ISOM 2700: Operations Management

Session review and practice.
Inventory Management

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Background information

- What is inventory

  It is a stock of goods awaiting consumption. Something the company has paid for but customers have not yet paid for.

- What are some of the main reasons for companies to carry inventory?

  regulation requirement; operation support; risk hedging; stock value…

- List the three types of costs related to inventory.

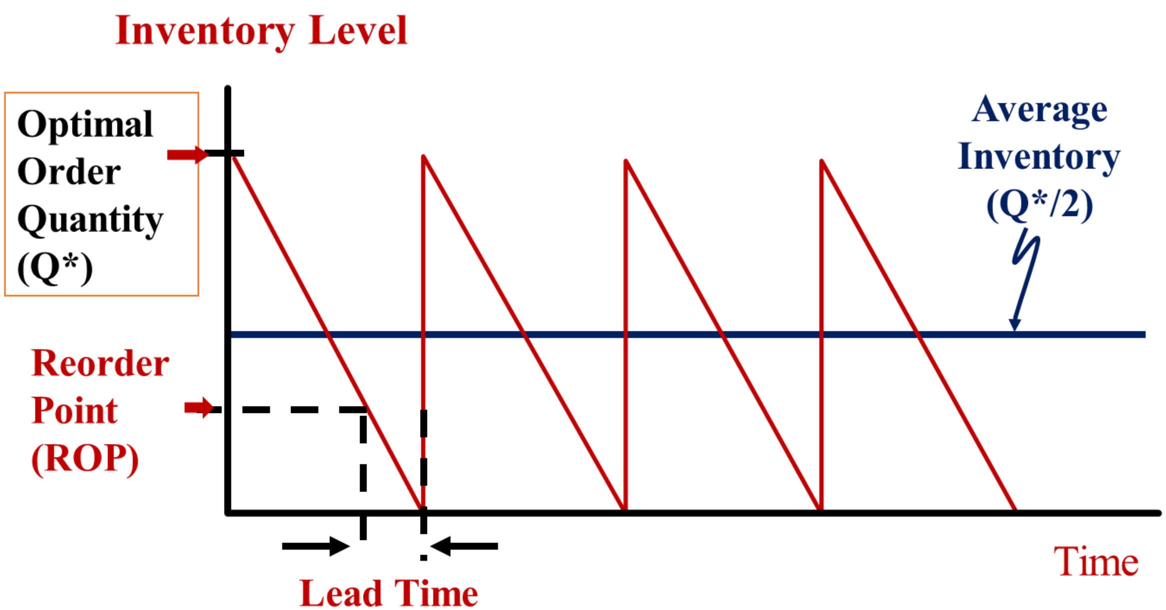
  Holding cost or carrying cost; Shortage or Overage cost ; Ordering cost.

Economic Order Quantity (EOQ) Model

- Applicable setting?

  long life cycle product; constant demand rate...

- Insight: Main trade-offs between ordering and holding cost



EOQ equation: setting

- Goal: minimize the total cost while satisfying demand

- Decision:
  - Q: Order quantities each time
  - ROP: When to order

- Given:
  - D - *Demand per unit of year*
  - S - *Setup or Order Cost ($/setup; $/order)*
  - H- *Marginal holding cost ($/per unit per unit of time)*
  - L - *Lead time in days*
  - d - *Demand per day* $=$ D/ *Working days per year*

EOQ equation: EOQ

Step (1): Construct the relationship between the decision variable and effectiveness measurements

(1.1) Frequency

Time between orders = Q/D

Order frequency per unit time = D/Q

(1.2) Cost

$$TC\ (Q) = \frac{DS}{Q} + \frac{QH}{2}$$

(1.3) Reordering point

$$ROP = d \cdot L$$

EOQ equation: EOQ

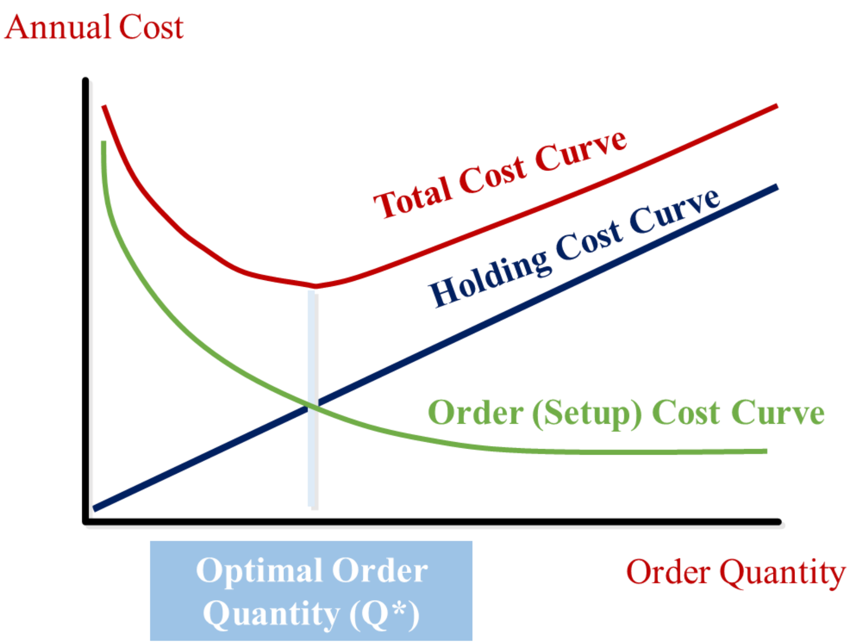
Step (2): Find the optimal Q at which the total cost is minimum

$$TC\ (Q) = \frac{DS}{Q} + \frac{QH}{2}$$

(2.1) $\frac{DS}{Q} = \frac{QH}{2}$

(2.2) $\frac{d\ TC(Q)}{d\ Q} = -\frac{DS}{Q^2} + \frac{H}{2} = 0$

$$Q^* = \sqrt{\frac{2DS}{H}}$$



EOQ: Cost calculation

Items purchased from a vendor cost $20 each, and the forecast for next year's demand is 1,000 units. If it costs $5 every time and order is placed for more units and the storage cost is $4 per unit per year.

- What quantity should be ordered each time?
- What is the total ordering cost for a year?
- What is the total holding cost for a year?

EOQ: Cost calculation answer

What is the total ordering cost for a year?

- *Given: D = 1,000 units/year; S = \$5/order; H = \$4/unit/year*
- *EOQ* $= \sqrt{2DS/H} = \sqrt{2(1000)(5)\ /\ 4}$ *= 50 units*
- *Total annual ordering cost = S (D/Q) = 5(1000/50) = \$100*

What is the total holding cost for a year?

- *Total annual holding cost = H (Q/2) = 4(50/2) = \$100*
- *Once again, total holding cost and ordering cost should be the same when the company is ordering the optimal economic quantity.*

EOQ : frequency calculation

Campus Publishing prints textbooks written by faculty and distribute around the world. The Business English book is particularly popular and has annual sales of 100,000 copies per year. Printing each copy of the book costs $60. The author is encouraged to come up with revisions (i.e., new editions) to cater to consumer needs. Each revision will require $250,000 for a new typeset of the book. After a new edition is released, its sales value is discounted at a rate of $2 every 3 months which means every three months, the value drops by $2.

- How frequently should new editions be published?

EOQ : frequency calculation answer

- Given: D = 100,000 copies/ year; S = $250,000/revision, H = $2(12/3) = $8/year

  Note: For H, we can find that within a year the total drop in value is 2+2+2+2 = $8. Because a book is not sold, there is a loss of $2 every three months, then in a year the total loss of holding the inventory becomes $2 x 4 = $8.

- $60 is a variable cost, EOQ model focuses on the fixed cost per batch order
- Sales of each edition: Q = $\sqrt{(2DS/H)}$= $\sqrt{2(100000)(250000)\ /\ 8}$≈ 79,057 copies
- Time between editions: T = Q/D = 79057 / 100000 ≈ 0.79 year ≈ 9.5 months

Newsvendor model

Decide order quantity Q

Actual sales = min (Q,D)

Purchasing cost: c*Q

Demand D is realized

Collecting sales revenue p*min(Q,D) and salvage value s*max(Q-D, 0)

Applicable setting?

Uncertain demand; shot life cycle product; no replenishment opportunities

Insight: Main trade-offs?

overage cost and underage cost

The mindset of Marginal Analysis

Newsvendor: setting

Goal: Maximize expected profits facing uncertain demand

Decisions:

- Q: Order quantities

Given:

- **Cost of over-stocking: C_o = cost − salvage value**
- **Cost of under-stocking: C_u = price − cost**

Newsvendor: step(1)

- Step (1): Construct the expected incremental profit from stocking the $Q+1^{st}$ unit

- Profit($Q \to Q+1$)

  = expected gain – expected loss

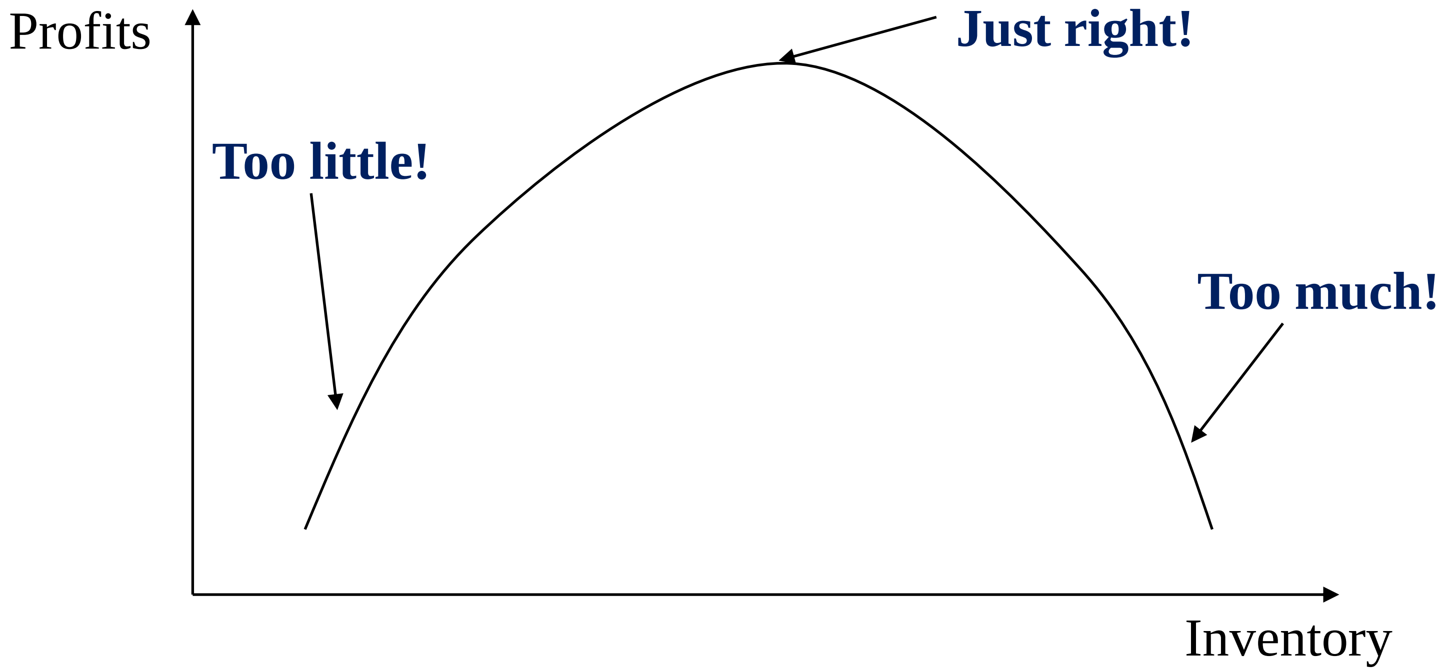
  = $\mathbf{C_u \times Pr\,(Demand > \mathit{Q}) - C_o \times Pr\,(Demand \leq \mathit{Q})}$

  = $\mathbf{C_u - (C_u + C_o) \times Pr\,(Demand \leq \mathit{Q})}$

Newsvendor: step(2)

- step(2). find optimal n using the key idea of marginal analysis

the expected value (profit) of ordering one more unit is ZERO.



Newsvendor: equations

Optimal stocking level

$$\Pr(\text{Demand} \leq Q^*) = \frac{c_u}{c_u + c_o}$$

Two cases:

- Special case of normally distributed demand
  - Standard normal probabilities table or NORM.INV function in excel
- Discrete probability distribution

Service level is the probability of no shortage

$$\Pr(\text{Demand} \leq Q) = \text{S}$$

Newsvendor: normal demand

- (1) Evaluate the critical ratio: $c_u/(c_u + c_o)$
  - Cost of over-stocking: c_o= cost – salvage value
  - Cost of under-stocking: c_u = price – cost

- (2) Demand follows Normal distribution with mean μ, standard deviation $\delta \Rightarrow$
  a) Find the z value in the standard normal distribution function table such that $\phi(z)$ is the same as the critical ratio.
     - Remember to round up!
  b) Convert z to the optimal $Q^* = \mu + z \times \sigma$

Newsvendor: discrete probability table

- (1) Evaluate the critical ratio: $c_u /(c_u + c_o)$

- (2) Construct the cumulative probabilistic function $F(D \leq Q)$

- (3) Find the order quantity in the table such that $F(D \leq Q) = c_u /(c_u + c_o)$. If the critical ratio is between two entries in the table, choose the entry with larger order quantity.

Newsvendor: Marginal Analysis 1

You have a date at 6pm You estimate that the average travel time from campus is 20 minutes but there is some variation due to traffic congestion during the rush hour. Although it is difficult to quantify the damage of each minute you are late, you feel that it would be 10 times more costly to be later than earlier. The travel time is normally distribution with mean 20 minutes and standard deviation 10 minutes. When should you set off from home?

a. Before 5:40pm
b. At 5:40pm sharp
c. After 5:40pm
d. Cannot be determined from the information given

Newsvendor: Marginal Analysis 2

You have a date at 6pm You estimate that the average travel time from campus is 30 minutes but there is some variation due to traffic congestion during the rush hour. Although it is difficult to quantify the damage of each minute you are late, you feel that it would be 10 times more costly to be earlier than later. The travel time is normally distribution with mean 30 minutes and standard deviation 10 minutes. When should you set off from home?

a. Before 5:30pm
b. At 5:30pm sharp
c. After 5:30pm
d. Cannot be determined from the information given

Newsvendor: discrete demand

A product is priced to sell at $100 per unit, and its cost is constant at $70 per unit. Each unsold unit has a salvage value of $20. Demand is expected to range between 30 and 40 units for the period. The demand probabilities and the associated cumulative probability distribution (P) for this situation are shown below.

- How many units should be ordered?

# of units demanded	Probability	Cumulative probability
35	0.10	0.10
36	0.15	0.25
37	0.25	0.50
38	0.25	0.75
39	0.15	0.90
40	0.10	1.00

Newsvendor: discrete demand answer

- Cu = \$100 - \$70 = \$30
- Co = \$70 - \$20 = \$50
- P = Cu/ (Cu + Co) = 30/ (30 + 50) = 0.375
- From the table, we will look for the cumulative probability and find closest one that is greater than the critical fractile, i.e. 0.5
- Q* is 37 units.

Newsvendor: normal demand

- Elite Couture, a high-end fashion goods store has to decide on the quantity of Luella Bartley handbags to sell during the Christmas season. The unit cost of the handbag is $28.5 and the handbag sells for $150. All handbags remaining unsold at the end of the season are purchased by a discounter for $20 each. Further, there is a significant inventory holding cost incurred for each unsold bag, which is 40% of the unit cost. Demand for bags is distributed normally with mean 150 and standard deviation 20.
- How many bags should be purchased to maximize expected profit?

Newsvendor: normal demand answer

- Cu = \$150 – \$28.5 = \$121.5
- Co = \$28.50 – \$20 + \$28.5*0.40 = \$8.5 + \$11.40 = \$19.9
- P = Cu/(Cu + Co) = 121.5/(121.5 + 19.9) = 0.8593
- F(1.07) = 0.8577, F(1.08) = 0.8599. Hence, z = 1.08
- $Q^* = \mu + z\sigma = 150 + 1.08*20 = 171.6$
- Hence, order 172 handbags

Newsvendor: service level

Goop Inc needs to order a raw material to make a special polymer. The demand for the polymer is forecasted to be normally distributed with a mean of 250 gallons and a standard deviation of 125 gallons. Goop sells the polymer for $25 per gallon. Goop's purchases raw material for $10 per gallon and Goop must spend $5 per gallon to dispose all unused raw material due to government regulations. (One gallon of raw material yields one gallon of polymer.) If demand is more than Goop can make, then Goop sells only what they made and the rest of demand is lost.

a) How many gallons should Goop purchase to maximize its expected profit?

a) Suppose Goop wants to ensure that there is a 92% probability that they will be able to satisfy the customer's entire demand. How many gallons of the raw material should they purchase?

a) Suppose Good purchases 150 gallons of raw material. What is the probability that they will run out of raw material?

Newsvendor: service level

How many gallons should Goop purchase to maximize its expected profit?

- Cu = \$25 – \$10 = \$15
- Co = \$10 + \$5 = \$15
- P = Cu/(Cu + Co) = 15/(15 + 15) = 0.5
- $z = 0$
- $Q^* = \mu + z\sigma = 250 + 0*125 = 250$

Newsvendor: service level

Suppose Goop wants to ensure that there is a 92% probability that they will be able to satisfy the customer's entire demand. How many gallons of the raw material should they purchase?

- Lookup 0.92 in the normal distribution table.
- $z = 1.41$
- $Q^* = \mu + z\sigma = 250 + 1.41*125 = 426.25 = 427$

Newsvendor: service level

Suppose Good purchases 150 gallons of raw material. What is the probability that they will run out of raw material?

- $z = (150 - 250)/125 = -0.8$.
- From the Distribution Function Table, $F(-0.8) = 0.2119$
- $P = \text{Prob}(D \le Q^*)$, which is the probability of no stock-out and demand can be fulfilled
- Hence, the risk of stocking out is $1 - 0.2119 = 78.81\%$