Characterization of Hyperelastic Models with Metaheuristic Algorithms

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Master's thesis presented in accordance with the requirements to obtain the degree of Civil Engineer in Mechanical Engineering and Master's degree in Engineering Sciences, mention in Mechanical Engineering.

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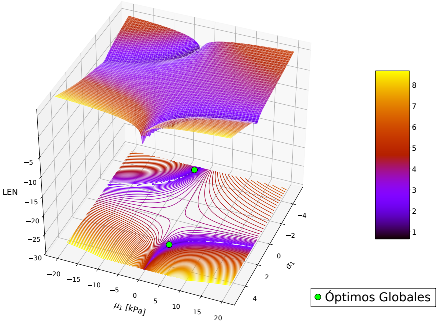
Motivation

Why should metaheuristic algorithms be used to characterize hyperelastic materials?

- Hyperelastic materials exhibit highly nonlinear behavior.
- The objective function of these constitutive models has valley regions, stability constraints and multiple local optima.
- Metaheuristic algorithms are oriented towards the global optimization of a problem
- It is also possible to optimize discontinuous objective functions with a domain bounded by multiple constraints.
- The algorithms are independent of gradients
- They are parallelizable, However high computational cost



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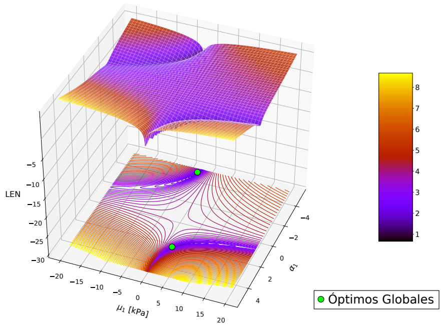

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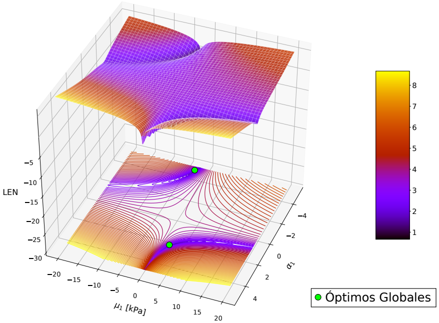

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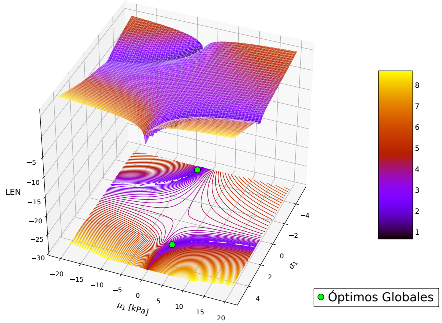

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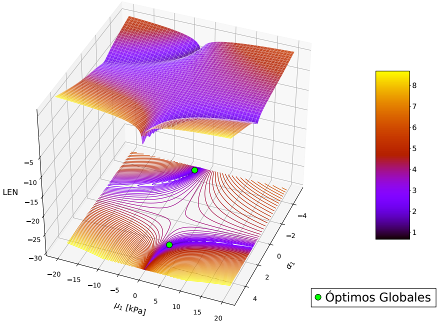

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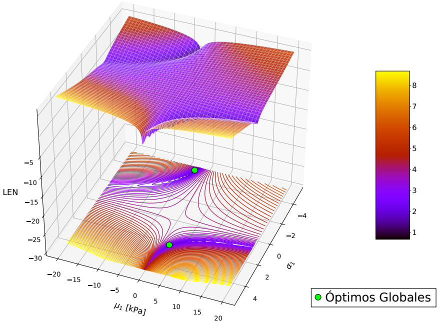

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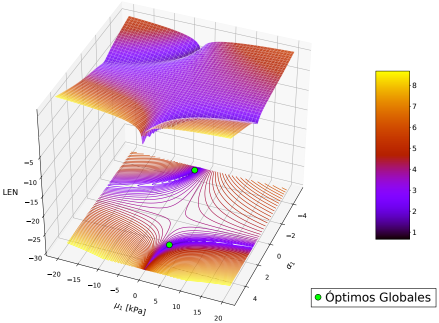

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Propose and analyze different metaheuristic algorithms to characterize hyperelastic materials, in order to evaluate their scope and conditions.

Specific Objectives

- To know metaheuristic algorithms to optimize and adjust parameters of nonlinear constitutive models.
- Analyze constitutive models suitable for the mechanical response of hyperelastic materials, applied to soft tissues (arteries) and elastomers..
- Implement and validate the different metaheuristic algorithms to solve the model characterization problem.
- Analyze the uniqueness and stability of the adjusted parameters.
- Characterize materials using experimental data from previous work and from this thesis.
- Characterizing soft tissue (arteries) through metaheuristic and finite element.

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Scope and Limitations

- Every material characterized is idealized as hyperelastic and is subject to an isotropic or anisotropic constitutive model.
- Any analytical or numerical expression is made under the assumption of hyperelasticity, homogeneity and continuity of the material.
- Three metaheuristic algorithms have been implemented: genetic algorithms, particle swarm optimization and evolutionary strategies.

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Metaheuristic Algorithms Implemented

Genetic Algorithm (GA)

Optimization algorithm based on the Darwinian process of survival. It represents its variables in discrete form.

Particle Swarm Optimization (PSO)

Optimization algorithm based on swarm intelligence. Represents its variables in a continuous form.

Evolutionary Strategy (ES)

This algorithm is based on the biological principle of evolution. The main difference with the genetic algorithm is that it uses two populations and continuous representation of variables.

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Isotropic Constitutive Models

Ogden's Incompressible Material

$$W = W(\lambda_1, \lambda_2, \lambda_3) = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3) \tag{1}$$

Mooney-Rivlin Model

$$W(I_1, I_2) = \sum_{i=0,j=0}^{\infty} C_{ij}(I_1 - 3)^i (I_2 - 3)^j \tag{2}$$

Yeoh Model

$$W(I_1) = \sum_{i=1}^{3} C_{i0}(I_1 - 3)^i \tag{3}$$

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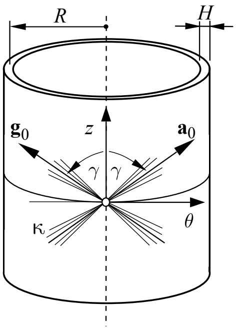
Gasser Model [1] $\longrightarrow W(\bar{\mathrm{C}}, \boldsymbol{a}_0 \otimes \boldsymbol{a}_0, \boldsymbol{g}_0 \otimes \boldsymbol{g}_0) = W(I_1, I_4, I_6)$

$$W = \frac{\mu}{2}(I_1 - 3) + \frac{k_1}{2k_2} \sum_{\alpha=4,6} \left[\exp\left(k_2[I_1\kappa + (1-3\kappa)I_\alpha - 1]^2\right) - 1\right] \quad (4)$$



- $W = W_{matrix} + W_{fiber}$
- $\mu,\ k_1$ y $k_2 \geq 0 \rightarrow$ material constant
- γ fiber directions $\rightarrow \boldsymbol{d} = \hat{\boldsymbol{z}}$
- $0 \leq \kappa \leq 1/3$
- Si $\kappa = 0 \rightarrow$ Gasser Model $\equiv$Holzapfel Model Holzapfel, G.A. & Gasser, C.T& Ogden, R.W.(2000) [2]

$$\begin{aligned} I_1 &= \operatorname{tr} \mathbf{C} \\ I_4 &= \boldsymbol{a}_0 \otimes \boldsymbol{a}_0 : \mathbf{C} = \lambda_{\mathbf{a}}^2 > 1 \\ I_6 &= \boldsymbol{g}_0 \otimes \boldsymbol{g}_0 : \mathbf{C} = \lambda_{\mathbf{g}}^2 > 1 \end{aligned}$$

Evaluation of Metaheuristic Algorithms

Uniaxial Tensile Adjustment

$$\min_{\mathbf{x}\in\mathbf{A^n}} f(\mathbf{x}) = \sum_{i=1}^{n} (\sigma_i - \sigma(\lambda_i, \mathbf{x}))^2 \qquad (5)$$

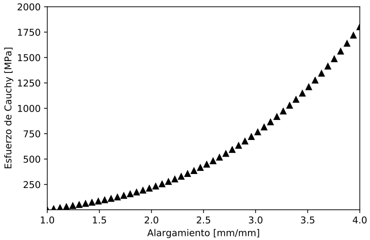


Figura 1: Curve generated from uniaxial test using known parameters of Mooney-Rivlin model $C_{10} = 10$ [MPa], $C_{01} = 20$[MPa] y $C_{11} = 5$ [MPa].

Evaluation of Metaheuristic Algorithms

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$$\min_{\mathbf{x}\in\mathbf{A^n}} f(\mathbf{x}) = \sum_{i=1}^{n}(\sigma_i - \sigma(\lambda_i,\mathbf{x}))^2 \qquad (6)$$

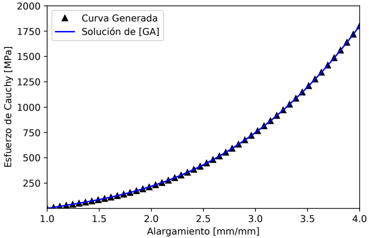

Figura 2: Curve generated from uniaxial test using known parameters of Mooney-Rivlin model $C_{10} = 10$ [MPa], $C_{01} = 20$[MPa] y $C_{11} = 5$ [MPa].

Parameters [kPa]	GA [3]	GA		PSO		ES elitist		ES non-elitist	
		μ	CI95 %	μ	CI,231	18,09	0,158		
$C_{10} = 10$	8,73	11,27	0,366	11,43	0,392	11,02	0,156	11,13	0,197
$C_{11} = 5$	5,078	4,87	0,031	4,89	0,029	4,92	7,8E-3	4,88	7,2E-2
TPR [s]	83	8,1		0,115		0,016		0,016	
MSE	0,24	0,26		0,22		0,16		0,21	

Cuadro 1: Comparison of ES, PSO and GA, replicating the conditions of J.R. Fernandez's publication. [3]

Assessment of Metaheuristic Algorithms

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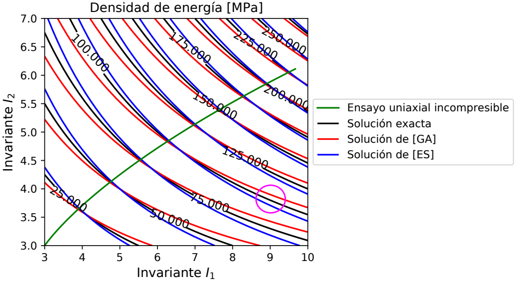

Figura 3: $W(I_1, I_2)$ for the theoretical values and the adjusted values.

Deformation path of uniaxial tensile test

$$I_1 = \lambda_1^2 + \frac{2}{\lambda_1}, I_2 = 2\lambda_1 + \frac{1}{\lambda_1^2} \qquad (8)$$

Deformation paths

Mechanical Test	I_1	I_2
Uniaxial	$\lambda_1^2 + \frac{2}{\lambda_1}$	$\lambda_1 + \frac{1}{\lambda_1^2}$
Biaxial	$\lambda_1^2 + \lambda_2^2 + \frac{1}{\lambda_1^2\lambda_2^2}$	$(\lambda_1\lambda_2)^2 + \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$
Equiaxial	$2\lambda_1^2 + \frac{1}{\lambda_1^4}$	$(\lambda_1)^4 + \frac{2}{\lambda_1^2}$
Presurization	$\lambda_r^2 + \lambda_\theta^2 + \lambda_z^2$	$\lambda_r^2\lambda_\theta^2 + \lambda_z^2\lambda_\theta^2 + \lambda_z^2\lambda_r^2$
Pure Shear	$\lambda^2 + \lambda^{-2} + 1$	$\lambda^2 + \lambda^{-2} + 1$
Simple Shear	$k^2 + 3$	$k^2 + 3$

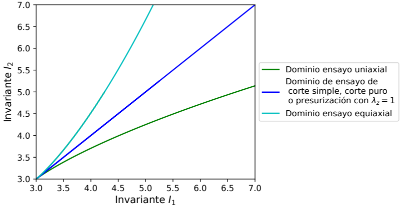

Figura 4: Strain paths for the different mechanical tests

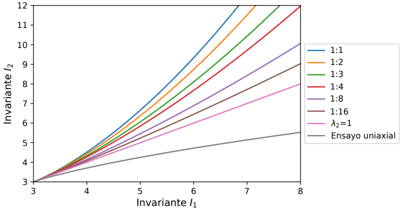

Figura 5: Biaxial test in the space of invariants

Characterization of multiple mechanical tests.

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Single-objective function multiple mechanical tests

$$\min_{\mathbf{x}\in\mathbf{A^n}} f(\mathbf{x}) = \sum_{i=1}^{n}(\sigma_i - \sigma(\lambda_i,\mathbf{x}))^2 + w_p^2\sum_{i=1}^{m}(P_i - P(\lambda_i^c,\mathbf{x}))^2 + w_e^2\sum_{i=1}^{q}(\sigma_i^e - \sigma^e(\lambda_i^e,\mathbf{x}))^2 \quad (9)$$

- S.C Mooney-Rivlin 2 Parameters $C_i = (20, 15)$
- S.C Mooney-Rivlin 3 Parameters $C_i = (20, 15, 12)$
- S.C Mooney-Rivlin 4 Parameters $C_i = (20, 15, 12, 7)$
- S.C Mooney-Rivlin 5 Parameters $C_i = (20, 15, 12, 7, 6)$

The parameters obtained have a relative error $< 0,5\,\%$?							
Curves		Uniaxial		Uniaxial-Presurization		Uniaxial- Presurization-Equiaxial	
Parameters Quantity of Mooney-Rivlin	2	YES	0,00 %	YES	0,00 %	YES	0,00 %
	3	YES	0,16 %	YES	0,00 %	YES	0,00 %
	4	NO	4,49 %	YES	0,25 %	YES	0,00 %
	5	NO	7,75 %	NO	0,84 %	YES	0,00 %

Cuadro 2: Qualitative evaluation of the fit of experimental data.

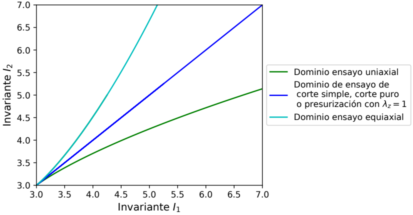

Figura 6: Strain paths for the different mechanical tests

Experimental Characterization (Latex).

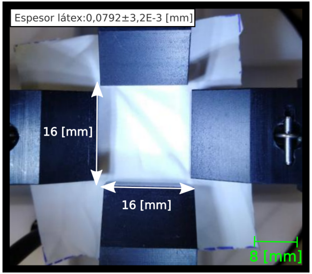

Figura 7: Latex test tube mounted on biaxial equipment.

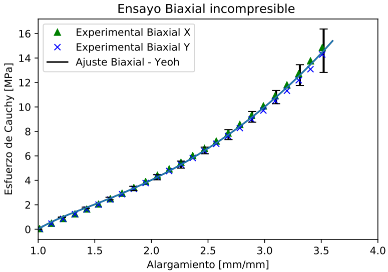

Figura 8: Fits to experimental data with Yeoh model

Parameters [MPa]	[ES] Elitista	
	μ	IC 95 %
C_1	4,881E-1	5,224E-4
C_2	10,21	3,712E-5
C_3	7,572E-6	1,009E-6
Standard Error	0,173	

Cuadro 3: Parameters retrieved with the Yeoh model for biaxial testing

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Characterization with uniaxial tensile test

Objective function anisotropic characterization.

$$\min_{\mathbf{x}\in\mathbf{A^n}} f(\mathbf{x}) = \sum_{i=1}^{n}\left(\frac{\sigma_i^{lon} - \sigma^{lon}(\lambda_i^{lon},\mathbf{x})}{n\sigma_n^{lon}}\right)^2 + \sum_{i=1}^{m}\left(\frac{\sigma_i^{cir} - \sigma^{cir}(\lambda_i^{cir},\mathbf{x})}{m\sigma_m^{cir}}\right)^2, \quad \mathbf{x} = (\mu,\kappa,k_1,k_2,\theta) \tag{10}$$

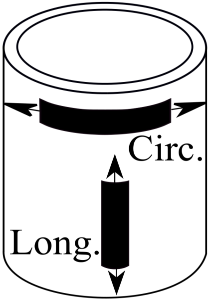


Figura 9: Graphical representation of the specimens obtained in an artery.

Evaluation of evolutionary strategies

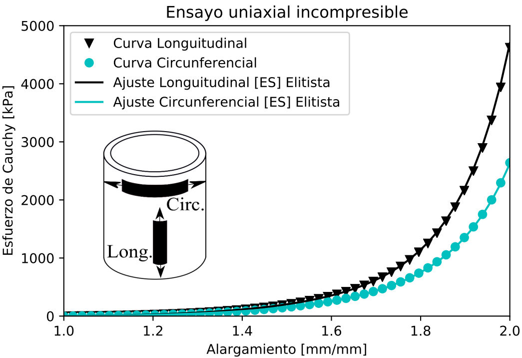


Figura 10: Fitting a known solution of an anisotropic material with evolutionary strategies

Parameters	Known Solution	Upper limit	Lower limit
$\mu[kPa]$	22,589	0,00	1,00E+05
$k_1[kPa]$	224,217	0,00	1,00E+06
k_2	2,464	0,00	1,00E+01
κ	0,285	0,00	0.3333
θ^o	39,624^o	0,0^o	90^o
Generations	150	Population	1140

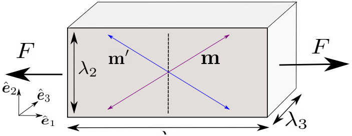
Cuadro 4: Search parameters of ES

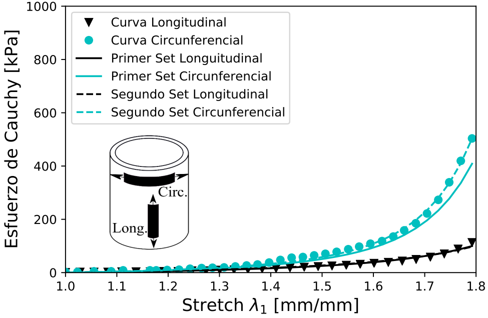
Parameters	ES Elitist	
	Average	95 % IC
μ [kPa]	2,2260E+01	8,3100E-02
k_1 [kPa]	2,1303E+02	2,3413E+00
k_2	2,3705E+00	1,9879E-02
κ	2,7391E-01	2,3596E-03
θ^o	4,0400E+01	1,7648E-01
Objective Function	5,06E-05	

Cuadro 5: Results of fitting a known solution using the Holzapfel-Gasser-Ogden model

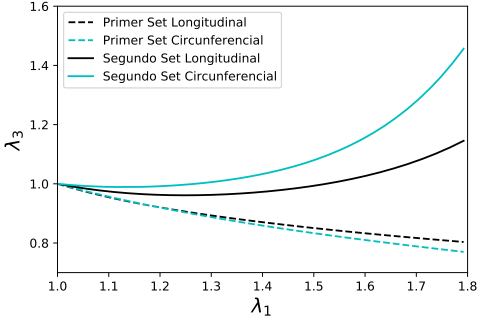
Transverse instability

x	μ	κ	k_1	k_2	θ
$\mathbf{x}_1$	10,4	0,27	30,6	3,94	1,07
$\mathbf{x}_2$	0,73	0,00	13,9	2,13	0,84





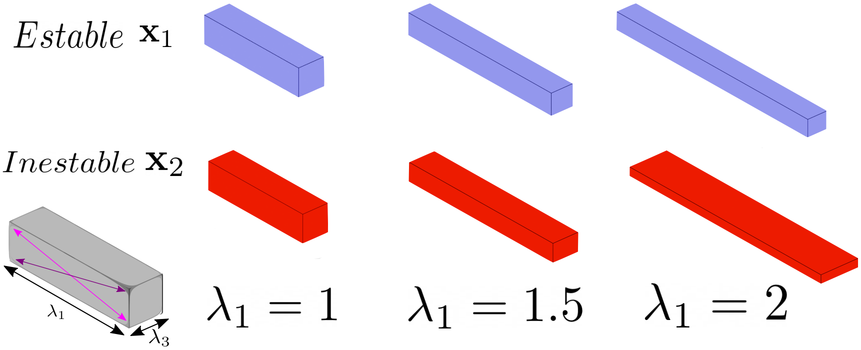
(a) Uniaxial tensile test



(b) Transverse stretch λ_3

Transverse Instability

$\mathbf{x}$	μ	κ	k_1	k_2	θ
$\mathbf{x}_1$	10,4	0,27	30,6	3,94	1,07
$\mathbf{x}_2$	0,73	0,00	13,9	2,13	0,84



Transverse elongations in uniaxial test

$$\lambda_3 = [\frac{W_1 + sin^2(\theta_1)W_4 + sin^2(\theta_2)W_6}{W_1}]^{1/4}\lambda_1^{-1/2} = \Psi^{1/4}\lambda_1^{-1/2} \quad (11)$$

Transverse instability occurs due to a disproportionate growth of the anisotropy with respect to the isotropy of the material.

Transverse elongations in uniaxial test

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Transverse instability occurs due to a disproportionate growth of the anisotropy with respect to the isotropy of the material.

Transverse stability criterion

Stability criterion is defined as

$$\frac{d\lambda_3}{d\lambda_1} \leq 0 \quad \textbf{Transverse stability criterion}\,. \tag{12}$$

For any $W(I_1, I_4, I_6)$ it must be fulfilled that

$$\frac{d\Psi}{d\lambda_1} \leq \frac{2\Psi}{\lambda_1}\,. \tag{13}$$

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Stability of [1] y Holzapfel[2] Model

- If we consider that there is a critical direction θ in which the fibers point and we evaluate the Gasser model in Ψ, we obtain that

$$\Psi(I_1, I_4) = 1 + \frac{k_1 E_4 (1 - 3\kappa) sin^2(\theta)}{\mu/2 + e^{-k_2 E_4^2} + k_1 \kappa E_4}. \tag{14}$$

If $\lambda_1 \to \infty$ the value of Ψ and it's derivative is:

$$\lim_{\lambda_1 \to \infty} \Psi = 1 + \frac{(1 - 3\kappa) sin^2(\theta)}{\kappa}; \quad \lim_{\lambda_1 \to \infty} \frac{d\Psi}{d\lambda_1} = 0 \quad \Leftrightarrow \quad \kappa \neq 0 \tag{15}$$

- Therefore, evaluating the transverse stability inequation we have that:

$$\lim_{\lambda_1 \to \infty} \frac{d\Psi}{d\lambda_1} \leq \lim_{\lambda_1 \to \infty} \frac{2\Psi}{\lambda_1} \quad \Rightarrow \quad 0 \leq 0 \tag{16}$$

Evaluating the transversal stability inequation we have that the Gasser model will be stable for $\lambda_1 \to \infty$, provided that $\kappa \neq 0$, i.e. the Holzapfel model is not.

Characterization with stabilizing term

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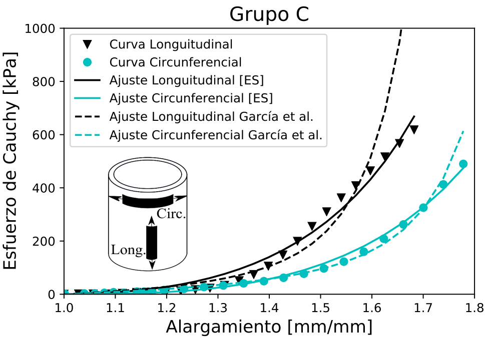
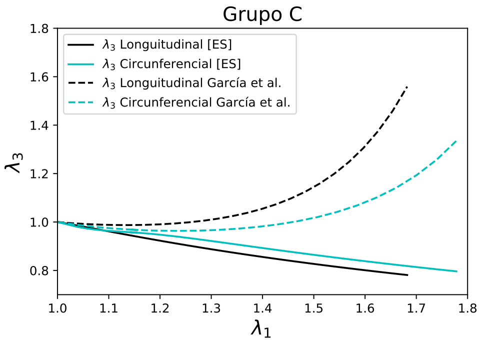
$$\min_{\mathbf{x}\in\mathbf{A^n}} f(\mathbf{x}) = \sum^{n}\left(\frac{\sigma_i^{lon} - \sigma^{lon}(\lambda_i^{lon}, \mathbf{x})}{n\sigma_n^{lon}}\right)^2 + \sum_{i=1}^{m}\left(\frac{\sigma_i^{cir} - \sigma^{cir}(\lambda_i^{cir}, \mathbf{x})}{m\sigma_m^{cir}}\right)^2 + g_1(\mathbf{x}, \lambda_i^{lon}, \lambda_i^{cir}) \tag{17}$$



(c) Uniaxial Tensile Test

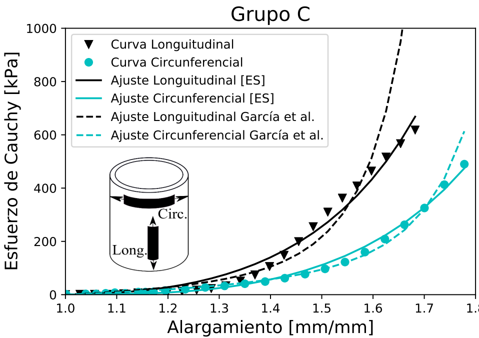


(d) Transverse Stretch λ_3

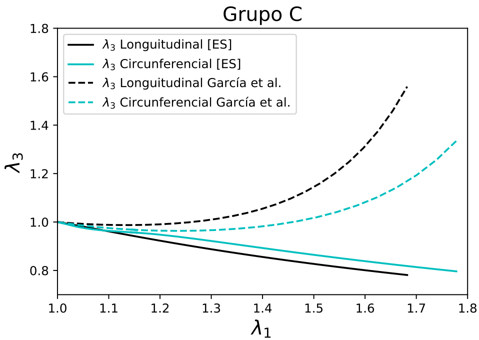
Garcia-Herrera, Claudio Celentano, Diego Cruchaga, Marcela Rojo, Francisco Atienza, José Guinea, Gustavo Goicolea, José. (2011). Mechanical characterisation of the human thoracic descending aorta: Experiments and modelling. Computer methods in biomechanics and biomedical engineering. 15. 185-93. 10.1080/10255842.2010.520704. [5]

Characterization with stabilizing term

$$\min_{\mathbf{x}\in\mathbf{A^n}} f(\mathbf{x}) = \sum_{i=1}^{n}\left(\frac{\sigma_i^{lon} - \sigma^{lon}(\lambda_i^{lon},\mathbf{x})}{n\sigma_n^{lon}}\right)^2 + \sum_{i=1}^{m}\left(\frac{\sigma_i^{cir} - \sigma^{cir}(\lambda_i^{cir},\mathbf{x})}{m\sigma_m^{cir}}\right)^2 + g_1(\mathbf{x},\lambda_i^{lon},\lambda_i^{cir}) \tag{17}$$



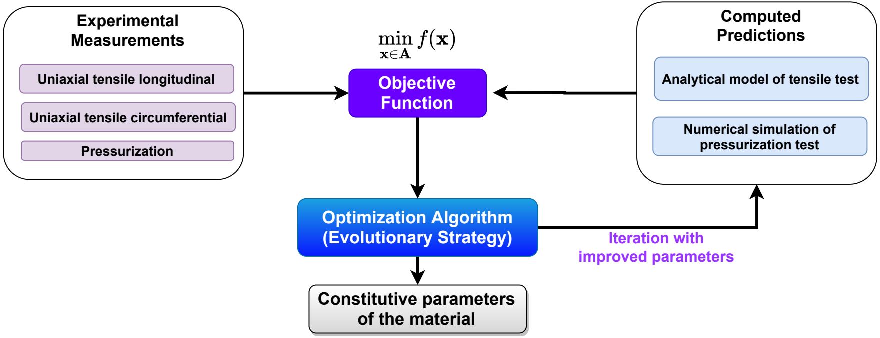
(e) Uniaxial Tensile Test



(f) Transverse Stretch λ_3

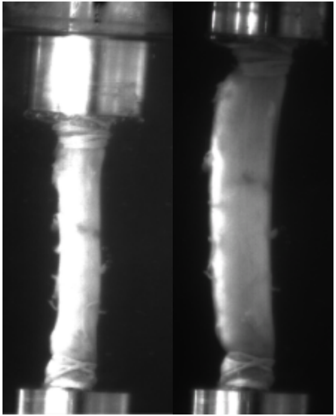
Garcia-Herrera, Claudio Celentano, Diego Cruchaga, Marcela Rojo, Francisco Atienza, José Guinea, Gustavo Goicolea, José. (2011). Mechanical characterisation of the human thoracic descending aorta: Experiments and modelling. Computer methods in biomechanics and biomedical engineering. 15. 185-93. 10.1080/10255842.2010.520704. [5]

Characterization of experimental data with FEM



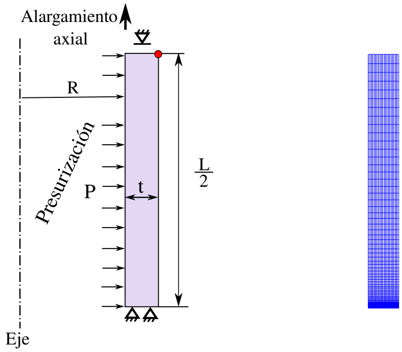
$$\min_{\mathbf{x}\in\mathbf{A}^n} f(\mathbf{x}) = \sum_{i=1}^{n}\left(\frac{\sigma_i^{lon} - \sigma^{lon}(\lambda_i^{lon}, \mathbf{x})}{n\sigma_n^{lon}}\right)^2 + \sum_{i=1}^{m}\left(\frac{\sigma_i^{cir} - \sigma^{cir}(\lambda_i^{cir}, \mathbf{x})}{m\sigma_m^{cir}}\right)^2 + 2\sum_{i=1}^{k}\left(\frac{\lambda_i^{\theta} - \lambda^{\theta}(P_i, \mathbf{x})}{k(\lambda_k^{\theta} - \lambda_1^{\theta})}\right)^2 + g_1(\mathbf{x}, \lambda_i^{lon}, \lambda_i^{cir}) \tag{18}$$

Pressurization Simulation



(a) (b)

Figura 11: Experimental assembly. (a) Unstretched thoracic aorta (b) Pressurized thoracic aorta with pre-stretching.



(a) (b)

Figura 12: (a) 2D representation of the boundary conditions for the simulation of the pressurization test. (b) Structured mesh of the finite element simulation with 2500 quadrilateral elements. Source: Own

Results

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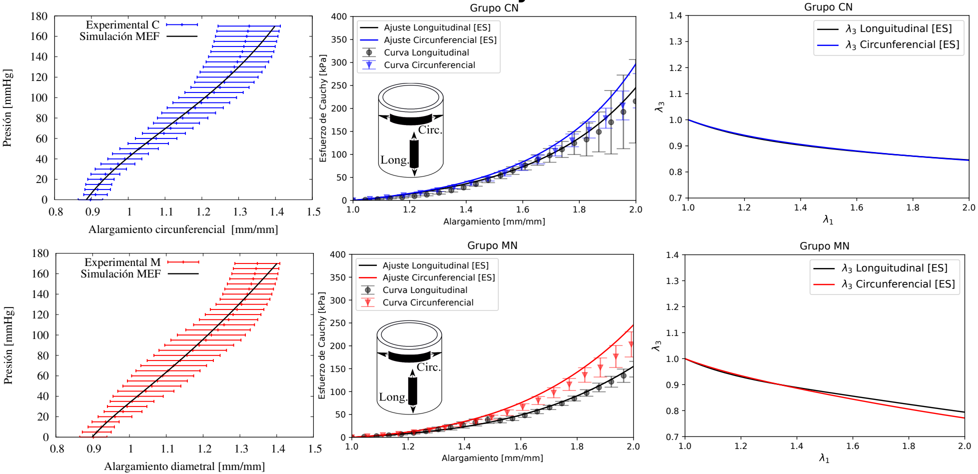
Presurización y Uniaxial



Conclusions

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Metaheuristic algorithms

- Allow characterization of hyperelastic material (isotropic and anisotropic)
- Successfully solve problems with multiple blues restrictions, multimodality, valley regions and discontinuities.
- Flexible and robust for different problems [6].

Hyperelastic Characterization

- Deformation paths.
- Degrees of freedom, bias and overfitting.

Transverse Stability

- A new criterion of anisotropic stabilization is established.
- It is shown that the Holzapfel model is unstable for large deformations and that the Gasser model is stable.
- A methodology to stabilize the mechanical response of the material is presented..

Inverse characterization FEM

- Characterization considering non-homogeneous deformation.
- Only option in the absence of an analytical model.

Future Lines of Work

- Extend and adapt characterization procedures to other types of complex materials.
- Propose a constitutive model with two preferred directions and meeting the transverse stability criterion.
- Inverse design metamaterials with metaheuristic optimization algorithms.
- Create a meta-model to reduce the computational cost of finite element simulations.

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