Practice Set 5 Inventory Management Answer

Question 1

A produce distributor uses 800 packing crates a month, which it purchases at a cost of $10 each. The manager has assigned as annual unit carrying cost of 35% of the purchase price per crate. Ordering cost is $28. Answer the following questions.

a) Currently the manager orders once a month. It takes one week (1 month = 4 weeks) for the order to arrive. What is the ordering policy, that is, order-size and reorder point?

*D = 800 per month = 12*800 per year = 9600 crates per year*
*H = 0.35*10 = $ 3.50 per crate per year*
S= $28
Current Policy: Manager orders once a month, i.e. cycle time T = 1 month
*Order- size: Q = D*T = 800 per month * 1 month = 800*
*Reorder Point: ROP = (800/4) crates per week * 1 week = 200*

b) What is the ordering and holding cost corresponding to order- size in (a)?

*Ordering + Holding Cost per year = (Q/2)*H+(D/Q)*S = (800/2)*3.50+(9600/800)*28 = $1736 per year*

c) What will be the optimal ordering policy?

Optimal Policy: Order EOQ = $\sqrt{(2DS/H)}$ *= 391.92*
*Reorder Point does not change = (800/4) crates per week * 1 week = 200*

d) What will be the annual cost saving of using optimal policy over current policy?

*Holding + Ordering Cost per year = (Q/2)*H+(D/Q)*S = (392/2)*3.50 + (9600/392)*28 = $1371.71 per year*
Savings: 1736-1371.71 = $364.29 per year

Question 2

HSBC has 2 ATMs located at different places on HKUST campus. The demand for cash from each ATM is a constant of $2,000 per day. It costs HSBC $50 to have a private security company go to each ATM and stock it with cash. The cost of capital that HSBC estimates for money is an annual rate of 15%.

a) What is the optimal amount of cash to be placed in each ATM during each trip to minimize total cost?

$$Q^* = \sqrt{2DS/H} = \sqrt{\frac{2(2000*365)50}{0.15*1}} = 22060.5$$

Note that h is 15% of the acquisition cost of a flow unit, which is $1 in this case.

b) How often should the ATM be replenished?

D/Q = 2000*365/22060.5 ≈33 times per year*

c) What will be the average amount of cash in the ATM?
$Q^*/2$ = 11030.25

d) A smart student suggests that HSBC shut down the 2 ATM's and have one new location that is easily accessible to all customers who use ATM's so there's no loss of demand. The new ATM still costs HSBC $50 to fill up, and the cost of capital remains at the same rate.

1) What is the optimal amount of cash to be placed in the new ATM during each trip?

$$Q^* = \sqrt{2DS/H} = \sqrt{\frac{2(4000*365)50}{0.15*1}} = 31198.3$$

2) How often should the ATM be replenished?
$D/Q^* = 4000*365/31198.3 \approx 47$

3) What will be the average amount of cash in the ATM?
$Q^*/2$ = 15599.15

e) What is the impact of the suggestion in part (d) on the holding and set-up cost?

$$Cost(Q^*)_{new} = \sqrt{2DSH} = \sqrt{2*(4000*365)*50*(0.15*1)} = 4679.74$$

$$Cost(Q^*)_{old} = 2\sqrt{2DSH} = 2\sqrt{2*(2000*365)*50*(0.15*1)} = 6618.16$$

Under the new system, total cost decrease by $1938.42.

Question 3

A newspaper has 500,000 subscribers who pay $4 per month for the paper. It costs the company $200,000 to bill its customers. Assume that the company can earn interest at a rate of 20% per year on all revenues. Determine how often the newspaper should bill its customers. (Hint: look at unpaid subscriptions as the inventoried good.

*D= $500,000*4*12=$24,000,000*
S= $200,000
*H= 0.2*1=0.2*

$$Q^* = \sqrt{2DS/H} = 6928203.23$$

Billing frequency = D/EOQ = 3.46 per year

Question 4

University of Florida football programs are printed 1 week prior to each home game. Attendance averages 90,000 screaming and loyal Gators fans, of whom two-thirds usually buy the program, following a normal distribution, for $4 each. Unsold programs are sent to a recycling center that pays only 10 cents per program. The standard deviation is 5,000 programs, and the cost to print each program is $1.

e) What is the cost of underestimating demand for each program?
C_u = $4 – $1 = $3

f) What is the overage cost per program?
C_o = $1 – $0.1 = $0.9

g) How many programs should be ordered per game?
$P = C_u / (C_u + C_o) = 3/(3 + 0.9) = 0.7692$
$F(0.73) = 0.7673$, $F(0.74) = 0.7704$, $z = 0.74$
Hence, $Q^* = \mu + z\sigma = 90000*2/3 + 0.74*5000 = 63{,}700$
(Only two-thirds of the fans would buy the program on average)

h) What is the stockout risk for this order size?
$P = Prob(D \leq Q^*)$, which is the probability of no stock-out and demand can be fulfilled
Hence, stockout risk = 1 – 0.7692 = 23.08%

Question 5

The local supermarket buys lettuce each day to ensure fresh produce. Each morning any lettuce that is left from the previous day is sold to a dealer that resells it to farmers who use it to feed their animals. This week the supermarket can buy fresh lettuce for $4.00 a box. The lettuce is sold for $10.00 a box and the dealer that sold lettuce is willing to pay $1.50 for a box of unsold lettuce at the end of the day. Past history says that demand for lettuce is normally distributed with a mean of 250 boxes with a standard deviation of 34 boxes. How many boxes of lettuce should the supermarket purchase?

$C_u = \$10 – \$4 = \$6$
$C_o = \$4 – \$1.5 = \$2.5$
$P = C_u/(C_u + C_o) = 6/(6 + 2.5) = 0.7059$
Hence, $z = 0.54$
$Q^* = \mu + z\sigma = 250 + 0.54*34 = 268.36$
Hence, purchase 268 boxes of lettuce

Question 6

Tom owns a small firm that manufactures "Tom Sunglasses." He has the opportunity to sell a particular seasonal model to Land's End. Tom offers Land's End two purchasing options:

- Option 1. Tom offers a price of $55 for each unit, but returns are no longer accepted. In this case, Land's End throws out unsold units at the end of the season.
- Option 2. Tom offers to set his price at $65 and agrees to credit Land's End $53 for each unit Land's End returns to Tom at the end of the season.

This season's demand for this model will be normally distributed with mean of 200 and standard deviation of 125. Land's End will sell those sunglasses for $110 each.

a) How much would Land's End buy if they choose option 1?
$C_u = \$110 – \$55 = \$55$
$C_o = \$55$
$P = C_u /(C_u + C_o) = 55/(55 + 55) = 1/2$
$z = 0$
$Q^* = \mu + z\sigma = 200 + 0*125 = 200$

b) How much would Land's End buy if they choose option 2? What is the probability that Land's End will return sunglasses to Tom at the end of the season?
$C_u = \$110 – \$65 = \$45$
$C_o = \$65 – \$53 = \$12$
$P = C_u /(C_o + C_u) = 45/(45 + 12) = 0.7894$
$F(0.80) = 0.7881$, $F(0.81) = 0.7910$. Hence, $z = 0.805$
$Q^* = \mu + z\sigma = 200 + 0.805*125 = 300.625$
Hence, purchase 301 sunglasses.

Prob(Land's End will return sunglasses) = Prob(D ≤ Q) = 0.7894*

Question 7

Dan McClure owns a thriving independent bookstore in artsy New Hope, Pennsylvania. He must decide how many copies to order of a new book, Power and Self-Destruction. The book's retail price is $20 and the wholesale prices is $12. The publisher will buy back the retailer's leftover copies at full refund, but McClure Books incurs $4 in shipping and handling costs for each book returned to the publisher. Dan believes his demand forecast can be represented by a normal distribution with mean 200 and standard deviation 80.

What would be the optimal order quantity for Dan?

$C_u = \$20 - \$12 = \$8$

$C_o = \$12 - (\$12 - \$4) = \$4$

(The salvage value is $8)

$P = C_u / (C_u + C_o) = 8/(8 + 4) = 0.667$

$F(0.43) = 0.6664$, hence, $z = 0.43$

$Q^* = 200 + 0.43*80 = 234.4$

Hence, order 234 copies

Question 8

A retailer sells a fashion product during a short sales season. Since there is a long production lead time, the retailer needs to purchase the product in advance and no further inventory replenishment is allowed. The product costs $70 per unit and the retail price is $100. For units that are not sold by the end of the main sales season, the retailer can sell the leftover units at a discounted price $30 through clearance sales. The demand is uncertain and the demand distribution is forecasted as follows.

Demand (units)	500	350	250	150
Probability	0.2	0.4	0.25	0.15

a) What is the underage cost?
   $C_u = \$100 - \$70 = \$30$

b) What is the overage cost?
   $C_o = \$70 - \$30 = \$40$

c) How many units should the retailer purchase in order to maximize the expected profit?
   $P = 30/(30 + 40) = 0.4286$
   Next, compute the cumulative probability and find closest one that is greater than the critical fractile, i.e. 0.8

Demand	*Probability*	*Cumulative Probability*
150	0.15	*0.15*
250	0.25	*0.40*
350	0.40	*0.80*
500	0.20	*1.00*

Q is 350*

Question 9

A product is priced to sell at $100 per unit, and its cost is constant at $70 per unit. Each unsold unit has a salvage value of $20. Demand is expected to range between 30 and 40 units for the period. The demand probabilities and the associated cumulative probability distribution (P) for this situation are shown below.

# of units demanded	*Probability*	*Cumulative probability*
35	0.10	0.10
36	0.15	0.25
37	0.25	0.50
38	0.25	0.75
39	0.15	0.90
40	0.10	1.00

How many units should be ordered?

C_u = $100 - $70 = $30

C_o = $70 - $20 = $50

$P = C_u / (C_u + C_o) = 30/ (30 + 50) = 0.375$

From the table, we will look for the cumulative probability and find closest one that is greater than the critical fractile, i.e. 0.5

Q is 37 units.*

To illustrate the problem, here is a full marginal analysis. Note that the cost is minimized when 37 units are purchased.

		# purchased					
# demanded	*Probability*	*35*	*36*	*37*	*38*	*39*	*40*
35	*0.1*	*0*	*50*	*100*	*150*	*200*	*250*
36	*0.15*	*30*	*0*	*50*	*100*	*150*	*200*
37	*0.25*	*60*	*30*	*0*	*50*	*100*	*150*
38	*0.25*	*90*	*60*	*30*	*0*	*50*	*100*
39	*0.15*	*120*	*90*	*60*	*30*	*0*	*50*
40	*0.1*	*150*	*120*	*90*	*60*	*30*	*0*
Expected total cost		*75*	*53*	*43*	*53*	*83*	*125*

Question 9

Sally's Silk Screening produces specialty T-shirts that are primarily sold at special events. She is trying to decide how many to produce for an upcoming event. During the event itself, which lasts one day, Sally can sell T-shirts for $20 a piece. However, when the event ends, any unsold T-shirts are sold for $4 a piece. It costs Sally $8 to make a specialty T-shirt. Using Sally's estimate of demand that follows, how many T-shirts should she produce for the upcoming event?

Demand	*Probability*
300	0.05
400	0.10
500	0.40
600	0.30
700	0.10
800	0.05

C_u = $20 - $8 = $12

C_o = $8 - $4 = $4

$P = C_u / (C_u + C_o) = 12/ (12 + 4) = 0.75$

Next, compute the cumulative probability and find closest one that is greater than the critical fractile, i.e. 0.85

Demand	*Probability*	*Cumulative probability*
300	0.05	*0.05*
400	0.10	*0.15*
500	0.40	*0.55*
600	0.30	*0.85*
700	0.10	*0.95*
800	0.05	*1.00*

Q is 600 units.*

Question 10

Famous Albert prides himself on being the Cookie King of the West. Small, freshly baked cookies are the specialty of his shop. Famous Albert has asked for help to determine the number of cookies he should make each day. From analysis of past demand he estimates demand for cookies as the following.

Demand	*Probability of demand*
1,800	0.05
2,000	0.10
2,200	0.20
2,400	0.30
2,600	0.20
2,800	0.10
3,000	0.05

Each dozen sells for \$0.69 and costs \$0.49, which includes handling and transportation. Cookies that are not sold at the end of the day are reduced to \$0.29 and sold the following day as day-old merchandise. What is the optimal number of cookies to make?

$C_u = \$0.69 – \$0.49 = \$0.2$

$C_o = \$0.49 – \$0.29 = \$0.2$

$P = C_u/ (C_u + C_o) = 0.2/ (0.2 + 0.2) = 0.5$

Next, compute the cumulative probability and find closest one that is greater than the critical fractile, i.e. 0.65

Demand	*Probability of demand*	*Cumulative probability*
1,800	0.05	*0.05*
2,000	0.10	*0.15*
2,200	0.20	*0.35*
2,400	0.30	*0.65*
2,600	0.20	*0.85*
2,800	0.10	*0.95*
3,000	0.05	*1.00*

Q is 2400 units*