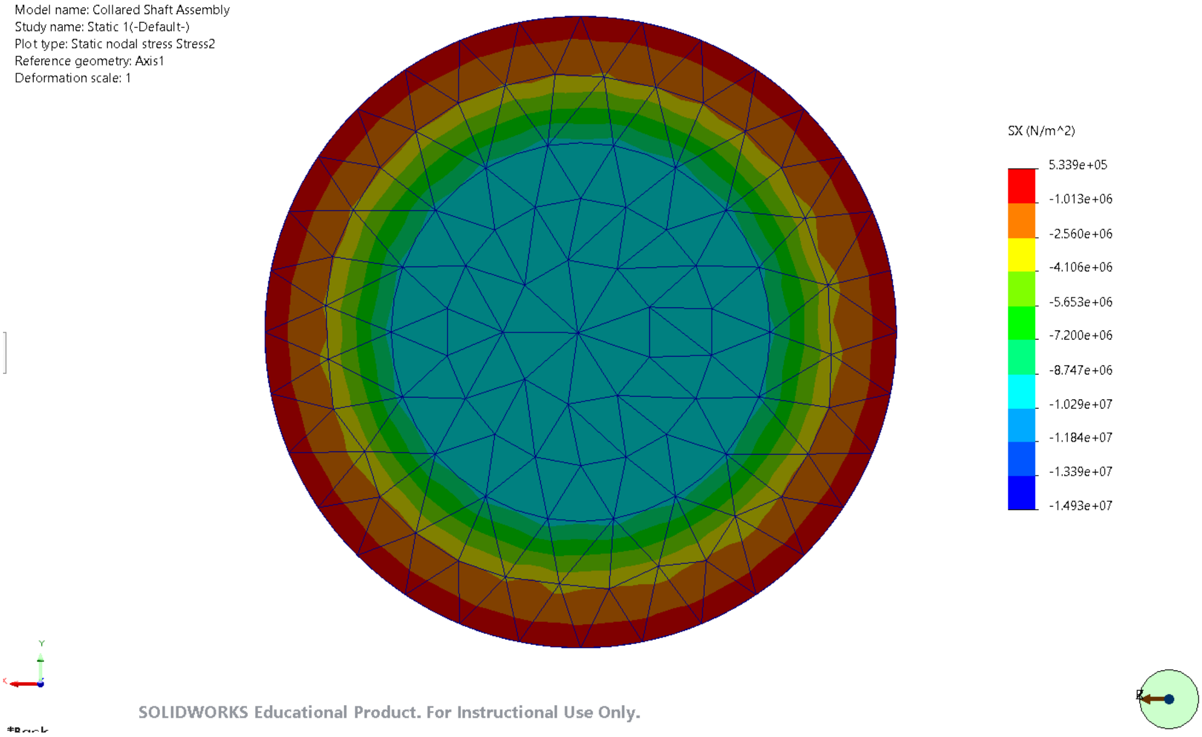
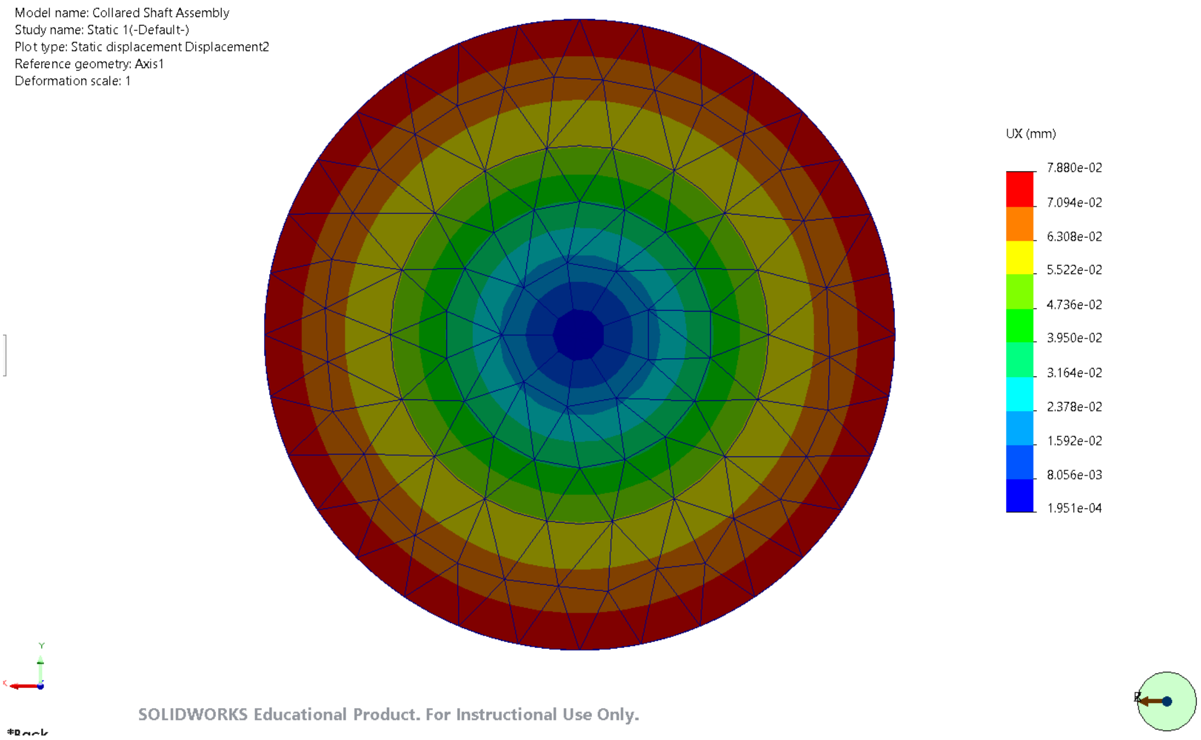
Problem: Shaft with shaft cover Thermal stresses:

Radial stress distribution contours for the assembly shown below:

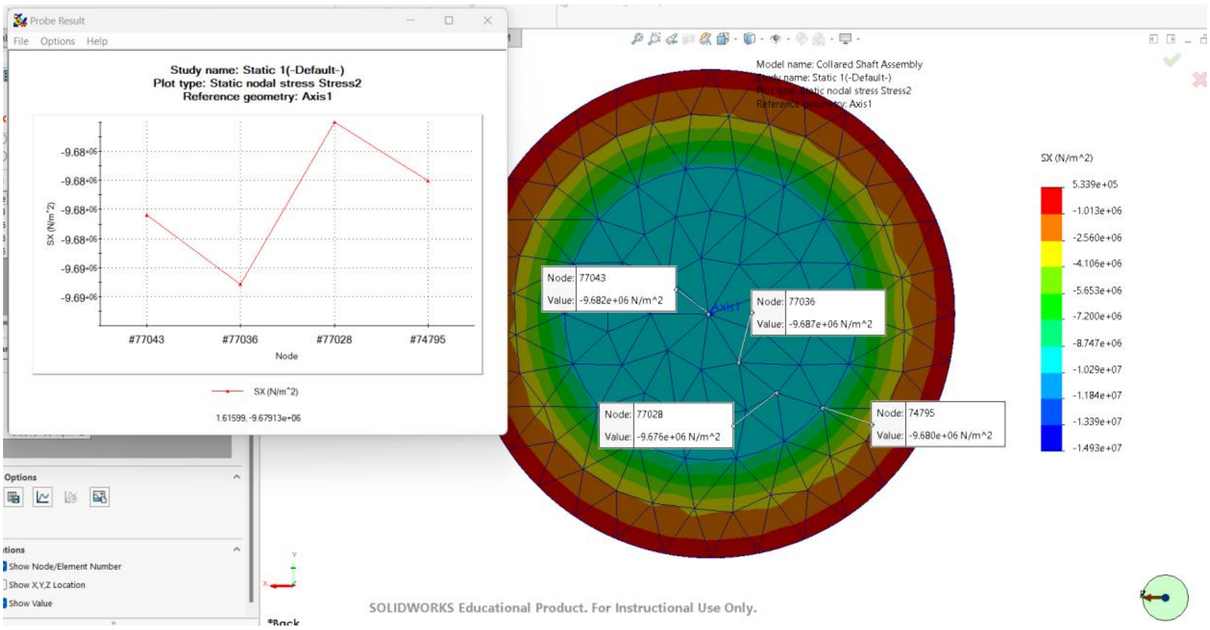


Radial displacement contours for the assembly shown below:

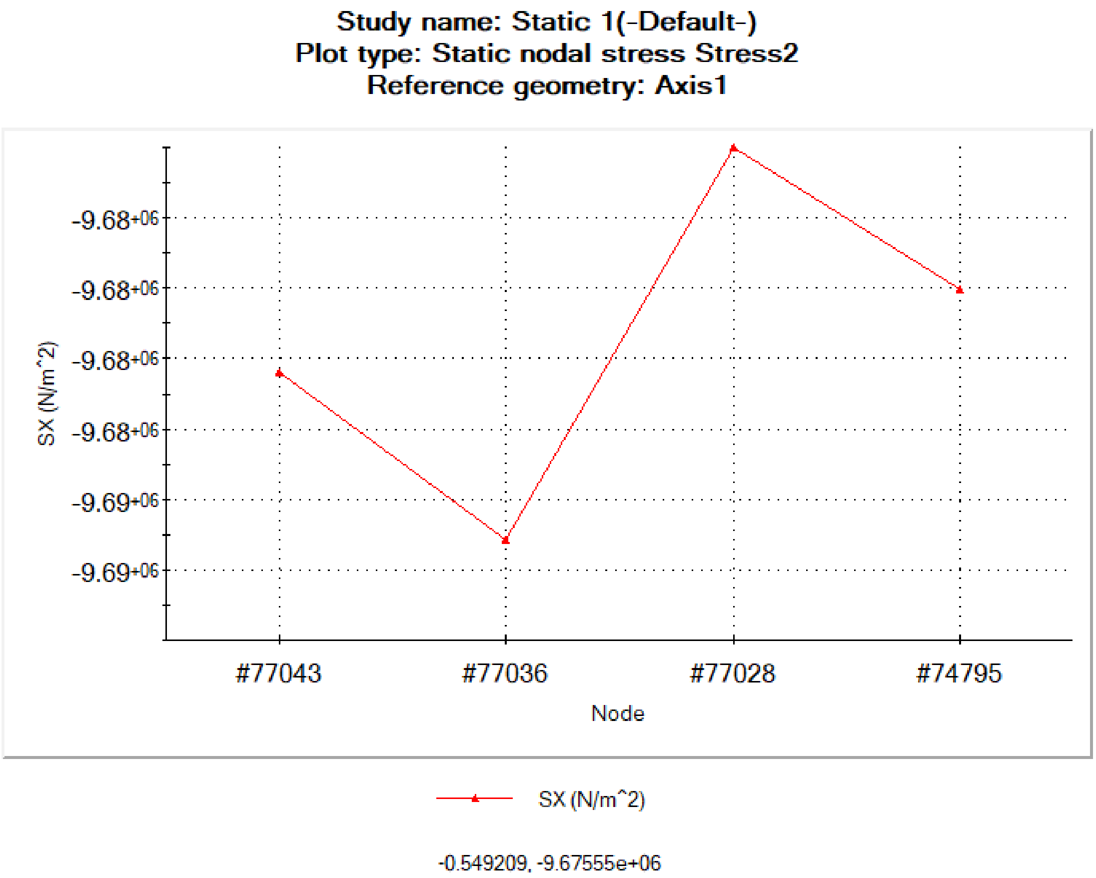


XY plots for shaft given below:

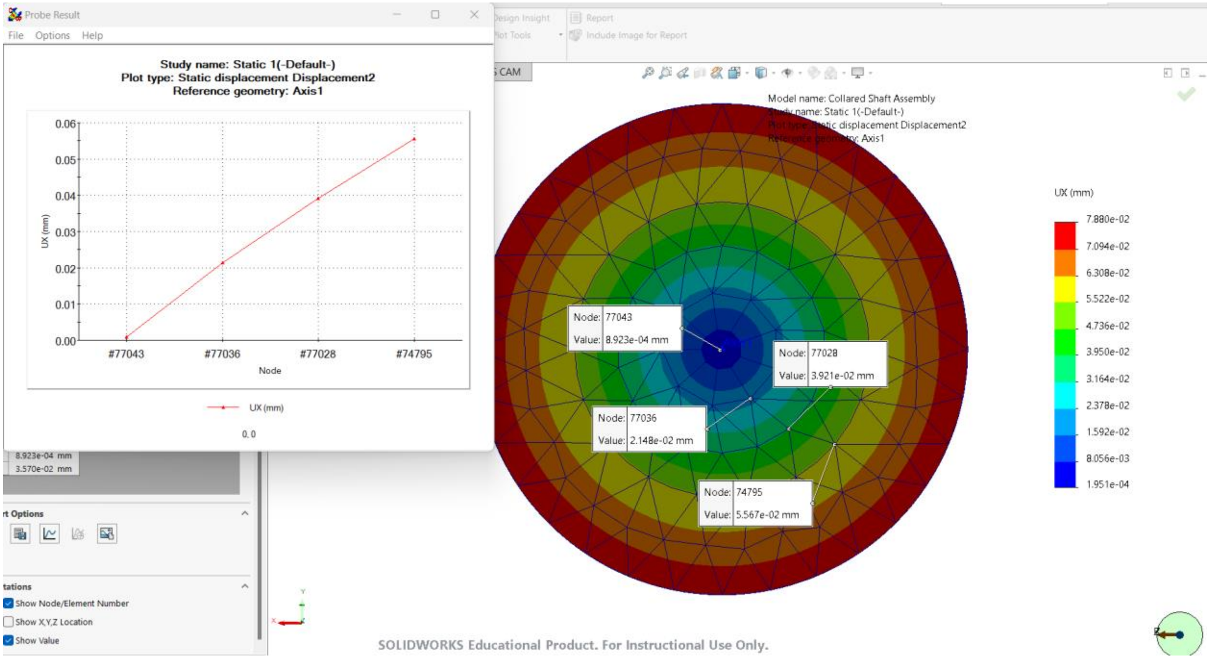
a. Radial Stress in shaft for probe locations from centre to its boundary below:



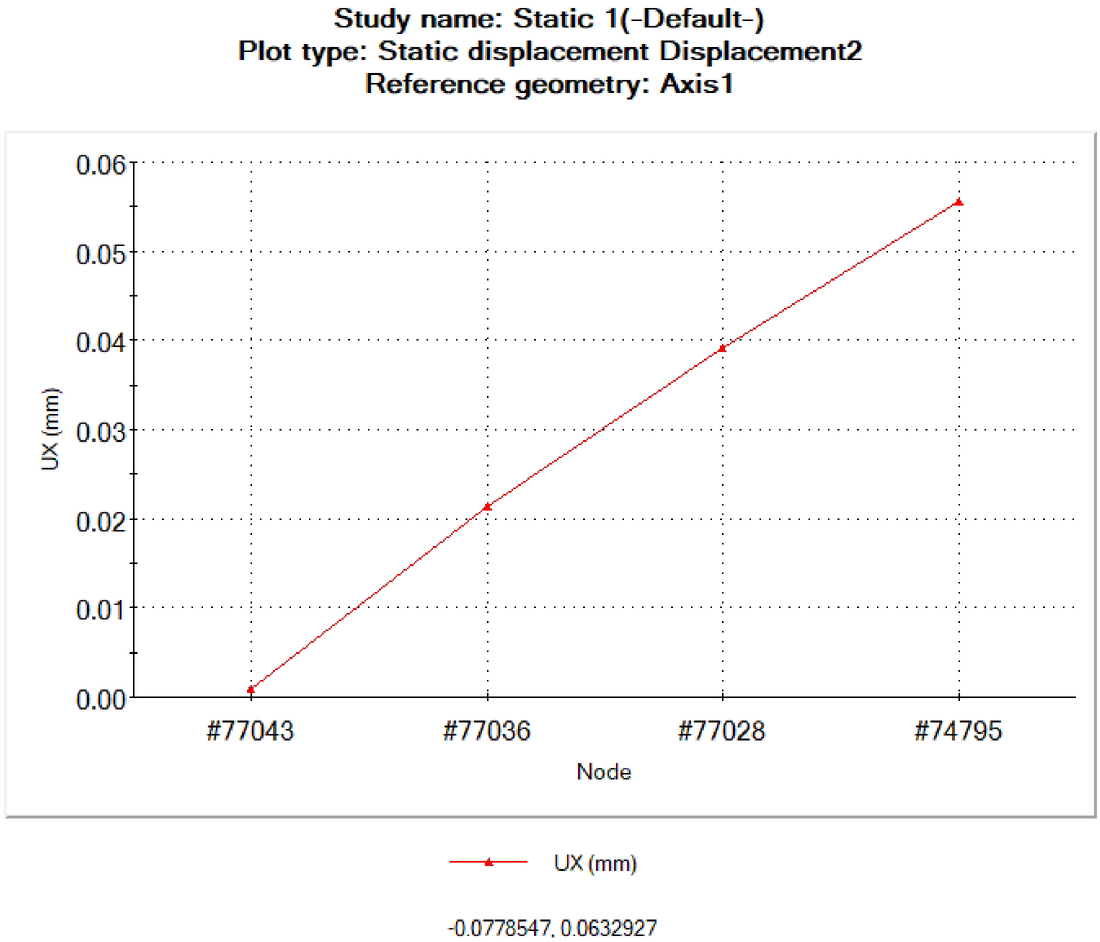
Larger view of the XY plot for radial stress in shaft from above is given below:



b. Radial displacement in shaft for probe locations from centre to its boundary below:

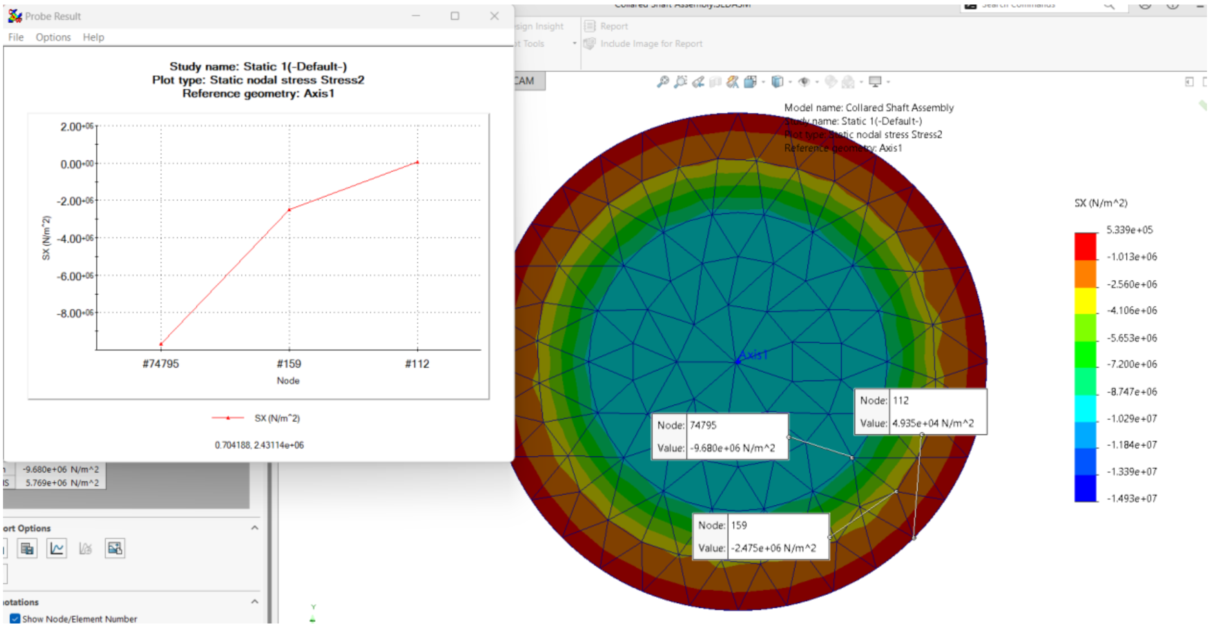


Larger view of the XY plot for radial displacement in shaft from above is given below:

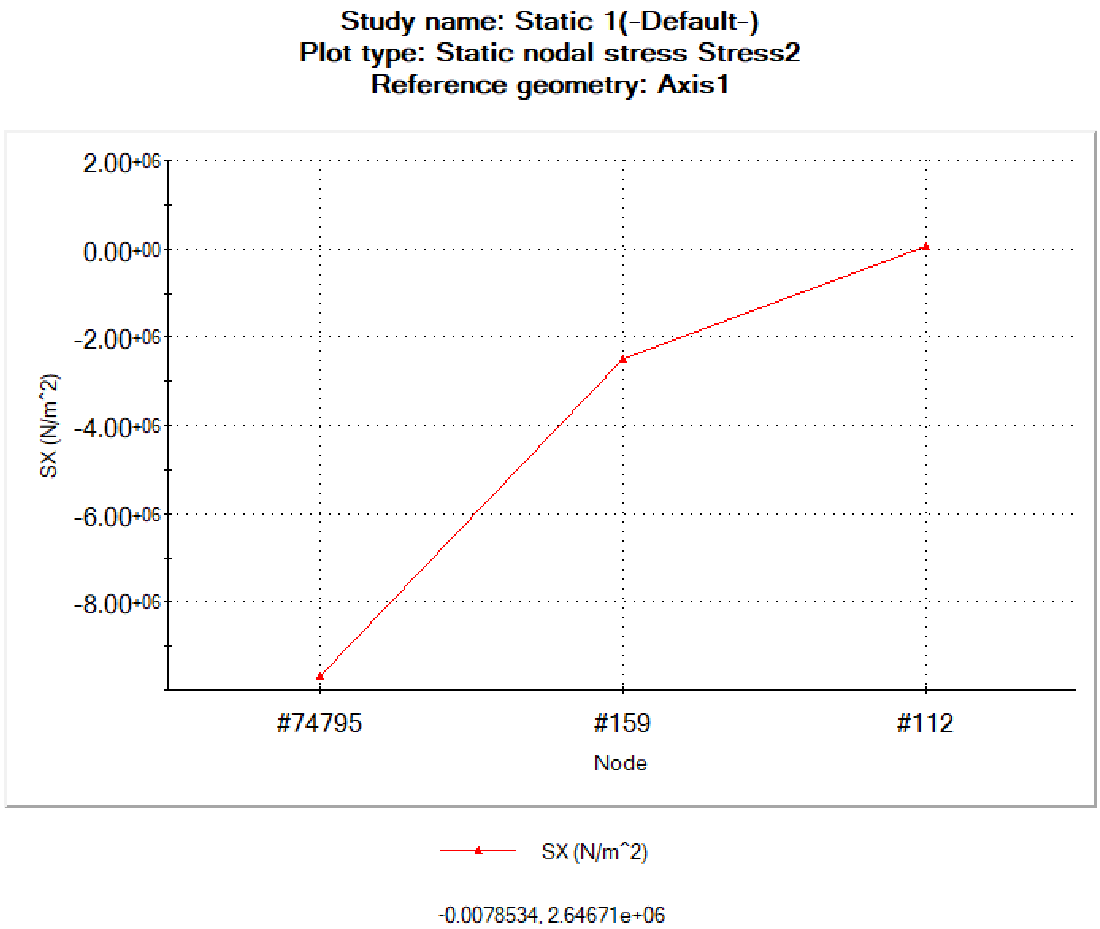


XY plots for shaft-cover given below:

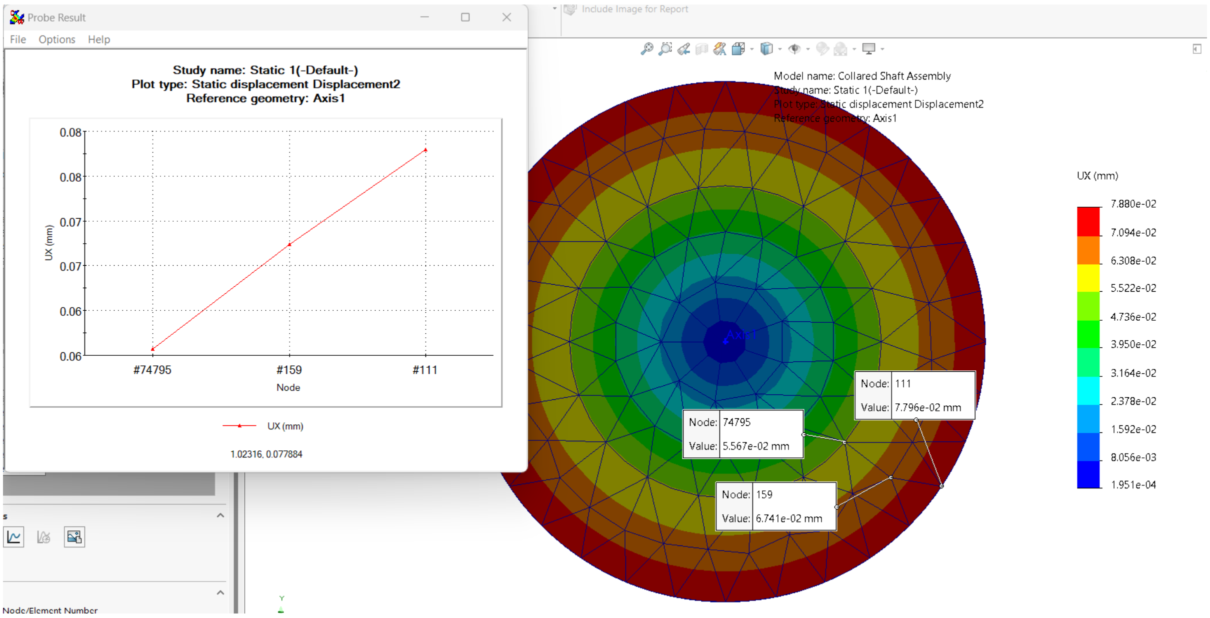
a. Radial Stress in shaft-cover for probe locations from inner to outer radius below:



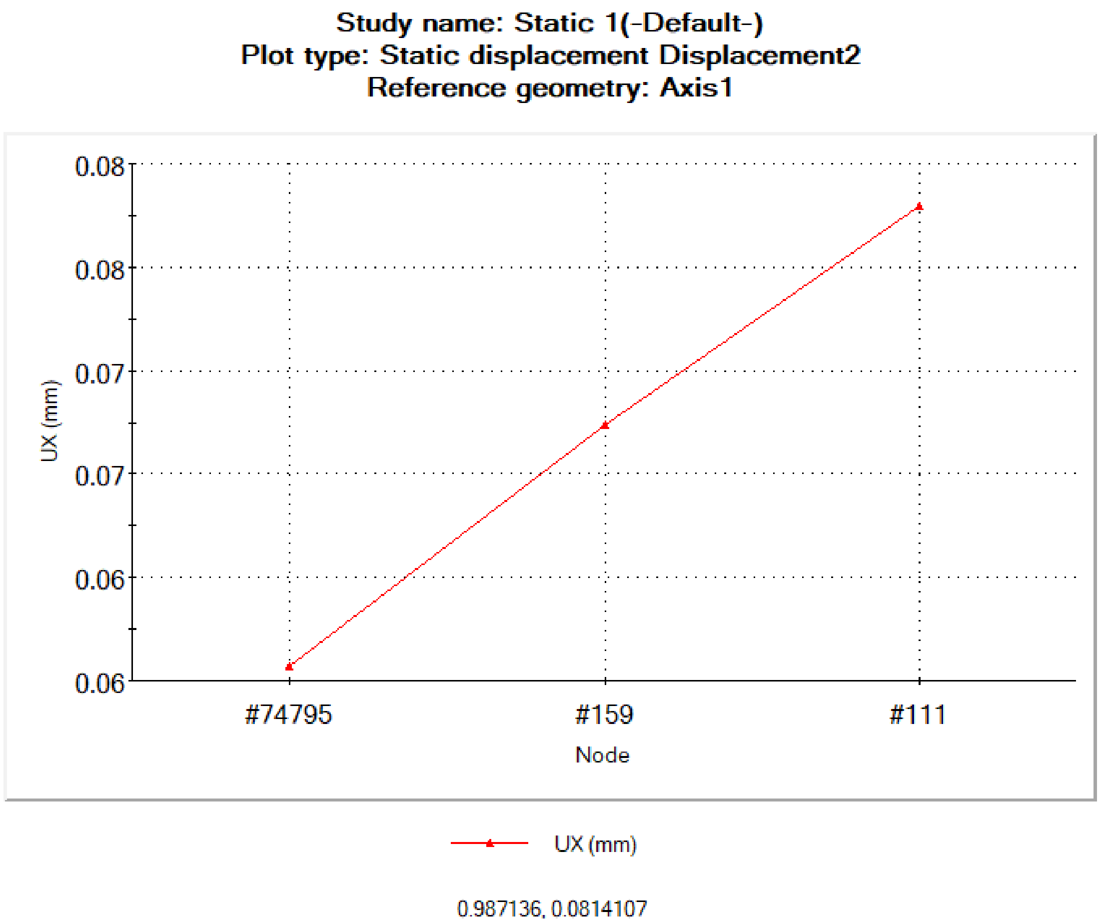
Larger view of the XY plot for radial stress in shaft-cover from above is given below:



b. Radial displacement in shaft-cover for probe locations from inner to outer radius below:

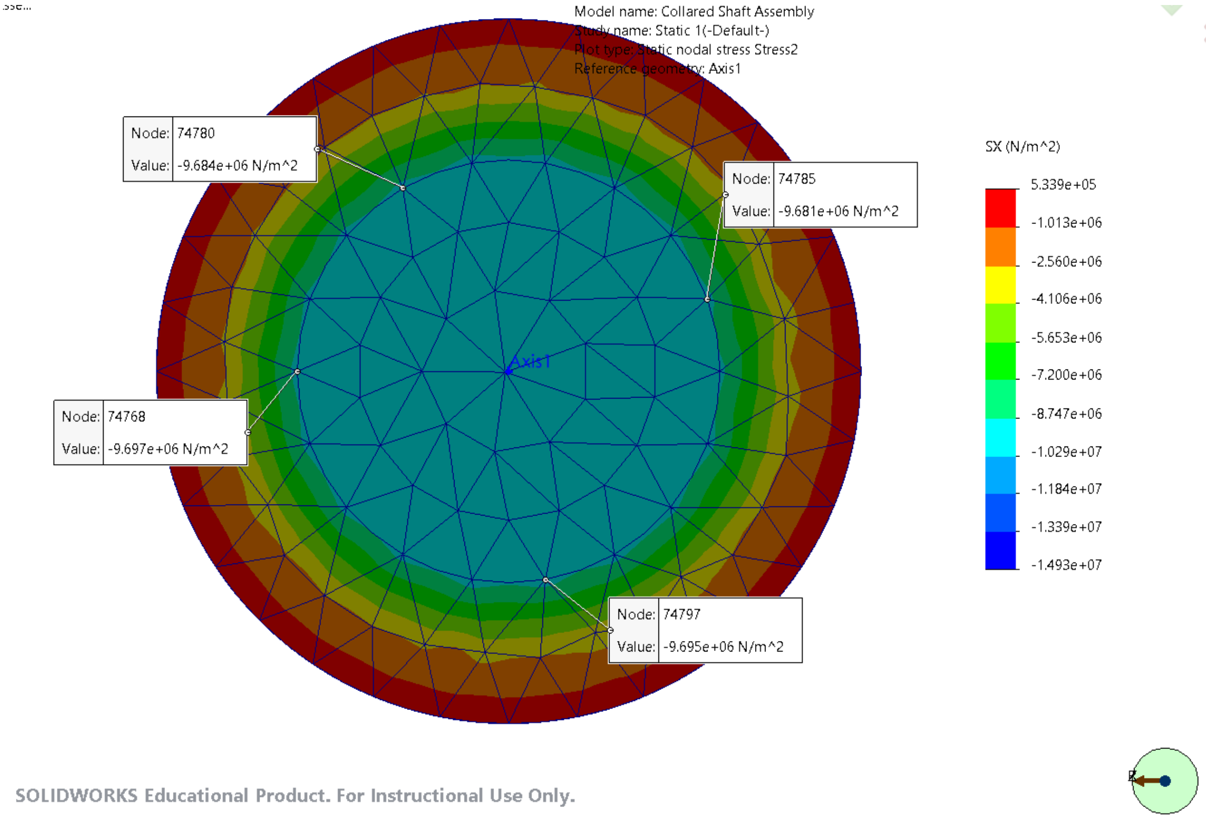


Larger view of the XY plot for radial displacement in shaft-cover from above is given below:



Validating the simulation results:

Radial thermal stress at boundary between shaft and shaft-cover from FEA shown below:



We can see, FEA simulation gives us a radial stress value of **9.68×10^6 to 9.7×10^6 N/m^2** (or Pa).

Calculations using classical equations given below:

$$P = \frac{(\alpha_s - \alpha_{sc})\Delta T}{\frac{1}{E_o}\left(\frac{r_o^2+R^2}{r_o^2-R^2} + \nu_o\right) + \frac{1}{E_i}\left(\frac{R^2+r_i^2}{R^2-r_i^2} - \nu_i\right)}$$

Shaft ⇒ 2014 Al alloy

$E_i = 7.3 \times 10^{10}$ N/m^2 [Pa] ≈ 73 GPa

$\nu_i = 0.33$

$\alpha_s = 2.3 \times 10^{-5}$ $1/K$

$r_i = 0$ mm

$R = 300$ mm $= 0.3$ m $\Delta T = 20$ K

Shaft collar ⇒ Alloy steel

$E_o = 2.1 \times 10^{11}$ N/m^2 [Pa] ≈ 210 GPa

$\nu_o = 0.28$

$\alpha_{sc} = 1.3 \times 10^{-5}$ $1/K$

$r_o = 500$ mm $= 0.5$ m

$$\therefore P = \frac{(2.3\times10^{-5} - 1.3\times10^{-5})(20)}{\frac{1}{2.1\times10^{11}}\left(\frac{0.5^2+0.3^2}{0.5^2-0.3^2} + 0.28\right) + \frac{1}{7.3\times10^{10}}\left(\frac{0.3^2+0}{0.3^2-0} - 0.33\right)}$$

$$= \frac{20\times10^{-5}}{\frac{1}{10^{10}}\left[\frac{1}{21}\left(\frac{0.34}{0.16} + 0.28\right) + \frac{1}{7.3}\left(1 - 0.33\right)\right]}$$

$$\therefore P = \frac{20\times10^{-5}\times10^{10}}{\frac{1}{21}(2.405) + \frac{1}{7.3}(0.67)}$$

$$= \frac{20\times10^{5}}{0.1145 + 0.0918}$$

$$\therefore P = 96.9462\times10^{5} \text{ Pa.}$$

$$\sigma_{thermal} = P\left(\frac{R^2+r_i^2}{R^2-r_i^2}\right)$$

$$= P\left(\frac{0.3^2+0}{0.3^2-0}\right)$$

$$\therefore \sigma_{thermal} = P = 9.69462\times10^{6} \text{ Pa}$$

From classical equations, we get thermal stress in radial direction as **9.695x10^6 N/m^2** (or Pa).

Conclusion:

Thus, we can infer that the FEA and classical calculations yield similar results for radial stress due to 20°C increase in surrounding temperature. And that the error is within 0.15% of the analytical solution.