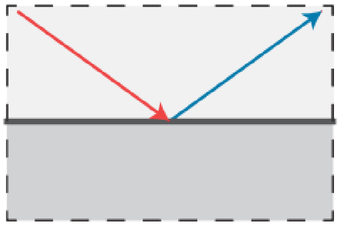
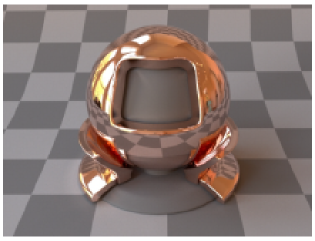
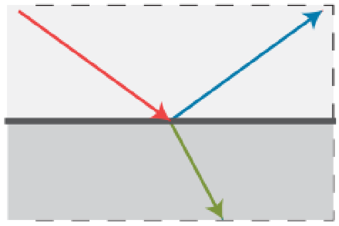
Appearance Modeling

Different materials have different interactions with the light. The interaction between light and materials is described by Bidirectional Reflectance Distribution Function.

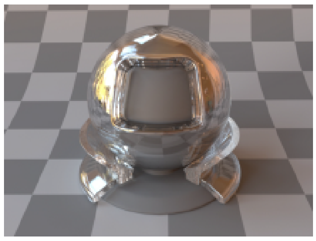


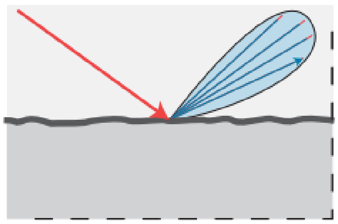
Smooth conducting material



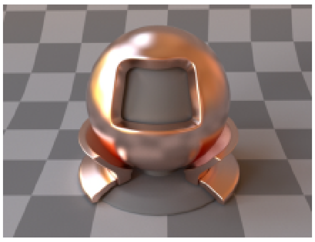


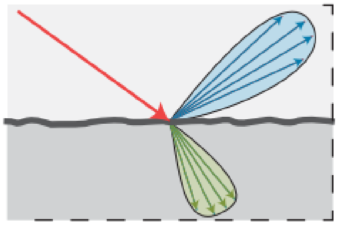
Smooth dielectric material



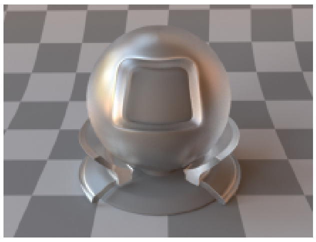


Rough conducting material





Rough dielectric material



BRDF

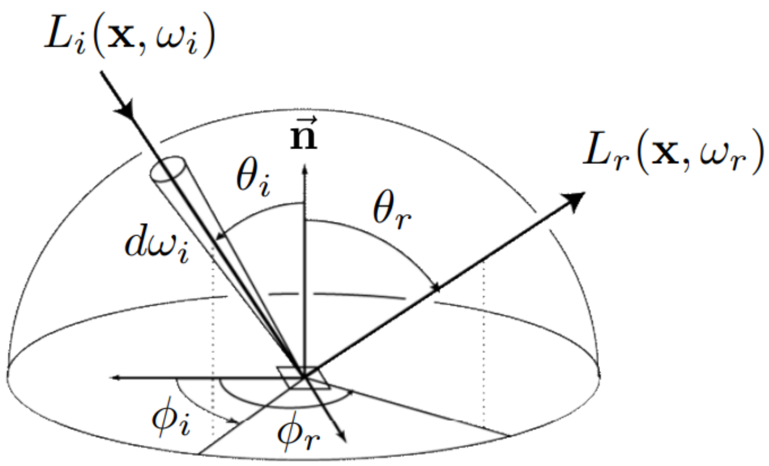
BRDF means Bidirectional Reflectance Distribution Function, which provides a relation between incident radiance and differential reflected radiance.

$$f_r(\mathbf{x}, \mathbf{w}_i, \mathbf{w}_r) = \frac{dL_r(\mathbf{x}, \mathbf{w}_r)}{dE_i(\mathbf{x}, \mathbf{w}_i)} = \frac{dL_r(\mathbf{x}, \mathbf{w}_r)}{L_i(\mathbf{x}, \mathbf{w}_i) \cos\theta_i d\mathbf{w}_i} \quad \left[\frac{1}{\mathrm{sr}}\right]$$

Reflection Equation

$$L_r(\mathbf{x}, \mathbf{w}_r) = \int_{H^2} f_r(\mathbf{x}, \mathbf{w}_i, \mathbf{w}_r) L_i(\mathbf{x}, \mathbf{w}_i) \cos\theta_i \, d\mathbf{w}_i$$

The reflection equation describes a local illumination model:



which results in the reflected radiance due to incident illumination from all directions.

BRDF Properties

Helmholtz Reciprocity

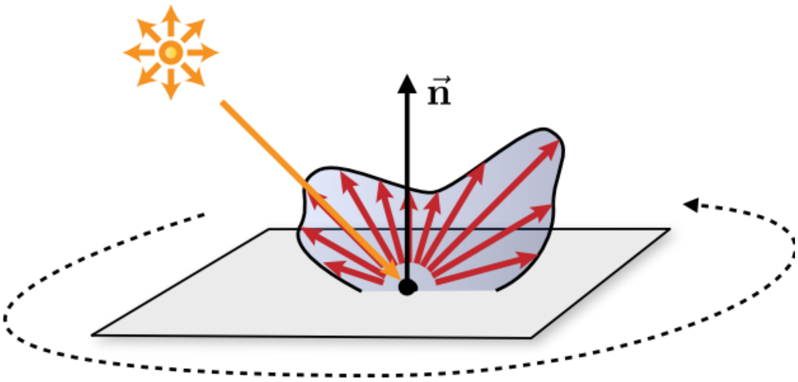
$$f_r(\mathbf{x}, \mathbf{w}_i, \mathbf{w}_r) = f_r(\mathbf{x}, \mathbf{w}_r, \mathbf{w}_i)$$

Energy Conservation

$$\int_{H^2} f_r(\mathbf{x}, \mathbf{w}_i, \mathbf{w}_r) \cos\theta_i \, d\mathbf{w}_i \leq 1$$

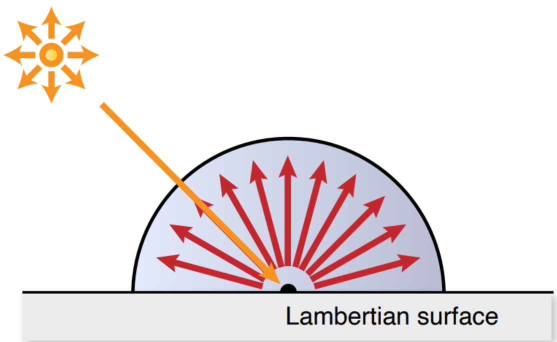
Isotropic vs. Anisotropic

If the BRDF is unchanged as the material is rotated around the normal, then it is isotropic, otherwise it is anisotropic. Isotropic BRDFs are functions of just 3 variables $(\theta_i, \theta_r, \Delta\phi)$.



Simple BRDF and BTDF Models

Lambertian Reflection Lambertian reflectance - Wikipedia



Lambertian reflectance is the property that defines an ideal "matte" or diffusely reflecting surface. The apparent brightness of a Lambertian surface to an observer is the same regardless of the observer's angle of view.

For Lambertian reflection, the BRDF is a constant. Thus we could write

$$\begin{aligned} L_r(\mathbf{w}, \mathbf{w}_r) &= \int_{H^2} f_r(\mathbf{x}, \mathbf{w}_i, \mathbf{w}_r) L_i(\mathbf{x}, \mathbf{w}_i) \cos\theta_i \, d\mathbf{w}_i \\ &= f_r \int_{H^2} L_i(\mathbf{x}, \mathbf{w}_i) \cos\theta_i \, d\mathbf{w}_i \\ &= f_r E(\mathbf{x}) \end{aligned}$$

If all incoming light is reflected, then

$$
\begin{aligned}
E(\mathbf{x}) &= B(\mathbf{x}) \\
B(\mathbf{x}) &= \int_{H^2} L_r(\mathbf{x}) \cos \, d\mathbf{w} \\
&= L_r(\mathbf{x}) \int_{H^2} \cos\theta \, d\mathbf{w} \\
&= L_r(\mathbf{x})\pi
\end{aligned}
$$

which leads to

$$f_r = \frac{1}{\pi}$$

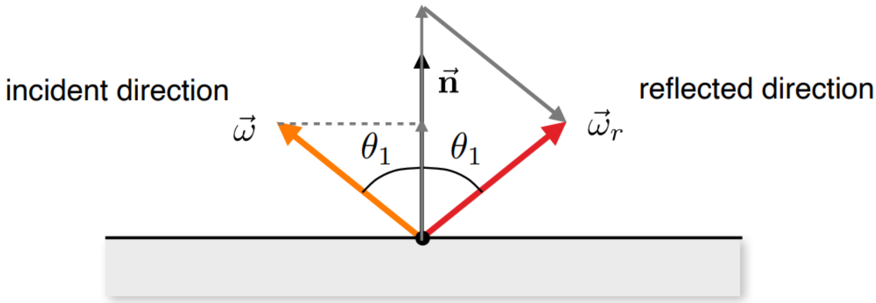
In resume, for Lambertian reflection, the reflected radiance is

$$L_r = \frac{\rho}{\pi} \int_{H^2} L_i(\mathbf{x}, \mathbf{w}_i) \cos\theta_i \, d\mathbf{w}_i$$

where $\rho \in [0, 1]$ is the diffuse reflectance:

- $\rho = 1$: all energy is (diffusely) reflected
- $\rho = 0$: all energy is absorbed

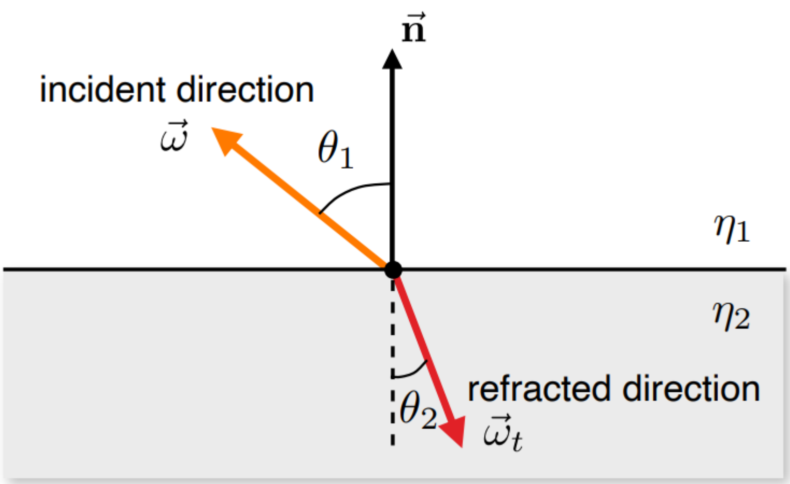
Ideal Specular Reflection



Every ray is reflected according to the geometrical rule without loss of energy. The BRDF is thus a delta function.
The reflected direction is

$$\mathbf{w}_r = 2(\mathbf{w}_i \cdot \mathbf{n})\mathbf{n} - \mathbf{w}_i$$

Ideal Specular Refraction



Every ray is refracted according to the geometrical rule without loss of energy. By Snell's law:

$$\eta_1 \sin\theta_1 = \eta_2 \sin\theta_2$$

The refracted direction is

$$\mathbf{w}_t = -\frac{\eta_1}{\eta_2}(\mathbf{w}_i - (\mathbf{w}_i \cdot \mathbf{n})\mathbf{n}) - \mathbf{n}\sqrt{1 - \left(\frac{\eta_1}{\eta_2}\right)^2 (1 - (\mathbf{w}_i \cdot \mathbf{n})^2)}$$

Material	Index of Refraction
Vacuum	1
Air at STP	1.00029
Ice	1.3
Water	1.33
Crown glass	1.52 - 1.65
Diamond	2.417

Reflection + Refraction

Fresnel Equations

$$\rho_{\parallel} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2}$$
$$\rho_{\perp} = \frac{\eta_1 \cos\theta_1 - \eta_2 \cos\theta_2}{\eta_1 \cos\theta_1 + \eta_2 \cos\theta_2}$$

The reflection and refraction factor are then

$$F_r = \frac{1}{2}\left(\rho_{\parallel}^2 + \rho_{\perp}^2\right)$$
$$F_t = 1 - F_r$$

> **Remark**
>
> The Fresnel coefficients are path-reversible, i.e. they are the same if we reverse the light path. This is easily proven by the fact that the s-polarized light has
>
> $$R_s = \left[\frac{\sin(\theta_t - \theta_i)}{\sin(\theta_t + \theta_i)}\right]^2$$
>
> and p-polarized light has
>
> $$R_p = \left[\frac{\tan(\theta_t - \theta_i)}{\tan(\theta_t + \theta_i)}\right]^2$$
>
> Only if we have the same θ_t and θ_i, we have the same coefficients.

BRDF of Specular Reflection

Dirac delta

Fresnel reflection

Reflection function
(flips about normal)

$$f_r(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = F_r(\vec{\omega}_i) \frac{\delta(\vec{\omega}_r - R(\vec{\omega}_i, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term
in the reflection equation
(Fresnel eq. account for it)

BRDF of Specular Refraction

Dirac delta

Fresnel reflection

Transmission
function

$$f_t(\mathbf{x}, \vec{\omega}_i, \vec{\omega}_r) = \frac{\eta_1^2}{\eta_2^2}(1 - F_r(\vec{\omega}_i)) \frac{\delta(\vec{\omega}_r - T(\vec{\omega}_i, \vec{\mathbf{n}}))}{\cos\theta_i}$$

to cancel the cosine term
in the reflection equation
(Fresnel eq. account for it)