Decision Making in Networks of Heterogeneous Drift-Diffusion Processes

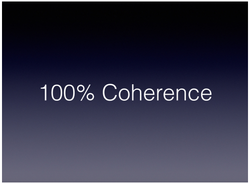
Cathy Chen
Advisor: Dr. Zahra Aminzare

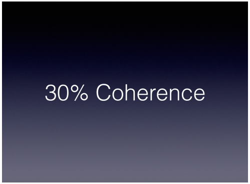
PACM Independent Work

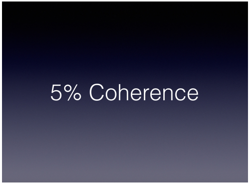
cathy.chen@princeton.edu

April 26, 2018

Overview







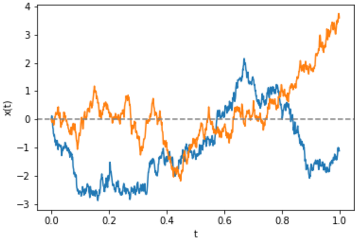
Newsome Lab, Stanford University School of Medicine. https://monkeybiz.stanford.edu/research.html.

Individual Decision Maker

Consider an individual choosing between two options, who accumulates information over time.

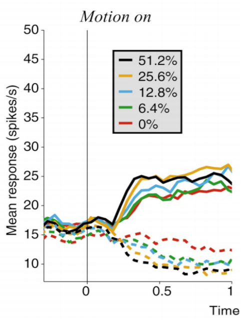
We can model their opinion at time t as $x(t)$ and their opinion updates $\dot{x}$ (Drift Diffusion Model)

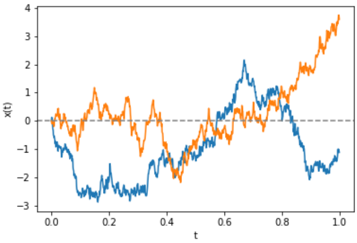
$$dx = \beta dt + \sigma dW$$



Two trials of a decision maker accumulating evidence in a noisy situation, with $\beta = 1$, $\sigma = 0.1$, $dt = 0.001$, and $x(0) = 0$

Individual Decision Maker, Experimental Evidence

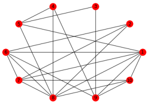




LIP population firing during dot motion task. Solid and dashed lines denote the direction of the monkey's selection.

Shadlen and Newsome, Journal of Neurophysiology. 2001

Networks of Decision Makers



Consider networks of decision makers.

We can model networks as a graph using a Laplacian matrix.

$$L_{ij} = \begin{cases} deg(v_i) & i = j \\ -\alpha_{ij} & i \neq j \end{cases}$$

We can model interactions as coupled drift-diffusion models.

$$d\mathbf{x}(t) = (\mathbf{B} - \mathbf{L}\mathbf{x}(t))dt + \sigma \mathbf{I_n} d\mathbf{W_n}(t)$$
$$\mathbf{x}(0) = \mathbf{x}_0$$

Srivastava and Leonard, IEEE Transactions on Control of Network Systems. 2014.

Goal

We model **networks** of decision makers who decide according to the **Drift Diffusion Model**, studying **individual opinion states** and **cohesion between individuals**, in **noise-free and noisy** networks.

Network Update Rules

Each individual has "opinion state" $x_i(t)$ and receives external information signal β_i.

Opinion state updates depend on network structure, current opinion states ($x_i(t)$), information signals (β_i) and noise.

$$d\mathbf{x}(t) = (\mathbf{B} - \mathbf{L}\mathbf{x}(t))dt + \sigma \mathbf{I_n} d\mathbf{W_n}(t)$$

$$\mathbf{x}(0) = \mathbf{x}_0$$

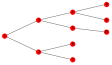
$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_n(t) \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_n \end{bmatrix}$$

Population and Network Structures

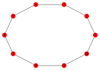
We allow heterogeneous networks, with leaders ($\beta \neq 0$) and followers ($\beta = 0$).

We specifically consider networks with one or two leaders.

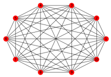
We assume the strength of information flow between any two connected individuals is the same (unweighted interactions).



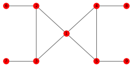
Tree



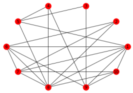
Circle



Complete



Srivastava-Leonard



Random

Fixed Total Information Accumulation

Proposition

Consider a network of individuals who accumulate information according to

$$d\mathbf{x}(t) = (\mathbf{B} - \mathbf{L}\mathbf{x}(t))dt + \sigma \mathbf{I_n} d\mathbf{W_n}(t)$$
$$\mathbf{x}(0) = \mathbf{x}_0$$

in a noise-free network ($\sigma = 0$). At each time t, the rate of information accumulated by all individuals is a constant that depends on β.

$$d\mathbf{x}(t) = (\mathbf{B} - \mathbf{L}\mathbf{x}(t))dt + \sigma \mathbf{I_n} d\mathbf{W_n}(t),\ \sigma = 0$$

$$\dot{\mathbf{x}} = \mathbf{B} - \mathbf{L}\mathbf{x}$$

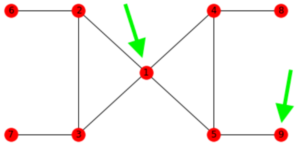
$$\dot{x}_i = \beta_i - \sum_{j=1}^{n} L_{ij} x_j$$

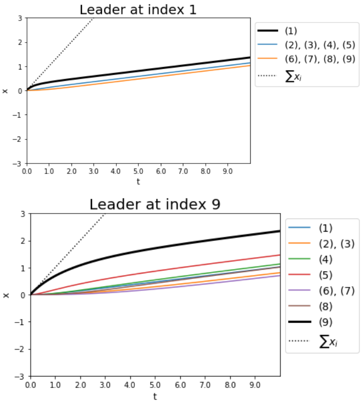
$$\dot{x}_i = \beta_i + \sum_{j \neq i} \alpha_{ij}(x_j - x_i)$$

$$\begin{aligned}
\sum_{i=1}^{n} \dot{x}_i &= \sum_{i=1}^{n} \left(\beta_i + \sum_{j \neq i} \alpha_{ij}(x_j - x_i) \right) \\
&= \sum_{i=1}^{n} \beta_i + \sum_{i=1}^{n} \sum_{j \neq i} \alpha_{ij}(x_j - x_i) \\
&= \sum_{i=1}^{n} \beta_i
\end{aligned}$$

Simulation

Trade-off between how much an individual influences other nodes' accumulation of information and how quickly the individual accumulates information for their own opinion state.





In addition to the opinion state of each individual, we study the **cohesion** between a leader and the individuals it communicates with.

Discrepancy Definitions

Pairwise Discrepancy

Pairwise discrepancy d_{ij} between individuals i and j

$$d_{ij}(t) = \begin{cases} (x_i(t) - x_j(t))^2 & \textit{if } i, j \textit{ connected} \\ 0 & \textit{else} \end{cases}$$

Discrepancy

Discrepancy d_i around individual i: $d_i(t) = \sum_{j=1}^{n} d_{ij}(t)$

Total Discrepancy

Total discrepancy d_L: the sum of all discrepancies

$$d_L(t) = \frac{\sum_{i=1}^{n} |\beta_i| d_i(t)}{\sum_{i=1}^{n} |\beta_i|}$$

Discrepancy Convergence

Proposition

Consider a network of individuals who accumulate information according to

$$d\mathbf{x}(t) = (\mathbf{B} - \mathbf{L}\mathbf{x}(t))dt + \sigma \mathbf{I_n} d\mathbf{W_n}(t)$$
$$\mathbf{x}(0) = \mathbf{x}_0$$

in a noise-free network ($\sigma = 0$). As $t \to \infty$, all pairwise discrepancies d_{ij} converge to values that depend on the network structure L and individual signals β.

Consider an incidence matrix of an arbitrary orientation on a graph

$$E[v,e] = \begin{cases} -1 & v = s(e) \\ 1 & v = t(e) \\ 0 & otherwise \end{cases}$$

Let $y = E^\top x$, which gives pairwise differences between individuals.

$$\begin{aligned} \dot{y} = (\dot{E^\top x}) = E^\top \dot{x} &= E^\top B - E^\top L x \\ &= E^\top B - E^\top (E E^\top) x \\ &= E^\top B - (E^\top E)(E^\top x) \\ &= E^\top B - E^\top E y \end{aligned}$$

which has a steady state at

$$E^\top B = (E^\top E) y^*$$

Resulting Pairwise Discrepancy Formulas

$$E^\top B = (E^\top E) y^*$$

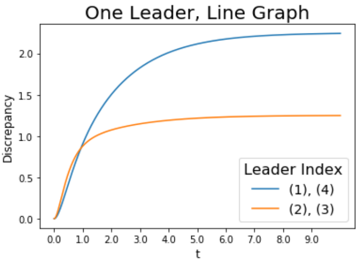
$$y^* = (E^\top E)^{-1} E^\top B \qquad \textit{tree graph } (E^\top E \textit{ invertible})$$

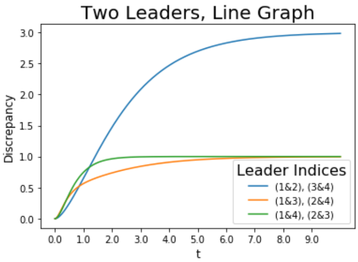
$$y^* = \frac{E^\top B}{n} \qquad \textit{complete graph } (EE^\top E = nE)$$

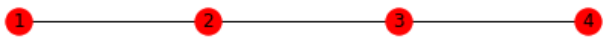
$$(E^\top E)^+ (E^\top E) y^* = (E^\top E)^+ E^\top B \qquad \textit{general graph}$$

Simulations of Discrepancy Convergence, Line Graph

$$y^* = (E^\top E)^{-1} E^\top B$$

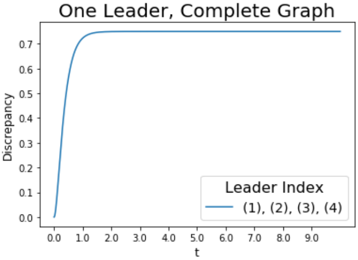


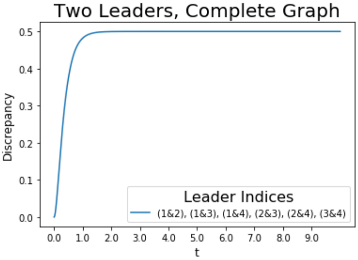


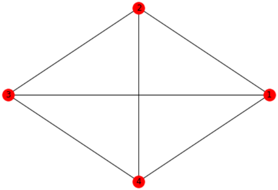


Simulations of Discrepancy Convergence, Complete Graph

$$y^* = \frac{E^\top B}{n}$$

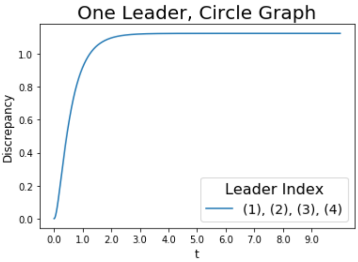


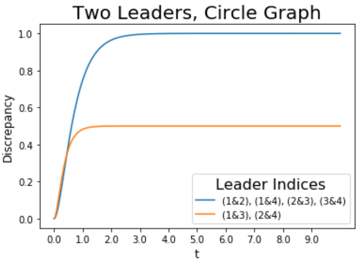


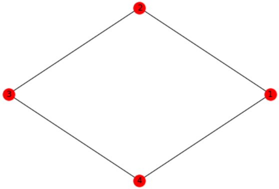


Simulations of Discrepancy Convergence, Circle Graph

$$(E^\top E)^+(E^\top E)y^* = (E^\top E)^+E^\top B$$

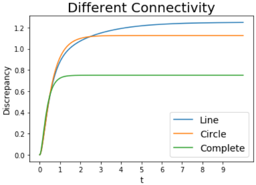




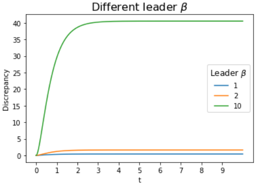


Minimizing Total Discrepancy

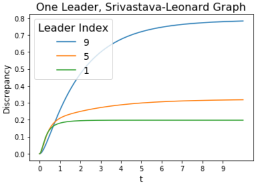
Increase network connectivity.

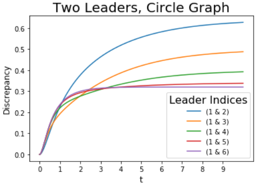


Decrease signal β.

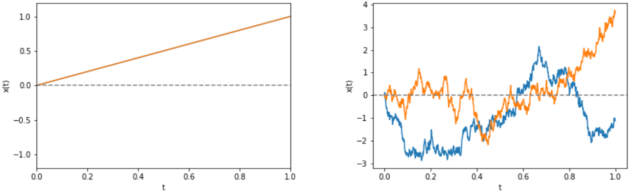


Maximize centrality.





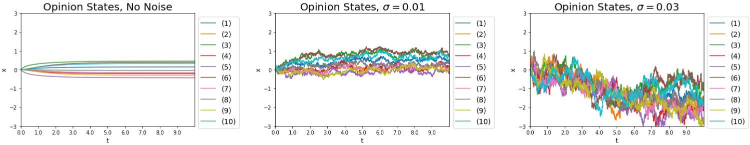
Simulations in Noisy Networks



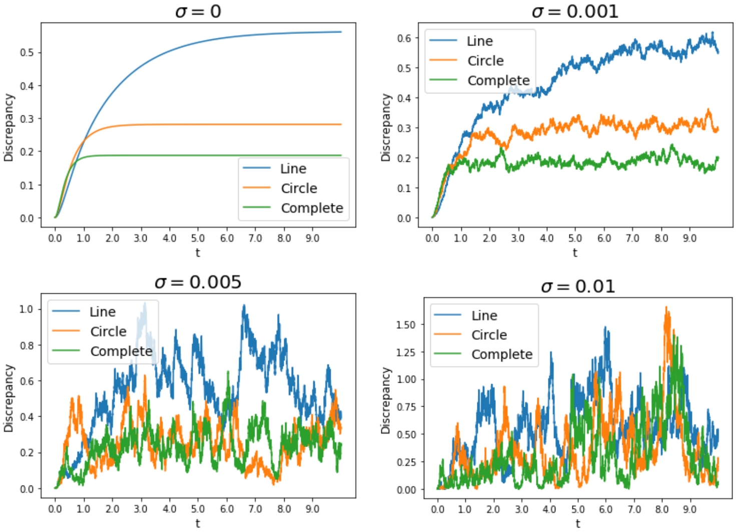
$$d\mathbf{x}(t) = (\mathbf{B} - \mathbf{L}\mathbf{x}(t))dt + \sigma \mathbf{I_n} d\mathbf{W_n}(t)$$
$$\mathbf{x}(0) = \mathbf{x}_0$$

Loss of Opinion State Convergence

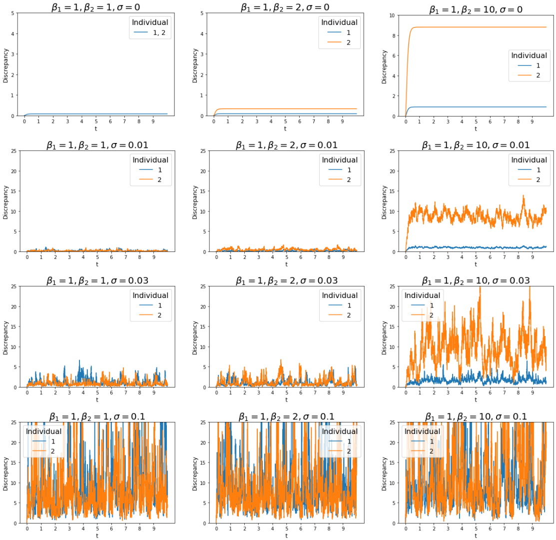
Simulation on random graph with two leaders, $\beta_{l1} = 1, \beta_{l2} = -1$



Discrepancy and Graph Connectivity

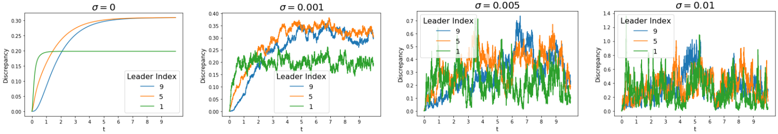


Discrepancy and Leader Signal Size

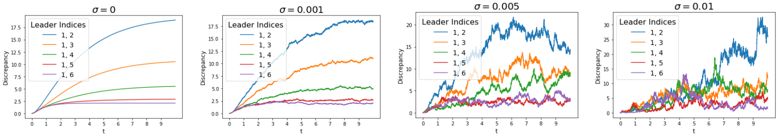


Simulations on a Complete Graph

Discrepancy and Centrality/Coverage



Srivastava-Leonard Graph with One Leader (Centrality).



Circle Graph with Two Leaders (Coverage).

Thank You!

Thank you to Dr. Zahra Aminzare to Professor Leonard for advising this project.

Conclusions:

- Fixed Total Information Accumulation.
- Converging Pairwise Discrepancy.
- Minimizing Total Discrepancy: increase connectivity, reduce leader β, trade off centrality and coverage in leader placement.
- Results hold with small amounts of noise.