Question 1:

$$\mathbf{A} = \begin{pmatrix} 4 & 7 & -2 \end{pmatrix} \mathbf{B} = \begin{pmatrix} 1 & -3 \\ 0 & 1 \\ 3 & -1 \end{pmatrix} \mathbf{C} = \begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix}$$

a)
i.

$$\mathbf{AB} = \begin{pmatrix} 4 \cdot 1 + 7 \cdot 0 + (-2) \cdot 2 & 4 \cdot (-3) + 7 \cdot 1 + -2 \cdot (-1) \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

1/2 Careful here the highlighted number should be 3 and you have evalauted the second element incorrectly, it should be -3

ii.
BA = This cannot be done, numbr of columns in **B** is not equal to the number of rows in **A**.

1/1

iii.

$$\mathbf{BC} = \begin{pmatrix} 1 & -3 \\ 0 & 1 \\ 3 & -1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \cdot 2 + (-3) \cdot (-1) & 1 \cdot 5 + (-3) \cdot 0 \\ 0 \cdot 2 + 1 \cdot (-1) & 0 \cdot 5 + 1 \cdot 0 \\ 3 \cdot 2 + (-1) \cdot (-1) & 3 \cdot 5 + (-1) \cdot 0 \end{pmatrix}$$

$$= \begin{pmatrix} 2+3 & 5+0 \\ 0-1 & 0+0 \\ 6+1 & 15-0 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 5 \\ -1 & 0 \\ 7 & 15 \end{pmatrix}$$ ✓ ✓

2/2

iv.

$$A^2 = \begin{pmatrix} a^2 + bc & ab + bd \\ ca + dc & cb + d^2 \end{pmatrix}$$

$$\mathbf{C}^2 = \begin{pmatrix} 2^2 + 5 \cdot -1 & 2 \cdot 5 + 5 \cdot 0 \\ -1 \cdot 2 + 0 \cdot -1 & -1 \cdot 5 + 0^2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 10 \\ -2 & -5 \end{pmatrix}$$ ✓

1/1

v.

$$\mathbf{4BC\text{-}3B} = \begin{pmatrix} 4 & -12 \\ 0 & 4 \\ 8 & -4 \end{pmatrix} \cdot \begin{pmatrix} 2 & 5 \\ -1 & 0 \end{pmatrix} - \begin{pmatrix} 3 & -9 \\ 0 & 3 \\ 6 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 4\cdot 2 + (-12)\cdot(-1) & 4\cdot 5 + (-12)\cdot 0 \\ 0\cdot 2 + 4\cdot(-1) & 0\cdot 5 + 4\cdot 0 \\ 8\cdot 2 + (-4)\cdot(-1) & 8\cdot 5 + (-4)\cdot 0 \end{pmatrix} - \begin{pmatrix} 3 & -9 \\ 0 & 3 \\ 6 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 20 & 20 \\ -4 & 0 \\ 16 & 20 \end{pmatrix} - \begin{pmatrix} 3 & -9 \\ 0 & 3 \\ 6 & -3 \end{pmatrix}$$

$$= \begin{pmatrix} 17 & 29 \\ -4 & -3 \\ 10 & 23 \end{pmatrix}$$

✓ ✗

1/2 The numbers highlighted have been evaluated incorrectly.
Please see below for the correct solution

Using the result for **BC** from part (iii),

$$4\mathbf{BC} - 3\mathbf{B} = 4\begin{pmatrix} 5 & 5 \\ -1 & 0 \\ 7 & 15 \end{pmatrix} - 3\begin{pmatrix} 1 & -3 \\ 0 & 1 \\ 3 & -1 \end{pmatrix}$$
$$= \begin{pmatrix} 20 & 20 \\ -4 & 0 \\ 28 & 60 \end{pmatrix} - \begin{pmatrix} 3 & -9 \\ 0 & 3 \\ 9 & -3 \end{pmatrix}$$
$$= \begin{pmatrix} 20-3 & 20+9 \\ -4-0 & 0-3 \\ 28-9 & 60+3 \end{pmatrix}$$
$$= \begin{pmatrix} 17 & 29 \\ -4 & -3 \\ 19 & 63 \end{pmatrix}$$

References

See activity 5 on page 227 of Book C (Unit 9).

b) you have omitted b) 0/1

b)

To find the inverse of a matrix
$$|\mathbf{A}| = \frac{1}{ad-bc}$$
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = |A| \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$2x - 6y = -12$$

$$3x - 7y = 10$$

In matrix notation $\mathbf{AX} = \mathbf{B}$

$$\begin{pmatrix} 2 & (-6) \\ 3 & (-7) \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -12 \\ 10 \end{pmatrix}$$

Hence $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{(-7)\,2 - (-6)\,3} \cdot \begin{pmatrix} (-7) & 6 \\ (-3) & 2 \end{pmatrix} \cdot \begin{pmatrix} (-12) \\ 10 \end{pmatrix}$$

$$= \frac{1}{(-14) + 18} \cdot \begin{pmatrix} -7 \cdot (-12) + 6 \cdot 10 \\ -3 \cdot (-12) + 2 \cdot 10 \end{pmatrix}$$

$$= \frac{1}{4} \cdot \begin{pmatrix} 144 \\ 56 \end{pmatrix}$$

$$= \begin{pmatrix} 36 \\ 14 \end{pmatrix}$$

6/6

therefore

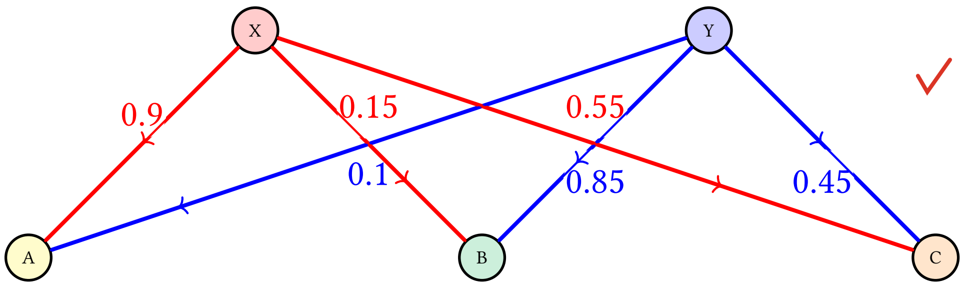
$$x = 36$$
$$y = 14$$

Q1 12/15

Question 2:

a)

i.



2/2

ii.

$$\begin{pmatrix} 0.9 & 0.1 \\ 0.15 & 0.85 \\ 0.55 & 0.45 \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \end{pmatrix}$$

2/2

iii.

Given the power distribution matrix

$$A = \begin{pmatrix} 0.9 & 0.1 \\ 0.15 & 0.85 \\ 0.55 & 0.45 \end{pmatrix}$$

and the total power consumption per city

$$B = \begin{pmatrix} 700 \\ 900 \\ 800 \end{pmatrix}$$

We can represent the system as

$$\begin{pmatrix} 0.9 & 0.1 \\ 0.15 & 0.85 \\ 0.55 & 0.45 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 700 \\ 900 \\ 800 \end{pmatrix}$$

You have written this incorrectly, however you have performed the correct evaluation below

ie, (700 900 800) x a)ii)

We compute the total contribution from each power station as

$$\begin{pmatrix} 0.9 \cdot 700 + 0.15 \cdot 900 + 0.55 \cdot 800 \\ 0.1 \cdot 700 + 0.85 \cdot 900 + 0.45 \cdot 800 \end{pmatrix}$$

$$= \begin{pmatrix} 630 + 135 + 440 \\ 70 + 765 + 360 \end{pmatrix}$$

$$= \begin{pmatrix} 1205 \\ 1195 \end{pmatrix}$$

Thus, the total power supplied by each station is

$$X = 1205\,\mathrm{GW\,h}, \quad Y = 1195\,\mathrm{GW\,h}$$

2/2

b)

i.

$$\begin{pmatrix} 0.2 & 0.6 \\ 0.8 & 0.4 \end{pmatrix}$$

1/1

ii.

$$\begin{pmatrix} 0.2 & 0.6 \\ 0.8 & 0.4 \end{pmatrix} \cdot \begin{pmatrix} 0.9 & 0.15 & 0.55 \\ 0.1 & 0.85 & 0.45 \end{pmatrix}$$

$$= \begin{pmatrix} 0.2 \cdot 0.9 + 0.6 \cdot 0.1 & 0.2 \cdot 0.15 + 0.6 \cdot 0.85 & 0.2 \cdot 0.55 + 0.6 \cdot 0.45 \\ 0.8 \cdot 0.9 + 0.4 \cdot 0.1 & 0.8 \cdot 0.15 + 0.4 \cdot 0.85 & 0.8 \cdot 0.55 + 0.4 \cdot 0.45 \end{pmatrix}$$

$$= \begin{pmatrix} 0.18 + 0.06 & 0.03 + 0.51 & 0.11 + 0.27 \\ 0.72 + 0.04 & 0.12 + 0.34 & 0.44 + 0.18 \end{pmatrix}$$

$$= \begin{pmatrix} 0.24 & 0.54 & 0.38 \\ 0.76 & 0.46 & 0.62 \end{pmatrix}$$

2/2

Town A's fossil fuel dependency:

$0.9 \times 0.8 + 0.4 \times 0.1 = 0.76 = 76\%$

Town B's fossil fuel dependency:

3/3

$0.15 \times 0.8 + 0.85 \times 0.4 = 0.46 = 46\%$

Town C's fossil fuel dependency:

$0.55 \times 0.8 + 0.45 \times 0.4 = 0.65 = 65\%$

Since Town A has the highest percentage of power derived from fossil fuels (76%), it is the most affected.

Q2 12/12

Question 3:

Let A be the quantities of onions, carrots and garlic cloves needed to make vegetable , minestrone and French onion soup respectively.

(%i8)
```
A: matrix(
[2,2,8],
[3,1,0],
[2,3,4]
);
```

(A) $\begin{pmatrix} 2 & 2 & 8 \\ 3 & 1 & 0 \\ 2 & 3 & 4 \end{pmatrix}$

Let x=vegtable, y=minestone and z=French onion.

(%i18)
```
B: matrix(
x
,
y
,
z
);
```

(B) $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

Let C be the quatities of the ingredients in the store cupboard.

(%i39)
```
C: matrix(
[40],
[10],
[25]
);
```

(C) $\begin{pmatrix} 40 \\ 10 \\ 25 \end{pmatrix}$

By using these matrices to represent the 3 simultaneous equation, 2x+2y+8z=40 3x+1y+0z=10 2x+3y+4z=25 The inverse of A is

(%i43)
```
A_inv:invert(A);
```

(A_ inv) $\begin{pmatrix} \frac{1}{10} & \frac{2}{5} & -\left(\frac{1}{5}\right) \\ -\left(\frac{3}{10}\right) & -\left(\frac{1}{5}\right) & \frac{3}{5} \\ \frac{7}{40} & -\left(\frac{1}{20}\right) & -\left(\frac{1}{10}\right) \end{pmatrix}$

By multiply both sides of the equations by the inverse of A

(% i42) A_inv.C;

(% o42) $\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ ✓

So we can mke 3 portions of vegtable soup, 1 minestone soup and 4 French onion soup.

Q3 5/5
I have awarded full marks,
however as part of GMC you should
show your simultaneous equations.
See below.

$$\begin{aligned} 2x + 2y + 8z &= 40 \\ 3x + y \qquad &= 10 \\ 2x + 3y + 4z &= 25 \end{aligned}$$

The matrix form of the three simultaneous equations is

$$\underbrace{\begin{pmatrix} 2 & 2 & 8 \\ 3 & 1 & 0 \\ 2 & 3 & 4 \end{pmatrix}}_{\mathbf{A}} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underbrace{\begin{pmatrix} 40 \\ 10 \\ 25 \end{pmatrix}}_{\mathbf{C}}$$

Question 4:

Let

$$x = 0.273273273\ldots$$

Then

$$1000x = 273.273273273\ldots$$

Subtracting the first equation from the second gives

$$999x = 273$$

Therefore

$$x = \frac{273}{999}$$

Simplifying the fraction gives

$$x = \frac{91}{333}$$

Q4 4/4

Question 5:

Given:

$$\left(\frac{x}{4} - 5\right)^7$$

We use the general formula for the binomial expansion:

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r,$$
$$\binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Substituting $a = \frac{x}{4}$, $b = -5$, and $n = 7$:

$$\begin{aligned}(a+b)^n &= \sum_{r=0}^{7} \binom{7}{r} \left(\frac{x}{4}\right)^{7-r} (-5)^r \\ &= \binom{7}{0}\left(\frac{x}{4}\right)^7 + \binom{7}{1}\left(\frac{x}{4}\right)^6 (-5) + \binom{7}{2}\left(\frac{x}{4}\right)^5 (-5)^2 \\ &\quad + \binom{7}{3}\left(\frac{x}{4}\right)^4 (-5)^3 + \binom{7}{4}\left(\frac{x}{4}\right)^3 (-5)^4 \\ &\quad + \binom{7}{5}\left(\frac{x}{4}\right)^2 (-5)^5 + \binom{7}{6}\left(\frac{x}{4}\right) (-5)^6 + \binom{7}{7}(-5)^7.\end{aligned}$$

4/4 Only the highlighted part is required

Expanding the binomial coefficients and simplifying:

$$= \frac{x^7}{16384} - \frac{35x^6}{4096} + \frac{525x^5}{1024} - \frac{4375x^4}{256} + \frac{21875x^3}{64} - \frac{65625x^2}{16} + \frac{109375x}{4} - 78125.$$

Hence the coefficient for $|x|^3$ is $\frac{21875}{64}$

Question 6:

2, $\quad -2.5,\quad 3.125,\quad -3.90625$

a)

$$a = 2$$

and

$$r = \frac{-2.5}{2}$$
$$= -1.25$$

3/4 The question asks for the recurrence system to be stated, see below

for

$$(n = 2, 3, 4, \dots)$$

So the recurrence system is:

$x_1 = 2,\ \ x_n = -1.25x_{n-1}\ \ (n = 2, 3, 4, \ldots).$

b)

The closed for for x_n is

$$x_n = 2\,(-1.25)^{n-1}$$

0.5/1 you must state the range,
ie n=1,2,3.....

c)

for the $32nd$ term

$$x_{32} = 2\,(-1.25)^{32-1}$$
$$= 2\,(-1.25)^{31}$$
$$= -2019.483917$$
$$= -2019.484$$

1/1 GMC alert! Careful the decimal does not terminate,
you must use ellipsis to show this
ie -2019.483917......

to 3 d.p

d)

For $|x_n| > 57,000$

$$\begin{aligned}
2\left(\frac{5}{4}\right)^n &= 57,000 \\
\left(\frac{5}{4}\right)^n &= \frac{57,000}{2} \\
&= 28,500 \\
n \ln\frac{5}{4} &= \ln 28,500 \\
n &= \frac{\ln 28,500}{\ln\frac{5}{4}} \\
&= 45.96888104
\end{aligned}$$

Therefore the smallest value of n for which $|x_n| > 57,000$

$$n = 46$$

2/4 The method of using logs is correct, however the power should be n-1.
By omitting this you have arranged at the wrong answer.
See the correct solution below.

$$\begin{aligned}
2\times(1.25)^{N-1} &= 57,000 \\
(1.25)^{N-1} &= 28,500 \\
(N-1)\ln 1.25 &= \ln(28,500) \\
N-1 &= \frac{\ln(28,500)}{\ln 1.25} \\
N-1 &= 45.9688\ldots \\
N &= 46.9688\ldots
\end{aligned}$$

Hence, the lowest value of n such that $|x_n| > 57,000$ is $n = 47$.

Q6 7/10

Question 7:

a)
Given the series

$$u_1 = 250 \quad u_2 = 280 \quad u_3 = 325 \quad u_4 = 385$$

The series of stamps added to the collecetion each year is;

$$30, 45, 60, 75, \ldots$$

closed form if an arimetic sequence is

$$x_n = a + (n-1)\,d$$

Hence the closed form of the series is

$$u_n = 30 + (25-1)\,15$$

1/1

For the $25th$ year

$$u_{25} = 30 + (25-1)\,15$$
$$= 30 + 24 \cdot 15$$
$$= 30 + 360$$
$$= 390$$

Therefore the number of stamps put into the colection in the 25^{th} years is 390

b)

The sum of an arrithmetic series is given by

$$\frac{1}{2}n\left(2a+(n-1)\,d\right)$$

or

$$\frac{n}{2}\left(a+l\right)$$

where l is the last term of the series.

The sum of the first 25 terms of the series is

$$\begin{aligned} S_{25} &= \frac{25}{2}\left(30+390\right) \\ &= \frac{25}{2}\cdot 420 \\ &= 5250 \end{aligned}$$

Therefore including the initial 250 stamps, the total number of stamps in the collection after 25 years is

$$\begin{aligned} total &= 5250+250 \\ &= 5500 \end{aligned}$$

4/4 GMC alert! You have clearly understood the formula, however you should state your values of a and l ahead of substituting.

Q7 5/5

Question 8:

a)

$$\sum_{n=0}^{\infty} 3\left(\frac{7}{11}\right)^n$$

For the sum of the series we can use,

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Using $a = 3r = \frac{7}{11}$

$$\sum_{n=0}^{\infty} 3\left(\frac{7}{11}\right)^n = \frac{3}{1-\frac{7}{11}}$$

$$= \frac{3}{\frac{4}{11}}$$

$$= \frac{3 \cdot 11}{4}$$

Therefore the sum of the series is

$$= \frac{33}{4}$$

2/2

b)

$$\sum_{n=12}^{36} \left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + 1\right)$$

Using the general formulae

$$\sum_{n=1}^{m} n^3 = \frac{m^2(m+1)^2}{4} - \frac{n^2(n+1)^2}{4}$$

$$\sum_{n=1}^{m} n^2 = \frac{m(m+1)(2m+1)}{6} - \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{n=1}^{m} 1 = m - n$$

Hence the sum of the series is

$$\sum_{n=12}^{36}\left(\frac{1}{3}n^3 + \frac{1}{2}n^2 + 1\right) =$$

Substitute into the equation

$$= \frac{1}{3}\left[\frac{36^2(36+1)^2}{4} - \frac{11^2(11+1)^2}{4}\right] + \frac{1}{2}\left[\frac{36(36+1)(2\cdot 36+1)}{6} - \frac{11(11+1)(2\cdot 11+1)}{6}\right] + 36 -$$

$$= \frac{1}{3}\left[\frac{36^2\cdot 37^2}{4} - \frac{11^2\cdot 12^2}{4}\right] + \frac{1}{2}\left[\frac{36\cdot 37\cdot 73}{6} - \frac{11\cdot 12\cdot 23}{6}\right] + 25$$

$$= \frac{1}{3}\left[\frac{36^2\cdot 37^2 - 11^2\cdot 12^2}{4}\right] + \frac{1}{2}\left[\frac{36\cdot 37\cdot 73 - 11\cdot 12\cdot 23}{6}\right] + 25$$

$$= \frac{1}{3}\left[\frac{1296\cdot 1369 - 121\cdot 144}{4}\right] + \frac{1}{2}\left[\frac{97236 - 3036}{6}\right] + 25$$

$$= \frac{1}{3}\left[\frac{1776864 - 17424}{4}\right] + \frac{1}{2}\left[\frac{94200}{6}\right] + 25$$

$$= \frac{1}{3}\left[\frac{1759440}{4}\right] + \frac{1}{2}[15700] + 25$$

$$= \frac{1}{3}[439860] + \frac{1}{2}[15700] + 25$$

$$= 146620 + 7850 + 25$$

$$= 154495$$

4/5 You have made an evaluation error here, once corrected you will the correct final answer is 154275

Q8 6/7

Question 9:

a)

$$z = 12\left(\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right)$$

and

$$w = 3\left(\cos\left(\frac{\pi}{3}\right) - i\sin\left(\frac{\pi}{3}\right)\right)$$

Euler's formula

$$e^{i\pi} = \cos\theta + i\sin\theta$$

From Euler's formula

$$z = 12e^{i\frac{7\pi}{12}}$$

$$w = 3e^{i\frac{\pi}{3}}$$

Hence

$$zw = 12 \cdot 3e^{i\left(\frac{7\pi}{12} - \frac{\pi}{3}\right)}$$

$$= 36e^{i\left(\frac{7\pi}{12} - \frac{4\pi}{12}\right)}$$

$$= 36e^{i\frac{3\pi}{12}}$$

$$= 36e^{i\frac{\pi}{4}}$$

Since $\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$$= 36\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$$

$$= 18\sqrt{2} + 18\sqrt{2}i$$

4/4

b)

$$z^4 = -3\left(\sqrt{3}i + 1\right)$$

Let $z = r\left(\cos\theta + i\sin\theta\right)$

and

$$-3\left(\sqrt{3}i + 1\right) = -3 - 3\sqrt{3}i$$

So r is

$$\begin{aligned} r &= \sqrt{\left(3\sqrt{3}\right)^2 + 3^2} \\ &= \sqrt{36} \\ &= 6 \end{aligned}$$

and ϕ is

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{3\sqrt{3}}{3}\right) \\ &= \tan^{-1}\left(\sqrt{3}\right) \\ &= \frac{\pi}{3} \end{aligned}$$

therefore

$$\begin{aligned} \theta &= \phi - \pi \\ &= \frac{\pi}{3} - \pi \\ &= -\frac{2\pi}{3} \end{aligned}$$

hence

$$\begin{aligned} -3\left(\sqrt{3}i + 1\right) &= 6\left(\cos\left(-\frac{2\pi}{3}\right) + \sin\left(-\frac{2\pi}{3}\right)i\right) \\ &= 6e^{i\left(-\frac{2\pi}{3}\right)} \end{aligned}$$

Therefore we can write

De Moivre's formula

$\left(r\left(\cos\theta + i\sin\theta\right)\right)^n = r^n\left(\cos n\theta + i\sin n\theta\right)$

or

$z^n = Re^{i\theta} \implies \sqrt[n]{R}e^{\left(i\frac{\theta + 2\pi k}{n}\right)}, k = 0, 1, ..., n-1$

$$
\begin{aligned}
(r(\cos\theta + i\sin\theta))^4 &= 6e^{i\left(-\frac{2\pi}{3}\right)} \\
&= \sqrt[4]{6}e^{i\frac{\frac{-2\pi}{3}+2\pi k}{4}} \\
&= \sqrt[4]{6}e^{i\frac{\pi k}{2}-\frac{\pi}{6}}
\end{aligned}
$$

Hence the solutions are

$$z = \sqrt[4]{6}e^{i\frac{\pi k}{2}-\frac{\pi}{6}}$$

where $k = 0, 1, 2, 3$

$k = 0$

$$= \sqrt[4]{6}e^{-\frac{\pi}{6}}$$

$k = 1$

$$
\begin{aligned}
&= \sqrt[4]{6}e^{\frac{\pi}{2}-\frac{\pi}{6}} \\
&= \sqrt[4]{6}e^{\frac{\pi}{3}}
\end{aligned}
$$

$k = 2$

$$
\begin{aligned}
&= \sqrt[4]{6}e^{\pi-\frac{\pi}{6}} \\
&= \sqrt[4]{6}e^{\frac{5\pi}{6}}
\end{aligned}
$$

$k = 3$

$$
\begin{aligned}
&= \sqrt[4]{6}e^{\frac{3\pi}{2}-\frac{\pi}{6}} \\
&= \sqrt[4]{6}e^{\frac{4\pi}{3}}
\end{aligned}
$$

Hence in polar form the solutions are,

$$z = \sqrt[4]{6}\left(\cos\left(\frac{-\pi}{6}\right) + \sin\left(\frac{-\pi}{6}\right)i\right)$$

$$z = \sqrt[4]{6}\left(\cos\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)i\right)$$

$$z = \sqrt[4]{6}\left(\cos\left(\frac{5\pi}{6}\right) + \sin\left(\frac{5\pi}{6}\right)i\right)$$

$$z = \sqrt[4]{6}\left(\cos\left(\frac{4\pi}{3}\right) + \sin\left(\frac{4\pi}{3}\right)i\right)$$

6/6

Q9 10/10

Question 10:

Define the complex numbers

–> z:17/4 + 2/5*%i;

(z) $\frac{2\%i}{5}+\frac{17}{4}$

–> w:23/4 - 1/2*%i;

(w) $\frac{23}{4}-\frac{\%i}{2}$

–> z*w;

(% o5) $\left(\frac{23}{4}-\frac{\%i}{2}\right)\left(\frac{2\%i}{5}+\frac{17}{4}\right)$

–> z/w;

(% o14) $\frac{\frac{2\%i}{5}+\frac{17}{4}}{\frac{23}{4}-\frac{\%i}{2}}$

4.5 Remember answers are required to 3dp

The modulus of zw is,

–> float(abs(z*w));

(% o13) 24.63812150408387

The principal argument of zw is,

–> float(carg(z*w));

(% o12) 0.0071028739533513935

The modulus of z/w is,

–> float(abs(z/w));

(% o10) 0.739605898809272

The pricipal argument of z/w is,

–> float(carg(z/w));

(% o11) 0.1805795513053216

B)

The modulus of zw is 24.638 and the principal argument of zw is 0.007 radians, correct to 3 decimal places.

The modulus of z/w is 0.740 and the principal argument of z/w is 0.181 radians, correct to 3 decimal places.

(% i1) solns:solve(4*z^ 6 + 20*z^ 5 + 53*z^ 4 + 100*z^ 3 +148*z^ 2 +120*z + 75 = 0, z);

✓

(solns)
$$\left[z=-\left(\frac{2\%i+1}{2}\right),z=\frac{2\%i-1}{2},z=-\left(\sqrt{3}\%i\right),z=\sqrt{3}\%i,z=-\%i-2,z=\%i-2\right]$$

So the solutions are -(2i+1)/(2), (2i-1)/(2), -sqrt3i, sqrt3i, -i-2 and i-2

(% i7) v:makelist(rhs(solns[k]), k, 1, length(solns));

✓

(v) $$\left[-\left(\frac{2\%i+1}{2}\right),\frac{2\%i-1}{2},-\left(\sqrt{3}\%i\right),\sqrt{3}\%i,-\%i-2,\%i-2\right]$$

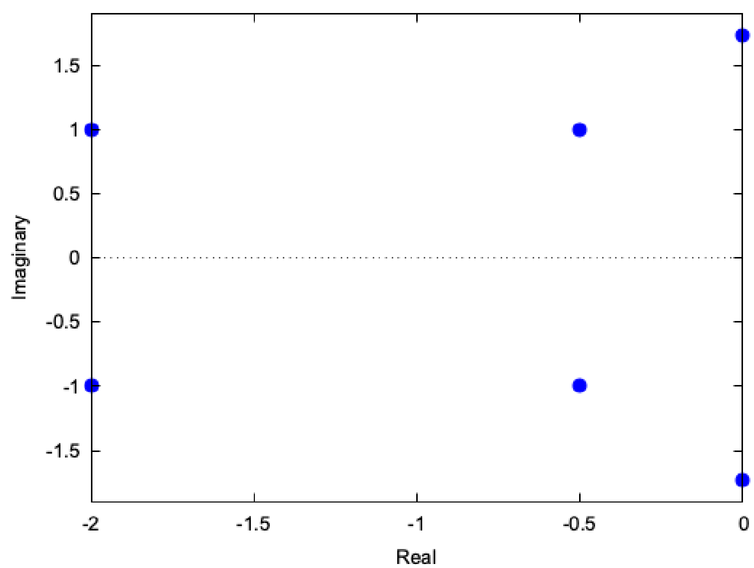
(% i8) pts:makelist([realpart(v[k]), imagpart(v[k])], k, 1, length(solns));

✓

(pts) $$\left[\left[-\left(\frac{1}{2}\right),-1\right],\left[-\left(\frac{1}{2}\right),1\right],\left[0,-\sqrt{3}\right],\left[0,\sqrt{3}\right],[-2,-1],[-2,1]\right]$$

(% i13) wxplot2d([discrete,pts], [style,points], [xlabel,"Real"], [ylabel,"Imaginary"]);

(% t13)



✓

4/5 You must list your solutions

The solutions to the equation $4z^6 + 20z^5 + 53z^4 + 100z^3 + 148z^2 + 120z + 75 = 0$ are

$$z = -\frac{1}{2} \pm i, \; z = \pm\sqrt{3}i, \text{ and } z = -2 \pm i.$$

Q10 9/10

Question 11:

a)

$$\left(\frac{2i}{3+3\sqrt{3}i}\right)^5$$

The modulus of the demominator is

$$\begin{aligned}\sqrt{3^2+3\sqrt{3}^2} &= \\ &= \sqrt{36} \\ &= 6\end{aligned}$$

The argument of the denominator is

$$\begin{aligned}\tan^{-1}\left(\frac{3\sqrt{3}}{3}\right) &= \\ &= \tan^{-1}\left(\sqrt{3}\right) \\ &= \frac{\pi}{3}\end{aligned}$$

Therefore, in exponetial form

$$= 6e^{i\frac{\pi}{3}}$$

Express the numerator in exponetial form

$$2i = 2e^{i\frac{\pi}{2}}$$

Writing both the numerator and denominator in exponential form

$$\left(\frac{2e^{i\frac{\pi}{2}}}{6e^{i\frac{\pi}{3}}}\right)^5 =$$

Using Index laws

$$= \left(\frac{2}{6}e^{i\frac{\pi}{2}-i\frac{\pi}{3}}\right)^5$$

$$= \left(\frac{1}{3}e^{i\frac{\pi}{6}}\right)^5$$

Using De Moivre's formula

$$= \left(\frac{1}{3}\right)^5 e^{i\frac{5\pi}{6}}$$

$$= \frac{1}{243}e^{i\frac{5\pi}{6}}$$

5/5

b)

Given

$$\sum_{k=0}^{n=1}\left(e^{\frac{2\pi i}{n}}\right)^{k-1}$$

formula for the sum of a geometric series

$$S_n = \frac{1-r^n}{1-r}$$

Using the sum of a geometric series

just n here

$$\sum_{k=0}^{n=1} e^{\frac{2\pi i}{n}} = \frac{1-\left(e^{\frac{2\pi i}{n}}\right)^n}{1-e^{\frac{2\pi i}{n}}}$$

k=1

Using Euler's formula

$$= \frac{1 - e^{i2\pi}}{1 - e^{\frac{i2\pi}{n}}}$$

3/3

From Eulers identity $e^{2\pi i} = 1$

$$= \frac{1 - 1}{1 - e^{\frac{i2\pi}{n}}}$$

$$= \frac{0}{1 - e^{\frac{i2\pi}{n}}}$$

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$$= 0$$

Question 12:

I think my time managment for this module was OK, as I have to get through all the material quickly in orer to move on to the next maths module, MST125. And still have enough time to complete the other module in astronomy and prepare for the begining of the IT course that has just started.

I struggled to get to all the tutorials live due to work commitments but always made sure I watched as many of the recording as I could.

I didn't find the forums as useful as I could have because I found I quickly became lost in all the threads because I didn't follow them from the start.

MAXIMA is a very powerful tool and I found it very useful for checking my answers. I still need to put more time into learnig how to use it efficiently, for my upcoming modules.

My further reading around maths has taken a bit of a back seat whilst I have been trying to study basically full time whist also working full time. I hope that next year when my study commitments are lestened I will be able to spend more time on this.

5/5 Thank you for sharing, all points
you can consider in your further studies

Question 13:

Q13 5/5 Well done, good use of GMC