

**Question 1:****a)**

The equation of a straight line has the general form

$$y = mx + c$$

Where m is the gradient and c is the intercept

Given our two coordinates (3, 5) and (5, 4):

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 5}{5 - 3}$$

$$= \frac{-1}{2}$$



3/3



Substituting this, and our coordinates:

$$mx + c = y$$

$$\frac{-1}{2}(3) + c = 5$$

$$-1.5 + c = 5$$

$$c = 6.5$$

Hence the equation of the line is;

$$y = \frac{-1}{2}x + 6.5$$

**b)**

Using:

$$ax^2 + bx + c = a(x + r)^2 + s$$

Given:

$$f(x) = x^2 - 6x - 4$$

By completing the square:

$$= (x - 3)^2 - 9 - 4$$

$$= (x - 3)^2 - 13$$



1/1

**c)**

Solve;

$$x^2 - 6x - 4 = 0$$

Using the completed square form;

$$(x - 3)^2 - 13 = 0$$

$$(x - 3)^2 = 13$$



$$x - 3 = \pm\sqrt{13}$$

$$x = 3 \pm \sqrt{13}$$



2/2

d)

For the line  $y = \frac{-1}{2}x + 6.5$ .

The  $y$  - *intercept* is when  $x = 0$  therefore  $y = 6.5$ .

The  $x$  - *intercept* is when  $y = 0$  therefore  $x = 13$ .

For the curve  $y = x^2 + 6x - 4$ ; The solutions are  $3 \pm \sqrt{13}$ .

The  $y$  - *intercept* is when  $x = 0$  therefore  $y = -4$ .

The turning point;

$$y = x^2 - 6x - 4$$

$$\frac{dy}{dx} = 2x - 6$$

$$2x - 6 = 0$$

$$2x = 6$$

$$x = 3$$



Substituting this back into the equation;

$$y = 3^2 - 6(3) - 4$$

$$y = -13$$

Therefore  $(3, -13)$

And the two curves meet at;

$$\frac{-1}{2}x + 6.5 = x^2 - 6x - 4$$

$$\frac{-1}{2}x = x^2 - 6x - \frac{21}{2}$$

$$0 = x^2 - 5.5x - \frac{21}{2}$$

$$= 2x^2 - 11x - 21$$

$$= (2x + 3)(x - 7)$$

the x-coordinates are  $x = \frac{-3}{2}$  and  $x = 7$

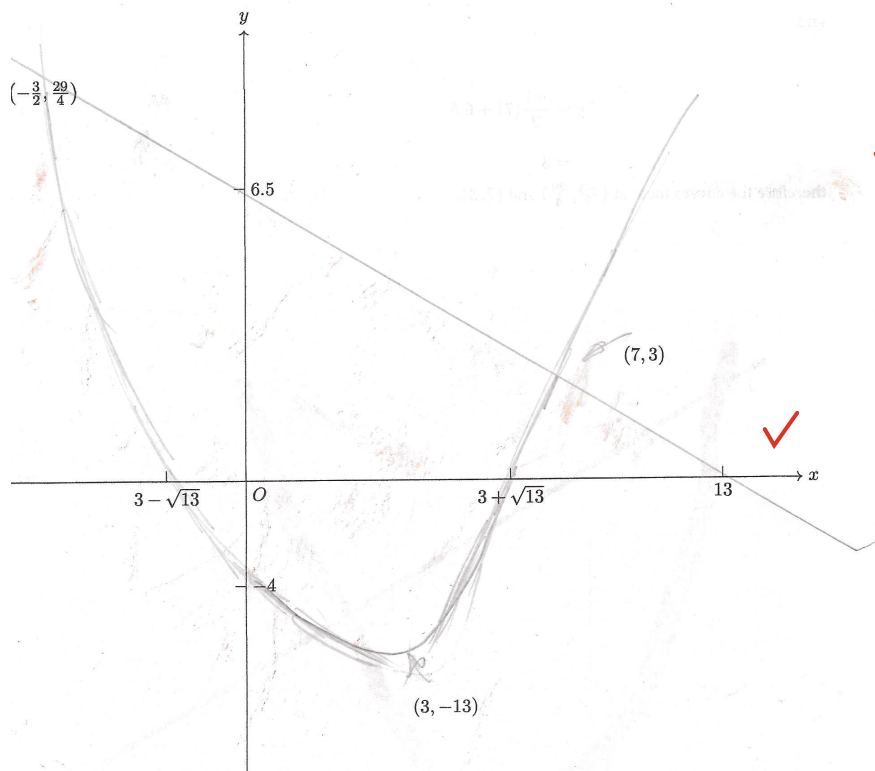
Substituting these back into our equation of the line;

$$\begin{aligned} y &= \frac{-1}{2} \left( \frac{-3}{2} \right) + 6.5 \\ &= \frac{29}{4} \end{aligned}$$

and

$$\begin{aligned} y &= \frac{-1}{2} (7) + 6.5 \\ &= 3 \end{aligned}$$

therefore the curves meet at  $(\frac{-3}{2}, \frac{29}{4})$  and  $(7, 3)$ .



3/3

Q1 9/9

**Question 2:**

Solve:

$$\frac{2x-7}{3x+1} \geq 0 \quad 3/4$$

The critical values the values which make either the denominator or the numerator 0;

So for  $2x - 7$  when;

$$x = 3.5$$

$$2(3.5) - 7 = 0$$

For  $3x + 1$ ;

$$x = \frac{-1}{3}$$

$$3\left(\frac{-1}{3} + 1\right) = 0$$

We can use a table of signs:

	$(-\infty, \frac{-1}{3})$	$\frac{-1}{3}$	$(\frac{-1}{3}, 3.5)$	3.5	$(3.5, \infty)$
$2x - 7$	—	—	—	0	+
$3x + 1$	—	0	+	+	+
$\frac{2x-7}{3x+1}$	+	*	—	0	+

As we want the values  $\geq 0$  we would use the + and 0 values of the table of signs;

	$(-\infty, \frac{-1}{3})$	$\frac{-1}{3}$	$(\frac{-1}{3}, 3.5)$	3.5	$(3.5, \infty)$
$2x - 7$	—	—	—	0	+
$3x + 1$	—	0	+	+	+
$\frac{2x-7}{3x+1}$	+	*	—	0	+

Using this we get:

$$x \in \left(-\infty, \frac{-1}{3}\right) \cup [3.5, \infty)$$

**Question 3:****a)**

We model the population size  $y$  over time  $t$  in months using the equation:

$$y = Ae^{kt}$$

Where  $A$  and  $k$  are constants.

Given that after 2 months there were 30 rabbits and after 5 months there were 80 rabbits, we can set up the following pair of simultaneous equations:

$$30 = Ae^{2k} \quad \checkmark \quad (1)$$

$$80 = Ae^{5k} \quad (2)$$

Dividing equation (2) by equation (1) gives:

$$\frac{80}{30} = \frac{Ae^{5k}}{Ae^{2k}}$$

Applying the index laws, this simplifies to:

$$\frac{80}{30} = e^{5k-2k}$$

$$\frac{8}{3} = e^{3k} \quad \checkmark$$

Taking the natural logarithm of both sides:

$$\ln\left(\frac{8}{3}\right) = 3k$$

Solving for  $k$ , we get:  $\checkmark$

$$k = \frac{1}{3} \ln\left(\frac{8}{3}\right)$$

*as required.*

Now, to find  $A$ , using equation (1):

$$30 = Ae^{2k}$$

$$A = \frac{30}{e^{2k}}$$

We substitute  $k = \frac{1}{3} \ln\left(\frac{8}{3}\right)$ :

$$A = \frac{30}{e^{2\frac{1}{3}\ln\left(\frac{8}{3}\right)}}$$

$$= \frac{30}{e^{\frac{2}{3}\ln\left(\frac{8}{3}\right)}}$$

Using the exponent rule  $e^{a \ln(b)} = b^a$ , we further simplify:

4/4

$$A = \frac{30}{\left(\frac{8}{3}\right)^{\frac{2}{3}}}$$

Calculating this, we find:



$$A \approx 15.6 \quad (\text{to 1 d.p.})$$

**b)**

Using these values gives us:

$$y = 15.6e^{t\left(\frac{1}{3}\ln\left(\frac{8}{3}\right)\right)}$$

To find the time taken for the population to exceed 1000:

$$15.6e^{t\left(\frac{1}{3}\ln\left(\frac{8}{3}\right)\right)} = 1000$$

2/2

Divide both sides by 15.6:

$$e^{t\left(\frac{1}{3}\ln\left(\frac{8}{3}\right)\right)} = \frac{1000}{15.6}$$

Taking the natural logarithms of both sides:



$$t\frac{1}{3}\ln\left(\frac{8}{3}\right) = \ln\left(\frac{1000}{15.6}\right)$$

Dividing both sides by  $\frac{1}{3}\ln\left(\frac{8}{3}\right)$ :

$$t = \frac{\ln\left(\frac{1000}{15.6}\right)}{\frac{1}{3}\ln\left(\frac{8}{3}\right)}$$

$$t \approx 12.72 \dots$$

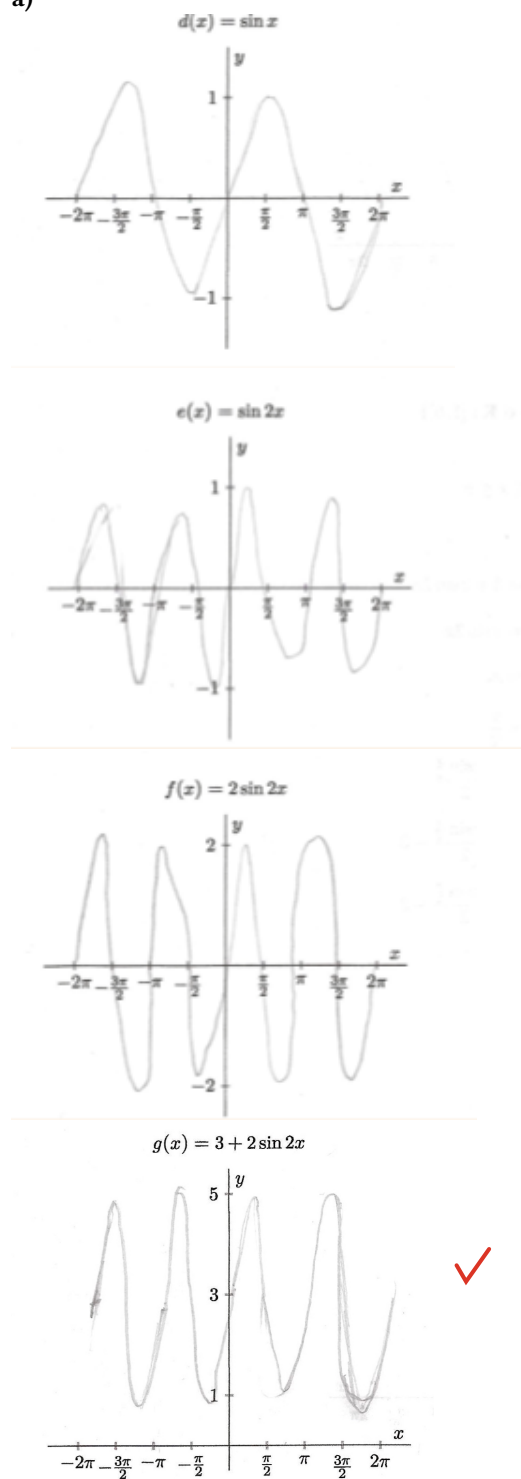


Q3 6/6

Therefore, using our model, it would take approximately 13 months for the population to exceed 1000.

**Question 4:**

a)



b)

The image set for  $g$  is;

$$g(x) = \{x \in \mathbb{R} : [1, 6]\}$$

✗

0/1 The image set only goes upto 5, therefore [1,5]

c)

$$-\pi \leq x \leq \pi$$



0/1 As a one-to-one function  $k=\pi/4$

d)

$$h(x) = 3 + 2 \sin 2x$$

$$3 + 2 = y \sin 2x$$



0/4 Your order of operations are mathematically incorrect.

$$3 + 2 \sin 2y = x$$

$$3 + \sin 2y = \frac{x}{2}$$



Please see the correct solution below

$$3 + y = \frac{\arcsin \frac{x}{2}}{2}$$



$$y = \frac{\arcsin \frac{x}{2}}{2} - 3$$

$$h^{-1}(x) = \frac{\arcsin \frac{x}{2}}{2} - 3$$



$$y = 3 + 2 \sin 2x$$

$$y - 3 = 2 \sin 2x$$

$$\frac{y - 3}{2} = \sin 2x$$

$$\sin^{-1} \left( \frac{y - 3}{2} \right) = 2x$$

$$\frac{1}{2} \sin^{-1} \left( \frac{y - 3}{2} \right) = x$$

$$h^{-1}(x) = \frac{1}{2} \sin^{-1} \left( \frac{x - 3}{2} \right)$$

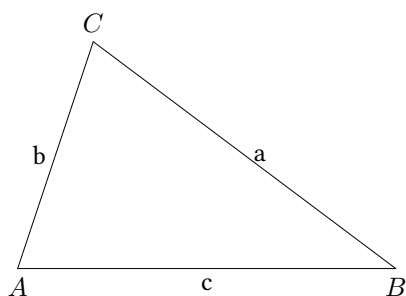
Image set of  $h^{-1}(x)$  = domain of  $h(x)$ :  $[-\frac{\pi}{4}, \frac{\pi}{4}]$ .

Domain of  $h^{-1}(x)$  = image set of  $h(x)$ :  $[1, 5]$ .

### References

Activity 36 page 260 of Book A (Unit 3).



**Question 5:****a)**Given  $a = 5$  cm,  $b = 6$  cm and  $C = 25^\circ$ . And using the cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$c = \sqrt{5^2 + 6^2 - 2(5)(6)(\cos 25)}$$

$$= \sqrt{25 + 36 - 60 \cos 25}$$

$$= \sqrt{61 - 54.378 \dots}$$

$$= 2.57 \text{ cm} \quad \text{to 2 d.p.}$$



2/2

**b)**

Using the Sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin A}{5} = \frac{\sin 25}{2.57}$$



$$\sin A = 5 \left( \frac{\sin 25}{2.57} \right)$$

$$B = \arcsin 5 \left( \frac{\sin 25}{2.57} \right)$$

$$= 55^\circ$$

To the nearest degree



You must find both the values angle A could be.

ie  $180 - 55.2042 \dots = 124.795 \dots$  degrees

$$\frac{\sin B}{6} = \frac{\sin 25}{2.57}$$

$$\sin B = 6 \left( \frac{\sin 25}{2.57} \right)$$

$$B = \arcsin 6 \left( \frac{\sin 25}{2.57} \right)$$

$$= 80^\circ$$

and To the nearest degree



So in this case we will need to consider the obtuse angle of  $100^\circ$  as  $b$  is the longest side therefore will have the largest angle and the total angles in a triangle must total  $180^\circ$ .



3/4 GMC alert! you must provide full answers prior to rounding. See the working I have provided above for angle A

Q5 5/6

**Question 6:**

$$\sin^2(2x) - \cos(2x) - 1 = 0$$

Using the trig identity

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2(2x) - \cos(2x) - 1 = 0$$

$$1 - \cos^2(2x) - \cos(2x) - 1 = 0 \quad \checkmark$$

$$-\cos^2(2x) - \cos(2x) = 0$$

$$\cos(2x)(\cos(2x) + 1) = 0 \quad \checkmark$$

Thus, the solutions are

$$\cos(2x) = 0$$

$$2x = \arccos 0$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots \quad \checkmark$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \dots \quad \checkmark$$

and

$$\cos(2x) + 1 = 0$$

Q6 6/6

$$\cos(2x) = -1$$

$$2x = \arccos -1$$

$$2x = \pi, 3\pi, \dots$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$$

So for the range

$$0 \leq x \leq \pi$$

The solutions are

$$x = \frac{\pi}{4}, \frac{3\pi}{4} \text{ and } \frac{5\pi}{4}, \frac{7\pi}{4} \quad \checkmark \quad \checkmark$$

**Question 7:****a)**

Using the general formula for a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

Where  $(x, y)$  are coordinates of a point on the circle, and  $(h, k)$  are the coordinates of the centre of the circle.

$$6x^2 + 24x + 6y^2 - 6y = 12$$

$$x^2 + 4x + y^2 - y = 2$$

$$(x + 2)^2 - 4 + (y - \frac{1}{2})^2 - \frac{1}{4} = 2$$

$$(x + 2)^2 + (y - \frac{1}{2})^2 = \frac{25}{4}$$

$$(x + 2)^2 + (y - \frac{1}{2})^2 = (\frac{5}{2})^2$$

4/4

Giving a circle whose radius,  $r = \frac{5}{2}$  and has a centre at  $(-2, \frac{1}{2})$ .**b)**The line  $y = -3x + 2$  intersects this circle at

$$(x + 2)^2 + (y - \frac{1}{2})^2 = \frac{25}{4}$$

First expand the  $y$  terms out and simplifying

$$(x + 2)^2 + y^2 - y + \frac{1}{4} =$$

$$(x + 2)^2 + y^2 - y = 6$$

By using substitution to solve the quadratic

$$(x + 2)^2 + (-3x + 2)^2 - (-3x + 2) =$$

$$(x^2 + 4x + 4) + (9x^2 - 12x + 4) + (3x - 2) =$$

$$10x^2 - 5x + 6 =$$

$$10x^2 - 5x = 0$$

Factorising

$$x(10x - 5) = 0$$

Thus the solutions are  $x = 0$  and  $x = \frac{1}{2}$ .

Substituting these back into the linear equation

$$y = -3x + 2$$

$$= -3(0) + 2$$

$$= 2$$

4/4

and

$$y = -3x + 2$$

$$= -3\left(\frac{1}{2}\right) + 2$$

$$= \frac{-3}{2} + 2$$

$$= \frac{1}{2}$$

Hence the line and circle intersect at  $(0, 2)$  and  $(\frac{1}{2}, \frac{1}{2})$ .

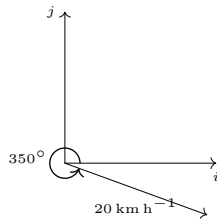


Q7 8/8 Excellent!

**Question 8:****a)**

The boat has a speed of  $20 \text{ km h}^{-1}$  and is traveling on a bearing of  $100^\circ$ , this is the equivalent of  $350^\circ$  in standard trig angles ( $\theta$ ) measured anti-clockwise with the positive  $x$ -direction. Therefore, we can use

$$\mathbf{v} = |\mathbf{v}| \cos \theta \mathbf{i} + |\mathbf{v}| \sin \theta \mathbf{j}$$

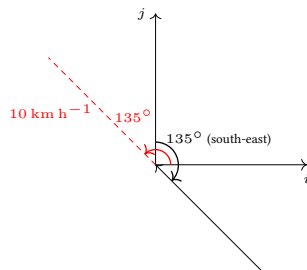


$$\mathbf{b} = 20 \cos 350^\circ \mathbf{i} + 20 \sin 350^\circ \mathbf{j}$$

$$= 19.70\mathbf{i} - 3.47\mathbf{j}$$

(to 2 d.p)

The current of the river is flowing at  $10 \text{ km h}^{-1}$  from a south-east direction, again using trig angles this gives  $\theta = 135^\circ$ .



7/7

$$\mathbf{b} = 10 \cos 135^\circ \mathbf{i} + 10 \sin 135^\circ \mathbf{j}$$

$$= -5\sqrt{2}\mathbf{i} + 5\sqrt{2}\mathbf{j}$$

$$= -7.07\mathbf{i} + 7.07\mathbf{j}$$

(to 2 d.p)

**b)**

The resultant vector,  $\mathbf{v}$ , of the boat with the current is

$$\mathbf{v} = \mathbf{b} + \mathbf{w}$$

$$= 19.70\mathbf{i} - 5\sqrt{2}\mathbf{i} - 3.47\mathbf{j} + 5\sqrt{2}\mathbf{j}$$

$$= 12.63\mathbf{i} + 3.60\mathbf{j}$$

(to 2 d.p)

3/3

c)

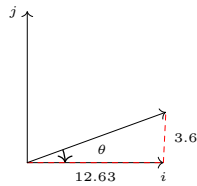
With a velocity of

$$|\mathbf{v}| = \sqrt{(12.62\dots)^2 + (3.59\dots)^2}$$

$$= 13.13 \text{ km h}^{-1}$$

(to 2 d.p)

With a direction of



$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

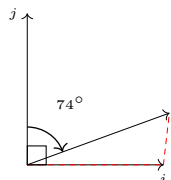
$$\theta = \arctan\left(\frac{v_y}{v_x}\right)$$

$$= \frac{3.59\dots}{12.62\dots}$$

$$= 15.91^\circ$$

(measured from the x-axis, to 2 d.p)

5/5



Thus the bearing of the boat is

$$90 - 15.88 = 74^\circ$$

(to the nearest degree)

Q9 15/15

A very well answered question!

**Question 9:**

$$f(x) = x^3 + 5x^2 + 8x - 1$$

3/4 GMC

**a)**

The stationary points of a curve are found at the points where the gradient is equal to 0.

$$f(x) = x^3 + 5x^2 + 8x - 1$$

$$f'(x) = 3x^2 + 10x + 8$$



factorising

$$0 = (3x + 4)(x + 2)$$



substituting these two solutions  $x = -2$  and  $x = -\frac{4}{3}$  back into our derivative



$$\begin{aligned} y &= x^3 + 5x^2 + 8x - 1 \\ &= (-2)^3 + 5(-2)^2 + 8(-2) - 1 \\ &= -8 + 20 - 16 - 1 \\ &= -5 \end{aligned}$$

and

$$\begin{aligned} &= \left(-\frac{4}{3}\right)^3 + 5\left(-\frac{4}{3}\right)^2 + 8\left(-\frac{4}{3}\right) - 1 \\ &= \frac{-64}{27} + \frac{80}{9} + \frac{-32}{3} - 1 \\ &= \frac{-139}{27} \end{aligned}$$

6/6

Hence the stationary points are

$$(-2, -5) \text{ and } \left(-\frac{4}{3}, \frac{-139}{27}\right)$$

**b)**

$$f(x) = x^3 + 5x^2 + 8x - 1$$

$$f'(x) = 3x^2 + 10x + 8$$

$$f''(x) = 6x + 10$$



Using the second derivative test

$$6(-2) + 10 = -2$$

Thus a maximum





and

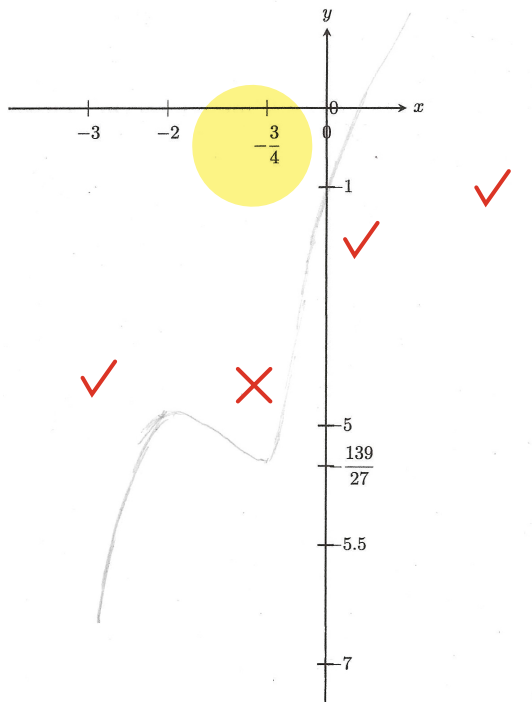
$$6\left(\frac{-4}{3}\right) + 10 = 2$$

✓

3/3

*Thus a minimum*

c)



3/4 It is best if you write on the coordinates of the turning points.

The lost of mark was due to the  $-3/4$  rather than  $-4/3$

d)

In the interval  $[-3, -1]$ ,  $-7 \geq x \geq -5$

✓

✓

2/2

Q9 14/15

**Question 10:**

Given the equation of an object moving along a straight line with  $s$  (in m) from a reference point and  $t$  (in s)

$$s = t^4 - 12t^3 + 38t^2 - 28t + 5$$

(where  $t \geq 0$ )

**a)**

Hence the velocity,  $\text{m s}^{-1}$  is given by

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= 4t^3 - 36t^2 + 76t - 28 \end{aligned}$$



2/2

And the acceleration,  $\text{m s}^{-2}$

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= 12t^2 - 72t + 76 \end{aligned}$$



**b)**

Thus at time,  $t = 2 \text{ s}$

$$\begin{aligned} v &= 4t^3 - 36t^2 + 76t - 28 \\ &= 4(2)^3 - 36(2)^2 + 76(2) - 28 \\ &= 12 \text{ m s}^{-1} \end{aligned}$$



and

$$\begin{aligned} a &= 12t^2 - 72t + 76 \\ &= 12(2)^2 - 72(2) + 76 \\ &= -20 \text{ m s}^{-2} \end{aligned}$$



2/2

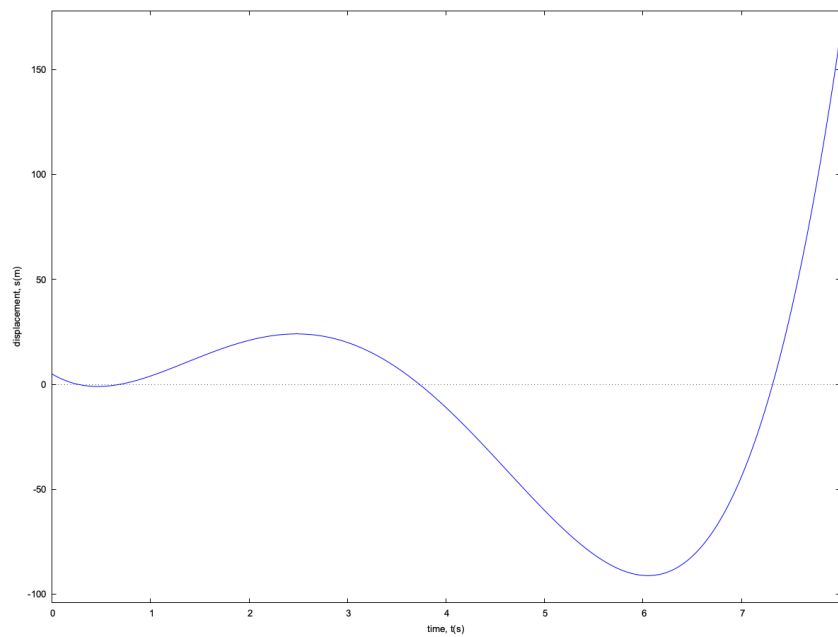
**c)**

```
(% i8) s: t^4 - 12*t^3 + 38*t^2 - 28*t + 5;
```

```
(s)  $t^4 - 12t^3 + 38t^2 - 28t + 5$ 
```

```
(% i47) wxplot2d([s],[t,0,8], [xlabel,"time, t(s)"], [ylabel,"displacement, s(m)"]);
```

```
(% t47)
```



✓

3/3

✓

✓

```
(% o47)
```

```
(% i31) factor(s);
```

```
(% o31)  $(t^2 - 8t + 5)(t^2 - 4t + 1)$ 
```

```
(% i32) solve(t^2 - 8*t + 5, t);
```

```
(% o32)  $[t = 4 - \sqrt{11}, t = \sqrt{11} + 4]$ 
```

```
(% i33) solve(t^2 - 4*t + 1, t);
```

```
(% o33)  $[t = 2 - \sqrt{3}, t = \sqrt{3} + 2]$ 
```

```
(% i37) root1 : 2 - sqrt(3);
      root2 : 4 - sqrt(11);
      root3 : 2 + sqrt(3);
      root4 : 4 + sqrt(11);
```

(root1)  $2 - \sqrt{3}$

(root2)  $4 - \sqrt{11}$

(root3)  $\sqrt{3} + 2$

(root4)  $\sqrt{11} + 4$

```
(% i42) float(root1);
```



4/4

(% o42) 0.2679491924311228

0.27 (to 2 d.p)

```
(% i43) float(root2);
```



(% o43) 0.6833752096446002

0.68 (to 2 d.p)

```
(% i44) float(root3);
```



Q10 11/11

(% o44) 3.732050807568877

3.73 (to 2 d.p)

```
(% i45) float(root4);
```



(% o45) 7.3166247903554

7.32 (to 2 d.p)

**Question 11:**

From the feedback I received for the last TMA, I focused on showing all steps clearly in my trigonometric solutions, as suggested. I also ensured that my graphs were clearly labeled as hand-drawn sketches and not computer-generated, which was another area noted.

I feel confident in the topics covered in TMA01 and most of TMA02. However, I recognize that I need to work further on bearings, particularly transitioning away from using trig angle methods, which I naturally find more intuitive. I plan to review this area before the next TMA to strengthen my understanding.

5/5 Well done on acting on the feedback given.

b) 5/5