UNICLab
泛在网络与智能计算研究组

西安電子科技大學
XIDIAN UNIVERSITY

Helmholtz Equation Informed Learning Methods for Radio Map Construction

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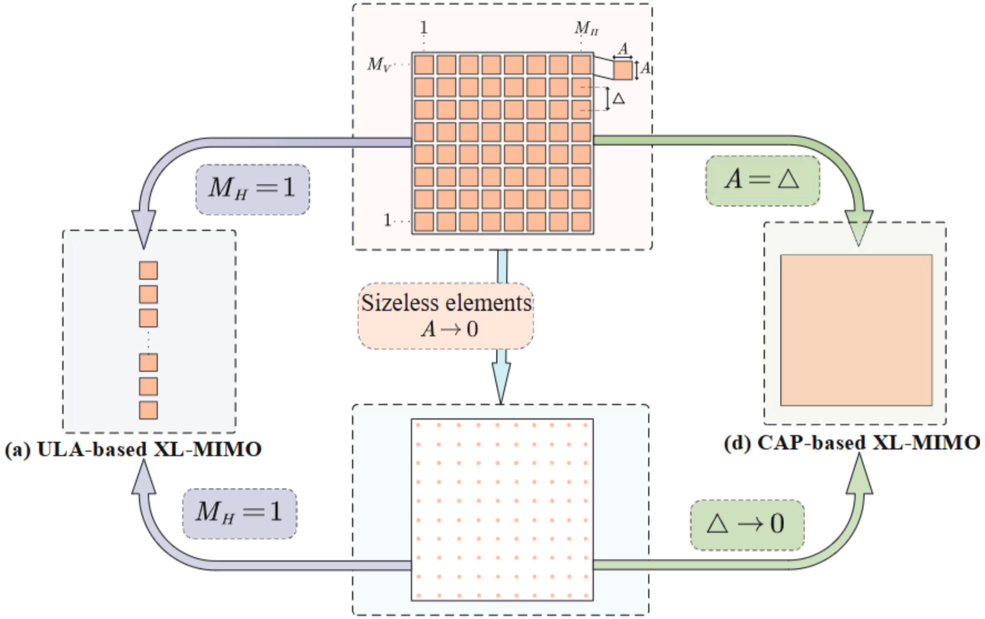
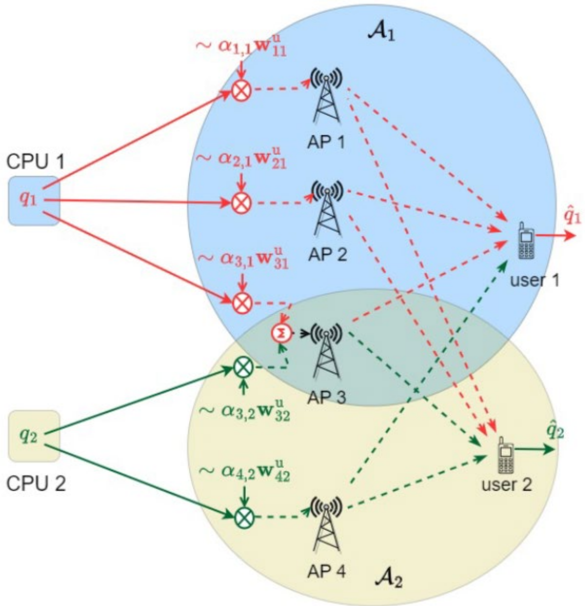
Mar 31, 2025

Challenge from 6G



(a) The scale of MIMO is growing from 64 into over 1024 antennas.



(b) The dense of network is also dramatically increasing.

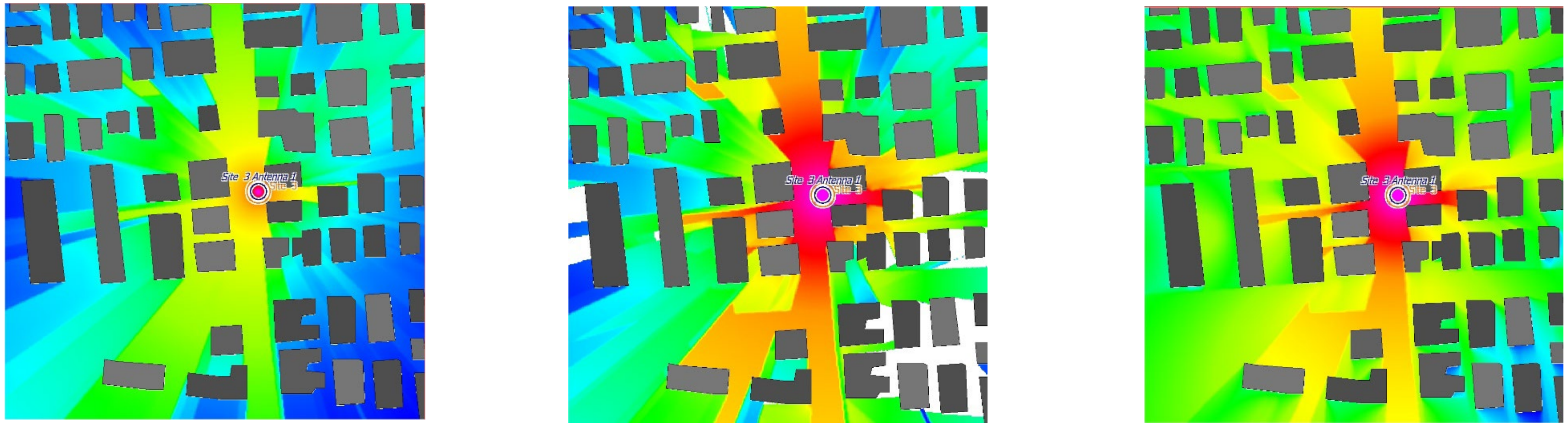
The time consumed by channel estimation will **exceed 100%** of the total slot resources[1].

It is the very time to propose **pilot-free/semi free** channel estimation method.

[1] Huawei. https://competition.huaweicloud.com/

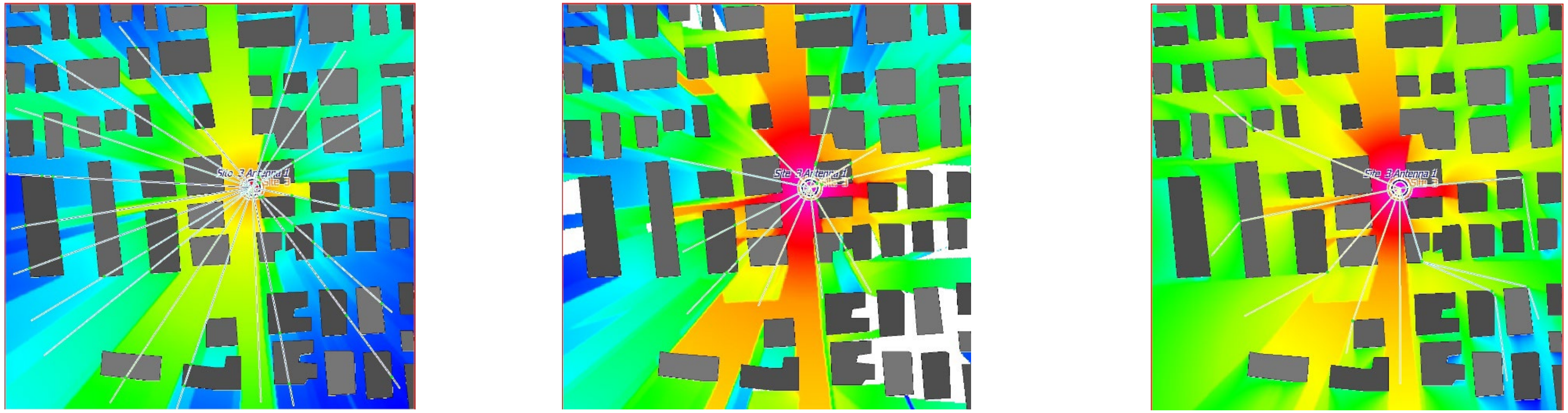
Why Radio Map

(1) Empirical channel model fail to preciously model the wireless channel features

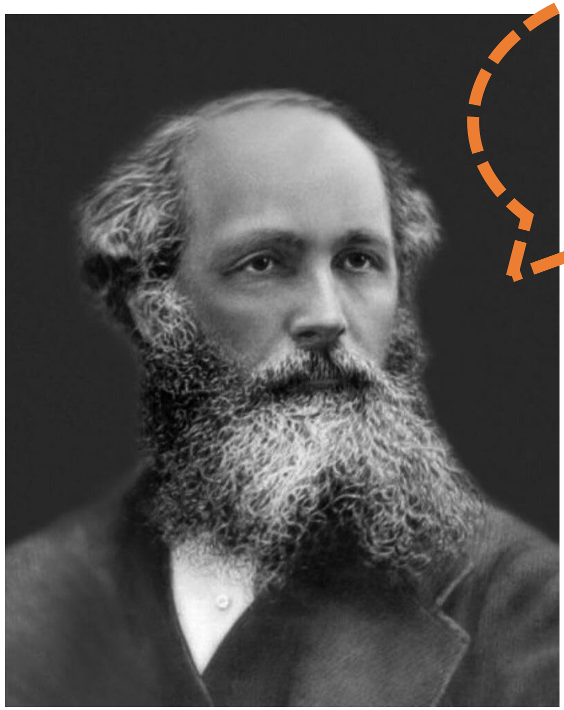


(a) COST 231 model (b) ITU P.1411 model (c) Ground truth

(2) Empirical channel model fail to obtain small-scale features



Is Radio Map Powerful Enough?



YES!

James Clerk Maxwell
(1831-1879)

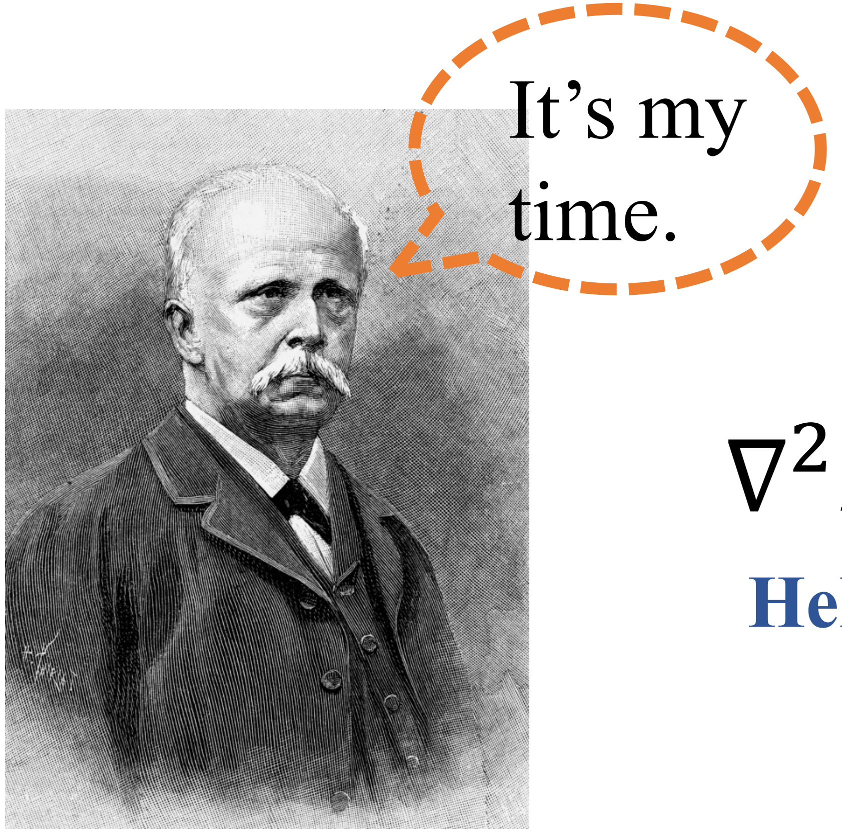
$$\nabla \times \boldsymbol{E} = iw\boldsymbol{B},$$
$$\nabla \times \boldsymbol{H} = -iw\boldsymbol{D},$$
$$\nabla \cdot \boldsymbol{D} = 0,$$
$$\nabla \cdot \boldsymbol{B} = 0.$$

All features of EM waves in **steady scenarios** can be calculated.

Is Radio Map Powerful Enough?

$$\nabla \times \boldsymbol{E} = iw\boldsymbol{B},$$
$$\nabla \times \boldsymbol{H} = -iw\boldsymbol{D},$$
$$\nabla \cdot \boldsymbol{D} = 0,$$
$$\nabla \cdot \boldsymbol{B} = 0.$$

Too hard to calculate



Hermann Ludwig Ferdinand
von Helmholtz
(1821-1894)

$$\nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0$$

Helmholtz Equation

Analysis of Helmholtz Equation

$$\nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0$$

(a) $k = w^2 \mu \epsilon$ for uniform medium

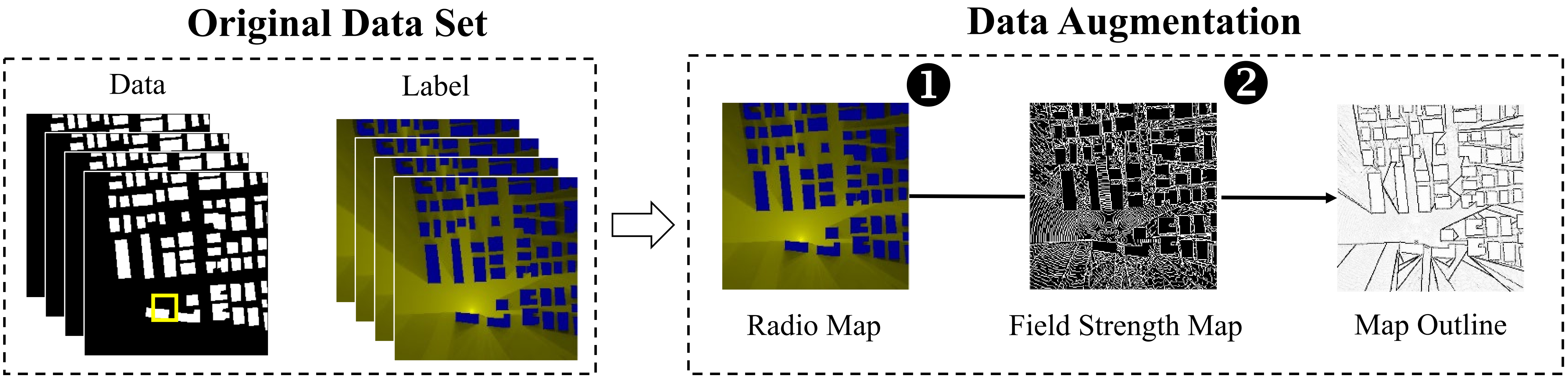
(b) $k^2 > 0$ for propagation wave

free space or waveguide.

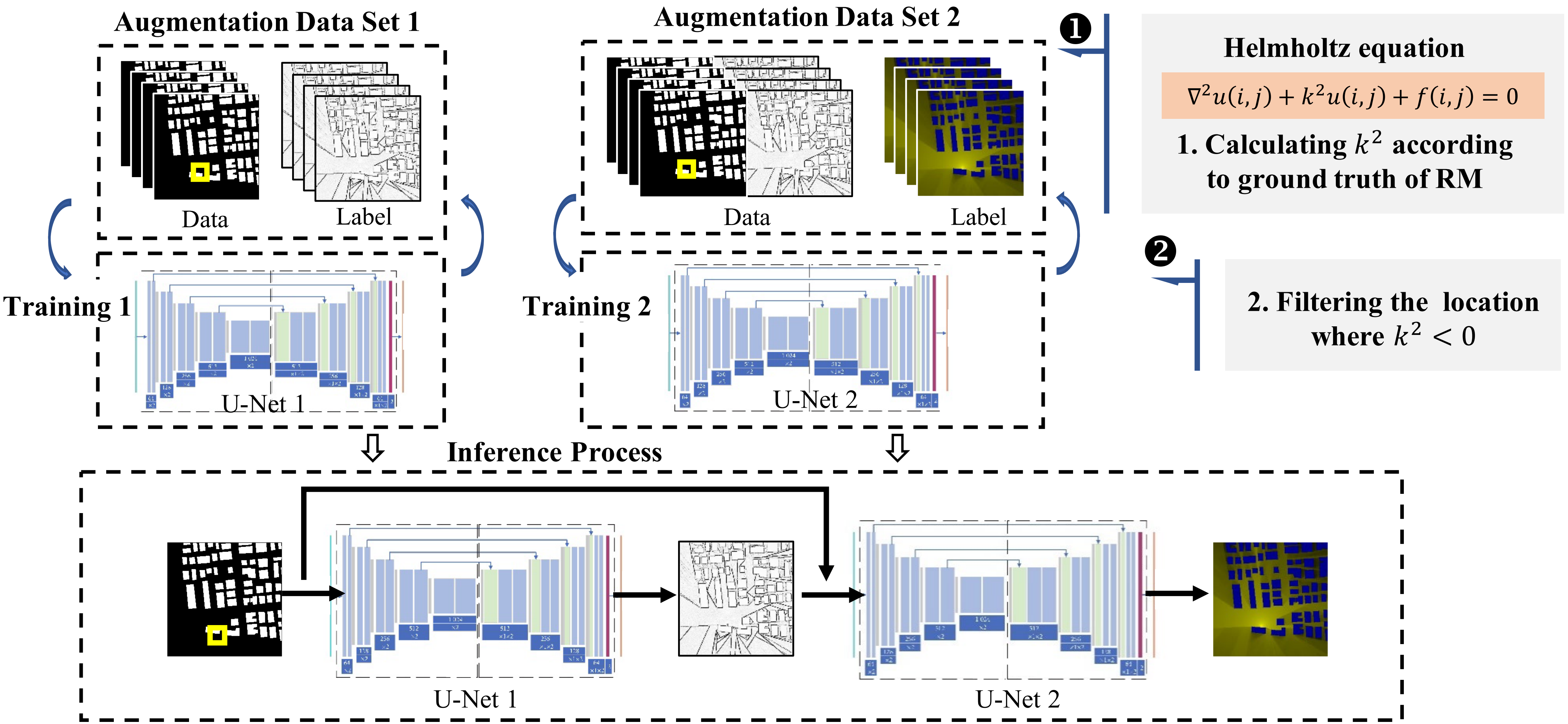
(c) $k^2 < 0$ for fleeting wave

waveguide cut-off, surface wave, tunneling wave.

Helmholtz Equation Informed NN



Helmholtz Equation Informed NN



Helmholtz Equation Informed NN

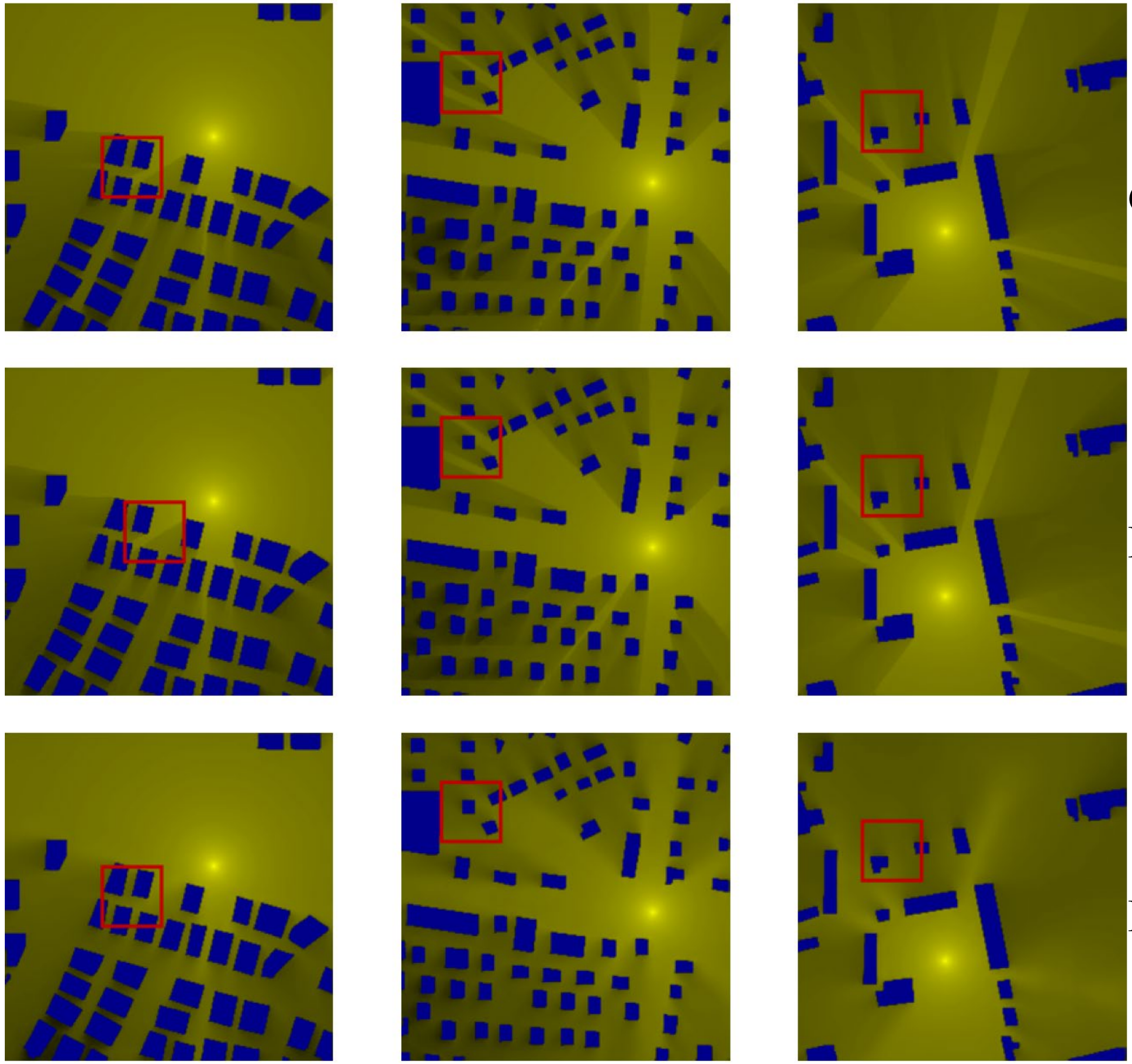


TABLE I: Performance Comparison on DPM.

Method	NMSE↓	RMSE↓	PSNR↑	SSIM↑
RME-GAN	0.0115	0.0303	30.54	0.9323
UVM-Net	0.0085	0.0304	30.34	0.9320
RadioDiff	0.0049	0.0190	35.13	0.9691
RadioUNet w/o PDE	0.0084	0.0272	31.66	0.9535
RadioUNet w/ PDE	0.0145	0.0368	28.90	0.9301
HEINN w/o PDE	**0.0025**	**0.0140**	**37.59**	**0.9888**
HEINN w/ PDE	0.0031	0.0161	36.25	0.9839

TABLE II: Performance Comparison on IRT.

Method	NMSE↓	RMSE↓	PSNR↑	SSIM↑
RadioUNet w/o PDE	0.0204	0.0415	28.02	0.9049
RadioUNet w/ PDE	0.0323	0.0536	25.71	0.8615
HEINN w/o PDE	**0.0102**	**0.0299**	**30.83**	**0.9473**
HEINN w/ PDE	0.0133	0.0348	29.40	0.9395

Analysis of Helmholtz Equation

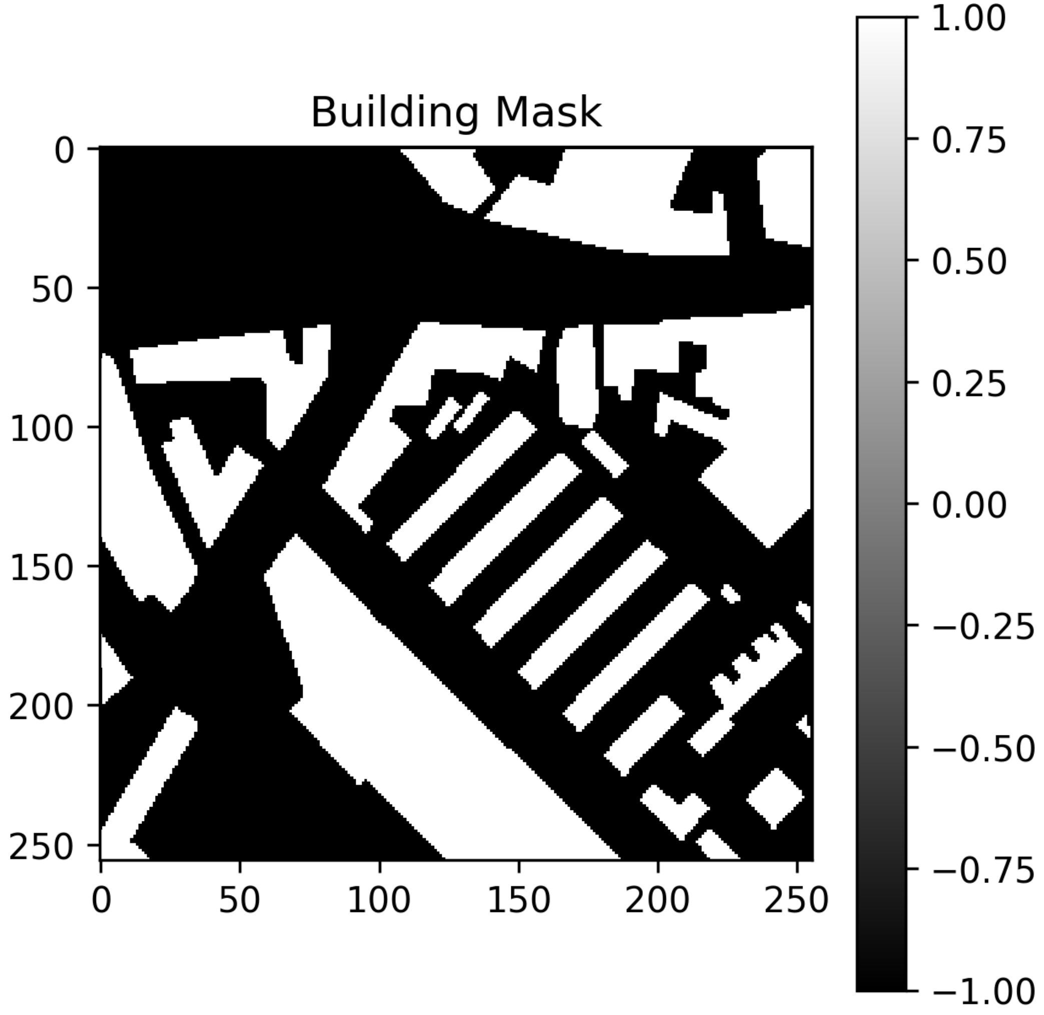
$$\nabla^2 \boldsymbol{E} + k^2 \boldsymbol{E} = 0$$

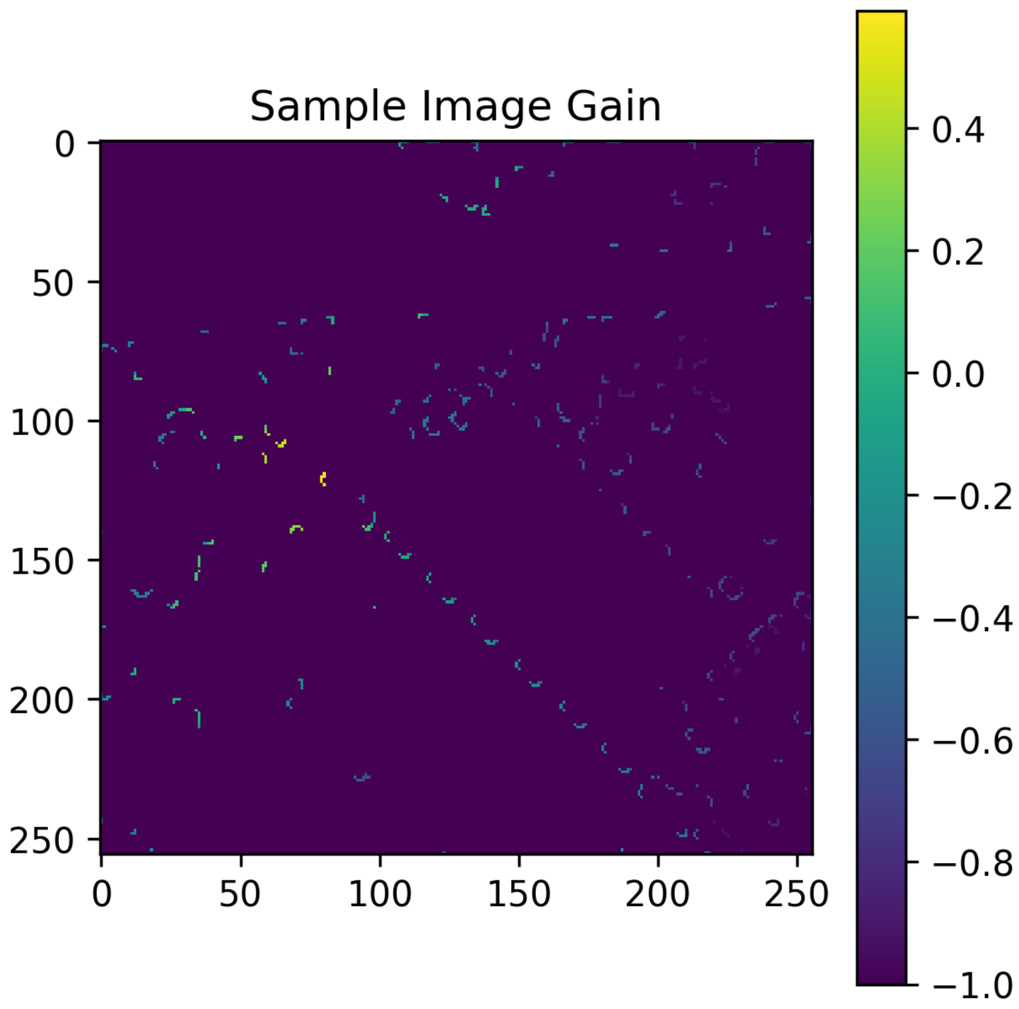
Johann Peter Gustav Lejeune Dirichlet
(1805-1859)

Dirichlet Boundary Theorem:

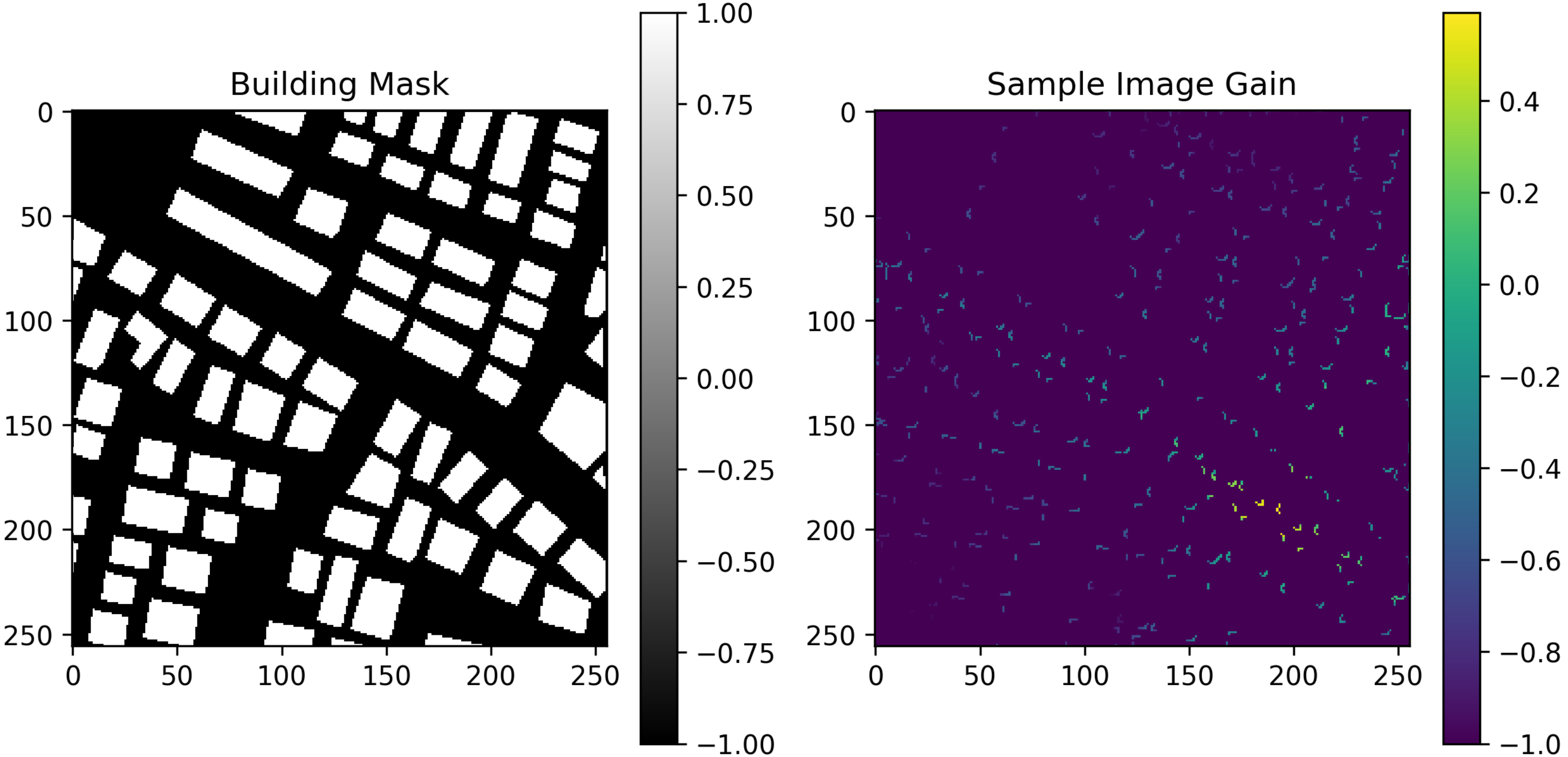
Give $u(i,j) = c(i,j)$, where $\boldsymbol{u} \in \boldsymbol{\Omega}$, the Helmholtz equation has unique solution.

Dirichlet Boundary Based Sampling





Dirichlet Boundary Based Sampling



Dirichlet Boundary Based Sampling

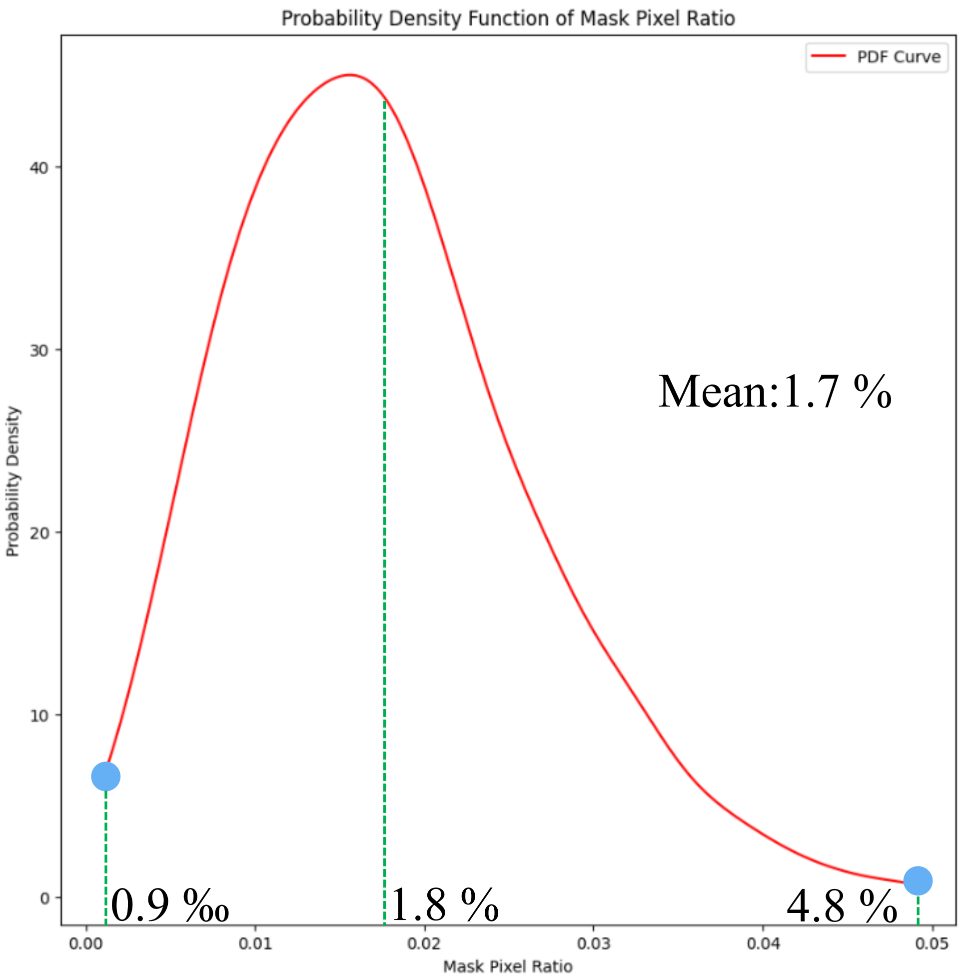


Fig.1 PDF of sampling ratio

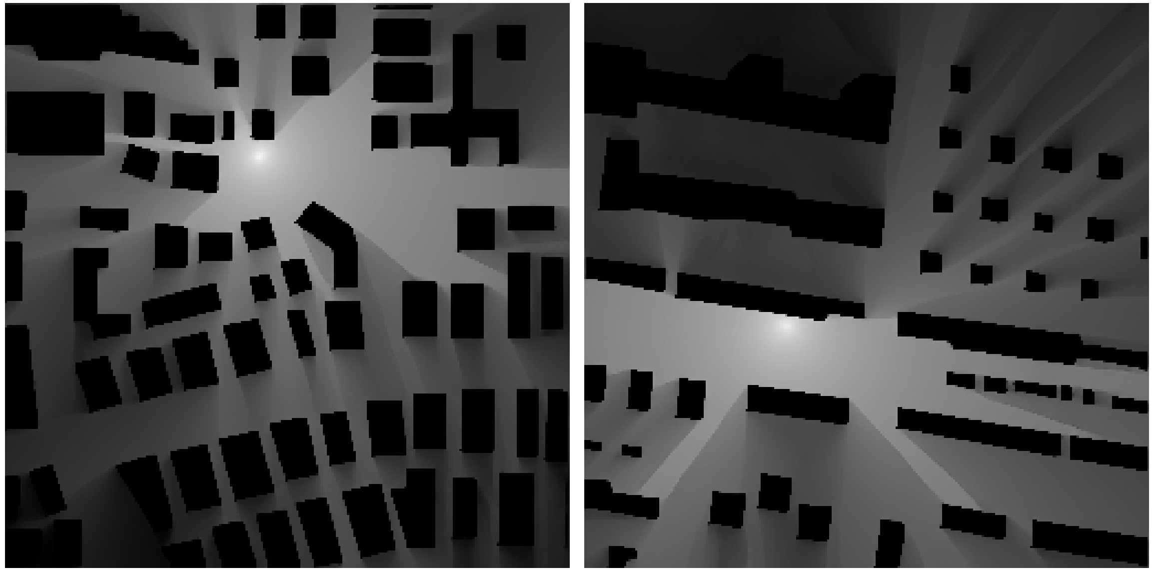


Fig.2 Generated RM

TABLE III: Performance Comparison

Method	NMSE	RMSE	PSNR	SSIM
Dirichlet Sampling	**0.0037**	**0.0215**	**33.34**	**0.9674**
RME-GAN	0.0115	0.0303	30.54	0.9323
UVM-Net	0.0085	0.0304	30.34	0.9320
RadioDiff	0.0049	0.0190	35.13	0.9691

Dirichlet Boundary Based Sampling

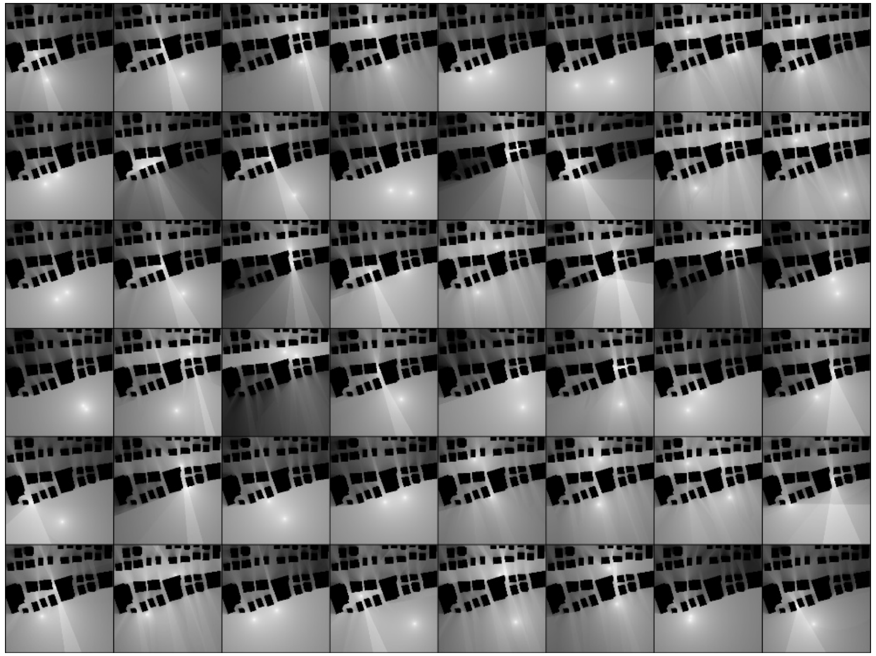


Fig.3 GT

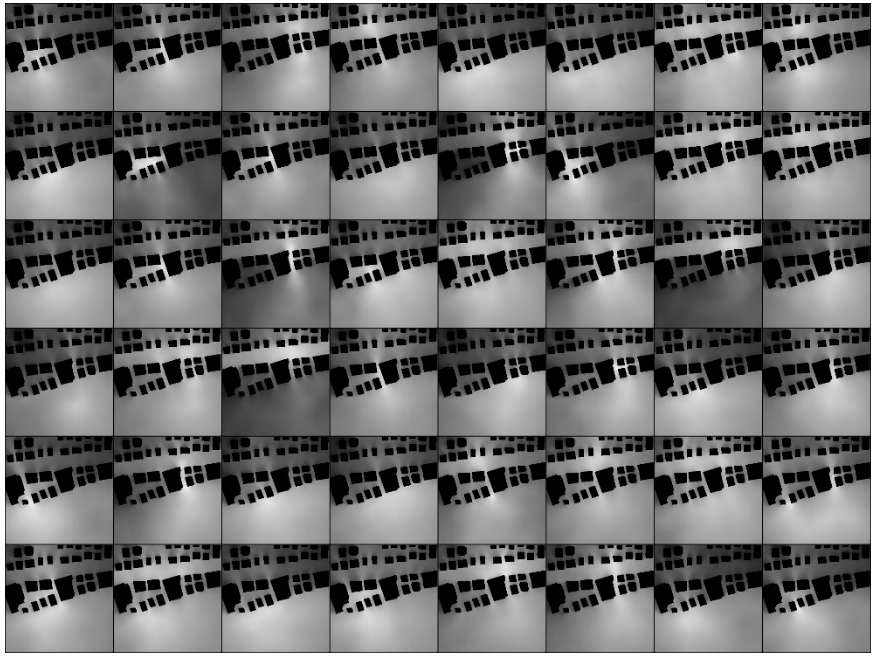


Fig.4 Predicted RM

TABLE III: Performance Comparison

Method	NMSE	RMSE	PSNR	SSIM
Dirichlet Sampling	**0.0060**	0.0371	28.61	**0.9549**
RME-GAN	0.0115	0.0303	30.54	0.9323
UVM-Net	0.0085	0.0304	30.34	0.9320
RadioDiff	0.0049	0.0190	35.13	0.9691

Thanks for Listening

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