HMNT Tryout - LHS Math Team
Monday October 3rd, 2011

1. [**3**] From a standard deck of 52 cards, 21 cards are drawn without replacement. What is the probability that the 15th card is an ace?

2. [**3**] Teams at HMNT consist of 4, 5, or 6 people. If there are 200 people at HMNT that are on a team, no two teams share a person, 11 teams have exactly 4 people, and 12 teams have exactly 5 people, how many teams have exactly six people?

3. [**4**] If $x^2 + x + 1 = 16$, evaluate $x^4 + 2x^3 + 3x^2 + 2x$.

4. [**4**] What is the sum of the squares of the divisors of 360?

5. [**4**] Find the volume of the region in space defined by

$$|x + y + z| + |x + y - z| \le 8,$$

   where $x, y, z \ge 0$.

6. [**5**] If n is a multiple of 4, evaluate the sum

$$1 + 2i + 3i^2 + ... + (n + 1)i^n,$$

   where $i^2 = -1$.

7. [**6**] Find all positive integers $0 \le x < 37$ such that $x^3 - 1$ is divisible by 37.

8. [**6**] Let ABC be a triangle with side lengths $AB = 65$, $BC = 70$, $CA = 75$. Point D is located on side $\overline{AC}$, point E is located on side $\overline{AB}$, and points F and G are located on side $\overline{BC}$ with G closer to C than to B. Given that $A = (0,0)$, $C = (75, 0)$, and $DEFG$ is a square, find the coordinates of point G.

9. [**7**] Let $T(n)$ be the number of ways to represent n as the sum of powers of 3, where each power of 3 may be used up to 5 times. For example, $T(5) = 2$ since

$$5 = 3^1 + 3^0 + 3^0 = 3^0 + 3^0 + 3^0 + 3^0 + 3^0.$$

   Similarly, $T(6) = 2$ since
$$6 = 3^1 + 3^1 = 3^1 + 3^0 + 3^0 + 3^0.$$

   Evaluate $T(2011)$.

10. [**8**] In triangle ABC, let D, E, and F be the points of tangency of the inscribed circle of ABC with sides $\overline{BC}$, $\overline{AC}$, and $\overline{AB}$, respectively. Let M be the midpoint of $\overline{BC}$, and let the intersection of $\overline{AM}$ with $\overline{EF}$ be P. Given that $m\angle B = 75°$ and $m\angle A = 68°$, find $m\angle DPM$.