The Open University

Faculty of Science, Technology, Engineering and Mathematics
MST124 Essential Mathematics 1

MST124

TMA 02 2024J

Covers Units 2 – 6 **Cut-off date 22 January 2025**

You will find information about TMAs in the 'Assessment' area of the MST124 website. Please read that information before beginning work on this TMA.

If you have a disability that makes it difficult for you to attempt any of these questions, then please contact your Student Support Team or your tutor for advice.

The work that you submit should include your working as well as your final answers.

Your solutions should not involve the use of Maxima, except in those parts of questions where this is explicitly required or suggested. Your solutions should not involve the use of any other mathematical software.

Your work should be written in a good mathematical style, as described in Section 6 of Unit 1, and as demonstrated by the example and activity solutions in the study units. Five marks (referred to as good mathematical communication, or GMC, marks) on this TMA are allocated for how well you do this.

Your score out of 5 for GMC will be recorded as part of Question 11.

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PLAGIARISM WARNING – the use of assessment help services and websites

The **work that you submit for any assessment/examination on any module should be your own**. Submitting work produced by or with another person, or a web service or an automated system, **as if it is your own** is cheating. It is **strictly forbidden** by the University.

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- provide any assessment question to a website, online service, social media platform or any individual or organisation, as this is an infringement of copyright.
- request answers or solutions to an assessment question on any website, via an online service or social media platform, or from any individual or organisation.
- use an automated system (other than one prescribed by the module) to obtain answers or solutions to an assessment question and submit the output as your own work.
- discuss examination questions with any other person, including your tutor.

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- using text obtained from assignment writing sites, organisations or private individuals.
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Question 1 – 9 marks

You should be able to answer this question after studying Unit 2.

(a) A straight line passes through $(3, 5)$ and $(5, 4)$.

Find the equation of the straight line. [3]

(b) Obtain the completed square form of $f(x) = x^2 - 6x - 4$. [1]

(c) **Hence (using your answer to (b))** solve the equation $x^2 - 6x - 4 = 0$ giving your solution in exact form. [2]

(d) **Sketch** by hand the graphs of the line in part (a) and the quadratic in part (b) on the same axes, making your method for this clear. [3]

Question 2 – 5 marks

You should be able to answer this question after studying Unit 3.

Use a table of signs to solve the inequality

$$\frac{2x-7}{3x+1} \geq 0.$$

Give your answer in interval notation. [5]

Question 3 – 6 marks

You should be able to answer this question after studying Unit 3.

The size of a population of rabbits (y) at time t months can be modelled by the equation $y = Ae^{kt}$, where A and k are constants.

After 2 months there were 30 rabbits. After 5 months there were 80 rabbits.

(a) Use this information to show that $k = \frac{1}{3}\ln(\frac{8}{3})$ and obtain the value of A to 1 decimal place. [4]

(b) After how many whole months does the rabbit population first exceed 1000? [2]

Question 4 – 9 marks

You should be able to answer this question after studying Units 3 and 4.

(a) By considering suitable transformations, **sketch** by hand the graph of the function $g(x) = 3 + 2\sin 2x$ for $-2\pi \leq x \leq 2\pi$. [3]

(b) Write down the image set of g, giving your answer in interval notation. [1]

(c) This part of the question concerns the function

$h(x) = 3 + 2\sin 2x$ for $-k \leq x \leq k$

By considering your graph from (a), or otherwise, state the largest value k can take if $h(x)$ is to be a one-to-one function. [1]

(d) Using your answer to (c), obtain the inverse function of $h(x)$ and state its domain and image set. [4]

Question 5 – 6 marks

You should be able to answer this question after studying Unit 4.

This part of the question uses the triangle ABC (with the usual notation, as used in Fig 44 on page 54 of Unit 4).

Side $a = 5$ cm, side $b = 6$ cm and angle $C = 25°$.

(a) Using the cosine rule, find the length of side c in cm, to 2 decimal places. [2]

(b) Without using the cosine rule again, find the remaining angles A and B, giving your answers to the nearest degree. [4]

Question 6 – 6 marks

You should be able to answer this question after studying Units 2 and 4.

Find all solutions to the equation

$$\sin^2(2x) - \cos(2x) - 1 = 0, \qquad 0 \leq x \leq \pi.$$

Consider how solving a quadratic equation may help here [6]

Question 7 – 8 marks

You should be able to answer this question after studying Unit 5.

(a) Obtain the centre and radius of the circle with equation
$6x^2 + 24x + 6y^2 - 6y = 12$. [4]

(b) Obtain the intersection points of this circle with the line $y = -3x + 2$. [4]

Question 8 – 15 marks

You should be able to answer this question after studying Unit 5.

A boat has a speed in calm water of 20 km h^{-1} and it is travelling on a bearing of 100° relative to the water. There is a current which has a speed of 10 kmh^{-1} **from** the south-east.

Take unit vectors **i** to point east and **j** to point north.

(a) Express the original velocity **b** of the boat relative to the water and the velocity **w** of the current in component form in km h^{-1} to two decimal places. [7]

(b) Express the resultant velocity **v** of the boat in component form in km h^{-1} to two decimal places. [3]

(c) Hence find the magnitude and direction of the resultant velocity **v** of the boat, giving the magnitude in km h^{-1} to one decimal place and the direction as a bearing to the nearest degree. [5]

Question 9 – 15 marks

You should be able to answer this question after studying Unit 6.

This question concerns the function $f(x) = x^3 + 5x^2 + 8x - 1$.

(a) Find the x- and y-coordinates of the stationary points of f, giving your answer in exact form. [*6*]

(b) Use the second derivative test to classify the stationary points that you found in part (a). [*3*]

(c) Sketch the graph of f, indicating the y-intercept and the points that you found in part (a). (You should draw this by hand, rather than using any software, and you can use different scales on the two axes if appropriate.) [*4*]

(d) State the greatest and least values that $f(x)$ takes on the interval $[-3, -1]$. [*2*]

Question 10 – 11 marks

You should be able to answer this question after studying Unit 6.

An object moves along a straight line. Its displacement s (in metres) from a reference point at time t (in seconds) is given by

$s = t^4 - 12t^3 + 38t^2 - 28t + 5$, where $t \geq 0$.

Answer parts (a) and (b) using calculus and algebra.

(a) Find expressions for the velocity v and the acceleration a of the object at time t. [*2*]

(b) Find the velocity and corresponding acceleration after 2 seconds. [*2*]

Answer parts (c) and (d) using Maxima.

(c) Use Maxima to obtain a graph of the displacement of the object against time. (You can use x and y as your variables in Maxima if this is easier; you do not have to use s and t.) [*3*]

(d) Use Maxima to find the times at which the object is at the origin, giving your answers to 2 decimal places.

Include a screenshot of your Maxima worksheet with your solutions. [*4*]

Question 11 – 10 marks

(a) Have a look at the feedback you received for TMA01. Explain how you intend to use this feedback in your MST124 studies. Please address the following points in your answer:

- What did you learn from the feedback you received?
- Have you reviewed any topics in Unit 1 as a result?
- Have you sought any further advice or additional resources?
- Has your confidence improved in any areas since TMA 01?
- Are there any topics from TMA 01 or TMA 02 in which you feel you need to build your confidence?

If you did not complete TMA01, please describe how you will use your feedback from TMA02 to help you with your MST124 studies. [5]

(b) A score out of 5 marks for good mathematical communication throughout TMA 02 will be recorded under Question 11 part (b). [5]
