Probability Using Geometry

LHS Math Team

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2-D Probability

1a. Two positive real numbers, each less than 3, are randomly chosen. What is the probability that the positive difference between the two numbers is less than 1?

1b. Two positive real numbers, each less than n, are randomly chosen. What is the probability, in terms of n, that the positive difference between the two numbers is less than 1?

2. John needs to catch a train. The train arrives randomly some time between 2:00 and 3:00, waits for 20 minutes, and then leaves. If John also arrives randomly between 2:00 and 3:00, what is the probability that the train will be there when John arrives?

3. Allen and Bethany each arrive at a party at a random time between 1:00 and 2:00. Each stays for 15 minutes, then leaves. What is the probability that Allen and Bethany see each other at the party?

4. Alex needs to catch a train. The train arrives randomly some time between 1:00 and 2:00, waits for 10 minutes, and then leaves. If Alex also arrives randomly between 1:00 and 2:00, what is the probability that the train will be there when Alex arrives?

5. [HMMT General 1, 2006] The train schedule in Hummut is hopelessly unreliable. Train A will enter Intersection X from the west at a random time between 9:00 am and 2:30 pm; each moment in that interval is equally likely. Train B will enter the same intersection from the north at a random time between 9:30 am and 12:30 pm, independent of Train A; again, each moment in the interval is equally likely. If each train takes 45 minutes to clear the intersection, what is the probability of a collision today?

6. [ARML Team Round Practice 3] Two people were in a store somewhere between 3:00 and 4:00 pm and each one stayed in the store for 10 minutes only during the hour. If it is equally probable that each person could have been in

the store at any time during this hour, what is the probability they were in the store together at some time during this hour?

7. Alice and Bob both go to a party which starts at 5:00. Each of them arrives at a random time between 5:00 and 6:00. What is the probability that the number of minutes Alice is late for the party plus the number of minutes Bob is late for the party is less than 45? Express your answer as a common fraction.

8. Alice and Bob each arrive at a party at a random time between 1:00 and 2:00. If Alice arrives after Bob, what is the probability that Bob arrived before 1:30?

3-D Probability

9. Three real numbers are picked, each between 0 and 1. What is the probability that they can form the sides of a triangle?

10. A boss plans a business meeting at Starbucks with the two engineers below him. However, he fails to set a time, and all three arrive at Starbucks at a random time between 2:00 and 4:00 p.m. When the boss shows up, if both engineers are not already there, he storms out and cancels the meeting. Each engineer is willing to stay at Starbucks alone for an hour, but if the other engineer has not arrived by that time, he will leave. What is the probability that the meeting takes place?

Answers

1a. $\frac{5}{9}$

1b. $1 - \frac{(n-1)^2}{n^2}$

2. $\frac{5}{18}$

3. $\frac{7}{16}$

4. $\frac{11}{72}$

5. $\frac{13}{48}$

6. $\frac{9}{25}$

7. $\frac{9}{32}$

8. $\frac{3}{4}$

9. $\frac{1}{2}$

10. $\frac{7}{24}$