

Extra New England Rounds – Generic Cover Sheet

10 minute rounds

Question n is worth n points

Round 1

Arithmetic and Number Theory

1. Express $0.\overline{108} + 0.\overline{108} + 0.\overline{108}$ as a fraction in lowest terms.

2. Each of the letters in the addition shown represents a distinct digit from 0 through 9. What is the minimum possible value for the sum ECFE?

$$\begin{array}{r} \text{ABA} \\ + \text{ACD} \\ \hline \text{ECFE} \end{array}$$

3. Find the sum of the distinct prime factors of $495 \cdot 509 + 24$.

Round 2

Algebra I

1. Given: $x + 7 = \sqrt{371 + x^2}$.

Find $\sqrt{x + 2}$.

2. Find all a such that $|5a - 8| + |4a + 5| > |6a - 2|$.

3. A group of students went to get dinner at ARML, which cost \$81.32. They only had \$80 worth of paper money, so to pay off the remaining amount, they used pennies, nickels, dimes, and quarters. Given that three-fourths of the coins were pennies and that there were twice as many dimes as quarters, how many nickels were used?

Round 3

Geometry

1. Given triangle ABC and points D, E on $\overline{AC}, \overline{AB}$, respectively.

$$AE:EB = 5:2 \text{ and } AD:DC = 3:4.$$

Compute the ratio of the area of ADE to the area of ABC .

2. Points T, A , and B lie on a circle, and point P lies outside the circle.

\overline{PT} is tangent to the circle, and P, A , and B are collinear with $PA < PB$.

Given $TA:TB = 3:4$ and $PT = 12$, find AB .

3. In rectangle $ABCD$, $AB = 10$.

Point P lies in $ABCD$ with $PA = 11$, $PB = 13$.

The bisector of $\angle DPC$ intersects \overline{CD} at E , and $DE = 4$.

Find PC .

Round 4

Algebra II

1. Solution A contains 20% alcohol and 80% water, while Solution B contains 50% of each.

Solution C is 70% Solution A and 30% Solution B.

What percent of Solution C is water?

2. Solve for x :

$$\log_x(x+3) - \log_{x+2} x = \log_x 3 - \log_2 x$$

3. Solve for x :

$$x(x+2) = \frac{18(x-2)}{x^3-8} + 5$$

Round 5

Analytic Geometry

1. For some value of a , the lines $L_1: 6x - 4y = 8$ and $L_2: ax - 3y = 3y - 2a + k$ are either parallel or the same line. Find the value of k such that they are the same line.

2. Three circles have equations

$$C_1: x^2 + y^2 - 64 = 0$$

$$C_2: x^2 + y^2 - 28x + 132 = 0$$

$$C_3: x^2 + y^2 - 10x - 24y + 105 = 0$$

Line L_1 goes through the intersection points of C_1 and C_3 .

Line L_2 goes through the intersection points of C_2 and C_3 .

Find the intersection point of lines L_1 and L_2 .

3. An ellipse has foci $C_1(2,3)$ and $C_2(8,11)$, and one of its vertices lies on the x -axis.

A line with slope $-3/4$ passes through C_2 and hits the ellipse at points A and B .

Find the length of segment \overline{AB} .

Round 6

Trig and Complex Numbers

1. In an arithmetic sequence of complex numbers, the sum of the first five terms is $35 - 15i$.

The product of the second and fourth terms is $40 - 34i$.

Find the first term of the sequence.

2. Solve for all angles $0 \leq x < 2\pi$: $\cos^2 x \tan x + \sin^2 x = 1$.

3. In triangle ABC , $\angle A = 30^\circ$, $\angle B = 105^\circ$, and $BC = \sqrt{2}$.

Points D and E are such that triangles ACD , ACE are equilateral.

Points F and G are such that triangles BCD , BCE are equilateral.

Compute $DF + DG + EF + EG$.