ISOM 2700: Operations Management
Session 7.2. Revenue Management: capacity based control

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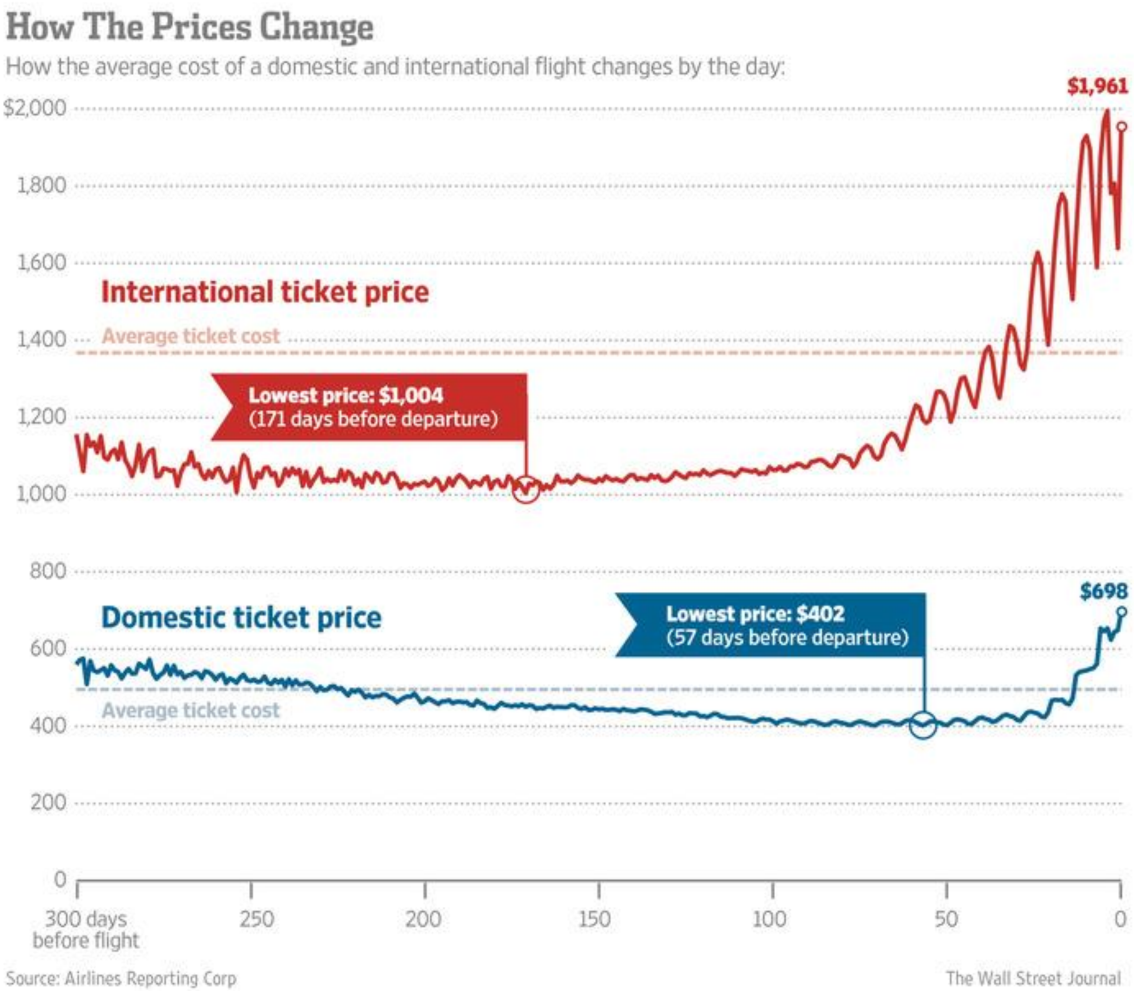
Why Prices Change Over Time?



Prices Differ for Customer Segmentations

Mon Nov 28 | Tue Nov 29 | **Departure** Wed Nov 30 from $479 | Thu Dec 01 from $479 | Fri Dec 02 from $479 | Sat Dec 03 from $412 | Sun Dec 04 from $412 | Next 3 Days

Departure Fares per person, in US Dollars (USD), based on Round-Trip purchase, additional taxes and fees will apply.

Sort By: # of Stops

Flight	Departure	Arrival	Economy Super Saver	Economy Saver	Economy Flexible	Instant Upgrade	Business Flexible	First Special	First Flexible
AA Non-Stops									
AA 177	05:40 PM JFK	09:10 PM SFO	$479	$622	$954	$2491	$2590	Not Available	$3138
+ Flight Details									
AA with Stops									
AA 773	05:05 PM LGA	08:25 PM DFW	$479	$622	$954	$1749	Not Available	$2099	$2229
AA 1575	09:20 PM DFW	11:05 PM SFO							
+ Flight Details * Economy is the only cabin offered on one or more flights									
AA 345	05:10 PM LGA	07:05 PM ORD	Not Available	$719	$954	Not Available	Not Available	$1913	$2137
AA 1521	08:15 PM ORD	10:50 PM SFO							
+ Flight Details * Economy is the only cabin offered on one or more flights									
AA 1377	05:15 PM JFK	07:20 PM ORD	Not Available	$719	$954	$2142	Not Available	Not Available	$2709
AA 1521	08:15 PM ORD	10:50 PM SFO							
+ Flight Details									

Customer Segmentation

- **Time-based differentiation**
  - Time-value of products for customers
  - E.g., regular selling season followed by markdown season

- **Different customer classes**
  - Airline products with different qualities (e.g., tickets that allow changes or refundability)
  - Group discounts, coupons
  - Shipping: same-day express, second-day shipping

Conditions Favoring RM

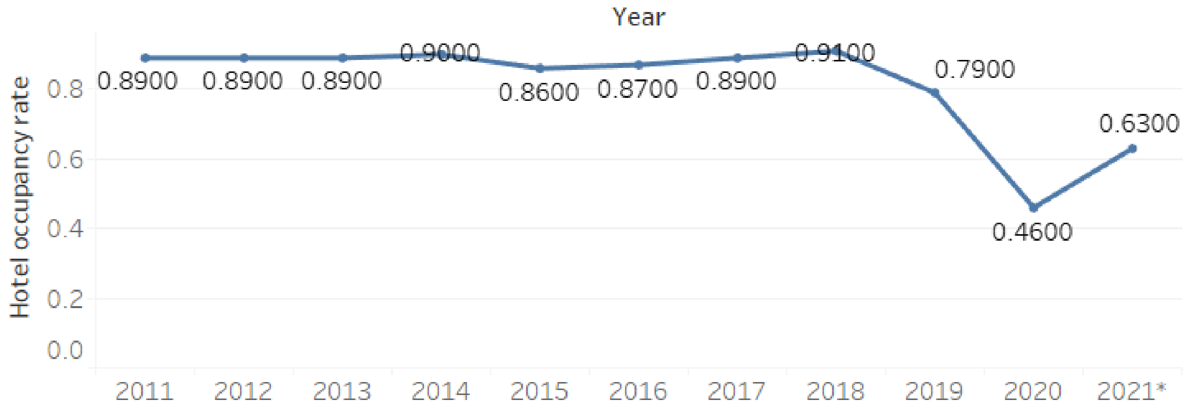
What is revenue management?

"Sell the Right product to the Right customer for the right time at the Right price."

- Customer heterogeneity: Target the right market segments
- Demand variability and uncertainty
- Fixed selling horizon / Perishable products
- Production inflexibility
- Price is not a signal for quality
- Data and IS infrastructure exist

Airline and Hotel Industries

- A typical airline operates with 73% of its seats filled but needs to fill 70% of its seats to break even
- Average hotel occupancy rate in HK is 63% in 2021



- Airline seats or Hotel rooms are highly perishable
  – E.g. Once the plane goes up, the airline gets absolutely no revenue for empty seats

Airline Industry Example

- **Key decisions**
  - How many seats to make available at each of the listed fares?
  - How many seats to make available to travel agents, and at what prices?
  - What contracts and prices to provide to corporations?

- **Possible decision criteria**
  - OD (origin-destination) pair
  - Time of year, time of week etc.
  - Remaining seats available
  - Remaining time until departure

Hotel Industry Example

- **Key decisions**
  - How much to charge for a room?
  - Other questions similar to the airline industry

- **Possible decision criteria**
  - Location
  - Type of room
  - Time of year and time of week
  - Special events (e.g., conference)
  - Duration of stay

How Revenue Management Works Today?

- Revenue management decision support system
  - A comprehensive computer system to maximize revenue for capacity-constrained services using reservation systems, overbooking, and partitioning demand

Selling the right product…	■ Airplane seats ■ Hotel rooms
to the right customer	■ Business travel ■ Leisure
for the right time	■ Same day purchase ■ 2 months in advance
at the right price	■ Full fare ■ Discount rate

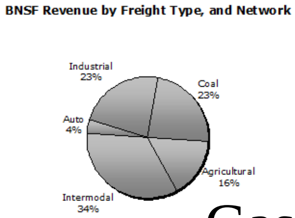
Current Industry Practice

Airlines

Hotel

Media & broadcasting

Car rental



Gas pipelines

Retailing

[Video: Marriott]

10

Learning objective

- Introduction to airline industry

- Revenue management capacity control
  - Airline industry: two class allocation problem
  - Hotel industry: use LP

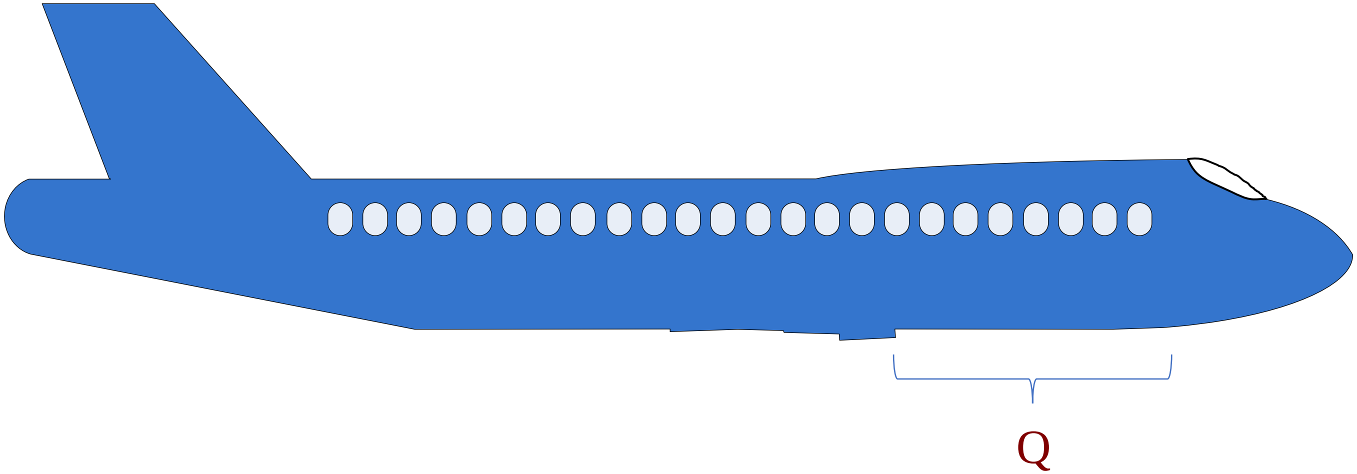
An Example with Two Classes

Two classes of passengers

- Leisure: Very price sensitive and buy their tickets in advance
- Business: Price insensitive and buy at the last minute

Business Strategy:

- Offer two fare classes $f_1 > f_2$
- Passengers that buy before a specific threshold pay f_2, otherwise, they pay f_1.



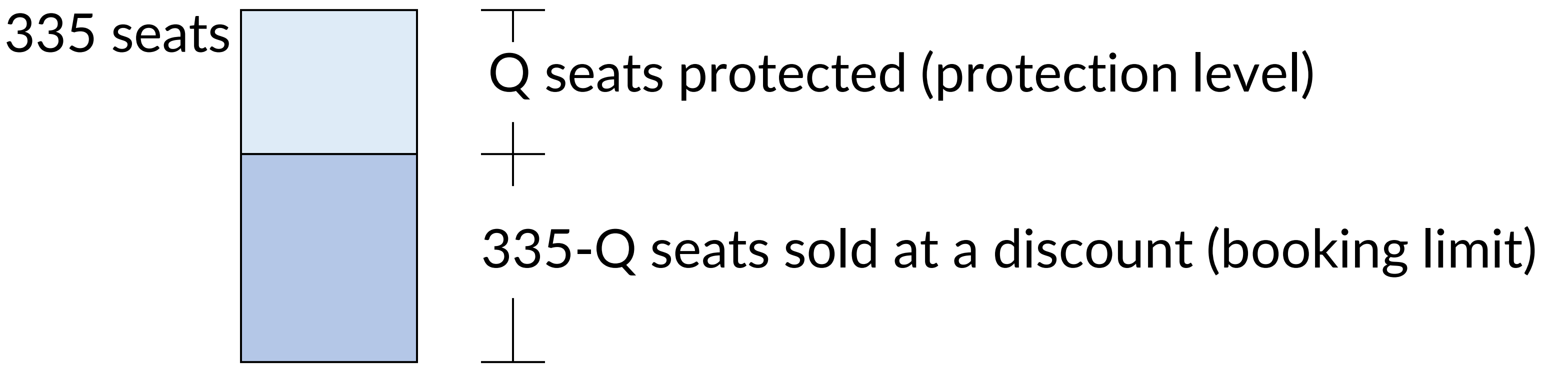
Q : Protection level for high-value or business travelers

Two-Class Allocation Problem

- An airline has started to sell tickets for the flight from Hong Kong to Cambridge on Dec. 24
- An airplane has 335 seats for passengers
- Two classes of passengers
  - Leisure: very sensitive to price and buy their tickets in advance
  - Business: price insensitive and buy at the last minute
- Two-price strategy
  - Offer two prices, **f_1 = \$7950 > f_2 = \$5250**
  - Discount price (f_2) targets leisure travelers
  - Full price (f_1) targets business travelers
- Assumption
  - The demand of leisure customers is ample. Discount tickets are always sold out first
  - Business travelers' demand is D_B~Normal (25, 52)

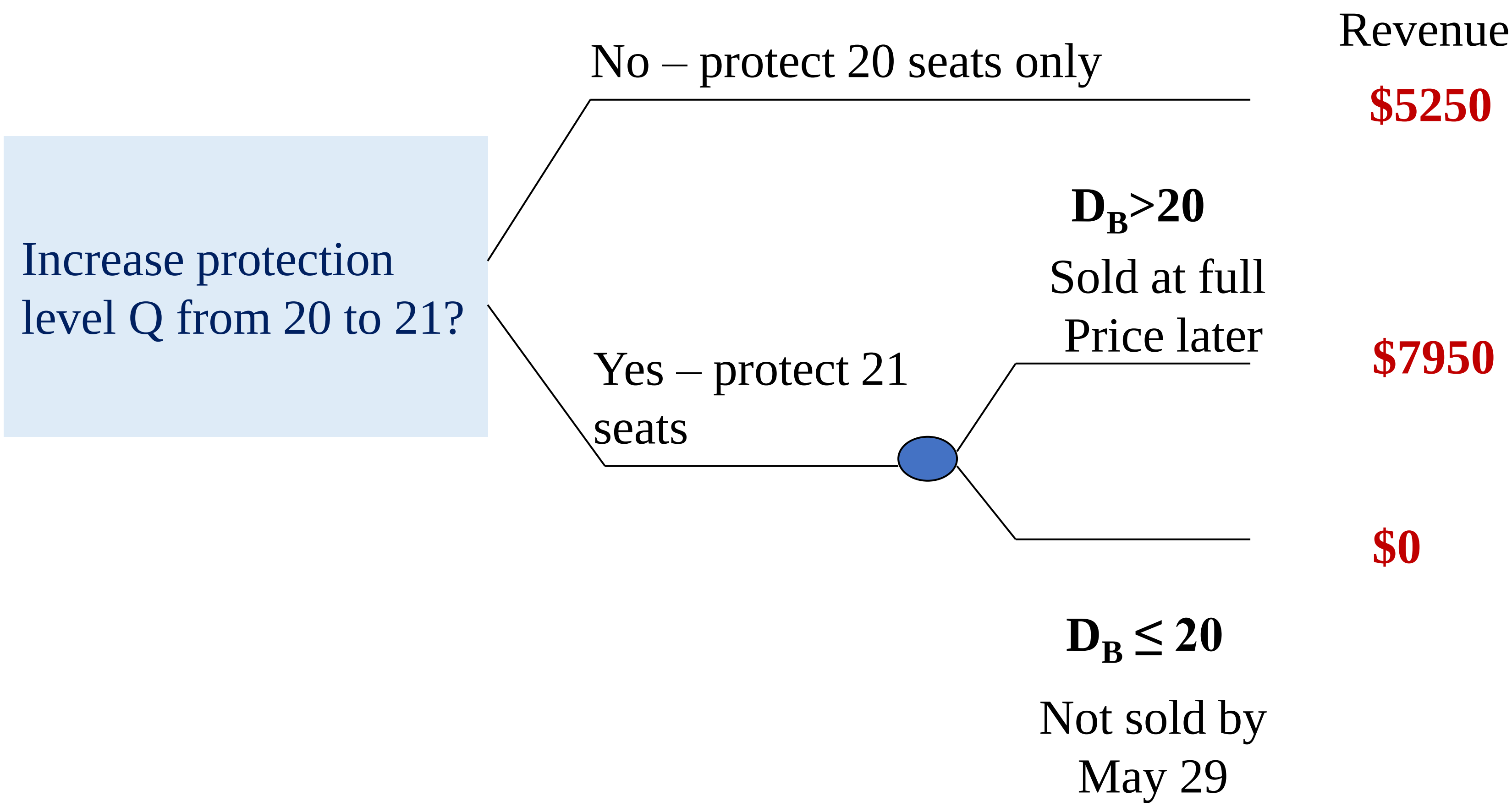
Two-Class Allocation Problem

- **Decision variable**: Protection level (Q)
  - How many seats (Q) should be reserved for business travelers?



- Booking limit: the maximum number of seats that may be sold at the discount price

Marginal Analysis



Marginal Analysis

- Solution: Use Newsvendor Formula
  - Overage cost (too high protection level Q)
    - $C_o = f_2 = \$5250$
  - Underage cost (too little protection level Q)
    - $C_u = f_1 – f_2 = \$7950 - \$5250 = \$2700$
- Optimal protection level satisfies

$$\Pr\left(D_B \leq Q\right) = \frac{C_u}{C_u + C_o} = \frac{2700}{2700+5250} = 0.339$$

- Since we assume that D_B is Normal(25, 5^2)

$$z^* \approx -0.41$$
$$Q^* = 25 + 5z^* \approx 23 \text{ seats}$$

- **Booking limit** = Capacity – protection level

Parallel with Newsvendor Model

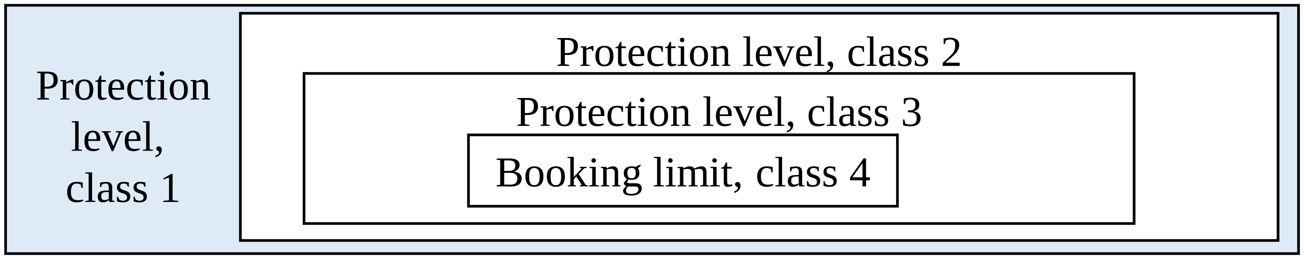
RM with capacity controls	Newsvendor
Decision: protection level for high fare	Decision: order quantity
Uncertain demand: Demand for high fare tickets	Uncertain demand: Demand for newspapers
Overstocking cost = discounted fare	Overstocking cost = purchase cost – salvage value
Understocking cost = full fare – discounted fare	Understocking cost = retail selling price – purchase cost

Generalization to Multiple Fare Classes

- There can be more than two fare classes, with

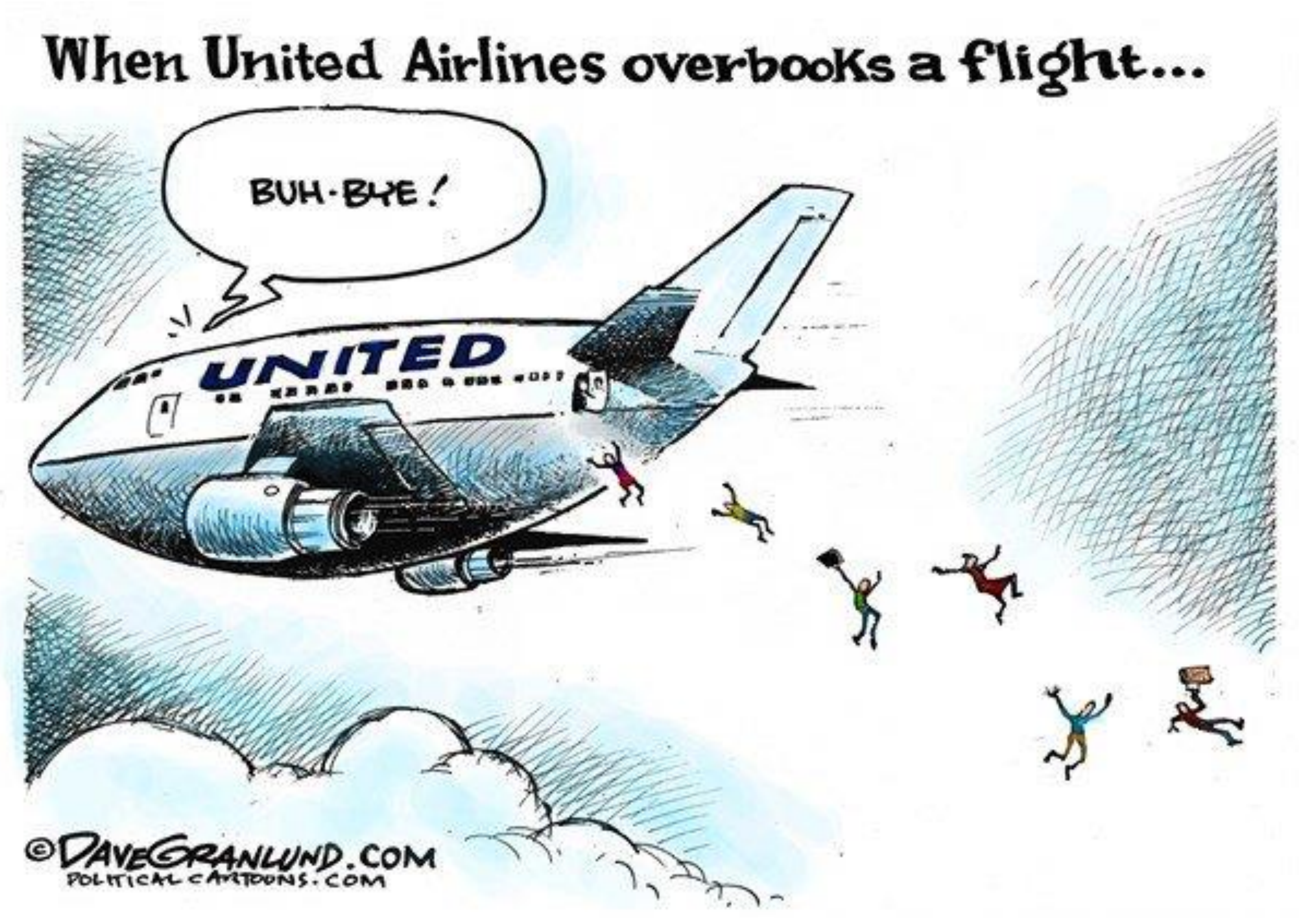
$$f_1 > f_2 > f_3 > \ldots > f_n$$

- Previous solution can be extended to multiple fare classes, using the nested structure



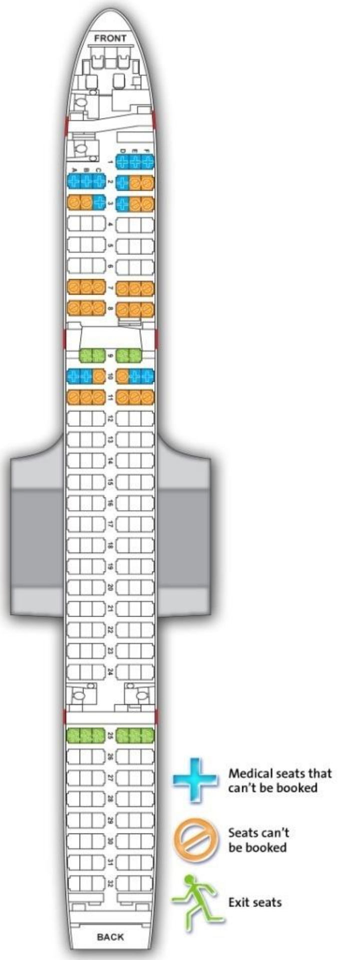
  - Class 1 vs. Classes 2-4: Determine protection level Q_1 for class 1
  - Class 2 vs. Classes 3-4: Determine protection level Q_2 for class 2
  - Class 3 vs. Class 4: Determine protection level Q_3 for class 3

Another common revenue management tactics: Overbooking



Overbooking Problem

- Suppose there are 100 seats on a flight from Hong Kong to Singapore
- The number of people who book tickets but do not show up: Normal $(20,10^2)$
- Air ticket price = \$105
- Cost of denied boarding: \$405
  - Arrangement for travel on another airline: \$200
  - Free air ticket: \$105
  - Ill-will cost: \$100
- **How many reservations should the airline take?**



Treatment of Overbooked Passengers

- Volunteers
  - First seek customers willing to take a later flight in return for compensation

- Involuntary denied boarding
  - Travel arrangement with a different flight or with another airline
  - Compensation depending on the arrival time (may include meal and lodging)

- Cost
  - Direct cost of the compensation
  - Travel arrangement cost
  - The ill-will cost

Determining the optimal overbooking level

Use marginal analysis to derive the optimal overbooking decision:

Cu = Cost of underestimating no-shows

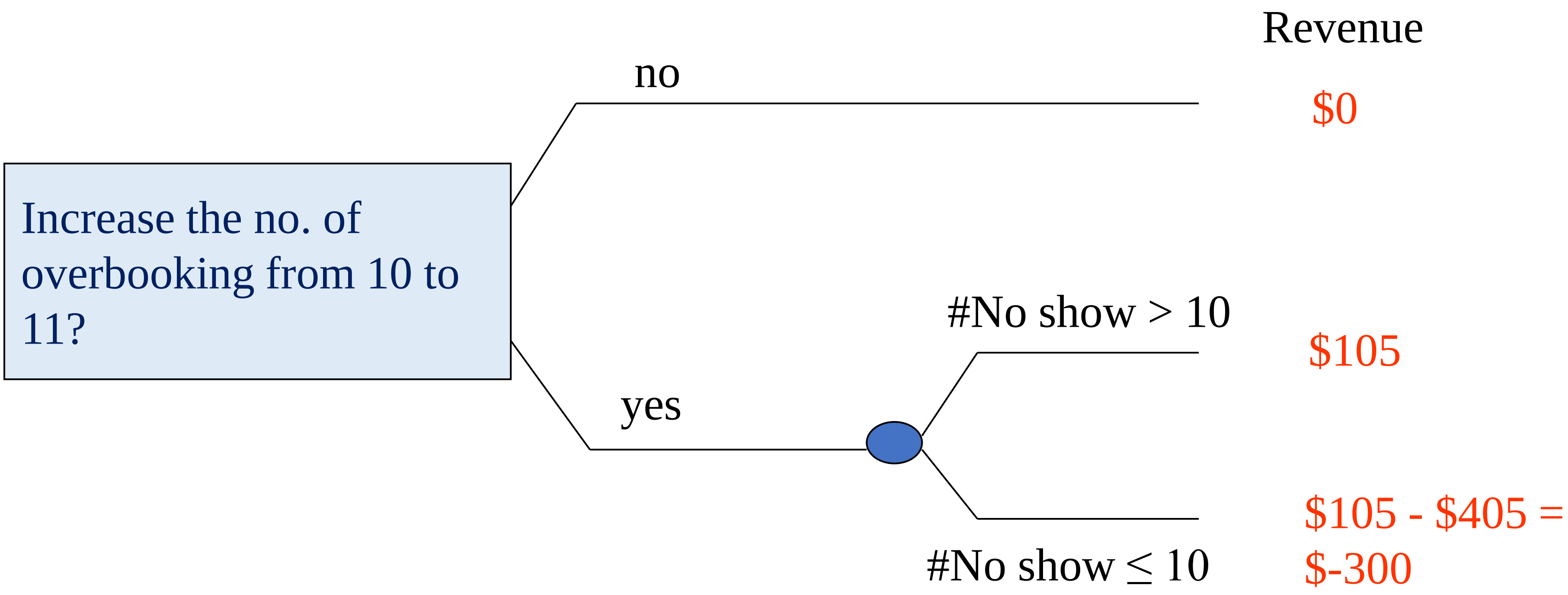
(when actual no-shows $\geq$ # of overbooked customers)

Co = Cost of overestimating no-shows

(when actual no-shows < # of overbooked customers)

Determining Optimal Overbooking Level

- Use marginal analysis to derive the optimal overbooking decision



Rephrasing the Problem into Newsvendor Model

How many seats should the airline overbook for this flight?

Air ticket fare = $105

Overbooking Penalty = $405

Underage Cost = $105

Overage Cost = $405-$105 = $300

Critical fractile = 105/(300+105) = 0.2592

From Standard Normal table: z = - 0.645

Optimal number of overbooked customers

= 20+(-0.645)(10)=13.5

Practice Problem

- The admissions office for BBA in OM at HKUST needs to decide how many offers to make for class size of 120
- Since some students will decide to pursue other opportunities, the office will admit more than 120
- In the upcoming year, the number of people who will not accept the offer is normally distributed with mean 10 and standard deviation 5
- (a) Suppose 120 were admitted, what is the probability that the class size will be less than or equal to 105?
- (b) It is 5 times more expensive to have a student in excess of 120 than to have fewer students accept. How many admission offers would you make?

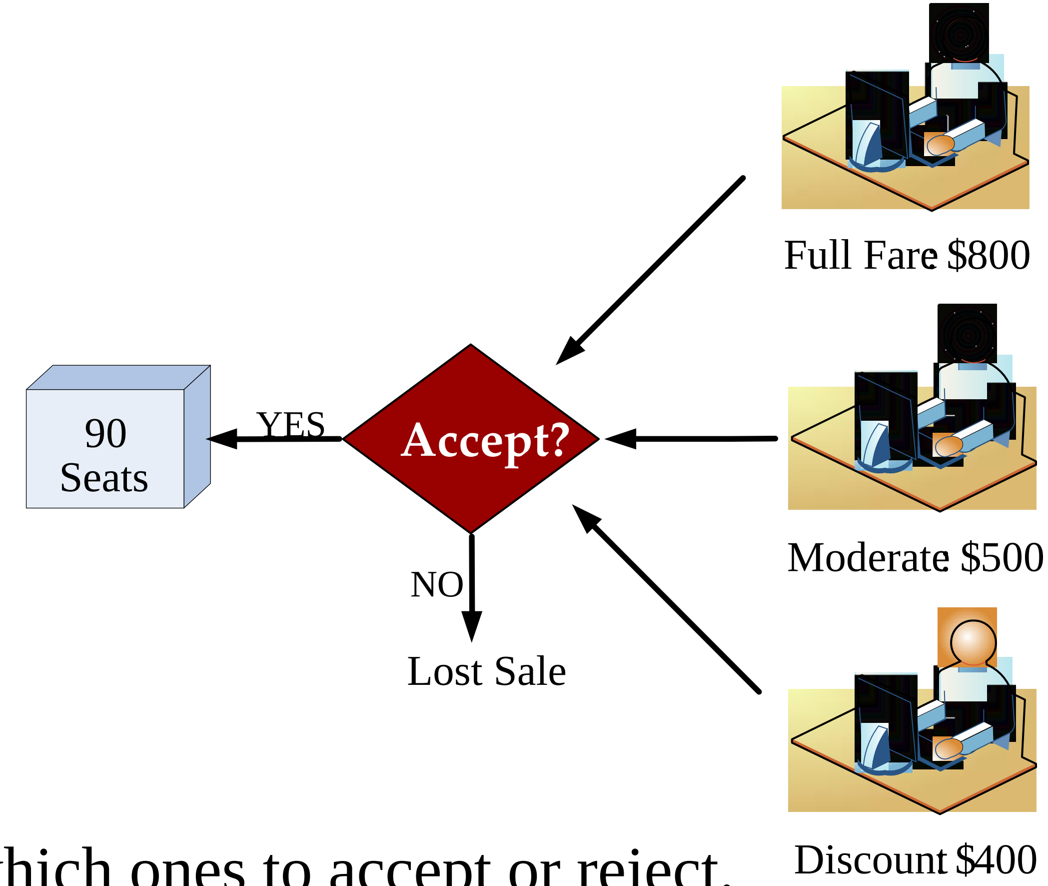
Takeaways

- Understand the analogy between the Newsvendor problem and the Revenue Management problem with capacity control

- Problem Walkthrough (Video)
  – https://www.youtube.com/watch?v=4SfMx3pVMgo&feature=youtu.be

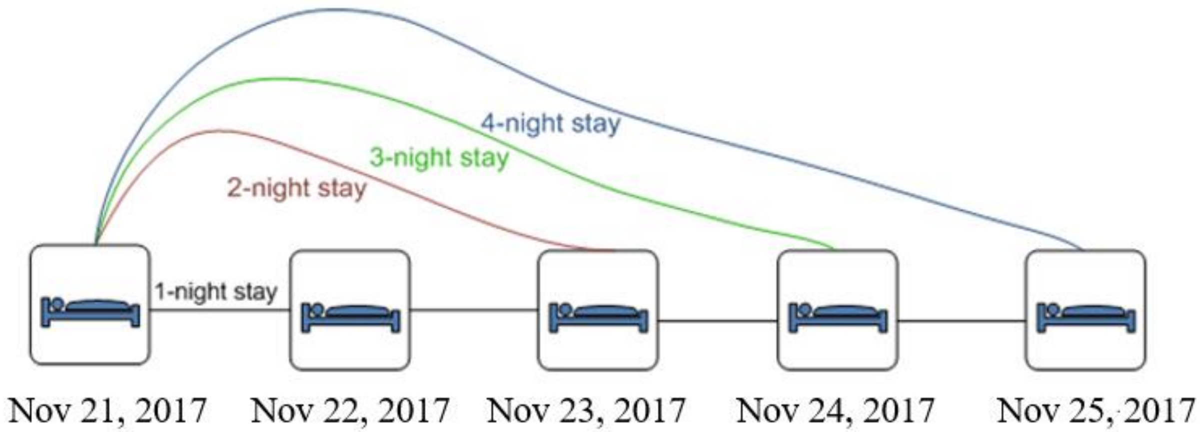
Single Product Capacity Control

- Traditional hotel quantity-based RM models:



... decide which ones to accept or reject.

Network Capacity Control: Hotel

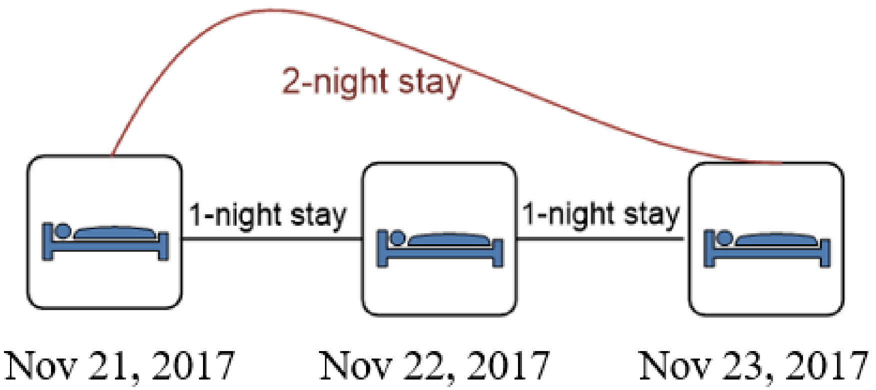


- Resources
  - Rooms in a particular day (e.g., a double room on Nov 22nd)
- Products
  - A collection / bundle of resources (e.g., a three-day stay in a double room from Nov 21st to Nov 24th)
- Problem
  - How to optimize revenues and resource allocation by accepting/denying requests for different products

Current Practice: Bid-price Control

- **Bid Prices**: Opportunity costs assigned to each resource.
- **Bid Prices Policy**: Accept a request if the price *exceeds* the sum of the *bid prices* of the resources consumed by the product.

Example: (2 resources and 3 products)



- Resources: Nov-21 and Nov-22
- Bid Prices: $95 and $ 90
- Products: Nov-21, Nov-22 and Nov-21&22
- Fares: $100, $100 and $180.

Strategy

Accept: 1-night Stays (**because 100>90, 100>95)**

Reject: 2-night Stays **(because 180<90+95)**

Use LP to Compute Bid Pricesc

Linear Programming

- C_i : Available capacity for resource i
- D_j : Average future demand for each product j
- A_{ij} : Units of resource i used by product j
- f_j : Fare (selling price) for product j
- x_j : Booking limit for product j

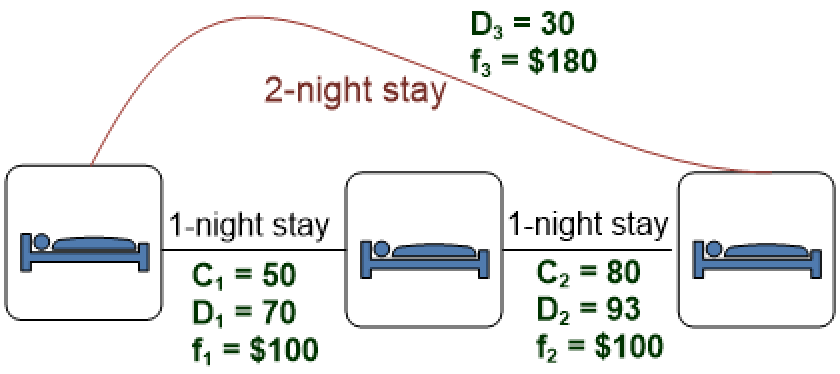
$$\max \quad Z = \sum_j f_j x_j$$

$$\text{Subject to} \quad \sum_j A_{ij} x_j \le C_i \quad \text{(for every i)}$$

$$0 \le x_j \le D_j \quad \text{(for every j)}$$

The bid price for resource i is equal to the shadow price of the constraint C_i

Example:



$$\max Z = 100x_1 + 100x_2 + 180x_3$$

$$x_1 + x_3 \le 50$$

$$x_2 + x_3 \le 80$$

$$0 \le x_1 \le 70$$

$$0 \le x_2 \le 93$$

$$0 \le x_3 \le 30$$

Solving LP to Compute Bid Prices

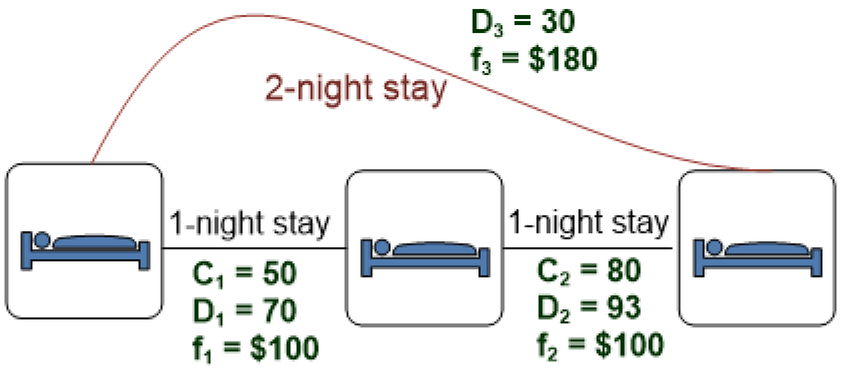
Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D22	X1	50	0	100	1E+30	20
E22	X2	80	0	100	1E+30	20
F22	X3	0	-20	180	20	1E+30

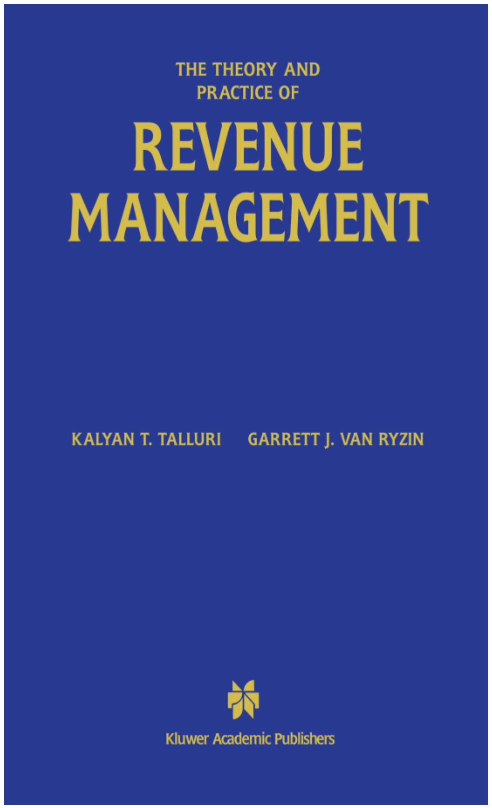
Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
L19	Resource C1	50	100	50	20	50
L20	Resource C2	80	100	80	13	80
L21	Demand D1	50	0	70	1E+30	20
L22	Demand D2	80	0	93	1E+30	13
L23	Demand D3	0	0	30	1E+30	30

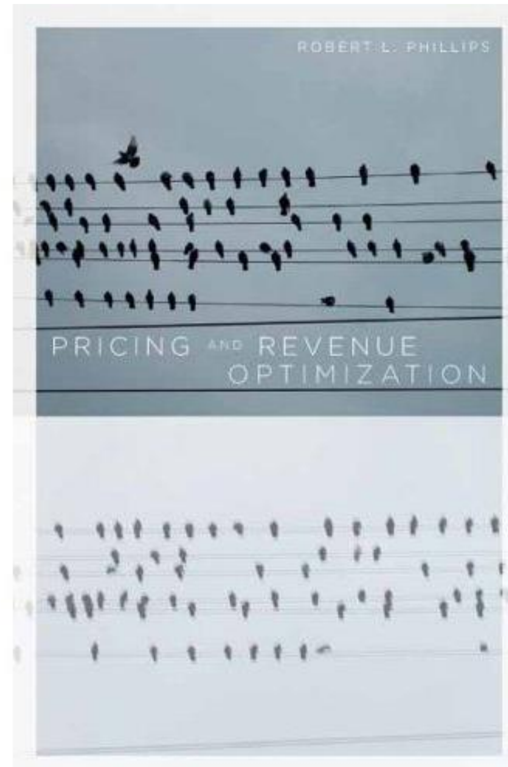
The bid price of Nov-21 stay = 100
The bid price of Nov-22 stay = 100
Hence, accept one-day stay, and reject two-day stay



Further Reading



“The Theory and Practice of Revenue Management”, by K. Talluri y G. van Ryzin, Kluwer Academic Publishers, 2004



“Pricing and Revenue Optimization”, by Robert Phillips, Stanford University Press, 2005.

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Network capacity control with multiple resources and multiple products

- **Bid price control: use linear programming to compute optimal bid prices**