

Instruction: Read and follow the Submission Policy carefully. You must show all your work clearly for credit. Partial credit will only be given to meaningful answers. You will be graded according to your approach to the problems, mathematical rigor, and quality of your solutions.

1. (15) In implementing an ADT S, an insert operation $\text{insert}(x, S)$ requires 0.5ms (10^{-3}s) to execute for $|S| = 1,000$. If the complexity of the insert operation is given by the following closed-form expressions, compute the time required to execute the operation when $|S| = 1,000,000$.
 - (a) $T(n) = 2\lg n$.
 - (b) $T(n) = n\lg n$.
 - (c) $T(n) = n^3$.
 - (d) $T(n) = n^2\lg n$.
 - (e) $T(n) = 4^n$.
2. (10) Prove the following statement using the definition of big-O and big- Ω :
$$\frac{n^4 - 168n^3 + 18n^2 - 15n}{18n^2 - 2n + 1024} = O(n^2) \text{ and } \frac{n^4 - 168n^3 + 18n^2 - 15n}{18n^2 - 2n + 1024} = \Omega(n^2).$$
3. (10) Prove or disprove the following statements:
 - (a) $3^n = O(2^{2n})$.
 - (b) $\lg^k n = O(n), k \in N = \text{set of all positive integers}$.
4. (10) Given an array $A = [a_1, \dots, a_n]$ and a key x . Assuming that $\Pr(x = a_i) = \frac{1}{2n+1}, \forall i$. Based on the number of comparisons between x and $A[i]$'s, compute $T_a(n)$ in closed-form if sequential search algorithm is used for searching for x in A (starting at $A[1]$).
5. (10) Do Problem 2.1 on Page 64.
6. (20) Prove or disprove the statements given in Problem 2.2 on Page 64. You must justify your answer for credit.
7. (10) Do Problem 2.12 on Page 66.
8. (15) Do Problem 2.14 on Page 66.