

# MST124

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## TMA04

## 2024J

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Covers Units 9, 10 and 12

Cut-off date 30 April 2025

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You will find information about TMAs in the ‘Assessment’ area of the MST124 website. Please read that information before beginning work on this TMA.

If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

The work that you submit should include your working as well as your final answers.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software.

Your work should be written in a good mathematical style, as described in Section 6 of Unit 1, and as demonstrated by the example and activity solutions in the study units. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded against Question 13. You do not have to submit any work for Question 13.

## **PLAGIARISM WARNING – the use of assessment help services and websites**

The work that you submit for any assessment/examination on any module should **be your own**. Submitting work produced by or with another person, or a web service or an automated system, **as if it is your own** is cheating. It is **strictly forbidden** by the University.

You should not:

- provide any assessment question to a website, online service, social media platform or any individual or organisation, as this is an infringement of copyright
- request answers or solutions to an assessment question on any website, via an online service or social media platform, or from any individual or organisation
- use an automated system (other than one prescribed by the module) to obtain answers or solutions to an assessment question and submit the output as your own work
- discuss examination questions with any other person, including your tutor.

The University actively monitors websites, online services and social media platforms for answers and solutions to assessment questions, and for assessment questions posted by students. Work submitted by students for assessment is also monitored for plagiarism.

A student who is found to have posted a question or answer to a website, online service or social media platform and/or to have used any resulting, or otherwise obtained, output as if it is their own work has committed a disciplinary offence under our [Code of Practice for Student Discipline](#). **This means the academic reputation and integrity of the University has been undermined.**

The Open University's [Academic Conduct Policy](#) defines plagiarism in part as:

- using text obtained from assignment writing sites, organisations or private individuals
- obtaining work from other sources and submitting it as your own.

**If it is found that you have used the services of a website, online service or social media platform, or that you have otherwise obtained the work you submit from another person, this is considered serious academic misconduct and you will be referred to the Central Disciplinary Committee for investigation.**

**Question 1** – 15 marks

*You should be able to answer this question after studying Unit 9.*

*Give sufficient details of your working to make it clear that you have not used any software or calculator that can perform matrix manipulation. For example, in part (a), show explicitly how you calculated at least one element of each matrix answer.*

(a) Let

$$\mathbf{A} = \begin{pmatrix} 4 & 7 & -2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 & -3 \\ 0 & 1 \\ 3 & -1 \end{pmatrix}, \text{ and } \mathbf{C} = \begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix}.$$

Evaluate each of the following expressions, if possible. Where evaluation is not possible, explain why not.

(i)  $\mathbf{AB}$

(ii)  $\mathbf{BA}$

(iii)  $\mathbf{BC}$

(iv)  $\mathbf{C}^2$

(v)  $4\mathbf{BC} - 3\mathbf{B}$ .

[8]

(b) Determine whether the following matrix is invertible, and justify your answer.

$$\begin{pmatrix} -8 & -12 \\ 4 & 6 \end{pmatrix}$$

[1]

(c) Use a matrix method to solve the following system of linear equations:

$$2x - 6y = -12$$

$$3x - 7y = 10.$$

Clearly show your full method.

[6]

**Question 2** – 12 marks

*You should be able to answer this question after studying Unit 9.*

*You can use Maxima to do the matrix arithmetic in this question, or you can do it by hand. If you use Maxima, then you do not need to include a printout or screenshot, but you must include sufficient details in your solution to make it clear what calculations Maxima has done.*

Three towns,  $A$ ,  $B$ , and  $C$  receive electricity from two suppliers,  $X$  and  $Y$ .  
During 2023:

Town  $A$  received 90% of its electricity from supplier  $X$  and 10% of its electricity from supplier  $Y$ .

Town  $B$  received 15% of its electricity from supplier  $X$  and 85% of its electricity from supplier  $Y$ .

Town  $C$  received 55% of its electricity from supplier  $X$  and 45% of its electricity from supplier  $Y$ .

- (a) (i) Draw a network diagram with input nodes  $A$ ,  $B$  and  $C$ , and output nodes  $X$  and  $Y$ , that represents the proportion of electricity each town received from its two energy suppliers. [2]
- (ii) Write down the matrix that represents this network. [2]
- (iii) Suppose that the total energy consumption in 2023 by towns  $A$ ,  $B$ , and  $C$  are 700, 900, and 800 Gigawatt hours (GWh) respectively. Using the matrix that you found in part (a)(ii), determine the total energy generated by each power supplier in 2023. [2]
- (b) Suppose that:
- Supplier  $X$  generates 20% of its electricity from renewable sources and 80% of its electricity from fossil sources.  
Supplier  $Y$  generates 60% of its electricity from renewable sources and 40% of its electricity from fossil sources.
- (i) Write down the matrix that represents the proportion of electricity each supplier generates from either a renewable or a fossil source. [1]
- (ii) Hence find the single matrix that represents the proportion of electricity each town  $A$ ,  $B$ , and  $C$  received from either a renewable or a fossil source. [2]
- (iii) There is a huge increase in the price of electricity generated by fossil fuels. State which town is most affected by this price increase and explain why. [3]

**Question 3** – 5 marks

*You should be able to answer this question after studying Unit 9.*

*You must use Maxima to answer this question. Include a printout or screenshot of your Maxima worksheet with your solution. Your solution should include a clear statement of the problem and the method used. Remember that for good mathematical communication you should present your answer clearly.*

Below is a list of ingredients for my favourite soups:

	Vegetable	Minestrone	French Onion
Onions	2	2	8
Carrots	3	1	0
Garlic cloves	2	3	4

Each set of ingredients will make just 1 portion of soup, for example I need 2 Onions, 3 Carrots, and 2 Garlic cloves to make 1 portion of Vegetable soup.

In my cupboards I have 40 Onions, 10 Carrots, and 25 cloves of garlic. How many portions of Vegetable, Minestrone, and French Onion soup can I make that will use up all these ingredients? (Give your answer as an integer.) [5]

You should model the problem as a system of three linear simultaneous equations, and then use matrices in Maxima to find the number of portions of Vegetable, Minestrone, and French Onion soup respectively.

**Question 4** – 4 marks

*You should be able to answer this question after studying Unit 10.*

Find a fraction equivalent to the recurring decimal

$$0.27\,32\,73\,27\,32\,73\ldots [4]$$

Give your answer in its simplest form.

**Question 5** – 4 marks

*You should be able to answer this question after studying Unit 10.*

Find the coefficient of  $x^3$  in the binomial expansion of

$$\left(\frac{x}{4} - 5\right)^7. [4]$$

Give your answer in exact form.

**Question 6** – 10 marks

*You should be able to answer this question after studying Unit 10.*

Let  $(x_n)$  be an infinite geometric sequence whose first four terms are

2, −2.5, 3.125, −3.90625, ...

- (a) Find the values of the first term  $x_1 = a$  and the common ratio  $r$ , and write down a recurrence system for this sequence. [4]
- (b) Find a closed form for  $x_n$ . [1]
- (c) Calculate the 32nd term of the sequence, giving your answer to three decimal places. [1]
- (d) What is the lowest value of  $n$  such that  $|x_n| > 57,000$ ? [4]

*Hint:* Find a closed form for  $|x_n|$ . The definition of  $|x|$  can be found on page 231 of Book A (Unit 3).

**Question 7** – 5 marks

*You should be able to answer this question after studying Unit 10.*

A mathematician decides to start collecting stamps. They start their collection with 250 stamps. In the first year they add another 30 stamps. Each subsequent year they add a number of stamps to their collection that is 15 more than the number of stamps they added the previous year. That is, in the second year they add 45 to their collection, in the third year they add 60 stamps to their collection, and so on.

- (a) How many stamps will be added to the collection in the twenty fifth year? [1]
- (b) What will be the total number of stamps in the collection at the end of the twenty fifth year? [4]

**Question 8** – 7 marks

*You should be able to answer this question after studying Unit 10.*

- (a) Find the sum of the infinite series

$$\sum_{n=0}^{\infty} 3 \left( \frac{7}{11} \right)^n$$

giving your answer in exact form. [2]

- (b) Find the sum of the finite series

$$\sum_{n=12}^{36} \left( \frac{1}{3}n^3 + \frac{1}{2}n^2 + 1 \right). \quad [5]$$

**Question 9** – 10 marks

*You should be able to answer this question after studying Unit 12.*

- (a) Given the complex numbers

$$z = 12 \left( \cos \left( \frac{7\pi}{12} \right) + i \sin \left( \frac{7\pi}{12} \right) \right)$$

and

$$w = 3 \left( \cos \left( \frac{\pi}{3} \right) - i \sin \left( \frac{\pi}{3} \right) \right),$$

find  $zw$ , giving your answer in Cartesian form.

[4]

- (b) Find all solutions of the following equation, giving your answer in polar form:

$$z^4 = -3(\sqrt{3}i + 1).$$

[6]

**Question 10** – 10 marks

*You should be able to answer this question after studying Unit 12.*

*You must use Maxima to answer both parts of this question. Include a printout or screenshot of your Maxima worksheets with your solution. You are not expected to annotate your Maxima worksheets with explanation. However, remember for good mathematical communication you should present your answer clearly.*

- (a) Given the complex numbers

$$z = \frac{17}{4} + \frac{2}{5}i \quad \text{and} \quad w = \frac{23}{4} - \frac{1}{2}i,$$

find the moduli and principal arguments of  $zw$  and  $\frac{z}{w}$ , giving your answers to 3 decimal places.

[5]

- (b) Solve the equation

$$4z^6 + 20z^5 + 53z^4 + 100z^3 + 148z^2 + 120z + 75 = 0.$$

State the solutions and plot them in the complex plane.

[5]

**Question 11** – 8 marks

*You should be able to answer this question after studying Unit 12.*

- (a) Find the exponential form of the complex number

$$\left( \frac{2i}{3 + 3\sqrt{3}i} \right)^5.$$

[5]

- (b) The  $n$ th roots of unity can be written in exponential form as  $e^{2\pi ki/n}$  for  $k = 0, 1, 2, \dots, n-1$ .

Using the formula for the sum of a finite geometric series, show that for any integer  $n \geq 2$  the sum of the  $n$ th roots of unity is 0.

[3]

*Hint:* Using de Moivre's formula in exponential form, the sum of the  $n$ th roots of unity can be written as

$$\sum_{k=1}^n \left( e^{2\pi i/n} \right)^{k-1}.$$

**Question 12** – 5 marks

Reflect on how you approached your study for the module, and if there is anything you might do differently for your next module. Areas for consideration could include:

- Time management;
- Completion of activities in the module workbooks;
- Attendance at, or engagement with, the tutorials;
- Use of the module forums;
- Using Maxima;
- Wider reading to gain a deeper understanding beyond the syllabus;
- What study skills have you gained, and what skills would you like to develop?

You may want to reflect on other areas of your study and you should NOT comment on all of these areas. You should aim to make five distinct reflective points, and make sure to highlight things you have done well!

You may find the Skills tab on the Mathematics and Statistics Study Site useful.

[5]

**Question 13** – 5 marks

A score out of 5 marks for good mathematical communication throughout TMA 04 will be recorded under Question 13.

[5]

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