

\documentclass[a4paper]{tufte-handout}

\usepackage{/Users/paulallen/OU/Maths/style}

\begin{document}

\tma{03}

%%%%%%%%%%Question one

\begin{question}

\qpart

\marginnote{

Product rule, for

\begin{align*}

$f(x) \cdot g(x)h(x)$

$f'(x) \cdot g(x)h(x) + f(x) \cdot g'(x)h(x) + f(x) \cdot g(x)h'(x)$

\end{align*}}

\begin{align*}

$f(x) = (x^5 + 3x^3 + 2x + 1)e^x$ [8pt]

\text{Using}

$g(x) = x^5 + 3x^3 + 2x + 1$ [8pt]

\text{and}

$h(x) = e^x$ [8pt]

\text{Differentiating}

$g'(x) = 5x^4 + 9x^2 + 2$ [8pt]

\text{and}

$h'(x) = e^x$ [8pt]

\text{Using the product rule}

$f'(x) = (5x^4 + 9x^2 + 2)e^x + e^x(x^5 + 3x^3 + 2x + 1)$ [8pt]

\text{Simplifying}

$= (x^5 + 5x^4 + 3x^3 + 9x^2 + 2x + 3)e^x$

\end{align*}

\vspace{5cm}

\qpart

\marginnote{

Chain rule, for

\begin{align*}

$g(y) = i(h(y))$ [8pt]

$g'(y) = i'(h(y))h'(y)$

\end{align*}}

\begin{align*}

$g(y) = \ln(y) + \sin(y)^6$ [8pt]

\text{Using}

$h(y) = \ln(y) + \sin(y)$ [8pt]

\text{and}

$i(h) = h^6$ [8pt]

\text{Differentiating}

$h'(y) = \frac{1}{y} + \cos(y)$ [8pt]

\text{and}

$i'(h) = 6h^5$ [8pt]

\text{Using the chain rule}

$g'(y) = 6\sqrt[5]{\ln(y) + \sin(y)} \cdot \left(\frac{1}{y} + \cos(y)\right)$

\end{align*}

\vspace{5cm}

\qpart

\marginnote{
Quotient rule, for

\begin{align*}
h(z) &= \frac{i(z)}{j(z)} \\ h^{\prime}(z) &= \frac{j(z)i^{\prime}(z) - i(z)j^{\prime}(z)}{(j(z))^2} \\ \end{align*}

\begin{align*}
h(z) &= \frac{e^{5z}}{(2 + \cos(10z))} \\ \text{Using} \\ i(z) &= e^{5z} \\ \text{and} \\ j(z) &= (2 + \cos(10z)) \\ \text{Differentiating} \\ i^{\prime}(z) &= 5e^{5z} \\ \text{and} \\ j^{\prime}(z) &= -10\sin(10z) \\ \text{Using the quotient rule} \\ h^{\prime}(z) &= \frac{(2 + \cos(10z))5e^{5z} - e^{5z}(-10\sin(10z))}{(2 + \cos(10z))^2} \\ \text{Simplifying} \\ &= \frac{5e^{5z}(2 + \cos(10z)) + 10e^{5z}\sin(10z)}{(2 + \cos(10z))^2} \\ \end{align*}

\vspace{5cm}

\qpart

\marginnote{
\begin{align*}
\text{Product rule: }
k(x) &= l(x)m(x) \\ k^{\prime}(x) &= l(x)m^{\prime}(x) + l^{\prime}(x)m(x) \\ \text{Chain rule} \\ m^{\prime}(x) &= u^{\prime}(v(x))v^{\prime}(x) \\ \end{align*}
}

\begin{align*}
k(x) &= x^2\sin(\cos{x}) \\ \text{Using} \\ & \& l(x) = x^2 \quad \text{and} \quad m(x) = \sin(\cos{x}) \\ \text{Differentiating using the product rule} \\ & \& l^{\prime}(x) = 2x \\ \text{Now using the chain rule to find } (m^{\prime}(x)), \text{ using } (u = \sin(x)) \text{ and } (v = \cos(x)) \\ u^{\prime} &= \cos(x) \\ \text{and} \\ v^{\prime} &= -\sin(x) \\ \text{Thus} \\ m^{\prime} &= \cos(\cos{x})(-\sin(x)) \\ \text{Applying the product rule} \\ k^{\prime}(x) &= x^2(\cos(\cos{x})(-\sin(x))) + 2x\sin(\cos{x}) \\ \text{Simplifying} \\ &= 2x\sin(\cos{x}) - x^2\sin(x)\cos(\cos{x}) \\ \end{align*}

\end{question}

\clearpage

%%%Question two

\begin{question}

Given the L-shaped enclosure

```
\begin{center}
\begin{tikzpicture}
\tkzDefPoint(-2,0){a}
\tkzDefPoint(4,0){b}
\tkzDefPoint(4,2){c}
\tkzDefPoint(0,2){d}
\tkzDefPoint(0,4){e}
\tkzDefPoint(-2,4){f}
\tkzDrawSegments(a,b b,c c,d d,e e,f f,a)
\tkzLabelSegment[below](a,b){$x+5$}
\tkzLabelSegment[right](b,c){$y$}
\tkzLabelSegment[above](c,d){$5$}
\tkzLabelSegment[right](d,e){$y$}
\tkzLabelSegment[above](e,f){$x$}
\tkzLabelSegment[left](f,a){$2y$}
\end{tikzpicture}
\end{center}
```

\qpart

```
\begin{align*}
&\text{Using the assumption that Steven uses all the fencing he has exactly the perimeter is } \backslash
(\text{SI}{74}{\text{metre}} \backslash)
\text{perimeter } \&= \backslash \text{rb}{x + 5} + y + 5 + y + x + 2y \backslash \backslash [8pt]
74 \&= 4y + 2x + 10 \backslash \backslash [8pt]
64 \&= 4y + 2x \backslash \backslash [8pt]
4y \&= 64 - 2x \backslash \backslash [8pt]
y \&= 16 - \frac{x}{2} \backslash \backslash [8pt]
\&= \frac{1}{2} \backslash \text{rb}{32 - x} \backslash \backslash [8pt]
&\text{\snote{as required}}
\end{align*}
```

\vspace{5cm}

\qpart

```
\begin{center}
\begin{tikzpicture}
\tkzDefPoint(-2,0){a}
\tkzDefPoint(4,0){b}
\tkzDefPoint(4,2){c}
\tkzDefPoint(0,2){d}
\tkzDefPoint(0,4){e}
\tkzDefPoint(-2,4){f}
\tkzDefPoint(0,0){g}
\tkzDrawSegments(a,b b,c c,d d,e e,f f,a e,g)
\tkzLabelSegment[right](b,c){$y$}
\tkzLabelSegment[above](c,d){$5$}
\tkzLabelSegment[above](e,f){$x$}
\tkzLabelSegment[left](f,a){$2y$}
\end{tikzpicture}
\end{center}
```

```
\end{tikzpicture}
\end{center}
```

The area of the L-shape is given by the total of the two shapes shown above.

```
\begin{align*}
A &= x^2 + 5x \\
&= x^2 + \frac{1}{2}x^2 + 5x + \frac{1}{2}x^2 \\
&= x^2 + 32 - x + 80 - \frac{5x}{2} \\
&= 32x - x^2 + 80 - \frac{5x}{2} \\
&\text{Multiply by } (2) \\
2A &= 160 + 64x - 5x - 2x^2 \\
&\text{Collect like terms} \\
&= 160 + 59x - 2x^2 \\
&\text{simplify} \\
A &= \frac{1}{2}(160 + 59x - 2x^2) \\
&\text{snote{as required}}
\end{align*}
```

```
\vspace{5cm}
```

```
\qpart
```

```
\begin{tikzpicture}
\begin{axis}[
width=12cm,
height=8cm,
xlabel={x},
ylabel={A(x)},
ymode=log,
log basis y={10},
ymin=0, ymax=300,
xmin=0, xmax=32,
domain=0:32,
samples=500,
grid=both,
grid style={line width=.1pt, draw=gray!10},
major grid style={line width=.2pt, draw=gray!50},
axis lines=middle,
legend pos=north west,
xlabel style={font=\large},
ylabel style={font=\large},
tick label style={font=\large},
]
\addplot [blue, thick] {0.5*(160 + 59*x - 2*x^2)};
\addlegendentry{$A(x) = \frac{1}{2}(160 + 59x - 2x^2)$}

% Mark the vertex
\fill (14.75,297.56) circle (3pt) node[above right] {Vertex};

% Mark the y-intercept
\fill (0,80) circle (3pt) node[above left] {(0,80)};

% Mark the asymptote at x=32
\draw[dashed, red] (32,10) -- (32,300) node[above] {Asymptote at $x=32$};
\end{axis}
\end{tikzpicture}
```

Based on the shape of the curve for this graph we need only consider the stationary point at which $\frac{dA}{dx} = 0$ to find the maximum area.

```

\begin{align*}
\text{\text{Given}}
A &= \frac{1}{2} \text{rb}\{160 + 59x - 2x^2\} \\\[8pt]
\text{\text{Differentiating}}
A^{\prime} &= -2x + \frac{59}{2} \\\[8pt]
\text{\text{Setting } ( A^{\prime} = 0 ) \text{ to find the stationary point}}
0 &= -2x + \frac{59}{2} \\\[8pt]
2x &= \frac{59}{2} \\\[8pt]
x &= \frac{59}{4} \\\[8pt]
\text{\text{Substituting this into the original equation}}
A &= \frac{1}{2} \text{rb}\{160 + 59x - 2x^2\} \\\[8pt]
&= \frac{1}{2} \text{rb}\{160 + 59 \text{rb}\{\frac{59}{4}\} - 2 \text{rb}\{\frac{59}{4}\}^2\} \\\[8pt]
&= 80 + \frac{3481}{8} - \text{rb}\{\frac{59}{4}\}^2 \\\[8pt]
&= 80 + \frac{3481}{8} - \text{rb}\{\frac{3481}{16}\} \\\[8pt]
&= \frac{1280}{16} + \frac{6962}{16} - \text{rb}\{\frac{3481}{16}\} \\\[8pt]
&= \frac{1280}{16} + \frac{3481}{16} \\\[8pt]
&= \frac{4761}{16} \\\[8pt]
\text{\text{Applying the second derivative test}}
A^{\prime} &= -2x + \frac{59}{2} \\\[8pt]
A^{\prime\prime} &= -2 \\\[8pt]
\text{\text{Showing that this a maximum stationary point}}
\end{align*}

```

Hence the maximum area of the L-shape is
 $[A = \frac{4761}{16} \text{unit}\{\text{metre}\text{squared}\}]$

\end{question}

\clearpage
 %%%Question three

\begin{question}

\qpart

```

\begin{align*}
f(x) &= x^2 + 2x + 5 \\\[8pt]
\text{\text{Then the indefinite integral is}}
F(x) &= \frac{x^3}{3} + x^2 + 5x + c \\\[8pt]
\end{align*}

```

\vspace{2cm}

\qpart

```

\begin{align*}
g(\theta) &= 5e^{\theta} + \frac{1}{5}\theta \\\[8pt]
\text{\text{Then the indefinite intergral is}}
G(\theta) &= 5e^{\theta} + \frac{\ln\{\theta\}}{5} + c \\\[8pt]
\end{align*}

```

\vspace{2cm}

\qpart

```

\begin{align*}
h(t) &= 2\sin\{t\} + \frac{1}{3} + 3t^2 + 3 \\\[8pt]
&= 2 \text{rb}\{\int \sin\{t\} dt\} + \frac{1}{3} \text{rb}\{\int \frac{1}{3} \{1 + t^2\}\} dt + 3 \\\[8pt]
\text{\text{Then the indefinite intergral is}}
\end{align*}

```

```

H(t) &= -2\cos{(t)} + \frac{1}{3}\taninv{(t)} + 3t + c\\[8pt]
&= \frac{1}{3}\operatorname{arctan}{(t)} - 6\cos{(t)} + 9t + c \}
\end{align*}

```

\clearpage

\qpart

```

\begin{align*}
j(y) &= \operatorname{arctan}{y - 2} + 3y - 2y^{\frac{-1}{2}} + 3\\[8pt]
&\text{\texttt{Expand the brackets}} \\
&= y^{\frac{1}{2}} + 3y - 2y^{\frac{-1}{2}} - 6\\[8pt]
&= y^{\frac{1}{2}} + \operatorname{arctan}{y} - \operatorname{arctan}{y^{\frac{-1}{2}}} - 6\\[8pt]
&\text{\texttt{Then the indefinite intergral is}} \\
J(y) &= \frac{1}{\frac{3}{2}}y^{\frac{3}{2}} + 3\operatorname{arctan}{y} - 2\operatorname{arctan}{y^{\frac{1}{2}}} \\
&\quad - 6y + c\\[8pt]
&= \frac{2}{3}y^{\frac{3}{2}} + 3\operatorname{arctan}{y} - 4\sqrt{y} - 6y + c
\end{align*}

```

\end{question}

\clearpage

%%%Question four

\begin{question}

$$f(x) = -x^2 + 4x + 12$$

\qpart

As the function is an inverted U parabola the x-intersection points will show where the curve crosses to below the x-axis.

```

\begin{align*}
f(x) &= -x^2 + 4x + 12\\[8pt]
&\text{\texttt{Substituting both } (-2) \text{ and } (6) \text{ into the equation}} \\
f(-2) &= -(-2)^2 + 4(-2) + 12\\[8pt]
&= -4 - 8 + 12\\[8pt]
&= 0\\[8pt]
&\text{\texttt{and}} \\
f(6) &= -(6)^2 + 4(6) + 12\\[8pt]
&= -36 + 24 + 12\\[8pt]
&= 0
\end{align*}

```

Hence the graph between and not including these points are above the x-axis.

\vspace{3cm}

\qpart

```

\begin{align*}
f(x) &= -x^2 + 4x + 12\\[8pt]
&= \int_1^3 (-x^2 + 4x + 12) dx\\[8pt]
&= \left[ -\frac{1}{3}x^3 + 4\frac{1}{2}x^2 + 12x \right]_1^3\\[8pt]
&= \left[ -\frac{1}{3}(3)^3 + 8(3)^2 + 12(3) \right] - \left[ -\frac{1}{3}(1)^3 + 8(1)^2 + 12(1) \right]
\end{align*}

```

\qpart

Using this to find the area under the curve between $(-2 < x < 6)$

```
\begin{align*}
f(x) &= -x^2 + 4x + 12 \\[8pt]
&= \int_{-2}^6 (-x^2 + 4x + 12) dx \\[8pt]
&= \left[ -\frac{1}{3}x^3 + 4\frac{1}{2}x^2 + 12x \right]_{-2}^6 \\[8pt]
&= \left( -\frac{1}{3}6^3 + 2 \cdot 6^2 + 12 \cdot 6 \right) - \left( -\frac{1}{3}(-2)^3 + 2 \cdot (-2)^2 + 12 \cdot (-2) \right) \\[8pt]
&= \left( -\frac{1}{3} \cdot 216 + 72 + 72 \right) - \left( -\frac{1}{3} \cdot (-8) + 8 - 24 \right) \\[8pt]
&= -72 + 72 + 72 - \left( -\frac{8}{3} + 8 - 24 \right) \\[8pt]
&= 72 - \left( -\frac{40}{3} \right) \\[8pt]
&\text{\textit{Hence the area under the curve between } } x=-2 \text{ \textit{ and } } x=6 \text{ \textit{ is } } \\[8pt]
&= \frac{256}{3} \\
\end{align*}
```

\end{question}

\clearpage

%%%Question five

\begin{question}

\qpart

$$\int \left(\frac{\cos 3x}{\sin 3x} - \sin 3x \right) (\sin 3x + \cos 3x)^2 dx$$

```
\begin{align*}
&\text{\textit{Substitute } } u=3x \text{ \textit{ and } } \frac{du}{dx}=3 \\[8pt]
&= \frac{1}{3} \int \left( \frac{\cos u}{\sin u} - \sin u \right) (\sin u + \cos u)^2 du \\[8pt]
&\text{\textit{Substitute } } v = \sin u + \cos u, \frac{dv}{du} = \cos u - \sin u \\[8pt]
&= \frac{1}{3} \int \frac{v^2}{v} dv \\[8pt]
&= \frac{1}{3} \left( -\frac{1}{2}v^2 \right) + C \\[8pt]
&\text{\textit{Substituting } } v \text{ \textit{ back in } } \\[8pt]
&= \frac{-1}{3} (\sin u + \cos u)^2 + C \\[8pt]
&\text{\textit{Substituting } } u \text{ \textit{ back in } } \\[8pt]
&= \frac{-1}{3} (\sin 3x + \cos 3x)^2 + C \\
\end{align*}
```

\clearpage

\qpart

$$\int_0^{\ln 5} e^{3x} \sqrt{e^{3x} + 2} dx$$

```
\begin{align*}
&\text{\textit{Substitute } } u = 3x, \frac{du}{dx} = 3 \\[8pt]
&= \frac{1}{3} \int_0^{\ln 5} e^u \sqrt{e^u + 2} du \\[8pt]
&\text{\textit{Substitute } } v = e^u + 2, \frac{dv}{du} = e^u \\[8pt]
&= \frac{1}{3} \int_0^{\ln 5} \sqrt{v} dv \\[8pt]
&\text{\textit{The integrand of } } v \text{ \textit{ is } } \frac{2}{3} v^{\frac{3}{2}} \\[8pt]
&= \frac{1}{3} \left[ \frac{2}{9} v^{\frac{3}{2}} \right] \\[8pt]
&\text{\textit{Substitute } } v \text{ \textit{ back in } } \\[8pt]
&= \frac{2}{9} (e^u + 2)^{\frac{3}{2}} \\[8pt]
&\text{\textit{Substitute } } u \text{ \textit{ back in } } \\[8pt]
&= \frac{2}{9} (e^{3x} + 2)^{\frac{3}{2}} \\
\end{align*}
```

It follows that

```
\begin{align*}
\int_0^{\frac{1}{3}\ln 5} e^{3x}\sqrt{e^{3x}+2} \, dx &= \sqrt{\frac{2}{9}} \left[ e^{3x} + 2 \right]^{\frac{3}{2}} \Big|_0^{\frac{1}{3}\ln 5} \\
&= \sqrt{\frac{2}{9}} \left[ e^{\ln 5} + 2 \right]^{\frac{3}{2}} - \sqrt{\frac{2}{9}} \left[ e^0 + 2 \right]^{\frac{3}{2}} \\
&= \sqrt{\frac{2}{9}} \left[ 5 + 2 \right]^{\frac{3}{2}} - \sqrt{\frac{2}{9}} \left[ 1 + 2 \right]^{\frac{3}{2}} \\
&= 4.115\ldots - 1.154\ldots \\
&= 2.96\ldots
\end{align*}
```

\end{question}

%%%Question six

\begin{question}

\marginnote[Integration by parts $\int f(x)g(x) \, dx = f(x)G(x) - \int f'(x)G(x) \, dx$]

\qpart

```
\begin{align*}
\int 81x^8 \ln(x) \, dx \\
\text{Let, } f(x) = \ln(x) \text{ and } g(x) = x^8 \\
\text{Then, } f'(x) = \frac{1}{x} \text{ and } G(x) = \frac{x^9}{9} \\
&= 81 \int x^8 \ln(x) \, dx \\
&= 81 \left[ \ln(x) \frac{x^9}{9} - \int \frac{x^9}{9} \cdot \frac{1}{x} \, dx \right] \\
&= 9 \left[ \ln(x) x^9 - \int x^8 \, dx \right] \\
&= 9 \left[ \ln(x) x^9 - \frac{x^9}{9} \right] \\
&= 9x^9 \ln(x) - x^9
\end{align*}
```

\vspace{5cm}

\qpart

```
\begin{align*}
\int e^{3y} \sin(2y) \, dy \\
\text{Let, } f(y) = \sin(2y) \text{ and } g(y) = e^{3y} \\
\text{Then, } f'(y) = 2\cos(2y) \text{ and } G(y) = \frac{e^{3y}}{3} \\
&= \frac{1}{3} e^{3y} \sin(2y) - \frac{2}{3} \int e^{3y} \cos(2y) \, dy \\
\text{Let, } h(y) = \cos(2y) \text{ and } i(y) = e^{3y} \\
\text{Then, } h'(y) = -2\sin(2y) \text{ and } I(y) = \frac{e^{3y}}{3} \\
&= \frac{1}{3} e^{3y} \sin(2y) - \frac{2}{3} \left[ \frac{e^{3y}}{3} \cos(2y) - \frac{2}{3} \int e^{3y} \sin(2y) \, dy \right] \\
&= \frac{1}{3} e^{3y} \sin(2y) - \frac{2}{9} e^{3y} \cos(2y) + \frac{4}{9} \int e^{3y} \sin(2y) \, dy \\
\text{Add } \frac{4}{9} \int e^{3y} \sin(2y) \, dy \text{ to both sides} \\
\frac{13}{9} \int e^{3y} \sin(2y) \, dy &= \frac{1}{3} e^{3y} \sin(2y) - \frac{2}{9} e^{3y} \cos(2y) \\
\text{Multiply both sides by } \frac{9}{13} \\
\int e^{3y} \sin(2y) \, dy &= \frac{9}{13} \left[ \frac{1}{3} e^{3y} \sin(2y) - \frac{2}{9} e^{3y} \cos(2y) \right] \\
&= \frac{3}{13} e^{3y} \sin(2y) - \frac{2}{13} e^{3y} \cos(2y)
\end{align*}
```

\clearpage

%%%Question seven

\begin{question}

\includepdf[pages=-]{/Users/paulallen/OU/Maths/TMA_03/question_7.pdf}

\end{question}

%%%Question eight

\clearpage

\begin{question}

\qpart

\qsubpart

\begin{fullwidth}

\begin{tabular}{p{1.5cm}}{p{1.5cm}}{p{1.5cm}}{p{1.5cm}}{p{1.5cm}}{p{1.5cm}}{p{1.5cm}}{}}

\hline

& Not at all confident & Slightly confident & Somewhat confident & Fairly confident & Very confident\\

\hline

Unit 1 &&&&\CheckmarkBold\\

\hline

Unit 2 &&&&\CheckmarkBold\\

\hline

Unit 3 &&&\CheckmarkBold&&\\

\hline

Unit 4 &&&&\CheckmarkBold&\\

\hline

Unit 5 &&&&\CheckmarkBold\\

\hline

Unit 6 &&&&\CheckmarkBold&\\

\hline

Unit 7 &&&\CheckmarkBold&&\\

\hline

Unit 8 &&&&\CheckmarkBold&\\

\hline

\end{tabular}

\end{fullwidth}

\qsubpart

I have a differnt room as my study, so I am separated from the rest of the house and all the distractions that comes with it.

I like to set out short 30 minute time slots with a 15 minute break over the course of a few hours.

I will need to work on the different methods of integration and Taylor polynomials.

\qpart

\qsubpart

Section 1 \ (2\% \times 25 = 0.5 \times 180 = 90)\\

Section 2 \ (3\% \times 10 = 0.3 \times 180 = 54)\\

Section 3 \ (4\% \times 5 = 0.2 \times 180 = 36)\\

For section A, I should be averaging about 3.6 minutes per question,

For section B, I should be averaging about 5.4 minutes per question,

For section C, I should be averaging about 7.2 minutes per question.

\qsubpart

```
\begin{itemize}
\item Review the material for the sections I am least confident in.
\item Some questions might take longer than others, so I should not spend too long on any one
question.
\item If I am struggling with a question, I should move on and come back to it later.
\item Keep track of the questions I do quickly , so I know how much i can spend on harder ones
\end{itemize}
```

\end{question}

\clearpage

%%%%%%%%Question nine

\begin{question}

\section{Section A}

\begin{exam_question}

A

\end{exam_question}

\begin{exam_question}

B

\end{exam_question}

\begin{exam_question}

C

\end{exam_question}

\begin{exam_question}

D

\end{exam_question}

\begin{exam_question}

E

\end{exam_question}

\section{Section B}

\begin{exam_question}

F

\end{exam_question}

\begin{exam_question}

\[$f^{\prime}(x) = 9x^2 - 4$ \]

The x-coordinates of one stationary point is at $(\textcolor{blue}{x=\frac{2}{3}})$. It is a $\textcolor{blue}{\text{local minimum}}$.

The x-coordinates of the other stationary point is at $(\textcolor{blue}{x=-\frac{2}{3}})$. It is a $\textcolor{blue}{\text{local maximum}}$.

\end{exam_question}

\section{Section C}

\begin{exam_question}

A. $(\frac{1}{8})$ \\[8pt]

B. $(\frac{1}{2})$ \\[8pt]

C. $(\frac{3}{8})$ \\

\end{exam_question}

\end{question}

\end{document}