Solving (Trigonometry) Problems That Don't Look Like Geometry with Euclidean Geometry

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Sometimes on math problems you have this very ugly expression with which you must do something with. The key to this skill is to notice that it is equivalent to something nice and use it to do the problem. Usually you would bash these problems, but no one really wants to do that.

Key points:

- Look for similarities between expressions and things we have seen previously.
- Inscribe triangles in a circle with diameter one. This provides elegant side lengths in relation with angles.

Problems:

1. Let $f(x) = \sqrt{(x^2-4)^2-(x-3)^2} + \sqrt{(x^2-2)^2-x^2}$. What is the minimum value of f(x).

2. Show that $(u-v)^2 + \left(\sqrt{2-u^2}+6-v\right)^2 \geq 8$

3. Prove that for all real x and y $\sqrt{x^2-3x+3} + \sqrt{y^2-3y+3} + \sqrt{y^2-\sqrt{3}xy+x^2} \geq \sqrt{6}$

4. Prove that $\sin^2(\alpha) + \sin^2(\beta) + \sin^2(\theta) = 2 + 2\cos(\alpha)\cos(\beta)\cos(\theta)$ if $\alpha+\beta+\theta=\pi$. From this we can also get $\cos^2(\alpha)+\cos^2(\beta)+\cos^2(\theta) = 1 - 2\cos(\alpha)\cos(\beta)\cos(\theta)$.

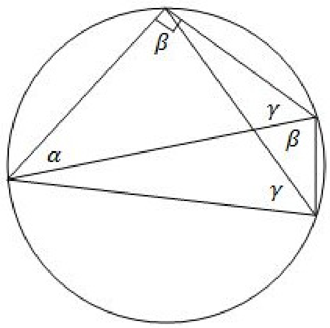
5. Find the value of $\cos^2 10 + \cos^2 50 - \sin 40 \sin 80$.

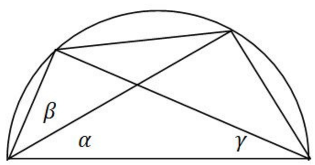
6. Prove that $\cot\frac{30}{2} + \cot\frac{50}{2} + \cot\frac{100}{2} = \frac{(\sin 30+\sin 50+\sin 100)^2}{2\sin 30\sin 50\sin 100}$

7. Prove that $\sin^2(\alpha)+\sin^2(\beta)+\sin^2(\theta)+2\sin(\alpha)\sin(\beta)\sin(\theta) = 1$ if $\alpha+\beta+\theta = \frac{\pi}{2}$.

8. Show that $\sin 20 + \sin 40 = \sin 80$.

Hints:

1. Maybe it looks like…Distance Formula!
2. Look above.
3. Does it look like law of cosines?
4. 

5. What is the relation between the numbers and trigonometry functions? See 3.
6. Notice the relation between the angles. What do the angles represent?
7. 

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