Question 1:

a)
The equation of a straight line has the general form

$$y = mx + c$$

Where m is the gradient and c is the intercept

Given our two coordinates $(3, 5)$ and $(5, 4)$:

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4-5}{5-3} \\ &= \frac{-1}{2} \end{aligned}$$

Substituting this, and our coordinates:

$$\begin{aligned} mx + c &= y \\ \frac{-1}{2}(3) + c &= 5 \\ -1.5 + c &= 5 \\ c &= 6.5 \end{aligned}$$

Hence the equation of the line is;

$$y = \frac{-1}{2}x + 6.5$$

b)
Using:

$$ax^2 + bx + c = a(x + r)^2 + s$$

Given:

$$f(x) = x^2 - 6x - 4$$

By completing the square:

$$\begin{aligned} &= (x-3)^2 - 9 - 4 \\ &= (x-3)^2 - 13 \end{aligned}$$

c)

Solve;

$$x^2 - 6x - 4 = 0$$

Using the completed square form;

$$(x-3)^2 - 13 = 0$$

$$(x-3)^2 = 13$$

$$x - 3 = \pm\sqrt{13}$$

$$x = 3 \pm \sqrt{13}$$

d)
For the line $y = \frac{-1}{2}x + 6.5$.
The $y - intercept$ is when $x = 0$ therefore $y = 6.5$.
The $x - intercept$ is when $y = 0$ therefore $x = 13$.
For the curve $y = x^2 + 6x - 4$; The solutions are $3 \pm \sqrt{13}$.
The $y - intercept$ is when $x = 0$ therefore $y = -4$.
The turning point;

$$y = x^2 - 6x - 4$$

$$\frac{dy}{dx} = 2x - 6$$

$$2x - 6 = 0$$

$$2x = 6$$

$$x = 3$$

Substituting this back into the equation;

$$y = 3^2 - 6(3) - 4$$

$$y = -13$$

Therefore $(3, -13)$

And the two curves meet at;

$$\frac{-1}{2}x + 6.5 = x^2 - 6x - 4$$

$$\frac{-1}{2}x = x^2 - 6x - \frac{21}{2}$$

$$0 = x^2 - 5.5x - \frac{21}{2}$$

$$= 2x^2 - 11x - 21$$

$$= (2x+3)(x-7)$$

the x-coordinates are $x = \frac{-3}{2}$ and $x = 7$

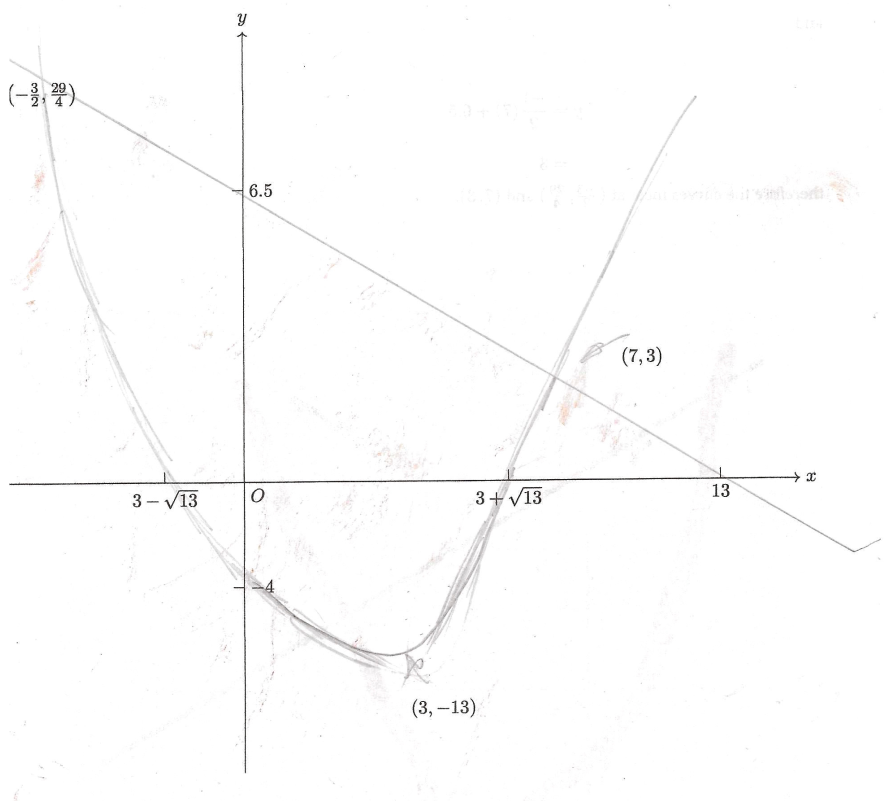
Substituting these back into our equation of the line;

$$\begin{aligned} y &= \frac{-1}{2}(\frac{-3}{2}) + 6.5 \\ &= \frac{29}{4} \end{aligned}$$

and

$$\begin{aligned} y &= \frac{-1}{2}(7) + 6.5 \\ &= 3 \end{aligned}$$

therefore the curves meet at $(\frac{-3}{2}, \frac{29}{4})$ and $(7, 3)$.



Question 2:

Solve:

$$\frac{2x-7}{3x+1} \geq 0$$

The critical values the values which make either the denominator or the numerator 0;

So for $2x-7$ when;

$$x = 3.5$$

$$2(3.5) - 7 = 0$$

For $3x+1$;

$$x = \frac{-1}{3}$$

$$3\left(\frac{-1}{3} + 1\right) = 0$$

We can use a table of signs:

	$(-\infty, \frac{-1}{3})$	$\frac{-1}{3}$	$(\frac{-1}{3}, 3.5)$	3.5	$(3.5, \infty)$
$2x-7$	$-$	$-$	$-$	0	$+$
$3x+1$	$-$	0	$+$	$+$	$+$
$\frac{2x-7}{3x+1}$	$+$	$*$	$-$	0	$+$

As we want the values $\geq\ 0$ we would use the $+$ and 0 values of the table of signs;

	$(-\infty, \frac{-1}{3})$	$\frac{-1}{3}$	$(\frac{-1}{3}, 3.5)$	3.5	$(3.5, \infty)$
$2x-7$	$-$	$-$	$-$	0	$+$
$3x+1$	$-$	0	$+$	$+$	$+$
$\frac{2x-7}{3x+1}$	$+$	$*$	$-$	0	$+$

Using this we get:

$$x \in \left(-\infty, \frac{-1}{3}\right) \cup [3.5, \infty)$$

Question 3:

a)
We model the population size y over time t in months using the equation:

$$y = Ae^{kt}$$

Where A and k are constants.

Given that after 2 months there were 30 rabbits and after 5 months there were 80 rabbits, we can set up the following pair of simultaneous equations:

$$30 = Ae^{2k} \quad (1)$$
$$80 = Ae^{5k} \quad (2)$$

Dividing equation (2) by equation (1) gives:

$$\frac{80}{30} = \frac{Ae^{5k}}{Ae^{2k}}$$

Applying the index laws, this simplifies to:

$$\frac{80}{30} = e^{5k-2k}$$

$$\frac{8}{3} = e^{3k}$$

Taking the natural logarithm of both sides:

$$\ln\left(\frac{8}{3}\right) = 3k$$

Solving for k, we get:

$$k = \frac{1}{3}\ln\left(\frac{8}{3}\right)$$

as required.

Now, to find A, using equation (1):

$$30 = Ae^{2k}$$

$$A = \frac{30}{e^{2k}}$$

We substitute $k = \frac{1}{3}\ln\left(\frac{8}{3}\right)$:

$$\begin{aligned} A &= \frac{30}{e^{2\frac{1}{3}\ln\left(\frac{8}{3}\right)}} \\ &= \frac{30}{e^{\frac{2}{3}\ln\left(\frac{8}{3}\right)}} \end{aligned}$$

Using the exponent rule $e^{a\ln(b)} = b^a$, we further simplify:

$$A = \frac{30}{\left(\frac{8}{3}\right)^{\frac{2}{3}}}$$

Calculating this, we find:

$$A \approx 15.6 \qquad \textit{(to 1 d.p.)}$$

b)
Using these values gives us:

$$y = 15.6e^{t\left(\frac{1}{3}\ln\left(\frac{8}{3}\right)\right)}$$

To find the time taken for the population to exceed 1000:

$$15.6e^{t\left(\frac{1}{3}\ln\left(\frac{8}{3}\right)\right)} = 1000$$

Divide both sides by 15.6:

$$e^{t\left(\frac{1}{3}\ln\left(\frac{8}{3}\right)\right)} = \frac{1000}{15.6}$$

Taking the natural logarithms of both sides:

$$t\frac{1}{3}\ln\left(\frac{8}{3}\right) = \ln\left(\frac{1000}{15.6}\right)$$

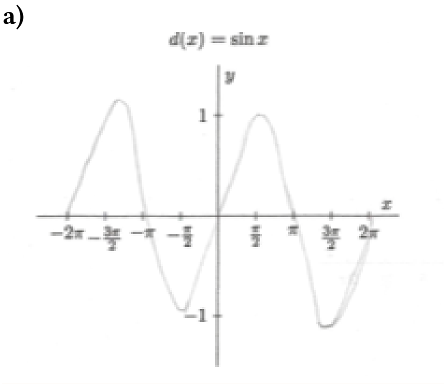
Dividing both sides by $\frac{1}{3}\ln\left(\frac{8}{3}\right)$:

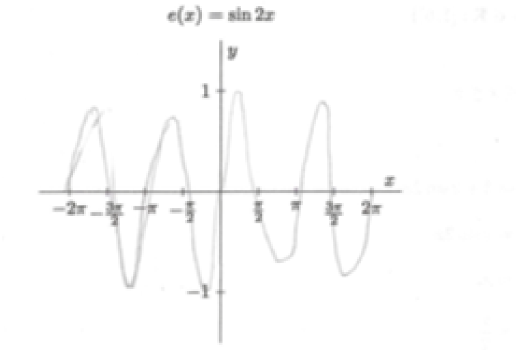
$$\begin{aligned} t &= \frac{\ln\left(\frac{1000}{15.6}\right)}{\frac{1}{3}\ln\left(\frac{8}{3}\right)} \\ t &\approx 12.72\ldots \end{aligned}$$

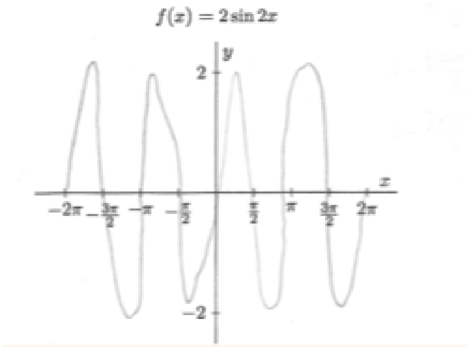
Therefore, using our model, it would take approximately 13 months for the population to exceed 1000.

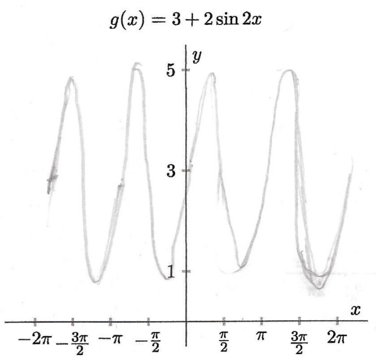
Question 4:

a)









b)

The image set for g is;

$$g(x) = \{x \in \mathbb{R} : [1, 6]\}$$

c)

$$-\pi \leq x \leq \pi$$

d)

$$h(x) = 3 + 2\sin 2x$$

$$3 + 2 = y \sin 2x$$

$$3 + 2\sin 2y = x$$

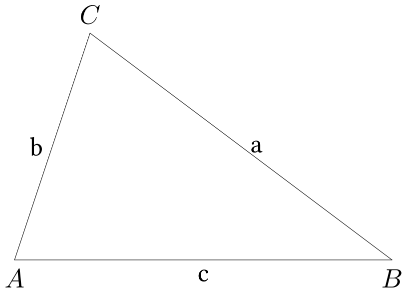
$$3 + \sin 2y = \frac{x}{2}$$

$$3 + y = \frac{\arcsin \frac{x}{2}}{2}$$

$$y = \frac{\arcsin \frac{x}{2}}{2} - 3$$

$$h^{-1}(x) = \frac{\arcsin \frac{x}{2}}{2} - 3$$

Question 5:



a)

Given $a = 5\,\text{cm}, b = 6\,\text{cm}$ and $C = 25°$. And using the cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c = \sqrt{5^2 + 6^2 - 2(5)(6)(\cos 25)}$$

$$= \sqrt{25 + 36 - 60\cos 25}$$

$$= \sqrt{61 - 54.378\ldots}$$

$$= 2.57\,\text{cm}$$ *to 2 d.p*

b)

Using the Sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{5} = \frac{\sin 25}{2.57}$$

$$\sin A = 5\left(\frac{\sin 25}{2.57}\right)$$

$$B = \arcsin 5\left(\frac{\sin 25}{2.57}\right)$$

$$= 55°$$ *To the nearest degree*

$$\frac{\sin B}{6} = \frac{\sin 25}{2.57}$$

$$\sin B = 6\left(\frac{\sin 25}{2.57}\right)$$

$$B = \arcsin 6\left(\frac{\sin 25}{2.57}\right)$$

$$= 80°$$ *and* $100°$ *To the nearest degree*

So in this case we will need to consider the obtuse angle of $100°$ as b is the longest side therefore will have the largest angle and the total angles in a triangle must total $180°$.

Question 6:

$$\sin^2(2x) - \cos(2x) - 1 = 0$$

Using the trig identity

$$\sin\theta + \cos\theta = 1$$

$$\begin{aligned}
\sin^2(2x) - \cos(2x) - 1 &= 0 \\
1 - \cos^2(2x) - \cos(2x) - 1 &= \\
\cos^2(2x)\cos(2x) &= \\
\cos(2x)(\cos(2x) + 1) &=
\end{aligned}$$

Thus, the solutions are

$$\begin{aligned}
\cos(2x) &= 0 \\
2x &= \arccos 0 \\
2x &= \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \\
x &= \frac{\pi}{4}, \frac{3\pi}{4}, \ldots
\end{aligned}$$

and

$$\begin{aligned}
\cos(2x) + 1 &= 0 \\
\cos(2x) &= -1 \\
2x &= \arccos -1 \\
2x &= \pi, 3\pi, \ldots \\
x &= \frac{\pi}{2}, \frac{3\pi}{2}, \ldots
\end{aligned}$$

So for the range

$$0 \leq x \leq \pi$$

The solutions are

$$x = \frac{\pi}{4}, \frac{\pi}{2} \text{ and } \frac{3\pi}{4}$$

Question 7:

a)
Using the general formula for a circle

$$(x-h)^2+(y-k)^2=r^2$$

Where $(x,y)$are coordinates of a point on the circle, and$(h,k)$are the coordinates of the centre of the circle.

$$6x^2+24x+6y^2-6y=12$$

$$x^2+4x+y^2-y=2$$

$$(x+2)^2-4+(y-\frac{1}{2})^2-\frac{1}{4}=2$$

$$(x+2)^2+(y-\frac{1}{2})^2=\frac{25}{4}$$

$$(x+2)^2+(y-\frac{1}{2})^2=(\frac{25}{2})^2$$

Giving a circle whose radius, $r=\frac{5}{2}$ and has a centre at $(-2,\frac{1}{2})$.

b)
The line $y=-3x+2$ intersects this circle at

$$(x+2)^2+(y-\frac{1}{2})^2=\frac{25}{4}$$

First expand the y terms out and simplifying

$$(x+2)^2+y^2-y+\frac{1}{4}=$$

$$(x+2)^2+y^2-y=6$$

By using substitution to solve the quadratic

$$(x+2)^2+(-3x+2)^2-(-3x+2)=$$

$$(x^2+4x+4)+(9x^2-12x+4)+(3x-2)=$$

$$10x^2-5x+6=$$

$$10x^2-5x=0$$

Factorising

$$x(10x-5)=0$$

Thus the solutions are $x=0$ and $x=\frac{1}{2}$.

Subsitituting these back into the linear equation

$$
\begin{aligned}
y &= -3x + 2 \\
&= -3(0) + 2 \\
&= 2
\end{aligned}
$$

and

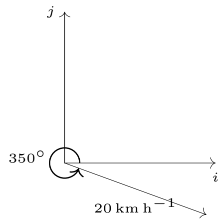
$$
\begin{aligned}
y &= -3x + 2 \\
&= -3(\frac{1}{2}) + 2 \\
&= \frac{-3}{2} + 2 \\
&= \frac{1}{2}
\end{aligned}
$$

Hence the line and circle interest at $(0, 2)$ and $(\frac{1}{2}, \frac{1}{2})$.

Question 8:

a)
The boat has a speed of $20\,\mathrm{km\,h^{-1}}$ and is traveling on a bearing of $100°$, this is the equivalent of $350°$ in standard trig angles (θ) measured anti-clockwise with the positive x-direction. Therefore, we can use

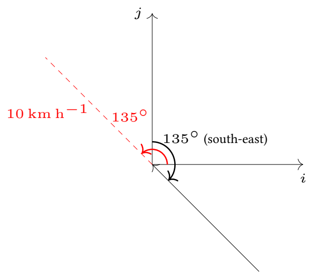
$$\mathbf{v} = |\mathbf{v}|\cos\theta\mathbf{i} + |\mathbf{v}|\sin\theta\mathbf{j}$$



$$\begin{aligned}\mathbf{b} &= 20\cos 350°\mathbf{i} + 20\sin 350°\mathbf{j}\\ &= 19.70\mathbf{i} - 3.47\mathbf{j}\end{aligned}$$

(to 2 d.p)

The current of the river is flowing at $10\,\mathrm{km\,h^{-1}}$ from a south-east direction, again using trig angles this gives $\theta = 135°$.



$$\begin{aligned}\mathbf{b} &= 10\cos 135°\mathbf{i} + 10\sin 135°\mathbf{j}\\ &= -5\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}\\ &= -7.07\mathbf{i} + 7.07\mathbf{j}\end{aligned}$$

(to 2 d.p)

b)
The resultant vector, $\mathbf{v}$, of the boat with the current is

$$\begin{aligned}\mathbf{v} &= \mathbf{b} + \mathbf{w} \\ &= 19.70\mathbf{i} - 5\sqrt{2}\mathbf{i} - 3.47\mathbf{j} + 5\sqrt{2}\mathbf{j} \\ &= 12.63\mathbf{i} + 3.60\mathbf{j}\end{aligned}$$

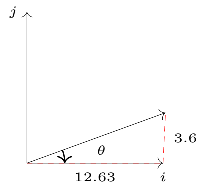
(to 2 d.p)

c)
With a velocity of

$$\begin{aligned}|\mathbf{v}| &= \sqrt{(12.62\ldots)^2 + (3.59\ldots)^2} \\ &= 13.13\,\mathrm{km\,h^{-1}}\end{aligned}$$

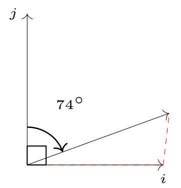
(to 2 d.p)

With a direction of



$$\begin{aligned}\tan\theta &= \frac{\text{opp}}{\text{adj}} \\ \theta &= \arctan\left(\frac{v_y}{v_x}\right) \\ &= \frac{3.59\ldots}{12.62\ldots} \\ &= 15.91^\circ\end{aligned}$$

(measured from the x-axis, to 2 d.p)



Thus the bearing of the boat is

$$90 - 15.88 = 74^\circ$$

(to the nearest degree)

Question 9:

$$f(x) = x^3 + 5x^2 + 8x - 1$$

a)
The stationary points of a curve are found at the points were the gradient is equal to 0.

$$f(x) = x^3 + 5x^2 + 8x - 1$$

$$f'(x) = 3x^2 + 10x + 8$$

factorising

$$0 = (3x + 4)(x + 2)$$

substituting these two solutions $x = -2$ and $x = \frac{-4}{3}$ back into our derivative

$$y = x^3 + 5x^2 + 8x - 1$$

$$= (-2)^3 + 5(-2)^2 + 8(-2) - 1$$

$$= -8 + 20 - 16 - 1$$

$$= -5$$

and

$$= \left(\frac{-4}{3}\right)^3 + 5\left(-\frac{-4}{3}\right)^2 + 8\left(\frac{-4}{3}\right) - 1$$

$$= \frac{-64}{27} + \frac{80}{9} + \frac{-32}{3} - 1$$

$$= \frac{-139}{27}$$

Hence the stationary points are

$$(-2, -5) \text{ and } \left(\frac{-3}{4}, \frac{-139}{27}\right)$$

b)

$$f(x) = x^3 + 5x^2 + 8x - 1$$

$$f'(x) = 3x^2 + 10x + 8$$

$$f''(x) = 6x + 10$$

Using the second derivative test

$$6(-2) + 10 = -2$$

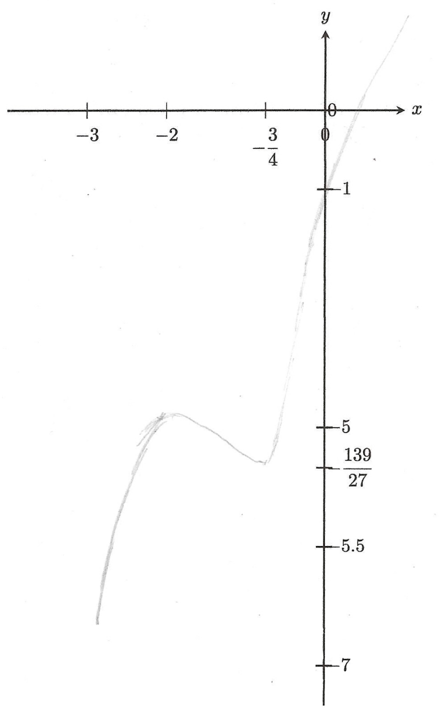
Thus a maximun

and

$$6\left(\frac{-4}{3}\right) + 10 = 2$$

Thus a minimum

c)



d)

In the interval $[-3, -1], . -7 \geq x \geq -5$

Question 10:

Given the equation of an object moving along a straight line with s (in m) from a reference point and t (in s)

$$s = t^4 - 12t^3 + 38t^2 - 28t + 5$$

(where $t \geq 0$)

a)

Hence the velocity, $v\,\mathrm{m\,s^{-1}}$ is given by

$$\begin{aligned} v &= \frac{\mathrm{d}s}{\mathrm{d}t} \\ &= 4t^3 - 36t^2 + 76t - 28 \end{aligned}$$

And the acceleration, $a\,\mathrm{m\,s^{-2}}$

$$\begin{aligned} a &= \frac{\mathrm{d}v}{\mathrm{d}t} \\ &= 12t^2 - 72t + 76 \end{aligned}$$

b)
Thus at time, $t = 2\,\mathrm{s}$

$$\begin{aligned} v &= 4t^3 - 36t^2 + 76t - 28 \\ &= 4(2)^3 - 36(2)^2 + 76(2) - 28 \\ &= 12\,\mathrm{m\,s^{-1}} \end{aligned}$$

and

$$\begin{aligned} a &= 12t^2 - 72t + 76 \\ &= 12(2)^2 - 72(2) + 76 \\ &= -20\,\mathrm{m\,s^{-2}} \end{aligned}$$

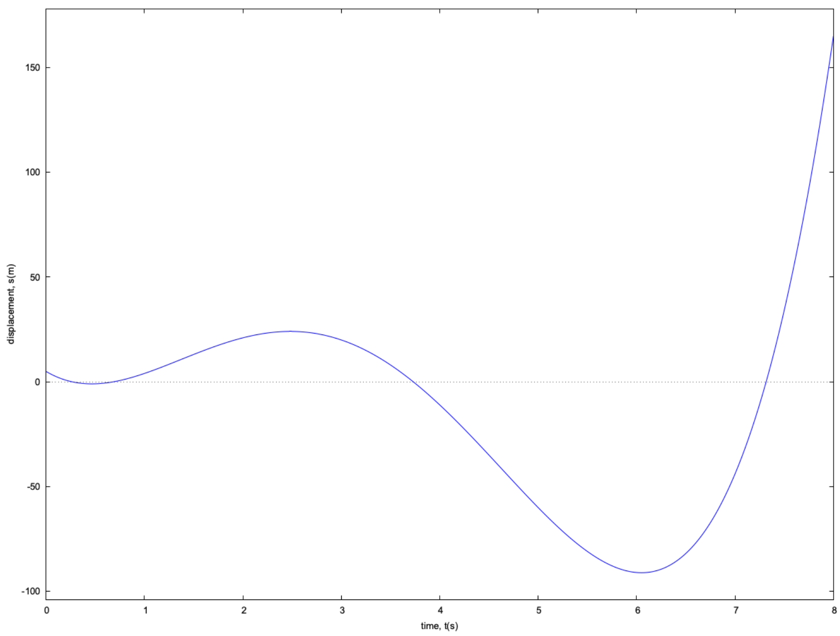
c)

(% i8) s: t^ 4 - 12*t^ 3 + 38*t^ 2 - 28*t + 5;

(s) $t^4 - 12t^3 + 38t^2 - 28t + 5$

(% i47) wxplot2d([s],[t,0,8], [xlabel,"time, t(s)"], [ylabel,"displacement, s(m)"]);

(% t47)



(% o47)

(% i31) factor(s);

(% o31) $\left(t^2 - 8t + 5\right)\left(t^2 - 4t + 1\right)$

(% i32) solve(t^ 2-8*t+5, t);

(% o32) $\left[t = 4 - \sqrt{11}, t = \sqrt{11} + 4\right]$

(% i33) solve(t^ 2 -4*t+1, t);

(% o33) $\left[t = 2 - \sqrt{3}, t = \sqrt{3} + 2\right]$

(% i37) root1 : 2 - sqrt(3);
root2 : 4 - sqrt(11);
root3 : 2 + sqrt(3);
root4 : 4 + sqrt(11);

(root1) $2-\sqrt{3}$

(root2) $4-\sqrt{11}$

(root3) $\sqrt{3}+2$

(root4) $\sqrt{11}+4$

(% i42) float(root1);

(% o42) 0.2679491924311228

0.27 (to 2 d.p)

(% i43) float(root2);

(% o43) 0.6833752096446002

0.68 (to 2 d.p)

(% i44) float(root3);

(% o44) 3.732050807568877

3.73 (to 2 d.p)

(% i45) float(root4);

(% o45) 7.3166247903554

7.32 (to 2 d.p)

Question 11:

From the feedback I received for the last TMA, I focused on showing all steps clearly in my trigonometric solutions, as suggested. I also ensured that my graphs were clearly labeled as hand-drawn sketches and not computer-generated, which was another area noted.

I feel confident in the topics covered in TMA01 and most of TMA02. However, I recognize that I need to work further on bearings, particularly transitioning away from using trig angle methods, which I naturally find more intuitive. I plan to review this area before the next TMA to strengthen my understanding.