MST124 Unit 5 Solutions to Tutorial Questions 07_01_2025

Differentiate the functions in question 1 to 8, simplifying your answers where possible.

1. $f(t) = e^{2t} \ln 4t$

$$f'(t) = e^{2t}\left(\frac{1}{t}\right) + (\ln 4t) \times 2e^{2t}$$

$$= e^{2t}\left(\frac{1}{t} + 2\ln 4t\right)$$

2. $y = \frac{4x^2+3x+5}{(x+1)^2}$

$$\frac{dy}{dx} = \frac{(x+1)^2(8x+3) - 2(4x^2+3x+5)(x+1)}{(x+1)^4}$$

$$= \frac{(x+1)[(x+1)(8x+3) - 2(4x^2+3x+5)]}{(x+1)^4}$$

$$= \frac{[(8x^2+11x+3) - (8x^2+6x+10)]}{(x+1)^3}$$

$$= \frac{5x-7}{(x+1)^3}$$

3. $g(x) = \sin(\ln x)$

$$g'(x) = \cos(\ln x)\frac{d}{dx}(\ln x)$$

$$= \frac{1}{x}\cos(\ln x)$$

MST124 Unit 5 Solutions to Tutorial Questions 07_01_2025

4. $p = (2x+3)(x^2+1)^4$

$$\frac{dp}{dx} = (2x+3) \times 4(x^2+1)^3 \times 2x + 2(x^2+1)^4$$

$$= 2(x^2+1)^3(4x(2x+3) + (x^2+1))$$

$$= 2(x^2+1)^3(9x^2+12x+1)$$

5. $h(x) = \sin^{-1}\left(\frac{1}{x}\right)$

$$h'(x) = -x^{-2}\left(\frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}}\right)$$

$$= -x^{-2}\left(\frac{1}{\sqrt{\frac{x^2-1}{x^2}}}\right)$$

$$= -\frac{1}{x^2}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

$$= -\frac{1}{x\sqrt{x^2-1}}$$

6. $k(x) = \ln\left(\frac{1+\sin x}{1-\sin x}\right)$

$$k'(x) = \frac{1}{\frac{1+\sin x}{1-\sin x}} \frac{d}{dx}\left(\frac{1+\sin x}{1-\sin x}\right)$$

$$= \frac{1-\sin x}{1+\sin x}\left(\frac{(1-\sin x)\cos x-(1+\sin x)(-\cos x)}{(1-\sin x)^2}\right)$$

$$= \frac{1-\sin x}{1+\sin x}\left(\frac{2\cos x}{(1-\sin x)^2}\right)$$

$$= \frac{2\cos x}{(1+\sin x)(1-\sin x)}$$

$$= \frac{2\cos x}{1-\sin^2 x}$$

$$= \frac{2\cos x}{\cos^2 x}$$

$$= \frac{2}{\cos x}$$

$$= 2\sec x$$

MST124 Unit 5 Solutions to Tutorial Questions 07_01_2025

7. $r = \frac{\sec\theta + \tan\theta}{\sec\theta - \tan\theta}$

$$\frac{dr}{d\theta} = \frac{(\sec\theta - \tan\theta)(\sec\theta\tan\theta + \sec^2\theta) - (\sec\theta + \tan\theta)(\sec\theta\tan\theta - \sec^2\theta)}{(\sec\theta - \tan\theta)^2}$$

$$= \frac{2\sec^3\theta - 2\sec\theta\tan^2\theta}{(\sec\theta - \tan\theta)^2}$$

$$= \frac{2\sec\theta(\sec^2\theta - \tan^2\theta)}{(\sec\theta - \tan\theta)^2}$$

$$= \frac{2\sec\theta(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)^2}$$

$$= \frac{2\sec\theta(\sec\theta + \tan\theta)}{(\sec\theta - \tan\theta)}$$

8. $y = \sin^2(\cos(9x))$

$$\frac{dy}{dx} = 2\sin(\cos(9x))\frac{d}{dx}(\sin(\cos(9x)))$$

$$= 2\sin(\cos(9x))\cos(\cos(9x))\frac{d}{dx}(\cos(9x))$$

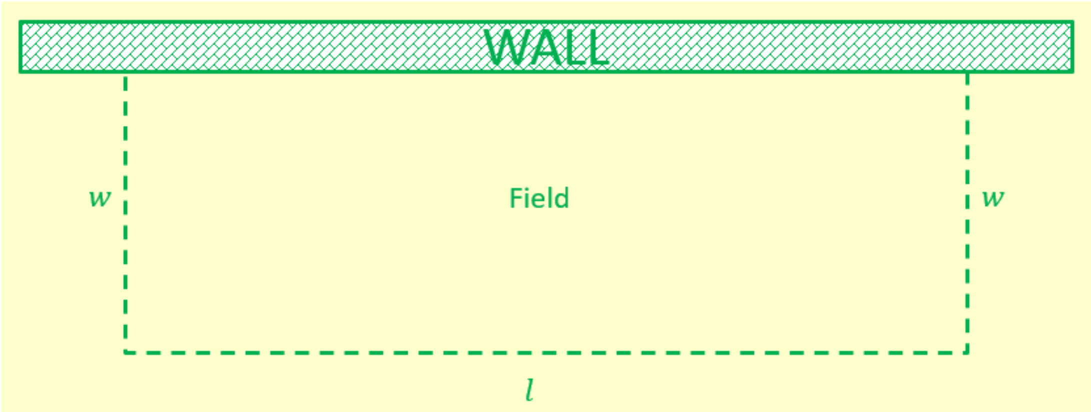
$$= 2\sin(\cos(9x))\cos(\cos(9x))(-9\sin(9x))$$

$$= -18\sin(\cos(9x))\cos(\cos(9x))\sin(9x)$$

MST124 Unit 5 Solutions to Tutorial Questions 07_01_2025

9. A farmer needs to enclose a rectangular area of a field with a fence. They have 500m of fencing material and there is a wall on one side of the field. What is the maximum area of field that can be enclosed?

Let w be the width, l the length and A the area of the field.



Since there is 500m of fencing material, then

$$2w + l = 500$$

$$l = 500 - 2w$$

$$A = w(500 - 2w)$$

$$= 500w - 2w^2$$

So $\frac{dA}{dw} = 500 - 4w$

When $\frac{dA}{dw} = 0$, $w = 125$

So, there is a stationary point when $w = 125$

This will be a maximum since $\frac{d^2A}{dw^2} = -4$ which is less than zero

When $w = 125$,

$$A = 125(500 - 250) = 31250$$

So, the maximum area of field that can be enclosed is 31250m^2

10.

A piece of wire 80cm in length is cut into three parts, two of which are bent into equal circles and the third into a square.
What is the radius of each of the circles if the total area enclosed by the three shapes is a minimum?
Give your answer in terms of π.

Let r be the radius of each circle and l be the length of a side of the square. Let A represent the combined area of the three shapes.

Since the length of the wire is 80cm, the circumference of each circle $2\pi r$ and the perimeter of the square $4l$ then

$$\begin{gathered}4\pi r + 4l = 80\\ \pi r + l = 20\\ l = 20 - \pi r\end{gathered}$$

The area of each circle is πr^2 and the area of the square is l^2.

So,

$$\begin{aligned}A &= 2\pi r^2 + (20 - \pi r)^2\\ &= 2\pi r^2 + 400 - 40\pi r + \pi^2 r^2\\ \frac{dA}{dr} &= 4\pi r - 40\pi + 2\pi^2 r\end{aligned}$$

When $\frac{dA}{dw} = 0$

$$\begin{aligned}4\pi r - 40\pi + 2\pi^2 r &= 0\\ r(4\pi + 2\pi^2) &= 40\pi\\ r &= \frac{40\pi}{2\pi(2+\pi)}\\ &= \frac{20}{2+\pi}\end{aligned}$$

This is a minimum since $\frac{d^2A}{dr^2} = 4\pi + 2\pi^2$, which is positive.

So, the radius of each of the circles if the total area enclosed by the three shapes is a minimum is $\frac{20}{2+\pi}$ cm.