VFDP: Visual Analysis of Flight Delay and Propagation on a Geographical Map

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***Abstract*—The propagation of flight delays is challenging to analyze because delay events depend on multiple variables. This phenomenon has become even worse with the increasing number of aircraft in China, and research into delay propagation has shown limited progress. In this paper, we design a visual analysis system for flight delay propagation. Unlike conventional flight delay research, this work focuses on the flight delay propagation trends in one region and representing the relationship of delays occurring in multiple airports. First, we construct a Bayesian network to analyze the delay parameters and select delay factors for visualization. Second, the system employs a series of visualization methods to present the propagation of flight delays, including density and flow visualizations. Third, the system combines multiple available visual representations for analyzing flight delays from different aspects. We demonstrate our methods with real data in multiple types of cases, and we evaluate our visual design through user studies. The results help identify several benefits of our system and confirm its usefulness for delay propagation analysis.**

***Index Terms*—Flight delay propagation, density visualization, delay propagation analysis.**

I. INTRODUCTION

FLIGHT delay research is a long-standing challenge in the field of civil aviation because delays have a massive impact on air traffic management and flight schedules. When air traffic controllers use route networks and airspace experts to optimize airspace, one of important standards is whether the airspace keeps a low delay level. The total number of flights in China has rapidly increased as more airports have been constructed; it is also becoming increasingly likely that flight delays will spread to other parts of the air route network. So far, the punctuality rate for flights is poor; for example, the rate for Beijing Capital Airport (ZBAA) is only 52.8%. The delays at Shanghai Pudong International Airport (ZSPD), Guangzhou Baiyun Airport (ZGGG), and the related airport clusters are all severe, and this phenomenon has attracted attention both academically and socially. According to a report from the Civil Aviation Administration of China (CAAC), the effect of human disturbance is expected to decrease with the development of technology.

Delay propagation refers to flight delay that occurs in an area, which may affect subsequent flights over time and cause more flight delays. The process is related to time and airport location. Understanding and forecasting delay propagation have been a focus of studies for many years. To achieve this goal, it is necessary to understand the propagation and connection of flight delays in different regions. For aviation experts, the delay propagation reflects the connectedness between multiple airports, which can be used to analyze the differences in different regions and then to optimize the aviation network according to the specific regional airspace conditions in the special region [1]. However, such work often contains a large amount of data and covers multiple levels of time ranges (hours, days, weeks), and many pieces of research are based on local aviation rules. For example, airport clusters tend to be larger in the US than in European areas, because Europe follows air traffic flow management rules, while US models are established based on the first-come, first-served protocol [2]. Another aviation domain analyses focus on the delay impact factor models and how to build a model for delay propagation [3], [4]. These analyses typically provide numerical conclusions or simple statistical charts, whose results are nonintuitive and challenging to compare. As a convenient presentation method, a visual analysis system for flight delays and propagation can support related domain research, which helps indicate the trends and relationships of flight delays in different regions.

The train schedule visualization systems has advanced in recent years. Such systems show the relationship of the trains by encoding the train system status with color and shape [5], [6]. Compared to train systems, flight schedules are difficult to describe for the following main reasons: 1) These systems mainly focus on the visualization of delay statistics for one station. 2) Compared with train delays, the frequency of flight delays is higher because such delays are easily caused by weather, events, and traffic control. 3) Flight delay propagation is a process with clear regional characteristics. The limited resources of airports and runways make flight delays easy to propagate, and alternate airports and air routes commonly influence the entire region.

The airport delay information we studied will mainly serve airspace users and researchers, such as mature employees worked in airlines and air traffic management department and professors in aviation university. In our experiments, all aviation experts and staffs are called as experts in this work, because they provide opinions from different perspectives. Ordinary passengers are concerned about the delay of a specific flight. In contrast, these professionals tend to observe the change process and trend of an airport delay from a macro perspective. Controllers can rearrange the sequencing of subsequent flights at the current airport according to the

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delay changes in an area, and airspace optimization experts can adjust the airspace structure and usage rules according to the regional delay changes, reducing flight delay among airports.

In this paper, we introduce VFDP (visual analysis of flight delay and propagation), a system to help users visually analyze the flight delays of a hub airport in China and understand the relationships between delay events. To encode variable data, probability theories are widely used in our system. We do not study accurate estimates of delay; the status and relationship of delay events are the focus in this work. Our main contributions are summarized as follows:

A pipeline for flight delay data transformation: Delay is a phenomenon that is influenced by continuous time, geographical position, and other impact factors. We first provide a basis for generating a pipeline for delay events to enable easier visualization of other related elements.

A hybrid approach for encoding multiple variables: To present the results from the different dimensions, we select and develop corresponding available styles based on the basic projection. By comprehensively considering the readability of visual styles and user friendliness, we develop a hybrid encoding method to help users analyze the delay propagation trend.

A visualization system for analyzing flight delay propagation: In this work, we analyze flight delay propagation based on multiple variables; the result of the visualization shows the change and evolution of delays during selected periods. This paper promotes a system with a hybrid visual analysis approach to analyze how the variables influence a single airport and multiple airports.

The rest of this paper is organized as follows: In Section II, we discuss the related background and existing work. Then, Section III describes our problem and summarizes the main requirements. Section IV presents the data processing of the system; this section explains how we choose the delay data model. The next section introduces and discusses the visual design of this work. In Section VI, we analyze the visual analysis of delay propagation though cases involving different aspects. To review the characteristics of our system, the delay system is evaluated through the user study. Then in Section VIII, we discuss some limitations of our system by these cases. Finally, we draw conclusions and future work in the last section.

II. Related Work

The visual analysis of flight delay data can be considered a research branch of flight visualization. In addition, works focusing on the causal relationships and propagation of events provide us more references [7], [8]. In order to realize our main contributions, we need to consider the following aspects. First, the flight delay data is a kind of uncertainty data, to transform flight delay data into a visual form, we need to refer to the uncertain data visualization. Secondly, due to the variables and elements involved in the delay data, these factors affect each other and ultimately cause the delay propagation, so visual analysis of the multiple information relationship is worth references. Finally, to realize a visualization system that analyzes flight delay propagation, we need to draw on other existing systems related to the transportation field. In related work we have summarized the following headings: 1) visualization of uncertainty, 2) visualization of information propagation, and 3) visualization of movement data.

A. *Visualization of uncertainty*

A flight delay is an event system composed of stochastic processes and probability events; this type of data needs to be designed in a special form.

Uncertain data visualization has been focused many years [9]. Networks can indicate the fuzzy relationship between members [10], [11], while air route network is too complex to show the relationship. These systems usually contain multiple visual components, in contrast, we want to express as much information as possible with as few views as possible.

In addition to network graphs, there are other visual forms. Görtler et al. [12] improved bubble treemaps based on hierarchy visualization; the application showed uncertain factors affecting many systems, but this visual form does not involve geographic maps. M. Wunderlich et al. [5] proposed and evaluated a system that was designed for train trip planning. As we introduced in Section I, the train delays visualization does not meet the needs of flight delay visualization.

B. *Visualization of information propagation*

Information propagation visualization provides a basic pipeline for visually analyzing the relationship of information sources.

A common scenario is the online network platform, which presents the relationship of multiple information sources [13]. Information theories can abstract topic groups as ensembles, for further modeling in these work [14], [15]. One of the main challenges we faced in our work was to translate delay-related information into visual elements. Animation can show the relationship between different data sets [16], and flight delay is a typical dynamic data, we are also inspired by some animation methods.

Propagation visualization has often been combined with time-varying features. Existing work compares the graph structure differences to show movement [17], and in our work we also try to present the differences in data changes. Inspired by conventional flow field visualization, the direction of flow presents information transfer, enabling users to understand the nature of diffusion [18], [19]. Compared to these works, we took advantage of flow field visualization in expressing dynamics, and designed more visual elements to provide more delay information.

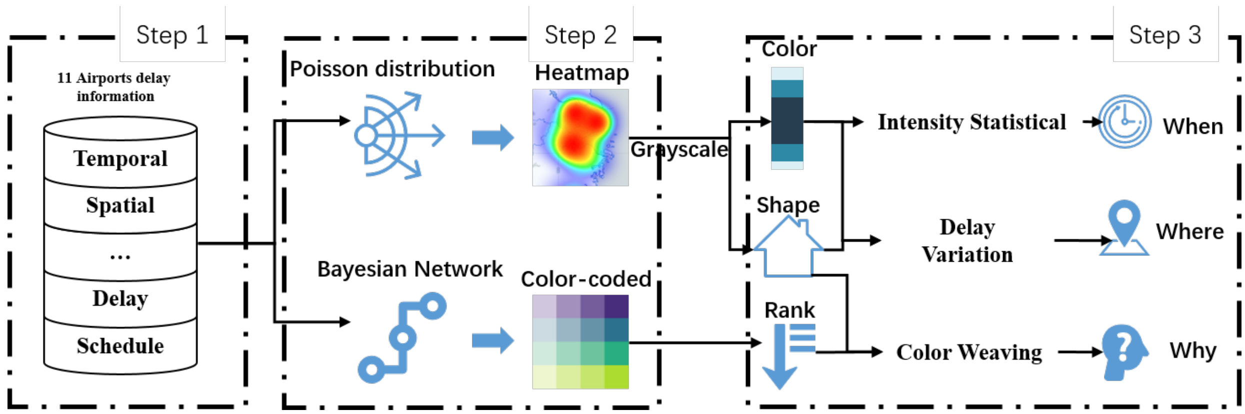

Fig. 1. System overview for delay propagation. **Step 1.** The data are classified according to properties such as temporal and spatial information and the delay time. **Step 2.** The data are processed from two aspects. We generate a delay map based on a Poisson distribution, representing the initialized delay views. The delay impact factors are processed by a Bayesian network (BN), and the results are encoded with VSUP. **Step 3.** The final views are extracted, and the main presentation consisting of when, where, and why.

C. *Visualization of movement data*

The visual analysis of movement is important for our system. An inescapable challenge is that we want to present the dynamic propagation in a geographical map.

With a certain research focus on urban mobility data [20], [21], flow and arrow glyphs are widely used in these systems. Regular movement data, such as human daily movement data, can be understood and utilized by interactive visualization systems [22]. In the transportation field, kernel density estimation (KDE) is widely used since the movement data are generally unpredictable [18], [23]. We were inspired by these visualizations and fuse them in several ways.

In aviation research, domain experts have focused on how to choose the factors that affect delays [2], [24], [25]; we rely in part on the rules obtained under these circumstances. Bayesian reasoning can also provide an understanding of uncertain factors; by controlling the amount of information, text and visualization, designs can increase overall accuracies [26], [27]. These works have provided valuable references for considering factors that affect the process of propagation. However, most of them lack visual expression and were designed for experts performing research in a specific domain area.

III. Problem Analysis

Region flight delay propagation visualization is a cross-domain, and our problem analysis involved the requirements of aviation and visualization. The system requirements come from civil aviation, while the solutions are provided from visualization. We first survey the delay problems from the civil aviation aspect and arrive at a preliminary conclusion about the system requirements. Then we analyze why aviation methods can not solve the problems and further determine the limitations for using the existing civil aviation online visualization systems, and to propose our solutions from the visualization aspect.

The first step is to survey our target users, airspace users (controllers and dispatchers) and researchers. As we introduced in Section I, these professionals analyze the regional airport delay from different perspectives. Users want to know the delay change every day for they need to judge how to arrange flight, while researchers focus more on the delay rules to get flight delay features in airspace for optimizing airspace. We first discussed the current regional flight delay propagation analysis requirements with users online, and then we exchanged face-to-face views with several experts. They provide us with several delay monthly reports as samples, these documents are the most common reference material for delay analysis in the civil aviation industry, we had provided a part of the sample in appendix II.

Combining with the materials provided by civil aviation, the experts have put forward some special domain requirements for the regional flight delay propagation. We have discussed the core **requirements** with the experts and summarized as follows:

R1: A clear judgment of delay severity in one airport such that users can visually feel the scale of the flight delay incident.

R2: Users can determine the geographical regions related to the current flight delay of the selected airport, which is one of the important value of the visualization system.

R3: It is easy to compare the delay change; particularly, the flight delay tends to increase or reduce in a region.

R4: They hope to compare delay impact factor in the system, and reflect the uncertainty of the impact factor.

A. *The domain problem of civil aviation*

In the second step, we tried to understand the difficulties of analyzing regional flight delay propagation with these common methods, and think about how to design a novel visualization system to solve these problems. We asked experts questions such as: How can flight delays propagate in spatial? How can flight delays propagate over time? What is the uncertainty of flight delay impact factor? By discussing these **difficulties**, we summarized the feedback from the experts:

1) The delay events occurring in one airport influence

others. There are regional relationships between origin and destination airports, which are difficult to describe by a digital table.
2) Flight delay propagation is a dynamic process occurring over time, and the notable features of how delays change are different across different time scales. The data report cannot support this dynamic comparison well.
3) In the civil aviation domain, the delay and the reasons are uncertain and varied, and it is challenging to describe the change in the delay from a spatiotemporal perspective and to construct visualizations of the delay information.

B. Investigating visualization systems

We conducted another investigation to complement the first part involving the survey. FlightStats [28] and FlightRadar24 [29] are two widely used flight information systems. For regional flight delay propagation visualization is a professional work, we have not found similar comparison systems. However, we hope to summarize how the existed visualization systems solve civil aviation problems, and compare our specific requirements then think about how to design visual elements.

Overall, the spatial distribution can accurately be shown in these systems based on the map, and users can check the real-time dynamic flight delay status of one airport in online systems. However, users cannot obtain the delay changes in multiple airports at the same time, and cannot determine how the delay influences other airports. It is challenging to analyze the relationship of delay events occurring in multiple airports or the propagation of delay. In addition, the cause of a flight delay is not described on these websites, which are designed for ordinary users. Therefore, we have to draw on the uncertainty visualization work and combine the necessary visual elements to design the visualization module of the flight delay impact factor.

C. Solutions

Based on the difficulties in the field of civil aviation, and draw on the successful concepts and modules in the visualization system, we have proposed the following **solutions** to meet the expert's requirements for regional flight delay visualization.

S1: Implementation of a visual form that can express the delayed data in space according to the time-series relationship, which can transform temporal information into a form that is straightforward to observe and compare.

S2: Visualize delay change at different times, showing the relationship between regions to determine and compare the propagation of delay.

S3: Definition of a form to present the delay impact factors to visualize uncertain weight of impact factors over time in different regions.

We designed the system to address the aforementioned requirements, combining multiple image methods and probability theory. Figure 1 presents an overview of our system and the data processing pipeline used.

IV. Data Description and Processing

Flight delay data are open to the public on many websites, in this work, delay data were obtained from FlightAware [28]. Such data we collected do not contain the activities of private aircraft, which account for a small percentage of Chinese flights. The samples that we chose were representative of the delays in Chinese major airports, the data covered Beijing, Shanghai, and Guangzhou airports, among others. Specifically, the total number of flights represented the top 11 airports distributed around China, totaling more than 3000 flights per day. It covers a half-year time span from September 2017 to March 2018. To reflect the research aim, we confined ourselves to domestic flights to avoid the influence of operations in different countries.

The main information in the data contained: flight number, departure airport, destination, estimated departure time, actual departure time, estimated arrival time, actual arrival time, delay time.

A. Model selection

The emphasis in this research is delay propagation, and delay impact factor selection is the first step in determining delay propagation. The number and type of final selected factors influence the visual design. For the current official reports, the delay impact factors are described by airlines and airports, which contain more domain rules and events. The delay factors are independent of each other according to the current reports. In fact, these factors jointly influence the current delay; each of these elements is fueled by the others, and each promotes the others. For example, a delay 30 minutes before would be affected by a delay that occurred 1 hour before. In the civil aviation field, the Bayesian network is an effective and common method for modeling delay propagation [27], [30], and our system analyzes delay impact factors with this method (T4, R3).

B. Bayesian network theory

BNs are commonly used to represent uncertain factors to solve decision problems [31]. The structural part of a Bayesian graphical model is a directed acyclic graph (DAG) (in which the nodes may be either discrete or continuous) that maps the elements that influence each other. We also construct a DAG in Section 4.3 to show how other factors impact delay.

The joint probability distribution $P(X_1, X_2, X_3 ... X_n)$ is represented as follows [32]:

$$P(X_1, X_2, X_3 ... X_n) = \prod_{i=1}^{n} P\left(X_i \mid X_{pa(i)}\right) \tag{1}$$

where $X_i (i = 1, 2...n)$ represents a series of influence factors. For each random variable X_i, $pa(i)$ means the parent nodes of i node, and $X_{pa(i)}$ is the probability depends only on the parent nodes.

Through the conditional probability distribution given above, we performed calculations for every variable of the studied data. The training data are $\mathfrak{D} = (D_1, D_2, ..., D_m)$, with $D_x = (X_1, X_2, ..., X_n)$, and the probability distribution is presented as:

$$P\left(D_x \mid \mathfrak{D}\right) = \prod_{i}^{n} P\left(X_i \mid \Pi\left(X_i\right), \mathfrak{D}\right) \tag{2}$$

In addition, the calculation results of a BN often constitute a probability distribution of factors, which is challenging to describe by visualization. To present the possibilities, the information entropy is the expected concept [33]. The average uncertainty of the source is described as follows:

$$H\left(X\right) = \sum_{i=1}^{n} P\left(x_i\right) I\left(x_i\right) = -\sum_{i=1}^{n} P\left(x_i\right) \log P\left(x_i\right) \tag{3}$$

where H is the entropy, $P\left(x_i\right)$ is calculated by the above equations, and $I\left(x_i\right)$ is the information content of $\{x_1, x_2, ...x_n\}$.

In our experiment, X represents the delay probability of one airport, and $H(X)$ tells us the uncertainty of the delay occurs with a probability of X. The higher value of $H(X)$, the higher uncertainty of the event if delay probability will be X. The daily flight delay data within a few months constitutes D_x, then we can obtain $P(x_i)$ by calculating the daily flight delay probability distribution. Through the calculation of Bayesian network, we have determined the uncertainty of the main impact factors of the airport delay, which is the basic work of the uncertainty visualization of the airport delay in the following text.

C. Graphical Model

For modeling based on a BN, constructing a graphical model is the key step. The aviation experts told us the flight delay situation in China, and more than 50% of the delays are attributable to factors related to flight sequence and weather. We take these two categories into account in our model because the detailed reasons influencing flight delay are not our focus. However, according to equation 3, the computational result is unexpected. The entropy (H) of weather can almost be ignored compared with the relationship between flights (details are shown in the appendix II). Civil aviation experts explained the result that the aircraft can adapt to more severe weather with improvements in technology, and the weather change is no longer the main cause of delay. Since we care more about uncertain changes in flight delays, the relationships of flights are selected as the main factors for constructing the graphical model.

Structure of the BN: In Figure 2, the figure represents the factors that affect the No.9 node, which represents the current delay at the airport. In order to clearly illustrate the structure of the graph, we list the meaning of each node described in Table I.

TABLE I
THE SYMBOL DEFINITIONS OF BN MODEL

Node	The event described by this node
No.1	The visibility of the departure airport
No.2	The visibility of the arrival airport
No.3	Probability of current airport departure delay
No.4	Probability of the previous flight delay
No.5	Visibility of the current airport at the time of the delay
No.6	The probability of a delay occurring 60 minutes ago
No.7	The probability of a delay occurring 30 minutes ago
No.8	The probability of a delay occurring 15 minutes ago
No.9	There is a delay at the current airport

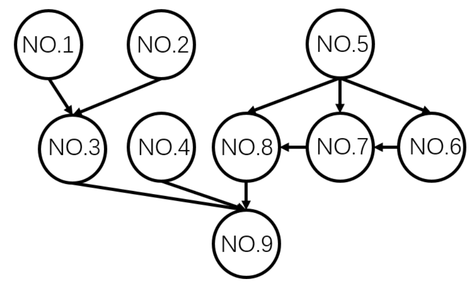

Fig. 2. Structure of the BN Model for All Segments.

V. VISUAL DESIGN

A. Representation of Delay Dynamics via Poisson Distribution

How to present the delay dynamics is one of the important tasks in our system (**S1**). We studied the knowledge of civil aviation and found that the Poisson distribution is an available method for describing flight delays, particularly for modeling the delays related to a single airport [34], [35]. In our research, the representation of the Poisson distribution is the basic design for S1 and S2. The Poisson distribution suits our particular scenario of delay propagation because the influence of delay factors is continuous in a relatively stable condition.

As the features of the Poisson distribution, we should confirm that the presented events in one distribution occurred in a similar scenario. We classified the delay events as **4 delay levels**: 15-30 minutes, 30-60 minutes, 1-2 hours, and more than 2 hours. The division of the delay helps us construct a representation of delay dynamics. We set a Poisson kernel as the center of the distribution, and the Poisson kernel is flexible and can move to convey orientation information. The final shape is generated according to the direction of Poisson kernel movement; the generated method is inspired by reasoning visualization [36], and the steps are shown in Figure 3.

Step 1. The distribution around a kernel is generated by fast Poisson disk sampling [37], and every data point indicates a delay event. To ensure a consistent distribution shape, we generate a Poisson disk as a template in advance to ensure that the data points in this work all follow the same Poisson distribution.

Step 2. The initial Poisson kernel is located at the destination/origin. When the delay level changes, the kernels move toward the delay sources, and the newly generated points are distributed around the updated kernel.

Step 3. Step 2 is performed for all related airports; each iteration either produces a new point based on the old kernel or generates a new point around the new kernel. The shift distance of the kernel is the same as the interval of the Poisson distribution, and it is necessary to check the positions of the new points to avoid overlapping.

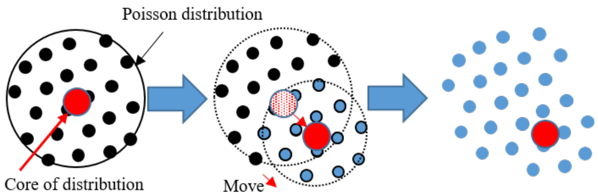

Fig. 3. Transition procedure of the Poisson kernel distribution.

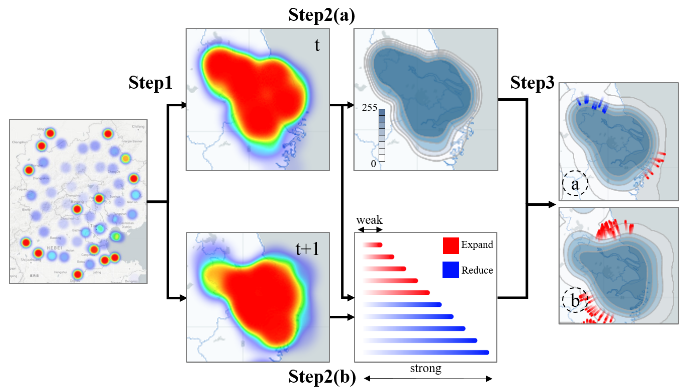

Fig. 4. Delay field generation steps. Variations between two delay maps are generated by using StreamMap [38].

B. Delay Field Generation

As described in the above subsection, we generate a Poisson distribution according to the flight delay series. The data distribution can be encoded by the kernel density estimate (KDE), as color variation is widely used to show comparisons in delay visualizations [5], [23]. However, in our work, there are two major drawbacks of the generated heat map:

1. It is challenging to confirm the spatial direction of a heat map, particularly observing slight changes in distribution. In our work, we apply streamlets to conveniently present the changing trend, and the user study confirms that the arrows can indicate the direction of delay sources.

2. As introduced above, the Poisson kernel would move if the delay level changes. We want to separate and show flight delay as **4 delay levels** in this work; however, the original heatmap is a continuous visual style for presenting data. Contour plots can transform a continuous visual into discrete types to meet this requirement, and the interval between contour lines can also mean a larger move increment.

We transform the colormap into a contour map, the contour plot reflects the distribution of the values of the data response. On the same contour, the values are always similar as shown in Figure 4. We also design streamlets to represent the flight delay variation (**S2**), the blue streamlet indicates the contraction, while the red streamlet represents the expansion. There are three main steps of the delay field generation.

Step 1. Generating the heat maps based on the dynamic transition Poisson kernel in a continuous time series.

Step 2(a). Rebuilding the contour plane based on the **4 delay levels** to show the distribution variation at time t. In our experiment, when the colormap is converted to a contour map, the data density is used as the response value.

Step 2(b). At the same time, compute the varying fields between two adjacent heat frames with StreamMap [38] and then show the fields by dot plots.

Step 3. Indicating the trend of the delay change and adjusting the vector fields to the contour plane. The density of field lines changes based on the length of the boundary, ensuring that the lines can clearly show directions.

The final design described in Step 3 features two typical results. Figure 4(a) shows that the edges of the contour are closely spaced and smooth, which means that the density distribution in this area is almost equal, therefore delays have always occurred regularly. The vector fields indicate that the contour map would move east. According to the Poisson distribution rule in the previous section, the data density distribution only changes while the center moves, that is, the delay level change. If the contours are sparse, it means that the delay level changes frequently. Relative to Figure 4(a), Figure 4(b) shows users an airport with frequent delay levels change; the blue arrows illustrate the delay events decreasing in the future.

C. Delay Variation Reconstruction

As we introduced in Section 5.2, the interval of the contour map indicates the difference in data density, which is related to the frequency of delay events. To summarize information on the delay change in one day, we set the color parameters of the contour map such that the levels of gray could be graded from 0 to 255. Inspired by the concept of a grayscale photograph, the color gamut is divided into 8 levels. Then, we construct a block filled with the color, and the fill ratio is determined by the grayscale value. The time line is an impressive and feasible visual design. The example in Figure 5(a) shows the delay change of an airport in one day. Heavy delays across the country are uncommon; therefore, the dark blue occupies the block in every time window, while the light part is clearly larger than the dark part.

For simply describing the design of impact factor presentation (**S3**), we abbreviated delay factors as **factor 1, factor 2, factor 3**. **Factor 1** indicates that there is a flight delay occurring 15 minutes before. **Factor 2** means that there is a flight delay 30 minutes before. **Factor 3** means there is a flight delay occurring 60 minutes before. The importance of different flight delay impact factors is investigated with a Bayesian network in Section IV, and the result can be described as ”the current delay is most likely caused when the occurrence frequency of factors reached percentage.”. We present visualization methods to show the impact factors in Figure 5(b).

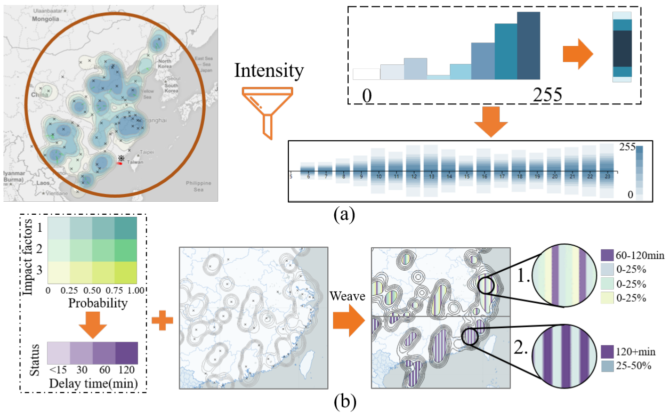

Fig. 5. For providing more information about delay, we present two visualization forms though intensity filling and factor encoding. (a) We extract colors according to the intensity of the delay map and construct the intensity statistic to show the delay change for one day. (b) The cause of delay impact factors and current time are encoded with VSUP. (The notes of impact factors are described in Section 5.3.)

In this work, we need to design a visualization method to show the multiple factors separately in one region. In our research, the multivariate data we need to express is also uncertain data. The reasonable combination of shape and color has always been the focus of uncertainty visualization. Weaving (rather than blending) is a conventional method to convey multivariate data with color [39], it provides us with a choice of shape. The color choice is an important task of visual perception, we refer to VSUP as a mature non-deterministic visual color scheme. In VSUP, a palette design is used to present uncertain data; this approach can encode the data value and uncertainty independently [40], [41]. The color scheme from VSUP has been proven to be readable and effective in many other works [42], [43]. We designed the chromatogram in Figure 5(b) in conjunction with the official non-deterministic visualization schemes related to flights. Considering symbol overlap in the view, we retain the boundary of the contour map and then weave color from palettes to fill the peak area of the contour map. As shown in Figure 5(b), the results show the influence of multiple factors.

The delay factors are encoded using VSUP palettes with the following rules in this paper. The purple bandwidth maps to the delay time length in a region, while the other three bandwidth maps present the factors influencing the current flight. To avoid confusion across visual elements, we select only the maximum probability value of the factors in one row and then weave them together with the delay time. For example, region 1 in Figure 5(b) shows a region in which most flight delays are less than 2 hours, and not more than 25% of the cause is **factor 1**. In contrast, region 2 indicates a region in which the delay time is heavier. We also designed interaction to choose the number of impact factors that users are interested in, and only one impact factor is chosen in region 2. This selection can be convenient if users are experienced experts who do not need too many indications, thereby reducing visual complexity.

VI. Case Studies

Usage scenario To envision a typical scenario, let us understand how VFDP helps users. Consider an aviation expert in the air traffic management department whose duties are to study delay statistics. At the end of each month, the expert produces a report about the delays in China, aiming to explore the external information about the delays and analyze the visualization results of the system.

We demonstrate three typical examples that are identified and analyzed with the VFDP, which show how our system solve the requirements described in Section III. At the beginning of each section, we describe the ability of our system that we hope to demonstrate through experiments.

A. *Case 1: Detecting Abnormal Delay*

The space expression ability is the first requirement of our system and the basis of the regional delay propagation visualization. In the following experiment, we can judge the delay relationship in space through the system display.

We initially randomly observed the daily delay information through the system. The delay variation field caught our attention, corresponding to a change in the next day. We quickly found two atypical patterns that produce a striking visualization contrast, as shown in Figure 6(a,b), both corresponding to the delay information of Shanghai Hongqiao Airport.

There are clearly more red arrows than blue arrows in Figure 6(a). On this day, delay surge propagated across almost the entire nation. In contrast, blue arrows are distributed everywhere the next day. According to the display of system, we determining that Figure 6(a) and (b) describes the delay situation on Jan. 24th, 2018, and Jan. 25th, 2018, respectively. In order to understand what happened, we checked the news events at the airport in those days. A heavy snow storm impacted East China in late January, and Shanghai Hongqiao Airport was closed due to the bad weather on Jan. 25th. The results show that the delay in Southern China was more influenced by snow, and we conclude that a large number of flights diverted to alternate airports that were generally close to the intended destination.

To further obtain external information, we compared the delay distributions near Beijing and Guangzhou. Figure 6(c) presents the delay change in the Pearl River Delta (PRD), and Figure 6(d) shows the delay around the Beijing region. The difference between the two views is that the blue arrows are distributed north of the PRD but south of Beijing. The opposite arrows point mostly to Shanghai, and we predict that the delay time associated with Hongqiao Airport was greatly reduced the next day.

The main delay factors of the two regions are all dark colors, which means that if the delay before appeared more than 75%, a delay would currently occur. The results perfectly explain the snow influence; flights are all delayed in their departure airports at all times. The rate of delay before is always 100%, and the delay is always the reason for the next one.

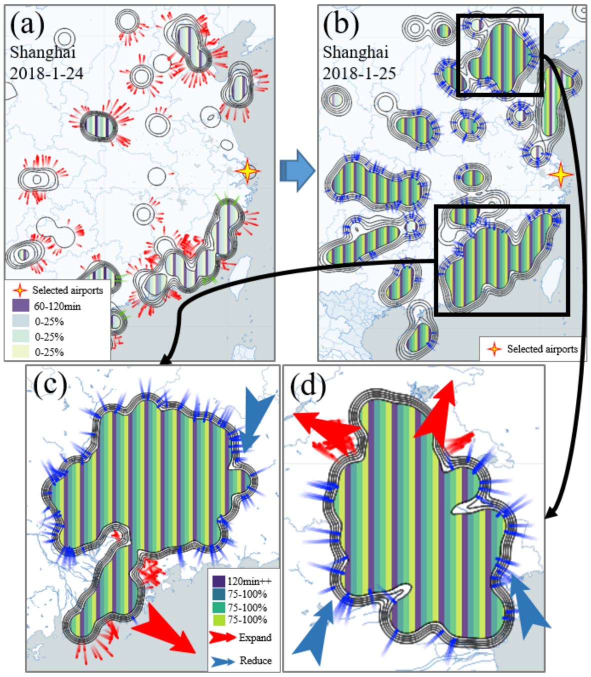

Fig. 6. Abnormal delay evolution when a heavy snowstorm hit the East China region. The blue arrow in the figure indicates the trend of decreasing delay, while the red arrow indicates the trend of increasing delay. The delays in both regions are changing in the opposite direction of Shanghai.

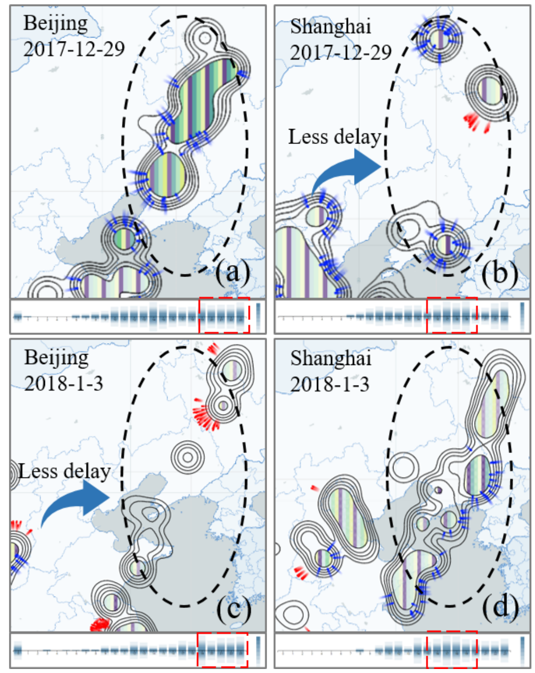

Fig. 7. Delay evolution in Northeast China around New Year's Day, where the flights are related to Beijing Airport (a,c) and Shanghai Airport (b,d). A large number of flights that fly to Beijing caused the delay before the New Year's Day, while the opposite delay happened for Shanghai.

B. Case 2: Comparing Delay Variables

The following experiment focuses on introducing that the system can show the delay dynamics that change over time, and our experiment includes a comparison of two airports, which is an important ability we hope to achieve.

In order to verify the effectiveness of the VFDP delay propagation display function, we selected a special scenario. Figure 7(a,b) shows how the delays of Beijing and Shanghai influence Northeast China around New Year's Day, and Figure 7(c,d) presents the information of the same region. The blue arrows in Figure 7(a) correspond to severe delays at Beijing Airport before New Year's Day. In contrast, Figure 7(b) shows that there were fewer delays in Shanghai on the same day. We searched for related news and consulted the relevant experts about the airport conditions around Jan. 1. Many New Year celebrations were organized in Beijing, and people tended to take their holiday around this time. A large number of flights that fly to Beijing caused the delay before the New Year's Day, and visitors postponed their journey until the following day.

Relative to Beijing, the delay at Shanghai was substantially different (as shown in Figure 7(c,d)). Figure 7(b) shows that less delay occurred at Shanghai on Dec. 29, and the fewer arrows indicate that the effects of the delay were not as large as in Figure 7(a). In contrast, Figure 7(d) contains more delays than Figure 7(c). As two hub airports, the opposite delay situation means that the airspace resources in this area cannot meet their needs at the same time. The comparison of these two airport delays tells us flights related Beijing are mainly influenced by major activities such as holidays, while Shanghai flight delays depend on daily flight activities.

Note that in the intensity statistics of every view, the Beijing delays occurred mainly at night, while most delays occurred in the afternoon at Shanghai. Hence, we can select a punctual flight according to the delay intensity map.

C. Case 3: Summarizing Delay Events

This experiment involves a long time span and includes major events. We hope to show the change of the delay factor in the system through this experiment, which is a major challenge for our work.

In addition to detecting abnormal events and comparing airports, we aim to characterize delay events to provide a reference for air traffic management. Spring Festival transportation is a large undertaking in China. Jan. 30th marked the beginning of Spring Festival travel in 2018 (as shown in Figure 8, which presents the process of delay change daily as the holiday unfolded).

One trend is that the delay area increased over time. The results support the general rule that more flights are scheduled near the Spring Festival because more passengers need to return home. A special phenomenon appears in the series: there are a few red arrows continuously generated in some parts of China in Figure 8(a,c); by contrast, a large number of red arrows appear in Figure 8(b). The change process of delay can be divided into three steps:

Step 1. The delays increase over time; it is clear that the main delays increase in East China, as shown in Figure 8(a), and the trend diminishes until Feb. 1st. Then, the delay in South China increases over the next two days.

Step 2. The state is stable on Feb. 3rd, and then the delay event encompasses the entire nation the following days, as shown in Figure 8(b), when the red arrows point in almost all directions.

Step 3. The delays no longer increase gradually, and the delay propagation ultimately overlays the entire country, as shown in Figure 8(c).

The phenomenon can be explained as follows: people who worked in developed cities began to leave the cities at the beginning of holiday, which are distributed mainly across East China. The capacity of the eastern airports quickly reached saturation, as shown in the red rectangle of Figure 8(a), and the delay propagated to the other regions. The maximum capacity of most airports was reached on Feb. 3rd with continuously increasing traffic. Many special policies were implemented after Feb. 4th, such as opening more air space, providing more flights, and adjusting flight schedules. With the help of policy, the flight delay can increase continuously, as did the delay is shown in the red rectangle of Figure 8(b). Finally, the capacity was fully filled by delay, and flights could not be added; the delay would fill the remaining region (the red rectangle in Figure 8(b)).

Another important feature is the uncertainty views. On most days, we determined that a delay would occur if few (0-25%) pre-delays appear, and this phenomenon continued for several days until Feb. 7th, as shown in Figure 8(b,c). Then, we would find delay factors change in part of China, as shown in Figure 8(c). The delay becomes heavier, and more frequencies of factor 2 and factor 3 lead to an inevitable current delay. We can conclude that the delay occurring 15 minutes before is more important compared with the others because few of factor 1 could cause the delay.

VII. User Studies

We have developed a delay propagation visual analysis system with multiple image processing methods that can be applied to real-world data. In this section, we discuss the user study results and limitations of our approach.

In order to comprehensively evaluate VFDP, avoid the influence of users' prior knowledge. We invited experts in the civil aviation field, visualization researchers, and non-professionals to participate in user study. Civil aviation experts are often experienced, they are also directly involved in part of our system designs. Visualization researchers would tend to provide views about visual perception and elements. The suggestions of non-professionals will help us evaluate the system more objectively from the perspective of freshmen. We divided the participants into two groups, group 1 and group 2 (**G1** and **G2**), and then we provided instructions (appendix I) to **G2** to explain our design. **G1** did not read the instructions in advance. The comparison of these two groups shows whether users can understand our system through a short training.

A. Study Design and Participants

We evaluated our methods mainly with questionnaires. Considering that VFDP is designed specifically for aviation experts, we invited aviation experts and company staff to analyze the presentation of delay information. Additionally, non-professionals were invited to provide feedback about the user-friendly aspect of VFDP. The questionnaire consists of 17 related questions **(Q1-Q17)**, of which the first three questions **(Q1-Q3)** investigate the user's basic information, and the last 14 questions **(Q4-Q17)** are related to our evaluation goals. These questions are shown in Appendix IV.

According to the results of Q1-Q3, we obtained 90 complete questionnaires from 90 respondents. The types of questionnaires include single choice and grading. There were 26 female participants. A total of 48.07% of the participants were visualization related researchers, and 34.14% of the participants worked in the civil aviation field. The age range was from 20 to 56. Particularly, Q3 survey results tell us that 83% of users understand the system within 20 minutes.

All the questions surround the following evaluation hypotheses, and our system is a comprehensive and efficient solution. We hope to confirm that users can understand the elements in our visualization in a short time through evaluation, and our system is available and easy to use. In order to conduct a comprehensive evaluation of VFDP, we divide questions (Q4-Q17) in the questionnaire according to the system functions into the following 4 categories as evaluation indicators:

- **Status** Users can use VFDP to correctly determine the region and severity of flight delays. **(Q4,Q5,Q6,Q7)**
- **Usability** Compared with existing systems and alternatives, VFDP shows valuable delay information that is different from common systems. **(Q8, Q9, Q17)**
- **Visually** With the help of interaction, Users can adjust the system to get the observation conclusion more conveniently. **(Q10, Q14, Q15)**
- **Trend** The system help users make a correct judgment about the trend of flight delay. **(Q11,Q12,Q13,Q16)**

Since the questions related to **Status** require users to determine the status of flight delays, these questions are different from other questions and they are all judgment questions. In order to establish a unified scoring system, we designed corresponding stars for the answers to the judgment questions. Finally, the answers to all questions can be classified range from 1 star to 5 stars, the corrected answer corresponding to 5 stars.

B. Evaluation Results

Since each indicator is composed of multiple questions, we take the mean scores of each question included in each indicator, the results are shown in Figure 9. As a preliminary summary of the survey results, we can see that each indicator exceeds the average score (3 stars). In addition, you can see that the score of **Status** is significantly better, for these

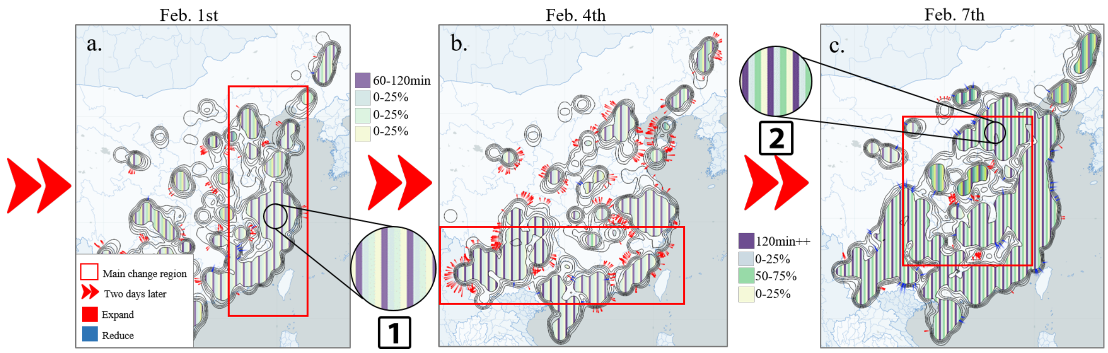

Fig. 8. Delay propagation across China during the Spring Festival (Jan. 30th to Feb. 7th)

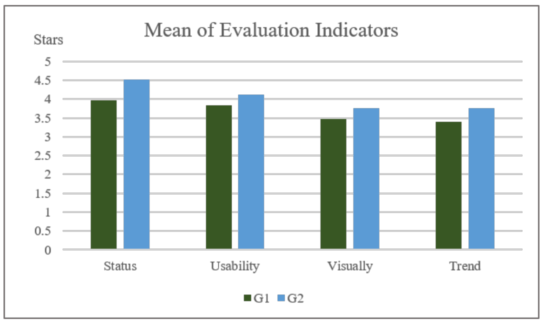

Fig. 9. We summarize the survey into four evaluation indicators. This figure shows the average score of each indicator. It can be seen that the performance of the G2 group is better than the G1 group for each category.

questions are converted from judgment questions, users can draw a definite answer. For the other three categories, the usability score is higher, confirming that the domain experts actually help us to improve the usability of the system during the design stage. The scores of interactions and trends are similar, we collected opinions from the users we surveyed in online group chats. Due to the differences in personal habits of experts, **Visually** and **Trend** judgments have a more personal tendency. A typical example, an experienced expert said that he like the simple heat maps type, combined with some text summaries or related news. He favors parts of our system, while he may want to further customize it.

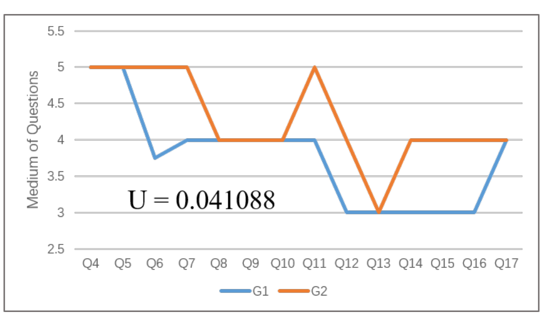

Fig. 10. The figure shows the median ranks for each question from Q4 to Q17. In addition, we also conducted the Mann-Whitney U test for the two groups of answers. According to U¡0.05, we can know that there are differences in the answers of the two groups.

According to Figure 9, we find that the performance of G2 is always better than G1. We guess that G2 has improved the understanding of the system through short-term learning. In order to accurately verify whether G2 actually performs better, some experts recommend using the median as a reference, for our questionnaire is discrete. We first extracted the median ranks of each question and the results are shown in Figure 10. It can be seen that it is similar to the distribution of the average indicators, and the median of G2 is always better than G1. In particular, Q8, Q9, and Q17 all belong to **Usability** category that we envisage, and their median are the same. We conclude it is available from **Usability** perspective regardless of whether the user has learned how to use VFDP. In contrast, **Visually** and **Trend** are learned through short-term learning, and users will show significant differences because they are more familiar with the current system. To make the argument more rigorous, we take the Mann–Whitney U test to compare the two sets of conclusions [44]. The final U value is lower than 5% level of significance, which shows that the answers of the two groups are significantly different, and G2 does get a better score by learning.

In addition to subjective user study, we also try to quantitatively evaluate the feedback delay ability of VFDP. In this work, we take heat map as the visual basis in the system, the delay information is established in the form of Poisson distribution. Therefore, we hope to evaluate the corresponding delay information by directly analyzing the image information of the heat map. We chose Beijing Airport as the observation airport for it is often the busiest airport in mainland China, the selected time range 30 days in November 2017. The system generated 30 delay maps correspond to date, we also kept a background picture without delay as a reference. By calculating the difference in pixel values between the delayed image of each day and the background image, we can obtain the pixel area occupied by each heat map. The relationship between the heat pixels area change and the delay change can reflect whether our system can correctly show the basic delay information. The parameters we selected include the number of delays per day (NUM), the total delay time (DELAY), and the area of the heat map (AREA) in the corresponding delay display. In order to observe correlation, we normalized each indicator as RATE based on the minimum value in the month, and the results are shown in Figure 11. It can be seen that in a month, the number of delays per day and the total delay time can always have a positive correlation with the heat map area. This shows that our system can indeed correctly feedback the changes and severity of delay information.

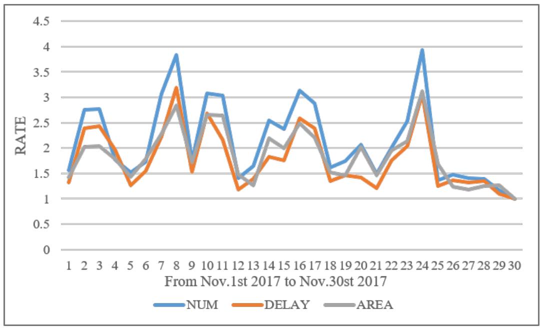

Fig. 11. We calculated the area of the heat map based on the pixel difference of the picture. It can be seen that there is a good positive correlation characteristic among the number of flight delays, the total delay time and the heat map proportion, which indicates our system can correctly reflect regional delays.

VIII. Discussion

We evaluated our design with a broad-based comparison that confirmed the feasibility of our designs. In our work, we originally designed several alternatives to meet the requirements. A broad combination of all these approaches created a large reading burden, which would have resulted in more required interactions with the system and increased the challenges of user interpretation. Therefore, we decided to abandon part of the delay-related design and simplified the design to show information in a single view to the greatest extent possible.

However, the single view is limited in the number of visual elements that it can obtain. Consequently, we present only limited impact factors in the system. In fact, besides the temporal delay, we think other factors can also be encoded. For example, if we take the visibility of each airport as delay impact factors, the same can be done according to Bayesian network. The visibility when the delay happened is used as the network node, the weight of the influence factor is converted into entropy through our method. Then a visual design similar to the impact factor variation reconstruction can be constructed according to the value of the entropy and the number of elements. We attempt to extend the system by considering other elements, as provided in appendix II. For civil aviation, the delay factors arise from various causes, and the system may be incapable of showing all of them.

A typical possible improvement is to reduce the number of colors according to experts' suggestions. However, it is difficult to determine how many and which color should be selected since the participants are limited. It is necessary to broadly collect suggestions from experts and company staff in the practice environment and then design an available color for most users. In our current work, we retain the color interaction for users as a choice.

The above discussion of the limitations confirms the versatility and scalability of our methods. These alternative schemes also allow VFDP to be easily applied to other regions and countries. The rules in this study are mostly designed according to the aviation regulations in China, such as the delay classification and the delay impact factors. If users want to apply it in other regions, they only need to replace the corresponding delay classification rules and the main impact factors. For additional necessary information displays, our spare visual design can be a supplement.

IX. Conclusion

We present a novel design for airport delay propagation that incorporates the delay impact of regional airports and the importance of potential factors. The major challenge was to create a design that is intuitively useful for aviation experts to study delay statistics. In our work, we learned that answering the needs of delay propagation visualization incorporates many design choices when incorporating uncertainty with regard to the information presented.

Our visualization is specifically designed for airport delay propagation. However, the obtained results could also be useful in other situations, typical scenarios such as subway delays and taxi congestion. As we said in Section I, there have been some successful cases in subway delay visualization. However, we believe that we can apply the uncertain related design to the subway scenario when discussing and designing delay impact factors. For taxi congestion data, due to the high flexibility of the road network, taxi activity visualization may not be suitable for our method. However, if we regard the important traffic intersections in the city as the airport, it may be possible to apply the region delay propagation of the airport to the delay changes of the traffic intersections. In general, some parts of our system and method can be applied to other traffic data scenarios.

An interesting direction for future work could be the visualization of additional information available from the aviation delay system in China. The research depends on the development of flight delays, combined with additional factors and different datasets. Our design could also be augmented by differentiating among aircraft types (e.g., B747 and A340) and studying the influence of the aircraft type on delay. Additionally, the interactive design might be incorporated within a full delay propagation application usable by the general public.

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