Faculty of Science, Technology, Engineering
and Mathematics
MST125 Essential Mathematics 2

MST125

TMA03 2025B

Covers Units 9, 10 and 11 **Cut-off date 13 August 2025**

You should submit this TMA electronically as a PDF file by using the University's online TMA/EMA service. Before starting work on it, please read the guidance for preparing and submitting TMAs, available from the 'Assessment' tab of the module website.

The work that you submit should include your working as well as your final answers.

You are expected to use the methods and results taught in MST125 rather than any alternative methods or results not covered in MST125.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not include the use of any other mathematical software. If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

Your work should be written in good mathematical style, as demonstrated by the example and activity solutions in the study units. You should explain your solutions carefully, using appropriate notation and terminology, and write in sentences. As usual, you should simplify algebraic answers where possible. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 9. You do not have to submit any work for Question 9.

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PLAGIARISM WARNING – the use of assessment help services and websites

The **work that you submit for any assessment/examination on any module should be your own**. Submitting work produced by or with another person, or a web service or an automated system, **as if it is your own** is cheating. It is **strictly forbidden** by the University.

You should not:

- provide any assessment question to a website, online service, social media platform or any individual or organisation, as this is an infringement of copyright
- request answers or solutions to an assessment question on any website, via an online service or social media platform, or from any individual or organisation
- use an automated system (other than one prescribed by the module) to obtain answers or solutions to an assessment question and submit the output as your own work
- discuss examination questions with any other person, including your tutor.

The University actively monitors websites, online services and social media platforms for answers and solutions to assessment questions, and for assessment questions posted by students. Work submitted by students for assessment is also monitored for plagiarism.

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The Open University's Academic Conduct Policy defines plagiarism in part as:

- using text obtained from assignment writing sites, organisations or private individuals
- obtaining work from other sources and submitting it as your own.

If it is found that you have used the services of a website, online service or social media platform, or that you have otherwise obtained the work you submit from another person, this is considered serious academic misconduct and you will be referred to the Central Disciplinary Committee for investigation.

Question 1 – 5 marks

You should be able to answer this question after studying Unit 9.

(a) You are told that the following two statements are true:

If it is Robin's birthday, then Robin eats cake.
Robin is eating cake.

Can you deduce from the statements above that the following statement is true?

It is Robin's birthday.

Justify your answer. [*3*]

(b) Show that the following statement is false by giving a counter-example:

For all positive integers n, we have $5^n \leq (n+1)^6$. [*2*]

Question 2 – 10 marks

You should be able to answer this question after studying Unit 9.

(a) Prove that the following statement is true for all real numbers $x \neq \frac{4}{3}$ and $y \neq \frac{4}{3}$, by using a sequence of equivalences:

$$y = \frac{4x-1}{3x-4} \quad \text{if and only if} \quad x = \frac{4y-1}{3y-4}.$$ [*3*]

(b) Prove that the following statement is true for all integers n:

$n+1$ is even if and only if $2(n+3)$ is a multiple of 4. [*7*]

Question 3 – 10 marks

You should be able to answer this question after studying Unit 9.

Use mathematical induction to prove that the following statement is true:

$(3n)! \geq (n!)^3$, for all $n \in \mathbb{N}$. [*10*]

Question 4 – 10 marks

You should be able to answer this question after studying Unit 9.

(a) Use proof by contradiction to show that there is no positive real number x such that

$$\frac{7x}{x+3} \leq \frac{x-3}{7x}.$$ [*4*]

(b) Use proof by contraposition to prove that the following statement is true for all positive integers n:

If $n^3 + 2n^2$ is not a multiple of 16, then n is odd. [*6*]

Hint: If k is an integer, then $k(k+1)$ is even, since one of k and $k+1$ is even.

Question 5 – 5 marks

You should be able to answer this question after studying Unit 10.

A bolt becomes detached from a panel on the side of a tower 300 m above the ground, and falls from rest in a straight, vertical line under the influence of gravity alone until it hits the ground. Take the magnitude of the acceleration due to gravity to be $g = 9.8\,\mathrm{m\,s^{-2}}$. Give numerical answers to two significant figures.

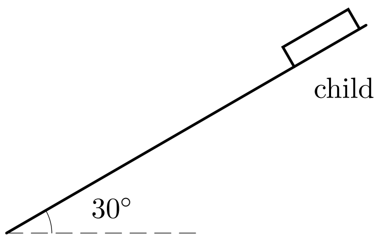
(a) Calculate the time taken by the bolt to hit the ground, and the final speed that it reaches before hitting the ground. [*3*]

(b) Draw a graph of the velocity v (in metres per second) of the bolt against the time t (in seconds), from the time at which the bolt starts falling until the instant before it hits the ground. [*2*]

Question 6 – 15 marks

You should be able to answer this question after studying Unit 10.

A water slide of length 30 m makes an angle of 30° with the horizontal. A child starts sliding from rest at the top of the slide, as shown in the diagram below.



The child has constant acceleration and reaches a speed of $9\,\mathrm{m\,s^{-1}}$ at the bottom of the slide. Let the mass of the child be m (in kg) and the coefficient of sliding friction between the child and the water slide be μ.

Model the child as a particle, and take the magnitude of the acceleration due to gravity to be $g = 9.8\,\mathrm{m\,s^{-2}}$.

(a) Draw a force diagram showing the three forces acting on the child when they are sliding down the water slide. Define the symbols that you use to denote the forces. Indicate the directions of the forces by marking suitable angles on your diagram. Choose the Cartesian unit vector $\mathbf{i}$ to be parallel to and pointing up the slide and the Cartesian unit vector $\mathbf{j}$ to be perpendicular to the slide and pointing upwards. Mark $\mathbf{i}$ and $\mathbf{j}$ on your diagram. [*3*]

(b) Show that the magnitude of the acceleration of the child is $1.35\,\mathrm{m\,s^{-2}}$. [*3*]

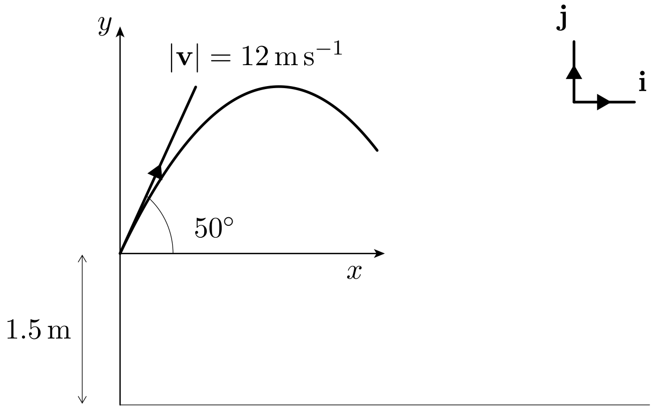
(c) State the forces from part (a) in component form and hence find the value of μ to two significant figures. [*9*]

Question 7 – 15 marks

You should be able to answer this question after studying Unit 10.

A rounders player uses their bat to hit a ball at an angle of 50° to the horizontal and at a speed of $12\,\mathrm{m\,s^{-1}}$. The ball is at a height of $1.5\,\mathrm{m}$ above the ground when it leaves the player's bat.

Set up coordinate axes at the point where the ball leaves the bat. Let $\mathbf{i}$ and $\mathbf{j}$ be the Cartesian unit vectors in the positive directions of the x- and y-axes respectively, as shown in the following diagram.



Model the ball as a particle and assume that the only force acting on it is its weight. Take the magnitude of the acceleration due to gravity to be $g = 9.8\,\mathrm{m\,s^{-2}}$.

Give numerical answers to two significant figures.

(a) Write down a vector expression for the acceleration $\mathbf{a}$ of the ball. [*1*]

(b) Starting with your expression for $\mathbf{a}$ show, by integrating twice, that the position $\mathbf{r}$ (in metres) of the ball t seconds after leaving the player's bat is given by

$$\mathbf{r} = 12t\cos 50^\circ\,\mathbf{i} + \left(12t\sin 50^\circ - \tfrac{1}{2}gt^2\right)\mathbf{j}.$$ [*7*]

(c) (i) Find the time that it takes for the ball to reach the ground. [*5*]

(ii) Determine the horizontal distance between the point at which the ball leaves the player's bat and the point where it reaches the ground. [*2*]

Question 8 – 25 marks

You should be able to answer this question after studying Unit 11.

Let $\mathbf{A} = \begin{pmatrix} 5 & 6 \\ 18 & 2 \end{pmatrix}$.

(a) Find the eigenvalues of $\mathbf{A}$ and, for each eigenvalue, find a corresponding eigenvector of the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where a is a positive number. [8]

(b) Hence express $\mathbf{A}$ in the form $\mathbf{PDP}^{-1}$, where $\mathbf{P}$ is an invertible matrix and $\mathbf{D}$ is a diagonal matrix, stating the matrices $\mathbf{P}$, $\mathbf{P}^{-1}$ and $\mathbf{D}$. [4]

(c) Use your answer to part (b) to calculate $\mathbf{A}^5$. [4]

(d) Use Maxima to check your answers to parts (a) and (c). Include a printout or screenshot of the Maxima output in your answer, and explain carefully how to interpret it. [5]

(e) Use your answer to part (a) to find the general solution of the following system of linear differential equations, writing the solution as an equation for x and an equation for y:

$$\begin{aligned} \dot{x} &= 5x + 6y, \\ \dot{y} &= 18x + 2y. \end{aligned}$$ [4]

Question 9 – 5 marks

Five marks on this assignment are allocated for good mathematical communication in Questions 1 to 8.

You do not have to submit any extra work for Question 9, but you are advised to check through your assignment carefully, making sure that you have explained your working clearly, used notation correctly, written in sentences and rounded answers as requested. [5]