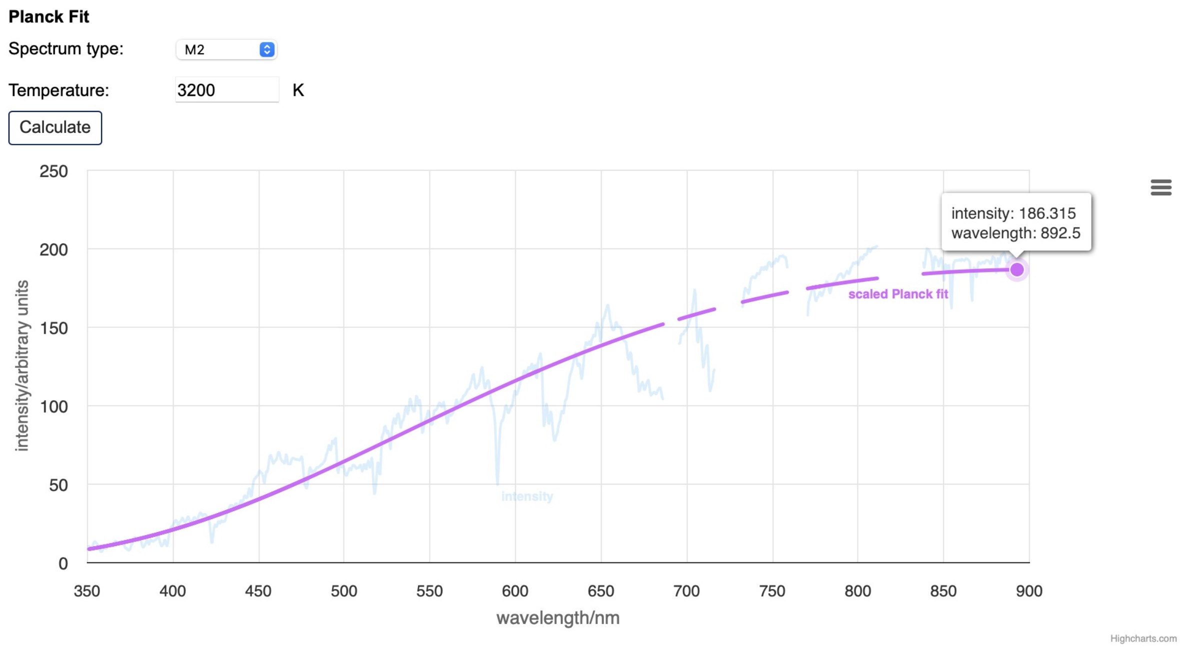
Question 1:

a)

i.



Accessed 25/11/2024

Figure 1: Screenshot of the best-fit Planck curve overlaid on the stellar spectrum

ii.

As shown in Figure 1, the Planck curve fits the spectrum well around 3200 K as it gives quite a good fit for most of the spectrum and so do temperatures of 3050 K to 3350 K, so we might estimate the temperature of this star to be $3200\,\text{K} \pm 150\,\text{K}$

iii.

The peak for the best-fitting Planck curve is at 892 nm with an intensity of 186.35, the temperature using Wien's law gives a wavelength of;

$$\lambda_{peak}(\mathrm{m}) = \frac{2.90 \times 10^{-3}\mathrm{K\,m}}{T/(\mathrm{K})}$$

$$= \frac{2.90 \times 10^{-3}\mathrm{K\,m}}{3200/\mathrm{K}}$$

$$= 9.0625 \times 10^{-7}\,\mathrm{m}$$

$$= 906\,\mathrm{nm}$$

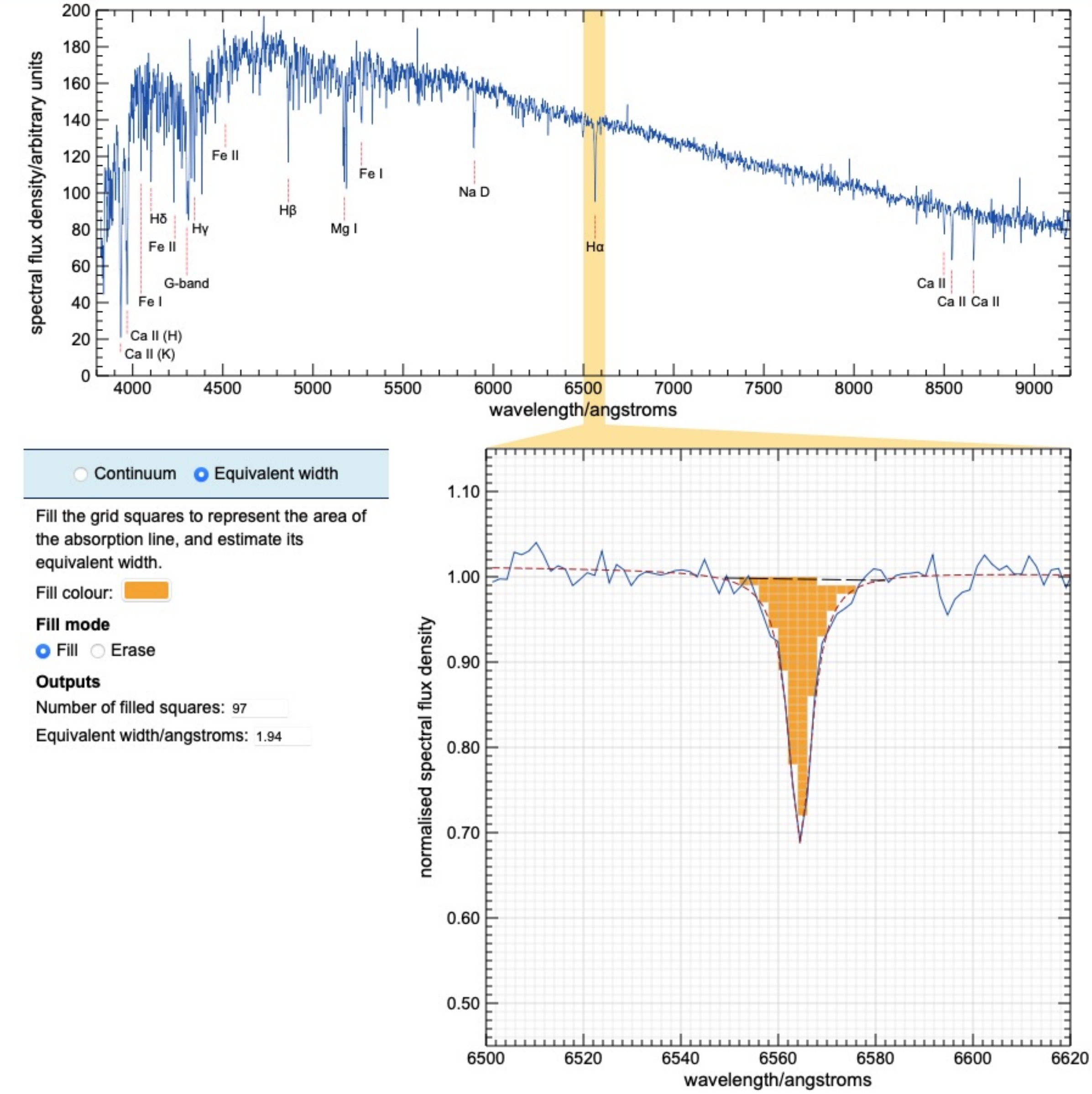
to 2 s.f

This calculated peak of 906 nm fits well with the observed Planck's curve peak. The difference could be due to absorption or emission features in the spectrum or observational noise.

This is quite different to the peak wavelength of the spectrum 811.0 nm with an intensity of 201.247., which would suggest a temperature of around 3600 K. This discrepancy highlights that stellar spectra are not perfect blackbodies due to absorption features (e.g., molecular bands) and emission from the outer layers of the star's atmosphere.

b)

i.



Accessed 25/11/2024

Figure 2: Screenshot of the stars $H\alpha$ line showing an estimation of its equivalent width.

ii.

As seen in figure 2, given the triple Ca II triplet at around 8600 Å, strong Ca II (H) and (K) at around 4000 Å and magnesium at around 5200 Å along with the equivalent width of 1.94 Å suggests an early G-class star.

Question 2:

a)
The orbital period of stars A and B are;

$$\begin{aligned} p_{orb} &= 15.678\,\mathrm{d} \\ &= 15.879 \times 86400 && = 1\,354\,579.2\,\mathrm{s} \\ &= \frac{1354579.2}{3.16 \times 10^7} && = 0.043\,\mathrm{yr} \end{aligned}$$

(to 3 s.f)

and the radial velocity amplitudes are;

$$V_{Rm1} = 162\,\mathrm{km\,s^{-1}}$$

and

$$V_{Rm2} = 214\,\mathrm{km\,s^{-1}}$$

respectively.

Using

$$d_m = \frac{(V_R)(P_{orb})}{2\pi}$$

$$\begin{aligned} d_{m1} &= \frac{162\,\mathrm{km\,s^{-1}} \times 1\,354\,597.2\,\mathrm{s}}{2\pi} \\ &= 3.49 \times 10^7 \mathrm{km} \end{aligned}$$

(to 3 s.f)

and

$$\begin{aligned} d_{m2} &= \frac{214\,\mathrm{km\,s^{-1}} \times 1\,354\,597.2\,\mathrm{s}}{2\pi} \\ &= 4.61 \times 10^7 \mathrm{km} \end{aligned}$$

(to 3 s.f)

Hence the total semi-major axis of the binary system is:

$$a = d_{m1} + d_{m2}$$

$$= 3.49 \times 10^7\,\mathrm{km} + 4.61 \times 10^7\,\mathrm{km}$$

$$= 8.11 \times 10^7\,\mathrm{km}$$

(to 3 s.f)

$$= \frac{8.11 \times 10^7\,\mathrm{km}}{1.50 \times 10^8}$$

$$= 0.540\,\mathrm{au}$$

(to 3 s.f)

Using Kepler's third law:

$$M_1 + M_2 = \frac{a^3}{p_{orb}^2}$$

$$= \frac{0.540^3}{0.043^2}$$

$$= 85.9$$

to 3 s.f

And:

$$\frac{M_2}{M_1} = \frac{4.61 \times 10^7\mathrm{km}}{3.49 \times 10^7\mathrm{km}}$$

$$= \frac{107}{81}$$

$$= 1.32$$

(to 3 s.f)

Using both these results:

$$M_1 = 1.32 M_2$$

and

$$M_1 = 85.9M_\odot - M_2$$

Equating both of these;

$$\begin{aligned}
1.32M_2 &= 85.9M_\odot - M_2 \\
1.32M_2 + M_2 &= 85.9M_\odot \\
M_2(1.32 + 1) &= 85.9M_\odot \\
M_2(2.32) &= 86.9M_\odot \\
M_2 &= \frac{85.9M_\odot}{2.32} \\
&= 37.0M_\odot \qquad \textit{(to 3 s.f)}
\end{aligned}$$

And using:

$$\begin{aligned}
\frac{M_1}{M_2} &= \frac{d_{m2}}{d_{m1}} \\
M_1 &= \left(\frac{d_{m2}}{d_{m1}}\right) M_2 \\
&= \left(\frac{107}{81}\right) \times 37.0 \\
&= 48.9M_\odot
\end{aligned}$$

(to 3 s.f)

These values might be lower than the actual masses due to observational noise in the radial velocity amplitudes, accuracy of the P_{orb} and other uncertainties in the modeling system of stellar atmospheres, which would compound errors in any of our derived values. Due to the uncertainty in the inclination, i, of the binary system, these values represent minimum masses. The closer i is to 90°, the larger the true masses would be compared to our calculated values.

b)
Given: A Cepheid star with a pulsation period of 13.6 days and a continuum spectrum peak at 517 nm.

Using Wiens law;

$$\lambda_{peak}(\mathrm{m}) = \frac{2.90 \times 10^{-3}\mathrm{m\,K}}{Temperature(\mathrm{K})}$$

$$T(\mathrm{K}) = \frac{2.90 \times 10^{-3}\mathrm{m\,K}}{\lambda_{peak}(\mathrm{m})}$$

$$= \frac{2.90 \times 10^{-3}\mathrm{m\,K}}{5.17 \times 10^{-7}\mathrm{m}}$$

$$= 5610\,\mathrm{K}$$

(to 3 s.f)

Calculating the absolute magnitude (M_v)) using the period-luminosity relation:

$$M_v = -2.43 \log_{10}(P) - 1.62$$

$$= -2.43 \log_{10}(13.6) - 1.62$$

$$= -4.37$$

(to 3 s.f)

Calculating the luminosity using the absolute magnitude to luminosity relationship:

$$M_v - M_{v\odot} = -2.5 \log_{10}\left(\frac{L}{L_\odot}\right)$$

$$L = L_\odot \times 10^{\frac{M_{v\odot} - M_v}{2.5}}$$

$$= 3.85 \times 10^{26}\mathrm{W} \times 10^{\frac{4.83+4.37}{2.5}}$$

$$= 1.85 \times 10^{30}\mathrm{W}$$

(to 3 s.f)

Calculating the radius (R) using the 'Stefan-Boltzmann law'

$$\frac{L}{L_\odot} = \left(\frac{R}{R_\odot}\right)^2 \times \left(\frac{T_\odot}{T}\right)^4$$

$$\frac{R}{R_\odot} = \sqrt{\left(\frac{L}{L_\odot}\right)} \times \left(\frac{T_\odot}{T}\right)^2$$

Now substituting our values:

$$= \sqrt{\left(\frac{1.85 \times 10^{30}\,\mathrm{W}}{3.84 \times 10^{26}\,\mathrm{W}}\right)} \times \left(\frac{5780\,\mathrm{K}}{5610\,\mathrm{K}}\right)^2$$

$$= \sqrt{4810} \times 1.03^2$$

$$= 69.3 \times 1.06$$

$$= 73.6 R_\odot$$

(to 3 s.f)

Using a H-R diagram[1]; with a radius of $73.6R_\odot$, a temperature of $5610\,\mathrm{K}$, luminosity of 1.85×10^{30}W and absolute magnitude of -4.37 suggests this is a low-end super giant or high-end giant. The radius is spot on for a super giant but the luminosity and absolute magnitude are a bit on the low side, but the temperature in the yellow-white also suggests super-giant. Cepheid variables are typically found in the instability strip of the H-R diagram, with the temperature and luminosities varying depending on the metallicity and mass of the star.

Using the H-R diagram for the classification of stellar objects contains uncertainties based on the input data, errors in luminosity, effective temperature, can make it difficult to assign a definitive position on the diagram.

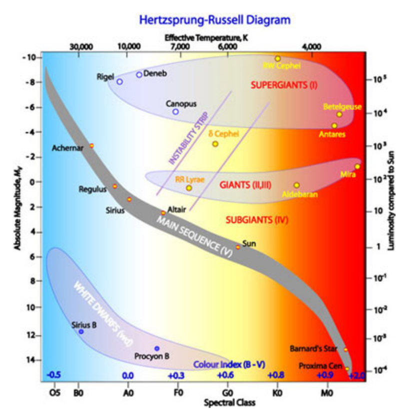


Figure 3: H-R diagram showing various stages of stellar evolution.

[1] CSIRO R. Hollow. COSMOS kernel description, 2024. URL `https://astronomy.swin.edu.au/cosmos/h/hertzsprung-russell+diagram`

References

CSIRO R. Hollow. COSMOS kernel description, 2024. URL https://astronomy.swin.edu.au/cosmos/h/hertzsprung-russell+diagram.