CO2005: Electronics I

Semiconductor Materials and Diodes

Silicon

- Electrons in the outermost shell are called **valence electrons**, and the chemical activity of a material is determined primarily by the number of such electrons.
- The valence electrons are shared between atoms, forming what are called covalent bonds.

□ Silicon:

At T=0K, each electron is in its lowest possible energy state, so each covalent bonding position is filled. If a small electric field is applied to this material, the electrons will not move. At T=0K, silicon is an **insulator**, no charge flows through it.

Table 1.1 A portion of the periodic table

		πι τν
— si —		В С
		Al Si Ga Ge
l		
Si Si Si		si
		s _i $==$ s _i $==$ s _i $$
Si		Si
(a)	(b)	(c)

Figure 1.1 Silicon atoms in a crystal matrix: (a) five noninteracting silicon atoms, each with four valence electrons, (b) the tetrahedral configuration, (c) a two-dimensional representation showing the covalent bonding

Conductors

- If the temperature increases, some of the valence electrons will gain enough thermal energy to break the covalent bond and move away from its original position, leaving a positively charged "empty state" or called "hole". The electrons will then be free to move within the crystal.
- More free electrons and positively empty states are created for more higher temperature.
- In order to break the covalent bond, a valence electron must gain a minimum energy Eg, called the **bandgap energy**.
- Materials that contain very large numbers of free electrons at room temperature are conductors.
- Insulator: Eg is about 3~6 eV, semiconductor: Eg is about 1eV
 An electron-volt (eV) is the energy of an electron that has been accelerated through a potential difference of 1 volt, and 1eV= 1.6×10⁻¹⁹ joules.

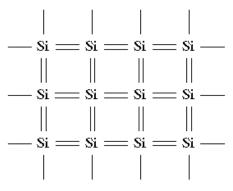


Figure 1.2 Two-dimensional representation of the silicon crystal at T = 0 °K

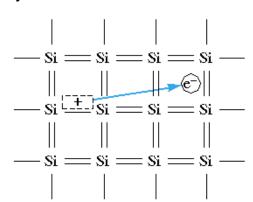


Figure 1.3 The breaking of a covalent bond for T > 0 °K

Current in Semiconductors

- A valence electron that has a certain thermal energy and is adjacent to a hole may move into that position, making it appear as if a positive charge is moving through the semiconductor.
- Two types of charged particles contribute to the current under a small electric field at room temperature:
 - ☐ the negatively charged free electron
 - ☐ the positively charged hole

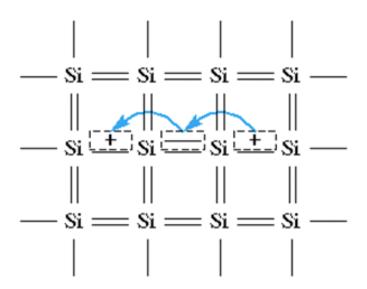


Figure 1.4 A twodimensional representation of the silicon crystal showing the movement of the positively charged hole

Intrinsic Semiconductors

- An intrinsic semiconductor is a single-crystal semiconductor material with no other type of atoms within the crystal.
- In an intrinsic semiconductor, the densities of electrons and holes are equal, since the thermally generated electrons and holes are the only source of such charged particles.
- lacktriangle Intrinsic carrier concentration n_i :

the concentration of the free electrons and that of the holes

$$n_i = BT^{3/2}e^{\left(-\frac{E_g}{2kT}\right)} = p_i$$

B is a constant related to the specific semiconductor material

 E_g is the bandgap energy (eV)

T is the absolute temperature (${}^{\circ}K$)

k is the Boltzman'n constant $(86 \times 10^{-6} \text{ eV/}^{\circ} K)$

Table 1.2 Semiconductor constants

Material	E_g (eV)	$B \text{ (cm}^{-3} \circ \text{K}^{-3/2})$
Silicon (Si) Gallium arsenide (GaAs) Germanium (Ge)	1.1 1.4 0.66	5.23×10^{15} 2.10×10^{14} 1.66×10^{15}

Example 1.1 Objective: Calculate the intrinsic carrier concentration in silicon at T = 300 °K.

Solution:

Example 1.1 Objective: Calculate the intrinsic carrier concentration in silicon at T = 300 °K.

Solution: For silicon at T = 300 °K, we can write

$$n_i = BT^{3/2} e^{\left(\frac{-E_g}{2kT}\right)}$$

$$= \left(5.23 \times 10^{15}\right) \left(300\right)^{3/2} e^{\left(\frac{-1.1}{2(86 \times 10^{-6})(300)}\right)}$$

or

$$n_i = 1.5 \times 10^{10} \, \text{cm}^{-3}$$

Comment: An intrinsic electron concentration of $1.5 \times 10^{10} \, \mathrm{cm}^{-3}$ may appear to be large, but it is relatively small compared to the concentration of silicon atoms, which is $5 \times 10^{22} \, \mathrm{cm}^{-3}$.

N-type Semiconductors

- Adding controlled amounts of certain impurities can greatly increase the concentration, then, the conductivity. The process to add impurities is called **doping**.
- A desirable impurity is one that enters the crystal lattice and replaces one of the semiconductor atoms. The impurities are from the group III and V elements.
- The materials containing impurity atoms are called extrinsic semiconductors.
- **□ Donor impurity**: phosphorus atom (group V element)
 - ✓ Four of its valence electron are used to satisfy the covalent bond requirements, the
 fifth valence electron is more loosely bound to the phosphorus atom.
 - ✓ At room temperature, this electron has enough thermal energy to break the bond, thus free to move through the crystal and contribute to the electron current.
 - ✓ The remaining phosphorus atom has a net positive charge, but the atom is immobile in the crystal and cannot contribute to the current.
 - ✓ A semiconductor containing donor impurity atoms is N-type semiconductor.

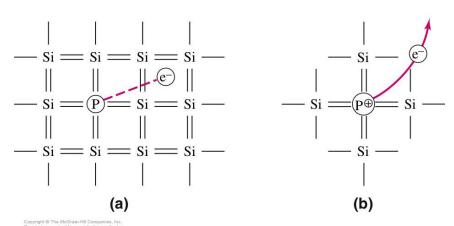


Figure 1.5 Two-dimensional representation of a silicon lattice doped with a phosphorus atom

P-type Semiconductors

- □ Acceptor impurity: boron atom (group III element)
 - ✓ Three valence electrons are used to satisfy the covalent bond requirements for three
 of four nearest silicon atoms. This leaves one bond position open.
 - ✓ At room temperature, adjacent silicon valence electrons have sufficient thermal energy to move into this position, thereby creating a hole.
 - ✓ The boron atom has a net negative charge, but cannot move, and a hole is created that can contribute to a hole current.
 - ✓ A semiconductor containing acceptor impurity atoms is P-type semiconductor.

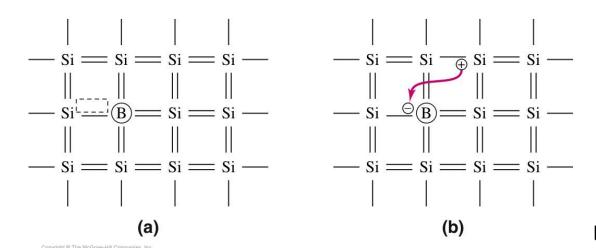


Figure 1.6 Two-dimensional representation of a silicon lattice doped with a boron atom

Thermal Equilibrium

■ Thermal Equilibrium: relationship between the electron and hole concentrations in a semiconductor

$$n_0 p_0 = n_i^2$$

where

 n_0 : the thermal equilibrium concentration of free electrons

 p_0 : the thermal equilibrium concentration of holes

 n_i : the intrinsic carrier concentration

- At room temperature,
 - lacksquare Donor Impurity: the donor concentration N_d is much larger than the intrinsic concentrations,

$$p_0 = n_i^2 / N_d$$

lacktriangle Acceptor Impurity: the acceptor concentration N_a is much larger than the intrinsic concentrations,

$$n_0 = n_i^2 / N_a$$

Example 1.2 Objective: Calculate the thermal equilibrium electron and hole concentrations.

Consider silicon at T = 300 °K doped with phosphorus at a concentration of $N_d = 10^{16} \, \mathrm{cm}^{-3}$. Recall from Example 1.1 that $n_i = 1.5 \times 10^{10} \, \mathrm{cm}^{-3}$.

Solution:

Example 1.2 Objective: Calculate the thermal equilibrium electron and hole concentrations.

Consider silicon at T = 300 °K doped with phosphorus at a concentration of $N_d = 10^{16} \, \mathrm{cm}^{-3}$. Recall from Example 1.1 that $n_i = 1.5 \times 10^{10} \, \mathrm{cm}^{-3}$.

Solution: Since $N_d \gg n_i$, the electron concentration is

$$n_o \cong N_d = 10^{16} \, \text{cm}^{-3}$$

and the hole concentration is

$$p_o = \frac{n_i^2}{N_d} = \frac{\left(1.5 \times 10^{10}\right)^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

Comment: In an extrinsic semiconductor, the electron and hole concentrations normally differ by many orders of magnitude.

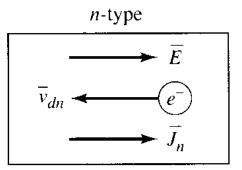
Drift Current

- N-type Semiconductor
 - ✓ Drift velocity of electrons

$$v_{dn} = -\mu_n E$$

 μ_n : electron mobility (cm²/V – s)

For low-doped silicon, $\mu_n \approx 1350 \, \mathrm{cm}^2 / \mathrm{V} - \mathrm{s}$.



✓ Drift current density

$$J_n = -env_{dn} = en\mu_n E \text{ (A/cm}^2)$$

n: electron concentration

e: the magnitude of the electronic charge

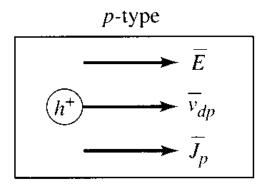
Drift Current

- □ P-type Semiconductor
 - ✓ Drift velocity of holes

$$v_{dp} = +\mu_p E$$

 μ_p : hole mobility (cm²/V – s)

For low-doped silicon, $\mu_p \approx 480 \, \mathrm{cm}^2 / \mathrm{V} - \mathrm{s}$.



✓ Drift current density

$$J_p = +epv_{dp} = +ep\mu_p E \quad (A/cm^2)$$

p : hole concentration

e: the magnitude of the electronic charge

Conductivity

☐ The total drift current density is the sum of the electron and hole components.

$$J = en\mu_n E + ep\mu_p E = \sigma E$$
$$\sigma = en\mu_n + ep\mu_p$$

where σ is the **conductivity** of the semiconductor.

The conductivity can be changed from strongly n-type, n>>p, by donor impurity doping to strongly p-type, p>>n, by acceptor impurity doping.

Diffusion Current

With diffusion, particles flow from a region of high concentration to a region of lower concentration.

$$J_n = eD_n \frac{dn(x)}{dx}, \quad J_p = -eD_p \frac{dp(x)}{dx}$$

 D_n : electron diffusion coefficient

 D_n : hole diffusion coefficient

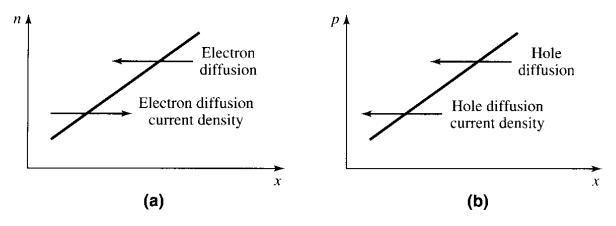


Figure 1.8 Current density caused by concentration gradients: (a) electron diffusion and corresponding current density and (b) hole diffusion and corresponding current density

Einstein Relation

■ Relationship between mobility values and diffusion coefficient values

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e} = V_T \cong 0.026 \text{ V}$$

■ The total current density is the sum of the drift and diffusion components. In most cases only one component dominates the current at any one time in a given region of a semiconductor.

PN Junction

- PN Junction: the entire semiconductor material is a single crystal, with one region doped to be p-type and the adjacent region doped to n-type.
- Initially, a large density gradient in both the hole and electron concentration occurs across this junction and causes a diffusion of holes and electrons.

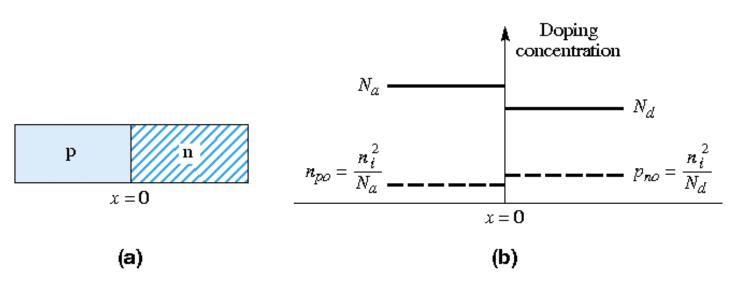


Figure 1.9 The pn junction: (a) simplified geometry of a pn junction and (b) doping profile of an ideal uniformly doped pn junction

Thermal Equilibrium

- If no voltage is applied to the pn junction, the diffusion of holes and electrons must eventually cease.
- ☐ The direction of the induced electric field will cause the resulting force to repel the diffusion of holes from the p-region and the diffusion of electrons from the n-region.
- Thermal Equilibrium: The force produced by the electric field and the force produced by the density gradient exactly balance.

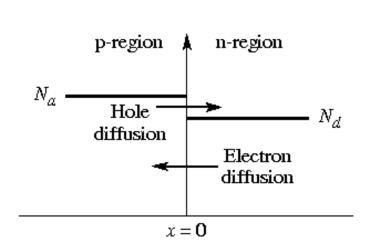


Figure 1.10 Initial diffusion of electrons and holes at the metallurgical junction, establishing thermal equilibrium

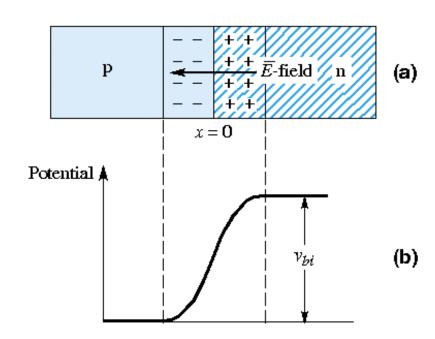


Figure 1.11 The pn junction in thermal equilibrium: (a) the space-charge region and electric field and (b) the potential through the junction

Depletion Region

- ☐ The positively charged region and the negatively charged region comprise the **depletion region** of the pn juction, in which there are essentially *no mobile electrons or holes*.
- ☐ There is a potential electric field difference across the region.
- ☐ This potential difference is called the built-in potential barrier,

$$V_{bi} = \frac{kT}{e} \ln \left(\frac{N_a N_d}{n_i^2} \right) = V_T \ln \left(\frac{N_a N_d}{n_i^2} \right)$$

- The parameter V_T is called the **thermal voltage** and is approximately $V_T = 0.026 \, \mathrm{V}$ at room temperature, $T = 300^{\mathrm{o}} \, \mathrm{K}$.
 - **Example 1.3 Objective:** Calculate the built-in potential barrier of a pn junction. Consider a silicon pn junction at T = 300 °K, doped at $N_a = 10^{16}$ cm⁻³ in the pregion and $N_d = 10^{17}$ cm⁻³ in the n-region.

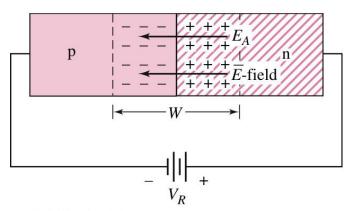
Solution: From the results of Example 1.1, we have $n_i = 1.5 \times 10^{10} \, \text{cm}^{-3}$ for silicon at room temperature. We then find

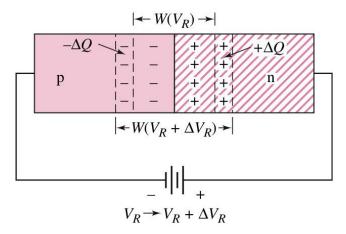
$$V_{bi} = V_T \ln(\frac{N_a N_d}{n_i^2}) = (0.026) \ln\left[\frac{(10^{16})(10^{17})}{(1.5 \times 10^{10})^2}\right] = 0.757 \text{ V}$$

Reverse-Biased PN Junction

- Reverse Bias: The magnitude of the electric field in the depletion region increases above the thermal equilibrium value.
- This increased electric field holds back the holes in the p-region and the electrons in the n-region, so there is essentially no current across the pn junction, so there is essentially no current across the pn junction.
- When the electric field in the depletion region increases, the number of positive and negative charges also increases.
- lacktriangle With an increasing reverse-bias voltage $V_{\scriptscriptstyle T}$, depletion width W also increases.
- Junction (depletion layer) capacitance:

$$C_j = C_{jo} \left(1 + \frac{V_R}{V_{bi}} \right)^{-1/2}$$





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Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display **Example 1.4 Objective:** Calculate the junction capacitance of a pn junction.

Consider a silicon pn junction at T = 300 °K, with doping concentrations of $N_a = 10^{16} \, \mathrm{cm}^{-3}$ and $N_d = 10^{15} \, \mathrm{cm}^{-3}$. Assume that $n_i = 1.5 \times 10^{10} \, \mathrm{cm}^{-3}$ and let $C_{jo} = 0.5 \, \mathrm{pF}$. Calculate the junction capacitance at $V_R = 1 \, \mathrm{V}$ and $V_R = 5 \, \mathrm{V}$.

Solution:

Example 1.4 Objective: Calculate the junction capacitance of a pn junction.

Consider a silicon pn junction at T = 300 °K, with doping concentrations of $N_a = 10^{16} \, \mathrm{cm}^{-3}$ and $N_d = 10^{15} \, \mathrm{cm}^{-3}$. Assume that $n_i = 1.5 \times 10^{10} \, \mathrm{cm}^{-3}$ and let $C_{jo} = 0.5 \, \mathrm{pF}$. Calculate the junction capacitance at $V_R = 1 \, \mathrm{V}$ and $V_R = 5 \, \mathrm{V}$.

Solution: The built-in potential is determined by

$$V_{bi} = V_T \ln(\frac{N_a N_d}{n_i^2}) = (0.026) \ln\left[\frac{(10^{16})(10^{15})}{(1.5 \times 10^{10})^2}\right] = 0.637 \text{ V}$$

The junction capacitance for $V_R = 1 \text{ V}$ is then found to be

$$C_j = C_{jo} \left(1 + \frac{V_R}{V_{bi}} \right)^{-1/2} = (0.5) \left(1 + \frac{1}{0.637} \right)^{-1/2} = 0.312 \text{ pF}$$

For $V_R = 5 \text{ V}$

$$C_j = (0.5) \left(1 + \frac{5}{0.637} \right)^{-1/2} = 0.168 \text{ pF}$$

Comment: The magnitude of the junction capacitance is usually at or below the picofarad range, and it decreases as the reverse-bias voltage increases.

Forward-Biased PN Junction

- Forward Bias: The net result is that the electric field in the depletion region is lower than the equilibrium value.
- Majority carrier electrons from the n-region diffuse into the p-region, and majority carrier holes from the p-region diffuse into the n-region.
- The diffusion process continues as long as the forward voltage v_D is applied, thus creating a current in the pn junction.

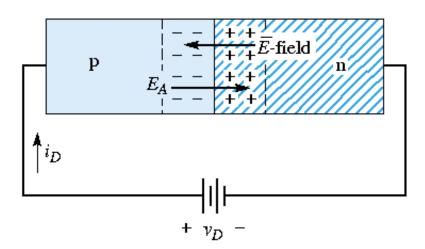


Figure 1.13 A pn junction with an applied forward-bias voltage, showing the direction of the electric field E_A induced by v_D and of the net space-charge electric field E

Minority Carrier Concentration

- As the majority carriers cross the opposite regions, they become minority carriers in those regions, causing the minority carrier concentrations to increase.
- There excess minority carriers diffuse into the neutral n- and p-regions, where they combine with majority carriers, thus establishing a steady-state condition with decaying concentration.

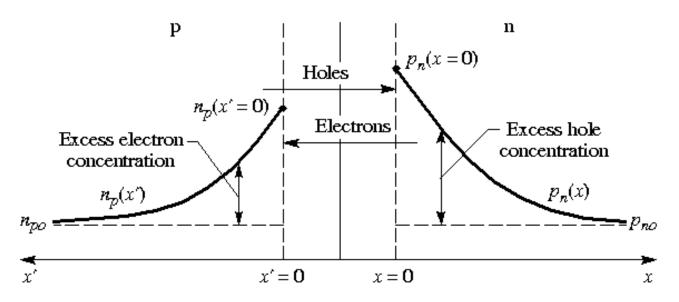


Figure 1.14 Steady-state minority carrier concentration in a pn junction under forward bias

Ideal Current-Voltage Relationship

■ The theoretical relationship between the voltage and the current in the pn junction

$$i_D = I_S(e^{\left(\frac{v_D}{nV_T}\right)} - 1)$$

The parameter I_S is the reversebias saturation current.

- For silicon pn junctions, typical value of I_S are in the range of 10^{-18} to 10^{-12} A.
- The parameter n is usually called the emission coefficient or ideality factor, and its value is in the range $1 \le n \le 2$.

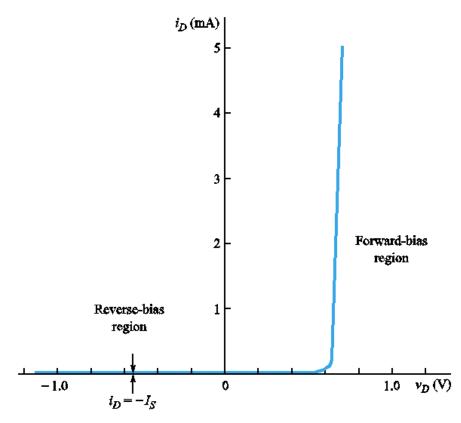


Figure 1.15 Ideal I-V characteristics of a pn junction diode for $I_S=10^{-14}\,\mathrm{A}$

Example 1.5 Objective: Determine the current in a pn junction.

Consider a pn junction at T = 300 °K in which $I_S = 10^{-14}$ A and n = 1. Find the diode current for $v_D = +0.70$ V and $v_D = -0.70$ V.

Solution:

Example 1.5 Objective: Determine the current in a pn junction.

Consider a pn junction at T = 300 °K in which $I_S = 10^{-14}$ A and n = 1. Find the diode current for $v_D = +0.70$ V and $v_D = -0.70$ V.

Solution: For $v_D = +0.70 \,\mathrm{V}$, the pn junction is forward-biased and we find

$$i_D = I_S \left[e^{\left(\frac{v_D}{V_T}\right)} - 1 \right] = (10^{-14}) \left[e^{\left(\frac{+0.70}{0.026}\right)} - 1 \right] \Longrightarrow 4.93 \text{ mA}$$

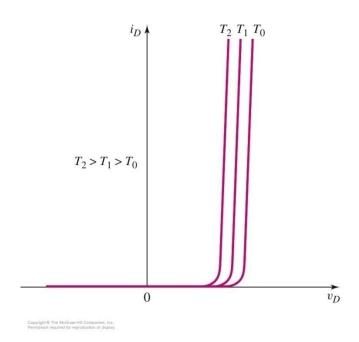
For $v_D = -0.70 \,\mathrm{V}$, the pn junction is reverse-biased and we find

$$i_D = I_S \left[e^{\left(\frac{v_D}{V_T}\right)} - 1 \right] = (10^{-14}) \left[e^{\left(\frac{-0.70}{0.026}\right)} - 1 \right] \cong 10^{-14} \text{ A}$$

Comment: Although I_S is quite small, even a relatively small value of forward-bias voltage can induce a moderate junction current. With a reverse-bias voltage applied, the junction current is virtually zero.

Temperature Effects

- lacktriangle Since both I_{S} and V_{T} are functions of temperature, the diode characteristics also vary with temperature.
- For a given current, the forward-bias voltage decreases as temperature increases. For silicon diodes, the change is approximately 2 mV/ °C.
- The value of I_S approximately doubles for every 5 °C increase in temperature. The actual reverse-bias diode current, as a general rule, doubles for every 10°C rise in temperature.



Ideal Diode Circuit Model

■ Symbol

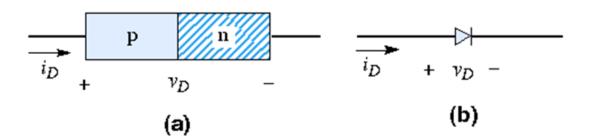


Figure 1.17 The basic pn junction diode: (a) simplified geometry and (b) circuit symbol, and conventional current direction and voltage polarity

I-V Characteristics of an Ideal Diode

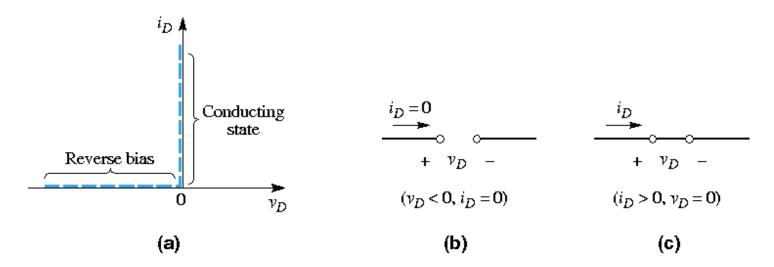


Figure 1.22 The ideal diode: (a) I-V characteristics, (b) equivalent circuit under reverse bias, and (c) equivalent circuit in the conducting state

Example: The Diode Rectifier

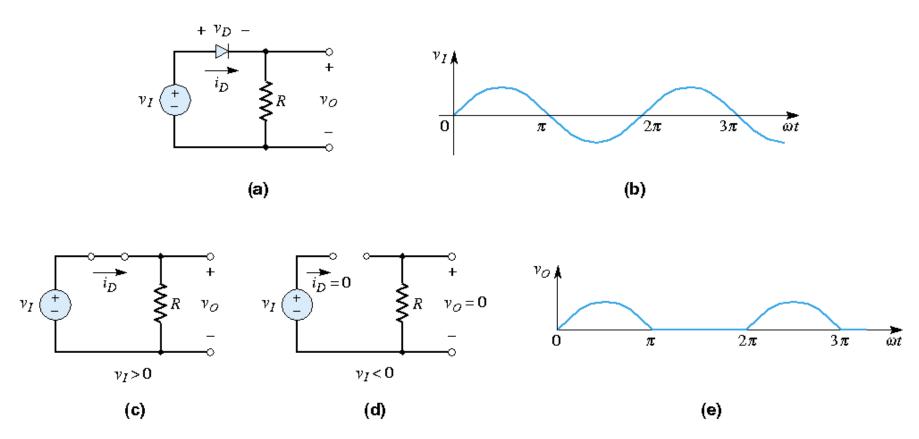


Figure 1.23 The diode rectifier: (a) circuit, (b) sinusoidal input signal, (c) equivalent circuit for $v_I > 0$, (d) equivalent circuit for $v_I < 0$, and (e) rectified output signal

A Simple Diode Circuit - DC Analysis

Diode Rectifier Circuit

By Kirchhoff's Voltage Law:

$$V_{PS} = I_D R + V_D$$

$$I_D = \frac{V_{PS}}{R} - \frac{V_D}{R}$$

By Diode I-V Characteristics:

$$I_D = I_S(e^{V_D/V_T} - 1)$$

You have to solve $\,V_{\scriptscriptstyle D}\,{\rm and}\,$ $\,I_{\scriptscriptstyle D}\,{\rm :}\,$

$$\begin{cases} I_D = \frac{V_{PS}}{R} - \frac{V_D}{R} \\ I_D = I_S (e^{V_D/V_T} - 1) \end{cases}$$

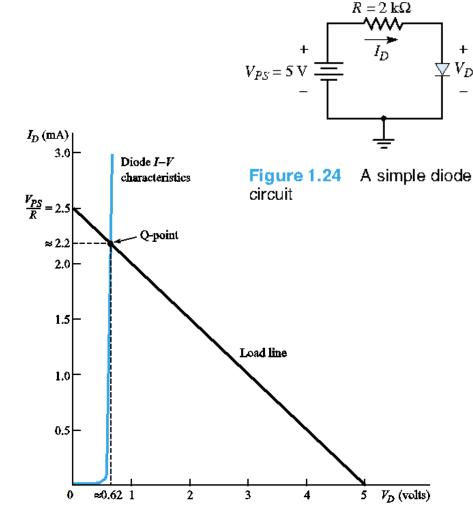
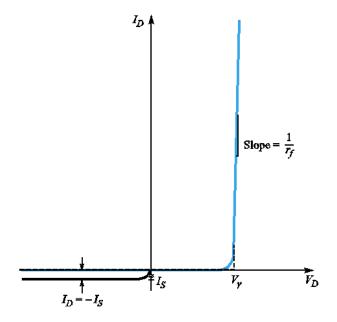


Figure 1.25 The diode and load line characteristics for the circuit shown in Figure 1.24

Piecewise Linear Model

lacksquare $V_{_{\gamma}}$ is the turn-on (cut-in) voltage of the diode, and $r_{_{\! f}}$ is the forward diode resistance.



In general case, $r_f \ll R_L$ The diode current is essentially independent of the value of r_f .

The calculated diode current is not a strong function of the cut-in voltage.

 I_D I

Figure 1.26 The ideal diode I–V characteristics and two linear approximations

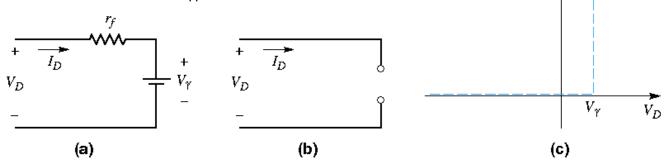


Figure 1.27 The diode equivalent circuit (a) in the "on" condition when $V_D > V_{\gamma}$, (b) in the "off" condition when $V_D < V_{\gamma}$, and (c) piecewise linear approximation when $r_f = 0$

Diode Circuit Solution by Piecewise Linear Model

Example 1.7 Objective: Determine the diode voltage and current in the circuit shown in Figure 1.24, using a piecewise linear model.

Assume piecewise linear diode parameters of $V_{\gamma} = 0.6 \,\mathrm{V}$ and $r_f = 10 \,\Omega$.

Solution:

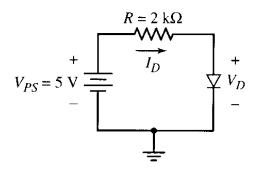


Figure 1.24 A simple diode circuit

Diode Circuit Solution by Piecewise Linear Model

Example 1.7 Objective: Determine the diode voltage and current in the circuit shown in Figure 1.24, using a piecewise linear model.

Assume piecewise linear diode parameters of $V_{\gamma} = 0.6 \,\mathrm{V}$ and $r_f = 10 \,\Omega$.

Solution:

Since $V_{PS} > V_r$, we have $V_{PS} = V_r + r_f I_D + RI_D$

$$I_D = \frac{V_{PS} - V_{\gamma}}{R + r_f} = \frac{5 - 0.6}{2 \times 10^3 + 10} \Rightarrow 2.19 \,\text{mA}$$

and the diode voltage is

$$V_D = V_{\gamma} + I_D r_f = 0.6 + (2.19 \times 10^{-3})(10) = 0.622 \text{ V}$$

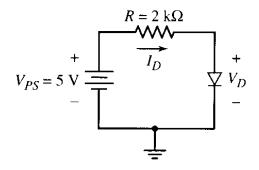


Figure 1.24 A simple diode circuit

Comment: This solution, obtained using the piecewise linear model, is nearly equal to the solution obtained in Example 1.6, in which the ideal diode equation was used. However, the analysis using the piecewise-linear model in this example is by far easier than using the actual diode I-V characteristics as was done in Example 1.6. In general, we are willing to accept some slight analysis inaccuracy for ease of analysis.

Reverse Biased Diode Circuit

■ The load line concept is also useful when the diode is reverse biased.

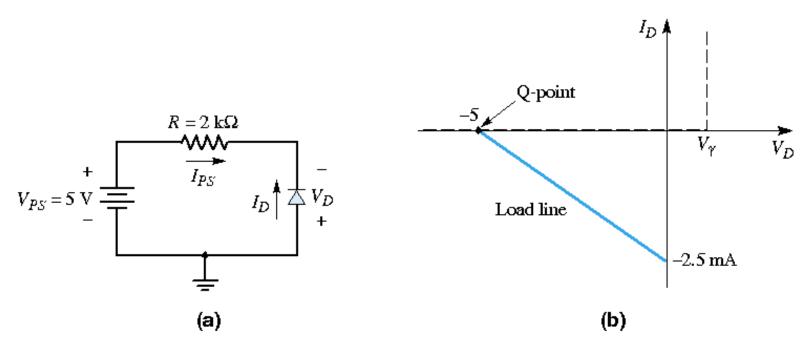


Figure 1.29 Reverse-biased diode (a) circuit and (b) piecewise linear approximation and load line

$$V_{PS} = I_{PS}R - V_D = -I_DR + V_D$$

$$\Rightarrow I_D = -\frac{V_{PS}}{R} - \frac{V_D}{R}$$

A Simple Diode Circuit – AC Analysis

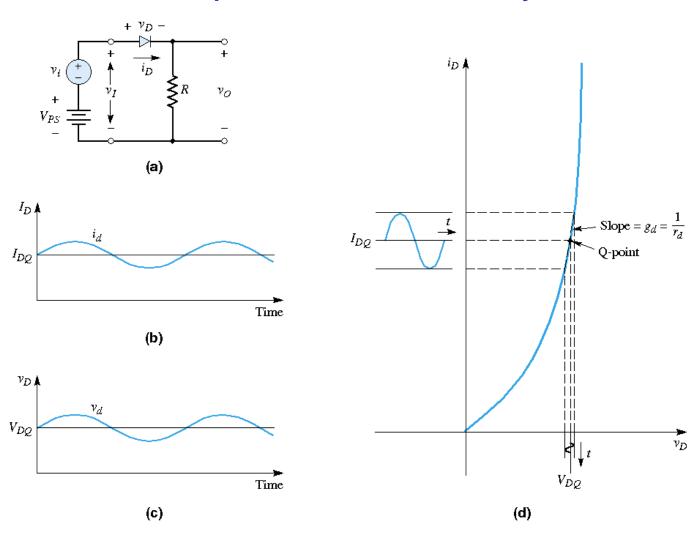


Figure 1.31 AC circuit analysis: (a) circuit with combined dc and sinusoidal input voltages, (b) sinusoidal diode current superimposed on the quiescent current, (c) sinusoidal diode voltage superimposed on the quiescent value, and (d) forward-biased diode I-V characteristics with a sinusoidal current and voltage superimposed on the quiescent values

A Simple Diode Circuit – AC Analysis

Find the relationship between I-V for AC component

The I-V relation by ideal diode I-V equation:

$$i_D = I_S(e^{v_D/V_T} - 1) \approx I_S e^{(V_{DQ} + v_d)/V_T} = I_S e^{V_{DQ}/V_T} e^{v_d/V_T} = I_{DQ} e^{v_d/V_T}$$

The Taylor series expansion: $e^{v_d/V_T} \approx 1 + v_d/V_T$

$$i_D = I_S(e^{v_D/V_T} - 1) = I_{DQ}(1 + v_d/V_T) = I_{DQ} + v_d \cdot I_{DQ}/V_T = I_{DQ} + i_d$$

The AC Component of the diode current:

$$i_d = v_d / r_d$$
 $r_d = V_T / I_{DO} = 1 / g_d$

 \mathcal{V}_d : the diode small-signal incremental resistance, diffusion resistance

 \mathcal{G}_d : the diode small-signal incremental conductance, diffusion conductance

Example 1.9 Objective: Analyze the circuit shown in Figure 1.31(a).

Assume circuit and diode parameters of $V_{PS} = 5 \text{ V}$, $R = 5 \text{ k}\Omega$, $V_{\gamma} = 0.6 \text{ V}$, and $v_i = 0.1 \sin \omega t(\text{V})$.

Solution:

Example 1.9 Objective: Analyze the circuit shown in Figure 1.31(a).

Assume circuit and diode parameters of $V_{PS} = 5 \text{ V}$, $R = 5 \text{ k}\Omega$, $V_{\gamma} = 0.6 \text{ V}$, and $v_i = 0.1 \sin \omega t(\text{V})$.

Solution: Divide the analysis into two parts: the dc analysis and the ac analysis. For the dc analysis, we set $v_i = 0$

$$I_{DQ} = \frac{V_{PS} - V_{\gamma}}{R} = \frac{5 - 0.6}{5} = 0.88 \,\text{mA}$$

$$V_O = I_{DO}R = (0.88)(5) = 4.4 \text{ V}$$

For the ac analysis, set $V_{PS} = 0$. The ac Kirchhoff voltage law (KVL) equation becomes

$$v_i = i_d r_d + i_d R = i_d \left(r_d + R \right)$$

where r_d is again the small-signal diode diffusion resistance.

$$r_d = \frac{V_T}{I_{DO}} = \frac{0.026}{0.88} = 0.0295 \text{k}\Omega$$

$$i_d = \frac{v_i}{r_d + R} = \frac{0.1 \sin \omega t}{0.0295 + 5} \Longrightarrow 19.9 \sin \omega t (\mu A)$$

$$v_o = i_d R = 0.0995 \sin \omega t(V)$$