Questions

1. Find the highest common factor of 132 and 168 and express this in the form 123v+168w where and are integers.

2. Find the least residue of -25+15 modulo 6.

3. Use Fermat's little theorem to find the least residue of 7^{50} modulo 13 .

4. Find a multiplicative inverse of 3 modulo 46.

5. Solve the linear congruence $47x \equiv -4 \pmod{18}$.

6. Find, in terms of x and y , the equation of the hyperbola in standard position with focus $(6, 0)$ and corresponding directrix $x = 3$.

7. The equation

$$4x^2 + 9y^2 - 36 = 0$$

represents an ellipse in standard position.

Find the coordinates of the foci and the equations of the directrices. Give your answers exactly.

8. Given that the equation

$$5x^2 + 6xy + 5y^2 - 26x - 22y + 21 = 0$$

represents a conic, determine what type of conic it is.

9. Two ships pursue straight-line courses at steady speeds. At each time (in hours, where $0 \leq t \leq 10$), the positions of the two ships are given by:

Ship A: $(20t + 35, 20t + 25)$, Ship B: .$(21t + 35, 21t + 15)$

These coordinates are with reference to a particular coordinate system, with distance measured in kilometres.

What is the closest distance to which the ships approach each other and when does this closest distance occur? Give you answers exactly.

10. Find a parametrisation of the hyperbola that is obtained by translating the hyperbola with equation

$$9x^2 - \frac{y^2}{16} = 1$$

by 2 unit(s) left and 4 unit(s) up.

answers

$$\begin{align}
&= -5 \cdot 132 + 4 \cdot 168 \tag{1}\\
&= 2 \pmod 6 \tag{2}\\
&= 10 \pmod{13} \tag{3}\\
&= 31 \tag{4}\\
&= 16 \tag{5}\\
&= a = 3\sqrt{2}, b = 3\sqrt{2}, x^2 - y^2 = 18 \tag{6}\\
&= foci : \left(\pm\sqrt{5}, 0\right), directrices : x = \pm\frac{9\sqrt{5}}{5} \tag{7}\\
&= ellipse \tag{8}\\
&= t = 10, d = \sqrt{10} \tag{9}\\
&= x = 4 + \frac{1}{3}\sec t, y = -2 + 4\tan t \tag{10}
\end{align}$$

$$d^2 = 2t^2 - 20t + 100$$

completing the square,

$$\begin{aligned}
&= 2\left((t-10)^2 - 100\right) + 100\\
&= 2\left(t-10\right)^2 + 100
\end{aligned}$$

The term $2\left(t-10\right)^2$ never takes a value less than 0, so the closest distance is 100 km and occurs at $t = 10$.

At this time $d^2 = 100$ so the distance is 10km.