Question 1:

a)
If if the following statments are true:

If it is Robin's birthday, then Robin eats cake. Robin is eating cake.

Let P be the statement "It is Robin's birthday" and Q be the statement "Robin eats cake". The first statement can be written as $P \implies Q$ and the second statement can be written as Q.

The statment;

It is Robin's birthday.

P does not imply Q. Robin could be eating cake for any number of reasons. This is an example of the formal fallacy affirming the consequences.

b)

$$\text{For all positive integers } n, \text{ We have } 5^n \leq (6+1)^6$$

This can be proved false with;

Using $n = 9$

Q1:
5/5

$$5^9 = 1953125$$

$$(9+1)^6 = 10^6 = 1000000$$

Thus

$$1953125 \leq 1000000 \text{ is false.}$$

Question 2:

a)

$$y = \frac{4x-1}{3x-4}$$

Note that instructions indicated to use a series of equivalences ($\Longleftrightarrow$). Instead, you have used $\Rightarrow$ and $\Leftarrow$ separately. Here's a sample solution.

Given $x \neq \frac{4}{3}$ and $y \neq \frac{4}{3}$

When $x = \frac{4y-1}{3y-4}$

$$y = \frac{4x-1}{3x-4}$$

$$= \frac{4\left(\frac{4y-1}{3y-4}\right)-1}{3\left(\frac{4y-1}{3y-4}\right)-4}$$

Let x and y be real numbers not equal to $\frac{4}{3}$. Then

$$\begin{aligned} y = \frac{4x-1}{3x-4} &\Leftrightarrow y(3x-4) = 4x-1 \quad (\text{since } x \neq \tfrac{4}{3}) \\ &\Leftrightarrow 3xy - 4y = 4x - 1 \\ &\Leftrightarrow 3xy - 4x = 4y - 1 \\ &\Leftrightarrow x(3y-4) = 4y-1 \\ &\Leftrightarrow x = \frac{4y-1}{3y-4} \quad (\text{since } y \neq \tfrac{4}{3}). \end{aligned}$$

References:
Example 14 on page 280 and Activity 31 on page 281 of Book C (Unit 9).
Exercise 9 in Exercise Booklet 9.

Lost 3 marks.

Distributing the 4 and 3

$$= \frac{\frac{16y-4}{3y-4}-1}{\frac{12y-3}{3y-4}-4}$$

Using the common denominator of $3y-4$

$$= \frac{\frac{16y-4-(3y-4)}{3y-4}}{\frac{12y-3-4(3y-4)}{3y-4}}$$

Simplifying the numerator and denominator

$$= \frac{\frac{16y-4-3y+4}{3y-4}}{\frac{12y-3-12y+16}{3y-4}}$$

$$= \frac{\frac{13y}{3y-4}}{\frac{13}{3y-4}}$$

Cancelling the common factor of $3y-4$

$$= \frac{13y}{13}$$

$$= y$$

Assume $x = \frac{4y-1}{3y-4}$
Then $y = \frac{4x-1}{3x-4}$

$$x = \frac{4y-1}{3y-4}$$

$$= \frac{4\left(\frac{4x-1}{3x-4}\right)-1}{3\left(\frac{4x-1}{3x-4}\right)-4}$$

Distributing the 4 and 3

$$= \frac{\frac{16x-4}{3x-4}-1}{\frac{12x-3}{3x-4}-4}$$

Using the common denominator of $3x-4$

$$= \frac{\frac{16x-4-(3x-4)}{3x-4}}{\frac{12x-3-4(3x-4)}{3x-4}}$$

Simplifying the numerator and denominator

$$= \frac{\frac{16x-4-3x+4}{3x-4}}{\frac{12x-3-12x+16}{3x-4}}$$

$$= \frac{\frac{13x}{3x-4}}{\frac{13}{3x-4}}$$

See comment above.

Cancelling the common factor of $3x-4$

$$= \frac{13x}{13}$$

$$= x$$

Thus the function is it's own inverse. Hence,

$$y = \frac{4x-1}{3x-4} \text{if and only if} x = \frac{4y-1}{3y-4}$$

for all real numbers such that $y \neq \frac{4}{3}$ and $x \neq \frac{4}{3}$.

b)
To prove

$$n + 1 \text{ is even if and only if } 2(n + 3) \text{ is a multiple of } 4$$

Assume $n + 1$ is even, therefore it can be writen as $n + 1 = 2k$ for some integer k.

It follows that we can write;

$$2(n + 3) = 2(n + 1) + 4$$

Substituting $n + 1 = 2k$

$$= 2(2k) + 4$$

$$= 4k + 4$$

$$= 4(k + 1)$$

and thus a multiple of 4

n+3, not n+1.

Conversly, assume $2(n + 1)$ is a multiple of 4, therefore it can be written as $2\boxed{(n + 1) =} 4k$ for some integer k.

$$2(n + 3) = 4k$$

Dividing both sides by 2

$$n + 3 = 2k$$

Rearranging gives

$$n = 2k - 3$$

$$n + 1 = 2k - 2$$

$$= 2(k - 1)$$

and thus an even number

Q2:
7/10

Question 3:

a)

$$(3n)! \geq (n!)^3, \text{ for all } n \in \mathbb{N}$$

Proof by induction.

Base case: $n = 1$

$$(3 \cdot 1)! = 3! = 6$$

$$(1!)^3 = 1^3 = 1$$

Thus

$$6 \geq 1 \text{ is true.}$$

Inductive step: Assume $(3n)! \geq (n!)^3$ is true for some $n \in \mathbb{N}$.

Use a different letter, say k, in the inductive step (that is, assume P(k) is true).

We need to show that $(3(n+1))! \geq ((n+1)!)^3$.

Since

$$\begin{aligned}(3(n+1))! &= (3n+3)! \\ &= (3n+3)(3n+2)(3n+1)\,(3n)!\end{aligned}$$

and

$$\begin{aligned}((n+1)!)^3 &= ((n+1)(n!))^3 \\ &= (n+1)^3(n!)^3\end{aligned}$$

Now using our assumption for the inductive step, it suffices to show:

$$(3n+3)(3n+2)(3n+1) \geq (n+1)^3$$

Expanding the LHS:

$$(3n+3)(3n+2) = 9n^2 + 15n + 6$$

$$(9n^2 + 15n + 6)(3n+1) = 27n^3 + 54n^2 + 33n + 6$$

Expanding the RHS:

$$(n+1)^3 = n^3 + 3n^2 + 3n + 1$$

Subtracting the RHS from the LHS:

$$(27n^3 + 54n^2 + 33n + 6) - (n^3 + 3n^2 + 3n + 1) = 26n^3 + 51n^2 + 30n + 5 \geq 0$$

This is true for all $n \in \mathbb{N}$.

This deduction is not right as you are assuming that this is already true when that is what you're trying to show. What you can do is to "go the other way around" in your proof and treat this part as a preliminary solution. That is, start with $26n^3 + 51n^2 + 30n + 5 \geq 0$ since $n \in \mathbb{N}$.

Lost 2 marks.

Alternative, here's a sample solution to the inductive step:

Now, for $k \geq 1$,

$$\begin{aligned}(3(k+1))! &= (3k+3)! \\ &= (3k+3)(3k+2)(3k+1)(3k)! \\ &\geq (3k+3)(3k+2)(3k+1)(k!)^3 \quad (\text{by } P(k)) \\ &\geq (k+1)(k+1)(k+1)(k!)^3 \\ &= ((k+1)!)^3,\end{aligned}$$

which means $P(k+1)$ holds.

You then need to write a conclusion to complete the proof. Here's a sample conclusion.

Thus $P(k+1)$ is true if $P(k)$ is true; that is,

$$P(k) \Rightarrow P(k+1), \text{ for } k = 1, 2, 3, \ldots .$$

Hence, by mathematical induction, $P(n)$ is true for all $n \in \mathbb{N}$.

Lost 2 marks

Q3:
6/10

Question 4:

a)
Prove that no such value of x exists such that x is a real positive number.

$$\frac{7x}{x+3} \leq \frac{x-3}{7x}$$

Assume that x is a positive real number.

$$\frac{7x}{x+3} \leq \frac{x-3}{7x}$$

Cross multiplying gives

You need to write why this can be done as cross multiplying may not be valid when you have inequalities. Lost 1 mark.

$$(7x)(7x) \leq (x-3)(x+3)$$

Expanding both sides

$$49x^2 \leq x^2 - 9$$

Rearranging gives

$$48x^2 + 9 \leq 0$$

This is not possible as $48x^2$ is always positive for all real numbers x and 9 is a positive constant.

Thus, we have a contradiction.

We can conclude that no such value of x exists such that x is a positive real number.

b)
Prove that:

If $n^3 + 2n^2$ is not a multiple of 16, then n is odd.

Let us consider the contraposition of this statement;

If n is even, then $n^3 + 2n^2$ is a multiple of 16.

Assume n is even, therefore it can be written as $n = 2k$ for some integer k.

$$\begin{aligned} n^3 + 2n^2 &= (2k)^3 + 2(2k)^2 \\ &= 8k^3 + 2(4k^2) \\ &= 8k^3 + 8k^2 \\ &= 8(k^3 + k^2) \\ &= 8k^2(k+1) \\ &= (8k)(k(k+1)) \end{aligned}$$

As $k(k+1)$ is even, we can write it as $2l$ for some integer l.

$$\begin{aligned} &= (8k)(2l) \\ &= 16kl \end{aligned}$$

Hence a multiple of 16

Thus by proof by contraposition:

If $n^3 + 2n^2$ is not a multiple of 16, then n is odd.

Q4:
9/10

Question 5:

a)

$$x_0 = 0\,\mathrm{m}, \quad x_1 = 300\,\mathrm{m}, \quad v_0 = 0\,\mathrm{m/s}, \quad \mathrm{a} = \mathrm{g} = 9.8\,\mathrm{m/s}$$

Using $x = v_0 t + \frac{1}{2}at^2$ to find t

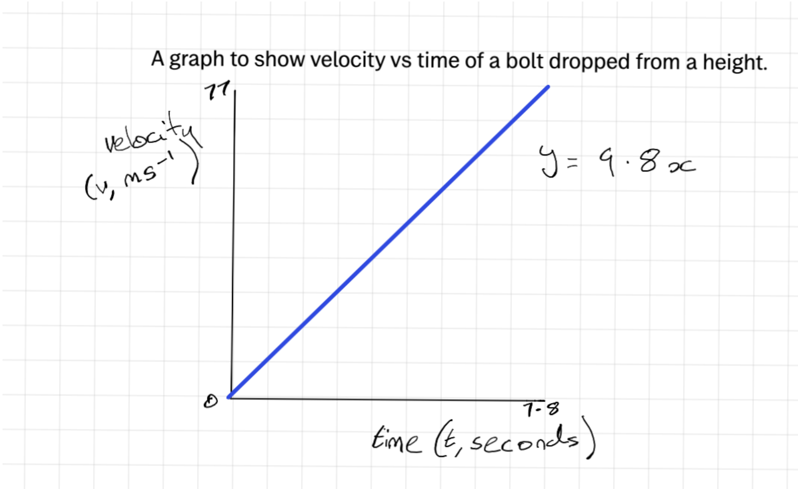
$$\begin{aligned} x_1 &= v_0 t + \frac{1}{2}at^2 \\ 300 &= \frac{1}{2}gt^2 \\ 600 &= gt^2 \\ t^2 &= \frac{600}{g} \\ t &= \sqrt{\frac{600}{g}} \\ &= \sqrt{\frac{600}{9.8}} \\ &= 7.824\ldots \\ &= 7.82\,\mathrm{s} \end{aligned}$$

to 2 s.f.

Using $v = v_0 + at$

$$\begin{aligned} v_1 &= v_0 + at \\ &= 0 + g\sqrt{\frac{600}{g}} \\ &= g\sqrt{\frac{600}{g}} \\ &= \sqrt{600g} \\ &= \sqrt{600 \times 9.8} \\ &= 77.46\ldots \\ &= 77\,\mathrm{m\,s^{-1}} \end{aligned}$$ *to 2 s.f*

b)


A graph to show velocity vs time of a bolt dropped from a height.
77
velocity
(v, ms⁻¹)
y = 9.8x
0
7.8
time (t, seconds)

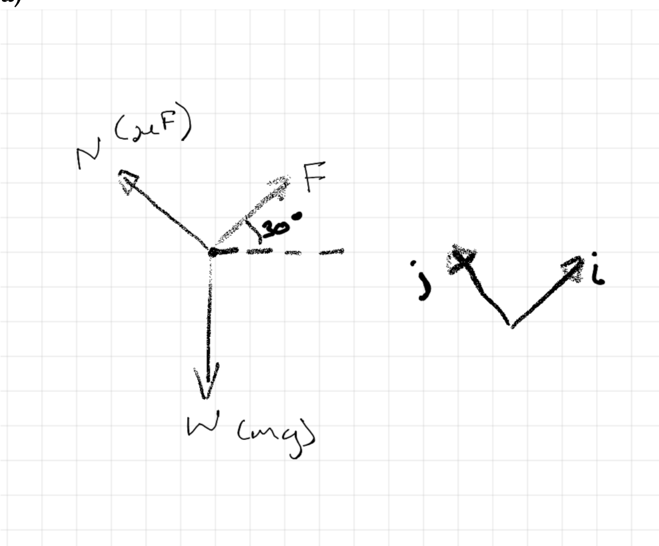
✓

Q5:
5/5

Question 6:

$$v_0 = 0\,\text{meter/s}, \quad x_0 = 0\,\text{m}, \quad v_1 = 9\,\text{m}\,\text{s}^{-1}, \quad x_1 = 30\,\text{m}$$

a)



Instructions indicate to define the forces acting on the object. Lost 1 mark.

Using

$$v_1 = v_0 + at$$

$$9 = at$$

and

$$x_1 = v_0 t + \frac{1}{2}at^2$$

$$30 = \frac{1}{2}at^2$$

$$60 = at^2$$

Substituting $at = 9$

It's actually quicker if you used the formula $v^2 = v_0{}^2 + 2ax$

$$\begin{aligned} 60 &= 9t \\ t &= \frac{60}{9} \\ &= \frac{20}{3} \\ &= 6.67\,\mathrm{s} \end{aligned}$$

and

$$\begin{aligned} a &= \frac{9}{t} \\ a &= \frac{9}{\frac{20}{3}} \\ a &= \frac{27}{20} \\ &= 1.35\,\mathrm{m\,s^{-2}} \end{aligned}$$

✓

b)

$$\begin{aligned} \mathbf{F} &= \mu|N| \\ \mathbf{N} &= |N| \\ \mathbf{W} &= -\sin(30)mg - \cos(30)mg \\ F_i &= \sin(30)mg - \mathbf{F} \\ &= \sin(30)mg - \mu|N| \\ N_j &= \cos(30)mg \end{aligned}$$

✓

thus

$$F = \sin 30mg - \mu\cos(30)mg$$

And using $F = ma$

$$ma = \sin(30)mg - \mu\cos(30)mg$$

Divinding through by m

$$a = \sin(30)g - \mu\cos(30)g$$

$$1.35 = \sin(30)g - \mu\cos(30)g$$

Rearranging

$$\mu\cos(30)g = \sin(30)g - 1.35$$

$$\mu = \frac{\sin(30)g - 1.35}{\cos(30)g}$$

$$= \frac{\sin(30)9.8 - 1.35}{\cos(30)9.8}$$

$$= \frac{4.9 - 1.35}{8.487}$$

$$= \frac{3.55}{8.487}$$

$$= 0.418\ldots$$

$$= 0.42$$

to 2 s.f.

Q6:
14/15

Question 7:

a)
The vector expression for the acceleration is

$$\mathbf{a} = -g\mathbf{j}$$

b)
The initial velocity vector is

$$\mathbf{v}_0 = 12\cos(50^\circ)\,\mathbf{i} + 12\sin(50^\circ)\,\mathbf{j}$$

Integrating the acceleration vector to find the velocity vector:

$$\begin{aligned}\mathbf{v}(t) &= \int \mathbf{a}\ \mathrm{d}t \\ &= \int -g\mathbf{j}\ \mathrm{d}t \\ &= -gt\,\mathbf{j} + \mathbf{C}_1\end{aligned}$$

Using the initial velocity to find the constant of integration:

$$\mathbf{v}(0) = \mathbf{v}_0 \Rightarrow \mathbf{C}_1 = \mathbf{v}_0$$

Thus, the velocity vector is:

$$\mathbf{v}(t) = \mathbf{v}_0 - gt\,\mathbf{j}$$

Integrating the velocity vector to find the position vector:

$$\begin{aligned}\mathbf{r}(t) &= \int \mathbf{v}(t)\ \mathrm{d}t \\ &= \int (\mathbf{v}_0 - gt\,\mathbf{j})\,\mathrm{d}t \\ &= \mathbf{v}_0 t - \frac{1}{2}gt^2\,\mathbf{j} + \mathbf{C}_2\end{aligned}$$

Taking the initial position as the origin:

$$\mathbf{r}(0) = \mathbf{0} \Rightarrow \mathbf{C}_2 = \mathbf{0}$$

Therefore

$$\mathbf{r}(t) = \mathbf{v}_0 t - \frac{1}{2} g t^2 \, \mathbf{j}$$

Substituting the expression for $\mathbf{v}_0$:

$$\mathbf{r} = (12t\cos(50^\circ))\,\mathbf{i} + \left(12t\sin(50^\circ) - \frac{1}{2}gt^2\right)\mathbf{j}$$ *(as required)*

c)

i.

Given the position vector $\mathbf{r}$ is from the orign, we want to find the time t for which the $\mathbf{j}$-component is -1.5, that is,

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^{2-4ac}}}{2a}$$

$$12t\sin(50^\circ) - \frac{1}{2}gt^2 = -1.5$$

Rewriting

$$-\frac{1}{2}gt^2 + 12t\sin(50^\circ) + 1.5 = 0$$

This is a quadratic equation in t:

$$\begin{aligned} t &= \frac{-12\sin(50) \pm \sqrt{(12\sin(50)^2 - 4(-\frac{1}{2}g)(1.5))}}{2(-\frac{1}{2}g)} \\ &= \frac{-12\sin(50) \pm \sqrt{(12\sin(50))^2 + \frac{147}{5}}}{-g} \\ &= -0.151\ldots \text{ and } 2.027\ldots \end{aligned}$$

since we can reject the negaive value for time, we have

$$\begin{aligned} &= 2.027\ldots \\ &= 2.0\,\mathrm{s} \end{aligned}$$

to 2 s.f.

ii.
The horizonal distance traveled by the ball is given by the **i**-component of the position vector **r** at time t:

$$\mathbf{r_i} = 12t\cos(50)\mathbf{i}$$

Substituting $t = 2.027\ldots$

$$\begin{aligned} &= 12(2.027\ldots)\cos(50)\mathbf{i} \\ &= 15.635\ldots \\ &= 16\,\mathrm{m} \end{aligned}$$

to 2 s.f.

$\boxed{\text{Q7: } 15/15}$

Question 8:

$$\mathbf{A} = \begin{pmatrix} 5 & 6 \\ 18 & 2 \end{pmatrix}$$

a)

The determinant of matrix $\mathbf{A}$ is given by:

$$\begin{aligned} \det(\mathbf{A}) &= 5 \cdot 2 - 6 \cdot 18 \\ &= 10 - 108 \\ &= -98 \end{aligned}$$

The trace of matrix $\mathbf{A}$ is given by the sum of the diagonal elements:

$$\begin{aligned} \operatorname{tr}(\mathbf{A}) &= 5 + 2 \\ &= 7 \end{aligned}$$

The characteristic equation of a 2×2 matrix $\mathbf{A}$ is given by:

$$\lambda^2 - (tr\mathbf{A})\lambda + \det \mathbf{A} = 0$$

where λ is the eigenvalue.

Hence, the characteristic equation of matrix $\mathbf{A}$ is:

$$\lambda^2 - 7\lambda - 98 = 0$$

$$(\lambda - 14)(\lambda + 7) = 0$$

Hence the eigenvalues are:

$$\lambda_1 = 14$$

and

$$\lambda_2 = -7$$

The corresponding eigenvectors can be found by solving the equation:

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = \mathbf{0}$$

$$\begin{pmatrix} 5-\lambda & 6 \\ 18 & 2-\lambda \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

For

$$\lambda_1 = 14:$$

$$\begin{pmatrix} 5-14 & 6 \\ 18 & 2-14 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -9 & 6 \\ 18 & -12 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This gives the system of equations:

$$-9x + 6y = 0$$

$$18x - 12y = 0$$

Hence

$$-9x + 6y = 18x - 12y$$

Rearranging gives

$$-27x + 18y = 0$$

$$18y = 27x$$

$$2y = 3x$$

This gives us the eigenvector:

$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

For

$$\lambda_2 = -7:$$

$$\begin{pmatrix} 5-(-7) & 6 \\ 18 & 2-(-7) \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 12 & 6 \\ 18 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This gives the system of equations:

$$12x + 6y = 0$$

$$18x + 9y = 0$$

Hence

$$12x + 6y = 18x + 9y$$

Rearranging gives

$$-6x - 3y = 0$$

$$-3y = 6x$$

$$-y = 2x$$

This gives us the eigenvector:

$$\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

b)

We can express

$$\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$$

Where $\mathbf{P}$ is $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$

and $\mathbf{D}$ is $\begin{pmatrix} 14 & 0 \\ 0 & -7 \end{pmatrix}$

and $\mathbf{P}^{-1}$ is the inverse of $\mathbf{P}$; $\begin{pmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{2}{7} \end{pmatrix}$

The inverse of a 2×2 matrix $\mathbf{P} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is given by:

$$\mathbf{P}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

where $ad - bc$ is the determinant of $\mathbf{P}$.

Hence, we can write:

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 14 & 0 \\ 0 & -7 \end{pmatrix} \begin{pmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{2}{7} \end{pmatrix}$$

Although not required in the question, it would be a good idea to multiply out to and see if you will obtain A.

For calculation purposes, it is better to leave 1/7 outside the matrix of the inverse.

c)

$$\mathbf{A}^5 = \mathbf{P}\mathbf{D}^5\mathbf{P}^{-1}$$

$$\begin{aligned}
\mathbf{A}^5 &= \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 14^5 & 0 \\ 0 & (-7)^5 \end{pmatrix} \begin{pmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{2}{7} \end{pmatrix} \\
&= \begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 537824 & 0 \\ 0 & -16807 \end{pmatrix} \begin{pmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{2}{7} \end{pmatrix} \\
&= \begin{pmatrix} 2 \cdot 537824 + -1 \cdot 0 & 2 \cdot 0 + -1 \cdot -16807 \\ 3 \cdot 537824 + 2 \cdot 0 & 3 \cdot 0 + 2 \cdot -16807 \end{pmatrix} \begin{pmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{2}{7} \end{pmatrix} \\
&= \begin{pmatrix} 1075648 & 16807 \\ 1613472 & -33614 \end{pmatrix} \begin{pmatrix} \frac{2}{7} & \frac{1}{7} \\ \frac{-3}{7} & \frac{2}{7} \end{pmatrix} \\
&= \begin{pmatrix} \frac{1075648 \cdot 2}{7} + \frac{16807 \cdot -3}{7} & \frac{1075648 \cdot 1}{7} + \frac{16807 \cdot 2}{7} \\ \frac{1613472 \cdot 2}{7} + \frac{-33614 \cdot -3}{7} & \frac{1613472 \cdot 1}{7} + \frac{-33614 \cdot 2}{7} \end{pmatrix} \\
&= \begin{pmatrix} \frac{2151296 - 50421}{7} & \frac{1075648 + 33614}{7} \\ \frac{3226944 + 100842}{7} & \frac{1613472 - 67228}{7} \end{pmatrix} \\
&= \begin{pmatrix} \frac{2100875}{7} & \frac{1109262}{7} \\ \frac{3327786}{7} & \frac{1546244}{7} \end{pmatrix} \\
&= \begin{pmatrix} 300125 & 158466 \\ 475398 & 220892 \end{pmatrix}
\end{aligned}$$

d)

Question 8 d)

Define the matrix

(%i1) `A:matrix([5,6],[18,2]);`

A $\begin{bmatrix} 5 & 6 \\ 18 & 2 \end{bmatrix}$

Find the eignevales and associated eigenvectors

(%i5) `eigenvalues(A);`

(%o5) $[[-7,14],[1,1]]$

The [1,1] being the multiplicities

(%i6) `eigenvectors(A);`

(%o6) $\left[[[-7,14],[1,1]],\left[[[1,-2]],\left[\left[1,\frac{3}{2}\right]\right]\right]\right]$

These are consistant with my results, Maxima uses the normalisation of having 1 as the first value.

Explain how the eigenvectors match your answers. Lost 1 mark.

Check the values of A^5

(%i4) `A^^5;`

(%o4) $\begin{bmatrix} 300125 & 158466 \\ 475398 & 220892 \end{bmatrix}$

e)

$$\dot{x} = 5x + 6y$$

$$\dot{y} = 18x + 2y$$

The general solution of a system of linear differential equations is given by:

$$\mathbf{x} = e^{\lambda_1 t}\mathbf{v}_1 + e^{\lambda_2 t}\mathbf{v}_2$$

where λ_1 and λ_2 are the eigenvalues, and $\mathbf{v}_1$ and $\mathbf{v}_2$ are the corresponding eigenvectors.

$$\mathbf{x} = Ce^{14t}\begin{pmatrix} 2 \\ 3 \end{pmatrix} + De^{-7t}\begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

hence we can write:

$$x = 2Ce^{14t} - De^{-7t}$$

$$y = 3Ce^{14t} + 2De^{-7t}$$

where C and D are constants determined by initial conditions.

Q8: 24/25

Question 9:

GMC: Your solutions demonstrate good mathematical communication. Well done!

Q9: 5/5

TMA: 90/100