HMNT Tryout - LHS Math Team

Monday October 15, 2012

1. [**3**] Isosceles triangle ABC, with $AB = AC = 10$, is inscribed in a circle. Triangle DAB is drawn with point D on the circle closer to point A than B. Triangle PAC, congruent to triangle DAB, is drawn with point P on the circle closer to point C than A. What is the length of $\overline{DP}$?

2. [**3**] Let a_1, a_2, a_3 be elements of the set $L = \{1, 2, 3, \ldots, 14\}$ such that $a_2 - a_1 \geq 3$ and $a_3 - a_2 \geq 3$. How many such ordered triplets (a_1, a_2, a_3) are there?

3. [**4**] Let $A = 666\ldots666$, where there are 2012 6's. Let $B = 333\ldots333$, where there are 2013 3's. What digits are in the number $A \times B$?

4. [**4**] Compute the largest positive integer n such that $\dfrac{(2012!)^n}{2012^{n!}}$ is an integer.

5. [**4**] Balls labeled from 1 through 9, inclusive, are seated equally around a table. The absolute differences between pairs of consecutive balls are written down. Given that the sum of these differences is a minimum, how many possible placements of the balls are there?

6. [**5**] In right triangle ABC, which has an area of 4, $AB = AC$. Let $\mathcal{S}$ be the set of all points P where the sum of the distances from P to sides $\overline{AB}$ and $\overline{AC}$ is equal to the distance from P to side $\overline{BC}$. The set $\mathcal{S}$ divides the triangle into two pieces. What is the area of the smaller piece?

7. [**6**] Find a triple (x, y, z) that satisfies

$$\begin{aligned} x + y + z &= 5 \\ x(y+z)^2 + y(x+z)^2 + z(x+y)^2 &= -14 \\ x^2(y+z) + y^2(x+z) + z^2(x+y) &= 34. \end{aligned}$$

8. [**6**] Find the coefficient of x^5 in the polynomial of minimal degree with leading coefficient 1 and integer coefficients that has $2^{1/2} + 3^{1/3}$ as a root.

9. [**7**] In triangle ABC, $\angle ABC$ is obtuse. Point D lies on side $\overline{AC}$ such that $\angle ABD$ is right, and point E lies on side $\overline{AC}$ between A and D such that $\overline{BD}$ bisects $\angle EBC$. Find CE, given that $AC = 35$, $BC = 7$, and $BE = 5$.

10. [**8**] How many 13-element subsets are there of the set $\{1, 2, 3, \ldots, 26\}$ where the sum of the elements is divisible by 13?