

1. Given $0^\circ \leq x < 360^\circ$, and $\tan x = 2 \sin x$, find all solutions for x .

Solution: We write $\tan x = \frac{\sin x}{\cos x} = 2 \sin x$, so either $\sin x = 0$ or $\frac{1}{\cos x} = 2$. In the first case, we get the solutions $x = 0^\circ$ and $x = 180^\circ$. In the second case, we get $\cos x = 1/2$, so drawing the unit circle, we get $x = 60^\circ$ and $x = 300^\circ$. These are the four solutions, after checking for division by zero.

2. Given $0^\circ \leq x < 360^\circ$ and $\sin 3x \cos 2x = 1 - \cos 3x \sin 2x$, find the sum of all solutions for x .

Solution: We move all trigonometric terms to the left and recognize the sine addition formula, $\sin(a + b) = \sin a \cos b + \sin b \cos a$, with $a = 3x$ and $b = 2x$, so $\sin 5x = 1$. From the restricted range of x , $0^\circ \leq 5x < 1800^\circ$. Since $\sin 5x = 1$, $5x = 90^\circ, 450^\circ, 810^\circ, 1170^\circ, 1530^\circ$. This gives us the five solutions $x = 18^\circ, 90^\circ, 162^\circ, 234^\circ, 306^\circ$, and their sum is 810.

3. Given $\sin(90^\circ + x) \cos(180^\circ + x) + \sec 300^\circ \cos(270^\circ + x) = \csc 210^\circ \csc(x - 180^\circ)$ and $0^\circ \leq x < 360^\circ$, find all solutions for x .

Solution: We use the unit circle to deduce that $\sin(90^\circ + x) = \cos x$, $\cos(180^\circ + x) = -\cos x$, $\sec 300^\circ = 2$, $\cos(270^\circ + x) = \sin x$, $\csc 210^\circ = -2$, and $\csc(x - 180^\circ) = -1/\sin x$, so the equation becomes $-\cos^2 x + 2 \sin x = 2/\sin x$. Moving the $2 \sin x$ term to the right, we get the equation $-\cos^2 x = 2 \left(\frac{1}{\sin x} - \sin x \right) = 2 \left(\frac{1 - \sin^2 x}{\sin x} \right) = 2 \cos^2 x / \sin x$, so either $\cos^2 x = 0$ or $-1 = 2/\sin x$. In the first case, using the unit circle, we get the solutions $x = 90^\circ$ and $x = 270^\circ$. In the second case, we get $\sin x = -2$, which is impossible since $|\sin x| \leq 1$. The two solutions in the first case are thus the only two solutions, after checking for division by zero.