Given the function

(% i1) f(x):=(3*x+15*x^ 2-x^ 4)/(9*x^ 2+1);

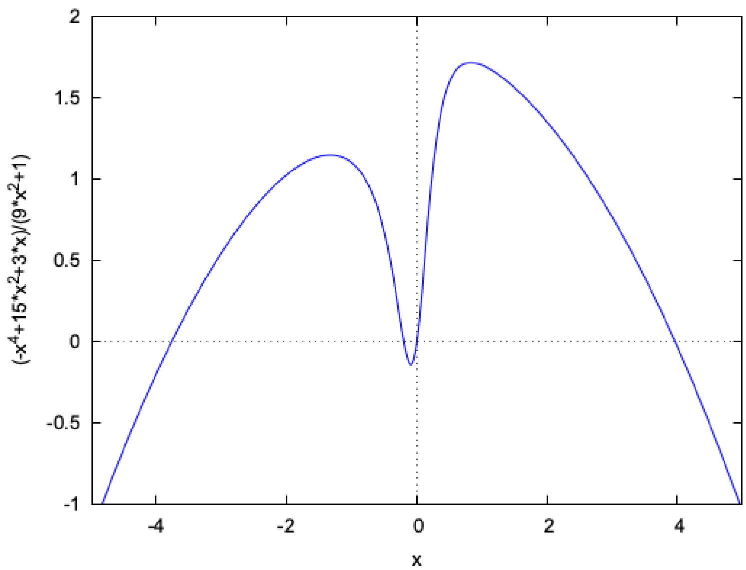
(% o1) $f(x) := \frac{3x + 15x^2 - x^4}{9x^2 + 1}$

(a) The graph of f(x) is

(% i2) wxplot2d(f(x), [x, -5, 5], [y, -1, 2]);

$plot2d : somevalueswillbeclipped.$

(% t2)



(% o2)

(b) the derivative of f(x) is

(% i3) df(x):=''(diff(f(x), x));

(% o3) $df(x) := \frac{-\left(4x^3\right) + 30x + 3}{9x^2 + 1} - \frac{18x\left(-x^4 + 15x^2 + 3x\right)}{\left(9x^2 + 1\right)^2}$

(c) The positive local maximum of f(x) is

(% i4) pos_root:find_root(df(x), x, 0, 5);

(pos_ root) 0.8288158368533624

Substituting this back into the f(x)

(% i5) f(pos_root);

(% o5) 1.71510439942526

Finding the second derivative of f(x)

(% i6) ddf(x):=''(diff(df(x), x));

(% o6)

$$ddf(x) := \frac{30-12x^2}{9x^2+1} - \frac{18\left(-x^4+15x^2+3x\right)}{\left(9x^2+1\right)^2} + \frac{648x^2\left(-x^4+15x^2+3x\right)}{\left(9x^2+1\right)^3} - \frac{36x\ \left(-\left(4x^3\right)+30x+3\right)}{\left(9x^2+1\right)^2}$$

And substitutiong our x variable

(% i7) ddf(0.829);

(% o7) -1.2681397331452517

As this is <0 the point is confirmed to be a local maximum of f(x) Hence the local maximum is at (0.828,1.715), to 3 d.p. (d) To find the root to the right of x=0

(% i8) float(realroots(f(x)));

(% o8)

$$[x=-3.7688187062740326\,,x=-0.20053765177726746\,,x=3.9693563878536224\,,x=0.0]$$

Therefore the grapg crosses the x-axis at 3.969, to 3 d.p. The area under the graph between x=0 and this point is

(% i10) quad_qags(f(x), x, 0, 3.969);

(% o10) $\left[4.342959640603124\,,2.562177785637564210^{-9}\,,105\,,0\right]$

Hence the area enclosed by the graph of f(x) between 0<=x<=3.969 is 4.343, to 3 d.p.