Problem Set 4 - Full
Lexington Math Team

Monday, October 22, 2012

1. Let ABC be a triangle with altitudes $\overline{CD}$ and $\overline{AE}$, with $BD = 3$, $DA = 5$, and $BE = 2$. Find EC.

2. Let $S(x)$ be the sum of the digits of the positive integer x in its decimal representation. Find the largest possible value of $S(x)/S(2x)$.

3. Evaluate
$$\frac{1}{2^{2012}} \sum_{n=0}^{1006} (-3)^n \binom{2012}{2n}.$$

4. In triangle ABC, $\overline{AE}$ is an angle bisector and $\overline{BH}$ is an altitude. Given that angle AEB measures 45 degrees, what is the measure of angle EHC?

5. A set T is called *even* if it has an even number of elements. Let n be a positive even integer, and let $S_1, S_2, \ldots, S_n$ be even subsets of the set $S = \{1, 2, \ldots, n\}$. Prove that there exist i and j, $1 \leq i < j \leq n$, such that $S_i \cap S_j$ is even.