Practice Set 6.2 Revenue Management

Question 1 (Newsvendor model)

Hong Kong Airport is interested in improving taxi service for arriving passengers who want to get a taxi to the city. To mitigate the variability in the number of passenger arrivals each day, the airport has decided to "book" a fixed number of taxis at 200 HKD (per day per taxi) every night for the next day's arrivals. Booked taxis arrive quickly when called upon by the airport authorities and are paid 200 HKD regardless of whether they are called upon or not. The daily demand for taxi service is Normally distributed with mean 10,000 taxis and standard deviation 2,000 taxis. The average taxi fare for each ride is 350 HKD. Assume that the arriving passengers wait until the booked taxi arrives at the airport. If there are no more booked taxis available, the airport authorities close down the taxi counter at the arrivals area.

a) Suppose the airport books 12,000 taxis each night. What is the probability that an arriving passenger finds the taxi counter closed?

   $Prob\ (D > 12000) = Prob\ (\frac{D-10000}{2000} > \frac{12000-10000}{2000}) = Prob(z > 1)$

   12000 taxis correspond to z = 1.

   From the normal distribution table, 0.8413 is probability arriving passenger will find a taxi.

   Therefore, the probability that an arriving passenger finds the taxi counter closed is 15.87%.

b) Suppose the airport wants to ensure that there is at least 90% service level. How many taxis should the airport book each night?

   Look up 0.9 in the Normal distribution table, z = 1.29.

   Q = 10000 + 1.29 *2000 = 12,580*

c) How many taxis should the airport book to maximize its expected profit?

   $C_u = \$350 - \$200 = \$150$

   $C_o = \$200$

   $P = C_u / (C_u + C_o) = 0.4286$

   $z = -0.18$

   $Q^* = 10000 + (-0.18)*2000 = 9{,}640$

Question 2 (Protection level)

HK4 is a television station that has 25 thirty-second advertising slots during each evening. It is early January and the station is selling advertising for Sunday, March 24. They could sell all of the slots right now for $4,000 each, but, because on that particular Sunday the station is televising the Oscar ceremonies, there will be an opportunity to sell slots during the week right before March 24 for a price of $10,000. For now, assume that all slots will be sold out for sure (either through advance booking or last-minute sales). To help make this decision, the salesforce has created the following probability distribution for last-minute sales:

Number of last-minute sales, x	*Pr(Exactly x last-minute sales)*
8	0.00
9	0.05
10	0.10
11	0.15
12	0.20

13	0.10
14	0.10
15	0.10
16	0.10
17	0.05
18	0.05
19	0.00

a) How many slots should HK4 sell in advance?

Q = protection level for last-minute sales

C_u (cost of reserving too few slots for last-minute sales) = \$10000 – \$4000 = \$6000

C_o (cost of reserving too many slots for last-minute sales) = \$4000

$P = C_u / (C_u + C_o) = 6000/ (6000 + 4000) = 0.6$

Next, compute the cumulative probability:

Number of last-minute sales, x	*Pr(Exactly x last-minute sales)*	*Cumulative probability*
8	0.00	*0*
9	0.05	*0.05*
10	0.10	*0.15*
11	0.15	*0.3*
12	0.20	*0.5*
13	0.10	*0.6*
14	0.10	*0.7*
15	0.10	*0.8*
16	0.10	*0.9*
17	0.05	*0.95*
18	0.05	*1*
19	0.00	*1*

From the table, the optimal protection quantity is 13. Therefore, HK4 should sell 25 – 13 = 12 slots in advance.

b) In practice, it is likely there will be empty slots available (i.e., slots that weren't sold in advance nor during the last week) and companies are willing to place standby advertising messages. Since there is no guarantee that such a slot will be available, standby messages are placed at a much lower cost. Now suppose that if a slot is not sold in advance and not sold during the last week, it will be used for a standby promotional message that costs advertisers \$2,500. Now how many slots should HK4 sell in advance?

C_u = \$10000 – \$4000 = \$6000

C_o = \$4000 – \$2500 = \$1500

$P = C_u / (C_u + C_o) = 6000/ (6000 + 1500) = 0.8$
From the table constructed for part a), the optimal protection quantity is 15.
Therefore, HK4 should sell 25 – 15 = 10 slots in advance.

c) Suppose HK4 chooses a booking limit of 10 slots on advanced sales. In this case, what is the probability there will be slots left over for stand-by messages?
From the table constructed for part a), if the booking limit is 10, there will be 15 slots for last-minute sales.
There will be standby messages only if there are 14 or fewer last-minute sales.
Hence, the probability there will be slots left over for stand-by messages is 70%.

Question 3 (Protection level & overbooking)

Doubletree in Austin has 150 standard rooms. Doubletree generally sells those rooms through two channels, one through their own website, call center and front desk usually at a high rate and the other through agencies like Priceline at a low rate. Suppose Doubletree charges a high rate at $120 per room per night through their own channel (they never mark down the price through their own explicit channel to avoid any bad gambling image which hurts reputation) while "implicitly" sells some rooms to the agencies at a low rate of $50 per room per night. Bargain customers who seek lower rate usually will book rooms much earlier than the premium customers through the agency channel.

To make it simple, suppose the customers always stay for one night, there is ample demand from the bargain customers for the low rate, however, the number of premium customers is uncertain, which is distributed according to the following table:

Number of premium customers	*Probability*
90	0.10
100	0.15
110	0.15
120	0.20
130	0.25
140	0.15

a) How many rooms shall Doubletree sell to the agencies like Priceline in advance?
Q = protection level for premium customers
C_u (the cost of reserving too few rooms for premium customers) = \$120 – \$50 = \$70
C_o (the cost of reserving too many rooms for premium customers) = \$50
$P = C_u / (C_u + C_o) = 70/ (70 + 50) = 0.5833$
Therefore, we want to find Q such that P (number of premium customers $\leq$ Q*) = 0.5833*
Next, compute the cumulative probability and find the closest one that is greater than the critical fractile, i.e. 0.6

Number of premium customers	*Probability*	*Cumulative probability*

90	0.10	*0.10*
100	0.15	*0.25*
110	0.15	*0.40*
120	0.20	*0.60*
130	0.25	*0.85*
140	0.15	*1.00*

From the table, the optimal protection level is 120 rooms.
Hence, Doubletree should sell 150 – 120 = 30 rooms in advance to the agencies.

b) The number of no-shows at Doubletree has a distribution as that in the following table. Doubletree estimates the net cost of bumping a customer is $200, which applies to both premium and bargain customer. Suppose overbooked rooms are only sold to the agency channel at $50, what is the optimal maximum number of reservations to accept per day?

Number of no-shows	*Probability*
0	0.05
1	0.05
2	0.10
3	0.15
4	0.15
5	0.20
6	0.20
7	0.10

Q = number of overbooking
C_u (cost of overbooking too few, i.e. loss sales) = $50
C_o (cost of overbooking too many, i.e. cost of bumping a customer) = $200
$P = C_u / (C_u + C_o) = 50/ (50 + 200) = 0.2$

Number of no-shows	*Probability*	*Cumulative probability*
0	0.05	*0.05*
1	0.05	*0.10*
2	0.10	*0.20*
3	0.15	*0.35*
4	0.15	*0.50*
5	0.20	*0.70*
6	0.20	*0.90*
7	0.10	*1.00*

We want to find Q such that P (number of no-shows $\leq Q^*$) = 0.2*
Next, compute the cumulative probability.
From the table, the corresponding no-shows is 2

Hence, Doubletree should overbook 2 rooms and accept a maximum of 152 reservations.

Question 4 (Overbooking)

Next week, Super Discount Airlines has a flight from New York to Los Angeles that will be booked to capacity. The airline knows from past history that an average of 25 customers (with a standard deviation of 15) cancel their reservation or do not show for the flight. Revenue from a ticket on the flight is $125. If the flight is overbooked, the airline has a policy of getting the customer on the next available flight and giving the person a free round-trip ticket on a future flight. The cost of this free round-trip ticket averages $250. Super Discount considers the cost of flying the plane from New York to Los Angeles a sunk cost. By how many seats should Super Discount overbook the flight?

C_u (cost of overbooking too few, i.e. loss sales) = $125

C_o (cost of overbooking too many, i.e. cost of bumping a customer) = $250

[Note: Since it is common for customer to pay in advance when booking flight. Flight company first received $125. If overbooking happened, the company need to get the customer on the next available flight (which cost $125 if price doesn't change) + an additional free ticket for future use (cost $250). so Co should be ($250+$125)-$125 =$250.]

$P = C_u / (C_u + C_o) = 125/(125 + 250) = 0.3333$

From the normal distribution table, F(-0.43) = 0.3336, F(-0.44) = 0.3300

Hence, Z = -0.43

$Q^* = \mu + z\sigma = 25 + (-0.43)*15 = 18.55$

The hotel should overbook 19 seats.

The optimal overbooking quantity can be further determined by comparing the expected total cost for Q = 18 and 19:

Objective: minimize the expected total cost = $P(D \leqslant Q)*C_o + P(D>Q)*C_u$

If Q = 18, Z = (18 – 25)/15 = -0.4667 and

*EV(Q = 18) = 0.3192*250 + 0.6808*125 = $164.9*

If Q = 19, Z = (19 – 25)/15 = -0.4 and

*EV(Q = 19) = 0.3446*250 + 0.6554*125 = $168.1*

Question 5 (WTP)

HKUST Center for the Arts is planning a concert by Hong Kong Philharmonic Orchestra. A recent survey from staffs and students of HKUST reveals the willingness-to-pay for the concert as follows:

WTP (HKD)	*Demand if price = WTP*
150	1
125	10
100	32
75	72
50	97
25	113

a) What is the revenue-maximizing price for the concert?

WTP (HKD)	*Demand if price = WTP*	*Revenue*
150	*1*	*150*
125	*10*	*1,250*
100	*32*	*3,200*
75	*72*	*5,400*
50	*97*	*4,850*
25	*113*	*2,825*

From the table, the revenue-maximizing price is 75 HKD.

b) HKUST Center for the Arts wishes to differentiate the price for staffs and students. The following table presents the detailed information about willingness-to-pay for staffs and students, respectively. What would be the revenue-maximizing prices for staffs and students based on the WTP information? Is it worthwhile to differentiate prices for staffs and students?

WTP (HKD)	*Staff demand if price = WTP*	*Students demand if price = WTP*
150	1	0
125	9	1
100	22	10
75	30	42
50	39	58
25	52	61

WTP (HKD)	*Staff demand if price = WTP*	*Revenue from Staff*	*Students demand if price = WTP*	*Revenue from Students*
150	*1*	*150*	*0*	*0*
125	*9*	*1,125*	*1*	*125*
100	*22*	*2,200*	*10*	*1,000*
75	*30*	*2,250*	*42*	*3,150*
50	*39*	*1,950*	*58*	*2,900*
25	*52*	*1,300*	*61*	*1,525*

For both staffs and students, the profit-maximizing price is 75 HKD.
Since the profit-maximizing price is the same with or without price differentiation, it wouldn't be worthwhile to differentiate prices for staffs and students if profit maximization is the only concern of Center for the Arts.

Question 6 (Linear Demand function)
Correct choice: More than 9

Standard Normal Probabilities

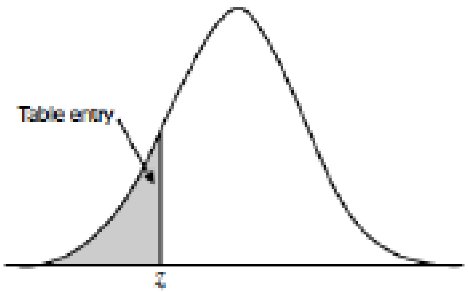
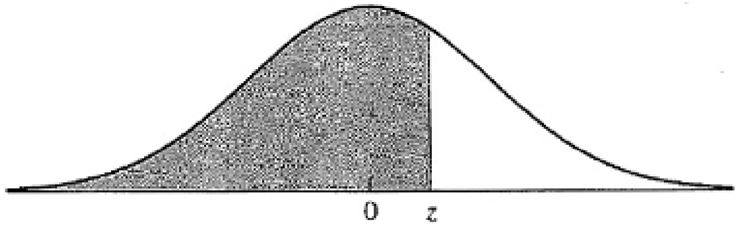


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
–3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
–3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
–3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
–3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
–3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
–2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
–2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
–2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
–2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
–2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
–2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
–2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
–2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
–2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
–2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
–1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
–1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
–1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
–1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
–1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
–1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
–1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
–1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
–1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
–1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
–0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
–0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
–0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
–0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
–0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
–0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
–0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
–0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
–0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
–0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999