ISOM 2700: Operations Management

Session 6.2 Newsvendor Model

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Inventory Models

- **EOQ Model**
  - Long lifecycle products
  - Tradeoff between setup and holding costs, driven by the frequency of ordering

- **Newsvendor Model**
  - **Short lifecycle** products
  - **Considerable demand uncertainty**
  - Tradeoff between costs of excess leftover inventory and excess demand

Newsvendor problem

- Every morning, a newsboy purchases newspapers to sell during the day

- How many units to order?
  - Purchase too much → money wasted on unsold items
  - Purchase too few → loses potential sales

- Not too much, not too little…

This is a Hard Problem…

- **Uncertain demand**
  - Demand is not known at the time of order
  - Demand forecast is available

- **Pre-commitment of order quantity**
  - Order quantity must be chosen before observing demand
  - No replenishment is possible

- **Perishable product**
  - Leftovers are costly: After the demand is realized, they can lose value, sometimes all of their value
  - Little or no salvage value

These three characteristics are found

in the newsvendor model

Examples of newsvendor problem

- How many Christmas trees to order before Christmas?
- How many pumpkins to order before Halloween?
- How much fashion goods (e.g., skiwear) to order?
- How much manufacturing capacity for short-life products should a company build?
- How many seats on a flight should be reserved for last-minute booking?

- Uncertain demand
- Perishable product
- Order has to be placed before the demand is realized

Newsvendor Example 1: Pumpkin

- **Demand information**

The retailer forecasts that the demand for pumpkins before Halloween is

- $D \leq 200$ with probability 0.5
- $D \leq 250$ with probability 0.75
- $D \leq 300$ with probability 0.9

- **Price & cost information**
  - The retailer buys pumpkins from a wholesaler at unit cost \$2
  - The retailer sells pumpkins at unit price \$5
  - Any unsold pumpkins has salvage value \$1

- **Question:**

How many pumpkins should the retailer buy so that she could maximize her expected profit?

A simple example

- Demand for the newspaper is 50 with probability 0.3 and 100 with probability 0.7
- Question:

  How many newspapers should the newsvendor order?

Marginal Analysis

Examination of the costs and benefits of a marginal (small) change in the order quantity.

So how many cookies will you buy if the cost is $7?

	Marginal benefit (WTP)
1st cookie	$ 20
2nd cookies	$12
3rd cookies	$6
4th cookies	$2

Marginal Analysis: Pumpkin Example

- Suppose the retailer has bought 200 pumpkins
- If the retailer buys one more pumpkin, what is the expected value of this 201^{st} pumpkin to the retailer?
  - Unit cost = \$2, Unit price = \$5, Salvage value = \$1
  - **Case 1: $D \leq 200$** (prob. 0.5)
    - The retailer cannot sell the 201^{st} pumpkin, and gets \$1 salvage value from it. Utility =\$1 – \$2= – \$1
  - **Case 2: $D > 200$** (prob. 0.5)
    - The retailer sells the 201^{st} pumpkin at \$5. Utility =\$5 – \$2= \$3
  - Expected value of the 201^{st} pumpkin

    $= 0.5 \times (-\$1) + 0.5 \times \$3 = \$1 > 0$

- Should the retailer buy the 201^{st} pumpkin?

  --Yes, because the expected value is positive!

Marginal Analysis: Pumpkin Example

- Suppose the retailer has bought 250 pumpkins
- If the retailer buys one more pumpkin, what is the expected value of this 251^{st} pumpkin to the retailer?
  - She needs to pay unit cost = \$2
  - **Case 1: $D \leq 250$** (prob. 0.75)
    - The retailer cannot sell the 251^{st} pumpkin, and gets \$1 salvage value from it. Utility =\$1 – \$2= – \$1
  - **Case 2: $D > 250$** (prob. 0.25)
    - The retailer sells the 251^{st} pumpkin at \$5. Utility =\$5 – \$2= \$3
  - Expected value of the 251^{st} pumpkin
    = 0.75 × (– \$1) + 0.25 × \$3 = 0
- Should the retailer buy the 251^{st} pumpkin?

  -- Indifferent, because the expected value is zero!

Marginal Analysis: Pumpkin Example

- Suppose the retailer has bought 300 pumpkins
- If the retailer buys one more pumpkin, what is the expected value of this 301^{st} pumpkin to the retailer?
  - She needs to pay unit cost = \$2
  - **Case 1: $D \leq 300$** (prob. 0.9)
    - The retailer cannot sell the 301^{st} pumpkin, and gets \$1 salvage value from it. Utility =\$1 – \$2= – \$1
  - **Case 2: $D > 300$** (prob. 0.1)
    - The retailer sells the 301^{st} pumpkin at \$5. Utility =\$5 – \$2= \$3
  - Expected value of the 301^{st} pumpkin
    $= 0.9 \times (-\$1) + 0.1 \times \$3 = -\$0.6 < 0$
- Should the retailer buy the 301^{st} pumpkin?

-- No, because the expected value is negative!

Critical Fractile Analysis

- **Cost of under-stocking: C_u = price − cost**
  - Cost of not having unit in stock when demand does materialize

- **Cost of over-stocking: C_o = cost − salvage value**
  - Cost of having unit in stock when demand does not materialize

Critical Fractile Analysis

- Suppose you have decided to stock/produce *n* units
- Consider the incremental decision to stock/produce one more, namely an $n+1^{st}$ unit

Expected underage cost of $\boldsymbol{n+1^{st}}$ unit

= underage cost × Pr($n+1^{st}$ unit sells)

= $\mathbf{C_u \times Pr\ (Demand > \mathit{n})}$

Expected gain of $n+1^{st}$ unit

Expected overage cost of $\boldsymbol{n+1^{st}}$ unit

= overage cost × Pr($n+1^{st}$ unit does not sell)

= $\mathbf{C_o \times Pr\ (Demand \leq \mathit{n})}$

Expected loss of $n+1^{st}$ unit

Critical Fractile Analysis

- What is the expected incremental profit from stocking the n+1st unit, i.e., Profit($n \rightarrow n$+1)?
- Profit($n \rightarrow n$+1)

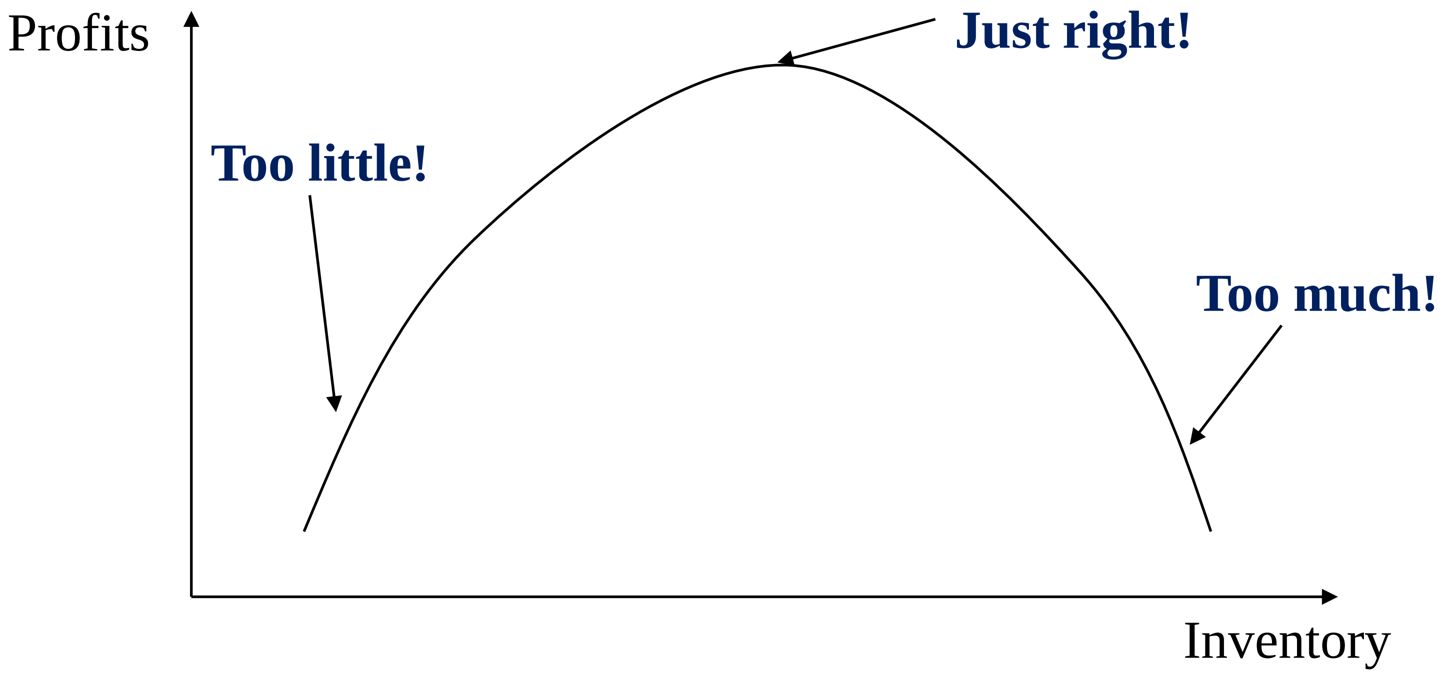
  = expected gain – expected loss

  = $\mathbf{C_u \times Pr\ (Demand > \boldsymbol{n}) - C_o \times Pr\ (Demand \leq \boldsymbol{n})}$

  = $\mathbf{C_u - (C_u + C_o) \times Pr\ (Demand \leq \boldsymbol{n})}$

Note: Pr(Demand > n) = 1 – Pr(Demand ≤ n)

Profit Curve: Pumpkin Example



Key idea of marginal analysis: At the optimal order quantity, the expected value (profit) of ordering one more unit is ZERO!!!

Trade-off between overage and underage costs

- As Q increases, total overage cost increases because it becomes more likely for inventory to exceed demand.

- Further, total underage cost decreases because it becomes less likely for inventory to fall short of demand.

- The optimal value of Q is determined by balancing the total overage cost with the total underage cost

Key idea of marginal analysis: At the optimal order quantity, the expected value (profit) of ordering one more unit is ZERO.

The Optimal Newsvendor Rule

Set the two costs equal to one another:

Overage cost × Pr (D ≤ Q) = Underage cost × Pr (D > Q)

Or,

Overage cost × Pr (D ≤ Q) = Underage cost × [1 – Pr (D≤ Q)]

Cost of over-stocking: c_o= cost – salvage value

cost of having unit in stock when demand does not materialize

Cost of under-stocking: c_u = price – cost

cost of not having unit in stock when demand does materialize

Newsvendor formula:

critical fractile

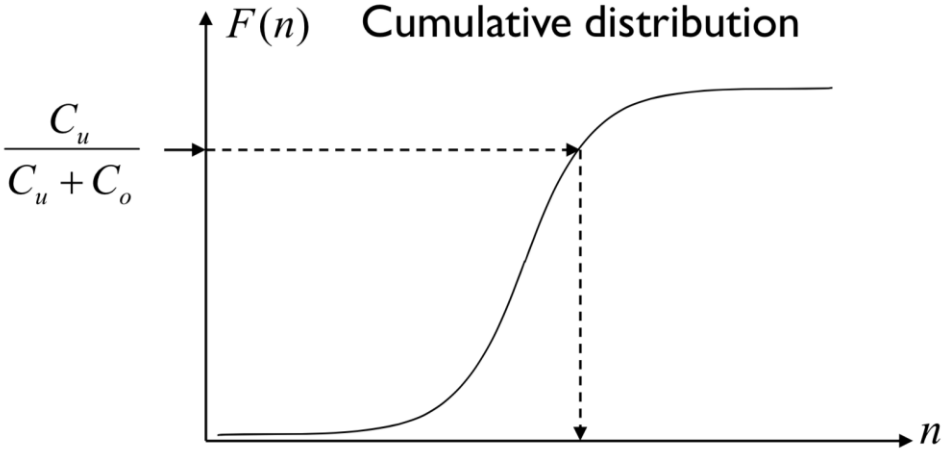
$$\mathbf{Pr\,(\,D \leq Q\,) = c_u\,/\,(c_u + c_o)}$$

Special case: normal demand

- If demand D is normally distributed with mean μ and standard deviation σ, what is the optimal stocking level

  $P(D \le Q) = c_u / (c_u + c_o)$; What is the value of Q?

Q = norminv(critical fractile, μ, σ)   in Excel

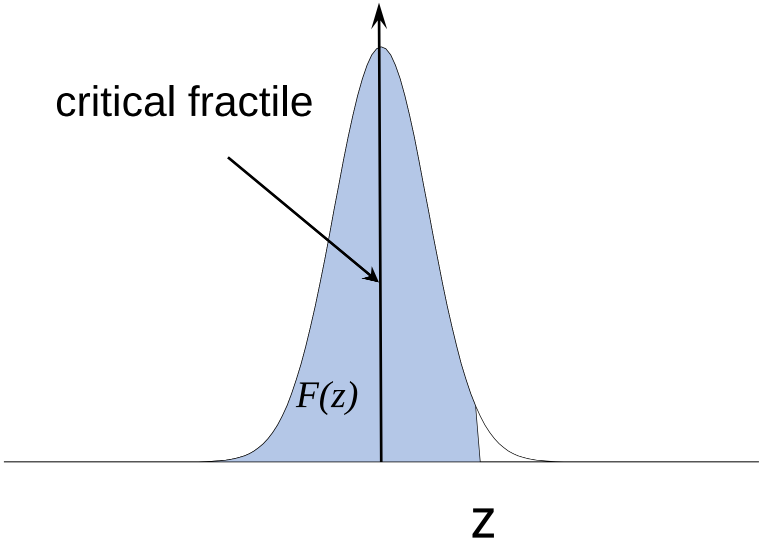


Special case: normal demand

- If demand D is normally distributed with mean μ and standard deviation σ, what is the optimal stocking level?

$P(D \leq Q) = c_u / (c_u + c_o)$; What is the value of Q?

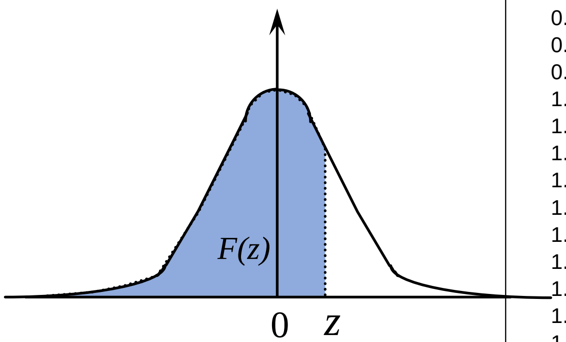
$Q = \mu + z\,\sigma$, where z is the value corresponding to the critical fractile (cumulative table)



$F(z) = \text{Prob}(N(0,1) \leq z)$

The Standard Normal Distribution

$F(z) = \text{Prob}(N(0,1) \leq z)$



α = 98.7% implies

$z^* = 2.23$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997

Standard Normal Distribution Function Table

How to start with a quantity and find the corresponding probability?

- Q: *What is the probability the outcome of a standard normal will be* ***z = 0.28*** *or smaller?*
  - Look for the intersection of the fourth row (with the header 0.2) and the ninth column (with the header 0.08) because 0.2+0.08 = 0.28, which is the *z* we are looking for

Standard Normal Distribution Function Table (continued), Φ(z)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

Standard Normal Distribution Function Table

How to start with a probability and find the corresponding quantity?

- Q: *For what z is there a* **69.90%** *chance that the outcome of a standard normal will be that z or smaller?*
  - Find the probability inside the table.
  - Add the row and column headers for that probability

Round-up rule!

Standard Normal Distribution Function Table (continued), $\Phi(z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
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0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

A procedure to find Q^* in newsvendor model

- (1) Evaluate the critical ratio: $c_u / (c_u + c_o)$
  - Cost of over-stocking: c_o= cost – salvage value
  - Cost of under-stocking: c_u = price – cost

- (2) Demand follows Normal distribution with mean μ, standard deviation $\delta \Rightarrow$
  a) Find the z value in the standard normal distribution function table such that $\phi(z)$ is the same as the critical ratio.
     - Remember to round up!
  b) Convert z to the optimal $Q^* = \mu + z \times \sigma$

Question for Students

- Suppose that the critical fraction is 0.4 for a newsvendor. What is the correct answer below?

1. Underage cost is greater than the overage cost
2. To choose the optimal ordering quantity, we only need to know the mean of the distribution
3. If the optimal quantity n is chosen, the newsvendor can satisfy all customer demand with 40% probability
4. None of the above

Back to Mr. Choi's problem

- Mr. Choi operates a news-stand and sells SCMP
  - orders copies of the newspaper from the publisher at a cost of \$4 per copy
  - sells at a retailing price of \$7
- The demand can be described with a normal distribution with mean 100 and standard deviation 12.
- How many copies should he order every day?

Critical Fractile

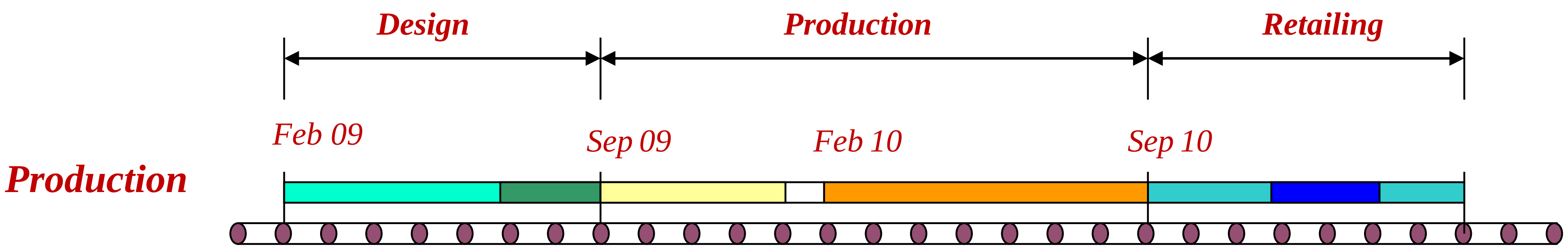
$= \$3 / (\$3 + \$4) = 0.4286$

$z = -0.18$

Optimal Quantity

$= 100 + (-0.18)(12) = 97.84$

Example: SnowTime Sporting Goods



Properties of SnowTime Sporting Goods:

- New designs are completed
- One production opportunity
- Based on past sales, knowledge of the industry, and economic conditions, the marketing department has a probabilistic forecast

SnowTime Revenue and Costs Information

- For each unit
  - c = production cost = $80
  - r = selling price = $125
  - s = salvage value of unsold unit = $20
- Recall
  - C_u = underage cost = marginal gain of extra unit sold
  - C_o = overage cost = marginal cost of unsold unit
- Hence,
  - $C_u = r – c = \$45$ and
  - $C_o = c – s = \$60$

Demand Scenario

Construct the cumulative probability: $F(D \leq Q)$

Demand	Probability	F(Q)
8,000	0.11	0.11
10,000	0.11	0.22
12,000	0.28	0.50
14,000	0.22	0.72
16,000	0.18	0.90
18,000	0.10	1

Optimal Solution for SnowTime

- Production cost per unit (C): $80
- Selling price per unit (S): $125
- Salvage value per unit (V): $20

Underage Cost

- if extra jacket sold, profit is $125-$80 = $45

Overage Cost

- if not sold, cost is $80-$20 = $60

$$\text{Prob}\,[D \leq Q^{*}]\,(\text{critical fractile}) = \frac{\$45}{\$45 + \$60} \approx 0.429$$

Optimal Solution for SnowTime

- The optimal production quantity n^* satisfies

$$\Pr(\text{Demand } \leq n^*) = \frac{C_u}{C_u + C_o} \approx 0.429$$

Demand	Probability	F(D)
8,000	0.11	0.11
10,000	0.11	0.22
12,000	0.28	0.50
14,000	0.22	0.72
16,000	0.18	0.90
18,000	0.10	1

Round-up rule ⇒ choose the larger one.

$Q^* = 12000$

Service Level

$$Prob[D \leq Q^*] = \frac{c_u}{c_u + c_o}$$

What is the probability that SnowTime does not experience a shortage (in other words, every customer who comes for a jacket is able to get a jacket) ?

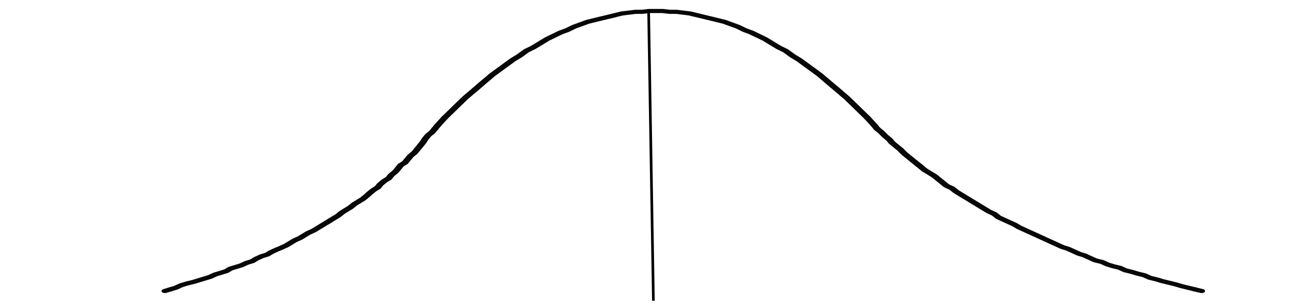
Service level

- Satisfy a customer service objective: a minimum in-stock probability.
- For a normal demand,
  a) Find the z value in the standard normal distribution function table such that $\phi(z) =$ in stock probability
     - Remember to round up!
  b) Convert z to the optimal $Q^* = \mu + z \times \sigma$
- For a discrete distribution function table, fin the order quantity such that $F(Q)$= in stock probability.

Properties of the Optimal Solution

$$Prob[D \leq Q^*] = \frac{c_u}{c_u + c_o} = \text{critical fractile}$$

Situation	Underage Cost ↑	Overage Cost ↑
Inventory Level	Increase	Decrease



$\overline{D}$, $Prob[D \leq \overline{D}] = 0.5$

Situation	$C_u < C_o$	$C_u > C_o$
Critical Fractile	<0.5	>0.5

Back to Mr. Choi's problem

- Mr. Choi operates a news-stand and sells SCMP
  - orders copies of the newspaper from the publisher at a cost of $4 per copy
  - sells at a retailing price of $7
- The demand can be described with a normal distribution with mean 100 and standard deviation 12.

How much should Mr. Choi order if he wants to ensure a minimum service level of 70%?

$$Z = 0.52$$

What if Mr. Choi wants to ensure a minimum service level of 10%?

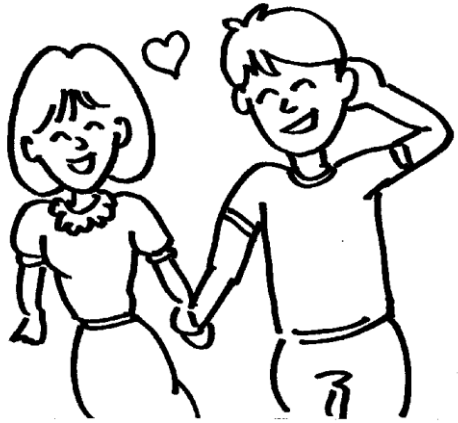
$$Z = -1.28$$

When to leave for a date?

You have a date at 6:00 p.m. You estimate that the average travel time from campus is 30 minutes but there is some variation due to traffic congestion during the rush hour. When should you set off from campus?

- *The travel time is normaly distribution with mean 30 minutes and standard deviation 10 minutes.*

(a) Leave at 5:30 pm
(b) Leave before 5:30 pm
(c) Leave after 5:30 pm
(d) Cannot be determined from the information given



Although it is difficult to quantify the damage of each minute you are late, you feel that **it would be 10 times more costly to be later than earlier**.

Summary

- Newsvendor Model
  - Probabilistic model
  - Perishable (short life cycle) product
  - Single selling period; no replenishment opportunities
  - The optimal order quantity is jointly decided by probability and profit margin
  - The marginal overage and underage costs are equal

$$Prob[D \leq Q^*] = \frac{c_u}{c_u + c_o} = \text{critical fractile}$$

Newsvendor Model Review

- Underage cost C_u
  - Marginal benefit when an additional unit is sold
- Overage cost C_o
  - Marginal cost when an additional unit is not sold

$$\Pr(\text{Demand} \le n^*) = \frac{C_u}{C_u + C_o} = \text{Critical Fractile}$$

- When demand is $N(\mu, \sigma^2)$

$$n^* = \mu + z^* \times \sigma$$

$$z^* = \text{normsinv}\left(\frac{C_u}{C_u + C_o}\right)$$

Service Level Requirement

- **Service level** is the probability of no shortage

- What is the order quantity n that guarantees the service level of S?

$$\Pr(\text{Demand} \leq n) = \text{S}$$

- When demand is $N(\mu, \sigma^2)$

$$n = \mu + z\sigma$$

$$z = \text{normsinv}(S)$$