Faculty of Science, Technology, Engineering and Mathematics
MST125 Essential mathematics 2

MST125

TMA 02 2025B

Covers Units 6, 7 and 8 **Cut-off date 25 June 2025**

You should submit this TMA electronically as a PDF file by using the University's online TMA/ EMA service. Before starting work on it, please read the guidance for preparing and submitting TMAs, available from the 'Assessment' tab of the module website.

The work that you submit should include your working as well as your final answers.

You are expected to use the methods and results taught in MST125 rather than any alternative methods or results not covered in MST125.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not include the use of any other mathematical software. If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

Your work should be written in a good mathematical style, as demonstrated by the example and activity solutions in the study units. You should explain your solutions carefully, using appropriate notation and terminology, and write in sentences. As usual, you should simplify algebraic answers where possible. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 10. You do not have to submit any work for Question 10.

WEB 15184 8
22.1

PLAGIARISM WARNING – the use of assessment help services and websites

The **work that you submit for any assessment/examination on any module should be your own**. Submitting work produced by or with another person, or a web service or an automated system, **as if it is your own** is cheating. It is **strictly forbidden** by the University.

You should not:

- provide any assessment question to a website, online service, social media platform or any individual or organisation, as this is an infringement of copyright
- request answers or solutions to an assessment question on any website, via an online service or social media platform, or from any individual or organisation
- use an automated system (other than one prescribed by the module) to obtain answers or solutions to an assessment question and submit the output as your own work
- discuss examination questions with any other person, including your tutor.

The University actively monitors websites, online services and social media platforms for answers and solutions to assessment questions, and for assessment questions posted by students. Work submitted by students for assessment is also monitored for plagiarism.

A student who is found to have posted a question or answer to a website, online service or social media platform and/or to have used any resulting, or otherwise obtained, output as if it is their own work has committed a disciplinary offence under our Code of Practice for Student Discipline. **This means the academic reputation and integrity of the University has been undermined.**

The Open University's Academic Conduct Policy defines plagiarism in part as:

- using text obtained from assignment writing sites, organisations or private individuals
- obtaining work from other sources and submitting it as your own.

If it is found that you have used the services of a website, online service or social media platform, or that you have otherwise obtained the work you submit from another person, this is considered serious academic misconduct and you will be referred to the Central Disciplinary Committee for investigation.

Question 1 – 15 marks

You should be able to answer this question after studying Unit 6.

(a) For each of the following linear transformations, write down its matrix and describe its type.

(i) $g(x, y) = (2x, 7y)$

(ii) $h(x, y) = (x, 4x + y)$

(iii) $k(x, y) = (y, x)$ [5]

(b) Use the matrices that you found in part (a) to show that the linear transformation $f = k \circ h \circ g$ is represented by the matrix

$$\mathbf{A} = \begin{pmatrix} 8 & 7 \\ 2 & 0 \end{pmatrix}.$$ [2]

(c) Show that f is invertible, and find the matrix that represents f^{-1}. [2]

(d) Find the equation of the image $f(\mathcal{C})$ of the unit circle $\mathcal{C}$ centred at $(0, 0)$, in the form

$$ax^2 + bxy + cy^2 = d,$$

where a, b, c and d are integers whose values you should state. [5]

(e) Calculate the area enclosed by $f(\mathcal{C})$. [1]

Question 2 – 10 marks

You should be able to answer this question after studying Unit 6.

(a) The affine transformation f maps the points $(0, 0)$, $(1, 0)$ and $(0, 1)$ to the points $(-3, 4)$, $(-2, 4)$ and $(-3, 5)$, respectively.

(i) Determine f in the form $f(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{a}$, where $\mathbf{A}$ is a 2×2 matrix and $\mathbf{a}$ is a column vector with two components. [2]

(ii) Find the fixed points (if any) of f, and state whether f is a translation, rotation, reflection or glide-reflection. [3]

(b) The transformation k is the reflection in the line $y = -x + 7$.

By using the translation h that maps the point $(0, 7)$ to the origin, and its inverse h^{-1}, find the affine transformation k in the form $k(\mathbf{x}) = \mathbf{B}\mathbf{x} + \mathbf{b}$, where $\mathbf{B}$ is a 2×2 matrix and $\mathbf{b}$ is a column vector with two components. [5]

Question 3 – 15 marks

You should be able to answer this question after studying Unit 7.

(a) Find the partial fraction expansion of the rational expression

$$\frac{5x^3 - 11x^2 - 99x - 72}{x^2 - 3x - 18}.$$ [10]

(b) Use the `partfrac` command in Maxima to verify your answer to part (a). Include a screenshot or printout of your Maxima worksheet in your answer. [1]

(c) Hence (without using Maxima) find the integral

$$\int \frac{5x^3 - 11x^2 - 99x - 72}{x^2 - 3x - 18}\,\mathrm{d}x.$$ [4]

Question 4 – 15 marks

You should be able to answer this question after studying Unit 7.

Consider the function $f(x) = \dfrac{2 - x}{x^2 + 21}$.

(a) Find the domain and intercepts of f. [2]

(b) Find $f'(x)$. [2]

(c) Find the coordinates of any stationary points of f. [1]

(d) Construct a table of signs for $f'(x)$, determine the intervals on which f is increasing and those on which f is decreasing, and determine the nature(s) of the stationary point(s). [4]

(e) Determine the equation of the asymptote of f. [2]

(f) Determine whether f is an even or odd function, or neither. [1]

(g) Hence sketch (by hand) the graph of f. [3]

Question 5 – 15 marks

You should be able to answer this question after studying Unit 7.

(a) Use integration by substitution and then a hyperbolic identity to find the integral

$$\int e^x \cosh^3(e^x)\,\mathrm{d}x.$$ [6]

(b) (i) Use suitable sum and difference identities for trigonometric functions to show that

$$\cos(2x)\cos(5x) = \frac{1}{2}(\cos(7x) + \cos(3x)).$$ [3]

(ii) Hence, using a half-angle identity, find

$$\int \sin^2 x \cos(5x)\,\mathrm{d}x.$$ [6]

Question 6 – 5 marks

You should be able to answer this question after studying Unit 8.

Consider the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{t^7}{(t^8+32)^{3/5}}.$$

(a) State the form of the differential equation and hence state which of the methods described in Unit 8 for finding solutions of differential equations you would use to solve this equation. [*1*]

(b) Find the general solution of the differential equation in explicit form. [*3*]

(c) Hence find the particular solution of the differential equation that satisfies the initial condition $x(0) = 1$. [*1*]

Question 7 – 5 marks

You should be able to answer this question after studying Unit 8.

Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\sqrt{1-y^2}}{t} \quad (t > 0, \quad -1 < y < 1).$$

(a) State the form of the differential equation and hence state which of the methods described in Unit 8 for finding solutions of differential equations you would use to solve this equation. [*1*]

(b) Find the general solution of the differential equation in explicit form. [*4*]

Question 8 – 5 marks

You should be able to answer this question after studying Unit 8.

Consider the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - 4y = x^5 \sinh x \quad (x > 0).$$

(a) State the form of the differential equation and hence state which of the methods described in Unit 8 for finding solutions of differential equations you would use to solve this equation. [*1*]

(b) Find the general solution of the differential equation in explicit form. [*4*]

Question 9 – 10 marks

You should be able to answer this question after studying Unit 8.

A tank contains 2000 litres of a salt-water mixture. Salt water containing 0.05 kilograms of salt per litre then enters the tank at 0.2 litre per second; the solution is kept thoroughly mixed, and drains from the tank at 0.2 litre per second.

The amount y (in kilograms) of salt in the tank contents at time t (in seconds) since the mixing process began is modelled by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1}{10\,000}(100 - y) \quad (t \geq 0, \quad 0 < y < 100).$$

(a) Find the general solution of this differential equation in explicit form. [*4*]

(b) The salt-water in the tank initially contains 30 kg of salt. Find the particular solution in this case. [*1*]

(c) How much salt is in the tank contents after 600 seconds? Give your answer to two significant figures. [*1*]

(d) What value does $y(t)$ approach as $t \to \infty$? What does this mean in the context of the problem? [*2*]

(e) Use Maxima to find the solution (in explicit form) of the initial value problem

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1}{10\,000}(100 - y) \quad \text{where } y(0) = 30\,, \qquad [2]$$

and check that it agrees with your answer to part (b).

Question 10 – 5 marks

Five marks on this assignment are allocated for good mathematical communication in Questions 1 to 9.

You do not have to submit any extra work for Question 10, but you are advised to check through your assignment carefully, making sure that you have explained your working clearly, used notation correctly, written in sentences and rounded answers as requested. [*5*]
