Spinor Algebra

DR basis

$$\gamma_0 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \qquad \gamma_1 = \begin{pmatrix} 0 & 0 & 0 & i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}$$

$\mathbb{1}$ $\qquad\qquad\qquad\qquad$ σ_1

$$\gamma_2 = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} \qquad \gamma_3 = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}$$

σ_2 $\qquad\qquad\qquad\qquad$ σ_3

Helicity spinors (z-direction boost, y-rotate)

$$u_+(p) = \begin{pmatrix} \sqrt{E-p}\begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix} \\ \sqrt{E+p}\begin{pmatrix} \cos\theta/2 \\ \sin\theta/2 \end{pmatrix} \end{pmatrix} \qquad u_-(p) = \begin{pmatrix} \sqrt{E+p}\begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix} \\ \sqrt{E-p}\begin{pmatrix} -\sin\theta/2 \\ \cos\theta/2 \end{pmatrix} \end{pmatrix}$$

when p along $+z$ axis

$$u_+(p) = \begin{pmatrix} \sqrt{E-p}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \sqrt{E+p}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} \qquad u_-(p) = \begin{pmatrix} \sqrt{E+p}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \sqrt{E-p}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix}$$

$$\langle N(p_f) | V_\mu(q^2) | N(p_i) \rangle$$

$$= \bar{u}(\vec{p}_f)\left[F_1(q^2)\gamma_\mu + i\sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2M} \right] u(\vec{p}_i)$$

$\boxed{V_4}$

$\mu = 4 \quad + \to + \qquad \sigma_{\mu\nu} = \frac{i}{2}\left[\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu\right]$

$$\left(\sqrt{E-p_f}\ \ 0\ \ \sqrt{E+p_f}\ \ 0\right) \gamma_4 \left[F_1 \gamma_4 + i\sigma_{43}\, q_3 \frac{F_2}{2M}\right] \begin{pmatrix} \sqrt{E-p_i} \\ 0 \\ \sqrt{E+p_i} \\ 0 \end{pmatrix}$$

set $p_f \equiv 0$ (fin state rest frame)

$$\left(\sqrt{M}\ \ 0\ \ \sqrt{M}\ \ 0\right) \gamma_4 \left[F_1 \gamma_4 - i\sigma_{43}\, q_3 \frac{F_2}{2M}\right] \begin{pmatrix} \sqrt{E-q} \\ 0 \\ \sqrt{E+q} \\ 0 \end{pmatrix}$$

$$\gamma_4 \gamma_4 = \mathbb{1}$$

$$-i\gamma_4 \sigma_{43} = \frac{1}{2}\gamma_4\left(\gamma_4\gamma_3 - \gamma_3\gamma_4\right)$$

$$= \frac{1}{2}\left(\gamma_3 - \gamma_3^\dagger\right) = 0$$

$$\Rightarrow F_1(q^2)\sqrt{M}\left(\sqrt{E-q} + \sqrt{E+q}\right)$$

$$\boxed{\frac{\partial F_1}{\partial q^2}\left(\ \ \right)\Big|_{q^2=0}} + F_1\sqrt{M}\left(\frac{1}{2}\frac{1}{\sqrt{E-q}}\left(-\frac{\partial q}{\partial q^2}\right) + \frac{1}{2}\frac{1}{\sqrt{E-q}}\left(\frac{\partial q}{\partial q^2}\right)\right)\Big|_{q=0}$$

$$= 2M \frac{dF_1}{dq^2}$$

negative helicity follows & same.

V_1 & V_2 gets qF_2 ×

$\boxed{V_3}$ $\left(\sqrt{M}\ \ 0\ \ \sqrt{M}\ \ 0\right) \gamma_4 F_1 \gamma_3 \begin{pmatrix} \sqrt{E-q} \\ 0 \\ \sqrt{E+q} \\ 0 \end{pmatrix} = i F_1(q^2)\sqrt{M} \times \left(-\sqrt{E-q} + \sqrt{E+q}\right)$

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \\ -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

$\Rightarrow$ diverges