PUMAC/HMMT Problem Set

LHS Math Team

September 19, 2014

1 "Can you go" Questions

Please answer the below questions even if you answered them last week. If you are not sure you can go, write "maybe" and get back to us ASAP.

Some people wanted more information. Here it is:

The PuMaC tournament is on November 22, 2014 (Saturday), at Princeton. The tournament lasts all day. We will drive up the Friday night before, and probably come back that Saturday night (maybe the next morning). It has a similar format to LMT, and is approximately the difficulty level of HMMT February (see below).

The HMMT November tournament is on November 15, 2014 (Saturday), at Harvard. The tournament lasts most of the day, and has a similar format to LMT.

The HMMT February tournament is on February 21, 2015 (Saturday), at MIT. The tournament lasts most of the day, and has a similar format to LMT. The questions are significantly more difficult than at November tournament.

The MML Meet 1 is on October 2, 2014 (Thursday), at Weston. It is directly after school. You take a bus there, and come back on the bus, at aboud 6:30pm. The format is similar to IMLEM.

Here are the questions. **Put the answers on the answer sheet!**

1. Can you go to PuMaC?
2. Can you go to HMMT November?
3. Can you go to HMMT February?
4. Can you go to MML Meet 1?

Note: you can change your answers to the above questions if you email us, just please do it ASAP.

2 Math Questions for PuMaC/HMMT

Some of the below questions are taken from PuMac 2008, HMMT 2000, and HMMT 2002.

1. (1 point) What is the radius of a circle of area 16?
2. (2 points) Consider a convex polygon P in space with perimeter 20 and rea 30. What is the volume of the locus of points that are at most 1 unit away from some point in the interior of P?
3. (3 points) Evaluate $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$.
4. (3 points) Find the sum of the even positive divisors of 1000.
5. (3 points) Find all integral solutions to $x^y - y^x = 1$.
6. (4 points) What is the polynomial of smallest degree that passes through (-2,2), (-1,1), (0,2), (1,-1), and (2, 10)?
7. (5 points) Equilateral triangle ABC of side length 2 is drawn. Three squares containing the triangle, $ABDE, BCFG, CAHI$, are drawn. What is the area of the smallest triangle that contains these squares?
8. (7 points) If $p(x)$ is a polynomial with integer coefficients, let $q(x) = \frac{p(x)}{x(1-x)}$. If $q(x) = q\left(\frac{1}{1-x}\right)$ for every $x \neq 0$, and $p(2) = -7, p(3) = -11$, find $p(10)$.
9. (0 points) (For calculus nerds.) A continuous real function f satisfies the property $f(2x) = 3f(x)$ for all x. If $\int_0^1 f(x)dx = 1$, what is $\int_1^2 f(x)dx$?