Problem Set 2 - Full
Lexington Math Team

Monday, October 1, 2012

1. Let ω be the circle determined by the equation $x^2 - 2x + y^2 - 4x = 4$. Point A has coordinates $(-2, -2)$ and a line that is tangent to ω is drawn through A. What is the length of the line segment from A to the point of tangency?

2. A ladder with n steps connects the Earth to the Moon. On each step of the ladder, there is an arrow pointing up or down. When Andre moves along, he follows the direction of the arrow of the step he is on. After he has passed the step, the direction of the arrow reverses. Is it possible that Andre is stuck on the ladder forever?

3. Suppose that n divides $11\ldots1$ (where there are n 1's). Prove that 3 must divide n.

4. The positive real numbers $x_0, x_1, \ldots, x_{2013}$ satisfy $x_0 = x_{2013}$ and

$$x_{i-1} + \frac{3}{x_{i-1}} = 3x_i + \frac{1}{x_i}$$

   for $i = 1, 2, \ldots, 2013$. Find the maximum value that x_0 can have.

5. Let ABC be a scalene triangle and let Ω be the circle through A, B, and C. Let the center of the inscribed circle to triangle ABC be I and let points D, E, and F be the points of tangency of this circle to the sides of the triangle opposite A, B, and C, respectively. The circle through points A, E, and F intersects Ω at A and K. Let $\overline{KI}$ and $\overline{EF}$ intersect at P. Prove that $\angle DPE$ is right.