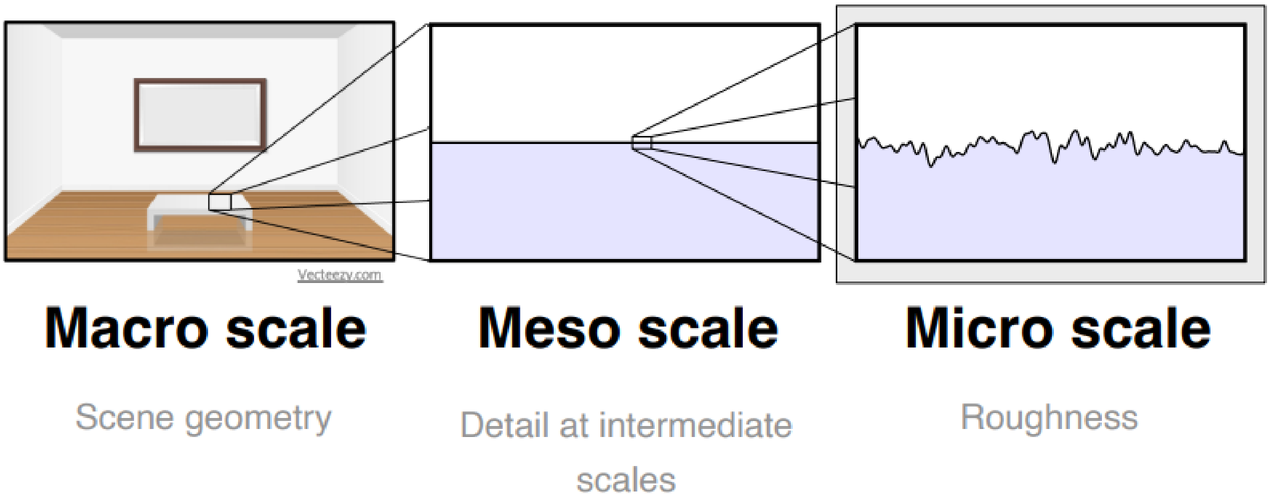
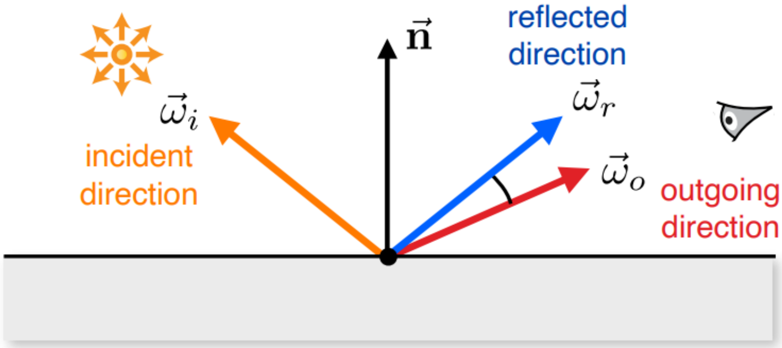
Advanced BRDF Models



Many surfaces are not perfectly smooth. We need an empirical model to account for this, which is simple and fast to evaluate.

Normalized Phong



Normalized exponentiated cosine lobe

$$f_r(\boldsymbol{\omega}_o, \boldsymbol{\omega}_i) = \frac{e+2}{2\pi}(\boldsymbol{\omega}_r \cdot \boldsymbol{\omega}_o)^e$$

where $\boldsymbol{\omega}_r = (2\mathbf{n}(\mathbf{n} \cdot \boldsymbol{\omega}_i) - \boldsymbol{\omega}_i)$.

- Blur the reflection rays in a cone about the mirror direction
- Perfect mirror reflection of a blurred light

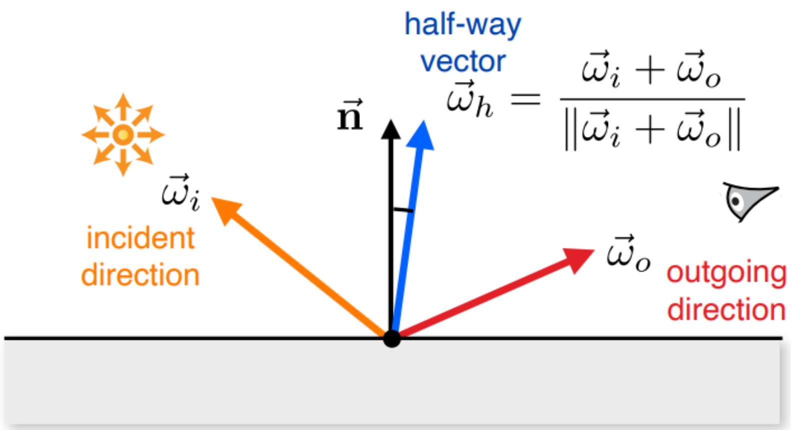
Blinn-Phong

Blur in normal domain instead of the reflection directions

$$f_r(\boldsymbol{\omega}_o, \boldsymbol{\omega}_i) = \frac{e+2}{2\pi}(\boldsymbol{\omega}_h \cdot \mathbf{n})^e$$

where $\boldsymbol{\omega}_h$ is the half-way vector

$$\boldsymbol{\omega}_h = \frac{\boldsymbol{\omega}_i + \boldsymbol{\omega}_o}{\|\boldsymbol{\omega}_i + \boldsymbol{\omega}_o\|}$$



Empirical glossy models have limitations:

- not physically-based
- (often) not reciprocal
- not energy-preserving (can be normalized)
- (often) no Fresnel effects
- cannot accurately model appearance of many glossy surfaces
- may generate directions below the surface

Microfacet Theory

Assumptions:

- surface consists of tiny facets
- the differential area being illuminated is relatively large compared to the size of microfacets
- a facet can be perfectly specular or diffuse

$$f(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o) = \frac{\overbrace{F(\boldsymbol{\omega}_h, \boldsymbol{\omega}_o)}^{\text{Fresnel coefficient}} \cdot \overbrace{D(\boldsymbol{\omega}_h)}^{\text{Microfacet distribution}} \cdot \overbrace{G(\boldsymbol{\omega}_i, \boldsymbol{\omega}_o)}^{\text{Shadowing/masking}}}{\underbrace{4|(\boldsymbol{\omega}_i \cdot \mathbf{n})(\boldsymbol{\omega}_o \cdot \mathbf{n})|}_{\text{Energy conservation}}}$$

where $\boldsymbol{\omega}_h$ is the halfway vector as in Blinn-Phong model.

Microfacet Distribution

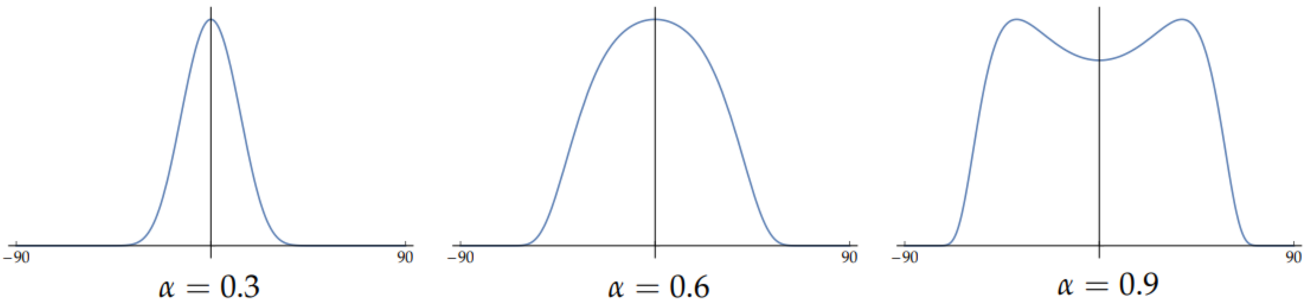
Probability density function over projected solid angle, which must be normalized:

$$\int_{H^2} D(\vec{\omega}_h) \cos\theta_h \, d\vec{\omega}_h$$

The Beckmann Distribution

The Beckmann distribution assumes that the slopes follow a Gaussian distribution.

$$D(\boldsymbol{\omega}_h) = \frac{1}{\pi\alpha^2 \cos^4\theta_h} e^{-\tan^2\theta_h/\alpha^2}$$

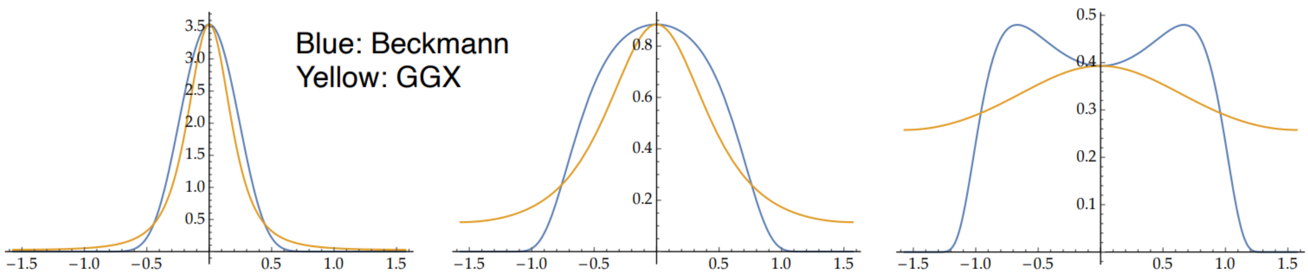


where α is the roughness parameter or diffuse parameter.

Blinn Distribution

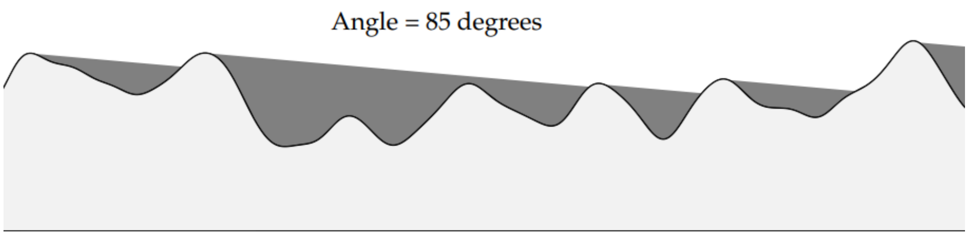
$$D(\vec{\omega}_h) = \frac{e+2}{2\pi}(\vec{\omega}_h \cdot \vec{n})^e$$

GGX distribution



Shadowing/masking

Microfacets can be shadowed and/or masked by other microfacets



Each microfacet distribution typically has its respective shadowing and masking term.

Beckmann distribution uses

$$G(\vec{\omega}) = \frac{2}{1 + \operatorname{erf}(s) + \frac{1}{s\sqrt{\pi}}e^{-s^2}}$$

where

$$s = \frac{1}{\alpha \tan\theta}$$

or in approximated version

$$G(\vec{\omega}) \simeq \begin{cases} \frac{3.535+2.181s^2}{1+2.276s+2.577s^2}, & s < 1.6 \\ 1, & \text{otherwise} \end{cases}$$

and

$$G(\vec{\omega}_i, \vec{\omega}_o) = G(\vec{\omega}_i) \cdot G(\vec{\omega}_o)$$

Blinn Distribution uses

$$G\left(\vec{\omega}_i, \vec{\omega}_o\right) = \min\left(1, \frac{2\left(\vec{n} \cdot \vec{\omega}_h\right)\left(\vec{n} \cdot \vec{\omega}_i\right)}{\left(\vec{\omega}_h \cdot \vec{\omega}_i\right)}, \frac{2\left(\vec{n} \cdot \vec{\omega}_h\right)\left(\vec{n} \cdot \vec{\omega}_o\right)}{\left(\vec{\omega}_h \cdot \vec{\omega}_o\right)}\right)$$

Sampling the Microfacet Model

$$f\left(\vec{\omega}_i, \vec{\omega}_o\right) = \frac{F\left(\vec{\omega}_h, \vec{\omega}_o\right) \cdot D\left(\vec{\omega}_h\right) \cdot G\left(\vec{\omega}_i, \vec{\omega}_o\right)}{4\left|\left(\vec{\omega}_i \cdot \vec{\mathbf{n}}\right)\left(\vec{\omega}_o \cdot \vec{\mathbf{n}}\right)\right|}$$

Note that the BRDF function is relevant with the half-way vector $\vec{\omega}_h$. We need to sample this vector to determine the BRDF function value. The general recipe is

- randomly generate a $\vec{\omega}_h$ with PDF proportional to D
- reflect incident direction $\vec{\omega}_i$ about $\vec{\omega}_h$ to obtain $\vec{\omega}_o$
- convert PDF($\vec{\omega}_h$) to PDF($\vec{\omega}_o$)

$$p(\vec{\omega}_o) = p(\vec{\omega}_h)\frac{d\vec{\omega}_h}{d\vec{\omega}_o} = p(\vec{\omega}_h)\frac{1}{4|\vec{\omega}_o \cdot \vec{\omega}_h|}$$

The Oren-Nayar Model

Assumes that the facets are diffuse

No analytical solution; fitted approximation

$$\begin{aligned}
&f_r\left(\vec{\omega}_o, \vec{\omega}_i\right) = \frac{\rho}{\pi}\left(A + B\max\left(0, \cos\left(\phi_i - \phi_o\right)\right)\sin\alpha\tan\beta\right) \\
&A = 1 - \frac{\sigma^2}{2\left(\sigma^2 + 0.33\right)} \quad B = \frac{0.45\sigma^2}{\sigma^2 + 0.09} \\
&\alpha = \max\left(\theta_i, \theta_o\right) \quad \beta = \min\left(\theta_i, \theta_o\right)
\end{aligned}$$

Ideal Lambertian is a special case of $\sigma = 0$.

Data-Driven BRDFs

The MERL Database

A data-driven reflectance model | ACM Transactions on Graphics
MERL – Mitsubishi Electric Research Laboratories

The RGL-EPFL Material Database

An adaptive parameterization for efficient material acquisition and rendering | ACM Transactions on Graphics
Material database | RGL (epfl.ch)