Faculty of Science, Technology, Engineering and Mathematics
MST125 Essential mathematics 2

MST125

TMA 01 2025B

Covers Units 2, 3, 4 and 5 **Cut-off date 16 April 2025**

You should submit this TMA electronically as a PDF file by using the University's online TMA/EMA service. Before starting work on it, please read the guidance for preparing and submitting TMAs, available from the 'Assessment' tab of the module website.

The work that you submit should include your working as well as your final answers.

You are expected to use the methods and results taught in MST125 rather than any alternative methods or results not covered in MST125.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not include the use of any other mathematical software. If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

Your work should be written in a good mathematical style, as demonstrated by the example and activity solutions in the study units. You should explain your solutions carefully, using appropriate notation and terminology, and write in sentences. As usual, you should simplify algebraic answers where possible. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 8. You do not have to submit any work for Question 8.

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PLAGIARISM WARNING – the use of assessment help services and websites

The **work that you submit for any assessment/examination on any module should be your own**. Submitting work produced by or with another person, or a web service or an automated system, **as if it is your own** is cheating. It is **strictly forbidden** by the University.

You should not:

- provide any assessment question to a website, online service, social media platform or any individual or organisation, as this is an infringement of copyright
- request answers or solutions to an assessment question on any website, via an online service or social media platform, or from any individual or organisation
- use an automated system (other than one prescribed by the module) to obtain answers or solutions to an assessment question and submit the output as your own work
- discuss examination questions with any other person, including your tutor.

The University actively monitors websites, online services and social media platforms for answers and solutions to assessment questions, and for assessment questions posted by students. Work submitted by students for assessment is also monitored for plagiarism.

A student who is found to have posted a question or answer to a website, online service or social media platform and/or to have used any resulting, or otherwise obtained, output as if it is their own work has committed a disciplinary offence under our Code of Practice for Student Discipline. **This means the academic reputation and integrity of the University has been undermined.**

The Open University's Academic Conduct Policy defines plagiarism in part as:

- using text obtained from assignment writing sites, organisations or private individuals
- obtaining work from other sources and submitting it as your own.

If it is found that you have used the services of a website, online service or social media platform, or that you have otherwise obtained the work you submit from another person, this is considered serious academic misconduct and you will be referred to the Central Disciplinary Committee for investigation.

Question 1 – 20 marks

You should be able to answer this question after studying Unit 2.

The purpose of this question is to allow you to demonstrate your mathematical typesetting skills. Therefore you *must* typeset your work for parts (a) and (b) of this question using one of LaTeX, a recent version of Word or LibreOffice.

If you are not typesetting the rest of the TMA, then please do *one* of the following.

(i) Print out your response to Question 1, scan it with the rest of your TMA, and save the scan as a single PDF file.

(ii) Combine the PDF file for Question 1 with the PDF file of the rest of the TMA into a single PDF file

Guidance on submitting single PDF files using the online TMA/EMA service is provided in the document *Student guidance for preparing and submitting TMAs* available via the 'TMA presentation and submission' link in the 'Assessment' area of the MST125 website.

(a) On the first line, type the heading 'MST125 TMA 01 Question 1', making it centred and bold.

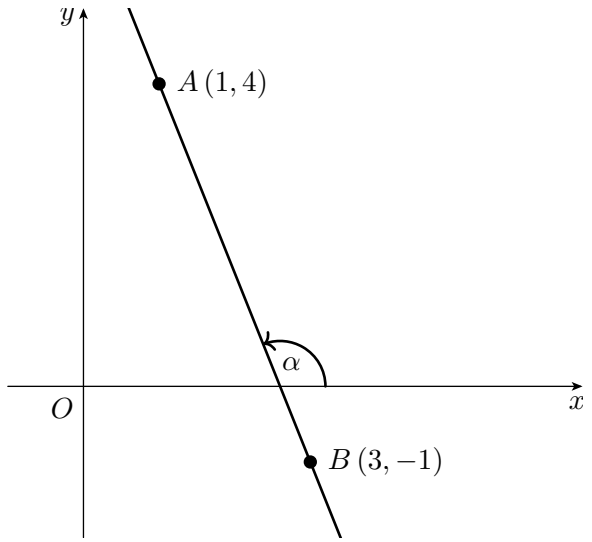
On the second line, type and centre your name and personal identifier.

On the third line, type and centre the name of the typesetting software that you are using. [*2*]

(b) A student is working on the following problem.

(Note that you are not asked to provide a solution to this problem.)

The diagram below shows the straight line through the points $A\,(1, 4)$ and $B\,(3, -1)$. Let α be the obtuse angle that this line makes with the positive direction of the x-axis, as shown. The scales on the two axes are equal.



1. Calculate the distance AB.
2. Calculate the gradient of the line through A and B.
3. Find the angle α in radians to two decimal places.

The student has produced the draft solution shown below.

1. The distance between $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Hence the distance between $A(1, 4)$ and $B(3, -1)$ is

$$AB = \sqrt{(3 - 1)^2 + (-1 - 4)^2}$$
$$= \sqrt{2^2 + (-5)^2}$$
$$= \sqrt{4 + 25}$$
$$= \sqrt{29}.$$

2. The gradient of the line through (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Hence the gradient of the line through $(1, 4)$ and $(3, -1)$ is

$$m = \frac{(-1) - 4}{3 - 1} = -\frac{5}{2}.$$

3. The gradient of the line is $-\frac{5}{2}$. Hence $\tan\alpha = -\frac{5}{2}$.

Let ϕ be the acute angle that the line makes with the negative direction of the x-axis. Then

$$\tan\phi = \frac{5}{2},$$

so

$$\phi = \tan^{-1}\left(\frac{5}{2}\right) = 1.190\ldots.$$

Hence

$$\alpha = \pi - 1.190\ldots = 1.951\ldots.$$

Therefore the angle α is 1.95 radians (to 2 d.p.).

Typeset this draft solution, using the same content and a similar layout. Take care to centre or indent each equation that is displayed on a line by itself, and to align equals signs where appropriate.

Marks will be awarded for

(i) layout and accuracy [9]

(ii) mathematical notation. [7]

(c) Do you intend to typeset your TMA solutions? Justify your decision briefly. [2]

Question 2 – 10 marks

You should be able to answer this question after studying Unit 3.

(a) Find the least residue of 405^{10} modulo 16. [*3*]

(b) Use Fermat's little theorem to find the least residue of 13^{494} modulo 83. [*3*]

(c) Let p be a prime number with $p \neq 13$. Use Fermat's little theorem to find a positive integer b such that $13^{6p-4} \equiv b \pmod{p}$. [*4*]

Question 3 – 15 marks

You should be able to answer this question after studying Unit 3.

(a) (i) Use Euclid's algorithm to find a multiplicative inverse of 29 modulo 80, and hence solve the linear congruence

$$29x \equiv 41 \pmod{80}.$$ [*5*]

(ii) Explain why the following linear congruence has no solutions:

$$12x \equiv 16 \pmod{54}.$$ [*2*]

(iii) Solve the linear congruence

$$12x \equiv 18 \pmod{54}.$$ [*4*]

(b) The affine cipher E is given by

$$E(x) \equiv 19x - 5 \pmod{26}.$$

The conversion table for letters and numbers is shown below.

A 0	B 1	C 2	D 3	E 4	F 5	G 6	H 7	I 8	J 9	K 10	L 11	M 12
N 13	O 14	P 15	Q 16	R 17	S 18	T 19	U 20	V 21	W 22	X 23	Y 24	Z 25

(i) Show that 11 is a multiplicative inverse of 19 modulo 26. [*1*]

(ii) Suppose that you receive the enciphered message

14, 11, 5.

Decipher the message and deduce the word that was sent. [*3*]

Question 4 – 15 marks

You should be able to answer this question after studying Unit 4.

(a) An ellipse in standard position has equation

$$4x^2 + 25y^2 - 9 = 0.$$

(i) Find the vertices of the ellipse. [*2*]

(ii) Find the eccentricity, foci and directrices of the ellipse. [*3*]

(iii) Sketch the ellipse, labelling its vertices with their coordinates. Include the foci and directrices on the sketch, clearly labelling them. [*3*]

(iv) Find a parametrisation of the part of this ellipse that lies in the third quadrant (not including any points that lie on the x-axis or y-axis). Make sure that you include the restrictions that are needed on the values of the parameters. [*2*]

(v) Suppose that the part of the ellipse that lies in the third quadrant is translated 4 units to the right and 2 units down. Give a parameterisation of this translated curve. [*1*]

(b) (i) State your personal identifier (PI) and let B be the last non-zero numeric character in your PI. For example, if your PI is F1234567 then $B = 7$ and if your PI is G123450X then $B = 5$.
The equation

$$-2x^2 + Bxy + 3y^2 - x + 3y - 5 = 0$$

represents a non-degenerate conic. Determine the type of the conic corresponding to your value of B. [*2*]

(ii) Plot your conic using Maxima, and provide a screenshot of your plot, including the command that you used to produce the plot. [*2*]

Question 5 – 10 marks

You should be able to answer this question after studying Unit 4.

Two objects, A and B, are moving along two different straight lines at constant speeds. With reference to a particular coordinate system in which distance is measured in metres, at time t (in minutes) the position of A is $(t-3, 3t+8)$ and the position of B is $(2t-5, 4t+5)$.

(a) Find the equation of the line along which A moves. [2]

(b) Let d be the distance between A and B at time t. Show that an expression for d^2 in terms of t is given by

$$d^2 = 2t^2 - 10t + 13.$$ [3]

(c) A student attempts to find the shortest distance between A and B as shown below. There are two lines where the working does not follow on from the previous line.

$$\begin{aligned} d^2 &= 2t^2 - 10t + 13 \\ &= 2\left(t^2 - 5t\right) + 13 \\ &= 2\left(t - \frac{5}{2}\right)^2 - \frac{25}{4} + 13 \\ &= 2\left(t - \frac{5}{2}\right)^2 + \frac{27}{4}. \end{aligned}$$

The minimum value of d^2 occurs when $t = \frac{5}{2} = 2.5$. Hence the minimum distance is $\frac{27}{4}$ m which is 6.75 m.

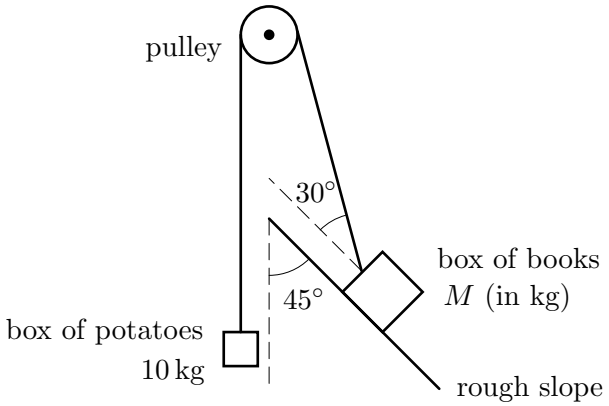
Find and describe the two mistakes.

Write out a correct solution, stating when the minimum value of d^2 occurs and the shortest distance between A and B. Give any non-exact answers to two significant figures. [5]

Question 6 – 20 marks

You should be able to answer this question after studying Unit 5.

A box of books of mass M (in kg) rests on a rough slope that makes an angle of 45° with the vertical. A rope is attached at one end to the box of books and passes over a pulley located above the top of the slope. The section of the rope between the box of books and the pulley makes an angle of 30° with the slope. The other end of the rope is attached to a box of potatoes of mass 10 kg which hangs freely. Both the box of potatoes and the box of books are in equilibrium.



Model the box of potatoes and the box of books as particles, the rope as a model string and the pulley as a model pulley. Let g denote the magnitude of the acceleration due to gravity (in $\mathrm{m\,s^{-2}}$).

Ensure that you underline all vectors if handwriting your work, or that you use bold for vectors if typesetting.

(a) (i) State the two forces that act on the box of potatoes and draw a force diagram to represent them. Label each vector on your force diagram and define the symbols that you use.
In this part of the question, we label the Cartesian unit vectors as $\mathbf{i}'$ and $\mathbf{j}'$. Take the vector $\mathbf{j}'$ to point vertically up. Draw this vector on your diagram. [3]

(ii) Express both of the forces acting on the box of potatoes in component form, using unknown magnitudes where appropriate. Hence find both forces in component form in terms of g. [3]

(b) (i) State all the forces that act on the box of books, and draw a force diagram to represent them. Indicate the directions of the forces by marking the sizes of appropriate angles on your diagram. Label each vector on your diagram and define the symbols that you use.
In this part of the question, we label the Cartesian unit vectors as $\mathbf{i}$ and $\mathbf{j}$. Take $\mathbf{i}$ and $\mathbf{j}$ to point parallel to the slope downwards and perpendicular to the slope upwards, respectively. Draw these vectors on your diagram. [4]

(ii) Express all of the forces acting on the box of books in component form, using unknown quantities where appropriate. Find these unknown quantities in terms of M and g, and hence give all forces in component form in terms of M and g. [6]

(iii) Taking the magnitude of the acceleration due to gravity to be $g = 9.8\,\mathrm{m\,s^{-2}}$, find the magnitude and direction of the friction force if $M = 12$ kg and if $M = 13$ kg. [4]

Question 7 – 5 marks

You should be able to answer this question after studying Unit 5.

Unit 5 introduces some key ideas in applied mathematics that you will use and build on in Unit 10. So it is important that you develop a good understanding of the ideas in this unit and also practise the techniques. This question is designed to assess your engagement with the Unit 5 practice quiz. This quiz can be found either on the MST125 study planner or in the 'Resources' area of the MST125 website. If you are unable to access it, please contact your tutor for advice.

You should not need to write more than 100 words in part (b) and 150 words in part (c).

(a) Complete the Unit 5 practice quiz and click on the 'Finish attempt ...' button so that your results are shown in the 'Unit 5 Practice quiz Summary of attempt' table. Remember that you may attempt the quiz as often as you wish. Provide a printout or take a screenshot from your computer that shows the 'Summary of attempt' table. [*1*]

(b) Based on your results from the practice quiz and your other work on Unit 5, state the ideas in this unit that you feel you now understand well and also any topics or questions that you still find difficult. [*2*]

(c) What further work (if any) do you plan to do to help your understanding of the ideas in Unit 5?

(If you feel that you do not need to do any further work, you should explain carefully why not, for example, by referring to your results on the practice quiz.) [*2*]

Question 8 – 5 marks

Five marks on this assignment are allocated for good mathematical communication in Questions 2 to 6.

You do not have to submit any extra work for Question 8, but you are advised to check through your assignment carefully, making sure that you have explained your working clearly, used notation correctly, written in sentences and rounded answers as requested. [*5*]
