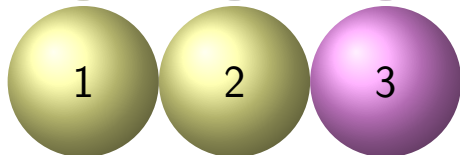
Example 3, continued



Choose a pivot, use last



Sorted, $R == 1$, $L == 0$



List is sorted

Extended example



Choose pivot, use middle

Extended example



Choose pivot, use middle



Place elements $<$ pivot on L, elements $>$ pivot on R

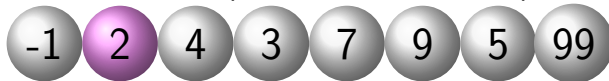
Extended example



Choose pivot, use middle



Place elements $<$ pivot on L, elements $>$ pivot on R



L is sorted, length == 1

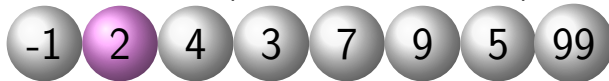
Extended example



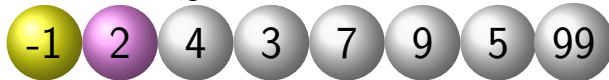
Choose pivot, use middle



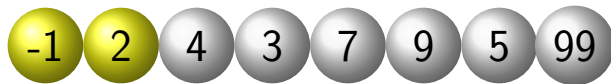
Place elements $<$ pivot on L, elements $>$ pivot on R



L is sorted, length == 1

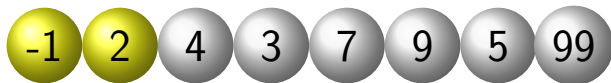


Extended example continued 1



Choose pivot, use middle

Extended example continued 1

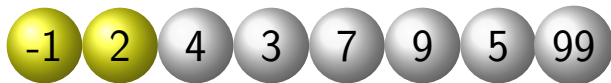


Choose pivot, use middle



Place elements $<$ pivot on L, elements $>$ pivot on R

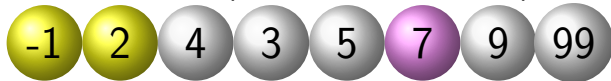
Extended example continued 1



Choose pivot, use middle

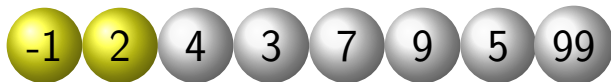


Place elements $<$ pivot on L, elements $>$ pivot on R



Pivot is in correct place (color is RED)

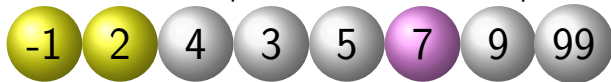
Extended example continued 1



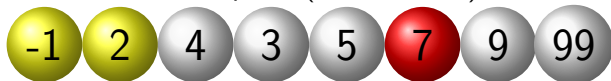
Choose pivot, use middle



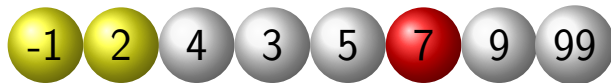
Place elements $<$ pivot on L, elements $>$ pivot on R



Pivot is in correct place (color is RED)

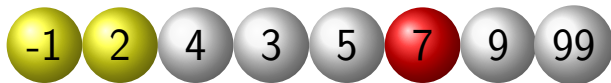


Extended example continued 2



Sorting L, Choose pivot, use middle

Extended example continued 2

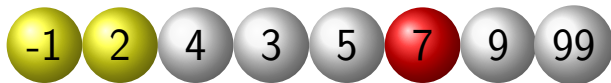


Sorting L, Choose pivot, use middle



Place elements $<$ pivot on L, elements $>$ pivot on R

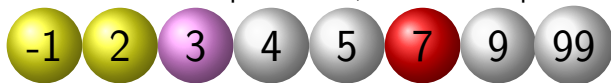
Extended example continued 2



Sorting L, Choose pivot, use middle

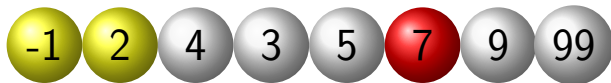


Place elements $<$ pivot on L, elements $>$ pivot on R



L is sorted, length == 0, sort R

Extended example continued 2



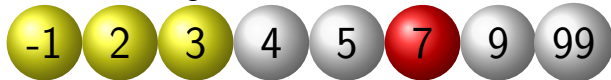
Sorting L, Choose pivot, use middle



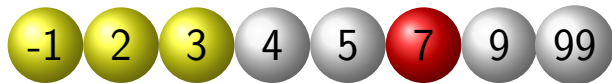
Place elements $<$ pivot on L, elements $>$ pivot on R



L is sorted, length == 0, sort R

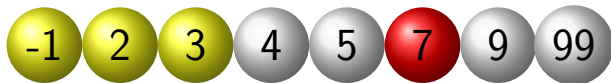


Extended example continued 3



Choose pivot, use middle

Extended example continued 3

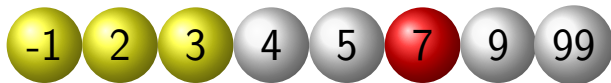


Choose pivot, use middle



L is sorted, length == 0

Extended example continued 3



Choose pivot, use middle

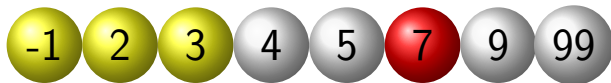


L is sorted, length == 0



R is sorted, length == 1

Extended example continued 3



Choose pivot, use middle



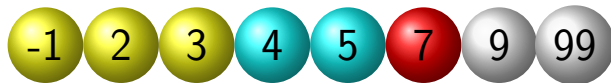
L is sorted, length == 0



R is sorted, length == 1



Extended example continued 4

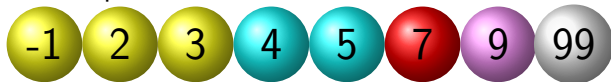


Choose pivot, use middle

Extended example continued 4



Choose pivot, use middle



L is sorted, length == 0

Extended example continued 4



Choose pivot, use middle



L is sorted, length == 0



R is sorted, length == 1

Extended example continued 4



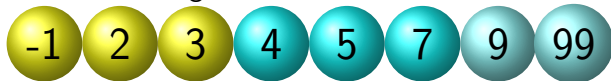
Choose pivot, use middle



L is sorted, length == 0



R is sorted, length == 1



Demonstration

Python code

```
def quick_sort(arr):  
    less = []  
    pivotList = []  
    more = []  
    if len(arr) <= 1:  
        return arr  
    else:  
        pivot = arr[(len(arr)-1) // 2]  
        for i in arr:  
            if i < pivot:  
                less.append(i)  
            elif i > pivot:  
                more.append(i)  
            else:  
                pivotList.append(i)  
        less = quick_sort(less)  
        more = quick_sort(more)  
        return less + pivotList + more
```

Python code comments, declare variables

```
def quick_sort(arr):  
    less = []  
    pivotList = []  
    more = []
```

Declare a function named *quick_sort* that takes one parameter, a list, which presumably is unsorted.

Declare a *less* list to hold the lesser-than items, a *more* list to hold the greater-than items, and a *pivotList* to hold the equal-to items.

Python code

```
if len(arr) <= 1:  
    return arr  
else:
```

If the length of the parameter list is 1 or 0, you are done. Stop the recursion.
Else, continue the recursion.

Python code

```

pivot = arr[(len(arr)-1) // 2]
for i in arr:
    if i < pivot:
        less.append(i)
    elif i > pivot:
        more.append(i)
    else:
        pivotList.append(i)

```

Select a pivot. It could be the first element ($arr[0]$), the last element ($arr[-1]$), the middle element ($arr[(len(arr)-1) // 2]$), or a random element.

Iterate through the parameter list, placing all lesser-than elements on the left, and all greater than elements on the right.

Python code

```
less = quick_sort(less)
more = quick_sort(more)
return less + pivotList + more
```

Call the function recursively on the left and right sides, passing each as a parameter. Return the three lists glued together.

QuickSort output 1

```
initial list is [4, 3, 7, 2, 9, 5, 99, -1]
calling quick_sort([4, 3, 7, 2, 9, 5, 99, -1])
  else branch, list is [4, 3, 7, 2, 9, 5, 99, -1]
    for: i = 4 and pivot = 2, i > pivot, more = [4]
    for: i = 3 and pivot = 2, i > pivot, more = [4, 3]
    for: i = 7 and pivot = 2, i > pivot, more = [4, 3, 7]
    for: i = 2 and pivot = 2, i == pivot, pivotList = [2]
    for: i = 9 and pivot = 2, i > pivot, more = [4, 3, 7, 9]
    for: i = 5 and pivot = 2, i > pivot, more = [4, 3, 7, 9, 5]
    for: i = 99 and pivot = 2, i > pivot, more = [4, 3, 7, 9, 5, 99]
    for: i = -1 and pivot = 2, i < pivot, less = [-1]
```

QuickSort output 1

```
calling quick_sort([-1])
  DONE if list length <= 1, returning [-1]
calling quick_sort([4, 3, 7, 9, 5, 99])
  else branch, list is [4, 3, 7, 9, 5, 99]
    for: i = 4 and pivot = 7, i < pivot, less = [4]
    for: i = 3 and pivot = 7, i < pivot, less = [4, 3]
    for: i = 7 and pivot = 7, i == pivot, pivotList = [7]
    for: i = 9 and pivot = 7, i > pivot, more = [9]
    for: i = 5 and pivot = 7, i < pivot, less = [4, 3, 5]
    for: i = 99 and pivot = 7, i > pivot, more = [9, 99]
```

QuickSort output 1

```
calling quick_sort([4, 3, 5])
  else branch, list is [4, 3, 5]
    for: i = 4 and pivot = 3, i > pivot, more = [4]
    for: i = 3 and pivot = 3, i == pivot, pivotList = [3]
    for: i = 5 and pivot = 3, i > pivot, more = [4, 5]
calling quick_sort([])
  DONE if list length <= 1, returning []
calling quick_sort([4, 5])
  else branch, list is [4, 5]
    for: i = 4 and pivot = 4, i == pivot, pivotList = [4]
    for: i = 5 and pivot = 4, i > pivot, more = [5]
calling quick_sort([])
  DONE if list length <= 1, returning []
```

QuickSort output 1

```
calling quick_sort([5])
  DONE if list length <= 1, returning [5]
<<<returning [] + [4] + [5]
<<<returning [] + [3] + [4, 5]
calling quick_sort([9, 99])
  else branch, list is [9, 99]
    for: i = 9 and pivot = 9, i == pivot, pivotList = [9]
    for: i = 99 and pivot = 9, i > pivot, more = [99]
calling quick_sort([])
  DONE if list length <= 1, returning []
calling quick_sort([99])
  DONE if list length <= 1, returning [99]
<<<returning [] + [9] + [99]
<<<returning [3, 4, 5] + [7] + [9, 99]
<<<returning [-1] + [2] + [3, 4, 5, 7, 9, 99]
end list is [-1, 2, 3, 4, 5, 7, 9, 99]
```

Conclusion and Questions

Time complexity of QuickSort

Best case

$$O = n \log n \quad (1)$$

Worst case

$$O = n^2 \quad (2)$$

History of QuickSort

Alternatives to QuickSort

Questions