Problem Set 3 - Full
Lexington Math Team

Monday, October 15, 2012

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Show that $f(x)$ can be expressed as the sum of an even function and an odd function.

2. Let $\overline{AC}$ be the diameter of semicircle ABC, and let arc BC be bisected at D. Let E be the foot of the perpendicular from D to $\overline{AC}$. Show that $CE = (AC - AB)/2$.

3. The points in 3D space are painted black and white. Determine whether or not they can be colored such that if any equilateral triangle is chosen in space, the vertices of it are not all black or all white.

4. Prove that any number consisting of 2^n identical digits has at least n distinct prime factors.

5. Given that a, b, and c are positive real numbers satisfying $a + b + c = 3$, prove that the minimum possible value of

$$\frac{a}{b^2+b+1} + \frac{b}{c^2+c+1} + \frac{c}{a^2+a+1}$$

is 1.