Question 1:

$$\int \frac{x^2}{\sqrt{x^2-9}}\,\mathrm{d}x$$

using the substitution $x = 3\cosh u$ and hence $dx = 3\sinh u\,\mathrm{d}u$

$$= \int \frac{(3\cosh u)^2}{\sqrt{(3\cosh u)^2 - 9}} \cdot 3\sinh u\,\mathrm{d}u$$

$$= \int \frac{9\cosh^2 u}{\sqrt{9\cosh^2 u - 9}} \cdot 3\sinh u\,\mathrm{d}u$$

$$= \int \frac{9\cosh^2 u}{\sqrt{9(\cosh^2 u - 1)}} \cdot 3\sinh u\,\mathrm{d}u$$

Using the identity $\cosh^2 u - 1 = \sinh^2 u$

$$= \int \frac{9\cosh^2 u}{\sqrt{9\sinh^2 u}} \cdot 3\sinh u\,\mathrm{d}u$$

$$= \int \frac{9\cosh^2 u}{3\sinh u} \cdot 3\sinh u\,\mathrm{d}u$$

$$= \int 9\cosh^2 u\,\mathrm{d}u$$

$$= 9\int \cosh^2 u\,\mathrm{d}u$$

Using the identity $\cosh^2 u = \frac{1+\cosh 2u}{2}$

$$= 9\int \frac{1+\cosh 2u}{2}\,\mathrm{d}u$$

$$= \frac{9}{2}\int (1+\cosh 2u)\,\mathrm{d}u$$

$$= \frac{9}{2}\left(u + \frac{\sinh 2u}{2}\right) + C$$

$$= \frac{9}{2}u + \frac{9}{4}\sinh 2u + C$$

Using the identity $\sinh 2u = 2\sinh u\cosh u$

$$= \frac{9}{2}u + \frac{9}{4}(2\sinh u\cosh u) + C$$

$$= \frac{9}{2}u + \frac{9}{2}\sinh u\cosh u + C$$

Back-substitute $\cosh u = \frac{x}{3},\ \sinh u = \frac{\sqrt{x^2-9}}{3},\ u = \left(\frac{x}{3}\right)$

$$= \frac{9}{2}\left(\frac{x}{3}\right) + \frac{9}{2}\left(\frac{\sqrt{x^2-9}}{3} \cdot \frac{x}{3}\right) + C$$

$$= \frac{9}{2}\left(\frac{x}{3}\right) + \frac{1}{2}\,x\sqrt{x^2-9} + C$$

Equivalently, using $t = \ln\left(t + \sqrt{t^2-1}\right)$

$$= \frac{1}{2}\,x\sqrt{x^2-9} + \frac{9}{2}\,\ln\left(x + \sqrt{x^2-9}\right) + C$$

$$= \boxed{\frac{1}{2}\,x\sqrt{x^2-9} + \frac{9}{2}\left(\frac{x}{3}\right) + C}$$