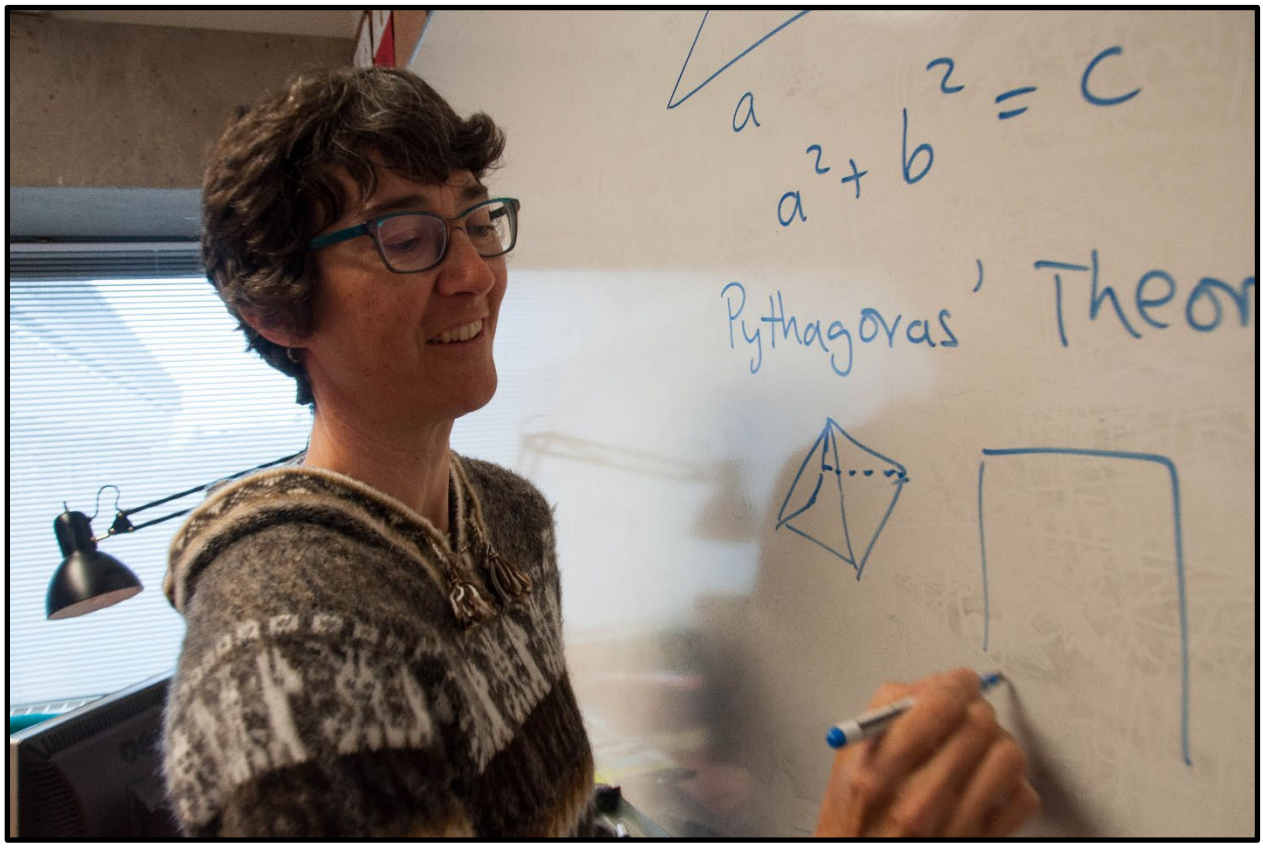
Teaching Portfolio

Petra Menz

Last Update: May 15, 2025

Welcome to my teaching portfolio! My name is Petra Menz, and my portfolio is a representation of what I have learned and accomplished as a teaching faculty member in the Department of Mathematics at Simon Fraser University. You will find that when I set out to develop material or am interested in a certain pedagogy, I dig deep and am engaged over a long period of time to gain expertise. Such is the case with two courses that are dear to me and are showcased in this portfolio: the departmental math-for-teachers course as well as the differential and integral calculus courses for social science students.

The portfolio is best navigated through the bookmarks. Take note that most of the material listed in the Table of Contents is contained within this portfolio except for a handful of publications which are hyperlinked, in particular the two open education resources that I published as PreTeXts.



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CV

Dr. Petra Menz / Senior Lecturer / Mathematics

Educational Background

2021		Professional Certificate Foundations of Intercultural Development 6-week Course in Equity, Diversity, and Inclusion hosted by Work Integrated Learning, Simon Fraser University
2015	Ph.D.	Joint degree in Mathematics and Mathematics Education, Simon Fraser University, Canada Unfolding of Diagramming and Gesturing between Mathematics Graduate Student and Supervisor during Research Meetings, Supervisors: Drs. Nathalie Sinclair (Faculty of Education, Mathematics Education) and Jonathan Jedwab (Department of Mathematics)
2001		Teaching Certificate New Brunswick, Canada
1995		Professional Certificate Secondary School, British Columbia College of Teachers, Canada
1995	B.Ed.	Mathematics and Computer Science, University of British Columbia, Canada
1994	M.Sc.	Mathematics, University of British Columbia, Canada An algorithm for computing the Riemann zeta function based on an analysis of Backlund's remainder estimate. Supervisor, Bill Casselman.
1992	B.Sc.	Mathematics and Computer Science, Scarborough College, University of Toronto, Canada

Other Professional Training

November 2021 - December 2021	6-wk online course for faculty: Foundations of Intercultural Development, with the learning goals of reconciliation, Indigenization, decolonization; addressing power, privilege and microaggression; and increasing EDI and intercultural self-awareness.
May 2021 - May 2021	Completed Respectful Working & Learning Environments training module to learn about the Bullying & Harassment Policy (GP-47).

Employment History at Academic Institutions

September 2010 - Current	Senior Lecturer, Department of Mathematics, Simon Fraser University
September 2004 - August 2010	Lecturer, Department of Mathematics, Simon Fraser University
August 2001 - June 2003	Lecturer (tenure-track), Department of Mathematics and Statistics, University of New Brunswick

Other Employment History

September 1995 - June 2001	Secondary Teacher and Math Department Head, Richmond School District Mathematics, Information Technology and Computer Science
September 1998 - June 2000	Writer and Developer of Math Material, Ministry of Education, BC Applied Mathematics and Integrated Resource Package Teams

Courses Taught at SFU

FAN X99 *Foundations of Analytical and Quantitative Reasoning*

MATH 100 *Precalculus*

MATH 150 *Calculus I with Review*

MATH 151 *Calculus I*

MATH 152 *Calculus II*

MATH 160 *Mathematics in Action*

MATH 157 *Calculus I for the Social Sciences*

MATH 158 *Calculus II for the Social Sciences*

MATH 190 *Principles of Mathematics for Teachers*

MATH 251 *Calculus III*

Coordination of Workshops (Math Help Centres) at SFU

Calculus Workshop	Services MATH 150, MATH 151, MATH 152, MATH 251
Applied Calculus Workshop	Services MATH 154, MATH 155, MATH 157, MATH 158
Quantitative Workshop	Services FAN X99, MATH 190

Courses Taught at UNB

MATH 1003 *Introduction to Calculus I*

MATH 1013 *Introduction to Calculus II*

MATH 1833 *Finite Mathematics for Management Sciences*

MATH 2503 *Calculus and Linear Algebra for Engineers I*

MATH 2513 *Calculus and Linear Algebra for Engineers II*

MATH 2633 *Fundamental Principles of Elementary School Mathematics*

Supervision of Research Assistants

September 2022 - November 2022	Part Time, Sheena Tan, Ph.D, Creator of online assignments, and marking scheme and Crowdmark comments for group assignments in MATH 190 **Funded by:** $1020 Online Course Improvement Funds, Department of Mathematics
January 2022 - April 2022	Part Time, Kailyn Pritchard, M.Sc, Proof-reader for restructured Canvas material and testing OA quizzes as OAs are transitioned from LON-CAPA to Canvas in MATH 190. **Funded by:** $750 Online Course Improvement Funds, Department of Mathematics
January 2022 - April 2022	Part Time, Sheena Tan, Ph.D, Concept-distiller from lightboard mini lectures, designer of online assignments as Canvas quizzes from Canvas Question Bank, and creator of questions for online assignments not yet transitioned from LON-CAPA to Canvas Question Bank in MATH 190. **Funded by:** $1740 Online Course Improvement Funds, Department of Mathematics
January 2021 - February 2021	Part Time, Hannah Sutton, M.Sc, Canvas online assignment question coder in MATH 190. **Funded by:** $2325 Online Course Improvement Funds, Department of Mathematics

August 2020 - August 2020	Part Time, Hannah Sutton, M.Sc, Canvas exam question coder in MATH 190. **Funded by:** Online Course Improvement Funds ($581), Department of Mathematics
August 2020 - August 2020	Part Time, Nicola Mulberry, Ph.D, PreTeXt-editor **Funded by:** $1500 from FIC Faculty Fund, Department of Mathematics.
September 2019 - August 2020	Part Time, Nicole Mulberry, M.Sc, Latex-editor, graphics productionist, and video-editor for MATH 157/158 comprehensive material development **Funded by:** $6480 from FIC Teaching & Learning Fund, Department of Mathematics.
January 2019 - August 2019	Part Time, Alamgir Hossain, M.Sc, Latex-editor and question-designer for MACM 316 clicker questions development **Funded by:** $5000 SFU OER Grant and $2050 FIC Teaching & Learning Fund, Department of Mathematics.
May 2018 - April 2019	Part Time, Nicola Mulberry, M.Sc, Latex-editor and graphics productionist for MATH 157/158 comprehensive material development **Funded by:** $5000 SFU OER Grant #2 and $3000 FIC Teaching & Learning Fund, Department of Mathematics.
May 2017 - April 2018	Part Time, Nicola Mulberry, B.Sc, Latex-editor and graphics production for MATH 157/158 comprehensive material development **Funded by:** $5000 SFU OER Grant #1 and $5000 FIC Teaching & Learning Fund, Department of Mathematics.

Course Preparation and Enhancement

2025	Updated FAN X99 *Foundations of Analytical and Quantitative Reasoning* lecture notes and student skeleton notes with visual aids, hands-on material, worksheets, and group activities to support scaffolding of concepts.
2023/24	Reconstructed all lecture material and assessments for section A320 of FAN X99 *Foundations of Analytical and Quantitative Reasoning* offered to students of the Cowichan tribe on Vancouver Island to incorporate Indigenous ways of learning. Although I was not the instructor, I mentored the instructor and attended Zoom lectures to cultivate relationships with the students and coach them through the course.
2022	Co-developed with Dr. Joanna Niezen three complete sets of group assignments with marking schemes and corresponding Crowdmark Comment Library for the blended and online versions of MATH 190 *Principles of Mathematics for Teachers*. Migrated the online assignments from LON-CAPA to Canvas Quizzes and restructured the Canvas container for the blended and online versions of MATH 190 *Principles of Mathematics for Teachers.* I led this major effort involving a team of stakeholders (undergraduate students, graduate students, instructors and technician).
2021	Completed construction of different Canvas Question Banks for each of the three semester online offerings of MATH 190 *Principles of Mathematics for Teachers*. All three midterm examinations are now hosted as online Canvas quizzes based on randomization in both the blended and online versions of the course.
2020	Created a 7-minute video by filming advice from TA Hannah Sutton given to MATH 190 *Principles of Mathematics for Teachers* students about how to approach studying for this course and making use of the associated Q Workshop. This video is now permanently installed in the Canvas container and one of the most popular resources.

Created Canvas Question Banks for the summer and fall online offering of MATH 190 *Principles of Mathematics for Teachers*. This was a massive undertaking with proof-reading done by Coordinator of Workshop Operations Adam Dyck.

Proofread the PreTeXt version of an extensively updated booklet "A Collection of Problems in Differential Calculus" by Veselin Jungić and four of his students. The original booklet was created in 2008 by Drs. Jungić, Menz and Pyke as online problems using the LON-CAPA system.

2019 Embarked on a string of workshops, talks and discussions hosted by SFU and other educational organizations to learn more about how I can respectfully and authentically achieve Indigenization and decolonization. Enriched MATH 190 *Principles of Mathematics for Teachers* with (1) discussion on Indigenous perspectives, knowledge and principles of learning based on relevant news articles; (2) activities such as Métis weaving, Coast Salish artwork and drum making; (3) bi-weekly Ojibwe poetry readings, and (4) a term-end talking stick ceremony as entries to culturally diverse mathematics.

2019 Filmed the remaining 20 lightboard videos ranging in length from 8 minutes to 14 minutes through CODE support to add to the 39 existing lightboard videos for MATH 190 *Principles of Mathematics for Teachers*.

2018/19 Developed Open Educational Resources for MATH 158 *Calculus II for the Social Sciences* through an SFU OER Grant and guided by educational consultant Dr. Cindy Xin that include the adaptation of a calculus text incorporating interactive examples and videos, creation of accompanying tablet lecture slides, skeleton student notes and construction of 10 assignments. This OER has also been adopted by post-secondary institutions in the lower mainland such as Douglas College and Alexander College.

Co-developed with Dr. Marie Loughin (Statistics and Actuarial Science) and Computer Systems Consultant Dave Carmean two online forms for the Department of Mathematics (later adopted by other science departments): (1) Grade Appeal form for students and (2) TA Availability form for workshop coordinators to efficiently schedule TAs.

2017 Updated MATH 190 *Principles of Mathematics for Teachers* course author notes and individual textbook assignments due to the publication of the vastly different 3rd edition of the course textbook "Reconceptualizing Mathematics for Elementary School Teachers" by Sowder, Sowder and Nickerson.

Filmed 39 lightboard videos ranging in length from 8 minutes to 14 minutes through CODE support for the online version of MATH 190 *Principles of Mathematics for Teachers*. Due to their success, these videos were adapted in the face-to-face offering of the course to turn it into a blended course.

Negotiated a pilot-testing of an e-learning platform in MATH 154/155 *Calculus I/II for the Biological Sciences* that was subsequentially adopted. Created sets of online assignments together with teaching colleague Dr. Natalia Kouzniak.

Developed Open Educational Resources for MATH 157 *Calculus I for the Social Sciences* through an SFU OER Grant and guided by educational consultant Dr. Cindy Xin that include the adaptation of a calculus text incorporating interactive examples and videos, creation of accompanying tablet lecture slides, skeleton student notes and construction of 10 assignments. This OER has also been adopted by post-secondary institutions in the lower mainland such as Douglas College and Alexander College.

2016 Formalized student-instructor meetings for MATH 190 *Principles of Mathematics for Teachers* and MATH 157 *Calculus I for the Social Sciences* because student feedback has been overwhelmingly positive claiming that this makes them try and study harder because I demonstrate a care for their learning.

Overhauled teaching and learning resources for MATH 190 *Principles of Mathematics for Teachers* and redesigned the associated Canvas container in consultation with Dr. Malgorzata Dubiel based on a student survey analysis. The surveys were constructed with the help of education consultant Dr. Cindy Xin, held for five consecutive semesters, and the analysis published in a 31-page document with Dr. Xin.

Co-piloted with Dr. Brenda Davison the e-learning platform WebAssign by Cengage to host weekly online assignments for in MATH 150 *Calculus I with Review* and MATH 151 *Calculus I*. The outcome was the adoption of WebAssign for MATH 150 *Calculus I with Review*, MATH 151 *Calculus I*, MATH 152 *Calculus II*, and MATH 251 *Calculus III*.

Proofread and updated course note drafts written by Drs. Matt Devos and Sophie Burrill for MATH 154/155 *Calculus I/II for the Biological Sciences*.

2015 Investigated the impact of weekly reflections in MATH 190 *Principles of Mathematics for Teachers* on students through an SFU Teaching and Learning Development Grant with Educational Consultant Dr. Cindy Xin and RA Jing Li from Education. The results revealed that indeed this reflective activity improves students' meta-cognition and initiates a positive change in attitude toward mathematics. The study was presented at the Society for Teaching and Learning in Higher Education (STLHE) Conference June 2015, the 13th International Conference of The Mathematics Education for the Future Project September 2015 and the International Congress on Mathematical Education (ICME) July 2016.

2014 Designed 15 group activities for MATH 190 *Principles of Mathematics for Teachers*. To facilitate these activities in the online offering of the course, 30-minute videos were produced by Nathan Douglas at CODE under the Program Director Dr. Kanthi Jayasundera, in which I introduce the activity and the learning objects followed by a student group as they live work through the activities.

Formalized the collection of graduate student exam dates for distribution among faculty to coordinate exam marking in workshops. This scheme became popular and is now managed by the Undergraduate Program Assistant.

Developed weekly reflections in MATH 190 *Principles of Mathematics for Teachers* in the hopes to enhance students' meta-cognition around mathematical learning and thereby orient them more positively towards mathematics.

2013 Redesigned from scratch the online version of MATH 190 *Principles of Mathematics for Teachers* with Dr. Malgorzata Dubiel as the faculty adviser and CODE Program Director Dr. Kanthi Jayasundera to implement the course on Canvas.

2011 Overhauled online assignments using LON-CAPA for MATH 190 *Principles of Mathematics for Teachers* to introduce randomization and hint features, correct syntactical and grammatical errors, enhance readability, and ensure that the problems are varied in levels and types.

Initiated the training of graduate students as workshop coordinators to free up lecturers from this role.

Wrote a three-page departmental guideline for Teaching Assistants regarding exam marking in workshops and tutorials.

2010 Created additional questions for the existing online assignments using LON-CAPA for MATH 190 *Principles of Mathematics for Teachers*.

Wrote the paper version of the online booklet *A Collection of Problems in Differential Calculus* with Drs. Veselin Jungić and Randall Pyke and as a resource for undergraduate students through the Math Student Union.

Wrote a five-page introduction *Recommendations for Success in Mathematics* to the paper booklet *A Collection of Problems in Differential Calculus* that later became a stand-alone article.

Edited (based on errata list) Tan, Menz, Ashlock's *Applied Calculus for the Managerial, Life, and Social Sciences*, First Canadian Edition, Thomson Nelson Publishers for reprinting of first edition.

2009 Revised the curriculum for MATH 154/155 *Calculus I/II for the Biological Sciences* together with Drs. Cedric Chauve and Natalia Kouzniak and input from the Department of Biological Sciences.

2008 Collaborated with Dr. Veselin Jungić in creating the online booklet *A Collection of Problems in Differential Calculus* using LON-CAPA.

2007 Participated in weekly 2-hour FAN X99 *Foundations of Analytical and Quantitative Reasoning* instructor meetings to collaborate on teaching and learning matters.

Proofread and edited Tan, Menz, Ashlock's *Applied Calculus for the Managerial, Life, and Social Sciences*, First Canadian Edition, Thomson Nelson Publishers published in the spring 2008.

2006 Co-authored chapters for Tan's 2005 *Applied Calculus for the Managerial, Life, and Social Sciences* for Thomson Nelson Publishers to be published as the first Canadian edition.

Collaborated with Dr. Veselin Jungić in creating online assignments using LON-CAPA for MATH 151 *Calculus I*.

Created online assignments using LON-CAPA for MATH 157 *Calculus I for the Social Science*s.

Piloted clickers MATH 157 *Calculus I for the Social Science* to investigate their potential use in all calculus courses.

Participated in weekly 2-hour FAN X99 *Foundations of Analytical and Quantitative Reasoning* instructor meetings to collaborate on teaching and learning matters.

2005 Developed the new course MATH160 *Mathematics in Action* as a combined Writing, Quantitative and Breadth course to investigate mathematical modeling and report writing.

2002 Developed the course MATH 2633 *Fundamental Principles of Elementary School Mathematics*, Education Faculty, UNB. This course is the equivalent of MATH 190 *Principles of Mathematics for Teachers* at SFU.

Teaching and Other Professional Development

2005-25 Participated semesterly at a variety of workshops, presentations and symposiums hosted by the **SFU body for teaching and learning** that supports instructors in developing innovative and inclusive teaching practices (2005-09, LIDC; 2010-18 TLC; 2019-present CEE), most notably "Explore Accessible Teaching" in 2025; "SoTL 101: Introduction to Scholarship of Teaching and Learning and Teaching + Learning Inquiry" in 2022; "Healing the Wound with the Weapon: University Instruction, Reconciliation and Healing" presented by Kevin Lamoureux in 2021; six workshops on online tools and approaches to cope with the COVID-induced switch to online teaching in 2020; "Indigenizing Course Outcomes: Updating your Syllabus to Use the Medicine Wheel as a Curriculum Design Framework" by Drs. Marcella LaFever and Justin Wilson in 2019; "Art & Design in Science Communication" hosted by 4D LABS Seminar in 2018; "The Art of Teaching: Best Practices from a Master Educator" by Dr. Patrick N. Allitt in 2012 and 2011; "The Engaged Learner: Stimulating Autonomy, Ensuring Support" by Dr. Alan Wright in 2006; and "Acting and Improv for SFU Instructors" by Jesai Jayhmes in 2005.

2005-25 Participated annually at workshops and presentations hosted by **other SFU bodies** such as Centre for Online and Distance Education; Sexual Violence Support and Prevention Office; the Research Hub of the Faculty of Education; Equity, Diversity and Inclusion ("With the new developments in Indigenous Education and Languages, what is the role of universities?" by Dr. Lorna Williams, former CRC Chair in Indigenous Knowledge and Learning in 2019); Centre for Imagination in Research, Culture & Education ("The Vital Role of Indigenous Imagination in Reconciliation" presented by Dr. Vicki Kelly in 2019) ; Centre for English Language Learning, Teaching and Research ("Teaching in a multilingual university" by Dr. Saskia Stille in 2016); and EdMedia (Expo in 2014).

2005-25 Participated annually at a variety of workshops, presentations and seminars hosted by the **SFU Department of Mathematics** or **Faculty of Science**, most notably "Special Faculty Session on Service" led by Dr. Aftab Erfan and Joel Harnest from the SFU Centre for Dialogue in 2025; "Recognizing and Interrupting Unconscious Bias in Academia: Practical tips for more inclusive cultures" by Dr. Maydianne Andrade; "Attending to Diversity, Equity, and Inclusion within Introductory College Mathematics Programs" presented by Dr. Jess Ellis Hagman in 2020; and "On the Horizon: Four Issues Impacting Teaching Faculty in the Next Five Years" by Dr. Russell Day in 2016.

2020-25 Participated bi-monthly at **FYMSiC** (First-Year Math and Stats in Canada) **Online Teaching Meetups** to discuss and learn about teaching and learning matters at the post-secondary level.

2014-23 Participated monthly at **SFU Teaching Matters Seminar**, which is an interdisciplinary series of panels, seminars and collaborative meetings on university teaching & learning.

2007-22 Participated **based on the theme of well-being** at a variety of workshops, presentations and events, most notably departmental EDI Learning Group (monthly discussions in 2022); Thriving Beyond Campuses - A Dialogue Series Connecting B.C. Post-Secondary ("The Impact of Mental Bandwidth Depletion on Student Mental Health and Well-Being" by Dr. Cia Verschelden in 2020);Pre-Symposium | Workshop for SFU Academic Leaders ("Using Mentorship and Coaching as a Lens to Support Academic Growth in Teaching and Learning" in 2019); SFU Health Promotion ("What role should faculty play in supporting student mental health?" in 2018), "Building Connections: Well-being & Teaching" in 2016); SFU LIDC ("Teaching Large Classes: Beyond Survival" by Dr. Russel Day in 2009); and Student Services ("Question, Persuade and Refer – Suicide Prevention Session" in 2007).

2016-21 Participated annually at workshops and presentations hosted by **other educational bodies** such as the ALM Virtual Seminar ("Numeracy vs. Numberacy" by Dr. Peter Liljedahl in 2021); Research in Undergraduate Mathematics Education ("Inquiry-Based Learning Workshop on Courses for Future Elementary Teachers" by Drs. Danielle Champney and Todd Grundmeier in 2020); Embodied Mathematical Imagination & Cognition ("Instructional Gestures for Classrooms and On-Line Mathematics Learning" by Drs. Mitchell J. Nathan, Martha W. Alibali and Rebecca Boncoddo in 2020); Cheakamus Centre, Squamish ("Place-Based Inquiry: Promising Practices & Inspiring Possibilities" by Squamish Nation members Alice Tsawaysia Spukwus Guss and Matthew Williams Siýám Ken in 2019); and Canadian Association of Research Libraries ("CARL Webinar: How to Create Inclusive and Accessible OER" by Josie Gray in 2017).

2006-18 Participated bi-annually at the **Canadian Mathematics Education Study Group** (CMESG) **annual meeting** in a variety of Working Groups, most notably "Confronting Colonialism in Mathematics and Mathematics Education. (K-16)" lead by Lisa Lunney Borden and Gale Russell in 2018; and "Mathematics in teacher education: What, how... and why" lead by Frédéric Gourdeau and Kathy Nolan in 2016.

2005-7 Participated monthly with the **SFU Rethinking Teaching Group** led by Dr. Cheryl Amundsen on teaching related matters.

Publications

Textbooks

Jungić, V., Menz, P., & Pyke, R. (December, 2023). Differential Calculus: Problems and Solutions from Fundamentals to Nuances. World Scientific Publishing Co Pte Ltd., Singapore. https://doi.org/10.1142/13324

Tan, Soo Tang, Petra Menz, and Dan Ashlock. Applied Calculus for the Managerial, Life, and Social Sciences. First Canadian Edition. Toronto: Nelson Education, 2009.

Open Educational Resources

Menz, Petra, and Nicola Mulberry. (July, 2020). Calculus Early Transcendentals Integral & Multi-Variable Calculus for Social Sciences, Open Educational Resources in PreTeXt, Department of Mathematics website: https://www.sfu.ca/math-coursenotes/Math%20158%20Course%20Notes/frontmatter-1.html

Menz, Petra, and Nicola Mulberry. (June, 2020). Calculus Early Transcendentals Differential & Multi-Variable Calculus for Social Sciences, Open Educational Resources in PreTeXt, Department of Mathematics website: https://www.sfu.ca/math-coursenotes/Math%20157%20Course%20Notes/frontmatter-1.html

Jungić, V., Menz, P., & Pyke, R. (2020). PreTeXt typeset: A Collection of Problems in Differential Calculus, Problems from Calculus I Final Examinations. Department of Mathematics, Simon Fraser University. Retrieved from https://www.sfu.ca/~vJungić/Zbornik2/book-1.html

Menz, Petra, and Nicola Mulberry. (May, 2018). Calculus Early Transcendentals Integral & Multi-Variable Calculus for Social Sciences, Open Educational Resources in LaTeX, SFU Summit.

Menz, Petra, and Nicola Mulberry. (November, 2017). Calculus Early Transcendentals Differential & Multi-Variable Calculus for Social Sciences, Open Educational Resources in LaTeX, SFU Summit.

Jungić, V., Menz, P., & Pyke, R. (2008). LaTeX typeset: A Collection of Problems in Differential Calculus, Problems from Calculus I Final Examinations. Department of Mathematics, Simon Fraser University.

Journal Papers

Hossain, M., Menz, P., & Stockie, J. (2022). An open-access clicker question bank for numerical analysis. PRIMUS: Problems, Resources, and Issues in Mathematics Undergraduate Studies, 32(8):858-880. DOI:10.1080/10511970.2021.1954113

Menz, P., & Xin, C. (2016). Making Students' Metacognitive Knowledge Visible through Reflective Writing in a Mathematics-for-Teachers Course. Collected Essays on Learning and Teaching, 9, 155-166. http://dx.doi.org/10.22329/celt.v9i0.4426

Menz, P. & Jungić, V. (2015). A University Math Help Centre as a Support Framework for Students, the Instructor, the Course, and the Department. Journal of University Teaching & Learning Practice (JUTLP), 13(1)

Jungić, V., Kent, D., & Menz, P. (2012). On Online Assignments in a Calculus Class. Journal of University Teaching & Learning Practice, 9(1), 1-15.

Jungić, V., Kent, D., Menz, P. (2006). Teaching Large Math Classes: Three Instructors - One Experience, International Electronic Journal of Mathematics Education, 1(1), 1-15.

Conference Proceedings

Menz, P. (2019). Open Source Differential and Integral Calculus Material Development to Support Student Accessibility and Learning, Proceedings of the 15th International Conference of The Mathematics Education for the Future Project, August 4-9, Maynooth University, Kildare, Ireland.

Menz, P., & Xin, C. (2016). Making Students' Metacognitive Knowledge Visible Through Reflective Writing in a Mathematics-For-Teachers Course. Proceedings of the 13-th International Congress on Mathematical Education, July 24-31, Hamburg, Germany.

Menz, P., & Sinclair, N. (2016). Diagramming and Gesturing During Mathematizing. Proceedings of the 13-th International Congress on Mathematical Education, July 24-31, Hamburg, Germany.

Menz, P. (2016). Diagramming and gesturing between mathematics graduate student and expert mathematicians. Proceedings of the Canadian Mathematics Education Study Group 40-th Annual Meeting, June 3-7, Kingston, Canada.

Menz, P., Xin, C. & Li, J. (2015). Design of Online Metacognitive Activity in a Post-Secondary Mathematics-for-Teachers Course, Proceedings of the 13th International Conference of The Mathematics Education for the Future Project, September 16-21, Sicily, Italy.

Menz, P., Xin, C. & Li, J. (2015). Using Reflective Activity to Improve Student Metacognition and Attitudes in Post-Secondary Education, Unpublished conference proceedings of the Society for Teaching and Learning in Higher Education Conference - Achieving Harmony: Tuning into Practice, June 16-19, Vancouver, Canada.

Menz, P. (2014). Diagramming Mobilizes Mathematics, Unpublished conference proceedings of the MEDS-Conference 2014, Education Faculty, SFU.

Jungić, V., Kent, D., & Menz, P. (2009). Creating and Utilizing Online Assignments in a Calculus Class. Proceedings of the 7th International Conference of The Mathematics Education for the Future Project (pp. 287-290), September 11-17, Dresden, Germany.

Jungić, V., Kent, D., & Menz, P. (2009). Creating and Utilizing Online Assignments in a Calculus Class. Proceedings of the Mathematics Education Into the 21st Century Project, 287-290.

Monograph

Menz, P. & Sinclair, N. (2017). Diagramming and gesturing during mathematizing: Kinesthetic and haptic interactions support mathematical ideation. In N. Presmeg, L. Radford, W.-M. Roth, G. Kadunz & L. Puig (Eds.), Monograph: Signs of Signification (pp. 315-334). Springer, Cham.

Theses

Menz, P. (2015). Unfolding of Diagramming and Gesturing between Mathematics Graduate Student and Supervisor during Research Meetings. (Doctoral dissertation, Simon Fraser University).

Menz, P. (1994). An algorithm for computing the Riemann zeta function based on an analysis of Backlund's remainder estimate (Masters dissertation, The University Of British Columbia).

Publications In Progress

Menz, P. & Niezen, J. (2025). Equitable Partners in Designing the Learning Environment of an Online Course, Proceedings of the 21st International Study Association on Teachers and Teaching Biennial Conference, June 30-July 4, Glasgow, Scotland.

Conferences and Workshops

Invited Speaker/Panelist

May 2024	PIMS Sharing the Math – Indigenizing the Delivery, Douglas College, New Westminster, Canada May 16. Presented "Indigenization - TWO (ánus) Stories" about working with indigenous students as the FAN X99 A320 coordinator and my efforts to indigenize and decolonize MATH 190 *Principles of Mathematics for Elementary Teachers*.
December 2024	Mainzer Mathe Akademie Program, Gutenberg University, Mainz, Germany, Dec. 9-10. Presented outreach program to Professor Dr. Manfred Lehn, Studies Manager & Lecturer Dr. Cynthia Hog-Angeloni and High School Head Dr. Martin Mattheis.
July 2023	13th Congress of the European Society for Research in Mathematics Education, Budapest, Hungary, July 10-14. Attended Thematic Working Group 28 "Collaborative Settings in Mathematics Teacher Education" led by Prof. Dr. Stéphane Clivaz, Lausanne University.
February 2022	Campus Stores Canada Annual Meeting, Vancouver, Canada, February 2. Panelist on OER Experiences in Post-Secondary Education with Dr. Laurie Prange (Capilano University) and Dr. Mirabell Tinio (Langara College) hosted by Imelda May.
April 2022	BIRS "Ted Lewis SNAP Math Fair Workshop" organized by Drs. Sean Graves and Trevor Pasanen (University of Alberta), Banff, Canada, April 29-May 1.
May 2021	Indigenizing Math and Science Curriculum NIC 2021 Virtual Workshop hosted by Dr. Dennis Lightfoot (Math/Science Department Chair, North Island College), Courtenay, Canada, May 17-21. Presented "A (mathematical) Journey into Indigenization & Discovery" about my efforts to indigenize and decolonize MATH 190 *Principles of Mathematics for Elementary Teachers.*
June 2021	Canadian Applied and Industrial Mathematics Society Virtual Meeting, June 21-24. Panelist on Indigenization of Mathematics - An institutional initiative that supports societal reconciliation along with Drs. Edward Doolittle (Mathematics Education Researcher, First Nations University of Canada), Kori Czuy (Indigenous Engagement Specialist, TELUS Spark Science Centre), and Nathan Rowbottom (Computer Studies Teacher, Six Nations Polytechnic)

May 2020	SFU CEE Remote Teaching Strategies for TAs and TMs, Virtual Meeting, May 6. Presented "Using a TA-ship as an Opportunity to Develop Soft Skills while Juggling, Building and Fostering Relationships" and participated on the panel "Advice from Experienced TAs and Faculty".
May 2019	PIMS 22nd Annual Changing the Culture, Vancouver, Canada, May 17. Panelist on "Should We Look at Teaching Math Around the World to Improve Teaching in Canada?"
June 2019	BIRS "Innovations in New Instructor Training" organized by Fok-Shuen Leung (University of British Columbia), Carmen Bruni (University of Waterloo), Shawn Desaulniers (University of Alberta) and Pam Sargent (Yale University), Banff, Canada, June 21-23.
November 2019	Professional Development Workshop at Douglas College, New Westminster, Canada, November 19. Co-presented with Dr. Kevin Lam (Biology) and three of our students on "Practical Ideas and Critical Considerations for a Compassionate Instructor".
May 2018	PIMS 21st Annual Changing the Culture, Vancouver, Canada, May 18. Panelist on "Outreach: Challenges and Rewards".
May 2018	SFU Open Education: Transforming Teaching & Learning, Burnaby, Canada, May 24. Panelist on "Open Education and OER in Action".
May 2016	PIMS 19th Annual Changing the Culture, Vancouver, Canada, May 13. Panelist on "*Active* Mathematics Class".
June 2016	Canadian Mathematics Education Study Group, Queen's University, Kingston, Canada, June 3 - 7. Presented my doctoral theses "Unfolding of Diagramming and Gesturing between Mathematics Graduate Student and Supervisor during Research Meetings" for the New PhD Session.
July 2016	International Congress on Mathematical Education, Hamburg, Germany, July 24-31. Participated in the Topic Study Group TSG 54 "Semiotics in Mathematics Education" and presented a co-authored paper on "Diagramming and Gesturing During Mathematizing" with Dr. Nathalie Sinclair.
June 2010	BCIT Professional Development Day for the Department of Mathematics, Burnaby, Canada, June 7. Co-presented with Dr. with Veselin Jungić "Using Technology to Motivate and Assess Students' Understanding".
April 2004	PIMS 7th Annual Changing the Culture, Vancouver, Canada, April 23. Panelist on "Reflections on Schools Mathematics Curriculum".

Presenter

June/July 2025	21st International Study Association on Teachers and Teaching Biennial Conference, June 30-July 4, Glasgow, Scotland. Presented "Equitable Partners in Designing the Learning Environment of an Online Course" co-written with Dr. Joanna Niezen.
February 2023	SFU Teaching Matters Seminar, Burnaby, Canada, February 13. Co-presented with Dr. Joanna Niezen "Partners in Design: Involving all Stakeholders in a Course Restructuring Process".
February 2023	FYMSiC (First-Year Math & Stats in Canada) Online Meet Up, February 28. Co-presented with Joanna Niezen "Partners in Design: Involving all Stakeholders in a Course Restructuring Process".
April 2022	13th Annual Teaching and Learning Conference - Fostering Partnerships in Pedagogy, University of Waterloo, Virtual Meeting, April 27-28. Co-presented "Partners In Design - Involving all Stakeholders in the Restructuring Process of an Online Course" with Drs. Sophie Burrill and Joanna Niezen, TA/RA Sheena Tan, and undergraduate student Marion Moldovan.
February 2021	SFU CEE Remote Teaching Forum 2021, Virtual Meeting, February 9. Presented a lightning talk on "Fostering Student Voices, Visibility and Accountability through Hooks and Group Activities".

March 2019	SFU Teaching Matters Seminar, Burnaby, Canada, March 4. Co-presented with Dr. Kevin Lam and three of our students "Some practical ideas and critical considerations for a compassionate instructor".
May 2019	SFU 17th Annual Symposium on Teaching and Learning: Assessing and Celebrating Teaching and Student Learning, Vancouver, Canada, May 15-16. Gave a Lightening Talk: "Emergence of a New Approach to Dichotomy between Curriculum and Assessment".
May 2019	PIMS 22nd Annual Changing the Culture, Vancouver, Canada, May 17, 2019. Co-led with Drs. Melania Alvarez (PIMS Education Coordinator), Nora Franzova (Langara College), Sonoko Nakano (Langara College) "Don't Change the Math, Change the Culture".
August 2019	15th International Conference of the Mathematics Education for the Future Project: Theory and Practice: An Interface or A Great Divide?, Maynooth, Ireland, August 4-9. Presented the paper "Open-Source Differential and Integral Calculus Material Development to Support Student Accessibility and Learning".
October 2019	British Columbia Association of Mathematics Teachers Fall Conference, Surrey, Canada, October 25. Presented "Exploring Tessellations and Discovering Them in Coast Salish Art".
February 2017	SFU Teaching Matters Seminar, Burnaby, Canada, February 6. Co-presented with Dr. Randall Pyke "Learning Objectives and Learning Outcomes in MATH 190 *Principles of Mathematics for Teachers* and MATH 232 *Applied Linear Algebra*".
March 2017	Building Connections: Well-being & Teaching, SFU Health Promotion, Burnaby, Canada, March 7. Co-presented with three MATH 190 students "Is getting personal too personal or does it open a window to both the student and instructor worlds and thereby support learning?".
October 2017	SFU Teaching Matters Seminar, Burnaby, Canada, October 30. Presented "Lightboard Videos for MATH 190 *Principles of Mathematics for Teachers*".
November 2017	SFU Teaching Matters Seminar, Burnaby, Canada, November 14. Panelist on "University Lecturer and the Collective Agreement".
October 2016	British Columbia Association of Mathematics Teachers Fall Conference, Vancouver, Canada, October 21. Presented "Explore, talk, visualize and conceptualize math through peer collaboration".
June 2015	Society for Teaching and Learning in Higher Education Conference - Achieving Harmony: Tuning into Practice, Vancouver, Canada, June 16-19. Co-presented with Dr. Cindy Xin and RA Jing Li on "Using Reflective Activity to Improve Student Metacognition and Attitudes in Post-Secondary Education".
August 2015	8th International Congress on Industrial and Applied Mathematics, Beijing, China, August 10-14. Presented the contributed paper "How Gestures and Diagrams Facilitate Emergence of Mathematical Creations in Supervisor-Graduate Student Research Meetings".
September 2015	13th International Conference of the Mathematics Education for the Future Project, Sicily, Italy, September 16-21. Presented the paper "Design of Online Metacognitive Activity in a Post-Secondary Mathematics-for-Teachers Course".
November 2015	Seminar for Mathematics Education at Universität Bremen, Bremen, Germany, November 3. Presented "Diagramming and Gesturing between Mathematics Graduate Student and Expert Mathematicians".
November 2015	Seminar for Mathematics Education at Universität Saarbrücken, Saarbrücken, Germany, November 17. Presented "Diagramming and Gesturing between Mathematics Graduate Student and Expert Mathematicians".
November 2014	SFU 9th Annual Mathematics Education Doctoral Students Conference, Burnaby, Canada, November 29. Presented "Diagramming Mobilizes Mathematics".

May 2014	Canadian Mathematics Education Study Group, Edmonton University, Edmonton, Canada, May 30 - June 3. Presented the poster "Unfolding of Diagramming and Gesturing between Mathematics Graduate Student and Supervisor during Research Meetings" on my thesis study at the Gallery Walk.
July 2014	39th International Conference Improving University Teaching, Vancouver, Canada, July 23 - 25. Presented "How Can a Flipped Classroom be Created Online Based on its Face-to-Face Counterpart?" in the Digital Showcase.
March 2011	SFU TLC Presentation Series, Burnaby, Canada, March 9. Co-presented with Dr. Veselin Jungić "Teaching large classes: Developing a model that works for you!".
July 2011	36th International Conference Improving University Teaching, Bielefeld, Germany, July 19 - 22. Participated in Round Table Discussion: "The Struggles of Shifting a Student's Perspective from being the Student in the Course to the Teacher they want to become Using Formative Assessment in a Content Mathematics Course leading to the Professional Development Program".
October 2011	British Columbia Association of Mathematics Teachers) Fall Conference, Burnaby, Canada, October 21. Co-presented with Dr. Natalia Kouzniak "Mathematical Connections and Educational Conversations: SFU Math Outreach Programs for Students and Teachers".
July 2009	34th International Conference Improving University Teaching, Vancouver, Canada, July 14 - 17. Presented "Instructor and Student Feedback on Clickers in Large Calculus Courses"
September 2009	6th International Conference of the Mathematics Education for the Future Project, Dresden, Germany, September 11-17. Presented "Creating, Implementing, and Evaluating Online Assignments for Post-Secondary Mathematics Courses"
October 2009	Northwest Mathematics Conference, Whistler, Canada, October 22-24. Co-presented with Dr. Veselin Jungić "Using Technology to Motivate and Assess Students' Understanding".
January 2008	ProD Event for Haus der Kinder Christ Koenig (Chair Dr. Heike Beck), Kaiserslautern, Germany, January 4. Presented and held a 2-hour workshop for parents "Das Kind und die Mathematik" (The Child and Mathematics").
January 2008	ProD Event for Kindertagesstaette Sankt Theresia (Chair Dr. Barbara Keller), Kaiserslautern, Germany, January 7. Presented and held a 2-hour workshop for parents "Das Kind und die Mathematik" (The Child and Mathematics")
April 2007	PIMS 10th Annual Changing the Culture, "Assessing What We Do", Vancouver, Canada, April 20. Co-led with Drs. Sue Harberger and Jamie Mulholland "Rethinking Precalculus Mathematics".
May 2007	SFU 9th Annual Symposium on Innovative Teaching, Burnaby, Canada, May 23. Co-presented with Drs. Malgorzata Dubiel, Ami Mamolo and Jamie Mulholland "Q-ing Students In: The Story of FAN X99 - Foundations of Analytical and Quantitative Reasoning".
October 2006	Mathematics Council of the Alberta Teachers Association Annual Conference, Jasper, Canada, October 12-14. Presented "Writing Projects in my Mathematics in Action University Course".

Participant

May 2025	First-Year Math and Stats in Canada Annual Conference, Virtual Meeting, May 8. Am I Convinced? Research and Evidence in Mathematics Education
May 2025	PIMS 27th Annual Changing the Culture, Vancouver, Canada, May 16.
May 2024	First-Year Math and Stats in Canada Annual Conference, Virtual Meeting, May 9. Why are we teaching mathematics today?

May 2024	Pearson Higher Education: Leading in Learning Webinar Series, Virtual Meeting, May 14. Honouring Indigenous Wisdom: Integrating Indigenous Perspectives into Course Curriculum presented by Justin Molander (CPA, CMA, Sapotaweyak Cree Nation, Kwantlen Polytechnic University and Langara College).
May 2024	PIMS 26th Annual Changing the Culture, Vancouver, Canada, May 17. Artificial Intelligence and Mathematics.
October 2024	Northwest Mathematics Conference, Whistler, Canada, October 24-26. Landscapes for Learning Mathematics: Inclusive approaches to mathematics education and Indigenous and place-based pedagogies in mathematics education.
May 2023	PIMS 25th Annual Changing the Culture, Vancouver, Canada, May 19, 2023. Geometry and Perseverance: The Importance of Open-Ended Problem-Solving.
May 2022	PIMS 24th Annual Changing the Culture, Vancouver, Canada, May 20, 2022. Logical Thinking, Mathematical Thinking, Computational Thinking: What is the Difference?
May 2022	SFU Symposium on Teaching and Learning, Burnaby, Canada, May 19. (Re)connecting Through Conversation.
May 2021	PIMS 23rd Annual Changing the Culture, Vancouver, Canada, May 14, 2021. How has Coronavirus changed the teaching of Mathematics?
May 2021	SFU Symposium on Teaching and Learning, Burnaby, Canada, May 19-20. Designing for Student Success in the Online and Blended Learning Environments.
September 2021	Online Symposium on Indigenising University Mathematics at Wollotuka Institute, University of Newcastle, Virtual Meeting, September 20-21. Challenges of how to Indigenize mathematical practice at universities, both in education and research.
December 2021	CMS Winter Meeting, Virtual Meeting, December 2-7. Attended “Mathematics and Mathematics Education: Two Communities in Conversation” co-organized by Veselin Jungić and Rina Zaskis; ““I don’t write in math, I just do it”: Emphasizing communication in mathematical practice” co-organized by Kseniya Garaschuk and Vanessa Radzimski; and “Indigenizing University Mathematics” co-organized by Veselin Jungić, Naomi Borwein and Florence Glanfield.
October 2020	British Columbia Association of Mathematics Teachers Fall Conference, Virtual Meeting, October 23. Transformations.
December 2020	CMS Winter Meeting, Virtual Meeting, December 3-8. Attended “Creative Assessments in the COVID-19 times” co-organized by Andie Burazin, Lauren DeDieu and Miroslav Lovric and COVID-19 panel with CMS and CMESG.
May 2019	First-Year Math and Stats in Canada, Edmonton, Canada, May 3-5. Time to Rethink our Curriculum?
May 2019	PIMS 22nd Annual Changing the Culture, Vancouver, Canada, May 17, 2019. Changing Culture for Indigenous Knowledges, Communities, and Mathematics Education.
May 2018	PIMS 21st Annual Changing the Culture, Vancouver, Canada, May 18, 2018. Thinking Outside the Box.
May 2017	PIMS 20th Annual Changing the Culture, Vancouver, Canada, May 19, 2017. Mathematics in the Common Core.
May 2016	PIMS 19th Annual Changing the Culture, Vancouver, Canada, May 13, 2016. Mathematical Habits of Mind.
May 2014	PIMS 17th Annual Changing the Culture, Vancouver, Canada, May 16, 2014. Fostering Curiosity.
October 2012	Northwest Mathematics Conference, Victoria, October 18-20. Starting With the Beginning.

April 2011	PIMS 14th Annual Changing the Culture, Vancouver, Canada, April 29. Through Our Teaching.
November 2011	PIMS, Office for Aboriginal Peoples and IRMACS Centre Seminar, Burnaby, Canada, November 11. Aboriginal Students in Math and Science: Situations and Solutions.
May 2010	88th Annual Meeting of British Columbia Committee on the Undergraduate Program in Mathematics and Statistics, Burnaby, Canada, May 18-19.
May 2010	PIMS 13th Annual Changing the Culture, Vancouver, Canada, May 21. Resources And How We Use Them (Or Not).
October 2010	British Columbia Association of Mathematics Teachers Fall Conference, October 22. We Connect.
April 2009	Canadian Mathematics Education Forum, Vancouver, Canada, April 30-May 3. The ways in which resources and assessment define, inform and mould curriculum.
April 2009	PIMS 12th Annual Changing the Culture, Vancouver, Canada, April 30. Integrating Mathematics.
October 2009	Northwest Mathematics Conference, Whistler, Canada, October 22-24. Going for Gold: Let the Mathematics Begin.
April 2008	PIMS 11th Annual Changing the Culture, Vancouver, Canada, April 18. Mathematics: Beauty and Utility.
May 2008	SFU 33rd McGraw-Hill Ryerson National Teaching, Learning & Technology Conference, Burnaby, Canada, May 13-15. Teaching for Learning: New Approaches for a New Generation
May 2008	LON-CAPA 10th Annual Conference & Workshop, SFU, Vancouver, Canada, May 22-24. For Assessment and Collaboration.
October 2008	British Columbia Association of Mathematics Teachers Fall Conference, October 24.
May 2006	Mini-Conference “Calculus: From the Classroom to the Concrete”, Bellingham, USA, May 4-7.
April 2005	SFU Rethinking Teaching: A Course Design Workshop for Professors, 5-day workshop led by Dr. Cheryl Amundsen, Burnaby, Canada, April 25-29.
May 2005	Canadian Mathematics Education Forum, Toronto, Canada, May 6-8. Why teach Mathematics?
May 2004	PIMS 7th Annual Changing the Culture, Vancouver, Canada, April 23. Mathematics Curriculum: Could Less Be More?

Outreach Activities

Presenter

May 2025	Canadian Association for Girls in Science (Grades 1-6), Virtual Workshop hosted by Tarveen Kaur, May 3. Held a hands-on workshop “Platonic Solids: Their Construction and Properties”.
June 2024	CMS Math Camp, Surrey, Canada, June 25. Held a hands-on workshop “Powerful, Playful and Pretty Platonic Solids”.
March 2022	Canadian Association for Girls in Science (Grades 1-6), Virtual Workshop hosted by Catherine McKenzie and Christine Song, March 5. Held a hands-on workshop “Platonic Solids and Graph Theory”.
March 2022	Canadian Association for Girls in Science (Grades 5-10), Virtual Workshop hosted by Catherine McKenzie and Christine Song, March 12. Held a hands-on workshop “Platonic Solids and Graph Theory”.

November 2022	SPARX podcast for high school math students (Elaine Li). Presented "Journey from German high school to Becoming a Math Instructor".
February 2020	SFU Let's Talk Science, a High School Monthly Career Series, at the Trottier Studio, Burnaby, Canada, February 27. Held a hands-on workshop "Creating Chaos ... sort of" with Grade 9-12 high school students.
July 2020	St. Michaels University School, Richmond, Canada. Assisted Mrs. Higinbotham with preparing a math unit on polygons and tessellations for her high school students.
April 2019	SFU Let's Talk Science, a High School Monthly Career Series, at the Trottier Studio, Burnaby, Canada, April 5. Held a hands-on workshop "Duality: Devilish or Desirable?" with Grade 9-12 high school students.
April 2018	Upper Lynn Elementary School, North Vancouver, Canada, April 18. Held a half-day hands-on workshop with four MATH 190 students for elementary teacher Kristine Chambers' grade 3 students on "1D, 2D and 3D Measurements".
April 2018	Upper Lynn Elementary School, North Vancouver, Canada, April 24. Held a half-day hands-on workshop with three MATH 190 students for elementary teacher Kristine Chambers' grade 3 students on "3D Measurements and Magic". Developed a self-guided fractal card instruction booklet geared at the grade 3-5 level for Kristine Chambers
May 2018	Upper Lynn Elementary School, North Vancouver, Canada, May 2. Held a half-day hands-on workshop with three MATH 190 students for elementary teacher Kristine Chambers' grade 3 students on "Making Fractal Cards".
June 2018	CMS Math Camp, Surrey, Canada, June 26. Held a hands-on workshop "Rotational and Reflective Symmetries of the Cube and its Dual".
February 2017	SFU Let's Talk Science, a High School Monthly Career Series, at the Trottier Studio, Burnaby, Canada, February 27. Held a hands-on workshop "Creating Chaos ... sort of" with Grade 9-12 high school students.
October 2017	SFU Science Spooktacular, Burnaby, Canada, October 29. Prepared mathematical activities and engaged families in exploring math.
June 2016	CMS Math Camp, Surrey, Canada, June 28. Held a hands-on workshop "Powerful, Playful and Pretty Platonic Solids".
October 2016	SFU Science Spooktacular, Burnaby, Canada, October 29. Prepared mathematical activities and engaged families in exploring math.
May 2015	École Sperling Elementary School, Burnaby, Canada, May 12. Held a 2-hour hands-on workshop "Fractal Cards and Dimensions" to a Grade 6 class.
June 2015	École Sperling Elementary School, Burnaby, Canada, June 9. Held a 2-hour hands-on workshop "Mondrian Multiplication Art and Eratosthenes Sieve for Primes" to a Grade 6 class.
June 2015	École Sperling Elementary School, Burnaby, Canada, June 16. Held a 2-hour hands-on workshop "Platonic Solids and their Symmetries" to a Grade 6 class.
April 2015	North Star German Academy, North Vancouver, Canada, April 28. Held a 2-hour hands-on workshop "Platonic Solids and their Symmetries" to a group of German-speaking high school students in German.
October 2014	SFU Science Spooktacular, Burnaby, Canada, December 10. Prepared mathematical activities and engaged families in exploring math.
December 2014	École Sperling Elementary School, Burnaby, Canada, December 10. Held a 2-hour hands-on workshop "Mondrian Multiplication Art" to a Grade 6 class.
December 2014	École Sperling Elementary School, Burnaby, Canada, December 12. Held a 2-hour hands-on workshop "Eratosthenes Sieve for Primes" to a Grade 6 class.

June 2014	CMS Math Camp, Surrey, Canada, June 25. Held a hands-on workshop "Constructing Chaos ... sort of".
May 2014	SFU Science Rendezvous and Astronomy Day, Burnaby, Canada, Saturday, May 10. Prepared mathematical activities and engaged families in exploring math.
July 2013	SFU Science Alive, Quark Summer Camp, Burnaby, Canada, July 15 and 29. Held a 2-hour hands-on workshop "Platonic Solids and their Symmetries".
June 2013	CMS Math Camp, Burnaby, Canada, June 24. Held a hands-on workshop "Constructing Chaos ... sort of".
June 2009	CMS Math Camp, Surrey, Canada, June 24. Held a hands-on workshop "The ABC's of Tessellation".
June 2006	SFU Open House, Burnaby, Canada, June 3. Prepared mathematical activities and engaged families in exploring math.
June 2005	CMS Math Camp, Vancouver, Canada, June 30. Held a hands-on workshop "The ABC's of Tessellation".
May 2003	CMS Math Camp, Fredericton, Canada, May 30. Held a hands-on workshop "Mod Art".
May 2002	CMS Math Camp, Fredericton, Canada, May 31. Held a hands-on workshop "The ABC's of Tessellation".

Organizer

September 2018	SFU MATH TA/TM Preparation Session 1, Burnaby, Canada, September 6. Co-organized with Dr. Randall Pyke.
September 2018	SFU MATH TA/TM Preparation Session 2, Burnaby, Canada, September 18. Co-organized with Dr. Randall Pyke.
December 2018	CMS Winter Meeting, Vancouver, Canada, December 7-10. Co-organized with Drs. Kseniya Garaschuk (University of the Fraser Valley) and Andrew Hare (Saint Mary's University) "Educational Resources in Mathematics".
April/May 2017	SFU Rethinking Teaching: A Course Design Workshop for Faculty. April 27–28, May 1–2. Co-organized with Dr. Cheryl Amundsen, co-facilitated workshop and presented "Connection Between Assessments and Teaching Strategies Through a Case Study".
September 2016	SFU MATH TA/TM Preparation Session, Burnaby, Canada, September 8. Co-organized with Dr. Randall Pyke.
September 2014	SFU MATH TA/TM Preparation Session, Burnaby, Canada, September 4. Co-organized with Dr. Daria Ahrensmeier.
April/May 2012	SFU Rethinking Teaching: A Course Design Workshop for Faculty. April 26, 27, 30 & May 1. Co-organized with Dr. Cheryl Amundsen and co-facilitated workshop.
March 2012	SFU Extracurricular Workshop for MATH 190 *Principles of Mathematics for Teachers*, Burnaby, Canada, March 12. Organized a visit by elementary teacher Carol Pettigrew to hold a 2-h hands-on workshop "Without Mathematics there is no Art and Without Art there should be no Mathematics" for the students of the course.
March 2011	SFU Extracurricular Workshop for MATH 190 *Principles of Mathematics for Teachers*, Burnaby, Canada, March 8. Organized a visit by elementary teacher Carol Pettigrew to hold a 2-h hands-on workshop "Without Mathematics there is no Art and Without Art there should be no Mathematics" for the students of the course.
September 2011	SFU Institute for the Study of Teaching and Learning in the Disciplines. Co-organized with Dr. Cheryl Amundsen (Director) two T&L Grant-Writing Development Workshops held September 23 and October 7.

Aug-Nov 2011	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungić and Stephen Price full day visits from ~200 high school students held November 18.
December 2011	SFU Department of Mathematics Ambassador Program, Burnaby, Canada, December 1. Organized a presentation and Q&A session by 8 SFU freshman, the Math Adviser and myself about studying mathematics to three different grade 12 classes at Burnaby Mountain School.
Aug-Nov 2010	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungić and Stephen Price full day visits from ~200 high school students held November 19.
Aug-Nov 2009	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungić and Stephen Price full day visits from ~240 high school students held November 13 and 20.
Aug-Nov 2009	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungić and Stephen Price full day visits from ~240 high school students held November 14.
Aug-Nov 2009	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungić and Stephen Price full day visits from ~240 high school students held November 14.
Jan-Mar 2008	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungić and Stephen Price full day visits from ~240 high school students held March 7.
May 2008	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Organized and hosted St. John's High School visit to the Department of Mathematics held May 16.
Jan-Mar 2007	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Dr. Veselin Jungić full day visits from ~300 high school students held March 9.
March 2007	Burnaby North School Science Fair Judge, Burnaby, Canada, March 15-16. Judge over 70 entries from grade 8 and 9 science students.
Jan-Mar 2006	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Dr. Veselin Jungić full day visits from ~250 high school students held March 31.
January 2005	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Organized and hosted St. John's High School visit to the Department of Mathematics held January 21.
Jan 2004-Jun 2009	Assisted Dr. Malgorzata Dubiel in the organization of CMS Math Camp.

Service Activities

Departmental Committees

January 2024 - December 2025	Member At-Large (Teaching Faculty), Chair’s Advisory Group (CAG)
April 2024 - March 2025	Participated in all renewal/tenure/promotion/salary review cases in the department, TPC (MATH)
September 2018 - August 2021	Special advisor on issues relating to graduate students in their role as TA or sessional instructor, GSC
May 2020 - April 2021	Participated in all renewal/tenure/promotion/salary review cases in the department, TPC (MATH)

September 2017 - August 2019	Provided permanent point of contact for the e-learning platform MathXL by Pearson used in MATH 154/155/157/158. Created an extensive document with screen shot and snappy instructions that outlines all tools necessary to operate MathXL.
May 2018 - April 2019	Participated in all renewal/tenure/promotion/salary review cases in the department, TPC (MATH)
May 2017 - April 2018	Participated in all renewal/tenure/promotion/salary review cases, TPC (STATS)
January 2017 - April 2017	Participated in promotion and salary review case of their lab instructor, TPC (STATS)
May 2016 - April 2017	Participated in all renewal/tenure/promotion/salary review cases among the group of lecturers, TPC (MATH)
January 2017 - February 2017	Represented the departmental teaching faculty on the RTP Document Development Committee, Department of Mathematics, SFU
May 2012 - April 2013	Participated in all renewal/tenure/promotion/salary review cases among the group of lecturers, TPC (MATH)
September 2004 - August 2008	Department of Mathematics Website Committee
March 2008 - June 2008	Calculus Challenge Exam 2008 Coordinator
February 2006 - May 2006	Applied Calculus Textbook Selection Committee
September 2004 - August 2005	Undergraduate Studies Committee
March 2005 - May 2005	Calculus Textbook Selection Committee

University Committees

February 2022 - March 2022	SLC Undergraduate Writing Contest. Judge in the First Year Category
July 2008 - October 2009	Task Force for Teaching and Learning established by the Associate VP, Academic. The outcome was the replacement of LIDC with TLC.

Other Departmental Service

February 2022 - February 2022	Presented Math Success Session, February 2.
September 2021 - September 2021	Presented Math Success Session, September 22 and 24.
January 2020 - January 2020	Presented Math Success Session, January 20.
September 2018 - September 2018	Presented Math Success Session, September 19.
July 2009 - May 2012	TA Assignment Director. Redesigned the workflow and automated the process for TA Assignments. This process and the documentation created are still in place today.
September 2007 - August 2011	Wrote and updated a handbook for Sessional Instructors in the Department of Mathematics.
February 2007 - February 2007	Organized live performance “Calculus: The Musical!” by Matheatre for first and second year calculus students at SFU, Department of Mathematics
June 2007 - July 2007	Co-wrote with Dr. Malgorzata Dubiel and Justin Gray a workshop pamphlet for first year students.
September 2005 - August 2010	Upkeep of the Department of Mathematics Website

Other University Service

September 2022	SFU Student Engagement and Retention. Was an "Ask Me" person during SFU Burnaby Welcome Days.
March 2019	Human Resources, Finance, and IT Services. Participated in the Focus Group on "myINFO 9.2 Upgrade", March 11.
January 2015	Task Force on Flexible Education. Participated in the World Cafe, January 26.

Reviewing Activities

February 2023	Online Learning System Achieve, Macmillan Learning Publisher (for mathematics).
July 2021	Chapter 15 - Further Topics in Algebra and Change, *Reconceptualizing Mathematics* by Sowder/Sowder/Nickerson, Macmillan Learning Publisher for the 4th edition.
May 2021	Chapter 6 - Some Conventional Ways of Computing, *Reconceptualizing Mathematics* by Sowder/Sowder/Nickerson, Macmillan Learning Publisher for the 4th edition.
March 2021	*Preservice mathematics teachers' perspectives during a mathematical letter writing exchange*, Journal of University Teaching & Learning Practice (JUTLP) hosted at the University of Wollongong, NSW, Australia.
May 2020	Approximately 200 new problems that have been added by undergraduate students to the written booklet *A Collection of Problems in Differential Calculus* by Drs. Veselin Jungić, Petra Menz and Randall Pyke.
February 2020	Proposal submissions to Building Connections: UBC & SFU Well-being in Learning Environments Symposium, May 7.
July 2019	Part III - Reasoning About Shapes and Measurements Chapters 16 – 26, *Reconceptualizing Mathematics* by Sowder/Sowder/Nickerson, Macmillan Learning Publisher.
March 2019	*Introductory Mathematical Analysis for Business, Economics, and the Life and Social Sciences* 15th Edition by Haeussler, Paul & Wood, Pearson Publishing Canada.
February 2019	*Introducing Problem Based Learning in Calculus subject of Engineering* to appear in For the Learning of Mathematics (FLM) published under the auspices of the Canadian Mathematics Education Study Group/Groupe Canadien d'étude en didactique des mathématiques (CMESG/GCEDM).
January 2019	*Counting with 10 fingers as a man, with 8 fingers as a hen or with 2 switches on a computer* for the conference proceedings of the 15th International Conference of The Mathematics Education for the Future Project published by WTM-Verlag, July 2019.
August 2013	*Calculus for the Life Sciences* by Stewart & Day for Brooks Cole Publishers.
May 2011	Video Clips Part I - Reasoning About Numbers and Quantities Chapters 1 - 11, *Reconceptualizing Mathematics* by Sowder/Sowder/Nickerson, Macmillan Learning Publisher.
February 2011	Part I - Reasoning About Numbers and Quantities Chapters 1 - 11, *Reconceptualizing Mathematics* by Sowder/Sowder/Nickerson, Macmillan Learning Publisher.
2005	*Applied Calculus for the Managerial, Life, and Social Sciences* by Tan, Thomson Nelson Publishers.
2002	*Finite Mathematics* by Maki and Thompson, McGraw-Hill Publishers.

Mentorship, Training and Advising

Mentoring Teaching Faculty

September 2021 - August 2022	Dr. Joanna Niezen, newly hired lecturer, by meeting in person once a week for 1-2 hours debriefing or foreshadowing teaching related matters, explaining the inner workings of the department, the faculty and SFU as a whole and making her aware of professional development opportunities and available teaching and learning organizations both locally and nationally

Mentoring Instructors

September 2024 - December 2024	PhD graduate student Sebastián Moraga Scheuermann as FAN X99 D300 Sessional Instructor, Department of Mathematics
September 2024 - December 2024	PhD graduate student Mahsa Ansari as FAN X99 D400 Sessional Instructor, Department of Mathematics
January 2024 - April 2024	Limited Term Lecturer Steve Cheung as Sessional Instructor of FAN X99 A320, a course for students of the Cowichan Tribe on Vancouver Island run online by the Department of Mathematics and offered through the Indigenous Languages Program (INLP). I provided all lecture material and oversaw assessment.
September 2023 - December 2023	Limited Term Lecturer Steve Cheung as Sessional Instructor of FAN X99 A320, a course for students of the Cowichan Tribe on Vancouver Island run online by the Department of Mathematics and offered through the Indigenous Languages Program (INLP). I provided all lecture material and oversaw assessment.
May 2019 - August 2019	PhD graduate student Arezou Valadkhani as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
May 2019 - August 2019	PhD graduate student Mahdieh Malekian as MATH 155 D100 Sessional Instructor, Department of Mathematics
January 2019 - April 2019	PhD graduate student Arezou Valadkhani as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
January 2019 - April 2019	PhD graduate student Nicola Mulberry as MATH 157 Sessional Instructor, Department of Mathematics
September 2018 - December 2018	PostDoc Michel Virgilio as MATH 157 D200 Sessional Instructor, Department of Mathematics
May 2018 - August 2018	PhD graduate student Arezou Valadkhani as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
January 2018 - April 2018	PhD graduate student Arezou Valadkhani as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
May 2017 - August 2017	MSc graduate student Jeremy Chiu as MATH 155 Sessional Instructor, Department of Mathematics
January 2017 - April 2017	MSc graduate student Jeremy Chiu as MATH 155 Sessional Instructor, Department of Mathematics
May 2016 - August 2016	PhD graduate student Sonoko Nakano as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
January 2016 - April 2016	PhD graduate student Oi-Lam Ng as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics

September 2015 - December 2015	PhD graduate student Oi-Lam Ng as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
May 2015 - August 2015	MSc graduate student Navid Alaei as MATH 155 Sessional Instructor, Department of Mathematics
May 2015 - August 2015	PhD graduate student Oi-Lam Ng as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics

Mentoring Workshop Coordinators

May 2020 - August 2020	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics, Department of Mathematics
January 2020 - April 2020	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics
May 2018 - August 2018	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics
January 2018 - April 2018	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics
May 2017 - August 2017	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics
May 2015 - August 2015	MSc graduate student Navid Alaei as ACW Workshop Coordinator, Department of Mathematics
January 2015 - April 2015	PhD graduate student Will Ko as ACW Workshop Coordinator, Department of Mathematics
May 2014 - August 2014	PhD graduate student Sonoko Nakano as ACW Workshop Coordinator, Department of Mathematics
January 2014 - April 2014	PhD graduate student Sonoko Nakano as ACW Workshop Coordinator, Department of Mathematics
May 2013 - August 2013	PhD graduate student Sonoko Nakano as ACW Workshop Coordinator, Department of Mathematics
January 2012 - April 2012	PhD graduate student Sonoko Nakano as ACW Workshop Coordinator, Department of Mathematics

Mentoring Students

September 2005 - December 2011	Presented information about studying math and was a point of contact throughout the fall semesters for international students in the Clan Mentorship Program, SFU Student Development & Programming Centre.
January 2011 - April 2011	PIMS and SFU Office for Aboriginal Peoples. Tutored and mentored at the Native Education College Wednesday 6-7:30 pm.
May 2009 - June 2010	SFU English Bridge Program. Presented semesterly “Studying at a University” to international students.
September 2008 - August 2009	Mentoring of 5 science students in their transition from high school to university, SFU Recruitment and Retention.

Training Workshop Coordinators

January 2019 - April 2019	MSc graduate student Alexandria Vassallo, Department of Mathematics
January 2017 - April 2017	PhD graduate student Abhinav Shantanam, Department of Mathematics

September 2014 - December 2014	MSc graduate student Navid Alaei, Department of Mathematics
September 2013 - December 2013	PhD graduate student Will Ko, Department of Mathematics
September 2011 - December 2011	PhD graduate student Sonoko Nakano, Department of Mathematics

Training MATH 190 C100 Course Supervisors

September 2017 - December 2017	PhD graduate student Arezou Valadkhani, Department of Mathematics
January 2015 - April 2015	PhD graduate student Oi-Lam Ng, Department of Mathematics

Ministry Advising

October 2003 - October 2003	Gave a presentation on the Calculus Challenge Examination and my responsibilities for on-going support in high schools, Provincial Enrichment Advisory Committee Meeting, Ministry of Education, NB
September 2001 - June 2003	Represented the Department of Mathematics and Statistics at Ministry of Education meetings, 2001 – 2003, Ministry of Education, NB

Membership in the Academic Community

First Year Math Courses in Canada (2018-Current)

Canadian Mathematics Society (2015-Current)

British Columbia Association of Mathematics Teachers (1995-Current)

Canadian Mathematics Education Study Group (2006-2023)

Society for Teaching and Learning in Higher Education (2015-2022)

Subscription “for the learning of mathematics”, CMESG (2008-2020)

MAA The Mathematical Association of America (2008-2011)

Subscription “Mathematics Magazine”, MAA (2008-2011)

International Commission on Mathematical Instruction (2009-2010)

National Council of Teachers of Mathematics (1995-2001)

In the News

March 2023	Math anxiety eased: SFU instructors collaborate with students to redesign a Canvas course, published through SFU CEE, https://www.sfu.ca/vpacademic/news/2023/03/math-anxiety-eased--sfu-instructors-collaborate-with-students-to.html
February 2021	Classroom Resource: An OER for Differential and Integral Calculus for Social Sciences by Petra Menz, published by FYMSiC (First-Year Math and Stats in Canada) Newsletter Issue 6, February 2021
January 2021	This math lecturer developed her own open textbook—now thousands of students are using it, by Chloe Riley, SFU Library, published through SFU CEE, https://www.sfu.ca/cee/news/this-math-lecturer-developed-her-own-textbook-now-thousands-of-students-are-using-it.html
October 2019	Can it be done? A math instructor attempts to indigenize her course, by Jackie Amsden, SFU Centre for Educational Excellence, published through SFU CEE, https://www.sfu.ca/cee/news/can-it-be-done-a-math-instructor-attempts-to-indigenize-her-course.html

June 2016 Love for math trounces adversity, by Allen Tung, published through SFU News, https://www.sfu.ca/content/sfu/sfunews/stories/2016/sfu-convocation-june-2016/love-for-math-trounces-adversity.html

Awards, Honors and Scholarships

2025 **Title:** PIMS Education Prize **Type:** Nominated
Organization: Pacific Institute for the Mathematical Sciences
Details: The results will be announced in May. Being nominated for this major teaching prize is a recognition of my decade-long work on the math-for-teachers course at SFU.

2024 **Title:** Celebration of Authors **Type:** Author
Organization: SFU Library
Details: Jungić, V., Menz, P., & Pyke, R. (December, 2023). Differential Calculus: Problems and Solutions from Fundamentals to Nuances. World Scientific Publishing Co Pte Ltd., Singapore. https://doi.org/10.1142/13324

2024 **Title:** 2023-2024 Faculty of Science Excellence in Teaching Award **Type:** Teaching
Organization: Faculty of Science

2021 **Title:** BCcampus Award for Excellence in Open Education **Type:** Award
Organization: BCcampus
Details: March 2021 recipient
https://bccampus.ca/2021/03/24/bccampus-award-for-excellence-in-open-education-petra-menz-and-nicola-mulberry/

2018 **Title:** Celebration of Authors **Type:** Author
Organization: SFU Library
Details: Menz, Petra, and Nicola Mulberry, “CALCULUS Early Transcendentals Differential & Multi-Variable Calculus for Social Sciences” and “CALCULUS Early Transcendentals Integral & Multi-Variable Calculus for Social Sciences”, SFU Summit, 2017 and 2018.

2017 **Title:** Champion for a Healthy Campus Community **Type:** Service
Organization: Health and Counselling Services
Details: SFU recognizes “champions” for their outstanding contributions to the health and well-being of the University as part of SFU’s Healthy Campus Community initiative. Petra Menz, a senior lecturer in Mathematics, has developed meaningful connections with hundreds of students through her Instructor-Student Meeting Initiative. Her work has had a remarkable impact on student well-being, and it is an example of how instructors participating in SFU's Well-being in Learning Environments (WILE) program are creating learning environments that boost well-being and academic performance.

2008 **Title:** Celebration of Authors **Type:** Author
Organization: SFU Library
Details: Tan, Soo Tang, Petra Menz, and Dan Ashlock. Applied Calculus for the Managerial, Life, and Social Sciences. First Canadian Edition. Toronto: Nelson Education, 2009.

2002 **Title:** Allen P. Stuart Award for Excellence in Teaching **Type:** Nominated
Organization: University of New Brunswick
Details: Being nominated for this major teaching prize is a recognition of my achievement in creating the first math-for-teachers course in the province of New Brunswick.

1995 **Title:** Dora S. Simpson Memorial Prize **Type:** Award
Organization: Faculty of Education, University of British Columbia

1992 **Title:** 2-year Postgraduate Scholarship **Type:** Scholarship
Organization: NSERC

1991 **Title:** Gilchrist Award in Computer Science **Type:** Award
Organization: Scarborough College, University of Toronto
Details: Top Computer Science Student

1991 **Title:** Undergraduate Student Research Award **Type:** Research
Organization: NSERC

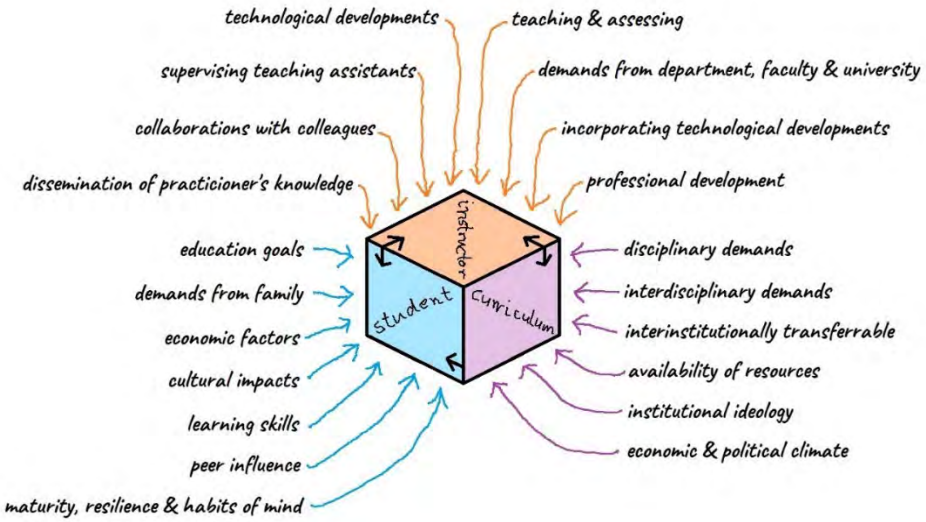
1991 **Title:** J.W. Billes **Type:** Scholarship
Organization: Scarborough College, University of Toronto

1991 **Title:** Samual Beatty In-Course Scholarship **Type:** Scholarship
Organization: Scarborough College, University of Toronto
Details: Top Math Student

Teaching and Learning Philosophy Statement

I have been teaching for almost 30 years and my philosophy of teaching has evolved primarily by gaining a deeper understanding of the learner.

Even more strongly than before, I see the macro world of teaching consisting of three fundamental elements: the instructor, the student, and the curriculum – each influencing the other while at the same time being influenced by both internal and external factors. The diagram below is an attempt to visualize these elements and factors.



In the micro world of teaching – my classroom – I make every decision based on the learner, understanding that each student is a complex person motivated by their own goals. I try to establish an environment that fosters inquisitiveness and mathematical play. Taking inspiration from my doctoral dissertation *Unfolding of Diagramming and Gesturing between Mathematics Graduate Student and Supervisor during Research Meetings*, I encourage my students to draw (diagrams, tables, graphs, …) and to manipulate (literally, objects by hand) to facilitate an embodied exploration of quantities and their relationships, which are at the heart of mathematics.

Learning is a life-long process. To teach something cannot mean to spoon-feed. To me, teaching means helping the learner develop patterns of intellectual behaviors or dispositions that will carry them beyond the classroom and through life. Over time, I have come to understand that the 16 habits of mind described by Costa and Kallick in their 2009 book *Habits of mind across the curriculum: Practical and creative strategies for teachers* are such patterns of intellectual behavior (e.g., persisting, thinking flexibly, questioning, creating, imagining and taking responsible risks). My students often think that they are predisposed to a specific learning style, yet there is no evidence that teaching to a specific learning style will improve learning. However, there is evidence that dispositions can be acquired and positively impact learning.

In 2017, I began in earnest to learn about Indigenous ways of knowing and what it might mean to indigenize and decolonize in mathematics. My conversations with Canadian Mohawk and mathematics educator Dr. Edward Doolittle during the 5-day 2018 CMESG working group *Confronting Colonialism in Mathematics and Mathematics Education* brought awareness that trying to impose the Canadian education system directly onto Indigenous people would simply

never work, yet, to not be afraid to get started in however small a measure. In the string of workshops and themed conferences, conversations with Elders and immersive reading that I undertook over a period of four years, a picture emerged of how I could start with just one course to respectfully and authentically achieve Indigenization and decolonization without appropriation.

Then in the fall semester 2023 and spring semester 2024, I was asked to coordinate a math section, which is offered under SFU's Indigenous Languages Program (INLP) to students from the Cowichan tribe on Vancouver Island, by establishing the curriculum and assessments and guiding the assigned instructor. It is difficult to put into words what unfolded over the tumultuous two semesters. There were many internal and external factors that were influencing the curriculum, the students, the instructor and myself. This was the steepest learning curve I have ever experienced, and my eyes were opened wide to the diversity of the needs and obstacles of the academic system faced by Indigenous learners. As a result, this experience was both challenging and rewarding. It was reassuring for me that falling back on patterns of intellectual behavior allowed me to completely revamp the assessments to allow some learners to be successful. What I have taken away from this experience is that implementing calls to action for reconciliation are hard work and must be done thoughtfully and carefully in small steps and through open communication among all stakeholders. This requires time and resources that need to be available to instructors and coordinators who are willing to walk this path. This also requires consideration that conventional academic deadlines may actually be a hindrance to learning in a culture that has a different relationship with time.

Lastly, I also practice life-long learning. I continuously strive to update my pedagogical and mathematical knowledge in a variety of ways: by giving and attending workshops (e.g., SFU Centre for Educational Excellence, SFU Teaching Matters Seminar, First-Year Math & Stats in Canada—FYMSIC); by attending conferences (e.g., Canadian Mathematics Education Study Group—CMESG, Canadian Mathematical Society—CMS), by reading relevant mathematics education research (e.g., Centre for Excellence in Learning and Teaching—CELT, For the Learning of Mathematics), collaborating with my colleagues (e.g., *Partners in Design* project), by writing teaching and learning related articles (e.g., Hossain, Menz and Stockie, *An open-access clicker question bank for numerical analysis*, *PRIMUS*, 2022), by authoring textbooks (e.g., Jungic, Menz and Pyke, *Differential Calculus: Problems and Solutions from Fundamentals to Nuances*, World Scientific Publishing Co Pte Ltd., Singapore, 2023, https://doi.org/10.1142/13324) and reviewing textbooks (*Reconceptualizing Mathematics* by Sowder, Sowder and Nickerson).

INTRODUCTION

This document highlights my achievements over the years in one course very dear to me, namely MATH 190 *Principles of Mathematics for Teachers*. While I have had many other accomplishments throughout my 30 years as an educator, my work for this math-for-teachers course is of particular importance because of its impact on future elementary teachers and their students' mathematical journeys.

My own journey as a mathematical educator began with my six years as a Secondary Mathematics Teacher in the Richmond School District, driven by my career goal to develop mathematical curricula and explore multifaceted ways of teaching and learning mathematics to a diverse group of students. Following that, I became a Lecturer at the University of New Brunswick, a position that was created specifically for me as the first such lectureship in the Mathematics and Statistics Department. During my two years there, I was invited by internationally renowned math education researcher Professor Marion Small to develop the first math-for-teachers course in the province, which sparked a lifelong interest in supporting undergraduate students wanting to become elementary teachers. I was fortunate to be able to continue this effort as a Senior Lecturer at Simon Fraser University, when I started to teach the equivalent course (MATH 190) in 2010 alongside 3M award winning educator Dr. Malgorzata Dubiel. After familiarizing myself with the SFU and BC contexts under her mentorship, she invited me in 2013 to take over the online course offering and completely rebuild it. In total, I have taught this course over 30 times in various formats including face-to-face, flipped, online, and blended.

This math-for-teachers course is especially complex for two reasons: first, it is an admission requirement for the Professional Development Program in the Education Faculty; and second, the student population varies widely in their academic levels, mathematical knowledge, relationship with math and accessible learning needs. So, I began a decade-long effort to systematically revamp the online course by incorporating sound pedagogical research on math education and cutting-edge technology offered through the Teaching and Learning Centre and Centre for Online and Distance Education. Because of its success, all developed materials have been adopted for the face-to-face course as well and were turned into a blended course. To fully understand the massive effort and consideration that went into building the course, an outline of its construction is provided below after my Teaching Philosophy statement. The Appendix showcases a sampling of group activities.

Two milestones were breaking the cycle of math-anxious education students and my progress with Indigenization and supporting Indigenous students. Math education giant Loewenberg Ball's poignant 2005 article *Knowing Mathematics for Teaching* had a snowball effect across North America for a simple reason: her research showed that an elementary teacher's mathematical knowledge is tightly linked to students' mathematical attainment. She describes a vicious cycle in which "we are simply failing to reach reasonable standards of mathematical proficiency with most of our students, and those students become the next generation of adults, some of them teachers". Subsequent math education research supports Loewenberg Ball's claim and policies around PDP have changed to include the requirement of a Mathematics for Teachers course. I have made it my mission to break this vicious cycle and work closely with various SFU units for teaching and learning to produce an engaging, informative, up-to-date and mathematically empowering course. Students of this course become secure in their own mathematical learning, attain mathematical knowledge, and realize how mathematics is an integral part in the fabric of society, all in an effort so that their own future students can become mathematically empowered.

My journey of indigenizing the MATH 190 curriculum and pedagogy began with SFU's call to indigenize and decolonize. My own home is full of Haida, Squamish and other Coast Salish art and my family has attended many native events over the years that celebrate their cultural uniqueness and raise awareness of the impact of residential schooling and governmental policies on land and resources. I knew I had to learn more to respectfully achieve indigenization and decolonization. Over a period of three years, I embarked on a series of workshops, talks and discussions led by Elders, indigenous educators and math education researchers in that area. My whole belief system was shaken up and to be frank, it was an emotional journey, as I questioned my actions and decisions I have made over the years. But I think this "shake-up"

was necessary for me to change. I also learned that my approach to teaching and learning overlaps with Indigenous values such as my students being actively engaged in pairs, small or large groups, at grouped tables or on the whiteboard; or respecting where knowledge is coming from and how it is fabricated, namely people and bringing this to the foreground; or situating the learning in stories. My development timeline lists the various ways in which I have indigenized MATH 190.

Along the way, I have disseminated my work locally, nationally and internationally through presentations, workshops and published papers; received a 2014 Teaching and Learning Development Grant of $5000 to formally investigate reflections and metacognition; mentored three colleagues to take on the teaching of this complex course; trained many graduate student TAs to support the fragile student population in the associated math help centre; and supported numerous previous MATH 190 students in reaching their career aspirations beyond the classroom.

DEVELOPMENT of MATH 190 *Principles of Mathematics for Teachers*

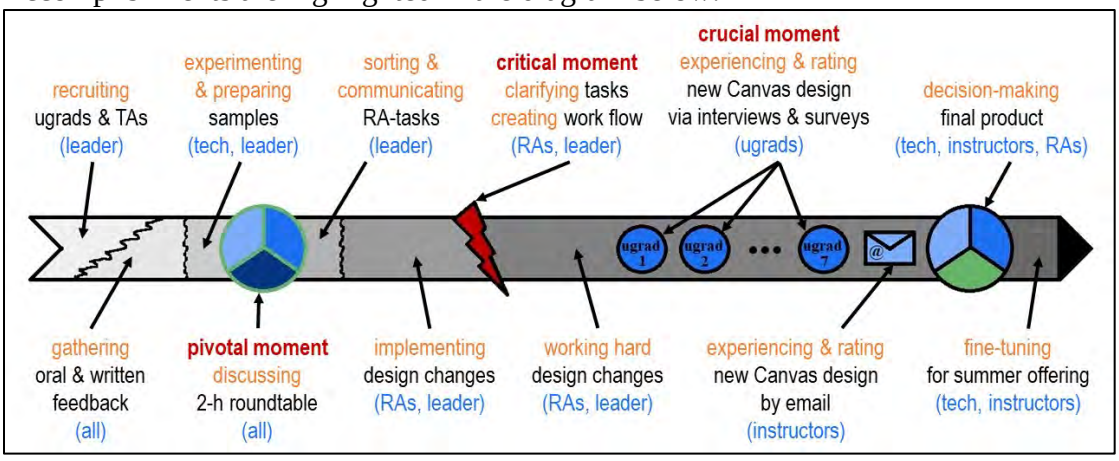
Below is a timeline of my accomplishments over the years in an effort to: (1) better serve the diverse student body and enhance student experience of and learning in MATH 190; (2) create an online version of MATH 190 that aligns with the learning and teaching goals of the face-to-face version; and (3) effectively transition from a lecture style face-to-face course to a blended learning course, in which students can explore mathematical concepts through well-designed group activities during class time.

- Spring 2009 to Fall 2019 – Instructor of MATH 190 D100 once each academic year
  - Mentored by Dr. Malgorzata Dubiel, who also taught this course once per year.
  - Introduced group assignments, which have become an **ANCHOR** for students (shared experiences support growth).
  - Redeveloped LON-CAPA online assignments to make full use of its technological capabilities.
  - Initiated formal outside-of-class instructor meetings to connect with students one-on-one and provide individual guidance, for which I have been named Champion for a Healthy Campus Community 2017.
  - Created visuals and manipulatives that are employed throughout the curriculum.
  - Reviewed of the accompanying textbook *Reconceptualizing Mathematics: for Elementary School Teachers* by Sowder et al. from the preliminary edition to the 4th edition.
- Summer/Fall 2013 – Completely redeveloped the content and delivery of MATH 190 C900 (distance version) to align with the goals of the face-to-face version with the support of Kanthi Jayasundera.
  - Made course transparent – explained EVERYTHING (this came back to haunt me, see *Partners in Design*).
  - Adopted group assignments and LON-CAPA online assignments from face-to-face version.
  - Created individual textbook assignments for routine exercises (this later proved to be too high a workload).
  - Introduced weekly reflections: Reflection Prompt – Student Response – Instructor Comment sequence.
  - Scaffolded learning of the 12 topics by creating Readings and Explorations pages for each topic.
  - Wrote 12 historical vignettes to align with each of the 12 topics with the help of our departmental and internationally recognized math historian Dr. Tom Archibald and his graduate student Brenda Davison.
  - Created online Blackboard Collaborative meetings to support live online mathematics.
- Spring 2014 to Spring 2017 – Course Supervisor for the distance version in almost every term tweaking the course
- Spring 2014
  - Completed the design of 14 hands-on activities to facilitate a flipped approach in the face-to-face version, which have become the **EXPLORATORY PLAYGROUND** for students.
  - Incorporated these activities into the distance version, CODE filmed 10 activities as they unfolded with student volunteers; embedded 2D and 3D mathematical visuals under my guidance

to enhance the viewing experience; student surveys were overwhelmingly positive, so clips are now firmly housed on CANVAS.

- Fall 2014 – Received a 2014 Teaching and Learning Development Grant with Educational Consultant Dr. C. Xin
  - Performed a data analysis of student responses to weekly reflections from three terms and showed that
    - students' meta-cognition in mathematical learning is enhanced; and
    - students are oriented more positively towards mathematics.
  - Enhanced standard instructor feedback based on categorizing student responses.
  - Ensuing presentations and publications:
    - STLHE Conference, June 16-19, 2015 (preliminary findings and research methodology).
    - ME 21st Century Conference, September 16-21, 2015 (design and rationale).
    - Menz, P., & Xin, C. (2016). Making Students' Metacognitive Knowledge Visible through Reflective Writing in a Mathematics-for-Teachers Course. *Collected Essays on Learning and Teaching, 9*, 155-166. http://dx.doi.org/10.22329/celt.v9i0.4426.
    - ICME-13, July 24-31, 2016 (pre-service education in mathematics) participation & paper presentation in study group on metacognition.
- Fall 2016 and Summer 2018 – CODE produced nearly 60 lightboard videos 8-12 minutes long, where I teach knowledge pieces bite-size and in a visual way; student surveys were again overwhelmingly positive and supported that these videos became the **BACKBONE** of the course.
- Spring 2017
  - Brought well-being and mindfulness into the course by first hosting an SFU Health & Counsellor presentation, which later expanded to student activities and are now embedded in the course material.
  - Adopted the CANVAS layout and all videos from the distance version in the face-to-face version to facilitate a blended learning approach – being ahead of the curve.
- Fall 2017 – Began efforts to indigenize and decolonize the curriculum and the classroom over a 2-year period
  - Developed several explorations (e.g., drum creation by Jorge Lewis from the Snuneymuxw First Nation in Nanaimo – fractions and patterning; weaving in Indigenous cultures with support from Métis 190-student Rebekah Trudel – transformations and symmetry, Coast Salish art by Dylan Thomas (Qwul'thilum) from the Lyackson First Nation of Valdes Island – tessellations).
  - Embedded relevant publications in Readings (e.g., *Mexico's Indigenous teachers fight to preserve ancient cultures in math class* by S. Nolan in the Globe and Mail, *First Peoples Principles of Learning* and *Authentic First Peoples Resources K-9* published by First Nations Education Steering Committee, *Indigenous Knowledge and Perspectives: Mathematics K–12* published by the BC Ministry of Education).
  - Engage students in discussions on Indigenous perspectives through weekly readings from *Embers: One Ojibway's Meditations* by Richard Wagamese.
  - Created an introductory and closing talking stick ceremonies to mark pivotal moments in the course.
  - Ensuing presentations and publications:
    - SFU News, October 2019, *Can it be done? A math instructor attempts to indigenize her course* published by Jackie Amsden, CEE, October 17, 2019.
    - CAIMS, June 21-24, 2021, panellist on Indigenization of Mathematics with Dr. Edward Doolittle
    - Presented at Indigenizing Math and Science Curriculum NIC 2021, May 17-21, 2021.
- Summer 2018 – Feedback from student surveys on workload and learning prompt changes
  - Overhauled 80 CANVAS pages to drastically reduce reading content by about 50% and streamline look.

  - Embedded the 14 activities in a scaffolded way in explorations to learn math through manipulatives.
  - Eliminated individual textbook assignments and created more focused discussion boards.
- Fall 2018 – MATH 190 C900 became MATH OL01, one of the first 8 instructor-led online course at SFU
- Fall 2019 and onward – instructor of MATH OL01
  - Replaced the final exam with a third midterm exam for all versions of MATH 190; i.e., after every third of the course a non-accumulative midterm exam is held.
- Fall 2020 – Replaced the three in-person midterm exams with online open-book exams using Canvas Quizzes.
- Spring 2022 – MATH 190 D100 officially became B100
- Spring 2021 – Led Partners in Design project
  - Design partners included 6 undergraduate students who had previously taken the course, 2 grad students in their roles as Q Workshop TAs and RAs, Coordinator of Workshop Operations Adam Dyck, and Drs. S. Burrill, J. Niezen, and myself as frequent MATH 190 instructors.
  - Outcomes were the migration of online assignments from LON-CAPA to Canvas Quizzes; a drastic restructuring of the Canvas course container by adopting modules for easy flow of content and assessments; the addition of learning outcomes to all lightboard videos for easy information finding; and the creation of a syllabus that was information-rich but not overwhelming by grouping information through colours and using expandable text behind appropriate headings (what haunted me above was finally addressed).
  - Accomplishments are highlighted in the diagram below:



  - Ensuing presentations and publications:
    - 13th Annual Teaching and Learning Conference *Fostering Partnerships in Pedagogy* at University of Waterloo, April 27-28, 2022, co-presented *Partners in Design - Involving all Stakeholders in the Restructuring Process of an Online Course*.
    - SFU Teaching Matters Seminar, February 13, 2023, co-presented with Dr. Joanna Niezen *Partners in Design: Involving all Stakeholders in a Course Restructuring Process*. (geared locally and additionally included course survey results from subsequent semesters)
    - FYMSiC (First-Year Math & Stats in Canada) Online Meet Up, February 28, 2023, co-presented with Joanna Niezen *Partners in Design: Involving all Stakeholders in a Course Restructuring Process*. (geared nationally and additionally included course survey results from subsequent semesters)
    - SFU News, March 23, 2023, *Math anxiety eased: SFU instructors collaborate with students to redesign a Canvas course.*
    - Paper for 21st ISATT Biennial Conference *Quality Teaching for a More Equitable World*, June 30 - July 4, 2025.

The MATH 190 course consists of 12 topics, for each of which I developed one to three activities, some of which are Indigenous-based and the rest purely mathematical. I have chosen two activities from each type as samples, because these activities are prime examples of how students are engaged in this course through a scaffolding of learning, historical snippets when appropriate, worksheets, interactive resources, and concept video clips. Students have mentioned numerous times how this approach helps them overcome their anxiousness about mathematics, because mathematical learning becomes accessible, rewarding and contextualized as real and useful.

SAMPLE 1

Métis Fingerweaving as Springboard for two Indigenous-based Activities

The first sample showcases how Métis fingerweavings are a springboard for two indigenous-based activities that literally weave through the course because they lend themselves well to the study of patterns and exhibit frieze patterns. In the topic 04 Whole Number Operations students engage with pattern recognition and analysis and in the topic 11 Transformations: Congruency & Similarity students perform and identify rigid motions (reflection, rotation, translation, glide reflections). Furthermore, these two activities reflect that mathematics instructors need to provide students with opportunities to physically and visually engage in mathematics to learn it. Highly influential math education researcher Boaler and neuroscientist Chen point out that “the neurobiological basis of mathematics cognition involves complicated and dynamic communication between the brain systems for memory, control and detection and the visual processing regions of the brain”, so it is of importance that students “be given opportunities to develop their own [gesture schemes]. We gesture because we see, experience and remember mathematics physically and visually, and greater emphasis on visual and physical mathematics will help students understand mathematics” (Boaler, J., Chen, L., Williams, C., & Cordero, M. (2016). *Seeing as understanding: The importance of visual mathematics for our brain and learning.* https://www.youcubed.org/wp-content/uploads/2017/03/Visual-Math-Paper-vF.pdf).

The activities were developed in the Fall of 2017 with the support of Rebekah Trudel, who is Métis and a former MATH 190 student. Rebekah further contributed to the course by also weaving a Métis sash that is now used to bring into the classroom for viewing and pattern analysis.

The headings in gray separate the two connected activities by topic. The activities are also interspersed with links to additional resources for students most of which I developed to support their learning but are not added to the Appendix.

Work through Explorations

- ▸ Activity - Matching Conceptualizations (Addition/Subtraction)
- ▸ Interactive Problem Solving
- ▸ Student(s)-Instructor Video Clip
- ▾ Activity - Discover Patterns and Rigid Motions in Métis Fingerweaving

The following activity guides you through the creation of a fingerweaving, pattern recognition, pattern recording, and identifications of rigid motions (reflection, rotation, translation, glide reflections) in fingerweaving patterns.

- **Broad Introduction to and History of Weaving**:
  - Native Americans cultivated weaving into a fine art over many thousands of years. For example, in Florida's peat bogs a 6,000 to 8,000 year old fingerwoven piece was unearthed. Materials such as wool, beads, hide, plant fibers or bark were used to create belts, sashes, pouches, headbands, baskets, mats and other clothing, art or household objects.
  - Weaving is differentiated by either using *fingers* or a *loom*.
  - For example, the Coast Salish, Ojibwe, Mohawk and Huron are well known for their weaving among the Indigenous people in Canada.
  - The Métis, a Canadian mixed race, practice both fingerweaving and loom weaving to create their traditional sashes, which are the focus of this activity. Look under *Readings >> Optional* to find out more about the cultural significance, uses, traditional colours, and historical background of sashes.
  - In an effort to revitalize the ancient tradition of fingerweaving, Métis textile artist Pat Adams creates sashes, artist Sharon Ensminger makes Cherokee-style instrument straps, and Ojibwe Dennis White known as Fancy Sky designs bags [inset image].
- **Finger- and Loom-Weaving Patterns**:
  - Patterns are classified as basic or advanced and the more advanced weaves incorporate a variety of elements.
  - Only watch the first 3 minutes of the following video and notice the three advanced patters that are being introduced: **arrowhead**, **diamond**, **diamond arrowhead**.

    Advanced Métis Finger Weaving

- **Create a Basic Fingerweaving, Record the Pattern, and do some Math:**
  - In many countries around the world, friendship bracelets are being fingerwoven.
  - Watch the video below and take note of all the mathematical terminology that Carol James is using while explaining the weaving before you check the solutions .
  - Basic Finger Weaving Method

- Prepare two pieces of masking tape, one short stick (popsicle stick, chop stick, etc.), and four 1-foot long strands of one colour wool and four 1-foot long strands of another colour wool.
- Watch the video again or use the handy instructions to follow along with your wool strands to make a friendship bracelet. Take particular note of how the strands travel so you can do the next activity.
- Print the table for recording strand positions . Label the red strands R1, R2, R3, R4 from left to right and similarly for the white strands W1, W2, W3, W4. Use the strand labels to record row by row the positions of the 8 strands after one strand has travelled to its new position and strands are rearranged according to Carol James' STAR-method. You can stop recording once all strands are back in their starting position and check against the recorded table .
- Answer the following questions based on your weaving and your recorded table before you check the solutions .

- **BC Curriculum Connections:**
  - Check out page 6 for possible mathematics curriculum connections through Indigenous weaving from the Aboriginal Mathematics Inquiry Team in the Burnaby School District.
  - Check out Jessica Johnson's Grade 2 lesson plan on Métis fingerweaving and math, which she developed as an Aboriginal Success Teacher from the North Vancouver School District. Notice the variety of ways that patterns can be recorded.

▸ Interactive Explorations
▸ Activity - Halloween Counting
▸ Movie: The Story of One

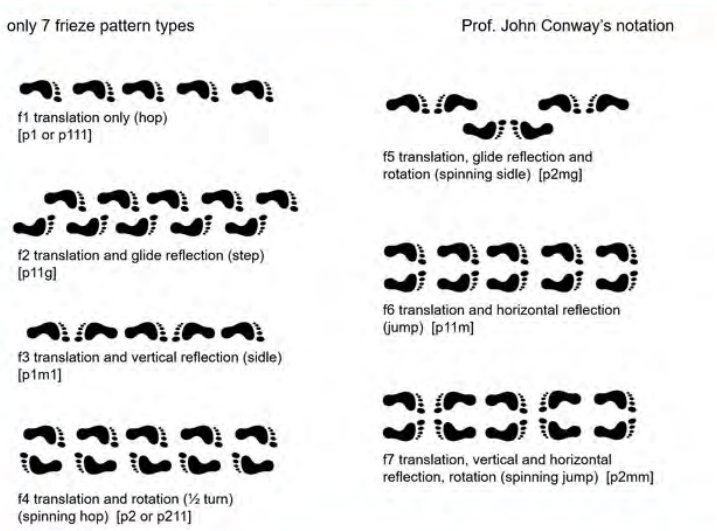
Transformations: Congruency & Similarity

Work through Explorations

▸ Interactive Explorations
▸ Video: All About Scaling
▸ Linear Transformations (Rigid Motions)
▸ Activity - Rigid Motions
▾ Activity - Discover Patterns and Rigid Motions in Métis Fingerweaving (revisited)

Recall the fingerweaving, pattern recognition, and pattern recording you performed in *Activity - Discover Patterns and Rigid Motions in Métis Fingerweaving*. The following activity guides you through identifications of rigid motions (reflection, rotation, translation, glide reflections) in fingerweaving patterns.

- **Study Fingerweaving Patterns for their Rigid Motions (great exam-level question):**
  - A fingerweave pattern is just like a mathematical frieze (an infinite strip of a repeating pattern). There are only 7 types of frieze patterns as shown with Professor John Conway's notation: (Side note: the abbreviations in the square brackets refer to the possible symmetry groups, but this is not part of the course and you can ignore that info; the important info is the visual and the description. However, here are more explanations, if you are curious.) Here is an example based on the arrowhead pattern.



  - Choose 2-3 patterns and suppose the patterns go on indefinitely like a frieze. Describe the rigid motions that exist in your pattern choices similarly to the above example.
- **BC Curriculum Connections** (reminder):
  - Check out page 6 for possible mathematics curriculum connections through Indigenous weaving from the Aboriginal Mathematics Inquiry Team in the Burnaby School District.
  - Check out Jessica Johnson's Grade 2 lesson plan on Métis fingerweaving and math, which she developed as an Aboriginal Success Teacher from the North Vancouver School District. Notice the variety of ways that patterns can be recorded.

▸ Activity - Tessellations

SAMPLE 2

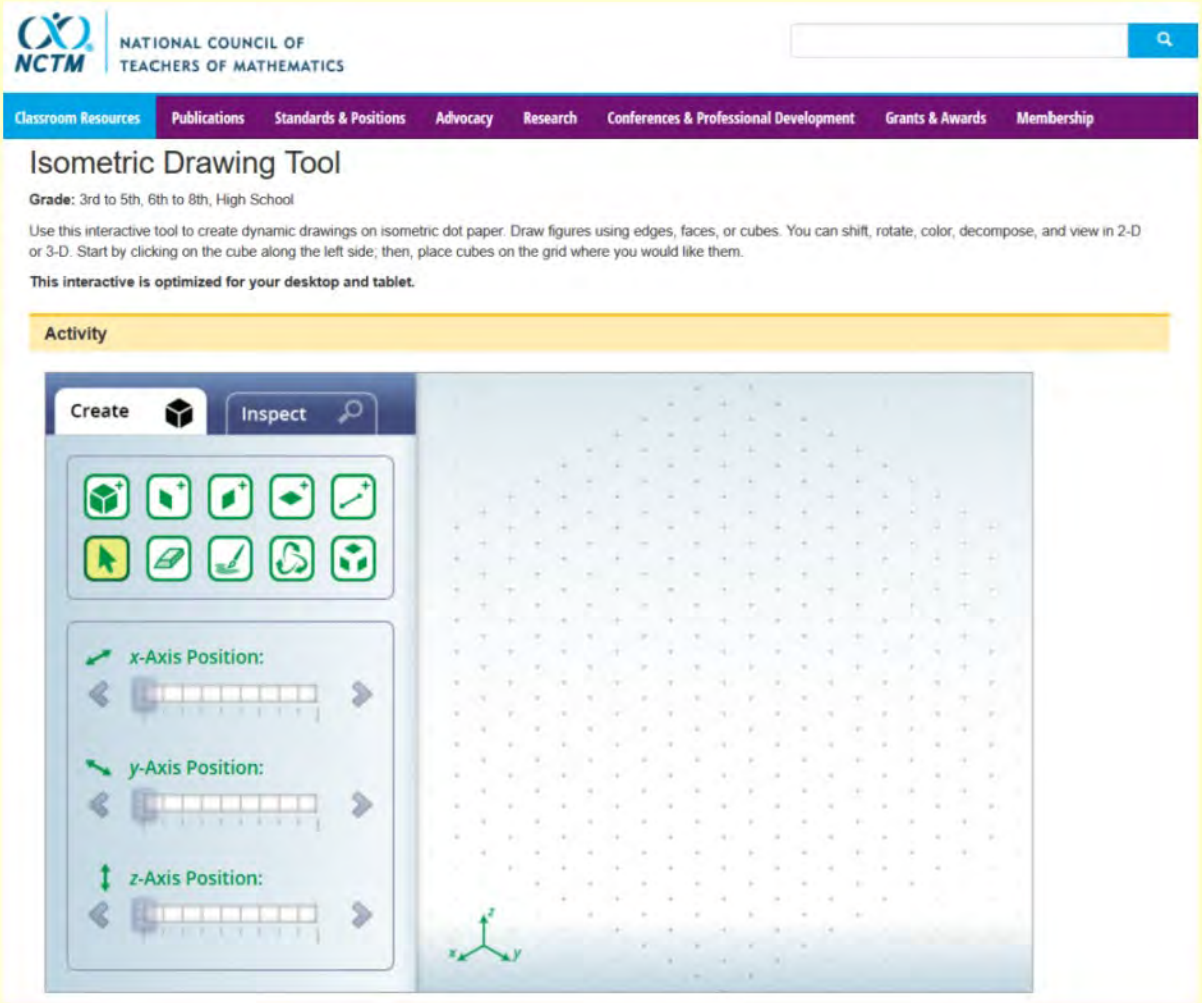
Activity – Centimetre and Isometric Dot Paper

The second sample is situated in the topic 09 Polyhedra and reiterates the structure of activities, which is an important design choice to set up students for routine approaches to support learning.

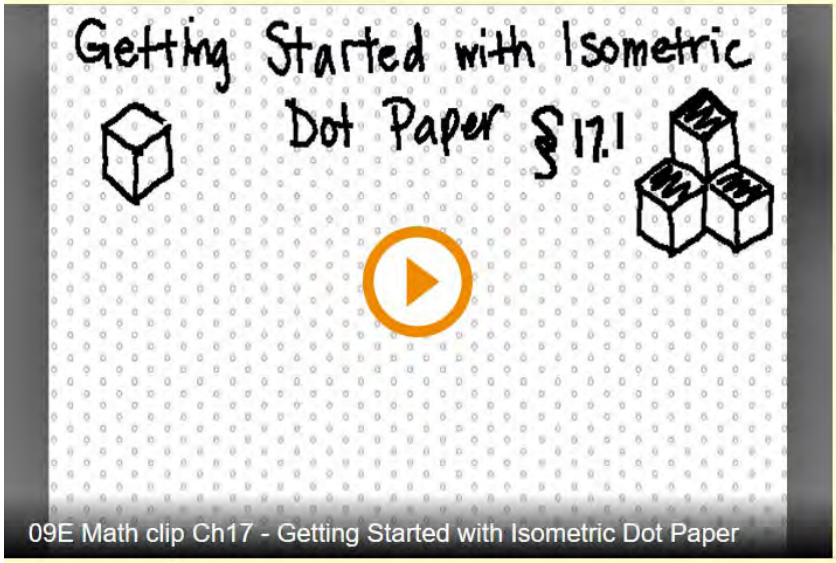
Furthermore, just like with the first sample, and in fact all my activities, you will notice how an embodied approach is taken to learning mathematical concepts by having students explore through manipulatives (the cubes) or a digital tool that simulates the manipulatives. Researchers of embodied cognition point out that when we engage in mathematics our gestures, body postures, and gazes are evidence of how the motor and perceptual areas of our brain process and retain mathematical ideas (Nemirovsky, R., Rasmussen, C., Sweeney, G., & Wawro, M. (2012). When the classroom floor becomes the complex plane: Addition and multiplication as ways of bodily navigation. *Journal of the Learning Sciences, 21*(2), 287-323. doi:10.1080/10508406.2011.611445). This approach is an outgrowth of my doctoral research completed in 2015 that is captured in my CV through 1 monograph, 3 journal publications, and 9 presentations.

Work through Explorations

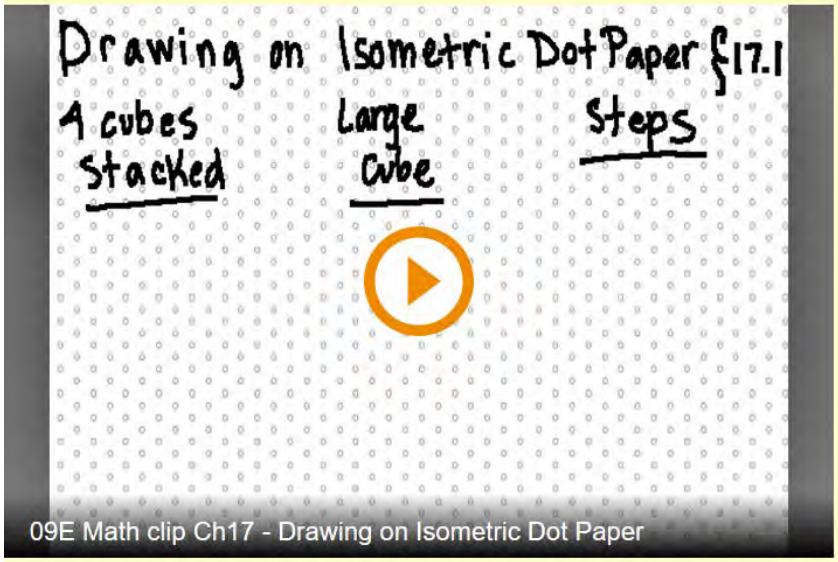
- ▸ Activity - Platonic Solids
- ▸ Interactive Explorations
- ▸ Flatland: The Movie
- ▾ Activity - Centimetre and Isometric Dot Paper
  - This activity is meant for you to practice 2D and 3D drawings of objects that are made up of cubes. You may be familiar with drawing cubes, but try this out on centimetre and isometric dot paper, where one needs to consider perspective as well.
  - Work through the Instructor Material on Practicing with Centimetre and Isometric Dot Paper .
  - Print Centimetre and Isometric Dot Paper and use it for this exploration.
  - Use this digital tool Illuminations: Isometric Drawing Tool or bring out your cube manipulatives to explore this activity.



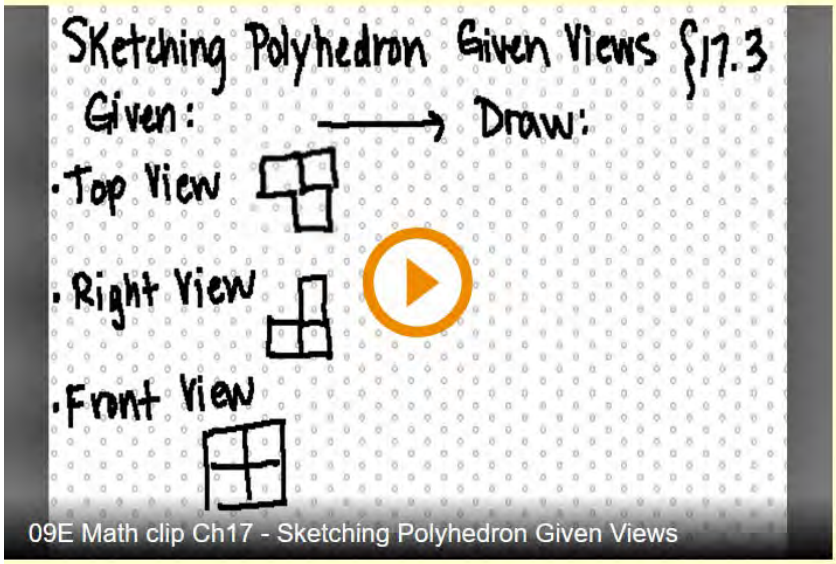
Math clip: Chapter 17 - Getting Started with Isometric Dot Paper



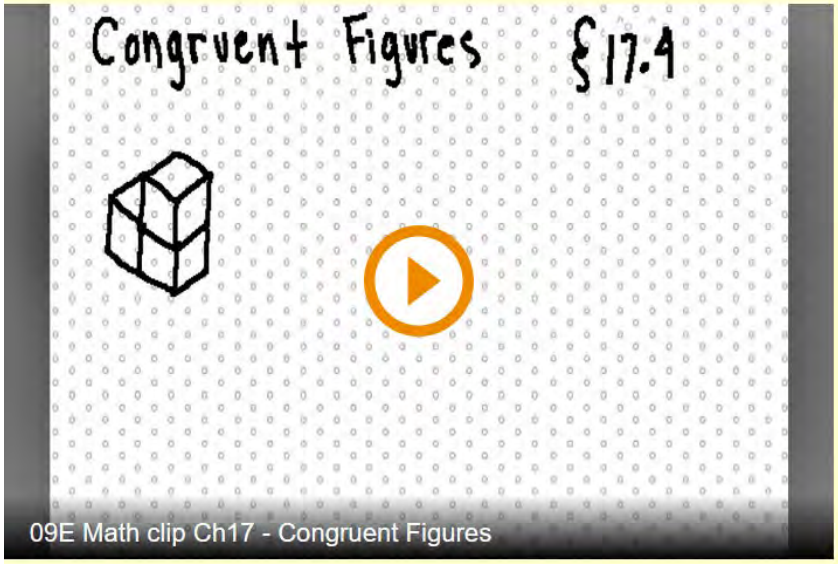
Math clip: Chapter 17 - Drawing on Isometric Dot Paper



Math clip: Chapter 17 - Sketching Polyhedron Given Views



Math clip: Chapter 17 - Congruent Figures



SFU

CENTRE FOR EDUCATIONAL EXCELLENCE

October 17, 2019

Can it be done? A math instructor attempts to indigenize her course

By Jackie Amsden, Centre for Educational Excellence

Petra Menz (senior lecturer, mathematics) revamped her first-year math course to integrate Indigenous knowledge and pedagogy as a way to enhance student learning and well-being.

"Math is math. What's there to indigenize?"

Petra Menz, a senior lecturer in the Department of Mathematics, admits that this was her first thought when she read the calls to indigenize and decolonize curriculum and teaching in the SFU Aboriginal Reconciliation Council's 2017 report, *Walk This Path With Us*.

Now, two years later, Menz regularly uses Métis weaving, Coast Salish artwork, Ojibwe poetry

Teachers course.

So, how did Menz shift her perspective from feeling reluctant about the idea of decolonizing and indigenizing her courses, to embracing it? Step. By. Step.

“As an instructor, I’m always asking myself how can I improve student learning and enhance their well-being? I wasn’t sure what decolonizing and indigenizing would mean for me, but I also knew that if there was a chance that it would contribute to those goals, I couldn’t just ignore it.”

Small steps, inside and outside the classroom

Menz says her journey began with modest steps.

“It can feel like a mountain at first, so I started small, with a land acknowledgement in class that was connected to some of the material we were working through.”

Menz also began collecting research articles and resources and attending events and workshops organized by groups in her field like the Canadian Mathematics Education Study Group.

“Going to these events was a very emotional experience because my whole belief system was shaken up. I would look back and question all of my past actions and decisions. This can be very scary, but I believe this is necessary for change.”

Menz explains that these experiences eventually helped her find a path forward.

“I remember someone suggesting I should just bring in a canoe and talk about the math of it, but that didn’t seem right. Instead, what I have done is weave Indigenous material and Indigenous ways of learning and teaching—such as group learning, interactivity and reflection—throughout the course, so that it is integrated deeply and not just on the surface.”

Menz points to the talking stick ceremony as one impactful example of what this integration looks like.

“We do the ceremony during the last class. Each student has a turn to hold the stick, which is actually a shell, and share the struggles they have overcome and what they have learned during

community and have taken a journey together."

An enriched learning space for students

The results for students, she notes, have been profound.

"Many of my students have expressed being deeply appreciative that Indigenous knowledge is out in the open and is not just being used in a token way here and there. I feel that they are all benefitting because bringing in other perspectives enriches the experience and equips them to be more creative in their problem-solving approaches."

Menz's next challenge? Decolonizing calculus.

"I'm teaching a calculus course in Fall semester and I know that is going to be more challenging to indigenize, not because of the complexity of content, but the complexity of the context. The classes are much larger, with 300 to 500 students in one lecture, and with more colleagues teaching the course that I will need to get on board."

Related links

- Petra Menz's faculty bio page

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Tags

Institutional Initiatives, Teaching Practice, Indigenizing and Decolonizing

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March 23, 2023

Math anxiety eased: SFU instructors collaborate with students to redesign a Canvas course

Ever wondered how to (re)design your course's Canvas shell so that it enhances learning and reduces confusion? Senior math lecturer Petra Menz did. To find the answer, she launched a Canvas redesign project for the blended and online offerings of her Math 190 course in collaboration with the people who had the most experience navigating them: her students.

"Students may not be experts in the course material, but they are experts in what they need. Involving them in identifying problems and developing solutions was eye-opening. And we are already seeing the impacts. Now when they come to the Help Centre, their questions are about content instead of where to find things. Seeing that change has been really satisfying," says math lecturer Joanna Niezen, who was also involved with the project.

The project engaged student feedback prior, throughout and following the redesign via surveys, roundtables and interviews. In addition to Menz and Niezen, the project team included lecturer Sophie Burrill, research assistants Sheena Tan and Kailyn Pritchard and a staff member, Adam

Menz selected Math 190 for redesign because it typically draws students from non-mathematical backgrounds who may already be struggling with math anxiety. She felt this group could most benefit from an improved online Canvas design.

What did the students have to say?

For Niezen, one of the simplest but most impactful insights was understanding how students interact with Canvas.

“Students were complaining they couldn’t find important content, like problem solutions, and I couldn’t understand why, until they pulled out their phones. I didn’t realize that so many of them are viewing our course on their phones and when they do that, they can’t see the homepage. Like it or not, students are engaging with our courses on smaller and smaller screens, and we have to cater to that.”

One of Menz’s key insights was the importance of keeping Canvas design consistent across the university.

“You might have an idea of a different way to lay things out in your course but that can cause unnecessary challenges for students who are taking multiple courses. It’s important that instructors follow some standardized layouts so that students find what they need more easily and focus on learning the material rather than spending time searching for it. For example, by putting all the content in modules that step students through their various learning material as the term progresses, an instructor not only helps students navigate through Canvas but also to create familiarity and routines.”

Menz adds that the process also revealed ways to design her Canvas shell that can better support students with learning disabilities.

“One of the students who has autism and struggles with time management said putting a table on the homepage outlining the course timeline really helped them keep up with assignments.”

The power of student voice

“Students can really be empowered by being asked to bring their voices and perspectives into the conversation. Having the students participate in the entire process allowed them to really engage with what we were trying to do. They gave thoughtful feedback and shared great ideas. By including them in the project, I believe we made impactful changes that address student needs and create a more inclusive and effective learning environment.”

If you are interested in redesigning your online or blended course, submissions to the Centre for Educational Excellence’s Full Course Design and Development intake close April 3, 2023.

If you are interested in conducting an inquiry project to examine the impact of teaching interventions, such as a course redesign, contact Transforming Inquiry into Learning and Teaching at tilt@sfu.ca to book a consultation.

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Excellence in Teaching Awards

November 06, 2024

SFU Faculty of Science is proud to announce this year's recipients of the 2024 Faculty of Science Excellence in Teaching awards, a list that includes faculty members and graduate students.

FACULTY TEACHING AWARDS (HONORARIUM OF $1000)

Petra Menz (Mathematics)

Dr. Menz has done exceptional work in developing the curriculum of and teaching MATH 190 to ensure that mathematics learning is accessible to students who will go on to become elementary school teachers. She is recognized for her efforts in respectfully and authentically decolonizing and Indigenizing her courses.

INTRODUCTION

This document is a summary of accomplishments over a three-year period from May 2017 to August 2020 creating an Open Education Resource (OER) for the following two courses offered by the SFU Department of Mathematics:

- MATH 157 *Calculus I for the Social Sciences* (Differential Calculus)
- MATH 158 *Calculus II for the Social Sciences* (Integral Calculus)

As a side note, from 2007 to 2008, I co-authored the textbook listed below, which subsequently was used in MATH 157/158 until the arrival of the OER that I created. However, over the years I became increasingly disillusioned with publishing companies seeking to profit from students. Starting around 2015, I became more and more aware of OERs but couldn't find a suitable OER for social science calculus. When SFU offered their first OER grants, I was ready to take on the challenge of adopting and majorly adapting an existing calculus OER.

- Tan, Soo Tang, Petra Menz, and Dan Ashlock. *Applied Calculus for the Managerial, Life, and Social Sciences*. First Canadian Edition. Toronto: Nelson Education, 2009.

FUNDING & ASSISTANCE

To cover the cost of hiring a research assistant (RA), I successfully secured the following funding. This is no small feat for a teaching faculty member as funding sources are scarce for this faculty group and the annual institutional Professional Development Reimbursement of $1910 allocated to each faculty member may not be used to pay RAs.

May 2017 - April 2018	$5000 SFU OER Grant $5000 FIC Teaching & Learning Fund, Department of Mathematics
May 2018 - April 2019	$5000 SFU OER Grant $3000 FIC Teaching & Learning Fund, Department of Mathematics
September 2019 - August 2020	$6480 FIC Teaching & Learning Fund, Department of Mathematics.
August 2020	$1500 FIC Teaching & Learning Fund, Department of Mathematics.
TOTAL	**$25,980**

From May 2017 to April 2019, I was given in-kind assistance from education consultant Dr. Cindy Xin, copyright specialist Jennifer Zerkee and teaching and learning librarian Hope Power. The expertise that each of them brought to the table ensured a high-quality product.

I selected Nicola Mulberry, then an M.Sc. graduate student in the Department of Mathematics at SFU, as the research assistant (RA) for this project because I supervised her as a Teaching Assistant and knew she was outstanding. Nicola was an extremely valuable collaborator on the OER developments. Because of her mathematical and LaTeX expertise, Nicola was able to find solutions for the many difficult aspects in the actual production of all material such as graphical insertions, creation of tables, consistent use of text elements such as example, definition, theorem and guidance boxes that are of high quality and bring the text closer to the reader. Furthermore, Nicola was also a creator of exercise material and the instructor behind the integral-based instruction videos that are embedded in the PreTeXt version.

ADOPTION & ADAPTATION

The OER is based on the adoption of Calculus Early Transcendentals an Open Text (VERSION 2017-REVISION A) by David Guichard through Lyryx Learning, which is an educational software company founded in 2000 at the University of Calgary, Alberta.

To produce LaTeX material for users in the mathematical community, the adaptation process required

1) streamlining the text, which was clearly written by multiple authors,
2) setting styles to be incorporated throughout the text,
3) editing to follow the suggested stylings and spellings for a "Canadian" textbook as outlined in Measurements and Canadian Spellings and Word List in the Self-Publishing Guide listed on the BCcampus website,
4) making it readable by the intended audience,
5) adding applications specific to the social science stream,
6) creating high quality graphs and visuals throughout the text,
7) enriching the limited exercises and their answers,
8) adding examples and their full solutions,
9) replacing American references with Canadian content and examples, and
10) removing and adding content to adhere to the departmental course curriculum and the guidelines set out by the British Columbia Committee on the Undergraduate Program in Mathematics and Statistics (BCCUPMS) for transferability among BC institutions.

Additionally, to create the first of its kind PreTeXt online calculus textbook based on the above LaTeX textbook, the production process required

11) making solutions to examples as well as exercises and their answers collapsable,
12) creating and embedding user-friendly interactive applets for exploration standard calculus concepts and
13) creating and embedding instructional videos that work through examples.

CALCULUS OER

Calculus Early Transcendentals: Differential & Multi-Variable Calculus for Social Sciences

Calculus Early Transcendentals: Integral & Multi-Variable Calculus for Social Sciences

Additionally, the following course materials were created tightly linked to the two textbooks (internally referred to as course notes).

- **Full Solution Manual:** The course notes for each course contain exercises and final answers to most (but not all) of the exercises, not including the full solution. It is my experience that students study with complete solutions not only to check their work but also to learn how to write up the mathematical solution process in its entirety, so a stand-alone solution manual accompanies the course notes.
- **Lecture notes:** These are ready-made slides that closely match the course notes, but in reduced format and with additional examples and applications, to be used by instructors to prepare for teaching a certain topic when a student-centered approach is chosen, to assign as reading for the students, or to lecture from. It was important to not force a certain teaching approach on to these notes, so that they are highly transferrable. The main ideas behind providing lecture notes in ready-made format, which can be adopted by any instructor, is (1) to maintain consistency of course curriculum coverage across terms and across locations of course offerings, and (2) to also support faculty new to these courses as well as sessional instructors, so that they can concentrate on the delivery of the courses rather than the development of notes as is typical. It is my hope that this will free the instructor to offer a student-centered approach such as a flipped classroom or use of clicker questions in their teachings.
- **Student notes:** These are skeleton notes that strongly match the lecture notes, but most importantly the Cornell note-taking system, studying guides, and comment space are super-imposed on to these student notes. Since the core participants in these courses are students who are fresh out of high

school, the idea behind these notes is to guide these students not only through the curriculum, but also to present to them from the outset a strategy for taking notes, how to reflect on these notes, and how to use them for studying purposes.

- **Assignments and solutions:** These consist of 12 weekly assignments and complete solutions per course that are not intended to be marked but rather aimed at practicing the calculus concepts and tools taught in that week.

Both texts, the lecture notes and students notes, including the art and illustrations, are available under the Creative Commons license (CC BY-NC-SA), allowing anyone to reuse, revise, remix and redistribute the text. For a more detailed description visit the copyright page of each product.

The unique features about this OER are four-fold:

1) Student notes are designed to teach students note-taking and studying skills in addition to the curricular content of the course.
2) Ready-made teaching package:
    a. New faculty or sessional instructors can step in to teach these two courses without being burdened by notes and resource developments but rather concentrate on their approach to teaching.
    b. Seasoned instructors can bring their application/modelling expertise to the foreground.
3) Tight link between course notes, lecture notes, student notes, and instructional videos as well as paper-and-pencil and online assignments:
    a. Allows an instructor to use any student-centered approach to their teaching such as a flipped classroom or blended learning.
    b. Ideal for construction of an online course around the notes, which is one of the next steps in this curriculum development and course enhancement project.
4) Through the adaptation, the original OER has been vastly improved in terms of flow, consistency, stylistically, use of colours throughout the text, and quality of visuals. In addition, the content has been adjusted for the social science stream. This sets our work apart from the few other calculus open resources currently available. Furthermore, our OER stands out by its coverage of the complete first year differential and integral calculus for the social sciences.

IMPACT

The average cost of a calculus textbook is $200. Since there are roughly 2200 students affected per year across Simon Fraser University and Fraser International College, this means savings of $440,000 per year.

This OER can be adopted by any post-secondary institution as well as secondary schools for their calculus courses for the following reasons:

1) BCCUPMS sets out well-established required and optional topics in a variety of mathematics courses that are adhered to by post-secondary intuitions across BC to ease transfer of these courses by students from one post-secondary institution to another in BC. This OER includes all required topics and provides a variety of optional topics so that this OER is attractive to any mathematics department that offers specialized calculus courses for their social science students.
2) Since calculus is a bridging course from high school to university/college, both stake holders the British Columbia Association of Mathematics Teachers (BCAMT) and the Department of Mathematics at Simon Fraser University want to ease transition for students. As a former high school teacher, I designed this high-quality open resource for ready use by any secondary and post-secondary teacher keeping very much in mind that the resource can also be readily used by students.

Fraser International College (a private educational institution in partnership with SFU, Burnaby), Columbia College (New Westminster), and Alexander College (Vancouver) have adopted this OER.

PRESENTATIONS AND PUBLICATIONS

- AMS MathFest, August 1-4, 2018, Nicola Mulberry presented *Entwining Research Expertise, Calculus Knowledge, and Teaching Skills* about her RA involvement in this OER project and the importance of OER for math education.
- CMS Winter Meeting, Vancouver, Canada, December 7-10, 2018, co-organized education session with Drs. Kseniya Garaschuk (University of the Fraser Valley) and Andrew Hare (Saint Mary's University) *Educational Resources in Mathematics*.
- CMS Winter Meeting, Vancouver, Canada, December 7-10, 2018, Nicola Mulberry presented *Experiences Creating and Teaching from Open Calculus Material* to showcase our OER and challenges we encountered.
- 15th International Conference of the Mathematics Education for the Future Project: Theory and Practice: An Interface or A Great Divide?, Maynooth, Ireland, August 4-9, 2019. Presented the paper *Open-Source Differential and Integral Calculus Material Development to Support Student Accessibility and Learning*.
- SFU News, January 2021 – *This math lecturer developed her own open textbook—now thousands of students are using it*
- FYMSiC News, February 2021 – *An OER for Differential and Integral Calculus for Social Sciences* (article in a national teaching newsletter)
- BCcampus Award for Excellence in Open Education, March 2021

January 29, 2021

This math lecturer developed her own open textbook—now thousands of students are using it

By Chloe Riley, SFU Library

Senior lecturer Petra Menz (left) and PhD student Nicola Mulberry adapted an open educational resource to produce an interactive textbook that is now used by thousands of students.

In 2018, **Petra Menz**, a senior lecturer in mathematics, was facing a problem: the textbook for two required courses in the social sciences calculus stream was going out of print.

Fueled by a desire to create a contemporary and free textbook for both students and instructors, Menz began working on an ambitious open education project, funded over two years by SFU

designed, technologically enriched open textbook and accompanying teaching materials.

An interactive textbook that students love

Menz and Mulberry worked with **Hope Power**, teaching and learning librarian, to identify an open educational resource that could be adapted for MATH 157 and MATH 158. They found the open textbook *Calculus Early Transcendentals*, which provided a framework for them to build on in several key ways.

Menz reorganized and substantially expanded the original textbook, increasing the emphasis on student-focused learning such as problem-solving and real-world applications. Menz and Mulberry adapted the written resource into an interactive online textbook and developed a colour-coded design that focused on readability, usability and consistency. They also collaborated on a suite of supporting materials, including lecture notes, student notes, exercises and assignments.

The integration of high-quality visual elements was essential to supporting students' understanding of mathematical concepts. In addition to figures and examples, interactive demonstrations embedded in the new learning materials allow students to explore and practice hands-on with 2D and 3D models of foundational mathematics concepts.

Student feedback on the new course material has been overwhelmingly positive, and students consistently rate the usefulness of the learning materials highly. "Extremely helpful and much more clear than my past experience," shared one student.

Menz and Mulberry also developed over 90 concept videos that are embedded alongside examples in the text, allowing students to further deepen their understanding.

An adaptable format and savings of $440,000 annually

From the beginning, Menz was interested in creating a resource that would be free and that could be used by multiple instructors at SFU and beyond.

The textbook was adopted by the mathematics department in Fall 2018. The average cost of a

Both Menz and Mulberry encourage their colleagues to explore options for open educational resources and emphasize the importance of building a network of support, including drawing on expertise from within the SFU Library and the Centre for Educational Excellence.

They also see the value of their open textbook extending beyond SFU.

“This format is so powerful in that way,” shared Mulberry. “We can put it out there, and other people can build on it.”

An enriching experience for the collaborators

The collaboration was a positive experience for both Menz and Mulberry.

In addition to building her technical skills, Mulberry found the experience developing and scaffolding educational materials invaluable in preparing her to teach the material as a sessional instructor.

Menz also found the experience rewarding. As teaching faculty, she rarely has opportunities to supervise graduate students. “Being able to guide and shape the learning of a graduate student and provide her with skills she can use for future employment is a satisfying experience for myself as a lecturer,” she said.

Related links

- View the textbook for MATH 157.
- View the textbook for MATH 158.
- Petra Menz’s faculty bio page.

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An OER for Differential and Integral Calculus for Social Sciences

Petra Menz, Simon Fraser University

An OER for Differential and Integral Calculus for Social Sciences has been published at SFU complete with teaching material by Senior Lecturer Dr. Petra Menz and mathematics PhD student Nicola Mulberry.

In 2017, we adapted an existing OER and substantially overhauled it. Over a period of almost three years we enriched the source by adding topics, exercises, interactive demonstrations and concept videos, and streamlined and colour-coded the writing with a focus on readability, usability and consistency. We also split the OER into two texts available in LaTex and PreTeXt to accommodate differential and integral calculus. In addition, we developed accompanying teaching materials: lecture slides and skeleton student slides. The differential OER has been in use since fall 2018 and the integral OER since spring 2019. All of the material is published under a Creative Commons license (CC BY-NC-SA). Contact Dr. Petra Menz for the source.

Resources: OER - Differential Calculus for Social Sciences
Read More: SFU published an article!

BCcampus Award for Excellence in Open Education: Petra Menz and Nicola Mulberry

MARCH 24, 2021 FISCAL 2020-2021, GENERAL, HOMEPAGE, OPEN EDUCATION

This month we are happy to present the BCcampus Award for Excellence in Open Education to *two* deserving recipients: Simon Fraser University's Petra Menz and Nicola Mulberry.

Nominated by Hope Power, teaching and learning librarian, Learning and Instructional Services, Simon Fraser University (SFU)

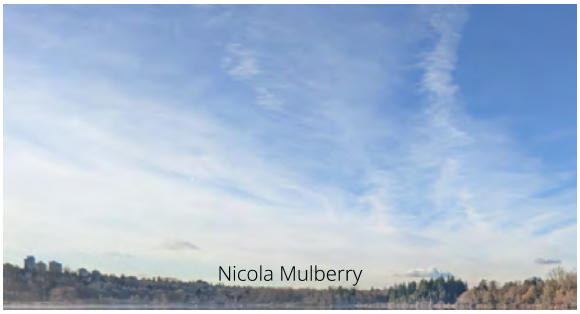
Petra Menz

Petra Menz, senior lecturer in mathematics at SFU, is a dedicated open-education leader and practitioner on campus. In collaboration with Nicola Mulberry, her research assistant and mathematics PhD student, Petra successfully adapted and expanded an open textbook that was recently adopted by the mathematics department for use in two large first-year calculus courses at SFU and Fraser International College (FIC). In addition to saving students thousands of dollars on required textbook costs each year through this open educational resources (OER) work, Petra is a notable faculty champion who encourages and supports colleagues with exploring and integrating OER in their own classrooms.

"This award needs to be shared with my RA, Nicola Mulberry," said Petra Menz. "While I had the vision and created the new mathematics material, Nicola was the one who coded all the information and later on created all the visuals. Furthermore, we shared production of all the videos: she took on the concept videos for MATH 157 while I tackled MATH 158. Without Nicola's technical expertise, I would not have been able to move this OER into such a visual, versatile, and interactive online presence.

This successful, high-impact open textbook adaptation project achieved the following:

- Awarded SFU OER grant funding (2017–2018) and matching funding from the department of Mathematics
- Substantially revised and expanded existing *Calculus Early Transcendentals* textbook in collaboration with Nicola Mulberry
- Revisions included adding interactive visual and video components to text and developing supporting materials such as lecture notes, student notes, exercises, and assignments
- Adopted for use in MATH 157 and 158 at SFU and FIC in 2018
- Estimated cost savings: The average cost of a calculus textbook is $200; with 2200 students per year in MATH 157 and 158, this open textbook saves SFU and FIC students approximately $440 000 annually.



Nicola Mulberry

"While we added a *lot* of new material, I should also emphasize that it is through the adaptation of an already existing OER that these two textbooks were possible," Menz added. "The original author is David Guichard, with a few more authors contributing in bits and pieces. Like Isaac Newton said, 'If I have seen a little further it's because I stand on the shoulders of giants.' Not that I am at all to be likened with Newton; I am just a lecturer who cares deeply about curriculum and student accessibility."

Both Menz and Mulberry hope their colleagues will explore options for open educational resources and stress the importance of building a network of support. They also see the value of their open textbook extending beyond SFU. “This format is so powerful in that way,” Mulberry has said. “We can put it out there, and other people can build on it.”

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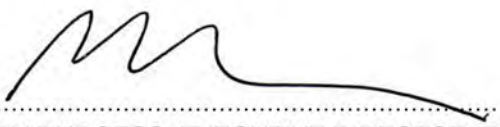

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DIAGRAMMING AND GESTURING DURING MATHEMATIZING

Petra Menz, Nathalie Sinclair

Simon Fraser University, Simon Fraser University

This paper focuses on the role of diagramming and gesturing in the process of mathematical invention. Based on data involving a group of mathematicians working on an open problem, we outline the different functions of diagramming during mathematizing. Our aim is not only to better understand the material forms of mathematical engagement, which are of interest to theories of embodied cognition, but to contribute to research on how to better support students in effectively using and making diagrams.

INTRODUCTION

Over the last two decades, researchers in anthropology, cognitive science, mathematics education and linguistics have focused explicitly on the role of gestures in cognition (Kita, 2000; Krummheuer, 2013; Lakoff & Núñez, 2000; McNeill, 2008; Radford, 2001). More recently, through the viewpoints of embodied cognition, alongside that of new materialism, gesturing has been shown to be crucial for doing and thinking mathematics (Bailly & Longo, 2011; Barany & MacKenzie, 2014; de Freitas & Sinclair, 2013; Greiffenhagen, 2014; Menz, 2015; Roth & Maheux 2015; Rotman, 2005; 2012). The innovative work of the philosopher of mathematics, Gilles Châtelet (2000), who studied the historical diagrams of mathematicians, highlights the complementary role of gesturing *and diagramming* in mathematics invention. In this paper, we apply Châtelet's theory to live mathematizing; and, furthermore, identify the different functions that diagramming play in the process of mathematical invention. These functions exemplify the material aspects of mathematizing, showing how diagrams do not merely function to represent known concepts, but become the concept through a *diagram life-cycle.*

Châtelet: Diagramming and Gesturing

In his foreword to Châtelet's (2000) book, Knoespel writes that the ancient Greek verb "Διαγραμμα in effect embodies a practice of figuring and defiguring" (p. xvi). Through this flexible view of figuring and defiguring, Châtelet brings new ideas to the study of gestures, diagrams and mathematics. By analyzing historical manuscripts of famous mathematicians such as Oresme, Leibniz and Hamilton and without access to video-recordings of these mathematicians' mathematizing, Châtelet traces their thoughts and actions from their writing and their diagramming. His fundamental insight is that the *virtual* is evoked in historical mathematical inventions "through diagramming experiments whose sources Châtelet can trace to mobile gestural acts" (Sinclair & de Freitas, 2014, p. 563). The virtual is that which is latent in the diagramming, but ontologically new. In this way, Châtelet challenges the disembodied, abstract conceptualization of mathematics. The key ideas of his theory are that (1) the diagram is never really fixed – it is erased, drawn over, reassembled, or redrawn – as it hovers between the actual and the virtual; (2) there is a mutual interaction between gestures and diagram; and (3) it is through the material interaction with the

diagram that a person understands or invents mathematics. As Roth and Maheux point out, "the virtual cannot be grasped but is that which allows grasping to occur" (2015, p. 236).

METHODOLOGY

The research subjects in this study comprise three male, Caucasian, research mathematicians in the field of Topological Graph Theory from two prominent North American universities, and are selected from a more detailed study (Menz, 2015) on diagramming and gesturing during mathematizing. The three mathematicians, referred to as Fred, Colin and Victor, met nine times over a period of three months to study the class of 2-regular directed graphs and how they embed in different surfaces. The research goal was to compile a list of obstructions for the projective plane in particular, although other surfaces such as the torus and Klein bottle were also explored at times, and to classify these obstructions for 2-regular directed graphs. All research meetings took place in one university's mathematics seminar room, well-equipped with blackboards on which all diagrams of this study were drawn. The first author was present in order to video-record these nine research meetings that varied in length from one to two hours; to capture digital still images of diagrams by the participants using a second camera; and to make field notes during the meeting that attended both to the mathematics that developed during the meeting and to observations regarding diagramming and gesturing.

Based on these observations and multiple viewings of the recorded research meetings that paid attention to the emergence of diagrams and how these three mathematicians create and engage with the diagrams, the 12 hours of video-recordings were broken up into time intervals that contained at least one diagram. A total of 122 time intervals were selected for further analysis varying in length from approximately thirty seconds to approximately six minutes, containing a total of 128 diagrams. For each diagram, a particular function was identified based on the words and gestures that were used, as well as on the broader context of the mathematicians' progress.

DIAGRAM LIFE-CYCLE

A diagram does not just simply get made, but ascertained though the data, the working on the diagram and the engagement with elements of the diagram are crucial in making the mathematics visible for the mathematicians. The mathematicians referred to the diagram by mathematical words such as "crossing land", "anti-digon", "surface" or by deictic words such as "this", "this guy", "this creature", "here", "it" to name but a few, which is evidence in itself that, for the mathematician, the diagram is not just a visual product, but also a mathematical object in its own right. While Châtelet only intimates at the whole process of diagramming during mathematical invention, the analysis of the 122 time intervals and the 128 diagrams provides a glimpse into the whole process and reveals how diagrams come into being and are engaged with. These findings enrich Châtelet's theories by enabling us to identify the different functions of interactions between mathematician and diagram in inventive mathematical processes, which are captured in the life-cycle of a diagram (see Figure 1).

The diagram life-cycle is divided into three phases, which concern themselves with (1) how the mathematician brings the diagram into being – *manufacturing phase*; (2) what the relationship is between the diagram and the mathematician – *communication phase*; and (3) how the mathematician resolves, in simplistic terms, whether the diagram stays or not – *dénouement phase*.

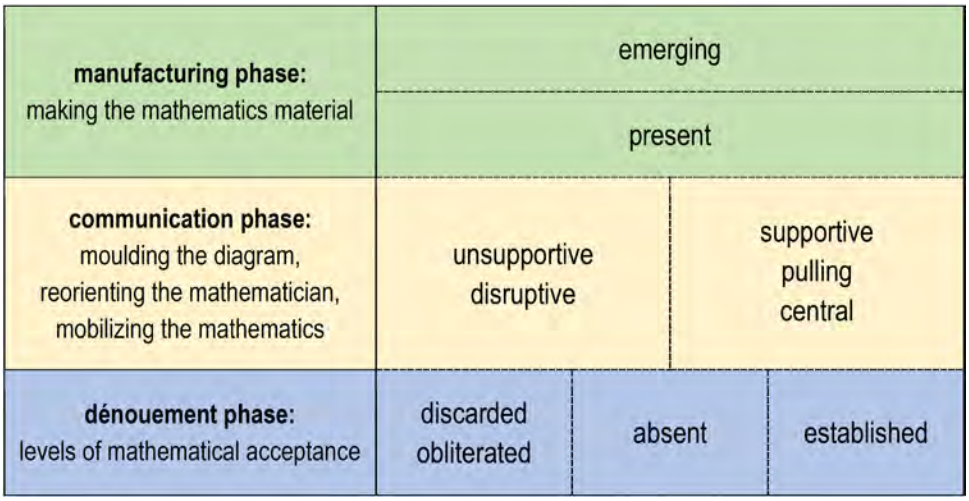

Figure 1: Episodes, phases and their relationships in the life-cycle of a diagram.

In each phase, so-called *episodes* describe a particular function of interaction between diagram and mathematician. The term *episode* is chosen because it alludes to both a period of time and a side-story being told. The analysis shows that throughout the life-cycle of a diagram, from its creation to its 'establishment' or 'obliteration', there are eleven distinct diagram episodes: *emerging*, *present*, *unsupportive*, *disruptive*, *supportive*, *pulling*, *central*, *discarded*, *obliterated*, *absent* or *established* (see Figure 1). In the interest of space, only two episodes are described here: *diagram-is-emerging* and *diagram-is-disruptive*. The remaining episodes are detailed in the work of Menz (2015).

Example 1: diagram-is-emerging

Every time a mathematician makes chalk marks on the blackboard that are not writings a diagram emerges. This diagram can consist of anything between just a few strokes to elaborate, colourful drawings. Once the diagramming mathematician steps or turns away from the diagram, the episode *diagram-is-emerging* is concluded and the episode *diagram-is-present* begins. If a mathematician adds to or erases from the original diagram, the emergence of another diagram is signaled. Five settings are identified when the episode diagram-is-emerging comes about: (1) the mathematician diagrams from scratch during his exploration, which is especially noticeable during sustained high diagramming activity such as in research meetings 3, 5 and 9; (2) the mathematician adds to or erases from an existing diagram, which alters the original diagram; (3) the mathematician re-figures a diagram by tracing the edges and vertices of an existing diagram (usually a graph), which occurred every time the mathematicians tried to find out if a given graph on the blackboard embeds in some surface; (4) the mathematician draws a known diagram by retrieving it from memory; and (5) the mathematician is directed to draw a diagram by another mathematician. Once a diagram has emerged, it is present on the blackboard, regardless of whether any of the mathematicians engage with or refer to it and no matter for how long a time period. When and how a diagram emerges and becomes present is of specific interest as it reveals how the diagram is interacted with from that moment onwards, which can lead to any of the other nine remaining episodes. In particular, the regularly occurring emergence as described in (3) throughout the nine research meetings provides evidence that the *re*-figuring of diagrams is a crucial element leading to mathematical inventions.

Figure 2 provides three different examples of diagram-is-emerging based on the same underpinning graph. Through exploration of the edges and vertices of one 2-regular orientation of C_6^2 (later

termed *reversed planar octahedron*), the three mathematicians discover an embedding of this particular octahedron in the torus. Next, they want to know if this embedding is unique and how C_6^2 can embed generally in a surface of Euler characteristic zero. For this, they need to understand how the faces of the embedding are connected to the edges of the original graph, which they refer to as identifying *closed walks*. First, Fred draws the reversed planar octahedron with yellow chalk on the blackboard from memory as described in (4) for diagram-is-emerging, but seems to be stuck finding the faces of the embedding. Then Colin joins Fred at the blackboard and asks, "may I please destroy this picture?" to which Fred replies "oh, sure". Colin adds red edges to the diagram on top of some of the yellow edges as described in (2) for diagram-is-emerging in order to identify closed walks (see Figure 2a). All the while, Fred is several times tracing through the diagram with a cupped right hand (see Figure 2d), which he places over each of the faces in the diagram as if his hand is mimicking the perimeter of the faces. During the times when he utters "red, yellow", he alternately moves his index and middle fingers up and down while at the same time moving his hand successively from face to face. Both of these are interesting gestures, as they not only embody the alternation between edge colourings and thereby virtually assemble the faces that the diagram holds, but the gestures also underline the content of Fred's utterances, which is a generalization of the process of finding the faces. Colin returns to his seat and a discussion about the closed walks ensues followed by silence. During this time, Fred labels the vertices of the red/yellow diagram and softly traces through the edges with red chalk, which can still be seen in Figure 2c.

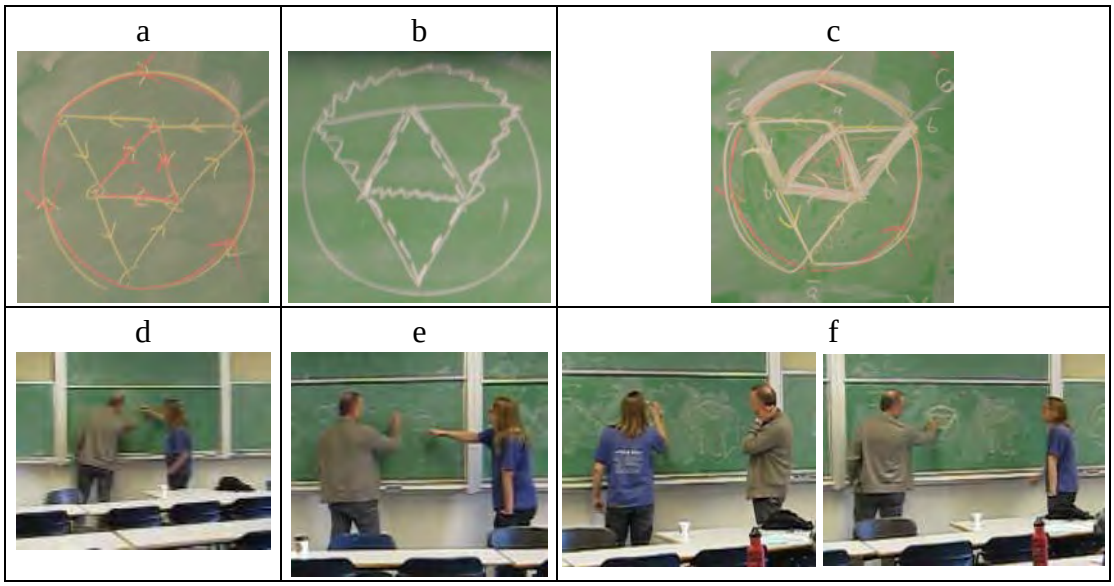

Figure 2: Examples of diagram-is-emerging.

About 20 minutes later, Colin again comes up to the blackboard and returns to exploring how these closed walks come about. As described in (3) for diagram-is-emerging, he re-figures the reversed planar octahedron with white chalk on the blackboard by covering some of its edges with squiggly and dashed lines (see Figure 2b). Again, Fred walks up to the diagram and this time traces with the index finger of the right hand over Colin's added lines (see Figure 2e). Another 10 minutes pass, during which time Fred and Colin explore the parity of the closed walks in the reversed planar octahedron using their two diagrams (see Figures 2a and 2b) to point to, trace through and touch the edges and vertices. Then Fred indicates that he finally understands how to argue that the embedding

is unique and turns to the diagram drawn in red and yellow. First he traces through the edges (see Figure 2f, left) and then adds thin and thick white lines as described in (2) for diagram-is-emerging exclaiming “let me make a total mess of this” (see Figure 2c). Afterwards, Colin traces through the edges of this diagram with his index finger to identify closed walks (see Figure 2f, right), while both him and Fred verbalize their satisfaction with their argument.

During all this time, the third mathematician, Victor, joins into the discussion at times, but also explores the graph on his own on paper. However, his diagram is ignored by the other two mathematicians. At the very end, Victor is asked by Colin “is that clear (.) so (.) Victor?’, to which Victor readily responds “yeah” and nods his head in agreement.

The diagrams in Figure 2 demonstrate the dynamic nature of diagramming in the hands of the mathematician. By drawing various types of lines (dashed, squiggly, coloured) on top of each other, the diagrams by Colin and Fred become layered, which creates a sense of plasticity. The smudges of chalk and finger prints on the blackboard within Fred’s diagram (see Figure 2c) are evidence of the erasing and touching of edges and faces, thereby speaking of the intimate engagement with the diagram. This is Châtelet’s figuring and defiguring that is embodied in the diagramming; and it is this defiguring which reorients the mathematician anew to the information that the diagram holds.

Interestingly, both diagrams (see Figures 2a and 2b) are drawn with the same orientation, which is not the case in their previous exploration and a subsequent exploration. Furthermore, Fred’s diagram has vertices labelled and edges are oriented, while Colin’s diagram is void of any labels and orientation. Therefore, the diagrams by Colin and Fred provide further evidence how uniquely virtual gestures are actualized. However, these personal actualizations also provide a common ground among the mathematicians to realize the possible mathematics as was discovered here.

Example 2: diagram-is-disruptive

The premise of a disruptive diagram is that the thinking track of a mathematician is interrupted through the information that the diagram reveals, which is either unexpected or in discord with the knowledge of the mathematician. Such episodes typically begin with speech repairs or exclamations such as “oh” and “ah” in the utterances of the mathematician who is engaged with the diagram, are usually quite tumultuous, and often end with laughter.

In the following example about a disruptive diagram the mathematicians are exploring the embedding of an unknown graph and are expecting the faces of the embedding to consist of four 3-cycles and two 4-cycles.Fred is working at the blackboard and attempting to find these particular cycles. After four minutes repeatedly walking to the blackboard, tracing edges (see Figure 3a), erasing edges, drawing over edges (see Figure 3b), stepping away from the blackboard, staring at the diagram (see Figure 3c), and otherwise being silent, he exclaims, “oh! It’s not three, three, three, three, four, four. It’s five threes and a five”. The data that the diagram reveals to the mathematician after his diagramming exploration comes as a surprise, and so, this is an episode of diagram-is-disruptive.

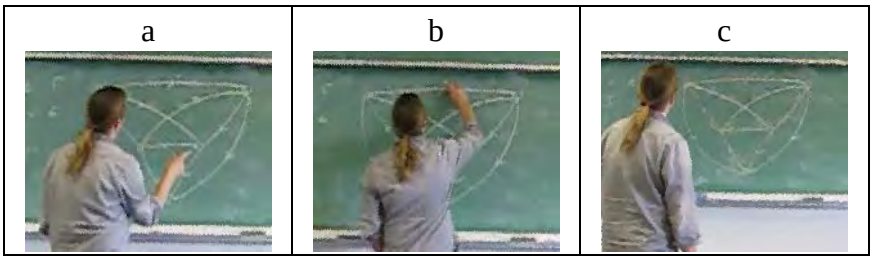

Figure 3: Mathematician tracing edges (a), drawing over edges (b), and staring at diagram (c).

DISCUSSION

One might see the discovery of uniqueness (for the embedding of the reversed planar octahedron in the torus using a parity argument of closed walks) as some kind of mental process, and as such an abstraction that does not partake of the physical world. However, the data contrast this point of view and provide evidence that the mathematical result derives from an embodied engagement with diagrams. Moreover, the diagrams are material entities (e.g., edges, vertices, graphs, embeddings) and relationships (e.g., closed walks) that are engaged with virtually as well as physically. The two mathematicians' abundant creation of diagrams on the blackboard, the subsequent direct, intimate and material engagement with the diagrams, and the prolific gestures that accompany speech and hint at the mathematical entities that are being explored through the diagram are evidence that the diagram is indeed a material object for these mathematicians.

Furthermore, we argue that the diagram is given a voice through the virtual and physical gestures of the mathematician, which speaks not only to the gesturing mathematician but all of its participating onlookers such as the third mathematician Victor. The two examples from the life-cycle of a diagram demonstrate that the diagram comes into existence (diagram-is-emerging) and forcefully announces information thus far unknown (diagram-is-disruptive). Considering all the episodes that are possible in the life-cycle of a diagram, the diagram is more than what it depicts: it constantly reassembles itself for the mathematician as he engages with the diagram virtually through gestures (e.g., points, traces, stares) or physically (e.g., adds, erases, draws over). Through the lenses of anthropology, archaeology, art and architecture, Tim Ingold offers the following insight from his study of drawings: "The drawing that tells is not an image, nor is it the expression of an image; it is the trace of a gesture. […] Thus the drawing is not the visible shadow of a mental event; *it is a process of thinking, not the projection of thought*" (2013, p. 128, *emphasis in original*). Ingold's conclusion equally serves this study of mathematical drawings, because ultimately, the diagram tells the story of how innovative mathematics comes into being.

Researchers such as Kita (2000), Krummheuer (2013), Lakoff and Núñez (2000), McNeill (2008) and Radford (2001) agree that gestures make thought visible. What is new in recent studies on gestures and thought "is the effort to identify causal and measureable relations and interactions between bodily behavior and hypothesized internal processes and to explain these within embracing and detailed theories of kinetic, communicative, cognitive, and symbolic systems" (Streeck, 2009, p. 172). As exemplified above, in the culture of mathematical research in Topological Graph Theory, gestures play a vital role in that they not only support communication among the mathematicians, but also kinesthetic and haptic experience with the diagram and the mathematical meaning that the diagram holds; and thereby support each mathematician in his ideational realm.

Aside from Châtelet's diagrammatical studies that cover Algebraic Manifolds, Calculus, Complex Numbers, Coordinate Geometry, Geometry, Infinite Sequences and Series, Ordinary Differential Equations, Partial Differential Equations, Symmetry, Classical Mechanics, Electricity, Magnetism, Quantum Mechanics and Relativity Theory, there are other studies that similarly investigate diagramming, gesturing and mathematizing. The sociologist Christian Greiffenhagen (2014) uses one mathematical lecture in Mathematical Logic as a paradigmatic example "to extend the 'material turn' to an instance of abstract thought" (p. 21). In his analysis, he similarly observes gestures of pointing and of mathematical entities during utterances and notes that while logicians are "predominately [sic] concerned with symbolic writing [diagrams] are nevertheless a crucial aspect of some parts of their practice" (p. 23). Barany and MacKenzie (2014) studied mathematicians engaged in partial differential equations and related topics while working on the blackboard. They draw awareness to the gesture space between the blackboard and the mathematician that lends itself to exhibit intuition, invention and discovery: "In the pregnant space between chalk and slate there reposes a germ of the bursts of inspiration, triumphs of logic, and leaps of intuition that dominate mindcentered accounts of mathematics" (pp. 10-11).

Lastly, we want to discuss some possible implications of these findings for the teaching and learning at all levels of acquiring mathematics. In their creative phase the mathematicians needed to *see* and *feel* objects and their relationships in order to gain an understanding about them or to explore how they could be altered or newly positioned to extract new insights in their field. How do we, as teachers in general and at all levels, allow our students to *see* and *feel* in the classroom or the office? How prevalent is the use of regurgitated visual products in order to explain, for example, what a parabola looks like instead of what a parabola is? How often do we create an environment that places each student in the position of a mathematician, where the student explores the relationship between input x and output $x^2 + c$, perhaps dynamically using geometry software, or drawing by hand the locus of points such that the distance to a given focus equals the distance to a given directrix? These are all environments, where the student is bound to point, touch, hold, trace, add and delete, which may lead to further material engagement with the curve and the curving that is the parabola.

CONCLUSION

While Châtelet only intimates at the whole process of diagramming during mathematical invention, the analysis of the 122 time intervals and the 128 diagrams allows a glimpse into the whole process and reveals how diagrams come into being and are engaged with. These findings enrich Châtelet's theories by being able to identify the different functions of interactions between mathematician and diagram in inventive mathematical processes. The explications of the two episodes above reveal that diagramming during mathematizing is a ritual kind of practice (done over and over again) that offers a physical arena for experimentation. We suggest that the life-cycle functions may also offer fruitful avenues for helping students incorporate diagramming into their own mathematical problem solving, if only by seeing how drawing lines on paper might initiate new gestures and new ideas.

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Chapter 17
Diagramming and Gesturing During Mathematizing: Kinesthetic and Haptic Interactions Support Mathematical Ideation

Petra Menz and Nathalie Sinclair

Abstract We focus on the role of diagramming and gesturing in mathematical practice. In most discussions of diagramming in mathematics, including mathematics education research, diagrams are seen as representations of mathematical objects and relations. This view was challenged by the philosopher and historian of mathematics, Gilles Châtelet, who argued that diagramming is a material practice of mathematical invention. His arguments are based on analyses of historical examples of diagrams associated with new mathematical ideas. Guided by his approach, we study the actual diagramming practices of mathematicians, as they work on unsolved problems. By doing so, we aim to identify the various roles that diagrams might play in mathematical invention as well as the material nature of diagramming practice. We hope that a more nuanced understanding of diagramming within mathematical practice will contribute to research on promoting and supporting student diagramming.

Keywords Diagram · Gestures · Materialism · Mathematical practice
Mathematical invention

17.1 Introduction

Diagramming is considered by many to be an essential strategy in mathematical problem solving (Grawemeyer and Cox 2008; Novick 2004; Stylianou and Silver 2004) as well as an essential component of mathematical discourse (O'Halloran

P. Menz (✉)
Department of Mathematics, Simon Fraser University,
8888 University Drive, Burnaby, BC V5A 1S6, Canada
e-mail: pmenz@sfu.ca

N. Sinclair
Faculty of Education, Simon Fraser University, 8888 University Drive,
Burnaby, BC V5A 1S6, Canada
e-mail: nathsinc@sfu.ca

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2005, 2011). In mathematics education research, many have used Peirce's semiotics to study the functioning of diagrams either as a special category of icons or in each of the iconic, indexical and symbolic registers (see Hoffmann 2006).

A critique of the semiotic approach to the study of diagrams relates to their assumed status as representations, either of mathematical objects/relations or as conceptions already formed through mental activity (de Freitas 2012). As representations, they can only be seen as copies of ideal mathematical entities—often flawed and misleading. Rotman (2005) has argued that such a view ignores the material, embodied nature of mathematical diagramming: instead of seeing them as mere representations, we could focus on "the physical activities themselves, the moving around, visualizing, talking, and gesturing involved in learning and communicating the subject" (p. 34).

In other words, the actual drawing of the line on a page that will produce a circle, is not just a representation of a geometric object, but also a mark-producing gesture in which the continuous application of lead on paper is led by the hand and guided by the eyes.

Gestures and diagrams have each received increased attention in mathematics education, especially within the recent emergence of embodied and semiotic perspectives. Châtelet's (2000) work, however, seeks to show, through examples from the history of mathematics, how gestures and diagrams play a pivotal role in mathematical invention. He wants to show how formal mathematics can be seen as continuously emerging from the material mobility of the human body. Diagrams are essential clues for him, since they provide traces of the moving hand, while also enabling—on the surface of the paper—the exploring and creating of new objects and dimensions. Châtelet thus brings together two hitherto-distinct areas of research —on gestures and diagrams, respectively—in pursuing his non-representational, non-dualistic account of mathematical thinking.

Our own material approach to semiotics also draws on Peirce, but with a particular focus on the index. The drawing of a circle can be seen as an icon, which operates according to likeness and resemblance between signifier and signified. Seen as an icon, however, it becomes a representation of a Platonic object (the circle) that is simply being copied, more or less faithfully. But what if we think of the drawing as an index, which has a more material link between signifier and signified? Indeed, unlike icons and symbols, indexical signs are bound to the context in important ways, as they "show something about things, on account of their being physically connected with them" (Peirce 1998, p. 5). The canonical example used by Peirce is that of smoke billowing from a chimney, which indicates that there is a fire in the fireplace so that the smoke indexes the fire. An index "refers to its object not so much because of any similarity or analogy with it, (…) because it is in dynamical (including spatial) connection both with the individual object, on the one hand, and with the senses or memory of the person for whom it serves as a sign, on the other" (Peirce 1932, 2.305). The gesturing hand that leaves a trace of congealed wax can therefore be seen as indexing a circle, producing a temporal and spatial record of circle-making. The circle drawing can thus be seen as an indexical sign that refers to the prior movement of the pencil. As Sinclair and de Freitas (2014) have argued,

> This latter indexical dimension is usually not emphasized in the semiotic study of mathematical meaning making, since we tend to focus on the completed trace and dislocate it from the labour that produced it. Such habits of focus have resulted in our neglect of how the activity of the body and various other material encounters factor in mathematical activity. (p. 356)

Since Châtelet's work was based on historical examples, he only had access to diagrams that have been completed and conserved, and not to actual making of the diagram, a process that would have a particular rhythm, weight, order and tempo that could help provide insight into the inventive process. In this chapter, we study the *live* process of mathematical diagramming. We aim first to corroborate Châtelet's claims in the context of contemporary mathematical practice. We further aim to better understand the embodied, material aspects of mathematical diagramming. In particular, we are interested in how diagrams, if not assumed to be mere representations, can be seen as indexical signs *producing* mathematical concepts.

In the next section, we provide a brief overview of Châtelet's work, which is central to our own approach. We also summarise other work in mathematics education that has adopted this approach. Where relevant, we connect to similar research on diagrams and, especially, gestures.

17.2 Châtelet on Diagramming and Gesturing

Through this flexible view of figuring and defiguring, Châtelet brings new ideas to the study of gestures, diagrams and mathematics. By analyzing historical manuscripts of famous mathematicians such as Oresme, Leibniz and Hamilton and without access to video-recordings of these mathematicians' mathematizing, Châtelet traces their thoughts and actions from their writing and their diagramming. His fundamental insight is that the *virtual* is evoked in historical mathematical inventions through diagramming experiments whose sources can be traced to mobile gestural acts. The virtual is that which is latent in the diagramming, but ontologically new. In this way, Châtelet challenges the disembodied, abstract conceptualization of mathematics. The key ideas of his theory are that (1) the diagram is never really fixed–it is erased, drawn over, reassembled, or redrawn—as it hovers between the actual and the virtual; (2) there is a mutual interaction between gestures and diagram; and (3) it is through the material interaction with the diagram that a person understands or invents mathematics. As Roth and Maheux (2015) point out, "the virtual cannot be grasped but is that which allows grasping to occur" (p. 236).

The crux of Châtelet's study is his revelation of how the virtual is evoked in historical mathematical inventions "through diagramming experiments whose sources Châtelet can trace to mobile gestural acts" (Sinclair and de Freitas 2014, p. 563). This trace of the gesture, which lingers in the creation of new mathematics, provokes and challenges the abstract nature that has come to be associated with mathematics.

Although the main focus of this chapter will be on diagramming, in light of the gestural nature of the production of diagramming coupled with the tight interplay hypothesized by Châtelet, it is worth examining how Châtelet's conception of gesture fits within the existing landscape, which we do in the following subsection.

17.2.1 Situating Châtelet's Conception of Gestures

The last two decades have been fertile ground for the study of gestures in mathematics from the anthropological (Rotman 2012), cognitive scientific (Lakoff and Núñez 2000), educational (Krummheuer 2013; Radford 2001), psycholinguistic (Levelt 1989; McNeill 1992, 2008), sociological (Greiffenhagen 2014), and philosophical (Châtelet 2000) points of view. The predominant line of research in gesture studies, especially within the Anglophone literature, focuses on movements of the body (especially the hand) and their interactions with speech in communication. For example, one of the groundbreaking studies in gesture theory asserts that "in a nutshell, […] *gestures are an integral part of language as much as are words, phrases, and sentences–gesture and language are one system*" (McNeill 1992, p. 2, emphasis in original). Drawing on Peirce's semiotics, this theory identified categories of gestures (iconic, metaphoric, deictic and beat) that distinguish different relationships between gesture and speech.

This type of gesture research emphasizes gesture as part of "the human capacity for language" and the study of gesture as "language in action" (Rossini 2012). In coding gesture only in terms of linguistic potential, such research can overlook the physicality of the hand movements of gestures. As Streeck (2009) indicates, "it is common to treat gesture as a medium of expression, which meets both informational and pragmatic or social-interactional needs, but whose 'manuality' is accidental and irrelevant" (p. 39). To counter this neglect, Streeck defines gesture: "[…] not as a code or symbolic system or (part of) language, but as a constantly evolving set of largely improvised, heterogeneous, partly conventional, partly idiosyncratic, and partly culture-specific, partly universal practices of using the hand to produce situated understanding" (p. 5).

With this definition, Streeck studies gestures for how they are "communicative action of the hands" (p. 4) and thus examines gesture for how it couples with and intervenes in the material world in non-representational ways, which is consistent with the approach of Châtelet. Indeed, for Streeck, distinctions between hand movements 'in the air' and hand movements 'on the page' become fuzzy. And it is in this fuzziness, that Châtelet can dwell on the back and forth interplay between gestures giving rise to diagrams that give rise to gestures.

In line with a French tradition of inquiry into gestures (by philosophers of mathematics such as Cavaillès, Desanti, Longo and Alunni, Châtelet sees the gesture as a tool for non-analytic reasoning (Maddalena 2015). Châtelet not only

notices the gesture and its link to the diagram, but he also interprets gesture as even more than a visible, non-verbal, bodily action that carries meaning; indeed, a gesture is the articulation between the virtual and the actual and as such is immediate and embodied. It is in this regard that a gesture is inseparable from where it came from and what it creates. This view led Châtelet to question the boundary between physics and mathematics that Aristotle drew with his interpretation that physics represents applications "that exist in Nature [… and are] mobile" whereas mathematics represents abstraction "which exists only by proxy through the wit" and is "immobile" (Châtelet 2000, p. 17).

Our own research investigates the gesture/diagram coupling in mathematical practice, where it should be possible to study the various hand movements—be they in the air or on the blackboard—made by mathematicians as they work on solving mathematical problems. Our goal is to better understand the material nature of their diagramming practices.

17.3 Methodology

The first author was given the opportunity to participate in a series of research meetings that a group of mathematicians had organized in order to pursue joint work on a particular problem. These meetings struck us as fortuitous settings for studying diagramming practices, since the discussion amongst the mathematicians would help us understand the evolution of their work without having to constantly prod them for explanation, as might be the case for a mathematician working on a problem alone. The research subjects in this study comprise three male, Caucasian, research mathematicians in the field of Topological Graph Theory from two prominent North American universities (see Menz 2015 for details of the full study).

The three mathematicians, referred to as Fred, Colin and Victor, met nine times over a period of three months to study the class of 2-regular directed graphs and how they embed in different surfaces. The mathematicians' goal was to compile a list of obstructions for the projective plane in particular, although other surfaces such as the torus and Klein bottle were also explored at times, and to classify these obstructions for 2-regular directed graphs. All research meetings took place in one university's mathematics seminar room, equipped with blackboards on which all diagrams of this study were drawn. The mathematicians never all sat at the tables in the room together; at least one of them was always standing, close to the blackboard. The first author was present in order to video-record these nine research meetings that varied in length from one to two hours; to capture digital still images of diagrams by the participants using a second camera; and to make field notes during the meeting that attended both to the mathematics that developed during the meeting and to observations regarding diagramming and gesturing.

Based on these observations and multiple viewings of the recorded research meetings, we looked for all diagramming events; an event is regarded to be when a

diagram is produced, enhanced, contemplated, referred to, changed or erased. In order to get an overall sense of the frequency and intensity of diagramming, we identified periods of time that contained at least one diagrammatic event. These diagrammatic events were viewed several times in order to describe the mathematics that was occurring, to track who was doing the talking, to identify the overall body language associated with the diagramming, to record spoken phrases, and to label the apparent mood. These descriptions enabled us to identify, more holistically, the different roles that the diagram played during the mathematizing.

Over the course of the 12 h of video-recordings, a total of 122 time intervals were selected for further analysis varying in length from approximately thirty seconds to approximately six minutes, containing a total of 128 distinct diagramming events. For each of these time intervals, we produced a transcript of the event as well as the accompanying gestures (in the air as well as on the blackboard). Our goal was to see whether we could identify different functions that the diagramming played over time, as the mathematicians worked on their problem, and whether some functions were more prevalent than others.

In the first section below, we analyze two different time intervals in order to highlight the material interactions at play, particularly the hand movements that preceded, accompanied and followed diagramming. In the following section, we look across the data set in order to propose different types of diagramming events that occurred, with a view to investigating how the diagramming event enabled invention, comprehension and communication.

17.4 Diagramming Matters

In his analysis, Châtelet focused on particular diagrams that he argued were instances of inventive mathematics and from which he hypothesized the significant role of gestures. In our analysis of the 122 time intervals and the 128 diagrams, we study not only the production of drawings on the blackboard, but also their evolution over the course of time—that is, we focus on the entire life-cycle of diagrams. In doing so, we were able to identify diverse types of diagramming events that occurred—not just the initial production of a diagram through the movement of the chalk on the blackboard, but also subsequent additions and deletions, as well as repetitions of drawings that had been made before. A glimpse into the whole process reveals how diagrams come into being and are engaged with.

Every time a mathematician makes chalk marks on the blackboard that are not writings a diagram emerges. This diagram can consist of anything from just a few strokes, to elaborate, colorful drawings. Once the diagramming mathematician steps or turns away from the diagram, the episode *diagram-is-emerging* is concluded and the episode *diagram-is-present* begins. If a mathematician adds to or erases from the original diagram, the emergence of another diagram is signaled. Five settings are identified when the episode diagram-is-emerging comes about: (1) the mathematician diagrams from scratch during his exploration, which is especially

noticeable during sustained high diagramming activity such as in research meetings 3, 5 and 9; (2) the mathematician adds to or erases from an existing diagram, which alters the original diagram; (3) the mathematician re-figures a diagram by tracing the edges and vertices of an existing diagram (usually a graph), which occurred every time the mathematicians tried to find out if a given graph on the blackboard embeds in some surface; (4) the mathematician draws a known diagram by retrieving it from memory; and (5) the mathematician is directed to draw a diagram by another mathematician.

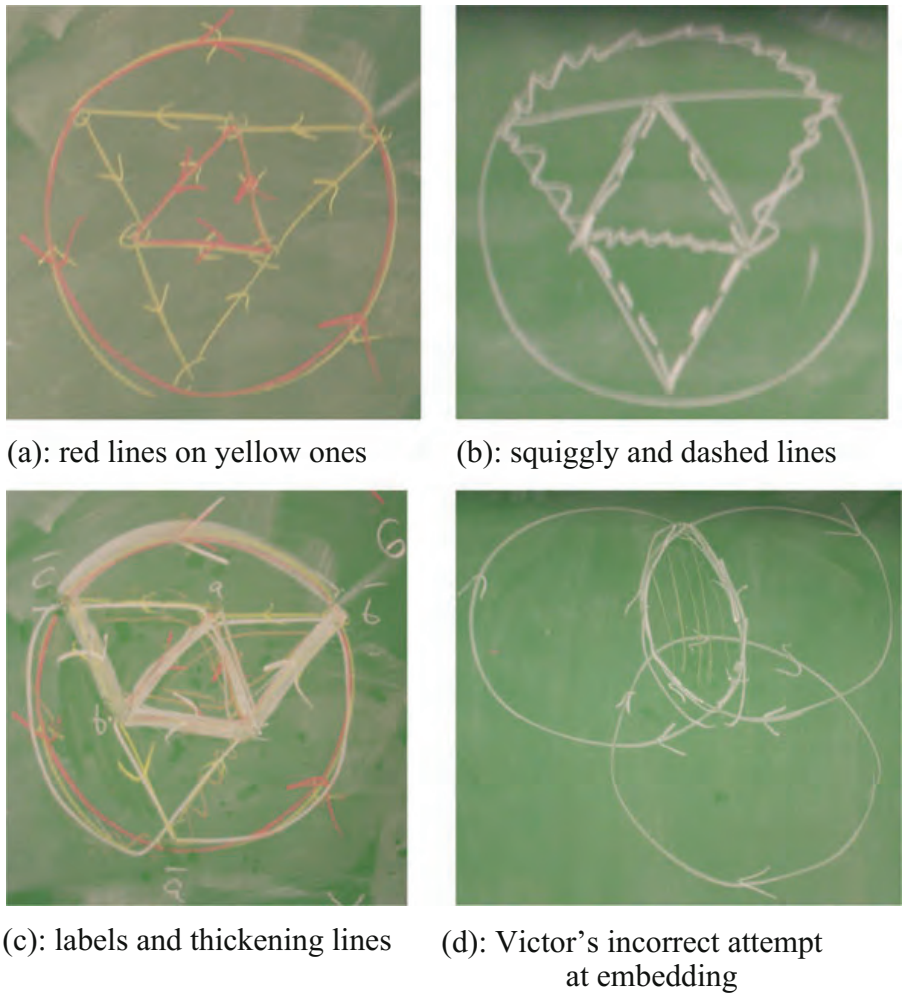
Once a diagram has emerged, it is present on the blackboard, regardless of whether any of the mathematicians engage with or refer to it and no matter for how long a time period. When and how a diagram emerges and becomes present is of specific interest as it reveals how the diagram is interacted with from that moment onwards. In particular, the regularly occurring emergence as described in (3) throughout the nine research meetings provides evidence that the *re*-figuring of diagrams is a crucial element leading to mathematical inventions. In the following example, we describe an instance of (3) in more detail.

17.4.1 Re-figuring Diagrams

Through exploration of the edges and vertices of one 2-regular orientation of C_6^2 (later termed *reversed planar octahedron*), the three mathematicians had discovered an embedding of this particular octahedron in the torus. But they wanted to know if this embedding was unique and how C_6^2 could embed, more generally, in a surface of Euler characteristic zero. For this, they needed to understand how the faces of the embedding were connected to the edges of the original graph, which they referred to as identifying *closed walks*.

The four diagrams in Fig. 17.1 were produced at different times during the research meetings. In some ways, they look ‘the same’ and could thus be considered representations of—or icons of—an embedding of an octahedron on the torus. Such an interpretation would ignore the actions with the hands that produces these diagrams, indexical actions that—as we show—include the mix of improvisation, heterogeneity, idiosyncrasy, and partly conventional, culturally specific, and universal practices evoked earlier by Streeck.

Fred began by drawing the reversed planar octahedron with yellow chalk on the blackboard from memory (they had worked on it in research meeting 2), but seemed to be stuck finding the faces of the embedding. He stood back from the blackboard, looking at it intently. Then Colin joined Fred at the blackboard and asked, “May I please destroy this picture?” to which Fred replied “Oh, sure.” Colin took the red chalk and drew lines on top of some of the yellow edges. He also added some new arrows, some going in directions opposite to the yellow ones. The direction of the lines/arrows corresponded to closed walks (see Fig. 17.1a). The fact that Colin re-drew the lines, right on top of the other lines, shows that the ‘walks’ were just



(a): red lines on yellow ones

(b): squiggly and dashed lines

(c): labels and thickening lines

(d): Victor's incorrect attempt at embedding

Fig. 17.1 **a–d** Different drawings of an octahedron on a torus

that, movements of the hands along the edges of the graphs, movements that started and stopped at the same place—actual closed walks. Had they been mere representations of a closed walk, Colin would have not re-drawn the lines; it would have been enough to change the arrows.

All the while, Fred was several times tracing through the diagram with a cupped right hand (see Fig. 17.2a), which he placed over each of the faces in the diagram as if his hand was mimicking the perimeter of the faces. During the times when he uttered "red, yellow", he alternately moved his index and middle fingers up and down while at the same time moving his hand successively from face to face. Both of these are interesting gestures, as they not only embody the alternation between edge colorings and thereby virtually assemble the faces that the diagram holds, but the gestures also underline the content of Fred's utterances, which is a generalization of the process of finding the faces.

Colin returned to his seat and a discussion about the closed walks ensued, followed by silence. During this time, Fred labelled the vertices of the red/yellow

(a): Fred cupping his hand over faces

(b): Mathematicians stare at diagram shown in Figure 17.1a for several seconds

Fig. 17.2 **a**, **b** Mathematicians interacting with diagram

diagram and softly traced through the edges with red chalk, which can be seen in Fig. 17.1c. It was not enough for him to see what was on the blackboard. Instead, he drew the lines again, his hand performing a walk along the graph. The repetition, by different mathematicians and then by the same mathematician, turns one idiosyncratic choreography of the hand into a convention and forges the closed walk on the imagined surface into an object. This is not done through abstraction, but through ritual action.

In the meantime, at the very beginning of Colin and Fred's exploration at the blackboard, Victor got up saying "I can't see it", but instead of joining Fred and Colin, he quietly drew his own version of the embedding on the blackboard (see Fig. 17.1d) until both Colin and Fred had stepped far away from their diagram. Next, Fred uttered "So that's (.) what am I doing? How did I get such funny numbers? (.) No, but that's perfect. That's, that's an embedding on the torus", during which time he stepped closer to the diagram and touch-pointed it. This statement raised Victor's interest, who stopped drawing and turned to Colin and Fred's diagram and stepped closer. Then, all the mathematicians stared at this diagram for several seconds (see Fig. 17.2b). Fred emphatically repeated "That's an embedding on the torus." Colin replied "Uh, so you have six vertices, twelve" and paused, so Fred finished with "Yah! Uh, uh, six vertices, twelve edges and six faces." Then Colin responded "Six faces is the torus, yah, yah", and Victor chimed in with "Yah, yah, that's what I was doing" and walked to his diagram. The next few minutes, the three mathematicians discussed their findings. Victor wanted to know which polygonal faces Colin and Fred used, both of whom responded by describing and counting the types of polygonal faces while touch-pointing them in the diagram.

During this episode, the diagramming, gesturing, positioning of the bodies, and verbalizing were entwined acts, which culminated in the three expert mathematicians forming a shared understanding of their discovery of the funny octahedron's embedding in the torus.

Next, Fred stated "I am curious how much symmetry of this you can keep when you take this funny embedding of the thing on the torus" and began to redraw the

embedding of the funny octahedron in the torus, which took him about two minutes to do. He used Fig. 17.1a to trace along the edges, which guided him in drawing the faces in his new version of the torus embedding using the same yellow and red chalk coloring (see Fig. 17.3). These diagrams were drawn by the mathematicians in order to understand the mathematical structure of the funny octahedron on the torus. It was necessary for Fred to trace along the edges and pause at the vertices of the diagram shown in Fig. 17.1a in order to produce a new version of the same graph. In Châtelet's terms, the constant back and forth between Figs. 17.1a and 17.3 helps actualize the virtual structure through this re-figuring.

Now that the mathematicians had discovered the embedding of C_6^2 in the torus, they wanted to know if this embedding is unique and how C_6^2 can embed generally in a surface of Euler characteristic zero. For this, they needed to understand how the faces of the embedding are connected to the edges of the original graph, which they referred to as closed walks. Fred noticed that "also around each, around each red edge you see a directed four cycle just taking those two faces", and half a minute later he stated "So if we say we have only quadrangles [sic], maybe we can work out, uh, what the possibilities are." Colin responded with "So we can try using no triangles, uh, uh, and then there would be only squares." During this utterance, Victor walked to the blackboard and joined Fred there tracing and touch-pointing at the diagram in Fig. 17.1a while exploring the connection between the edges and faces. After about a minute, Fred exclaimed: "Wait! I don't understand. Are you taking, are you taking these two?" A silence of 11 s ensued, then Colin again came up to the blackboard and picked up the white chalk. He covered some of the edges of the reversed planar octahedron with squiggly and dashed lines (see Fig. 17.1b). Again, Fred walked up to the diagram and this time used the index finger of his right hand to trace over Colin's added lines (see Fig. 17.4a). This is a literal sense in which Colin's gesture/drawing gave rise to a new gesture by Fred, which re-produced the closed path, thus re-figuring the diagram yet again.

Another 10 min passed, during which time Fred and Colin explored the parity of the closed walks in the reversed planar octahedron. They did this by referring to their two diagrams (Fig. 17.1a, b), pointing to them, tracing over them and touching

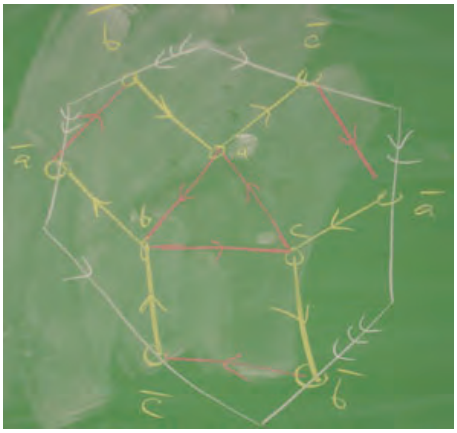


Fig. 17.3 Re-figuring Fig. 17.1a

Fig. 17.4 **a–c** Examples of diagram-is-emerging

the edges and vertices. Then Fred indicated that he finally understood how to argue that the embedding is unique and turned to the diagram drawn in red and yellow. First he traced through the edges (see Fig. 17.4b) and then added thin and thick white lines. He then exclaimed, "Let me make a total mess of this" (producing Fig. 17.1c). Afterwards, Colin traced through the edges of this diagram with his index finger to identify closed walks (see Fig. 17.4c), while both he and Fred verbalized their satisfaction with their argument.

During all this time, the third mathematician, Victor, joined the discussion at times, but also explored the graph on his own on paper. However, his diagramming was ignored by the other two mathematicians. At the very end, Victor was asked by Colin "Is that clear (.) so (.) Victor?', to which Victor readily responded "yeah" and nodded his head in agreement.

Far from being fixed representations of mathematical objects or relations, the diagrams shown in Fig. 17.1—despite their static nature on the page—unfolded over time, changing and growing with added marks, traces and touches. The continuous lines made in both red and yellow chalk involved quite different hand movements than the dashed, squiggly and thick lines that were piled on top of the original, thin lines—resulting in a layering of new closed walks that were sometimes tentative, and sometimes needing to be insisted upon. The smudges of chalk and finger prints on the blackboard within Fred's diagram (see Fig. 17.1c) were evidence of the erasing and touching of edges and faces, thereby speaking of the intimate engagement with the objects and relations that were coming into being.

Interestingly, all three diagrams by each of the mathematicians (see Fig. 17.1a, b, d) were drawn with the same orientation, which was not the case in previous and a subsequent explorations of the mathematicians. Their diagrams also display rotational symmetry with a 120-degree angle of rotation. While Fred and Colin's diagrams are based on an existing representation of the funny octahedron, albeit in the projective plane, Victor's diagram does not resemble any previous representation of the funny octahedron and employs only one color, although two edges stand out more because he draws over them so often when he is explaining his method to the mathematicians. At a fleeting glance, one can argue that these are still the same diagrams, by just morphing the curved edges of Fig. 17.1a into straight lines and rotating the entire diagram by a 180-degree angle of rotation through the

center. Yet, Victor's explanations, which he offers a little while later, elucidate that he is using the idea of Venn diagrams to represent the inner triangle, outer triangle, hexagon and the three quadrilaterals of the embedding. When Fred and Colin are curious how the faces align, they almost immediately discover that Victor's diagram is actually an embedding of the funny octahedron in the Klein bottle rather than the torus. Furthermore, Fred's diagram had vertices labelled and edges oriented, Colin's diagram was void of any labels and orientation, and Victor's diagram had only the orientation of edges. Therefore, the diagrams by the three mathematicians provided further evidence of how uniquely virtual gestures are actualized. However, these personal actualizations are also indicative of a common ground among the mathematicians to realize the possible mathematics, as was discovered here.

In the above example, which occurred over a period of about thirty minutes, the diagram is being brought into being, which Menz (2015) refers to as the *manufacturing phase*. The diagram is present, available both to the mathematician who has drawn it, as well as to his colleagues. This manufacturing phase is one of intense material interaction, but once the diagram is on the blackboard, as when Fig. 17.1a was made, additional material interactions can occur, as in the re-drawing, tracing and touching that was seen above. These interactions serve the purpose of bringing relations and objects into being; providing a site for repeated gestures that can eventually become captured into, in this case, a closed walk; highlighting, bracketing or connecting certain objects and relations for further attention. Each of these actions further shapes the diagram, as it shapes the mathematicians' reasoning.

In the next example, we analyze a similar situation in terms of the material interaction, but where the diagram plays a different function than the ones described above.

17.4.2 Example 2: The Diagram Reveals

During meeting 2, the mathematicians were exploring the embedding of an unknown graph and were expecting the faces of the embedding to consist of four 3-cycles and two 4-cycles. Fred was working at the blackboard and attempting to find these particular cycles. Fred spent about four minutes standing in front of the blackboard, most often with his arms crossed, but several times stepping forward to draw or gesture. The first time, he approached the board, saying "I can go here (tapping on top left vertex, then following arc to top right vertex), here (tapping second vertex, then following an arc to bottom vertex) here" (see Fig. 17.5a). The three instances of "here" were accompanied by the three sounds of taps on the board. No new lines were drawn, but his arms remade the circuit.

A couple of minutes later, standing back from the board, his right hand made quicker and smaller scale, in-the-air movements, that involved more turning of the hand than translating of the hand along lines. Then he approached the board and

Fig. 17.5 Mathematician tracing edges (**a**), drawing over edges (**b**), and staring at diagram (**c**)

drew a label on the top line (see Fig. 17.5b). He then made several more small, quick in-the-air hand gestures, paused and then erased several of the arrows on the lines with his fingers. This was followed by yet more small, quick gestures and then he started to draw new arcs wrapping around the vertices of the graph. After drawing the arcs, his right hand, which was holding the chalk, traced out yet another circuit, this time returning to a larger gesture. He then stood back (see Fig. 17.5c) and came forward several times, then approached the board and moved his chalk in the air, drawing virtual lines close to the surface of the blackboard. After drawing a few more arcs on the board, he stepped away from the blackboard and exclaimed, "Oh! It's not three, three, three, three, four, four. It's five threes and a five."

There were three types of gestures that occurred in this four-minute interval: the large arm movements that created a virtual circuit and were close to the board; the smaller hand movements that were further from the board but also created virtual circuits; and, the curved hand movements that left a trace on the blackboard around the vertices of the graph. These all followed, evidently, the creating of the actual graph (drawing) on the blackboard, as well as the re-drawing of the graph that we discussed earlier. The drawing, re-drawing and large gesturing involve the same movements of the arm, all at the scale of the graph, with the large gesturing being evidence of Châtelet's sprouting of gestures from diagrams, gestures that begin to actualize a cycle. In the shift from large to small gestures, from arms to hands, the process of actualization continued, now turning a longer traversing of the graph into a quick, settled cycle. The last gestures, which left their traces on the board, carved out the corners of the cycles, inscribing the turns of the wrist that featured in the small gestures—and in this sense, the resulting drawing captures the gesture, as Châtelet theorized.

It is tempting to say that the diagram, as a representation of the embedding, contains the structure required to reveal everything about its cycles. But this would ignore the extended labor involved in drawing and re-drawing edges, in gesturing across from vertex to vertex, as the embedding takes shape, and embedding that is presumed to have a certain property. It is not a question of inferring something from a given diagram since the diagram is being shaped along with the realization of the cycles.

17.5 Characterizing Diagramming Practice Over Time

There were 179 distinct diagrams recorded with the picture camera during the research meetings; however, this number does not include the erased and drawn-over diagrams that were not captured but that are recorded on the videos of the research meetings. Very rarely, a mathematician would place just a few chalk marks on the blackboard and immediately erase them, which occurred at most once in each research meeting. Furthermore, during sustained high diagramming activity such as in research meetings 3, 5 and 9, for every two diagrams of which a digital photo was taken, another diagram was either drawn over or partly erased. Since there is no clear demarcation between the beginning and ending of diagramming, there was a fine balance between getting the data and not disturbing the meeting. Based on the viewing of the video recordings by the first author, there are an estimated twenty diagrams that are recorded only on video and not as digital photographs. Therefore, there are about 200 diagrams drawn during the nine research meetings.

By looking at each one of the diagramming intervals, we were able to identify different functions of interactions between mathematician and diagram in inventive mathematical processes, which are captured in the life-cycle of Fig. 17.6. We were interested in doing this in part because Châtelet provided little insight into how the diagramming process evolved from suggested movements of the hand to more refined, shared and dependable diagrams that could appear as part of the formal mathematical discourse. We propose that the diagram life-cycle is divided into three phases, which concern themselves with (1) how the mathematician brings the diagram into being–*manufacturing phase*; (2) what the relationship is between the diagram and the mathematician–*communication phase*; and (3) how the mathematician resolves, in simplistic terms, whether the diagram stays or not–*dénouement phase*.

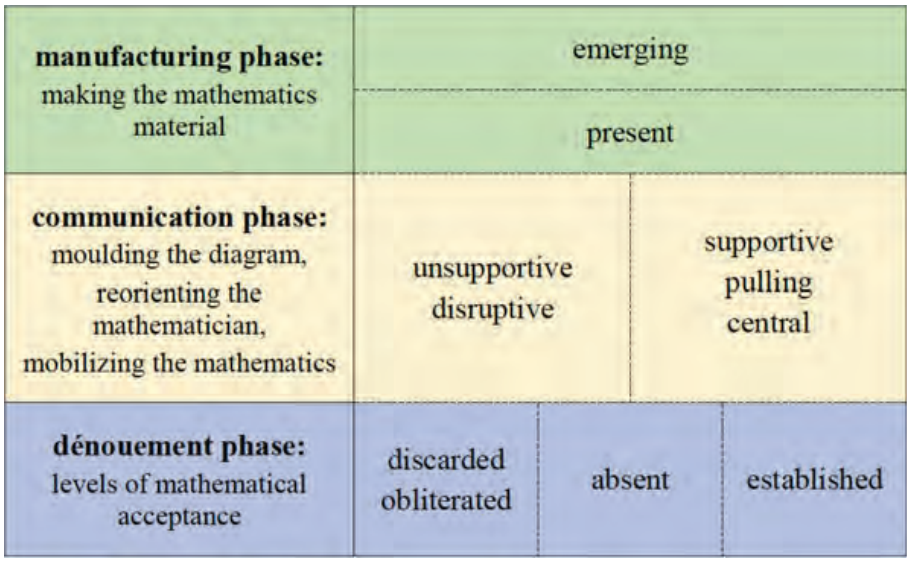


Fig. 17.6 Episodes, phases and their relationships in the life-cycle of a diagram

In each phase, so-called *episodes* describe a particular function of interaction between diagram and mathematician. The term *episode* is chosen because it alludes to both a period of time and a side-story being told. The analysis shows that throughout the life-cycle of a diagram, from its creation to its 'establishment' or 'obliteration', there are eleven distinct diagram episodes: *emerging*, *present*, *unsupportive*, *disruptive*, *supportive*, *pulling*, *central*, *discarded*, *obliterated*, *absent* or *established* (see Menz 2015 for descriptions and examples of each). It is important to point out that not all of these episodes need to occur in the life-cycle of a particular diagram, except for diagram-is-emerging followed by diagram-is-present, since its occurrence is dependent on the relationship the mathematician has with the diagram.

As indicated in the methodology section, 122 time intervals of interest for further analysis were selected, which contain a total of 128 diagrams. Only the first occurrence of a diagram was counted. In other words, if a diagram in a particular time interval had already been counted in any of the previous time intervals, then this diagram was not counted again. This means that of the estimated total of 200 diagrams, the data analysis contains 64% of the diagrams. Since each diagram was identified with an episode, we were able to conduct a quantitative analysis of the eleven diagram episodes, the result of which is shown in Fig. 17.7. Even though only 94 out of the 128 diagrams are shown to emerge in the selection of 122 time intervals, the episode diagram-is-emerging is still regarded to be 100% of the total number of diagrams, because every one of these diagrams was created by the mathematicians at some point during the nine research meetings. Once a diagram has emerged, however sketchily or incompletely drawn, it is present on the blackboard, no matter for how short or long a time period. Therefore, the episode diagram-is-present also occurs as 100% of the total number of diagrams. The remaining nine types of diagram episodes were counted and their percentage was calculated compared to the 128 total diagrams. Every so often, more than one episode in the life-span of a diagram was identified, and therefore, the percentages in Fig. 17.7 for the bottom nine types of episodes do not add up to 100%.

The two most common of the nine episode types from the communication and dénouement phases are diagram-is-supportive and diagram-is-central at 37 and 38%, respectively. This does not come as a surprise, because the diagram is chosen by the mathematicians in order to explore the mathematics of their research. In other words, a diagram is the quintessential playground, where a mathematician's intuition and experience shape the mathematics that is under exploration, since "one is infused with the gesture before knowing it" (Châtelet 2000, p. 10). This becomes even more evident in the mathematicians who gestured a diagram during speech before the chalk marks created it on the blackboard. It is in the same sense that de Freitas and Sinclair (2012) emphasize that "[the potential] marks that which is latent or ready in a body. In the case of the diagram, the potential is the virtual motion or mobility that is presupposed in an apparently static figure—and that was central to its creation in the first place" (p. 139). Perhaps this also explains why the episode

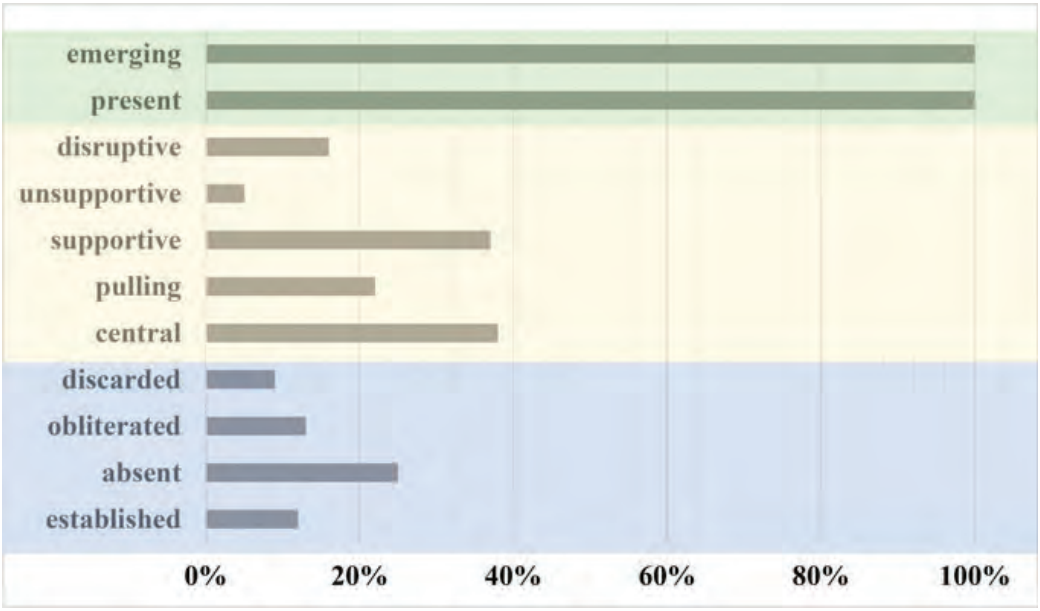


Fig. 17.7 Quantitative analysis of diagram episodes

diagram-is-unsupportive is the least common type of episode with only a 5% occurrence. The mathematician's physical and mathematical intuition shapes his gestures that actualize the diagram, thereby realizing the mathematics that is possible rather than the mathematics that is not.

17.6 Discussion

One might see the discovery of uniqueness (for the embedding of the reversed planar octahedron in the torus using a parity argument of closed walks) as some kind of mental process, and as such an abstraction that does not partake of the physical world. However, the data we have presented and analyzed contrast this point of view and provide evidence that the mathematical result derives from an embodied, material engagement with diagrams. The diagrams involve material objects (e.g., edges, vertices, graphs, embeddings) and relationships (e.g., closed walks) that are made by the hand in time, manipulated and experimented with, and actualized through gestures and drawings. The two mathematicians' abundant production of material, indexical signs on the blackboard, their subsequent direct and intimate engagement with these marks, and the prolific gestures that accompany speech and hint at the mathematical entities that are being explored through the diagram are all evidence that the diagram is not merely a representation of a known object, nor merely a fixed platform on which to experiment. The diagram is the mathematical becoming.

Furthermore, we argue that the diagram is given a voice through the virtual and physical gestures of the mathematician, which speaks not only to the gesturing mathematician but all of its participating onlookers such as the third mathematician Victor. Considering all the episodes that are possible in the life-cycle of a diagram, the diagram is more than what it depicts: it constantly reassembles itself for the mathematician as he engages with the diagram virtually through gestures (e.g., points, traces, stares) or physically (e.g., adds, erases, draws over). Through the lenses of anthropology, archaeology, art and architecture, Ingold (2013) offers the following insight from his study of drawings: "The drawing that tells is not an image, nor is it the expression of an image; it is the trace of a gesture. […] Thus the drawing is not the visible shadow of a mental event; *it is a process of thinking, not the projection of thought*" (p. 128, emphasis in original). Ingold's conclusion equally serves this study of mathematical drawings, because ultimately, the diagram tells the story of how innovative mathematics comes into being.

Researchers such as Kita (2000), Krummheuer (2013), Lakoff and Núñez (2000), McNeill (2008) and Radford (2001) agree that gestures make thought visible. What is new in recent studies on gestures and thought "is the effort to identify causal and measureable relations and interactions between bodily behavior and hypothesized internal processes and to explain these within embracing and detailed theories of kinetic, communicative, cognitive, and symbolic systems" (Streeck 2009, p. 172). As exemplified above, in the culture of mathematical research in Topological Graph Theory, gestures play a vital role in that they not only support communication among the mathematicians, but also kinesthetic and haptic experience with the diagram and the mathematical meaning that the diagram holds; and thereby they support each mathematician in his ideational realm.

Aside from Châtelet's diagrammatical studies that cover Algebraic Manifolds, Calculus, Complex Numbers, Coordinate Geometry, Geometry, Infinite Sequences and Series, Ordinary Differential Equations, Partial Differential Equations, Symmetry, Classical Mechanics, Electricity, Magnetism, Quantum Mechanics and Relativity Theory, there are other studies that similarly investigate diagramming, gesturing and mathematizing. The sociologist Christian Greiffenhagen (2014) uses one mathematical lecture in Mathematical Logic as a paradigmatic example "to extend the 'material turn' to an instance of abstract thought" (p. 21). In his analysis, he similarly observes gestures of pointing and of mathematical entities during utterances and notes that while logicians are "predominately [sic] concerned with symbolic writing [diagrams] are nevertheless a crucial aspect of some parts of their practice" (p. 23). Barany and MacKenzie (2014) studied the activities of mathematicians engaged in partial differential equations and related topics while working on the blackboard. They draw awareness to the gesture space between the blackboard and the mathematician that lends itself to exhibit intuition, invention and discovery: "In the pregnant space between chalk and slate there reposes a germ of the bursts of inspiration, triumphs of logic, and leaps of intuition that dominate mind centered accounts of mathematics" (pp. 10–11). We cannot say with certainty that every mathematical field is prone to so much diagramming and gesturing as witnessed in Topological Graph Theory through the research meetings.

Nonetheless, the findings suggest that awareness needs to be raised to the acts of diagramming in particular as indexical signs of engaging with and creating mathematics.

Lastly, we want to discuss some possible implications of these findings for the teaching and learning at all levels of acquiring mathematics. In their creative phase the mathematicians needed to *see* and *feel* objects and their relationships in order to gain an understanding about them or to explore how they could be altered or newly positioned to extract new insights in their field. How do we, as teachers in general and at all levels, allow our students to *see* and *feel* in the classroom or the office? How prevalent is the use of regurgitated visual products in order to explain, for example, what a parabola looks like instead of what a parabola is? How often do we create an environment that places each student in the position of a mathematician, where the student explores the relationship between input x and output $x^2 + c$, perhaps dynamically using geometry software, or drawing by hand the locus of points such that the distance to a given focus equals the distance to a given directrix? These are all environments, where the student is bound to point, touch, hold, trace, add and delete, which may lead to further material engagement with the curve and the curving that is the parabola.

17.7 Conclusion

While Châtelet only hints at the whole process of diagramming during mathematical invention, the analysis of the 122 time intervals and the 128 diagrams allows a glimpse into the whole process and reveals how diagrams—seen from a material semiotic perspective—come into being and are engaged with. These findings enrich Châtelet's theories by being able to identify the different functions of interactions between mathematician and diagram in inventive mathematical processes. The explications of the two episodes above reveal that diagramming during mathematizing is a ritual kind of practice (done over and over again) that offers a physical arena for experimentation. We suggest that the life-cycle functions may also offer fruitful avenues for helping students incorporate diagramming into their own mathematical problem solving, if only by seeing how drawing lines on paper might initiate new gestures and new ideas.

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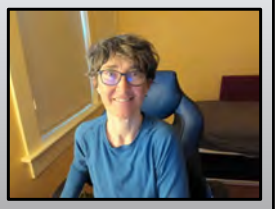
We created our presentation on the ancestral, traditional, and unceded lands of the Coast Salish Peoples, including the territories of the xwməθkwəy̓əm (Musqueam), Skwxwú7mesh (Squamish), Stó:lō,and Səl̓ílwətaʔ/Selilwitulh (Tsleil-Waututh) Nations.

Department of Mathematics
Faculty of Science
Burnaby, BC, Canada

SFU



PARTNERS IN DESIGN

Involving all Stakeholders in the Restructuring Process of an Online Course

1 technician

6 undergraduates

2 graduates as TAs & RAs

3 instructors

Image curtesy of Clipart Library http://clipart-library.com

PETRA - video:
Welcome! My name is Petra Menz and I am the project leader. In honour of our theme, Partners in Design, this presentation was put together by including all stakeholders via audio, video, or text. We are excited to share our journey re-designing a distance education course by first acknowledging the ancestral, traditional, and unceded Coast Salish lands on which we reside.

MATH 190 Principles of Math for Teachers

Delivery (1 LMS + 2 VLE)

- Canvas
  - → institutional LMS
  - → content (readings, multimedia, activities)
  - → reflections, participation, midterm exams
  - → gradebook
- Crowdmark
  - → online collaborative grading platform
  - → group assignments
- LON-CAPA
  - → randomized questions
  - → discussion board per question
  - → online assignments

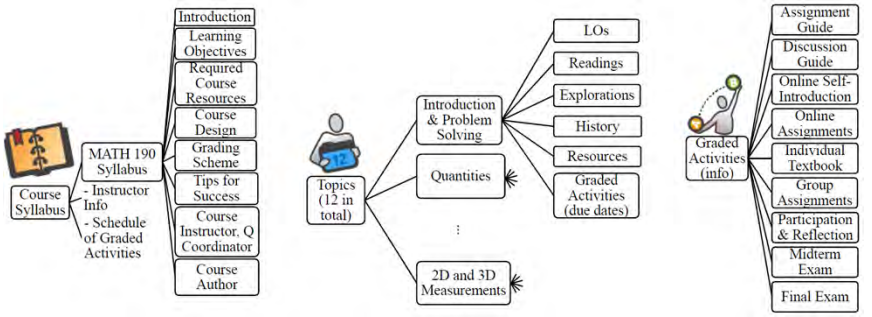
JOANNA - video:
Canvas is our institutional Learning Management System. In 2013, the Canvas container for the distance education version of the course was created under the guidance of the Teaching and Learning Centre. The learning tools that were developed became quite popular with the students, so the Canvas container was adopted for the face-to-face version. Our courses also depend on two other Virtual Learning Environments for assignment submissions and feedback. However, as the number of resources increased, the layout negatively impacted the learning experience.

Canvas in the Way of Content

Content is really great BUT…

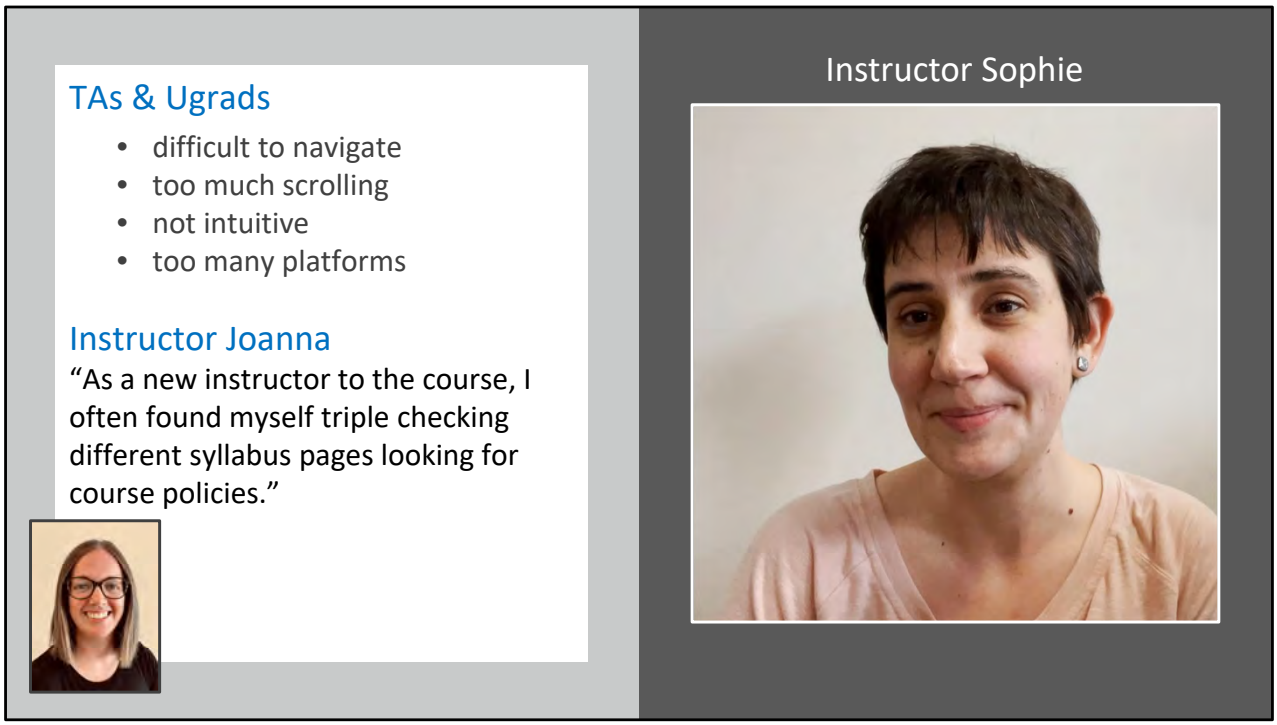
Ugrad Damian – F2F

even a road map didn't significantly reduce the layout's complexity.



increased complexity => steep learning curve

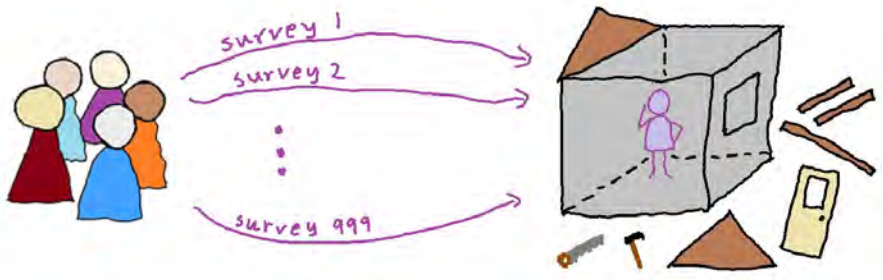
DAMIAN - video:
Hi, so last semester I took MATH190 and I had a terrible experience to begin with trying to parse the Canvas system. I downloaded different pages trying to look for information, where it didn't feel like it should have been. I had to ask classmates for help, professor for help. And eventually I figured it out and it was systemic. The learning curve was incredibly steep to begin with, and that felt like a huge barrier to get over.

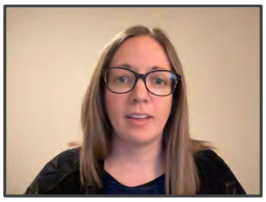


SOPHIE - video:

My name is Sophie Burrill and I am a lecturer in the Math Department at Simon Fraser University. I taught MATH190 a few times and in a few different contexts including in person and online. I notably have used the old Canvas container that is currently being redeveloped right now. There is a ton of resources that have been developed for the students. Very much laid out here are the learning outcomes for this particular topic. Here are all of the readings you need to do and all the videos you need to watch. There are some explorations you can do to kind of like get your own hands onto this subject matter. There is history that's all collected and attached to the right like appropriate area. It is a really great Canvas container. Also, I have felt a little sheepish showing up on day 1 with this Canvas container. That happens in some sense in all of our math classes. They show up on day 1. You feel a little like you sprang the students with a firehose of information. There is just so much in the Canvas container and it can be a little bit overwhelming for the student to figure out where do they need to go to learn about all the very important aspects of this course. There were ways for them to get onboarded into this Canvas container but it always seemed like an extra step. If you need to have someone show you where you need to go there is something lacking in intuition.

How to Restructure Canvas Container?



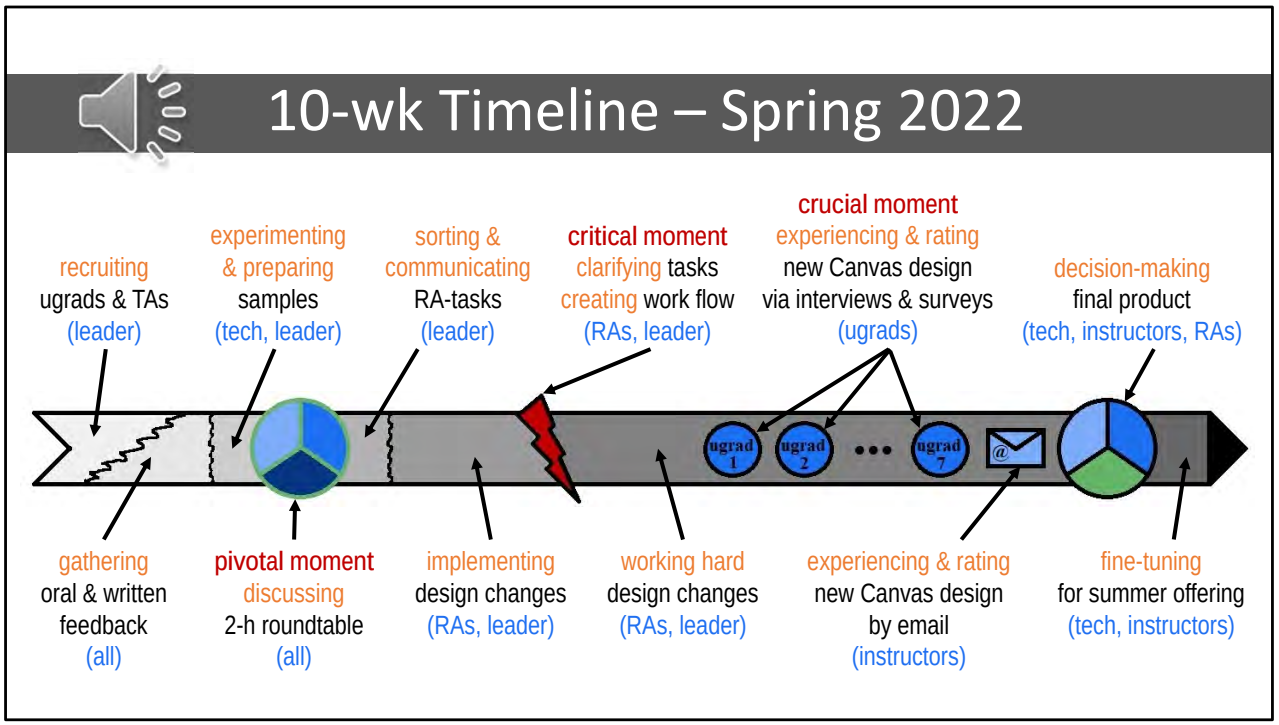


Students are the system's principal stakeholders and we need to hear from them, [...] let's get more students to the decision-making tables in higher education. (Flynn, 2020)

⇒ ugrads
⇒ TAs/RAs
⇒ tech

must be on board with instructors

JOANNA - video:
Even though student surveys initiated a number of revisions, it had become clear that while surveys capture issues students are experiencing, they are not able to solicit student ideas for solutions in a meaningful way. Instead, open dialog was needed with full involvement of all stakeholders.



PETRA – voice only:
I invite you to glance over our project timeline starting at the top left.

There is a lot of work to get a collaborative project going, such as obtaining funding, finding willing undergrads to participate, hiring competent and invested research assistants, getting all instructors on board, securing a knowledgeable technician, and most importantly, keeping communication channels open in all directions.

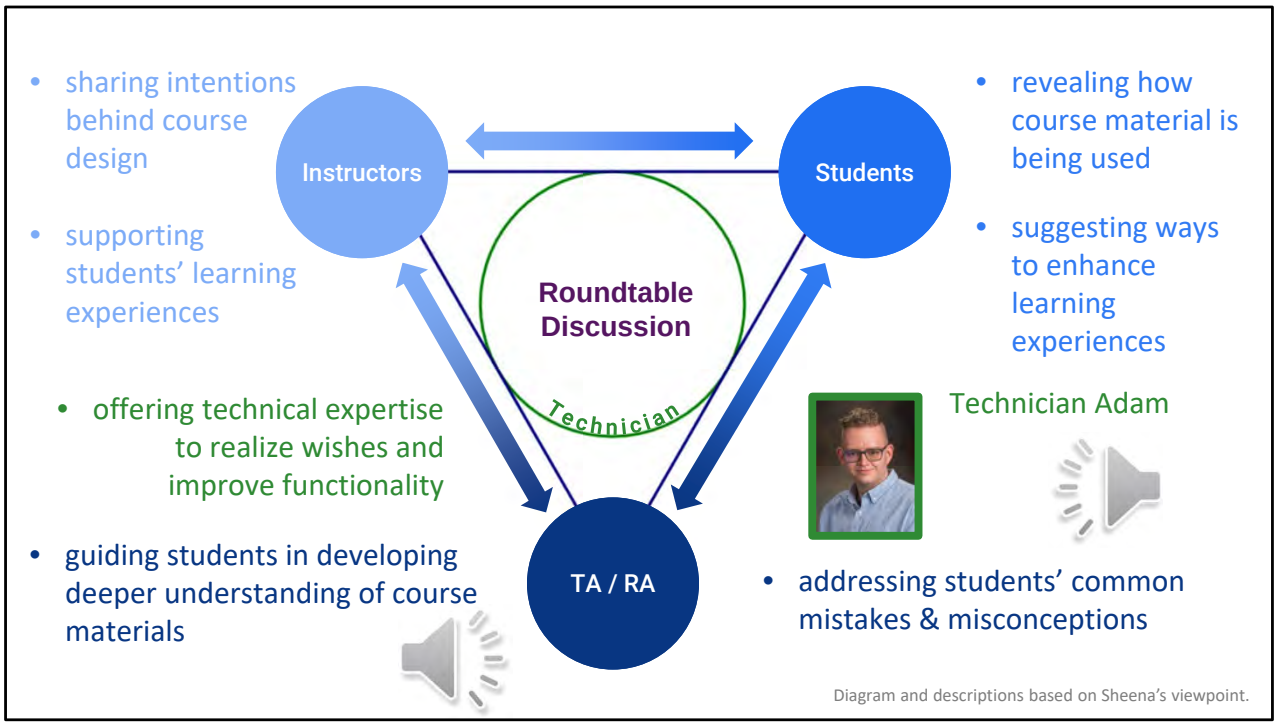
The graphic conveys 3 key moments in our journey. The **critical** moment near the centre alludes to the fact that the project wasn't all smooth sailing. Our plan of the two research assistants going back and forth between each other during the implementation phase simply didn't work, because situations arose that needed decision-making by an instructor. The plan was then simply altered and work re-distributed. Interviewing and surveying the undergraduate students while experiencing the new Canvas container towards the latter half of the project provided a **crucial** moment. We were now at a cross-road that brought us back to the beginning. Did we hear the ugrads correctly and did they like what we did? Hence, it was **pivotal** for all stakeholders to meet and exchange viewpoints at the beginning, so we will highlight the roundtable discussion shortly through the eyes of the teaching and research assistant Sheena and the technician Adam.

TA /RA Sheena – Many Viewpoints

As a Graduate Student	As a Teaching Assistant	As a Research Assistant
● Prior, mathematics teacher ● Currently, Mathematics Education PhD program ● Interested in → developing students' conceptual understanding in Mathematics → building teachers' mathematical knowledge for teaching	● MATH 190 – 3 terms ● Student-interaction → in help centre → through grading → common mistakes and misconceptions ● Gained familiarity with → course material → Canvas container	● Tapped on experience gained as a 190-TA ● Created 36 quizzes ● Coded questions for quizzes ● Distilled concepts from 84 video lectures

SOPHIE - voice:

We showcase Sheena, who brings in a rich background as a prior mathematics teacher, current mathematics education PhD-student, and 3-time teaching assistant for this course. It was quite evident that she cared about improving both pedagogical & functional aspects of the students' learning experience.



SOPHIE - voice:
This diagram is Sheena's interpretation of the project in her role as a teaching and research assistant. The diagram perfectly describes the dialog at the roundtable discussion connecting all stakeholders with one another and creating pivotal awareness of each other's viewpoints.

ADAM - voice:
As the coordinator of workshop operations in the Math Department at SFU, I do a lot of work with Canvas. This includes experimenting with different features and designs, hence my involvement with this project. Prior to our roundtable discussions, I made some prototypes for the course based on the existing design and following that discussion, I did many of the initial high-level structural changes to the Canvas container. To that end, the partners in design approach is really a holistic approach to revamping the Canvas container as everyone interacts with the online learning environment in a different way: I look at it from a technical perspective, while the instructors and TAs have a pedagogical mindset, and the students come at it with the intend to learn the material. By incorporating these different viewpoints, we were able to find an equilibrium design that is appealing to everyone involved.

TA/RA Participation Overview

- Written feedback as former TA
- Round-table: shared issues & ideas
- Implemented plan
- Proofed implementations

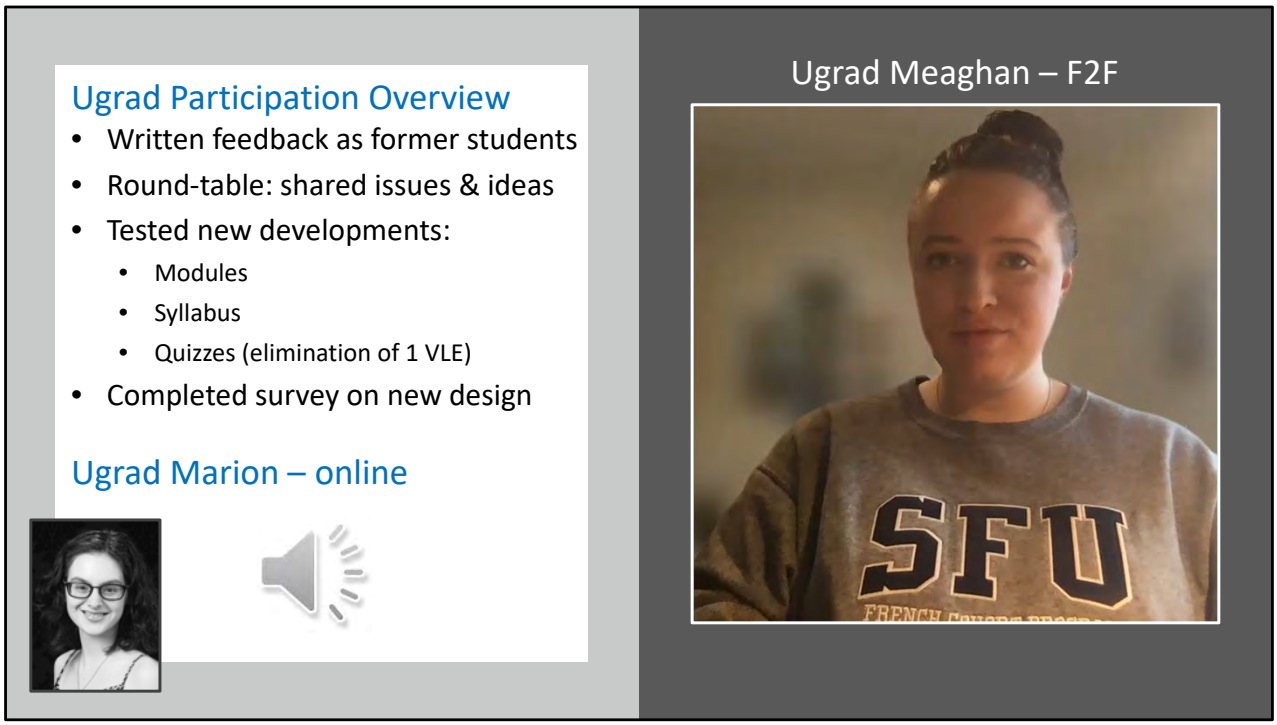
TA/RA Sheena

- Really enjoyed working on project:
  - → Learnt how students used materials
  - → Discovered instructors' intent on design
  - → Focused on pedagogical and functional aspects to enhance students' learning
  - → Aligns with interests and career goals

TA/RA Kailyn

KAILYN - video:

My name is Kailyn Pritchard and I am a Masters student in the Department of Mathematics at Simon Fraser University. I was a research assistant on this project and have been a teaching assistant for the MATH190 course for four semesters now. I really appreciate the opportunity that the roundtable discussion regarding the Canvas container provided at the beginning of the project to get an idea how different folks were interacting with the container and the problems we saw within it. If someone were to do a similar project as this, I would really recommend some mindfulness just about the power dynamics that can occur in these kind of spaces and conversations between different stakeholders and different roles in the university setting. My role in this project was to provide feedback on the new Canvas container as it was being updated. And again, if someone were to do a similar project in the future, I would recommend having some sort of collective space where communication can occur between different members of the project. So, as things were passed back and forth it can be communicated what about that feedback is working really well and what sorts of adjustments need to be made in order for us to be even more effective and to collaborate more effectively.



MARION - video:
My name is Marion Maldoven. I am an SFU education student and I hope to become an elementary school teacher. It made me feel heard to be part of this project. I felt like my feedback and input were taken seriously. I think this was a great opportunity because it allowed me to share my thoughts as a former student how the Canvas container could be improved and streamlined for future students. The main thing that I like about the new Canvas container is the fact that there are more modules instead of separate pages. I find the module view more convenient as I can minimize things that I do not need and still have an overview all in one place. I also really like the sample weekly schedule that is provided on the homepage. As a student with autism, I appreciate having schedules or suggested schedules easily accessible and built into a course.

MEAGHAN - video:
Hi, my name is Meaghan Laycock. I am a French major and political science minor in my fifth year and I was in the MATH190 course in person in fall 2021. I felt that the process for providing feedback was very fair and I always felt that my input was being heard. I really appreciated the 2-hour meeting where I got to see where the course was at with the changes they already made and provide my feedback from there. And being able to go through the new Canvas course I felt was a really great opportunity to look at it from a student's perspective having already taken the course to see what changes have been made and if they would work. My initial thought of the new Canvas container was that it was much more clear. The syllabus was really easy to read and follow and the modules had a really great layout with the timeline of the assignments you had to do but also the work you had to learn beforehand.

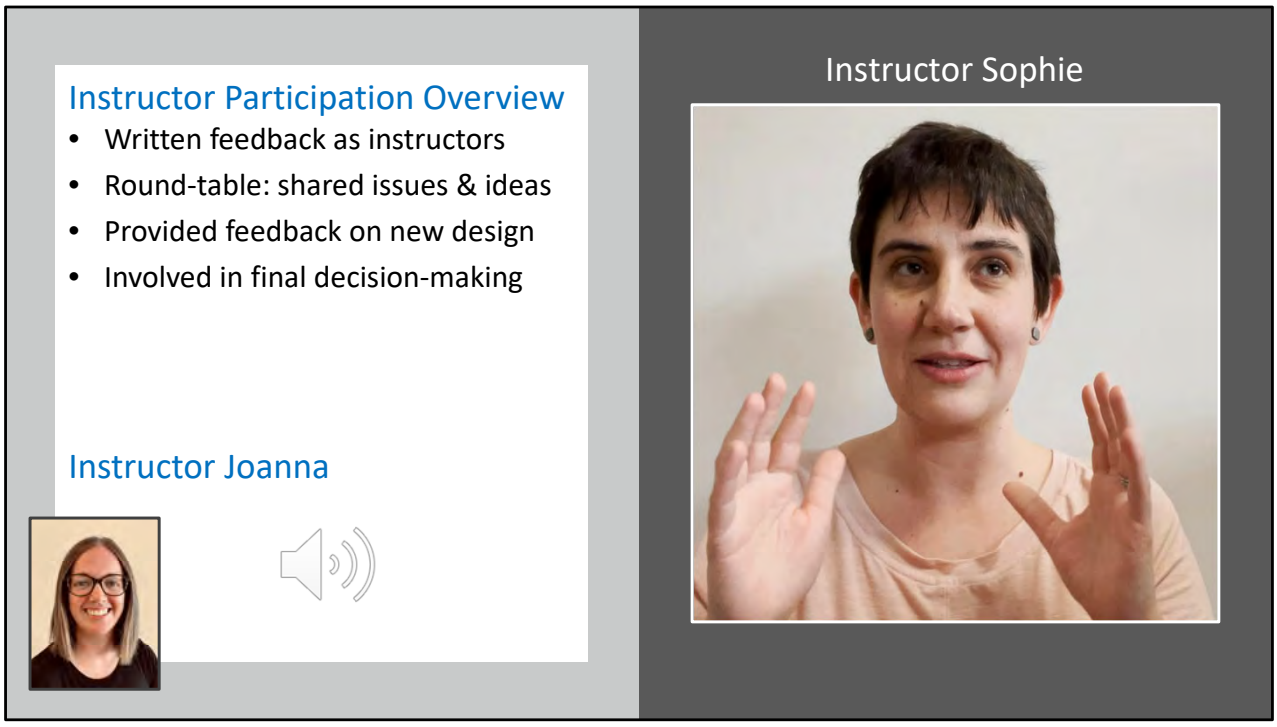
Ugrads

- Syllabus
  - easy to navigate
  - bite-size information
  - intuitive
- Modules
  - similar to other courses
  - guided learning
  - info at a glance
- Online Assignments
  - similar to exams
  - better exam preparation
  - one less platform

Ugrad Maitreyee – online

MAITREYEE - video:

Okay, so syllabus page, it's great. It looks amazing. Everything I need is right there. You don't need to click through one link to get to another link. Love that! Because every other course that I have done on Canvas, it heavily relies on modules and the MATH190 one didn't. And so that's why it was a little harder to, I guess, because it not being there. That was the learning curve. But now that everything is in modules, I think it would be a lot easier for a newcomer to navigate through the course. That's always great. So, because the exams are open book, sometimes if I was unsure about an answer, I would always go back to the lightboard video and now that I know what is in each video I would have to do a lot less scrolling and stuff.



JOANNA - voice:

At the roundtable it was great to hear the perspectives of the students in a serious and engaged way. There is what we think the students think, there is what they tell us of the top of their head when polled and what they come up with when they are empowered and fully engaged in a project. Seeing their involvement at the roundtable was so cool. It was also a great chance to discuss the pedagogical reasons behind the certain styles of choices with all the different stakeholders. It was nice to feel a part of the project. Now when I teach this course, I feel more ownership with the course design. I am happy with the user-friendliness of the result and I am excited to test it out this summer.

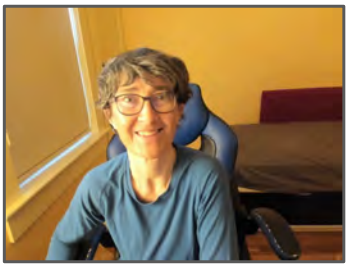
SOPHIE - video:

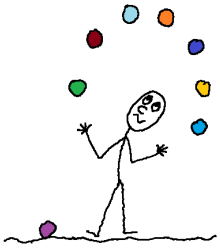
The reorganization was very, it just felt like this was the thing that we needed to do. Someone needed to do. And with Dr. Menz taking this on I was so grateful. And she really heard my feedback. And I have now looked through the Canvas container and I am so excited about it. I am so excited about it. The layout is just so much clearer. I like there is a little welcome message, which is something I always like to do. There is a snapshot of the weeks so that the students can right away figure out what a typical week is going to look like. And then the modules, I am really really very excited about the modules. There is a next button and they do this, and there is a next button and they do that. The syllabus looks great, I just, with the dropdown menus. I feel like the learning curve is going to be, the start and end of it is going to be, when you see that little triangle, then go click on it so you see the information below. Everything else is super duper intuitive at least from where I am sitting. And I am really excited for it to be tested out and I then to make use of it myself.

Nuts & Bolts of Project

Project Leader Overview

- Obtain funding
- **Get everyone on board**
- Collect ideas
- Organize meetings
- **Be goal-oriented**
- **Create a (flexible) plan**
- Train RAs
- Run after ugrads
- **Coordinate feedback loops**
- Deal with obstacles
- **Make decisions**
- Don't drown in emails ☺
- **Breathe & Smile**





PROS

- Truly collaborative
- Ideas appear
- Satisfaction way up
- Valuable product

CONS

- Limitations of LMS
- Not enough $
- System deficiencies
- Misunderstandings
- Not enough

PETRA – video:
We hope we conveyed how thrilled all stakeholders are with the outcome of this project. It was no small feat to accomplish all the changes in just 10 weeks, thanks in part to the two research assistants, who were well chosen for their roles. It really helped to be goal-oriented and have created a plan, whereby one RA did the heavy lifting, while the other RA functioned as a proof-reader, with my role being the go-between and decision-maker.

Most importantly, by coordinating a number of feedback loops, we were assured that many of the needs of the undergraduate students and instructors were heard, and that satisfaction and user-friendliness was high. It was an incredible experience to get authentic involvement by the undergraduate students. Their voices brought another reality to the discussions and interviews that positively shaped the final product.

This project did feel like I was juggling a lot of balls and I sure dropped a few of them. Mostly, this was around mis-communicating and mis-understanding. My advice to anyone doing a similar project would be to make the time to speak to people in person in between the mega flow of emails. It takes maturity to see redirections as places of growth rather than critique when provided in written form. Communicating essential changes in person would perhaps have made for a smoother journey.

After some reflection, my parting wisdom is that even collaborative projects will – in the end – need someone to coordinate and make hard decisions, so breathe and smile.

Next Steps Summer 2022

- Teaching from newly designed Canvas
- Surveying new students
  - Syllabus
  - Modules
  - Online quizzes

Thank You

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JOANNA – voice:
The project will continue next semester when the course is tested out for the first time. Students will be surveyed on their experience with the syllabus, modules and newly incorporated quizzes. Thank you.

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EQUITABLE PARTNERS IN DESIGNING THE LEARNING ENVIRONMENT OF AN ONLINE COURSE

Petra Menz, Joanna Niezen

Simon Fraser University

This paper describes the rationale, implementation and assessment of restructuring an online course, where all stakeholders—students, instructors, teaching assistants and a technician—had equitable input throughout the process. The course is designed for post-secondary students aspiring to be elementary teachers. Being offered tri-semesterly, online and asynchronously with students self-led through the material, the course's learning environment is of paramount importance. Since its creation in Fall 2013, with the advancement of technology, the learning environment of the course has become rich with material offering active and comprehensive learning. However, as the course has grown, its structural complexity grew as well making navigation increasingly intricate. Furthermore, the students taking this course are not only comprised of a variety of ethnic backgrounds in Canada, a multicultural nation, but often identify with disability-related needs (~15%). When student surveys started to indicate that the learning and accessibility of the course were hindered by its navigation, the instructors realized that these undergraduate students needed to be involved in the restructuring process along with the graduate student teaching assistants and the implementing technician. After combining and organizing feedback that was collected both informally and via surveys, representatives from all stakeholder groups met to decide on various design choices and map out a plan to improve the course layout. Undergraduate and graduate students contributed equally to this restructuring plan. Not only were the undergraduate students' insights thoughtful, but their unique viewpoints were invaluable, leading the discussion as experts in user experience and ultimately guiding many design choices. Moreover, the graduate-undergraduate student relationship that develops within the help centre was leveraged to speak to the common misconceptions that undergraduates face in this course. Student surveys from three subsequent semesters were analyzed. Ultimately, the inclusion of all stakeholders led to a richer and more accessible online learning environment.

INTRODUCTION

This paper recounts the successful restructuring process of an online learning environment (OLE) by fully including instructors, teaching assistants, recently successful students and a technician in the project. Despite financial barriers, time constraints and a wide variety of viewpoints, this stakeholder-inclusive approach led to exceptional results in a math course for pre-service elementary school teachers. We share the rationale, implementation and assessment of our project, Partners in Design, in the hopes that this paper will inspire similar projects in education. As Chemistry and Biomolecular Sciences Professor and 3M National Teaching Fellow Flynn (2020, p. 2) points out, "the results are better when students are at the table: the work goes faster, and students think of ideas that professors would not have thought of alone".

The next three sections describe our course, its student population and its OLE, as these are important components that led to the redesign of the OLE. The restructuring process is then outlined, introducing the stakeholders and describing the data collected prior to implementation. Three significant moments in the timeline of events are included due to their impact on the project. Following the implementation of the new restructured course, the reactions of the instructors and undergraduate students are shared. Lastly, student survey results are provided, which were collected over the three semesters following the implementation of the restructured OLE and from which conclusions are drawn.

About the Course

MATH 190, *Principles of Mathematics for Teachers* (math-for-teachers), is offered through the Department of Mathematics at Simon Fraser University (SFU) in Vancouver, Canada. This math-for-teachers course is delivered five times a year in every semester of a tri-semester university, either online and asynchronously (capped at 40 students per semester) or synchronously with a blend of online and in-person in a flipped classroom setting (capped at 60 students per semester). Furthermore, the course is designed as a gateway for students applying to the Education Faculty's Professional Development Program to become elementary school teachers, instructing Kindergarten through Grade 7 or ages 5 to 12 years old. Three teaching faculty members regularly instruct this course.

Additionally, the math-for-teachers course is serviced by a drop-in help centre, an open and welcoming study space for students to meet in groups or individually and to get timely help in all aspects of the course. The help centre is run by instructors and teaching assistants. Graduate students from both the Department of Mathematics and the Faculty of Mathematics Education are hired as teaching assistants not only to staff the help centre but also grade assessments. These graduate students are typically chosen not only for their mathematical expertise, but also for their kind and caring manner towards students who struggle with mathematics. It is often the case that these graduate students are interested in education and seek professional development in teaching themselves.

About the Undergraduate Students Taking the Course

Students of the math-for-teachers course vary widely in their academic levels (from first year to graduated), mathematical knowledge (from almost none to shaky and rarely fully meeting the prerequisites), relationship with math (from phobia to indifference and rarely an appreciation), accessible learning needs (approximately 15% disability-related needs) and English language ability in multicultural Canada (from weak communication ability to fluent native speakers). Furthermore, Ball's poignant 2005 article *Knowing Mathematics for Teaching* had a snowball effect across North America for a simple reason: her research showed that an elementary school teacher's mathematical knowledge is tightly linked to their students' mathematical attainment. Additionally, teachers' habits, beliefs and confidence influence their learning practice (Beilock et al., 2010; Clarke & Hollingsworth, 2002), while students' mathematical beliefs are correlated with achievement (Mason & Scrivani, 2009). As a result, when the online course was first created in 2013, it was the first author's goal not only to enrich the mathematical knowledge of these teachers-to-be, but to increase their confidence in their mathematical abilities.

About the Online Learning Environment

The online course is housed in Canvas—the institutional Learning Management System—and was created in 2013 by the first author and the SFU Teaching and Learning Centre to parallel the face-to-

face version in pedagogy, content and assessment. To encourage student engagement and learning in online courses, it is critical to carefully choose design elements and embed instructional activities (Graham et al., 2001; Hegeman, 2015; Robinson & Hullinger, 2008; Simpson, 2013; Twigg, 2003). To achieve this goal, the online material was fine-tuned through multiple student surveys. As a result, the online material became rich, offering active, comprehensive and student-centred learning through 57 Lightboard videos of 8-10 minutes in length, hands-on activities, virtual explorations and topical instructor summaries. Like Hegeman (2015, p. 83), the first author found that through instructor-generated videos "procedural learning was deemphasized and the importance of conceptual understanding in the learning of mathematics was highlighted". Reflections, group assignments and real-time feedback assignments were used for formative assessment to provide a weekly routine and facilitate both collaborative and individual learning. In terms of summative assessment, three non-cumulative examinations wrap up a sequence of topics approximately every four weeks. Grades are not scaled, and the grading scheme and letter grade points are published from the outset, making the course transparent to the students. The grade point average of both versions of the course is consistently between 75% and 80% or a B and B+.

The online learning tools became quite popular with students, evident from surveys and unsolicited feedback, so the face-to-face course adopted the OLE of the online course and was turned into a blended flipped classroom, with the activities and discussions primarily taking place in the classroom instead of online. As online learning expert Twigg points out, in a blended course, "instructors are able to reduce class time spent on topics that the students clearly understand, increase time spent on problem areas, and target individual students for remedial help" (2003, p. 32).

By 2021, both versions of the course depended on two additional virtual learning platforms. LON-CAPA was selected for the online assignments because it was the state-of-the-art automated and randomized assessment tool at the time. Crowdmark was chosen for hand-graded group assignments because the department adopted this virtual mechanism for written assignment submission and grader feedback. Over time, navigating the OLE became complex. Rickwood (1998) concluded that students can be overwhelmed and confused when a course uses multiple media types and often prefer only a limited set of media exposure.

Multiple instructor-led and institutional student experience surveys supported that the teaching and learning material, assessment model and support structure are effective both inside and outside of the classroom. However, these surveys also brought the underlying theme of the complexity of the student experience to the surface, negatively impacting the learning experience. One student, diagnosed with ADHD and achieving an A+ in the course, echoed the anonymous student surveys succinctly, indicating that they "had a terrible experience at the beginning trying to parse the Canvas online system [which] felt like a huge barrier to get over". The online education researcher Simpson emphasizes "the critical need to support students holistically—as people with more than just cognitive needs—and that, indeed, quite often it is organizational and emotional support that is most effective in helping them to success, rather than focusing on their intellectual characteristics" (2013, p. 14). Emotional and technical support are provided through the drop-in help centre, but students' time devoted to the course is better spent on the content. The access and presentation of information in an online course should not detract from time spent on the material. It had become apparent that the previous approach of using student surveys to guide periodic peripheral course upgrades exacerbated the complexity of the OLE.

Literature supports that instead of seeing students as consumers, whose voices are listened to, they need to be involved as partners to create transformation in curricular design and pedagogy (Bengtson et al., 2017; Dunne & Zandstra, 2011; Healey & Healey, 2019; Gravett et al., 2020). After all, "students can identify and propose/design solutions to potential issues before they arise within the course itself" (Flynn, 2020, pp. 2-3). The time was ripe to adopt this viewpoint wholeheartedly and bring all stakeholders—instructors, Canvas technical support, teaching assistants and undergraduate students—to the table for an inclusive approach to restructure the OLE.

RESTRUCTURING PROCESS

Drawing on the enthusiasm of the involved instructors to change the OLE of the course, the first author secured funding for two research assistantships and honoraria for six undergraduate students, so that the Canvas container could be redesigned over the course of 10 weeks starting in January 2022 with the collected stakeholder data. The project timeline is depicted in Figure 1. Absolutely no elements were preselected for redesign, but rather, the process was approached organically, driven by the voices of all stakeholders as captured under the section **Pivotal Moment** below.

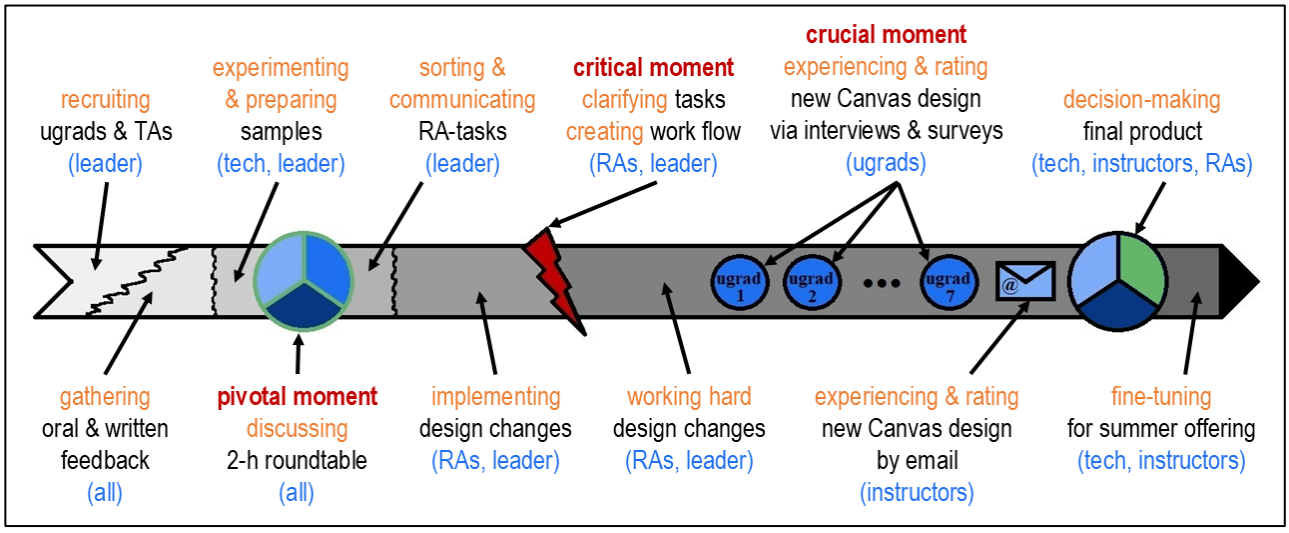

Figure 1: 10-week restructuring project timeline in the spring of 2022

Recruiting the Stakeholders

The recruitment of all stakeholders as partners in design marks the start of the restructuring process, as depicted on the left of the event timeline in Figure 1. The words of the departmental technical support person (TECH) are shared below:

> "The partners in design approach is really a holistic approach to revamping the Canvas container as everyone interacts with the online learning environment in a different way: I look at it from a technical perspective, while the instructors and TAs have a pedagogical mindset, and the students come at it with the intend to learn the material. By incorporating these different viewpoints, we were able to find an equilibrium design that is appealing to everyone involved."

Rather than view the TECH as the implementor of a new design, as is typical, the TECH was included in the restructuring process as an equal partner in design. Given their expertise with Canvas, the TECH was able to bring implementation ideas, as well as their own ideas and approaches, to the table.

Three instructors regularly teach both versions of our course, all of whom are invested in offering a pedagogically sound approach in their classrooms and the OLE. All three of these instructors came to the design table.

Flynn indicates that teaching assistants (TAs) can be collaborators in creating a new course format and developing materials (2020). For this project, including TAs was important for several reasons: First and foremost, TAs are in direct contact with undergraduate students of the course in the help centre, including helping students with navigation and administrative questions, so that they hear and troubleshoot firsthand accounts of students' struggles with elements of the OLE. Secondly, in their role as assessment graders, they interact with the OLE and have a good understanding of its shortcomings. Finally, as TAs are graduate students themselves, they are more familiar with the learning experience of students than instructors and therefore may have ideas for better design elements that support learning. For all the above reasons, the TAs hired for the course are the perfect candidates to act as research assistants (RAs) for the project. Ultimately, one master's student in the Department of Mathematics (MSc RA) and one mathematics education doctoral candidate from the Faculty of Education (PhD RA) were selected as the RAs for this project, both with the desire to become educators themselves.

Finally, six undergraduate students (UGRADs) who were recently successful in the course were invited to participate as partners, since "[t]hey are not experts in the subject matter, but they are experts in the system" (Flynn, 2020, p. 1) and would therefore know what elements were not working and have ideas to improve their learning and navigation in the OLE. To ensure that these UGRADs represent the typical student makeup of the course in our multicultural nation, Canada, the six UGRADs were comprised of one male and five females, two students with diagnosed learning challenges (ADHD and autism) and three different ethnic backgrounds including two English as an additional language speakers.

Stakeholder Data Collection

The stakeholder data collection took place in three phases throughout the 10-week redesign project.

The first phase involved gathering informal oral feedback from the six UGRADs, three instructors, and the TECH, as well as open-ended written feedback from both RAs and two UGRADs, listed as the second event on the timeline in Figure 1. Both types of feedback described OLE elements that worked and didn't work as well as suggestions for improvement. The TECH used the suggestions relevant to technical aspects to experiment with Canvas features and design elements to come up with several prototypes, while the first author prepared a summary of the data, all of which formed a springboard for the next step.

The second phase of stakeholder data collection emerged organically through the coming together of all stakeholders at a 2-hour roundtable discussion (the third event on the timeline in Figure 1), which is elaborated on below under **Pivotal Moment**, because it marked the genesis of all design goals for the OLE restructuring.

The third phase consisted of interviewing five of the UGRADs as they visited the redesigned OLE (the ninth event on the timeline in Figure 1). Interviews were conducted and recorded via Zoom. An RA conducted the interviews by first observing and documenting each UGRAD's unguided exploration of the OLE, since navigation had been heavily criticized in the old OLE. After that, the RA answered any

questions and asked for the UGRADs' critical opinion on all new OLE elements. The UGRADs' manner of navigation and reactions were then summarized by the RA in written form. Additionally, the interviews were immediately followed by an emailed survey with detailed questions on each new OLE element. Having the RAs host the interviews ended up being a great way to encourage honest feedback from the students. The two instructors who were not involved in the implementation event were invited to browse the redesigned OLE on their own and provide their feedback via film or audio recording (the tenth event on the timeline in Figure 1). The recording of both UGRADs and instructors was done to bring their voices and facial expressions into context and for use in any future presentations. The feedback determined whether the restructured OLE was ready for trial in a live offering of the course. This event is expanded on below under **Crucial Moment**.

Pivotal Moment – Emerging Redesign Goals

During the 2-hour roundtable discussion among the stakeholders, UGRADs and RAs contributed equally to the restructuring plan (the third event on the timeline in Figure 1). Not only were the students' insights thoughtful, but their unique viewpoint was invaluable. Not only did these students participate in the discussion, but they also led the discussion as experts in user experience and ultimately guided many design choices. Similarly, the roundtable was a great chance for the instructors to share with the UGRADs and TAs the pedagogical reasons behind certain choices. After the discussion, the PhD RA shared her insight on the dialogue via the visual representation given in Figure 2, which perfectly describes the role each stakeholder held at the roundtable and the vital connections that were created between all stakeholders to establish the necessary awareness of each other's viewpoints.

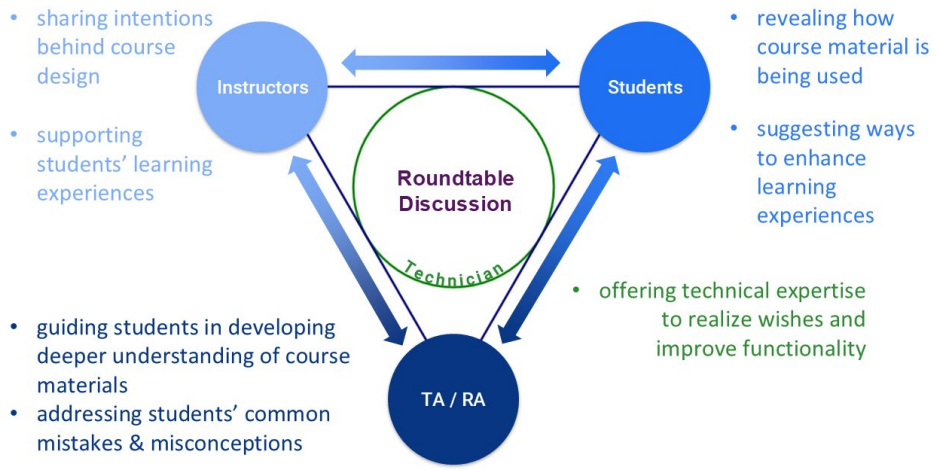

Figure 2: Roundtable dialogue from the viewpoint of the PhD RA

The unfolding open and equitable communication and engagement created an evolving flow of ideas. Through this organic process, the following six goals emerged for the OLE restructuring, with specific tasks assigned to the people given in brackets:

1. Provide a welcoming front page that briefly points to the main features of the course and depicts a snapshot of weekly tasks (all three instructors).
2. Design a comprehensible syllabus containing all administrative items in accordion style, thematically grouped by colour (TECH and one instructor, the first author).

3. Present all readings, media, activities and assessments in a topic-based modular design, making use of Canvas' module feature, to guide students through a sequential learning path and to provide a weekly routine (TECH).
4. Replace the typed, verbose instructor summaries with handwritten, colour-coded one-page study sheets on each topic, which depict a model for students to make their own mathematical summary notes. Moreover, these summaries provide a topic roadmap for the students and TAs alike (two instructors, the authors).
5. List the learning outcomes in accordion style for each of the 57 lightboard videos to make content appear more manageable and less overwhelming, and to help students and TAs locate and relocate learning topics (PhD RA).
6. Migrate the LON-CAPA online assignments over to Canvas by recreating them as Canvas quizzes, to do away with one virtual learning platform (both RAs and one instructor, the first author).

Critical Moment – Bumpy Communication

The actual restructuring of the OLE took place over eight weeks. It would have been unrealistic to assume that everything would roll out smoothly during this time. While most goals were accomplished effectively, a critical moment came about two weeks into the workflow of Goal 6 (the red lightning moment in the seventh event on the timeline in Figure 1). After many weeks spent closely working together, emotions ran high when feedback was misunderstood, as outlined below, which, upon reflection, simply had to do with poor communication.

As a part of the move of the online assignments to Canvas (Goal 6), tasks were distributed among the two RAs and the instructor to create a workflow. The PhD RA did the heavy lifting by coding the online assignment questions into various Canvas Question Banks based on the existing LON-CAPA questions, while the MSc RA functioned as a proofreader and question-tester, writing a report for each online assignment. The instructor used the information in the report to make edits, acting as the final decision-maker and advising the PhD RA as needed. Initially, wanting to provide autonomy to the RAs, the instructor did not detail how the questions were to be coded nor how the reports should be written. Since the PhD RA was not a native English speaker, the questions needed massaging. Being a math history major, the MSc RA wrote overly detailed paragraphs that were heavy on critiquing the questions and justifying the critique rather than providing a list of edits and coding errors that could be quickly considered and acted upon. In the interest of time and RA money, the instructor stepped in by writing a brief factual email to each RA redirecting their tasks, not realizing that meeting in person would have been conducive to a dialogue and consensus rather than being interpreted as a top-down reprimand. The MSc RA verbalized their dismay to one of the other instructors and a meeting was called to clear the air. The lesson learned is that instructors must act with care to overcome the inherent power structure to provide equitable collaboration among all stakeholders, especially those perceived as lower in a power dynamic. The authors emphasize the importance of ongoing face-to-face communication with all stakeholders as the vehicle for overcoming miscommunication and power imbalances.

Crucial Moment – How did we do?

Eight weeks into the project, with two weeks left, the feedback collected via the Zoom interviews of UGRADs, written surveys from UGRADs and recorded reactions of the instructors provided a crucial moment where the efforts of the stakeholders could be evaluated against the project's goals. The reactions and feedback were overwhelmingly positive, indicating that the project's goals had largely been met. Below is a transcript from one of the instructors, who had voiced dismay about the old OLE:

> "She really heard my feedback. [...] The layout is so much clearer. I like that there is a little welcome message, which is something I always like to do. There is a snapshot of the weeks so that the students can right away figure out what a typical week is going to look like. And then the modules, I am really, really very excited about the modules. There is a next button and they do this, and there is a next button and they do that. The syllabus looks great, I just mean, with the dropdown menus. [...] Everything else is super duper intuitive at least from where I am sitting."

The UGRADs specifically commented on the following features: the new syllabus is intuitive, easy to navigate and only contains bite-size information; the modules support guided learning, provide information at a glance and are like other courses they experience at SFU and so build on familiarity; and the online assignments no longer had the hurdle of learning an additional platform and are now in a similar format to exams improving exam preparation and reducing exam anxiety. The voice of one UGRAD echoes the feedback from all UGRADs:

> "I always felt that my input was being heard. I really appreciated the 2-hour meeting where I got to see where the course was at with the [potential design ideas for change] and provide my feedback from there. And being able to go through the new Canvas course I felt was a really great opportunity to look at it from a student's perspective having already taken the course to see what changes have been made and if they would work. My initial thought of the new Canvas container was that it was much more clear. The syllabus was really easy to read and follow and the modules had a really great layout with the timeline of the assignments you had to do but also the work you had to learn beforehand."

The feedback also included several improvements suggested specifically by the UGRADs, which were discussed at a follow-up roundtable with the instructors, RAs and TECH for decision-making about the final product (the second last event on the timeline in Figure 1). This led to some fine-tuning of the OLE for its first implementation of the redesigned course in the subsequent summer semester (the final event on the timeline in Figure 1).

COURSE DATA RESULTS AFTER RESTRUCTURING

Despite promising outcomes from students familiar with earlier iterations of the course, it was important to understand how the course would be viewed by students stepping into the course for the first time with the restructured layout of the OLE. A survey was created with the help of the Centre for Educational Excellence, a learning and teaching support department at SFU with experience in survey writing, to ensure the survey format and questions would gather the desired data and be easy for students to answer. In the three semesters following the completed project, students were surveyed to gauge if the goals outlined in **Pivotal Moment** had been met. In Summer 2022, the online version of the course ran. In Fall 2022 and Spring 2023, both the online and blended versions of the course ran. In all three surveyed semesters, one of the three regular course instructors was involved in the course. The response rates in each semester were 62% in Summer 2022, 86% in Fall 2022, and 87%

in Spring 2023. The course data results are grouped into Navigation, Online Assignments and Learning, Online Assignments Format, Instructor Summaries and Organization and discussed next. All stated percentages in this section will be calculated as averages over the three surveyed semesters.

Course Data Results on Navigation

Students were asked how easy it was to navigate Canvas based on ease of locating six key features: policies in the Syllabus (Goal 2), assignment and quiz deadlines (Goal 1), relevant discussion boards (Goal 3), the weekly readings and videos (Goal 3), help centre schedule (Goal 1), and solutions to graded activities (Goal 1).

The results of this topic are depicted in Figure 3. The first four questions had strong results, where students responded positively ("agree" or "strongly agree") with 80%, 91%, 78% and 92% respectively across the three semesters. One student commented, "Everything was extremely well laid out." and another said, "I really appreciated how organized everything was on Canvas. There is a lot of different material, so I found the colour coding to be very helpful as well." Of note were 5% of students adding freeform comments across the three semesters, asking for a shortcut to the discussion boards outside of the modules, which goes to show that familiarity and consistency are more important than design for some students.

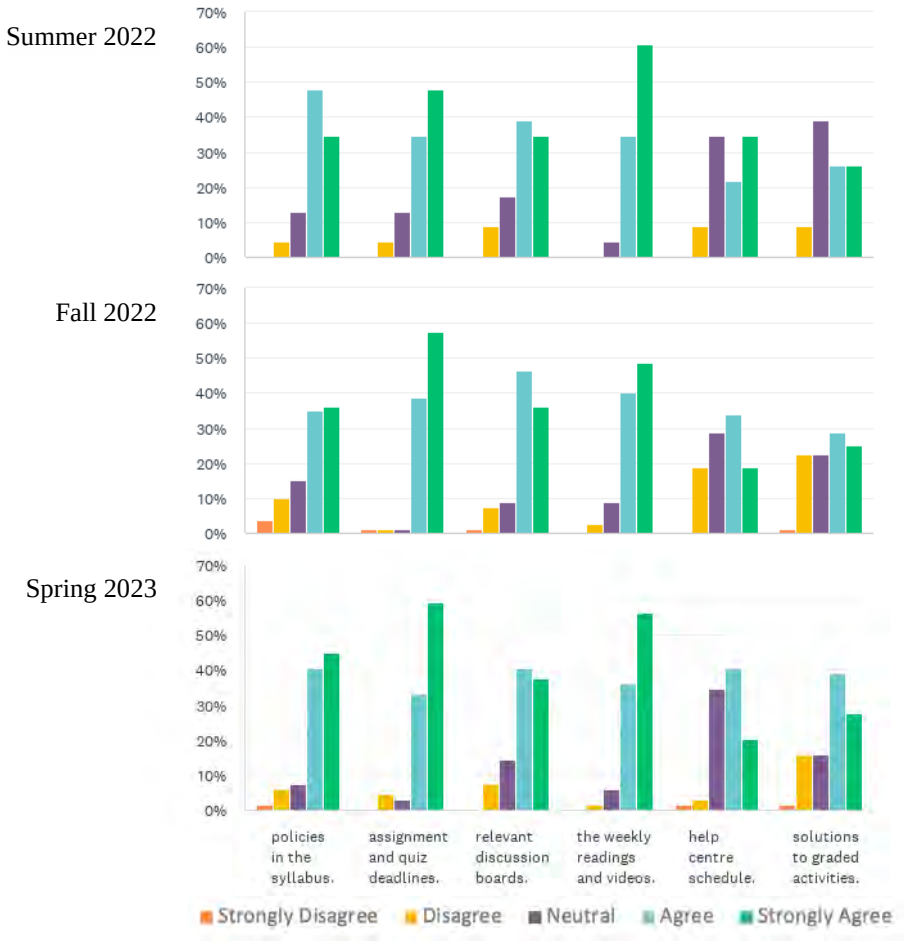

Figure 3: Course data results: "Navigating Canvas: It was easy to locate the…"

Responses for locating the help centre schedule and solutions to graded activities were still positive 57% and 58% respectively. Both of these questions had to do with material posted primarily on the

course's homepage. One theory is that students are not checking the homepage for weekly information, a conclusion that is supported by the increased use of the Canvas phone application, where the homepage is not shown by default. To improve visibility of the solutions to graded activities in subsequent semesters, links to this page were added to each assignment's instructions. Similarly, links to the help centre schedule were added in multiple locations throughout the course.

Course Data Results on Online Assignments and Learning

The newly migrated online assignments were assessed in their perceived aid in student learning (Goal 6). Students were asked if the online assignments had aided them in understanding the course content, pinpointing areas to study more, knowing what types of questions to expect on the midterm exams, and knowing what level of questions to expect on the midterm exams. While the content of the online assignment questions had generally stayed the same from its LON-CAPA to Canvas migration, the goal was to see whether student perception of this support had improved with the reduced access barrier and the increased similarity to the format of the midterm exam. The results of this topic are depicted in Figure 4.

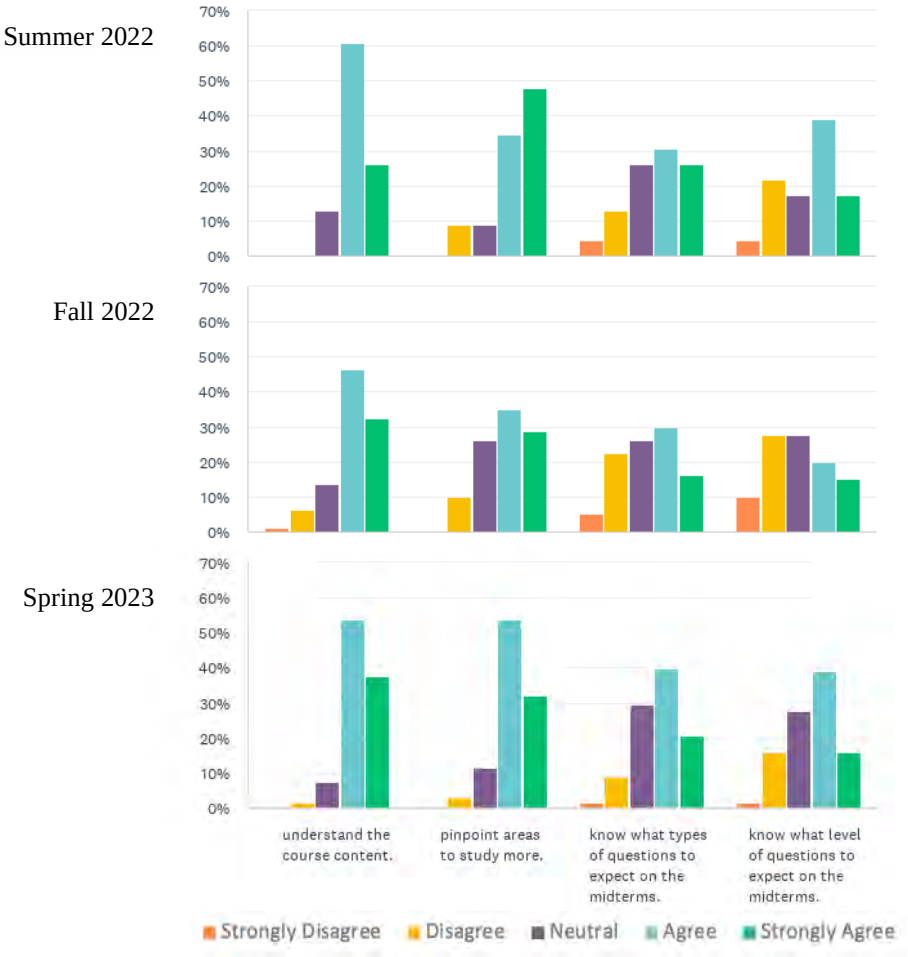

Figure 4: Course data results: "I found the weekly Online Assignments helped me to..."

The first two questions, "understanding the content" and "pinpointing study areas", had excellent results with positive responses 86% and 77% respectively. One student commented, "OAs are one of the most helpful resources for this course, I appreciate how they are laid out." However, when asked whether the online assignments helped them to "know what type of questions to expect on the midterm

exams" and "know what level of questions to expect on the midterm exams", students responded positively only 54% and 49% respectively.

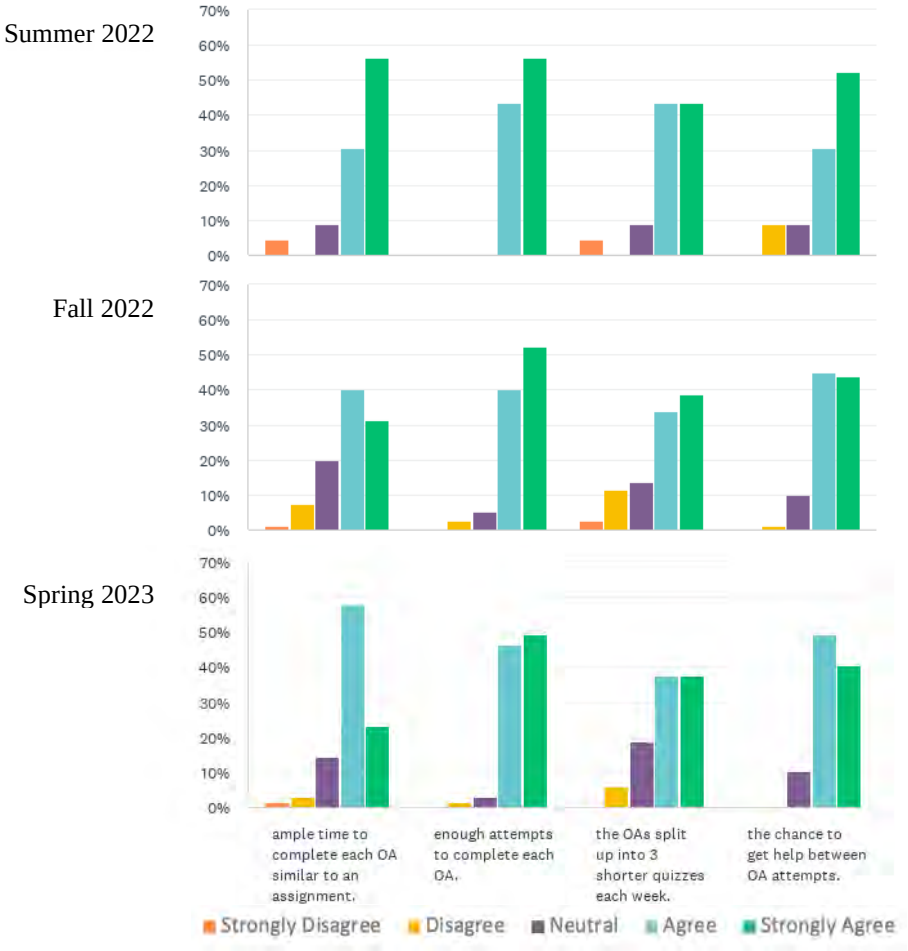

Figure 5: Course data results: "Online Assignments (OAs) Format: I liked having...

While initially surprised by this result, the instructor team understands that since the online assignments have multiple attempts and the midterm exams do not, it is easy for students to view the material on the midterm exams as more difficult to complete. Similarly, while students are given ample time to learn the content during online assignments (why they are called "assignments" despite being set up as Canvas Quizzes), they are not provided this luxury in the summative midterm exams. While the instructors still believe that making the format and question style more closely aligned to exams is beneficial for the students, it was clear that additional exposition was required for students to have realistic expectations for exams based on their online assignment results, and to help students turn their online assignment results into exam study tools. Following the survey results in Fall 2022, information regarding the differences between online assignments and midterms exams were added to the syllabus and discussed with students at the beginning of the course. These small additions improved survey results for the Spring 2023 semester, bringing the number of positive results up for these two survey questions slightly to 60% and 55% respectively.

Course Data Results on Online Assignments Format

In addition to their support as learning tools, students were also prompted for their opinions on the format of the online assignments. This aspect of Goal 6 was assessed via four questions, whether

students liked: the ample amount of time to complete each online assignment, enough attempts, having online assignments split into three smaller Canvas quizzes, and the chance to get help between attempts. All four categories had strong results, with positive responses 80%, 96%, 78% and 97% respectively. Some students noted that the multiple deadlines were not ideal for them, relating to the slightly lower average for the splitting of the online assignments into three shorter quizzes. The complete results of this topic are depicted in Figure 5.

Course Data Results on Instructor Summaries

Fifteen new instructor summaries were added for each course topic to address Goal 4. To measure the impact of the instructor summaries, students were asked whether the summaries were helpful: to learn the required definitions and formulas, by providing additional examples, by highlighting what was important from the readings, to reference while studying, and to reference during the open-book midterm exams. The results for all categories were strong, with positive responses 92%, 79%, 81%, 87% and 79% respectively. The results are depicted in Figure 6.

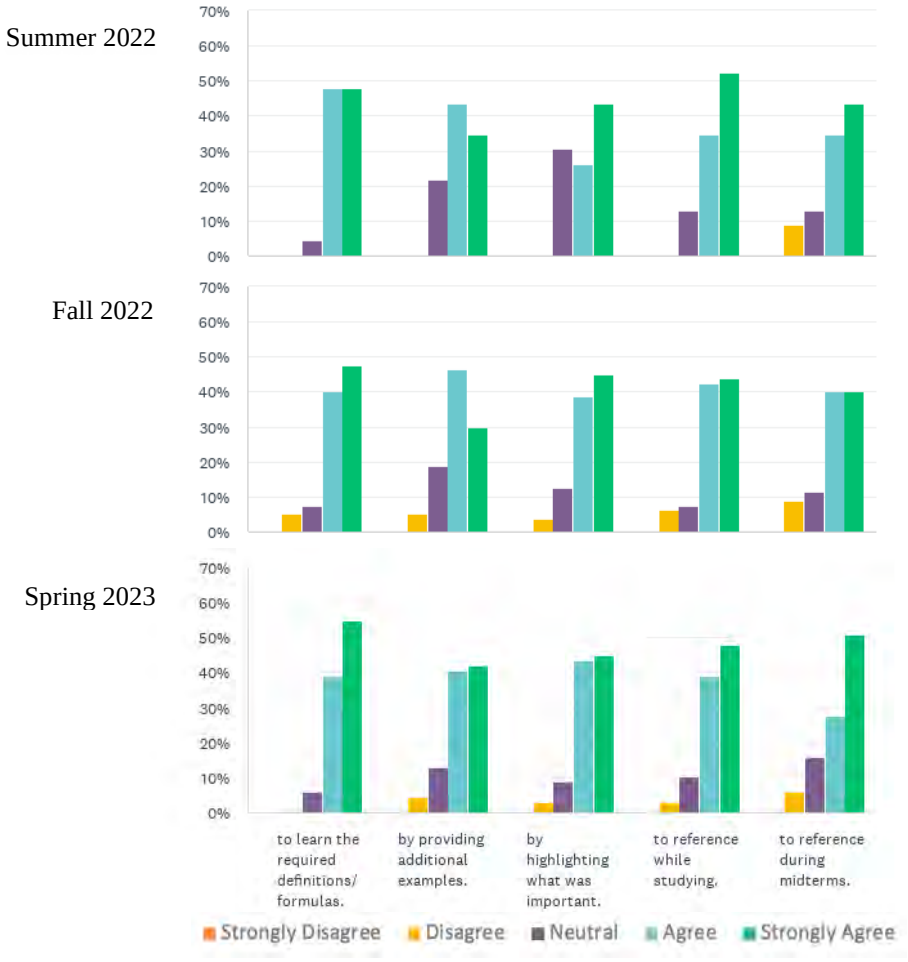

Figure 6: Course data results: “I found the instructor summaries helpful…”

Positive free-form comments included, “I found the Instructor Summaries even more helpful than the textbook.” and “I found the instructor summaries quite useful, it helped me know the key things I should know and what to look out for when studying.”, as well as one student who verbalized why they prefer the summaries to the textbook saying, “I loved these. I get distracted and stressed out easily when reading long texts. And these quick and easy notes made it easy to understand the week's

materials. The formulas and examples of concepts really helped as visuals. Also the colours really helped as well".

Course Data Results on Organization

The final question group focused on Goals 3 and 5 by investigating the organization of the learning materials and assessments. Students were asked which of the following course components helped them to stay on top of the weekly material: content organized into weekly modules, online assignments due over two days, regular announcements, the weekly snapshot on the Canvas homepage, and the hand-written group assignments weekly timeline. The results of this topic are depicted in Figure 7.

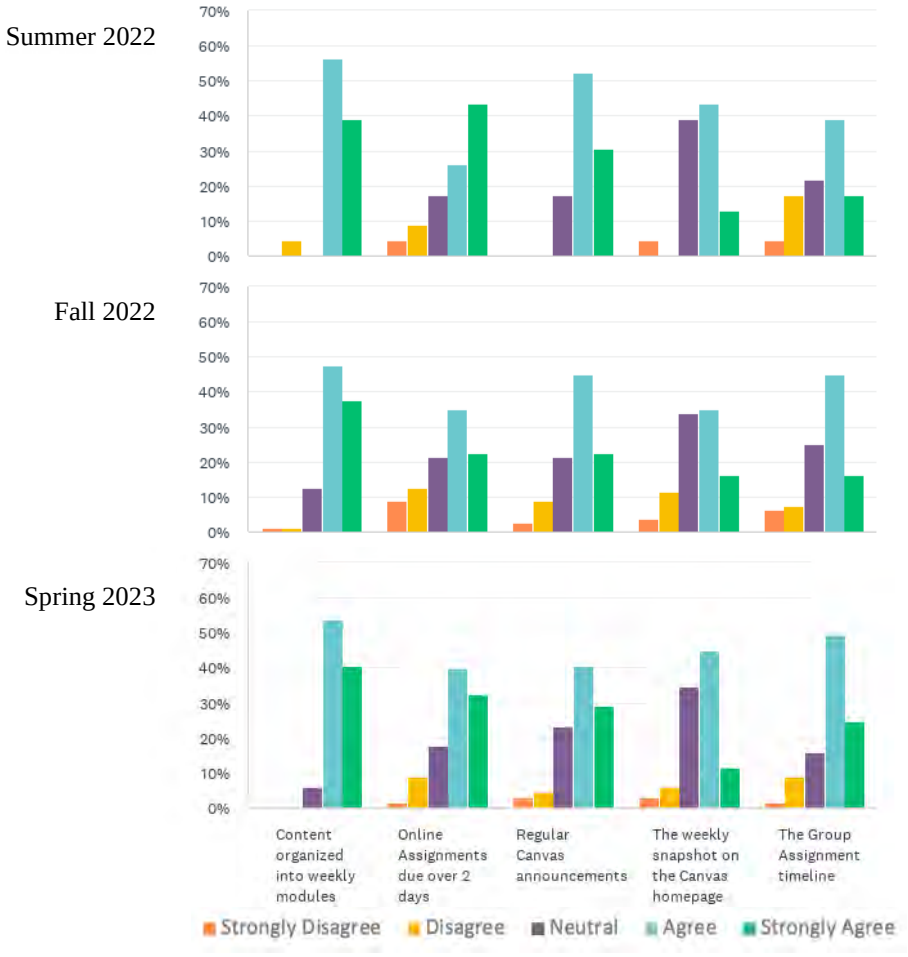

Figure 7: Course data results: "The following aspects helped keep me on top of the weekly material."

The biggest overall Canvas change is summarized by "content organized into weekly modules" and Goal 3. This response received an overwhelmingly positive result 92%. Student free-form comments included, "This course has been better than I expected it to be, likely due to the well-organized learning features. Thank you!" and "The changes from the last time I took this course have made things a lot easier to understand. I was quite stressed from some personal matters last year and still am managing it, but this new layout has helped find things a lot easier as everything is on canvas." The latter comment, coming from a student with the unique ability to compare the course in its previous form, shows how monumentally the new formatting improved the flow for students.

The four subsequent questions received more mixed results, with positive results 66%, 73%, 55% and 64% respectively. The lowest score corresponding to the weekly snapshot on the Canvas homepage

echoes the conclusion in the section **Course Data Results on Navigation** that students need many access points.

CONCLUSION

It was no small feat to accomplish all the changes to the OLE in 10 short weeks. The project was possible thanks in part to the two carefully selected research assistants and the six UGRADs who wanted to make a difference for future students taking the course. In the words of one UGRAD about the inclusion in the restructuring project:

> "It made me feel heard to be part of this project. I felt like my feedback and input were taken seriously. I think this was a great opportunity because it allowed me to share my thoughts as a former student how the Canvas container could be improved and streamlined for future students. [...] I also really like the sample weekly schedule that is provided on the homepage. As a student with autism, I appreciate having schedules or suggested schedules easily accessible and built into a course."

It was an incredible experience to get authentic involvement from the UGRADs. Their voices brought another reality to the discussions and interviews that positively shaped the final product. Furthermore, "[r]esearch has found that students leave partnerships more engaged, motivated, and take more ownership for their learning" (Flynn, 2020, p. 2).

The results utilizing this Partners in Design approach were well-received by students taking the math-for-teachers course in subsequent terms. Involving all stakeholders in the design process aided in the success of addressing all six design and functionality goals outlined in **Pivotal Moment**. Student survey data collected following the redesign demonstrates that satisfaction was high among students in all areas of the restructured OLE. One surveyed student commented, "…Overall, the course feels very organized, particularly the Canvas page.", while another student said, "Overall, I was very impressed by the level of organization of the class. […] This has definitely been one of the most well-run courses that I have taken!". Additionally, the survey results highlighted a few small areas for further improvement in future iterations. The ongoing feedback loop from students in the course, past and present, allowed the instructors to iterate on the updated design, efficiently leading to a great result.

Upon reflection, the first author has the following advice to give for anyone who wants to take on such a project as project leader: Obtain funding, leave time to get everyone on board and be prepare to run after undergraduate students as they lead busy lives, collect ideas first, organize face-to-face meetings, be goal-oriented, create a flexible plan and be prepared to change it to allow for the nature of the organic process, deal with obstacles in a mindful way, don't be afraid to make decisions as even a collaborative project needs a leader, don't let yourself drown in emails, and breathe and smile.

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March 6, 2017 11:55 PM

Hello Petra,

Thank you again for inviting me to your classroom. It was an eye-opening experience for me. Having worked with you over the years, I anticipated that your classroom would be interactive; however, I never expected to experience a classroom to be a stage for a symphony of learning in which you were a master conductor and composer. That's how I felt. As I said in an earlier email to you right after the class, if I have to use one word to describe it, it is togetherness. You, your students, the activities, the materials used, the dialogue between you and your students, the discussions among themselves, the timing, the pace, and the space: everything came together and flowed together. There is an effortless fluency in how you conducted the class, but I know that apparent effortlessness is a result of hours of preparation and years of refinement.

I arrived in your class seven minutes before it started. The classroom was already half full. Right away I sensed a healthy level of energy. Students were sitting in groups chatting lively. Their notebooks, pencil cases, and computers were spread out on their desks. They appeared that they were ready to go.

All your problem-solving activities were thoughtfully designed, thoroughly prepared, and impeccably executed. I strive to teach in the same way and I know how much effort goes into this kind of teaching. I admire how you were able to roll out one activity after another, keep students alert and working hard throughout the class, and made them laugh from time to time. You have a way of letting go of them so they have to think hard and struggle and then calling them back to your safe harbour. I sensed a real connection between you and your students. You kept them with you all the time. I started my observation tracking the tos-and-fros between you and your students, but I had to give it up because there were so many. I still wanted to have some kind of statistics, so I managed to record the number of questions you asked your students during a 12-minute period when you engaged them in solving the "200 objects" problem. There were 28. I could tell these questions were not pre-determined, rather they emerged from assessing students' learning as it was unfolding. And not just one or two, or three or four students answered those questions; you insisted that everyone get it at the end, "Come on, everybody."

I particularly enjoyed seeing the first problem-solving activity when you asked students to come up with a problem in the form of a story based on a set of facts presented in a graphic. So often we ask students to come up with answers to

problems constructed for them rather than the other way around. I think there is real value in training students to compose problems and ask questions. You went a step further. Not only must they come up with different problems, they must also come up with different *types* of problems based on the same set of facts. Insisting on learning through variations on various themes helps students discern what is relevant, what is different, and what stays the constant in each situation. This, I believe, is how deep understanding is achieved.

I can't help noticing the wide array of materials, tools, and objects you used in supporting learning in the classroom. Some of these were prepared, such as work sheets, individual white boards, markers, a novel (*Flatland*), transparencies and projector. Some of these were spontaneous such as pencil cases (you picked two off students' desks and used them for demonstration) and coins from students' pockets. I admire your ability to move from one thing to the next and come up with one thing after another. You made me feel that each was just the right thing at just the right moment. This was part of the fluency I meant above.

I really enjoyed your reading of *Flatland* to introduce the concept of dimension. It was the first time I saw a teacher standing on top of her desk to orchestrate the whole class in interpretative dances of dimension!

I wanted to talk to the students and ask them how they liked the class, but they were so fully engaged in the activities that I couldn't bear to interrupt them. Nevertheless, I managed to slip in a few questions when I was sitting with various groups. The handful of students I talked to said that your class was "very engaging" and "it was hard." One of them said that he didn't expect that there would be so much work in a first-year course but he wanted to become a teacher so he stuck to it. I asked him if they had a choice would he rather not take this course. He thought about it, "Maybe, but I've learned a lot from this class."

Was there a break? The class was so vigorously conducted, I can't remember whether the students got a break. Did they want one? No one made any noise about it so maybe they didn't care.

Oh, I must mention the hand clapping. Pa-pa, pa-pa-pa. The first time I heard it, I asked myself, "What is that?" and I noticed the magical effect of getting everyone's attention all at once and pulling the class back into a whole from the heated small group discussions. It was like a secret handshake between you and your class. It really worked. It made me chuckle.

If I have one wish for your class, that is I wish more instructors would visit it and experience it.

Cheers,
Cindy

Cindy Xin, PhD | Educational Consultant
Teaching and Learning Centre | Simon Fraser University
Education Building 7560 | 8888 University Drive | Burnaby BC | V5A 1S6
T: 778.782.7453 | F: 778.782.4900 | cxin@sfu.ca
www.sfu.ca/tlcentre

Math 157 - class observation notes (Fri Nov 23, 2018)

Gaitri Yapa

Sat 2018-11-24 2:43 PM

To:Petra Menz <pmenz@sfu.ca>;

Hi Petra,

Thank you for having me observe your class on Friday! It was very helpful for me as I plan to teach my Spring classes (one in the same lecture hall, with potentially the same number of students, and interest level!).

Here are notes that might be useful for you, based on ~10 students I observed directly around me (I was at the last row in the very centre of the room), and the rest of the class in general:

Engagement:
i.e. listening to instructor, working on a problem shown on screen, answering questions posed by instructor, discussing with neighbours when directed by instructor

- Student engagement was high throughout the lecture, even in the back of the class where I was sitting.
- The students who seemed not as engaged appeared to be those who had completed the problems that were discussed from the Wed class. There were a few (say 1 out of 10) who appeared to be taking notes to look at later, as they hadn't kept up with the course material perhaps
- Most (about 8 out of 10) students were either taking notes on paper/tablets/laptops
- Only about 2 out of 10 appeared to be distracted with electronics such as phones or laptops from time to time

Technology us:
i.e. overhead projector, wireless mic, laser pointer, writing directly on slides projected - from personal laptop

- Some images/writing on projected slides were not as clearly visible from the back rows, but did not affect the learning, as the skeleton slides were available to students ahead of class
- Having the wireless mic helped to have the instructor not be standing in one place, and available to answer individual questions during discussions

Peer-instruction and large group discussions:
i.e. think-pair-share or small group discussions followed by instructor led whole class discussions

- giving time to discuss in pairs/groups at regular intervals was very helpful for students to stop and absorb concepts being taught
- time allowed for discussion appeared adequate, given the amount of material that needed to be "covered" during the 50 min
- majority of students appeared to be making use of the opportunity to discuss (say 75% but probably more)
- instructor walking around lecture hall to join small group discussions or answer student questions was very helpful (and probably encouraged students to discuss more with peers?)

Only thing I noticed as a possible item to consider changing:
- when walking up the isles to the middle of the class, it seemed the instructor only walked on one isle in particular. This could be because the observer did not see the other occasions, or due to the need to guide specific students or due to a particular reason which the instructor was aware of

Hope this helps, Petra.

Thank you again, and I appreciate you sharing words of wisdom with me regarding possible mishaps too.
Gaitri

March 6, 2017 11:55 PM

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Reflections on Observations of MATH 190,
a course by Dr. Petra Menz, Spring 2019

Context & Summary:

My name is Dr. Sophie Burrill and I am a Lecturer in the Department of Mathematics at Simon Fraser University. In the Spring 2019 semester I had the privilege of being invited into Dr. Petra Menz's MATH 190: Principles of Mathematics for Teachers. Having co-taught MATH 190 myself, I was initially only interested in getting a sense of how Dr. Menz ran her class, and intended on attending just one or two lectures. That said, within the first hour I realized I needed to rethink this plan and make better use of her open invitation; Dr. Menz's MATH 190 was incredibly impressive. As such, I cleared my own schedule and attended well over half of the classes.

In Dr. Menz, I observed an incredibly thoughtful, bold and caring instructor who expects and receives a lot from her students. She is always exceptionally prepared and organized, and at the same time remarkably innovative, able to shift and adapt as surprising questions and events inevitably arose in the duration of a 13 week course. Her students are lucky to have her; Simon Fraser University is as well.

Preparation:

Dr. Menz's MATH 190 was exceptionally well prepared. Her plan for the course was laid out clearly on Canvas before the first day of class. This plan included well thought-out readings and activities for the students to complete before each class, clearly articulated assessments, and engaging lessons. Such a lesson typically included at least one well planned activity, some time to work on problems individually, in pairs and/or in groups, plus some lecturing, which always involved questions from the class. She often brought worksheets and/or manipulatives, or was prepared to make use of everyday objects in the room in order to illustrate a concept. She prepared her students well in advance of every event, giving plenty of advance notice when they needed to create a fraction kit or even just bring a pair of scissors. This level of preparation must have been reassuring to those students who were anxious about taking a mathematics course.

Organization:

I am particularly in awe of Dr. Menz's organizational abilities. She asked me to substitute for a class one month in advance, and when I happily agreed she immediately handed me all of the content (slides, activities, and instructions) she hoped I would cover. When a weather cancellation disrupted this plan, she immediately adjusted and gave me an updated version within days. I aspire to be that organized.

In the classroom, this manifested in a class so well run that students were audibly reassured; several times I overheard one ask another, "when is — happening?" and another reply that, "I don't remember, but it's on Canvas". Indeed, from the very first class, students had access to all important dates, the nature of their assignments and grade breakpoints/descriptions. If a particular lesson would benefit from a different room configuration, students were messaged in advance, which allowed class to begin promptly on time. Planned activities with multiple pages to be distributed to students were always colour coded. Transitions between activities were always smooth, and disruption was minimal and motivated.

Lesson Structures:

A typical lesson in Dr. Menz's MATH 190 was never simply a lecture. Even on the occasional day when lecturing made up the majority of class time, students were always involved in the material through group discussions and/or breaks to work on questions that arose. During such discussions, Dr. Menz made full use of the room, using all three walls with whiteboards, and meandering around clusters of students to great effect. More often than not, such a lecture was accompanied by at least one planned activity, and problem solving (individually, in pairs or in groups). Depending on what Dr. Menz had planned for that class, worksheets, manipulatives, whiteboard use and movement might all be incorporated into the lesson. The result was a classroom consistently filled with engaged students whose time was meaningfully spent learning the material.

Knowledge of Math 190 material

It was clear to me that Dr. Menz had a complete mastery of the material in MATH 190. She had thought deeply about how best to facilitate her students' learning, and chose lessons to reflect it. Her class's engagement would have exposed any weaknesses, as her students were constantly probing both their own and her knowledge. On the contrary, her proficiency with the material meant that she could anticipate when content would be particularly challenging and develop activities to better support her students. Further, her mathematical background allowed her to place the MATH 190 content in broader contexts, both within the scope of the course and into the larger field of mathematics.

Knowledge of MATH 190 students

From the beginning of the semester, Dr. Menz was explicit that she understood many of her students did not have a science background and that many may experience math anxiety, in addition to leading full lives outside of her classroom. This was not just an assumption; students communicated this through Canvas submitted introductions, class discussions, and student-instructor meetings (which I did not observe, but did discuss with her). Dr. Menz took this information to heart and proceeded with care, compassion and determination to help build students' mathematical confidence and change their preconceptions about mathematics. This was embodied in many ways: her generous use of group work fostered a community of mathematical learners; the wide variety of planned activities allowed students to experience math in ways they likely had not seen before; she consistently emphasized justification over final answer, and was quick to highlight and clarify confusing terminology (eg. "remainder of zero" is preferable to "no remainder"). When midterms drew near, she patiently answered anxious question after anxious question, until the noticeably calmer class ran out of queries. Dr. Menz also sought outside support for her students; she invited Health and Counselling into her classroom to present about mental health and resources around campus. I know her efforts had a positive effect on her students; at the end of the semester; on two separate occasions, I saw students speak to how meaningful it was to know Dr. Menz cared and understood them.

Planned activities

The activities Dr. Menz planned for her classes varied widely in scope over the semester. Some were powerfully simple: she had students perform word dumps from their assigned reading, or spend several minutes crafting their lingering questions on a subject they were about to move on from. The reflective nature of these activities allowed Dr. Menz to probe her students's understanding and react accordingly, which helped make class time more valuable.

Other activities were much more involved, consisting of different stages, manipulatives and instructions. These were generally aimed at leading students' learning through content that was much newer and/or challenging. For example, when different conceptualizations of whole number operations are introduced, many students struggle to remember the new terminology and apply it correctly. Through laminated tables, small movable entries and erasable markers, Dr. Menz stripped away many of the details that initially impede understanding, and gave them a valuable opportunity to get started immediately with identifying conceptualizations. This level of planning and preparation for an activity was by no means rare, and leant itself to a vibrant, excited atmosphere week after week.

Handling of ad hoc questions

One of Dr. Menz's most impressive skills was her ability to anticipate student questions and navigate the unexpected. From my seat at the back of the class, I was envious of her ability to have students ask natural questions - it indicated they were both paying attention and comfortable enough to speak up. Such questions tended to launch into fuller class discussions that seemed like she had been planning all along - or perhaps she was just able to make it appear that way. Other times, students asked questions that were more surprising. Dr. Menz always answered thoughtfully. On the one occasion that she was unsure (I was as well), Dr. Menz admitted as much and followed up during the next class. Her students appreciated seeing this side of her, and clearly valued that she never brushed aside their questions.

Collaborations and discussions

Dr. Menz built confidence and community in her MATH 190 class through an emphasis on collaboration and discussion. Often this took the form of students working independently, before scaffolding them up through paired, group and then entire class discussions, to explore a topic. This allowed students to investigate their own intuition/understanding, and grapple with explaining themselves to a growing audience. The resulting class discussions were always very fruitful after this progression, with students communicating their fleshed out ideas more clearly.

A wonderful byproduct of this was that MATH 190 developed a strong sense of community. Students were not just friendly with each other in class; I heard them make plans for the weekend and would see them walking across campus together. Maybe they were simply a friendly group, but I was left with the impression that much could be attributed to the culture of group collaboration that Dr. Menz fostered. Group composition changed frequently, sometimes multiple times in a single class. Time was consistently allotted for introductions, and it was expected that students would learn with their peers. Students' rose to this expectation, practicing both how to explain themselves, and ask better questions. I am sure this was not accidental.

Whiteboard usage

Students in MATH 190 used whiteboards regularly. Dr. Menz would bring a class set of mini whiteboards which allowed students to work individually, in pairs, or in groups, depending on the context. The mini whiteboards facilitated groups working together, and could be used to anonymize solutions for comment and critique by first collecting several and then displaying them all together and inviting comment. One of my favourite strategies Dr. Menz used was to pose a question, and have the students work in pairs on a single mini whiteboard with no talking. The silent vigour that emerged from the class (initially hesitant and uncomfortable to express themselves mathematically) was a delight to behold.

With three walls filled with whiteboards, the entire class could also (tightly) work on classroom whiteboards as well. Typically this meant students were standing around the perimeter of the room working in small groups on one of several problems. Dr. Menz made this as orderly as possible, labelling each space with a number and counting students off into groups. Problems were taped to the board, and students were eager to get to work. The excitement that resulted made for a boisterous room, and no one could stay stuck or comfortable for long: soon enough, students were instructed to shift so that each group was infused with a new mind to either help or convince. She built in enough time and shifts so that every student would have exposure to every problem. It was very impressive.

Interactions with students

Before observing Dr. Menz, I knew she had a reputation for caring about her students. Watching her throughout the term though, I was repeatedly inspired by how deep and layered that care extended. She quickly learned every name, and later invited every student into her office for a meeting. She noticed and followed up when a student's pattern changed, and devoted as much time as was necessary to explaining higher stakes testing until nerves were settled. Often she was held up after class by students seeking guidance on a particular issue, and she was generous with her time and understanding. Several weeks into the semester when many students were feeling the pressures of their degrees, she told the class that she always has a 'carte blanche' ready for extenuating circumstances. At the time I was taken aback; it seemed too bold of a statement to make to the entire class. But Dr. Menz is a bold teacher and it paid off: you could feel tensions ease in the room and that she had earned even more of their trust.

Dr. Menz further built that trust through being both brave and vulnerable with her class. She periodically offered parts of her own life story, with both trials and triumphs. Beyond this, she partook in some of the bravest, most vulnerable teaching I have ever witnessed, with her weaving of Indigenous ways of knowing into the course. In particular, this culminated in a ceremony near the end of the course to celebrate the students' transformations. As she served bread and fruit, students one-by-one shared stories of their own trials and triumphs. It was a truly moving experience. Such moments may be more common in other faculties and departments, but in a math course they are virtually unheard of. In light of my own observations, I was not at all surprised that the thread that arose over and over was one of great gratitude that the students had had Dr. Menz as their teacher for MATH 190.

COLLEGIAL COMMENTS

March 2, 2017, Educational Consultant Dr. Cindy Xin sent this email:
I just want to drop you a quick note to say that it was a fantastic class. If I have to use one word to describe it, it is togetherness. You, the students, the activities, the materials used, the dialogue between you and the students, the discussions among themselves, the time, and the space, everything seemed to come together and flow together. I wish more instructors were there to see it. I don't know how much time and effort you have spent to come to this point. The result is that it looked effortless from your side as the instructor. You were like a super conductor.

January 12, 2021, English University Lecturer Dr. Nicky Didicher sent this email:
I was just talking to a new student in my poetry class who took Math 190 with you and said it was the best online course she's had so far and you were a great teacher.

March 20, 2023, Mathematics Lecturer Dr. Andrew Hare of Saint Mary's University, Halifax, reviewed the reshaped Canvas container from the *Partners in Design* project:
There is no question to me that your students are very lucky indeed to have a teacher who has thought through from beginning to end what it feels like to take the course.

UNSOLICITED STUDENT COMMENTS

April 17, 2010, Siya Mishra student of MATH 190 D100:
I would just like to simply thank you for everything you've done. You probably may not know but you've taught me things that don't just apply to mathematics or teaching but also to everyday life and striving for more. You are an inspiration to all teachers! Even though I took this course in order to fulfill my Q requirements, I feel as though I made a very good choice in choosing to take math 190 with you. Your lectures were insightful and the curriculum is made to challenge at any level. I look forward to the final exam and I want to wish you the best of luck in the future.

April 14, 2012, Tristan Taylor student of MATH 190 D100:
I just wanted to say thanks for the help this semester. I really feel like a lot more confident in mathematics and a lot of it was thanks to how you ran your class. It was a lot of work but it really helped me from falling behind and pushed me to challenge myself. At the beginning I looked at the class as just something that I need to get through so that I could get into my teaching program. But you really taught me how important it is for kids to have a solid foundation in math and it changed my perspective on the subject, and how I will go about teaching it and making sure that I know it very well for their sake. Anyway, thanks for being very available for extra help all semester, you are a great prof!

October 15, 2014, Cristina Moretti student of MATH 190 CODE and Anthropology Instructor at SFU:
I would like to take this opportunity to tell you that I am enjoying the course. I particularly like the mix of slides, readings, video clips, exploration activities, and the variety of assignments which provide many ways of working with the material for different learning styles.

April 7, 2017, Jennifer Sylte student of MATH 190 D100:
I just wanted to let you know a few things as the course wraps up and we are all preparing for the final! First off, It was a pleasure having you as our prof this semester. Like I've said before, you are really a caring and genuine teacher and have done a fantastic job! Your passion and dedication, as well as your love for what you do really shows! Ive said it once or twice, but this is the first time where I have ever actually learned valuable and useful concepts and skills in a class at SFU, and this course quickly became my favourite by far, this semester.. maybe even throughout my entire degree! This is largely because of you. In the beginning of the semester I thought to myself, "Oh no.. she is tough.. she is so strict and this is going to be so hard." I wasn't sure that I could do it. But then, from that day on you showed more engagement and interest in our learning that I knew right away you may have came off that way, but it was only because you wanted us to do well and work hard and you knew we had the potential. Also, you wanted us to learn, because that is why we are at university. Your dedication to making the future teachers that much more

educated and knowledgeable is inspiring. You truly are an inspiration and I hope many people let you know that, all of the time!

Leaving this course, not only have you taught me valuable concepts, tools and skills in mathematics, but you have taught me how to be a caring, kind and inspiring teacher. You taught us many life lessons and made my eyes water in class more than once! In addition to those things, you have taught me to strive to be better, to do what you love and you have made me feel more important (somehow!) and it is a refreshing feeling as a student. I will always think back on your class and your teaching as a motivator and an inspiration to be better and do better as a teacher of my own! Please keep doing what you're doing because you are making a significant impact on many future educators!

Thank you for everything this semester, I hope to stay in touch with you when I am a teacher :) :) I found out I have been accepted for PDP at SFU this September, so I am working my way towards my dream and my goal of being a teacher so I can inspire others that same way teachers like you have inspired me!

Thank you endlessly.

January 24, 2018, Teanna Lackner student of MATH 190 D100:
I wanted to congratulate you on being awarded SFU's 2017 Champion for a Healthy Campus Community! I am a former Math 190 student of yours and I just wanted to say that Math 190 has been the most memorable and enriching class of my academic journey thus far. I was wondering if you teach any other mathematics courses, as I would love to take another class of yours.

March 26, 2019, Caelen Dalmer student of MATH 190 D100:
I want to tell you how wonderful this class has been for me. This is my last semester at SFU and I have never experienced a class and classroom culture like yours. The intimacy of our class and the smaller groups within it have brought about a learning style I have never experienced in a university setting. I would not say that I ever considered myself horrible at math, however when I first started university I took Calculus 1 and 2 and did very poorly in those classes, partly overwhelmed by the environment and the uncertainty I felt about communicating with my professors. The organization of your class and the many different methods of learning brought back some of the confidence I had lost during my first 2 years at SFU.

April 15, 2020, Julie Schiller student of MATH 190 C900:
I just wanted to email and thank you personally for your support during this course. Not just to me, your compassion and caring for your students is evident even through a distance education course. Your passion for math and for teaching is unquestionable. You know already I was struggling through this course, as were many students I'm sure, and I am truly thrilled with the final result. I know it's not the best result, but I worked hard for it, and I didn't give up despite the obstacles I was facing. I have never been so relieved, nor so proud of myself as I am at the end of this course (and I graduated SFU back in 2007 so that's truly saying something!). I proved to myself I could do it, but I don't know if I would have continued if not for you reaching out to me at what was really a low point. It was a small gesture that had a huge impact.

April 20, 2021, Chrystal Yang student of MATH 190 B100:
I want to thank you for such an amazing term. I started off very hesitant and quite scared to take a math course again but I finished this term beginning to enjoy math and believing in myself more. You definitely inspire me to become the type of teacher who has a vast variety of ways to teach each student as well as putting in the effort and care.

May 7, 2021, Julia Kayda student of MATH 190 OL01:
I don't think I ever got to say a proper thank you, so here we are! Thank you SO much for such a lovely semester. Not only did I thoroughly enjoy the contents of your course, but I really appreciated your guidance throughout it too! It was really refreshing to see such an organized and transparent approach to a course taught remotely. All of your course contents were incredibly well integrated, which made it that much easier to get a full picture of whatever concept we were learning. It was all really well done and that made a big difference to my learning - especially as someone who comes from a strong (yet somewhat shaky) foundation in math!

You really are an incredible professor and I'm lucky to have had the pleasure of learning from you! Thank you for answering all of my questions and allowing me the space to reflect on myself as a learner. Math 190 is not an easy course, but it is definitely worth the effort. This course made me realize that I'm actually really excited to teach math myself and I wasn't sure I'd be saying that, haha! I've definitely gained a lot more confidence surrounding my math abilities, which makes me feel like I'm leaving with far more than a shiny new grade on my transcript. It was nice to learn from a professor who not only enjoys what they are teaching, but genuinely cares for those that they are teaching too! Thank you for all of it!

I hope you're able to take some time for yourself before we roll into yet another semester of online learning! Enjoy some time in your garden and with family!

December 7, 2023, Clayton Webster student of MATH 190 B100:
Thank you very much for your time, effort and commitment to this course. I appreciated the immediate responses, helpful resources for the future, and the focus on mental perseverance. I will carry many of the lessons / teachings forward through my careers and life.

Thanks for a great semester! - megan Bleasdale

My love for math is still not congruent to your love for math, but I feel like I learnt a lot! Thank you! - sydney Dumore

If Christmas 2018 was on a Tuesday, What day was Christmas on in 1966?

Petra! Thank you for caring about all of us! It means a lot. ♡ -Nancy Larosa

Thank you for bringing me one step closer to loving math and inspiring me to be a teacher again! -Maiya Chen

SFU Math Department Calculus Courses (# of times taught)

- (1) MATH 100 *Precalculus*
- (1) MATH 150 *Calculus I with Review*
- (1) MATH 151 *Calculus I*
- (2) MATH 152 *Calculus II*
- (1) MATH 251 *Calculus III*
- (11) MATH 157 *Calculus I for the Social Sciences*
- (2) MATH 158 *Calculus II for the Social Sciences*

COLLEGIAL COMMENTS

April 5, 2006, Math Department Chair Dr. Tom Archibald sent this email:
I am impressed by the fact that you can get students to have such attitudes without sacrificing standards. The "popular" prof is frequently the one that makes life easy, but you manage to convince them that even if things are difficult they can be interesting and fun. It's a rare quality, and we value it (need it!) a lot.

January 16, 2019, Senior Lecturer and Instructor of MATH 157 Dr. Natalia Kouzniak sent this email:
Just a quick view proves that you did a tremendous job [on Calculus OER]! My sincere admiration!

April 8, 2019, Professor and Instructor of MATH 157 Dr. Ladislav Stacho sent this email:
Overall I was very impressed how well the notes [Calculus OER] were done and teaching based on these notes was straightforward. All lectures were designed with right level and amount of material and included good selection of examples. I appreciated that the examples varied from purely theoretical to nice real life problems with economy flavour. Few lectures which consisted of more that 10-11 slides were a bit too dense.

One suggestion that I would propose is that if the statements of definitions, theorems, examples, notes, and all introductory text for them is completely typeset (there are no white spaces for instructor to fill in) and leave the solutions to examples either completely empty for the instructor to fill in. If it is too much writing for some instructors, they can always pre-fill their notes before their lecture to get the speed on track.

(There are now two sets of lecture/student notes available for instructors: skeleton version and def/thm filled but examples blank.)

Other than this I was very happy with the notes and I think you did a tremendous job to put these notes together!

December 6, 2019, Professor and Instructor of MATH 157 Dr. Stephen Choi sent this email:
In general, I think it is very good to have standard slides and notes (even assignments) for all sections and semesters. This makes it easier to maintain the uniform standard among all sections. I think we are doing quite a good job this time. However, since it is the first time using the slide, I try to finish all the slides every lecture but it is quite difficult. It will be better next time if I can skip some slides if the time is short.

(Some examples are now marked as OPTIONAL to provide more time for the instructor.)

Overall, I think the new lectures are very good and surely cover all the necessary topics. In general, the approach/layout is very nice and suitable although I personally prefer to use examples with simple and nice numbers in the calculation because students tend to get distracted or frustrated by the nasty calculations such as cubic root. Difficulty and topic breadth are quite appropriate. The length after adjustment should be better and suitable. The slides are segmented appropriately and very well-organized.

March 26, 2021, Professor and Instructor of MATH 158 Dr. Michael Monagan sent this email:
Very nice Petra. I poked around the book [Calculus OER] (both courses) a bit. Congrats!

November 14, 2023, Math Professor at Howard University Dr. Roberto De Leo sent this email:
I recently discovered your open-source textbook "Calculus Early Trascendentals" and it has been useful to me as a source of examples, thanks!

UNSOLICITED STUDENT COMMENTS

December 12, 2005, Kaveh Naziripour student of MATH 251:
I wanted to thank you for all your effort this semester. You are a great instructor and i'll gladly take any course that would be offered by you that is of my program. Your geneorusity helped me out alot and i'm very thankful to you. Hope to see you in future courses.

December 14, 2005, Brittany student of MATH 157:
Although you were the most intimidating prof on the first day of classes, I would like to tell you that you are a terrific math teacher. Because you pushed the understanding behind the concepts, gave us easy to hard examples, listened and commented on our feedback, and (most importantly) told us what you expected of us, you enabled us to do well if we put the enough effort into it. I loved how all your tests were very fair. Also, I think it is a great idea that you do not use the bell-curve. I think teachers who use it have not taught the material correctly or have not composed fair exams.

As a student who is generally weak at math and who has never taken calculus before, you are honestly the best math teacher I have ever had. Don't change anything about your teaching style...really!

January 8, 2006, Douglas Tam student of MATH 152:
Thank You for such a fair and well written midterm. I believe you should not have hinted at the quadrant 1 question as I thought it was wonderfully asked. Just a tad tricky to see who really pays attention to the details. You're just too nice when it comes to preparing us for midterms. Anyways, just wanted to give thanks to you for a excellent job on the midterm. Nothing new and was very well prepared.

p.s I can easily see you care about us, so cheer up, you're a wonderful prof that really knows how to teach

April 2, 2006, Keelan student of MATH 152:
I would just like to let you know that I think you are an awesome math teacher and a very kind person. I admire your sincere care for your students and your passion for math; it makes the material seem so much more tangible. After reading the last email, (thanks, by the way, I had a great Sunday in the sun, and hope you did too) I knew I had to respond, as I would never be able to leave the mark I felt necessary in those evaluations. Your (152) lectures, though early, are my favorite and give me hope in continued pursuit of math.

November 11, 2008, anonymous student of MATH 157:
I am not going to put my name in this email because I do not want you to think that I am writing it for any ulterior motive.

I am not sure how often you get these emails and I hope that this will not be a bother to you, but I wanted to tell you that although I do not do particularly well in your class (at least not as well as I would hope!) I find everything that you do to be very fair and effective. More importantly, I really appreciate the effort you put into our class as well as how much you seem to care about our success.

Having never taken a university math course before, I was nervous that it would affect my decision to become a high school math teacher. But, after this course, I have made the formal decision to transfer from the faculty of business to science for a degree in mathematics.

March 13, 2009, Leah Harrison student of MATH 157:
I came to see you about a month ago and we went over the first 157 midterm and you sent me some links for more practice for the course. I just wanted to say a big THANK YOU because my score literally doubled on the second midterm, and I am sure it had a lot to do with the extra practice those links gave me. They were a huge help!

April 17, 2009, Trevor Woods student of MATH 100:
I just wanted to say thanks for a really great semester (in this course, at least; my other professor was downright horrid). Not to blow a bit too much smoke, but you've got a great teaching style and hopefully I'll be able to study under you again. On that note, which other courses do you teach? Anyway, just wanted to

let you know you're doing a great job and I hope they give you a big pay raise, or tenure or whatever it is they give good profs.

December 9, 2010, Antone Darakjian student of MATH 157:
Mrs.Menz, I would like to formally thank you for making me work my butt off this term, it has given me a good look at the work ethic that is needed to succeed in University. [...] Anyways all I wanted to say was thanks and Merry Christmas!

December 9, 2012, Jordan Binotto student of MATH 157:
Just wanted to let you know that I ended up with a B- in your 157 class. As I said this class was a big stepping stone and challenge for me and I succeeded! I wanted to thank you for your help, time and attention. I learned more in this class then any other math class I have been in and I have garnered new confidence in areas where before it may have been lacking.

Thank you for all of your help and your passion. The way you taught and the care you put into you teaching really showed strongly and encouraged myself and other to go the extra mile.

Have a great holiday season and maybe see you around in another math class....maybe....haha!

November 25, 2016, Amneet Brar student of MATH 150:
I just wanted to say thank you! Thank you very much for opening up during this Friday's lecture. It's really nice to see that even people like you who look to have their life together and a great career, have gone through so much. its nice to know that it's not just me! I found your story to be very interesting and in some ways very relatable. I too, everything i do, i think of my family. Before starting school my thoughts are always what should i do? Something that makes my parents proud. I've learned a little on the way and have changed my perspective, i will do what i love, because they want me to be happy! I also probably call them 3 times a day just to ask what they're doing and hear their voices! My little brother, 14 years old, feels like my child, someone i've watched grow up been a vital part of his life. It's amazing how important and how great of an impact our family makes on us. What i also related to was the rough patches of life. Your stories were an eye opener, don't judge a book by its cover, right? I'm sorry you had a rough time in the past, no one deserves that but after knowing you a semester (not long) I've learned that you are a strong, independent and amazing woman who is a role model to a lot of women, especially when were told were not good enough, or that we can't do it. Thats something i've struggle with ALOT. I too have had my share of troubles. Last year i was going through a tough time, dealing with depression, i still am. I always though i was just weird and no one understood me. I was not like that a few months before. I was the athletic, smart, outgoing and passionate young woman, but things changed. What i also didn't know at the time was i was dealing with lot of anxiety, something that comes along with depression and coming from the Okanagan its a hard transition. But i know its nice to know that this is my rough patch. I know i'll get through it. It's comforting to know that you, me and everyone will eventually make it out of this rough patch. So i'd just like to say thank you. I know this is weird and its weird for me to email you regarding this but it really hit home for me. Although Math isn't my favourite class, you are one of my favourite teachers this semester. You always find a way to make us laugh even though we want to cry and break down, you get us through. Thank you, i really appreciate you.

December 18, 2018, Taylor Jackson student of MATH 157:
Just wanted to send you a quick email to thank you for this semester! I started the class with a 33% on the diagnostics test, and ended up getting an 82% on the final exam. I hadn't done pre-calculus since high school (seven years ago), and was very nervous coming into the class. Thank you so much for all the resources you offered: ACW, PDF Textbook, practice exams etc... In unison with the lectures, these resources that you provide set up me to succeed and helped me find my footing for calculus. Thank you for challenging me - I worked really hard to prove myself I could get my mark up and couldn't have done it without you and your TA staff.

December 18, 2018, Francesco Guzzo student of MATH 157:
I was not sure about the beginning of this semester in regards to your math 157 class, but i went in trying my best i studied for long hours to pass your diagnostic test and was confident walking into your class that

day. I got 2/15 on it... haha. What most students would have done is drop the course to maybe take it another time with a different professor (upon hearing that most students who failed the test end up failing the class...and i bombed it). Long story short i went to everyone of your lectures, used the workshop when i needed extra help and did all of the work i could to get a passing mark in your class. And to my surprise i finished with 80%. I just wanted to thank you for guiding me through this class, making this semester a great semester. I really enjoyed it no matter how much i wanted to kill Related Rates

April 19, 2019, Chengzhuo Zhang student of MATH 158:
To be honest, this course is very difficult for me. I'm not major in math, my major is economics. I've learned Math 157 before(in FIC, Fraser International college), and I find that 158 is more difficult than 157...I guess that's because my math skill is not good enough, and I hope that I can pass the course. Although I'm not doing well in Math 158, I have to say that you are the best professor that I have ever met. You always send emails to tell us how to learn well in this course and you try to help us as much as you can; you care about students and you don't allow us to stand close to you when you have a cold... I'm so glad to have you to be my professor.

September 8, 2021, Tracy Than student of MATH 157:
My professor mentioned that you wrote the math 157 textbook with a grad student and I wanted to thank you for making it free.

September 9, 2021, Ava Naseri student of MATH 157:
Allow me to introduce myself, I am a new undergraduate student studying Business and this term I am taking Math 157. During the summer I viewed my course syllabus and came across the open source textbook you have made freely available for us, and at first I thought it must have been the class notes but when Mr. Jedwab told us that this is the textbook we will be using for our course I was shocked! I wanted to show my appreciation for all the work you and Nicola Mulberry have put towards this amazing resource! I have had the chance to briefly look over the material and it is all so well organized and I cannot wait to begin this course! Thank you once again!

October 31, 2022, Jack Zhou student of MATH 150:
I wanted to say my thank you's. Among many things I have taken from MATH 150 that doesn't involve calculus, there is one thing in particular that I am thankful for. Specifically, you delivered the first lecture in my degree, and that sat the tone for the rest of my academic career. In that lecture, you said to us (in broad strokes) to own what we learned, and the coursework to be graded on are ways for us to demonstrate our learning, not as much to test our performance.

I have certainly taken this to heart. In the years since, whenever I felt lost or overwhelmed with unknowns and new information, I remembered what you said in that lecture, and that gives me the courage to stand my ground on what I know, and to step forward firmly and steadily. To this day, I wouldn't say I know a lot of things, but the things I do know, I am confident and I own them. As part of the process to teach myself, I've become somewhat of an educator, and to the many that I have taught, tutored, and influenced, I've done my best to pass this idea on too.

17 April 2017

To Whom It May Concern:

I was Petra Menz's tutor marker for the Math 190 distance course in the Fall 2016 and Spring 2017 semesters. In the Fall, we met a week before the semester began and I could already tell from that one meeting that she would be an excellent mentor. The tutor marker position for Math 190 is a lot of work, with the added learning curve of working with students online instead of face-to-face. Petra was upfront about this and provided me with very organized resources to get me up to speed. She was available and supportive in helping me become familiar with my tasks, and this support continued any time I had a question throughout the two terms. Whenever conflicts came up, Petra would respond quickly to the student and offer me advice for similar situations I may face in the future.

I have been a teaching assistant for two and a half years at Simon Fraser University, and for two years in my undergraduate degree at the University of Regina. Comparing my experience with Petra as my supervisor to what is typical supervision for mathematics teaching assistants, I can say with confidence that Petra is miles ahead of her colleagues. At the beginning of the term she gave me a complete schedule of what is expected from the tutor mentor (right down to the very day!), as well as a document with general advice from previous tutor mentors of Math 190.

As the tutor mentor for Math 190 is the main point of contact for students in the course, it is important to have a tutor mentor that cares about the material and well-being of the students. Petra's dedication to teaching is infectious and inspiring. It trickles down to have positive effects on the students. It was my absolute pleasure to work for Petra. If only all tutor mentors could be so lucky as to work for her, the post-secondary math world would be a much better place!

Thank you,

Tara Petrie, MSc.

April 26, 2017

To whom it may concern:

It is my pleasure to write a reference for Petra Menz regarding the Math 190 course.

I was the TM for the Math 190 distance course for three semesters: Fall 2015, Spring 2016, and Summer 2016. I was also the TA for the Math 190 in-person course for five semesters: Fall 2014, Spring 2015, Summer 2015, Fall 2016, and Spring 2017. When I was a TM for the distance course, each semester had a course supervisor other than Petra; however, the course content was always of Petra's creation and she was still involved in the course from an administrative perspective. Petra was the lecturer for the in-person course for three of the semesters where I was the TA.

Petra did an excellent job preparing me to be the TM for the distance course. We had a substantial meeting prior to my first appointment to go over the Canvas container. Here she also described, in detail, my role as TM and answered any questions I had. After this, I met with the course supervisor to discuss how the semester would unfold. Throughout the semester, Petra was always an email away. Several times the course supervisor and I had questions that we would pose to Petra, and she was always very timely and helpful in her responses. Petra made being the TM for the distance course quite pleasant.

Working with Petra as a TA for the in-person course has been fantastic. She is very clear about what is required from the TA, and very helpful in assisting in order to make it happen. She also cares deeply about the course. One specific instance of this is the role she appoints herself in the TA's task of evaluating the students' group assignments. Each week the TA marks the group assignment and sends a report to Petra on their performance. Petra, in turn, would always respond to the report; this gives the TA validation of their work as well as provides the opportunity for TA/instructor dialog, be it feedback pertaining to the assignment, progression of the course, the TA's work, etc. Petra is also wonderful during exam marking. She provides very clear solutions to the problems, and she is also open to discussing the inevitable deviations from the prepared solutions that arise.

Petra is, in general, very professional, organized, and efficient. I will continue to seek TA/TM appointments with her.

Sincerely,
Stefan Hannie

May 28, 2019

Re: Evaluation of Supervision and Mentorship by Petra Menz

To Whom It May Concern:

I have worked under the supervision of Dr. Menz as a Teaching Assistant in the Applied Calculus Workshop (Fall 2017, Spring 2017, Fall 2019) and as a Research Assistant developing Open Educational Resources (Summer 2017–ongoing). Dr. Menz furthermore served as a faculty mentor for my first sessional instructor appointment in Spring 2019.

The Applied Calculus Workshop under Dr. Menz was organized and efficient. The expectations for teaching assistants were clearly laid out at the start of the semester, with follow-up throughout. During the semester, there was ample communication from Dr. Menz which helped me to stay on top of my duties. These qualities were particularly helpful as an incoming graduate student—having Dr. Menz in my first year as a graduate student set me up for success in future teaching assistant positions.

For the past two years, I have worked as a Research Assistant under Dr. Menz's supervision. This experience provided multiple opportunities for my growth as a graduate student. I have received constant feedback on my communication skills throughout this project; thanks to this feedback, I have developed a better sense of how to clearly communicate with students, and to be more careful with my presented work. Although I received frequent feedback and communication from Dr. Menz, I was given independence on the technical aspects of the project. I have found the skills I have developed from this to be immensely useful in other aspects of my graduate studies. Additionally, Dr. Menz encouraged my attendance at two mathematical conferences (AMS MathFest 2018 and CMS 2018) to give talks about this project—these were my first experiences presenting at conferences, and have positively impacted my experience as a student.

Dr. Menz furthermore encouraged me to apply for a sessional position when the opportunity arose to teach from the resources we developed, and then served as my faculty mentor for this appointment. Dr. Menz shared teaching resources and tools that I found useful while preparing my lectures. Before the start of the semester, she encouraged me to develop the majority of the assessments for the class—for which I was later grateful—and then provided me with significant feedback on this material. During the semester, Dr. Menz was always quick to respond to my many questions on how to deal with different situations and student questions, and gave advice on how to get through my workload efficiently. My overall experience with the position was made much more positive and significantly less overwhelming thanks to Dr. Menz's mentorship.

In closing, my experience as a graduate student has been greatly enhanced thanks to the supervision and mentorship of Dr. Menz. My experience working with her as both a Teaching and Research Assistant has encouraged my academic development, and has led to opportunities that I would not have otherwise had.

Best Regards,

Nicola Mulberry
nmulberr@sfu.ca

March 10, 2025
To: Pacific Institute for the Mathematical Sciences
Re: Letter of Support – PIMS Education Prize

Dear Committee of the PIMS Education Prize,

Thank you for gifting your time to read this letter of support for Petra Menz - mathematical genius, course designer, facilitator, coach, mom and amazing human. I am honoured to have an opportunity to share Petra's generosity of knowledge, wisdom and community-building from the perspective of a new teacher.

On the surface, Petra is an award-winning university instructor and lecturer. Beneath the surface, she is tirelessly committed to enhancing the fundamental needs of our society – mastery, independence, community and generosity *(Martin Brokenleg, Circle of Courage, 2003)* – through mathematical teaching and learning. The facilitation strategies that Petra embeds into her courses reach far beyond math and model instructional strategies for teachers to apply at all levels of learning.

I was fortunate to know Petra through sport, prior to enrolling in SFU's Education program. She guided me toward Math 190. The course transformed my way of thinking about math and validated my commitment to "making learning meaningful" to all learners. I have since applied Math 190 teaching strategies, including the First People's Principles of Learning, to literacy, science, social studies, physical education and social-emotional learning.

Always eager to share her wisdom, Petra not only guided my learning through Teacher Education, but visited my first classroom to inspire grade three students in what they labelled as "totally FUN math!" I was astounded at how many children exhibited negative attitudes and low self-confidence in math. Petra inspired me to break through the fear of "teaching math correctly" and embrace methods that began nudging students' attitudes toward wonder, appreciation and growth-mindset. She donated math tools, activities, games and manipulatives with a generous heart… AND a list of activities connected to each item.

As a new teacher, I constantly relied on Petra's advice to differentiate concepts and reach diverse learners. Despite a full schedule of family, work and volunteer coaching, she never refused my questions. After our discussions, I would return to the classroom armed with stacks of ideas, files and books of new pathways to explore with my students. Since that time, I have shared many of her 'tricks of the trade' with colleagues and pre-service teachers. Her ripples of math enthusiasm extend far beyond a university course.

"Learning is embedded in history and story." *(First Peoples' Principles of Learning)*
I will end this letter with a true tale from February, 2025. We were finishing a Petra-inspired math activity and my students were, quite literally, jumping with excitement at the strategies they "discovered" and shared. In our closing circle I told them, "I never learned math like this until I took my teaching degree. An AMAZING person completely changed my view about math and planted seeds for all this EXCITEMENT!!" I dismissed the students, turned on my computer, and saw Petra's request for a letter of support. One story + one story = an infinity of inspiring stories….and learning!

Petra uses one of our most powerful universal languages – math – to make our communities more welcoming places for ALL learners. There is no one I can think of who is more deserving of an award that recognizes enduring, positive effects of higher education on our larger world.

Kind regards,

Kristine Chambers | Primary Teacher | Upper Lynn Elementary
604.903.3820 | kchambers@sd44.ca
I am honoured to have the opportunity to learn, teach, play and explore on the unceded, ancestral territories of the xʷməθkʷəy̓əm (Musqueam), səlilwətaɬ (Tsleil-Waututh) and Skwxwú7mesh (Squamish) Nations.

DESCRIPTION

In 2009, the department found itself in the unusual situation of having no Department Manager, who—among other jobs—also assigns teaching assistantships to graduate students and postdocs. Coupled with the mounting dissatisfaction of the teaching faculty—who have the highest contact with teaching assistants (TAs) —the Chair created the position of TA Assignment Director in order to create a time-sensitive workflow and documentation, where processes are automated as much as possible. In my five years in the department, I had already stood out for my organizational skills and automating other processes, having a major not just in mathematics but also in computing science, and additionally being intimately familiar with the needs of all stakeholders in TA assignments: graduate students, workshop coordinators, instructors and departmental administrators.

In the summer semester of 2009, I took on the huge task of creating a TA assignment procedure that followed a specific timeline, so that all stakeholders would be timely informed, graduate students would meet their financial needs through TA-ships and the workshop and course machinery could be enacted in a timely manner. Any communication was standardized and task-oriented and the pain-staking calculation of base units and preparation time associated with various courses was automated with the creation of Excel sheets. Additionally, I met with TSSU, the teaching and support staff labour union at SFU, to finetune various aspects of TA assignments, which also included the introduction of a TA evaluation form.

Over 8 semesters (fall 2009 to spring 2012), I acted as TA Assignment Director testing the procedure of assigning TA-ships to graduate students and informing workshop coordinators, instructors and the financial assistant. This allowed me the necessary length of time to iron out any wrinkles in the system.

The TA Assignment procedure that I created is still in place today. I recently had the opportunity of on-boarding the newly hired Department Manager Maureen Curtin and the Chair to the TA assignment procedure and was pleasantly surprised with the ease of transitioning back into that position and how quickly Maureen and the Chair picked up the required skills.

Prepared by Petra Menz 06/09/2011

1117 based on Sep. 2 enrollment & projections	207.94 = Total workshop BU needs	+	9 = CSS BU
	219.71 = Total workshop BU appointed		

current BU entitlement listed in grey heading (enrollment/18)

TA written in red means offer is still in the works

Under Shortfall: a positive number means there is a shortfall and a negative number means there is a surplus!

Burnaby Workshop APPOINTMENTS

AW	40.72	prep	CW	55.44	prep	ACW	61.67	prep	QSW	13.83	prep
Andrew Adams	5.17	1.17	Bedanta Chhetri	4.30	0.30	Navid Alaei	4.30	0.30	Pinar Colak	1.22	0.22
Justin Chan	2.30	0.30	Dawei Deng	5.17	1.17	Bahar Behsaz	2.00	0.00	Sarah Kok	3.22	0.22
Iain Crump	5.30	0.30	Daryl Funk	2.00	0.00	Tom Boothby	2.67	0.67	Stephanie Langille	5.22	0.22
Richard Leyland	5.17	1.17	Gordon Hiscott	5.30	0.30	Reanne Bowlby	5.30	0.30	Jemma Lorenat	2.22	0.22
Patrick McMahon	5.17	1.17	Bamdad Hosseini	5.17	1.17	Chiaka Drakes	5.30	0.30	Emily McAlister	2.72	0.72
Steve Melczer	4.30	0.30	Bebart Janbek	5.30	0.30	Haowen Fang	5.30	0.30	Maria Tamayo	2.72	0.72
Carl Qian	3.17	1.17	Harpreet Kaur	5.30	0.30	Sudeshna Ghosh	3.30	0.30			
James Ratcliffe	1.50	0.00	Masood Masjoody	5.30	0.30	Soo Go	5.30	0.30			
Aleks Vlasev	5.17	1.17	Justin Meskas	2.30	0.30	William Ko	5.67	0.67			
Tim Wu	5.30	0.30	Parousia Rockstroh	5.17	1.17	Graham Lea	5.67	0.67			
			M. Vinayagam	5.17	1.17	Nicolas Marie	2.30	0.30			
			Yi Zhong	5.30	0.30	Sonoko Nakano	3.30	0.30			
						Ashok Rajaraman	4.30	0.30			
						Amir Taghavi	5.30	0.30			
						Larry (Xin) Yang	4.67	0.67			
Total BU	**42.55**		**Total BU**	**55.78**		**Total BU**	**64.68**		**Total BU**	**17.32**	
{} = prep	7.05		{} = prep	6.78		{} = prep	5.68		{} = prep	2.32	
% prep	16.6		% prep	12.2		% prep	8.8		% prep	13.4	
CSS	2.00		CSS	0.00		CSS	3.00		CSS+J200 mrk	3.00	
Shortfall	**0.17**		Shortfall	**-0.34**		Shortfall	**-0.01**		Shortfall	**-0.49**	
WC=J. Mulholland			LC =J. Gray			LC = P. Menz			LC = M. Dubiel		

Notes	Notes	Notes	Notes
Ratcliffe marking only	Funk marking only	Behsaz marking only	Colak LON-CAPA only
Melczer 2 BU CSS		Alaei 1 BU CSS	Lorenat 2 BU CSS
		Bowlby 2 BU CSS	Langille 1 BU FAN J200 marking

Surrey Workshop APPOINTMENTS

IMW	5.33	prep	PCW	11.61	prep	ACAW	19.33	prep
M. Malekesmaeili	2.43	0.43	M. Malekesmaeili	2.00	0.00	T. Deshpande	4.38	0.38
Piyashat Sripratak	3.43	0.43	Krishna T. Malladi	5.17	1.17	Xiaorui Li	4.02	1.02
			Xueying Shen	4.17	1.17	Farzana Sultana	4.38	0.38
						Tim Yusun	4.02	1.02
						Jong Zhang	5.38	0.38
Total BU	**5.86**		**Total BU**	**11.34**		**Total BU**	**22.18**	
{} = prep	0.86		{} = prep	2.34		{} = prep	3.18	
% prep	14.7		% prep	20.6		% prep	14.3	
			CSS	0.00		CSS	2.00	
Shortfall	**-0.53**		Shortfall	**0.27**		Shortfall	**-0.85**	
WC=N. Kouzniak								

Notes	Notes	Notes
	Malekesmaeili marking only	
		Zhang 2 BU CSS

Prepared by Cedric Chauve / Maureen Curtin (April 17, 2025)

1254 based on Apr 2 enrollment & projections	**71.11 = Total workshop BU needs (excluding prep)**
	58.00 = Total workshop BU appointed (excluding prep)

current BU entitlement listed in grey heading (enrollment/18)

TA written in red means offer is still in the works

Under Shortfall: a positive number means there is a shortfall and a negative number means there is a surplus!

Burnaby Workshop APPOINTMENTS

AW	**4.11**	**prep**	**CW**	**30.39**	**prep**	**ACW**	**18.22**	**prep**	**QSW**	**4.22**	**prep**
Suozer, Deniz	1.00	0.00	Huang, Nick/Zi Yuan	3.50	0.57	Chaudhury, Samara	2.50	0.57	So, Karolyn/Win	1.00	0.00
Yuzawa, Hirotoshi	2.00	0.57	Kumar, Hitesh	3.00	0.57	Saeidi Mobarakeh, Niloufa	3.00	0.57	Houle, James	3.00	0.57
		0.00	Mohammadi, Negarin/Zahr	2.00	0.57	Patil, Kshitij	1.00	0.00			0.00
		0.00	Obison, Tracy	3.00	0.57	Ndayisenga, Benitha	1.00	0.00			0.00
		0.00	Song, Yexuan	3.00	0.57	Prachchhak, Ishit	1.00	0.00			0.00
		0.00	Sadeghian, Mohammad	3.50	0.57	Roy, Rene	2.00	0.57			0.00
		0.00	Yin, Ming	1.00	0.00	Wang, Sherril/Xuemeng	2.50	0.67			0.00
		0.00	Carr, MacKenzie	1.00	0.00	Salehzadeh, Mahdi	2.50	0.57			0.00
		0.00	Valizadeh, Javad	3.50	0.57			0.00			0.00
		0.00	Sarmah, Pijush	1.00	0.00			0.00			0.00
		0.00			0.00			0.00			0.00
Total BU	**3.00**		**Total BU**	**24.50**		**Total BU**	**15.50**		**Total BU**	**4.00**	
{} = prep	0.57		{} = prep	3.99		{} = prep	2.38		{} = prep	0.57	
% prep	16.0		% prep	14.0		% prep	13.3		% prep	12.5	
Shortfall	**0.54**		Shortfall	**1.90**		Shortfall	**0.34**		Shortfall	**-0.35**	
WC=N/A			WC=JF Williams			WC=S. Rutherford			WC=J. Niezen		

Notes	**Notes**	**Notes**	**Notes**
Marking only: Suozer	Marking only: Yin, Carr, Mohammadi, Sarmah	Marking only: Patil, Ndayisenga, Prachchhak	Marking only: So
	Away: Huang Jun 12-16, Song May 5-16, Yin all Jul,	Away: Roy all May, Wang Jun 4-12	
	Sarmah 3 weeks in July, Valizadeh May 1-Jun 23		

Surrey Workshop APPOINTMENTS

IMW	**0.00**	**prep**	**PCW**	**6.33**	**prep**	**ACAW**	**7.83**	**prep**
		0.00	Pragada, Shivaramakrishna	3.00	0.57	Firoozy, Peyman	3.00	0.57
		0.00	Pender, Thomas	1.00	0.00	Mane, Aniket	2.00	0.57
		0.00	Potgieter	1.00	0.00	Narayanan, Krishna	1.00	0.00
		0.00			0.00			0.00
		0.00			0.00			0.00
Total BU	**0.00**		**Total BU**	**5.00**		**Total BU**	**6.00**	
{} = prep	0.00		{} = prep	0.57		{} = prep	1.14	
% prep	#####		% prep	10.2		% prep	16.0	
Shortfall	**0.00**		Shortfall	**0.76**		Shortfall	**0.69**	
WC=								

Notes	**Notes**	**Notes**
	Marking only: Pender	Marking only: Narayanan
	Away: Narayanan May 12-13, Potgieter July 28-Aug 1,	

MY HISTORY WITH MENTORSHIP AND WELL-BEING

- Through the Professional Development Program at UBC, I became a BC High School Teacher, which included being guided by a mentor. Later, I underwent mentorship training and in turn became a mentor to beginning teachers for several years.
- From 2005 to 2011, I provided mentorship to numerous SFU students (international students through Clan Mentorship Program and local students through Recruitment and Retention).
- From 2007 to 2022, I participated at a variety of workshops, presentations and events **based on the theme of well-being**, most notably the departmental EDI Learning Group (monthly discussions in 2022); Thriving Beyond Campuses - A Dialogue Series Connecting B.C. Post-Secondary (*The Impact of Mental Bandwidth Depletion on Student Mental Health and Well-Being* by Dr. Cia Verschelden in 2020); Pre-Symposium | Workshop for SFU Academic Leaders (*Using Mentorship and Coaching as a Lens to Support Academic Growth in Teaching and Learning* in 2019); SFU Health Promotion (*What role should faculty play in supporting student mental health?* in 2018); *Building Connections: Well-being & Teaching* in 2016); SFU LIDC (*Teaching Large Classes: Beyond Survival* by Dr. Russel Day in 2009); and Student Services (*Question, Persuade and Refer – Suicide Prevention Session* in 2007).
- In 2011, I tutored and mentored Aboriginal adult learners enrolled in math and science classes at the Native Education College.
- As part of my PhD from 2011 to 2015, I studied the research on graduate and supervisor mentorship. There is strong evidence that the success of novices is not just through the acquisition of skills, but also by becoming part of the community. A mentorship model provides this so-called enculturation.
- From 2012 to 2020, I mentored 4 graduate students—some of them multiple times—during 11 semesters in their role as Coordinator of the Applied Calculus Workshop (Math Study Centre).
- From 2015 to 2024, I mentored 9 graduate students, 1 postdoc and 1 limited term lecturer—some of them multiple times—during 15 semesters in their role as sessional instructor, tutor marker or course supervisor.
- In 2017, I was awarded Champion for a Healthy Campus Community by SFU Health and Counselling Services in recognition of outstanding contributions to the health and well-being of the University.
- In 2019, I connected with Biology Lecturer Dr. Kevin Lam because independently of each other, we created well-being slides for our students. We combined our ideas and presented *Some practical ideas and critical considerations for a compassionate instructor* locally at the SFU Teaching Matters Seminar and in the Professional Development Workshop at Douglas College in New Westminster.
- From September 2021 to August 2022, I took it upon myself to mentor newly hired lecturer Dr. Joanna Niezen by meeting in person once a week for 1-2 hours debriefing or foreshadowing teaching related matters, explaining the inner workings of the department, the faculty and SFU as a whole and making her aware of professional development opportunities and available teaching and learning organizations both locally and nationally. At the time, the department still had no formal mentoring mechanism or recognition of mentoring as service.

THOUGHTS ON MENTORSHIP AND WELL-BEING

I value mentorship as a process for enculturation and strongly support well-being in all its forms (physical, emotional, mental, social, spiritual and what I call academic). As my history list indicates, my relationship with mentorship and well-being is on-going and ever evolving. We are all trying to navigate life, which at times means trying to manage obstacles. The ones that I have encountered made me appreciate when a friendly mentor provided crucial guidance. I also learned through my own obstacles that one’s well-being ties into self-efficacy and resilience and so has a major impact on how one is able to manage life and its challenges. Figure 1 depicts visuals on how I inform myself about mentorship and well-being.

Figure 1: Mentoring visual from *https://www.liberty.edu/casas/blog/the-power-of-peer-mentoring/* and Well-Being visual from *https://www.sfu.ca/healthycampuscommunity/learningenvironments/rationale.html*

From early on, I sought ways to be involved in mentorship and well-being, which first were oriented around students. As an immigrant and English as a Second Language speaker, I understood quite well what barriers international students face and so it was natural for me to start providing mentorship through the SFU Clan Mentorship Program shortly after my arrival at SFU. Aside from a presentation to this group of students, I also met 2-3 times with students throughout the term and was their email contact for any academic challenge they encountered. In 2010, out of this experience and with student well-being in mind, I developed a 5-page document *Recommendations to Thrive in Mathematics* for first-year mathematics students, which was first published through the Math Student Union, and which since has evolved into an 8-page Chapter 1 in the textbook *Differential Calculus: Problems and Solutions from Fundamentals to Nuances* published by World Scientific Publishing.

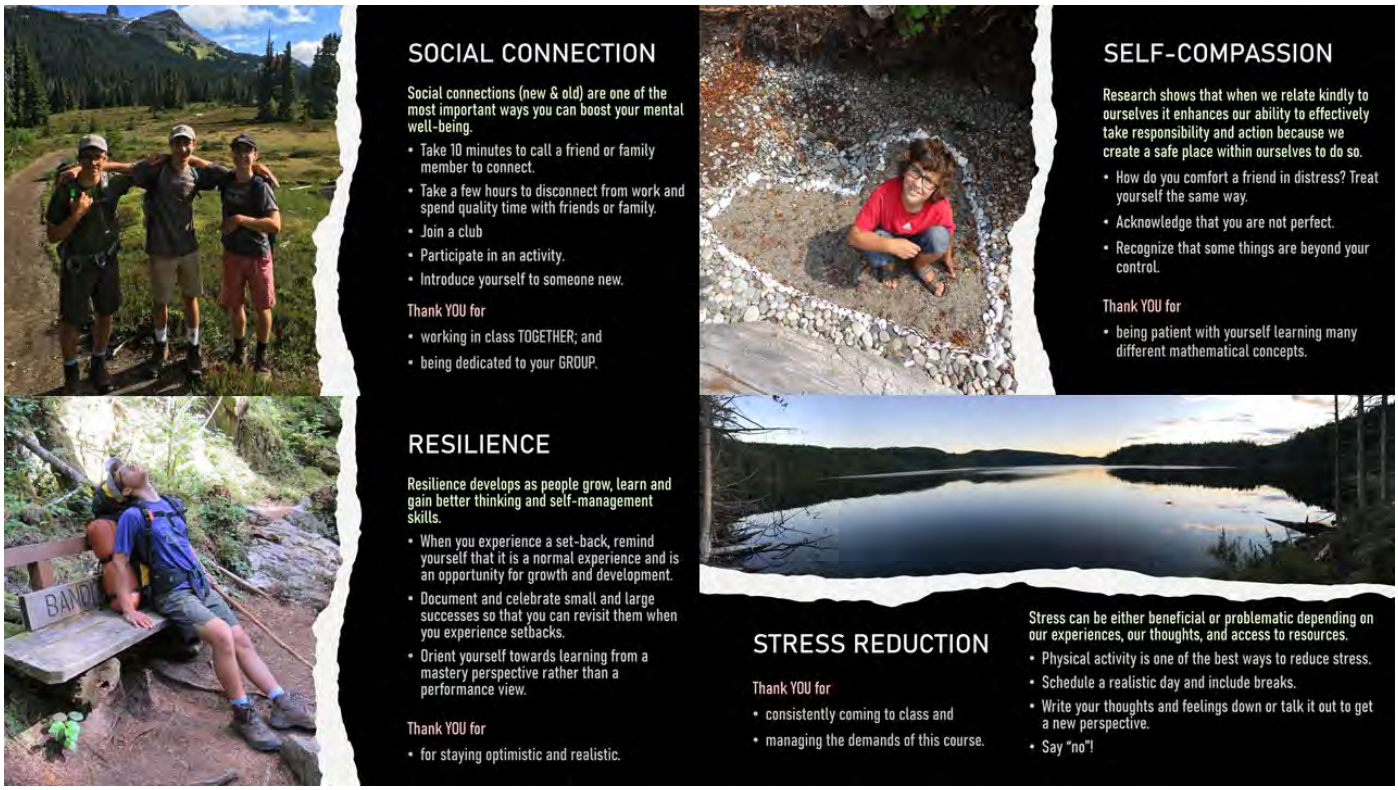


Figure 2 Four samples from my own developed well-being slides for my students

I also connected with the SFU Health and Counselling Service and found an immediate connection with Health Promotion Specialist Alisa Stanton. I made space in each one of my courses for a 30-minute well-being workshop led by Alisa or others from her unit. Countless students told me how valuable this experience was for them. I also approached Alisa about ways to offer well-being to graduate students in their role as teaching assistant (TA). She developed 12 brief email messages around a specific well-being

theme, which were sent to TAs each week as a reminder for their own well-being, but which could also be incorporated into their tutorials for their own students. My group of TAs were the test-group and subsequently this well-being messaging program was adopted for future semesters. Unfortunately, the health promotion unit was abolished several years ago due to financial constraints. So, in 2019, I developed my own well-being slides, which I have been using since in all my courses, see Figure 2 for sample slides.

At the departmental level, I have been an advocate for a formal mentorship program—for a long time as a solitary voice—since my arrival two decades ago because I firmly believe that such a program supports new faculty, sessional instructors and graduate students in becoming part of our academic community along with our departmental and university norms by directly tapping into existing knowledge, skills, experience, and expertise. Furthermore, being a mentor is a service not just to the mentee but the whole community and should be recognized as such. My advocacy for mentorship slowly gained traction in the department and 2023 saw the formalization of a mentorship program that is now recognized as service.

MENTEES AS LISTED ON CV

Mentoring Teaching Faculty

September 2021 - August 2022	Dr. Joanna Niezen, newly hired lecturer, by meeting once a week for 1-2 hours in person debriefing or foreshadowing teaching related matters, explaining the inner workings of the department, the faculty and SFU as a whole and making her aware of professional development opportunities and available teaching and learning organizations both locally and nationally

Mentoring Instructors

September 2024 - December 2024	PhD graduate student Sebastián Moraga Scheuermann as FAN X99 D300 Sessional Instructor, Department of Mathematics
September 2024 - December 2024	PhD graduate student Mahsa Ansari as FAN X99 D400 Sessional Instructor, Department of Mathematics
January 2024 - April 2024	Limited Term Lecturer Steve Cheung as Sessional Instructor of FAN X99 A320, a course for students of the Cowichan Tribe on Vancouver Island run online by the Department of Mathematics and offered through the Indigenous Languages Program (INLP). I provided all lecture material and oversaw assessment.
September 2023 - December 2023	Limited Term Lecturer Steve Cheung as Sessional Instructor of FAN X99 A320, a course for students of the Cowichan Tribe on Vancouver Island run online by the Department of Mathematics and offered through the Indigenous Languages Program (INLP). I provided all lecture material and oversaw assessment.
May 2019 - August 2019	PhD graduate student Arezou Valadkhani as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
May 2019 - August 2019	PhD graduate student Mahdieh Malekian as MATH 155 D100 Sessional Instructor, Department of Mathematics
January 2019 - April 2019	PhD graduate student Arezou Valadkhani as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
January 2019 - April 2019	PhD graduate student Nicola Mulberry as MATH 157 Sessional Instructor, Department of Mathematics
September 2018 - December 2018	PostDoc Michel Virgilio as MATH 157 D200 Sessional Instructor, Department of Mathematics
May 2018 - August 2018	PhD graduate student Arezou Valadkhani as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
January 2018 - April 2018	PhD graduate student Arezou Valadkhani as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
May 2017 - August 2017	MSc graduate student Jeremy Chiu as MATH 155 Sessional Instructor, Department of Mathematics

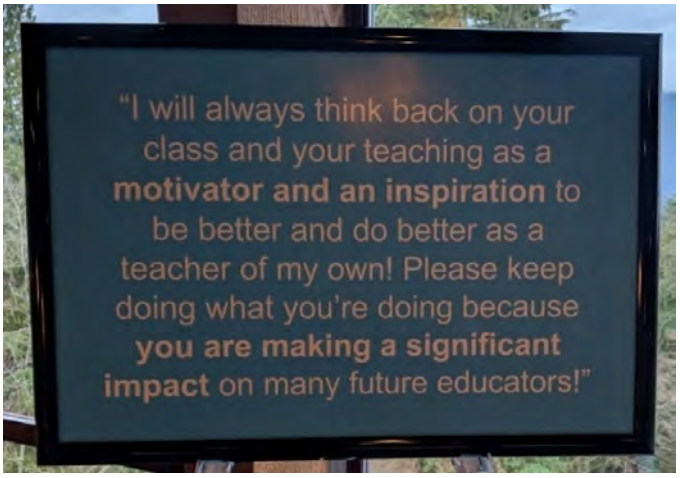
January 2017 - April 2017	MSc graduate student Jeremy Chiu as MATH 155 Sessional Instructor, Department of Mathematics
May 2016 - August 2016	PhD graduate student Sonoko Nakano as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
January 2016 - April 2016	PhD graduate student Oi-Lam Ng as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
September 2015 - December 2015	PhD graduate student Oi-Lam Ng as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics
May 2015 - August 2015	MSc graduate student Navid Alaei as MATH 155 Sessional Instructor, Department of Mathematics
May 2015 - August 2015	PhD graduate student Oi-Lam Ng as MATH 190 C100 Tutor Marker and Course Supervisor, Department of Mathematics

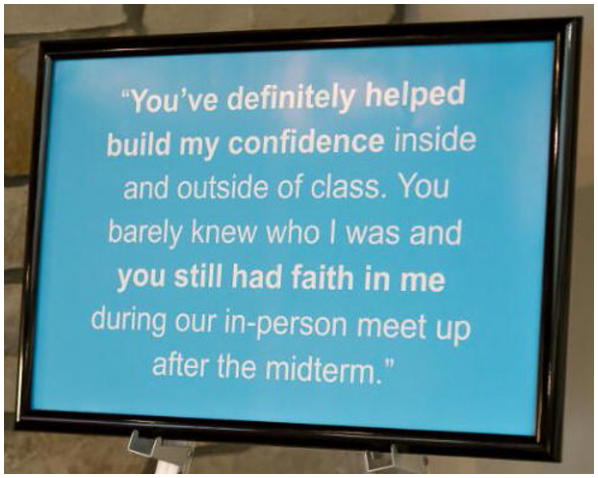
Mentoring Workshop Coordinators

May 2020 - August 2020	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics, Department of Mathematics
January 2020 - April 2020	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics
May 2018 - August 2018	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics
January 2018 - April 2018	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics
May 2017 - August 2017	PhD graduate student Abhinav Shantanam as ACW Workshop Coordinator, Department of Mathematics
May 2015 - August 2015	MSc graduate student Navid Alaei as ACW Workshop Coordinator, Department of Mathematics
January 2015 - April 2015	PhD graduate student Will Ko as ACW Workshop Coordinator, Department of Mathematics
May 2014 - August 2014	PhD graduate student Sonoko Nakano as ACW Workshop Coordinator, Department of Mathematics
January 2014 - April 2014	PhD graduate student Sonoko Nakano as ACW Workshop Coordinator, Department of Mathematics
May 2013 - August 2013	PhD graduate student Sonoko Nakano as ACW Workshop Coordinator, Department of Mathematics
January 2012 - April 2012	PhD graduate student Sonoko Nakano as ACW Workshop Coordinator, Department of Mathematics

Mentoring Students

September 2005 - December 2011	Presented information about studying math and was a point of contact throughout the fall semesters for international students in the Clan Mentorship Program, SFU Student Development & Programming Centre.
January 2011 - April 2011	PIMS and SFU Office for Aboriginal Peoples. Tutored and mentored at the Native Education College Wednesday 6-7:30 pm.
May 2009 - June 2010	SFU English Bridge Program. Presented semesterly “Studying at a University” to international students.
September 2008 - August 2009	Mentoring of 5 science students in their transition from high school to university, SFU Recruitment and Retention.





Mathematics professor Petra Menz named SFU's 2017 healthy campus champion

JANUARY 16, 2018

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Mathematics professor Petra Menz has received SFU's 2017 Champion for a Healthy Campus Community award for the remarkable impact her teaching methods have on student well-being.

Each year, the university recognizes a person, group or program that has made an outstanding contribution to student well-being.

Menz's teaching approach aims to foster meaningful social connections with and between her students, creating a positive learning environment that cultivates students' self-esteem, resilience and engagement. She invites students to meet with her individually and in small groups so that she can get to know them, understand their needs, and support their success.

"Because of this personal connection that I create, students become bigger risk takers," says Menz. "They learn that it's okay if they are wrong in the classroom, and they are more engaged in the discussions because they know each other."

Indeed, research has revealed that creating connections in the learning environment not only boosts students' well-being but also their academic performance.

"I think Petra's one-on-one meetings really helped me be more confident," says student Diamond Huynh. "She totally believed in me. I told her straight up that I was very scared in the course because I'm not strong in math but she made sure that I really believed in myself."

Says another student, Hassan Ahmed, "The French would say, 'students are not a vase that you fill, they are a fire that you set alight' and this is what Petra has done. She made this connection; she stimulates you and gets the best out of you."

SFU's Health Promotion Team and the Teaching and Learning Centre are collaborating on a Wellbeing in Learning Environments project that supports teaching staff with evidenced-based strategies for fostering well-being in classrooms and lecture halls.

The Well-being in Learning Environments project is part of SFU's progressive Healthy Campus Community Initiative. It aligns with the internationally recognized, and SFU-adopted, Okanagan Charter for Health-Promoting Universities and Colleges.

2017 CHAMPION FOR A HEALTHY CAMPUS COMMUNITY

is awarded to

Dr. Petra Menz

Senior Lecturer, Mathematics

January 16, 2018

DR. PETER KELLER
VICE PRESIDENT, ACADEMIC
AND PROVOST

DR. NANCY JOHNSTON
VICE PROVOST STUDENTS AND
INTERNATIONAL, PRO TEM

SFU SIMON FRASER UNIVERSITY
HEALTH PROMOTION

Chapter 1

Recommendations to Thrive in Mathematics

The following is a list of recommendations concerning habits around learning, exam preparations, and well-being. This list was put together for all students who are thinking about their actions, values, and choices that affect their studies in academia. By making a commitment to regularly assess your values and choices in life, you not only discover what you value and how you choose, but also how these choices impact your educational goals and their attainment, which hopefully bring about the right actions. Bear in mind that your values and choices are bound to change as you go through your various life stages. However, by actively engaging in this sort of reflection and assessment exercise, you learn to manage change in your academic career and beyond.

Tips for Reading these Recommendations:

- Do not be overwhelmed with the number of items on this list. You may not want to read the whole chapter at once. Instead, choose some recommendations that appeal to you.

- You may want to make changes in your habits and study approaches after reading the recommendations. Our advice is to take small steps. Small changes are easier to make and chances are those changes will stick with you and become part of your habits.

- Take the time to reflect on the recommendations. Look at the people in your life you respect and admire for their groundedness and accomplishments. Do you believe the recommendations are reflected in their approach to life?

Habits of a Thriving Student:

- **Act responsibly and respectfully:**
  - Be accountable for your own behavior.
  - Respect diversity in people, ideas, and opinions.
  - Read documents that are passed on by the instructor and act on them, such as the syllabus, a lecture outline, or an assignment guideline.
  - Take an active role in your education, for example, regularly attend class and come prepared, complete assignments to the best of your ability, and seek out an academic advisor for course and program guidance.
  - Uphold and be committed to honesty, trust, fairness, respect, responsibility, and courage, which are the fundamental values set out by the International Center for Academic Integrity, even when situations become tough or stressful. In other words, do not cheat and encourage academic integrity in others.
- **Set goals:**
  - Set attainable goals based on specific information, such as the syllabus, the academic calendar, or academic advisor.
  - Be motivated to reach your educational goals.
  - Be committed to thrive.
  - Understand that your physical, mental, and emotional well-being influences how well you can perform academically.
- **Be reflective:**
  - Understand that deep learning comes out of reflective activities.
  - Reflect on your learning by revisiting assignments, midterm exams, and quizzes and comparing them against posted solutions.
  - Reflect on why certain concepts and knowledge are more readily or less readily acquired.
  - Reflect on your achievements and your failures to bring about change.
- **Be inquisitive:**
  - Be active in a course and ask questions that aid your learning and build your knowledge base.
  - Seek out your instructor after class and during office hours to clarify concepts and content and to find out more about the subject area.
  - Show an interest in your program of studies that drives you to do well.
- **Practice communication:**

  - Articulate questions. This is one of the best ways to probe your own understanding and gain new knowledge.
  - Speak about the subject matter of your courses, for example, by explaining concepts to your friends.
  - Take good notes that pay attention to detail but still give a holistic picture.
  - Observe how mathematics is written and attempt to use a similar style in your written work.
  - Pay attention to new terminology and use it in your written and oral work.

- **Find ways to enjoy learning:**
  - Be passionate about your program of study.
  - Be able to cope with a course you don't like because you see the bigger picture.
  - Be a student because you made a positive choice to be one.
  - Have a habit of reviewing, such as study notes, lecture material, or the textbook.
  - Work through assignments individually at first and way before the due date.
  - Do extra problems as needed.
  - Read unassigned material related to the course with an eye spend on time.

- **Be resourceful:**
  - Use the resources made available by the course and instructor, such as the virtual course container or website, textbook, tutorial, or drop-in help centre.
  - Trust that the course is well constructed and contains all necessary material and activities to attain your learning goals.
  - Use the library and internet thoughtfully and purposefully to find additional resources for a certain area of study, but do not gorge yourself on outside sources thereby wasting time.
  - Research support structures offered through your post-secondary institution – often through the library, teaching and learning centre, or health and counselling services.

- **Be organized:**
  - Employ a calendar to organize class time, due dates, exam dates, and other time-sensitive activities.
  - Create a weekly to-do list to stay on top of required tasks.
  - Adopt a particular method for organizing lecture notes and extra material that aids your way of thinking and learning. And if you have not figured this out yet, then experiment to find out.
  - Organize your learning space as an inviting place for studying.

- **Manage your time effectively:**

  - Be in control of your time.
  - Follow a schedule that not only includes time for study and research, but also time for eating, social, and physical activities.

- **Be involved:**
  - Be informed about your program of study and your courses and take an active role in them.
  - Visit the instructor or academic advisor to get help.
  - Join a study group and use the support that is being offered in your program.
  - See the bigger picture and find ways to be involved in more than just studies by joining clubs offered at your post-secondary institution.
  - Look for volunteer opportunities, for example, by becoming a peer tutor.

- **Be resilient:**
  - Be kind to yourself when you face challenges and setbacks.
  - Withstand stressful experiences by addressing them and by adjusting your mode of operating and perhaps also the environment that you live and study in.
  - Navigate challenges by trying new approaches and by reaching out to others.
  - Bounce back from setbacks by reminding yourself of your accomplishments. We often forget our achievements when we feel disappointed or rejected.
  - Realize that challenges and setbacks are a normal part of life and opportunities to assess your actions, values, and choices. By actively engaging in this assessment, you not only learn to manage ups and downs across the course of your life, but you also become resilient.

How to Prepare for Exams:

- Start preparing for an exam on the FIRST DAY of class!
- Come to all classes and listen for where the instructor stresses material or points to classical mistakes. Make a note about these pointers.
- Treat each chapter with equal importance, but distinguish among items within a chapter.
- Study your lecture notes in conjunction with the textbook because it was chosen for a reason.
- Pay particular attention to technical terms from each lecture. Understand them and use them appropriately yourself. The more you use them, the more fluent you will become.
- Pay particular attention to definitions, theorems, and formulas from each class. Know the major ones by heart.

- Create a "study sheet" that summarizes terminology, definitions, theorems, and formulas. You should think of a study sheet as a condensed form of lecture notes that organizes the material to aid *your* understanding. (However, you may not take this study sheet into an exam unless the instructor specifically says so.)

- Check your assignments against posted solutions. Be critical and compare how you wrote up a solution versus the instructor/textbook. Do not be afraid to question a solution. This is a sign of wanting to understand.

- Read through or even work through the assignments and quizzes (if any) a second time. However, be selective the second time around and zero in on material that is of higher importance or where you need another go at it to improve your understanding.

- Study the examples in your lecture notes in detail. Ask yourself why they were offered by the instructor.

- Work through some of the examples in your textbook. Then compare your solution to the detailed solution offered by the textbook, if any, or seek out the instructor or a teaching assistant.

- Does your textbook come with a review section for each chapter or grouping of chapters? Make use of it. This may be a good starting point for a study sheet. There may also be additional practice questions.

- Practice writing exams by doing old midterm and final exams under the same constraints as a real midterm or final exam: strict time limit, no interruptions, no notes, and no other aides unless specifically allowed.

- Study how old exams are set up! How many questions are there on average? What types of questions are being asked? What would be a topic header for each question? Rate the level of difficulty of each question. Now come up with an exam of your own making and have a study partner do the same. Exchange your created exams, write them, and then discuss the solutions.

Getting and Staying Connected:

- Stay in touch with family and friends:
  - A network of family and friends can provide security, stability, support, encouragement, and wisdom.
  - This network may consist of people that live nearby or far away. Technology – in the form of phones and social media – allows us to stay connected no matter where we are. However, it is up to us at times to reach out and stay connected.

  - Do not be afraid to talk about your accomplishments and difficulties with people that are close to you and you feel safe with to get different perspectives.

- Create a study group or join one:
  - Both the person being explained to and the person doing the explaining benefit from this learning exchange.
  - Study partners are great resources! They can provide you with notes and important information if you miss a class. They may have found a great book, website, or other resource for your studies.
  - Most importantly, you will learn that other students also encounter struggles and difficulties, so you are not alone in this journey. Perhaps you will even get some tips how to overcome some of the struggles and difficulties.

- Go to your faculty or department and find out what student groups there are:
  - Is there a math student union or club that promotes student interests within the Department of Mathematics? This is often a great place to find like-minded people and to get connected within mathematics.
  - Student groups or unions may also provide you with connections after you complete your program and are seeking either employment or further areas of study.

- Go to your faculty or department and find out what undergraduate outreach programs there are:
  - Is there an organized group in the Department of Mathematics that prepares students for the William Lowell Putnam Mathematical Competition held annually the first Saturday in December?
  - Are there opportunities to apply for an undergraduate research assistant-ship? These are typically offered through government agencies and supported in the Department of Mathematics (or other science departments and faculties) such as the Canadian NSERC USRA or US NSF REU. These are great opportunities to not only earn some money, but to also embark on some mathematics research being guided by a professor.
  - Is there an undergraduate seminar that presents a variety of topics concerning mathematics? This is a great way to learn about mathematics in action either in research, industries, business, or other math-driven endeavors.

Staying Healthy:

A healthy mind, body, and soul make you thrive. Create a healthy lifestyle by taking an active role in this lifelong process.

- Mentally:

  - Feed your intellectual hunger! Choose a program of study that suits your talents and interests. You can benefit from visiting with an academic advisor.
  - Take breaks from studying! This clears your mind and energizes you.

- Physically:

  - Eat well! Have regular meals and make them nutritious.
  - Exercise! You may want to get involved in a recreational sport.
  - Get out, rain or shine! Your body needs sunshine to produce vitamin D, which is important for healthy bones.
  - Sleep well! Have a bed time routine that will relax you so that you get good sleep and enough of it. Get enough sleep so that you are energized. Brain functions such as memory maintenance and building depend on sleep.

- Socially:

  - Be respectful, inclusive, and supportive of others, and appreciate diversity.
  - Stay in touch with loved ones.
  - Make friends! Friends are good for listening, help you to study, and make you feel connected.
  - Get involved! Join a university club or student union.
  - Manage challenging relationships.

- Emotionally:

  - Accept yourself for who you are. We all have strengths and weaknesses.
  - Look after yourself and treat yourself to something enjoyable.
  - Ask your instructor, student colleagues, family, friends, or institutional resources for help when you need it.
  - Recognize and manage stressful experiences.
  - Be optimistic when facing stressful experiences, this helps you cope with them.

- Spiritually:

  - Be in touch with your own values and beliefs.
  - Respect the values and beliefs of others.
  - Be present, mindful, and aware how you breathe.
  - Connect the complex interrelationship between your mind and body for improved mental and physical functionality, for example, through yoga.

Resources:

- How to Ace Calculus: The Streetwise Guide by Colin Adams, Abigail Thompson, and Joel Hass published by Macmillan.
- Paul's Online Notes by Paul Dawkins: https://tutorial.math.lamar.edu/
- Desmos Graphing Calculator: https://www.desmos.com/calculator
- Symbolab Math Solver - Step by Step calculator: https://www.symbolab.com/
- WolframAlpha – Computational Knowledge Engine: https://www.wolframalpha.com/examples/mathematics/
- 16 Habits of Mind (1-page summary): https://www.chsvt.org/wdp/Habits_of_Mind.pdf

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DESCRIPTION

In 2003, during the day-long interview for the lectureship that I currently hold in the department, I met with Dr. Veselin Jungić (Veso) on the teaching faculty. He asked me to elaborate on my outreach ideas, which featured in my teaching portfolio at the time. As a grade 12 teacher for six years followed by a post-secondary lectureship for two years, when I only taught first-year calculus courses, I noticed the stark contrast in study behaviours between the two groups of students, which in age are often only one year apart from each other. Research shows this has to do with providing structure versus needing to be in charge of yourself, size of community 1000 students versus 10,000 students, emphasis on formative assessment versus summative assessment and many more factors that contribute to secondary institutions being based on academic dependence, while secondary institutions demand academic independence. One of my ideas was to bring high school math students to campus not only to experience the sheer size of a post-secondary institution, but to also take part in a typical math lecture to experience what it is like to be a student in a lecture hall among hundreds of other students rather than in a high school class of 30 students.

In 2005, Veso and I created an event called *Math Meet and Greet* for local high school students and their math teacher with the support of SFU Community Outreach and the SFU High School Liaison Committee. At first, this event was only hosted once a year and included a presentation by a research faculty geared to the audience followed by a meet-and-greet with the research faculty and volunteer SFU calculus students, participation in a live first-year calculus lecture and a campus tour. The event was instantaneously popular and saw attendance between 200 and 300 high school students, and we even accepted requests for single high school visits. Because of the popularity, other departments and faculty were interested in showcasing their subject and so an additional session was included in the event alternating between Physics, Chemistry and Engineering. In 2012, the event was taken over by SFU Community Outreach and grew beyond a *Math Meet and Greet* into a Science Outreach event.

EVENTS AS LISTED ON CV

Aug-Nov 2011	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungic and Stephen Price full day visits from ~200 high school students held November 18.
Aug-Nov 2010	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungic and Stephen Price full day visits from ~200 high school students held November 19.
Aug-Nov 2009	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungic and Stephen Price full day visits from ~240 high school students held November 13 and 20.
Aug-Nov 2009	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungic and Stephen Price full day visits from ~240 high school students held November 14.
Aug-Nov 2009	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungic and Stephen Price full day visits from ~240 high school students held November 14.
Jan-Mar 2008	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Drs. Veselin Jungic and Stephen Price full day visits from ~240 high school students held March 7.
May 2008	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Organized and hosted St. John's High School visit to the Department of Mathematics held May 16.
Jan-Mar 2007	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Dr. Veselin Jungic full day visits from ~300 high school students held March 9.
Jan-Mar 2006	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Co-organized with Dr. Veselin Jungic full day visits from ~250 high school students held March 31.
January 2005	SFU Community Outreach & High School Liaison Committee, Burnaby, Canada. Organized and hosted St. John's High School visit to the Department of Mathematics held January 21.

Math Meet and Greet

Schedule of the Day for Friday November 18, 2011

Time	Group A school – contact (#) Volunteers TBA		Group B school – contact (#) Volunteers TBA	
8:30AM	**HOST: Petra** **Presenter: Matt** Title Spherical Geometry	IRMACS Theatre ASB10900	**Instructor: Jamie** **MATH 150** Calculus I with Review	AQ 3182
			Instructor: Brenda **MATH 151** Calculus I	B 9201
			Instructor: Nils **MATH 154** Calculus I for the Biological Sciences	C9001
9:30AM	Inside Stats/ Quantitative Careers **Presenter: Dale (and others)**	IRMACS Theatre ASB10900	Computing and Engineering	TASC 9204
10:30AM	Computing and Engineering	TASC 9204	Inside Stats/ Quantitative Careers **Presenter: Dale (and others)**	IRMACS Theatre ASB10900
11:30AM	**Instructor: Petra** **MATH 157** Calculus I for Social Sciences	C9001	**HOST: Veso** **Presenter: Matt** Title Spherical Geometry	IRMACS Theatre ASB10900

Organizers: Stephen Price, cell: 778-833-2792 (event day only), Dr. Veselin Jungic, Petra Menz

Math Meet and Greet

About our Programs

Professors and You

One of the great strengths of the Mathematics Department is the availability of your professors and a dedicated undergraduate advisor. With over 200 students in our programs and nearly 40 professors, you will find a professor who can assist you in your exploration of Mathematics.

Student Life

The Math Student Union and Math Lounge is a great place to meet friends and find other students who like Math. With events ranging from undergraduate conferences to informal social nights, Math students always find a community passionate about Math.

Navigating Your Degree

Choosing an education that allows you to pursue your passion and meet your needs is most important when choosing your degree. The Mathematics Undergraduate Advisor will be happy to schedule a time to discuss with you the variety of career options for graduates with a mathematics degree and how our programs may help you pursue your career and life goals.

Math Awards

We like to promote our best students with Math Awards. Each year we give out a variety of awards and monetary prizes to recognize the achievements of great students. In addition, the department offers research and Mathematics' competition awards for outstanding achievement.

Career Options

With Work Integrated Learning, SFU provides you with a variety of options for getting work experience during your degree. Among the best in the country, SFU's Coop Program will give you the greatest amount of experience and help build your career skills. Coop and career programs are available to students across all disciplines at SFU.

Student Success

When you arrive, make sure you attend the workshops, tutorials and review sessions that we offer to help make you successful. The most important thing to remember is to work hard, ask questions and seek help – academic support is available whether it's to go deeper or to keep up.

Workshops

Our math workshops provide a stimulating environment where you can meet students and discuss course material with you instructor, workshop coordinator and teaching assistants (TAs). Here you can find help with assignments, problems, and exam preparation. Workshops also provide an opportunity for you to meet fellow classmates and form study groups.

Math Programs

www.math.sfu.ca

The strength of the mathematics programs at SFU is that you can study mathematics in the purest form of simply thinking about numbers and how they describe our world and tackle specific, real-world problems whose solutions lie only through expanded understanding of complex mathematical models and careful application of mathematical skills.

Applied Mathematics - Major/Honours

Ever wondered what people actually do with math? Applied Mathematics is all about finding good uses for mathematics, and focuses on continuous models from the physical, life and financial sciences. Courses emphasize theory, model development and scientific computing and can lead to work or research in many fields. For example, students could build a model to show the impacts of global warming or solve the issue of how to make bus systems more efficient. Graduates work in roles that use high performance computing to solve real-world problems.

Industrial Mathematics - Honours/Major/Minor

This program combines a broad education in mathematics with courses in computing science, engineering, and business. A branch of applied mathematics, Industrial Mathematicians use logic, abstraction and analytical skills to tackle problems faced by industry. Graduates solve logistical problems, help managers make good decisions and guide new technologies. For example graduates could use math to determine how a company like FedEx could deliver a package most efficiently. When you deliver 1 million packages, this can mean millions of dollars saved.

Mathematics - Major/Honours/Minor

Whether you want to solve problems or explore the beauty of the mathematical world, the mathematics program allows you the greatest flexibility to choose courses that suit your interests. Each day, mathematics graduates use the work of Pythagoras, Gauss and modern day discrete mathematicians to solve issues and push the boundaries of how we think about quantity, space and change. Using an SFU math degree, you can work in business and industry or study mathematics at a higher level to pursue a teaching and research career.

Mathematics and Computing Science – Honours/Major

The computer has revolutionized the way we do mathematics and mathematics forms the basis of how computers operate. The MACM program is an excellent choice for students who want to study mathematics and computing science. This program is an excellent choice for students looking to work in the high-tech sector or for students looking to pursue graduate degrees in Mathematics or Computer Science.

Mathematical Physics - Honours

This program lets you pursue a passion for both Physics and Mathematics. You can study the interplay of Physics and Mathematics to understand topics like black holes, string theory and space time. Because you have learned the combined theoretical and lab skills in close contact with your professors, you can go on to work in industry, or conduct research in both fields. You could also write for Sci-Fi TV shows.

Statistics and Actuarial Science Programs

www.stat.sfu.ca

Statistics – Major/Honours/Minor

Every day we are bombarded by statistics, but what do those numbers really mean? Statistics uses quantitative tools to solve problems in just about every field. From business to industry to government to health, statisticians provide the backbone of information that informs good policies and good decisions. At SFU our statistics department has great strength in health statistics and in statistics relating to environmental management. The BSc in Statistics is also where you start at SFU on your way to entry into Actuarial Science.

Actuarial Science – Major/Honours

Risk enters into every personal and business decision we make. Actuarial scientists are interested in the aggregates of all of those individual decisions and how to help businesses like insurance companies manage that risk. Actuaries assess the possibility that a future event will happen, they design mechanisms to reduce the chances that an undesirable event occurs and they put in place systems to minimize the impact of negative events that do occur. Actuarial Sciences is a professional program leading to certification with the Canadian Institute of Actuaries.

Management and System Science - Major/Honours

www.students.surrey.sfu.ca/science/mssc

This interdisciplinary program based at our SFU Surrey Campus combines analytical insight, computational skills, and business savvy. Using the skills learned in this program, you will be able to make businesses more effective and profitable in areas as diverse as data-driven marketing, finance, systems analysis, and optimizing production practices. Graduates have gone on to leadership positions at firms like HSBC, Telus, and Crystal Decisions.

Learn More:

Check out our websites for detailed information:
General Info – www.sfu.ca/prospectiveundergrads
Faculty of Science - www.sfu.ca/science
Faculty of Applied Sciences – fas.sfu.ca
Math Department - www.math.sfu.ca
Statistics Department – www.stat.sfu.ca
Physics Department – www.physics.sfu.ca
Coop/Work Integrated Learning - www.sfu.ca/wil
Calculus Challenge Exam - www.math.sfu.ca/outreach/schools/challenge

Fun Fact | Math Girl at SFU

SFU is the only university with its own Math Superhero – search "Math Girl" on YouTube and learn how **mathematics really can save the world.**
http://en.wikipedia.org/wiki/Math_Girl

ROCKRIDGE SECONDARY SCHOOL

5350 Headland Drive
West Vancouver, BC V7W 3H2
Telephone (604) 981-1300 Fax (604) 981-1301

Marne Owen – Principal

Craig Chubb – Acting Vice-Principal
John Crowley – Vice-Principal

SCHOLARSHIP ISSUE
The Newsletter of Rockridge School

www.sd45.bc.ca/rockridge

Issue No. 3 **December 2009**

PARENT / TEACHER CONFERENCES
THURSDAY, JANUARY 21, 2010

SEE PAGE 3 FOR DETAILS!

NEXT NEWSLETTER – FEBRUARY 2010

INSIDE THIS ISSUE:

UPCOMING EVENTS

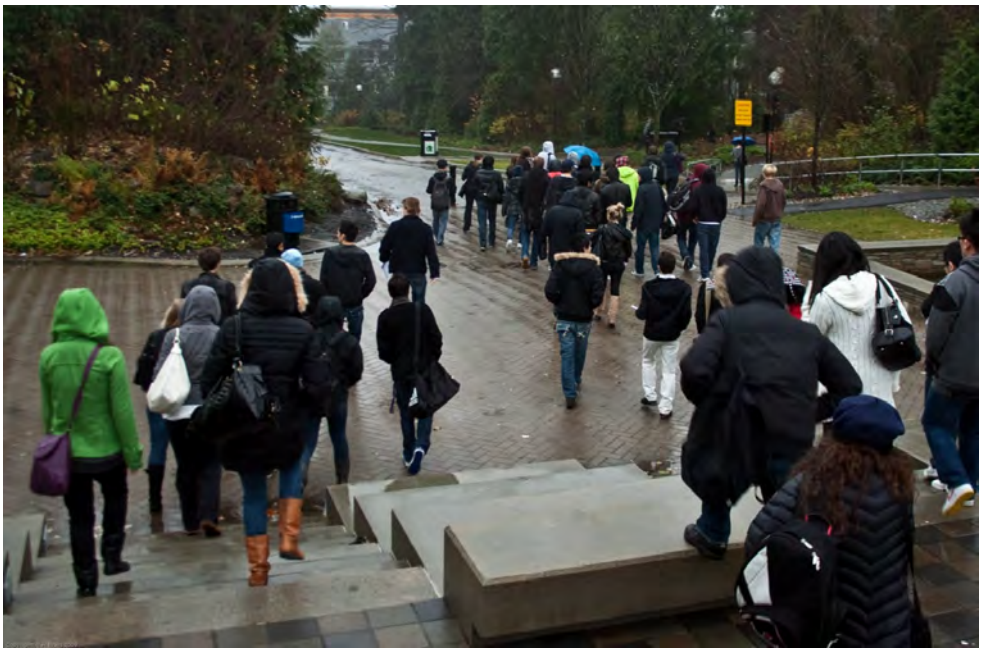
Dec. 9-11 - School Theatre Performance – 7:00 pm
Dec. 11 - Report Cards/Interview Requests Distributed
Dec. 14 - MADD Presentation – 8:30 am – Grades 8/9/10
Dec. 18 - Santa's Breakfast – 7:30 am
Dec. 18 - Last day of classes prior to Winter Break
Early Dismissal – 12:30 pm (Bowen - 11:30 am)
Dec. 19 - **WINTER BREAK STARTS**
Jan. 4 - School re-opens for 2010
Jan. 11 - Parent/Teacher Conference Schedules Handed Out
Jan. 11 - BC Business Challenge – Richmond
Jan. 14 - Grade 7 into 8 Info Evening – 6:30 pm
Jan. 19 - RSPAC Meeting – Library – 7:00 pm
Jan. 21 - Immunizations – Grade 9 Girls – 8:30 am
Jan. 21 - Parent/Teacher Conferences - 1:00 – 4:00 pm
and 6:00 – 8:00 pm – Early Dismissal - 11:45 am
Jan. 21-24 - Sun Peaks Ski Trip
Jan. 22 - PROFESSIONAL DEVELOPMENT DAY
Jan. 27 - Grades 8-10 – Internet Safety – 8:30 am
Jan. 27 - Parent Info Session - Internet Safety– 7:00 pm
Jan. 28 - Grades 9-12 Programming Evening – 7:00 pm
Feb. 2 - Jazz Café - 6:30- 8:00 pm
Feb. 3-4 - Math Mid-year Exams

MARK YOUR CALENDAR!

Calculus Field Trip to SFU: Exploring Mathematics

Correct the following equation using a single straight line: **5 + 5 + 5 = 550**

(You may not touch the "=" or "550")



The calculus field trip to SFU gave us a different outlook on math and the real world. We took the day off from high school and got a taste of university life instead. Shortly after arriving on campus, we were welcomed into a first year calculus class. Once we got over the embarrassment of having all the students turn around to stare at us little grade 12's, we managed to pick up on parts of the lesson.

After the lecture, we participated in a trivia game where it became obvious that math and physics aren't just about formulas, but about logic and forcing yourself to think about things differently (as demonstrated in the example above).

In the second half of the morning, various professors and students spoke to us about the applications of math. They also did some cool demos; for instance, the physics professor demonstrated levitation abilities when you cool a super conductor with liquid nitrogen and send it flying through the air around a track. A statistician focused our attention on real life scenarios that can be solved with math. Mathematicians, statisticians etc. collaborate with other professions to come up with things like "Smart Grid" power. BC Hydro will implement this in the years to come and it will save energy and money by showing residents how much power they are using at the exact moment and how much it is costing.

(Lecture Hall)

Math is often looked down upon but in reality, it plays a huge role in our world. Although most of us will not pursue a degree in math, we will still use math in day to day life and work. Regardless of whether we become physicists, mathematicians or statisticians, the field trip was great because it gave us a glimpse into not only higher level math, but into university itself. After a day at SFU, it sure felt weird to return to our little high school.

(Answer: 5 + 545 = 550) **Laurel Stephens**

A frigid, damp morning was complimented by steaming coffee and a bright yellow school bus crammed full of Ms. Neacs' Calculus 12 classes. The "Math Meet and Greet" at SFU was our portal to peer into the busy life of a Statistics, Physics, Engineering or Computing student. Upon arriving at SFU we saw hundreds of students in the Applied Sciences building looking far too aloof to be university level math pupils. Once inside we immediately disintegrated into the student body to attend a Calculus 1 lecture. The short and concise lecture had the regular "scholastic" feel. However, the lab followed by the lecture was anything, but "scholastic". Comprised of a series of awe-inspiring experiments (with seemingly unexplainable results), the Professors introduced the class to the endless opportunities Mathematics can provide. The class was informed of *The Wall Street Journal's* top 200 jobs, of which, the majority were math related (especially the top 10). Who would have thought North America's money makers were mathematicians?

(Seminar Room)

With the use of modern technology, the real world applications of math no longer have the "ugh math class!" feel. The majority of businesses now collaborate with Mathematicians in order to predict, plan, innovate and more importantly, succeed.

Aaron Case

Going to SFU was a really great experience for me because I had the opportunity to learn what Math is really all about. Math has always fascinated me but I've never really thought of becoming a mathematician. The Field Trip to SFU changed my opinion completely, because I was able to see how many different jobs involve math: architects, doctors, accountants, researchers, bankers, ecologist, physician, engineers and even attorneys. The list basically goes on to infinity. One of the professors at SFU told us that "almost all of the top fifty jobs in the top 250 best jobs list involve

mathematical reasoning and knowledge". That's when I realized if I someday want to have one of the top 50 jobs I need to be eminently familiar with math. The greater understanding I have of math, the easier it will be to pursue the job of my dreams because I know math will most likely be part of my future profession.

Céline Grootes

(IRMACS Centre)