The Calculus Bee at MIT

Sample problems

Problem Contributors.

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Instructions

Enclosed are three integration problems, three short-answer problems, and one proof-level problem. These are not official and they will not appear on any CalcBee contest but they should give some insights into the difficulty level of the problems on the contest.

Integration problems.

1. *(Easy.)* Find $\displaystyle\int (1 + \ln|x|)\,\mathrm{d}x$.
2. *(Medium.)* Find $\displaystyle\int \sinh\left(\sqrt{x}\right)\,\mathrm{d}x$.
3. *(Hard.)* Find $\displaystyle\int \frac{1-x^2}{1+3x^2+x^4}\,\mathrm{d}x$.

Short-answer problems.

1. *(Easy.)* A car has instantaneous velocity function $v(t) = \frac{1}{2t-3}$. Given that the position at time $t = 0$ is 0, what is the position at time $t = 2$? If indeterminate, say so.
2. *(Medium.)* Consider the polynomial $f(x) = ax^2 + 20x + 15$. If $f'(t) = f''(t)$, then t is an integer. Compute the sum of all possible values of a.
3. *(Hard.)* Find the volume of the solid resulting from the rotation of the region enclosed by $y = x^m$ and $y = x^n$ about the line $x = 1$.

Proof problem.

1. Compute, with proof, the limit

$$\lim_{n\to\infty} \sqrt[n]{\left|\ln\left|1 + \left(\frac{1}{n\cdot\log n}\right)^k\right|\right|}.$$