Question 1:

a)

Product rule, for

$$f(x) = g(x)h(x)$$
$$f'(x) = g(x)h'(x) + h(x)g'(x)$$

$$f(x) = (x^5 + 3x^3 + 2x + 1)e^x$$

Using

$$g = x^5 + 3x^3 + 2x + 1$$

and

$$h = e^x$$

Differentiating

$$g' = 5x^4 + 9x^2 + 2$$

and

$$h' = e^x$$

Using the product rule

$$f'(x) = (5x^4 + 9x^2 + 2)e^x + e^x(x^5 + 3x^3 + 2x + 1)$$

Simplifying

$$= (x^5 + 5x^4 + 3x^3 + 9x^2 + 2x + 3)e^x$$

b)

Chain rule, for

$$g(y) = i(h(y))$$

$$g'(y) = i'(h(y))h'(y)$$

$$g(y) = (\ln(y) + \sin(y))^6$$

Using

$$h(y) = \ln(y) + \sin(y)$$

and

$$i(h) = h^6$$

Differentiating

$$h'(y) = \left(\frac{1}{y} + \cos y\right)$$

and

$$i'(h) = 6h^5$$

Using the chain rule

$$g'(y) = 6\left[\ln(y) + \sin(y)\right]^5 \left(\frac{1}{y} + \cos(y)\right)$$

c)

Quotient rule, for

$$h(z) = \frac{i(z)}{j(z)}$$

$$h'(z) = \frac{j(z)i'(z) - i(z)j'(z)}{(j(z))^2}$$

$$h(z) = \frac{e^{5z}}{(2 + \cos(10z))}$$

Using

$$i(z) = e^{5z}$$

and

$$j(z) = (2 + \cos(10z))$$

Differentiating

$$i'(z) = 5e^{5z}$$

and

$$j'(z) = -10\sin(10z)$$

Using the quotient rule

$$h'(z) = \frac{(2 + \cos(10z))5e^{5z} - e^{5z}(-10\sin(10z))}{(2 + \cos(10z))^2}$$

Simplifying

$$= \frac{5e^{5z}\left((2 + \cos(10z)) + (2\sin(10z))\right)}{(2 + \cos(10z))^2}$$

d)

Product rule:

$$k(x) = l(x)m(x)$$

$$k'(x) = l(x)m'(x) + l'(x)m(x)$$

Chain rule

$$m'(x) = u'(v(x))v'(x)$$

$$k(x) = x^2 \sin(\cos x)$$

Using

$$l(x) = x^2 \quad \text{and} \quad m(x) = \sin(\cos x)$$

Differentiating using the product rule

$$l'(x) = 2x$$

Now using the chain rule to find $m'(x)$, using $u = \sin(x)$ and $v = \cos(x)$

$$u' = \cos(x)$$

and

$$v' = -\sin(x)$$

Thus

$$m' = \cos(\cos x)\,(-\sin(x))$$

Applying the product rule

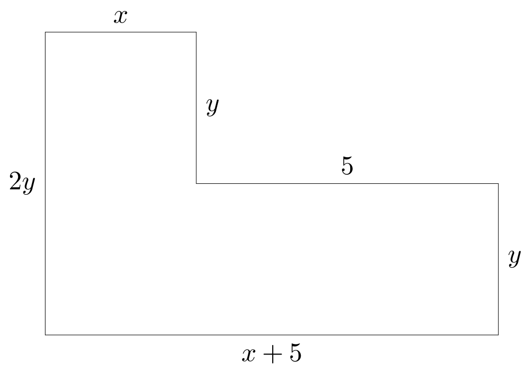
$$k'(x) = x^2(\cos(\cos x)\,(-\sin(x))) + 2x\sin(\cos x)$$

Simplifying

$$= 2x\sin(\cos x) - x^2\sin(x)\cos(\cos x)$$

Question 2:

Given the L-shaped enclosure



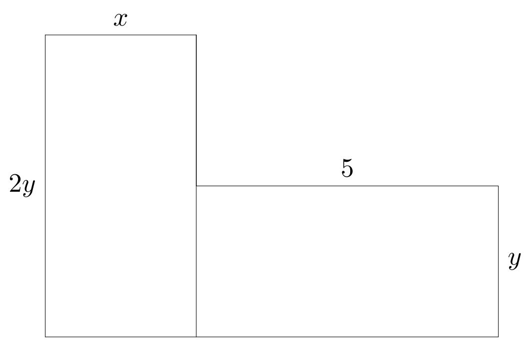
a)

Using the assumption that Steven uses all the fencing he has exactly the perimeter is $74\,\mathrm{m}$

$$\begin{aligned} perimeter &= (x+5) + y + 5 + y + x + 2y \\ 74 &= 4y + 2x + 10 \\ 64 &= 4y + 2x \\ 4y &= 64 - 2x \\ y &= 16 - \frac{x}{2} \\ &= \frac{1}{2}(32 - x) \end{aligned}$$

as required

b)



The area of the L-shape is given by the total of the two shapes shown above.

$$
\begin{aligned}
A &= x\left(2y\right) + 5\left(y\right) \\
&= x\left(2\left(\frac{1}{2}\left(32-x\right)\right)\right) + 5\left(\frac{1}{2}\left(32-x\right)\right) \\
&= x\left(32-x\right) + \left(80 - \frac{5x}{2}\right) \\
&= 32x - x^2 + 80 - \frac{5x}{2}
\end{aligned}
$$

Multiply by 2

$$2A = 160 + 64x - 5x - 2x^2$$

Collect like terms

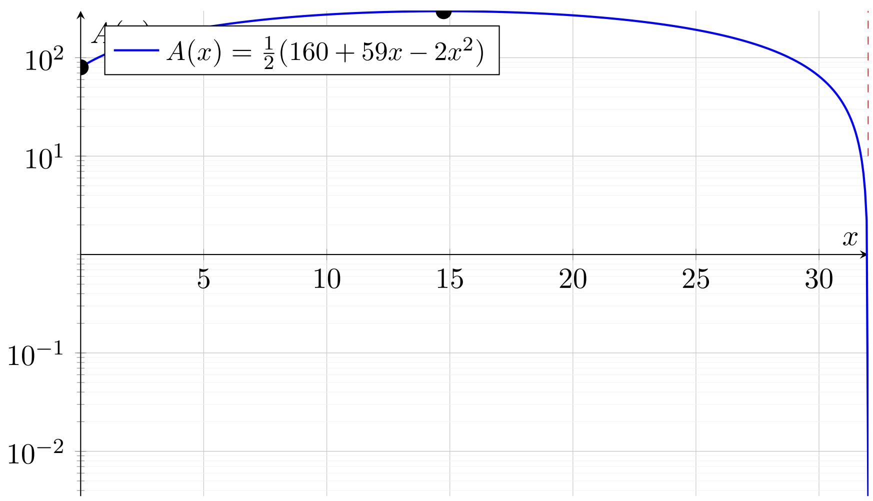
$$= 160 + 59x - 2x^2$$

simplify

$$A = \frac{1}{2}\left(160 + 59x - 2x^2\right)$$

as required

c)



Based on the shape of the curve for this graph we need only consider the stationary point at which $dA/dx = 0$ to find the maximum area.

Given

$$A = \frac{1}{2}\left(160 + 59x - 2x^2\right)$$

Differentiating

$$A' = -2x + \frac{59}{2}$$

Setting $A' = 0$ to find the stationary point

$$0 = -2x + \frac{59}{2}$$

$$2x = \frac{59}{2}$$

$$x = \frac{59}{4}$$

Substituting this into the original equation

$$\begin{aligned}
A &= \frac{1}{2}\left(160 + 59x - 2x^2\right) \\
&= \frac{1}{2}\left(160 + 59\left(\frac{59}{4}\right) - 2\left(\frac{59}{4}\right)^2\right) \\
&= 80 + \frac{3481}{8} - \left(\frac{59}{4}\right)^2 \\
&= 80 + \frac{3481}{8} - \left(\frac{3481}{16}\right) \\
&= \frac{1280}{16} + \frac{6962}{16} - \left(\frac{3481}{16}\right) \\
&= \frac{1280}{16} + \frac{3481}{16} \\
&= \frac{4761}{16}
\end{aligned}$$

Applying the second derivative test

$$\begin{aligned}
A' &= -2x + \frac{59}{2} \\
A'' &= -2
\end{aligned}$$

Showing that this a maximum stationary point

Hence the maximum area of the L-shape is

$$A = \frac{4761}{16}\mathrm{m}^2$$

Question 3:

a)

$$f(x) = x^2 + 2x + 5$$

Then the indefinite integral is

$$F(x) = \frac{x^3}{3} + x^2 + 5x + c$$

b)

$$g(\theta) = 5e^\theta + \frac{1}{5\theta}$$

Then the indefinite intergral is

$$G(\theta) = 5e^\theta + \frac{\ln\theta}{5} + c$$

c)

$$\begin{aligned} h(t) &= 2\sin(t) + \frac{1}{3+3t^2} + 3 \\ &= 2\left(\int \sin(t)\right) dt + \frac{1}{3}\left(\int \frac{1}{1+t^2}\right) dt + 3 \end{aligned}$$

Then the indefinite intergral is

$$\begin{aligned} H(t) &= -2\cos(t) + \frac{1}{3}\tan^{-1}(t) + 3t + c \\ &= \frac{1}{3}\left(\tan^{-1}(t) - 6\cos(t) + 9t + c\right) \end{aligned}$$

d)

$$j(y) = (y-2)\left(y^{\frac{-1}{2}}\right) + 3)$$

Expand the brackets

$$= y^{\frac{1}{2}} + 3y - 2y^{\frac{-1}{2}} - 6$$

$$= y^{\frac{1}{2}} + \left(3\int y\right)dy - \left(2\int y^{\frac{-1}{2}}\right)dy - 6$$

Then the indefinite intergral is

$$J(y) = \frac{1}{\frac{3}{2}}y^{\frac{3}{2}} + 3\left(\frac{1}{2}y^2\right) - 2\left(\frac{1}{\frac{1}{2}}y^{\frac{1}{2}}\right) - 6y + c$$

$$= \frac{2t^{\frac{3}{2}}}{3} + \frac{3y^2}{2} - 4\sqrt{y} - 6y + c$$

Question 4:

$$f(x) = -x^2 + 4x + 12$$

a)
As the function is an inverted U parabola the x-intersection points will show were the curve crosses to below the x-axis.

$$f(x) = -x^2 + 4x + 12$$

Substituting both -2 and 6 into the equation

$$\begin{aligned} f(-2) &= -(-2)^2 + 4(-2) + 12 \\ &= -4 - 8 + 12 \\ &= 0 \end{aligned}$$

and

$$\begin{aligned} f(6) &= -(6)^2 + 4(6) + 12 \\ &= -36 + 24 + 12 \\ &= 0 \end{aligned}$$

Hence the graph between and not including these points are above the x-axis.

b)

$$\begin{aligned} f(x) &= -x^2 + 4x + 12 \\ &= \int_1^3 \left(-x^2 + 4x + 12\right) dx \\ &= \left(-\int x^2 + 4\int x + 12\int 1\right) dx \\ &= \left(\frac{-1}{3}x^3 + 4\left(\frac{1}{2}x^2\right) + 12(x)\right) \\ &= \left[\frac{-1}{3}x^3 + 8x^2 + 12x\right]_1^3 \end{aligned}$$

c)

Using this to find the area under the curve between $-2 < x < 6$

$$f(x) = -x^2 + 4x + 12$$

$$= \int_{-2}^{6} \left(-x^2 + 4x + 12\right) dx$$

$$= \left[\frac{-1}{3}x^3 + 4\left(\frac{1}{2}\right)x^2 + 12x\right]_{-2}^{6}$$

$$= \left(\frac{-1}{3}6^3 + 2\left(6\right)^2 + 12\left(6\right)\right) - \left(\frac{-1}{3}\left(-2\right)^3 + 2\left(-2\right)^2 + 12\left(-2\right)\right)$$

$$= \left(\frac{-1}{3}\left(216\right) + \left(2\right)36 + 72\right) - \left(\frac{-1}{3}\left(-8\right) + \left(2\right)4 - 24\right)$$

$$= \left(-72 + 72 + 72\right) - \left(\frac{8}{3} + 8 - 24\right)$$

$$= 72 - \left(\frac{-40}{3}\right)$$

Hence the area under the curve between $x = -2$ and $x = 6$ is

$$= \frac{256}{3}$$

Question 5:

a)

$$\int \frac{\cos(3x) - \sin(3x)}{(\sin(3x) + \cos(3x))^2}$$

substitute $u = 3x$ and $du = 3\,dx$

$$= \frac{1}{3} \int \frac{\cos(u) - \sin(u)}{(\sin(u) + \cos(u))^2}\,du$$

Substitute $v = \sin(u) + \cos(u)$, $dv = \cos(u) - \sin(u)\,du$

$$= \frac{1}{3} \int \frac{1}{v^2}\,dv$$
$$= \frac{1}{3}\left(-\frac{1}{v}\right) + C$$

Substituting v back in

$$= \frac{-1}{3(\sin u + \cos u)} + C$$

Substituting u back in

$$= \frac{-1}{3(\sin 3x + \cos 3x)} + C$$

b)

$$\int_0^{\frac{1}{3}\ln 5} e^{3x}\sqrt{e^{3x}+2}\,dx$$

Substitute $u = 3x$, $du = 3\,dx$

$$= \frac{1}{3}\int_0^{\ln 5} e^{u}\sqrt{e^{u}+2}\,du$$

Substitute $v = e^u + 2$, $dv = e^u\,du$

$$= \frac{1}{3}\int_0^{\ln 5} \sqrt{v}\,dv$$

The integrand of v is $\frac{2}{3}v^{\frac{3}{2}}$

$$= \frac{1}{3}\left(\frac{2}{3}v^{\frac{3}{2}}\right)$$
$$= \frac{2}{9}v^{\frac{3}{2}}$$

Substitute v back in

$$= \frac{2}{9}\left(e^{u}+2\right)^{\frac{3}{2}}$$

Substitute u back in

$$= \frac{2}{9}\left(e^{3x}+2\right)^{\frac{3}{2}}$$

It follows that

$$\begin{aligned}
\int_0^{\frac{1}{3}\ln 5} e^{3x}\sqrt{e^{3x}+2}\,dx &= \left[\frac{2}{9}\left(e^{3x}+2\right)^{\frac{3}{2}}\right]_0^{\frac{1}{3}\ln 5} \\
&= \left[\frac{2}{9}\left(e^{3\left(\frac{1}{3}\ln 5\right)}+2\right)^{\frac{3}{2}}\right] - \left[\frac{2}{9}\left(e^{3(0)}+2\right)^{\frac{3}{2}}\right] \\
&= \left[\frac{2}{9}\left(5+2\right)^{\frac{3}{2}}\right] - \left[\frac{2}{9}\left(1+2\right)^{\frac{3}{2}}\right] \\
&= 4.115\ldots - 1.154\ldots \\
&= 2.96
\end{aligned}$$

to 2 d.p

Question 6:

Integration py parts

$$\int f(x)g(x)\,dx = f(x)G(x) - \int f'G(x)\,d(x)$$

a)

$$\int 81x^8 \ln(x)\,dx$$

Let, $f(x) = \ln((x))$ and $g(x) = x^8$

Then, $f'(x) = \frac{1}{x}$ and $G(x) = \frac{x^9}{9}$

$$= 81 \int x^8 \ln(x)\,dx$$

$$= 81 \left[\ln(x)\frac{x^9}{9} - \int (\frac{x^9}{9x})\,dx\right]$$

$$= 9 \left(\ln((x))x^9 - \int x^8\,dx\right)$$

$$= 9 \left(\ln(x)x^9 - \frac{x^9}{9}\right)$$

$$= 9\ln(x)x^9 - x^9$$

$$= 9x^9\left(\ln(x) - 1\right)$$

b)

$$\int e^{3y} \sin(2y)\,dy =$$

Let, $f(y) = \sin(2y)$ and $g(y) = e^{3y}$

Then, $f'(y) = 2\cos(2y)$ and $G(y) = \frac{e^{3y}}{3}$

$$= \frac{1}{3}e^{3y}\sin 2y - \frac{2}{3}\int e^{3y}\cos(2y)\,dy$$

Let, $h(y) = \cos(2y)$ and $i(y) = e^{3y}$

Then, $h'(y) = -2\sin(2y)$ and $I(y) = \frac{e^{3y}}{3}$

$$= \frac{1}{3}e^{3y}\sin 2y - \frac{2}{3}\left[\frac{e^{3y}\cos(2y)}{3} - \frac{2}{3}\int e^{3y}\sin(2y)\,dy\right]$$

$$= \frac{1}{3}e^{3y}\sin(2y) - \frac{2}{9}e^{3y}\cos(2y) - \frac{4}{9}\int e^{3y}\sin(2y)\,dy$$

Add $\frac{4}{9}\int e^{3y}\sin(2y)\,dy$ to both sides

$$\frac{13}{9}\int e^{3y}\sin(2y)\,dy = \frac{1}{3}e^{3y}\sin 2y - \frac{2}{9}e^{3y}\cos(2y)$$

Multiply both sides by $\frac{9}{13}$

$$\int e^{3y}\sin(2y)\,dy = \frac{9}{13}\left[\frac{1}{3}e^{3y}\sin(2y) - \frac{2}{9}e^{3y}\cos(2y)\right]$$

$$= \frac{3}{13}e^{3y}\sin(2y) - \frac{2}{13}e^{3y}\cos(2y)$$

$$= \frac{e^{3y}}{13}\left(3\sin(2y) - 2\cos(2y)\right)$$

Question 7:

Given the function

(% i1) f(x):=(3*x+15*x^ 2-x^ 4)/(9*x^ 2+1);

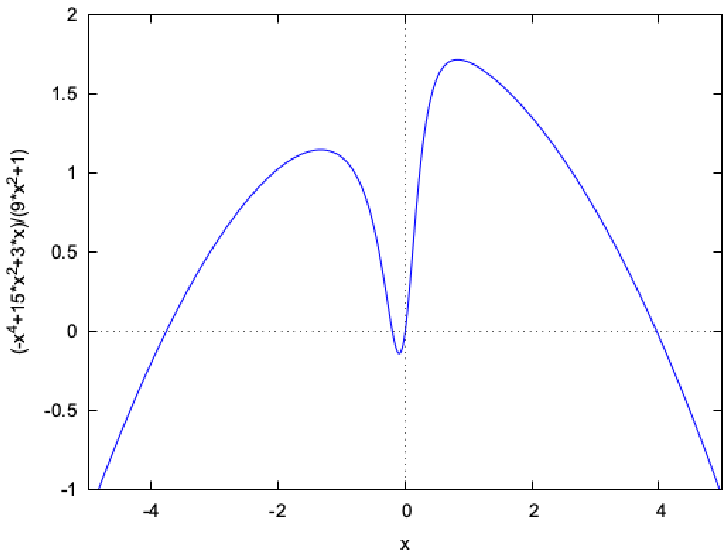
(% o1) $f(x) := \frac{3x + 15x^2 - x^4}{9x^2 + 1}$

(a) The graph of f(x) is

(% i2) wxplot2d(f(x), [x, -5, 5], [y, -1, 2]);

$plot2d : somevalueswillbeclipped.$

(% t2)



(% o2)

(b) the derivative of f(x) is

(% i3) df(x):=''(diff(f(x), x));

(% o3) $df(x) := \frac{-\left(4x^3\right) + 30x + 3}{9x^2 + 1} - \frac{18x\ \left(-x^4 + 15x^2 + 3x\right)}{\left(9x^2 + 1\right)^2}$

(c) The positive local maximum of f(x) is

(% i4) pos_root:find_root(df(x), x, 0, 5);

(pos_ root) 0.8288158368533624

Substituting this back into the f(x)

(% i5) f(pos_root);

(% o5) 1.71510439942526

Finding the second derivative of f(x)

(% i6) ddf(x):=''(diff(df(x), x));

(% o6)

$$ddf(x):=\frac{30-12x^2}{9x^2+1}-\frac{18\left(-x^4+15x^2+3x\right)}{\left(9x^2+1\right)^2}+\frac{648x^2\left(-x^4+15x^2+3x\right)}{\left(9x^2+1\right)^3}-\frac{36x\left(-\left(4x^3\right)+30x+3\right)}{\left(9x^2+1\right)^2}$$

And substitutiong our x variable

(% i7) ddf(0.829);

(% o7) -1.2681397331452517

As this is <0 the point is confirmed to be a local maximum of f(x) Hence the local maximum is at (0.828,1.715), to 3 d.p. (d) To find the root to the right of x=0

(% i8) float(realroots(f(x)));

(% o8)

$$[x=-3.7688187062740326\,,x=-0.20053765177726746\,,x=3.9693563878536224\,,x=0.0]$$

Therefore the grapg crosses the x-axis at 3.969, to 3 d.p. The area under the graph between x=0 and this point is

(% i10) quad_qags(f(x), x, 0, 3.969);

(% o10) $\left[4.342959640603124\,,2.562177785637564210^{-9}\,,105\,,0\right]$

Hence the area enclosed by the graph of f(x) between 0<=x<=3.969 is 4.343, to 3 d.p.

Question 8:

a)
i.

	Not at all confident	Slightly confident	Somewhat confident	Fairly confident	Very confident
Unit 1					✔
Unit 2					✔
Unit 3			✔		
Unit 4				✔	
Unit 5					✔
Unit 6				✔	
Unit 7			✔		
Unit 8				✔	

ii.
I have a differnt room as my study, so I am separated from the rest of the house and all the distractions that comes with it. I like to set out short 30 minute time slots with a 15 minute break over the course of a few hours. I will need to work on the different methods of integration and Taylor polynomials.

b)
i.
Section 1 $2\% \times 25 = 0.5 \times 180 = 90$
Section 2 $3\% \times 10 = 0.3 \times 180 = 54$
Section 3 $4\% \times 5 = 0.2 \times 180 = 36$
For section A, I should be averaging about 3.6 minutes per question,
For section B, I should be averaging about 5.4 minutes per question, For section C, I should be averaging about 7.2 minutes per question.

ii.

- Review the material for the sections I am least confident in.
- Some questions might take longer than others, so I should not spend too long on any one question.
- If I am struggling with a question, I should move on and come back to it later.
- Keep track of the questions I do quickly , so I know how much i can spend on harder ones

Question 9:

Section A

Question 1: A

Question 2: B

Question 3: C

Question 4: D

Question 5: E

Section B

Question 6: F

Question 7:

$$f'(x) = 9x^2 - 4$$

The x-coordinates of one stationary point is at $x = \frac{2}{3}$. It is a local minimum. The x-coordinates of the other stationary point is at $x = -\frac{2}{3}$. It is a local maximum.

Section C

Question 8:

A. $\frac{1}{8}$

B. $\frac{1}{2}$

C. $\frac{3}{8}$