Question 1:

a)

MST125 TMA 01 Question 1
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LaTeXcompiled with TeXLive 2019

b)
The distance between $A\left(x_1, y_1\right)$ and $B\left(x_2, y_2\right)$ is given by

$$AB = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$

Hence the distance between $A\left(1, 4\right)$ and $B\left(3, -1\right)$ is

$$\begin{aligned} AB &= \sqrt{\left(1-3\right)^2 + \left(4-(-1)\right)^2} \\ &= \sqrt{\left(-2\right)^2 + \left(5\right)^2} \\ &= \sqrt{4+25} \\ &= 29 \end{aligned}$$

c)
The gradient of the line thriough $\left(x_1, y_1\right)$
and x_2, y_2 is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Hence the gradient of the line through
$\left(1, 4\right)$ and $\left(3, -1\right)$ is

$$m = \frac{(-1) - 4}{3 - 1} = -\frac{5}{2}$$

d)
The gradient of the line is $-\frac{5}{2}$ Hence
$\tan\alpha = -\frac{5}{2}$.

Let ϕ be the acute angle that the line
makes with the negative direction of the $x - axis$.

Then

$$\tan\phi = \frac{5}{2},$$

so

$$\phi = \tan^{\text{-1}}\left(\frac{5}{2}\right) = 1.950\ldots$$

Hence

$$\alpha = \pi - 1.190\ldots = 1.951\ldots$$

Therefore the angle α is 1.95 *radians*

(to 2 d.p.)

.

Question 2: **a)**

$$405 \equiv 5 \pmod{16}$$

It follows that

$$405^{10} \equiv 5^{10} \pmod{16}$$

$$5^2 = 25 \equiv 9 \pmod{16}$$

$$5^4 = 25^2 \equiv 9^2 = 81 \equiv 1 \pmod{16}$$

$5^{10} = 5^8 \cdot 5^2$

$$5^8 = \left(5^4\right)^2 \equiv 1^2 = 1 \pmod{16}$$

So

$$5^{10} \equiv 1 \cdot 5^2 \equiv 9 \pmod{16}$$

Hence

$$405^{10} \equiv 9 \pmod{16}$$

$$a^{p-1} \equiv 1 \pmod{p}$$

b)

Using Fermat's Little Theorem, as 83 is prime

$$13^{82} \equiv 1 \pmod{83}$$

That implies

$$13^{164} \equiv \left(13^{82}\right)^2 \equiv 1^2 \equiv 1 \pmod{83}$$

$$13^{328} \equiv \left(13^{82}\right)^4 \equiv 1^4 \equiv 1 \pmod{83}$$

$$13^{492} \equiv \left(13^{82}\right)^6 \equiv 1^6 \equiv 1 \pmod{83}$$

Hence, for 13^{494}

$$13^{494} = 13^{492} \cdot 13^2 \equiv 1 \cdot 13^2 \pmod{83}$$

So

$$= 169 \pmod{83}$$

$$= 3 \pmod{83}$$

Hence the least residue of $13^{494} \pmod{83}$ is 3

c)
Let p be a prime $p \neq 13$ find a positive integer b such that

$$13^{6p-4} \equiv b \pmod{5}$$

Let p=5

$$13^{(6(5)-4)} \equiv b \pmod{5}$$

$$13^{26} \equiv b \pmod{5}$$

$$13^2 = 169 \equiv 4 \pmod{5}$$

Using Fermat's Little Theorem

$$13^4 \equiv 1 \pmod{5}$$

$$13^{16} equiv 1 \pmod{5}$$

$$13^{26} = 13^{16} \cdot 13^4 \cdot 13^2 \equiv 1 \cdot 1 \cdot 4 \equiv 4 \pmod{5}$$

Question 3:

a)
i.

Find the multiplicative inverse of $29 \pmod{80}$ using Euclid's algorithm

$$80 = 2 \cdot 29 + 22$$

$$29 = 1 \cdot 22 + 7$$

$$22 = 3 \cdot 7 + 1$$

$$7 = 1 \cdot 7 + 0$$

As the second to last remainder is 1 (Thus are co-prime) we can work backwards to find the multiplicative inverse.

Backward substitution gives

$$\begin{aligned}
1 &= 22 - 3 \cdot 7 \\
&= 22 - 3 \cdot \left(29 - 22\right) \\
&= 22 - 3 \cdot 29 + 3 \cdot 22 \\
&= 4 \cdot 22 - 3 \cdot 29 \\
&= 4\left(80 - 2 \cdot 29\right) - 3 \cdot 29 \\
&= 4 \cdot 80 - 8 \cdot 29 - 3 \cdot 29 \\
&= 4 \cdot 80 - 11 \cdot 29
\end{aligned}$$

Hence

$$-11 \equiv 69 \pmod{80}$$

Hence the multiplicative inverse of $29 \pmod{80}$ is 69

To solve $29x \equiv 41 \pmod{80}$ multiply both sides by the multiplicative inverse

$$69 \cdot 29x \equiv 69 \cdot 41 \pmod{80}$$

Since

$$69 \cdot 29 \equiv 1 \pmod{80}$$

We have

$$x \equiv 69 \cdot 41 \pmod{80}$$
$$x \equiv 2829 \pmod{80}$$
$$x \equiv 29 \pmod{80}$$

Hence the solution is $x \equiv 29 \pmod{80}$

ii.

Given

$$12x \equiv 16 \pmod{54}$$

As 12 and 54 are not co-prime, we cannot use Euclid's algorithm to find the multiplicative inverse.

Instead, we can divide both sides by the greatest common divisor of 12 and 54

The greatest common divisor of 12 and 54 is 6

But, 6 does not divide 16

Hence, the equation has no solution.

iii.

Given

$$12x = 18 \pmod{54}$$

As 12 and 54 are not co-prime, we cannot use Euclid's algorithm to find the multiplicative inverse.

Instead, we can divide both sides by the greatest common divisor of 12 and 54

The greatest common divisor of 12 and 54 is 6

Dividing both sides by 6 gives

$$2x \equiv 3 \pmod{9}$$

As the numbers are small we can try all possible values of x to find the solution.

$$\begin{aligned} 2 \times 1 &\equiv 2 \pmod 9 \\ 2 \times 2 &\equiv 4 \pmod 9 \\ 2 \times 3 &\equiv 6 \pmod 9 \\ 2 \times 4 &\equiv 8 \pmod 9 \\ 2 \times 5 &\equiv 1 \pmod 9 \\ 2 \times 6 &\equiv 3 \pmod 9 \end{aligned}$$

Hence the solution is $x \equiv 6 \pmod 9$

b)

$$E\left(x\right) \equiv 19x - 5 \pmod{26}$$

A 0	B 1	C 2	D 3	E 4	F 5	G 6	H 7	I 8	J 9	K 10	L 11	M 12
N 13	O 14	P 15	Q 16	R 17	S 18	T 19	U 20	V 21	W 22	X 23	Y 24	Z 25

So we can write the rule for deciphering as

$$D\left(y\right) \equiv v\left(y + 5\right) \pmod{26}$$

(Were v is the multiplicative inverse of $19 \pmod{26}$)

i.

Using the Euclidean Algorithm to find the multiplicative inverse of 19 $\pmod{26}$

$$\begin{aligned} 26 &= 1 \cdot 19 + 7 \\ 19 &= 2 \cdot 7 + 5 \\ 7 &= 1 \cdot 5 + 2 \\ 5 &= 2 \cdot 2 + 1 \\ 2 &= 2 \cdot 1 + 0 \end{aligned}$$

As the second to last remainder is 1 (Thus co-prime)

Backward substitution gives

$$\begin{aligned}
1 &= 5 - 2 \cdot 2 \\
&= 5 - 2 \cdot (7 - 5) \\
&= 5 - 2 \cdot 7 + 2 \cdot 5 \\
&= 5 - 2 \cdot 7 + 2 \cdot (19 - 2 \cdot 7) \\
&= 5 - 2 \cdot 7 + 2 \cdot 19 - 4 \cdot 7 \\
&= 5 - 6 \cdot 7 + 2 \cdot 19 \\
&= 5 - 6 \cdot 26 - 19 + 2 \cdot 19 \\
&= 5 - 6 \cdot 26 + 6 \cdot 19 + 2 \cdot 19 \\
&= 5 - 6 \cdot 26 + 8 \cdot 19 \\
&= 19 - 2 \cdot 7 + 8 \cdot 19 \\
&= 19 - 2 \cdot (26 - 19) + 8 \cdot 19 \\
&= 19 - 2 \cdot 26 + 2 \cdot 19 + 8 \cdot 19 \\
&= 11 \cdot 19 - 2 \cdot 26
\end{aligned}$$

Hence the multiplicative inverse of $19 \pmod{26}$ is 11

As required

ii.

Hence we can use

$$D(y) \equiv 11(y + 5) \pmod{26}$$

to decipher the message $14, 11, 5$

$$\begin{aligned}
D\,(14) &\equiv 11\,(14+5) \pmod{26} \\
&\equiv 11 \cdot 19 \pmod{26} \\
&\equiv 209 \pmod{26} \\
&\equiv 1 \pmod{26} \\
&\equiv B \\
D\,(11) &\equiv 11\,(11+5) \pmod{26} \\
&\equiv 11 \cdot 16 \pmod{26} \\
&\equiv 176 \pmod{26} \\
&\equiv 20 \pmod{26} \\
&\equiv U \\
D\,(5) &\equiv 11\,(5+5) \pmod{26} \\
&\equiv 11 \cdot 10 \pmod{26} \\
&\equiv 110 \pmod{26} \\
&\equiv 6 \pmod{26} \\
&\equiv G
\end{aligned}$$

Hence the message is BUG

Question 4:

Given the elipse

$$4x^2 + 25y^2 - 9 = 0$$

a)

i.

When $x = 0$

$$25y^2 = 9$$

$$y^2 = \frac{9}{25}$$

$$y = \pm\frac{3}{5}$$

When $y = 0$

$$4x^2 = 9$$

$$x^2 = \frac{9}{4}$$

$$x = \pm\frac{3}{2}[8pt]$$

Hence the vertices are

$$\left(\pm\frac{3}{2}, 0\right)$$

,

$$\left(0, \pm\frac{3}{5}\right)$$

ii.

To figure out the eccentricity of the elipse, we can use,

$$e = \sqrt{1 - \frac{b^2}{a^2}}$$

Using the values of a and b from the equation of the elipse

$$\begin{aligned} e &= \sqrt{1 - \frac{\left(\frac{3}{5}\right)^2}{\left(\frac{3}{2}\right)^2}} \\ &= \sqrt{1 - \frac{9}{25} \cdot \frac{4}{9}} \\ &= \sqrt{1 - \frac{4}{25}} \\ &= \sqrt{\frac{21}{25}} \\ &= \frac{\sqrt{21}}{5} \end{aligned}$$

To find the foci of the elipse, we can use,

The foci of the elipse are at $(\pm ae, 0)$

Using the values above, we have

$$\begin{aligned} foci &= \left(\pm\frac{3}{2} \cdot \frac{\sqrt{21}}{5}, 0\right) \\ &= \left(\pm\frac{3\sqrt{21}}{10}, 0\right) \end{aligned}$$

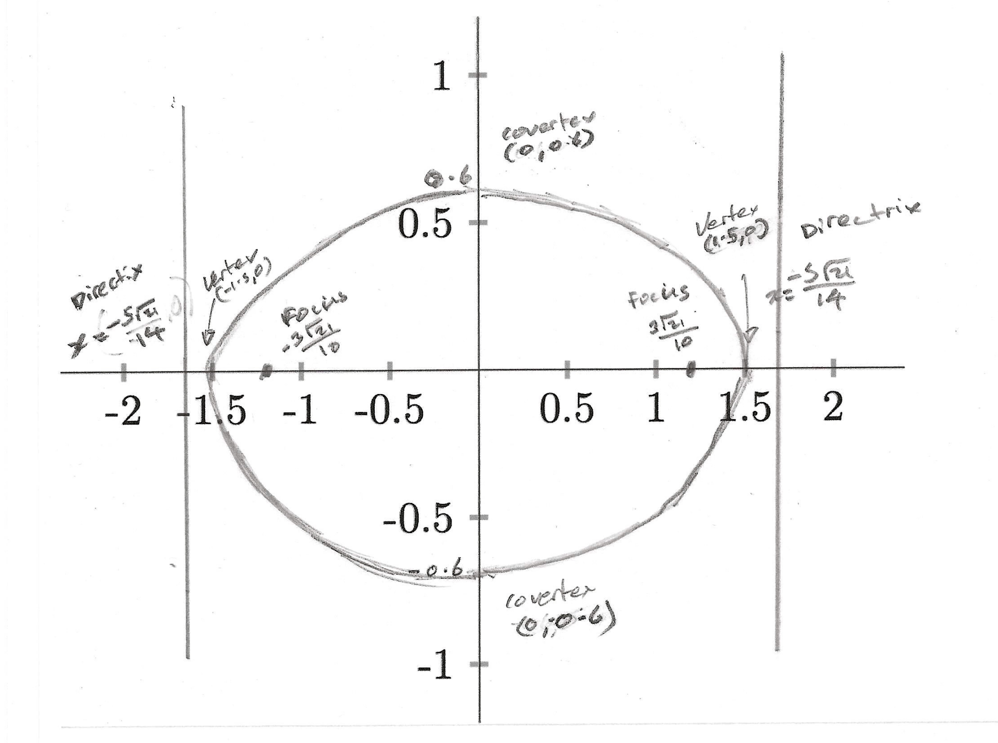
The equation of the directrices is

$$x = \pm\frac{a}{e}$$

Using the values of a and e from the equation of the elipse, the directories are

$$\begin{aligned} x &= \pm\frac{\frac{3}{2}}{\frac{\sqrt{21}}{5}} \\ &= \pm\frac{5\sqrt{21}}{14} \end{aligned}$$

iii.



iv.

Using the standard form of the equation of an elipse

$$4^2 + 25^2 - 9 = 0$$
$$4x^2 + 25y^2 = 9$$

Dividing by 9

$$\frac{4x^2}{9} + \frac{25y^2}{9} = 1$$

Hence the equation of the elipse is

$$\frac{x^2}{\left(\frac{3}{2}\right)^2} + \frac{y^2}{\left(\frac{3}{5}\right)^2} = 1$$

Therefore the parameterisation of the elispe in the third quadrand is

$$x = \frac{3}{2}\cos(t), y = \frac{3}{5}\sin(t)$$

Where $\pi < t < \frac{3}{2}\pi$

v.

When this part of the conic is moved 4 units to the right and 2 units down, it would have parameterisation of

$$x = 4 + \frac{2}{3}\cos(t), y = -2 + \frac{5}{3}\sin(t)$$

b)

i.

Using the general formula for the equation for a conic, and my OU student number being Y362220x, hense the last nono zero digit is 2 therefore $B = 2$

General formula for a conic

$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

$$-2x^2 + Bxy + 3y^2 - x + 3y - 5 = 0$$
$$-2x^2 + 2xy + 3y^2 - x + 3y - 5 = 0$$

Therefore $A = -2, B = 2$ and $c = 3$

evaluating $B^2 - 4AC$

$$\begin{aligned} 2^2 - 4 \cdot -2 \cdot 3 &= 4 + 24 \\ &= 28 \\ 28 &> 0 \end{aligned}$$

Hence the equation represents a hyperbola

ii.

Using the general formula for a conic .of Ax2̂+Bxy+C2̂+Dx+Ey+f=0 My OU number is Y362220X, hence the last non zero digit is 2, therefore B=2.

(% i1) conic:-2*x^ 2+2*x*y+3*y^ 2-x+3*y-5=0;

(conic) $3y^2+2xy+3y-2x^2-x-5=0$

Drawing the graph of this conic

(% i3)
```
load("draw")$
draw2d(
implicit(
conic,
x, -5, 5,
y, -5, 5
)
);
```

(% o3)

$[gr2d\,(\text{implicit})]\; qt.qpa.fonts : Populating font family aliases took 107ms. Replace uses of missing font family"S$

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Warning:  slow font initialization2025-03-03 09:38:42.161 gnuplot_
qt[5615:55353974] +[IMKClient subclass]:  chose IMKClient_ Modern
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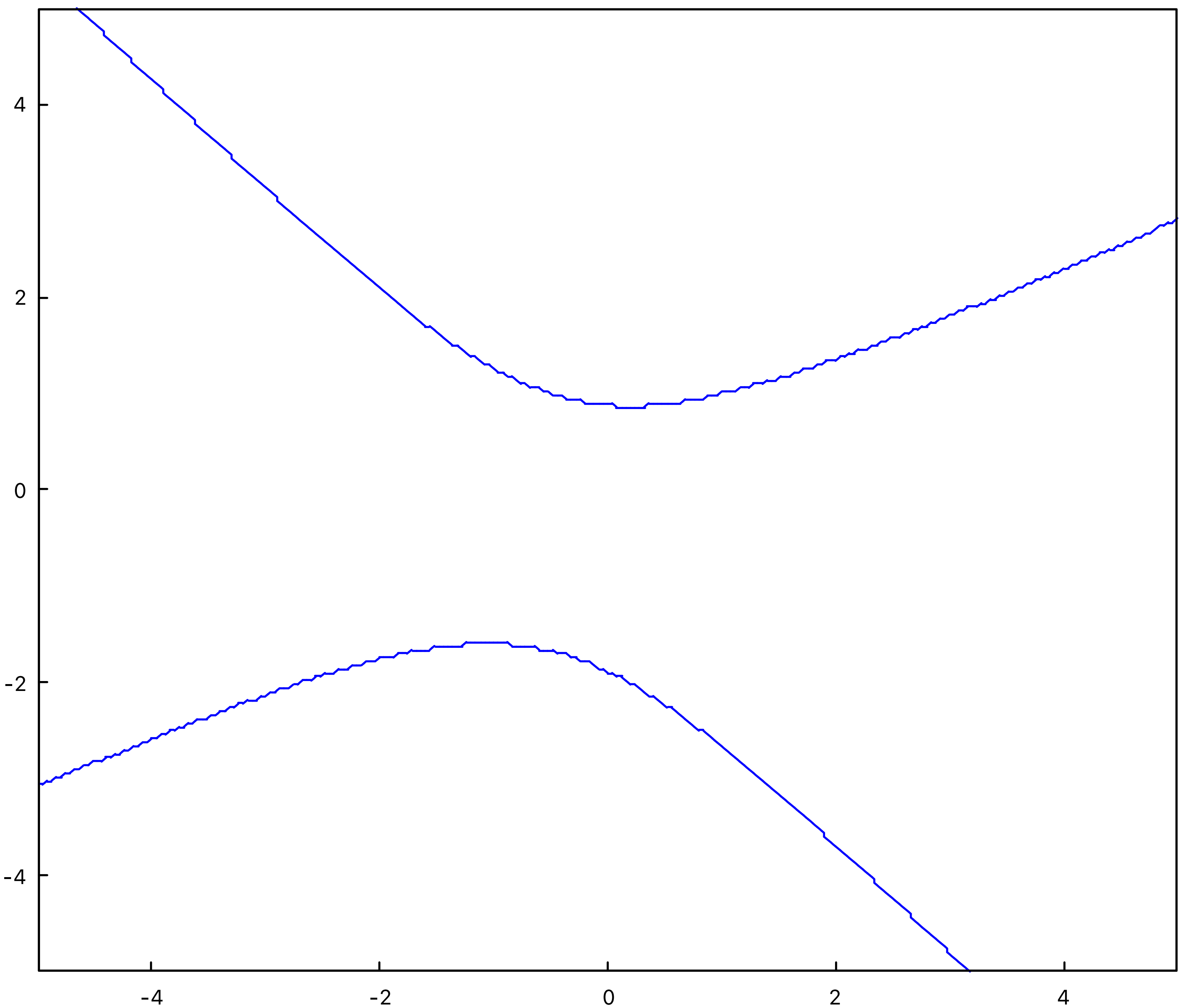
$2025-03-0309:38:42.161 gnuplot\_qt[5615:55353974]+[IMKInputSession subclass]: choseIMKInputSes$



4
2
0
-2
-4
-4
-2
0
2
4

Question 5:

a)
The position of **A** is $(t-3, 3t+8)$

$$x = t - 3$$
$$t = x + 3$$

Hence the equation fo the line is given by

$$\begin{aligned} y &= 3(x+3) + 8 \\ &= 3x + 9 + 8 \\ &= 3x + 17 \end{aligned}$$

b)
The position of **B** is $(2t-5, 4t+5)$

The distance between two points is given by
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Substituting our position of **A** and **B** into the distance formula

$$\begin{aligned} d^2 &= (2t - 5 - (t-3))^2 + (4t + 5 - (3t+8))^2 \\ &= (t-2)^2 + (t-3)^2 \\ &= t^2 - 4t + 4 + t^2 - 6t + 9 \\ &= 2t^2 - 10t + 13 \end{aligned}$$
as required

c)

$$
\begin{aligned}
d^2 &= 2t^2 - 10t + 13 \\
&= 2\left(t^2 - 5t\right) + 13 \\
&= 2\left(t - \frac{5}{2}\right)^2 - \frac{25}{4} + 13
\end{aligned}
$$

Note 1

$$= 2\left(t - \frac{5}{2}\right)^2 + \frac{27}{4}$$

The mimimum value of d^2 occurs when $t = \frac{5}{2} = 2.5$. Hence the minium distance is $\frac{27}{4}$

The first mistake that was made was when completing the square, the expression should have been

$$
\begin{aligned}
&= 2\left(t - \frac{5}{2} - \frac{25}{4}\right) + 13 \\
&= 2\left(t - \frac{5}{2}\right) - \frac{25}{2} + 13
\end{aligned}
$$

.

If we contiue from this error, the second mistake was when solving for the distance, the expression should have been

$$d^2 = \frac{27}{4}$$

hence

$$d = \sqrt{\frac{27}{4}} = 2.59\ldots$$

My soloution is

$$
\begin{aligned}
d^2 &= 2t^2 - 10t + 13 \\
&= 2\left(t^2 - 5t\right) + 13
\end{aligned}
$$

Completing the square

$$= 2\left(\left(t - \frac{5}{2}\right)^2 - \frac{25}{4}\right) + 13$$

simplyfying

$$= 2\left(t - \frac{5}{2}\right)^2 - \frac{25}{2} + 13$$

$$= 2\left(t - \frac{5}{2}\right)^2 + \frac{1}{2}$$

The minimum value of d^2 occurs when $t = \frac{5}{2}$

$$= \frac{1}{2}$$

$$= 0.5$$

to 2 s.f

Hence

$$d = \sqrt{\frac{1}{2}}$$

$$= \frac{\sqrt{2}}{2}$$

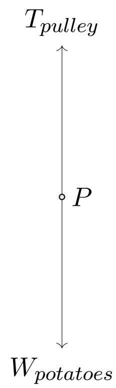
$$= 0.7071\ldots$$
$$= 0.71$$

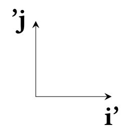
to 2 s.f

Question 6:

a)

i.





As the system is in equalibrium,

$$T + W = 0$$

Or

$$T = -W$$

With

$$W = 10g$$

where g is the acceleration due to gravity

$$T = -10g$$

ii.

In component from

$$T = 10g\textbf{j'}$$

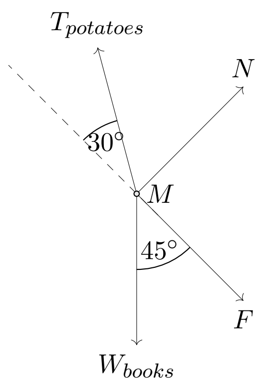
and

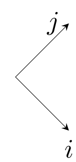
$$W = -10g\textbf{j'}$$

b)

i.

Force diagram of a mass on a slope.





ii.

The forces acting on the box of books in component form are

$$\mathbf{W_{books}} = Mg\cos 45\mathbf{i} - Mg\sin 45\mathbf{j}$$

$$\mathbf{N} = N\mathbf{j}$$

$$\mathbf{T_{potatoes}} = -10g\cos 30\mathbf{i} + 10g\sin 30\mathbf{j}$$

$$\mathbf{F} = F\mathbf{i}$$ *Were g is the acceleration due to gravity*

iii.

As the whole system is in Equlilibrium

$$\mathbf{T_{potatoes}} + \mathbf{W_{books}} + N + F = 0$$

iv.

Resolving this is the $\mathbf{i} - direction$

$$-10g\cos 30 + Mg\cos 45 + F = 0$$

$$F = 10g\cos 30 - Mg\cos 45$$

When $12\,\text{kg}$

$$\begin{aligned} F &= 10 \cdot 9.8\cos 30 - 12 \cdot 9.8\cos 45 \\ &= 49\sqrt{3} - 83.155\ldots \\ &= -1.7\text{N} \end{aligned}$$

to 2 s.f

when $13\,\text{kg}$

$$\begin{aligned} F &= 10 \cdot 9.8\cos 30 - 13 \cdot 9.8\cos 45 \\ &= 49\sqrt{3} - 90.085\ldots \\ &= 5.2\text{N} \end{aligned}$$

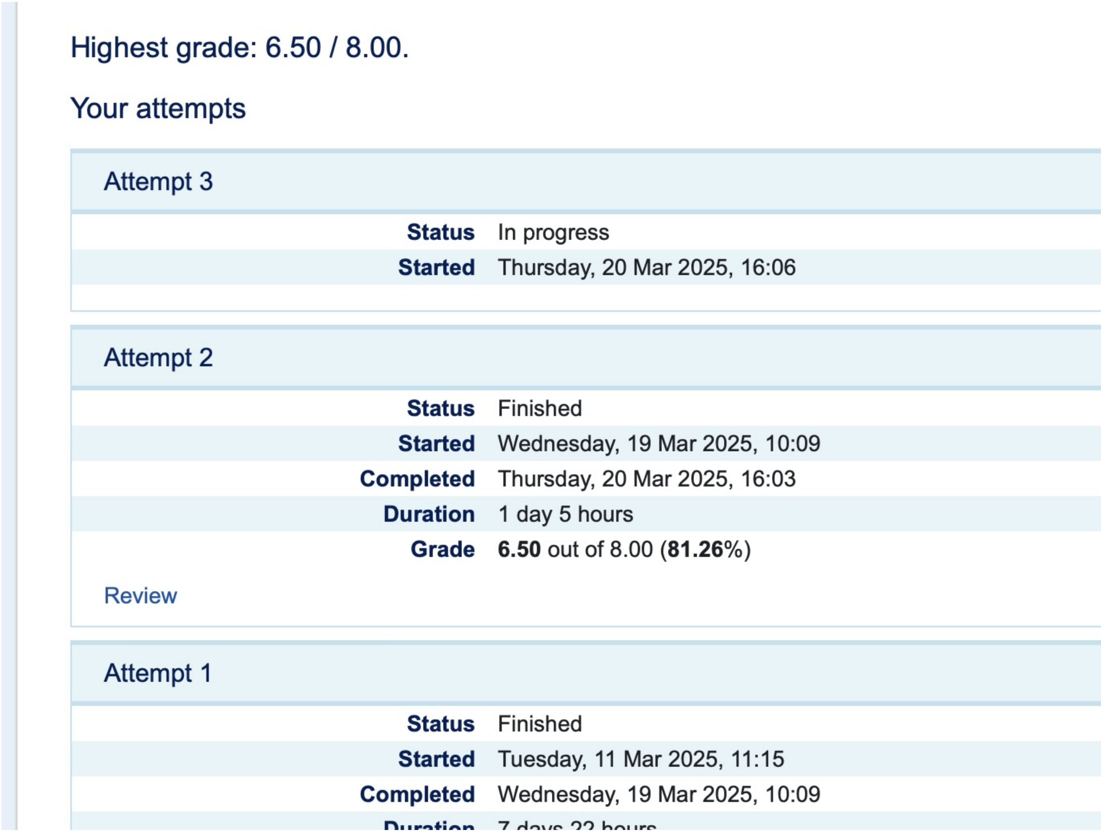
to 2 s.f

So when the mass of the books is $12\,\mathrm{kg}$ the force imparted by friction is $-1.7\mathrm{N}$ and is running up the slope in the $\mathbf{i}-direction$.

When the mass of the books is $13\,\mathrm{kg}$ the force imparted by friction is $5.2\mathrm{N}$ and is running down the slope in the $-\mathbf{i}-direction$.

Question 7:

a)



Highest grade: 6.50 / 8.00.

Your attempts

Attempt 3

Status	In progress
Started	Thursday, 20 Mar 2025, 16:06

Attempt 2

Status	Finished
Started	Wednesday, 19 Mar 2025, 10:09
Completed	Thursday, 20 Mar 2025, 16:03
Duration	1 day 5 hours
Grade	**6.50** out of 8.00 (**81.26**%)

Review

Attempt 1

Status	Finished
Started	Tuesday, 11 Mar 2025, 11:15
Completed	Wednesday, 19 Mar 2025, 10:09
Duration	7 days 22 hours

b)

Based on my work in Unit 5 and the practice quiz, I feel confident in resolving forces in a system and finding the resultant force acting on an object. My prior experience with mechanics has definitely helped. However, I still struggle with keeping track of signs when resolving forces. I often miss a negative sign, especially when transcribing my handwritten work into LaTeX. This is partly due to my dyslexia, which makes spotting transcription errors difficult. While I can catch these mistakes when I print my work, it's something I'll need to be especially careful of during exams.

c)

To improve my understanding of Unit 5, I plan to revisit my previous Level 2 mechanics material, which I believe will reinforce the current content. I also aim to practise more problem-solving to strengthen my intuition and accuracy, particularly in sign conventions. Doing this will help me reduce mistakes and improve confidence. Additionally, I'll review my printed drafts more carefully, as this method helps me catch errors more effectively. I scored well on the practice quiz overall, but these small mistakes could cost me marks under timed conditions. That's why I think continued practice and review are essential.