Faculty of Science, Technology, Engineering and Mathematics
MST124 Essential mathematics 1

MST124

TMA 03 2024J

Covers Units 7 and 8 **Cut-off date 12 March 2025**

You will find information about TMAs in the 'Assessment' area of the MST124 website. Please read that information before beginning work on this TMA.

If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

The work that you submit should include your working as well as your final answers.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software.

Your work should be written in a good mathematical style, as described in Section 6 of Unit 1, and as demonstrated by the example and activity solutions in the study units. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 10. You do not have to submit any work for Question 10.

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PLAGIARISM WARNING – the use of assessment help services and websites

The **work that you submit for any assessment/examination on any module should be your own**. Submitting work produced by or with another person, or a web service or an automated system, **as if it is your own** is cheating. It is **strictly forbidden** by the University.

You should not:

- provide any assessment question to a website, online service, social media platform or any individual or organisation, as this is an infringement of copyright
- request answers or solutions to an assessment question on any website, via an online service or social media platform, or from any individual or organisation
- use an automated system (other than one prescribed by the module) to obtain answers or solutions to an assessment question and submit the output as your own work
- discuss examination questions with any other person, including your tutor.

The University actively monitors websites, online services and social media platforms for answers and solutions to assessment questions, and for assessment questions posted by students. Work submitted by students for assessment is also monitored for plagiarism.

A student who is found to have posted a question or answer to a website, online service or social media platform and/or to have used any resulting, or otherwise obtained, output as if it is their own work has committed a disciplinary offence under our Code of Practice for Student Discipline. **This means the academic reputation and integrity of the University has been undermined.**

The Open University's Academic Conduct Policy defines plagiarism in part as:

- using text obtained from assignment writing sites, organisations or private individuals
- obtaining work from other sources and submitting it as your own.

If it is found that you have used the services of a website, online service or social media platform, or that you have otherwise obtained the work you submit from another person, this is considered serious academic misconduct and you will be referred to the Central Disciplinary Committee for investigation.

Question 1 – 15 marks

You should be able to answer this question after studying Unit 7.

Differentiate the following functions, simplifying your answers as far as possible.

(a) $f(x) = (x^5 + 3x^3 + 2x + 1)e^x$ [*3*]

(b) $g(y) = (\ln(y) + \sin(y))^6$ [*3*]

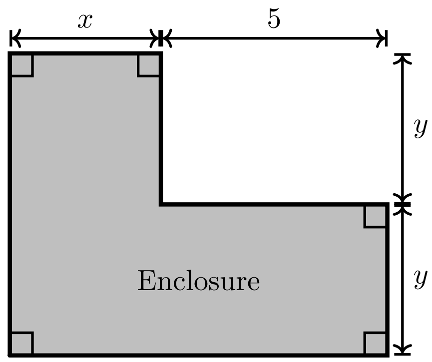
(c) $h(z) = \dfrac{e^{5z}}{(2 + \cos(10z))}$ [*3*]

(d) $k(x) = x^2 \sin(\cos(x))$ [*6*]

Question 2 – 10 marks

You should be able to answer this question after studying Unit 7.

Steven is making an L-shaped enclosure from some fencing. The enclosure is shown on the diagram below with the lengths of the sides of the enclosure given in metres. One side has a fixed length of 5 m but the values of both x and y can be varied. In total Steven has 74 m of fencing to use.



(Not to scale. All distances measured in metres.)

(a) Show that

$$y = \frac{1}{2}(32 - x).$$ [*2*]

(b) Let A denote the area of the enclosure measured in m^2. Show that

$$A = \frac{1}{2}\left(160 + 59x - 2x^2\right).$$ [*2*]

(c) Find the stationary point of A and use the second derivative test to show that it is a maximum. Hence identify the maximum area of the enclosure in m^2. Give your answer as an exact value in the form of a fraction. [*6*]

Question 3 – 10 marks

You should be able to answer this question after studying Unit 7.

For this question you may find the table of 'Standard indefinite integrals' on page 7 of the MST124 Handbook helpful.

Find the indefinite integrals of the following functions, simplifying your answers as far as possible.

(a) $f(x) = x^2 + 2x + 5$ [*2*]

(b) $g(\theta) = 5e^{\theta} + \dfrac{1}{5\theta}$ [*2*]

(c) $h(t) = 2\sin(t) + \dfrac{1}{3+3t^2} + 3$ [*3*]

(d) $j(y) = (y-2)(y^{-1/2}+3)$

Hint for (d): Try expanding the brackets first. [*3*]

Question 4 – 5 marks

You should be able to answer this question after studying Unit 8.

This question is about the function

$$f(x) = -x^2 + 4x + 12.$$

(a) Explain why the graph of $f(x)$ lies above the x-axis for $-2 < x < 6$. [*1*]

(b) Write down a definite integral, that gives the area between the graph of f and the x-axis, from $x = 1$ to $x = 3$. [*1*]

(c) Hence find the area described in part (b), giving the exact answer. [*3*]

Question 5 – 10 marks

You should be able to answer this question after studying Unit 8.

(a) Use integration by substitution to find the indefinite integral

$$\int \frac{\cos(3x) - \sin(3x)}{(\sin(3x) + \cos(3x))^2}\,\mathrm{d}x.$$ [*4*]

(b) Use integration by substitution to evaluate the definite integral

$$\int_0^{\frac{1}{3}\ln 5} e^{3x}\sqrt{e^{3x}+2}\,\mathrm{d}x.$$

Give your answer correct to two decimal places. [*6*]

Question 6 – 10 marks

You should be able to answer this question after studying Unit 8.

(a) Use integration by parts to find the indefinite integral

$$\int 81x^8 \ln(x)\,\mathrm{d}x.$$ [*4*]

(b) Use integration by parts twice to find the indefinite integral

$$\int e^{3y}\sin(2y)\,\mathrm{d}y.$$ [*6*]

Question 7 – 10 marks

You should be able to answer this question after studying Unit 8.

Include a printout or screenshot of your Maxima worksheet for this question. You are not expected to annotate your Maxima worksheet with explanation. However, remember that for good mathematical communication you should present your answers clearly.

This question is about the function

$$f(x) = \frac{3x + 15x^2 - x^4}{9x^2 + 1}.$$

Use Maxima to do each of parts (a)–(d).

(a) Plot the graph of f, choosing ranges of values on the x- and y-axes to make all **three** of its stationary points clearly visible. [*3*]

(b) Find the derivative of f. [*1*]

(c) Calculate the x- and y-coordinates of the local maximum of f that has a positive x coordinate. Give your answers to three decimal places. [*3*]

(d) The graph of f crosses the x-axis at $x = 0$. Calculate the value of x where the graph first crosses the x-axis to the right of $x = 0$. Find the area enclosed by the graph of f and the x-axis, between $x = 0$ and this value. Give your answer to three decimal places. [*3*]

Question 8 – 5 marks

You should be able to answer this question after studying Units 1 to 8.

In this question, you are asked to consider how you will prepare for the exam and how you will manage your time whilst writing the exam.

Before attempting this question we advise you to look at the revision resources available in the help centre:

https://help.open.ac.uk/browse/assessments-and-exams/revision

and the resources on preparing for a remote exam in maths available from the Mathematics and Statistics study site:

https://learn2.open.ac.uk/mod/oucontent/view.php?id=1757721

(a) (i) Looking back over each of the units 1 – 8 and your TMAs, broadly how confident do you feel about the topics covered in each units? Indicate this with a tick in the appropriate column.

	not at all confident	slightly confident	somewhat confident	fairly confident	very confident
Unit 1					
Unit 2					
Unit 3					
Unit 4					
Unit 5					
Unit 6					
Unit 7					
Unit 8					

[*1*]

(ii) Thinking about where you feel most and least confident, write a brief explanation of how you will prepare for your exam. This could include:

- where and when you will revise.
- the revision techniques you will use.
- the units or topics you find most challenging.
- the units or topics you want to focus on.
- the sources of help available to you. [*2*]

(b) This part of the question requires you to consider how you will manage your time during the exam.

The MST124 exam consists of 40 computer-marked questions divided into three sections:

- Section 1 consists of 25 questions each worth 2% of the total exam mark,
- Section 2 consists of 10 questions, each worth 3% of the total exam mark,
- Section 3 consists of 5 questions, each worth 4% of the total exam mark.

The exam is scheduled to take 3 hours to complete.

(i) Assuming that the time should be allocated in proportion to the marks for each question, approximately how long should you spend on answering a question in each of the three sections? [*1*]

(ii) List any other factors that you may need to take into account when estimating how long you should spend on each question. [*1*]

Question 9 – 20 marks

You should be able to answer this question after studying Units 1 to 8.

Complete the mini examination paper on pages 8–10. Include your answers in your TMA.

You can either annotate the question paper and include it in your TMA submission, or handwrite/type the answers within your TMA document. You are not required to include any working. [*20*]

Question 10 – 5 marks

Your score out of 5 marks for good mathematical communication in Questions 1 to 7 will be recorded under Question 10.

You do not need to submit any work for this question. [*5*]

MST124 Mini examination paper

This paper has **THREE** sections. You should attempt **ALL** questions in each section.

Section A has 5 questions, each worth 2 marks.

Section B has 2 question, each worth 3 marks.

Section C has 1 question, worth 4 marks.

Each question in Section A and one question in Section B is multiple-choice, with **ONE** correct answer.

Answer each multiple choice question by circling **ONE** of the options given. No marks will be deducted for incorrectly answered questions.

For the other question in Section B, and for Section C, write your answers in the boxes provided. Do not include any working; only fully correct answers will be awarded the marks for a question. No marks will be deducted for incorrectly answered questions.

SECTION A

Question 1

Which of the following is equivalent to $(1 + 2x)^2 - (1 + x)(1 - x)$?

A $5x^2 + 4x + 2$

B $5x^2 + 4x$

C $3x^2 + 4x$

D $3x^2 + 4x + 2$

E $5x^2$

Question 2

How many values of θ in radians between -2π and 2π satisfy $\tan(\theta) = 1$?

A 3

B 4

C 5

D 6

E 8

Question 3

What is the derivative of $x^2 \ln\left(\frac{x}{2}\right)$?

A $2x \ln\left(\frac{x}{2}\right)$

B $2x$

C $2x \ln\left(\frac{x}{2}\right) + x$

D $2x \ln\left(\frac{x}{2}\right) + 2x$

E 2

Question 4

On which open interval is the function $f(x) = (2+3x)^2 - 9$ decreasing?

A $(\frac{2}{3}, \infty)$

B $(-\frac{2}{3}, \infty)$

C $(-\infty, \frac{2}{3})$

D $(-\infty, -\frac{2}{3})$

E $(-\frac{5}{3}, \frac{1}{3})$

Question 5

Which of the following is equal to

$$\int_1^4 \left(\frac{1}{\sqrt{x}} + \frac{2}{x}\right) \mathrm{d}x?$$

A $\ln 2 + 2\ln 4$

B $1 + \ln 16$

C $-2 + \ln 16$

D $\frac{1}{2} + \ln 16$

E $2 + \ln 16$

SECTION B

Question 6

Which of the following gives the solution set to the inequality
$\dfrac{3x^2-1}{2x-1} > x+1$?

A $(1, \infty)$

B $(0, \frac{1}{2})$

C $(-\infty, 0) \cup (1, \infty)$

D $(-\infty, 0) \cup (\frac{1}{2}, 1)$

E $(-\infty, -1) \cup (0, \frac{1}{2})$

F None of the above

Question 7

A function f has derivative $f'(x) = 9x^2 - 4$. What are the x-coordinates of the stationary points of f and the natures of the stationary points? The

x-coordinate of one stationary point is at $x =$ ___________

It is a ______________________

(options: local maximum, local minimum or horizontal point of inflection).

The x-coordinate of the other stationary point is at $x =$ ___________

It is a ______________________

(options: local maximum, local minimum or horizontal point of inflection).

SECTION C

Question 8

Find the values of the constants A, B and C in the
expression $\cos^4 x = A\cos(4x) + B\cos(2x) + C$.

$A =$______________________

$B =$______________________

$C =$______________________

[END OF QUESTION PAPER]